

# Service Robot Project

This project focuses on a 4 Degree of Freedom (DOF) robotic arm with a gripper. The arm has four revolute joints, allowing it to rotate at each joint and perform various tasks with flexibility. The link lengths are:

- L1=10 units
- L2=20 units
- L3=20 units
- L4=10 units

These lengths define the structure of the arm, balancing reach and stability. The gripper acts as the end effector, making the arm capable of picking and placing objects. This setup is suitable for tasks requiring precise motion and multi-axis movement.

In this project, forward kinematics was used, applying a set of 4 basic transformation matrices. These include rotation about the Z-axis, translation along the Z-axis, rotation about the X-axis, and translation along the X-axis.

$$Trans_z = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Trans_x = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

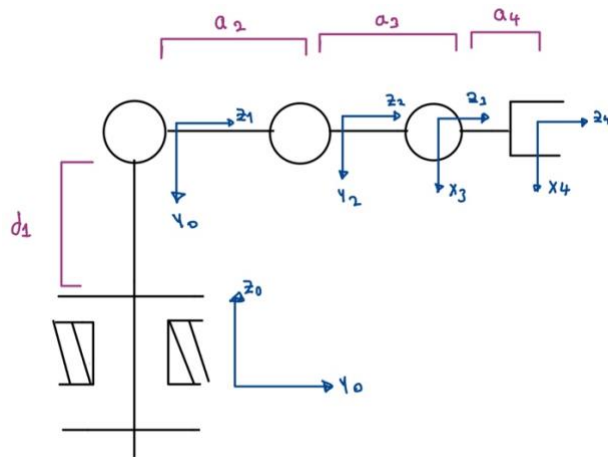
$$Rot_x \alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha) & -\sin(\alpha) & 0 \\ 0 & \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Rot_z \theta = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The fundamental four parameters  $a_i$ ,  $\alpha_i$ ,  $d_i$  and  $\theta_i$  are associated with link  $i$  and joint  $i$ . These parameters, as represented in the transformation matrices, are referred to in the following order:

$a_i$	-	Length of a member
$\alpha_i$	-	Rotation of a member
$d_i$	-	Offset of a joint
$\theta_i$	-	Angle of a joint

## 1-Forward Kinematics



Joint	$\theta_i$	$d_i \text{ mm}$	$a_i \text{ mm}$	$\alpha_i$
1	$\theta_{var=0}$	D1	-	-90
2	-	-	-	$A2_{var=0}$
3	-	D2	-	$A3_{var=0}$
4	-	D3	-	$A4_{var=0}$
5	-	D4	-	-

$$A1 = Rotz\theta. Transz. Rotx\alpha =$$

$$\begin{bmatrix} \cos(0) & -\sin(0) & 0 & 0 \\ \sin(0) & \cos(0) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

ans =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2 = Rotx\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3=TranszRotx\alpha=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0. & 0 \\ 0 & \cos 0 & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A4=Transz\ Rotx\alpha=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0. & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

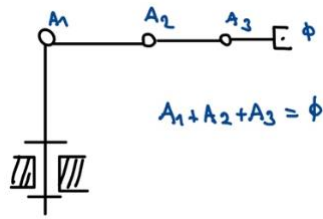
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A5=Transz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T=A1.A2.A3.A4.A5=$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 50 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

# 1-Inverse Kinematic



$$T[1,1] = \cos(\theta_1) = 1$$

$$\theta_1 = 0$$

## Simplified equations

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 50$$

$$T[3,2] = -\cos(a_2 + a_3 + a_4) = -1$$

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 10$$

We are summing first equation and second equation for simplifying.

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 50$$

$$10 \cdot -\cos(a_2 + a_3 + a_4) = -10 \text{ (we expand this equation with 10)}$$

We observe this equation from previous calculation.

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) = 40$$

$$\cos(a_2) = 1$$

$$\cos(a_2 + a_3) = 1$$

We found and put the proper values.

$$a_2 = 0$$

$$a_3 = 0$$

We put the values to first equation to get  $a_4$ .

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 50$$

$$a_4 = 0$$

For proving our values, We are testing the values with third equation.

