



# **Filtering in SAR Interferometry to Avoid Doppler Effect**

## **《The Wavenumber Shift in SAR Interferometry》**

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# **Part 1**

## **Filtering in SAR Interferometry to Avoid Doppler Effect**



# Outline

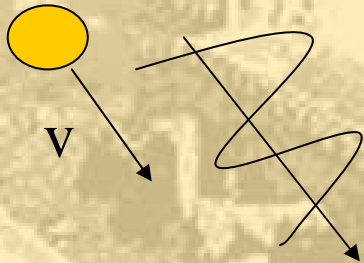
1. A brief introduction of Doppler effect in INSAR
2. Purpose: to reduce the decorrelation caused by Doppler effect
3. Our method: filtering
4. Problems remained: Some conflicts in theory and practice

## A brief introduction of Doppler effect in INSAR

- A physics phenomenon.
- Caused by a velocity relative to the object
- The frequency of the scattered signal is different from the frequency of the signal sent by radar
- In image, this causes a shift of central frequency
- The shifted central frequency is Doppler central frequency
- it is varied for different SAR images.

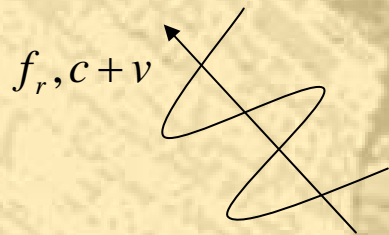
## An overview in physics

satellite



- The radar frequency is:  $f_0 = \frac{c}{\lambda}$

- When the relative speed  $v$  is considered, the frequency of the received signal is:



scatter

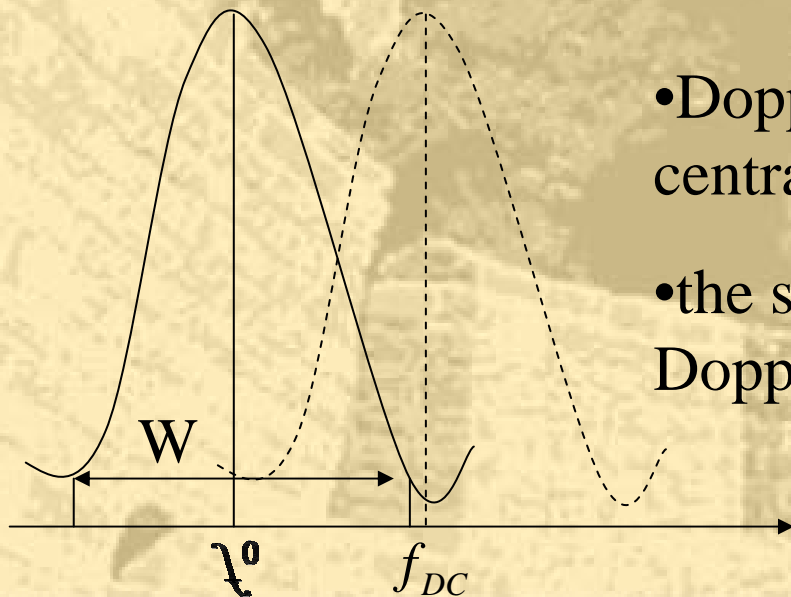


$$f_r = \frac{c + v}{\lambda}$$

- Conclusion: the frequency of the scattered signal is different from that of the signal sent by radar.

