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Game Theory

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Summary

- Introduction to Game Theory
- Classical Game Theory Techniques
  - Best Response Strategies
  - Dominant Strategies
  - Pareto Optimality
  - Nash Equilibrium
  - Max-Min Strategies
- The Prisoner's Dilemma
- The Iterated Prisoner's Dilemma
- Evolutionary Game Theory

Introduction

Game theory studies of mathematical models of competition and cooperation among intelligent rational decision-makers, and it is mainly used in economics, political science, psychology, biology, e-Commerce or artificial intelligence (MAS) among other fields.

In this presentation we will consider a game scenario modeled by a payoff matrix with several options to do, and how a **rational agent** will act?

Intuitively the agent will try to maximize its expected payoff (self-interested point of view). But in most cases this is unfeasible because the individual best strategy depends on the choices done by others (**multi-agent** point of view).

We will consider players as **self-interested agents**, that have their own preferences and desires over the states of the world (i.e., non-cooperative game theory).

Introduction

Games can be divided in:

- **zero sum games** (left matrix), where the gains of one agent are the losses of the other, and;
- **non-zero sum games** (right matrix), where both agents can take mutual benefits from the game.

	Agent 2 does C	Agent 2 does D
Agent 1 does C	2, −2	−3, 3
Agent 1 does D	3, −3	−1, 1

	Agent 2 does C	Agent 2 does D
Agent 1 does C	3, 3	0, 5
Agent 1 does D	5, 0	1, 1

Introduction

There are a set of classical techniques that can be used to try to find what's the best option from the agent point of view:

- Best Response
- Dominant strategies
- Pareto optimality
- Nash equilibrium
- Max-min Strategy

Next, we will give a quick look to all these techniques.

Best Response

Given the player 2 will play strategy S2, the best response of agent 1 to S2 is the strategy S1 that provides agent 1 the highest payoff.

Example:

Agent 2 plays C --> Agent 1 best response --> D

Agent 2 plays D --> Agent 1 best response --> C

	Agent 2 does C	Agent 2 does D
Agent 1 does C	1, 1	3, 2
Agent 1 does D	3, 3	1, 4

Dominant Strategies

A strategy S1* is dominant for agent 1 if, whatever strategy S2 agent 2 chooses, player 1 will do at least as well with S1* as it does with any other one.

Strategy **S1* is dominant if it is the best response to all agent 2 strategies**.

In the left matrix below, D is dominant for 1. Dominated strategies can be discarded, and then there is a new dominant strategy for agent 2.

	Agent 2: C	Agent 2: D
Agent 1: C	2, 3	1, 2
Agent 1: D	3, 3	2, 2

	Agent 2: C	Agent 2: D
Agent 1: D	3, 3	2, 2

Pareto Optimality

A set of strategies for a game are denoted as **Pareto optimal** (or Pareto efficient) if there is no other set that makes one agent better off, without making another agent worse off.

Therefore, if a set of strategies Q is not Pareto optimal, then there is another set P that gets at least a better payoff for one agent, and no worst for the rest.

"**Rational**" agents should agree to move to P in this case.

Example: Pareto Optimality

Find the dominant strategy and the Pareto optimal points for this matrix:

	Agent 2 does A	Agent 2 does B	Agent 2 does C	Agent 2 does D
Agent 1 does A	4, 2	3, 1	1, 3	1, 2
Agent 1 does B	3, 2	1, 4	6, 7	3, 2
Agent 1 does C	5, 5	7, 6	1, 2	6, 4
Agent 1 does D	2, 3	4, 1	1, 4	2, 2

Nash Equilibrium

Two pure strategies S1* and S2* are a Nash equilibrium IFF:

a) under the assumption that agent 1 plays S1*, agent 2 can not do better than playing S2*;

AND

b) under the assumption that agent 2 plays S2*, agent 1 can not do better than play S1*.

Agents have no incentives to deviate from Nash equilibrium points, that represent a “rational” outcome of a game played by selfish agents.

Nash Equilibrium

Unfortunately not every interaction scenario has a Nash equilibrium, while some other have more than one.

Nash's Theorem (1950): Every game with a finite number of players, and a finite set of possible strategies, has at least one Nash equilibrium in mixed strategies.

A **mixed strategy**, as a difference with a pure one, means that an agents creates a new strategy by selecting, using a probability distribution, among several pure strategies.

Nash Equilibrium: Battle of Sexes

A boy and his girlfriend wish to go to the cinema, but they doubt between going to see a "War Movie (W)" or a "Love Movie (L)". They much prefer to go together, although the girlfriend prefers the love movie (L) and the boyfriend prefers the war movie (W).

	Boyfriend: W	Boyfriend: L
Girlfriend: W	2, 3	0, 0
Girlfriend: L	1, 1	3, 2

Hint: to find a pure-strategy Nash equilibrium in a payoff matrix, just take the cell where the first number is the maximum of the column, and check if the second number is the maximum of that row.

