Stochastic Modeling for Hostage Rescue Operations

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Abstract

This work seeks to establish optimal resource allocation policies for region-specific hostage rescue missions under different cost considerations on resource allocation. A queuing model is used to design an operational system, and a Markov decision process (MDP) identifies optimal policies for rescue team allocations given the cost. The policies are then evaluated against a range of different hostage taking conditions to measure for performance and flexibility. Several assumptions are made regarding rescue operations. A rescue operation is assumed to be successful if it completes before the hostage incident. The operational system does not consider reorganization after the rescue team usage. Proposed model considers one rescue team for each hostage incident.

Keywords: Terrorism, Hostage rescue operations, Queuing theory, Markov decision processes.

1 Introduction

The word “terrorism” was first used in 1795 by a British writer and politician Edmund Burke (Webb, 2017). Even though there is no international consensus for the meaning of terrorism (Ellis, 2008), it is defined in the Code of Federal Regulations as “the unlawful use of violence and/or intimidation by nongovernment individuals or organizations, especially against civilians, in the pursuit of political and/or religious aims” (28 CFR Section 0.85, 1969). Today, terrorism continues to evolve and adapt. Specifically, terrorism is expected to become more diverse and more fragmented with actors who will use advanced technology (Clarke and Al Aqeedi, 2021). Also, attacks are carried out by individuals or small groups that may, or may not, have training, yet their activities will have a significant impact on citizens and countries worldwide (Mooney, 2017).

Since the September 11, 2001 attacks conducted by Al-Qaeda targeting the Twin Towers and the Pentagon, a large amount of academic research and books have been published with interest in terrorism and conflict, including quantitative work. However, it does not mean that the quality of research has been improved drastically. Indeed, Youngman (2020) highlights methodological challenges to working with datasets on terrorism and notes there are practical limitations for quantitative studies (Ellis, 2008). And Merari (1991) asserts that terrorism’s sheer diversity makes it difficult to generalize research conclusions; Ellis (2008) echoes these sentiments. In addition, even though researchers have applied various data gathering techniques and used historical data to develop insights, the use of statistical models remains relatively low in this field. Schuurman (2020) assesses terrorism studies over the past decades (2007–2016) and pointed out that statistics were used about 20–30% of the time, and inferential statistics which help us come to an understanding of the larger population are seldom used. Furthermore, among a few exceptions which used statistical analysis, the majority of their articles on terrorism research have attempted to identify the causes or influences of terrorism using traditional statistical methods (Ruiz Estrada, 2020; Young, 2019). But due to data limitations and validation difficulties, researchers have not developed models to study their impact (van der Zwet et al., 2022). Therefore, more encompassing, multi-disciplinary approaches are needed for more detailed analysis on terrorism and interventions.

Considering cutting edge statistical analysis is commonly used to understand social, political, and other fields (LaFree and Freilich, 2012), it is sensible that statistical techniques will improve the quality of terrorism research. Moreover, we need to consider how to overcome data limitations and implement models for military operations. Given that understanding a changing situation is critical to the use of military forces effectively, designing a computational model based on statistical analysis of enemy and ally behavior will be meaningful even if limitations exist.

In order to conduct effective quantitative analysis, a proper database is crucial. There are several prominent terrorism databases including the Global Terrorism Database (GTD) (National Consortium for the Study of Terrorism and Responses to Terrorism (START), 2019) and the International Terrorism: Attributes of Terrorist Events (ITERATE) database (Fleming et al., 2010; Flemming et al., 2011; ITERATE website, 2016; Mickolus, 2018). Several recent studies provide general analysis and attempt quantitative modeling using the GTD data (Li et al., 2021; Pan, 2021).

For decision-makers, there are some important questions that must be addressed when designing a hostage-rescue strategy. For example, what is the proper number of rescue teams to be organized for a hostage rescue operation? Or, if a hostage-taking occurs, how many rescue teams should prepare for the operation? In reality, however, it is difficult to adequately answer these questions because of the numerous factors impacting hostage rescue operations such as rescue force size, national budget, geographical restrictions, hostage-taking risk level, etc. To address this complexity in decision-making, we suggest a mathematical hostage rescue model as a decision-making tool for rescue team allocation based on the cost of terrorist events and rescue team deployment. For our analysis, we used hostage taking data from the Global Terrorism Database related to the Taliban terrorist group in Afghanistan.

In response to a hostage taking incident, the military usually decides how many rescue forces will be needed. Although hostage rescue operations have relatively shorter durations compared to other military operations, decision makers aim to be prepared ahead of time with an adequate number of rescue teams considering the high collateral damages associated with such events. It is reasonable to assume that assigning more rescue teams increases the probability of a successful rescue operation. However, that is not the case in actual practice. Using more rescue teams is costlier and does not necessarily guarantee more effective rescue operations. It is actually possible for the cost incurred from excessive rescue team usage to exceed the long term cost of a terrorist incident. So, cost consideration bears a lot of importance regarding optimal allocation of rescue teams.

The rest of the paper is organized as follows: Section 2 reviews applicable literature and identifies limitations, Section 3 presents a stochastic approach to model rescue operations and evaluate the system, and Section 4 provides closing remarks and extensions for future work.

2 Literature Review

In quantitative terrorism research, statistics is commonly used to understand the significant factors in a terrorist event (Chen et al., 2008; Wilson et al., 2006) though some studies rely on systems dynamics models (Hanrahan, 2003). For example, correspondence analysis is used to identify the relationship between fatality and factors, countries, regions, weapons, attack types and targets (Guohui et al., 2014). Regression analysis is used to measure the effectiveness of US efforts in the war on terror (Goepner, 2016) and to clarify the economical sanction effect on international terrorism (whan Choi and Luo, 2013). Also, probability distributions have been used to model aspects of terrorism. The exponential distribution is used for small terrorism event intervals to investigate the impact of current events on future events (Jenkins et al., 2016), and Guo (2019) find that the attack frequency in urban cities follows the exponential distribution and the deaths per attack follows the Power-Law distribution. We cannot guarantee that these patterns and trends are commonly applicable, but they do provide insight into terror incidents.

Since the main topic of this paper is hostage taking incidents with quantitative approaches, we primarily focus on the details of hostage taking resolutions. Time is the most important factor in handling hostage taking (Sen, 1989). From this point of view, both negotiation and rapid rescue operations are good resolutions for the hostage taking case, since negotiation delays the hostage taker's decision and the rapid rescue operations interdict the hostage taker's intended action. Despite significant data challenges, researchers exploit quantitative methods in hostage negotiation, rescue operations, and other military operations where there are political and other nuances not captured by available data.

