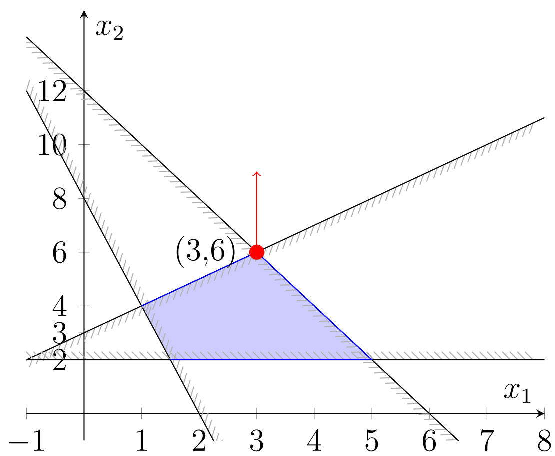
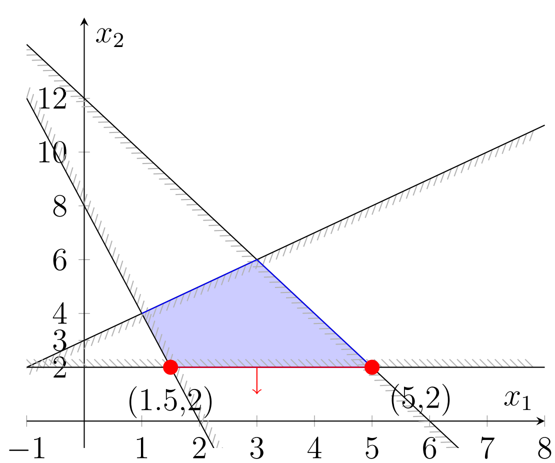
## Problem 1: Graphical solution of a linear program

$$
\begin{array}{llrcrcr}
\max & & & & x_2 & & \\
\text{s.t} & - & 4x_1 & - & x_2 & \leq & -8 \\
 & - & x_1 & + & x_2 & \leq & 3 \\
 & & & - & x_2 & \leq & -2 \\
 & & 2x_1 & + & x_2 & \leq & 12 \\
 & & x_1 & & & \geq & 0 \\
 & & & & x_2 & \geq & 0
\end{array}
$$

**a** *Determine all optimal solutions graphically.*




**b** *Instead of maximizing the objective function, minimize it and graphically determine all optimal solutions of the minimization problem.*

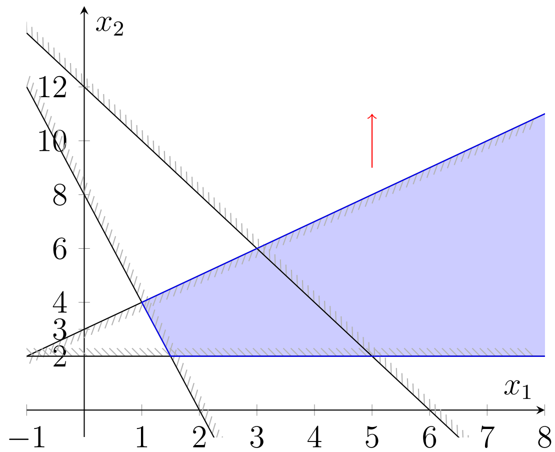



**c** *Change the fourth constraint $2x_1 + x_2 \leq 12$ such that*

*(i) the point (5, 2) still satisfies the fourth constraint with equality, and*

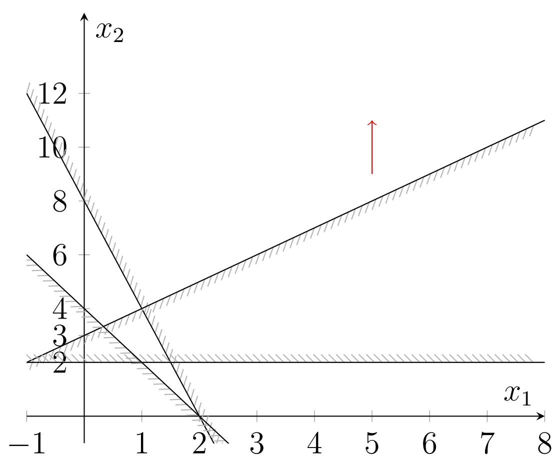
*(ii) the maximization problem becomes unbounded.*

A simple change would be to make the constraint unnecessary by changing $\leq$ to $\geq$. The maximization problem becomes unbounded since there is always a bigger (better) maximum value ($\infty$).




**d** *Change the right-hand side of the fourth constraint $2x_1+x_2 \leq 12$ (i.e., the number 12) such that the maximization problem becomes infeasible. Is the corresponding minimization problem infeasible, as well?*

Any value $< 5$ will do, the minimization problem will become infeasible too since there is no solution space.




**e** *Prove that the linear programs that you wrote in parts (c) and (d) are unbounded and infeasible, respectively. How would you prove optimality of the solutions you found in parts (a) and (b)?*

- unbounded: there is no constraint limiting $x_2$ from growing. It can grow infinitly on the constraint $-x_1 + x_2 \leq 3$.
- infeasible: if there is no solution area, there cannot be a solution.
- optimality in a and b: i could use the simplex algorithm and obtain the same solution.