

Migros Cross-Math Solver

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January 14, 2020

1 Problem

The Cross-Math puzzle is a puzzle with 9 numbers to fill. The rules are simple:

- Mathematical operation ordering does not count, simply calc from left to right and from top to bottom
- Each cell is filled with a number between 1 - 9
- Each number can only appear once

The puzzle presented is the following:

	-		*		=	3
-		+		*		
	+		*		=	20
+		/		*		
	+		+		=	20
=		=		=		
13		1		20		

2 QCP Mathematical formulation

In this section i present some quadratic constraint programming solutions

2.1 With Big- M

$$\begin{aligned}
\min \quad & 0 & (1) \\
\text{s.t.} \quad & x_i & \geq 1, & \forall i \in \{1, \dots, 9\} & (2) \\
& x_i & \leq 9, & \forall i \in \{1, \dots, 9\} & (3) \\
& x_i - x_j & \leq -\epsilon + y_{i,j}M, & \forall i, j \in \{1, \dots, 9\} & (4) \\
& x_i - x_j & \geq \epsilon - (1 - y_{i,j})M, & \forall i, j \in \{1, \dots, 9\} & (5) \\
& (x_0 - x_1)x_2 & = 3 & (6) \\
& (x_3 + x_4)x_5 & = 20 & (7) \\
& x_6 + x_7 + x_8 & = 20 & (8) \\
& x_0 - x_3 + x_6 & = 13 & (9) \\
& \frac{(x_1 + x_4)}{x_7}, & = 1 & (10) \\
& x_2x_5x_8 & = 20 & (11) \\
& x_i & \in \mathbb{N}, & \forall i \in \{1, \dots, 9\} & (12) \\
& y_{i,j} & \in \{0, 1\}, & \forall i, j \in \{1, \dots, 9\} & (13) \\
& & & & (14)
\end{aligned}$$

M is very big and ϵ is a very small number.

Equation 2 – Equation 5 are the basic requirements for the corss-math puzzle. Equation 6 – Equation 11 are the specific equations for this problem.

2.2 One Hot Matrix

The idea behind a one hot matrix is to add a binary variable for each possibility. In our case this means to add 9 binary variables for every field.

$$\min \quad 0 \quad (15)$$

$$\text{s.t.} \quad \sum_{j=1}^{j \leq 9} x_{i,j} = 1, \quad \forall i \in \{1, \dots, 9\} \quad (16)$$

$$\sum_{i=1}^{i \leq 9} x_{i,j} = 1, \quad \forall j \in \{1, \dots, 9\} \quad (17)$$

$$y_i = \sum_{j=1}^{j \leq 9} j x_{i,j}, \quad \forall i \in \{1, \dots, 9\} \quad (18)$$

$$(y_0 - y_1)y_2 = 3 \quad (19)$$

$$(y_3 + y_4)y_5 = 20 \quad (20)$$

$$y_6 + y_7 + y_8 = 20 \quad (21)$$

$$y_0 - y_3 + y_6 = 13 \quad (22)$$

$$\frac{(y_1 + y_4)}{y_7} = 1 \quad (23)$$

$$y_2 y_5 y_8 = 20 \quad (24)$$

$$x_{i,j} \in \{0, 1\}, \quad \forall i, j \in \{1, \dots, 9\} \quad (25)$$

$$(26)$$

The constraints Equation 16 and Equation 17 define the uniqueness of a number in the problem. With Equation 18 a linear expression y_i is defined that expresses the actual value of the cell i . This linear expression is then used in Equation 19 – Equation 24 to define the actual problem. These constraints are the same as Equation 6 – Equation 11.

3 Backtracking

The puzzle is small and can easily be solved using a naive backtracking algorithm. A pseudo code algorithm is shown here:

```
int[] field = new int[9];
Func<int,int,int,bool> predicates;
Solve(int currentField, int[] availableNumbers)
    if (!checkAllPredicates) return false;
    if (currentField > 8) return true

    foreach(number in availableNumbers) do
        field[currentField] = number
        if (Solve(currentField + 1, availableNumbers without number))
            return true
    field[currentField] = 0
    return false
```

4 Solution

9	−	6	*	1	=	3
−		+		*		
3	+	2	*	4	=	20
+		/		*		
7	+	8	+	5	=	20
=		=		=		
13		1		20		