Nash Equilibrium & Max-min Strategy

The max-min strategy for player 1 is the strategy S1 that maximizes agent 1 payoff in the worst case. Min-max is the dual strategy for the agent 2.

From the player 1 point of view:

- If he plays C, the min payoff is 1.
- If he plays D, the min payoff is 0.

so player 1 chooses C, and thinking the same player 2 chooses C too.

	Agent 2: C	Agent 2: D
Agent 1: C	1, -1	2, -2
Agent 1: D	0, 0	3, -3

Nash Equilibrium & Max-min Strategy

The “max-min solution” is (C, C), and it is also the (only) Nash equilibrium of this game.

Min-max Theorem: In any finite zero-sum game for two players: max-min, min-max and Nash equilibrium coincide.

	Agent 2: C	Agent 2: D
Agent 1: C	1, −1	2, −2
Agent 1: D	0, 0	3, −3

Example: Nash equilibrium

Find the Pareto, Nash equilibrium and max-min points for this matrix:

	Agent 2 does A	Agent 2 does B	Agent 2 does C	Agent 2 does D
Agent 1 does A	5, 4	3, 1	2, 1	1, 2
Agent 1 does B	3, 1	1, 4	6, 7	3, 2
Agent 1 does C	4, 2	8, 8	5, 5	6, 4
Agent 1 does D	2, 3	4, 1	1, 4	2, 2

The Prisoner's Dilemma (PD)

Two men are collectively charged with a crime and held in separate cells, with no way of meeting or communicating. They are told that (see left figure):

- if one confesses (defeating the colleague: D) and the other does not, the confessor will be freed, and the other one will be jailed for five years;
- if both confess (D), then each will be jailed for four years.
- if they don't confess (cooperate: C), then each will be jailed for two years.

Right figure is similar, but without negative values, and it is the classical PD matrix.

	Agent 2: C	Agent 2: D
Agent 1: C	–2, –2	–5, 0
Agent 1: D	0, –5	–4, –4

	Agent 2: C	Agent 2: D
Agent 1: C	3, 3	0, 5
Agent 1: D	5, 0	1, 1

The Prisoner's Dilemma

We can realize that:

- D is a dominant strategy.
- (D, D) is the only Nash equilibrium.
- All outcomes except (D,D) are Pareto optimal.

	Agent 2: C	Agent 2: D
Agent 1: C	3, 3	0, 5
Agent 1: D	5, 0	1, 1

Game theorists' conclusions: In the worst case D guarantees a highest payoff so D is for both players the rational move to make with a payoff of (1,1).

In the one-shot prisoner's dilemma the individual rational choice is defection (D).

Why dilemma? They both know they could make better off by both cooperating and each get payoff (3,3). But cooperation is too dangerous if the other plays D.

The Iterated Prisoner's Dilemma (IPD)

If the PD is repeated a number of times (meeting the opponents again) then the incentive to Defect can decrease and Cooperation can finally emerge.

Robert Axelrod organized in 1980 and 1981 two computer tournaments, inviting participants to submit their strategies to play the IPD. The length of the game was unknown to the players and 15 strategies submitted in the first tournament, to compete among them.

Some of the best strategies were:

- ALL-D: Always defect.
- SPITEFUL: Cooperate until receive a defection, then always defect.
- MISTRUST: Defect on 1st round, then play the last opponent’s move.
- TIT FOR TAT: Cooperate on 1st round, then play last opponent's move.

The Iterated Prisoner's Dilemma

TIT_FOR_TAT (TFT) won the tournament for that year and the next one, where 61 strategies were submitted !!! Still today it is one of the best and simplest strategies, together with the Pavlov strategy (Win-Stay / Lose-Shift) in noisy environments.

Even though TFT was the best strategy in the Axelrod's tournament, it doesn't mean it is the Optimal Strategy for the IPD; because it depends on the set of strategies used by the opponents.

Axelrod's Theorem: If the game is long enough, and the players do care about their future encounters, then a universal optimal strategy for the Iterated Prisoner's Dilemma does not exist.

Axelrod's Results

Axelrod also suggested the following rules for a successful strategy to play the IPD:

- **Don't be envious**: Remember the PD is not a zero-sum game. Don't play as you would strike your opponent!
- **Be nice**: Never be the first to defect.
- **Retaliate appropriately**: There must be a proper measure in rewarding cooperation and punishing defection.
- **Don't be too clever**: The performance of a strategy are not necessarily related to its complexity.

The Finite vs. Infinite IPD

Infinite IPD: if the number of PD interactions is infinite, then there can be time for cooperation to emerge.