Negotiation Pattern analysis is commonly used to quantitatively study the impacts of negotiation in hostage scenarios, along with more policy-oriented approaches (Hyatt, 2016). Terrorist behavior along with its implications in hostage negotiation has been analyzed for three categories of events: aerial hijackings, barricade siege, and hijacking incidents (Wilson, 2000). Silke (2001) examines three mathematical models for hostage situations and tests them using three hostage situations in 1995–1999. Significant factors impacting duration (length of terrorist hostage-taking events) are analyzed using regression and hazard function (Kim and Sandler, 2021). Also, based on the hostage taking dataset, a logistic regression model identifies the factors that determine terrorist negotiation success (Gaibulloev and Sandler, 2009; Michaud et al., 2008). Due to the lack of available historical data, researchers have resorted to surveying 188 negotiators from various countries via an online survey to perform frequency analysis on the results (Johnson et al., 2018), and conducting live simulated hostage negotiation exercises to study negotiator decisions (Claudia van den Heuvel et al., 2012).

Hostage taking studies regarding negotiations have focused on finding significant factors. Ruiz Estrada (2020) assesses that a multi-disciplinary approach is needed to improve terrorism research. With this point of view, various approaches should be considered to study negotiations in depth.

Rescue operations There are limited quantitative studies of hostage rescue operations, though qualitative studies do exist. Researchers often focus on historical case studies and analyze past operations through the lenses of special operations doctrine (Adriansyah and Suntoro, 2015; Pérez, 2004; Rapanu, 2006). Notably, Pérez (2004) introduces a hostage crisis biorhythm model in his analysis of historical cases. Relevant statistics that may drive modeling efforts clearly vary both by situation, and across time (Jenkins, 1976, 2014). These operations are also quite complex (Jenkins, 2015; Pérez, 2004). The optimal resource allocation in military operations is important (Fauske, 2008). In this point of view, hostage rescue operations have been studied to improve the efficiency despite data limitations. Examples include distributing limited resources to multiple targets to minimize the total expected loss (Zhang and Zhuang, 2019) and designing a communication system to enhance the situational awareness between participating actors for successful operations (Chehade et al., 2019). We know that there is not sufficient terrorism data to study rescue operation processes thoroughly (Zhang and Zhuang, 2019). For this reason, further analysis for rescue operation models, designed in those studies, should be conducted as to whether their works are effective in new cases.

Other military operations Even though it is hard to analyze the operational effort on counter terrorism policy due to sufficient data being generally unavailable (Goepner, 2016), quantitative methods are well used to understand and improve the efficiency in other military operations. Given the available transportation, planning, and management procedures for US noncombatant evacuation operations in South Korea was suggested to improve the efficiency (Kearby et al., 2020), assessing uncertainty and risk in military logistic networks was conducted to develop a system more practical (McConnell et al., 2021), the effectiveness of unmanned aerial vehicle (UAV) reconnaissance during military protection activities was verified (Drozd et al., 2021). Also, military airlift in Iraq and Afghanistan is optimized (Brown et al., 2013) and Replenishment at Sea Planner for Combat Logistics Force is solved with integer linear optimization (Brown et al., 2018). These papers might not reflect the real system exactly, however, they are still useful in providing insight for decision makers.

Discussion Due to lack of available data and useful analytic models, terrorism research on effective counter measures has been limited. It should be pointed out that making public the data regarding counter terrorism tactics and policies can cause terrorists to develop new strategies. Therefore, it is reasonable to say that a generalized and simplified rescue operational system using reliable statistical trends would be needed to overcome data limitations and to improve its applicability. Multi-disciplinary approaches such as statistical analysis coupled with analytical queuing models, not used in hostage taking studies before (Ruiz Estrada, 2020), can help us to design a generalized operational system.

3 Modeling

3.1 Approach

We design a stochastic model to investigate the hostage rescue operational system. The events in the model include: hostage taking occurrence, killing of hostages, and successful rescue operations. Each of the events is described below.

- Hostage taking events occur in our territory according to rate λ. A new hostage taking event is independent of the results of other hostage taking events.
- Once a hostage taking event occurs, the government and/or military may initiate a hostage rescue operation. The total time for a successful rescue operation includes becoming aware of the hostage taking event, planning, rehearsals, and executing the rescue operation. On average this time is μ^{-1}, where rate of a successful rescue operation is defined as μ. No hostages are killed by hostage takers if the rescue operation is successful.
- At any time the hostage takers may kill the hostages; this occurs at rate ε and concludes the hostage taking event. We assume the terrorists have no intention of negotiating with the government. Therefore, if the hostage incident ends before the rescue operation is completed, it is assumed that the hostage takers kill all hostages.

Koo (2022) uses historical data to demonstrate that hostage taking, rescue operations and hostage killing can be well-described by Poisson processes (i.e. inter-event times are exponentially distributed) with the associated occurrence rates: hostage taking, $\lambda = 0.2763$; successful rescue operation, $\mu = 0.2829$; and hostage killing, $\varepsilon = 0.1719$. Figure 1 shows the modeling concept.

