1. How many subspaces does R^2 have?

$\left[0, \text{ lines through origin}, R^2\right]$

In 4 option, 0, $\underbrace{0 \times R, \ R \times 0}_{\text{through Origin}}$, R^2 is correct.

Option infinitely many is wrong. [finitely many means correct]

~~Or~~ [infinitely many lines passing through origin is correct]

2. Dimension of W

$$W = \{(x_1, x_2, \ldots x_{10}) \in R^{10} : x_n = x_{n-1} + x_{n-2} \text{ for } 3 \le n \le 10\}$$

$x_3 = x_2 + x_1$; $x_4 = x_3 + x_2 = x_2 + x_1 + x_2 = 2x_2 + x_1$ and so on

$\therefore$ Basis $= \{(x_1, x_2, x_1 + x_2 \ \ldots)\}$

$\therefore$ $\dim(W) = 2$

3. (a) $T(x) = x^2$

$T(cx + y) = (cx + y)^2 = c^2x^2 + y^2 + 2cxy$ — (1)

$cT(x) + T(y) = cx^2 + y^2$ — (2)

(1) $\neq$ (2)

(b) $T(a_1, a_2) = (a_1 - 2a_2,\ a_1 - 3,\ 2a_1 - 5a_2)$

$x = (a_1, a_2) \quad y = (b_1, b_2)$

$cx + y = (ca_1 + b_1,\ ca_2 + b_2)$

$$T(cx+y) = \Big((ca_1+b_1) - 2(ca_2+b_2),\ \overset{ca_1+b_1}{\cancel{ca_1}} - 3,\ 2(ca_1+b_1) - 5(ca_2+b_2)\Big)$$

$$= \Big(c(a_1 - 2a_2) + b_1 - 2b_2,\ ca_1^{+b_1} - 3,\ c(2a_1 - 5a_2) + 2b_1 - 5b_2\Big)$$

$$= c\left[a_1 - 2a_2,\ a_1,\ 2a_1 - 5a_2\right] + (b_1 - 2b_2,\ b_1 - 3,\ 2b_1 - 5b_2)$$

$$\neq cT(x) + T(y)$$

(c) $T(x) = ax + b$

$T(cx+y) = a[cx+y] + b$

$= acx + ay + b \quad -(1)$

$cT[x) + T(y) = c[ax+b] + ay + b \quad -(2)$

$(1) \neq (2)$

(d) $T(ax+b) = \dfrac{ax^2}{2} + bx$

$T(cx+y) = \dfrac{cx^2}{2} + yx$

$cT(x) = c\dfrac{x^2}{2} \quad ; \quad T(y) = yx$

$\therefore \quad T(cx+y) = cT(x) + T(y)$

$$A = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

The no. of non zero rows is 2

$\therefore$ rank $= 2$

rank (T) + Nullity of T = dim V

$2 +$ Nullity $= 3$

Nullity $= 1$