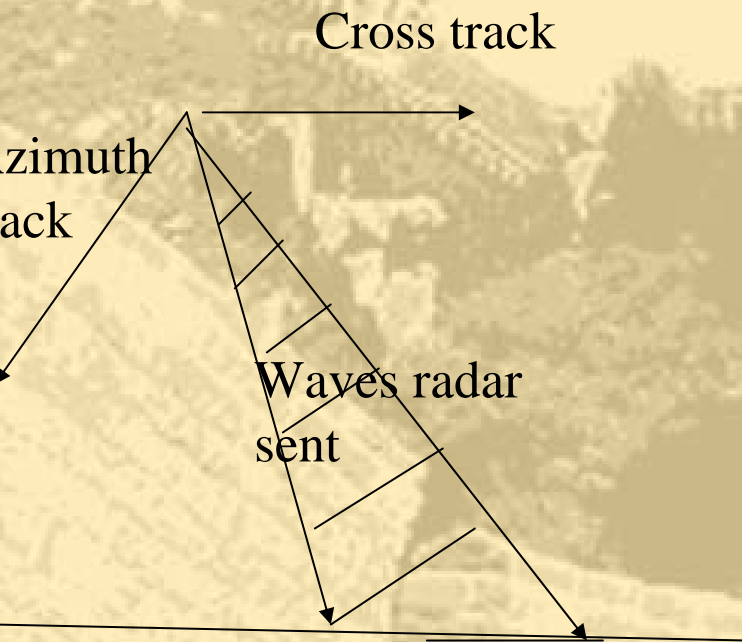
## Doppler central frequency

- No Doppler effect case: bandwidth  $W$  and central frequency  $f_0$




- Doppler effect case: bandwidth  $W$   
central frequency  $f_{DC}$

- the shifted frequency center is  
Doppler central frequency



This figure shows how radar sends waves when it is flying.



Purpose: To reduce the decorrelation of  
Doppler effect

To do SAR interferometry, we must create the interferogram. Interferogram is very important in INSAR processing and it is, for example, the basis of phase unwrapping, from which we can get the real altitude of the ground object.



## Interferogram generation

- $v_1$  and  $v_2$  are the two registered images.

$$v_1(x, y) = A_1(x, y) e^{j\phi_1(x, y)}$$

$$v_2(x, y) = A_2(x, y) e^{j\phi_2(x, y)}$$

$$\begin{aligned} v(x, y) &= v_1(x, y) * v_2^*(x, y) = A_1(x, y) A_2(x, y) e^{j(\phi_1 - \phi_2)} \\ &= A(x, y) e^{j\phi(x, y)} \end{aligned}$$

- $v$  is a third image, with an amplitude  $A(x, y)$ , which is called coherence image, and a phase term  $\phi(x, y)$ , which is called interferogram

## Correlation

- The correlation of the two images decide the quality of the interferogram.
- To get a high coherence between the two images is a key point



## Decorrelation factors

Decorrelation factors lead to phase noise in the interferogram.

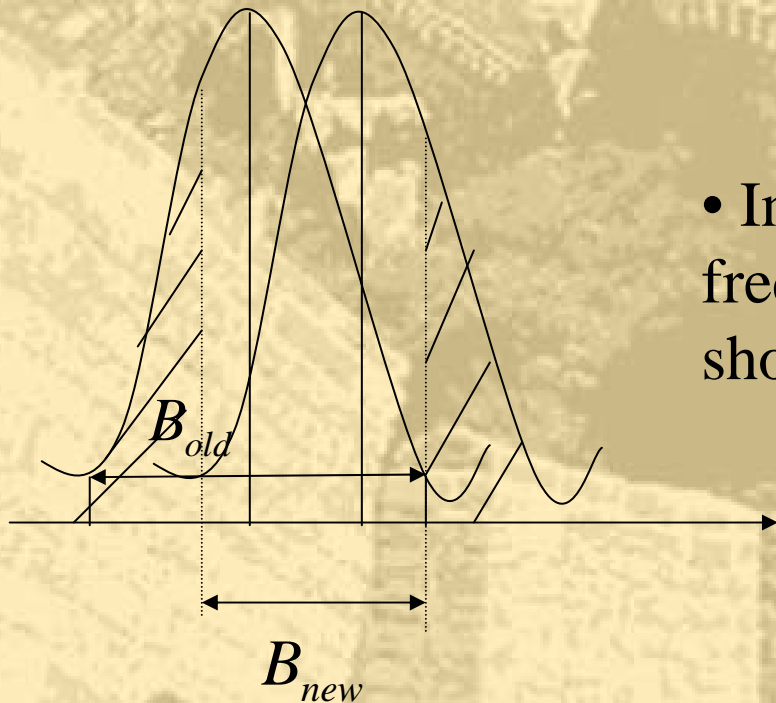
The de-correlation factors include:

1. System noise
2. Terrain decorrelation
3. Image misregistration
4. Approximate focusing
5. Geometry decorrelation
6. Doppler effect decorrelation

## Doppler effect decorrelation

- Some factors mentioned above are random such as 1,2,and 4
- Some factors can be calculated such as Geometry decorrelation
- Doppler effect decorrelation: our emphasis

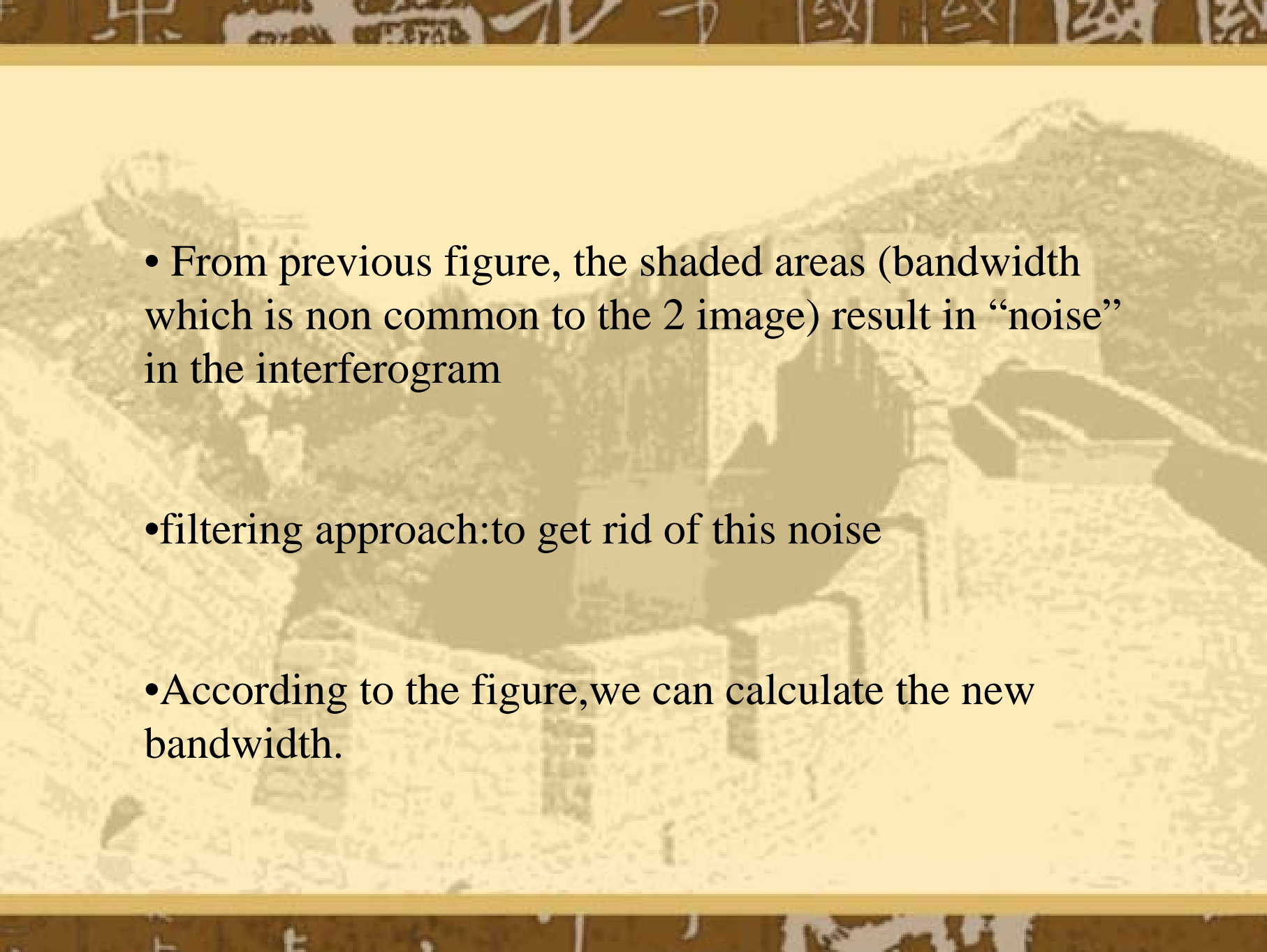
$f_{DC1}$   $f_{DC2}$



- the Doppler central frequencies in the two images  $f_{DC1}$ ,  $f_{DC2}$  are, most of the time, not the same.