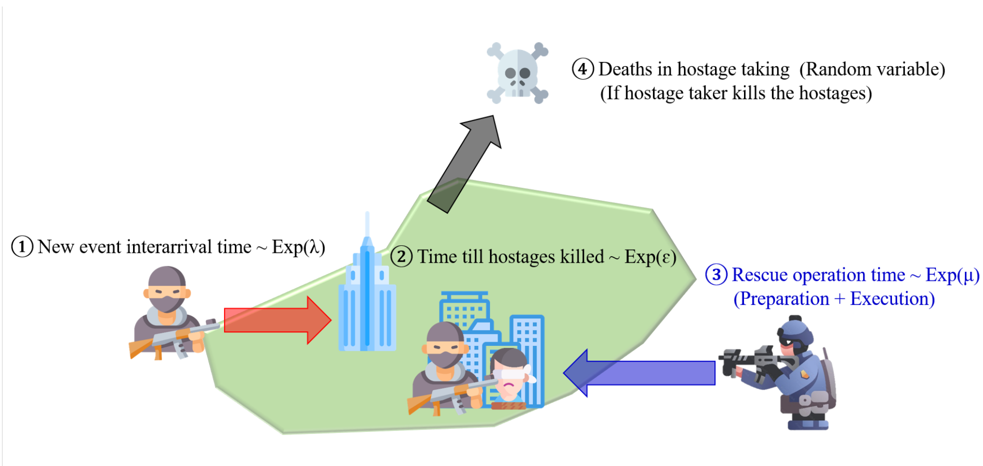


Figure 1: Model concept

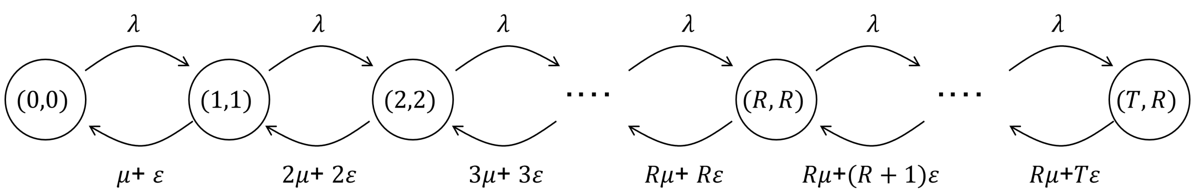
3.2 Model 1. Queuing model

Approach A Continuous Time Markov Chain (CTMC) queuing model is used to model the terrorist activities. Properties of exponential distribution are utilized to govern inter-arrival and incident duration within the model. We consider R as the total number of available rescue teams capable of tackling a total of T hostage incidents in the system ($T = R$), under the assumption that one rescue team can tackle one hostage incident at a time. In the model, we define the state of the system as (i, r), where $0 \leq i \leq T$ is the number of active hostage taking events currently in progress, and $0 \leq r \leq min(i, R)$ is the number of teams currently conducting rescue operations. The upper limit on r implies if the total number of hostage events T exceeds R, then for any $R \leq i \leq T$ all assets (R) will be allocated. In other words, the number of active rescue teams is constant if the rescue team's capacity is reached, while the number of hostage taking events in progress can continue to increase beyond R. We assume that any required reorganization of a rescue team is completed immediately after the hostage taking event concludes. Therefore, the case that some of the rescue teams cannot be used to conduct the next operation due to rest, refit, and recovery is not considered. With each new hostage taking event, a new rescue team is assigned. Figure 2 shows the corresponding transition rate diagram for a $M/M/R+T//T$ queuing model (Stewart, 2009).

Each state (i, r) in the model has a different departure or service rate that is dependant on the number of active hostage events (i) and the number of rescue teams deployed (r). Considering ϵ = average completion rate for a successful rescue operation, μ = average completion rate for a hostage incident, and λ = average arrival rate of hostage incidents ; the transition rate from any state (i, r) to another state (j, r) for any $0 \leq r \leq R$ within the model can be defined as follows:

$$a_{i,j}^{r} = \begin{cases} \lambda & ; j = i+1, i \in \{0, 1, 2...., T-1\} \\ (min\{i, r\} * \mu) + (i * \epsilon) & ; j = i-1, i \in \{1, 2, 3, ..., T\} \\ -(a_{i,i-1}^{r} + a_{i,i+1}^{r}) & ; j = i, i \in \{1, 2, 3, ..., T-1\} \\ -a_{0,1}^{r} & ; j = i, i = 0 \\ -a_{T,T-1}^{r} & ; j = i, i = T \\ 0 & ; otherwise \end{cases}$$

It is to be noted that transition rate from state (i, r) to state $(i-1, r)$ within the model is defined as $(min\{i, r\} * \mu) + (i * \epsilon)$. However, transition rate from state (i, r) to state $(i+1, r)$ is fixed at λ. These are to imply that we have no control over hostage takers and when they might strike, but the outcome of such hostage events can be dictated by the total number and efficiency of the rescue teams deployed. Only one-step transitions have positive rates as each rescue team can tackle one hostage event at a time.



Note: (i, r): i active hostage taking events in progress with r rescue teams conducting rescue operations

Figure 2: Queuing model transition rate diagram

Analysis Table 1 shows the steady state probability distribution of the number of active hostage taking events for different numbers of rescue teams. We consider T=10 and calculate steady state probabilities for each state i in the model against $1 \leq R \leq T$. It is worth noting that the steady state distribution converges once there are six rescue teams. Given that P_i is the probability that there are i active hostage events, the expected number of active hostage events would be:

$$\mathrm{E}[active\ Hostage\ events] = \sum_{i=0}^{T} P_i * i \tag{1}$$

and the number of active rescue teams is:

$$\mathrm{E}[active\ Rescue\ teams] = \sum_{i=0}^{T} P_i * R \tag{2}$$

The index "*Operational performance*" which measures the model performance is given by:

$$Operational\ performance = \frac{\mathrm{E}[active\ Rescue\ teams]}{\mathrm{E}[active\ Hostage\ events]}. \tag{3}$$

The operational performance value would be less than or equal to 1.

Table 1: Steady state probability for $R = 1, 2, \dots, 10$.

Rescue teams, R	Steady state probability for i = number of active hostage taking events										
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$
1 team	0.7129669	0.2166184	0.05535669	0.01220675	0.002367156	0.0004096488	6.400473e-05	9.114786e-06	1.192431e-06	1.442634e-07	1.745336e-08
2 teams	0.7254089	0.2203986	0.04464198	0.008030848	0.001299364	0.0001910158	2.572889e-05	3.197738e-06	3.689235e-07	3.971386e-08	4.275116e-09
3 teams	0.7266378	0.220772	0.04471761	0.006793202	0.0009429126	0.0001204804	1.42613e-05	1.572385e-06	1.622414e-07	1.573108e-08	1.525301e-09
4 teams	0.7267465	0.220805	0.0447243	0.006794219	0.000825706	9.32984e-05	9.849967e-06	9.758492e-07	9.106871e-08	8.032653e-09	7.085146e-10
5 teams	0.7267552	0.2208076	0.04472483	0.006794299	0.0008257158	8.362484e-05	7.967428e-06	7.166484e-07	6.104635e-08	4.938545e-09	3.995198e-10
6 teams	0.7267558	0.2208078	0.04472487	0.006794305	0.0008257165	8.362491e-05	7.25928e-06	5.978892e-07	4.684441e-08	3.499746e-09	2.61466e-10
7 teams	0.7267558	0.2208078	0.04472487	0.006794305	0.0008257165	8.362491e-05	7.25928e-06	5.513908e-07	3.999298e-08	2.775561e-09	1.926272e-10
8 teams	0.7267558	0.2208078	0.04472487	0.006794305	0.0008257165	8.362491e-05	7.25928e-06	5.513908e-07	3.722828e-08	2.412273e-09	1.563075e-10
9 teams	0.7267558	0.2208078	0.04472487	0.006794305	0.0008257166	8.362491e-05	7.25928e-06	5.513908e-07	3.722828e-08	2.262189e-09	1.374627e-10
10 teams	0.7267558	0.2208078	0.04472487	0.006794305	0.0008257166	8.362491e-05	7.25928e-06	5.513908e-07	3.722828e-08	2.262189e-09	1.294111e-10

Note: Probabilities may not sum to one due to rounding.

Table 2: Operational performance values for $T = 10$.

Rescue teams, R	1 team	2 teams	3 teams	4 teams	5 teams	6 teams	7 teams	8 teams	9 teams	10 teams
Operation performance	0.935	0.992	0.999	0.9999	0.99999	0.999999	0.99999997	0.999999998	0.999999999897	1.0

Table 2 shows the operational performance for various numbers of rescue teams. As an example, 0.935 is the operational performance value when there is one rescue team, and the upper bound of terrorist teams is ten. Intuitively, operational performance is non-decreasing with the number of rescue teams, and can be used to target the number of rescue teams to have on stand-by.

