

## Analysis of the Algorithm

Kruskal's algorithm is a greedy algorithm used to find the Minimum Spanning Tree (MST) of a connected, undirected graph with weighted edges. The algorithm follows these steps:

### 1. Initialization:

- Create a graph with a specified number of vertices.
- Store edges in a list, where each edge is represented as a tuple of two vertices and a weight.

### 2. Sorting Edges:

- Sort all edges in non-decreasing order based on their weights. This is crucial because the algorithm processes the smallest edges first to ensure that the MST remains minimal.

### 3. Union-Find Structure:

- Use a union-find (disjoint-set) data structure to keep track of which vertices are in which components. This helps in efficiently checking whether adding an edge would create a cycle.
- The **find** function implements path compression to flatten the structure of the tree whenever **find** is called, which speeds up future operations.
- The **union** function uses union by rank to attach smaller trees under larger trees, keeping the overall structure balanced.

### 4. Building the MST:

- Iterate through the sorted edges and for each edge, check if the vertices of the edge belong to different components using the **find** function.
- If they do, add the edge to the MST and unite the two components using the **union** function.
- Stop when the number of edges in the MST equals  $V - 1$ , where  $V$  is the number of vertices.

### 5. Output:

- Print the edges included in the MST and the total cost of the MST.

## Complexity Analysis

### • Time Complexity:

- Sorting the edges takes  $O(E \log E)$ , where  $E$  is the number of edges.
- Each union and find operation takes nearly constant time,  $O(\alpha(V))$ , where  $\alpha$  is the inverse Ackermann function, which grows very slowly.
- Overall, the time complexity is  $O(E \log E)$ .

- **Space Complexity:**
  - The space complexity is  $O(V + E)$