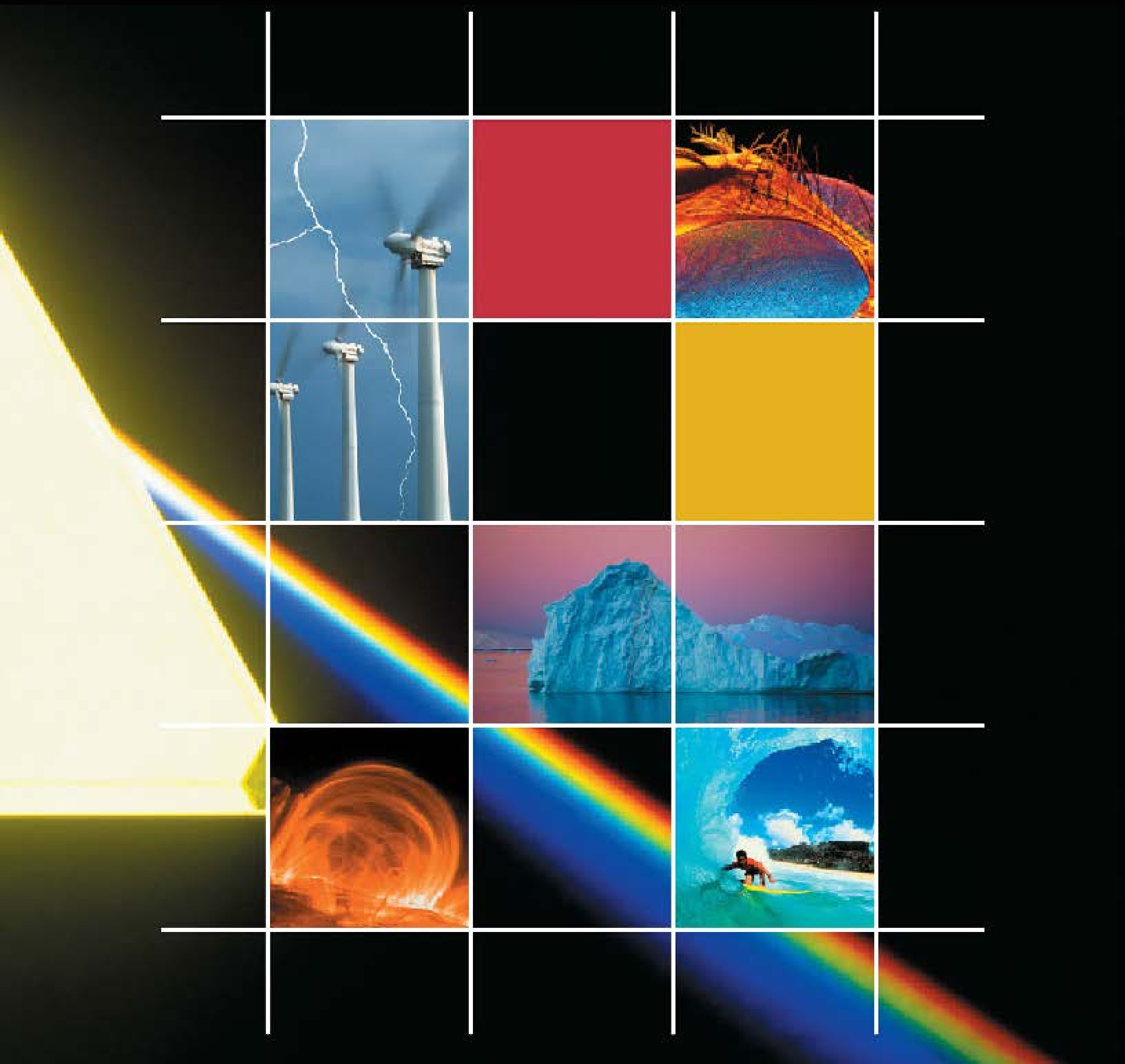
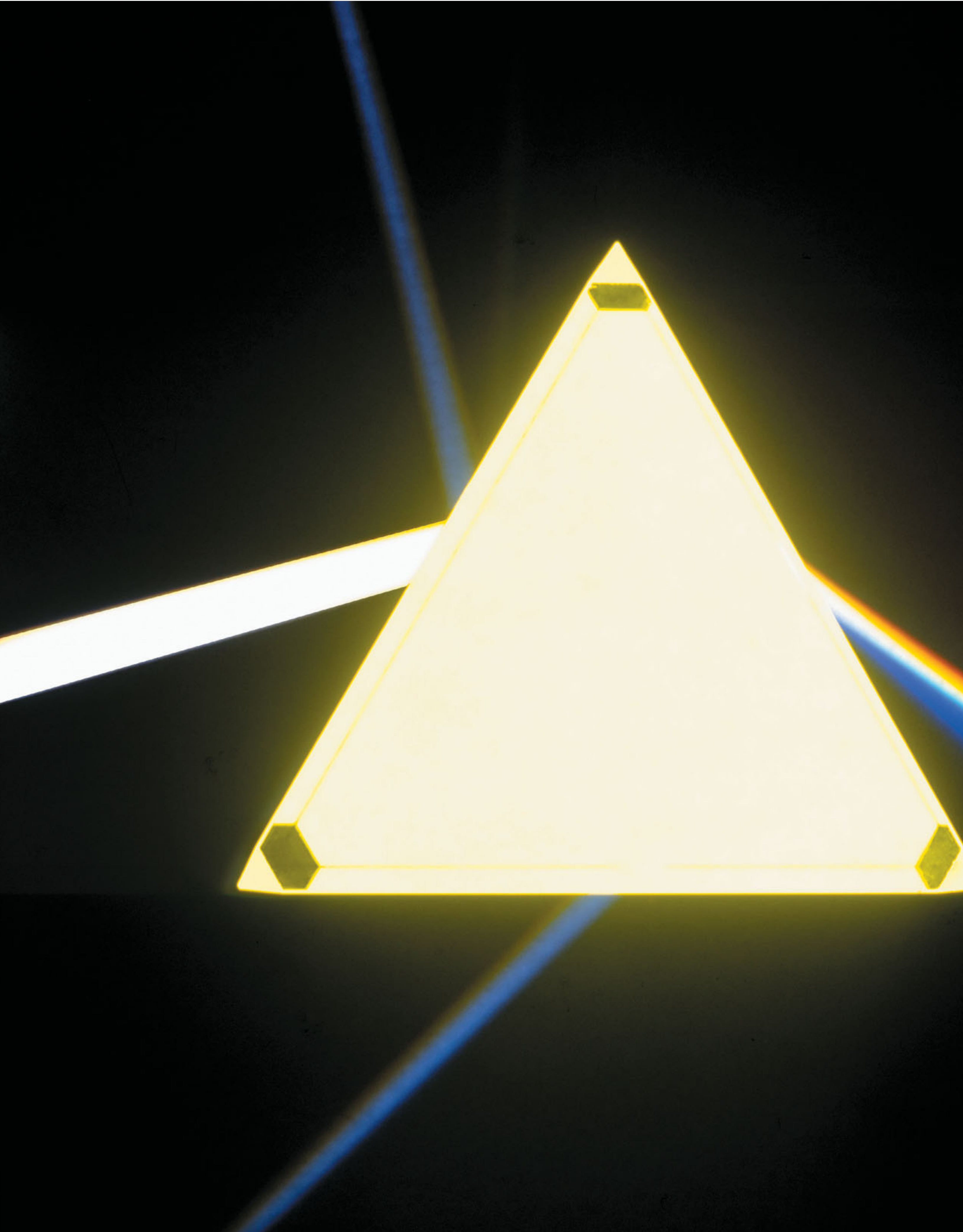
FOURTH EDITION

PHYSICS



JAMES S. WALKER

PHYSICS

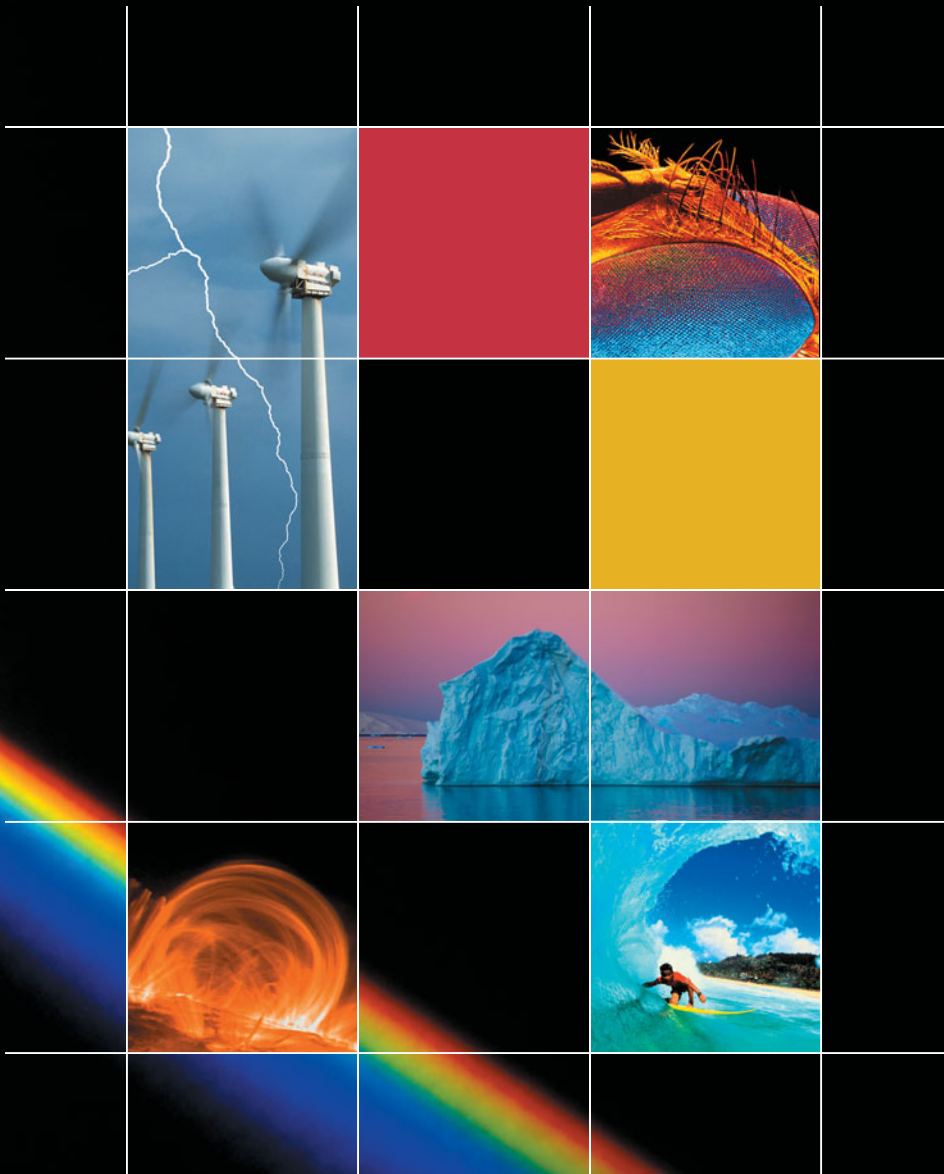


FOURTH EDITION

PHYSICS

JAMES S. WALKER

Western Washington University



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THIS BOOK IS DEDICATED TO MY PARENTS, IVAN AND JANET WALKER, AND TO MY WIFE, BETSY.

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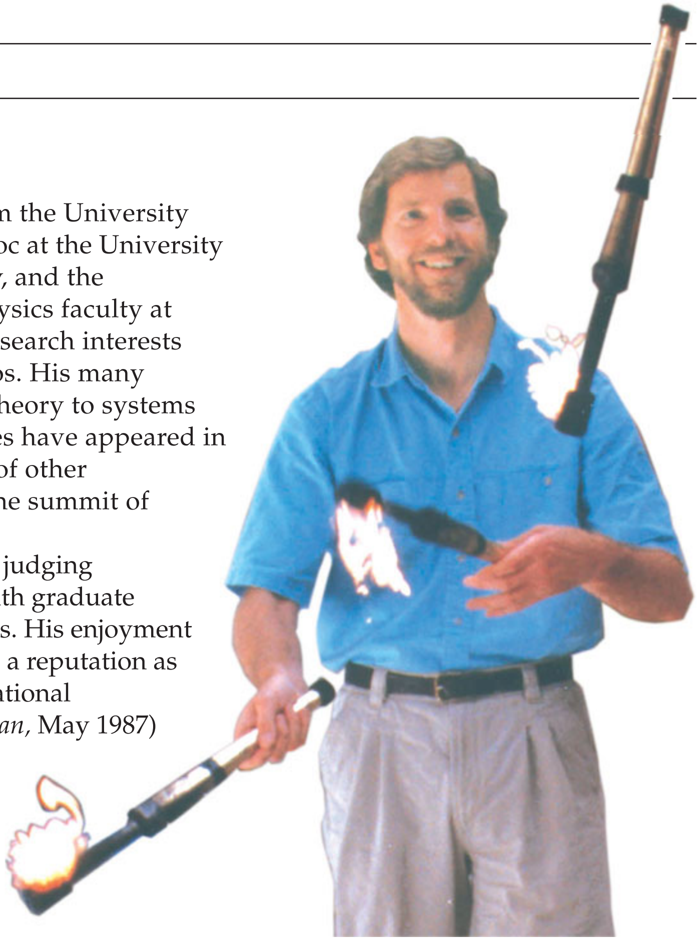
About the Author



JAMES S. WALKER

James Walker obtained his Ph.D. in theoretical physics from the University of Washington in 1978. He subsequently served as a post-doc at the University of Pennsylvania, the Massachusetts Institute of Technology, and the University of California at San Diego before joining the physics faculty at Washington State University in 1983. Professor Walker's research interests include statistical mechanics, critical phenomena, and chaos. His many publications on the application of renormalization-group theory to systems ranging from absorbed monolayers to binary-fluid mixtures have appeared in *Physical Review, Physical Review Letters, Physica,* and a host of other publications. He has also participated in observations on the summit of Mauna Kea, looking for evidence of extra-solar planets.

Jim Walker likes to work with students at all levels, from judging elementary school science fairs to writing research papers with graduate students, and has taught introductory physics for many years. His enjoyment of this course and his empathy for students have earned him a reputation as an innovative, enthusiastic, and effective teacher. Jim's educational publications include "Reappearing Phases" (*Scientific American,* May 1987) as well as articles in the *American Journal of Physics* and *The Physics Teacher.* In recognition of his contributions to the teaching of physics at Washington State University, Jim was named the Boeing Distinguished Professor of Science and Mathematics Education for 2001–2003. He currently teaches at Western Washington University.

When he is not writing, conducting research, teaching, or developing new classroom demonstrations and pedagogical materials, Jim enjoys amateur astronomy, eclipse chasing, bird and dragonfly watching, photography, juggling, unicycling, boogie boarding, and kayaking. Jim is also an avid jazz pianist and organist. He has served as ballpark organist for a number of Class A minor league baseball teams, including the Bellingham Mariners, an affiliate of the Seattle Mariners, and the Salem-Keizer Volcanoes, an affiliate of the San Francisco Giants. He can play "Take Me Out to the Ball Game" in his sleep.

About the Cover

The photographs on the cover of this book are a reminder of the wide "spectrum" of physics applications that are a part of our everyday lives.

Wind Turbines and Lightning Bolt: Wind turbines convert the mechanical energy of moving air into electrical energy to power our homes and cities. Electrical energy is also produced by nature, and occasionally unleashed in impressive bolts of lightning.

Scanning Electron Micrograph: Though electrons are usually thought of as "particles," they also have wave-like properties similar to light. The image of a fly's eye was taken with a beam of electrons.

Iceberg in the Errera Channel: A floating iceberg is a visual demonstration that ice has a lower density than liquid water.

Solar Coronal Loops: Magnetic storms often rage on the surface of the Sun. These glowing loops of ionized gas follow the curved lines of the magnetic field.

Surfer in the "Tube" on the North Shore of Oahu: The laws of physics determine the motion of the wave this surfer is riding.

As you study the material in this book, your understanding of physics will deepen, and your appreciation for the world around you will increase as you come to recognize the fundamental physical principles on which all of our lives are based.

Brief Contents

Foundations for Student Success

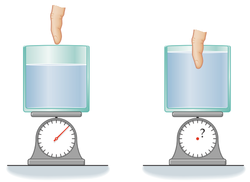
Walker's *Physics* has always been known for its integrated, coherent approach to teaching students the skills to solve problems successfully.

CONCEPTUAL CHECKPOINTS ➤ help students to master key ideas and relationships in a nonquantitative setting.

The end-of-chapter Conceptual Questions, Conceptual Exercises, and Predict/Explain problems further develop students' conceptual understanding.

CONCEPTUAL CHECKPOINT 15–3 HOW IS THE SCALE READING AFFECTED?

A flask of water rests on a scale. If you dip your finger into the water, without touching the flask, does the reading on the scale **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

Your finger experiences an upward buoyant force when it is dipped into the water. By Newton's third law, the water experiences an equal and opposite reaction force acting downward. This downward force is transmitted to the scale, which in turn gives a higher reading.

Another way to look at this result is to note that when you dip your finger into the water, its depth increases. This results in a greater pressure at the bottom of the flask, and hence a greater downward force on the flask. The scale reads this increased downward force.

ANSWER

(a) The reading on the scale increases.

EXERCISES ➤ present brief calculations which illustrate the application of important new relationships.

EXERCISE 7–1

One species of Darwin's finch, *Geospiza magnirostris*, can exert a force of 205 N with its beak as it cracks open a *Tribulus* seed case. If its beak moves through a distance of 0.40 cm during this operation, how much work does the finch do to get the seed?

SOLUTION

$W = Fd = (205\text{ N})(0.0040\text{ m}) = 0.82\text{ J}$

EXAMPLES ➤ model and explain how to solve a particular type of problem.

All Examples use a consistent strategy:

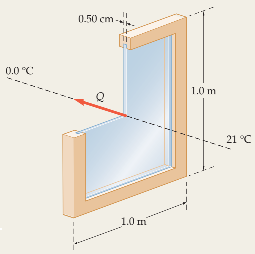
Picture the Problem
Strategy
Solution
Insight

EXAMPLE 16–6 WHAT A PANE!

One of the windows in a house has the shape of a square 1.0 m on a side. The glass in the window is 0.50 cm thick. **(a)** How much heat is lost through this window in one day if the temperature in the house is 21 °C and the temperature outside is 0.0 °C? **(b)** Suppose all the dimensions of the window—height, width, thickness—are doubled. If everything else remains the same, by what factor does the heat flow change?

PICTURE THE PROBLEM

The glass from the window is shown in our sketch, along with its relevant dimensions. Heat flows from the 21 °C side of the window to the 0.0 °C side.



STRATEGY

a. The heat flow is given by $Q = kA(\Delta T/L)t$ (Equation 16–16). Note that the area is $A = (1.0\text{ m})^2$ and that the length over which heat is conducted is, in this case, the thickness of the glass. Thus, $L = 0.0050$ m. The temperature difference is $\Delta T = 21\text{ C}° = 21$ K, and the thermal conductivity of glass (from Table 16–3) is $0.84\text{ W/(m}\cdot\text{K)}$. Also, recall from Section 7–4 that 1 W = 1 J/s.

b. Doubling all dimensions increases the thickness by a factor of 2 and increases the area by a factor of 4; that is, $L \to 2L$ and $A \to (2 \times \text{height}) \times (2 \times \text{width}) = 4A$. Use these results in $Q = kA(\Delta T/L)t$.

SOLUTION

Part (a)

1. Calculate the heat flow for a given time, t:

$$Q = kA\left(\frac{\Delta T}{L}\right)t$$

$$= [0.84\text{ W/(m}\cdot\text{K)}](1.0\text{ m})^2\left(\frac{21\text{ K}}{0.0050\text{ m}}\right)t = (3500\text{ W})t$$

2. Substitute the number of seconds in a day, 86,400 s, for the time t in the expression for Q:

$$Q = (3500\text{ W})t = (3500\text{ W})(86{,}400\text{ s}) = 3.0 \times 10^8\text{ J}$$

Part (b)

3. Replace L with $2L$ and A with $4A$ in Step 1. The result is a doubling of the heat flow, Q:

$$Q = kA\left(\frac{\Delta T}{L}\right)t \to k(4A)\left[\frac{\Delta T}{(2L)}\right]t \to 2\left[kA\left(\frac{\Delta T}{L}\right)t\right] = 2Q$$

INSIGHT

Q is a sizable amount of heat, roughly equivalent to the energy released in burning a gallon of gasoline. A considerable reduction in heat loss can be obtained by using a double-paned window, which has an insulating layer of air (actually argon or krypton) sandwiched between the two panes of glass. This is discussed in more detail later in this section, and is explored in Homework Problems 53 and 91.

PRACTICE PROBLEM

Suppose the window is replaced with a plate of solid silver. How thick must this plate be to have the same heat flow in a day as the glass? [**Answer:** The silver must have a thickness of $L = 2.5$ m.]

Some related homework problems: Problem 49, Problem 50

In response to user feedback, selected examples throughout the Fourth Edition are now more challenging.

Unique two-column layout helps the students relate the strategy to the math.

Examples end with a related **Practice Problem.**

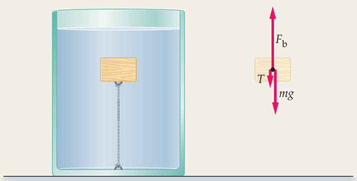
ACTIVE EXAMPLES ➤ provide a skeleton solution that the student must flesh out, helping to bridge the gap from the Examples to the end-of-chapter problems.

ACTIVE EXAMPLE 15–1 FIND THE TENSION IN THE STRING

A piece of wood with a density of 706 kg/m³ is tied with a string to the bottom of a water-filled flask. The wood is completely immersed, and has a volume of 8.00×10^{-6} m³. What is the tension in the string?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply Newton's second law to the wood: $F_b - T - mg = 0$
2. Solve for the tension, T: $T = F_b - mg$
3. Calculate the weight of the wood: $mg = 0.0554$ N
4. Calculate the buoyant force: $F_b = 0.0785$ N
5. Subtract to obtain the tension: $T = 0.0231$ N



INSIGHT

Since the wood floats in water, its buoyant force when completely immersed is greater than its weight.

YOUR TURN

What is the tension in the string if the piece of wood has a density of 822 kg/m³?

(Answers to **Your Turn** *problems can be found in the back of the book.)*

New to the Fourth Edition

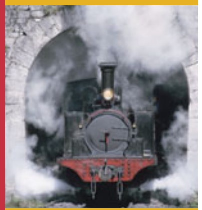
PHYSICS IN PERSPECTIVE ➤
Located at key junctures in the book, **Physics in Perspective** two-page spreads focus on the core ideas developed in the preceding several chapters.

Looking back over several chapters, the spreads show unifying perspectives that the students are only now equipped to see.

For instance, the Physics in Perspective spread illustrated here, located after the final thermodynamics chapter, uses the second law to unify and explain ideas that initially had to be presented from a different perspective.

The **Big Picture** feature at the end of each chapter (not shown here) performs a similar function on a chapter level.

PHYSICS IN



PERSPECTIVE

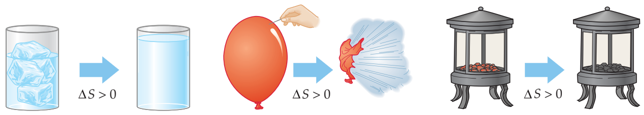
Entropy and Thermodynamics

The behavior of heat engines may seem unrelated to the fate of the universe. However, it led physicists to discover a new physical quantity: entropy. The future of the universe is shaped by the fact that the total entropy can only increase. Our fate is sealed.

1 Spontaneous processes cannot cause a decrease in entropy

Fundamentally, entropy (S) is randomness or disorder. A process that occurs spontaneously—without a driving input of energy—cannot result in a net increase in order (decrease in entropy).

Irreversible processes: $\Delta S > 0$
An *irreversible* process runs spontaneously in just one direction—for instance, ice melts in warm water; warm water doesn't spontaneously form ice cubes. Irreversible processes always cause a net increase in entropy.



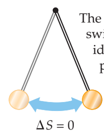
Ice melts in warm water

Air leaves a popped balloon

Cooling embers heat their surroundings

Reversible processes: $\Delta S = 0$
If a process can run spontaneously in either direction—so that a movie of it would look equally realistic run forward or backward—it is *reversible* and causes zero entropy change.

In practice, reversibility is an idealization—real processes are never completely reversible.

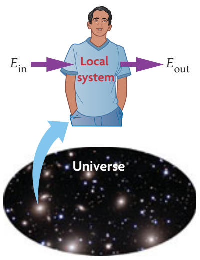


The to-and-fro swinging of an ideal frictionless pendulum is a reversible process

2 Entropy can decrease locally but must increase overall

An input of energy can be used to drive *nonspontaneous* processes that reduce disorder (entropy). That is what your body does with the energy it gains from food.

However, the universe as a whole cannot gain or lose energy, so its total entropy cannot decrease. This means that every process that decreases entropy locally must cause a larger entropy increase elsewhere.



Local system:
Input of energy can drive a decrease in entropy: $\Delta S < 0$.

Universe:
$\Delta E = 0$ (energy is conserved), so
$\Delta S > 0$ (total entropy can only change by increasing)

3 The second law puts entropy in thermodynamic terms

The second law of thermodynamics—that heat moves from hotter to colder objects—actually implies all that we've said about entropy. In fact, the change in entropy ΔS can be defined in terms of the thermodynamic quantities heat Q and temperature T:

$$\Delta S = \frac{Q}{T}$$

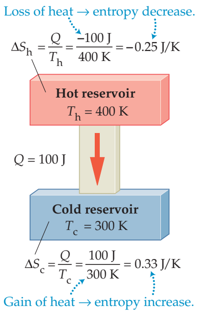
Change in system's entropy

Heat entering or leaving system (positive if heat enters system)

System's temperature

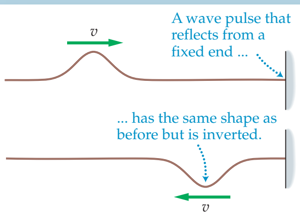
As the example at right shows, the fact that temperature T is in the denominator means that the transfer of a given amount of heat Q causes a greater magnitude of entropy change for a colder object than for a hotter one.

Therefore, a flow of heat from a hotter to a colder object causes a net increase in entropy—as we would predict from the fact that this process is spontaneous and irreversible.

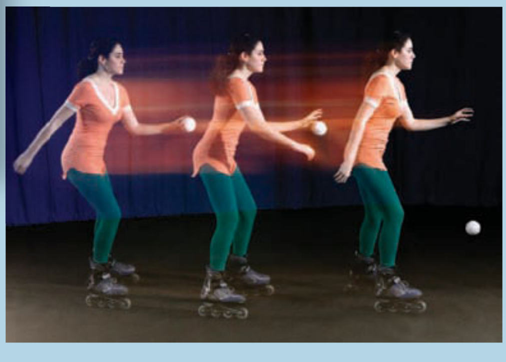


Annotated equations help students to see the meaning in the math.

ANNOTATED FIGURES ➤
Blue explanatory annotations help students to read complex figures and to integrate verbal and visual knowledge.



▲ **FIGURE 14–7 A reflected wave pulse: fixed end**
A wave pulse on a string is inverted when it reflects from an end that is tied down.



PHYSICS DEMONSTRATION PHOTOS ➤
use high-speed time-lapse photography to illustrate phenomena that illuminate physical principles.

Building on strong pedagogic foundations, the Fourth Edition adds features that help students see beyond the mathematical details to the underlying ideas of physics.

PHYSICS IN PERSPECTIVE

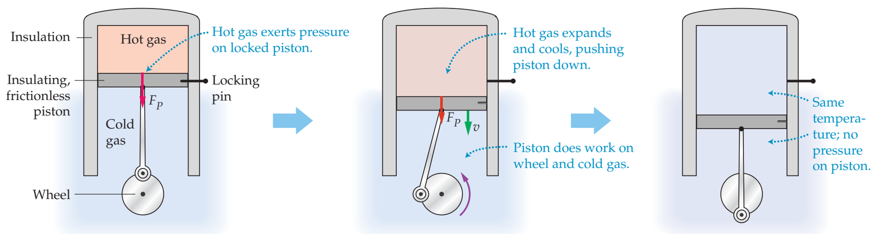
4 A temperature difference can be exploited to do work ...

The tendency of hotter and colder objects to come to the same temperature can be tapped to do work, as in this example:

Initial state: Gases at different temperatures are separated by a locked piston.

Piston unlocked: Pressure difference causes piston to move, doing work on wheel.

Final state: Gases at same temperature; no more work can be done.



The expansion shown above is a single process, not a cycle, so this piston-cylinder does not constitute a heat engine.

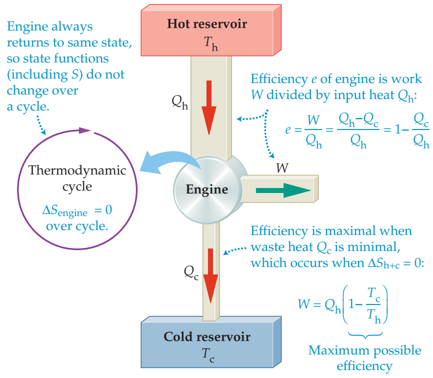
5 ... but entropy sets the limit of efficiency for a heat engine

A heat engine is a device that converts part of a heat flow into work. Entropy sets an absolute limit on the efficiency of this process.

To see why, we start with the fact that a heat engine operates on a thermodynamic cycle—it starts in a particular state, goes through a series of proc esses involving heat and work, and returns to its original state. (Think of the cyclic operation of a cylinder in a car engine.)

Because entropy S is a state function, the engine's entropy returns to it s original value at the end of each cycle—so over the course of a cycle, the entropy change ΔS_{engine} of a heat engine is zero. Therefore, the entropy of the engine's *environment*—specifically, of the hot and cold reservoir (S_{h+c})—must increase or stay the same ($\Delta S_{h+c} \geq 0$).

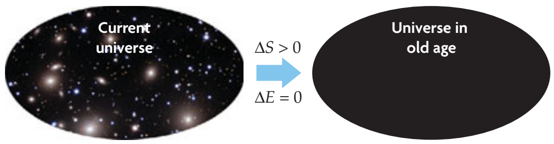
The engine will have the highest efficiency $e = W/Q_h$ when $\Delta S_{h+c} = 0$, because higher values of ΔS_{h+c} entail more waste heat (Q_c) and thus yield less work W. To be more efficient than this, an engine would have to cause a net *decrease* in entropy, which is impossible. Actual engines all have $\Delta S_{h+c} \geq 0$.



6 Entropy spells the death of the universe

The night sky shows us a universe of stars and galaxies separated by cold, nearly empty space. Over time, the inexorable growth of entropy will erase these differences, leaving a universe that is uniform in temperature and density—unable ever again to create stars or give rise to life.

Nevertheless, the energy content of the universe will remain the same as at its birth.



The Physics in Perspective spreads blend words, equations, and pictures into an integrated and highly visual presentation.

Blue explanatory annotations guide the student through the diagrams.

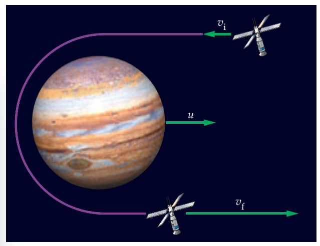
NEW END-OF-CHAPTER PROBLEM TYPES

PASSAGE PROBLEMS ➤
offer a reading passage followed by a set of multiple-choice questions (the format used by most MCAT questions), testing students' ability to apply what they've learned to a real-world situation.

PASSAGE PROBLEMS

Navigating in Space: The Gravitational Slingshot

Many spacecraft navigate through space these days by using the "gravitational slingshot" effect, in which a close encounter with a planet results in a significant increase in magnitude and change in direction of the spacecraft's velocity. In fact, a space-



▲ **FIGURE 9–31** Problems 97, 98, 99, and 100

PREDICT/EXPLAIN PROBLEMS ➤
consist of two linked multiple choice questions — the first asking the student to *predict* the outcome of a situation and the second asking for its physical *explanation*.

15. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the acceleration experienced by the car greater than, less than, or equal to the acceleration experienced by the truck? **(b)** Choose the *best explanation* from among the following:
   I. The truck exerts a larger force on the car, giving it the greater acceleration.
   II. Both vehicles experience the same magnitude of force, therefore the lightweight car experiences the greater acceleration.
   III. The greater force exerted on the truck gives it the greater acceleration.

Applications in the Text

Note: This list includes applied topics that receive significant discussion in the chapter text or a worked Example, as well as topics that are touched on in end-of-chapter Conceptual Questions, Conceptual Exercises, and Problems. Topics of particular relevance to the life sciences or medicine are marked **BIO**. Topics related to Passage Problems are marked **PP**.

ActivPhysics OnLine™ Activities

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1.1 Analyzing Motion Using Diagrams
1.2 Analyzing Motion Using Graphs
1.3 Predicting Motion from Graphs
1.4 Predicting Motion from Equations
1.5 Problem-Solving Strategies for Kinematics
1.6 Skier Races Downhill
1.7 Balloonist Drops Lemonade
1.8 Seat Belts Save Lives
1.9 Screeching to a Halt
1.10 Pole-Vaulter Lands
1.11 Car Starts, Then Stops
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2.3 Lowering a Crate
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2.8 Skier and Rope Tow
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2.10 Truck Pulls Two Crates
2.11 Modified Atwood Machine
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Preface: To the Instructor

Teaching introductory algebra-based physics can be a most challenging—and rewarding—experience. Students enter the course with a wide range of backgrounds, interests, and skills and we, the instructors, strive not only to convey the basic concepts and fundamental laws of physics but also to give students an appreciation of its relevance and appeal.

I wrote this book to help with that task. It incorporates a number of unique and innovative pedagogical features that evolved from years of teaching experience. The materials have been tested extensively in the classroom and in focus groups, and refined based on comments from students and teachers who used the earlier editions of the text. The enthusiastic response I received from users of the first three editions was both flattering and motivating. The fourth edition has been improved in response to this feedback.

Learning Tools in the Text

A key goal of this text is to help students make the connection between a conceptual understanding of physics and the various skills necessary to solve quantitative problems. One of the chief means to that end is the replacement of traditional "textbook" Examples with an integrated system of learning tools: fully worked Examples with Solutions in Two-Column Format, Active Examples, Conceptual Checkpoints, and Exercises. Each of these tools is specialized to meet the needs of students at a particular point in the development of a chapter.

These needs are not always the same. Sometimes students require a detailed explanation of how to tackle a particular problem; at other times, they must be allowed to take an active role and work out the details for themselves. Sometimes it is important for them to perform calculations and concentrate on numerical precision; at other times it is more fruitful for them to explore a key idea in a conceptual context. And sometimes, all that is required is practice using a new equation or definition.

This text attempts to emulate the teaching style of successful instructors by providing the right tool at the right place and the right time.

Perspective Across Chapters

It's easy for students to miss the forest for the trees—to overlook the unifying concepts that are central to physics and that will make the details easier to learn and retain. To address this difficulty, the fourth edition adds two features. At key junctures in the text are six **Physics in Perspective** features, two-page spreads that take a highly visual look at core ideas whose significance students are now prepared to understand. For instance, after working through the energy chapters, do students *really* understand how conservation of energy relates to conservation of mechanical energy, and the role of work done by dissipative and nondissipative forces? And after working through the chapters on electricity and magnetism, do they have a clear view of how electric and magnetic forces relate to each other? These are two of the topics on which the Physics in Perspective pages focus. Each chapter now ends with a **Big Picture box** that links ideas covered in the chapter to related material from earlier and later chapters in the text.

NEW

NEW

WORKED EXAMPLES WITH SOLUTIONS IN TWO-COLUMN FORMAT

Examples model the most complete and detailed method of solving a particular type of problem. The Examples in this text are presented in a format that focuses on the basic strategies and thought processes involved in problem solving. This focus on the intimate relationship between conceptual insights and problem-solving techniques encourages students to view the ability to solve problems as a logical outgrowth of conceptual understanding rather than a kind of parlor trick.

Each Example has the same basic structure:

- **Picture the Problem** This first step discusses how the physical situation can be represented visually and what such a representation can tell us about how to analyze and solve the problem. At this step, always accompanied by a figure, we set up a coordinate system where appropriate, label important quantities, and indicate which values are known.
- **Strategy** The Strategy addresses the commonly asked question, "How do I get started?" by providing a clear overview of the problem and helping students to identify the relevant physical principles. It then guides the student in using known relationships to map a step-by-step path to the solution.
- **Solution in Two-Column Format** In the step-by-step Solution of the problem, each of the steps is presented with a prose statement in the left-hand column and the corresponding mathematical implementation in the right-hand column. Each step clearly translates the idea described in words into the appropriate equations.
- **Insight** Each Example wraps up with an Insight—a comment regarding the solution just obtained. Some Insights deal with possible alternative solution techniques, others with new ideas suggested by the results.
- **Practice Problem** Following the Insight is a Practice Problem, which gives the student a chance to practice the type of calculation just presented. The Practice Problems, always accompanied by their answers, provide students with a valuable check on their understanding of the material. Finally, each Example ends with a reference to some related end-of-chapter Problems to allow students to test their skills further.

ACTIVE EXAMPLES

Active Examples serve as a bridge between the fully worked Examples, in which every detail is fully discussed and every step is given, and the homework Problems, where no help is given at all. In an Active Example, students take an active role in solving the problem by thinking through the logic of the steps described on the left and checking their answers on the right. Students often find it useful to practice problem solving by covering one column of an Active Example with a sheet of paper and filling in the covered steps as they refer to the other column. Follow-up questions, called Your Turns, ask students to look at the problem in a slightly different way. Answers to Your Turns are provided at the end of the book.

CONCEPTUAL CHECKPOINTS

Conceptual Checkpoints help students sharpen their insight into key physical principles. A typical Conceptual Checkpoint presents a thought-provoking question that can be answered by logical reasoning based on physical concepts rather than by numerical calculations. The statement of the question is followed by a detailed discussion and analysis in the section titled Reasoning and Discussion, and the Answer is given at the end of the checkpoint for quick and easy reference.

EXERCISES

Exercises present brief calculations designed to illustrate the application of important new relationships, without the expenditure of time and space required by a fully worked Example. Exercises generally give students an opportunity to practice the use of a new equation, become familiar with the units of a new physical quantity, and get a feeling for typical magnitudes.

PROBLEM-SOLVING NOTES

Each chapter includes a number of Problem-Solving Notes in the margin. These practical hints are designed to highlight useful problem-solving methods while helping students avoid common pitfalls and misconceptions.

End-of-Chapter Learning Tools

The end-of-chapter material in this text also includes a number of innovations, along with refinements of more familiar elements.

- Each chapter concludes with a **Chapter Summary** presented in an easy-to-use outline style. Key concepts, equations, and important figures are organized by topic for convenient reference.
- A unique feature of this text is the **Problem-Solving Summary** at the end of the chapter. This summary addresses common sources of misconceptions in problem solving, and gives specific references to Examples and Active Examples illustrating the correct procedures.
- The homework for each chapter begins with a section of **Conceptual Questions**. Answers to the odd-numbered Questions can be found in the back of the book. Answers to even-numbered Conceptual Questions are available in the online Instructor Solutions Manual.
- **Conceptual Exercises (CE)** have been integrated into the homework section at the end of the chapter and consist of multiple-choice and ranking questions. These questions have been carefully selected and written for maximum effectiveness when used with classroom-response systems (clickers). Answers to the odd-numbered Exercises can be found in the back of the book. Answers to even-numbered Conceptual Exercises are available in the online Instructor Solutions Manual.
- **Predict/Explain problems** are new to this edition. These problems ask the student to predict what will happen in a given physical situation and then to choose an explanation for their prediction. NEW
- Also new to this edition, **Passage Problems** are similar to those found on MCAT exams, with associated multiple-choice questions. NEW
- **Interactive Problems** are based on the animations and simulations associated with the Interactive Figures and are found within MasteringPhysics.
- A popular feature within the homework section is the **Integrated Problems (IP).** These problems, labeled with the symbol IP, integrate a conceptual question with a numerical problem. Problems of this type, which stress the importance of reasoning from basic principles, show how conceptual insight and numerical calculation go hand in hand in physics.
- In addition, a section titled **General Problems** presents a variety of problems that use material from two or more sections within the chapter, or refer to material covered in earlier chapters.
- **Problems of special biological or medical relevance** are indicated with the symbol BIO.

Scope and Organization

TABLE OF CONTENTS

The presentation of physics in this text follows the standard practice for introductory courses, with only a few well-motivated refinements.

- First, note that Chapter 3 is devoted to **vectors and their application to physics.** My experience has been that students benefit greatly from a full discussion of vectors early in the course. Most students have seen vectors and trigonometric functions before, but rarely from the point of view of physics. Thus, including vectors in the text sends a message that this is important material, and it gives students an opportunity to brush up on their math skills.
- Note also that **additional time is given to some of the more fundamental aspects of physics,** such as Newton's laws and energy. Presenting such

material in two chapters gives the student a better opportunity to assimilate and master these crucial topics. Sections considered optional are marked with an asterisk.

REAL-WORLD PHYSICS

REAL-WORLD PHYSICS

REAL-WORLD PHYSICS: BIO

Since physics applies to everything in nature, it is only reasonable to point out applications of physics that students may encounter in the real world. Each chapter presents a number of discussions focusing on "Real-World Physics." Those of general interest are designated by a globe icon in the margin. Applications that pertain more specifically to biology and medicine are indicated by a green frog icon in the margin.

The Illustration Program

DRAWINGS

Many physics concepts are best conveyed by graphic means. Figures do far more than illustrate a physics text—often, they bear the main burden of the exposition. Accordingly, great attention has been paid to the figures in this book, with the primary emphasis always on the clarity of the analysis. Color has been used consistently throughout the text to reinforce concepts and make the diagrams easier for students to understand. New to this edition, helpful **annotations in blue** are included on select figures to help guide students in "reading" graphs and other figures. This technique emulates what instructors do at the chalkboard when explaining figures.

NEW

PHOTOGRAPHS

One of the most fundamental ways in which we learn is by comparing and contrasting. Many **companion photos** are presented in groups of two or three that contrast opposing physical principles or illustrate a single concept in a variety of contexts. Grouping carefully chosen photographs in this way helps students to see the universality of physics. In this edition, we have added new **demonstration photos** that use high-speed time-lapse photography to dramatically illustrate topics, such as standing waves, static versus kinetic friction, and the motion of center of mass, in a way that reveals physical principles in the world around us.

NEW

Resources

The fourth edition is supplemented by an ancillary package developed to address the needs of both students and instructors.

FOR THE INSTRUCTOR

Instructor Solutions Manual by Kenneth L. Menningen (University of Wisconsin–Stevens Point) is available online at the Instructor Resource Center:
www.pearsonhighered.com/educator
You will find detailed, worked solutions to every Problem and Conceptual Exercise in the text, all solved using the step-by-step problem-solving strategy of the in-chapter Examples (Picture the Problem, Strategy, two-column Solutions, and Insight). The solutions also contain answers to the even-numbered Conceptual Questions.

Instructor Resource Manual with Notes on ConcepTest Questions
Available at the Instructor Resource Center: www.pearsonhighered.com/educator, this online manual consists of two parts. The first part, prepared by Katherine Whatley and Judith Beck (both of University of North Carolina, Asheville), contains sample syllabi, lecture outlines, notes, demonstration suggestions, readings, and additional references and resources. The second part, prepared by Cornelius Bennhold and Gerald Feldman (both of George Washington University) contains an overview of the development and implementation of ConcepTests, as well as instructor notes for each ConcepTest found in the Instructor Resource Center and available on the Instructor Resource DVD.

Test Bank Available at the Instructor Resource Center:
www.pearsonhighered.com/educator
Written by Delena Bell Gatch (Georgia Southern University), this online, cross-platform test bank contains approximately 3000 multiple-choice, short-answer, and true/false questions, many conceptual in nature. All are referenced to the corresponding text section and ranked by level of difficulty.

Instructor Resource DVD (ISBN 0-321-60193-9)
This cross-platform DVD provides virtually every electronic asset you'll need in and out of the classroom. The DVD is organized by chapter and includes all text illustrations and tables from *Physics,* Fourth Edition, in jpeg and PowerPoint formats. The IRDVD also contains the Interactive Figures, chapter-by-chapter lecture outlines in PowerPoint, ConcepTest "Clicker" Questions in PowerPoint, editable Word files of all numbered equations, the eleven "Physics You Can See" demonstration videos, and pdf files of the *Instructor Resource Manual with Notes on ConcepTest Questions.*

MasteringPhysics™ www.masteringphysics.com
This homework, tutorial, and assessment system is designed to assign, assess, and track each student's progress using a wide diversity of tutorials and extensively pre-tested problems. All the end-of-chapter problems from the text and the Interactive Figures are available in MasteringPhysics. MasteringPhysics provides instructors with a fast and effective way to assign uncompromising, wide-ranging online homework assignments of just the right difficulty and duration. The tutorials coach 90% of students to the correct answer with specific wrong-answer feedback. The powerful post-assignment diagnostics allow instructors to assess the progress of their class as a whole or to quickly identify individual student's areas of difficulty.

NEW

myeBook is available through MasteringPhysics either automatically when Mastering-Physics is packaged with new books, or available as a purchased upgrade online. Allowing students access to the text wherever they have access to the Internet, myeBook comprises the full text, including figures that can be enlarged for better viewing. Within myeBook, students are also able to pop up definitions and terms to help with vocabulary and the reading of the material. Students can also take notes in myeBook using the annotation feature at the top of each page.

NEW

ActivPhysics OnLine™ (accessed through the Self Study area within www.masteringphysics.com) provides a comprehensive library of more than 420 tried and tested *ActivPhysics* applets. In addition, it provides a suite of applet-based tutorials developed by education pioneers Alan Van Heuvelen and Paul D'Alessandris. The online exercises are designed to encourage students to confront misconceptions, reason qualitatively about physical processes, experiment quantitatively, and learn to think critically. They cover all topics from mechanics to electricity and magnetism and from optics to modern physics. The *ActivPhysics OnLine* companion workbooks help students work through complex concepts and understand them more clearly.

FOR THE STUDENT

Student Study Guide with Selected Solutions by David Reid (University of Chicago) Volume 1: ISBN 0-321-60200-5; Volume 2: ISBN 0-321-60199-8
The print study guide provides the following for each chapter:

Objectives; Warm-Up Questions from the Just-in-Time Teaching (JiTT) method by Gregor Novak and Andrew Gavrin (Indiana University–Purdue University, Indianapolis); Chapter Review with two-column Examples and integrated quizzes; Reference Tools & Resources (equation summaries, important tips, and tools); Puzzle Questions (also from Novak & Gavrin's JiTT method); Selected Solutions for several end-of-chapter questions and problems.

