The Pol-InSAR Course

Synthetic Aperture Radar (SAR)

Part 2: Azimuth Resolution

Prepared by DLR-HR's Pol-InSAR Team

German Aerospace Center (DLR), Microwaves & Radar Institute (HR), Pol-InSAR Research Group

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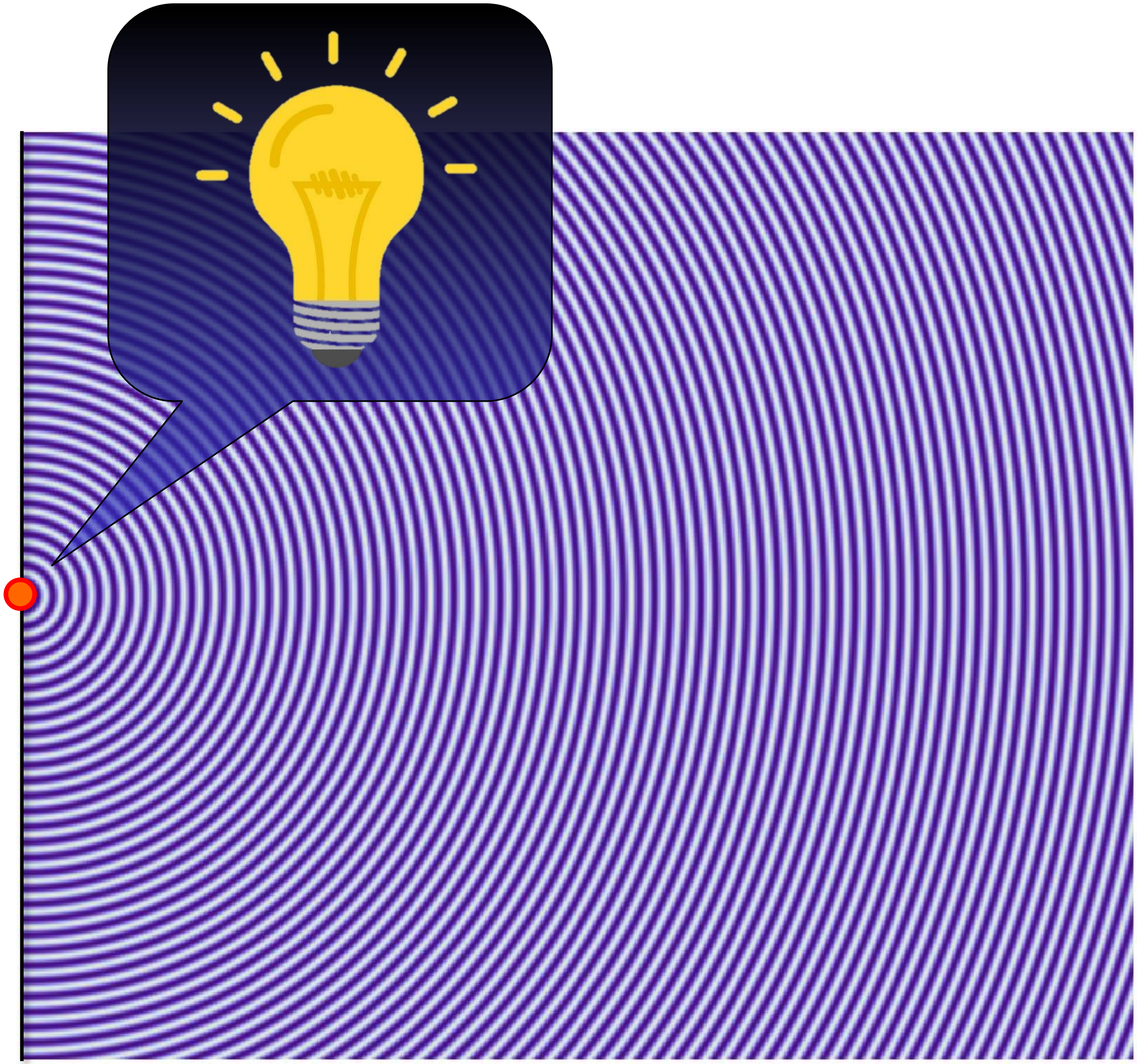
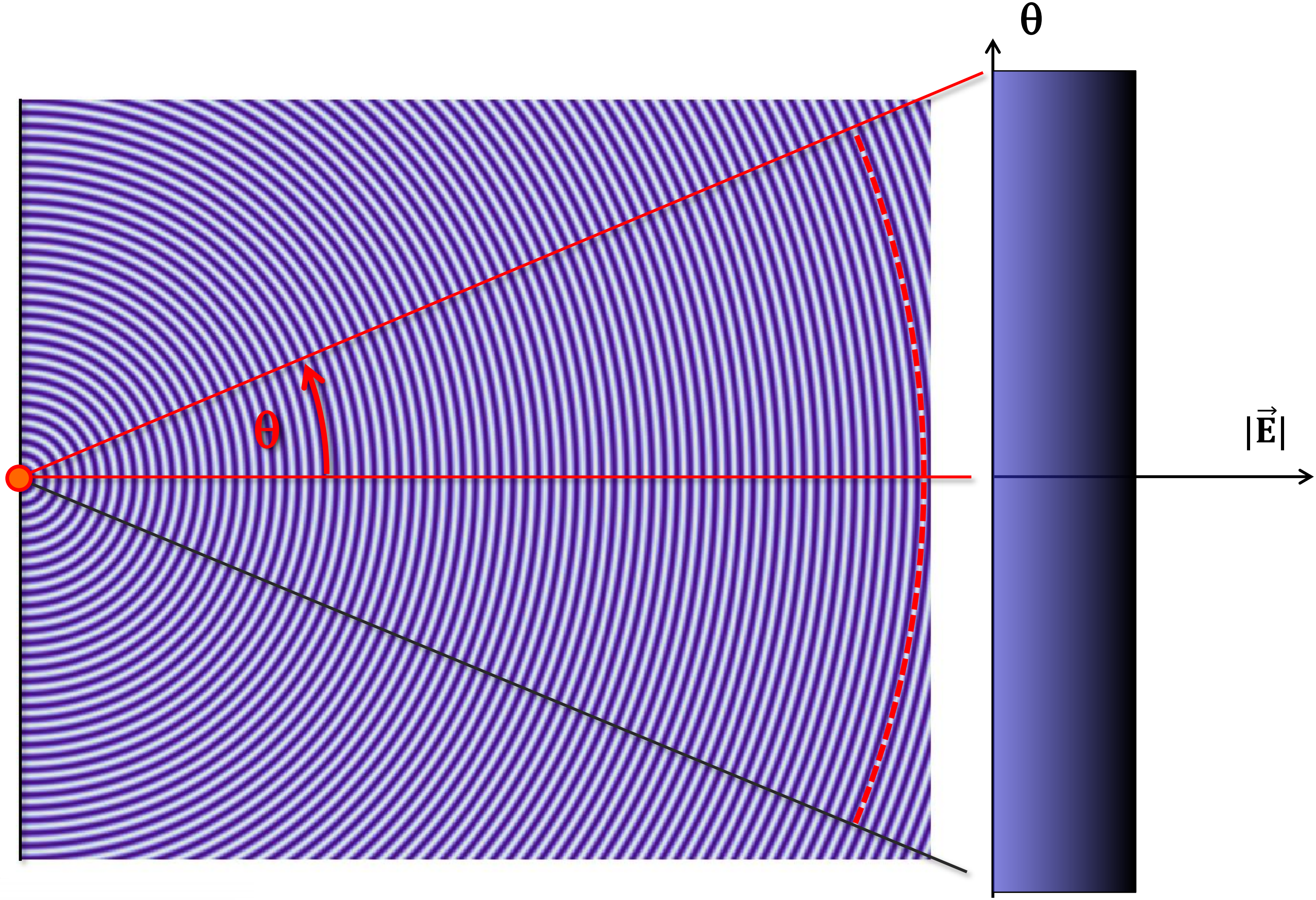
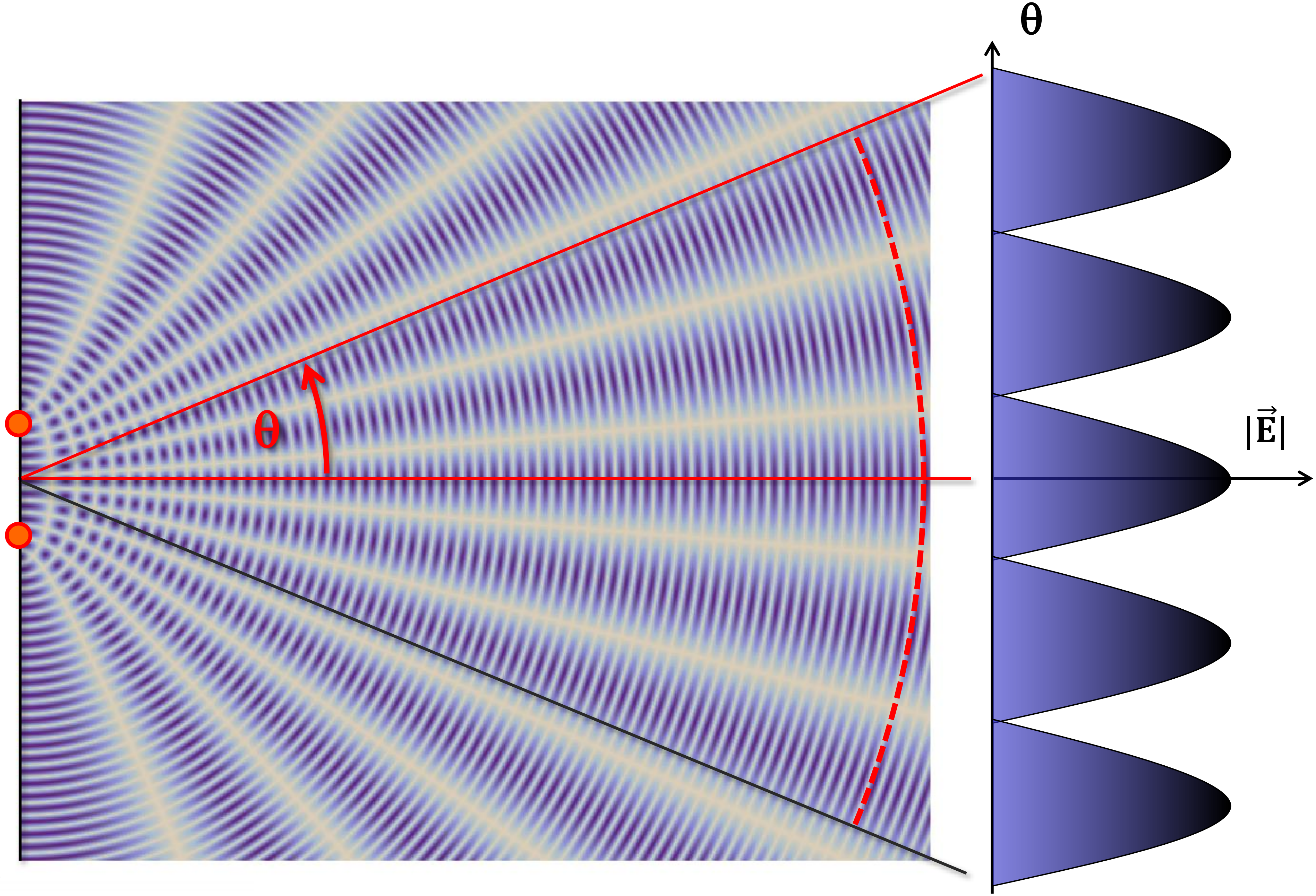
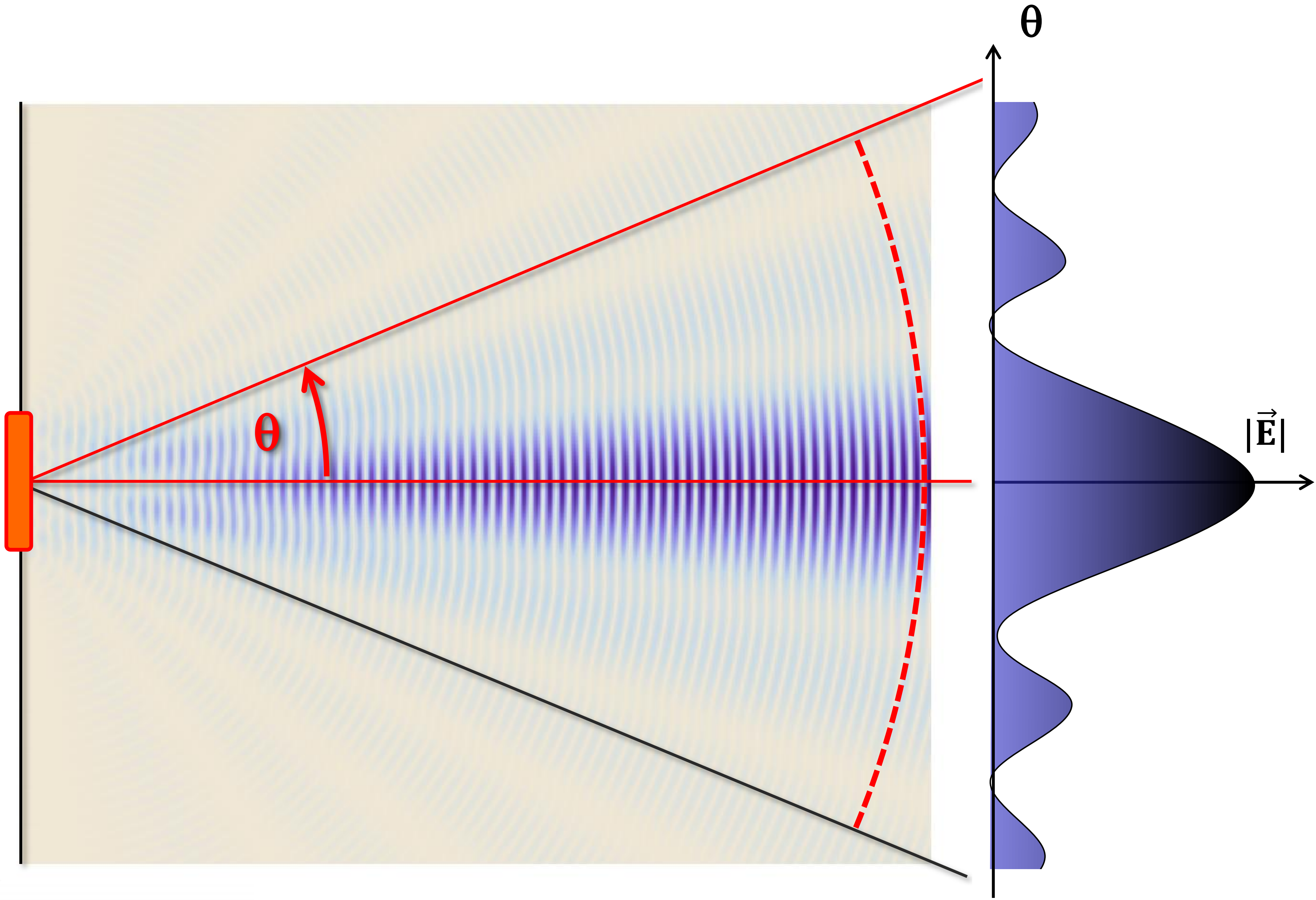
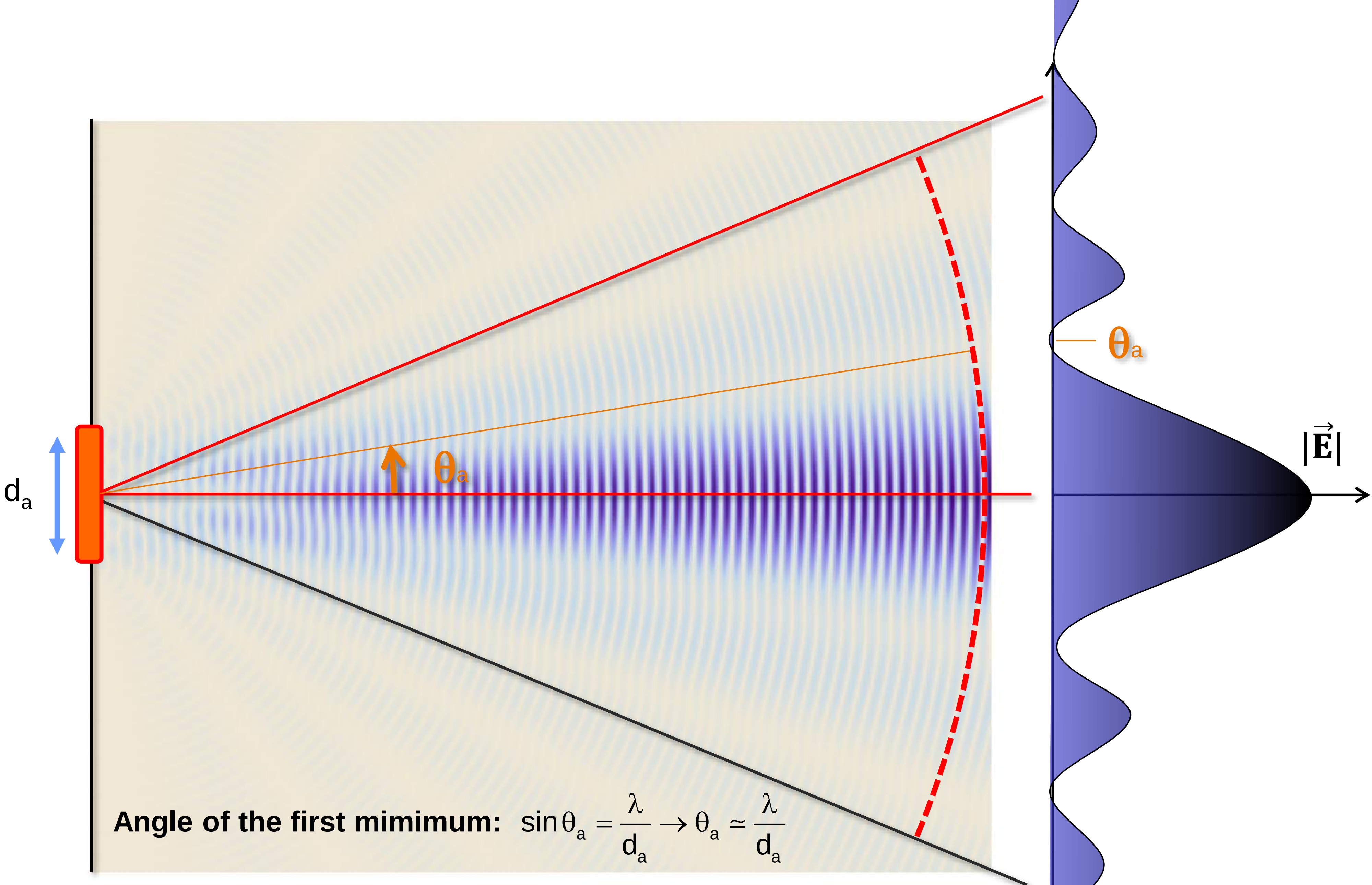
Azimuth Resolution