- Interfere conditions in physics: frequency bandwidth,  $B_1 B_2$  should be the same.  $\rightarrow B_{new}$

In INSAR: the same frequency range



- From previous figure, the shaded areas (bandwidth which is non common to the 2 image) result in “noise” in the interferogram

- filtering approach: to get rid of this noise

- According to the figure, we can calculate the new bandwidth.


$$B_{new} = B_{old} - |f_{DC1} - f_{DC2}|$$

- in general, we should have:

$$\Delta f = |f_{DC1} - f_{DC2}| < B_{old}$$

- Problem:

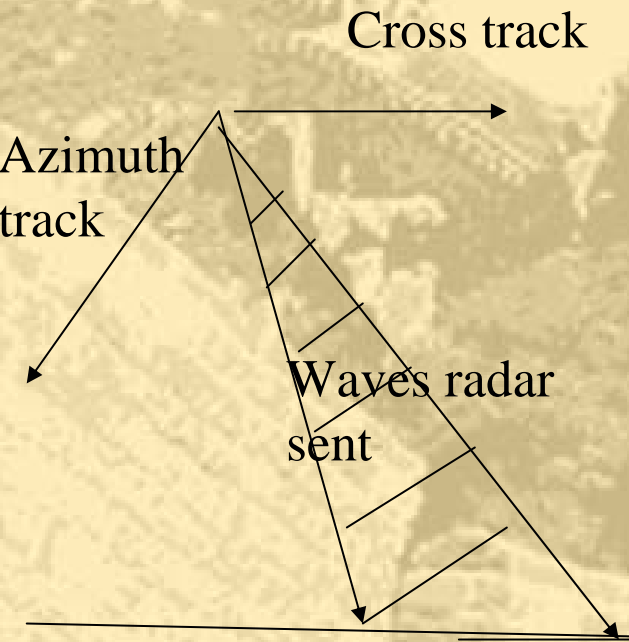
- What will happen if this rule is not obeyed?
- -> In theory: correlation would be extremely low
- -> In practice: Is it true? I will demonstrate later.



Our method:filtering



## Doppler Central frequency:



- for each line
- Varied for different lines

- A formula:

$$f_{DC}(i) = f_{DC0} + f_{DC1} * (i / RSR) + f_{DC2} * (i / RSR)^2$$

- I is the line number.
- parameters are from SLCI leader file data.

## Calculating the central frequencies

- To calculate along the azimuth track
- Lines should be correspondent
- “correspondent” means:
  1. Same lines in two images
  2. Considering the matching errors in line

Calculate the new bandwidth:

$$B_{new} = B_{old} - |f_{DC1} - f_{DC2}|$$

Benefit:

Design filters according to  $B_{new}$

# Filtering approach

1. 1-D Fourier transform on lines
2. Design filter
3. Inverse 1-D Fourier transform

A simple filter:

$$filter(u) = \begin{cases} 1(\text{if } |u| < B_{new}) \\ 0(\text{others}) \end{cases}$$

## The whole procedure:

Get parameters from SLCI files, such as fDC0, fDC1, RSR. They are included in the leader file

Along the azimuth track calculate Doppler central frequency in each line according to the formula.

Do 1-D Fourier transform of each line.

Design filter and filtering.

Do the inverse 1-D Fourier transform in each line and we get the final filtered complex image.