MasteringPhysics™ (www.masteringphysics.com)
This homework, tutorial, and assessment system is based on years of research into how students work physics problems and precisely where they need help. Studies show that students who use MasteringPhysics significantly increase their final scores compared to hand-written homework. MasteringPhysics achieves this improvement by providing students with instantaneous feedback specific to their wrong answers, simpler sub-problems upon request when they get stuck, and partial credit for their method(s) used. This individualized, 24/7 Socratic tutoring is recommended by nine out of ten students to their peers as the most effective and time-efficient way to study.

NEW

myeBook is available through MasteringPhysics either automatically when MasteringPhysics is packaged with new books, or available as a purchased upgrade online. Allowing students access to the text wherever they have access to the Internet, myeBook comprises the full text, including figures that can be enlarged for better viewing. Within myeBook, students are also able to pop up definitions and terms to help with vocabulary and the reading of the material. Students can also take notes in myeBook using the annotation feature at the top of each page.

NEW

ActivPhysics OnLine™ (accessed via www.masteringphysics.com) provides students with a suite of highly regarded applet-based self-study tutorials (see description on previous page). The following workbooks provide a range of tutorial problems designed to use the *ActivPhysics OnLine* simulations, helping students work through complex concepts and understand them more clearly:

- ***ActivPhysics OnLine Workbook*** Volume 1: Mechanics • Thermal Physics • Oscillations & Waves (ISBN 0-8053-9060-X)
- ***ActivPhysics OnLine Workbook*** Volume 2: Electricity & Magnetism • Optics • Modern Physics (ISBN 0-8053-9061-8)

Pearson Tutor Services (www.pearsontutorservices.com) Each student's subscription to MasteringPhysics also contains complimentary access to Pearson Tutor Services, powered by Smarthinking, Inc. By logging in with their MasteringPhysics ID and password, they will be connected to highly qualified e-structors™ who provide additional, interactive online tutoring on the major concepts of physics. Some restrictions apply; offer subject to change.

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Reviewers

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REVIEWERS OF THE FOURTH EDITION

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Kenneth L. Menningen, *University of Wisconsin–Stevens Point*
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Gaylon Ross, *University of Central Arkansas*
Lawrence G. Rowan, *University of North Carolina at Chapel Hill*
Gerald Royce, *Mary Washington College*
Wolfgang Rueckner, *Harvard University*
Misa T. Saros, *Viterbo University*
C. Gregory Seab, *University of New Orleans*
Mats Selen, *University of Illinois*
Bartlett Sheinberg, *Houston Community College*
Peter Shull, *Oklahoma State University*
Christopher Sirola, *Tri-County Technical College*
Daniel Smith, *South Carolina State University*
Leigh M. Smith, *University of Cincinnati*
Soren Sorensen, *University of Tennessee–Knoxville*
Mark W. Sprague, *East Carolina University*
George Strobel, *University of Georgia*
Carey E. Stronach, *Virginia State University*
Irina Struganova, *Barry University*
Daniel Stump, *Michigan State University*
Leo Takahashi, *Penn State University–Beaver*
Harold Taylor, *Richard Stockton College*
Frederick Thomas, *Sinclair Community College*
Jack Tuszynski, *University of Alberta*
Lorin Vant Hull, *University of Houston*
John A. Underwood, *Austin Community College, Rio Grande*
Karl Vogler, *Northern Kentucky University*
Desmond Walsh, *Memorial University of Newfoundland*
Toby Ward, *College of Lake County*
Richard Webb, *Pacific Union College*
Lawrence Weinstein, *Old Dominion University*
Jeremiah Williams, *Illinois Wesleyan University*
Linda Winkler, *Moorhead State University*
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STUDENT REVIEWERS

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Gina Hedberg
Kyle Kazsinas
Ty Keeney
Justin Kvenzi
Tannia Lau
Ann MaKarewicz
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Philip Dagostino
Andrew D. Fisher
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Khang Tran
Michael Vasquez
Jerod Williams
Nathan Witwer
Alexander Wood
Melissa Wright

ATTENDEE: Lynda Klein, *California State University–Chico*

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Preface: To the Student

As a student preparing to take an algebra-based physics course, you are probably aware that physics applies to absolutely everything in the natural world, from raindrops and people to galaxies and atoms. Because physics is so wide-ranging and comprehensive, it can sometimes seem a bit overwhelming. This text, which reflects nearly two decades of classroom experience, is designed to help you deal with a large body of information and develop a working understanding of the basic concepts in physics. Now in its fourth edition, it incorporates many refinements that have come directly from interacting with students using the first three editions. As a result of these interactions, I am confident that as you develop a deeper understanding of physics, you will also enrich your experience of the world in which you live.

Now, I must admit that I like physics, and so I may be a bit biased in this respect. Still, the reason I teach and continue to study physics is that I enjoy the insight it gives into the physical world. I can't help but notice—and enjoy—aspects of physics all around me each and every day. As I always tell my students on the first day of class, I would like to share some of this enjoyment and delight in the natural world with you. It is for this reason that I undertook the task of writing this book.

To assist you in the process of studying physics, this text incorporates a number of learning aids, including Two-Column Examples, Active Examples, and Conceptual Checkpoints. These and other elements work together in a unified way to enhance your understanding of physics on both a conceptual and a quantitative level—they have been developed to give you the benefit of what we know about how students learn physics, and to incorporate strategies that have proven successful to students over the years. The pages that follow will introduce these elements to you, describe the purpose of each, and explain how they can help you.

As you progress through the text, you will encounter many interesting and intriguing applications of physics drawn from the world around you. Some of these, such as magnetically levitated trains or the satellite-based Global Positioning System that enables you to determine your position anywhere on Earth to within a few feet, are primarily technological in nature. Others focus on explaining familiar or not-so-familiar phenomena, such as why the Moon has no atmosphere, how sweating cools the body, or why flying saucer shaped clouds often hover over mountain peaks even when the sky is clear. Still others, such as countercurrent heat exchange in animals and humans or the use of sound waves to destroy kidney stones, are of particular relevance to students of biology and the other life sciences.

In many cases, you may find the applications to be a bit surprising. Did you know, for example, that you are shorter at the end of the day than when you first get up in the morning? (This is discussed in Chapter 5.) That an instrument called the ballistocardiograph can detect the presence of a person hiding in a truck, just by registering the minute recoil from the beating of the stowaway's heart? (This is discussed in Chapter 9.) That if you hum next to a spider's web at just the right pitch you can cause a resonance effect that sends the spider into a tizzy? (This is discussed in Chapter 13.) That powerful magnets can exploit the phenomenon of diamagnetism to levitate living creatures? (This is discussed in Chapter 22.)

Writing this textbook was a rewarding experience for me. I hope using it will prove equally rewarding to you, and that it will inspire an interest in and appreciation of physics that will last a lifetime.

James S. Walker

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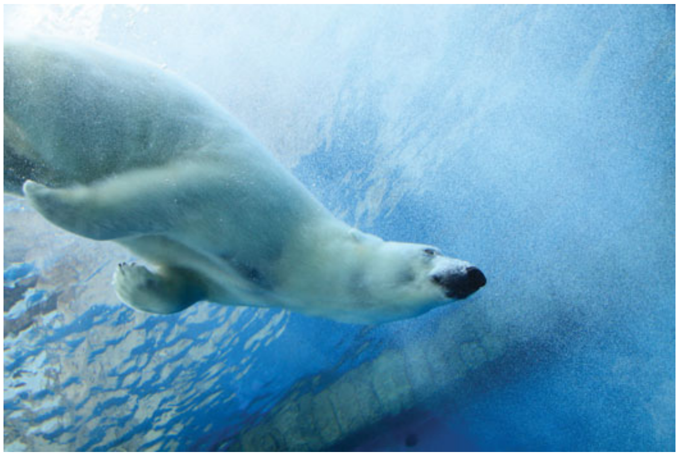
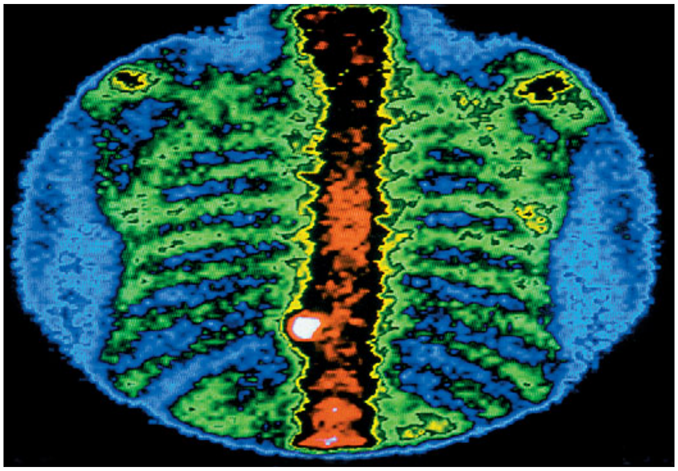
Detailed Contents



PART IV LIGHT AND OPTICS

PART V MODERN PHYSICS



1 Introduction to Physics

Physics is a quantitative science, based on careful measurements of quantities such as mass, length, and time. In the measurement shown here, a baby elephant is found to have a mass of approximately 425 kilograms, corresponding to a weight of about 935 pounds. Measurements of length and time indicate that the elephant's height is 1.25 meters, and its age is eleven months.

The goal of physics is to gain a deeper understanding of the world in which we live. For example, the laws of physics allow us to predict the behavior of everything from rockets sent to the Moon, to integrated chips in computers, to lasers used to perform eye surgery. In short, everything in nature—from atoms and subatomic particles to solar systems and galaxies—obeys the laws of physics.

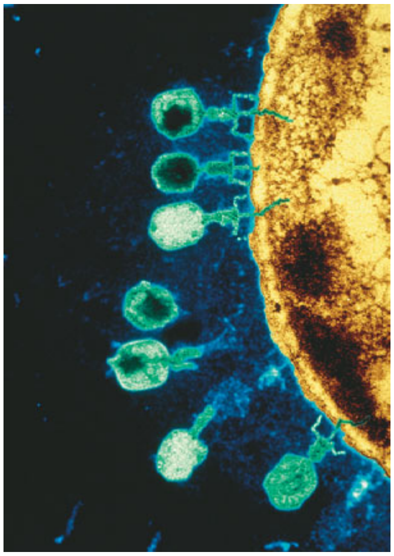
As we begin our study of physics, it is useful to consider a range of issues that underlies everything to follow. One of the most fundamental of these is the system of units we use when we measure such things as the mass of an object, its length, and the time between two events. Other equally important issues include methods for handling numerical calculations and basic conventions of mathematical notation. By the end of the chapter we will have developed a common "language" of physics that will be used throughout this book and probably in any science that you study.

1–1 Physics and the Laws of Nature

Physics is the study of the fundamental laws of nature, which, simply put, are the laws that underlie all physical phenomena in the universe. Remarkably, we have found that these laws can be expressed in terms of mathematical equations. As a result, it is possible to make precise, quantitative comparisons between the predictions of theory—derived from the mathematical form of the laws—and the observations of experiments. Physics, then, is a science rooted equally firmly in theory and experiment, and, as physicists make new observations, they constantly test and—if necessary—refine the present theories.

What makes physics particularly fascinating is the fact that it relates to everything in the universe. There is a great beauty in the vision that physics brings to our view of the universe; namely, that all the complexity and variety that we see in the world around us, and in the universe as a whole, are manifestations of a few fundamental laws and principles. That we can discover and apply these basic laws of nature is both astounding and exhilarating.

For those not familiar with the subject, physics may seem to be little more than a confusing mass of formulas. Sometimes, in fact, these formulas can be the trees that block the view of the forest. For a physicist, however, the many formulas of physics are simply different ways of expressing a few fundamental ideas. It is the forest—the basic laws and principles of physical phenomena in nature—that is the focus of this text.



▲ The size of these viruses, seen here attacking a bacterial cell, is about 10^{-7} m.

▲ The diameter of this typical galaxy is about 10^{21} m. (How many viruses would it take to span the galaxy?)

1–2 Units of Length, Mass, and Time

To make quantitative comparisons between the laws of physics and our experience of the natural world, certain basic physical quantities must be measured. The most common of these quantities are **length** (L), **mass** (M), and **time** (T). In fact, in the next several chapters these are the only quantities that arise. Later in the text, additional quantities, such as temperature and electric current, will be introduced as needed.

We begin by defining the units in which each of these quantities is measured. Once the units are defined, the values obtained in specific measurements can be expressed as multiples of them. For example, our unit of length is the **meter** (m). It follows, then, that a person who is 1.94 m tall has a height 1.94 times this unit of length. Similar comments apply to the unit of mass, the **kilogram,** and the unit of time, the **second.**

The detailed system of units used in this book was established in 1960 at the Eleventh General Conference of Weights and Measures in Paris, France, and goes by the name Système International d'Unités, or SI for short. Thus, when we refer to **SI units,** we mean units of meters (m), kilograms (kg), and seconds (s). Taking the first letter from each of these units leads to an alternate name that is often used—the **mks system.**

In the remainder of this section we define each of the SI units.

Length

Early units of length were often associated with the human body. For example, the Egyptians defined the cubit to be the distance from the elbow to the tip of the middle finger. Similarly, the foot was at one time defined to be the length of the royal foot of King Louis XIV. As colorful as these units may be, they are not particularly reproducible—at least not to great precision.

In 1793 the French Academy of Sciences, seeking a more objective and reproducible standard, decided to define a unit of length equal to one ten-millionth the distance from the North Pole to the equator. This new unit was named the metre (from the Greek *metron* for "measure"). The preferred spelling in the United States is *meter*. This definition was widely accepted, and in 1799 a "standard" meter was produced. It consisted of a platinum-iridium alloy rod with two marks on it one meter apart.

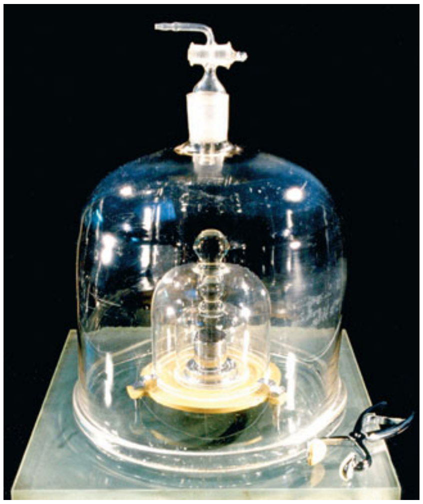
TABLE 1–1 Typical Distances

Distance from Earth to the nearest large galaxy (the Andromeda galaxy, M31)	2×10^{22} m
Diameter of our galaxy (the Milky Way)	8×10^{20} m
Distance from Earth to the nearest star (other than the Sun)	4×10^{16} m
One light-year	9.46×10^{15} m
Average radius of Pluto's orbit	6×10^{12} m
Distance from Earth to the Sun	1.5×10^{11} m
Radius of Earth	6.37×10^{6} m
Length of a football field	10^{2} m
Height of a person	2 m
Diameter of a CD	0.12 m
Diameter of the aorta	0.018 m
Diameter of a period in a sentence	5×10^{-4} m
Diameter of a red blood cell	8×10^{-6} m
Diameter of the hydrogen atom	10^{-10} m
Diameter of a proton	2×10^{-15} m

Since 1983 we have used an even more precise definition of the meter, based on the speed of light in a vacuum. In particular:

> One meter is defined to be the distance traveled by light in a vacuum in 1/299,792,458 of a second.

No matter how its definition is refined, however, a meter is still about 3.28 feet, which is roughly 10 percent longer than a yard. A list of typical lengths is given in Table 1–1.



▲ The standard kilogram, a cylinder of platinum and iridium 0.039 m in height and diameter, is kept under carefully controlled conditions in Sèvres, France. Exact replicas are maintained in other laboratories around the world.

Mass

In SI units, mass is measured in kilograms. Unlike the meter, the kilogram is not based on any natural physical quantity. By convention, the kilogram has been defined as follows:

> The kilogram, by definition, is the mass of a particular platinum-iridium alloy cylinder at the International Bureau of Weights and Standards in Sèvres, France.

To put the kilogram in everyday terms, a quart of milk has a mass slightly less than 1 kilogram. Additional masses, in kilograms, are given in Table 1–2.

Note that we do not define the kilogram to be the *weight* of the platinum-iridium cylinder. In fact, weight and mass are quite different quantities, even though they are often confused in everyday language. Mass is an intrinsic, unchanging property of an object. Weight, in contrast, is a measure of the gravitational force acting on an object, which can vary depending on the object's location. For example, if you are fortunate enough to travel to Mars someday, you will find that your weight is less than on Earth, though your mass is unchanged. The force of gravity will be discussed in detail in Chapter 12.

TABLE 1–2 Typical Masses

Galaxy (Milky Way)	4×10^{41} kg
Sun	2×10^{30} kg
Earth	5.97×10^{24} kg
Space shuttle	2×10^{6} kg
Elephant	5400 kg
Automobile	1200 kg
Human	70 kg
Baseball	0.15 kg
Honeybee	1.5×10^{-4} kg
Red blood cell	10^{-13} kg
Bacterium	10^{-15} kg
Hydrogen atom	1.67×10^{-27} kg
Electron	9.11×10^{-31} kg

Time

Nature has provided us with a fairly accurate timepiece in the revolving Earth. In fact, prior to 1956 the mean solar day was defined to consist of 24 hours, with 60 minutes per hour, and 60 seconds per minute, for a total of $(24)(60)(60) = 84{,}400$ seconds. Even the rotation of the Earth is not completely regular, however.

Today, the most accurate timekeepers known are "atomic clocks," which are based on characteristic frequencies of radiation emitted by certain atoms. These

▲ This atomic clock, which keeps time on the basis of radiation from cesium atoms, is accurate to about three millionths of a second per year. (How long would it take for it to gain or lose an hour?)

TABLE 1–3 Typical Times

Age of the universe	5×10^{17} s
Age of the Earth	1.3×10^{17} s
Existence of human species	6×10^{13} s
Human lifetime	2×10^{9} s
One year	3×10^{7} s
One day	8.6×10^{4} s
Time between heartbeats	0.8 s
Human reaction time	0.1 s
One cycle of a high-pitched sound wave	5×10^{-5} s
One cycle of an AM radio wave	10^{-6} s
One cycle of a visible light wave	2×10^{-15} s

clocks have typical accuracies of about 1 second in 300,000 years. The atomic clock used for defining the second operates with cesium-133 atoms. In particular, the second is defined as follows:

> One second is defined to be the time it takes for radiation from a cesium-133 atom to complete 9,192,631,770 cycles of oscillation.

A range of characteristic time intervals is given in Table 1–3.

The nation's time and frequency standard is determined by a *cesium fountain atomic clock* developed at the National Institute of Standards and Technology (NIST) in Boulder, Colorado. The fountain atomic clock, designated NIST-F1, produces a "fountain" of cesium atoms that are projected upward in a vacuum to a height of about a meter. It takes roughly a second for the atoms to rise and fall through this height (as we shall see in the next chapter), and during this relatively long period of time the frequency of their oscillation can be measured with great precision. In fact, the NIST-F1 will gain or lose no more than one second in every 20 million years of operation.

Atomic clocks are almost commonplace these days. For example, the satellites that participate in the Global Positioning System (GPS) actually carry atomic clocks with them in orbit. This allows them to make the precision time measurements that are needed for an equally precise determination of position and speed. Similarly, the "atomic clocks" that are advertised for use in the home, while not atomic in their operation, nonetheless get their time from radio signals sent out from the atomic clocks at NIST in Boulder. You can access the official U.S. time on your computer by going to http://time.gov on the Web.

Other Systems of Units and Standard Prefixes

Although SI units are used throughout most of this book and are used almost exclusively in scientific research and in industry, we will occasionally refer to other systems that you may encounter from time to time.

For example, a system of units similar to the mks system, though comprised of smaller units, is the **cgs system,** which stands for centimeter (cm), gram (g), and second (s). In addition, the British engineering system is often encountered in everyday usage in the United States. Its basic units are the slug for mass, the foot (ft) for length, and the second (s) for time.

Finally, multiples of the basic units are common no matter which system is used. Standard prefixes are used to designate common multiples in powers of ten. For example, the prefix *kilo* means one thousand, or, equivalently, 10^3. Thus, 1 kilogram is 10^3 grams, and 1 kilometer is 10^3 meters. Similarly, *milli* is the prefix for one thousandth, or 10^{-3}. Thus, a millimeter is 10^{-3} meter, and so on. The most common prefixes are listed in Table 1–4.

EXERCISE 1–1

a. A minivan sells for 33,200 dollars. Express the price of the minivan in kilodollars and megadollars.

b. A typical *E. coli* bacterium is about 5 micrometers (or microns) in length. Give this length in millimeters and kilometers.

SOLUTION

a. 33.2 kilodollars, 0.0332 megadollars

b. 0.005 mm, 0.000000005 km

1–3 Dimensional Analysis

In physics, when we speak of the **dimension** of a physical quantity, we refer to the *type* of quantity in question, regardless of the units used in the measurement. For example, a distance measured in cubits and another distance measured in light-years both have the same dimension—length. The same is true of compound units such as velocity, which has the dimensions of length per unit time (length/time). A velocity measured in miles per hour has the same dimensions—length/time—as one measured in inches per century.

Now, any valid formula in physics must be **dimensionally consistent;** that is, each term in the equation must have the same dimensions. It simply doesn't make sense to add a distance to a time, for example, any more than it makes sense to add apples and oranges. They are different things.

To check the dimensional consistency of an equation, it is convenient to introduce a special notation for the dimension of a quantity. We will use square brackets, [ ], for this purpose. Thus, if x represents a distance, which has dimensions of length [L], we write this as $x = [\mathrm{L}]$. Similarly, a velocity, v, has dimensions of length per time [T]; thus we write $v = [\mathrm{L}]/[\mathrm{T}]$ to indicate its dimensions. Acceleration, a, which is the change in velocity per time, has the dimensions $a = ([\mathrm{L}]/[\mathrm{T}])/[\mathrm{T}] = [\mathrm{L}]/[\mathrm{T}^2]$. The dimensions of some common physical quantities are summarized in Table 1–5.

Let's use this notation to check the dimensional consistency of a simple equation. Consider the following formula:

$$x = x_0 + vt$$

In this equation, x and x_0 represent distances, v is a velocity, and t is time. Writing out the dimensions of each term, we have

$$[\mathrm{L}] = [\mathrm{L}] + \frac{[\mathrm{L}]}{[\mathrm{T}]}[\mathrm{T}]$$

It might seem at first that the last term has different dimensions than the other two. However, dimensions obey the same rules of algebra as other quantities. Thus the dimensions of time cancel in the last term:

$$[\mathrm{L}] = [\mathrm{L}] + \frac{[\mathrm{L}]}{[\cancel{\mathrm{T}}]}[\cancel{\mathrm{T}}] = [\mathrm{L}] + [\mathrm{L}]$$

As a result, we see that each term in this formula has the same dimensions. This type of calculation with dimensions is referred to as **dimensional analysis.**

EXERCISE 1–2

Show that $x = x_0 + v_0t + \frac{1}{2}at^2$ is dimensionally consistent. The quantities x and x_0 are distances, v_0 is a velocity, and a is an acceleration.

SOLUTION

Using the dimensions given in Table 1–5, we have

$$[\mathrm{L}] = [\mathrm{L}] + \frac{[\mathrm{L}]}{[\cancel{\mathrm{T}}]}[\cancel{\mathrm{T}}] + \frac{[\mathrm{L}]}{[\cancel{\mathrm{T}}]^2}[\cancel{\mathrm{T}}^2] = [\mathrm{L}] + [\mathrm{L}] + [\mathrm{L}]$$

Note that $\frac{1}{2}$ is ignored in this analysis because it has no dimensions.

Later in this text you will derive your own formulas from time to time. As you do so, it is helpful to check dimensional consistency at each step of the derivation. If at any time the dimensions don't agree, you will know that a mistake has been made, and you can go back and look for it. If the dimensions check, however, it's not a guarantee the formula is correct—after all, dimensionless factors, like 1/2 or 2, don't show up in a dimensional check.

1–4 Significant Figures

When a mass, a length, or a time is measured in a scientific experiment, the result is known only to within a certain accuracy. The inaccuracy or uncertainty can be caused by a number of factors, ranging from limitations of the measuring device itself to limitations associated with the senses and the skill of the person performing the experiment. In any case, the fact that observed values of experimental

TABLE 1–4 Common Prefixes

Power	Prefix	Abbreviation
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f

TABLE 1–5 Dimensions of Some Common Physical Quantities

Quantity	Dimension
Distance	[L]
Area	$[\mathrm{L}^2]$
Volume	$[\mathrm{L}^3]$
Velocity	[L]/[T]
Acceleration	$[\mathrm{L}]/[\mathrm{T}^2]$
Energy	$[\mathrm{M}][\mathrm{L}^2]/[\mathrm{T}^2]$

quantities have inherent uncertainties should always be kept in mind when performing calculations with those values.

▲ Every measurement has some degree of uncertainty associated with it. How precise would you expect this measurement to be?

Suppose, for example, that you want to determine the walking speed of your pet tortoise. To do so, you measure the time, t, it takes for the tortoise to walk a distance, d, and then you calculate the quotient, d/t. When you measure the distance with a ruler, which has one tick mark per millimeter, you find that $d = 21.2$ cm, with the precise value of the digit in the second decimal place uncertain. Defining the number of **significant figures** in a physical quantity to be equal to the number of digits in it that are known with certainty, we say that d is known to *three* significant figures.

Similarly, you measure the time with an old pocket watch, and as best you can determine it, $t = 8.5$ s, with the second decimal place uncertain. Note that t is known to only *two* significant figures. If we were to make this measurement with a digital watch, with a readout giving the time to 1/100 of a second, the accuracy of the result would still be limited by the finite reaction time of the experimenter. The reaction time would have to be predetermined in a separate experiment. (See Problem 77 in Chapter 2 for a simple way to determine your reaction time.)

Returning to the problem at hand, we would now like to calculate the speed of the tortoise. Using the above values for d and t and a calculator with eight digits in its display, we find $(21.2 \text{ cm})/(8.5 \text{ s}) = 2.4941176 \text{ cm/s}$. Clearly, such an accurate value for the speed is unjustified, considering the limitations of our measurements. After all, we can't expect to measure quantities to two and three significant figures and from them obtain results with eight significant figures. In general, the number of significant figures that result when we multiply or divide physical quantities is given by the following rule of thumb:

> The number of significant figures after multiplication or division is equal to the number of significant figures in the *least* accurately known quantity.

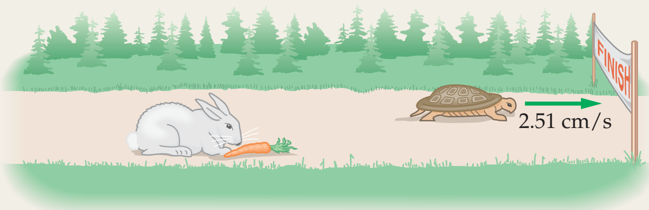
In our speed calculation, for example, we know the distance to three significant figures, but the time to only two significant figures. As a result, the speed should be given with just two significant figures, $d/t = (21.2 \text{ cm})/(8.5 \text{ s}) = 2.5 \text{ cm/s}$. Note that we didn't just keep the first two digits in 2.4941176 cm/s and drop the rest. Instead, we "rounded up"; that is, because the first digit to be dropped (9 in this case) is greater than or equal to 5, we increase the previous digit (4 in this case) by 1. Thus, 2.5 cm/s is our best estimate for the tortoise's speed.

EXAMPLE 1–1 IT'S THE TORTOISE BY A HARE

A tortoise races a rabbit by walking with a constant speed of 2.51 cm/s for 12.23 s. How much distance does the tortoise cover?

PICTURE THE PROBLEM

The race between the rabbit and the tortoise is shown in our sketch. The rabbit pauses to eat a carrot while the tortoise walks with a constant speed.



STRATEGY

The distance covered by the tortoise is the speed of the tortoise multiplied by the time during which it walks.

SOLUTION

1. Multiply the speed by the time to find the distance d:

$$d = (\text{speed})(\text{time})$$
$$= (2.51 \text{ cm/s})(12.23 \text{ s}) = 30.7 \text{ cm}$$

INSIGHT

Notice that if we simply multiply 2.51 cm/s by 12.23 s, we obtain 30.6973 cm. We don't give all of these digits in our answer, however. In particular, because the quantity that is known with the least accuracy (the speed) has only three significant figures, we give a result with three significant figures. Note, in addition, that the third digit in our answer has been rounded up from 6 to 7.

PRACTICE PROBLEM

How long does it take for the tortoise to walk 17 cm? [**Answer:** $t = (17\text{ cm})/(2.51\text{ cm/s}) = 6.8\text{ s}$]

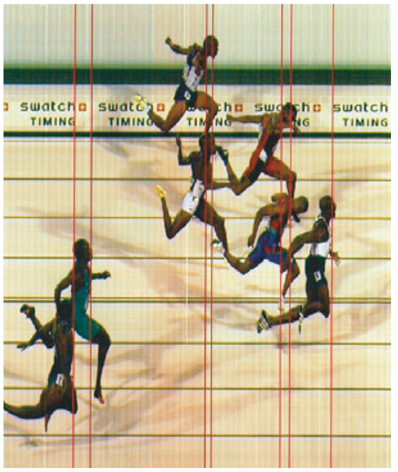
Some related homework problems: Problem 14, Problem 18

Note that the distance of 17 cm in the Practice Problem has only two significant figures because we don't know the digits to the right of the decimal place. If the distance were given as 17.0 cm, on the other hand, it would have three significant figures.

When physical quantities are added or subtracted, we use a slightly different rule of thumb. In this case, the rule involves the number of decimal places in each of the terms:

> The number of decimal places after addition or subtraction is equal to the smallest number of decimal places in any of the individual terms.

Thus, if you make a time measurement of 16.74 s, and then a subsequent time measurement of 5.1 s, the total time of the two measurements should be given as 21.8 s, rather than 21.84 s.



▲ The finish of the 100-meter race at the 1996 Atlanta Olympics. This official timing photo shows Donovan Bailey setting a new world record of 9.84 s. (If the timing had been accurate to only tenths of a second—as would probably have been the case before electronic devices came into use—how many runners would have shared the winning time? How many would have shared the second-place and third-place times?)

EXERCISE 1–3

You and a friend pick some raspberries. Your flat weighs 12.7 lb, and your friend's weighs 7.25 lb. What is the combined weight of the raspberries?

SOLUTION

Just adding the two numbers gives 19.95 lb. According to our rule of thumb, however, the final result must have only a single decimal place (corresponding to the term with the smallest number of decimal places). Rounding off to one place, then, gives 20.0 lb as the acceptable result.

Scientific Notation

The number of significant figures in a given quantity may be ambiguous due to the presence of zeros at the beginning or end of the number. For example, if a distance is stated to be 2500 m, the two zeros could be significant figures, or they could be zeros that simply show where the decimal point is located. If the two zeros are significant figures, the uncertainty in the distance is roughly a meter; if they are not significant figures, however, the uncertainty is about 100 m.

To remove this type of ambiguity, we can write the distance in **scientific notation**—that is, as a number of order unity times an appropriate power of ten. Thus, in this example, we would express the distance as 2.5×10^3 m if there are only two significant figures, or as 2.500×10^3 m to indicate four significant figures. Likewise, a time given as 0.000036 s has only two significant figures—the preceding zeros only serve to fix the decimal point. If this quantity were known to three significant figures, we would write it as 3.60×10^{-5} s to remove any ambiguity. See Appendix A for a more detailed discussion of scientific notation.

EXERCISE 1–4

How many significant figures are there in **(a)** 21.00, **(b)** 21, **(c)** 2.1×10^{-2}, **(d)** 2.10×10^{-3}?

SOLUTION

(a) 4, **(b)** 2, **(c)** 2, **(d)** 3

Round-Off Error

Finally, even if you perform all your calculations to the same number of significant figures as in the text, you may occasionally obtain an answer that differs in its last digit from that given in the book. In most cases this is not an issue as far as understanding the physics is concerned—usually it is due to **round-off error.**

Round-off error occurs when numerical results are rounded off at different times during a calculation. To see how this works, let's consider a simple example. Suppose you are shopping for knickknacks, and you buy one item for $2.21, plus 8 percent sales tax. The total price is $2.3868, or, rounded off to the nearest penny, $2.39. Later, you buy another item for $1.35. With tax this becomes $1.458 or, again to the nearest penny, $1.46. The total expenditure for these two items is $2.39 + $1.46 = $3.85.

Now, let's do the rounding off in a different way. Suppose you buy both items at the same time for a total before-tax price of $2.21 + $1.35 = $3.56. Adding in the 8% tax gives $3.8448, which rounds off to $3.84, one penny different from the previous amount. This same type of discrepancy can occur in physics problems. In general, it's a good idea to keep one extra digit throughout your calculations whenever possible, rounding off only the final result. But while this practice can help to reduce the likelihood of round-off error, there is no way to avoid it in every situation.

1–5 Converting Units

It is often convenient to convert from one set of units to another. For example, suppose you would like to convert 316 ft to its equivalent in meters. Looking at the conversion factors on the inside front cover of the text, we see that

$$1\text{ m} = 3.281\text{ ft} \tag{1–1}$$

Equivalently,

$$\frac{1\text{ m}}{3.281\text{ ft}} = 1 \tag{1–2}$$

Now, to make the conversion, we simply multiply 316 ft by this expression, which is equivalent to multiplying by 1:

$$(316\ \cancel{\text{ft}})\left(\frac{1\text{ m}}{3.281\ \cancel{\text{ft}}}\right) = 96.3\text{ m}$$

Note that the conversion factor is written in this particular way, as 1 m divided by 3.281 ft, so that the units of feet cancel out, leaving the final result in the desired units of meters.

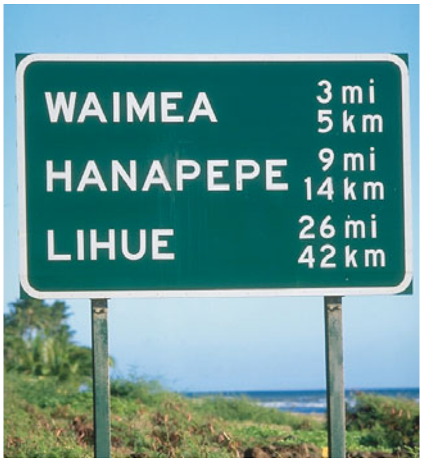
Of course, we can just as easily convert from meters to feet if we use the reciprocal of this conversion factor—which is also equal to 1:

$$1 = \frac{3.281\text{ ft}}{1\text{ m}}$$

For example, a distance of 26.4 m is converted to feet by canceling out the units of meters, as follows:

$$(26.4\ \cancel{\text{m}})\left(\frac{3.281\text{ ft}}{1\ \cancel{\text{m}}}\right) = 86.6\text{ ft}$$

Thus, we see that converting units is as easy as multiplying by 1—because that's really what you're doing.



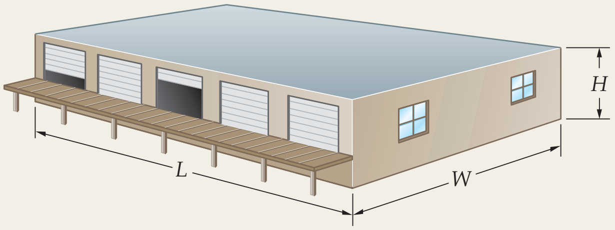
▲ From this sign, you can calculate factors for converting miles to kilometers and vice versa. (Why do you think the conversion factors seem to vary for different destinations?)

EXAMPLE 1–2 A HIGH-VOLUME WAREHOUSE

A warehouse is 20.0 yards long, 10.0 yards wide, and 15.0 ft high. What is its volume in SI units?

PICTURE THE PROBLEM

In our sketch we picture the warehouse, and indicate the relevant lengths for each of its dimensions.



STRATEGY

We begin by converting the length, width, and height of the warehouse to meters. Once this is done, the volume in SI units is simply the product of the three dimensions.

SOLUTION

1. Convert the length of the warehouse to meters: $L = (20.0 \text{ yard})\left(\frac{3 \text{ ft}}{1 \text{ yard}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 18.3 \text{ m}$
2. Convert the width to meters: $W = (10.0 \text{ yard})\left(\frac{3 \text{ ft}}{1 \text{ yard}}\right)\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 9.14 \text{ m}$
3. Convert the height to meters: $H = (15.0 \text{ ft})\left(\frac{1 \text{ m}}{3.281 \text{ ft}}\right) = 4.57 \text{ m}$
4. Calculate the volume of the warehouse: $V = L \times W \times H = (18.3 \text{ m})(9.14 \text{ m})(4.57 \text{ m}) = 764 \text{ m}^3$

INSIGHT

We would say, then, that the warehouse has a volume of 764 cubic meters—the same as 764 cubical boxes that are 1 m on a side.

PRACTICE PROBLEM

What is the volume of the warehouse if its length is one-hundredth of a mile, and the other dimensions are unchanged? [**Answer:** $V = 672 \text{ m}^3$]

Some related homework problems: Problem 20, Problem 21

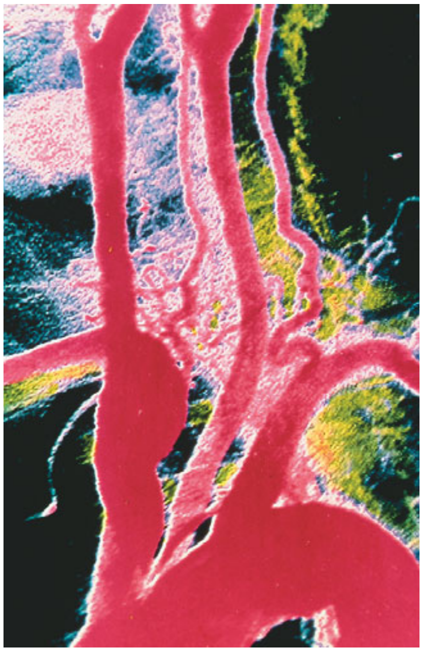
Finally, the same procedure can be applied to conversions involving any number of units. For instance, if you walk at 3.00 mi/h, how fast is that in m/s? In this case we need the following additional conversion factors:

$$1 \text{ mi} = 5280 \text{ ft} \qquad 1 \text{ h} = 3600 \text{ s}$$

With these factors at hand, we carry out the conversion as follows:

$$(3.00 \cancel{\text{mi}}/\cancel{\text{h}})\left(\frac{5280 \cancel{\text{ft}}}{1 \cancel{\text{mi}}}\right)\left(\frac{1 \text{ m}}{3.281 \cancel{\text{ft}}}\right)\left(\frac{1 \cancel{\text{h}}}{3600 \text{ s}}\right) = 1.34 \text{ m/s}$$

Note that in each conversion factor the numerator is equal to the denominator. In addition, each conversion factor is written in such a way that the unwanted units cancel, leaving just meters per second in our final result.



▲ Major blood vessels branch from the aorta (bottom), the artery that receives blood directly from the heart.

ACTIVE EXAMPLE 1–1 FIND THE SPEED OF BLOOD

Blood in the human aorta can attain speeds of 35.0 cm/s. How fast is this in **(a)** ft/s and **(b)** mi/h?

SOLUTION

(Test your understanding by performing the calculations indicated in each step.)

Part (a)

1. Convert centimeters to meters and then to feet: 1.15 ft/s

CONTINUED ON NEXT PAGE

CONTINUED FROM PREVIOUS PAGE

Part (b)

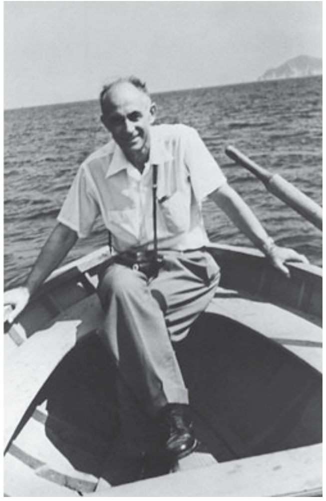
2. First, convert centimeters to miles:	2.17×10^{-4} mi/s
3. Next, convert seconds to hours:	0.783 mi/h

INSIGHT

Of course, the conversions in part (b) can be carried out in a single calculation if desired.

YOUR TURN

Find the speed of blood in units of km/h. *(Answers to* **Your Turn** *problems are given in the back of the book.)*



▲ Enrico Fermi (1901–1954) was renowned for his ability to pose and solve interesting order-of-magnitude problems. A winner of the 1938 Nobel Prize in physics, Fermi would ask his classes to obtain order-of-magnitude estimates for questions such as "How many piano tuners are there in Chicago?" or "How much is a tire worn down during one revolution?" Estimation questions like these are known to physicists today as "Fermi Problems."

1–6 Order-of-Magnitude Calculations

An **order-of-magnitude** calculation is a rough "ballpark" estimate designed to be accurate to within a factor of about 10. One purpose of such a calculation is to give a quick idea of what order of magnitude should be expected from a complete, detailed calculation. If an order-of-magnitude calculation indicates that a distance should be on the order of 10^4 m, for example, and your calculator gives an answer on the order of 10^7 m, then there is an error somewhere that needs to be resolved.

For example, suppose you would like to estimate the speed of a cliff diver on entering the water. First, the cliff may be 20 or 30 feet high; thus in SI units we would say that the order of magnitude of the cliff's height is 10 m—certainly not 1 m or 10^2 m. Next, the diver hits the water something like a second later—certainly not 0.1 s later nor 10 s later. Thus, a reasonable order-of-magnitude estimate of the diver's speed is $10 \text{ m}/1 \text{ s} = 10 \text{ m/s}$, or roughly 20 mi/h. If you do a detailed calculation and your answer is on the order of 10^4 m/s, you probably entered one of your numbers incorrectly.

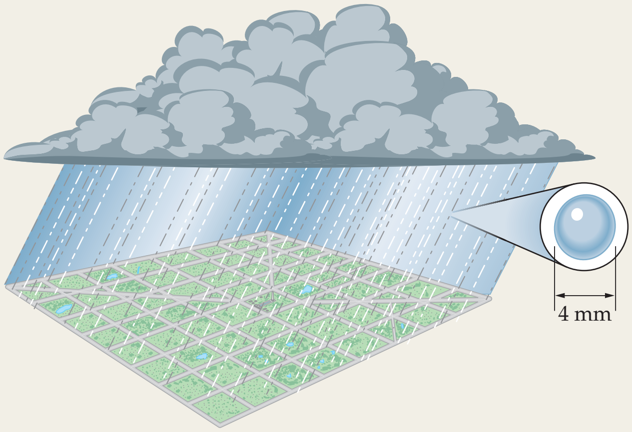
Another reason for doing an order-of-magnitude calculation is to get a feeling for what size numbers we are talking about in situations where a precise count is not possible. This is illustrated in the following Example.

EXAMPLE 1–3 ESTIMATION: HOW MANY RAINDROPS IN A STORM

A thunderstorm drops half an inch ($\sim$0.01 m) of rain on Washington D.C., which covers an area of about 70 square miles ($\sim 10^8 \text{ m}^2$). Estimate the number of raindrops that fell during the storm.

PICTURE THE PROBLEM

Our sketch shows an area $A = 10^8 \text{ m}^2$ covered to a depth $d = 0.01$ m by rainwater from the storm. Each drop of rain is approximated by a small sphere with a diameter of 4 mm.



STRATEGY

To find the number of raindrops, we first calculate the volume of water required to cover 10^8 m^2 to a depth of 0.01 m. Next, we calculate the volume of an individual drop of rain, recalling that the volume of a sphere of radius r is $4\pi r^3/3$. We estimate the diameter of a raindrop to be about 4 mm. Finally, dividing the volume of a drop into the volume of water that fell during the storm gives the number of drops.

SOLUTION

1. Calculate the order of magnitude of the volume of water, V_{water}, that fell during the storm:

$$V_{water} = Ad = (10^8\ m^2)(0.01\ m) \approx 10^6\ m^3$$

2. Calculate the order of magnitude of the volume of a drop of rain, V_{drop}. Note that if the diameter of a drop is 4 mm, its radius is $r = 2\ mm = 0.002\ m$:

$$V_{drop} = \frac{4}{3}\pi r^3 \approx \frac{4}{3}\pi(0.002\ m)^3 \approx 10^{-8}\ m^3$$

3. Divide V_{drop} into V_{water} to find the order of magnitude of the number of drops that fell during the storm:

$$\textit{number of raindrops} \approx \frac{V_{water}}{V_{drop}} \approx \frac{10^6\ m^3}{10^{-8}\ m^3} = 10^{14}$$

INSIGHT

Thus the number of raindrops in this one small storm is roughly 100,000 times greater than the current population of the Earth.

PRACTICE PROBLEM

If a storm pelts Washington D.C. with 10^{15} raindrops, how many inches of rain fall on the city? [**Answer:** About 5 inches]

Some related homework problems: Problem 36, Problem 38

Appendix B provides a number of interesting "typical values" for length, mass, speed, acceleration, and many other quantities. You may find these to be of use in making your own order-of-magnitude estimates.

1–7 Scalars and Vectors

Physical quantities are sometimes defined solely in terms of a number and the corresponding unit, like the volume of a room or the temperature of the air it contains. Other quantities require both a numerical value *and* a direction. For example, suppose a car is traveling at a rate of 25 m/s in a direction that is due north. Both pieces of information—the rate of travel (25 m/s) and the direction (north)—are required to fully specify the motion of the car. The rate of travel is given the name **speed;** the rate of travel combined with the direction is referred to as the **velocity.**

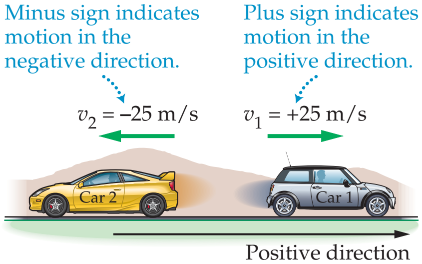
In general, quantities that are specified by a numerical value only are referred to as **scalars;** quantities that require both a numerical value and a direction are called **vectors:**

- A scalar is a numerical value, expressed in terms of appropriate units. An example would be the temperature of a room or the speed of a car.
- A vector is a mathematical quantity with both a numerical value and a direction. An example would be the velocity of a car.

All the physical quantities discussed in this text are either vectors or scalars. The properties of numbers (scalars) are well known, but the properties of vectors are sometimes less well known—though no less important. For this reason, you will find that Chapter 3 is devoted entirely to a discussion of vectors in two and three dimensions and, more specifically, to how they are used in physics.