3.3 Model 2. Markov decision process

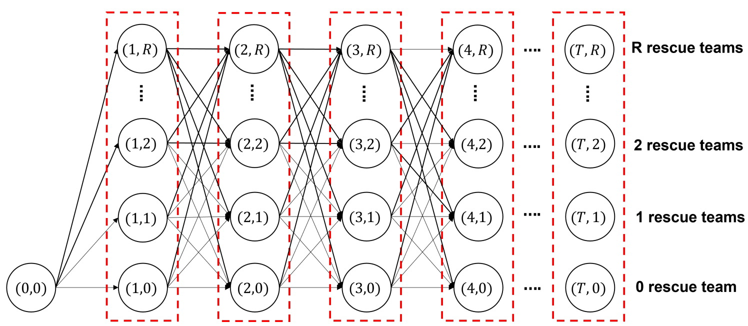


Figure 3: MDP model concept

Approach We employ a Markov decision process model (MDP) (Howard, 1960) to optimize the number of rescue teams to be allocated to minimize the total system cost given the number of active hostage taking events. Figure 3 shows the concept of the MDP model. The MDP chooses how many rescue teams will be used in view of the cost efficiency in each state. When a decision has been specified for all states, the "policy" has been determined. Policy can be understood as the set of best decisions for each state, and the optimal policy minimizes the cost of the system. Policy iteration is used in this model, and the gain value (g) is the average cost per unit time (Howard, 1960). The state space is defined the same as in Model 1. For details on the policy iteration algorithm used in this paper, see Koo (2022).

Decision space The decision space defines how many rescue teams can be allocated in each state. Figure 3 illustrates the MDP model. The decision space is a function of the state space, since the number of possible rescue teams to allocate is governed by the number of the active hostage events. To be precise, if there are i active hostage events, the decision space is $r \in \{0, 1, \dots, R\}$ if $i > 0$ and trivially $r \in \{0\}$ if $i = 0$.

Cost design The model considers two cost components: cost of rescue team deployement per unit time (C_R) and total cost of damages incurred from a hostage incident (C_T). The hostage event-induced cost may include physical damages to infrastructure, human casualties, or economic impacts. The cost of rescue team deployment can vary according to the emergency and requirements of the hostage incidents at hand. Considering the numerous cost impacts and also due to limited data, defining general values for both types of costs can be difficult. For our calculations, we have considered a normalized fixed cost of 1 for each hostage incident incurred. The cost of damages incurred from hostage incidents is assumed to be linearly proportional to the total number of hostage incidents. Cost of each rescue team deployment is decided using the following relation:

$$C_R = \alpha * C_T * (\mu + \epsilon) \tag{4}$$

Here α signifies a non-negative "multiplier" value which can dictate the cost of rescue team deployment while the other parameters are fixed. For example, choosing $\alpha = 1$ would yield $C_R = 0.4547$ against $C_T = 1$ and the estimated completion rates for rescue team operations ($\mu = 0.2829$) and hostage incidents ($\epsilon = 0.1719$). This implies that it would cost around 45% of the estimated cost of damages from one hostage incident to keep a rescue team deployed for each unit of time. Thus by considering different α values, the decision maker can consider different deployment costs with a given estimate on hostage incident cost. Values for μ and ϵ would stay fixed and are region-specific (i.e. in our study we consider Taliban hostage incidents in Afghanistan and the efficiency of rescue teams deployed to specifically resolve said incidents.)

Three separate kinds of cost considerations have been assumed for additional rescue team deployment costs under Linear, Square root and Quadratic cost structures. A linear cost structure considers that additional rescue team deployment costs remain fixed, and total deployment cost is linearly proportional to the total number of rescue teams deployed. A square root cost structure implies that additional deployments incur lower unit costs, and as such the total deployment cost would be lower than the total cost under the linear structure for the same number of teams deployed. This is opposite to the quadratic cost structure assumption where additional deployments incur higher unit costs, and as such total cost would be higher than the linear deployment cost for a given number of rescue teams. Cost of damages incurred from additional hostage incidents is assumed to follow the linear cost structure.

Under these conditions, we define a state cost per unit time q_i^r for any state (i, r) following the principals of Continuous time Markov Chains (Howard, 1960). For any $0 \leq i \leq T(= R)$ and $0 \leq r \leq min(i, R)$, q_i^r can be defined as follows:

Linear cost increase:

$$q_i^r = (r * C_R) + (i * \epsilon * C_T) \tag{5}$$

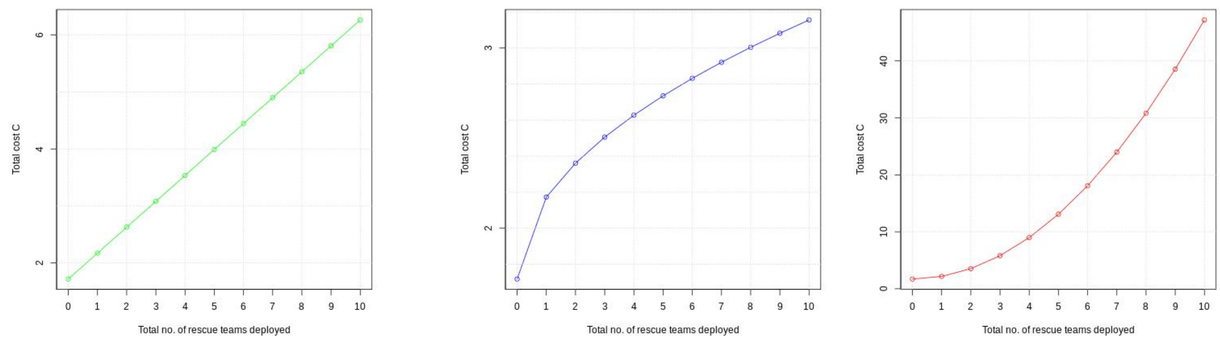
Square root cost increase:

$$q_i^r = (\sqrt{r} * C_R) + (i * \epsilon * C_T) \tag{6}$$

Quadratic cost increase:

$$q_i^r = (r^2 * C_R) + (i * \epsilon * C_T) \tag{7}$$

In Figure 4, cost rate q_i^r is plotted for all three cost increase structures. We consider $\alpha = 1$, C_T=1, and use estimated service rates for successful rescue operations ($\mu = 0.2829$) and hostage incidents ($\epsilon = 0.1719$). C_R is calculated using equation 4. For T=R=10, we calculate q_i^r for deploying $0 \leq r \leq R$ rescue teams to tackle $i = T$ hostage incidents. As expected, the graphs show that the Quadratic cost structure yields the highest cost per unit time ($q_T^R = 47.19$) for $(i, r) \equiv (T = 10, R = 10)$ under the given parameters, followed by the Linear cost structure ($q_T^R = 6.27$) and the Square root cost structure ($q_T^R = 3.16$).