## Problems remained: Some conflicts in theory and practice

1. In theory, the difference in Doppler central frequency should be smaller than the bandwidth in azimuth track.
2. if it is not so, The correlation of the two images would be zero and we can not get the interferogram.

examples that does not follow this concept:

1. In Beijing 10,16,2002 and 9,11,2002,the difference between their central frequencies is large .While they can also generate interferogram.
2. In data in 10,16,2002 and 8,23,2003,their central frequencies are very close.While they can only get poor interferogram.

What is wrong?

图像获取时间	0911	1016	0827
常数项	710.711975	-856.237976	-744.388977
一次项	1657131.000000	-1074299.000000	-194918.000000
二次项	-2752999936.00000	-646000000.00000	-2036999936.0000

表: 多普勒系数





## **Part 2**

# **《The Wavenumber Shift in SAR Interferometry》**

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I will introduce this paper according to the authors thinking:

.Introduction

.Interferogram generation:The Monochromatic and Spectral Approach

.Several Applications

.Summary

# Introduction

1. Simple case: The radar waves's bandwidth is so small as to be monochromatic.
2. Real case: A Bandwidth exists.
3. A spectra shift for the reason of different looking angles
4. This can be used in several applications

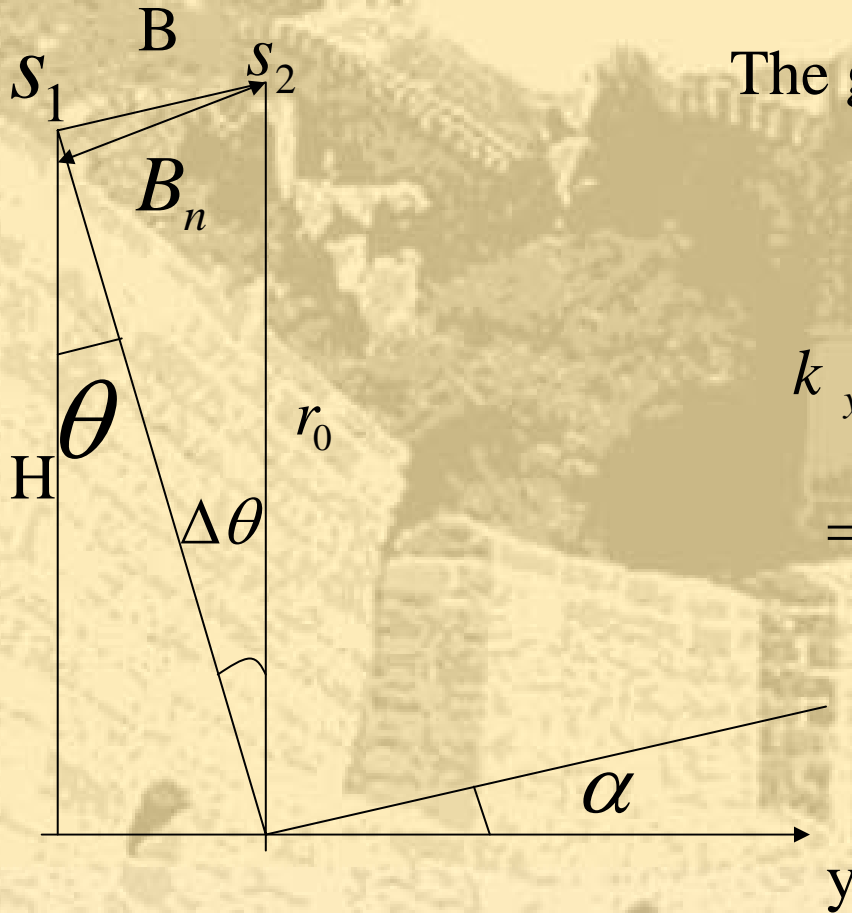
## .Interferogram generation:The Monochromatic and Spectral Approach

- Monochromatic approach

$$\phi = \frac{4\pi}{\lambda} \Delta r_0$$

# The spectral approach:

The ground range wavenumber is



$$\begin{aligned}
 k_y &= \frac{4\pi}{\lambda} \sin(\theta - \alpha) \\
 &= \frac{4\pi f}{c} \sin(\theta - \alpha) \quad (1)
 \end{aligned}$$