The rather straightforward special case of vectors in one dimension is discussed in Chapter 2. There, we see that the direction of a velocity vector, for example, can only be to the left or to the right, up or down, and so on. That is, only two choices are available for the direction of a vector in one dimension. This is illustrated in **Figure 1–1**, where we see two cars, each traveling with a speed of 25 m/s. We also see that the cars are traveling in opposite directions, with car 1 moving to the right and car 2 moving to the left. We indicate the direction of travel with a plus sign for motion to the right, and a negative sign for motion to the left. Thus, the velocity of car 1 is written $v_1 = +25\ m/s$, and the velocity of car 2 is $v_2 = -25\ m/s$. The speed of each car is the absolute value, or **magnitude,** of the velocity; that is, speed $= |v_1| = |v_2| = 25\ m/s$.

Whenever we deal with one-dimensional vectors, we shall indicate their direction with the appropriate sign. Many examples are found in Chapter 2 and, again, in later chapters where the simplicity of one dimension can again be applied.



▲ **FIGURE 1–1 Velocity vectors in one dimension**

The two cars shown in this figure have equal speeds of 25 m/s, but are traveling in opposite directions. To indicate the direction of travel, we first choose a positive direction (to the right in this case), and then give appropriate signs to the velocity of each car. For example, car 1 moves to the right, and hence its velocity is positive, $v_1 = +25\ m/s$; the velocity of car 2 is negative, $v_2 = -25\ m/s$, because it moves to the left.

1–8 Problem Solving in Physics

Physics is a lot like swimming—you have to learn by doing. You could read a book on swimming and memorize every word in it, but when you jump into a pool the first time you are going to have problems. Similarly, you could read this book carefully, memorizing every formula in it, but when you finish, you still haven't learned physics. To learn physics, you have to go beyond passive reading; you have to interact with physics and experience it by doing problems.

In this section we present a general overview of problem solving in physics. The suggestions given below, which apply to problems in all areas of physics, should help to develop a systematic approach.

We should emphasize at the outset that there is no recipe for solving problems in physics—it is a creative activity. In fact, the opportunity to be creative is one of the attractions of physics. The following suggestions, then, are not intended as a rigid set of steps that must be followed like the steps in a computer program. Rather, they provide a general guideline that experienced problem solvers find to be effective.

- **Read the problem carefully** Before you can solve a problem, you need to know exactly what information it gives and what it asks you to determine. Some information is given explicitly, as when a problem states that a person has a mass of 70 kg. Other information is implicit; for example, saying that a ball is dropped from rest means that its initial speed is zero. Clearly, a *careful* reading is the essential first step in problem solving.
- **Sketch the system** This may seem like a step you can skip—but don't. A sketch helps you to acquire a physical feeling for the system. It also provides an opportunity to label those quantities that are known and those that are to be determined. All Examples in this text begin with a sketch of the system, accompanied by a brief description in a section labeled "Picture the Problem."
- **Visualize the physical process** Try to visualize what is happening in the system as if you were watching it in a movie. Your sketch should help. This step ties in closely with the next step.
- **Strategize** This may be the most difficult, but at the same time the most creative, part of the problem-solving process. From your sketch and visualization, try to identify the physical processes at work in the system. Ask yourself what concepts or principles are involved in this situation. Then, develop a strategy—a game plan—for solving the problem. All Examples in this book have a "Strategy" spelled out before the solution begins.
- **Identify appropriate equations** Once a strategy has been developed, find the specific equations that are needed to carry it out.
- **Solve the equations** Use basic algebra to solve the equations identified in the previous step. Work with symbols such as x or y for the most part, substituting numerical values near the end of the calculations. Working with symbols will make it easier to go back over a problem to locate and identify mistakes, if there are any, and to explore limits and special cases.
- **Check your answer** Once you have an answer, check to see if it makes sense: (i) Does it have the correct dimensions? (ii) Is the numerical value reasonable?
- **Explore limits/special cases** Getting the correct answer is nice, but it's not all there is to physics. You can learn a great deal about physics and about the connection between physics and mathematics by checking various limits of your answer. For example, if you have two masses in your system, m_1 and m_2, what happens in the special case that $m_1 = 0$ or $m_1 = m_2$? Check to see whether your answer and your physical intuition agree.

The **Examples** in this text are designed to deepen your understanding of physics and at the same time develop your problem-solving skills. They all have the same basic structure: Problem Statement; Picture the Problem; Strategy; Solution, presenting the flow of ideas and the mathematics side-by-side in a two-column format; Insight; and a Practice Problem related to the one just solved. As you work through the Examples in the chapters to come, notice how the basic problem-solving guidelines outlined above are implemented in a consistent way.

In addition to the Examples, this text contains a new and innovative type of worked-out problem called the **Active Example,** the first one of which appears on page 9. The purpose of Active Examples is to encourage active participation in the solution of a problem and, in so doing, to act as a "bridge" between Examples—where each and every detail is worked out—and homework problems—where you are completely on your own. An analogy would be to think of Examples as like a tricycle, with no balancing required; homework problems as like a bicycle, where balancing is initially difficult to master; and Active Examples as like a bicycle with training wheels that give just enough help to prevent a fall. When you work through an Active Example, keep in mind that the work you are doing as you progress step-by-step through the problem is just the kind of work you'll be doing later in your homework assignments.

Finally, it is tempting to look for shortcuts when doing a problem—to look for a formula that seems to fit and some numbers to plug into it. It may seem harder to think ahead, to be systematic as you solve the problem, and then to think back over what you have done at the end of the problem. The extra effort is worth it, however, because by doing these things you will develop powerful problem-solving skills that can be applied to unexpected problems you may encounter on exams—and in life in general.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The three physical dimensions introduced in this chapter—mass, length, time—are the only ones we'll use until Chapter 19, when we introduce electric charge. Other quantities found in the next several chapters, like force, momentum, and energy, are combinations of these three basic dimensions.

In this chapter we discussed the idea of a vector in one spatial dimension and showed how the direction of the vector can be indicated by its sign. These concepts are developed in more detail in Chapter 2.

LOOKING AHEAD

Dimensional analysis is used frequently in the coming chapters to verify that each term in an equation has the correct dimensions. See, for example, the discussion following Equation 2–7, where we show that each term has the dimensions of velocity. We also use dimensional analysis to help derive some results, such as the speed of waves on a string in Section 14–2.

Vectors are extended to two and three spatial dimensions in Chapter 3. After that, they are a standard tool throughout mechanics, and they appear again in electricity and magnetism.

CHAPTER SUMMARY

1–1 PHYSICS AND THE LAWS OF NATURE

Physics is based on a small number of fundamental laws and principles.

1–2 UNITS OF LENGTH, MASS, AND TIME

Length
One meter is defined as the distance traveled by light in a vacuum in 1/299,792,458 second.

Mass
One kilogram is the mass of a metal cylinder kept at the International Bureau of Weights and Standards.

Time
One second is the time required for a particular type of radiation from cesium-133 to undergo 9,192,631,770 oscillations.

1–3 DIMENSIONAL ANALYSIS

Dimension
The dimension of a quantity is the type of quantity it is, for example, length [L], mass [M], or time [T].

Dimensional Consistency
An equation is dimensionally consistent if each term in it has the same dimensions. All valid physical equations are dimensionally consistent.

Dimensional Analysis
A calculation based on the dimensional consistency of an equation.

1–4 SIGNIFICANT FIGURES

Significant Figures
The number of digits reliably known, excluding digits that simply indicate the decimal place. For example, 3.45 and 0.0000345 both have three significant figures.

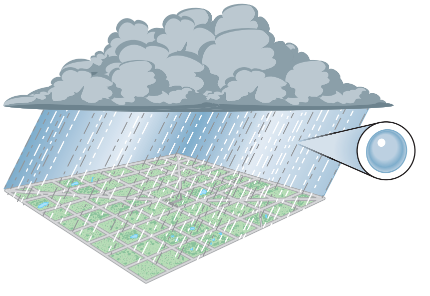
Round-off Error
Discrepancies caused by rounding off numbers in intermediate results.

1–5 CONVERTING UNITS

Multiply by the ratio of two units to convert from one to another. As an example, to convert 3.5 m to feet, you multiply by the factor (1 ft/0.3048 m).

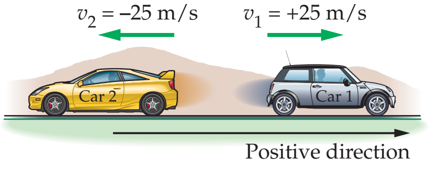
1–6 ORDER-OF-MAGNITUDE CALCULATIONS

A ballpark estimate designed to be accurate to within the nearest power of ten.



1–7 SCALARS AND VECTORS

A physical quantity that can be represented by a numerical value only is called a scalar. Quantities that require a direction in addition to the numerical value are called vectors.



1–8 PROBLEM SOLVING IN PHYSICS

A good general approach to problem solving is as follows: read; sketch; visualize; strategize; identify equations; solve; check; explore limits.

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Can dimensional analysis determine whether the area of a circle is πr^2 or $2\pi r^2$? Explain.

2. If a distance d has units of meters, and a time T has units of seconds, does the quantity $T + d$ make sense physically? What about the quantity d/T? Explain in both cases.

3. Is it possible for two quantities to **(a)** have the same units but different dimensions or **(b)** have the same dimensions but different units? Explain.
4. Give an order-of-magnitude estimate for the time in seconds of the following: **(a)** a year; **(b)** a baseball game; **(c)** a heartbeat; **(d)** the age of the Earth; **(e)** the age of a person.
5. Give an order-of-magnitude estimate for the length in meters of the following: **(a)** a person; **(b)** a fly; **(c)** a car; **(d)** a 747 airplane; **(e)** an interstate freeway stretching coast-to-coast.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 1–2 UNITS OF LENGTH, MASS, AND TIME

1. • **Spiderman** The movie *Spiderman* brought in \$114,000,000 in its opening weekend. Express this amount in **(a)** gigadollars and **(b)** teradollars.

2. • **BIO The Thickness of Hair** A human hair has a thickness of about 70 μm. What is this in **(a)** meters and **(b)** kilometers?

3. • The speed of light in a vacuum is approximately 0.3 Gm/s. Express the speed of light in meters per second.

4. • **A Fast Computer** IBM has a computer it calls the Blue Gene/L that can do 136.8 teracalculations per second. How many calculations can it do in a microsecond?

SECTION 1–3 DIMENSIONAL ANALYSIS

5. • **CE** Which of the following equations are dimensionally consistent? **(a)** $x = vt$, **(b)** $x = \frac{1}{2}at^2$, **(c)** $t = (2x/a)^{1/2}$.

6. • **CE** Which of the following quantities have the dimensions of a distance? **(a)** vt, **(b)** $\frac{1}{2}at^2$, **(c)** $2at$, **(d)** v^2/a.

7. • **CE** Which of the following quantities have the dimensions of a speed? **(a)** $\frac{1}{2}at^2$, **(b)** at, **(c)** $(2x/a)^{1/2}$, **(d)** $(2ax)^{1/2}$.

8. • Velocity is related to acceleration and distance by the following expression: $v^2 = 2ax^p$. Find the power p that makes this equation dimensionally consistent.

9. • Acceleration is related to distance and time by the following expression: $a = 2xt^p$. Find the power p that makes this equation dimensionally consistent.

10. • Show that the equation $v = v_0 + at$ is dimensionally consistent. Note that v and v_0 are velocities and that a is an acceleration.

11. •• Newton's second law (to be discussed in Chapter 5) states that acceleration is proportional to the force acting on an object and is inversely proportional to the object's mass. What are the dimensions of force?

12. •• The time T required for one complete oscillation of a mass m on a spring of force constant k is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Find the dimensions k must have for this equation to be dimensionally correct.

SECTION 1–4 SIGNIFICANT FIGURES

13. • The first several digits of π are known to be $\pi = 3.14159265358979\ldots$. What is π to **(a)** three significant figures, **(b)** five significant figures, and **(c)** seven significant figures?

14. • The speed of light to five significant figures is 2.9979×10^8 m/s. What is the speed of light to three significant figures?

15. • A parking lot is 144.3 m long and 47.66 m wide. What is the perimeter of the lot?

16. • On a fishing trip you catch a 2.35-lb bass, a 12.1-lb rock cod, and a 12.13-lb salmon. What is the total weight of your catch?

17. •• How many significant figures are there in **(a)** 0.000054 and **(b)** 3.001×10^5?

18. •• What is the area of a circle of radius **(a)** 14.37 m and **(b)** 3.8 m?

SECTION 1–5 CONVERTING UNITS

19. • **BIO Mantis Shrimp** Peacock mantis shrimps (*Odontodactylus scyllarus*) feed largely on snails. They shatter the shells of their prey by delivering a sharp blow with their front legs, which have been observed to reach peak speeds of 23 m/s. What is this speed in **(a)** feet per second and **(b)** miles per hour?

20. • **(a)** The largest building in the world by volume is the Boeing 747 plant in Everett, Washington. It measures approximately 631 m long, 707 yards wide, and 110 ft high. What is its volume in cubic feet? **(b)** Convert your result from part (a) to cubic meters.

21. • The Ark of the Covenant is described as a chest of acacia wood 2.5 cubits in length and 1.5 cubits in width and height. Given that a cubit is equivalent to 17.7 in., find the volume of the ark in cubic feet.

22. • How long does it take for radiation from a cesium-133 atom to complete 1.5 million cycles?

23. • **Angel Falls** Water going over Angel Falls, in Venezuela, the world's highest waterfall, drops through a distance of 3212 ft. What is this distance in km?

24. • An electronic advertising sign repeats a message every 7 seconds, day and night, for a week. How many times did the message appear on the sign?

25. • **BIO Blue Whales** The blue whale (*Balaenoptera musculus*) is thought to be the largest animal ever to inhabit the Earth. The longest recorded blue whale had a length of 108 ft. What is this length in meters?

26. • **The Star of Africa** The Star of Africa, a diamond in the royal scepter of the British crown jewels, has a mass of 530.2 carats, where 1 carat = 0.20 g. Given that 1 kg has an approximate weight of 2.21 lb, what is the weight of this diamond in pounds?

27. • **IP** Many highways have a speed limit of 55 mi/h. **(a)** Is this speed greater than, less than, or equal to 55 km/h? Explain. **(b)** Find the speed limit in km/h that corresponds to 55 mi/h.

28. • What is the speed in miles per hour of a beam of light traveling at 3.00×10^8 m/s?

29. • **BIO** **Woodpecker Impact** When red-headed woodpeckers (*Melanerpes erythrocephalus*) strike the trunk of a tree, they can experience an acceleration ten times greater than the acceleration of gravity, or about 98.1 m/s^2. What is this acceleration in ft/s^2?

30. •• **A Jiffy** The American physical chemist Gilbert Newton Lewis (1875–1946) proposed a unit of time called the "jiffy." According to Lewis, 1 jiffy = the time it takes light to travel one centimeter. **(a)** If you perform a task in a jiffy, how long has it taken in seconds? **(b)** How many jiffys are in one minute? (Use the fact that the speed of light is approximately 2.9979×10^8 m/s.)

31. •• **The Mutchkin and the Noggin** **(a)** A mutchkin is a Scottish unit of liquid measure equal to 0.42 L. How many mutchkins are required to fill a container that measures one foot on a side? **(b)** A noggin is a volume equal to 0.28 mutchkin. What is the conversion factor between noggins and gallons?

32. •• Suppose 1.0 cubic meter of oil is spilled into the ocean. Find the area of the resulting slick, assuming that it is one molecule thick, and that each molecule occupies a cube 0.50 μm on a side.

33. •• **IP** **(a)** A standard sheet of paper measures 8 1/2 by 11 inches. Find the area of one such sheet of paper in m^2. **(b)** A second sheet of paper is half as long and half as wide as the one described in part (a). By what factor is its area less than the area found in part (a)?

34. •• **BIO** **Squid Nerve Impulses** Nerve impulses in giant axons of the squid can travel with a speed of 20.0 m/s. How fast is this in **(a)** ft/s and **(b)** mi/h?

35. •• The acceleration of gravity is approximately 9.81 m/s^2 (depending on your location). What is the acceleration of gravity in feet per second squared?

SECTION 1–6 ORDER-OF-MAGNITUDE CALCULATIONS

36. • Give a ballpark estimate of the number of seats in a typical major league ballpark.

Shea Stadium, in New York. How many fans can it hold? (Problem 36)

37. • Milk is often sold by the gallon in plastic containers. **(a)** Estimate the number of gallons of milk that are purchased in the United States each year. **(b)** What approximate weight of plastic does this represent?

38. •• New York is roughly 3000 miles from Seattle. When it is 10:00 A.M. in Seattle, it is 1:00 P.M. in New York. Using this information, estimate **(a)** the rotational speed of the surface of Earth, **(b)** the circumference of Earth, and **(c)** the radius of Earth.

39. •• You've just won the $12 million cash lottery, and you go to pick up the prize. What is the approximate weight of the cash if you request payment in **(a)** quarters or **(b)** dollar bills?

GENERAL PROBLEMS

40. • **CE** Which of the following equations are dimensionally consistent? **(a)** $v = at$, **(b)** $v = \frac{1}{2}at^2$, **(c)** $t = a/v$, **(d)** $v^2 = 2ax$.

41. • **CE** Which of the following quantities have the dimensions of an acceleration? **(a)** xt^2, **(b)** v^2/x, **(c)** x/t^2, **(d)** v/t.

42. • **BIO** **Photosynthesis** The light that plants absorb to perform photosynthesis has a wavelength that peaks near 675 nm. Express this distance in **(a)** millimeters and **(b)** inches.

43. • **Glacial Speed** On June 9, 1983, the lower part of the Variegated Glacier in Alaska was observed to be moving at a rate of 210 feet per day. What is this speed in meters per second?

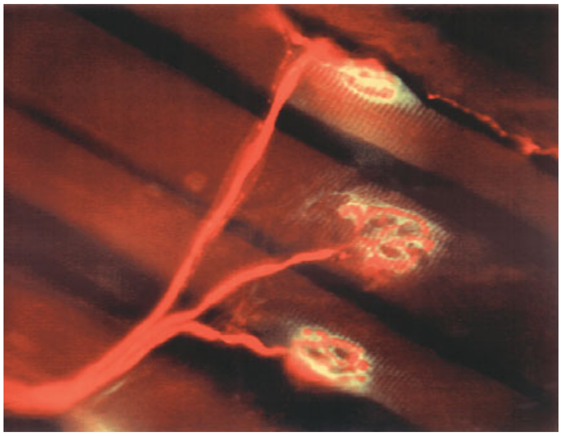
Alaska's Variegated Glacier (Problem 43)

44. •• **BIO** **Mosquito Courtship** Male mosquitoes in the mood for mating find female mosquitoes of their own species by listening for the characteristic "buzzing" frequency of the female's wing beats. This frequency is about 605 wing beats per second. **(a)** How many wing beats occur in one minute? **(b)** How many cycles of oscillation does the radiation from a cesium-133 atom complete during one mosquito wing beat?

45. •• **Ten and Ten** When Coast Guard pararescue jumpers leap from a helicopter to save a person in the water, they like to jump when the helicopter is flying "ten and ten," which means it is 10 feet above the water and moving forward with a speed of 10 knots. What is "ten and ten" in SI units? (A knot is one nautical mile per hour, where a nautical mile is 1.852 km.)

46. •• **IP** A Porsche sports car can accelerate at 14 m/s^2. **(a)** Is this acceleration greater than, less than, or equal to 14 ft/s^2? Explain. **(b)** Determine the acceleration of a Porsche in ft/s^2. **(c)** Determine its acceleration in km/h^2.

47. •• **BIO Human Nerve Fibers** Type A nerve fibers in humans can conduct nerve impulses at speeds up to 140 m/s. **(a)** How fast are the nerve impulses in miles per hour? **(b)** How far (in meters) can the impulses travel in 5.0 ms?



The impulses in these nerve axons, which carry commands to the skeletal muscle fibers in the background, travel at speeds of up to 140 m/s. (Problem 47)

48. •• **BIO Brain Growth** The mass of a newborn baby's brain has been found to increase by about 1.6 mg per minute. **(a)** How much does the brain's mass increase in one day? **(b)** How long does it take for the brain's mass to increase by 0.0075 kg?

49. •• **The Huygens Probe** NASA's Cassini mission to Saturn released a probe on December 25, 2004, that landed on the Saturnian moon Titan on January 14, 2005. The probe, which was named Huygens, was released with a gentle relative speed of 31 cm/s. As Huygens moved away from the main spacecraft, it rotated at a rate of seven revolutions per minute. **(a)** How many revolutions had Huygens completed when it was 150 yards from the mother ship? **(b)** How far did Huygens move away from the mother ship during each revolution? Give your answer in feet.

50. ••• Acceleration is related to velocity and time by the following expression: $a = v^p t^q$. Find the powers p and q that make this equation dimensionally consistent.

51. ••• The period T of a simple pendulum is the amount of time required for it to undergo one complete oscillation. If the length of the pendulum is L and the acceleration of gravity is g, then T is given by

$$T = 2\pi L^p g^q$$

Find the powers p and q required for dimensional consistency.

52. ••• Driving along a crowded freeway, you notice that it takes a time t to go from one mile marker to the next. When you increase your speed by 7.9 mi/h, the time to go one mile decreases by 13 s. What was your original speed?

PASSAGE PROBLEMS

BIO Using a Cricket as a Thermometer

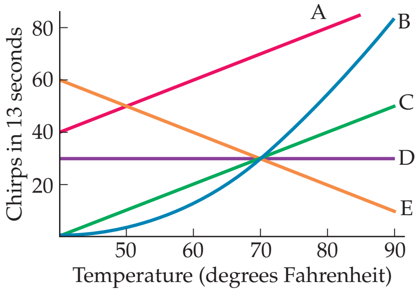
All chemical reactions, whether organic or inorganic, proceed at a rate that depends on temperature—the higher the temperature, the higher the rate of reaction. This can be understood in terms of molecules moving with increased energy as the temperature is increased, and colliding with other molecules more frequently. In the case of organic reactions, the result is that metabolic processes speed up with increasing temperature.

An increased or decreased metabolic rate can manifest itself in a number of ways. For example, a cricket trying to attract a mate chirps at a rate that depends on its overall rate of metabolism. As a result, the chirping rate of crickets depends directly on temperature. In fact, some people even use a pet cricket as a thermometer.

The cricket that is most accurate as a thermometer is the snowy tree cricket (*Oecanthus fultoni* Walker). Its rate of chirping is described by the following formula:

$$\begin{aligned} N &= \text{number of chirps per 13.0 seconds} \\ &= T - 40.0 \end{aligned}$$

In this expression, T is the temperature in degrees Fahrenheit.



▲ **FIGURE 1–2** Problem 53

53. • Which plot in **Figure 1–2** represents the chirping rate of the snowy tree cricket?

A B C D E

54. • If the temperature is 43 degrees Fahrenheit, how long does it take for the cricket to chirp 12 times?

A. 12 s **B.** 24 s **C.** 43 s **D.** 52 s

55. •• Your pet cricket chirps 112 times in one minute (60.0 s). What is the temperature in degrees Fahrenheit?

A. 41.9 **B.** 47.0 **C.** 64.3 **D.** 74.7

56. •• Suppose a snowy cricket is chirping when the temperature is 65.0 degrees Fahrenheit. How many oscillations does the radiation from a cesium-133 atom complete between successive chirps?

A. 7.98×10^7 **B.** 3.68×10^8

C. 4.78×10^9 **D.** 9.58×10^9

2 One-Dimensional Kinematics

These sprinters, crossing the finish line of the 100-m dash, illustrate one-dimensional motion. At the moment shown in the photograph the runners are moving with constant velocity at a speed of approximately 10 m/s.

We begin our study of physics with **mechanics,** the area of physics perhaps most apparent to us in our everyday lives. Every time you raise an arm, stand up or sit down, throw a ball, or open a door, your actions are governed by the laws of mechanics. Basically, mechanics is the study of how objects move, how they respond to external forces, and how other factors, such as size, mass, and mass distribution, affect their motion. This is a lot to cover, and we certainly won't try to tackle it all in one chapter.

Furthermore, in this chapter we treat all physical objects as *point particles;* that is, we consider all the mass of the object to be concentrated at a single point. This is a common practice in physics. For example, if you are interested in calculating the time it takes the Earth to complete a revolution about the Sun, it is reasonable to consider the Earth and the Sun as simple particles. In later chapters, we extend our studies to increasingly realistic situations, involving motion in more than one dimension and physical objects with shape and size.

2–1 Position, Distance, and Displacement

The first step in describing the motion of a particle is to set up a **coordinate system** that defines its position. An example of a coordinate system in one dimension is shown in **Figure 2–1**. This is simply an x axis, with an origin (where $x = 0$) and an arrow indicating the positive direction—the direction in which x increases. In setting up a coordinate system, we are free to choose the origin and the positive direction as we like, but once we make a choice we must be consistent with it throughout any calculations that follow.

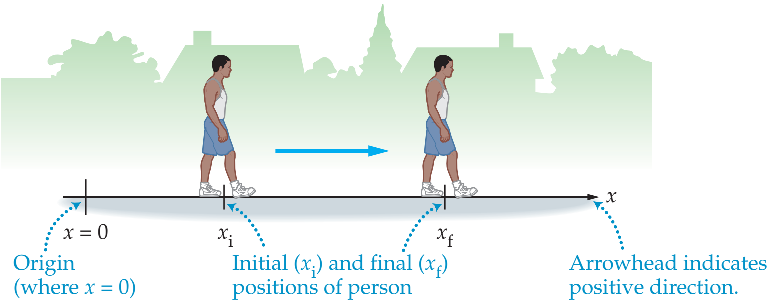


FIGURE 2–1 A one-dimensional coordinate system
You are free to choose the origin and positive direction as you like, but once your choice is made, stick with it.

The particle in Figure 2–1 is a person who has moved to the right from an initial position, x_i, to a final position, x_f. Because the positive direction is to the right, it follows that x_f is greater than x_i; that is, $x_f > x_i$.

Now that we've seen how to set up a coordinate system, let's use one to investigate the situation sketched in **Figure 2–2**. Suppose that you leave your house, drive to the grocery store, and then return home. The **distance** you've covered in your trip is 8.6 mi. In general, distance is defined as follows:

Definition: Distance

distance = total length of travel

SI unit: meter, m

Using SI units, the distance in this case is

$$8.6\text{ mi} = (8.6\text{ mi})\left(\frac{1609\text{ m}}{1\text{ mi}}\right) = 1.4 \times 10^4\text{ m}$$

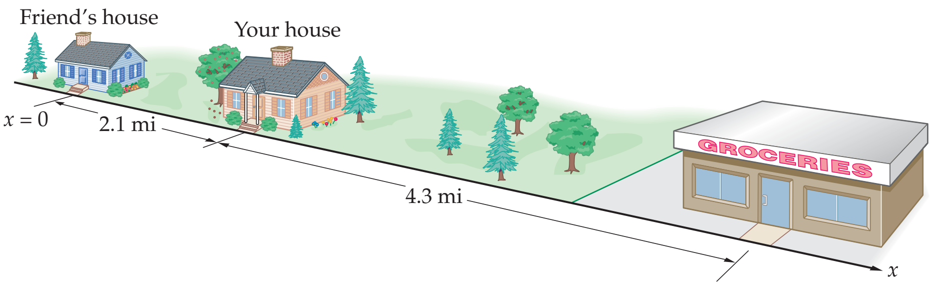


FIGURE 2–2 One-dimensional coordinates
The locations of your house, your friend's house, and the grocery store in terms of a one-dimensional coordinate system.

In a car, the distance traveled is indicated by the odometer. Note that distance is always positive and, because it has no direction associated with it, it is a scalar, as discussed in Chapter 1.

Another useful way to characterize a particle's motion is in terms of the **displacement,** Δx, which is simply the change in position.

Definition: Displacement, Δx

displacement = change in position = final position − initial position

$$\text{displacement} = \Delta x = x_f - x_i \qquad 2\text{–}1$$

SI unit: meter, m

Notice that we use the delta notation, Δx, as a convenient shorthand for the quantity $x_f - x_i$. (See Appendix A for a complete discussion of delta notation.) Also, note that Δx can be positive (if the final position is to the right of the initial position, $x_f > x_i$), negative (if the final position is to the left of the initial position, $x_f < x_i$), or zero (if the final and initial positions are the same, $x_f = x_i$). In fact, the displacement is a one-dimensional vector, as defined in Chapter 1, and its direction (right or left) is given by its sign (positive or negative, respectively).

The SI units of displacement are meters—the same as for distance—but displacement and distance are really quite different. For example, in the round trip from your house to the grocery store and back the distance traveled is 8.6 mi, whereas the displacement is zero because $x_f = 2.1\ \text{mi} = x_i$. Suppose, instead, that you go from your house to the grocery store and then to your friend's house. On this trip the distance is 10.7 mi, but the displacement is

$$\Delta x = x_f - x_i = (0) - (2.1\ \text{mi}) = -2.1\ \text{mi}$$

As mentioned in the previous paragraph, the minus sign means your displacement is in the negative direction, that is, to the left.

ACTIVE EXAMPLE 2–1 FIND THE DISTANCE AND DISPLACEMENT

Calculate **(a)** the distance and **(b)** the displacement for a trip from your friend's house to the grocery store and then to your house.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Add the distances for the various parts of the total trip: $2.1\ \text{mi} + 4.3\ \text{mi} + 4.3\ \text{mi} = 10.7\ \text{mi}$

Part (b)

2. Determine the initial position for the trip, using Figure 2–2: $x_i = 0$
3. Determine the final position for the trip, using Figure 2–2: $x_f = 2.1\ \text{mi}$
4. Subtract x_i from x_f to find the displacement: $\Delta x = 2.1\ \text{mi}$

YOUR TURN

Suppose we choose the origin in Figure 2–2 to be at your house, rather than at your friend's house. In this case, find **(a)** the distance and **(b)** the displacement for the trip from your friend's house to the grocery store and then to your house.
(Answers to **Your Turn** *problems are given in the back of the book.)*

2–2 Average Speed and Velocity

The next step in describing motion is to consider how rapidly an object moves. For example, how long does it take for a Randy Johnson fastball to reach home plate? How far does an orbiting space shuttle travel in one hour? How fast do your eyelids move when you blink? These are examples of some of the most basic questions regarding motion, and in this section we learn how to answer them.

The simplest way to characterize the rate of motion is with the **average speed:**

$$\text{average speed} = \frac{\text{distance}}{\text{elapsed time}} \qquad 2\text{–}2$$

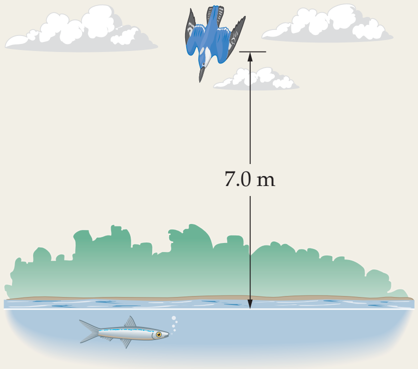
The dimensions of average speed are distance per time or, in SI units, meters per second, m/s. Both distance and elapsed time are positive; thus average speed is always positive.

EXAMPLE 2–1 THE KINGFISHER TAKES A PLUNGE

A kingfisher is a bird that catches fish by plunging into water from a height of several meters. If a kingfisher dives from a height of 7.0 m with an average speed of 4.00 m/s, how long does it take for it to reach the water?

PICTURE THE PROBLEM

As shown in the sketch, the kingfisher moves in a straight line through a vertical distance of 7.0 m. The average speed of the bird is 4.00 m/s.



STRATEGY

By rearranging Equation 2–2 we can solve for the elapsed time.

SOLUTION

1. Rearrange Equation 2–2 to solve for elapsed time: $$\text{elapsed time} = \frac{\text{distance}}{\text{average speed}}$$

2. Substitute numerical values to find the time: $$\text{elapsed time} = \frac{7.0\ \cancel{\text{m}}}{4.00\ \cancel{\text{m}}/\text{s}} = \frac{7.0}{4.00}\text{s} = 1.8\ \text{s}$$

INSIGHT

Note that Equation 2–2 is not just a formula for calculating the average speed. It relates speed, time, and distance. Given any two of these quantities, Equation 2–2 can be used to find the third.

PRACTICE PROBLEM

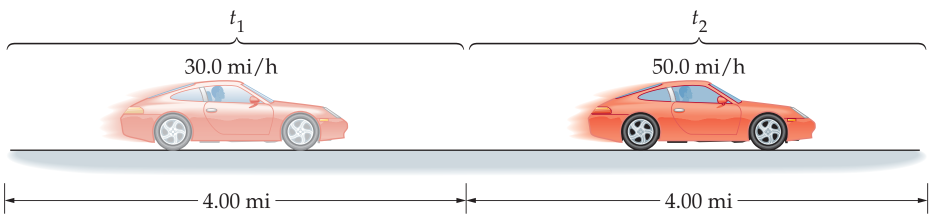
A kingfisher dives with an average speed of 4.6 m/s for 1.4 s. What was the height of the dive? [**Answer:** distance = (average speed) (elapsed time) = (4.6 m/s) (1.4 s) = 6.4 m]

Some related homework problems: Problem 13, Problem 15

Next, we calculate the average speed for a trip consisting of two parts of equal length, each traveled with a different speed.

CONCEPTUAL CHECKPOINT 2–1 AVERAGE SPEED

You drive 4.00 mi at 30.0 mi/h and then another 4.00 mi at 50.0 mi/h. Is your average speed for the 8.00-mi trip **(a)** greater than 40.0 mi/h, **(b)** equal to 40.0 mi/h, or **(c)** less than 40.0 mi/h?



REASONING AND DISCUSSION

At first glance it might seem that the average speed is definitely 40.0 mi/h. On further reflection, however, it is clear that it takes more time to travel 4.00 mi at 30.0 mi/h than it does to travel 4.00 mi at 50.0 mi/h. Therefore, you will be traveling at the lower speed for a greater period of time, and hence your average speed will be *less* than 40.0 mi/h—that is, closer to 30.0 mi/h than to 50.0 mi/h.

ANSWER

(c) The average speed is less than 40.0 mi/h.

To confirm the conclusion of the Conceptual Checkpoint, we simply apply the definition of average speed to find its value for this trip. We already know that the distance traveled is 8.00 mi; what we need now is the elapsed time. On the first 4.00 mi the time is

$$t_1 = \frac{4.00 \text{ mi}}{30.0 \text{ mi/h}} = (4.00/30.0) \text{ h}$$

The time required to cover the second 4.00 mi is

$$t_2 = \frac{4.00 \text{ mi}}{50.0 \text{ mi/h}} = (4.00/50.0) \text{ h}$$

Therefore, the elapsed time for the entire trip is

$$t_1 + t_2 = (4.00/30.0) \text{ h} + (4.00/50.0) \text{ h} = 0.213 \text{ h}$$

This gives the following average speed:

$$\text{average speed} = \frac{8.00 \text{ mi}}{0.213 \text{ h}} = 37.6 \text{ mi/h} < 40.0 \text{ mi/h}$$

Note that a "guess" will never give a detailed result like 37.6 mi/h; a systematic, step-by-step calculation is required.

In many situations, there is a quantity that is even more useful than the average speed. It is the **average velocity,** v_{av}, and it is defined as displacement per time:

Definition: Average velocity, v_{av}

$$\text{average velocity} = \frac{\text{displacement}}{\text{elapsed time}}$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad 2\text{–}3$$

SI unit: meter per second, m/s

Not only does the average velocity tell us, on average, how fast something is moving, it also tells us the *direction* the object is moving. For example, if an object moves in the positive direction, then $x_f > x_i$, and $v_{av} > 0$. On the other hand, if an object moves in the negative direction, it follows that $x_f < x_i$, and $v_{av} < 0$. As with displacement, the average velocity is a one-dimensional vector, and its direction is given by its sign. Average velocity gives more information than average speed; hence it is used more frequently in physics.

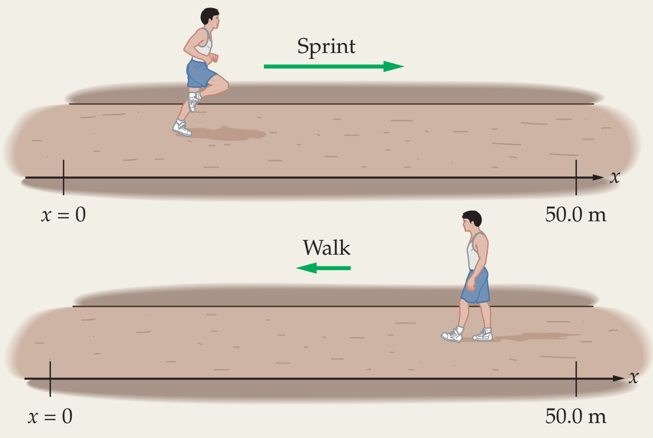
In the next Example, pay close attention to the positive and negative signs.

EXAMPLE 2–2 SPRINT TRAINING

An athlete sprints 50.0 m in 8.00 s, stops, and then walks slowly back to the starting line in 40.0 s. If the "sprint direction" is taken to be positive, what are **(a)** the average sprint velocity, **(b)** the average walking velocity, and **(c)** the average velocity for the complete round trip?

PICTURE THE PROBLEM

In our sketch we set up a coordinate system with the sprint going in the positive x direction, as described in the problem. For convenience, we choose the origin to be at the starting line. The finish line, then, is at $x = 50.0$ m.



STRATEGY

In each part of the problem we are asked for the average velocity and we are given information for times and distances. All that is needed, then, is to determine $\Delta x = x_f - x_i$ and $\Delta t = t_f - t_i$ in each case and apply Equation 2–3.

SOLUTION

Part (a)

1. Apply Equation 2–3 to the sprint, with $x_f = 50.0\text{ m}$, $x_i = 0$, $t_f = 8.00\text{ s}$, and $t_i = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{50.0\text{ m} - 0}{8.00\text{ s} - 0} = \frac{50.0}{8.00}\text{ m/s} = 6.25\text{ m/s}$$

Part (b)

2. Apply Equation 2–3 to the walk. In this case, $x_f = 0$, $x_i = 50.0\text{ m}$, $t_f = 48.0\text{ s}$, and $t_i = 8.00\text{ s}$:

$$v_{av} = \frac{x_f - x_i}{t_f - t_i} = \frac{0 - 50.0\text{ m}}{48.0\text{ s} - 8.00\text{ s}} = -\frac{50.0}{40.0}\text{ m/s} = -1.25\text{ m/s}$$

Part (c)

3. For the round trip, $x_f = x_i = 0$; thus $\Delta x = 0$:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{0}{48.0\text{ s}} = 0$$

INSIGHT

Note that the sign of the velocities in parts (a) and (b) indicates the direction of motion; positive for motion to the right, negative for motion to the left. Also, notice that the average *speed* for the entire 100.0-m trip $(100.0\text{ m}/48.0\text{ s} = 2.08\text{ m/s})$ is nonzero, even though the average velocity vanishes.

PRACTICE PROBLEM

If the average velocity during the walk is -1.50 m/s, how long does it take the athlete to walk back to the starting line? [**Answer:** $\Delta t = \Delta x/v_{av} = (-50.0\text{ m})/(-1.50\text{ m/s}) = 33.3\text{ s}$]

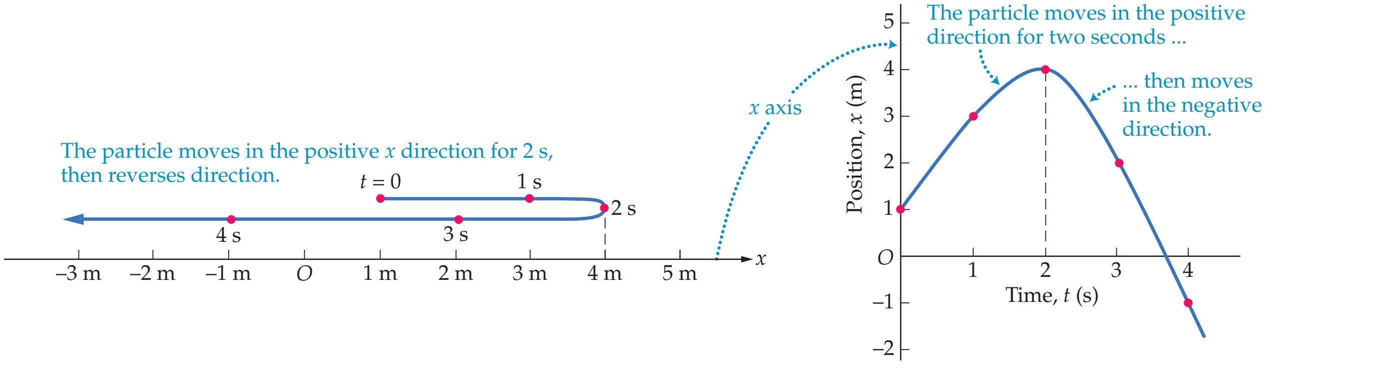
Some related homework problems: Problem 9, Problem 17, Problem 18

Graphical Interpretation of Average Velocity

It is often useful to "visualize" a particle's motion by sketching its position as a function of time. For example, consider a particle moving back and forth along the x axis, as shown in **Figure 2–3 (a)**. In this plot, we have indicated the position of a particle at a variety of times.

This way of keeping track of a particle's position and the corresponding time is a bit messy, though, so let's replot the same information with a different type of graph. In **Figure 2–3 (b)** we again plot the motion shown in Figure 2–3 (a), but this time with the vertical axis representing the position, x, and the horizontal axis representing time, t. An **x-versus-t graph** like this makes it considerably easier to visualize a particle's motion.

An x-versus-t plot also leads to a particularly useful interpretation of average velocity. To see how, suppose you would like to know the average velocity of the particle in Figures 2–3 (a) and 2–3 (b) from $t = 0$ to $t = 3\text{ s}$. From our definition of average velocity in Equation 2–3, we know that $v_{av} = \Delta x/\Delta t = (2\text{ m} - 1\text{ m})/(3\text{ s} - 0) = +0.3\text{ m/s}$. To relate this to the

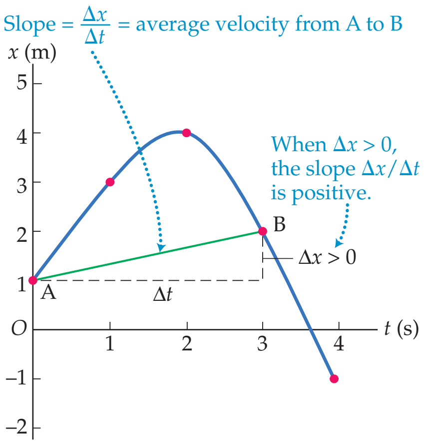


(a) The particle's path shown on a coordinate axis

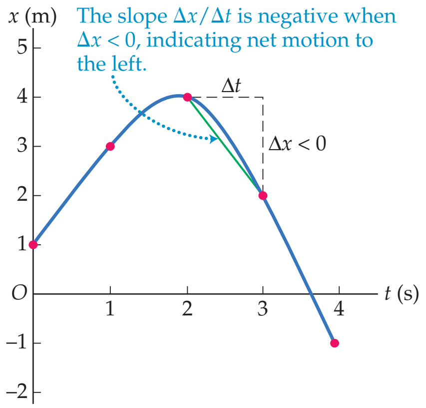
(b) The same path as a graph of position x versus time t

FIGURE 2–3 Two ways to visualize one-dimensional motion

Although the path in (a) is shown as a "U" for clarity, the particle actually moves straight back and forth along the x axis.



(a) Average velocity between $t = 0$ and $t = 3$ s



(b) Average velocity between $t = 2$ s and $t = 3$ s

▲ **FIGURE 2–4 Average velocity on an *x*-versus-*t* graph**

The slope of a straight line between any two points on an x-versus-t graph equals the average velocity between those points. Positive slopes indicate net motion to the right; negative slopes indicate net motion to the left.

▲ A speedometer indicates the instantaneous speed of a car. Note that the speedometer gives no information about the *direction* of motion. Thus, the speedometer is truly a "speed meter," not a velocity meter.

x-versus-t plot, draw a straight line connecting the position at $t = 0$ (call this point A) and the position at $t = 3$ s (point B). The result is shown in **Figure 2–4 (a)**.

The slope of the straight line from A to B is equal to the rise over the run, which in this case is $\Delta x/\Delta t$. But $\Delta x/\Delta t$ is the average velocity. Thus we see that:

- The slope of a line connecting two points on an x-versus-t plot is equal to the average velocity during that time interval.

As an additional example, let's calculate the average velocity between times $t = 2$ s and $t = 3$ s in Figure 2–3 (b). A line connecting the corresponding points is shown in **Figure 2–4 (b)**.

The first thing we notice about this line is that it has a negative slope; thus $v_{av} < 0$ and the particle is moving to the left. We also note that it is inclined more steeply than the line in Figure 2–4 (a), hence the magnitude of its slope is greater. In fact, if we calculate the slope of this line we find that $v_{av} = -2$ m/s for this time interval.

Thus, connecting points on an x-versus-t plot gives an immediate "feeling" for the average velocity over a given time interval. This type of graphical analysis will be particularly useful in the next section.

2–3 Instantaneous Velocity

Though average velocity is a useful way to characterize motion, it can miss a lot. For example, suppose you travel by car on a long, straight highway, covering 92 mi in 2.0 hours. Your average velocity is 46 mi/h. Even so, there may have been only a few times during the trip when you were actually driving at 46 mi/h. You may have sped along at 65 mi/h during most of the time, except when you stopped to have a bite to eat at a roadside diner, during which time your average velocity was zero.

To have a more accurate representation of your trip, you should average your velocity over shorter periods of time. If you calculate your average velocity every 15 minutes, you have a better picture of what the trip was like. An even better, more realistic picture of the trip is obtained if you calculate the average velocity every minute or every second. Ideally, when dealing with the motion of any particle, it is desirable to know the velocity of the particle at each instant of time.

This idea of a velocity corresponding to an instant of time is just what is meant by the **instantaneous velocity.** Mathematically, we define the instantaneous velocity as follows:

Definition: Instantaneous Velocity, *v*

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad 2\text{–}4$$

SI unit: meter per second, m/s

In this expression the notation $\lim_{\Delta t \to 0}$ means "evaluate the average velocity, $\Delta x/\Delta t$, over shorter and shorter time intervals, approaching zero in the limit." Note that the instantaneous velocity can be positive, negative, or zero, just like the average velocity—and just like the average velocity, the instantaneous velocity is a one-dimensional vector. The magnitude of the instantaneous velocity is called the **instantaneous speed.** In a car, the speedometer gives a reading of the vehicle's instantaneous speed.

