1. *Problem 3.1: Cardinality bounds on maximal matchings*

Let G be a graph (not necessarily bipartite) and let M_1, M_2 be two maximal matchings in G .Show that$|M_1| \leq 2|M_2|$.

Proof. Assume the contrary that there exists an $|M_1| > 2|M_2|$ with both M_1, M_2 being maximal matchings in G.

Let V_{M_1} be the matched vertices in M_1 and V_{M_2} the same for M_2. Then there exists at least one $e \in M_1$ with both endpionts not in V_{M_2}. We could add this edge to M_2. But since both M_1 and M_2 are maximal this leads to a contradiciton. ■