θ
θ
|E⃗|


θ
θ
$|\vec{E}|$

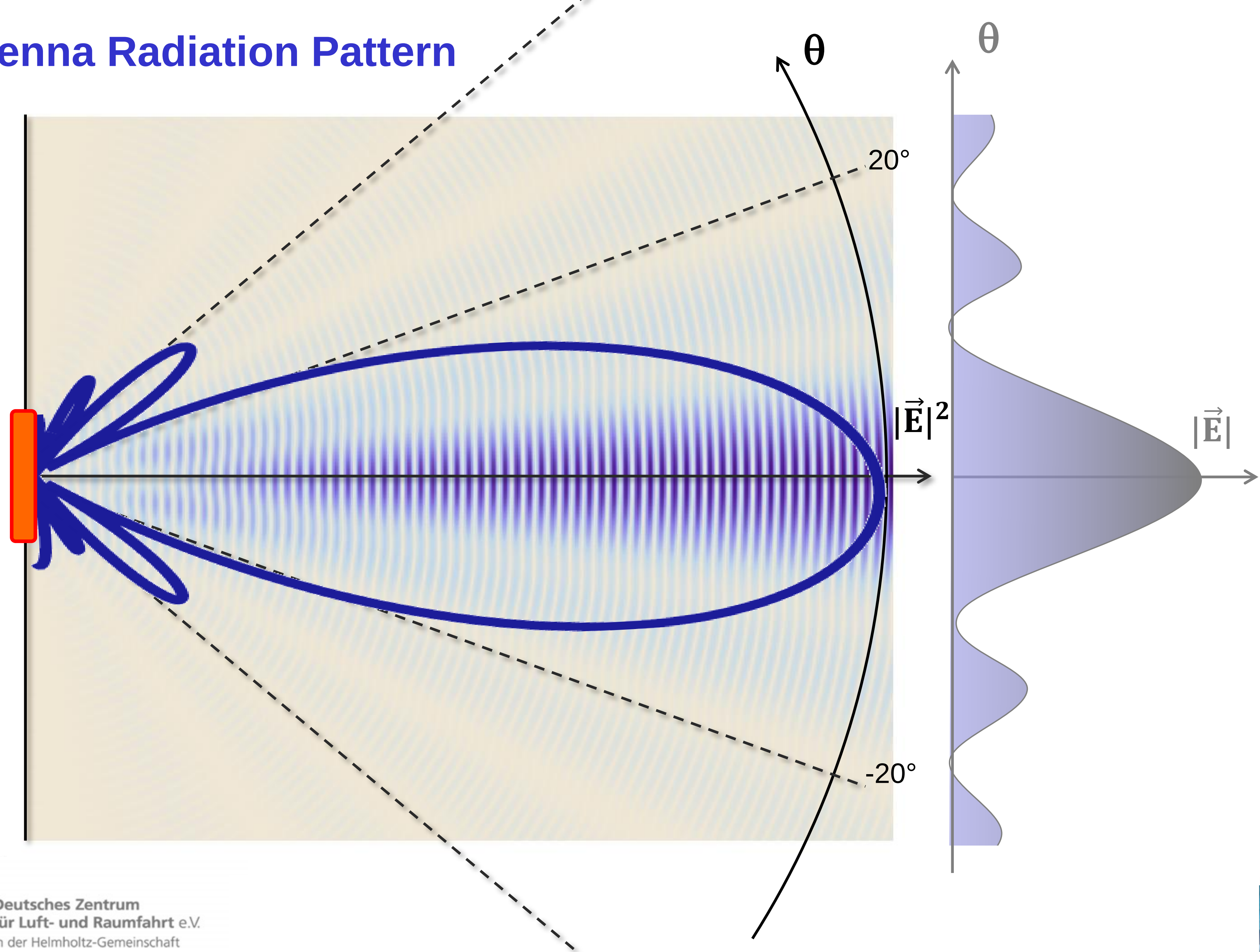

θ
θ
$|\vec{E}|$



Angle of the first mimimum: $\sin\theta_a = \dfrac{\lambda}{d_a} \rightarrow \theta_a \simeq \dfrac{\lambda}{d_a}$

d_a is the aperture's length, λ the wavelength, θ ithe angle relative to the "original" direction of wave propagation.

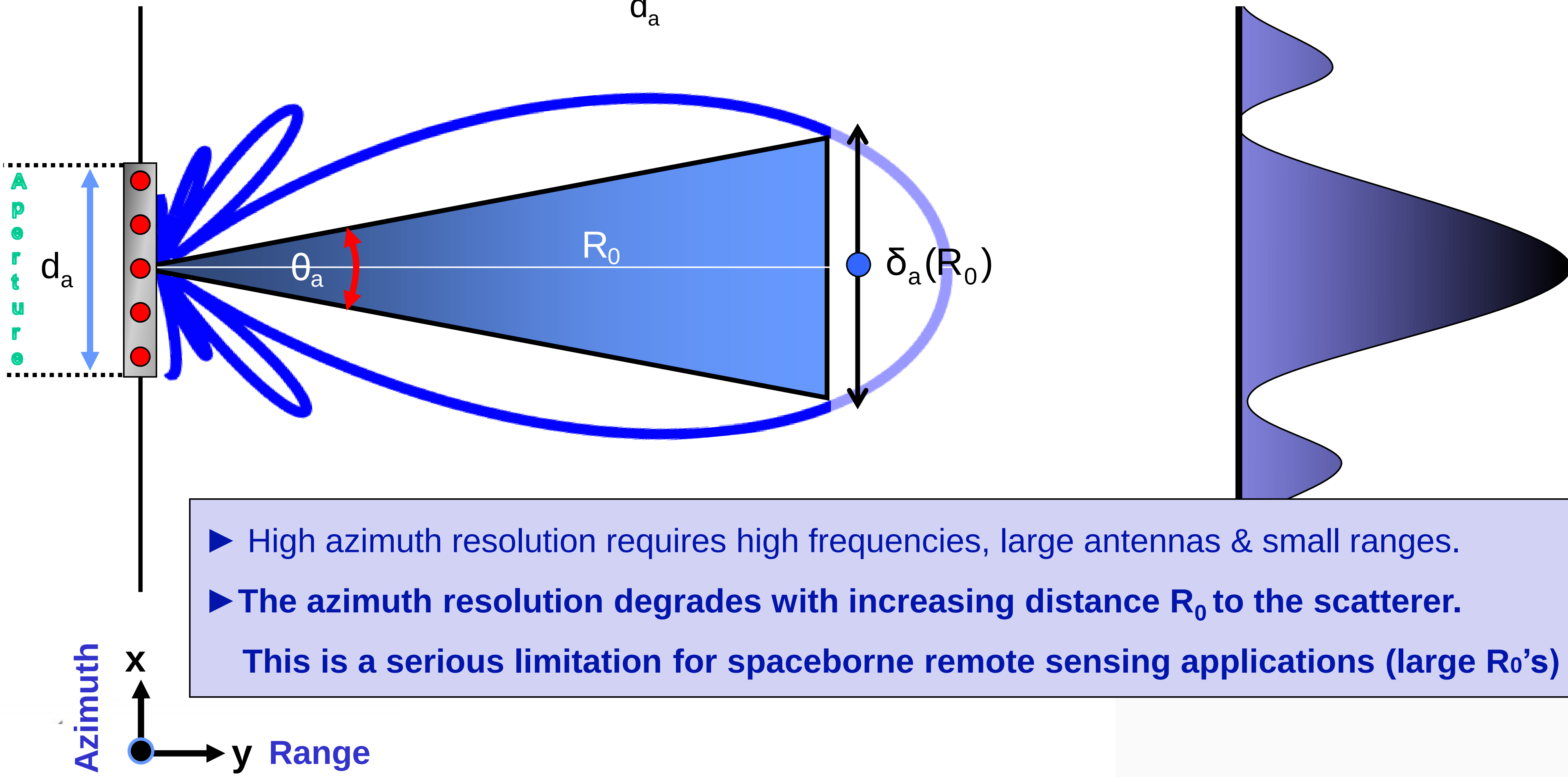
Antenna Radiation Pattern



Real Aperture Radar

Angular resolution: $\theta_a \approx \frac{\lambda}{d_a} = \frac{\textbf{Wavelength}}{\textbf{(Real) Antenna Aperture}}$