Finite IPD: but if the number of times (n) that the IPD is played is finite, then at the last round both have an incentive to play D, so this can be considered for round (n-1) and by induction D becomes the rational choice for all the rounds.

Real life interactions are in between both cases, as the number of rounds if finite, but usually unknown by the players.

PD and IPD in Network interactions

We can consider as particular applications of these techniques:

- eBuy models for auctions.
- P2P systems and models to incentive peers for the sharing of resources: eMule, BitTorrent (uses TFT), etc.
- Automatic stock exchange electronic programs and algorithms.
- Game modeling on the Internet: either two player or multi-player models.
- Network traffic management, based on contracts nets or negotiations for resource allocation.
- Wireless networks communications.
- Vehicular networks and urban traffic management.

Evolutionary Game Theory (EGT)

The quest was to find a realistic model to predict how animals or species behave when competing for a resource.

The answer led to **evolutionary game theory**, as a way to describe animal contests as games with strategies.

EGT has become a major vehicle to help to understand some fundamental questions in biology like group selection, sexual selection, altruism, parental care, co-evolution, and ecological dynamics.

Evolutionary Game Theory (EGT)

EGT does not need that players act rationally, instead, the notion of **rationality is replaced by the concept of reproductive success**.

EGT differs from classical game theory by focusing more on the **dynamics of strategy change**, depending on the competing strategies, and also on the effect of the frequency of those competing strategies over the whole population.

Under this model, the **payoff utility is measured in fitness units**, that describe the reproductive success: strategies that are successful on average will be used more frequently, and prevail in the end.

EGT: Hawk-Dove Game (HD)

Hawk: individual playing a fighter strategy.

Dove: individual playing a fighter-avoider strategy.

V: value of a resource

C: cost of fighting for the resource

Hawk-Dove Game	Agent 2: Hawk	Agent 2: Dove
Agent 1: Hawk	(V-C)/2, (V-C)/2	V, 0
Agent 1: Dove	0, V	V/2, V/2

EGT: HD → PD Game

If V > C then we have the one-shot PD game, but not the IPD. Why?

Suppose that V=5 and C=3, then we have the next payoff matrix.

What are the Nash equilibrium points?

PD Game	Agent 2: Hawk	Agent 2: Dove
Agent 1: Hawk	1, 1	5, 0
Agent 1: Dove	0, 5	2.5, 2.5

EGT: HD → Snowdrift or Chicken Game

Usually, it is assumed that C > V > 0, and the game has been also denoted as the *Snowdrift* or the *Chicken* game.

Suppose that V=5 and C=7, then we have the next payoff matrix.

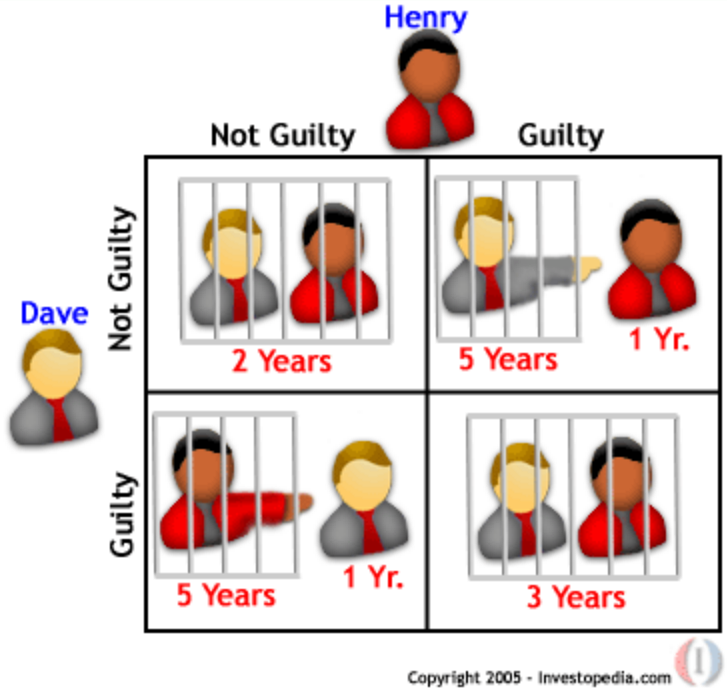
What are the Nash equilibrium points?

Chicken Game	Agent 2: Hawk	Agent 2: Dove
Agent 1: Hawk	-1, -1	5, 0
Agent 1: Dove	0, 5	2.5, 2.5

Bibliography

- Michael Wooldridge. An Introduction to Multiagent Systems. John Wiley & Sons. 2009.
- Robert Axelrod. The Evolution of Cooperation. 1984.
- Game Theory online course, accessible at: https://class.coursera.org/gametheory/auth/welcome
- Prisoner's Dilemma game simulator, accessible at: http://www.iterated-prisoners-dilemma.net/

Game Theory



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