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 10$$

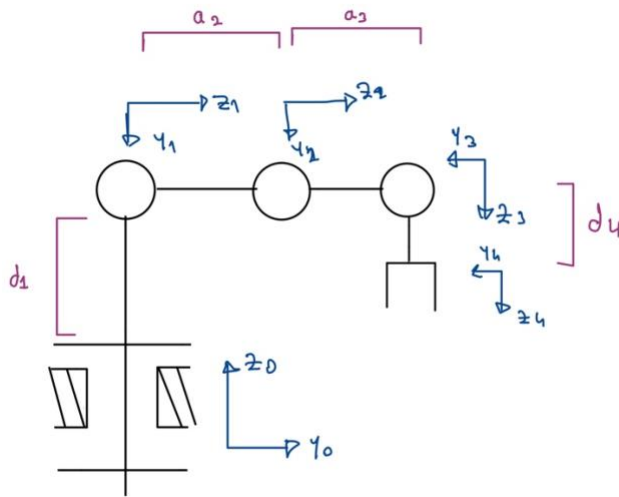
$$T[3,4] = 20 \cdot \sin(0 + 0) + 20 \cdot \sin(0) + 10 \cdot \sin(0 + 0 + 0) + 10 = 10$$

$$10 = 10$$

Our values are correct

$$A_1 + a_2 + a_3 = 0$$

## 2-Forward Kinematics



Joint	$\theta_i$	$d_i \text{ mm}$	$a_i \text{ mm}$	$\alpha_i$
1	$\theta \text{ var}=0$	D1	-	-90
2	-	-	-	$A2 \text{ var}=0$
3	-	D2	-	$A3 \text{ var}=0$
4	-	D3	-	$A4 \text{ var}=-90$
5	-	D4	-	-

$$A1 = \text{Rotz}\theta. \text{Transz}. \text{Rotx}\alpha =$$

$$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

ans =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2 = \text{Rotx}\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3=TranszRotx\alpha=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0. & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A4=Transz\ Rotx\alpha=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0. & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}=$$

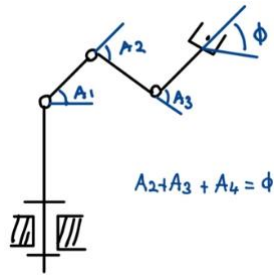
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A5=Transz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T=A1.A2.A3.A4.A5$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 40 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## 2-Inverse kinematics



$$T[1,1] = \cos(\theta_1) = 1$$

$$\theta_1 = 0$$

### Simplified equations

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$T[3,2] = -\cos(a_2 + a_3 + a_4) = 0$$

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 0$$

We are summing first equation and second equation for simplifying.

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$10 \cdot -\cos(a_2 + a_3 + a_4) = 0 \text{ (we expand this equation with 10)}$$

We observe this equation from previous calculation

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) = 40$$

$$\cos(a_2) = 1$$

$$\cos(a_2 + a_3) = 1$$

We found and put the proper values.

$$a_2 = 0$$

$$a_3 = 0$$

We put the values to first equation to get  $a_4$ .

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$a_4 = -90 \text{ or } a_4 = 90$$

For proving our values, We are testing the values with third equation.

First we try  $a_4 = -90$ .

$$T[3,4] = 20 \cdot \sin(0 + 0) + 20 \cdot \sin(0) + 10 \cdot \sin(0 + 0 - 90) + 10 = 0$$

$$0 = 0$$

Our values are correct

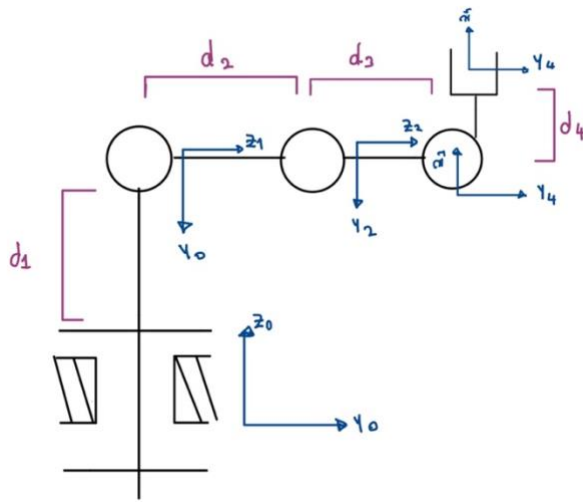
Secondly, we try  $a_4 = 90$

$$20 = 0$$

So  $a_4 = 90$  is not correct

$$A_1 + a_2 + a_3 = -90$$

### 3-Forward Kinematics



Joint	$\theta_i$	$d_i \text{ mm}$	$a_i \text{ mm}$	$\alpha_i$
1	$\theta_{var}=0$	D1	-	-90
2	-	-	-	$A2_{var}=0$
3	-	D2	-	$A3_{var}=0$
4	-	D3	-	$A4_{var}=90$
5	-	D4	-	-

$$A1 = \text{Rot}z\theta. \text{Trans}z. \text{Rot}x\alpha =$$

$$\begin{bmatrix} \cos(0) & -\sin(0) & 0 & 0 \\ \sin(0) & \cos(0) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

ans =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2 = \text{Rot}x\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3=TranszRotx\alpha=\begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0. & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A4=Transz\ Rotx\alpha=\begin{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0. & 0 \\ 0 & \cos(90) & -\sin(90) & 0 \\ 0 & \sin(90) & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{bmatrix}=$$

