1. *Problem 1: Simplex Algorithm*

Consider the following LP in canonical form:

$$
\begin{array}{lrcrcrcr}
\max & & x_1 & + & x_2 & & \\
\text{s.t.} & & x_1 & & & \leq & 4 \\
& & & & x_2 & \leq & 4 \\
& & x_1 & + & x_2 & \leq & 7 \\
& - & x_1 & - & x_2 & \leq & -3 \\
& & x_1 & & & \geq & 0 \\
& & & & x_2 & \geq & 0
\end{array}
$$

1.a *Transform the given linear program into standard form, using slack variables $y1, \ldots, y4$ corresponding to the four constraints (excluding non-negativity constraints).*

$$
\begin{array}{lrcrcrcrcrcrcrcr}
\max & & x_1 & + & x_2 & & & & & & & & & & \\
\text{s.t.} & & x_1 & & & + & y_1 & & & & & & & = & 4 \\
& & & & x_2 & & & + & y_2 & & & & & = & 4 \\
& & x_1 & + & x_2 & & & & & + & y_3 & & & = & 7 \\
& - & x_1 & - & x_2 & & & & & & & + & y_4 & = & -3 \\
& & \multicolumn{13}{l}{x_i, y_j \geq 0 \quad \forall i \in [1,2], \forall j \in [1,2,3,4]}
\end{array}
$$

1.b *Write down the short tableau with basis $B = (y1, y2, y3, y4)$. Is it feasible or infeasible? Why? What is the basic solution in the original space corresponding to this tableau?*

	x_1	x_2	1
z	-1	-1	0
y_1	1	0	4
y_2	0	1	4
y_3	1	1	7
y_4	-1	-1	-3

This tableau is infeasible since there is a negative b value. The basic solution is $(x_1, x_2, y_1, y_2, y_3, y_4) = (0, 0, 4, 4, 7, -3)$ which violates $y_4 \geq 0$

1.c *Run phase I of the Simplex Method to certify feasibility of the problem. From the tableau you obtain, extract a feasible tableau for the given LP.*

	x_1	x_2	x_0	(1)
$\tilde{z}$	0	0	1	0
z	-1	-1	0	0
y_1	1	0	-1	4
y_2	0	1	-1	4
y_3	1	1	-1	7
y_4	-1	-1	$\boxed{-1}$	-3

most negative row

$\Rightarrow$

	x_1	x_2	y_4	(1)
$\tilde{z}$	-1	-1	1	-3
z	-1	-1	0	0
y_1	2	1	-1	7
y_2	1	2	-1	7
y_3	2	2	-1	10
x_0	$\boxed{1}$	1	-1	3

row x_1 Bland's rule
col x_0 quotient rule

$\Rightarrow$

	x_0	x_2	y_4	(1)
$\tilde{z}$	1	0	0	0
z	1	0	-1	3
y_1	-2	-1	1	1
y_2	-1	1	0	4
y_3	-2	0	1	4
x_1	1	1	-1	3

since x_0 is no longer in the basis we can delete this column and continue with Simplex phase II

1.d *Starting from the tableau obtained in 1.c, run phase II of the Simplex Method to obtain anoptimal tableau for the given LP, and argue why it is optimal. Read the corresponding solutionand its value from the tableau.*

	x_2	y_4	(1)
z	0	-1	3
y_1	-1	$\boxed{1}$	1
y_2	1	0	4
y_3	0	1	4
x_1	1	-1	3

$\Rightarrow$

	x_2	y_1	(1)
z	-1	1	4
y_4	-1	1	1
y_2	1	0	4
y_3	$\boxed{1}$	-1	3
x_1	1	1	4

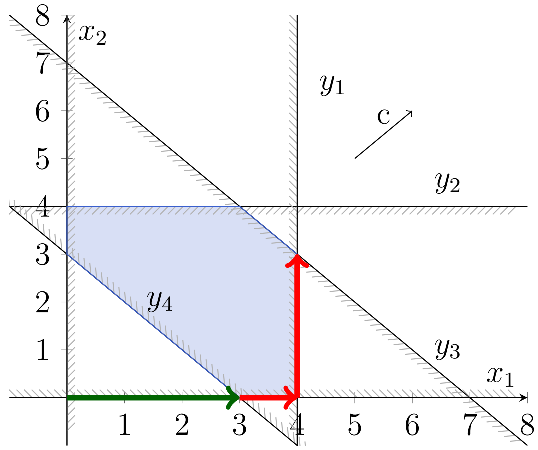
$\Rightarrow$

	y_3	y_1	(1)
z	1	0	7
y_4	1	0	4
y_2	-1	1	1
x_2	1	-1	3
x_1	0	1	4

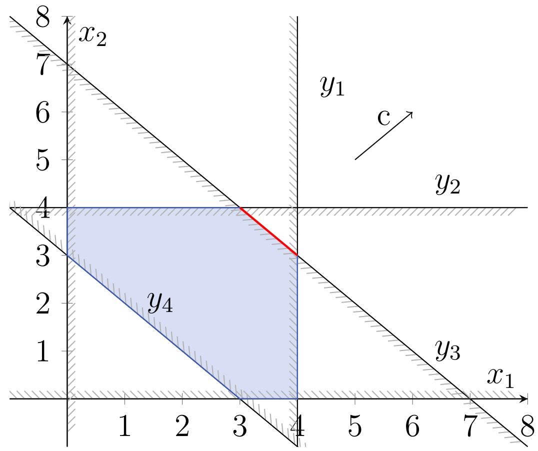
This solution is optimal since all coefficients in the objective row (z) are non-negative and the tableau is feasible. The optimal solution is 7 with $(x_1, x_2, y_1, y_2, y_3) = (4, 3, 0, 1, 9, 4)$.

1.e *Draw the feasible region of the initial canonical linear program, mark all solutions that you visited when executing phase I and phase II of the Simplex Method. Use the graphic solution method to double-check that phase II ended at an optimal solution.*

Simplex-vertices visited in phase I and phase II.



Graphical solution



1.f *In the drawing from 1.e, you might have observed that there exist infinitely many optimal solutions. Can you argue that this is indeed true by only looking at the optimal tableau obtained in part 1.d?*

Yes, by performing a pivot to bring y_1 into basis and y_2 out of basis the tableau remains optimal and feasible.

	y_3	y_1	(1)
z	1	0	7
y_4	1	0	4
y_2	-1	$\boxed{1}$	1
x_2	1	-1	3
x_1	0	1	4

(Copied from 1.d)

$\Rightarrow$

	y_3	y_2	(1)
z	1	0	7
y_4	1	0	4
y_1	-1	1	1
x_2	0	1	4
x_1	1	-1	3

2. *Problem 2:Certifying infeasibility from phase I of the Simplex Method*