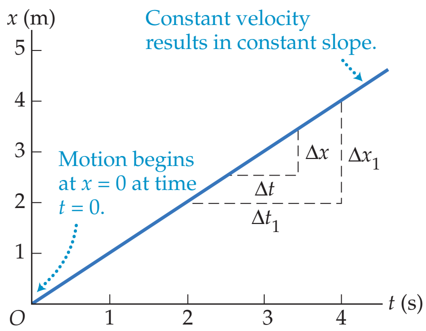
As Δt becomes smaller, Δx becomes smaller as well, but the ratio $\Delta x/\Delta t$ approaches a constant value. To see how this works, consider first the simple case of a particle moving with a constant velocity of $+1$ m/s. If the particle starts at $x = 0$ at $t = 0$, then its position at $t = 1$ s is $x = 1$ m, its position at $t = 2$ s is $x = 2$ m, and so on. Plotting this motion in an x-versus-t plot gives a straight line, as shown in **Figure 2–5**.

Now, suppose we want to find the instantaneous velocity at $t = 3$ s. To do so, we calculate the average velocity over small intervals of time centered at 3 s, and let the time intervals become arbitrarily small, as shown in the Figure. Since x-versus-t is a straight line, it is clear that $\Delta x/\Delta t = \Delta x_1/\Delta t_1$, no matter how small the time interval Δt. As Δt becomes smaller, so does Δx, but the ratio $\Delta x/\Delta t$ is simply the slope of the line, 1 m/s. Thus, the instantaneous velocity at $t = 3$ s is 1 m/s.

Of course, in this example the instantaneous velocity is 1 m/s for any instant of time, not just $t = 3$ s. Therefore:

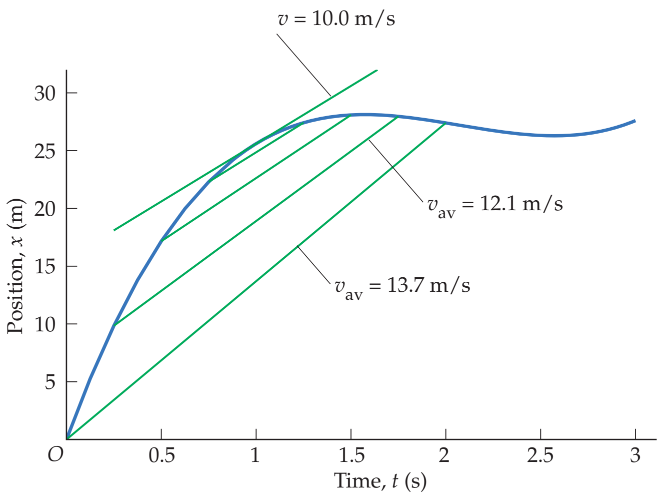
- When velocity is constant, the average velocity over any time interval is equal to the instantaneous velocity at any time.

In general, a particle's velocity varies with time, and the x-versus-t plot is not a straight line. An example is shown in **Figure 2–6**, with the corresponding numerical values of x and t given in Table 2–1.



▲ FIGURE 2–5 Constant velocity corresponds to constant slope on an *x*-versus-*t* graph

The slope $\Delta x_1/\Delta t_1$ is equal to $(4\text{ m} - 2\text{ m})/(4\text{ s} - 2\text{ s}) = (2\text{ m})/(2\text{ s}) = 1$ m/s. Because x-versus-t is a straight line, the slope $\Delta x/\Delta t$ is also equal to 1 m/s for any value of Δt.



▲ FIGURE 2–6 Instantaneous velocity

An x-versus-t plot for motion with variable velocity. The instantaneous velocity at $t = 1$ s is equal to the slope of the tangent line at that time. The average velocity for a small time interval centered on $t = 1$ s approaches the instantaneous velocity at $t = 1$ s as the time interval goes to zero.

In this case, what is the instantaneous velocity at, say, $t = 1.00$ s? As a first approximation, let's calculate the average velocity for the time interval from $t_i = 0$ to $t_f = 2.00$ s. Note that this time interval is centered at $t = 1.00$ s. From Table 2–1 we see that $x_i = 0$ and $x_f = 27.4$ m, thus $v_{av} = 13.7$ m/s. The corresponding straight line connecting these two points is the lowest straight line in Figure 2–6.

The next three lines, in upward progression, refer to time intervals from 0.250 s to 1.75 s, 0.500 s to 1.50 s, and 0.750 s to 1.25 s, respectively. The corresponding average velocities, given in Table 2–2, are 12.1 m/s, 10.9 m/s, and 10.2 m/s. Table 2–2 also gives results for even smaller time intervals. In particular, for the interval from 0.900 s to 1.10 s the average velocity is 10.0 m/s. Smaller intervals also give 10.0 m/s. Thus, we can conclude that the instantaneous velocity at $t = 1.00$ s is $v = 10.0$ m/s.

The uppermost straight line in Figure 2–6 is the tangent line to the x-versus-t curve at the time $t = 1.00$ s; that is, it is the line that touches the curve at just a single point. Its slope is 10.0 m/s. Clearly, the average-velocity lines have slopes that

TABLE 2–1
***x*-versus-*t* Values for Figure 2–6**

t (s)	x (m)
0	0
0.25	9.85
0.50	17.2
0.75	22.3
1.00	25.6
1.25	27.4
1.50	28.1
1.75	28.0
2.00	27.4

TABLE 2–2 Calculating the Instantaneous Velocity at $t = 1$ s

t_i (s)	t_f (s)	Δt (s)	x_i (m)	x_f (m)	Δx (m)	$v_{av} = \Delta x/\Delta t$ (m/s)
0	2.00	2.00	0	27.4	27.4	13.7
0.250	1.75	1.50	9.85	28.0	18.2	12.1
0.500	1.50	1.00	17.2	28.1	10.9	10.9
0.750	1.25	0.50	22.3	27.4	5.10	10.2
0.900	1.10	0.20	24.5	26.5	2.00	10.0
0.950	1.05	0.10	25.1	26.1	1.00	10.0

approach the slope of the tangent line as the time intervals become smaller. This is an example of the following general result:

- The instantaneous velocity at a given time is equal to the slope of the tangent line at that point on an x-versus-t graph.

Thus, a visual inspection of an x-versus-t graph gives information not only about the location of a particle, but also about its velocity.

CONCEPTUAL CHECKPOINT 2–2 INSTANTANEOUS VELOCITY

Referring to Figure 2–6, is the instantaneous velocity at $t = 0.500$ s **(a)** greater than, **(b)** less than, or **(c)** the same as the instantaneous velocity at $t = 1.00$ s?

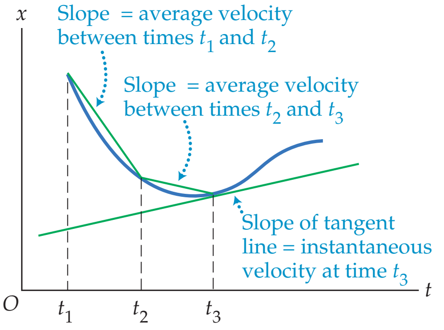
REASONING AND DISCUSSION

From the x-versus-t graph in Figure 2–6 it is clear that the slope of a tangent line drawn at $t = 0.500$ s is greater than the slope of the tangent line at $t = 1.00$ s. It follows that the particle's velocity at 0.500 s is greater than its velocity at 1.00 s.

ANSWER

(a) The instantaneous velocity is greater at $t = 0.500$ s.

In the remainder of the book, when we say velocity it is to be understood that we mean *instantaneous* velocity. If we want to refer to the average velocity, we will specifically say average velocity.



▲ **FIGURE 2–7 Graphical interpretation of average and instantaneous velocity**

Average velocities correspond to the slope of straight-line segments connecting different points on an x-versus-t graph. Instantaneous velocities are given by the slope of the tangent line at a given time.

Graphical Interpretation of Average and Instantaneous Velocity

Let's summarize the graphical interpretations of average and instantaneous velocity on an x-versus-t graph:

- Average velocity is the slope of the straight line connecting two points corresponding to a given time interval.
- Instantaneous velocity is the slope of the tangent line at a given instant of time.

These relations are illustrated in **Figure 2–7**.

▲ The space shuttle *Discovery* accelerates upward on the initial phase of its journey into orbit. During this time the astronauts on board the shuttle experience an approximately linear acceleration that may be as great as 20 m/s^2.

2–4 Acceleration

Just as velocity is the rate of change of *displacement* with time, **acceleration** is the rate of change of *velocity* with time. Thus, an object accelerates whenever its velocity *changes,* no matter what the change—it accelerates when its velocity increases, it accelerates when its velocity decreases. Of all the concepts discussed in this chapter, perhaps none is more central to physics than acceleration. Galileo, for example, showed that falling bodies move with constant acceleration. Newton showed that acceleration and force are directly related, as we shall see in Chapter 5. Thus, it is particularly important to have a clear, complete understanding of acceleration before leaving this chapter.

We begin, then, with the definition of **average acceleration:**

Definition: Average Acceleration, a_{av}

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad \text{2–5}$$

SI unit: meter per second per second, m/s^2

Note that the dimensions of average acceleration are the dimensions of velocity per time, or (meters per second) per second:

$$\frac{\text{meters per second}}{\text{second}} = \frac{m/s}{s} = \frac{m}{s^2}$$

This is generally expressed as meters per second squared. For example, the acceleration of gravity on the Earth's surface is approximately 9.81 m/s^2, which means that the velocity of a falling object changes by 9.81 meters per second (m/s) every second (s). In addition, we see that the average acceleration can be positive, negative, or zero. In fact, it is a one-dimensional vector, just like displacement, average velocity, and instantaneous velocity. Typical magnitudes of acceleration are given in Table 2–3.

TABLE 2–3 Typical Accelerations (m/s^2)

Ultracentrifuge	3×10^6
Bullet fired from a rifle	4.4×10^5
Batted baseball	3×10^4
Click beetle righting itself	400
Acceleration required to deploy airbags	60
Bungee jump	30
High jump	15
Acceleration of gravity on Earth	9.81
Emergency stop in a car	8
Airplane during takeoff	5
An elevator	3
Acceleration of gravity on the Moon	1.62

EXERCISE 2–1

a. Saab advertises a car that goes from 0 to 60.0 mi/h in 6.2 s. What is the average acceleration of this car?

b. An airplane has an average acceleration of 5.6 m/s^2 during takeoff. How long does it take for the plane to reach a speed of 150 mi/h?

SOLUTION

a. average acceleration $= a_{av} = (60.0\ \text{mi/h})/(6.2\ \text{s})$

$$= (26.8\ \text{m/s})/(6.2\ \text{s}) = 4.3\ \text{m/s}^2$$

b. $\Delta t = \Delta v/a_{av} = (150\ \text{mi/h})/(5.6\ \text{m/s}^2) = (67.0\ \text{m/s})/(5.6\ \text{m/s}^2) = 12\ \text{s}$

Next, just as we considered the limit of smaller and smaller time intervals to find an instantaneous velocity, we can do the same to define an **instantaneous acceleration:**

Definition: Instantaneous Acceleration, *a*

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \qquad \text{2–6}$$

SI unit: meter per second per second, m/s^2

As you might expect, the instantaneous acceleration is a one-dimensional vector, just like the average acceleration, and its direction is given by its sign. For simplicity, when we say acceleration in the future we are referring to the instantaneous acceleration.

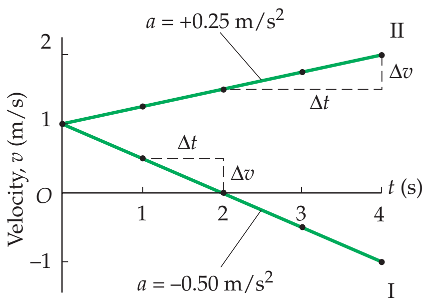
One final note before we go on to some examples. If the acceleration is constant, it has the same value at all times. Therefore:

- When acceleration is constant, the instantaneous and average accelerations are the same.

We shall make use of this fact when we return to the special case of constant acceleration in the next section.

Graphical Interpretation of Acceleration

To see how acceleration can be interpreted graphically, suppose that a particle has a constant acceleration of $-0.50\ \text{m/s}^2$. This means that the velocity of the particle *decreases* by 0.50 m/s each second. Thus, if its velocity is 1.0 m/s at $t = 0$, then at $t = 1$ s its velocity is 0.50 m/s, at $t = 2$ s its velocity is 0, at $t = 3$ s its velocity is -0.50 m/s, and so on. This is illustrated by curve I in **Figure 2–8**, where we see that a plot of v-versus-t results in a straight line with a negative slope. Curve II in Figure 2–8 has a positive slope, corresponding to a constant acceleration of $+0.25\ \text{m/s}^2$. Thus, in terms of a v-versus-t plot, a constant acceleration results in a straight line with a slope equal to the acceleration.



▲ **FIGURE 2–8 *v*-versus-*t* plots for motion with constant acceleration**
Curve I represents the movement of a particle with constant acceleration $a = -0.50\ \text{m/s}^2$. Curve II represents the motion of a particle with constant acceleration $a = +0.25\ \text{m/s}^2$.

CONCEPTUAL CHECKPOINT 2–3 SPEED AS A FUNCTION OF TIME

The speed of a particle with the v-versus-t graph shown by curve II in Figure 2–8 increases steadily with time. Consider, instead, a particle whose v-versus-t graph is given by curve I in Figure 2–8. As a function of time, does the speed of this particle **(a)** increase, **(b)** decrease, or **(c)** decrease and then increase?

REASONING AND DISCUSSION

Recall that speed is the *magnitude* of velocity. In curve I of Figure 2–8 the speed starts out at 1.0 m/s, then *decreases* to 0 at $t = 2$ s. After $t = 2$ s the speed *increases* again. For example, at $t = 3$ s the speed is 0.50 m/s, and at $t = 4$ s the speed is 1 m/s.

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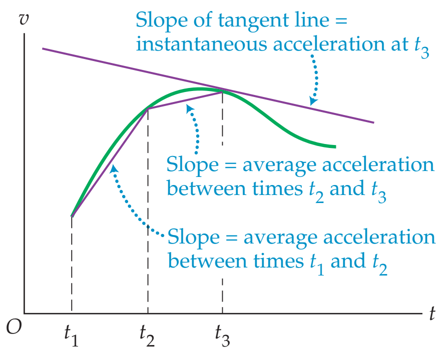


▲ **FIGURE 2–9 Graphical interpretation of average and instantaneous acceleration**

Average accelerations correspond to the slope of straight-line segments connecting different points on a v-versus-t graph. Instantaneous accelerations are given by the slope of the tangent line at a given time.

CONTINUED FROM PREVIOUS PAGE

Did you realize that the particle represented by curve I in Figure 2–8 changes direction at $t = 2$ s? It certainly does. Before $t = 2$ s the particle moves in the positive direction; after $t = 2$ s it moves in the negative direction. At precisely $t = 2$ s the particle is momentarily at rest. However, regardless of whether the particle is moving in the positive direction, moving in the negative direction, or instantaneously at rest, it still has the same constant acceleration. Acceleration has to do only with the way the velocity is *changing* at a given moment.

ANSWER

(c) The speed decreases and then increases.

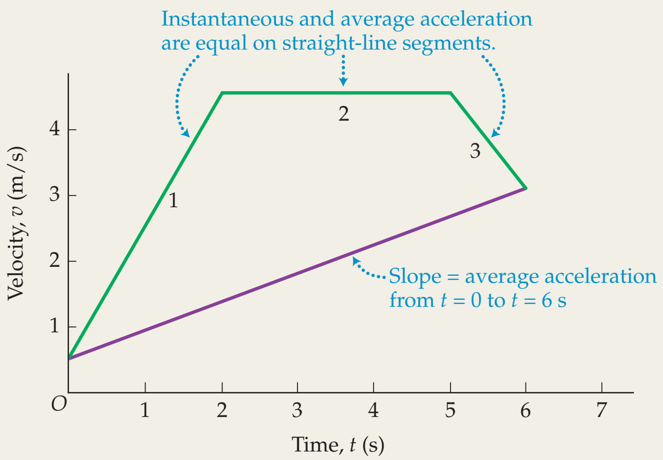
The graphical interpretations for velocity presented in Figure 2–7 apply equally well to acceleration, with just one small change: Instead of an x-versus-t graph, we use a v-versus-t graph, as in **Figure 2–9**. Thus, the average acceleration in a v-versus-t plot is the slope of a straight line connecting points corresponding to two different times. Similarly, the instantaneous acceleration is the slope of the tangent line at a particular time.

EXAMPLE 2–3 AN ACCELERATING TRAIN

A train moving in a straight line with an initial velocity of 0.50 m/s accelerates at 2.0 m/s^2 for 2.0 seconds, coasts with zero acceleration for 3.0 seconds, and then accelerates at -1.5 m/s^2 for 1.0 second. **(a)** What is the final velocity of the train? **(b)** What is the average acceleration of the train?

PICTURE THE PROBLEM

We begin by sketching a v-versus-t plot for the train. The basic idea is that each interval of constant acceleration is represented by a straight line of the appropriate slope. Therefore, we draw a straight line with the slope 2.0 m/s^2 from $t = 0$ to $t = 2.0$ s, a line with zero slope from $t = 2.0$ s to $t = 5.0$ s, and a line with the slope -1.5 m/s^2 from $t = 5.0$ s to $t = 6.0$ s. The line connecting the initial and final points determines the average acceleration.



STRATEGY

During each period of constant acceleration the change in velocity is $\Delta v = a_{av}\Delta t = a\Delta t$.

a. Adding the individual changes in velocity gives the total change, $\Delta v = v_f - v_i$. Since v_i is known, this expression can be solved for the final velocity, v_f.

b. The average acceleration can be calculated using Equation 2–5, $a_{av} = \Delta v/\Delta t$. Note that Δv has been obtained in part (a), and that the total time interval is $\Delta t = 6.0$ s, as is clear from the graph.

SOLUTION

Part (a)

1. Find the change in velocity during each of the three periods of constant acceleration:

$$\Delta v_1 = a_1\Delta t_1 = (2.0\ \text{m/s}^2)(2.0\ \text{s}) = 4.0\ \text{m/s}$$
$$\Delta v_2 = a_2\Delta t_2 = (0)(3.0\ \text{s}) = 0$$
$$\Delta v_3 = a_3\Delta t_3 = (-1.5\ \text{m/s}^2)(1.0\ \text{s}) = -1.5\ \text{m/s}$$

2. Sum the change in velocity for each period to obtain the total Δv:

$$\Delta v = \Delta v_1 + \Delta v_2 + \Delta v_3$$
$$= 4.0\ \text{m/s} + 0 - 1.5\ \text{m/s} = 2.5\ \text{m/s}$$

3. Use Δv to find v_f, recalling that $v_i = 0.50$ m/s:

$$\Delta v = v_f - v_i$$
$$v_f = \Delta v + v_i = 2.5\ \text{m/s} + 0.50\ \text{m/s} = 3.0\ \text{m/s}$$

Part (b)

4. The average acceleration is $\Delta v/\Delta t$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{2.5 \text{ m/s}}{6.0 \text{ s}} = 0.42 \text{ m/s}^2$$

INSIGHT

Note that the average acceleration for these six seconds is not simply the average of the individual accelerations, 2.0 m/s^2, 0 m/s^2, and -1.5 m/s^2. The reason is that different amounts of time are spent with each acceleration. In addition, the average acceleration can be found graphically, as indicated in the v-versus-t sketch on the previous page. Specifically, the graph shows that Δv is 2.5 m/s for the time interval from $t = 0$ to $t = 6.0$ s.

PRACTICE PROBLEM

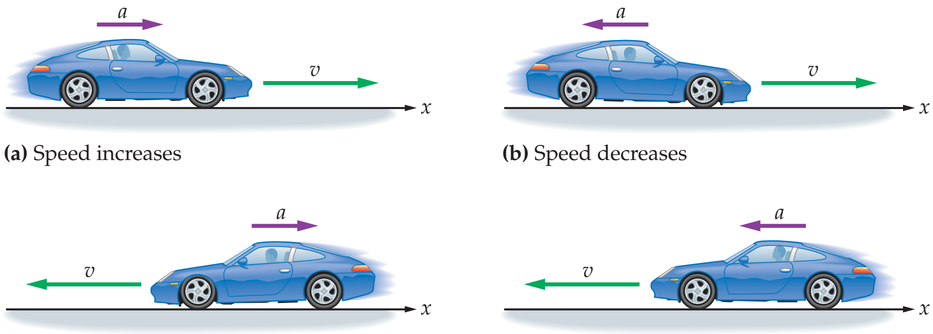
What is the average acceleration of the train between $t = 2.0$ s and $t = 6.0$ s?
[**Answer:** $a_{av} = \Delta v/\Delta t = (3.0 \text{ m/s} - 4.5 \text{ m/s})/(6.0 \text{ s} - 2.0 \text{ s}) = -0.38 \text{ m/s}^2$]

Some related homework problems: Problem 36, Problem 38

In one dimension, nonzero velocities and accelerations are either positive or negative, depending on whether they point in the positive or negative direction of the coordinate system chosen. Thus, the velocity and acceleration of an object may have the same or opposite signs. (Of course, in two or three dimensions the relationship between velocity and acceleration can be much more varied, as we shall see in the next several chapters.) This leads to the following two possibilities:

- When the velocity and acceleration of an object have the same sign, the speed of the object increases. In this case, the velocity and acceleration point in the same direction.
- When the velocity and acceleration of an object have opposite signs, the speed of the object decreases. In this case, the velocity and acceleration point in opposite directions.

These two possibilities are illustrated in **Figure 2–10**. Notice that when a particle's speed increases, it means either that its velocity becomes more positive, as in **Figure 2–10 (a)**, or more negative, as in **Figure 2–10 (d)**. In either case, it is the magnitude of the velocity—the speed—that increases.



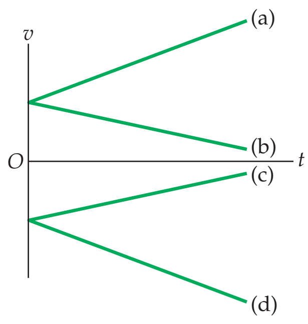
(a) Speed increases **(b)** Speed decreases

(c) Speed decreases **(d)** Speed increases

FIGURE 2–10 Cars accelerating or decelerating

A car's speed increases when its velocity and acceleration point in the same direction, as in cases **(a)** and **(d).** When the velocity and acceleration point in opposite directions, as in cases **(b)** and **(c),** the car's speed decreases.

The winner of this race was traveling at a speed of 313.91 mi/h at the end of the quarter-mile course. Since the winning time was just 4.607 s, the *average* acceleration during this race was approximately three times the acceleration of gravity (Section 2–7).



▲ **FIGURE 2–11** ***v*-versus-*t* plots with constant acceleration**

Four plots of v versus t corresponding to the four situations shown in Figure 2–10. Note that the speed increases in cases **(a)** and **(d),** but decreases in cases **(b)** and **(c).**

When a particle's speed decreases, it is often said to be *decelerating*. A common misconception is that deceleration implies a negative acceleration. This is not true. Deceleration can be caused by a positive or a negative acceleration, depending on the direction of the initial velocity. For example, the car in **Figure 2–10 (b)** has a positive velocity and a negative acceleration, while the car in **Figure 2–10 (c)** has a negative velocity and a positive acceleration. In both cases, the speed of the car decreases. Again, all that is required for deceleration in one dimension is that the velocity and acceleration have *opposite signs*; that is, they must point in *opposite directions*, as in parts (b) and (c) of Figure 2–10.

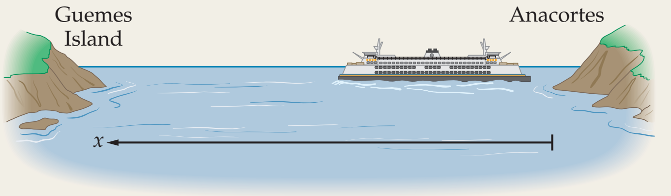
Velocity-versus-time plots for the four situations shown in Figure 2–10 are presented in **Figure 2–11**. In each of the four plots in Figure 2–11 we assume constant acceleration. Be sure to understand clearly the connection between the v-versus-t plots in Figure 2–11 and the corresponding physical motions indicated in Figure 2–10.

EXAMPLE 2–4 THE FERRY DOCKS

A ferry makes a short run between two docks; one in Anacortes, Washington, the other on Guemes Island. As the ferry approaches Guemes Island (traveling in the positive x direction), its speed is 7.4 m/s. **(a)** If the ferry slows to a stop in 12.3 s, what is its average acceleration? **(b)** As the ferry returns to the Anacortes dock, its speed is 7.3 m/s. If it comes to rest in 13.1 s, what is its average acceleration?

PICTURE THE PROBLEM

Our sketch shows the locations of the two docks and the positive direction indicated in the problem. Note that the distance between docks is not given, nor is it needed.



STRATEGY

We are given the initial and final velocities (the ferry comes to a stop in each case, so its final speed is zero) and the relevant times. Therefore, we can find the average acceleration using $a_{av} = \Delta v/\Delta t$, being careful to get the signs right.

SOLUTION

Part (a)

1. Calculate the average acceleration, noting that $v_i = 7.4$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 7.4\text{ m/s}}{12.3\text{ s}} = -0.60\text{ m/s}^2$$

Part (b)

2. In this case, $v_i = -7.3$ m/s and $v_f = 0$:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{0 - (-7.3\text{ m/s})}{13.1\text{ s}} = 0.56\text{ m/s}^2$$

INSIGHT

In each case, the acceleration of the ferry is opposite in sign to its velocity; therefore the ferry decelerates.

PRACTICE PROBLEM

When the ferry leaves Guemes Island, its speed increases from 0 to 5.8 m/s in 9.25 s. What is its average acceleration? [**Answer:** $a_{av} = -0.63\text{ m/s}^2$]

Some related homework problems: Problem 34, Problem 35

2–5 Motion with Constant Acceleration

In this section, we derive equations describing the motion of particles moving with **constant acceleration.** These "equations of motion" can be used to describe a wide range of everyday phenomena. For example, in an idealized world with no air resistance, falling bodies have constant acceleration.

As mentioned in the previous section, if a particle has constant acceleration—that is, the same acceleration at every instant of time—then its instantaneous acceleration, a, is equal to its average acceleration, a_{av}. Recalling the definition of average acceleration, Equation 2–5, we have

$$a_{av} = \frac{v_f - v_i}{t_f - t_i} = a$$

where the initial and final times may be chosen arbitrarily. For example, let $t_i = 0$ for the initial time, and let $v_i = v_0$ denote the velocity at time zero. For the final time and velocity we drop the subscripts to simplify notation; thus we let $t_f = t$ and $v_f = v$. With these identifications we have

$$a_{av} = \frac{v - v_0}{t - 0} = a$$

Therefore,

$$v - v_0 = a(t - 0) = at$$

or

Constant-Acceleration Equation of Motion: Velocity as a Function of Time

$$v = v_0 + at \qquad 2\text{–}7$$

Note that Equation 2–7 describes a straight line on a v-versus-t plot. The line crosses the velocity axis at the value v_0 and has a slope a, in agreement with the graphical interpretations discussed in the previous section. For example, in curve I of Figure 2–8, the equation of motion is $v = v_0 + at = (1\ \text{m/s}) + (-0.5\ \text{m/s}^2)t$. Also, note that $(-0.5\ \text{m/s}^2)t$ has the units $(\text{m/s}^2)\ (\text{s}) = \text{m/s}$; thus each term in Equation 2–7 has the same dimensions (as it must to be a valid physical equation).

EXERCISE 2–2

A ball is thrown straight upward with an initial velocity of $+8.2$ m/s. If the acceleration of the ball is $-9.81\ \text{m/s}^2$, what is its velocity after

a. 0.50 s, and **b.** 1.0 s?

SOLUTION

a. Substituting $t = 0.50$ s in Equation 2–7 yields

$$v = 8.2\ \text{m/s} + (-9.81\ \text{m/s}^2)(0.50\ \text{s}) = 3.3\ \text{m/s}$$

b. Similarly, using $t = 1.0$ s in Equation 2–7 gives

$$v = 8.2\ \text{m/s} + (-9.81\ \text{m/s}^2)(1.0\ \text{s}) = -1.6\ \text{m/s}$$

Next, how far does a particle move in a given time if its acceleration is constant? To answer this question, recall the definition of average velocity:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

Using the same identifications given previously for initial and final times, and letting $x_i = x_0$ and $x_f = x$, we have

$$v_{av} = \frac{x - x_0}{t - 0}$$

Thus,

$$x - x_0 = v_{av}(t - 0) = v_{av}t$$

or

$$x = x_0 + v_{av}t \qquad 2\text{–}8$$

Now, Equation 2–8 is fine as it is. In fact, it applies whether the acceleration is constant or not. A more useful expression, for the case of constant acceleration, is obtained by writing v_{av} in terms of the initial and final velocities. This can be done by referring to **Figure 2–12 (a)**. Here the velocity changes linearly (since a is

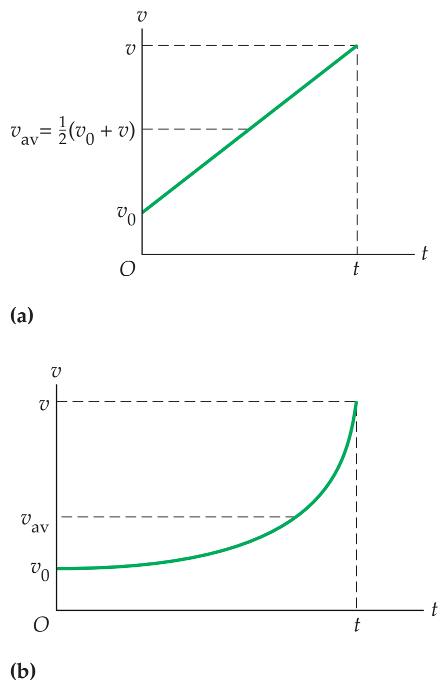


FIGURE 2–12 The average velocity
(a) When acceleration is constant, the velocity varies linearly with time. As a result, the average velocity, v_{av}, is simply the average of the initial velocity, v_0, and the final velocity, v. **(b)** The velocity curve for nonconstant acceleration is nonlinear. In this case, the average velocity is no longer midway between the initial and final velocities.

constant) from v_0 at $t = 0$ to v at some later time t. The average velocity during this period of time is simply the average of the initial and final velocities; that is, the sum of the two velocities divided by two:

Constant-Acceleration Equation of Motion: Average Velocity

$$v_{av} = \tfrac{1}{2}(v_0 + v) \qquad 2\text{–}9$$

The average velocity is indicated in the figure. Note that if the acceleration is not constant, as in **Figure 2–12 (b)**, this simple averaging of initial and final velocities is no longer valid.

Substituting the expression for v_{av} from Equation 2–9 into Equation 2–8 yields

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \qquad 2\text{–}10$$

This equation, like Equation 2–7, is valid *only* for constant acceleration. The utility of Equations 2–7 and 2–10 is illustrated in the next Example.

PROBLEM-SOLVING NOTE

"Coordinate" the Problem

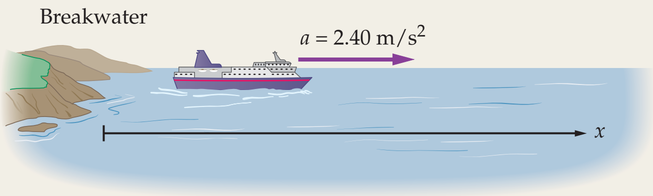
The first step in solving a physics problem is to produce a simple sketch of the system. Your sketch should include a coordinate system, along with an origin and a positive direction. Next, you should identify quantities that are given in the problem, such as initial position, initial velocity, acceleration, and so on. These preliminaries will help in producing a mathematical representation of the problem.

EXAMPLE 2–5 FULL SPEED AHEAD

A boat moves slowly inside a marina (so as not to leave a wake) with a constant speed of 1.50 m/s. As soon as it passes the breakwater, leaving the marina, it throttles up and accelerates at 2.40 m/s^2. **(a)** How fast is the boat moving after accelerating for 5.00 s? **(b)** How far has the boat traveled in this time?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the breakwater, and the positive x direction to be the direction of motion. With this choice the initial position is $x_0 = 0$, and the initial velocity is $v_0 = 1.50$ m/s.



STRATEGY

The acceleration is constant, so we can use Equations 2–7 to 2–10. In part (a) we want to relate velocity to time, so we use Equation 2–7, $v = v_0 + at$. In part (b) our knowledge of the initial and final velocities allows us to relate position to time using Equation 2–10, $x = x_0 + \frac{1}{2}(v_0 + v)t$.

SOLUTION

Part (a)

1. Use Equation 2–7 with $v_0 = 1.50$ m/s and $a = 2.40$ m/s^2:

$$v = v_0 + at = 1.50\ \text{m/s} + (2.40\ \text{m/s}^2)(5.00\ \text{s})$$
$$= 1.50\ \text{m/s} + 12.0\ \text{m/s} = 13.5\ \text{m/s}$$

Part (b)

2. Apply Equation 2–10, using the result for v obtained in part (a):

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$
$$= 0 + \tfrac{1}{2}(1.50\ \text{m/s} + 13.5\ \text{m/s})(5.00\ \text{s})$$
$$= (7.50\ \text{m/s})(5.00\ \text{s}) = 37.5\ \text{m}$$

INSIGHT

Since the boat has a constant acceleration between $t = 0$ and $t = 5.00$ s, its velocity-versus-time curve is linear during this time interval. As a result, the average velocity for these 5.00 seconds is the average of the initial and final velocities, $v_{av} = \frac{1}{2}(1.50\ \text{m/s} + 13.5\ \text{m/s}) = 7.50$ m/s. Multiplying the average velocity by the time, 5.00 s, gives the distance traveled—which is exactly what Equation 2–10 does in Step 2.

PRACTICE PROBLEM

At what time is the boat's speed equal to 10.0 m/s? [**Answer:** $t = 3.54$ s]

Some related homework problems: Problem 47, Problem 48

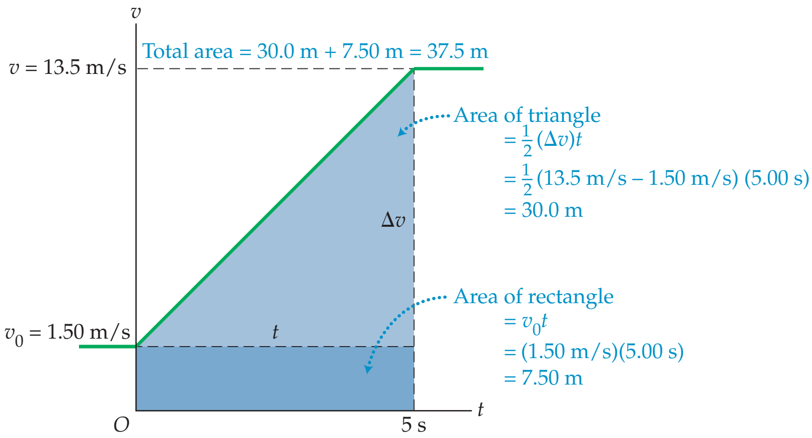


FIGURE 2–13 Velocity versus time for the boat in Example 2–5

The distance traveled by the boat between $t = 0$ and $t = 5.00$ s is equal to the corresponding area under the velocity curve.

The velocity of the boat in Example 2–5 is plotted as a function of time in **Figure 2–13**, with the acceleration starting at time $t = 0$ and ending at $t = 5.00$ s. We will now show that the *distance* traveled by the boat from $t = 0$ to $t = 5.00$ s *is equal to the corresponding area under the velocity-versus-time curve*. This is a general result, valid for any velocity curve and any time interval:

- The distance traveled by an object from a time t_1 to a time t_2 is equal to the area under the velocity curve between those two times.

In this case, the area is the sum of the areas of a rectangle and a triangle. The rectangle has a base of 5.00 s and a height of 1.50 m/s, which gives an area of $(5.00\text{ s})(1.50\text{ m/s}) = 7.50$ m. Similarly, the triangle has a base of 5.00 s and a height of $(13.5\text{ m/s} - 1.50\text{ m/s}) = 12.0$ m/s, for an area of $\frac{1}{2}(5.00\text{ s})(12.0\text{ m/s}) = 30.0$ m. Clearly, the total area is 37.5 m, just as found in Example 2–5.

Staying with Example 2–5 for a moment, let's repeat the calculation of part (b), only this time for the general case. First, we use the final velocity from part (a), calculated with $v = v_0 + at$, in the expression for the average velocity, $v_{av} = \frac{1}{2}(v_0 + v)$. Symbolically, this gives the following:

$$\tfrac{1}{2}(v_0 + v) = \tfrac{1}{2}[v_0 + (v_0 + at)] = v_0 + \tfrac{1}{2}at \qquad \textit{(constant acceleration)}$$

Next, we substitute this result into Equation 2–10, which yields

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = x_0 + \left(v_0 + \tfrac{1}{2}at\right)t$$

Multiplying through by t gives the following result:

Constant-Acceleration Equation of Motion: Position as a Function of Time

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \qquad 2\text{–}11$$

Here we have an expression for position versus time that is explicitly in terms of the acceleration, a.

Note that each term in Equation 2–11 has the same dimensions, as they must. For example, the velocity term, $v_0 t$, has the units $(\text{m/s})(\text{s}) = \text{m}$. Similarly, the acceleration term, $\frac{1}{2}at^2$, has the units $(\text{m/s}^2)(\text{s}^2) = \text{m}$.

EXERCISE 2–3

Repeat part (b) of Example 2–5 using Equation 2–11.

SOLUTION

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + (1.50\text{ m/s})(5.00\text{ s}) + \tfrac{1}{2}(2.40\text{ m/s}^2)(5.00\text{ s})^2 = 37.5\text{ m}$$

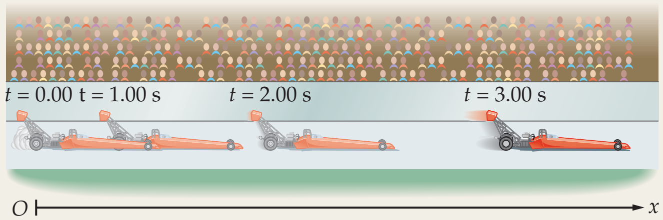
The next Example gives further insight into the physical meaning of Equation 2–11.

EXAMPLE 2–6 PUT THE PEDAL TO THE METAL

A drag racer starts from rest and accelerates at $7.40\ \mathrm{m/s^2}$. How far has it traveled in **(a)** 1.00 s, **(b)** 2.00 s, **(c)** 3.00 s?

PICTURE THE PROBLEM

We set up a coordinate system in which the drag racer starts at the origin and accelerates in the positive x direction. With this choice, it follows that $x_0 = 0$ and $a = +7.40\ \mathrm{m/s^2}$. Also, since the racer starts from rest, its initial velocity is zero, $v_0 = 0$. Incidentally, the positions of the racer in the sketch have been drawn to scale.



STRATEGY

Since this problem gives the acceleration, which is constant, and asks for a relationship between position and time, we use Equation 2–11.

SOLUTION

Part (a)

1. Evaluate Equation 2–11 with $a = 7.40\ \mathrm{m/s^2}$ and $t = 1.00$ s:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = 0 + 0 + \tfrac{1}{2}at^2 = \tfrac{1}{2}at^2$$

$$x = \tfrac{1}{2}(7.40\ \mathrm{m/s^2})(1.00\ \mathrm{s})^2 = 3.70\ \mathrm{m}$$

Part (b)

2. From the calculation in part (a), Equation 2–11 reduces to $x = \frac{1}{2}at^2$ in this situation. Evaluate $x = \frac{1}{2}at^2$ at $t = 2.00$ s:

$$x = \tfrac{1}{2}at^2$$
$$= \tfrac{1}{2}(7.40\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2 = 14.8\ \mathrm{m} = 4(3.70\ \mathrm{m})$$

Part (c)

3. Repeat with $t = 3.00$ s:

$$x = \tfrac{1}{2}at^2$$
$$= \tfrac{1}{2}(7.40\ \mathrm{m/s^2})(3.00\ \mathrm{s})^2 = 33.3\ \mathrm{m} = 9(3.70\ \mathrm{m})$$

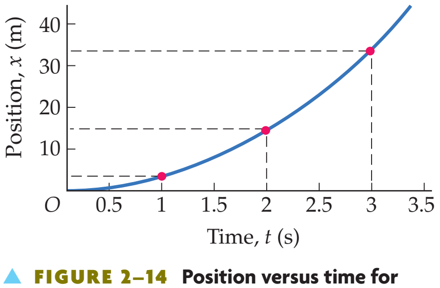
INSIGHT

This Example illustrates one of the key features of accelerated motion—position does not change uniformly with time when an object accelerates. In this case, the distance traveled in the first two seconds is 4 times the distance traveled in the first second, and the distance traveled in the first three seconds is 9 times the distance traveled in the first second. This kind of behavior is a direct result of the fact that x depends on t^2 when the acceleration is nonzero.

PRACTICE PROBLEM

In one second the racer travels 3.70 m. How long does it take for the racer to travel $2(3.70\ \mathrm{m}) = 7.40$ m? [**Answer:** $t = \sqrt{2}\ \mathrm{s} = 1.41\ \mathrm{s}$]

Some related homework problems: Problem 49, Problem 64



▲ **FIGURE 2–14 Position versus time for Example 2–6**

The upward-curving, parabolic shape of this x-versus-t plot indicates a positive, constant acceleration. The dots on the curve show the position of the drag racer in Example 2–6 at the times 1.00 s, 2.00 s, and 3.00 s.

Figure 2–14 shows a graph of x-versus-t for Example 2–6. Notice the parabolic shape of the x-versus-t curve, which is due to the $\frac{1}{2}at^2$ term, and is characteristic of constant acceleration. In particular, if acceleration is positive ($a > 0$), then a plot of x-versus-t curves upward; if acceleration is negative ($a < 0$), a plot of x-versus-t curves downward. The greater the magnitude of a, the greater the curvature. In contrast, if a particle moves with constant velocity ($a = 0$) the t^2 dependence vanishes, and the x-versus-t plot is a straight line.

Our final equation of motion with constant acceleration relates velocity to position. We start by solving for the time, t, in Equation 2–7:

$$v = v_0 + at \quad \text{or} \quad t = \frac{v - v_0}{a}$$

Next, we substitute this result into Equation 2–10, thus eliminating t:

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = x_0 + \tfrac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right)$$

Noting that $(v_0 + v)(v - v_0) = v_0 v - v_0{}^2 + v^2 - v v_0 = v^2 - v_0{}^2$, we have

$$x = x_0 + \frac{v^2 - v_0{}^2}{2a}$$

Finally, a straightforward rearrangement of terms yields

Constant-Acceleration Equation of Motion: Velocity in Terms of Displacement

$$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x \qquad \text{2–12}$$

This equation allows us to relate the velocity at one position to the velocity at another position, without knowing how much time is involved. The next Example shows how Equation 2–12 can be used.

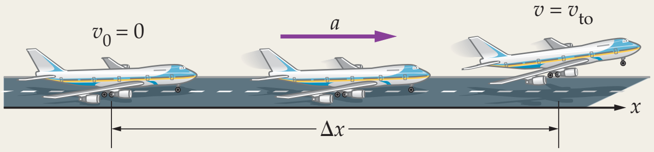
EXAMPLE 2–7 TAKEOFF DISTANCE FOR AN AIRLINER

REAL-WORLD PHYSICS

Jets at JFK International Airport accelerate from rest at one end of a runway, and must attain takeoff speed before reaching the other end of the runway. **(a)** Plane A has acceleration a and takeoff speed v_{to}. What is the minimum length of runway, Δx_A, required for this plane? Give a symbolic answer. **(b)** Plane B has the same acceleration as plane A, but requires twice the takeoff speed. Find Δx_B and compare with Δx_A. **(c)** Find the minimum runway length for plane A if $a = 2.20\ \text{m/s}^2$ and $v_{to} = 95.0\ \text{m/s}$. (These values are typical for a 747 jetliner.)

PICTURE THE PROBLEM

In our sketch, we choose the positive x direction to be the direction of motion. With this choice, it follows that the acceleration of the plane is positive, $a = +2.20\ \text{m/s}^2$. Similarly, the takeoff velocity is positive as well, $v_{to} = +95.0\ \text{m/s}$.



STRATEGY

From the sketch it is clear that we want to express Δx, the distance the plane travels in attaining takeoff speed, in terms of the acceleration, a, and the takeoff speed, v_{to}. Equation 2–12, which relates distance to velocity, allows us to do this.

SOLUTION

Part (a)

1. Solve Equation 2–12 for Δx. To find Δx_A, set $v_0 = 0$ and $v = v_{to}$:

$$\Delta x = \frac{v^2 - v_0^2}{2a} \qquad \Delta x_A = \frac{v_{to}^2}{2a}$$

Part (b)

2. To find Δx_B, simply change v_{to} to $2v_{to}$ in part (a):

$$\Delta x_B = \frac{(2v_{to})^2}{2a} = \frac{4v_{to}^2}{2a} = 4\Delta x_A$$

Part (c)

3. Substitute numerical values into the result found in part (a):

$$\Delta x_A = \frac{v_{to}^2}{2a} = \frac{(95.0\ \text{m/s})^2}{2(2.20\ \text{m/s}^2)} = 2050\ \text{m}$$

INSIGHT

For purposes of comparison, the shortest runway at JFK International Airport is 04R/22L, which has a length of 2560 m.

This Example illustrates the fact that there are many advantages to obtaining symbolic results before substituting numerical values. In this case, we find that the takeoff distance is proportional to v^2; hence, we conclude immediately that doubling v results in a fourfold increase of Δx.

PRACTICE PROBLEM

Find the minimum acceleration needed for a takeoff speed of $v_{to} = (95.0\ \text{m/s})/2 = 47.5\ \text{m/s}$ on a runway of length $\Delta x = (2050\ \text{m})/4 = 513\ \text{m}$. [**Answer:** $a = v_{to}^2/2\Delta x = 2.20\ \text{m/s}^2$]

Some related homework problems: Problem 55, Problem 57

Finally, all of our constant-acceleration equations of motion are collected for easy reference in Table 2–4.

TABLE 2–4 Constant-Acceleration Equations of Motion

Variables Related	Equation	Number
velocity, time, acceleration	$v = v_0 + at$	2–7
initial, final, and average velocity	$v_{av} = \frac{1}{2}(v_0 + v)$	2–9
position, time, velocity	$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10
position, time, acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11
velocity, position, acceleration	$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x$	2–12

2–6 Applications of the Equations of Motion

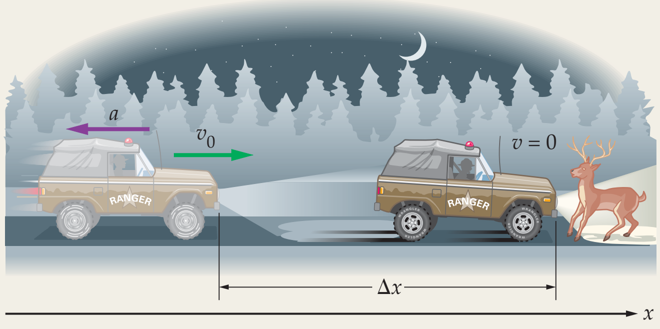
We devote this section to a variety of examples further illustrating the use of the constant-acceleration equations of motion. In our first Example, we consider the distance and time needed to brake a vehicle to a complete stop.

