1. *Problem 2.1: Night shift planning*

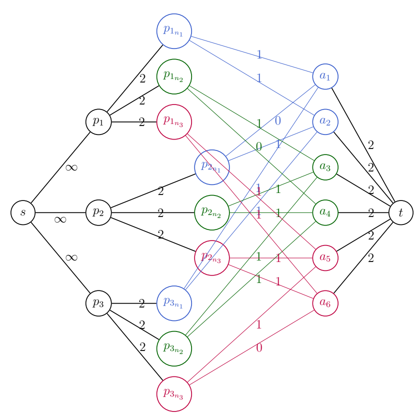
1.a *Design an efficient algorithm that decides whether there is a feasible assignment, and returns one if it exists. The running time of the algorithm should be in $O(n^c)$ for some constant $c \in \mathbb{Z} \geq 0$. Prove that your algorithm is correct and that it obeys the required running time bound.*

1.a.1 algorithm and construction of the graph

First we construct a graph using the information given as following:

1. s is the source,
2. for each physicians create a node p_i and connect s to all p_i with capacity ∞
3. for each physicians create nodes for each night-block (called $p_{i_{N_l}}$) and connect p_i to N_l with capacity 2
4. for each night create a node a_i and connect the corresponding $p_{i_{N_l}}$ to them with capacity 1 if the physician p_i is available on this night
5. create a node t (sink) and connect all a_i with capacity 2

An example of such a graph is given below. To get a better visual understanding, the night-blocks are colored.



We then apply the max s-t flow algorithm.

1.a.2 runtime analysis

Running time analysis for the graph construction steps:

1. $O(1)$
2. $O(|P|)$
3. $O(|P| * l)$ (l = number of night-blocks)
4. $O(|N| * l)$
5. $O(|N|)$

The running time for the Edmonds-Karp s-t algorithm is $O(nm^2)$. We have $|P| + |P| \cdot l + |N| + 2$ nodes and $|P| + |P| \cdot l + l \cdot |N| + |N|$ edges. Therefore our algorithm runs in polinomial time.

1.a.3 proof of correctness