Migros Cross-Math Solver

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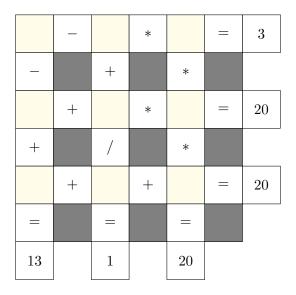
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1 Problem

The Cross-Math puzzle is a puzzle with 9 numbers to fill. The rules are simple:

- Mathematical operation ordering does not count, simply calc from left to right and from top to bottom
- Each cell is filled with a number between 1 - 9
- Each number can only appear once

The puzzle presented is the following:

	−		*		=	3
−		+		*		
	+		*		=	20
+		/		*		
	+		+		=	20
=		=		=		
13		1		20		

2 QCP Mathematical formulation

In this section i present some quadratic constraint programming solutions

2.1 With Big-M

$$\min \quad 0 \tag{1}$$

$$\text{s.t.} \quad x_i \quad \geq 1, \qquad \forall i \in \{1, \ldots, 9\} \tag{2}$$

$$x_i \quad \leq 9, \qquad \forall i \in \{1, \ldots, 9\} \tag{3}$$

$$x_i - x_j \quad \leq -\varepsilon + y_{i,j}M, \qquad \forall i, j \in \{1, \ldots, 9\} \tag{4}$$

$$x_i - x_j \quad \geq \varepsilon - (1 - y_{i,j})M, \qquad \forall i, j \in \{1, \ldots, 9\} \tag{5}$$

$$(x_0 - x_1)x_2 \quad = 3 \tag{6}$$

$$(x_3 + x_4)x_5 \quad = 20 \tag{7}$$

$$x_6 + x_7 + x_8 \quad = 20 \tag{8}$$

$$x_0 - x_3 + x_6 \quad = 13 \tag{9}$$

$$\frac{(x_1 + x_4)}{x_7}, \quad = 1 \tag{10}$$

$$x_2 x_5 x_8 \quad = 20 \tag{11}$$

$$x_i \quad \in \mathbb{N}, \qquad \forall i \in \{1, \ldots, 9\} \tag{12}$$

$$y_{i,j} \quad \in \{0, 1\}, \qquad \forall i, j \in \{1, \ldots, 9\} \tag{13}$$

$$\tag{14}$$

M is very big and ε is a very small number.

Equation 2 – Equation 5 are the basic requirements for the corss-math puzzle. Equation 6 – Equation 11 are the specific equations for this problem.

2.2 One Hot Matrix

The idea behind a one hot matrix is to add a binary variable for each possibility. In our case this means to add 9 binary variables for every field.

$$
\begin{align}
\min \quad & 0 \tag{15} \\
\text{s.t.} \quad & \sum_{j=1}^{j \leq 9} x_{i,j} = 1, && \forall i \in \{1, \ldots, 9\} \tag{16} \\
& \sum_{i=1}^{i \leq 9} x_{i,j} = 1, && \forall j \in \{1, \ldots, 9\} \tag{17} \\
& y_i = \sum_{j=1}^{j \leq 9} j x_{i,j}, && \forall i \in \{1, \ldots, 9\} \tag{18} \\
& (y_0 - y_1) y_2 = 3 \tag{19} \\
& (y_3 + y_4) y_5 = 20 \tag{20} \\
& y_6 + y_7 + y_8 = 20 \tag{21} \\
& y_0 - y_3 + y_6 = 13 \tag{22} \\
& \frac{(y_1 + y_4)}{y_7} = 1 \tag{23} \\
& y_2 y_5 y_8 = 20 \tag{24} \\
& x_{i,j} \in \{0, 1\}, && \forall i, j \in \{1, \ldots, 9\} \tag{25} \\
& \tag{26}
\end{align}
$$

The constraints Equation 16 and Equation 17 define the uniqueness of a number in the problem. With Equation 18 a linear expression y_i is defined that expresses the actual value of the cell i. This linear expression is then used in Equation 19 – Equation 24 to define the actual problem. These constraints are the same as Equation 6 – Equation 11.

3 Backtracking

The puzzle is small and can easily be solved using a naive backtracking algorithm. A pseudo code algorithm is shown in Listing 1. The function checkAllPredicates checks if all conditions are met.

Listing 1: Backtrack algorithm

```
int[] field = new int[9];
Func<int,int,int,bool> predicates;
Solve(int currentField, int[] availableNumbers)
  if (!checkAllPredicates) return false;
  if (currentField > 8) return true

  foreach(number in availableNumbers) do
    field[currentField] = number
      if (Solve(currentField + 1, availableNumbers without number))
        return true
  field[currentField] = 0
  return false
```

4 Solution

9	−	6	*	1	=	3
−		+		*		
3	+	2	*	4	=	20
+		/		*		
7	+	8	+	5	=	20
=		=		=		
13		1		20		