1.Our basis is:

- a slight change of looking angle  $\Delta\theta$

2.From (1) we can get

$$\Delta k_y = \frac{4\pi f_0}{c} \cos(\theta - \alpha) \quad (2)$$

3.by differentiation of (1) and according to (2),we can get the change of f:

$$\Delta f = - \frac{f_0 \Delta \theta}{\tan(\theta - \alpha)}$$

From geometry, we have:  $\Delta\theta = \frac{B_n}{r_0}$  So we have:

$$\Delta f = -\frac{cB_n}{r_0\lambda \tan(\theta - \alpha)} \quad (3)$$

- Conclusion:

If the two SARs are separated by an angle  $\Delta\theta$   
the same spectral of the first signal will be found in the  
second shifted by  $\Delta f$

- Bistatic case:

The basic principle is the same

# The concept of critical baseline

1. When  $\Delta f$  is larger, the correlation will be smaller
2. So the decorrelation will be larger with the increase of baseline
3. The critical baseline is the largest baseline allowed, and it can be computed as:

$$B_{nc} = \left| \frac{W r_0 \tan(\theta - \alpha)}{c} \right|$$





## .Several Applications

Based on equation (3),the authors gave several applications.

## A.Improvement if the Inreferometry SNR

Signals:

- W:the bandwidth
- $f_1, f_2$  :The central frequencies is
- $W_f$  :The filter's bandwidth is

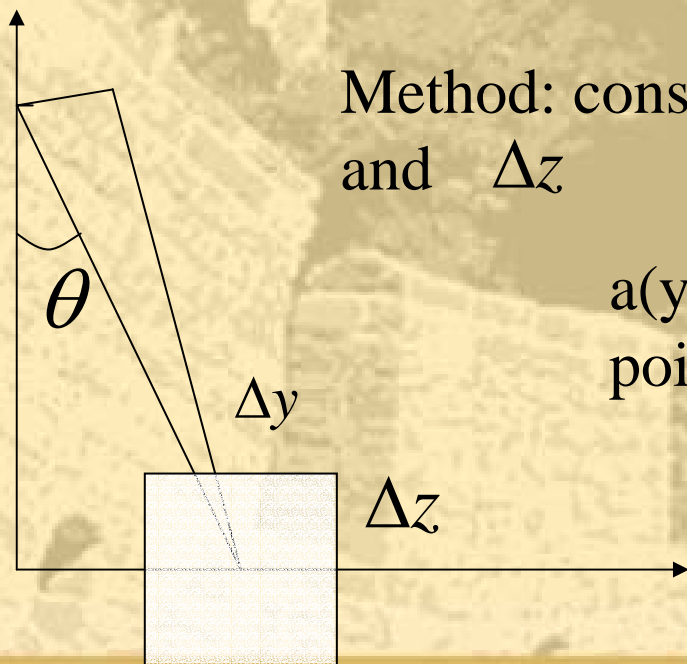
We have equations:

$$W_{f1} = W_{f2} = W - \Delta f$$

$$f_1 = \frac{\Delta f}{2} \quad f_2 = -\frac{\Delta f}{2}$$

## B. Effect of volume scattering: The baseline as an additional dimension

- In general case: we only consider surface scattering.
- In real case: volume scattering does exist



Method: consider a box with the dimensions  $\Delta y$  and  $\Delta z$

$a(y, z)$  is the complex reflectivity at point  $(y, z)$

We can get the received signal from the following equation:

$$A(\omega, \theta) = \iint a(y, z) \exp[-j \frac{2\omega}{c} (y \sin \theta - z \cos \theta)] dy dz$$

Noting that:

$$k_y = \frac{2\omega}{c} \sin \theta \quad k_z = \frac{2\omega}{c} \cos \theta \quad \text{We can get:}$$

$$A(k_y, k_z) = \iint a(y, z) \exp[-j(k_y y + k_z z)] dy dz \quad (4)$$

Now consider the looking angle change. From (3) we know to get the same  $k_y$ , there should be a spectral shift:  $\Delta\omega = -\omega\Delta\theta / \tan\theta$

