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Mathematical Optimization – Problem set 1

https://moodle-app2.let.ethz.ch/course/view.php?id=12839

General remarks

- Every Tuesday, a new problem set will be published on the moodle webpage. Problem sets consist of both theoretical problems and programming exercises, usually with a focus on theoretical problems. The first problem set is an exception to that rule. ☺
- In the exercise classes, teaching assistants discuss the problem sets, lecture contents, and any of your questions. We strongly encourage you to take advantage of the opportunity to discuss any questions with your assistants and to create an interactive environment that you can profit from in the best way possible. Your teaching assistant will also answer questions via email.
- You can hand in written solutions to the problems at the exercise class in the week after their publication to get feedback on your write-ups. Complete solutions to all problems will be made available online.
- You can also get feedback on your implementations of the Python problems if you submit them via the upload tool `https://sam-up.math.ethz.ch/`
- There is no extra credit for the problem sets or the programming exercises.
- For general course-related questions, send an email to `math.opt@ifor.math.ethz.ch` . We also offer office hours upon request (on zoom or in-person).

Problem 1: Graphical solution of a linear program

Consider the linear program

$$
\begin{array}{lrcrcr}
\max & & & x_2 & & \\
\text{s.t.} & -4x_1 & - & x_2 & \leq & -8 \\
 & -x_1 & + & x_2 & \leq & 3 \\
 & & - & x_2 & \leq & -2 \\
 & 2x_1 & + & x_2 & \leq & 12 \\
 & x_1 & & & \geq & 0 \\
 & & & x_2 & \geq & 0\,.
\end{array}
$$

(a) Determine all optimal solutions graphically.

(b) Instead of maximizing the objective function, minimize it and graphically determine all optimal solutions of the minimization problem.

(c) Change the fourth constraint $2x_1 + x_2 \leq 12$ such that

  (i) the point $(5, 2)$ still satisfies the fourth constraint with equality, and

  (ii) the maximization problem becomes unbounded.

(d) Change the right-hand side of the fourth constraint $2x_1 + x_2 \leq 12$ (i.e., the number 12) such that the maximization problem becomes infeasible. Is the corresponding minimization problem infeasible, as well?

(e) Prove that the linear programs that you wrote in parts (c) and (d) are unbounded and infeasible, respectively. How would you prove optimality of the solutions you found in parts (a) and (b)?

Programming exercises

(a) Install the Anaconda Distribution as well as the extra package PuLP on your machine. To this end, you can follow the guidelines given on the course moodle page (see the link in the title).

(b) Work through the notebook `01_introduction.ipynb`, which introduces both Jupyter Notebooks and basic programming in Python. To this end, open a JupyterLab environment as described at the end of the installation instructions, and open the notebook `01_introduction.ipynb` from JupyterLab's file navigation pane.

Remark: In case you are familiar with Python and Jupyter Notebooks, you can skip this step.

(c) *Linear programming:*

- Work through the linear programming tutorial `01_tutorialLP.ipynb`.
- Solve the regression problems in `01_regression.ipynb` using linear programs.

(d) *Working with graphs:*

- Work through the tutorials on dealing with graphs in python using the NetworkX library, namely `01_tutorialGraphDrawing.ipynb`, `01_tutorialDigraphs.ipynb`, as well as `01_tutorialEdgeAttributes.ipynb`.
- Work through `01_maxFlow.ipynb`, where you implement a maximum flow problem solver using linear programs.