Spatial resolution (Az): $\delta_a = \theta_a R_0 \approx \frac{\lambda}{d_a} R_0$ where R_0 is the (closest) distance to the scatterer



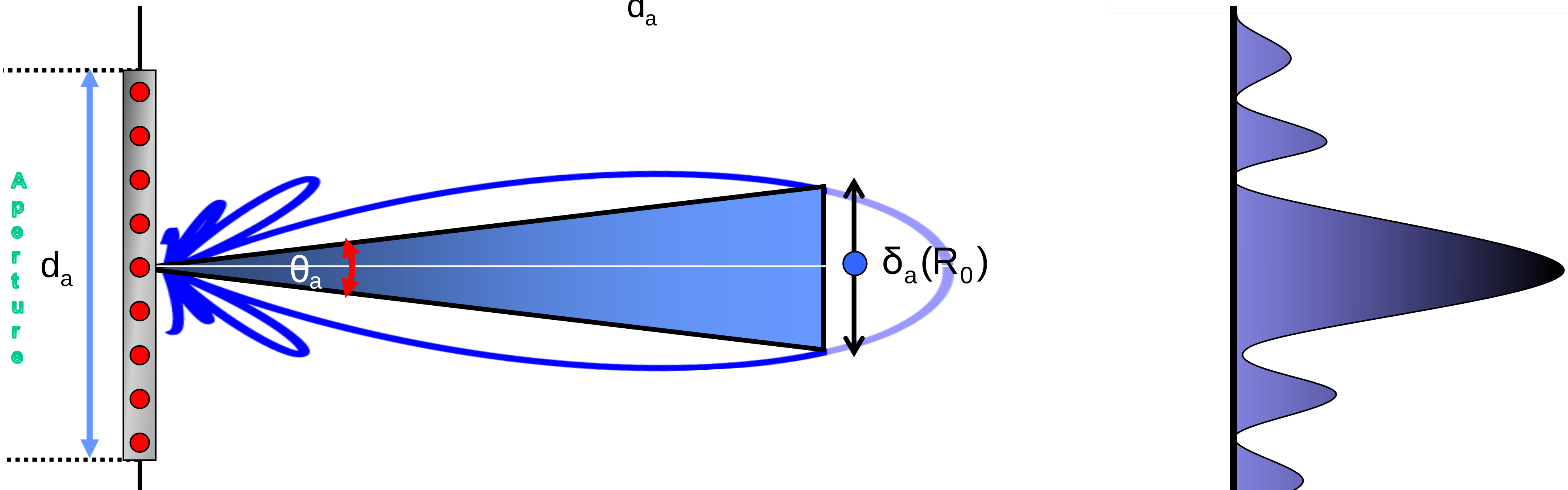
- ► High azimuth resolution requires high frequencies, large antennas & small ranges.
- ►**The azimuth resolution degrades with increasing distance R_0 to the scatterer.**

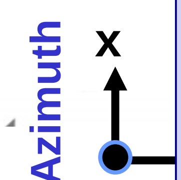
  This is a serious limitation for spaceborne remote sensing applications (large R_0's)

Real Aperture Radar: Increasing the Antenna

Angular resolution: $\theta_a \approx \dfrac{\lambda}{d_a} = \dfrac{\textbf{Wavelength}}{\textbf{(Real) Antenna Aperture}}$

Spatial resolution (Az): $\delta_a = \theta_a R_0 \approx \dfrac{\lambda}{d_a} R_0$ where R_0 is the (closest) distance to the scatterer





Example 1: X-Band Airborne system, 50MHz bandwidth, 3m antenna, 3000m range

$\delta_r = 3$ m $\qquad\qquad \delta_a = 30$ m

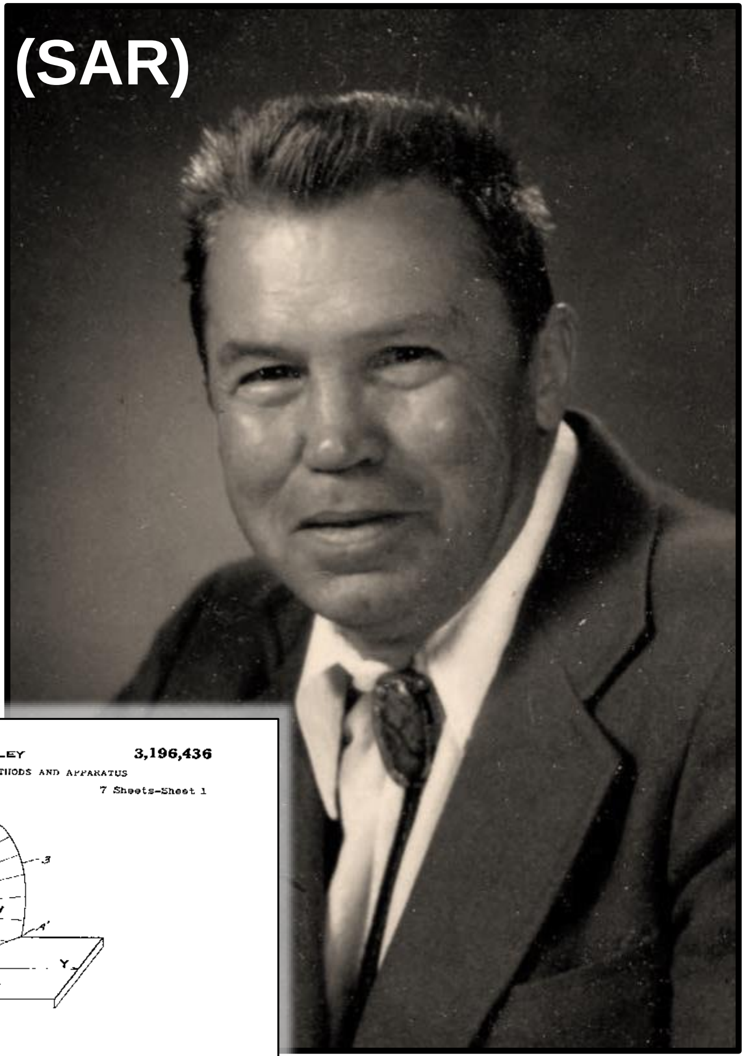
Example 2: X-Band Satellite system, 50MHz bandwidth, 12m antenna, 800km range

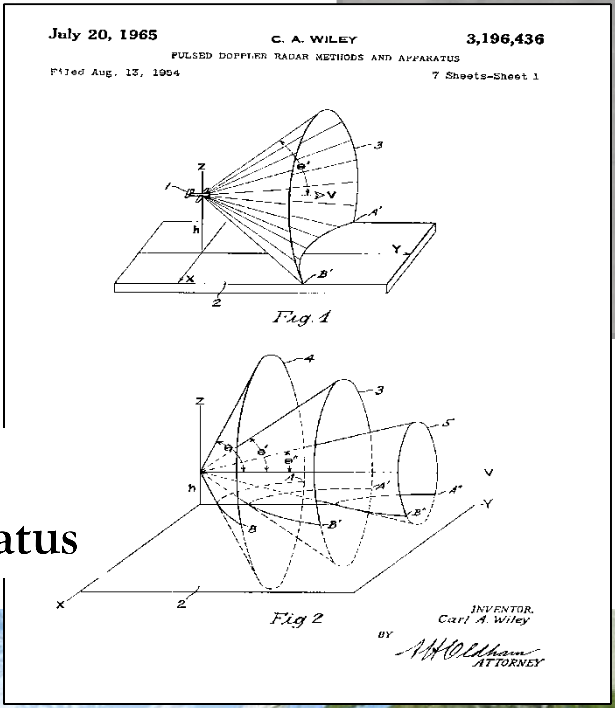
$\delta_r = 3$ m $\qquad\qquad \delta_a = 2000$ m !

The Solution: Synthetic Aperture Radar (SAR)

Carl A. Wiley, 1951:

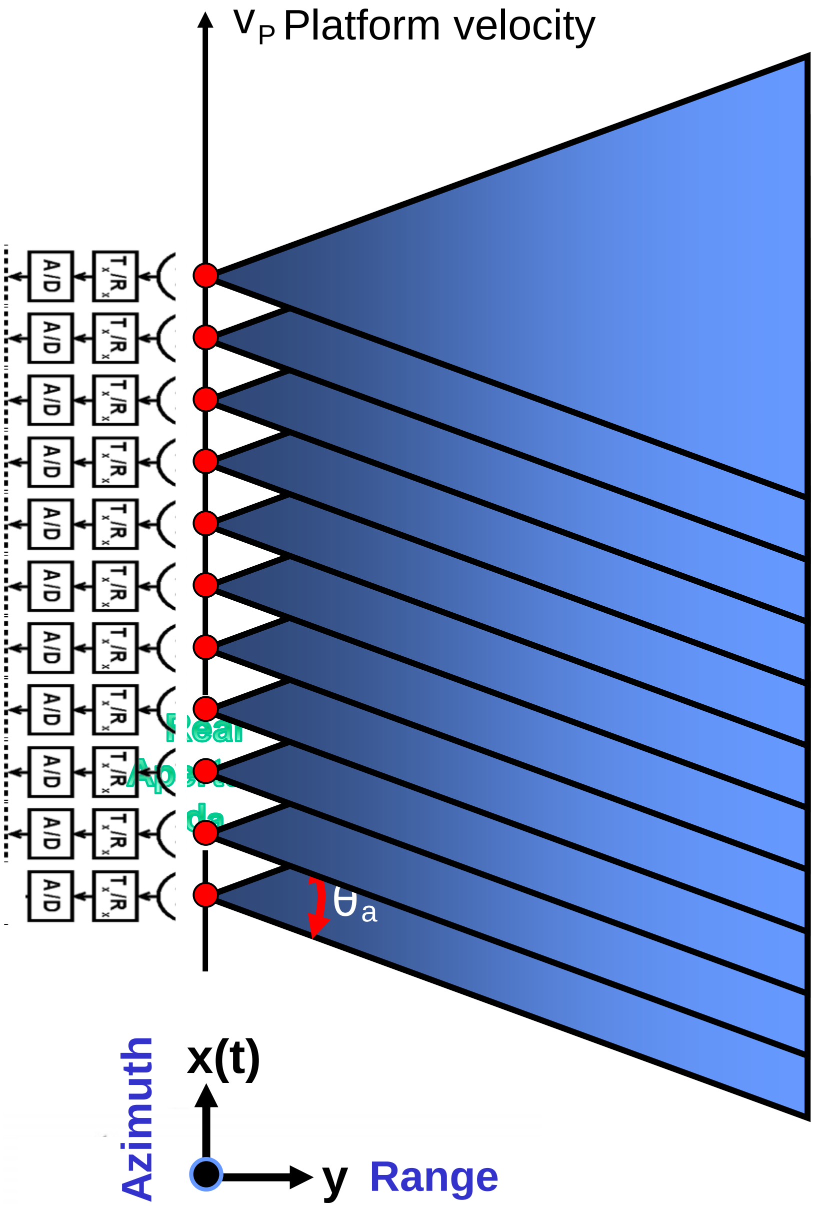
- **A large antenna** - required for achieving a high spatial resolution in azimuth - is “synthesised” using an array of small antennas working together.
- This array of small antennas is formed by using a **real aperture antenna**
  - mounted on a moving (with velocity v_P) platform
  - operated in a pulsed mode (with a Pulse Repetition Frequency given by PRF).