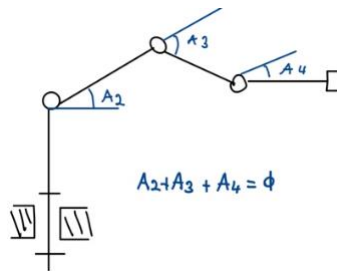
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A5=Transz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T=A1.A2.A3.A4.A5$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3-Inverse Kinematics



$$T[1,1] = \cos(\theta_1) = 1$$

$$\theta_1 = 0$$

#### Simplified equations

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$T[3,2] = -\cos(a_2 + a_3 + a_4) = 0$$

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 20$$

**We are summing first equation and second equation for simplifying.**

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$10 \cdot -\cos(a_2 + a_3 + a_4) = 0 \quad (\text{we expand this equation with 10})$$

**We observe this equation from previous calculation**

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) = 40$$

$$\cos(a_2) = 1$$

$$\cos(a_2 + a_3) = 1$$

**We found and put the proper values.**

$$a_2 = 0$$

$$a_3 = 0$$

**We put the values to first equation to get  $a_4$ .**

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$a_4 = 90$$

or

$$a_4 = -90$$

For proving our values, We are testing the values with third equation.

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 20$$

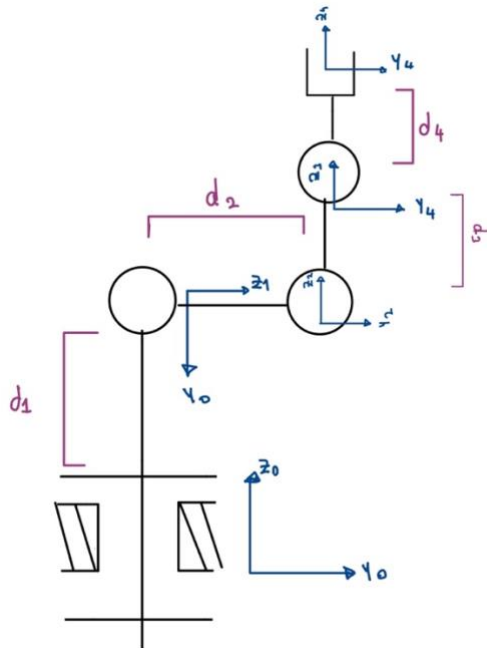
$$T[3,4] = 20 \cdot \sin(0 + 0) + 20 \cdot \sin(0) + 10 \cdot \sin(0 + 0 + 90) + 10 = 20$$

$$20 = 20$$

**Our values are correct**

$$A_1 + a_2 + a_3 = 90$$

## 4-Forward Kinematics



Joint	$\theta_i$	$d_i \text{ mm}$	$a_i \text{ mm}$	$\alpha_i$
1	$\theta_{var=0}$	D1	-	-90
2	-	-	-	$A2var=0$
3	-	D2	-	$A3var=90$
4	-	D3	-	$A4var=0$
5	-	D4	-	-

$A1 = Rotz\theta. Transz. Rotx\alpha =$

$$\begin{bmatrix} \cos(0) & -\sin(0) & 0 & 0 \\ \sin(0) & \cos(0) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

ans =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2=Rotx\alpha=\begin{bmatrix}1 & 0 & 0. & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A3=TranszRotx\alpha=\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0. & 0 \\ 0 & \cos(90) & -\sin(90) & 0 \\ 0 & \sin(90) & \cos(90) & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}.$$

$$\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A4=TranszRotx\alpha=\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1 & 0 & 0. & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1\end{bmatrix}.$$

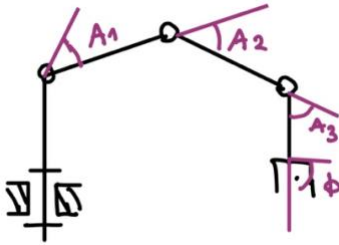
$$\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$A5=Transz=\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$T=A1.A2.A3.A4.A5$$

$$\begin{bmatrix}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 40 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1\end{bmatrix}$$