EXAMPLE 2–8 HIT THE BRAKES!

A park ranger driving on a back country road suddenly sees a deer "frozen" in the headlights. The ranger, who is driving at 11.4 m/s, immediately applies the brakes and slows with an acceleration of 3.80 m/s^2. **(a)** If the deer is 20.0 m from the ranger's vehicle when the brakes are applied, how close does the ranger come to hitting the deer? **(b)** How much time is needed for the ranger's vehicle to stop?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion. With this choice it follows that $v_0 = +11.4$ m/s. In addition, the fact that the ranger's vehicle is slowing down means its acceleration points in the *opposite* direction to that of the velocity [see Figure 2–10 (b) and (c)]. Therefore, the vehicle's acceleration is $a = -3.80$ m/s^2. Finally, when the vehicle comes to rest its velocity is zero, $v = 0$.



STRATEGY

The acceleration is constant, so we can use the equations listed in Table 2–4. In part (a) we want to find a distance when we know the velocity and acceleration, so we use a rearranged version of Equation 2–12. In part (b) we want to find a time when we know the velocity and acceleration, so we use a rearranged version of Equation 2–7.

SOLUTION

Part (a)

1. Solve Equation 2–12 for Δx: $$\Delta x = \frac{v^2 - v_0^{\,2}}{2a}$$

2. Set $v = 0$, and substitute numerical values: $$\Delta x = -\frac{v_0^{\,2}}{2a} = -\frac{(11.4 \text{ m/s})^2}{2(-3.80 \text{ m/s}^2)} = 17.1 \text{ m}$$

3. Subtract Δx from 20.0 m to find the distance between the stopped vehicle and the deer: $$20.0 \text{ m} - 17.1 \text{ m} = 2.9 \text{ m}$$

Part (b)

4. Set $v = 0$ in Equation 2–7 and solve for t: $$v = v_0 + at = 0$$ $$t = -\frac{v_0}{a} = -\frac{11.4 \text{ m/s}}{(-3.80 \text{ m/s}^2)} = 3.00 \text{ s}$$

INSIGHT

Note the difference in the way t and Δx depend on the initial speed. If the initial speed is doubled, for example, the time needed to stop also doubles, but the distance needed to stop increases by a factor of four. This is one reason why speed on the highway has such a great influence on safety.

PRACTICE PROBLEM

Show that using $t = 3.00$ s in Equation 2–11 results in the same distance needed to stop.
[**Answer:** $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + (11.4 \text{ m/s})(3.00 \text{ s}) + \frac{1}{2}(-3.80 \text{ m/s}^2)(3.00 \text{ s})^2 = 17.1$ m, as expected.]

Some related homework problems: Problem 57, Problem 58

In Example 2–8, we calculated the distance necessary for a vehicle to come to a complete stop. But how does v vary with distance as the vehicle slows down? The next Conceptual Checkpoint deals with this topic.

CONCEPTUAL CHECKPOINT 2–4 STOPPING DISTANCE

The ranger in Example 2–8 brakes for 17.1 m. After braking for only half that distance, $\frac{1}{2}(17.1\ \text{m}) = 8.55$ m, is the ranger's speed **(a)** equal to $\frac{1}{2}v_0$, **(b)** greater than $\frac{1}{2}v_0$, or **(c)** less than $\frac{1}{2}v_0$?

REAL-WORLD PHYSICS

The stopping distance of a car

REASONING AND DISCUSSION

As pointed out in the Insight for Example 2–8, the fact that the stopping distance, Δx, depends on v_0^2 means that this distance increases by a factor of four when the speed is doubled. For example, the stopping distance with an initial speed of v_0 is four times the stopping distance when the initial speed is $v_0/2$.

To apply this observation to the ranger, suppose that the stopping distance with an initial speed of v_0 is Δx. It follows that the stopping distance for an initial speed of $v_0/2$ is $\Delta x/4$. This means that as the ranger slows from v_0 to 0, it takes a distance $\Delta x/4$ to slow from $v_0/2$ to 0, and the remaining distance, $3\Delta x/4$, to slow from v_0 to $v_0/2$. Thus, at the halfway point the ranger has not yet slowed to half of the initial velocity—the speed at this point is greater than $v_0/2$.

ANSWER

(b) The ranger's speed is greater than $\frac{1}{2}v_0$.

Clearly, v does not decrease uniformly with distance. A plot showing v as a function of x for Example 2–8 is shown in **Figure 2–15**. As we can see from the graph, v changes more in the second half of the braking distance than in the first half.

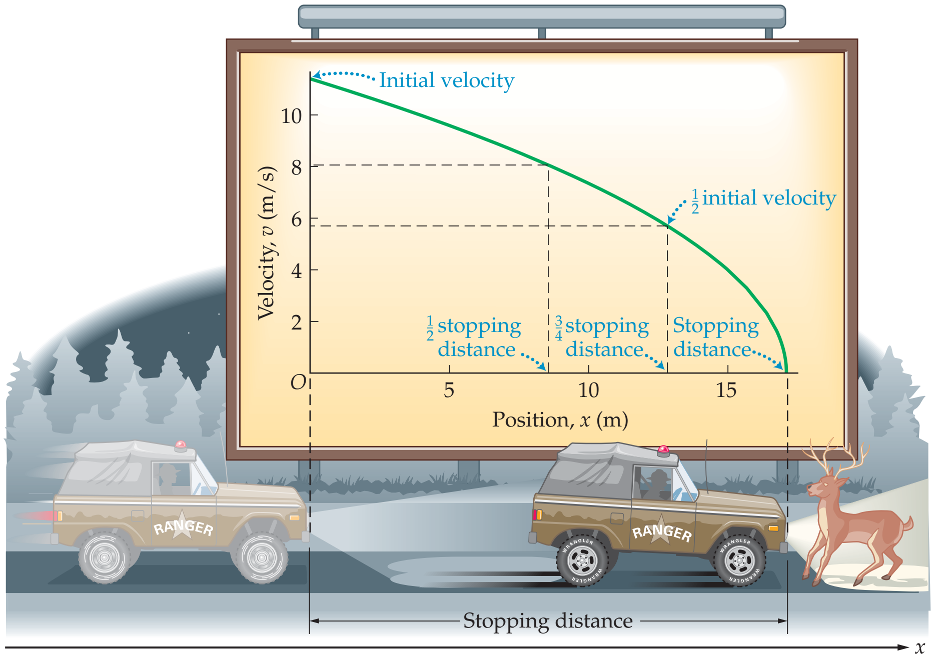


FIGURE 2–15 Velocity as a function of position for the ranger in Example 2–8
The ranger's vehicle in Example 2–8 comes to rest with constant acceleration, which means that its velocity decreases uniformly with time. The velocity *does not* decrease uniformly with distance, however. In particular, note how rapidly the velocity decreases in the final one-quarter of the stopping distance.

We close this section with a familiar, everyday example: a police car accelerating to overtake a speeder. This is the first time that we use two equations of motion for two different objects to solve a problem—but it won't be the last. Problems of this type are often more interesting than problems involving only a single object, and they relate to many types of situations in everyday life.

PROBLEM-SOLVING NOTE

Strategize

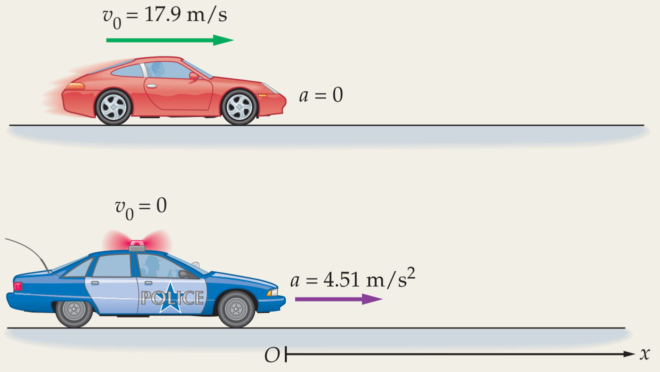
Before attempting to solve a problem, it is a good idea to have some sort of plan, or "strategy," for how to proceed. It may be as simple as saying, "The problem asks me to relate velocity and time, therefore I will use Equation 2–7." In other cases the strategy is a bit more involved. Producing effective strategies is one of the most challenging—and creative—aspects of problem solving.

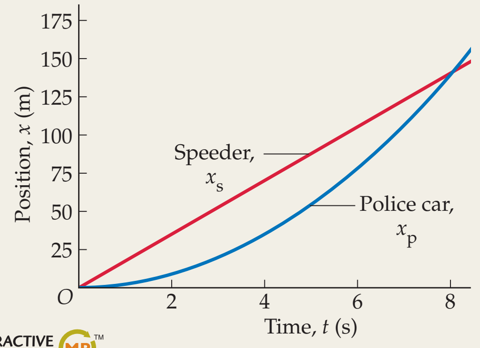
EXAMPLE 2–9 CATCHING A SPEEDER

A speeder doing 40.0 mi/h (about 17.9 m/s) in a 25 mi/h zone approaches a parked police car. The instant the speeder passes the police car, the police begin their pursuit. If the speeder maintains a constant velocity, and the police car accelerates with a constant acceleration of 4.51 m/s^2, **(a)** how long does it take for the police car to catch the speeder, **(b)** how far have the two cars traveled in this time, and **(c)** what is the velocity of the police car when it catches the speeder?

PICTURE THE PROBLEM

Our sketch shows the two cars at the moment the speeder passes the resting police car. At this instant, which we take to be $t = 0$, both the speeder and the police car are at the origin, $x = 0$. In addition, we choose the positive x direction to be the direction of motion; therefore, the speeder's initial velocity is given by $v_s = +17.9\ \text{m/s}$, and the police car's initial velocity is zero. The speeder's acceleration is zero, but the police car has an acceleration given by $a = +4.51\ \text{m/s}^2$. Finally, our plot shows the linear x-versus-t plot for the speeder, and the parabolic x-versus-t plot for the police car.





INTERACTIVE FIGURE MP™

STRATEGY

To solve this problem, first write down a position-versus-time equation for the police car, x_p, and a separate equation for the speeder, x_s. Next, we find the time it takes the police car to catch the speeder by setting $x_p = x_s$ and solving the resulting equation for t. Once the catch time is determined, it is straightforward to calculate the distance traveled and the velocity of the police car.

SOLUTION

Part (a)

1. Write equations of motion for the two vehicles. For the police car, $v_0 = 0$ and $a = 4.51\ \text{m/s}^2$. For the speeder, $v_0 = 17.9\ \text{m/s} = v_s$ and $a = 0$:

$$x_p = \tfrac{1}{2}at^2$$
$$x_s = v_s t$$

2. Set $x_p = x_s$ and solve for the time:

$$\tfrac{1}{2}at^2 = v_s t \text{ or } \left(\tfrac{1}{2}at - v_s\right)t = 0$$

$$\text{two solutions: } t = 0 \quad \text{or} \quad t = \frac{2v_s}{a}$$

3. Clearly, $t = 0$ corresponds to the initial conditions, because both vehicles started at $x = 0$ at that time. The time of interest is obtained by substituting numerical values into the other solution:

$$t = \frac{2v_s}{a} = \frac{2(17.9\ \text{m/s})}{4.51\ \text{m/s}^2} = 7.94\ \text{s}$$

Part (b)

4. Substitute $t = 7.94$ s into the equations of motion for x_p and x_s. Note that $x_p = x_s$, as expected:

$$x_p = \tfrac{1}{2}at^2 = \tfrac{1}{2}(4.51\ \text{m/s}^2)(7.94\ \text{s})^2 = 142$$
$$x_s = v_s t = (17.9\ \text{m/s})(7.94\ \text{s}) = 142\ \text{m}$$

Part (c)

5. To find the velocity of the police car use Equation 2–7, which relates velocity to time:

$$v_p = v_0 + at = 0 + (4.51\ \text{m/s}^2)(7.94\ \text{s}) = 35.8\ \text{m/s}$$

INSIGHT

When the police car catches up with the speeder, its velocity is 35.8 m/s, which is exactly twice the velocity of the speeder. A coincidence? Not at all. When the police car catches the speeder, both have traveled the same distance (142 m) in the same time (7.94 s), therefore they have the same average velocity. Of course, the average velocity of the speeder is simply v_s. The average velocity of the police car is $\frac{1}{2}(v_0 + v)$, since its acceleration is constant, and thus $\frac{1}{2}(v_0 + v) = v_s$. Since $v_0 = 0$ for the police car, it follows that $v = 2v_s$. Notice that this result is independent of the acceleration of the police car, as we show in the following Practice Problem.

PRACTICE PROBLEM

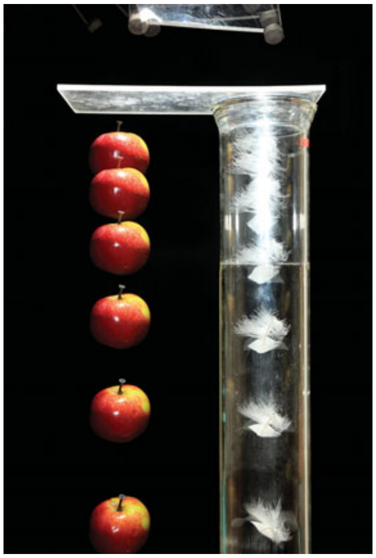
Repeat this Example for the case where the acceleration of the police car is $a = 3.25\ \text{m/s}^2$. **[Answer: (a)** $t = 11.0$ s, **(b)** $x_p = x_s = 197$ m, **(c)** $v_p = 35.8$ m/s]

Some related homework problems: Problem 54, Problem 65

2–7 Freely Falling Objects

The most famous example of motion with constant acceleration is **free fall**—the motion of an object falling freely under the influence of gravity. It was Galileo (1564–1642) who first showed, with his own experiments, that falling bodies move with constant acceleration. His conclusions were based on experiments done by rolling balls down inclines of various steepness. By using an incline, Galileo was able to reduce the acceleration of the balls, thus producing motion slow enough to be timed with the rather crude instruments available.

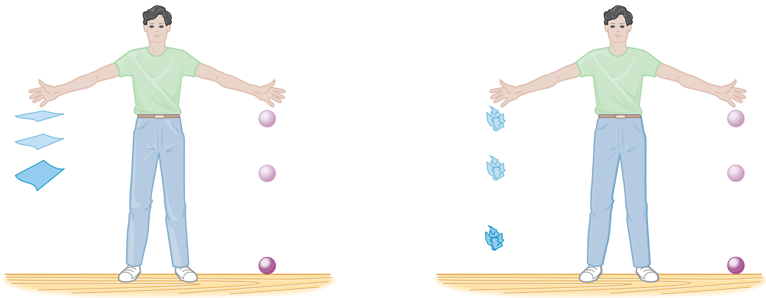
Galileo also pointed out that objects of different weight fall with the *same* constant acceleration—provided air resistance is small enough to be ignored. Whether he dropped objects from the Leaning Tower of Pisa to demonstrate this fact, as legend has it, will probably never be known for certain, but we do know that he performed extensive experiments to support his claim.



▲ In the absence of air resistance, all bodies fall with the same acceleration, regardless of their mass.

Today it is easy to verify Galileo's assertion by dropping objects in a vacuum chamber, where the effects of air resistance are essentially removed. In a standard classroom demonstration, a feather and a coin are dropped in a vacuum, and both fall at the same rate. In 1971, a novel version of this experiment was carried out on the Moon by astronaut David Scott. In the near-perfect vacuum on the Moon's surface he dropped a feather and a hammer and showed a worldwide television audience that they fell to the ground in the same time.

To illustrate the effect of air resistance in everyday terms, consider dropping a sheet of paper and a rubber ball (**Figure 2–16**). The paper drifts slowly to the ground, taking much longer to fall than the ball. Now, wad the sheet of paper into a tight ball and repeat the experiment. This time the ball of paper and the rubber ball reach the ground in nearly the same time. What was different in the two experiments? Clearly, when the sheet of paper was wadded into a ball, the effect of air resistance on it was greatly reduced, so that both objects fell almost as they would in a vacuum.



◀ **FIGURE 2–16 Free fall and air resistance**

(a) Dropping a sheet of paper and a rubber ball

(b) Dropping a wadded-up sheet of paper and a rubber ball

Before considering a few examples, let's first discuss exactly what is meant by "free fall." To begin, the word *free* in free fall means free from any effects other than gravity. For example, in free fall we assume that an object's motion is not influenced by any form of friction or air resistance.

- Free fall is the motion of an object subject *only* to the influence of gravity.

Though free fall is an idealization—which does not apply to many real-world situations—it is still a useful approximation in many other cases. In the following examples we assume that the motion may be considered as free fall.

▲ Whether she is on the way up, at the peak of her flight, or on the way down, this girl is in free fall, accelerating downward with the acceleration of gravity. Only when she is in contact with the blanket does her acceleration change.

Next, it should be realized that the word *fall* in free fall does not mean the object is necessarily moving downward. By free fall, we mean *any* motion under the influence of gravity alone. If you drop a ball, it is in free fall. If you throw a ball upward or downward, it is in free fall as soon as it leaves your hand.

- An object is in free fall as soon as it is released, whether it is dropped from rest, thrown downward, or thrown upward.

Finally, the acceleration produced by gravity on the Earth's surface (sometimes called the gravitational strength) is denoted with the symbol g. As a shorthand

TABLE 2–5 Values of g at Different Locations on Earth (m/s^2)

Location	Latitude	g
North Pole	90° N	9.832
Oslo, Norway	60° N	9.819
Hong Kong	30° N	9.793
Quito, Ecuador	0°	9.780

name, we will frequently refer to g simply as "the acceleration due to gravity." In fact, as we shall see in Chapter 12, the value of g varies according to one's location on the surface of the Earth, as well as one's altitude above it. Table 2–5 shows how g varies with latitude.

In all the calculations that follow in this book, we shall use $g = 9.81\ m/s^2$ for the acceleration due to gravity. Note, in particular, that g always stands for $+9.81\ m/s^2$, never $-9.81\ m/s^2$. For example, if we choose a coordinate system with the positive direction upward, the acceleration in free fall is $a = -g$. If the positive direction is downward, then free-fall acceleration is $a = g$.

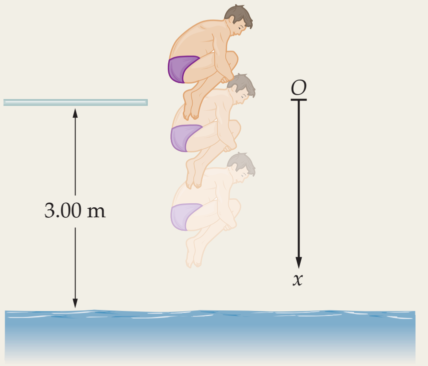
With these comments, we are ready to explore a variety of free-fall examples.

EXAMPLE 2–10 DO THE CANNONBALL!

A person steps off the end of a 3.00-m-high diving board and drops to the water below. **(a)** How long does it take for the person to reach the water? **(b)** What is the person's speed on entering the water?

PICTURE THE PROBLEM

In our sketch we choose the origin to be at the height of the diving board, and we let the positive direction be downward. With these choices, $x_0 = 0$, $a = g$, and the water is at $x = 3.00$ m. Of course, $v_0 = 0$ since the person simply steps off the board.



STRATEGY

We can neglect air resistance in this case and model the motion as free fall. This means we can assume a constant acceleration equal to g and use the equations of motion in Table 2–4. For part (a) we want to find the time of fall when we know the distance and acceleration, so we use Equation 2–11. For part (b) we can relate velocity to time by using Equation 2–7, or we can relate velocity to position by using Equation 2–12. We will implement both approaches and show that the results are the same.

SOLUTION

Part (a)

1. Write Equation 2–11 with $x_0 = 0$, $v_0 = 0$, and $a = g$: $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

2. Solve for the time, t, and set $x = 3.00$ m: $t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(3.00\ m)}{9.81\ m/s^2}} = 0.782\ s$

Part (b)

3. Use the time found in part (a) in Equation 2–7: $v = v_0 + gt = 0 + (9.81\ m/s^2)(0.782\ s) = 7.67\ m/s$

4. We can also find the velocity without knowing the time by using Equation 2–12 with $\Delta x = 3.00$ m: $v^2 = v_0^{\,2} + 2a\Delta x = 0 + 2g\Delta x$

$$v = \sqrt{2g\Delta x} = \sqrt{2(9.81\ m/s^2)(3.00\ m)} = 7.67\ m/s$$

INSIGHT

Let's put these results in more common, everyday units. If you step off a diving board 9.84 ft (3.00 m) above the water, you enter the water with a speed of 17.2 mi/h (7.67 m/s).

PRACTICE PROBLEM

What is your speed on entering the water if you step off a 10.0-m diving tower? [**Answer:** $v = \sqrt{2(9.81\ m/s^2)(10.0\ m)} = 14.0\ m/s = 31.4\ mi/h$]

Some related homework problems: Problem 71, Problem 83

The special case of free fall from rest occurs so frequently, and in so many different contexts, that it deserves special attention. If we take x_0 to be zero, and positive to be downward, then position as a function of time is $x = x_0 + v_0 t + \frac{1}{2}gt^2 = 0 + 0 + \frac{1}{2}gt^2$, or

$$x = \tfrac{1}{2}gt^2 \qquad 2\text{–}13$$

Similarly, velocity as a function of time is

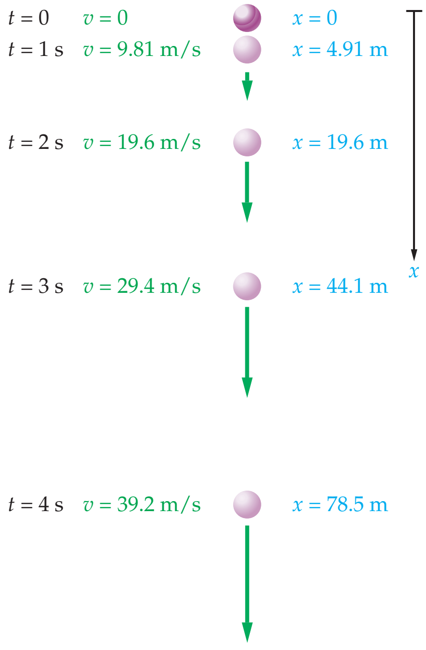
$$v = gt \qquad 2\text{–}14$$

and velocity as a function of position is

$$v = \sqrt{2gx} \qquad \text{2–15}$$

The behavior of these functions is illustrated in **Figure 2–17**. Note that position increases with time squared, whereas velocity increases linearly with time.

Next we consider two objects dropped from rest, one after the other, and discuss how their separation varies with time.

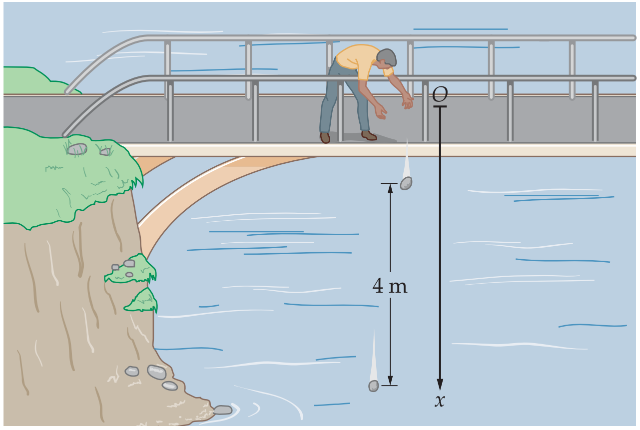


▲ **FIGURE 2–17 Free fall from rest**
Position and velocity are shown as functions of time. It is apparent that velocity depends linearly on t, whereas position depends on t^2.

CONCEPTUAL CHECKPOINT 2–5

FREE-FALL SEPARATION

You drop a rock from a bridge to the river below. When the rock has fallen 4 m, you drop a second rock. As the rocks continue their free fall, does their separation **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

It might seem that since both rocks are in free fall, their separation remains the same. This is not so. The rock that has a head start always has a greater velocity than the later one; thus it covers a greater distance in any interval of time. As a result, the separation between the rocks increases.

ANSWER

(a) The separation between the rocks increases.

PROBLEM-SOLVING NOTE

Check Your Solution

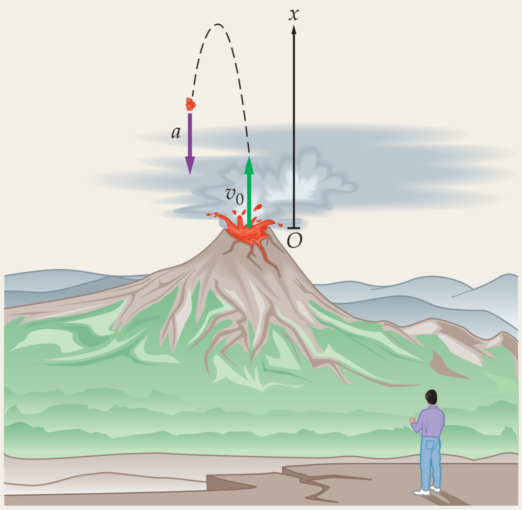
Once you have a solution to a problem, check to see whether it makes sense. First, make sure the units are correct; m/s for speed, m/s^2 for acceleration, and so on. Second, check the numerical value of your answer. If you are solving for the speed of a diver dropping from a 3.0-m diving board and you get an unreasonable value like 200 m/s ($\approx$450 mi/h), chances are good that you've made a mistake.

An erupting volcano shooting out fountains of lava is an impressive sight. In the next Example we show how a simple timing experiment can determine the initial velocity of the erupting lava.

EXAMPLE 2–11 BOMBS AWAY: CALCULATING THE SPEED OF A LAVA BOMB

REAL-WORLD PHYSICS

A volcano shoots out blobs of molten lava, called lava bombs, from its summit. A geologist observing the eruption uses a stopwatch to time the flight of a particular lava bomb that is projected straight upward. If the time for it to rise and fall back to its launch height is 4.75 s, and its acceleration is 9.81 m/s^2 downward, what is its initial speed?



PICTURE THE PROBLEM

Our sketch shows a coordinate system with upward as the positive x direction. For clarity, we offset the upward and downward trajectories slightly. In addition, we choose $t = 0$ to be the time at which the lava bomb is launched. With these choices it follows that $x_0 = 0$ and the acceleration is $a = -g = -9.81$ m/s^2. The initial speed to be determined is v_0.

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STRATEGY

Once again, we can neglect air resistance and model the motion of the lava bomb as free fall—this time with an initial upward velocity. We know that the lava bomb starts at $x = 0$ at the time $t = 0$ and returns to $x = 0$ at the time $t = 4.75$ s. This means that we know the bomb's position, time, and acceleration $(a = -g)$, from which we would like to determine the initial velocity. A reasonable approach is to use Equation 2–11 and solve it for the one unknown it contains, v_0.

SOLUTION

1. Write out $x = x_0 + v_0 t + \frac{1}{2}at^2$ with $x_0 = 0$ and $a = -g$. Factor out a time, t, from the two remaining terms:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 = v_0 t - \tfrac{1}{2}gt^2 = \left(v_0 - \tfrac{1}{2}gt\right)t$$

2. Set x equal to zero, since this is the position of the lava bomb at $t = 0$ and $t = 4.75$ s:

$x = \left(v_0 - \frac{1}{2}gt\right)t = 0$ two solutions:

(i) $t = 0$

(ii) $v_0 - \frac{1}{2}gt = 0$

3. The first solution is simply the initial condition; that is, $x = 0$ at $t = 0$. Solve the second solution for the initial speed:

$$v_0 - \tfrac{1}{2}gt = 0 \quad \text{or} \quad v_0 = \tfrac{1}{2}gt$$

4. Substitute numerical values for g and the time the lava bomb lands:

$$v_0 = \tfrac{1}{2}gt = \tfrac{1}{2}(9.81\text{ m/s}^2)(4.75\text{ s}) = 23.3\text{ m/s}$$

INSIGHT

A geologist can determine a lava bomb's initial speed by simply observing its flight time. Knowing the lava bomb's initial speed can help geologists determine how severe a volcanic eruption will be, and how dangerous it might be to people in the surrounding area.

PRACTICE PROBLEM

A second lava bomb is projected straight upward with an initial speed of 25 m/s. How long is it in the air? [**Answer:** $t = 5.1$ s]

Some related homework problems: Problem 73, Problem 86

▲ In the absence of air resistance, these lava bombs from the Kilauea volcano on the big island of Hawaii would strike the water with the same speed they had when they were blasted into the air.

What is the speed of a lava bomb when it returns to Earth; that is, when it returns to the same level from which it was launched? Physical intuition might suggest that, in the absence of air resistance, it should be the same as the initial speed. To show that this hypothesis is indeed correct, write out Equation 2–7 for this case:

$$v = v_0 - gt$$

Substituting numerical values, we find

$$v = v_0 - gt = 23.3\text{ m/s} - (9.81\text{ m/s}^2)(4.75\text{ s}) = -23.3\text{ m/s}$$

Thus, the velocity of the lava when it lands is just the negative of the velocity it had when launched upward. Or put another way, when the lava lands, it has the same speed as when it was launched; it's just traveling in the opposite direction.

It is instructive to verify this result symbolically. Recall from Example 2–11 that $v_0 = \frac{1}{2}gt$, where t is the time the bomb lands. Substituting this result into Equation 2–7 we find

$$v = \tfrac{1}{2}gt - gt = -\tfrac{1}{2}gt = -v_0$$

The advantage of the symbolic solution lies in showing that the result is not a fluke—no matter what the initial velocity, no matter what the acceleration, the bomb lands with the velocity $-v_0$.

These results hint at a symmetry relating the motion on the way up to the motion on the way down. To make this symmetry more apparent, we first solve for the time when the lava bomb lands. Using the result $v_0 = \frac{1}{2}gt$ from Example 2–11, we find

$$t = \frac{2v_0}{g} \qquad \textit{(time of landing)}$$

Next, we find the time when the velocity of the lava is zero, which is at its highest point. Setting $v = 0$ in Equation 2–7, we have $v = v_0 - gt = 0$, or

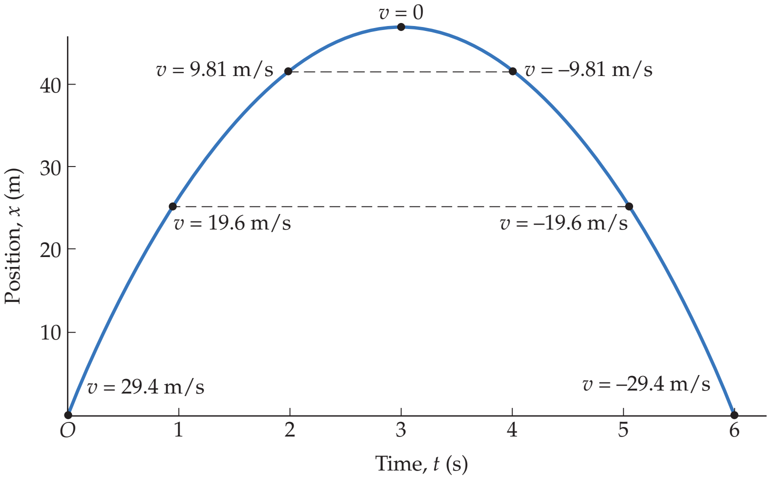
$$t = \frac{v_0}{g} \qquad \textit{(time when } v = 0\textit{)}$$

Note that this is exactly half the time required for the lava to make the round trip. Thus, the velocity of the lava is zero and the height of the lava is greatest exactly halfway between launch and landing.

This symmetry is illustrated in **Figure 2–18**. In this case we consider a lava bomb that is in the air for 6.00 s, moving without air resistance. Note that at $t = 3.00$ s the lava is at its highest point and its velocity is zero. At times equally spaced before and after $t = 3.00$ s, the lava is at the same height, has the same speed, but is moving in opposite directions. As a result of this symmetry, a movie of the lava bomb's flight would look the same whether run forward or in reverse.

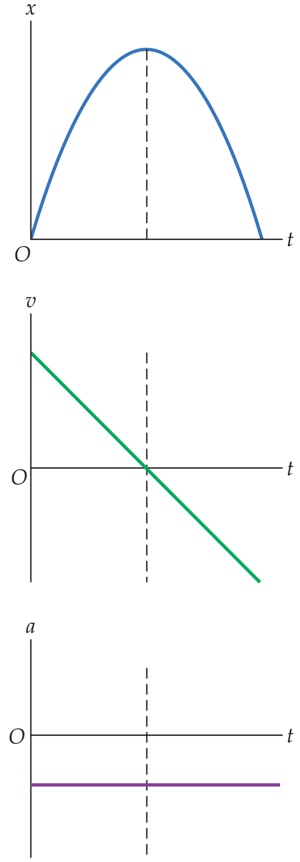
Figure 2–19 shows the time dependence of position, velocity, and acceleration for an object in free fall without air resistance after being thrown upward. As soon as the object is released, it begins to accelerate downward—as indicated by the negative slope of the velocity-versus-time plot—though it isn't necessarily moving downward. For example, if you throw a ball *upward* it begins to accelerate *downward* the moment it leaves your hand. It continues moving upward, however, until its speed diminishes to zero. Since gravity is causing the downward acceleration, and gravity doesn't turn off just because the ball's velocity goes through zero, the ball continues to accelerate downward even when it is momentarily at rest.

Similarly, in the next Example we consider a sandbag that falls from an ascending hot-air balloon. This means that before the bag is in free fall it was moving upward—just like a ball thrown upward. And just like the ball, the sandbag continues moving upward for a brief time before momentarily stopping and then moving downward.



▲ **FIGURE 2–18 Position and velocity of a lava bomb**

This lava bomb is in the air for 6 seconds. Note the symmetry about the midpoint of the bomb's flight.



▲ **FIGURE 2–19 Position, velocity, and acceleration of a lava bomb as functions of time**

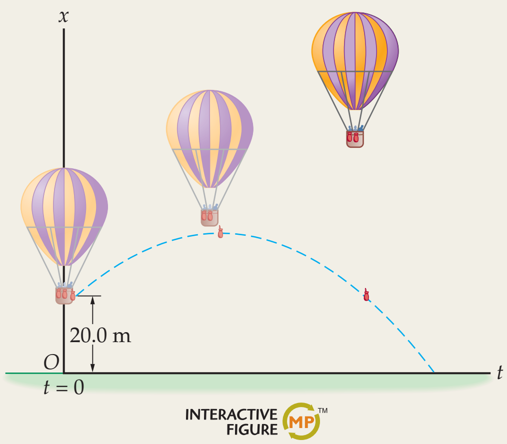
The fact that x versus t is curved indicates an acceleration; the downward curvature shows that the acceleration is negative. This is also clear from v versus t, which has a negative slope. The constant slope of the straight line in the v-versus-t plot indicates a constant acceleration, as shown in the a-versus-t plot.

EXAMPLE 2–12 LOOK OUT BELOW! A SANDBAG IN FREE FALL

A hot-air balloon is rising straight upward with a constant speed of 6.5 m/s. When the basket of the balloon is 20.0 m above the ground, a bag of sand tied to the basket comes loose. **(a)** How long is the bag of sand in the air before it hits the ground? **(b)** What is the greatest height of the bag of sand during its fall to the ground?

PICTURE THE PROBLEM

We choose the origin to be at ground level and positive to be upward. This means that, for the bag, we have $x_0 = 20.0$ m, $v_0 = 6.5$ m/s, and $a = -g$. Our sketch also shows snapshots of the balloon and bag of sand at three different times, starting at $t = 0$ when the bag comes loose. Note that the bag is moving upward with the balloon at the time it comes loose. It therefore continues to move upward for a short time after it separates from the basket, exactly as if it had been thrown upward.



STRATEGY

The effects of air resistance on the sandbag can be ignored. As a result, we can use the equations in Table 2–4 with a constant acceleration $a = -g$.

In part (a) we want to relate position and time—knowing the initial position and initial velocity—so we use Equation 2–11. To find the time the bag hits the ground, we set $x = 0$ and solve for t.

For part (b) we have no expression that gives the maximum height of a particle—so we will have to come up with something on our own. We can start with the fact that $v = 0$ at the greatest height, since it is there the bag momentarily stops as it changes direction. Therefore, we can find the time t when $v = 0$ by using Equation 2–7, and then substitute t into Equation 2–11 to find x_{max}.

SOLUTION

Part (a)

1. Apply Equation 2–11 to the bag of sand, where x_0 and v_0 have the values given. Set $x = 0$:

$$x = x_0 + v_0 t - \tfrac{1}{2}gt^2 = 0$$

2. Note that we have a quadratic equation for t in the form $v_f = v_i$ where $A = -\frac{1}{2}g$, $B = v_0$, and $C = x_0$. Solve this equation for t. The positive solution, 2.78 s, applies to this problem: (Quadratic equations and their solutions are discussed in Appendix A. In general, one can expect two solutions to a quadratic equation.)

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(-\frac{1}{2}g\right)(x_0)}}{2\left(-\frac{1}{2}g\right)}$$

$$= \frac{-(6.5\ \mathrm{m/s}) \pm \sqrt{(6.5\ \mathrm{m/s})^2 + 2(9.81\ \mathrm{m/s^2})(20.0\ \mathrm{m})}}{(-9.81\ \mathrm{m/s^2})}$$

$$= \frac{-(6.5\ \mathrm{m/s}) \pm 20.8\ \mathrm{m/s}}{(-9.81\ \mathrm{m/s^2})} = 2.78\ \mathrm{s},\ -1.46\ \mathrm{s}$$

Part (b)

3. Apply Equation 2–7 to the bag of sand, then find the time when the velocity equals zero:

$$v = v_0 + at = v_0 - gt$$

$$v_0 - gt = 0 \qquad \text{or} \qquad t = \frac{v_0}{g} = \frac{6.5\ \mathrm{m/s}}{9.81\ \mathrm{m/s^2}} = 0.66\ \mathrm{s}$$

4. Use $t = 0.66$ s in Equation 2–11 to find the maximum height:

$$x_{max} = 20.0\ \mathrm{m} + (6.5\ \mathrm{m/s})\ (0.66\ \mathrm{s}) - \tfrac{1}{2}(9.81\ \mathrm{m/s^2})\ (0.66\ \mathrm{s})^2$$
$$= 22\ \mathrm{m}$$

INSIGHT

The positive solution to the quadratic equation is certainly the one that applies here, but the negative solution is not completely without meaning. What physical meaning might it have? Well, if the balloon had been *descending* with a speed of 6.5 m/s, instead of rising, then the time for the bag to reach the ground would have been 1.46 s. Try it! Let $v_0 = -6.5$ m/s and repeat the calculation given in part (a).

PRACTICE PROBLEM

What is the velocity of the bag of sand just before it hits the ground? [**Answer:** $v = v_0 - gt = (6.5\ \mathrm{m/s}) - (9.81\ \mathrm{m/s^2})(2.78\ \mathrm{s}) = -20.8\ \mathrm{m/s}$; the minus sign indicates the bag is moving downward.]

Some related homework problems: Problem 90, Problem 107

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In this chapter we have made extensive use of the sign conventions for one-dimensional vectors—positive for one direction, negative for the opposite direction—as introduced in Chapter 1. See, for example, the positive and negative velocities in Figure 2–18.

We have been careful to check the dimensional consistency of our equations in this chapter. For example, the discussion following Equation 2–11 shows that all the terms in that equation have the dimensions of length.

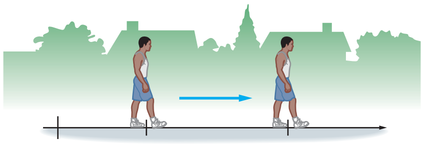
LOOKING AHEAD

The distinctions developed in this chapter between velocity and acceleration will play a key role in our understanding of Newton's laws of motion in Chapters 5 and 6, and everywhere else that Newton's laws are used throughout the text.

The equations developed for motion with constant acceleration in this chapter (Equations 2–7, 2–10, 2–11, and 2–12) will be used again with slightly different symbols when we study rotational motion in Chapter 10. See, in particular, Equations 10–8, 10–9, 10–10, and 10–11.

CHAPTER SUMMARY

2–1 POSITION, DISTANCE, AND DISPLACEMENT



Distance
Total length of travel, from beginning to end. The distance is always positive.

Displacement
Displacement, Δx, is the change in position:

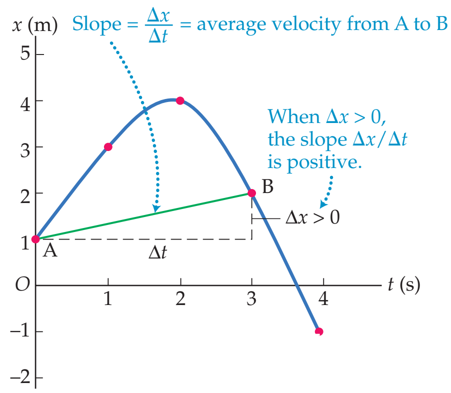
$$\Delta x = x_f - x_i \qquad 2\text{–}1$$

When calculating displacement, it is important to remember that it is always *final* position minus *initial* position—never the other way. Displacement can be positive, negative, or zero.

Positive and Negative Displacement
The *sign* of the displacement indicates the *direction* of motion. For example, suppose we choose the positive direction to be to the right. Then $\Delta x > 0$ means motion to the right, and $\Delta x < 0$ means motion to the left.

Units
The SI unit of distance and displacement is the meter, m.

2–2 AVERAGE SPEED AND VELOCITY



Average Speed
Average speed is *distance* divided by elapsed time:

$$\text{average speed} = \text{distance/time} \qquad 2\text{–}2$$

Average speed is never negative.

Average Velocity
Average velocity, v_{av}, is *displacement* divided by time:

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \qquad 2\text{–}3$$

Average velocity is positive if motion is in the positive direction, and negative if motion is in the negative direction.

Graphical Interpretation of Velocity
In an x-versus-t plot, the average velocity is the slope of a line connecting two points.

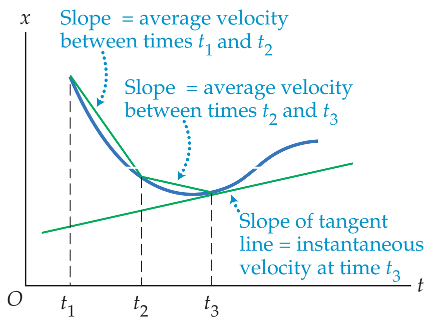
Units
The SI unit of speed and velocity is meters per second, m/s.

2–3 INSTANTANEOUS VELOCITY

The velocity at an instant of time is the limit of the average velocity over shorter and shorter time intervals:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{2–4}$$

Instantaneous velocity can be positive, negative, or zero, with the sign indicating the direction of motion.



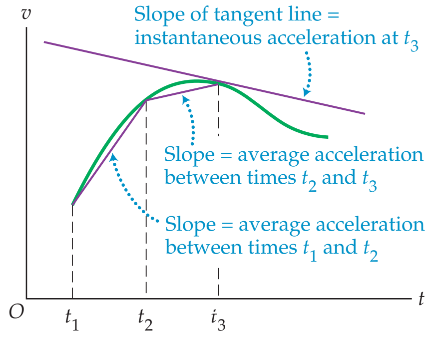
Constant Velocity
When velocity is constant, the instantaneous velocity is equal to the average velocity.

Graphical Interpretation
In an x-versus-t plot, the instantaneous velocity at a given time is equal to the slope of the tangent line at that time.

2–4 ACCELERATION

Average Acceleration
Average acceleration is the change in velocity divided by the change in time:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \tag{2–5}$$



Average acceleration is positive if $v_f > v_i$, is negative if $v_f < v_i$, and is zero if $v_f = v_i$.

Instantaneous Acceleration
Instantaneous acceleration is the limit of the average acceleration as the time interval goes to zero:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} \tag{2–6}$$

Instantaneous acceleration can be positive, negative, or zero, depending on whether the velocity is becoming more positive, more negative, or is staying the same. Knowing the sign of the acceleration *does not* tell you whether an object is speeding up or slowing down, and it *does not* give the direction of motion.

Constant Acceleration
When acceleration is constant, the instantaneous acceleration is equal to the average acceleration.

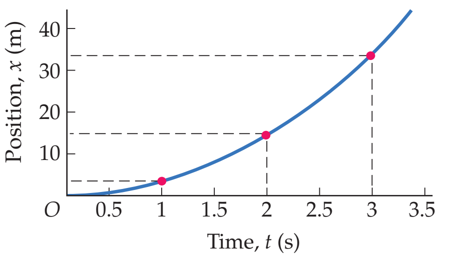
Deceleration
An object whose speed is decreasing is said to be decelerating. Deceleration occurs whenever the velocity and acceleration have opposite signs.

Graphical Interpretation
In a v-versus-t plot, the instantaneous acceleration is equal to the slope of the tangent line at a given time.

Units
The SI unit of acceleration is meters per second per second, or m/s^2.

2–5 MOTION WITH CONSTANT ACCELERATION

Several different "equations of motion" describe particles moving with constant acceleration. Each equation relates a different set of variables:



Velocity as a Function of Time

$$v = v_0 + at \tag{2–7}$$

Initial, Final, and Average Velocity

$$v_{av} = \tfrac{1}{2}(v_0 + v) \tag{2–9}$$

Position as a Function of Time and Velocity

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t \tag{2–10}$$

Position as a Function of Time and Acceleration

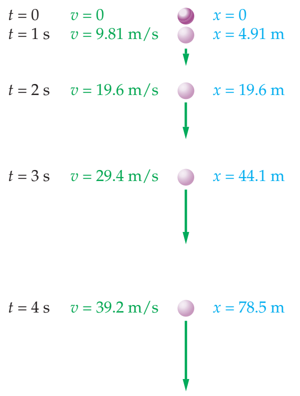
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2 \tag{2–11}$$

Velocity as a Function of Position

$$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x \tag{2–12}$$

2–7 FREELY FALLING OBJECTS

Objects in free fall move under the influence of gravity alone. An object is in free fall as soon as it is released, whether it is thrown upward, thrown downward, or released from rest.