C.A. Wiley / US Patent 3,196,436 / 1951
Pulsed Doppler Radar Methods and Apparatus

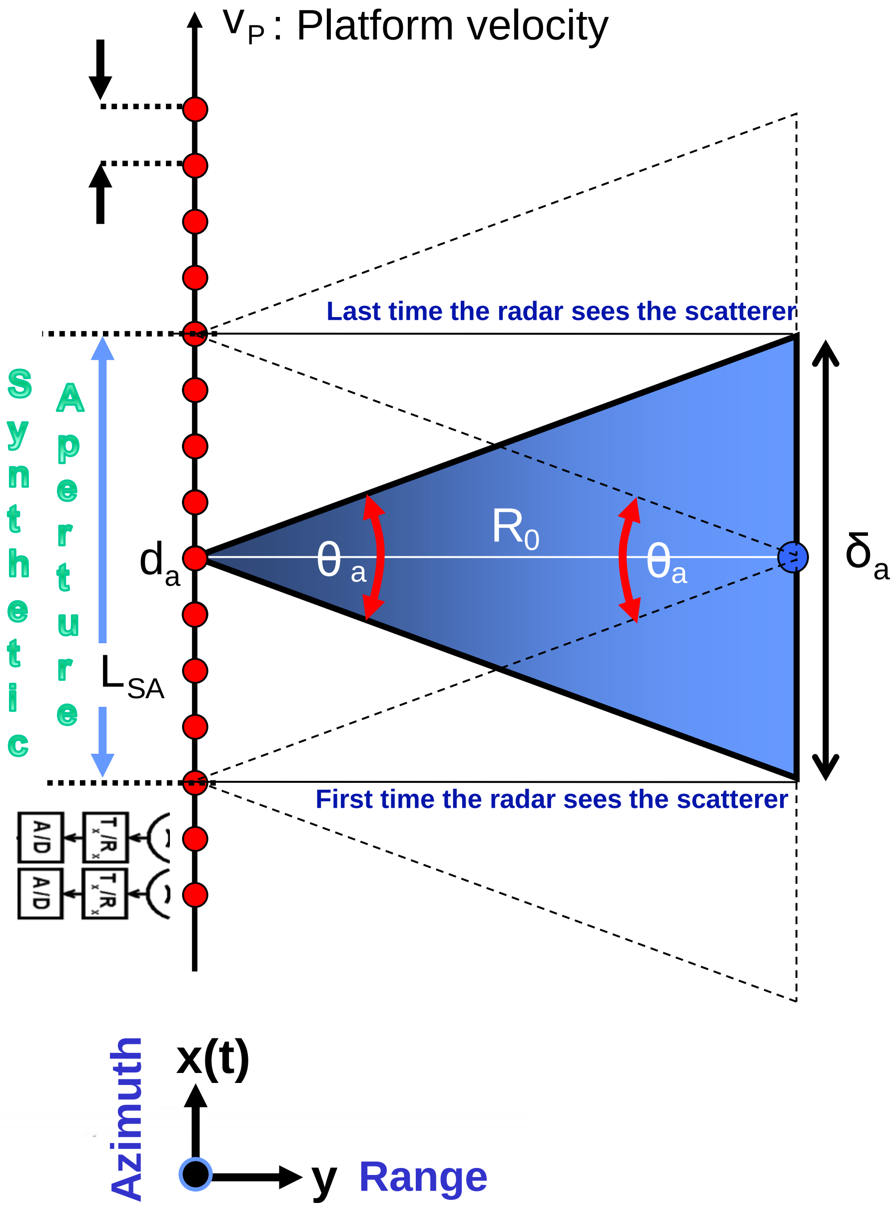
SAR Azimuth Resolution



The real aperture antenna is moving forward with velocity v_P and operates in a pulsed mode

Azimuth resolution of the real aperture at distance R_0: $\delta_a = \theta_a R_0 = \dfrac{\lambda}{d_a} R_0$

SAR Azimuth Resolution

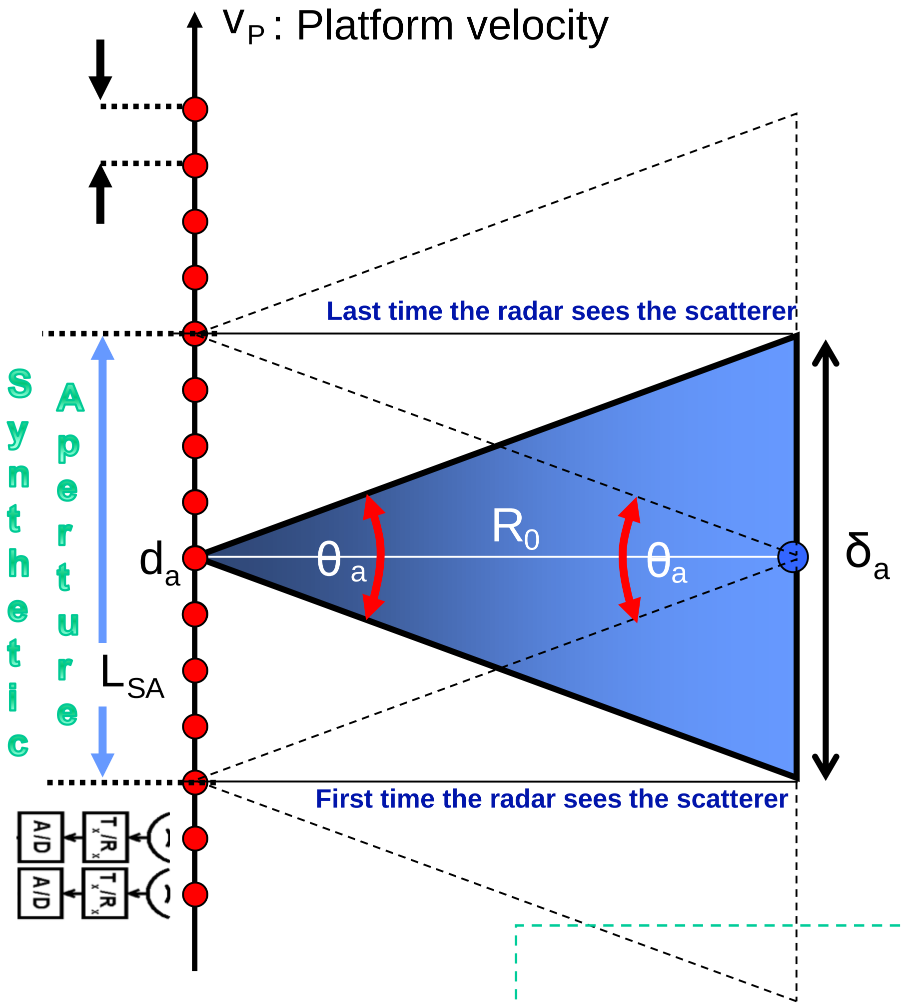


Looking on a scatterer at distance R_0

All echoes received from the scatterer between the first and last echo can be coherently combined to "synthesise" a Synthetic Aperture

Azimuth resolution of the real aperture at distance R_0: $\delta_a = \theta_a R_0 = \frac{\lambda}{d_a} R_0$

SAR Azimuth Resolution



- Angular resolution of synthetic antenna:

$$\theta_{SA} = \frac{\lambda}{2L_{SA}} = \frac{d_a}{2R_0}$$ decreases with distance R_0

- Length of the synthetic aperture (at distance R_0):

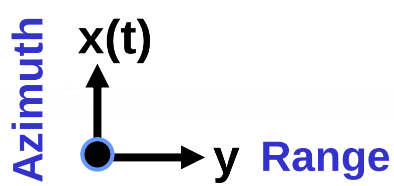
$$L_{SA} = \delta_a = \theta_a R_0 = \frac{\lambda}{d_a} R_0$$ increases with R_0

- Illumination time of an scatterer at distance R_0 :

$$t_{SA} = \frac{L_{SA}}{v_P} = \frac{\lambda}{d_a}\frac{R_0}{v_P}$$ increases with R_0

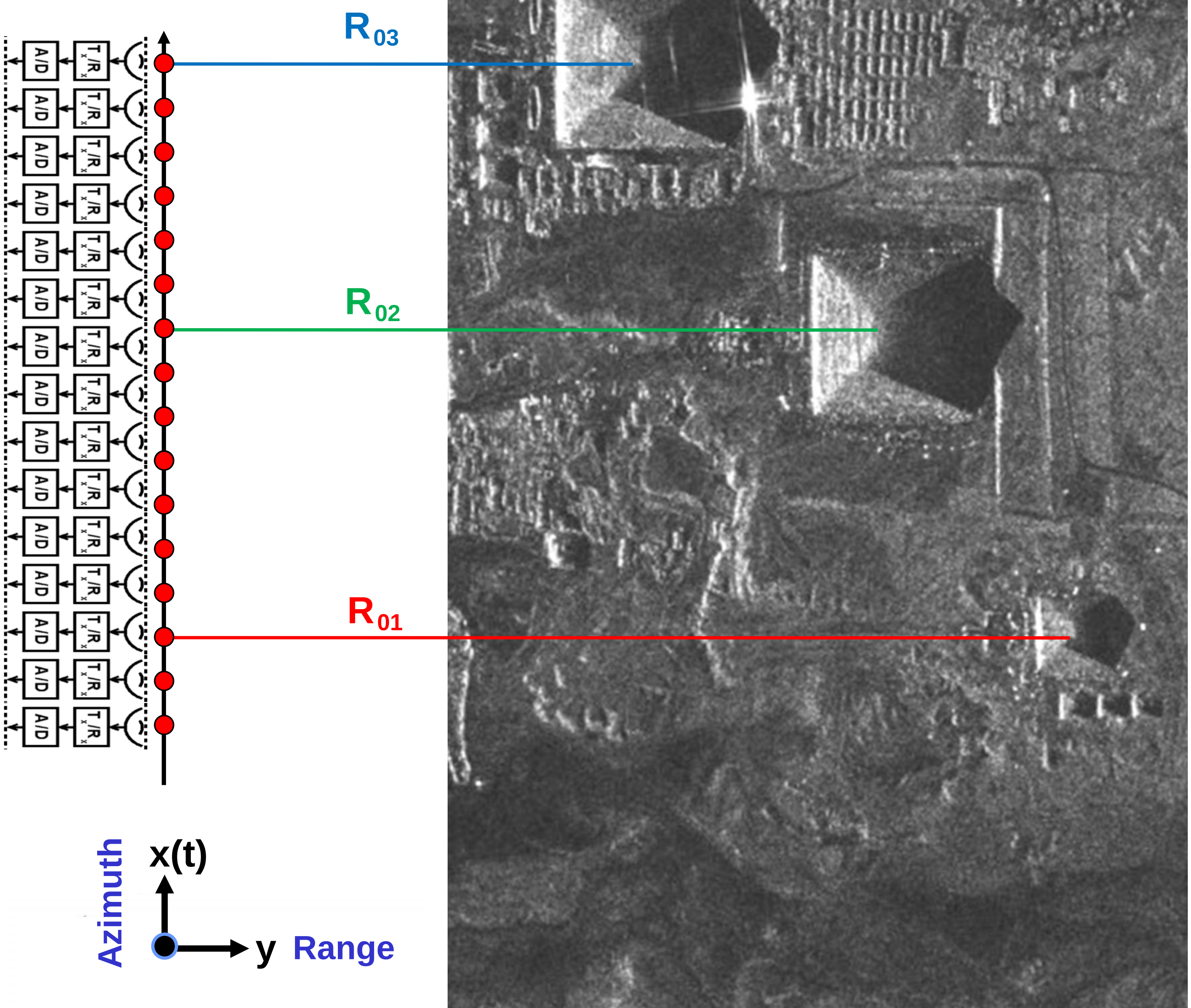
- Azimuth resolution: δ_a v_P: velocity of the platform

$$\delta_{SA} = \theta_{SA} R_0 = \frac{d_a}{2}$$ = Half (Real) Antenna Length!

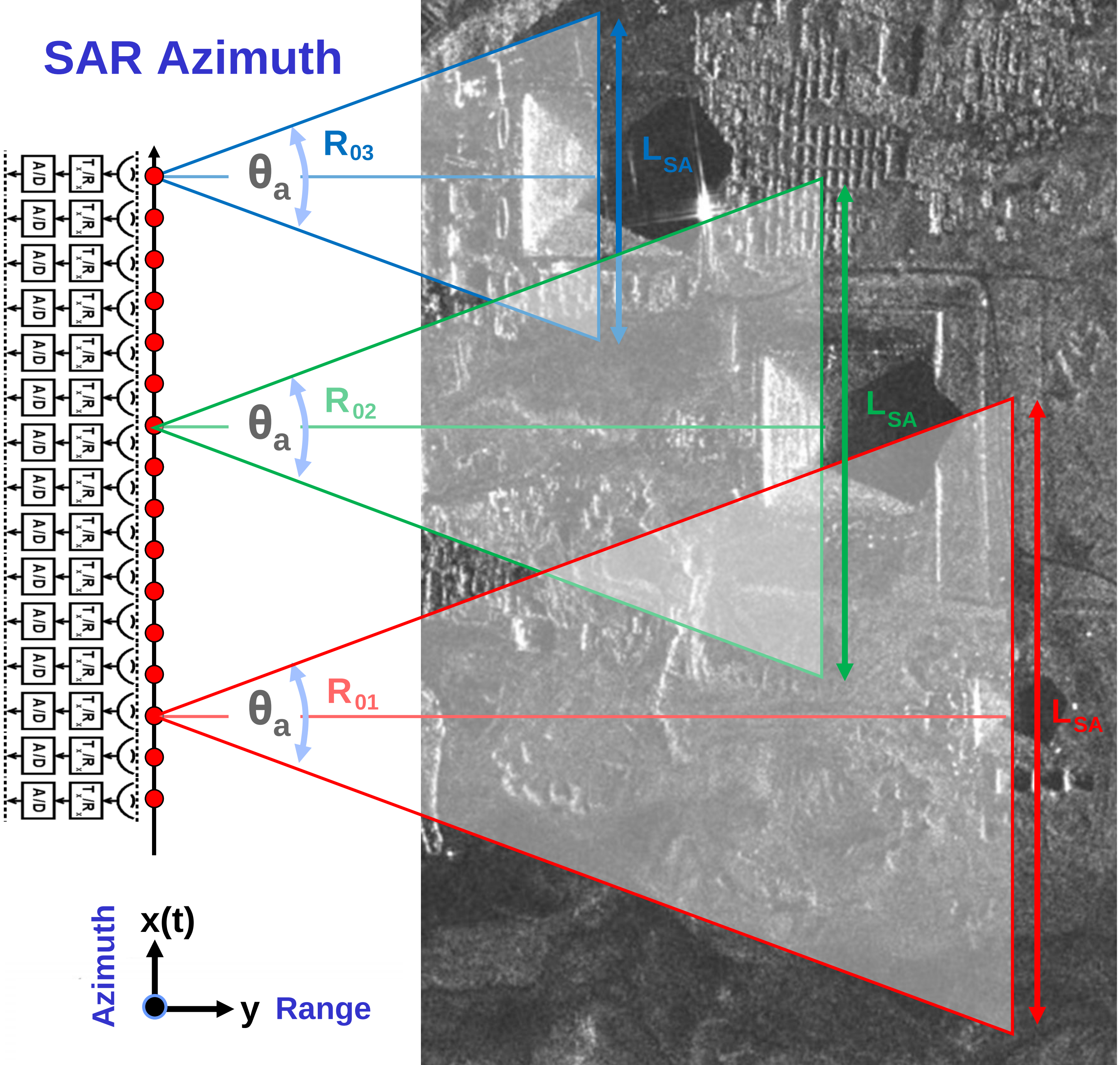


The azimuth resolution is independent on range (i.e. the distance $\mathbf{R_0}$ to the scatterer): **The (linear) degrease of spatial resolution with range distance $\mathbf{R_0}$ is compensated by the longer illumination time (i.e. length of the synthetic aperture) which increases (linear) with $\mathbf{R_0}$.**