## 4-Inverse Kinematics



$$T[1,1] = \cos(\theta_1) = 1$$

$$\theta_1 = 0$$

### Simplified equations

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$T[3,2] = -\cos(a_2 + a_3 + a_4) = 0$$

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 20$$

**We are summing first equation and second equation for simplifying.**

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$10 \cdot -\cos(a_2 + a_3 + a_4) = 0. \quad (\text{we expand this equation with } 10)$$

**We observe this equation from previous calculation**

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) = 40$$

$$\cos(a_2) = 1$$

$$\cos(a_2 + a_3) = 1$$

**We found and put the proper values.**

$$a_2 = 0$$

$$a_3 = 0$$

**We put the values to first equation to get  $a_4$ .**

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 40$$

$$a_4 = 90$$

or

$$a_4 = -90$$

**For proving our values, We are testing the values with third equation.**

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 20$$

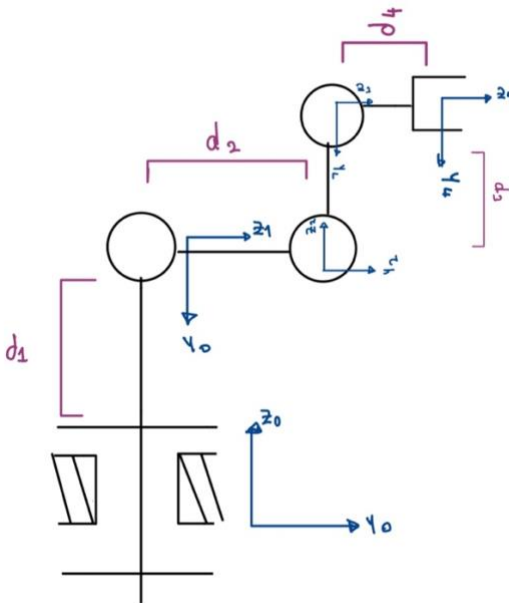
$$T[3,4] = 20 \cdot \sin(0 + 0) + 20 \cdot \sin(0) + 10 \cdot \sin(0 + 0 + 90) + 10 = 20$$

$$20 = 20$$

**Our values are correct**

$$A_1 + a_2 + a_3 = 90$$

## 5-Forward Kinematics



Joint	$\theta_i$	$d_i$ mm	$a_i$ mm	$\alpha_i$
1	$\theta_{var}=0$	D1	-	-90
2	-	-	-	$A2var=0$
3	-	D2	-	$A3var=90$
4	-	D3	-	$A4var=-90$
5	-	D4	-	-

$$A1 = Rotz\theta. Transz. Rotx\alpha =$$

$$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

ans =

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A2 = Rotx\alpha = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(0) & -\sin(0) & 0 \\ 0 & \sin(0) & \cos(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A3=TranszRotx\alpha=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(90) & -\sin(90) & 0 \\ 0 & \sin(90) & \cos(90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}=.$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A4=Transz\ Rotx\alpha=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(-90) & -\sin(-90) & 0 \\ 0 & \sin(-90) & \cos(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}=$$

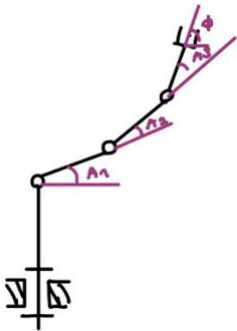
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A5=Transz=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T=A1.A2.A3.A4.A5$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 30 \\ 0 & -1 & 0 & 30 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

## 5-Inverse Kinematics



$$T[1,1] = \cos(\theta_1) = 1$$

$$\theta_1 = 0$$

### Simplified equations

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 30$$

$$T[3,2] = -\cos(a_2 + a_3 + a_4) = -1$$

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 30$$

We are summing first equation and second equation for simplifying.