Acceleration Due to Gravity

The acceleration due to gravity on the Earth's surface varies slightly from place to place. In this book we shall define the acceleration of gravity to have the following magnitude:

$$g = 9.81\ \text{m/s}^2$$

Note that g is always a positive quantity. If we choose the positive direction of our coordinate system to be downward (in the direction of the acceleration of gravity), it follows that the acceleration of an object in free fall is $a = +g$. On the other hand, if we choose our positive direction to be upward, the acceleration of a freely falling object is in the negative direction; hence $a = -g$.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Relate velocity to time.	In motion with uniform acceleration a, the velocity changes with time as $v = v_0 + at$ (Equation 2–7).	Examples 2–5, 2–8, 2–9, 2–10, 2–11, 2–12
Relate velocity to position.	If an object with an initial velocity v_0 accelerates with a uniform acceleration a for a distance Δx, the final velocity, v, is given by $v^2 = v_0{}^2 + 2a\Delta x$ (Equation 2–12).	Examples 2–7, 2–8, 2–10
Relate position to time.	The position of an object moving with constant acceleration a varies with time as follows: $x = x_0 + \frac{1}{2}(v_0 + v)t$ (Equation 2–10) or equivalently $x = x_0 + v_0t + \frac{1}{2}at^2$ (Equation 2–11).	Examples 2–5, 2–6, 2–9, 2–10, 2–11, 2–12

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)
(The effects of air resistance are to be ignored in this chapter.)

1. You and your dog go for a walk to a nearby park. On the way, your dog takes many short side trips to chase squirrels, examine fire hydrants, and so on. When you arrive at the park, do you and your dog have the same displacement? Have you traveled the same distance? Explain.
2. Does an odometer in a car measure distance or displacement? Explain.
3. Can you drive your car in such a way that the distance it covers is **(a)** greater than, **(b)** equal to, or **(c)** less than the magnitude of its displacement? In each case, give an example if your answer is yes, explain why not if your answer is no.
4. An astronaut orbits Earth in the space shuttle. In one complete orbit, is the magnitude of the displacement the same as the distance traveled? Explain.
5. After a tennis match the players dash to the net to congratulate one another. If they both run with a speed of 3 m/s, are their velocities equal? Explain.
6. Does a speedometer measure speed or velocity? Explain.
7. Is it possible for a car to circle a race track with constant velocity? Can it do so with constant speed? Explain.
8. Friends tell you that on a recent trip their average velocity was +20 m/s. Is it possible that their instantaneous velocity was negative at any time during the trip? Explain.
9. For what kind of motion are the instantaneous and average velocities equal?
10. If the position of an object is zero, does its speed have to be zero? Explain.
11. Assume that the brakes in your car create a constant deceleration, regardless of how fast you are going. If you double your driving speed, how does this affect **(a)** the time required to come to a stop, and **(b)** the distance needed to stop?
12. The velocity of an object is zero at a given instant of time. **(a)** Is it possible for the object's acceleration to be zero at this time? Explain. **(b)** Is it possible for the object's acceleration to be nonzero at this time? Explain.
13. If the velocity of an object is nonzero, can its acceleration be zero? Give an example if your answer is yes, explain why not if your answer is no.
14. Is it possible for an object to have zero average velocity over a given interval of time, yet still be accelerating during the interval? Give an example if your answer is yes, explain why not if your answer is no.
15. A batter hits a pop fly straight up. **(a)** Is the acceleration of the ball on the way up different from its acceleration on the way down? **(b)** Is the acceleration of the ball at the top of its flight different from its acceleration just before it lands?

16. A person on a trampoline bounces straight upward with an initial speed of 4.5 m/s. What is the person's speed when she returns to her initial height?

17. After winning a baseball game, one player drops a glove, while another tosses a glove into the air. How do the accelerations of the two gloves compare?

18. A volcano shoots a lava bomb straight upward. Does the displacement of the lava bomb depend on **(a)** your choice of origin for your coordinate system, or **(b)** your choice of a positive direction? Explain in each case.

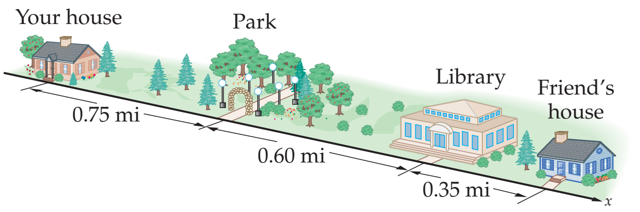
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(The effects of air resistance are to be ignored in this chapter.)

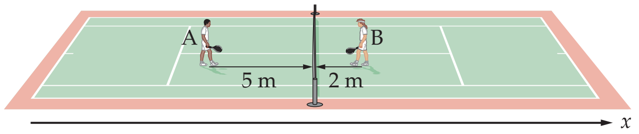
SECTION 2–1 POSITION, DISTANCE, AND DISPLACEMENT

1. • Referring to **Figure 2–20**, you walk from your home to the library, then to the park. **(a)** What is the distance traveled? **(b)** What is your displacement?



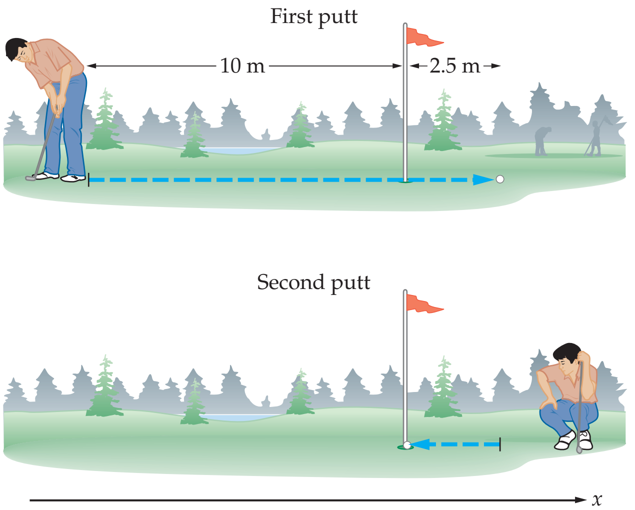
▲ **FIGURE 2–20** Problems 1 and 4

2. • The two tennis players shown in **Figure 2–21** walk to the net to congratulate one another. **(a)** Find the distance traveled and the displacement of player A. **(b)** Repeat for player B.



▲ **FIGURE 2–21** Problem 2

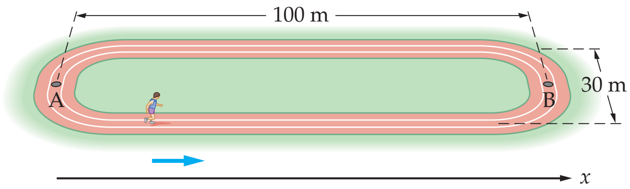
3. • The golfer in **Figure 2–22** sinks the ball in two putts, as shown. What are **(a)** the distance traveled by the ball, and **(b)** the displacement of the ball?



▲ **FIGURE 2–22** Problem 3

4. • In Figure 2–20, you walk from the park to your friend's house, then back to your house. What is your **(a)** distance traveled, and **(b)** displacement?

5. • A jogger runs on the track shown in **Figure 2–23**. Neglecting the curvature of the corners, **(a)** what is the distance traveled and the displacement in running from point A to point B? **(b)** Find the distance and displacement for a complete circuit of the track.



▲ **FIGURE 2–23** Problem 5

6. •• **IP** A child rides a pony on a circular track whose radius is 4.5 m. **(a)** Find the distance traveled and the displacement after the child has gone halfway around the track. **(b)** Does the distance traveled increase, decrease, or stay the same when the child completes one circuit of the track? Explain. **(c)** Does the displacement increase, decrease, or stay the same when the child completes one circuit of the track? Explain. **(d)** Find the distance and displacement after a complete circuit of the track.

SECTION 2–2 AVERAGE SPEED AND VELOCITY

7. • **CE Predict/Explain** You drive your car in a straight line at 15 m/s for 10 kilometers, then at 25 m/s for another 10 kilometers. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:
   I. More time is spent at 15 m/s than at 25 m/s.
   II. The average of 15 m/s and 25 m/s is 20 m/s.
   III. Less time is spent at 15 m/s than at 25 m/s.

8. • **CE Predict/Explain** You drive your car in a straight line at 15 m/s for 10 minutes, then at 25 m/s for another 10 minutes. **(a)** Is your average speed for the entire trip more than, less than, or equal to 20 m/s? **(b)** Choose the *best* explanation from among the following:
   I. More time is required to drive at 15 m/s than at 25 m/s.
   II. Less distance is covered at 25 m/s than at 15 m/s.
   III. Equal time is spent at 15 m/s and 25 m/s.

9. • Joseph DeLoach of the United States set an Olympic record in 1988 for the 200-meter dash with a time of 19.75 seconds. What was his average speed? Give your answer in meters per second and miles per hour.

10. • In 1992 Zhuang Yong of China set a women's Olympic record in the 100-meter freestyle swim with a time of 54.64 seconds. What was her average speed in m/s and mi/h?

11. • **BIO** Kangaroos have been clocked at speeds of 65 km/h. **(a)** How far can a kangaroo hop in 3.2 minutes at this speed? **(b)** How long will it take a kangaroo to hop 0.25 km at this speed?

12. • **Rubber Ducks** A severe storm on January 10, 1992, caused a cargo ship near the Aleutian Islands to spill 29,000 rubber ducks and other bath toys into the ocean. Ten months later hundreds of rubber ducks began to appear along the shoreline near Sitka, Alaska, roughly 1600 miles away. What was the approximate average speed of the ocean current that carried the ducks to shore in **(a)** m/s and **(b)** mi/h? (Rubber ducks from the same spill began to appear on the coast of Maine in July 2003.)

13. • Radio waves travel at the speed of light, approximately 186,000 miles per second. How long does it take for a radio message to travel from Earth to the Moon and back? (See the inside back cover for the necessary data.)

14. • It was a dark and stormy night, when suddenly you saw a flash of lightning. Three-and-a-half seconds later you heard the thunder. Given that the speed of sound in air is about 340 m/s, how far away was the lightning bolt?

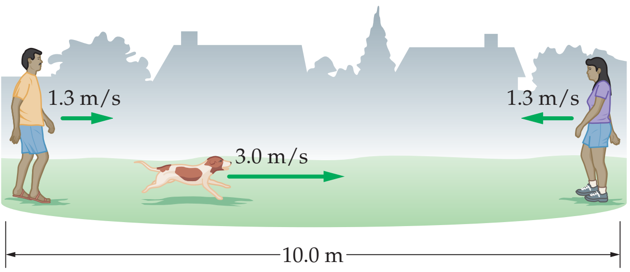
15. • **BIO** **Nerve Impulses** The human nervous system can propagate nerve impulses at about 10^2 m/s. Estimate the time it takes for a nerve impulse generated when your finger touches a hot object to travel to your brain.

16. • Estimate how fast your hair grows in miles per hour.

17. •• A finch rides on the back of a Galapagos tortoise, which walks at the stately pace of 0.060 m/s. After 1.2 minutes the finch tires of the tortoise's slow pace, and takes flight in the same direction for another 1.2 minutes at 12 m/s. What was the average speed of the finch for this 2.4-minute interval?

18. •• You jog at 9.5 km/h for 8.0 km, then you jump into a car and drive an additional 16 km. With what average speed must you drive your car if your average speed for the entire 24 km is to be 22 km/h?

19. •• A dog runs back and forth between its two owners, who are walking toward one another (**Figure 2–24**). The dog starts running when the owners are 10.0 m apart. If the dog runs with a speed of 3.0 m/s, and the owners each walk with a speed of 1.3 m/s, how far has the dog traveled when the owners meet?



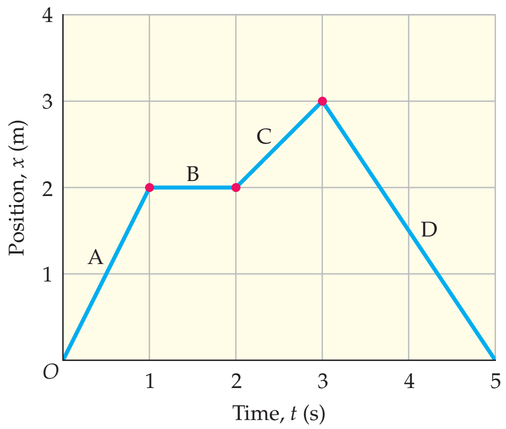
▲ **FIGURE 2–24** Problem 19

20. •• **IP** You drive in a straight line at 20.0 m/s for 10.0 minutes, then at 30.0 m/s for another 10.0 minutes. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

21. •• In heavy rush-hour traffic you drive in a straight line at 12 m/s for 1.5 minutes, then you have to stop for 3.5 minutes, and finally you drive at 15 m/s for another 2.5 minutes. **(a)** Plot a position-versus-time graph for this motion. Your plot should extend from $t = 0$ to $t = 7.5$ minutes. **(b)** Use your plot from part (a) to calculate the average velocity between $t = 0$ and $t = 7.5$ minutes.

22. •• **IP** You drive in a straight line at 20.0 m/s for 10.0 miles, then at 30.0 m/s for another 10.0 miles. **(a)** Is your average speed 25.0 m/s, more than 25.0 m/s, or less than 25.0 m/s? Explain. **(b)** Verify your answer to part (a) by calculating the average speed.

23. •• **IP** An expectant father paces back and forth, producing the position-versus-time graph shown in **Figure 2–25**. Without performing a calculation, indicate whether the father's velocity is positive, negative, or zero on each of the following segments of the graph: **(a)** A, **(b)** B, **(c)** C, and **(d)** D. Calculate the numerical value of the father's velocity for the segments **(e)** A, **(f)** B, **(g)** C, and **(h)** D, and show that your results verify your answers to parts (a)–(d).

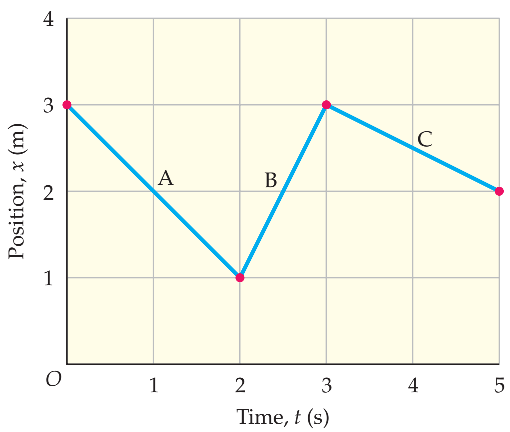


▲ **FIGURE 2–25** Problem 23

24. •• The position of a particle as a function of time is given by $x = (-5\text{ m/s})t + (3\text{ m/s}^2)t^2$. **(a)** Plot x versus t for $t = 0$ to $t = 2$ s. **(b)** Find the average velocity of the particle from $t = 0$ to $t = 1$ s. **(c)** Find the average speed from $t = 0$ to $t = 1$ s.

25. •• The position of a particle as a function of time is given by $x = (6\text{ m/s})t + (-2\text{ m/s}^2)t^2$. **(a)** Plot x versus t for $t = 0$ to $t = 2$ s. **(b)** Find the average velocity of the particle from $t = 0$ to $t = 1$ s. **(c)** Find the average speed from $t = 0$ to $t = 1$ s.

26. •• **IP** A tennis player moves back and forth along the baseline while waiting for her opponent to serve, producing the position-versus-time graph shown in **Figure 2–26. (a)** Without performing a calculation, indicate on which of the segments of the graph, A, B, or C, the player has the greatest speed. Calculate the player's speed for **(b)** segment A, **(c)** segment B, and **(d)** segment C, and show that your results verify your answers to part (a).

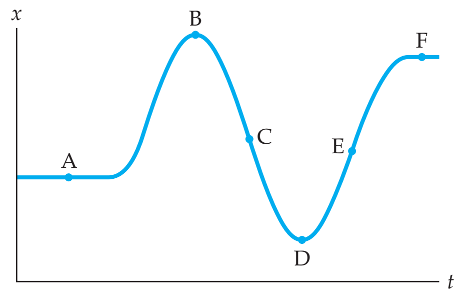


▲ **FIGURE 2–26** Problem 26

27. ••• On your wedding day you leave for the church 30.0 minutes before the ceremony is to begin, which should be plenty of time since the church is only 10.0 miles away. On the way, however, you have to make an unanticipated stop for construction work on the road. As a result, your average speed for the first 15 minutes is only 5.0 mi/h. What average speed do you need for the rest of the trip to get you to the church on time?

SECTION 2–3 INSTANTANEOUS VELOCITY

28. • **CE** The position-versus-time plot of a boat positioning itself next to a dock is shown in **Figure 2–27**. Rank the six points indicated in the plot in order of increasing value of the velocity v, starting with the most negative. Indicate a tie with an equal sign.



▲ **FIGURE 2–27** Problem 28

29. •• The position of a particle as a function of time is given by $x = (2.0\text{ m/s})t + (-3.0\text{ m/s}^3)t^3$. **(a)** Plot x versus t for time from $t = 0$ to $t = 1.0$ s. **(b)** Find the average velocity of the particle from $t = 0.35$ s to $t = 0.45$ s. **(c)** Find the average velocity from $t = 0.39$ s to $t = 0.41$ s. **(d)** Do you expect the instantaneous velocity at $t = 0.40$ s to be closer to 0.54 m/s, 0.56 m/s, or 0.58 m/s? Explain.

30. •• The position of a particle as a function of time is given by $x = (-2.00\text{ m/s})t + (3.00\text{ m/s}^3)t^3$. **(a)** Plot x versus t for time from $t = 0$ to $t = 1.00$ s. **(b)** Find the average velocity of the particle from $t = 0.150$ s to $t = 0.250$ s. **(c)** Find the average velocity from $t = 0.190$ s to $t = 0.210$ s. **(d)** Do you expect the instantaneous velocity at $t = 0.200$ s to be closer to -1.62 m/s, or -1.66 m/s? Explain.

SECTION 2–4 ACCELERATION

31. • **CE Predict/Explain** Two bows shoot identical arrows with the same launch speed. To accomplish this, the string in bow 1 must be pulled back farther when shooting its arrow than the string in bow 2. **(a)** Is the acceleration of the arrow shot by bow 1 greater than, less than, or equal to the acceleration of the arrow shot by bow 2? **(b)** Choose the *best explanation* from among the following:

I. The arrow in bow 2 accelerates for a greater time.
II. Both arrows start from rest.
III. The arrow in bow 1 accelerates for a greater time.

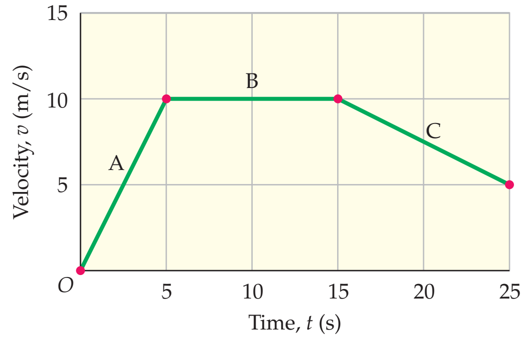
32. • A 747 airliner reaches its takeoff speed of 173 mi/h in 35.2 s. What is the magnitude of its average acceleration?

33. • At the starting gun, a runner accelerates at 1.9 m/s^2 for 5.2 s. The runner's acceleration is zero for the rest of the race. What is the speed of the runner **(a)** at $t = 2.0$ s, and **(b)** at the end of the race?

34. • A jet makes a landing traveling due east with a speed of 115 m/s. If the jet comes to rest in 13.0 s, what are the magnitude and direction of its average acceleration?

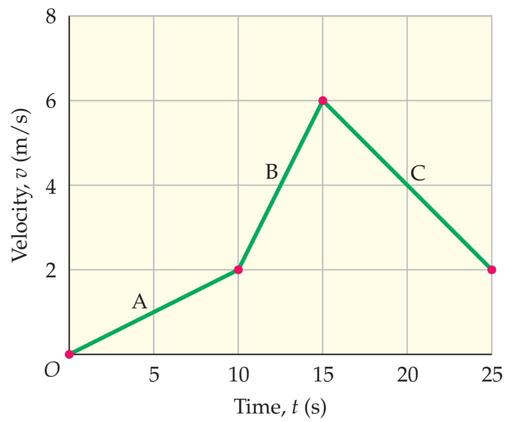
35. • A car is traveling due north at 18.1 m/s. Find the velocity of the car after 7.50 s if its acceleration is **(a)** 1.30 m/s^2 due north, or **(b)** 1.15 m/s^2 due south.

36. •• A motorcycle moves according to the velocity-versus-time graph shown in **Figure 2–28**. Find the average acceleration of the motorcycle during each of the following segments of the motion: **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 2–28** Problem 36

37. •• A person on horseback moves according to the velocity-versus-time graph shown in **Figure 2–29.** Find the displacement of the person for each of the following segments of the motion: **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 2–29** Problem 37

38. •• Running with an initial velocity of +11 m/s, a horse has an average acceleration of -1.81 m/s^2. How long does it take for the horse to decrease its velocity to +6.5 m/s?

39. •• **IP** Assume that the brakes in your car create a constant deceleration of 4.2 m/s^2 regardless of how fast you are driving. If you double your driving speed from 16 m/s to 32 m/s, **(a)** does the time required to come to a stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping times for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

40. •• **IP** In the previous problem, **(a)** does the distance needed to stop increase by a factor of two or a factor of four? Explain. Verify your answer to part (a) by calculating the stopping distances for initial speeds of **(b)** 16 m/s and **(c)** 32 m/s.

41. •• As a train accelerates away from a station, it reaches a speed of 4.7 m/s in 5.0 s. If the train's acceleration remains constant, what is its speed after an additional 6.0 s has elapsed?

42. •• A particle has an acceleration of $+6.24\text{ m/s}^2$ for 0.300 s. At the end of this time the particle's velocity is +9.31 m/s. What was the particle's initial velocity?

SECTION 2–5 MOTION WITH CONSTANT ACCELERATION

43. • Landing with a speed of 81.9 m/s, and traveling due south, a jet comes to rest in 949 m. Assuming the jet slows with constant acceleration, find the magnitude and direction of its acceleration.

44. • When you see a traffic light turn red, you apply the brakes until you come to a stop. If your initial speed was 12 m/s, and you were heading due west, what was your average velocity during braking? Assume constant deceleration.

45. CE •• A ball is released at the point $x = 2$ m on an inclined plane with a nonzero initial velocity. After being released, the ball moves with constant acceleration. The acceleration and initial velocity of the ball are described by one of the following four cases: case 1, $a > 0, v_0 > 0$; case 2, $a > 0, v_0 < 0$; case 3, $a < 0, v_0 > 0$; case 4, $a < 0, v_0 < 0$. **(a)** In which of these cases will the ball definitely pass $x = 0$ at some later time? **(b)** In which of these cases is more information needed to determine whether the ball will cross $x = 0$? **(c)** In which of these cases will the ball come to rest momentarily at some time during its motion?

46. •• Suppose the car in Problem 44 comes to rest in 35 m. How much time does this take?

47. •• Starting from rest, a boat increases its speed to 4.12 m/s with constant acceleration. **(a)** What is the boat's average speed? **(b)** If it takes the boat 4.77 s to reach this speed, how far has it traveled?

48. •• IP BIO A cheetah can accelerate from rest to 25.0 m/s in 6.22 s. Assuming constant acceleration, **(a)** how far has the cheetah run in this time? **(b)** After sprinting for just 3.11 s, is the cheetah's speed 12.5 m/s, more than 12.5 m/s, or less than 12.5 m/s? Explain. **(c)** What is the cheetah's average speed for the first 3.11 s of its sprint? For the second 3.11 s of its sprint? **(d)** Calculate the distance covered by the cheetah in the first 3.11 s and the second 3.11 s.

SECTION 2–6 APPLICATIONS OF THE EQUATIONS OF MOTION

49. • A child slides down a hill on a toboggan with an acceleration of 1.8 m/s^2. If she starts at rest, how far has she traveled in **(a)** 1.0 s, **(b)** 2.0 s, and **(c)** 3.0 s?

50. • The Detonator On a ride called the Detonator at Worlds of Fun in Kansas City, passengers accelerate straight downward from rest to 45 mi/h in 2.2 seconds. What is the average acceleration of the passengers on this ride?

The Detonator (Problem 50)

51. • Air Bags Air bags are designed to deploy in 10 ms. Estimate the acceleration of the front surface of the bag as it expands. Express your answer in terms of the acceleration of gravity g.

52. • Jules Verne In his novel *From the Earth to the Moon* (1866), Jules Verne describes a spaceship that is blasted out of a cannon, called the *Columbiad*, with a speed of 12,000 yards/s. The *Columbiad* is 900 ft long, but part of it is packed with powder, so the spaceship accelerates over a distance of only 700 ft. Estimate the acceleration experienced by the occupants of the spaceship during launch. Give your answer in m/s^2. (Verne realized that the "travelers would . . . encounter a violent recoil," but he probably didn't know that people generally lose consciousness if they experience accelerations greater than about $7g \sim 70$ m/s^2.)

53. •• BIO Bacterial Motion Approximately 0.1% of the bacteria in an adult human's intestines are *Escherichia coli*. These bacteria have been observed to move with speeds up to 15 μm/s and maximum accelerations of 166 μm/s^2. Suppose an *E. coli* bacterium in your intestines starts at rest and accelerates at 156 μm/s^2. How much **(a)** time and **(b)** distance are required for the bacterium to reach a speed of 12 μm/s?

54. •• Two cars drive on a straight highway. At time $t = 0$, car 1 passes mile marker 0 traveling due east with a speed of 20.0 m/s. At the same time, car 2 is 1.0 km east of mile marker 0 traveling at 30.0 m/s due west. Car 1 is speeding up with an acceleration of magnitude 2.5 m/s^2, and car 2 is slowing down with an acceleration of magnitude 3.2 m/s^2. **(a)** Write x-versus-t equations of motion for both cars, taking east as the positive direction. **(b)** At what time do the cars pass next to one another?

55. •• A Meteorite Strikes On October 9, 1992, a 27-pound meteorite struck a car in Peekskill, NY, leaving a dent 22 cm deep in the trunk. If the meteorite struck the car with a speed of 130 m/s, what was the magnitude of its deceleration, assuming it to be constant?

56. •• A rocket blasts off and moves straight upward from the launch pad with constant acceleration. After 3.0 s the rocket is at a height of 77 m. **(a)** What are the magnitude and direction of the rocket's acceleration? **(b)** What is its speed at this time?

57. •• IP You are driving through town at 12.0 m/s when suddenly a ball rolls out in front of you. You apply the brakes and begin decelerating at 3.5 m/s^2. **(a)** How far do you travel before stopping? **(b)** When you have traveled only half the distance in part (a), is your speed 6.0 m/s, greater than 6.0 m/s, or less than 6.0 m/s? Support your answer with a calculation.

58. •• IP You are driving through town at 16 m/s when suddenly a car backs out of a driveway in front of you. You apply the brakes and begin decelerating at 3.2 m/s^2. **(a)** How much time does it take to stop? **(b)** After braking half the time found in part (a), is your speed 8.0 m/s, greater than 8.0 m/s, or less than 8.0 m/s? Support your answer with a calculation. **(c)** If the car backing out was initially 55 m in front of you, what is the maximum reaction time you can have before hitting the brakes and still avoid hitting the car?

59. •• IP BIO A Tongue's Acceleration When a chameleon captures an insect, its tongue can extend 16 cm in 0.10 s. **(a)** Find the magnitude of the tongue's acceleration, assuming it to be constant. **(b)** In the first 0.050 s, does the tongue extend 8.0 cm, more than 8.0 cm, or less than 8.0 cm? Support your conclusion with a calculation.

It's not polite to reach! (Problem 59)

60. •• **IP** Coasting due west on your bicycle at 8.4 m/s, you encounter a sandy patch of road 7.2 m across. When you leave the sandy patch your speed has been reduced by 2.0 m/s to 6.4 m/s. **(a)** Assuming the sand causes a constant acceleration, what was the bicycle's acceleration in the sandy patch? Give both magnitude and direction. **(b)** How long did it take to cross the sandy patch? **(c)** Suppose you enter the sandy patch with a speed of only 5.4 m/s. Is your final speed in this case 3.4 m/s, more than 3.4 m/s, or less than 3.4 m/s? Explain.

61. •• **BIO Surviving a Large Deceleration** On July 13, 1977, while on a test drive at Britain's Silverstone racetrack, the throttle on David Purley's car stuck wide open. The resulting crash subjected Purley to the greatest "g-force" ever survived by a human—he decelerated from 173 km/h to zero in a distance of only about 0.66 m. Calculate the magnitude of the acceleration experienced by Purley (assuming it to be constant), and express your answer in units of the acceleration of gravity, $g = 9.81\ \mathrm{m/s^2}$.

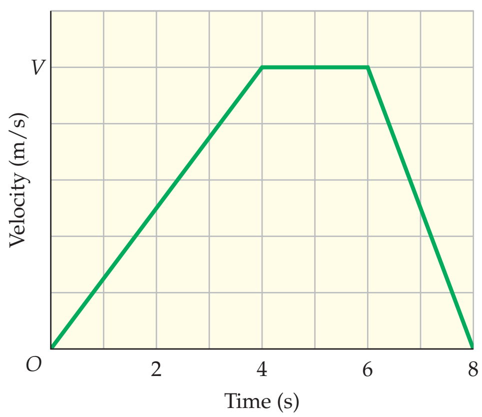
62. •• **IP** A boat is cruising in a straight line at a constant speed of 2.6 m/s when it is shifted into neutral. After coasting 12 m the engine is engaged again, and the boat resumes cruising at the reduced constant speed of 1.6 m/s. Assuming constant acceleration while coasting, **(a)** how long did it take for the boat to coast the 12 m? **(b)** What was the boat's acceleration while it was coasting? **(c)** When the boat had coasted for 6.0 m, was its speed 2.1 m/s, more than 2.1 m/s, or less than 2.1 m/s? Explain.

63. •• A model rocket rises with constant acceleration to a height of 3.2 m, at which point its speed is 26.0 m/s. **(a)** How much time does it take for the rocket to reach this height? **(b)** What was the magnitude of the rocket's acceleration? **(c)** Find the height and speed of the rocket 0.10 s after launch.

64. •• The infamous chicken is dashing toward home plate with a speed of 5.8 m/s when he decides to hit the dirt. The chicken slides for 1.1 s, just reaching the plate as he stops (safe, of course). **(a)** What are the magnitude and direction of the chicken's acceleration? **(b)** How far did the chicken slide?

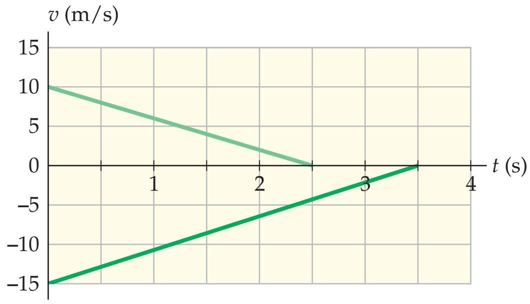
65. •• A bicyclist is finishing his repair of a flat tire when a friend rides by with a constant speed of 3.5 m/s. Two seconds later the bicyclist hops on his bike and accelerates at $2.4\ \mathrm{m/s^2}$ until he catches his friend. **(a)** How much time does it take until he catches his friend? **(b)** How far has he traveled in this time? **(c)** What is his speed when he catches up?

66. •• A car in stop-and-go traffic starts at rest, moves forward 13 m in 8.0 s, then comes to rest again. The velocity-versus-time plot for this car is given in **Figure 2–30.** What distance does the car cover in **(a)** the first 4.0 seconds of its motion and **(b)** the last 2.0 seconds of its motion? **(c)** What is the constant speed V that characterizes the middle portion of its motion?



▲ **FIGURE 2–30** Problem 66

67. ••• A car and a truck are heading directly toward one another on a straight and narrow street, but they avoid a head-on collision by simultaneously applying their brakes at $t = 0$. The resulting velocity-versus-time graphs are shown in **Figure 2–31.** What is the separation between the car and the truck when they have come to rest, given that at $t = 0$ the car is at $x = 15$ m and the truck is at $x = -35$ m? (Note that this information determines which line in the graph corresponds to which vehicle.)



▲ **FIGURE 2–31** Problem 67

68. ••• In a physics lab, students measure the time it takes a small cart to slide a distance of 1.00 m on a smooth track inclined at an angle θ above the horizontal. Their results are given in the following table.

θ	**10.0°**	**20.0°**	**30.0°**
time, s	1.08	0.770	0.640

(a) Find the magnitude of the cart's acceleration for each angle. **(b)** Show that your results for part (a) are in close agreement with the formula, $a = g \sin\theta$. (We will derive this formula in Chapter 5.)

SECTION 2–7 FREELY FALLING OBJECTS

(Problems 71 and 72)

69. • **CE** At the edge of a roof you throw ball 1 upward with an initial speed v_0; a moment later you throw ball 2 downward with the same initial speed. The balls land at the same time. Which of the following statements is true for the instant just before the balls hit the ground? **A.** The speed of ball 1 is greater than the speed of ball 2; **B.** The speed of ball 1 is equal to the speed of ball 2; **C.** The speed of ball 1 is less than the speed of ball 2.

70. • Legend has it that Isaac Newton was hit on the head by a falling apple, thus triggering his thoughts on gravity. Assuming the story to be true, estimate the speed of the apple when it struck Newton.

71. • The cartoon shows a car in free fall. Is the statement made in the cartoon accurate? Justify your answer.

72. • Referring to the cartoon in Problem 71, how long would it take for the car to go from 0 to 30 mi/h?

73. • **Jordan's Jump** Michael Jordan's vertical leap is reported to be 48 inches. What is his takeoff speed? Give your answer in meters per second.

74. • **BIO** Gulls are often observed dropping clams and other shellfish from a height to the rocks below, as a means of opening the shells. If a seagull drops a shell from rest at a height of 14 m, how fast is the shell moving when it hits the rocks?

75. • A volcano launches a lava bomb straight upward with an initial speed of 28 m/s. Taking upward to be the positive direction, find the speed and direction of motion of the lava bomb **(a)** 2.0 seconds and **(b)** 3.0 seconds after it is launched.

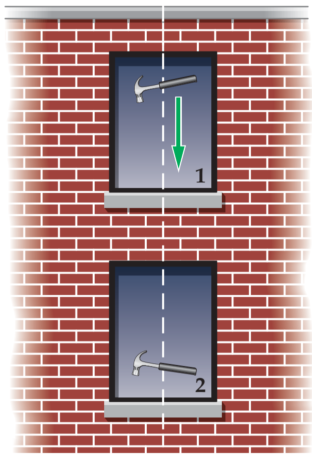
76. • **An Extraterrestrial Volcano** The first active volcano observed outside the Earth was discovered in 1979 on Io, one of the moons of Jupiter. The volcano was observed to be ejecting material to a height of about 2.00×10^5 m. Given that the acceleration of gravity on Io is $1.80\ \text{m/s}^2$, find the initial velocity of the ejected material.

77. • **BIO Measure Your Reaction Time** Here's something you *can* try at home—an experiment to measure your reaction time. Have a friend hold a ruler by one end, letting the other end hang down vertically. At the lower end, hold your thumb and index finger on either side of the ruler, ready to grip it. Have your friend release the ruler without warning. Catch it as quickly as you can. If you catch the ruler 5.2 cm from the lower end, what is your reaction time?

How fast are your reactions? (Problem 77)

78. •• **CE Predict/Explain** A carpenter on the roof of a building accidentally drops her hammer. As the hammer falls it passes two windows of equal height, as shown in **Figure 2–32.** **(a)** Is the *increase* in speed of the hammer as it drops past window 1 greater than, less than, or equal to the *increase* in speed as it drops past window 2? **(b)** Choose the *best explanation* from among the following:

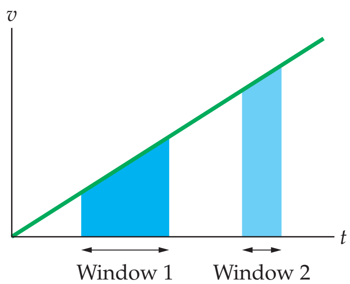
I. The greater speed at window 2 results in a greater increase in speed.
II. Constant acceleration means the hammer speeds up the same amount for each window.
III. The hammer spends more time dropping past window 1.



▲ **FIGURE 2–32** Problem 78

79. •• **CE Predict/Explain Figure 2–33** shows a v-versus-t plot for the hammer dropped by the carpenter in Problem 78. Notice that the times when the hammer passes the two windows are indicated by shaded areas. **(a)** Is the area of the shaded region corresponding to window 1 greater than, less than, or equal to the area of the shaded region corresponding to window 2? **(b)** Choose the *best explanation* from among the following:

I. The shaded area for window 2 is higher than the shaded area for window 1.
II. The windows are equally tall.
III. The shaded area for window 1 is wider than the shaded area for window 2.



▲ **FIGURE 2–33** Problem 79

80. •• **CE** A ball is thrown straight upward with an initial speed v_0. When it reaches the top of its flight at height h, a second ball is thrown straight upward with the same initial speed. Do the balls cross paths at height $\frac{1}{2}h$, above $\frac{1}{2}h$, or below $\frac{1}{2}h$?

81. •• Bill steps off a 3.0-m-high diving board and drops to the water below. At the same time, Ted jumps upward with a speed of 4.2 m/s from a 1.0-m-high diving board. Choosing the origin to be at the water's surface, and upward to be the positive x direction, write x-versus-t equations of motion for both Bill and Ted.

82. •• Repeat the previous problem, this time with the origin 3.0 m above the water, and with downward as the positive x direction.

83. •• On a hot summer day in the state of Washington while kayaking, I saw several swimmers jump from a railroad bridge into the Snohomish River below. The swimmers stepped off the bridge, and I estimated that they hit the water 1.5 s later. **(a)** How high was the bridge? **(b)** How fast were the swimmers moving when they hit the water? **(c)** What would the swimmers' drop time be if the bridge were twice as high?

84. •• **Highest Water Fountain** The world's highest fountain of water is located, appropriately enough, in Fountain Hills, Arizona. The fountain rises to a height of 560 ft (5 feet higher than the Washington Monument). **(a)** What is the initial speed of the water? **(b)** How long does it take for water to reach the top of the fountain?

85. •• Wrongly called for a foul, an angry basketball player throws the ball straight down to the floor. If the ball bounces straight up and returns to the floor 2.8 s after first striking it, what was the ball's greatest height above the floor?

86. •• To celebrate a victory, a pitcher throws her glove straight upward with an initial speed of 6.0 m/s. **(a)** How long does it take for the glove to return to the pitcher? **(b)** How long does it take for the glove to reach its maximum height?

87. •• **IP** Standing at the edge of a cliff 32.5 m high, you drop a ball. Later, you throw a second ball downward with an initial speed of 11.0 m/s. **(a)** Which ball has the greater *increase* in speed when it reaches the base of the cliff, or do both balls speed up by the same amount? **(b)** Verify your answer to part (a) with a calculation.

88. •• You shoot an arrow into the air. Two seconds later (2.00 s) the arrow has gone straight upward to a height of 30.0 m above its launch point. **(a)** What was the arrow's initial speed? **(b)** How long did it take for the arrow to first reach a height of 15.0 m above its launch point?

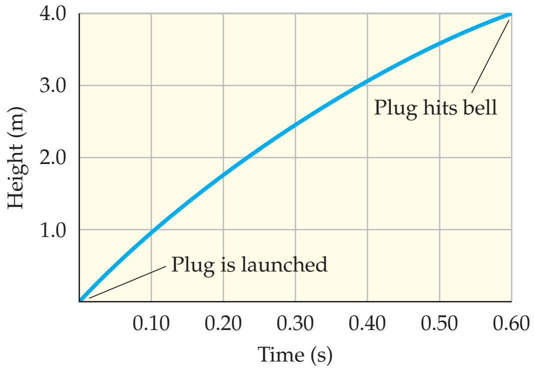
89. •• While riding on an elevator descending with a constant speed of 3.0 m/s, you accidentally drop a book from under your arm. **(a)** How long does it take for the book to reach the elevator floor, 1.2 m below your arm? **(b)** What is the book's speed relative to you when it hits the elevator floor?

90. •• A hot-air balloon is descending at a rate of 2.0 m/s when a passenger drops a camera. If the camera is 45 m above the ground when it is dropped, **(a)** how long does it take for the camera to reach the ground, and **(b)** what is its velocity just before it lands? Let upward be the positive direction for this problem.

91. •• **IP** Standing side by side, you and a friend step off a bridge at different times and fall for 1.6 s to the water below. Your friend goes first, and you follow after she has dropped a distance of 2.0 m. **(a)** When your friend hits the water, is the separation between the two of you 2.0 m, less than 2.0 m, or more than 2.0 m? **(b)** Verify your answer to part (a) with a calculation.

92. •• A model rocket blasts off and moves upward with an acceleration of 12 m/s^2 until it reaches a height of 26 m, at which point its engine shuts off and it continues its flight in free fall. **(a)** What is the maximum height attained by the rocket? **(b)** What is the speed of the rocket just before it hits the ground? **(c)** What is the total duration of the rocket's flight?

93. ••• **Hitting the "High Striker"** A young woman at a carnival steps up to the "high striker," a popular test of strength where the contestant hits one end of a lever with a mallet, propelling a small metal plug upward toward a bell. She gives the mallet a mighty swing and sends the plug to the top of the striker, where it rings the bell. **Figure 2–34** shows the corresponding position-versus-time plot for the plug. Using the information given in the plot, answer the following questions: **(a)** What is the average speed of the plug during its upward journey? **(b)** By how much does the speed of the plug decrease during its upward journey? **(c)** What is the initial speed of the plug? (Assume the plug to be in free fall during its upward motion, with no effects of air resistance or friction.)



▲ **FIGURE 2–34** Problem 93

94. ••• While sitting on a tree branch 10.0 m above the ground, you drop a chestnut. When the chestnut has fallen 2.5 m, you throw a second chestnut straight down. What initial speed must you give the second chestnut if they are both to reach the ground at the same time?

GENERAL PROBLEMS

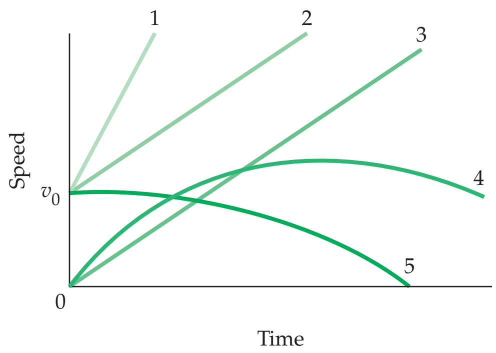
95. • In a well-known Jules Verne novel, Phileas Fogg travels around the world in 80 days. What was Mr. Fogg's approximate average speed during his adventure?

96. • An astronaut on the Moon drops a rock straight downward from a height of 1.25 m. If the acceleration of gravity on the Moon is 1.62 m/s^2, what is the speed of the rock just before it lands?

97. • You jump from the top of a boulder to the ground 1.5 m below. Estimate your deceleration on landing.

98. • **A Supersonic Waterfall** Geologists have learned of periods in the past when the Strait of Gibraltar closed off, and the Mediterranean Sea dried out and become a desert. Later, when the strait reopened, a massive saltwater waterfall was created. According to geologists, the water in this waterfall was supersonic; that is, it fell with speeds in excess of the speed of sound. Ignoring air resistance, what is the minimum height necessary to create a supersonic waterfall? (The speed of sound may be taken to be 340 m/s.)

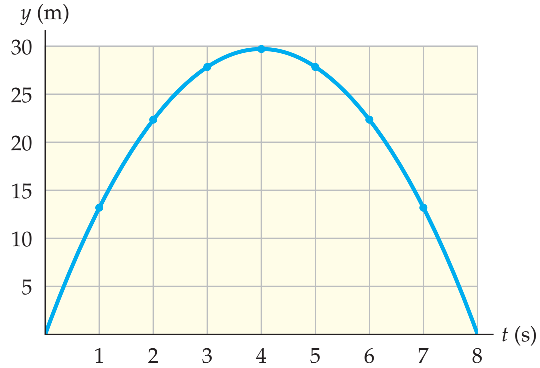
99. •• **CE** At the edge of a roof you drop ball A from rest, and then throw ball B downward with an initial velocity of v_0. Is the increase in speed just before the balls land more for ball A, more for ball B, or the same for each ball?



▲ **FIGURE 2–35** Problem 100

100. •• **CE** Suppose the two balls described in Problem 99 are released at the same time, with ball A dropped from rest and ball B thrown downward with an initial speed v_0. Identify which of the five plots shown in **Figure 2–35** corresponds to **(a)** ball A and **(b)** ball B.

101. •• Astronauts on a distant planet throw a rock straight upward and record its motion with a video camera. After digitizing their video, they are able to produce the graph of height, y, versus time, t, shown in **Figure 2–36. (a)** What is the acceleration of gravity on this planet? **(b)** What was the initial speed of the rock?



▲ **FIGURE 2–36** Problem 101

102. •• **Drop Tower** NASA operates a 2.2-second drop tower at the Glenn Research Center in Cleveland, Ohio. At this facility, experimental packages are dropped from the top of the tower, on the 8th floor of the building. During their 2.2 seconds of free fall, experiments experience a microgravity environment similar to that of a spacecraft in orbit. **(a)** What is the drop distance of a 2.2-s tower? **(b)** How fast are the experiments traveling when they hit the air bags at the bottom of the tower? **(c)** If the experimental package comes to rest over a distance of 0.75 m upon hitting the air bags, what is the average stopping acceleration?

103. •• **IP** A youngster bounces straight up and down on a trampoline. Suppose she doubles her initial speed from 2.0 m/s to 4.0 m/s. **(a)** By what factor does her time in the air increase? **(b)** By what factor does her maximum height increase? **(c)** Verify your answers to parts (a) and (b) with an explicit calculation.

104. •• At the 18th green of the U.S. Open you need to make a 20.5-ft putt to win the tournament. When you hit the ball, giving it an initial speed of 1.57 m/s, it stops 6.00 ft short of the hole. **(a)** Assuming the deceleration caused by the grass is constant, what should the initial speed have been to just make the putt? **(b)** What initial speed do you need to make the remaining 6.00-ft putt?

105. •• **IP** A popular entertainment at some carnivals is the blanket toss (see photo, p. 39). **(a)** If a person is thrown to a maximum height of 28.0 ft above the blanket, how long does she spend in the air? **(b)** Is the amount of time the person is above a height of 14.0 ft more than, less than, or equal to the amount of time the person is below a height of 14.0 ft? Explain. **(c)** Verify your answer to part (b) with a calculation.

106. •• Referring to Conceptual Checkpoint 2–5, find the separation between the rocks at **(a)** $t = 1.0$ s, **(b)** $t = 2.0$ s, and **(c)** $t = 3.0$ s, where time is measured from the instant the second rock is dropped. **(d)** Verify that the separation increases linearly with time.