SAR Azimuth Resolution



SAR Azimuth Resolution



The deterioration of spatial resolution δ_{SA} with increasing range distance R_0

$$\delta_{SA} = \theta_{SA} R_0 = \frac{\lambda}{2\,L_{SA}} R_0$$

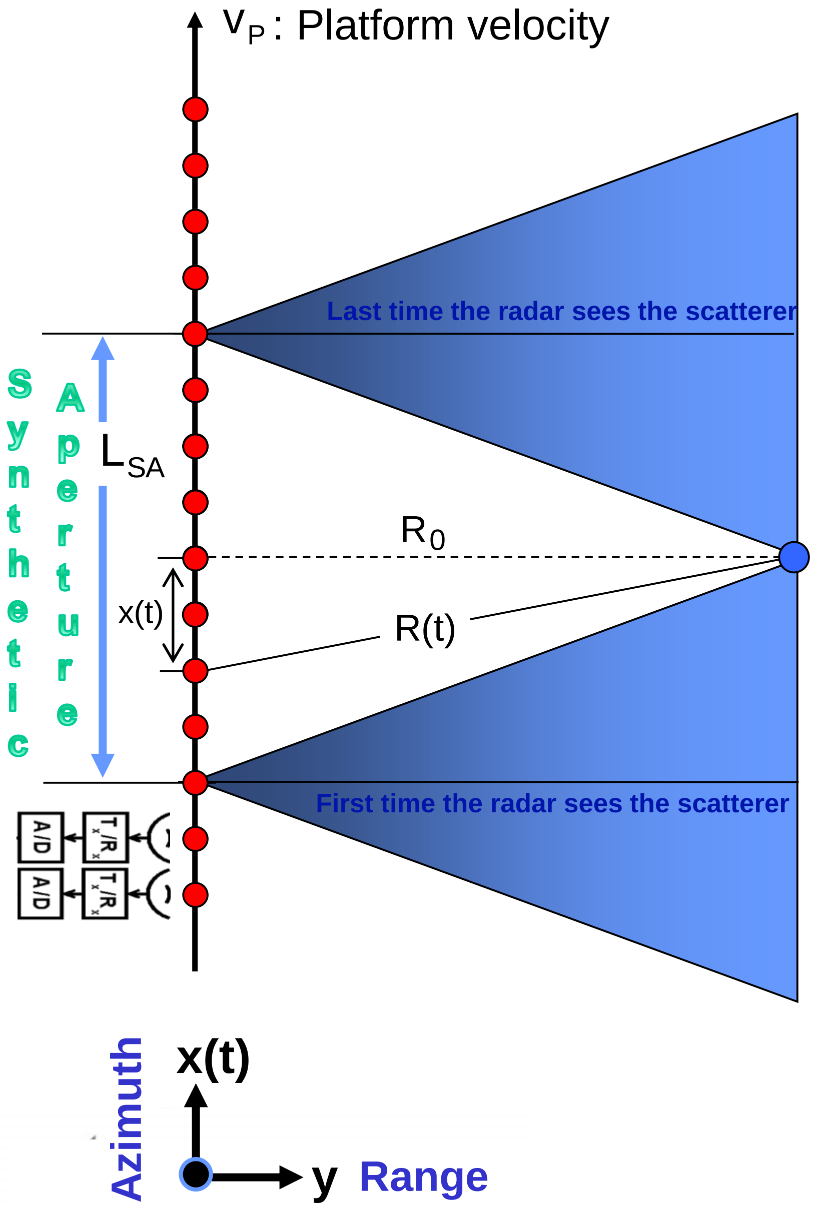
is compensated with the increase of the synthetic aperture L_{SA}

$$L_{SA} = \frac{\lambda}{d_a} R_0$$

with the increase of the time t_{SA} the scatterer is seen by the SAR):

$$t_{SA} = \frac{L_{SA}}{v_P} = \frac{\lambda}{d_a v_P} R_0$$

SAR Azimuth Signal Reconstruction



- Radar-Scatterer distance variation along the aperture:

$$R(t) = \sqrt{R_0^2 + x(t)^2}$$

$$= R_0 \sqrt{1 + \frac{x(t)^2}{R_0^2}} \approx R_0 \left(1 + \frac{x(t)^2}{2R_0^2}\right) = R_0 + \frac{x(t)^2}{2R_0}$$

$$R(t) = R_0 + \frac{v_P^2 t^2}{2R_0} \qquad \text{with} \qquad x(t) = v_P t$$

- Phase variation along the aperture:

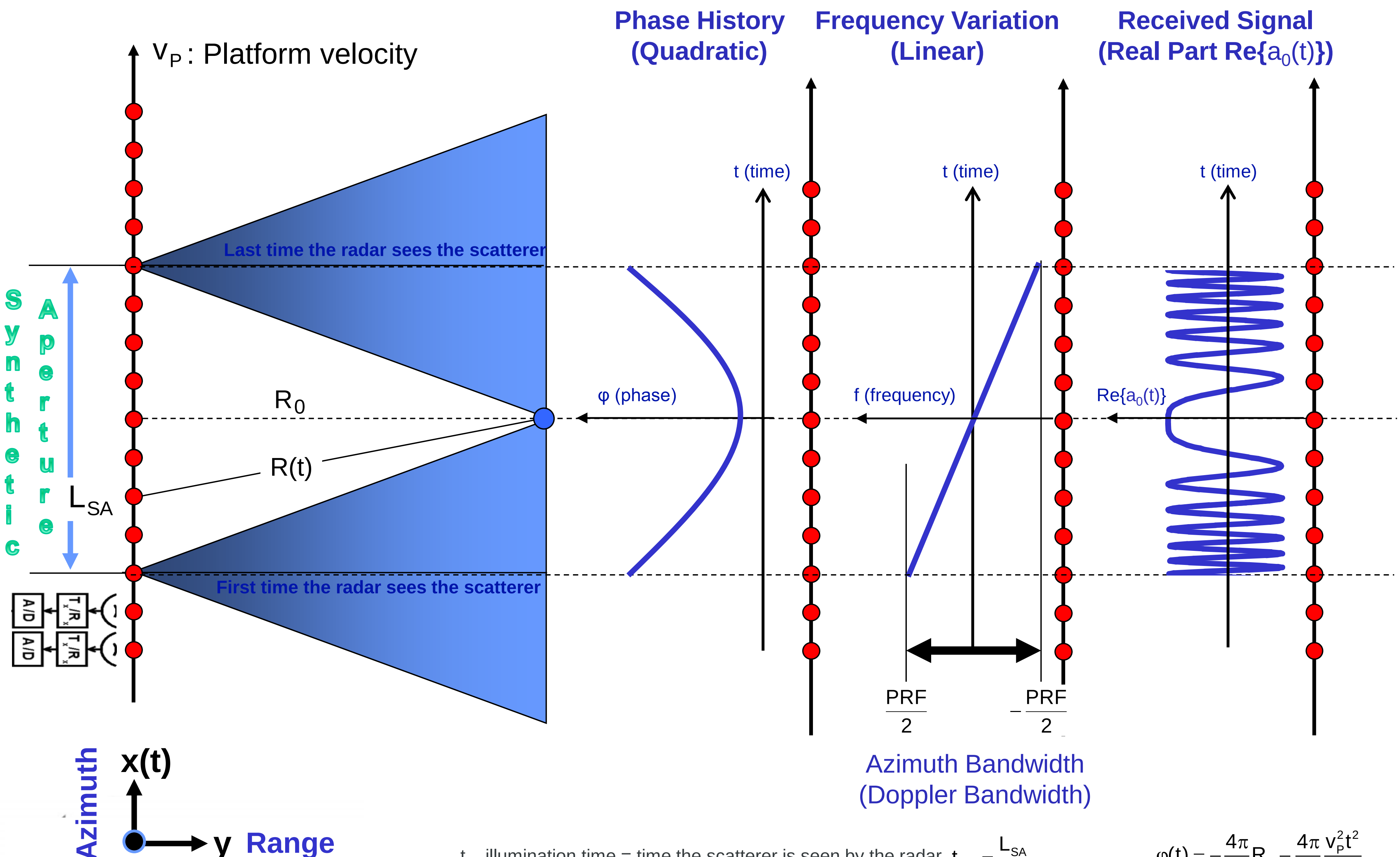
$$\varphi(t) = -2\frac{2\pi}{\lambda} R(t) = -\frac{4\pi}{\lambda} R_0 - \frac{4\pi}{\lambda} \frac{v_P^2 t^2}{2R_0}$$

- Frequency variation along the aperture:

$$f(t) = \frac{1}{2\pi} \frac{d\varphi}{dx} = -\frac{2}{\lambda} \frac{v_P^2 t}{R_0}$$

SAR Azimuth Signal Reconstruction

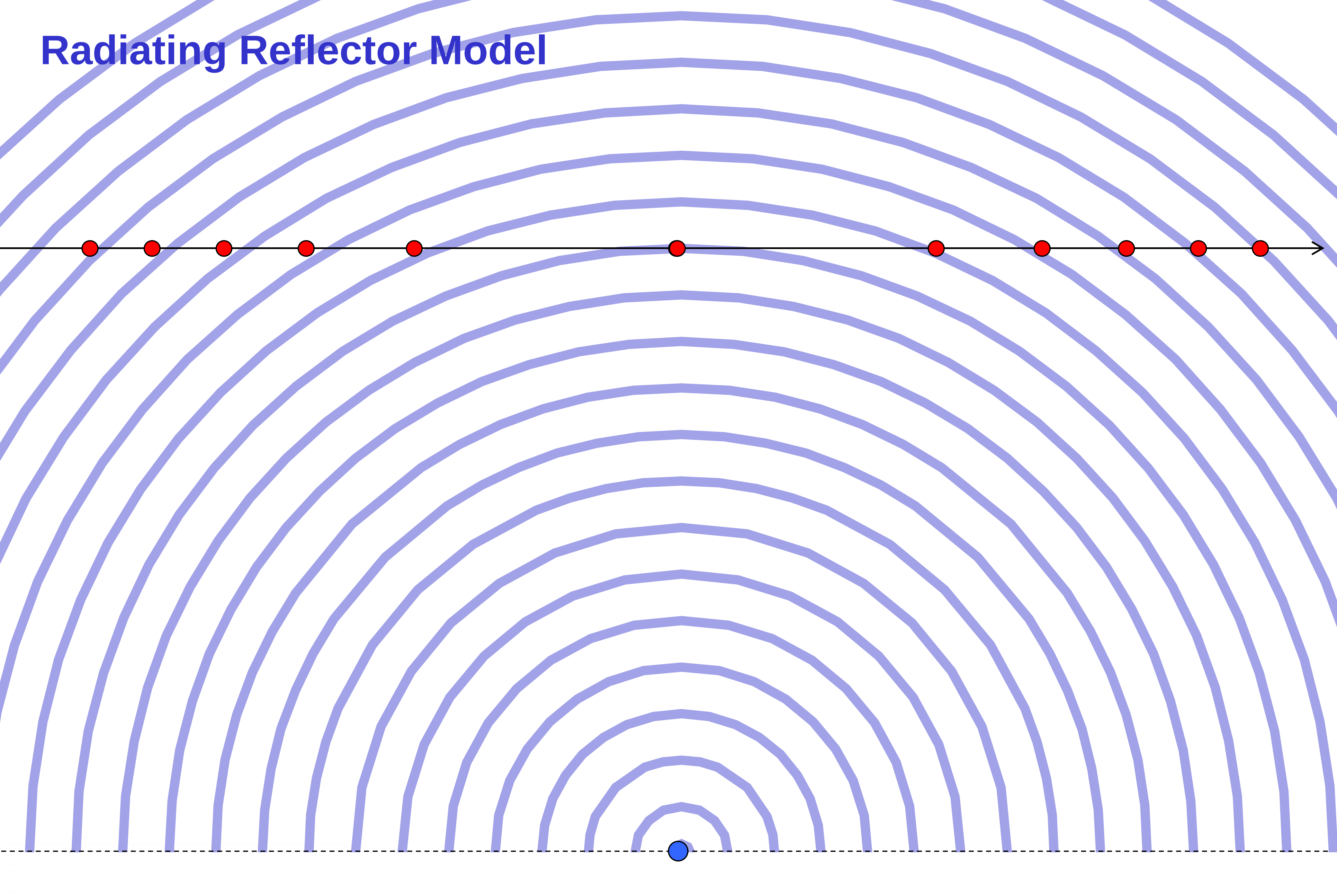
$$a_0(t) = A\exp(i\varphi(t))\ \mathrm{rect}\left(\frac{t}{t_{SA}}\right)$$