(a) Linear cost structure

(b) Square root cost structure

(c) Quadratic cost structure

Figure 4: Different cost considerations for additional rescue team deployment.

For a set of i where $0 \leq i \leq T$, a policy can be defined as a set of value decisions on r for states (i, r). Following (Howard, 1960), a given policy set is evaluated by solving equations 8 and 9 for g_i= Average cost per unit time for i and v_j= Expected total cost of j with known q_i and a_{ij}.

Policy evaluation:

$$g_i = q_i + \sum_{j=1}^{T} a_{ij} v_j \quad ; \quad i = 1, 2, 3....T \tag{8}$$

$$\sum_{j=1}^{T} a_{ij} g_j \quad ; \quad i = 1, 2, 3....T \tag{9}$$

v_j calculated from the policy evaluation step is thus used for policy improvement under equation 10. Iterations are continued till policy convergence is found, which yields the optimal policy.

Policy improvement:

$$min \;\; q_i^r + \sum_{j=1}^{T} a_{ij}^r v_j \quad ; \quad i = 1, 2, 3....T \tag{10}$$

Analysis We now explore different values of α in equation 4 under all three cost increase considerations to obtain optimal policies from our proposed MDP model over a range of different cost scenarios. The results are summarized in tables 3,4,and 5.

α is changed between [0,3] with an increment of 0.01 to define different deployment cost rates (C_R) against a fixed total cost of damages from hostage incidents (C_T=1). Service rates used for successful rescue operations ($\mu = 0.2829$) and hostage incidents ($\epsilon = 0.1719$) are estimated using historical data (Koo (2022)). We consider the total number of available rescue teams to be $R = 10$, and as such allow $T = 10$ to be the maximum number of hostage incidents within the system. This follows from an assumption that each available rescue team is sufficient for resolving one hostage incident at a time, and as such $R = 10$ teams can at most engage in $T = 10$ separate hostage incidents. For any i number of active hostage incidents in the system where $0 \leq i \leq 10$, we deploy $0 \leq r \leq min(i, 10)$ rescue teams.

Considering $\alpha = 0$ implies that there are no cost constraints on deploying rescue teams, and as such we are free to deploy as many rescue teams as there are active hostage incidents in the system under any of the three cost increase structures. Increasing the value of α would define higher costs per rescue team deployment. It is to be noted that Tables 3, 4, 5 records all the unique policies obtained over $\alpha \in$ [0,3]. This implies that increasing rescue team deployment cost doesn't necessarily change the optimal policy unless the increase is substantial enough.

When cost per rescue team deployment increases linearly (see Table 3), deploying larger numbers of rescue teams becomes less and less economically feasible till deploying no teams at all is the optimal decision. For square root increase in deployment costs, however, it is more optimal to deploy comparatively larger numbers of rescue teams as deployment cost increases (see Table 4). Quadratic cost increase yields policies similar to those from linear cost increase (see Table 5), where deploying larger numbers of rescue teams become less optimal with increasing costs. However, rate of increase in gain (average cost per unit time) is much slower with quadratic cost increase and as such there are more unique policies compared to linear cost increase.

Table 3: Optimal policies for linear cost structure on rescue team deployment.

α	Optimal policy											Gain
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$	
0	0	1	2	3	4	5	6	7	8	9	10	0.1043949
0.54	0	1	2	3	4	5	6	7	8	9	0	0.2535969
0.61	0	1	2	3	4	5	6	7	8	0	0	0.2729379
0.62	0	1	2	3	4	5	6	7	0	0	0	0.2757009
0.63	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 4: Optimal policies for square root cost structure on rescue team deployment.

α	Optimal policy											Gain
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$	
0	0	1	2	3	4	5	6	7	8	9	10	0.1043949
0.5	0	0	2	3	4	5	6	7	8	9	10	0.2213795
0.78	0	0	0	3	4	5	6	7	8	0	10	0.2568958
1.01	0	0	0	0	4	5	6	7	8	9	10	0.2703092
1.2	0	0	0	0	0	5	6	7	8	9	10	0.2746707
1.36	0	0	0	0	0	0	6	7	8	9	10	0.2759051
1.5	0	0	0	0	0	0	0	7	8	9	10	0.2762147
1.63	0	0	0	0	0	0	0	0	8	9	10	0.2762839
1.66	0	0	0	0	0	0	0	0	8	9	0	0.2762874
1.75	0	0	0	0	0	0	0	0	0	9	0	0.2762969
1.82	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 5: Optimal policies for quadratic cost structure on rescue team deployment.

α	Optimal policy											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
0	0	1	2	3	4	5	6	7	8	9	10	0.1043949
0.02	0	1	2	3	4	5	6	7	8	9	9	0.1132787
0.03	0	1	2	3	4	5	6	6	6	7	6	0.1177207
0.04	0	1	2	3	4	5	5	5	5	5	5	0.1221616
0.05	0	1	2	3	4	4	4	4	5	5	4	0.1265929
0.06	0	1	2	3	3	4	4	4	4	4	4	0.1309942
0.07	0	1	2	3	3	3	3	3	3	4	3	0.1353385
0.08	0	1	2	3	3	3	3	3	3	3	3	0.1396802
0.09	0	1	2	2	3	3	3	3	3	3	3	0.1437832
0.1	0	1	2	2	2	2	3	3	3	3	2	0.1477042
0.11	0	1	2	2	2	2	2	2	2	2	2	0.1515986
0.16	0	1	1	2	2	2	2	2	2	2	2	0.1703508
0.17	0	1	1	1	2	2	2	2	2	2	1	0.1731827
0.18	0	1	1	1	1	2	2	2	2	2	1	0.1756083
0.19	0	1	1	1	1	1	1	2	2	2	1	0.1779065
0.2	0	1	1	1	1	1	1	1	1	1	1	0.1801834
0.53	0	1	1	1	1	1	1	1	1	1	0	0.2553156
0.61	0	1	1	1	1	1	1	1	1	0	0	0.2735294
0.62	0	1	1	1	1	1	1	1	0	0	0	0.2758061
0.63	0	0	0	0	0	0	0	0	0	0	0	0.2762982

For any state (i, r) in the model, the probability of success (p) for the deployed rescue teams (r) against the active hostage incidents (i) can be quantified using the following relation:

$$p = \frac{r * \mu}{(r * \mu) + (i * \epsilon)} \quad ; \quad i = 1, 2, 3....T, \quad r = 1, 2, 3....R \tag{11}$$