107. •• **IP** A glaucous-winged gull, ascending straight upward at 5.20 m/s, drops a shell when it is 12.5 m above the ground. **(a)** What are the magnitude and direction of the shell's acceleration just after it is released? **(b)** Find the maximum height above the ground reached by the shell. **(c)** How long does it take for the shell to reach the ground? **(d)** What is the speed of the shell at this time?

108. •• A doctor, preparing to give a patient an injection, squirts a small amount of liquid straight upward from a syringe. If the liquid emerges with a speed of 1.5 m/s, **(a)** how long does it take for it to return to the level of the syringe? **(b)** What is the maximum height of the liquid above the syringe?

109. •• A hot-air balloon has just lifted off and is rising at the constant rate of 2.0 m/s. Suddenly one of the passengers realizes she has left her camera on the ground. A friend picks it up and tosses it straight upward with an initial speed of 13 m/s. If the passenger is 2.5 m above her friend when the camera is tossed, how high is she when the camera reaches her?

110. ••• In the previous problem, what is the minimum initial speed of the camera if it is to just reach the passenger? (*Hint:* When the camera is thrown with its minimum speed, its speed on reaching the passenger is the same as the speed of the passenger.)

111. ••• **Old Faithful** Watching Old Faithful erupt, you notice that it takes a time t for water to emerge from the base of the geyser and reach its maximum height. **(a)** What is the height of the geyser, and **(b)** what is the initial speed of the water? Evaluate your expressions for **(c)** the height and **(d)** the initial speed for a measured time of 1.65 s.

112. ••• **IP** A ball is thrown upward with an initial speed v_0. When it reaches the top of its flight, at a height h, a second ball is thrown upward with the same initial velocity. **(a)** Sketch an x-versus-t plot for each ball. **(b)** From your graph, decide whether the balls cross paths at $h/2$, above $h/2$, or below $h/2$. **(c)** Find the height where the paths cross.

113. ••• Weights are tied to each end of a 20.0-cm string. You hold one weight in your hand and let the other hang vertically a height h above the floor. When you release the weight in your hand, the two weights strike the ground one after the other with audible thuds. Find the value of h for which the time between release and the first thud is equal to the time between the first thud and the second thud.

114. ••• A ball, dropped from rest, covers three-quarters of the distance to the ground in the last second of its fall. **(a)** From what height was the ball dropped? **(b)** What was the total time of fall?

115. ••• A stalactite on the roof of a cave drips water at a steady rate to a pool 4.0 m below. As one drop of water hits the pool, a second drop is in the air, and a third is just detaching from the stalactite. **(a)** What are the position and velocity of the second drop when the first drop hits the pool? **(b)** How many drops per minute fall into the pool?

116. ••• You drop a ski glove from a height h onto fresh snow, and it sinks to a depth d before coming to rest. **(a)** In terms of g and h, what is the speed of the glove when it reaches the snow? **(b)** What are the magnitude and direction of the glove's acceleration as it moves through the snow, assuming it to be constant? Give your answer in terms of g, h, and d.

117. ••• To find the height of an overhead power line, you throw a ball straight upward. The ball passes the line on the way up after 0.75 s, and passes it again on the way down 1.5 s after it was tossed. What are the height of the power line and the initial speed of the ball?

118. ••• Suppose the first rock in Conceptual Checkpoint 2–5 drops through a height h before the second rock is released from rest. Show that the separation between the rocks, S, is given by the following expression:

$$S = h + (\sqrt{2gh})t$$

In this result, the time t is measured from the time the second rock is dropped.

119. ••• An arrow is fired with a speed of 20.0 m/s at a block of Styrofoam resting on a smooth surface. The arrow penetrates a certain distance into the block before coming to rest relative to it. During this process the arrow's deceleration has a magnitude of 1550 m/s^2 and the block's acceleration has a magnitude of 450 m/s^2. **(a)** How long does it take for the arrow to stop moving with respect to the block? **(b)** What is the common speed of the arrow and block when this happens? **(c)** How far into the block does the arrow penetrate?

120. ••• Sitting in a second-story apartment, a physicist notices a ball moving straight upward just outside her window. The ball is visible for 0.25 s as it moves a distance of 1.05 m from the bottom to the top of the window. **(a)** How long does it take before the ball reappears? **(b)** What is the greatest height of the ball above the top of the window?

121. ••• **The Quadratic Formula from Kinematics** In this problem we show how the kinematic equations of motion can be used to derive the quadratic formula. First, consider an object with an initial position x_0, an initial velocity v_0, and an acceleration a. To find the time when this object reaches the position $x = 0$ we can use the quadratic formula, or apply the following two-step procedure: **(a)** Use Equation 2–12 to show that the velocity of the object when it reaches $x = 0$ is given by $v = \pm\sqrt{v_0^2 - 2ax_0}$. **(b)** Use Equation 2–7 to show that the time corresponding to the velocity found in part (a) is $t = \dfrac{-v_0 \pm \sqrt{v_0^2 - 2ax_0}}{a}$. **(c)** To complete our derivation, show that the result of part (b) is the same as applying the quadratic formula to $x = x_0 + v_0 t + \frac{1}{2}at^2 = 0$.

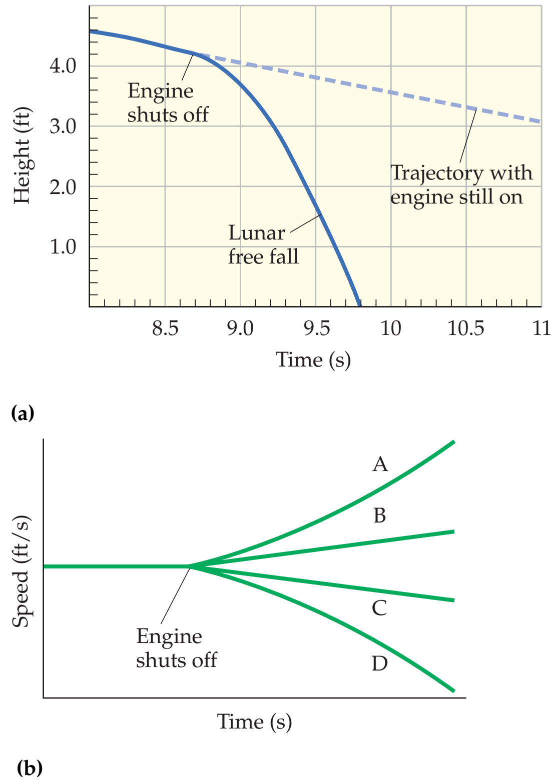
PASSAGE PROBLEMS

Bam!—*Apollo 15* Lands on the Moon

The first word spoken on the surface of the Moon after *Apollo 15* landed on July 30, 1971, was "Bam!" This was James Irwin's involuntary reaction to their rather bone-jarring touchdown. "We did hit harder than any of the other flights!" says Irwin. "And I was startled, obviously, when I said, 'Bam!'"

The reason for the "firm touchdown" of *Apollo 15,* as pilot David Scott later characterized it, was that the rocket engine was shut off a bit earlier than planned, when the lander was still 4.30 ft above the lunar surface and moving downward with a speed of 0.500 ft/s. From that point on the lander descended in lunar free fall, with an acceleration of 1.62 m/s^2. As a result, the landing speed of *Apollo 15* was by far the largest of any of the *Apollo* missions. In comparison, Neil Armstrong's landing speed on *Apollo 11* was the lowest at 1.7 ft/s—he didn't shut off the engine until the footpads were actually on the surface. *Apollos 12, 14,* and *17* all landed with speeds between 3.0 and 3.5 ft/s.

To better understand the descent of *Apollo 15,* we show its trajectory during the final stages of landing in **Figure 2–37 (a)**. In **Figure 2–37 (b)** we show a variety of speed-versus-time plots.



▲ **FIGURE 2–37** Problems 122, 123, 124, and 125

122. • How long did it take for the lander to drop the final 4.30 ft to the Moon's surface?

A. 1.18 s **B.** 1.37 s

C. 1.78 s **D.** 2.36 s

123. •• What was the impact speed of the lander when it touched down? Give your answer in feet per second (ft/s), the same units used by the astronauts.

A. 2.41 ft/s **B.** 6.78 ft/s

C. 9.95 ft/s **D.** 10.6 ft/s

124. • Which of the speed-versus-time plots in Figure 2–37 (b) correctly represents the speed of the *Apollo 15* lander?

A B C D

125. • Suppose, instead of shutting off the engine, the astronauts had increased its thrust, giving the lander a small, but constant, upward acceleration. Which speed-versus-time plot in Figure 2–37 (b) would describe this situation?

A B C D

INTERACTIVE PROBLEMS

126. •• **Referring to Example 2–9** Suppose the speeder (red car) is traveling with a constant speed of 25 m/s, and that the maximum acceleration of the police car (blue car) is 3.8 m/s^2. If the police car is to start from rest and catch the speeder in 15 s or less, what is the maximum head-start distance the speeder can have? Measure time from the moment the police car starts.

127. •• **Referring to Example 2–9** The speeder passes the position of the police car with a constant speed of 15 m/s. The police car immediately starts from rest and pursues the speeder with constant acceleration. What acceleration must the police car have if it is to catch the speeder in 7.0 s? Measure time from the moment the police car starts.

128. •• **IP Referring to Example 2–12** **(a)** In Example 2–12, the bag of sand is released at 20.0 m and reaches a maximum height of 22 m. If the bag had been released at 30.0 m instead, with everything else remaining the same, would its maximum height be 32 m, greater than 32 m, or less than 32 m? **(b)** Find the speed of the bag just before it lands when it is released from 30.0 m.

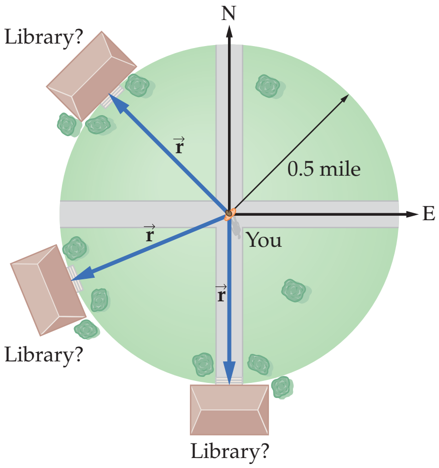
129. •• **Referring to Example 2–12** Suppose the balloon is descending with a constant speed of 4.2 m/s when the bag of sand comes loose at a height of 35 m. **(a)** How long is the bag in the air? **(b)** What is the speed of the bag when it is 15 m above the ground?

3 Vectors in Physics

The points of the compass have long been used as a framework for indicating directions. The compass shown here was produced by Gowin Knight (1713–1772), whose improved designs were adopted by the Royal Navy in 1752. In physics, we more frequently indicate directions with x and y rather than N, S, E, and W. Either way, specifying a direction as well as a magnitude is essential to defining one of the physicist's basic tools, the vector.

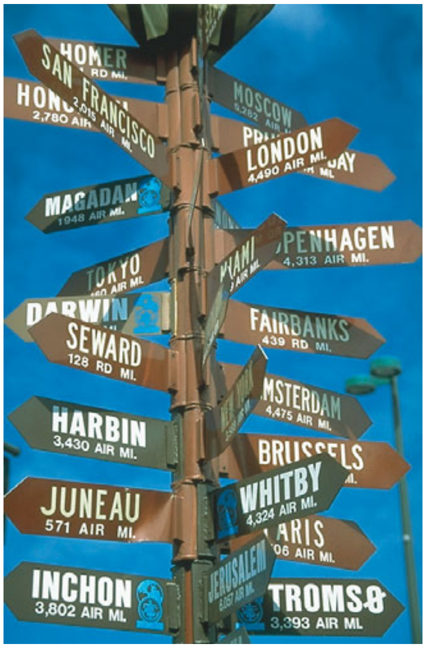
Of all the mathematical tools used in this book, perhaps none is more important than the vector. In the next chapter, for example, we use vectors to extend our study of motion from one dimension to two dimensions. More generally, vectors are *indispensable* when a physical quantity has a direction associated with it. Suppose, for example, that a pilot wants to fly from Denver to Dallas. If the air is still, the pilot can simply head the plane toward the destination. If there is a wind blowing from west to east, however, the pilot must use vectors to determine the correct heading so that the plane and its passengers will arrive in Dallas and not Little Rock.

In this chapter we discuss what a vector is, how it differs from a scalar, and how it can represent a physical quantity. We also show how to find the components of a vector and how to add and subtract vectors. All of these techniques are used time and again throughout the book. Other useful aspects of vectors, such as how to multiply them, will be presented in later chapters when the need arises.



▲ **FIGURE 3–1 Distance and direction**

If you know only that the library is 0.5 mi from you, it could lie anywhere on a circle of radius 0.5 mi. If, instead, you are told the library is 0.5 mi northwest, you know its precise location.



▲ The information given by this sign includes both a distance and a direction for each city. In effect, the sign defines a displacement vector for each of these destinations.

3–1 Scalars Versus Vectors

Numbers can represent many quantities in physics. For example, a numerical value, together with the appropriate units, can specify the volume of a container, the temperature of the air, or the time of an event. In physics, a number with its units is referred to as a **scalar:**

- A scalar is a number with units. It can be positive, negative, or zero.

Sometimes, however, a scalar isn't enough to adequately describe a physical quantity—in many cases, a direction is needed as well. For example, suppose you're walking in an unfamiliar city and you want directions to the library. You ask a passerby, "Do you know where the library is?" If the person replies "Yes," and walks on, he hasn't been too helpful. If he says, "Yes, it is half a mile from here," that is more helpful, but you still don't know where it is. The library could be anywhere on a circle of radius one-half mile, as shown in **Figure 3–1**. To pin down the location, you need a reply such as, "Yes, the library is half a mile northwest of here." With both a distance *and* a direction, you know the location of the library.

Thus, if you walk northwest for half a mile you arrive at the library, as indicated by the upper left arrow in Figure 3–1. The arrow points in the direction traveled, and its **magnitude,** 0.5 mi in this case, represents the distance covered. In general, a quantity that is specified by both a *magnitude* and a *direction* is represented by a **vector:**

- A vector is a mathematical quantity with both a direction and a magnitude.

In the example of walking to the library, the vector corresponding to the trip is the displacement vector. Other examples of vector quantities are the velocity and the acceleration of an object. For example, the magnitude of a velocity vector is its speed, and its direction is the direction of motion, as we shall see later in this chapter.

When we indicate a vector on a diagram or a sketch, we draw an arrow, as in Figure 3–1. To indicate a vector with a written symbol, we use **boldface** for the vector itself, with a small arrow above it to remind us of its vector nature, and *italic* for its magnitude. Thus, for example, the upper-left vector in Figure 3–1 is designated by the symbol $\vec{\mathbf{r}}$, and its magnitude is $r = 0.5$ mi. (When we represent a vector in a graph, we sometimes label it with the corresponding boldface symbol, and sometimes with the appropriate magnitude.) It is common in handwritten material to draw a small arrow over the vector's symbol, which is very similar to the way vectors are represented in this text.

3–2 The Components of a Vector

When we discussed directions for finding a library in the previous section, we pointed out that knowing the magnitude and direction angle—0.5 mi northwest—gives its precise location. We left out one key element in actually *getting* to the library, however. In most cities it would not be possible to simply walk in a straight line for 0.5 mi directly to the library, since to do so would take you through buildings where there are no doors, through people's backyards, and through all kinds of other obstructions. In fact, if the city streets are laid out along north–south and east–west directions, you might instead walk west for a certain distance, then turn and proceed north an equal distance until you reach the library, as illustrated in **Figure 3–2**. What you have just done is "resolved" displacement vector $\vec{\mathbf{r}}$ between you and the library into east–west and north–south "components."

In general, to find the components of a vector we need to set up a coordinate system. In two dimensions we choose an origin, O, and a positive direction for both the x and the y axes, as in **Figure 3–3**. If the system were three-dimensional, we would also indicate a z axis.

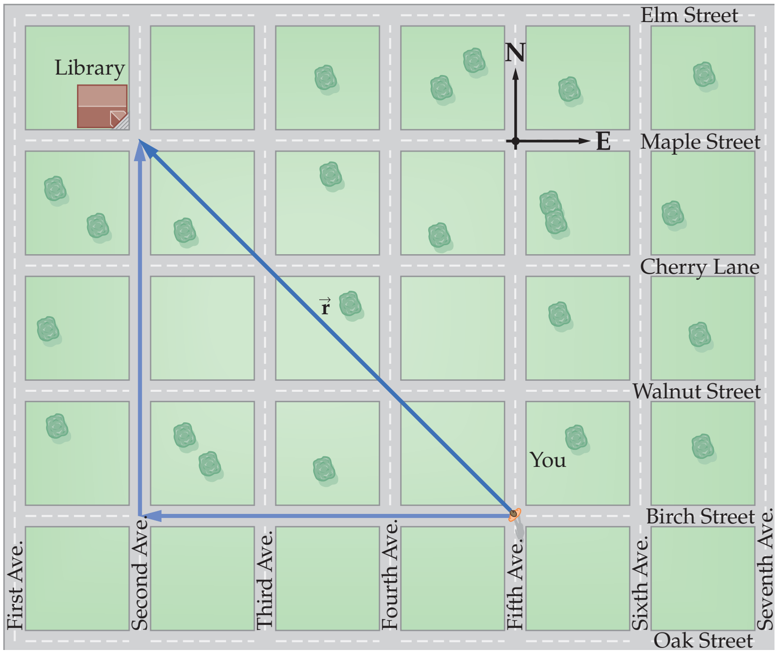


FIGURE 3–2 A walk along city streets to the library

By taking the indicated path, we have "resolved" the vector $\vec{\mathbf{r}}$ into east–west and north–south components.

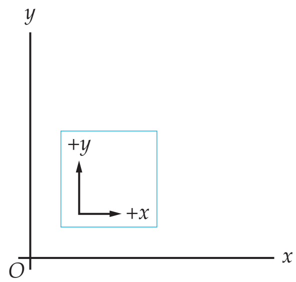


FIGURE 3–3 A two-dimensional coordinate system

The positive x and y directions are indicated in this shorthand form.

Now, a vector is defined by its magnitude (indicated by the length of the arrow representing the vector) and its direction. For example, suppose an ant leaves its nest at the origin and, after foraging for some time, is at the location given by the vector $\vec{\mathbf{r}}$ in **Figure 3–4 (a)**. This vector has a magnitude $r = 1.50$ m and points in a direction $\theta = 25.0°$ above the x axis. Equivalently, $\vec{\mathbf{r}}$ can be defined by saying that it extends a distance r_x in the x direction and a distance r_y in the y direction, as shown in **Figure 3–4 (b)**. The quantities r_x and r_y are referred to as the x and y **scalar components** of the vector $\vec{\mathbf{r}}$.

We can find r_x and r_y by using standard trigonometric relations, as summarized in the Problem-Solving Note on this page. Referring to Figure 3–4 (b), we see that

$$r_x = r\cos 25.0° = (1.50\text{ m})(0.906) = 1.36\text{ m}$$

and

$$r_y = r\sin 25.0° = (1.50\text{ m})(0.423) = 0.634\text{ m}$$

Thus, we can say that the ant's final displacement is equivalent to what it would be if the ant had simply walked 1.36 m in the x direction and then 0.634 m in the y direction.

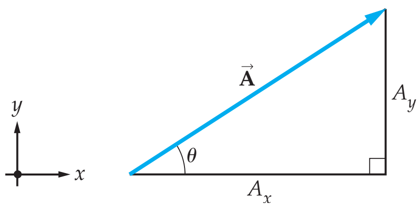
To show the equivalence of these two ways of describing a vector, let's start with the components of $\vec{\mathbf{r}}$, as determined previously, and use them to calculate the magnitude r and the angle θ. First, note that r_x, r_y, and r form a right triangle with r as the hypotenuse. Thus, we can use the Pythagorean theorem (Appendix A) to find r in terms of r_x and r_y. This gives

$$r = \sqrt{r_x^2 + r_y^2} = \sqrt{(1.36\text{ m})^2 + (0.634\text{ m})^2} = \sqrt{2.25\text{ m}^2} = 1.50\text{ m}$$

PROBLEM-SOLVING NOTE

A Vector and Its Components

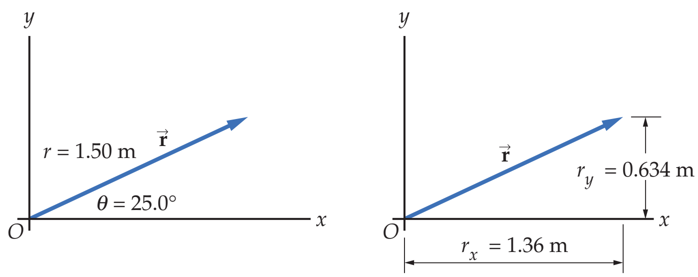
Given the magnitude and direction of a vector, find its components:



$$A_x = A\cos\theta$$
$$A_y = A\sin\theta$$

Given the components of a vector, find its magnitude and direction:

$$A = \sqrt{A_x^2 + A_y^2}$$
$$\theta = \tan^{-1}\frac{A_y}{A_x}$$



(a) A vector defined in terms of its length and direction angle

(b) The same vector defined in terms of its x and y components

FIGURE 3–4 A vector and its scalar components

(a) The vector $\vec{\mathbf{r}}$ is defined by its length ($r = 1.50$ m) and its direction angle ($\theta = 25.0°$) measured counterclockwise from the positive x axis. **(b)** Alternatively, the vector $\vec{\mathbf{r}}$ can be defined by its x component, $r_x = 1.36$ m, and its y component, $r_y = 0.634$ m.

as expected. Second, we can use any two sides of the triangle to obtain the angle θ, as shown in the next three calculations:

$$\theta = \sin^{-1}\left(\frac{0.634\text{ m}}{1.50\text{ m}}\right) = \sin^{-1}(0.423) = 25.0°$$

$$\theta = \cos^{-1}\left(\frac{1.36\text{ m}}{1.50\text{ m}}\right) = \cos^{-1}(0.907) = 25.0°$$

$$\theta = \tan^{-1}\left(\frac{0.634\text{ m}}{1.36\text{ m}}\right) = \tan^{-1}(0.466) = 25.0°$$

In some situations we know a vector's magnitude and direction; in other cases we are given the vector's components. You will find it useful to be able to convert quickly and easily from one description of a vector to the other using trigonometric functions and the Pythagorean theorem.

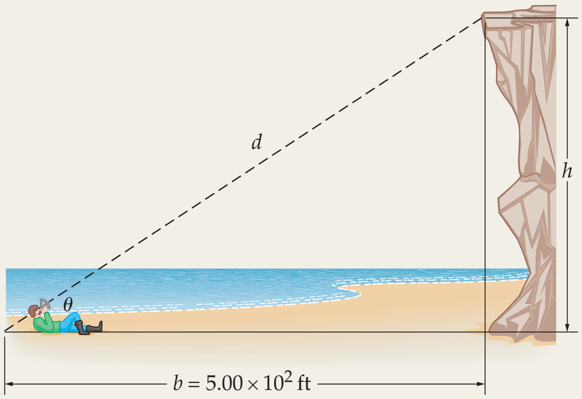
EXAMPLE 3–1 DETERMINING THE HEIGHT OF A CLIFF

REAL-WORLD PHYSICS

In the Jules Verne novel *Mysterious Island*, Captain Cyrus Harding wants to find the height of a cliff. He stands with his back to the base of the cliff, then marches straight away from it for 5.00×10^2 ft. At this point he lies on the ground and measures the angle from the horizontal to the top of the cliff. If the angle is 34.0°, **(a)** how high is the cliff? **(b)** What is the straight-line distance from Captain Harding to the top of the cliff?

PICTURE THE PROBLEM

Our sketch shows Cyrus Harding making his measurement of the angle, $\theta = 34.0°$, to the top of the cliff. The relevant triangle for this problem is also indicated. Note that the opposite side of the triangle is the height of the cliff, h; the adjacent side is the distance from the base of the cliff to Harding, $b = 5.00 \times 10^2$ ft; and finally, the hypotenuse is the distance, d, from Harding to the top of the cliff.



STRATEGY

The tangent of θ is the height of the triangle divided by the base: $\tan\theta = h/b$. Since we know both θ and the base, we can find the height using this relation. Similarly, the distance from Harding to the top of the cliff can be obtained by solving $\cos\theta = b/d$ for d.

SOLUTION

Part (a)

1. Use $\tan\theta = h/b$ to solve for the height of the cliff, h:

$$h = b\tan\theta = (5.00 \times 10^2\text{ ft})\tan 34.0° = 337\text{ ft}$$

Part (b)

2. Similarly, use $\cos\theta = b/d$ to solve for the distance d from Captain Harding to the top of the cliff:

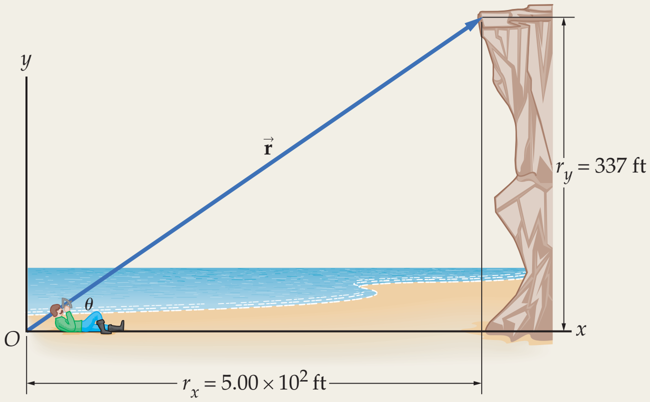
$$d = \frac{b}{\cos\theta} = \frac{5.00 \times 10^2\text{ ft}}{\cos 34.0°} = 603\text{ ft}$$

INSIGHT

An alternative way to solve part (b) is to use the Pythagorean theorem:

$$d = \sqrt{h^2 + b^2} = \sqrt{(337\text{ ft})^2 + (5.00 \times 10^2\text{ ft})^2} = 603\text{ ft}$$

Thus, if we let $\vec{\mathbf{r}}$ denote the vector from Cyrus Harding to the top of the cliff, as shown here, its magnitude is 603 ft and its direction is 34.0° above the x axis. Alternatively, the x component of $\vec{\mathbf{r}}$ is 5.00×10^2 ft and its y component is 337 ft.



PRACTICE PROBLEM

What angle would Cyrus Harding have found if he had walked 6.00×10^2 ft from the cliff to make his measurement? **[Answer:** $\theta = 29.3°$**]**

Some related homework problems: Problem 5, Problem 17

EXERCISE 3–1

a. Find A_x and A_y for the vector $\vec{\mathbf{A}}$ with magnitude and direction given by $A = 3.5$ m and $\theta = 66°$, respectively.

b. Find B and θ for the vector $\vec{\mathbf{B}}$ with components $B_x = 75.5$ m and $B_y = 6.20$ m.

SOLUTION

a. $A_x = 1.4$ m, $A_y = 3.2$ m

b. $B = 75.8$ m, $\theta = 4.69°$

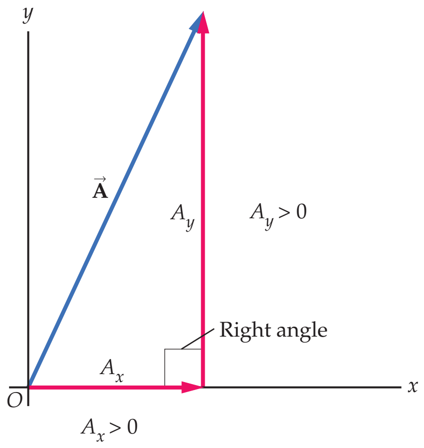


FIGURE 3–5 **A vector whose x and y components are positive**

Next, how do you determine the correct signs for the x and y components of a vector? This can be done by considering the right triangle formed by A_x, A_y, and $\vec{\mathbf{A}}$, as shown in **Figure 3–5**. To determine the sign of A_x, start at the tail of the vector and move along the x axis toward the right angle. If you are moving in the positive x direction, then A_x is positive ($A_x > 0$); if you are moving in the negative x direction, then A_x is negative ($A_x < 0$). For the y component, start at the right angle and move toward the tip of the arrow. A_y is positive or negative depending on whether you are moving in the positive or negative y direction.

For example, consider the vector shown in **Figure 3–6 (a)**. In this case, $A_x > 0$ and $A_y < 0$, as indicated in the figure. Similarly, the signs of A_x and A_y are given in **Figure 3–6 (b, c, d)** for the vectors shown there. Be sure to verify each of these cases by applying the rules just given. As we continue our study of physics, it is important to be able to find the components of a vector *and* to assign to them the correct signs.

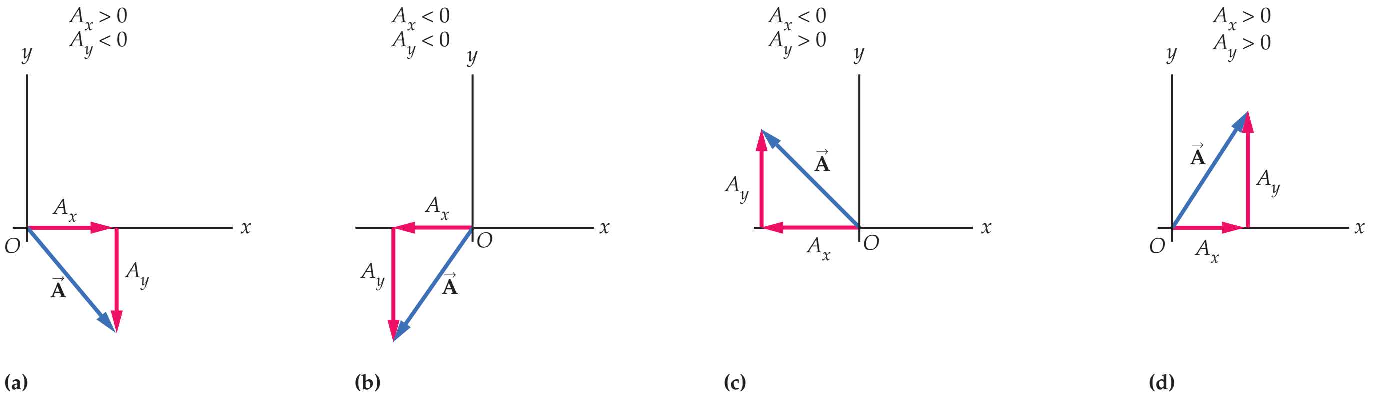


FIGURE 3–6 **Examples of vectors with components of different signs**

To determine the signs of a vector's components, it is only necessary to observe the direction in which they point. For example, in part (a) the x component points in the positive direction; hence $A_x > 0$. Similarly, the y component in part (a) points in the negative y direction; therefore $A_y < 0$.

EXERCISE 3–2

The vector $\vec{\mathbf{A}}$ has a magnitude of 7.25 m. Find its components for direction angles of

a. $\theta = 5.00°$ **c.** $\theta = 245°$

b. $\theta = 125°$ **d.** $\theta = 335°$

SOLUTION

a. $A_x = 7.22$ m, $A_y = 0.632$ m

b. $A_x = -4.16$ m, $A_y = 5.94$ m

c. $A_x = -3.06$ m, $A_y = -6.57$ m

d. $A_x = 6.57$ m, $A_y = -3.06$ m

Be careful when using your calculator to determine the direction angle, θ, because you may need to add 180° to get the correct angle, as measured counterclockwise from the positive x axis. For example, if $A_x = -0.50$ m and $A_y = 1.0$ m, your calculator will give the following result:

$$\theta = \tan^{-1}\left(\frac{1.0\text{ m}}{-0.50\text{ m}}\right) = \tan^{-1}(-2.0) = -63°$$

Does this angle correspond to the specified vector? The way to check is to sketch $\vec{\mathbf{A}}$. When you do, your drawing is similar to Figure 3–6 (c), and thus the direction angle of $\vec{\mathbf{A}}$ should be between 90° and 180°. To obtain the correct angle, add 180° to the calculator's result:

$$\theta = -63° + 180° = 117°$$

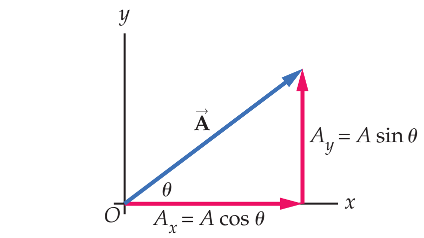
This, in fact, is the direction angle for the vector $\vec{\mathbf{A}}$.

EXERCISE 3–3

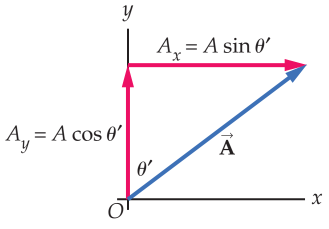
The vector $\vec{\mathbf{B}}$ has components $B_x = -2.10$ m and $B_y = -1.70$ m. Find the direction angle, θ, for this vector.

SOLUTION

$\tan^{-1}[(-1.70\text{ m})/(-2.10\text{ m})] = \tan^{-1}(1.70/2.10) = 39.0°, \theta = 39.0 + 180° = 219°$



(a)



(b)

▲ **FIGURE 3–7 Vector direction angles**
Vector $\vec{\mathbf{A}}$ and its components in terms of **(a)** the angle relative to the x axis and **(b)** the angle relative to the y axis.

Finally, in many situations the direction of a vector $\vec{\mathbf{A}}$ is given by the angle θ, measured relative to the x axis, as in **Figure 3–7 (a)**. In these cases we know that

$$A_x = A\cos\theta$$

and

$$A_y = A\sin\theta$$

On the other hand, we are sometimes given the angle between the vector and the y axis, as in **Figure 3–7 (b)**. If we call this angle θ', then it follows that

$$A_x = A\sin\theta'$$

and

$$A_y = A\cos\theta'$$

These two seemingly different results are actually in complete agreement. Note that $\theta + \theta' = 90°$, or $\theta' = 90° - \theta$. If we use the trigonometric identities given in Appendix A, we find

$$A_x = A\sin\theta' = A\sin(90° - \theta) = A\cos\theta$$

and

$$A_y = A\cos\theta' = A\cos(90° - \theta) = A\sin\theta$$

EXERCISE 3–4

If a vector's direction angle relative to the x axis is 35°, then its direction angle relative to the y axis is 55°. Find the components of a vector $\vec{\mathbf{A}}$ of magnitude 5.2 m in terms of

a. its direction relative to the x axis, and

b. its direction relative to the y axis.

SOLUTION

a. $A_x = (5.2\text{ m})\cos 35° = 4.3\text{ m}, A_y = (5.2\text{ m})\sin 35° = 3.0\text{ m}$

b. $A_x = (5.2\text{ m})\sin 55° = 4.3\text{ m}, A_y = (5.2\text{ m})\cos 55° = 3.0\text{ m}$

3–3 Adding and Subtracting Vectors

One important reason for determining the components of a vector is that they are useful in adding and subtracting vectors. In this section we begin by defining vector addition graphically, and then show how the same addition can be performed more concisely and accurately with components.

Adding Vectors Graphically

One day you open an old chest in the attic and find a treasure map inside. To locate the treasure, the map says that you must "Go to the sycamore tree in the backyard, march 5 paces north, then 3 paces east." If these two displacements are represented by the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in **Figure 3–8**, the total displacement from the tree to the treasure is given by the vector $\vec{\mathbf{C}}$. We say that $\vec{\mathbf{C}}$ is the *vector sum* of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$; that is, $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. In general, vectors are added graphically according to the following rule:

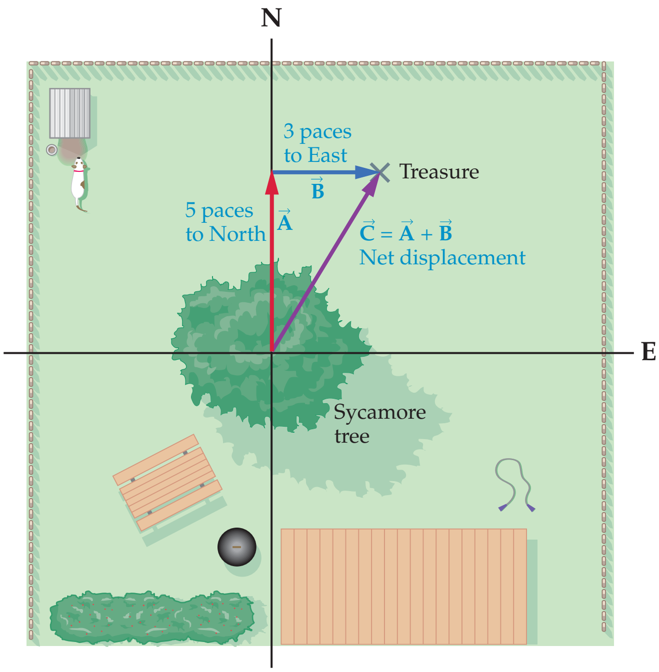
- To add the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, place the tail of $\vec{\mathbf{B}}$ at the head of $\vec{\mathbf{A}}$. The sum, $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, is the vector extending from the tail of $\vec{\mathbf{A}}$ to the head of $\vec{\mathbf{B}}$.

If the instructions to find the treasure were a bit more complicated—5 paces north, 3 paces east, then 4 paces southeast, for example—the path from the sycamore tree to the treasure would be like that shown in **Figure 3–9**. In this case, the total displacement, $\vec{\mathbf{D}}$, is the sum of the three vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$; that is, $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$. It follows that to add more than two vectors, we just keep placing the vectors head-to-tail, head-to-tail, and then draw a vector from the tail of the first vector to the head of the last vector, as in Figure 3–9.

In order to place a given pair of vectors head-to-tail, it may be necessary to move the corresponding arrows. This is fine, as long as you don't change their length or their direction. After all, a vector is defined by its length and direction; if these are unchanged, so is the vector.

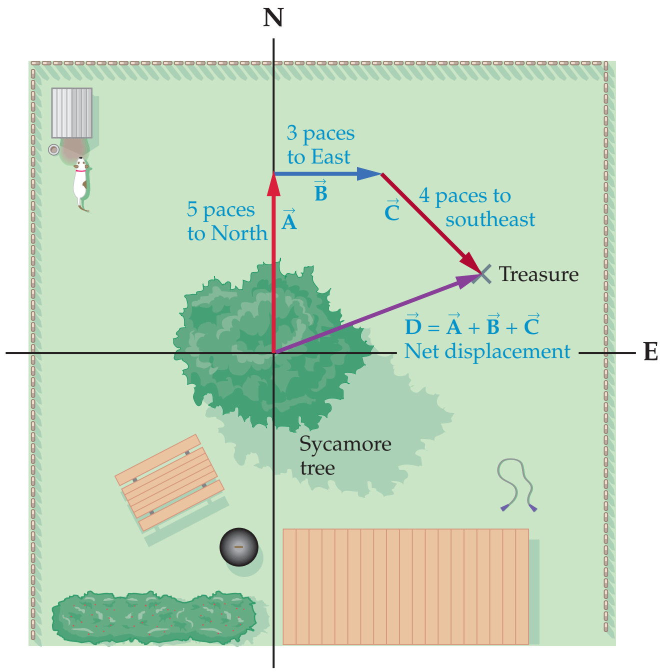
- A vector is defined by its magnitude and direction, regardless of its location.

▲ To a good approximation, these snow geese are all moving in the same direction with the same speed. As a result, their velocity vectors are equal, even though their positions are different.



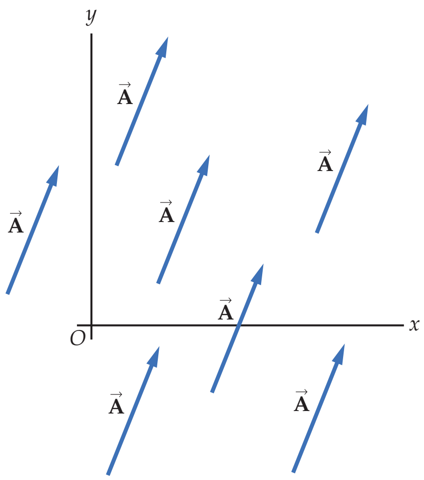
▲ **FIGURE 3–8 The sum of two vectors**

To go from the sycamore tree to the treasure, one must first go 5 paces north ($\vec{\mathbf{A}}$) and then 3 paces east ($\vec{\mathbf{B}}$). The net displacement from the tree to the treasure is $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$.



▲ **FIGURE 3–9 Adding several vectors**

Searching for a treasure that is 5 paces north ($\vec{\mathbf{A}}$), 3 paces east ($\vec{\mathbf{B}}$), and 4 paces southeast ($\vec{\mathbf{C}}$) of the sycamore tree. The net displacement from the tree to the treasure is $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$.



▲ **FIGURE 3–10 Identical vectors $\vec{\mathbf{A}}$ at different locations**

A vector is defined by its direction and length; its location is immaterial.

For example, in **Figure 3–10** all of the vectors are the same, even though they are at different locations on the graph.

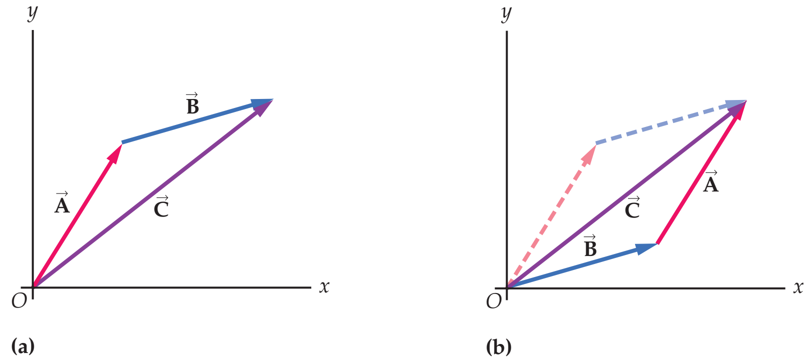
As an example of moving vectors, consider two vectors, $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, and their vector sum, $\vec{\mathbf{C}}$:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$

as illustrated in **Figure 3–11 (a)**. By moving the arrow representing $\vec{\mathbf{B}}$ so that its tail is at the origin, and moving the arrow for $\vec{\mathbf{A}}$ so that its tail is at the head of $\vec{\mathbf{B}}$, we obtain the construction shown in **Figure 3–11 (b)**. From this graph we see that $\vec{\mathbf{C}}$, which is $\vec{\mathbf{A}} + \vec{\mathbf{B}}$, is also equal to $\vec{\mathbf{B}} + \vec{\mathbf{A}}$:

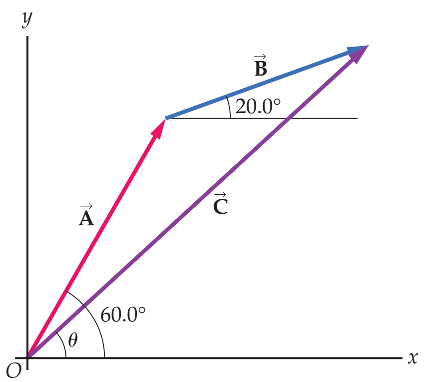
$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$$

That is, the sum of vectors is independent of the order in which the vectors are added.



▲ **FIGURE 3–11 $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$**

The vector $\vec{\mathbf{C}}$ is equal to **(a)** $\vec{\mathbf{A}} + \vec{\mathbf{B}}$ and **(b)** $\vec{\mathbf{B}} + \vec{\mathbf{A}}$. Note also that $\vec{\mathbf{C}}$ is the diagonal of the parallelogram formed by the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. For this reason, this method of vector addition is referred to as the "parallelogram method."



▲ **FIGURE 3–12 Graphical addition of vectors**

The vector $\vec{\mathbf{A}}$ has a magnitude of 5.00 m and a direction angle of 60.0°; the vector $\vec{\mathbf{B}}$ has a magnitude of 4.00 m and a direction angle of 20.0°. The magnitude and direction of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ can be measured on the graph with a ruler and a protractor.

Now, suppose that $\vec{\mathbf{A}}$ has a magnitude of 5.00 m and a direction angle of 60.0° above the x axis, and that $\vec{\mathbf{B}}$ has a magnitude of 4.00 m and a direction angle of 20.0° above the x axis. These two vectors and their sum, $\vec{\mathbf{C}}$, are shown in **Figure 3–12**. The question is: What are the length and direction angle of $\vec{\mathbf{C}}$?

A graphical way to answer this question is to simply measure the length and direction of $\vec{\mathbf{C}}$ in Figure 3–12. With a ruler, we find the length of $\vec{\mathbf{C}}$ to be approximately 1.75 times the length of $\vec{\mathbf{A}}$, which means that $\vec{\mathbf{C}}$ is roughly 1.75 (5.00 m) = 8.75 m. Similarly, with a protractor we measure the angle θ to be about 45.0° above the x axis.

Adding Vectors Using Components

The graphical method of adding vectors yields approximate results, limited by the accuracy with which the vectors can be drawn and measured. In contrast, exact results can be obtained by adding $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ in terms of their components. To see how this is done, consider **Figure 3–13 (a)**, which shows the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, and **Figure 3–13 (b)**, which shows the components of $\vec{\mathbf{C}}$. Clearly,

$$C_x = A_x + B_x$$

and

$$C_y = A_y + B_y$$

Thus, to add vectors, you simply add the components.

Returning to our example in Figure 3–12, the components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are

$$A_x = (5.00\text{ m})\cos 60.0° = 2.50\text{ m} \quad A_y = (5.00\text{ m})\sin 60.0° = 4.33\text{ m}$$

and

$$B_x = (4.00\text{ m})\cos 20.0° = 3.76\text{ m} \quad B_y = (4.00\text{ m})\sin 20.0° = 1.37\text{ m}$$

Adding component by component yields the components of $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$:

$$C_x = A_x + B_x = 2.50 \text{ m} + 3.76 \text{ m} = 6.26 \text{ m}$$

and

$$C_y = A_y + B_y = 4.33 \text{ m} + 1.37 \text{ m} = 5.70 \text{ m}$$

With these results, we can now find *precise* values for C, the magnitude of vector $\vec{\mathbf{C}}$, and its direction angle θ. In particular,

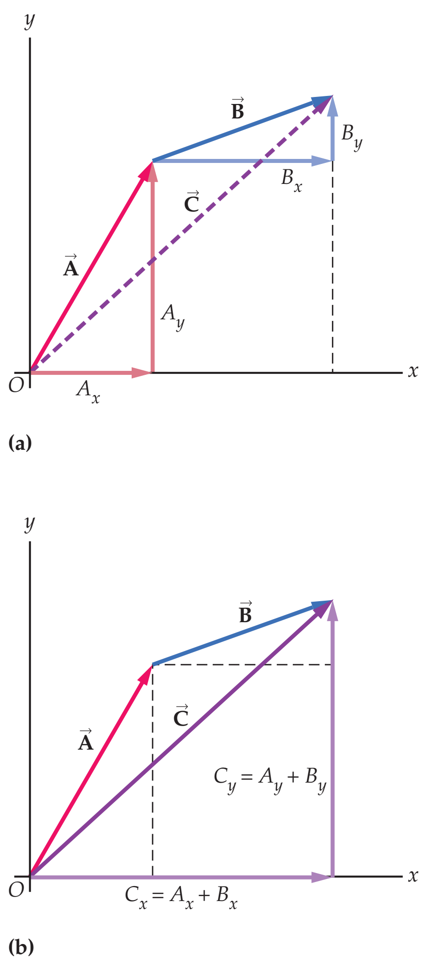
$$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(6.26 \text{ m})^2 + (5.70 \text{ m})^2} = \sqrt{71.7 \text{ m}^2} = 8.47 \text{ m}$$

and

$$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{5.70 \text{ m}}{6.26 \text{ m}}\right) = \tan^{-1}(0.911) = 42.3°$$

Note that these exact values are in rough agreement with the approximate results found by graphical addition.