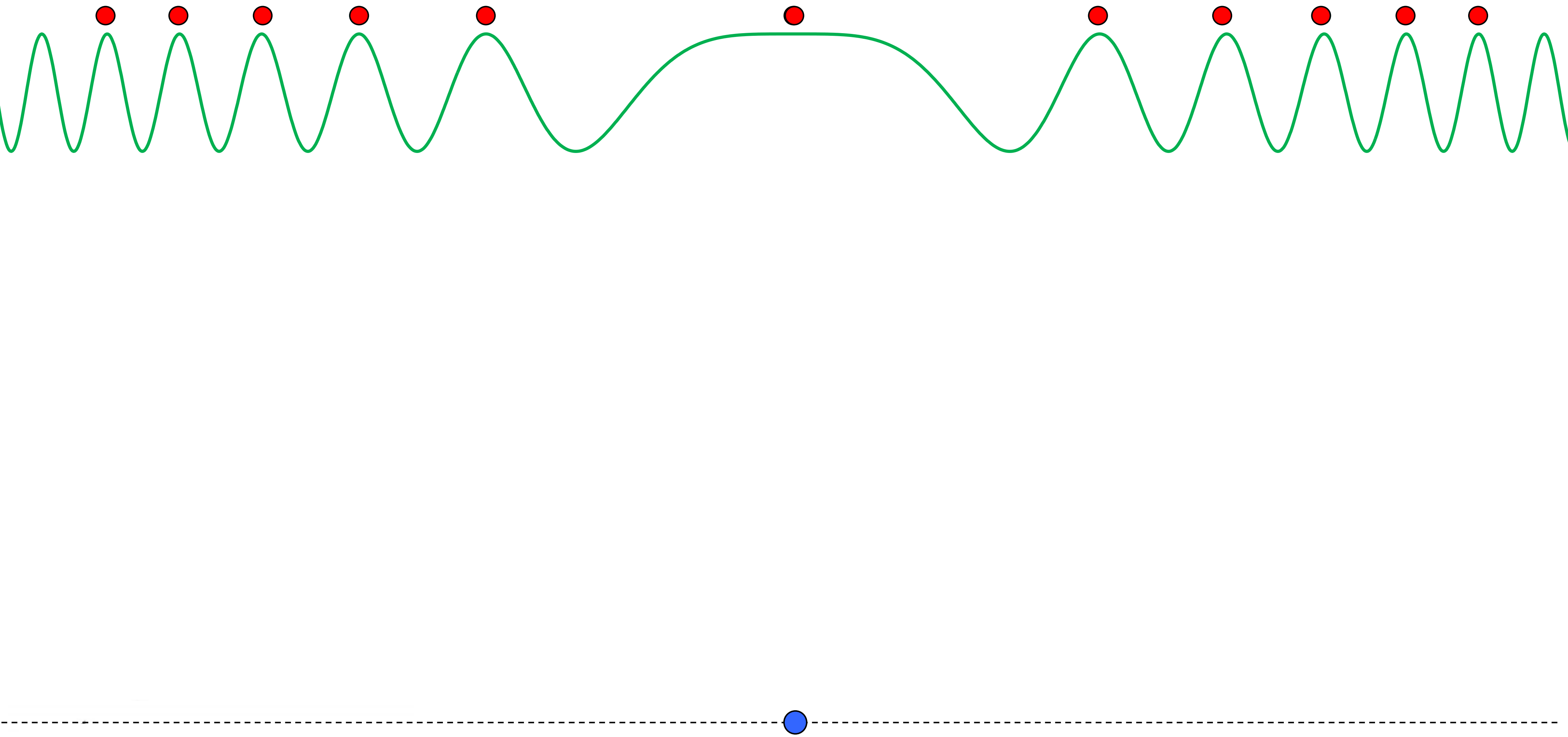
t_{SA} illumination time = time the scatterer is seen by the radar $t_{SA} = \frac{L_{SA}}{v_P}$

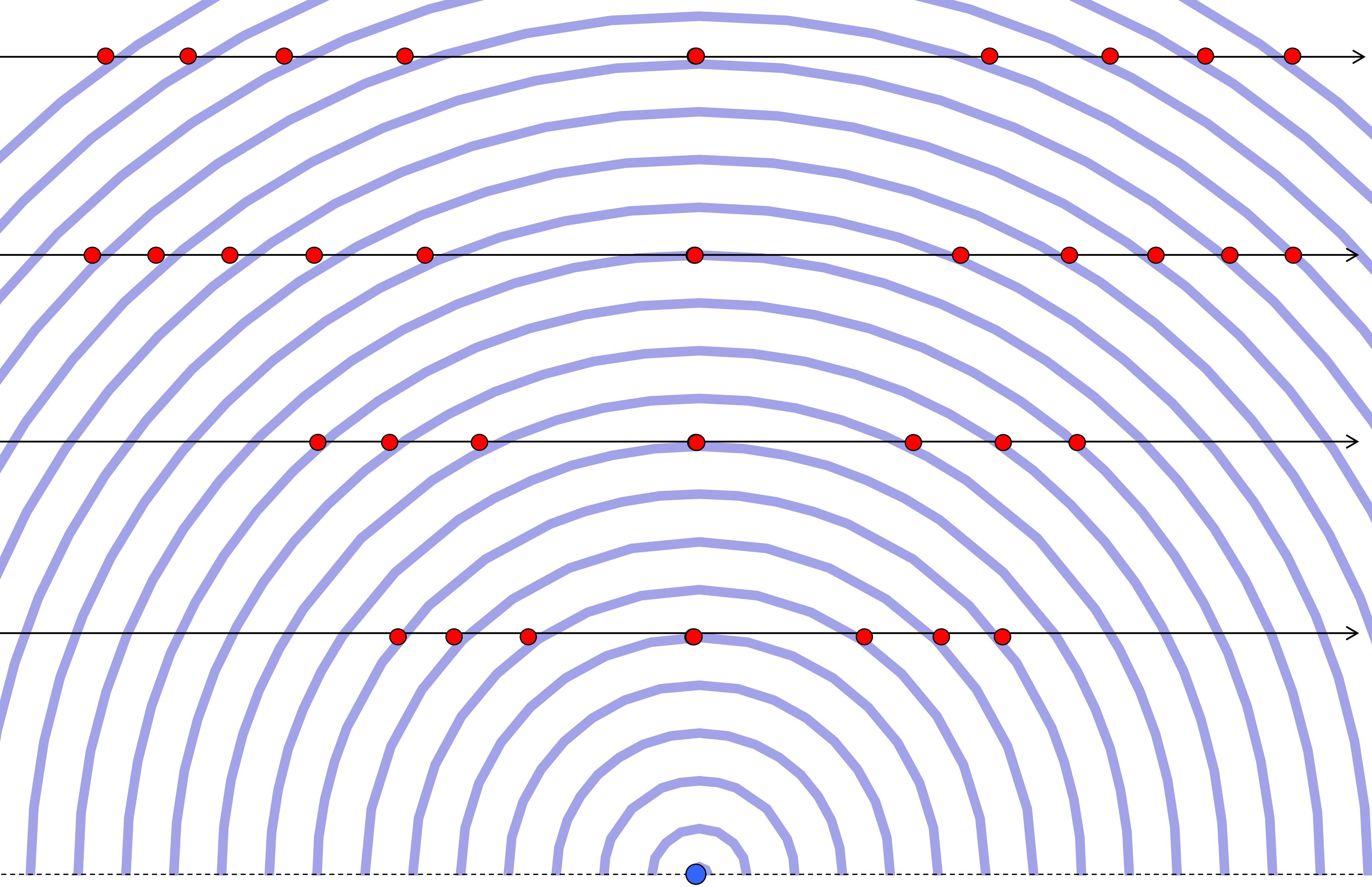
$$\varphi(t) = -\frac{4\pi}{\lambda}R_0 - \frac{4\pi}{\lambda}\frac{v_P^2 t^2}{2R_0}$$

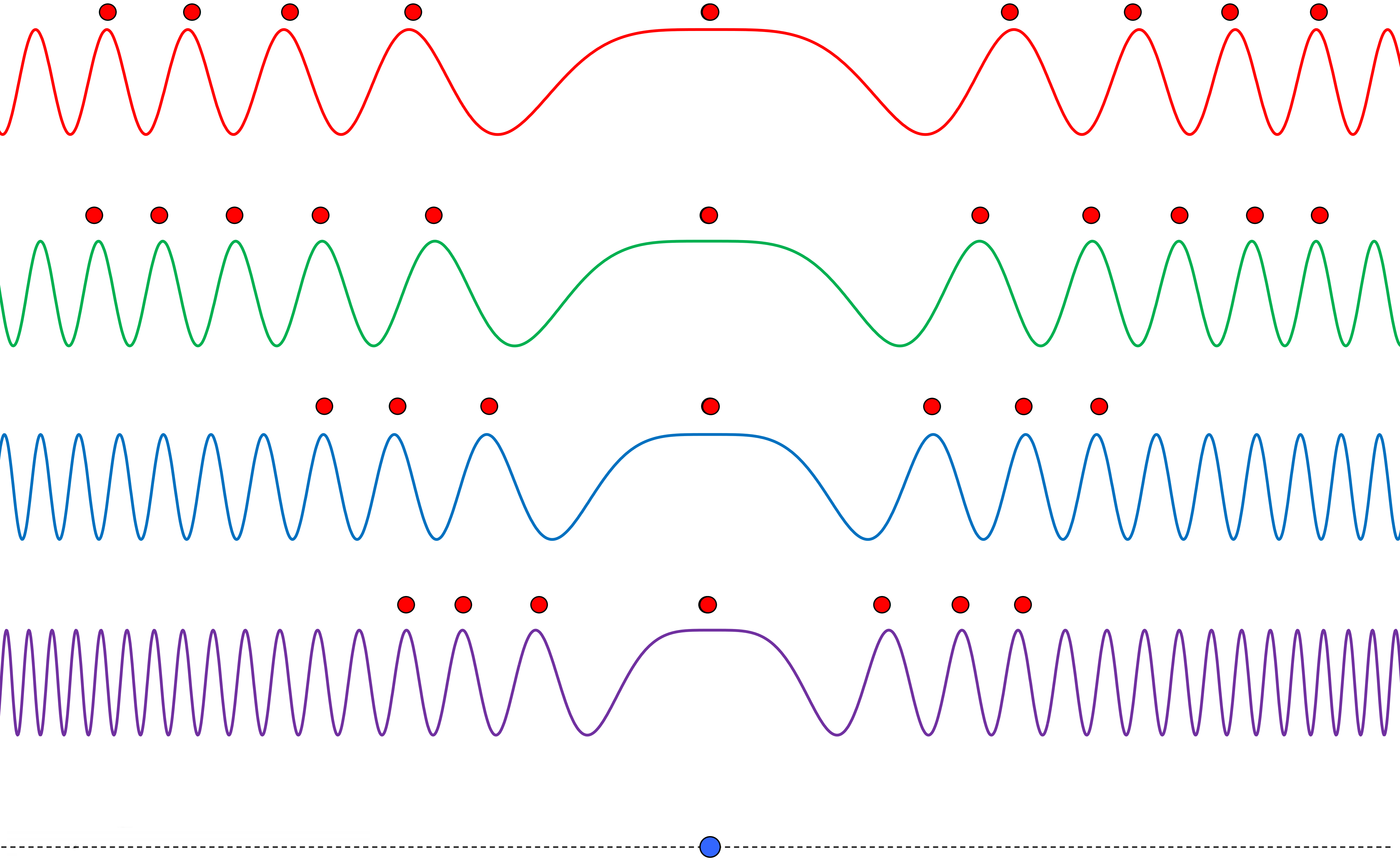
Radiating Reflector Model



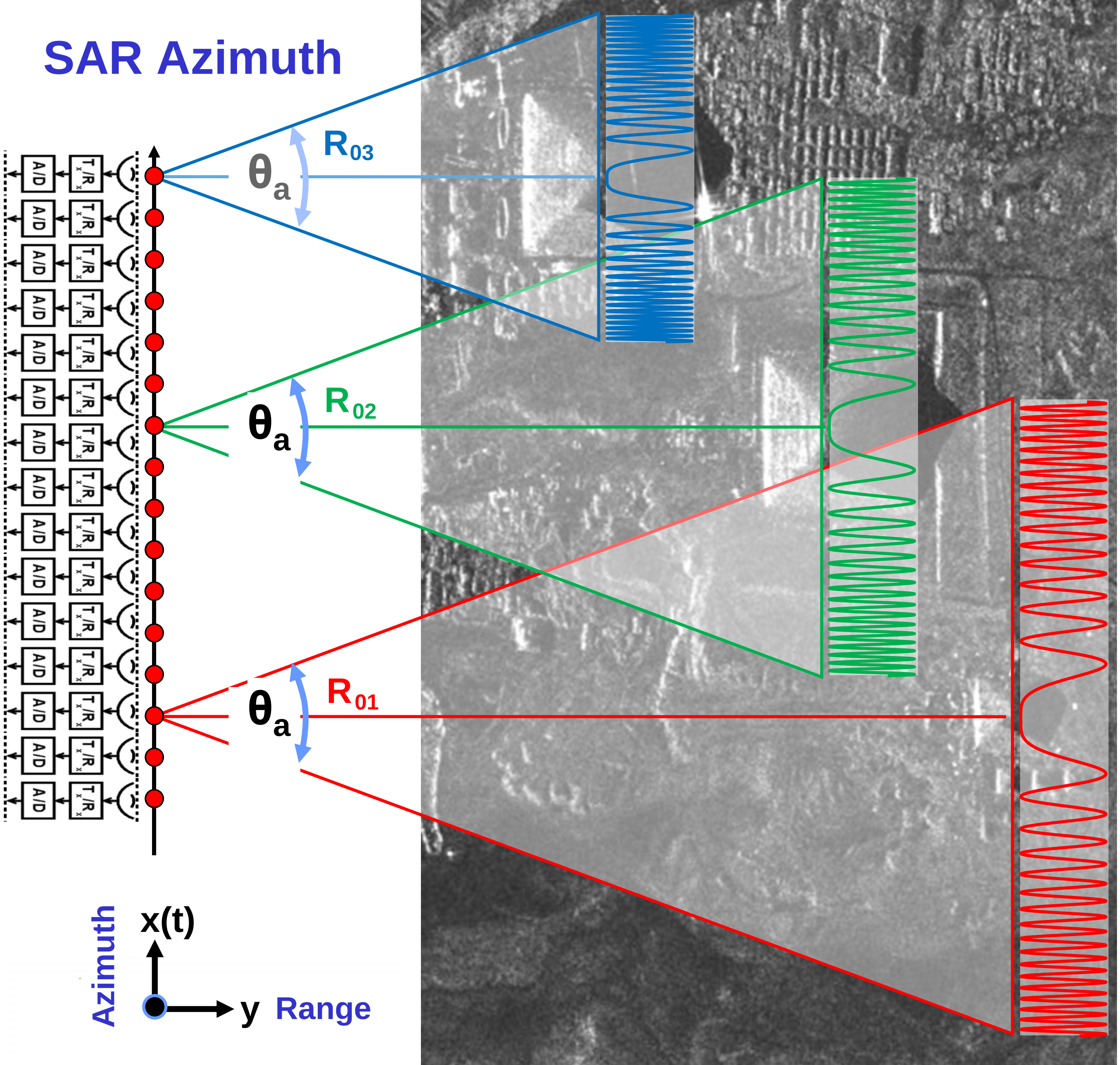
Radiating Reflector Model







SAR Azimuth Resolution



The deterioration of spatial resolution δ_{SA} with increasing range distance R_0