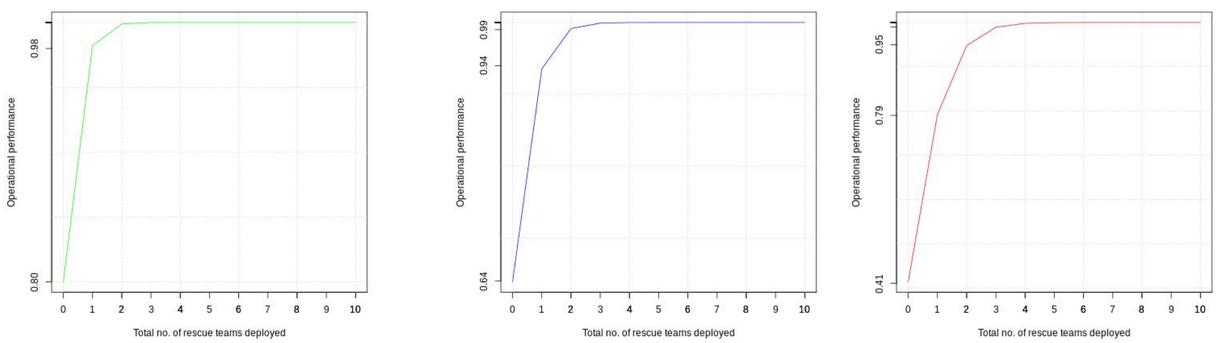
As such, Tables 6, 7, and 8 quantify probabilities of success for the obtained optimal policies over all three cost increase considerations.

Table 6: Probability of success for optimally allocated rescue teams under linear cost structure.

α	p											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.1043949
Policy 2	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0	0.2535969
Policy 3	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0	0	0.2729379
Policy 4	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0	0	0	0.2757009
Policy 5	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 7: Probability of success for optimally allocated rescue teams under Square root cost structure.

α	p											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.1043949
Policy 2	0	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.2213795
Policy 3	0	0	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.2568958
Policy 4	0	0	0	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.2703092
Policy 5	0	0	0	0	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.2746707
Policy 6	0	0	0	0	0	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.2759051
Policy 7	0	0	0	0	0	0	0	0.6221685	0.6221685	0.6221685	0.6221685	0.2762147
Policy 8	0	0	0	0	0	0	0	0	0.6221685	0.6221685	0.6221685	0.2762839
Policy 9	0	0	0	0	0	0	0	0	0.6221685	0.6221685	0	0.2762874
Policy 10	0	0	0	0	0	0	0	0	0	0.6221685	0	0.2762969
Policy 11	0	0	0	0	0	0	0	0	0	0	0	0.2762982



(a) 0.5λ arrival rate (b) λ arrival rate (c) 2λ arrival rate

Figure 5: Different arrival rate considerations for evaluating operational performance.

Table 8: Probability of success for optimally allocated rescue teams under Quadratic cost structure.

α	p											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.1043949
Policy 2	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.5971014	0.1132787
Policy 3	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.5853103	0.552575	0.5615483	0.4969842	0.1177207
Policy 4	0	0.6221685	0.6221685	0.6221685	0.6221685	0.6221685	0.5784566	0.5404837	0.5071892	0.4777586	0.4515563	0.1221616
Policy 5	0	0.6221685	0.6221685	0.6221685	0.6221685	0.5684718	0.5233074	0.4847914	0.5071892	0.4777586	0.3971084	0.1265929
Policy 6	0	0.6221685	0.6221685	0.6221685	0.552575	0.5684718	0.5233074	0.4847914	0.4515563	0.4225857	0.3971084	0.1309942
Policy 7	0	0.6221685	0.6221685	0.6221685	0.552575	0.4969842	0.4515563	0.4137376	0.3817642	0.4225857	0.330658	0.1353385
Policy 8	0	0.6221685	0.6221685	0.6221685	0.552575	0.4969842	0.4515563	0.4137376	0.3817642	0.3543781	0.330658	0.1396802
Policy 9	0	0.6221685	0.6221685	0.5233074	0.552575	0.4969842	0.4515563	0.4137376	0.3817642	0.3543781	0.330658	0.1437832
Policy 10	0	0.6221685	0.6221685	0.5233074	0.4515563	0.3971084	0.4515563	0.4137376	0.3817642	0.3543781	0.247745	0.1477042
Policy 11	0	0.6221685	0.6221685	0.5233074	0.4515563	0.3971084	0.3543781	0.3199502	0.2916194	0.2678977	0.247745	0.1515986
Policy 12	0	0.6221685	0.4515563	0.5233074	0.4515563	0.3971084	0.3543781	0.3199502	0.2916194	0.2678977	0.247745	0.1703508
Policy 13	0	0.6221685	0.4515563	0.3543781	0.4515563	0.3971084	0.3543781	0.3199502	0.2916194	0.2678977	0.1413864	0.1731827
Policy 14	0	0.6221685	0.4515563	0.3543781	0.2916194	0.3971084	0.3543781	0.3199502	0.2916194	0.2678977	0.1413864	0.1756083
Policy 15	0	0.6221685	0.4515563	0.3543781	0.2916194	0.247745	0.215346	0.3199502	0.2916194	0.2678977	0.1413864	0.1779065
Policy 16	0	0.6221685	0.4515563	0.3543781	0.2916194	0.247745	0.215346	0.1904409	0.1706993	0.1546662	0.1413864	0.1801834
Policy 17	0	0.6221685	0.4515563	0.3543781	0.2916194	0.247745	0.215346	0.1904409	0.1706993	0.1546662	0	0.2553156
Policy 18	0	0.6221685	0.4515563	0.3543781	0.2916194	0.247745	0.215346	0.1904409	0.1706993	0	0	0.2735294
Policy 19	0	0.6221685	0.4515563	0.3543781	0.2916194	0.247745	0.215346	0.1904409	0	0	0	0.2758061
Policy 20	0	0	0	0	0	0	0	0	0	0	0	0.2762982

3.4 Sensitivity

Considering the many uncertainties related to hostage incidents, we explore the proposed model's performance and robustness over different possible scenarios by changing relevant parameters in our model.

The frequency of hostage incidents can vary based on a lot of different factors including geographical location, international relations, political and economic instability. The variance in the frequency of hostage event occurrences can be addressed by considering different values of the arrival rate λ, where a larger λ dictates more frequent occurrences and vice versa. We test our model's operational performance against 0.5λ, λ, and 2λ ; where λ=0.2763 is considered (Koo (2022)).

Figure 5 illustrates operational performances for $0 \leq R \leq T$ against $T = 10$ over these different arrival rates. Operational performances converges to 1 for all three rates as R gets closer to T.

Tables 9 to 17 explore the necessary service rates required from the rescue teams to achieve given probabilities of success (5%,50%,95%) over all three deployment cost considerations.

