Contents

***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

1 Stability

2 Routh Hurwitz Criterion

2.1. consider a standard negative feedback transfer function configuration with

$$G(s) = \frac{1}{(s+1)(s+2)} \tag{2.1.1}$$

and

$$H(s) = \frac{s+\alpha}{s} \tag{2.1.2}$$

the closed loop system to have poles on the imaginary axis,the value of α should be equal to **Solution:** The transfer function for negative

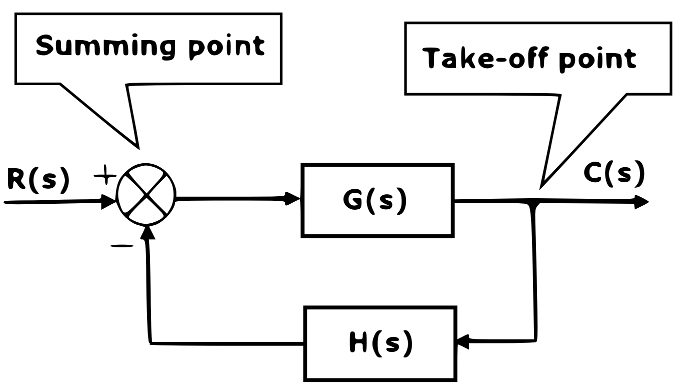

Fig. 2.1

feedback is given by

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)H(s)} \tag{2.1.3}$$

where

$$G(s) = \frac{1}{(s+1)(s+2)} H(s) = \frac{s+\alpha}{s} \tag{2.1.4}$$

Characteristic equation is..,

$$1 + G(s)H(s) = 0 \tag{2.1.5}$$

$$=> 1 + \left[\frac{1}{(s+1)(s+2)}\right]\left[\frac{s+\alpha}{s}\right] = 0 \tag{2.1.6}$$

$$=> s^3 + 3s^2 + 3s + \alpha = 0 \tag{2.1.7}$$

The Routh hurwitz criterion:- This criterion is based on arranging the coefficients of characteristic equation into an array called Routh array. For any characteristic equation q(s),

$$q(s) = a_0 s^n + a_1 s^{n-1} + ... + a_{n-1}s + a_n = 0 \tag{2.1.8}$$

Routh array can be constructed as follows..,

$$\begin{pmatrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{pmatrix} \begin{pmatrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ b_1 & b_2 & b_3 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \ldots \end{pmatrix}$$

where

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1} \tag{2.1.9}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1} \tag{2.1.10}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} \tag{2.1.11}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1} \tag{2.1.12}$$

For poles to lie on imaginary axis any one entire row of hurwitz matrix should be zero. Constructing the routh array for the characteristic equation obtained in equation(**??**)

$$s^3 + 3s^2 + 3s + \alpha = 0 \tag{2.1.13}$$

$$\begin{pmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 3 & \alpha \\ \frac{9-\alpha}{3} & 0 \\ \alpha & 0 \end{pmatrix} \tag{2.1.14}$$

For poles on jω axis any one of the row should be zero.

$$\frac{9-\alpha}{3} = 0 \ or \ \alpha = 0 \tag{2.1.15}$$

But when $\alpha = 0$ poles does not lie on the imaginary axis

$$9 - \alpha = 0 \tag{2.1.16}$$

$$\alpha = 9 \tag{2.1.17}$$

put α in the characteristic equation we get

$$s^3 + 3s^2 + 3s + 9 = 0 \tag{2.1.18}$$

poles are

$$s = -3, -j\sqrt{3}, j\sqrt{3} \tag{2.1.19}$$

3 Compensators

4 Nyquist Plot