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is compensated with the increase of the synthetic aperture L_{SA}

$$L_{SA} = \frac{\lambda}{d_a} R_0$$

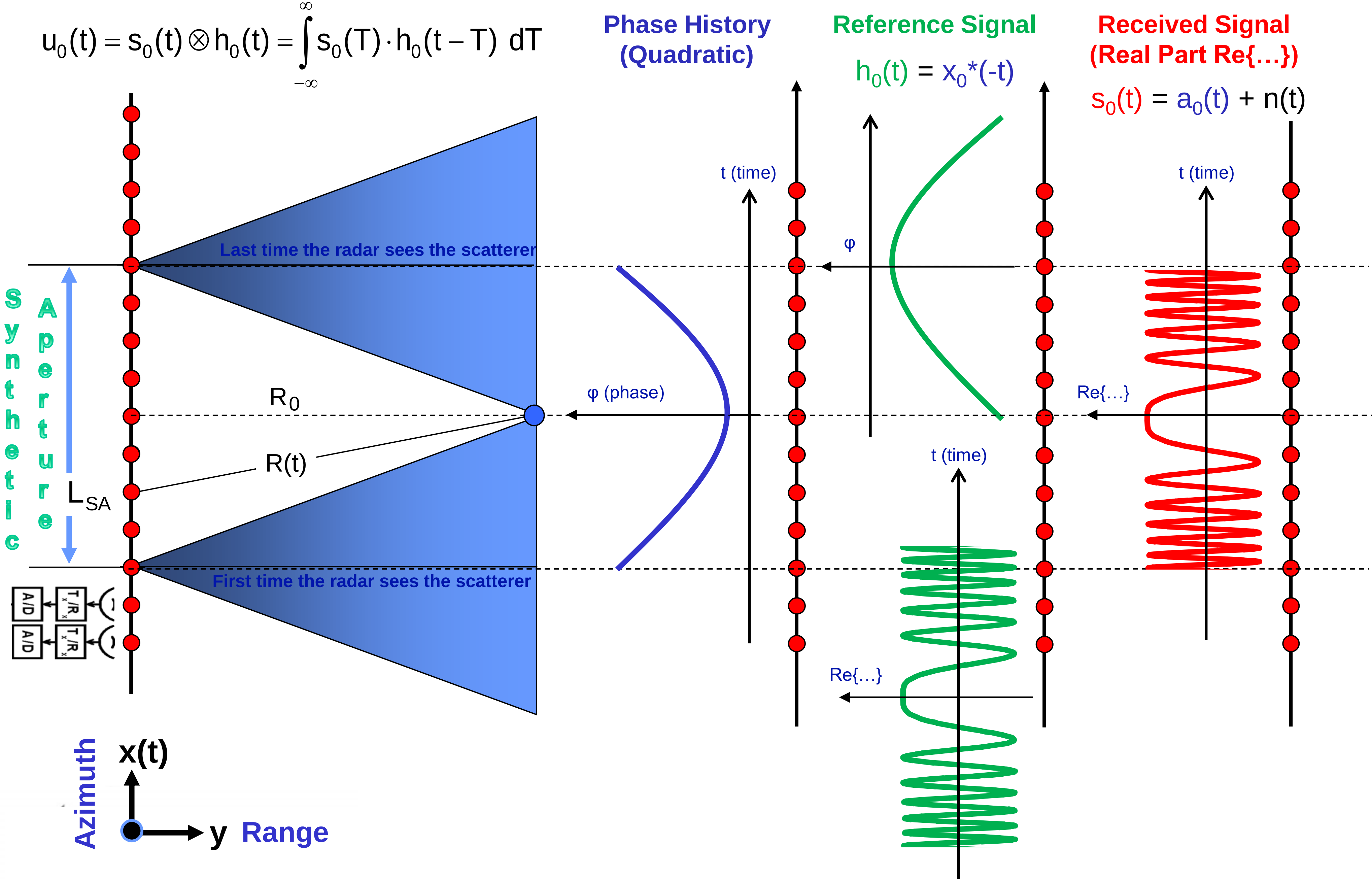
with the increase of the time t_{SA} the scatterer is seen by the SAR):

$$t_{SA} = \frac{L_{SA}}{v_P} = \frac{\lambda}{d_a v_P} R_0$$

SAR Azimuth Compression

Matched Filter:

$$u_0(t) = s_0(t) \otimes h_0(t) = \int_{-\infty}^{\infty} s_0(T) \cdot h_0(t - T)\, dT$$



SAR Azimuth Compression

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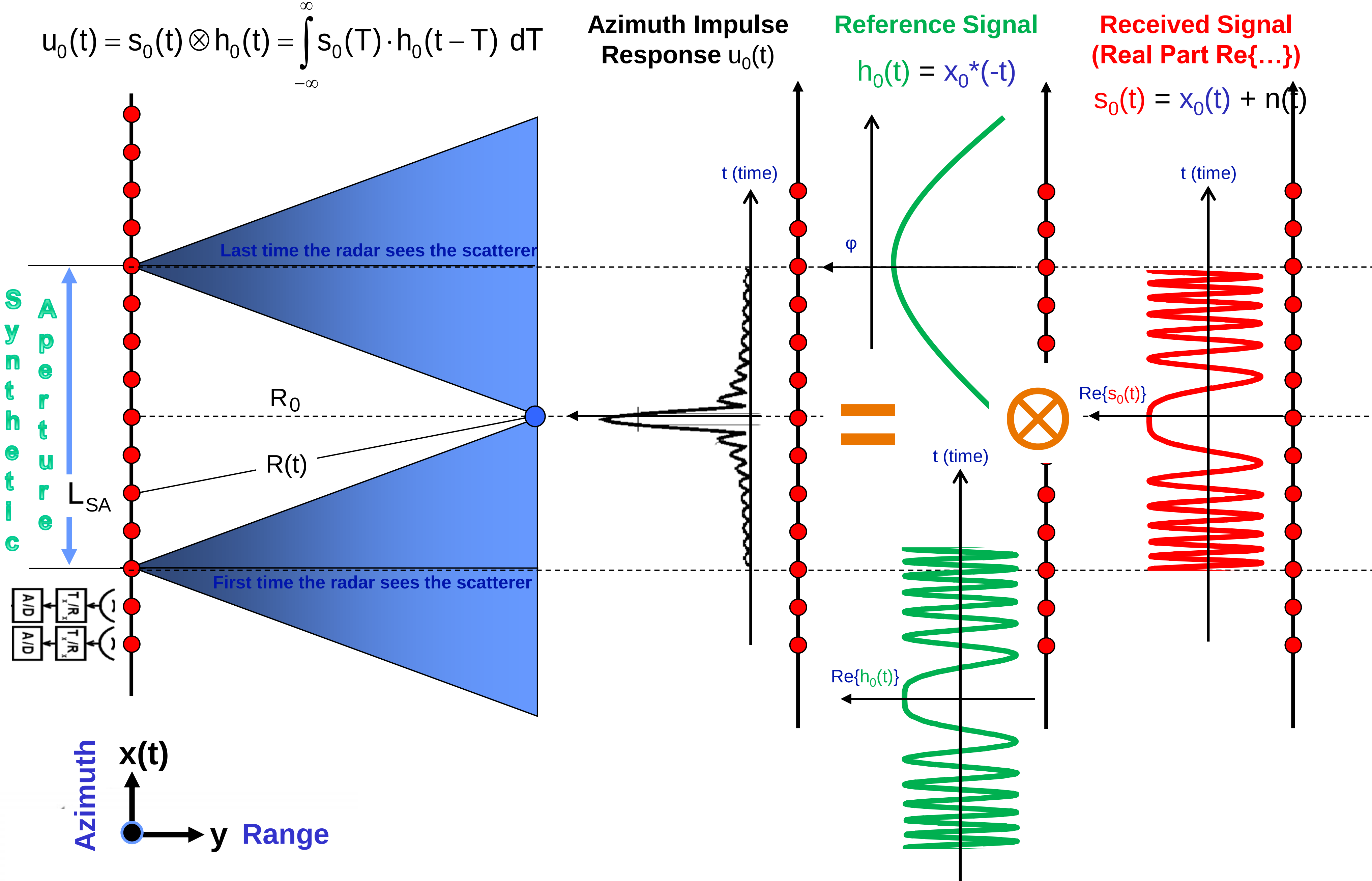
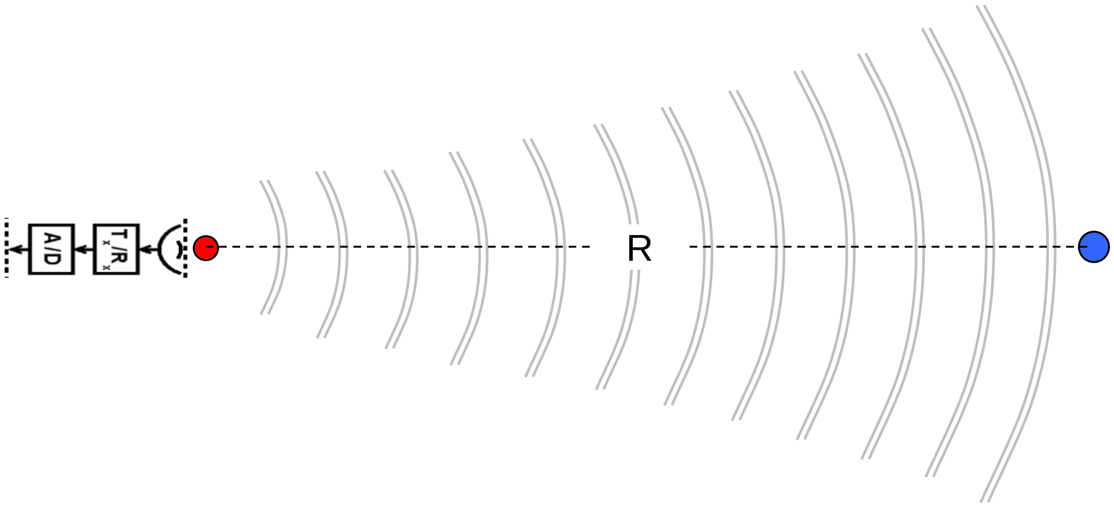