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 30$$

$$10 \cdot -\cos(a_2 + a_3 + a_4) = -10. \quad (\text{we expand this equation with } 10)$$

We observe this equation from previous calculation

$$20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) = 20$$

$$\cos(a_2) = 1$$

$$\cos(a_2 + a_3) = 0$$

or

$$\cos(a_2) = 0$$

$$\cos(a_2 + a_3) = 1$$

We found and put the proper values.

$$a_2 = 0 \quad a_3 = 90$$

or

$$a_2 = 90 \quad a_3 = 90$$

We put the values to first equation to get  $a_4$ .

$$T[2,4] = 20 \cdot \cos(a_2 + a_3) + 20 \cdot \cos(a_2) + 10 \cdot \cos(a_2 + a_3 + a_4) = 30$$

$$a_4 = -90$$

When we put  $a_2 = 90$  and  $a_3 = 90$ . They do not fit the equation because;

$$T[2,4] = -20 + 0 + 10 \cdot \cos(180 + a_4) = 30$$

$$10 \cdot \cos(180 + a_4) = 40 \quad (\text{This calculation can not be done.})$$

For proving our values, We are testing the values with third equation.

$$T[3,4] = 20 \cdot \sin(a_2 + a_3) + 20 \cdot \sin(a_2) + 10 \cdot \sin(a_2 + a_3 + a_4) + 10 = 20$$

$$T[3,4] = 20 \cdot \sin(0 + 90) + 20 \cdot \sin(0) + 10 \cdot \sin(0 + 90 + -90) + 10 = 30$$

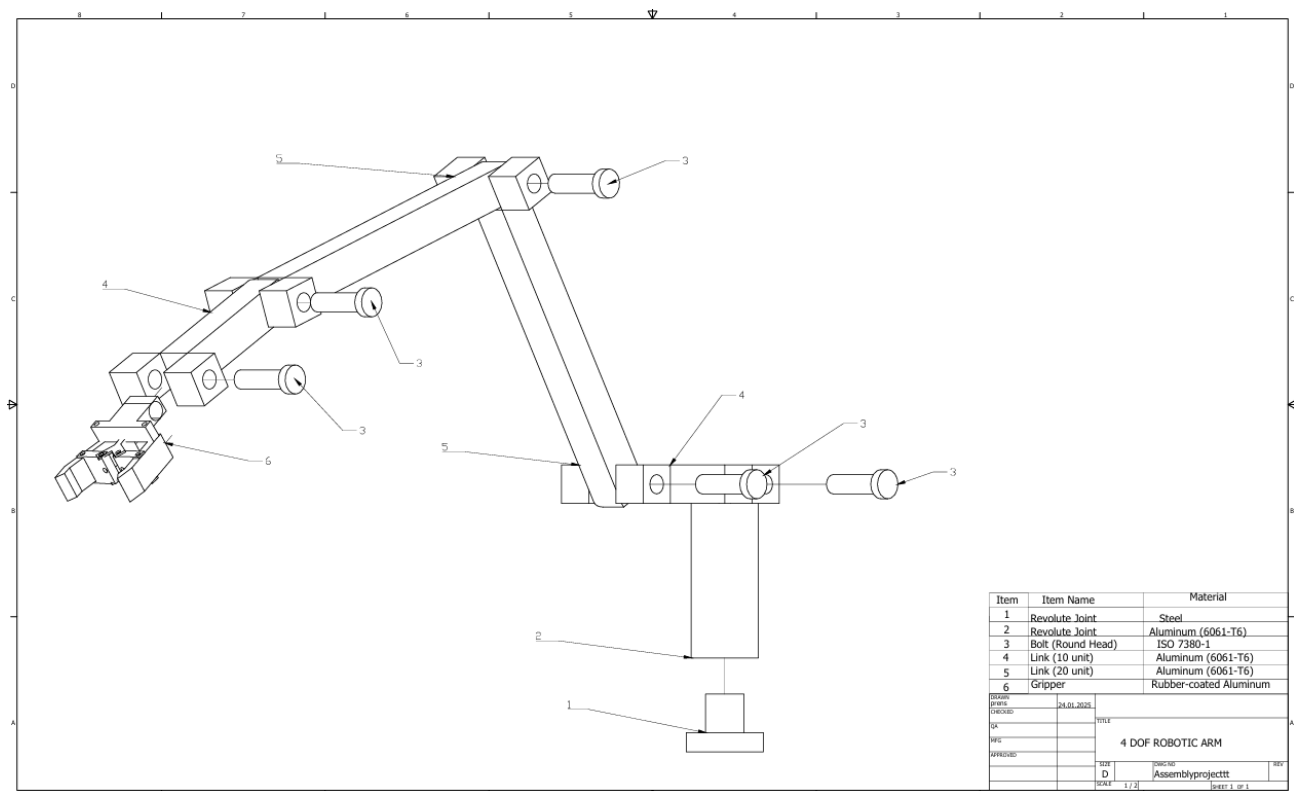
$$30 = 30$$

Our values are correct

$$A_1 + a_2 + a_3 = 0$$



# Assembly Drawing



# Manufacturing Drawing