And this can lead to an equivalent wavenumber shift:

$$\Delta k_z = 2\omega\Delta\theta / c \sin\theta \quad (5)$$

Two very important conclusions can derive from this:

. The decorrelation caused by the volume scattering:

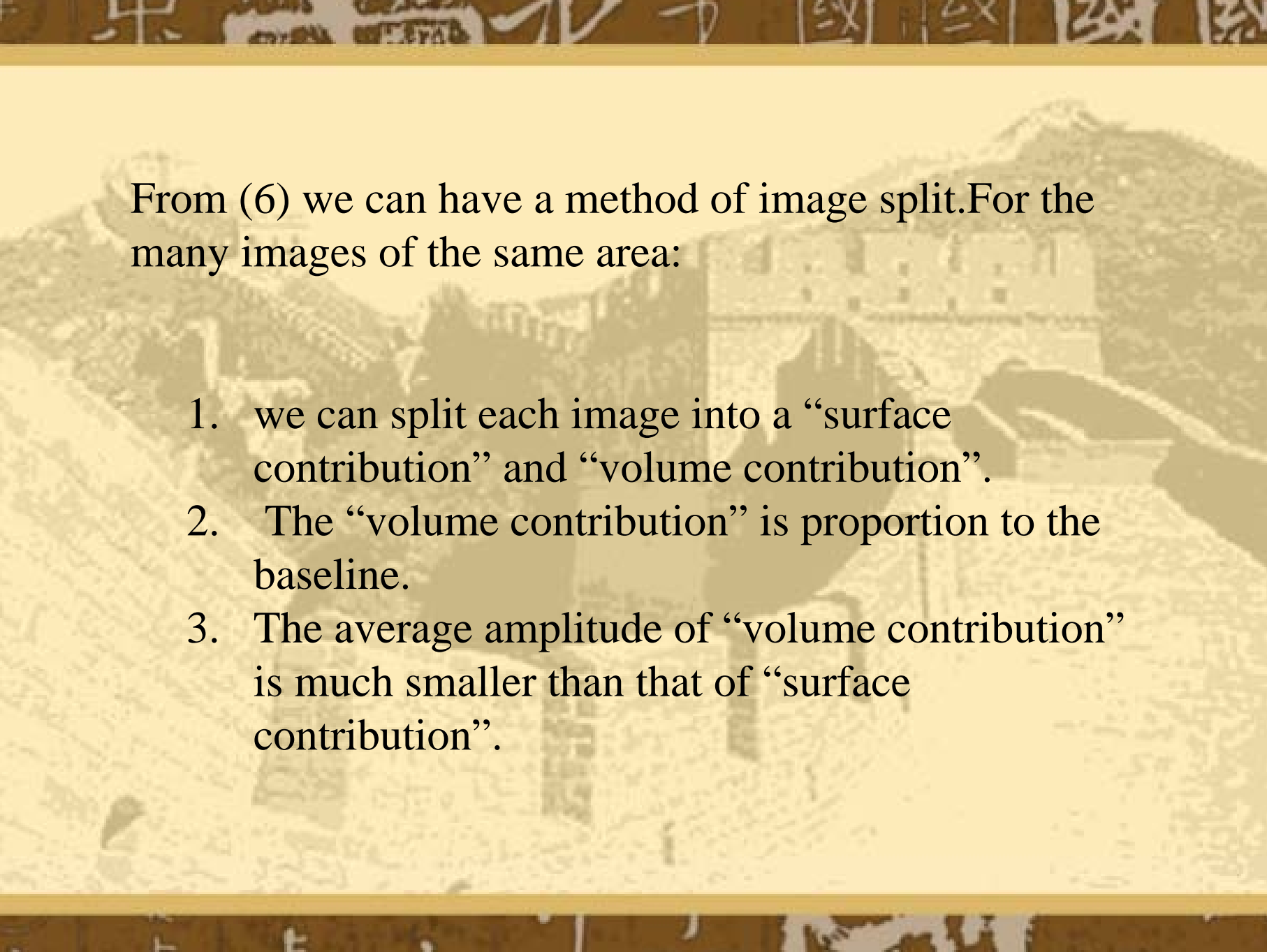
1. When  $\Delta z = 0$  this sort of correlation will be 1.

