Think Math Puzzle 1

Can you spin the table?

Pascal Lüscher

March 26, 2020

1 Problem

The problem can be found in Matt Parker's Maths Puzzles , it's the first one. In his video Matt explains the problem very well, so I'm not going to repeat it here. Thanks Matt for the puzzle!

2 Solution

While there are always many ways to solve a problem, I opted for an ILP (integer linear program). In this section i present the mathematical formulation and explain it a bit

$$
\begin{aligned}
\min \quad & 0 & & & (1)\\
\text{s.t.} \quad & \sum_{i=0}^{n-1} x_{i,j} & = 1 & \quad \forall j \in \{0,\dots,n-1\} & (2)\\
& \sum_{i=0}^{n-1} x_{j,i} & = 1 & \quad \forall j \in \{0,\dots,n-1\} & (3)\\
& b_{i,j} & = 1 - x_{(j-i \mod n),j} & \quad \forall i,j \in \{0,\dots,n-1\} & (4)\\
& \sum_{i=0}^{n-1} b_{j,i} & = n-1 & \quad \forall j \in \{0,\dots,n-1\} & (5)\\
& x_{0,0} & = 1 & & (6)\\
& x_{i,j} \in \{0,1\} & & \quad \forall i,j \in \{0,\dots,n-1\} & (7)
\end{aligned}
$$

The basic idea is to create a one-hot encoding for the investors(x) initial-seat (in our case an investor can sit on place 0 to n) In the Equation 2 it is ensured that each place is assigned only once per investor. In the Equation 3 the one-hot encoding is ensured, so investor 1 can only obtain one seat. The Equation 4 introduces an expression b which value ranges from 0 to 1. It is used to determine if the place matches the investor on the table. For example if the investor 3 sits on place 3 ($x_{3,3} = 1$) the value of $b_{0,3}$ becomes 0. The first index is a "round" counter, so if the table gets turned once, the index increases. In Equation 5 the sum of all non-matching investors is summed up and it has to be or equal to $n-1$, so only 1 investor is happy. The last constraint, Equation 6 is just a symmetry breaking constraint, it strengthens the formulation so the solution 0123456 is equal to 6012345 (shifted by 1).