1. *Problem 1: Carathéodory's theorem*

Let $P \subseteq \mathbb{R}^{\times}$ be a non-empty polytope and let $x \in P$. We prove Carathéodory's theorem, namely that x is a convex combination of at most $n+1$ many vertices of P.

1.a *Let $V = vertices(P)$ and define*

$$Q := \left\{ \lambda \in \mathbb{R}^V_{\geq 0} : \sum_{v \in V} \lambda_v v = x \quad \textit{and} \quad \sum_{v \in V} \lambda_v = 1 \right\}$$

Show that Q is a non-empty polytope.