SAR Azimuth Signal Reconstruction

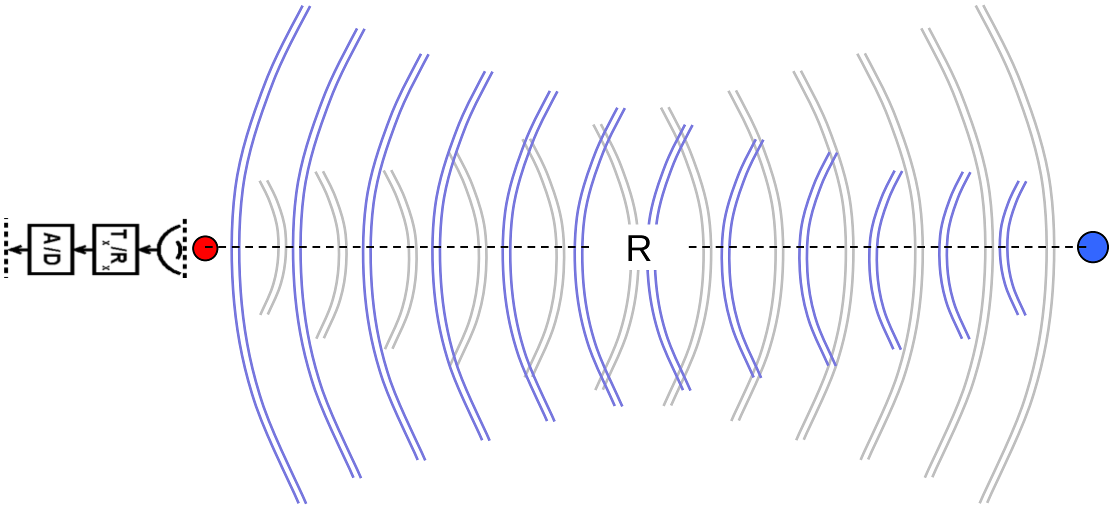


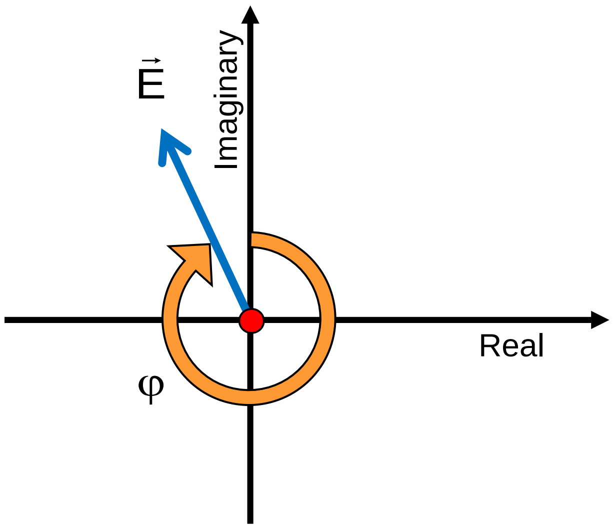
SAR Azimuth Signal Reconstruction

Received Electric Field: $\vec{E} = A \exp(i\varphi)$

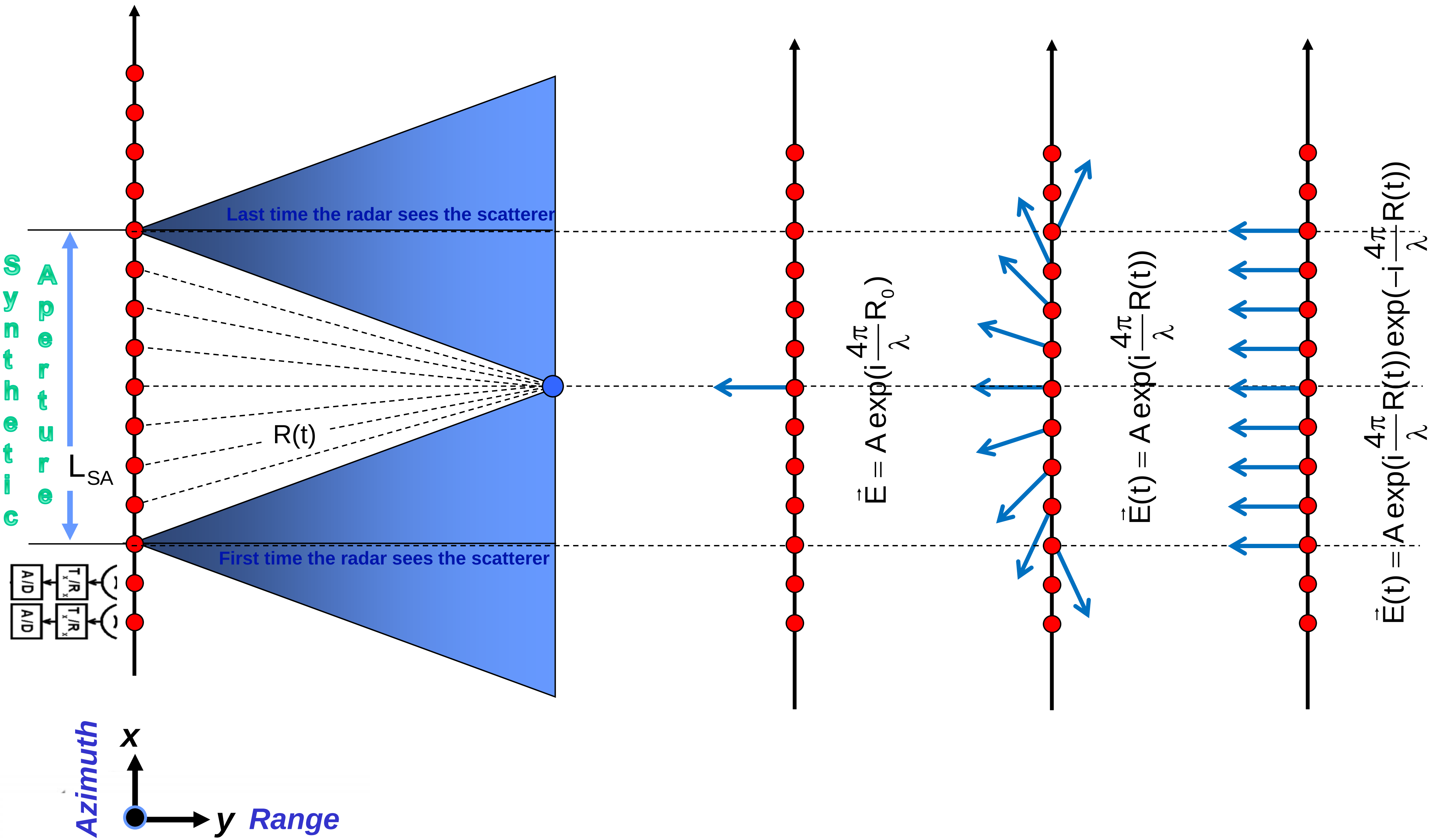
(Scattering) Amplitude: $|\vec{E}| = A$

Phase: $\varphi := 2\frac{2\pi}{\lambda}R$

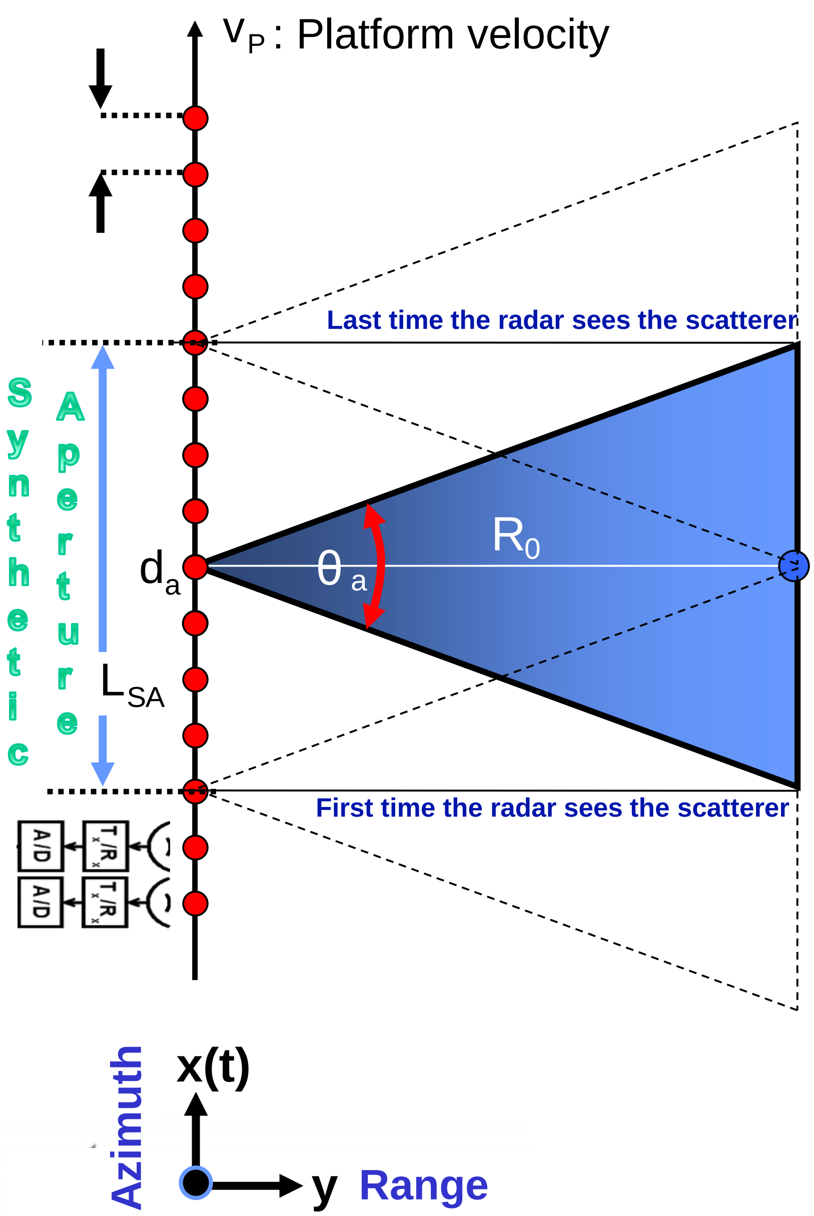




SAR Azimuth Signal Reconstruction



SAR Azimuth Resolution: An example



Antenna length: $d_a = 3$ m

Platform velocity: $v_P = 100$ m/s

Scatterer distance: $R_0 = 8570$ m

Wavelength P-band: $\lambda = 0.7$m (P-band)

Max azimuth resolution: $\delta_{SA} = \dfrac{d_a}{2}$

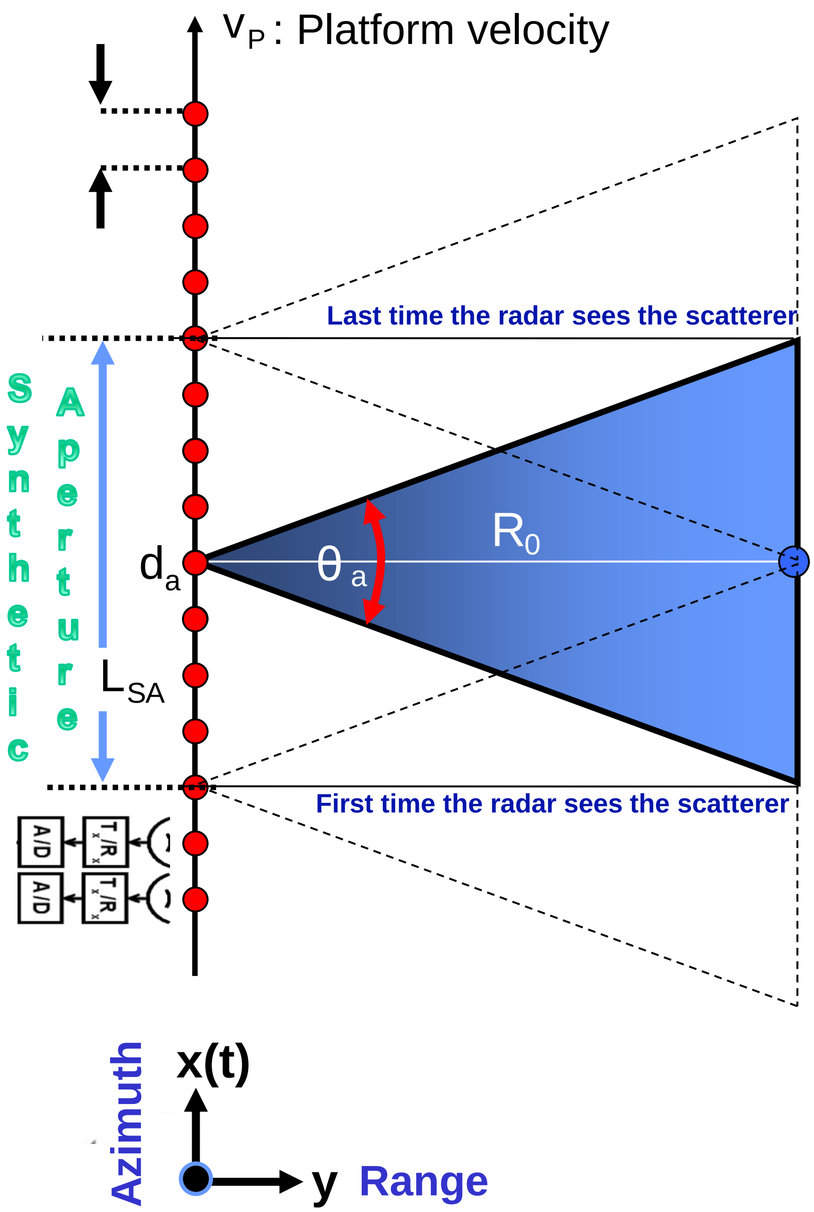
Synthetic aperture length: $L_{SA} = \dfrac{\lambda}{d_a} R_0$

Illumination time: $t_{SA} = \dfrac{L_{SA}}{v_P} = \dfrac{\lambda}{d_a}\dfrac{R_0}{v_P}$

Azimuth Signal: $a_0(t) = A \exp(i\varphi(t)) \operatorname{rect}(\dfrac{t}{t_{SA}})$

with: $\varphi(t) = -\dfrac{4\pi}{\lambda} R_0 - \dfrac{4\pi}{\lambda}\dfrac{v_P^2 t^2}{2R_0}$

SAR Azimuth Resolution: An example



Antenna length: $d_a = 3$ m

Platform velocity: $v_P = 100$ m/s

Scatterer distance: $R_0 = 8570$ m

Wavelength P-band: $\lambda = 0.7$m (P-band)

Azimuth resolution: $\delta_{SA} = 5$m

Equiv. real aperture: $\delta_{SA} = \frac{d_{a'}}{2} \rightarrow d_{a'} = 2\delta_{SA}$

Synthetic aperture length: $L_{SA} = \frac{\lambda}{d_{a'}} R_0$

Illumination time: $t_{SA} = \frac{L_{SA}}{v_P} = \frac{\lambda}{d_{a'}} \frac{R_0}{v_P}$

Azimuth Signal: $a_0(t) = A \exp(i\varphi(t)) \ \mathrm{rect}(\frac{t}{t_{SA}})$

with: $\varphi(t) = -\frac{4\pi}{\lambda} R_0 - \frac{4\pi}{\lambda} \frac{v_P^2 t^2}{2R_0}$

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