1. *Problem 1: Relaxation of the matchin polytope*

Let $G = (V, E)$ be a simple undirected graph. We are interested in studying a polytope P that may be a candidate for the matching polytope $P_{\mathcal{M}}$. Recall that the familiy of all matchis is

$$\mathcal{M} = \{M \subseteq E : e_1 \cap e_2 = \emptyset \quad \forall e_1, e_2 \in M, e_1 \neq e_2\},$$

and that the matching polytope is, by definition, $P_{\mathcal{M}} := \mathrm{conv}(\{\chi^M : M \in \mathcal{M}\})$. Consider the polytope

$$P := \{x \in \mathbb{R}^E_{\geq 0} : x(\delta(v)) \leq 1 \quad \forall v \in V\}.$$

Show that there are graphs G for which P does not describe the matching polytope, i.e., $P \neq P_{\mathcal{M}}$.

Solution Let's look at the Graph G drawn in Figure 1. $P_{\mathcal{M}}$ is defined as $\{(1,0,0)^\top, (0,1,0)^\top, (0,0,1)^\top, (0,0,0$ where $(e_1, e_2, e_3)^\top$ corresponds to the vectors.

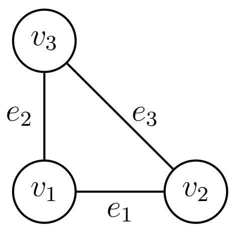

Figure 1: Graph G where $P \neq P_{\mathcal{M}}$