Table 9: Required completion rates (μ) for 5% probability of a successful rescue mission under Linear cost increase structure

Policies	μ											Gain
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$	
Policy 1	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.1043949
Policy 2	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0	0.2535969
Policy 3	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0	0	0.2729379
Policy 4	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0	0	0	0.2757009
Policy 5	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 10: Required completion rates (μ) for 50% probability of a successful rescue mission under Linear cost increase structure

Policies	μ											Gain
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$	
Policy 1	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1043949
Policy 2	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0	0.2535969
Policy 3	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0	0	0.2729379
Policy 4	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0	0	0	0.2757009
Policy 5	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 11: Required completion rates (μ) for 95% probability of a successful rescue mission under Linear cost increase structure

Policies	μ											Gain
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$	
Policy 1	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.1043949
Policy 2	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0	0.2535969
Policy 3	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0	0	0.2729379
Policy 4	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0	0	0	0.2757009
Policy 5	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 12: Required completion rates (μ) for 5% probability of a successful rescue mission under Square root cost increase structure

Policies	μ											Gain
	$i = 0$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$	$i = 10$	
Policy 1	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.1043949
Policy 2	0	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.2213795
Policy 3	0	0	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.2568958
Policy 4	0	0	0	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.2703092
Policy 5	0	0	0	0	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.2746707
Policy 6	0	0	0	0	0	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.2759051
Policy 7	0	0	0	0	0	0	0	0.009042105	0.009042105	0.009042105	0.009042105	0.2762147
Policy 8	0	0	0	0	0	0	0	0	0.009042105	0.009042105	0.009042105	0.2762839
Policy 9	0	0	0	0	0	0	0	0	0.009042105	0.009042105	0	0.2762874
Policy 10	0	0	0	0	0	0	0	0	0	0.009042105	0	0.2762969
Policy 11	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 13: Required completion rates (μ) for 50% probability of a successful rescue mission under Square root cost increase structure

Policies	μ											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1043949
Policy 2	0	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.2213795
Policy 3	0	0	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.2568958
Policy 4	0	0	0	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.2703092
Policy 5	0	0	0	0	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.2746707
Policy 6	0	0	0	0	0	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.2759051
Policy 7	0	0	0	0	0	0	0	0.1718	0.1718	0.1718	0.1718	0.2762147
Policy 8	0	0	0	0	0	0	0	0	0.1718	0.1718	0.1718	0.2762839
Policy 9	0	0	0	0	0	0	0	0	0.1718	0.1718	0	0.2762874
Policy 10	0	0	0	0	0	0	0	0	0	0.1718	0	0.2762969
Policy 11	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 14: Required completion rates (μ) for 95% probability of a successful rescue mission under Square root cost increase structure

Policies	μ											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.1043949
Policy 2	0	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.2213795
Policy 3	0	0	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.2568958
Policy 4	0	0	0	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.2703092
Policy 5	0	0	0	0	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.2746707
Policy 6	0	0	0	0	0	0	3.2642	3.2642	3.2642	3.2642	3.2642	0.2759051
Policy 7	0	0	0	0	0	0	0	3.2642	3.2642	3.2642	3.2642	0.2762147
Policy 8	0	0	0	0	0	0	0	0	3.2642	3.2642	3.2642	0.2762839
Policy 9	0	0	0	0	0	0	0	0	3.2642	3.2642	0	0.2762874
Policy 10	0	0	0	0	0	0	0	0	0	3.2642	0	0.2762969
Policy 11	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 15: Required completion rates (μ) for 5% probability of a successful rescue mission under Quadratic cost increase structure

Policies	μ											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.1043949
Policy 2	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.01004678	0.1132787
Policy 3	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.01054912	0.01205614	0.01162556	0.01507018	0.1177207
Policy 4	0	0.009042105	0.009042105	0.009042105	0.009042105	0.009042105	0.01085053	0.01265895	0.01446737	0.01627579	0.01808421	0.1221616
Policy 5	0	0.009042105	0.009042105	0.009042105	0.009042105	0.01130263	0.01356316	0.01582368	0.01446737	0.01627579	0.02260526	0.1265929
Policy 6	0	0.009042105	0.009042105	0.009042105	0.01205614	0.01130263	0.01356316	0.01582368	0.01808421	0.02034474	0.02260526	0.1309942
Policy 7	0	0.009042105	0.009042105	0.009042105	0.01205614	0.01507018	0.01808421	0.02109825	0.02411228	0.02034474	0.03014035	0.1353385
Policy 8	0	0.009042105	0.009042105	0.009042105	0.01205614	0.01507018	0.01808421	0.02109825	0.02411228	0.02712632	0.03014035	0.1396802
Policy 9	0	0.009042105	0.009042105	0.01356316	0.01205614	0.01507018	0.01808421	0.02109825	0.02411228	0.02712632	0.03014035	0.1437832
Policy 10	0	0.009042105	0.009042105	0.01356316	0.01808421	0.02260526	0.01808421	0.02109825	0.02411228	0.02712632	0.04521053	0.1477042
Policy 11	0	0.009042105	0.009042105	0.01356316	0.01808421	0.02260526	0.02712632	0.03164737	0.03616842	0.04068947	0.04521053	0.1515986
Policy 12	0	0.009042105	0.01808421	0.01356316	0.01808421	0.02260526	0.02712632	0.03164737	0.03616842	0.04068947	0.04521053	0.1703508
Policy 13	0	0.009042105	0.01808421	0.02712632	0.01808421	0.02260526	0.02712632	0.03164737	0.03616842	0.04068947	0.09042105	0.1731827
Policy 14	0	0.009042105	0.01808421	0.02712632	0.03616842	0.02260526	0.02712632	0.03164737	0.03616842	0.04068947	0.09042105	0.1756083
Policy 15	0	0.009042105	0.01808421	0.02712632	0.03616842	0.04521053	0.05425263	0.03164737	0.03616842	0.04068947	0.09042105	0.1779065
Policy 16	0	0.009042105	0.01808421	0.02712632	0.03616842	0.04521053	0.05425263	0.06329474	0.07233684	0.08137895	0.09042105	0.1801834
Policy 17	0	0.009042105	0.01808421	0.02712632	0.03616842	0.04521053	0.05425263	0.06329474	0.07233684	0.08137895	0	0.2553156
Policy 18	0	0.009042105	0.01808421	0.02712632	0.03616842	0.04521053	0.05425263	0.06329474	0.07233684	0	0	0.2735294
Policy 19	0	0.009042105	0.01808421	0.02712632	0.03616842	0.04521053	0.05425263	0.06329474	0	0	0	0.2758061
Policy 20	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 16: Required completion rates (μ) for 50% probability of a successful rescue mission under Quadratic cost increase structure

