Migros Cross-Math Solver

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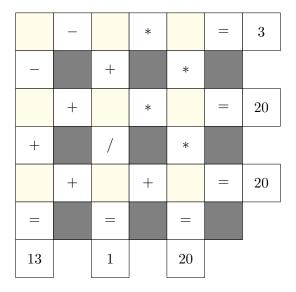
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1 Problem

The Cross-Math puzzle is a puzzle with 9 numbers to fill. The rules are simple:

- Mathematical operation ordering does not count, simply calc from left to right and from top to bottom
- Each cell is filled with a number between 1 9
- Each number can only appear once

The puzzle presented is the following:



2 QCP Mathematical formulation

In this section i present some quadratic constraint programming solutions

2.1 With Big-M

M is very big and ϵ is a very small number.

Equation 2 – Equation 5 are the basic requirements for the corss-math puzzle. Equation 6 – Equation 11 are the specific equations for this problem.

2.2 One Hot Matrix

 $x_{i,j}$

The idea behind a one hot matrix is to add a binary variable for each possibility. In our case this means to add 9 binary variables for every field.

min 0 (15)
s.t.
$$\sum_{j=1}^{j \le 9} x_{i,j} = 1, \qquad \forall i \in \{1, \dots, 9\}$$
(16)

$$\sum_{i \le 9} x_{i,j} = 1, \qquad \forall j \in \{1, \dots, 9\}$$
(17)

$$y_i = \sum_{j=1}^{j \le 9} j x_{i,j}, \qquad \forall i \in \{1, \dots, 9\}$$
(18)

$$(y_0 - y_1) y_2 = 3$$
(19)

$$(y_3 + y_4) y_5 = 20$$
(20)

$$y_6 + y_7 + y_8 = 20$$
(21)

$$y_0 - y_3 + y_6 = 13$$
(22)

$$\frac{(y_1 + y_4)}{y_7} = 1$$
(23)

$$y_2 y_5 y_8 = 20$$
(24)

$$x_{i,j} \in \{0, 1\}, \qquad \forall i, j \in \{1, \dots, 9\}$$
(25)

The constraints Equation 16 and Equation 17 define the uniqueness of a number in the problem. With Equation 18 a linear expression y_i is defined that expresses the actual value of the cell i. This linear expression is then used in Equation 19 – Equation 24 to define the actual problem. These constraints are the same as Equation 6 – Equation 11.

(26)

3 Backtracking

The puzzle is small and can easily be solved using a naive backtracking algorithm. A pseudo code algorithm is shown in Listing 1. The function checkAllPredicates checks if all conditions are met.

Listing 1: Backtrack algorithm

```
int[] field = new int[9];
Func<int,int,int,bool> predicates;
Solve(int currentField, int[] availableNumbers)
  if (!checkAllPredicates) return false;
  if (currentField > 8) return true

foreach(number in availableNumbers) do
   field[currentField] = number
    if (Solve(currentField + 1, availableNumbers without number))
     return true
  field[currentField] = 0
  return false
```

4 Solution

9		6	*	1	=	3
_		+		*		
3	+	2	*	4	=	20
+		/		*		
7	+	8	+	5	=	20
=		=		=		
13	-	1		20		