2. In general , The correlation is:  $\frac{\sin(\Delta k_z \Delta z / 2)}{(\Delta k_z \Delta z / 2)}$

3. when  $\Delta z \ll \frac{2\pi}{\Delta k_z}$ , the correlation will be nearly 1, and this type of decorrelation can be ignored

According to (4) and (5), we can get:

$$\begin{aligned} A(k_y, k_z + \Delta k_z) &\approx A(k_y, k_z) + \frac{\partial A}{\partial k_z} \Delta k_z \\ &= A(k_y, k_z) - \frac{j 2 \omega \Delta \theta}{c \sin \theta} \iint z a(y, z) \cdot \exp[-j(k_y y + k_z z)] dy dz \\ &= A(k_y, k_z) - \frac{j 4 \pi B_n}{H \lambda \tan \theta} V(k_y, k_z) \end{aligned} \quad (6)$$



From (6) we can have a method of image split. For the many images of the same area:

1. we can split each image into a “surface contribution” and “volume contribution”.
2. The “volume contribution” is proportion to the baseline.
3. The average amplitude of “volume contribution” is much smaller than that of “surface contribution”.



## C. Separation of layover areas

Study (3) again, we know:

- When the local slope  $\alpha$  approaches the off-nadir angle  $\theta$ , the spectral shift will become infinite.
- If the local slope continue to increase, the absolute value of the shift decreases, but the sign is changed.
- we can refer to areas of layover and nonlayover according to the frequency shift direction.

## D.TINSAR

- TINSAR:A tunable interferometric SAR
- Object:To reduce the decorrelation of large baselines.

Here we will talk about:

1. monostatic case
2. bistatic case
3. advantages

Monostatic: Shift the central system frequency of the second survey.

Bistatic:

- one transmitter and two receives.
- The transmitter's bandwidth must be larger than receiver's bandwidth  $W$
- The overlapped bandwidth of two receivers should be  $W$
- It is centered around two receivers' central frequencies.

# Advantages of TINSAR

In TINSAR we can use large baselines!

The largest local slope angle is  $\alpha$ , and the largest frequency shift allowed is  $W/4$ . According to (3), we can get the largest baseline in common case:

$$B_n = \frac{r_0 \lambda W}{2c} \tan(\theta - \alpha) \quad (7)$$

In TINSAR, noting the effect of local slope change,(3) is modified as follows:

$$\begin{aligned}\Delta f &= \frac{f_0 \Delta \theta}{2 \tan(\theta - \alpha)} - \frac{f_0 \Delta \theta}{2 \tan(\theta)} \\ &= \frac{cB_n}{2r_0\lambda} \left( \frac{1}{\tan(\theta - \alpha)} - \frac{1}{\tan \theta} \right)\end{aligned}$$

Similarly we can get the largest baseline in TINSAR case:

$$B_{nT} = \frac{r_0 \lambda W}{2c} \frac{\tan(\theta - \alpha) \tan \theta}{\tan \theta - \tan(\theta - \alpha)} \quad (8)$$

$$B_n = \frac{r_0 \lambda W}{2c} \tan(\theta - \alpha) \quad (7)$$

## E.Improvement of the slant range resolution

1. If the reflectivity's bandwidth is larger, the across-track resolution could be improved.
2. We can get a larger bandwidth by combining SAR surveys of different passes.
3. Conflicts: The larger the frequency shift, the larger cross track resolution we can get, while the phase estimation is less robust, for noise is more.

## .Summary

This paper has discussed the relationship between angular separation and spectral shift. It has also introduced techniques based on this relationship. Among these the idea of TINSAR is very important. And the other techniques are also very useful.