Policies	μ											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1043949
Policy 2	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.1908889	0.1132787
Policy 3	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.1718	0.2004333	0.2290667	0.2208857	0.2863333	0.1177207
Policy 4	0	0.1718	0.1718	0.1718	0.1718	0.1718	0.20616	0.24052	0.27488	0.30924	0.3436	0.1221616
Policy 5	0	0.1718	0.1718	0.1718	0.1718	0.21475	0.2577	0.30065	0.27488	0.30924	0.4295	0.1265929
Policy 6	0	0.1718	0.1718	0.1718	0.2290667	0.21475	0.2577	0.30065	0.3436	0.38655	0.4295	0.1309942
Policy 7	0	0.1718	0.1718	0.1718	0.2290667	0.2863333	0.3436	0.4008667	0.4581333	0.38655	0.5726667	0.1353385
Policy 8	0	0.1718	0.1718	0.1718	0.2290667	0.2863333	0.3436	0.4008667	0.4581333	0.5154	0.5726667	0.1396802
Policy 9	0	0.1718	0.1718	0.2577	0.2290667	0.2863333	0.3436	0.4008667	0.4581333	0.5154	0.5726667	0.1437832
Policy 10	0	0.1718	0.1718	0.2577	0.3436	0.4295	0.3436	0.4008667	0.4581333	0.5154	0.859	0.1477042
Policy 11	0	0.1718	0.1718	0.2577	0.3436	0.4295	0.5154	0.6013	0.6872	0.7731	0.859	0.1515986
Policy 12	0	0.1718	0.3436	0.2577	0.3436	0.4295	0.5154	0.6013	0.6872	0.7731	0.859	0.1703508
Policy 13	0	0.1718	0.3436	0.5154	0.3436	0.4295	0.5154	0.6013	0.6872	0.7731	1.718	0.1731827
Policy 14	0	0.1718	0.3436	0.5154	0.6872	0.4295	0.5154	0.6013	0.6872	0.7731	1.718	0.1756083
Policy 15	0	0.1718	0.3436	0.5154	0.6872	0.859	1.0308	0.6013	0.6872	0.7731	1.718	0.1779065
Policy 16	0	0.1718	0.3436	0.5154	0.6872	0.859	1.0308	1.2026	1.3744	1.5462	1.718	0.1801834
Policy 17	0	0.1718	0.3436	0.5154	0.6872	0.859	1.0308	1.2026	1.3744	1.5462	0	0.2553156
Policy 18	0	0.1718	0.3436	0.5154	0.6872	0.859	1.0308	1.2026	1.3744	0	0	0.2735294
Policy 19	0	0.1718	0.3436	0.5154	0.6872	0.859	1.0308	1.2026	0	0	0	0.2758061
Policy 20	0	0	0	0	0	0	0	0	0	0	0	0.2762982

Table 17: Required completion rates (μ) for 95% probability of a successful rescue mission under Quadratic cost increase structure

Policies	μ											Gain
	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=10$	
Policy 1	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	0.1043949
Policy 2	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.626889	0.1132787
Policy 3	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.2642	3.808233	4.352267	4.196829	5.440333	0.1177207
Policy 4	0	3.2642	3.2642	3.2642	3.2642	3.2642	3.91704	4.56988	5.22272	5.87556	6.5284	0.1221616
Policy 5	0	3.2642	3.2642	3.2642	3.2642	4.08025	4.8963	5.71235	5.22272	5.87556	8.1605	0.1265929
Policy 6	0	3.2642	3.2642	3.2642	4.352267	4.08025	4.8963	5.71235	6.5284	7.34445	8.1605	0.1309942
Policy 7	0	3.2642	3.2642	3.2642	4.352267	5.440333	6.5284	7.616467	8.704533	7.34445	10.88067	0.1353385
Policy 8	0	3.2642	3.2642	3.2642	4.352267	5.440333	6.5284	7.616467	8.704533	9.7926	10.88067	0.1396802
Policy 9	0	3.2642	3.2642	4.8963	4.352267	5.440333	6.5284	7.616467	8.704533	9.7926	10.88067	0.1437832
Policy 10	0	3.2642	3.2642	4.8963	6.5284	8.1605	6.5284	7.616467	8.704533	9.7926	16.321	0.1477042
Policy 11	0	3.2642	3.2642	4.8963	6.5284	8.1605	9.7926	11.4247	13.0568	14.6889	16.321	0.1515986
Policy 12	0	3.2642	6.5284	4.8963	6.5284	8.1605	9.7926	11.4247	13.0568	14.6889	16.321	0.1703508
Policy 13	0	3.2642	6.5284	9.7926	6.5284	8.1605	9.7926	11.4247	13.0568	14.6889	32.642	0.1731827
Policy 14	0	3.2642	6.5284	9.7926	13.0568	8.1605	9.7926	11.4247	13.0568	14.6889	32.642	0.1756083
Policy 15	0	3.2642	6.5284	9.7926	13.0568	16.321	19.5852	11.4247	13.0568	14.6889	32.642	0.1779065
Policy 16	0	3.2642	6.5284	9.7926	13.0568	16.321	19.5852	22.8494	26.1136	29.3778	32.642	0.1801834
Policy 17	0	3.2642	6.5284	9.7926	13.0568	16.321	19.5852	22.8494	26.1136	29.3778	0	0.2553156
Policy 18	0	3.2642	6.5284	9.7926	13.0568	16.321	19.5852	22.8494	26.1136	0	0	0.2735294
Policy 19	0	3.2642	6.5284	9.7926	13.0568	16.321	19.5852	22.8494	0	0	0	0.2758061
Policy 20	0	0	0	0	0	0	0	0	0	0	0	0.2762982

4 Conclusion

Using the hostage events data from Taliban terrorist acts in Afghanistan from 2015–2019, we design an analytical Continuous Time Markov Chain model—equivalently, a $M/M/R+T//T$ queuing model—using hostage taking frequency and time until hostage execution. We study the optimal rescue team employment strategy obtained from a Continuous Time Markov Decision Process seeking to minimize system costs. Analysis shows the system benefits from increases in the maximum rescue capacity if expected costs from terrorist actions exceed rescue costs. This effect varies as a function of the number of hostage events but is independent of the expected time to complete a rescue operation.

Future work Hostage rescue operations are inherently risky and difficult missions (Jenkins, 2014). As is, this model is not suitable for immediate use by government and/or military counterterrorism policy makers.

Because our model simplified the operational procedure and conditions (Federal Bureau of Investigation, 2022; Sturcke, 2006), it has serious limitations. We did not consider geographic factors in rescue team employment or in hostage incidents. Further, this research assumes all rescue teams have identical capabilities. However, military and law enforcement assets capable of counterterrorism and/or hostage rescue vary in their areas of specialty. Future research should include a wider variety of operational scenarios and must model additional problem features such as unsuccessful rescue operations and the possibility of terrorists releasing hostages without executions. Using cost to motivate the rescue team allocation is difficult due to the operational complexity and lack of validated cost models. It is also likely misguided. While cost is a consideration, future research must incorporate hostage deaths as the driving objective. Risk models that consider hostage deaths, collateral damage, and/or rescuer casualties would also be of interest.

Disclaimer

The views expressed in this paper are those of the authors and do not reflect the official policy or position of the Republic of Korea Army, the Ministry of National Defense, or the Republic of Korea Government.

Statement on Competing Interests

The authors report no competing interests.

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