In the future, we will always add vectors using components—graphical addition is useful primarily as a rough check on the results obtained with components.



▲ **FIGURE 3–13 Component addition of vectors**
(a) The x and y components of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. **(b)** The x and y components of $\vec{\mathbf{C}}$. Notice that $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

ACTIVE EXAMPLE 3–1 **TREASURE HUNT: FIND THE DIRECTION AND MAGNITUDE**

What are the magnitude and direction of the total displacement for the treasure hunt illustrated in Figure 3–9? Assume each pace is 0.750 m in length.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

To define a convenient notation, let the first 5 paces be represented by $\vec{\mathbf{A}}$, the next 3 paces by $\vec{\mathbf{B}}$, and the final 4 paces by $\vec{\mathbf{C}}$. The total displacement, then, is $\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$.

1. Find the components of $\vec{\mathbf{A}}$:	$A_x = 0$, $A_y = 3.75$ m
2. Find the components of $\vec{\mathbf{B}}$:	$B_x = 2.25$ m, $B_y = 0$
3. Find the components of $\vec{\mathbf{C}}$:	$C_x = 2.12$ m, $C_y = -2.12$ m
4. Sum the components of $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ to find the components of $\vec{\mathbf{D}}$:	$D_x = 4.37$ m, $D_y = 1.63$ m
5. Determine D and θ:	$D = 4.66$ m, $\theta = 20.5°$

YOUR TURN

If the length of each pace is decreased by a factor of two, to 0.375 m, by what factors do you expect D and θ to change? Verify your answers with a numerical calculation.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Subtracting Vectors

Next, how do we subtract vectors? Suppose, for example, that we would like to determine the vector $\vec{\mathbf{D}}$, where

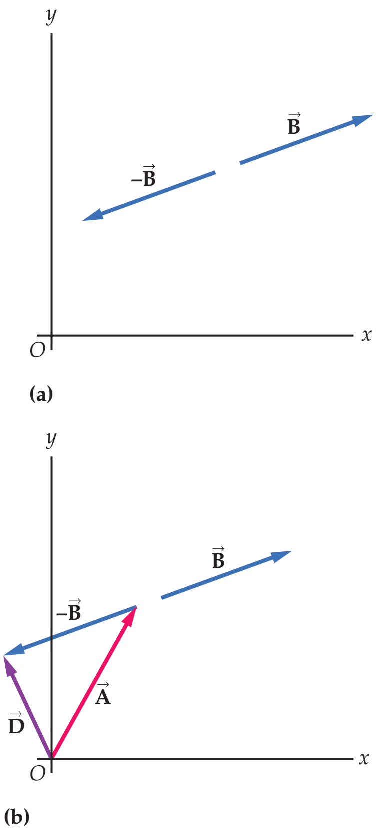
$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$$

and $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are the vectors shown in Figure 3–12. To find $\vec{\mathbf{D}}$, we start by rewriting it as follows:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + (-\vec{\mathbf{B}})$$

That is, $\vec{\mathbf{D}}$ is the sum of $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$. Now the negative of a vector has a very simple graphical interpretation:

- The negative of a vector is represented by an arrow of the same length as the original vector, but pointing in the opposite direction. That is, multiplying a vector by minus one *reverses its direction*.



▲ **FIGURE 3–14 Vector subtraction**
(a) The vector $\vec{\mathbf{B}}$ and its negative $-\vec{\mathbf{B}}$. **(b)** A vector construction for $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$.

For example, the vectors $\vec{\mathbf{B}}$ and $-\vec{\mathbf{B}}$ are indicated in **Figure 3–14 (a)**. Thus, to subtract $\vec{\mathbf{B}}$ from $\vec{\mathbf{A}}$, simply reverse the direction of $\vec{\mathbf{B}}$ and add it to $\vec{\mathbf{A}}$, as indicated in **Figure 3–14 (b)**.

In terms of components, you subtract vectors by simply subtracting the components. For example, if

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$$

then

$$D_x = A_x - B_x$$

and

$$D_y = A_y - B_y$$

Once the components of $\vec{\mathbf{D}}$ are found, its magnitude and direction angle can be calculated as usual.

EXERCISE 3–5

a. For the vectors given in Figure 3–12, find the components of $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$.
b. Find D and θ and compare with the vector $\vec{\mathbf{D}}$ shown in Figure 3–14 (b).

SOLUTION

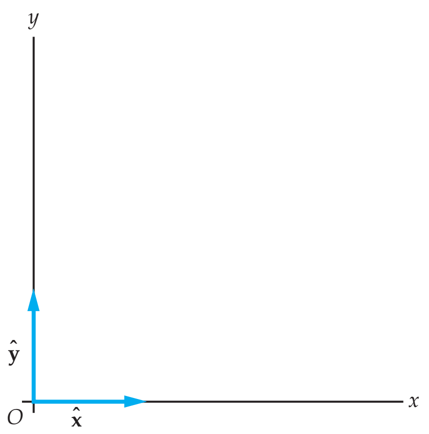
a. $D_x = -1.26$ m, $D_y = 2.96$ m
b. $D = 3.22$ m, $\theta = -66.9° + 180° = 113°$. In Figure 3–14 (b) we see that $\vec{\mathbf{D}}$ is shorter than $\vec{\mathbf{B}}$, which has a magnitude of 4.00 m, and its direction angle is somewhat greater than 90°, in agreement with our numerical results.

3–4 Unit Vectors

Unit vectors provide a convenient way of expressing an arbitrary vector in terms of its components, as we shall see. But first, let's define what we mean by a unit vector. In particular, the unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ are defined to be dimensionless vectors of unit magnitude pointing in the positive x and y directions, respectively:

- The x unit vector, $\hat{\mathbf{x}}$, is a dimensionless vector of unit length pointing in the positive x direction.
- The y unit vector, $\hat{\mathbf{y}}$, is a dimensionless vector of unit length pointing in the positive y direction.

Figure 3–15 shows $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ on a two-dimensional coordinate system. Since unit vectors have no physical dimensions—like mass, length, or time—they are used to specify direction only.



▲ **FIGURE 3–15 Unit vectors**
The unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ point in the positive x and y directions, respectively.

Multiplying Unit Vectors by Scalars

To see the utility of unit vectors, consider the effect of multiplying a vector by a scalar. For example, multiplying a vector by 3 increases its magnitude by a factor of 3, but does not change its direction, as shown in **Figure 3–16**. Multiplying by -3 increases the magnitude by a factor of 3 *and* reverses the direction of the vector. This is also shown in Figure 3–16. In the case of unit vectors—which have a magnitude of 1 and are dimensionless—multiplication by a scalar results in a vector with the same magnitude and dimensions as the scalar.

For example, if a vector $\vec{\mathbf{A}}$ has the scalar components $A_x = 5$ m and $A_y = 3$ m, we can write it as

$$\vec{\mathbf{A}} = (5\text{ m})\hat{\mathbf{x}} + (3\text{ m})\hat{\mathbf{y}}$$

We refer to the quantities $(5\text{ m})\hat{\mathbf{x}}$ and $(3\text{ m})\hat{\mathbf{y}}$ as the x and y **vector components** of the vector $\vec{\mathbf{A}}$. In general, an arbitrary two-dimensional vector $\vec{\mathbf{A}}$ can always be written as the sum of a vector component in the x direction and a vector component in the y direction:

$$\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$$

This is illustrated in **Figure 3–17 (a)**. An equivalent way of representing the vector components of a vector is illustrated in **Figure 3–17 (b)**. In this case we see that the vector components are the *projection* of a vector onto the x and y axes. The sign of the vector components is positive if they point in the positive x or y direction, and negative if they point in the negative x or y direction. This is how vector components will generally be shown in later chapters.

Finally, note that vector addition and subtraction are straightforward with unit vector notation:

$$\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}}$$

and

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = (A_x - B_x)\hat{\mathbf{x}} + (A_y - B_y)\hat{\mathbf{y}}$$

Clearly, unit vectors provide a useful way to keep track of the x and y components of a vector.

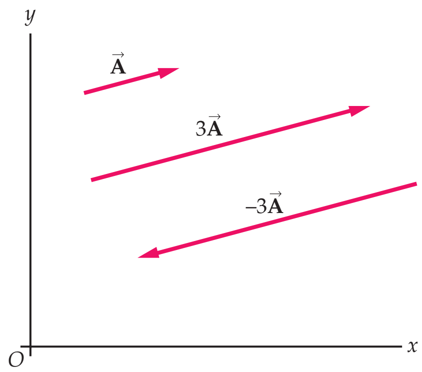


FIGURE 3–16 Multiplying a vector by a scalar

Multiplying a vector by a positive scalar different from 1 will change the length of the vector but leave its direction the same. If the vector is multiplied by a negative scalar its direction is reversed.

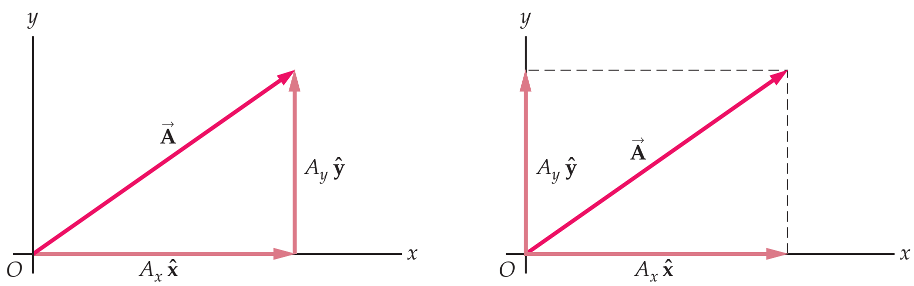


FIGURE 3–17 Vector components

(a) A vector $\vec{\mathbf{A}}$ can be written in terms of unit vectors as $\vec{\mathbf{A}} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}}$. **(b)** Vector components can be thought of as the projection of the vector onto the x and y axes. This method of representing vector components will be used frequently in subsequent chapters.

3–5 Position, Displacement, Velocity, and Acceleration Vectors

In Chapter 2 we discussed four different one-dimensional vectors: position, displacement, velocity, and acceleration. Each of these quantities had a direction associated with it, indicated by its sign; positive meant in the positive direction, negative meant in the negative direction. Now we consider these vectors again, this time in two dimensions, where the possibilities for direction are not so limited.

Position Vectors

To begin, imagine a two-dimensional coordinate system, as in **Figure 3–18**. Position is indicated by a vector from the origin to the location in question. We refer to the position vector as $\vec{\mathbf{r}}$; its units are meters, m.

Definition: Position Vector, $\vec{\mathbf{r}}$

position vector $= \vec{\mathbf{r}}$ 3–1

SI unit: meter, m

In terms of unit vectors, the position vector is simply $\vec{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$.

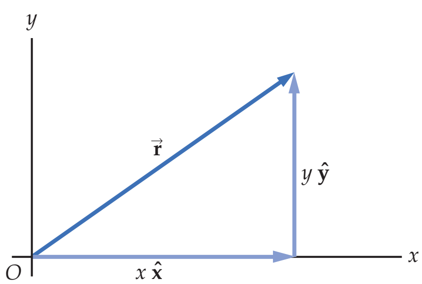


FIGURE 3–18 Position vector

The position vector $\vec{\mathbf{r}}$ points from the origin to the current location of an object. The x and y vector components of $\vec{\mathbf{r}}$ are $x\hat{\mathbf{x}}$ and $y\hat{\mathbf{y}}$, respectively.

▲ A map can be used to determine the direction and magnitude of the displacement vector from your initial position to your destination.

Displacement Vectors

Now, suppose that initially you are at the location indicated by the position vector $\vec{\mathbf{r}}_i$, and that later you are at the final position represented by the position vector $\vec{\mathbf{r}}_f$. Your displacement vector, $\Delta\vec{\mathbf{r}}$, is the change in position:

Definition: Displacement Vector, $\Delta\vec{\mathbf{r}}$

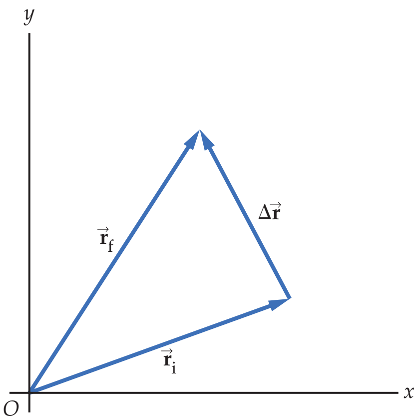
$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i \qquad 3\text{–}2$$

SI unit: meter, m

Rearranging this definition slightly, we see that

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$$

That is, the final position is equal to the initial position plus the change in position. This is illustrated in **Figure 3–19**, where we see that $\Delta\vec{\mathbf{r}}$ extends from the head of $\vec{\mathbf{r}}_i$ to the head of $\vec{\mathbf{r}}_f$.



▲ **FIGURE 3–19 Displacement vector**

The displacement vector $\Delta\vec{\mathbf{r}}$ is the change in position. It points from the head of the initial position vector $\vec{\mathbf{r}}_i$ to the head of the final position vector $\vec{\mathbf{r}}_f$. Thus $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$ or $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$.

Velocity Vectors

Next, the average velocity vector is defined as the displacement vector $\Delta\vec{\mathbf{r}}$ divided by the elapsed time Δt.

Definition: Average Velocity Vector, $\vec{\mathbf{v}}_{av}$

$$\vec{\mathbf{v}}_{av} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad 3\text{–}3$$

SI unit: meter per second, m/s

Since $\Delta\vec{\mathbf{r}}$ is a vector, it follows that $\vec{\mathbf{v}}_{av}$ is also a vector; it is the vector $\Delta\vec{\mathbf{r}}$ times the scalar $(1/\Delta t)$. Thus $\vec{\mathbf{v}}_{av}$ is parallel to $\Delta\vec{\mathbf{r}}$ and has the units m/s.

EXERCISE 3–6

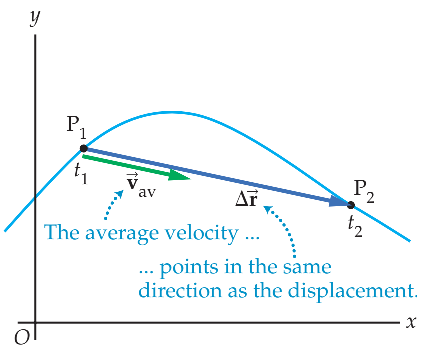
A dragonfly is observed initially at the position $\vec{\mathbf{r}}_i = (2.00\text{ m})\hat{\mathbf{x}} + (3.50\text{ m})\hat{\mathbf{y}}$. Three seconds later it is at the position $\vec{\mathbf{r}}_f = (-3.00\text{ m})\hat{\mathbf{x}} + (5.50\text{ m})\hat{\mathbf{y}}$. What was the dragonfly's average velocity during this time?

SOLUTION

$$\begin{aligned}\vec{\mathbf{v}}_{av} &= (\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i)/\Delta t = [(-5.00\text{ m})\hat{\mathbf{x}} + (2.00\text{ m})\hat{\mathbf{y}}]/(3.00\text{ s})\\ &= (-1.67\text{ m/s})\hat{\mathbf{x}} + (0.667\text{ m/s})\hat{\mathbf{y}}\end{aligned}$$

To help visualize $\vec{\mathbf{v}}_{av}$, imagine a particle moving in two dimensions along the blue path shown in **Figure 3–20**. If the particle is at point P_1 at time t_1, and at P_2 at time t_2, its displacement is indicated by the vector $\Delta\vec{\mathbf{r}}$. The average velocity is parallel to $\Delta\vec{\mathbf{r}}$, as indicated in Figure 3–20. It makes sense physically that $\vec{\mathbf{v}}_{av}$ is parallel to $\Delta\vec{\mathbf{r}}$; after all, on *average* you have moved in the direction of $\Delta\vec{\mathbf{r}}$ during the time from t_1 to t_2. To put it another way, a particle that starts at P_1 at the time t_1 and moves with the velocity $\vec{\mathbf{v}}_{av}$ until the time t_2 will arrive in precisely the same location as the particle that follows the blue path.

By considering smaller and smaller time intervals, as in **Figure 3–21**, it is possible to calculate the instantaneous velocity vector:



▲ **FIGURE 3–20 Average velocity vector**

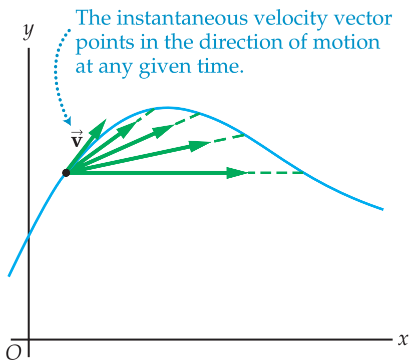
The average velocity, $\vec{\mathbf{v}}_{av}$, points in the same direction as the displacement, $\Delta\vec{\mathbf{r}}$, for any given interval of time.

Definition: Instantaneous Velocity Vector, $\vec{\mathbf{v}}$

$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t} \qquad 3\text{–}4$$

SI unit: meter per second, m/s

As can be seen in Figure 3–21, the instantaneous velocity at a given time is tangential to the path of the particle at that time. In addition, the magnitude of the velocity vector is the speed of the particle. Thus, the instantaneous velocity vector tells you both how fast a particle is moving and in what direction.



▲ **FIGURE 3–21 Instantaneous velocity vector**

The instantaneous velocity vector $\vec{\mathbf{v}}$ is obtained by calculating the average velocity vector over smaller and smaller time intervals. In the limit of vanishingly small time intervals, the average velocity approaches the instantaneous velocity, which points in the direction of motion.

EXERCISE 3–7

Find the speed and direction of motion for a rainbow trout whose velocity is $\vec{\mathbf{v}} = (3.7\text{ m/s})\hat{\mathbf{x}} + (-1.3\text{ m/s})\hat{\mathbf{y}}$.

SOLUTION

speed $= v = \sqrt{(3.7\text{ m/s})^2 + (-1.3\text{ m/s})^2} = 3.9\text{ m/s}, \theta = \tan^{-1}\left(\dfrac{-1.3\text{ m/s}}{3.7\text{ m/s}}\right) = -19°$,

that is, 19° below the x axis.

Acceleration Vectors

Finally, the average acceleration vector over an interval of time, Δt, is defined as the change in the velocity vector, $\Delta\vec{\mathbf{v}}$, divided by the scalar Δt.

Definition: Average Acceleration Vector, $\vec{\mathbf{a}}_{av}$

$$\vec{\mathbf{a}}_{av} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} \qquad 3\text{–}5$$

SI unit: meter per second per second, m/s^2

An example is given in **Figure 3–22**, where we show the initial and final velocity vectors corresponding to two different times. Since the change in velocity is defined as

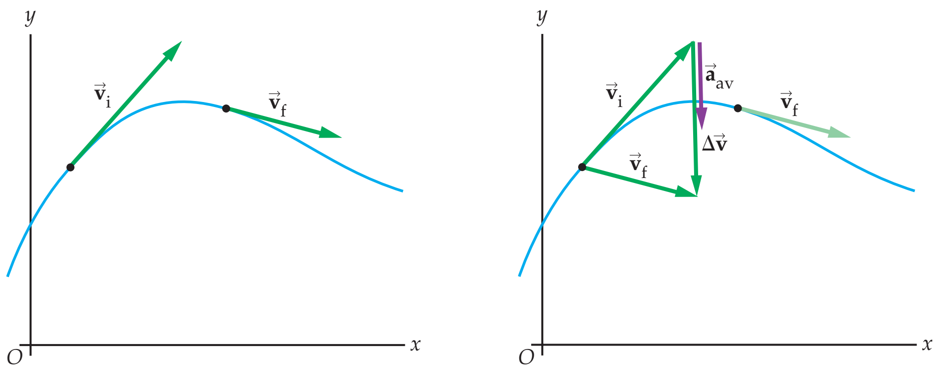
$$\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$$

it follows that

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \Delta\vec{\mathbf{v}}$$

as indicated in Figure 3–22. Thus, $\Delta\vec{\mathbf{v}}$ is the vector extending from the head of $\vec{\mathbf{v}}_i$ to the head of $\vec{\mathbf{v}}_f$, just as $\Delta\vec{\mathbf{r}}$ extends from the head of $\vec{\mathbf{r}}_i$ to the head of $\vec{\mathbf{r}}_f$ in Figure 3–19. The direction of $\vec{\mathbf{a}}_{av}$ is the direction of $\Delta\vec{\mathbf{v}}$, as shown in **Figure 3–22(b)**.

Can an object accelerate if its speed is constant? Absolutely—if its direction changes. Consider a car driving with a constant speed on a circular track, as

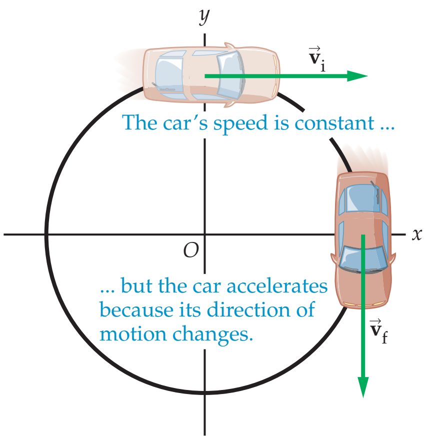


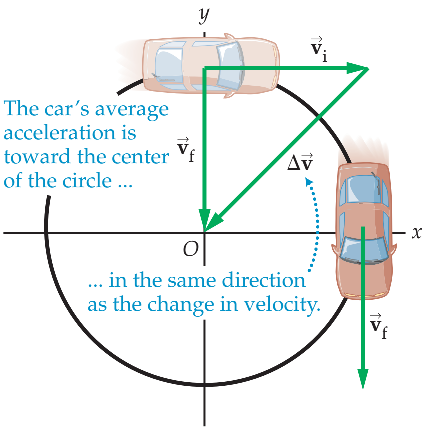
(a) The instantaneous velocity at two different times

(b) The average acceleration points in the same direction as the change in velocity

▲ **FIGURE 3–22 Average acceleration vector**

(a) As a particle moves along the blue path its velocity changes in magnitude and direction. At the time t_i the velocity is $\vec{\mathbf{v}}_i$; at the time t_f the velocity is $\vec{\mathbf{v}}_f$. **(b)** The average acceleration vector $\vec{\mathbf{a}}_{av} = \Delta\vec{\mathbf{v}}/\Delta t$ points in the direction of the change in velocity vector $\Delta\vec{\mathbf{v}}$. We obtain $\Delta\vec{\mathbf{v}}$ by moving $\vec{\mathbf{v}}_f$ so that its tail coincides with the tail of $\vec{\mathbf{v}}_i$, and then drawing the arrow that connects the head of $\vec{\mathbf{v}}_i$ to the head of $\vec{\mathbf{v}}_f$. Note that $\vec{\mathbf{a}}_{av}$ need not point in the direction of motion, and in general it doesn't.





▲ **FIGURE 3–23 Average acceleration for a car traveling in a circle with constant speed**
Although the speed of this car never changes, it is still accelerating—due to the change in its direction of motion. For the time interval depicted, the car's average acceleration is in the direction of $\Delta\vec{v}$, which is toward the center of the circle. (As we shall see in Chapter 6, the car's acceleration is always toward the center of the circle.)

shown in **Figure 3–23**. Suppose that the initial velocity of the car is $\vec{v}_i = (12\text{ m/s})\hat{x}$, and that 10.0 s later its final velocity is $\vec{v}_f = (-12\text{ m/s})\hat{y}$. Note that the speed is 12 m/s in each case, but the velocity is different because the *direction* has changed. Calculating the average acceleration, we find a nonzero acceleration:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{10.0\text{ s}}$$

$$= \frac{(-12\text{ m/s})\hat{y} - (12\text{ m/s})\hat{x}}{10.0\text{ s}} = (-1.2\text{ m/s}^2)\hat{x} + (-1.2\text{ m/s}^2)\hat{y}$$

Thus, a change in direction is just as important as a change in speed in producing an acceleration. We shall study circular motion in detail in Chapter 6.

Finally, by going to infinitesimally small time intervals, $\Delta t \to 0$, we can define the instantaneous acceleration:

Definition: Instantaneous Acceleration Vector, $\vec{a}$

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} \tag{3–6}$$

SI unit: meter per second per second, m/s^2

ACTIVE EXAMPLE 3–2 FIND THE AVERAGE ACCELERATION

A car is traveling northwest at 9.00 m/s. Eight seconds later it has rounded a corner and is now heading north at 15.0 m/s. What are the magnitude and direction of its average acceleration during those 8.00 seconds?

Let the positive x direction be east, and the positive y direction be north.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write out $\vec{v}_i$:	$\vec{v}_i = (-6.36\text{ m/s})\hat{x} + (6.36\text{ m/s})\hat{y}$
2. Write out $\vec{v}_f$:	$\vec{v}_f = (15.0\text{ m/s})\hat{y}$
3. Calculate $\Delta\vec{v}$:	$\Delta\vec{v} = (6.36\text{ m/s})\hat{x} + (8.64\text{ m/s})\hat{y}$
4. Find $\vec{a}_{av}$:	$\vec{a}_{av} = (0.795\text{ m/s}^2)\hat{x} + (1.08\text{ m/s}^2)\hat{y}$
5. Determine a_{av} and θ:	$a_{av} = 1.34\text{ m/s}^2$, $\theta = 53.6°$ north of east

YOUR TURN
Find the magnitude and direction of the average acceleration if the same change in velocity occurs in 4.00 s rather than 8.00 s.
(Answers to **Your Turn** *problems are given in the back of the book.)*

▶ The velocities of these cyclists change in both magnitude and direction as they slow to negotiate a series of sharp curves and then speed up again. Both kinds of velocity change involve an acceleration.

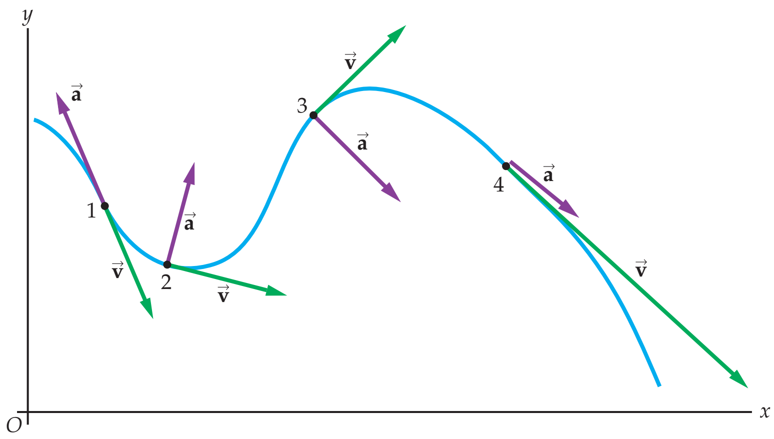


FIGURE 3–24 Velocity and acceleration vectors for a particle moving along a winding path
The acceleration of a particle need not point in the direction of motion. At point (1) the particle is slowing down, at (2) it is turning to the left, at (3) it is turning to the right, and, finally, at point (4) it is speeding up.

Note carefully the following critical distinctions between the velocity vector and the acceleration vector:

- The velocity vector, $\vec{\mathbf{v}}$, is always in the direction of a particle's motion.
- The acceleration vector, $\vec{\mathbf{a}}$, can point in directions other than the direction of motion, and in general it does.

An example of a particle's motion, showing the velocity and acceleration vectors at various times, is presented in **Figure 3–24**.

Note that in all cases the velocity is tangential to the motion, though the acceleration points in various directions. When the acceleration is perpendicular to the velocity of an object, as at points (2) and (3) in Figure 3–24, its speed remains constant while its direction of motion changes. At points (1) and (4) in Figure 3–24 the acceleration is antiparallel (opposite) or parallel to the velocity of the object, respectively. In such cases, the direction of motion remains the same while the speed changes. Throughout the next chapter we shall see further examples of motion in which the velocity and acceleration are in different directions.

3–6 Relative Motion

A good example of the use of vectors is in the description of relative motion. Suppose, for example, that you are standing on the ground as a train goes by at 15.0 m/s, as shown in **Figure 3–25**. Inside the train, a free-riding passenger is walking in the forward direction at 1.2 m/s relative to the train. How fast is the passenger moving relative to you? Clearly, the answer is 1.2 m/s + 15.0 m/s = 16.2 m/s. What if the passenger had been walking with the same speed, but toward the back of the train? In this case, you would see the passenger going by with a speed of −1.2 m/s + 15.0 m/s = 13.8 m/s.

Let's generalize these results. Call the velocity of the *t*rain relative to the *g*round $\vec{\mathbf{v}}_{tg}$, the velocity of the *p*assenger relative to the *t*rain $\vec{\mathbf{v}}_{pt}$, and the velocity of the *p*assenger relative to the *g*round $\vec{\mathbf{v}}_{pg}$. As we saw in the previous paragraph, the velocity of the *p*assenger relative to the *g*round is

$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg} \qquad 3\text{–}7$$

This vector addition is illustrated in **Figure 3–26** for the two cases we discussed.

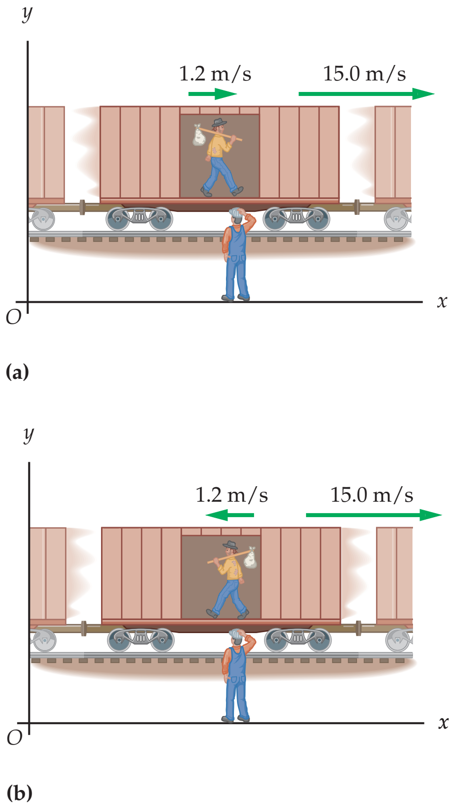


FIGURE 3–25 Relative velocity of a passenger on a train with respect to a person on the ground
(a) The passenger walks toward the front of the train. **(b)** The passenger walks toward the rear of the train.

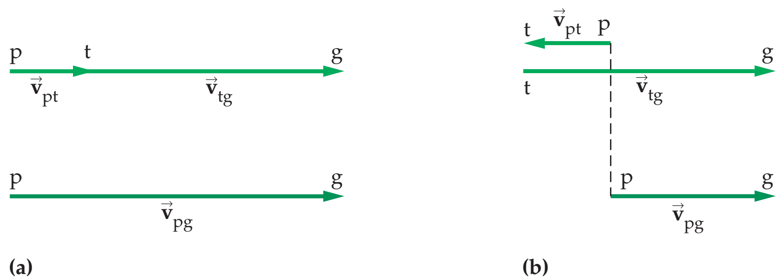
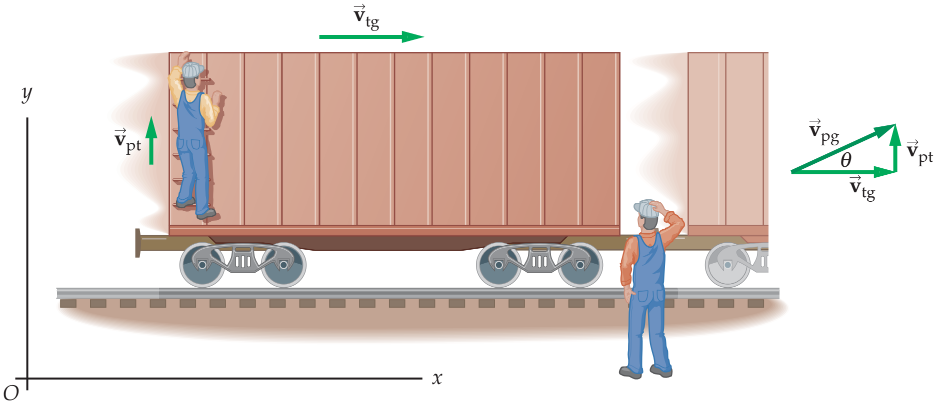


FIGURE 3–26 Adding velocity vectors
Vector addition to find the velocity of the passenger with respect to the ground for **(a)** Figure 3–25 (a) and **(b)** Figure 3–25 (b).



▲ **FIGURE 3–27 Relative velocity in two dimensions**

A person climbs up a ladder on a moving train with velocity $\vec{\mathbf{v}}_{pt}$ relative to the train. If the train moves relative to the ground with a velocity $\vec{\mathbf{v}}_{tg}$, the velocity of the person on the train relative to the ground is $\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$.

Though this example dealt with one-dimensional motion, Equation 3–7 is valid for velocity vectors pointing in arbitrary directions. For example, instead of walking on the car's floor, the passenger might be climbing a ladder to the roof of the car, as in **Figure 3–27**. In this case $\vec{\mathbf{v}}_{pt}$ is vertical, $\vec{\mathbf{v}}_{tg}$ is horizontal, and $\vec{\mathbf{v}}_{pg}$ is simply the vector sum $\vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$.

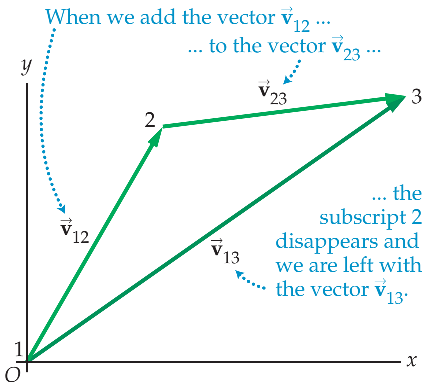
EXERCISE 3–8

Suppose the passenger in Figure 3–27 is climbing a vertical ladder with a speed of 0.20 m/s, and the train is slowly coasting forward at 0.70 m/s. Find the speed and direction of the passenger relative to the ground.

SOLUTION

$$\vec{\mathbf{v}}_{pg} = (0.70\ \text{m/s})\hat{\mathbf{x}} + (0.20\ \text{m/s})\hat{\mathbf{y}};\ \text{thus}$$

$$v_{pg} = \sqrt{(0.70\ \text{m/s})^2 + (0.20\ \text{m/s})^2} = 0.73\ \text{m/s},\ \theta = \tan^{-1}(0.20/0.70) = 16°$$



▲ **FIGURE 3–28 Vector addition used to determine relative velocity**

Note that the subscripts in Equation 3–7 follow a definite pattern. On the left-hand side of the equation we have the subscripts pg. On the right-hand side we have two sets of subscripts, pt and tg; note that a pair of t's has been inserted between the p and the g. This pattern always holds for any relative motion problem, though the subscripts will be different when referring to different objects. Thus, we can say quite generally that

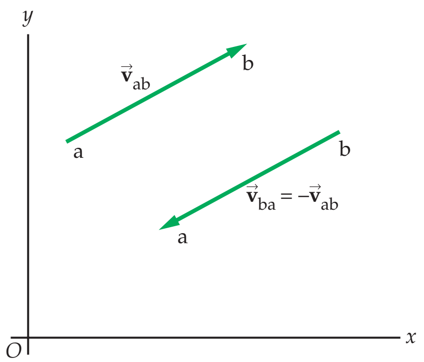
$$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23} \qquad 3\text{–}8$$

where, in the train example, we can identify 1 as the *p*assenger, 2 as the *t*rain, and 3 as the *g*round.

The vector addition in Equation 3–8 is shown in **Figure 3–28**. For convenience in seeing how the subscripts are ordered in the equation, we have labeled the tail of each vector with its first subscript and the head of each vector with its second subscript.

One final note about velocities and their subscripts: Reversing the subscripts reverses the velocity. This is indicated in **Figure 3–29**, where we see that

$$\vec{\mathbf{v}}_{ba} = -\vec{\mathbf{v}}_{ab}$$



▲ **FIGURE 3–29 Reversing the subscripts of a velocity reverses the corresponding velocity vector**

Physically, what we are saying is that if you are riding in a car due *north* at 20 m/s relative to the ground, then the ground, relative to you, is moving due *south* at 20 m/s.

Let's apply these results to a two-dimensional example.

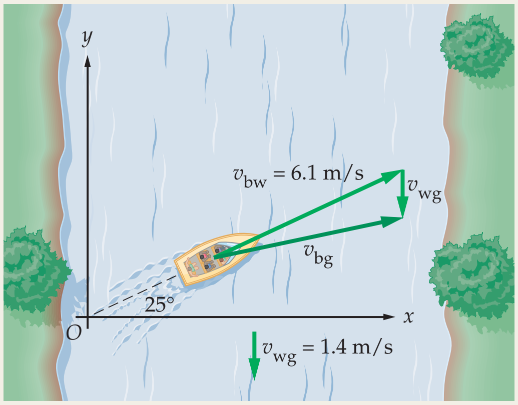
EXAMPLE 3–2 CROSSING A RIVER

REAL-WORLD PHYSICS

You are riding in a boat whose speed relative to the water is 6.1 m/s. The boat points at an angle of 25° upstream on a river flowing at 1.4 m/s. **(a)** What is your velocity relative to the ground? **(b)** Suppose the speed of the boat relative to the water remains the same, but the direction in which it points is changed. What angle is required for the boat to go straight across the river?

PICTURE THE PROBLEM

We choose the x axis to be perpendicular to the river, and the y axis to point upstream. With these choices the velocity of the boat relative to the water is 25° above the x axis. In addition, the velocity of the water relative to the ground has a magnitude of 1.4 m/s and points in the negative y direction.



INTERACTIVE FIGURE

STRATEGY

If the water were still, the boat would move in the direction in which it is pointed. With the water flowing downstream, as shown, the boat will move in a direction closer to the x axis. **(a)** To find the velocity of the boat we use $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with 1 referring to the boat (b), 2 referring to the water (w), and 3 referring to the ground (g). **(b)** To go "straight across the river" means that the velocity of the boat relative to the ground should be in the x direction. Thus, we choose the angle θ that cancels the y component of velocity.

SOLUTION

Part (a)

1. Rewrite $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with $1 \to$ b, $2 \to$ w, and $3 \to$ g: $\quad \vec{\mathbf{v}}_{bg} = \vec{\mathbf{v}}_{bw} + \vec{\mathbf{v}}_{wg}$

2. From our sketch we see that the water flows at 1.4 m/s in the negative y direction relative to the ground: $\quad \vec{\mathbf{v}}_{wg} = (-1.4\ \mathrm{m/s})\hat{\mathbf{y}}$

3. The velocity of the boat relative to the water is given in the problem statement:
$$\vec{\mathbf{v}}_{bw} = (6.1\ \mathrm{m/s})\cos 25°\,\hat{\mathbf{x}} + (6.1\ \mathrm{m/s})\sin 25°\,\hat{\mathbf{y}} = (5.5\ \mathrm{m/s})\hat{\mathbf{x}} + (2.6\ \mathrm{m/s})\hat{\mathbf{y}}$$

4. Carry out the vector sum in Step 1 to find $\vec{\mathbf{v}}_{bg}$:
$$\vec{\mathbf{v}}_{bg} = (5.5\ \mathrm{m/s})\hat{\mathbf{x}} + (2.6\ \mathrm{m/s} - 1.4\ \mathrm{m/s})\hat{\mathbf{y}} = (5.5\ \mathrm{m/s})\hat{\mathbf{x}} + (1.2\ \mathrm{m/s})\hat{\mathbf{y}}$$

Part (b)

5. To cancel the y component of $\vec{\mathbf{v}}_{bg}$, we choose the angle θ that gives 1.4 m/s for the y component of $\vec{\mathbf{v}}_{bw}$: $\quad (6.1\ \mathrm{m/s})\sin\theta = 1.4\ \mathrm{m/s}$

6. Solve for θ. With this angle, we see that the y component of $\vec{\mathbf{v}}_{bg}$ in Step 4 will be zero: $\quad \theta = \sin^{-1}(1.4/6.1) = 13°$

INSIGHT

(a) Note that the speed of the boat relative to the ground is $\sqrt{(5.5\ \mathrm{m/s})^2 + (1.2\ \mathrm{m/s})^2} = 5.6\ \mathrm{m/s}$, and the direction angle is $\theta = \tan^{-1}(1.2/5.5) = 12°$ upstream. **(b)** The speed of the boat in this case is equal to the x component of its velocity, since the y component is zero. Therefore, its speed is $(6.1\ \mathrm{m/s})\cos 13° = 5.9\ \mathrm{m/s}$.

PRACTICE PROBLEM

Find the speed and direction of the boat relative to the ground if the river flows at 4.5 m/s. [**Answer:** $v_{bg} = 5.8\ \mathrm{m/s}$, $\theta = -19°$. In this case, a person on the ground sees the boat going slowly downstream, even though the boat itself points upstream.]

Some related homework problems: Problem 50, Problem 53, Problem 55

Suppose the problem had been to find the velocity of the boat relative to the water so that it goes straight across the river at 5.0 m/s. That is, we want to find $\vec{\mathbf{v}}_{bw}$ such that $\vec{\mathbf{v}}_{bg} = (5.0\ \mathrm{m/s})\hat{\mathbf{x}}$. One approach is to simply solve $\vec{\mathbf{v}}_{bg} = \vec{\mathbf{v}}_{bw} + \vec{\mathbf{v}}_{wg}$ for $\vec{\mathbf{v}}_{bw}$, which gives

$$\vec{\mathbf{v}}_{bw} = \vec{\mathbf{v}}_{bg} - \vec{\mathbf{v}}_{wg} \qquad 3\text{–}9$$

Another approach is to go back to our general relation, $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ and choose 1 to be the *boat*, 2 to be the *ground*, and 3 to be the *water*. With these substitutions we find

$$\vec{\mathbf{v}}_{bw} = \vec{\mathbf{v}}_{bg} + \vec{\mathbf{v}}_{gw}$$

This is the same as Equation 3–9, since $\vec{\mathbf{v}}_{gw} = -\vec{\mathbf{v}}_{wg}$. In either case, the desired velocity of the boat relative to the water is

$$\vec{\mathbf{v}}_{bw} = (5.0\ \mathrm{m/s})\hat{\mathbf{x}} + (1.4\ \mathrm{m/s})\hat{\mathbf{y}}$$

which corresponds to a speed of 5.2 m/s and a direction angle of 16° upstream.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In Chapter 2 we indicated direction with + and – signs, since only two directions were possible. With the results from this chapter we can now deal with quantities that point in any direction at all.

The vector quantities we have considered so far are position, displacement, velocity, and acceleration. These quantities are important throughout our study of mechanics.

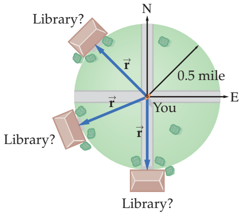
LOOKING AHEAD

In Chapter 4 we will consider kinematics in two dimensions. As we shall see, the vectors developed in this chapter will play a key role in that study. In particular, vectors will allow us to analyze two-dimensional motion as a combination of two completely independent one-dimensional motions.

In Chapter 5 we will introduce one of the most important concepts in all of physics—force. It is a vector quantity. Other important vector quantities to be introduced in later chapters include linear momentum (Chapter 9), angular momentum (Chapter 11), electric field (Chapter 19), and magnetic field (Chapter 22).

CHAPTER SUMMARY

3–1 SCALARS VERSUS VECTORS



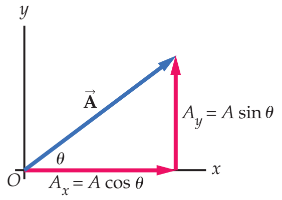
Scalar

A number with appropriate units. Examples of scalar quantities include time and length.

Vector

A quantity with both a magnitude and a direction. Examples include displacement, velocity, and acceleration.

3–2 THE COMPONENTS OF A VECTOR



x Component of Vector $\vec{\mathbf{A}}$

$A_x = A\cos\theta$, where θ is measured relative to the x axis.

y Component of Vector $\vec{\mathbf{A}}$

$A_y = A\sin\theta$, where θ is measured relative to the x axis.

Sign of the Components

A_x is positive if $\vec{\mathbf{A}}$ points in the positive x direction, and negative if it points in the negative x direction. Similar remarks apply to A_y.

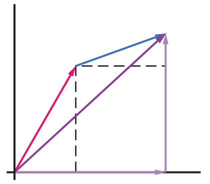
Magnitude of Vector $\vec{\mathbf{A}}$

The magnitude of $\vec{\mathbf{A}}$ is $A = \sqrt{A_x^2 + A_y^2}$.

Direction Angle of Vector $\vec{\mathbf{A}}$

The direction angle of $\vec{\mathbf{A}}$ is $\theta = \tan^{-1}(A_y/A_x)$, where θ is measured relative to the x axis.

3–3 ADDING AND SUBTRACTING VECTORS

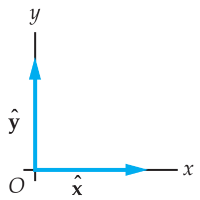


Graphical Method
To add $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, place them so that the tail of $\vec{\mathbf{B}}$ is at the head of $\vec{\mathbf{A}}$. The sum $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ is the arrow from the tail of $\vec{\mathbf{A}}$ to the head of $\vec{\mathbf{B}}$. See Figure 3–8.

To find $\vec{\mathbf{A}} - \vec{\mathbf{B}}$, place $\vec{\mathbf{A}}$ and $-\vec{\mathbf{B}}$ head-to-tail and draw an arrow from the tail of $\vec{\mathbf{A}}$ to the head of $-\vec{\mathbf{B}}$. See Figure 3–14.

Component Method
If $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, then $C_x = A_x + B_x$ and $C_y = A_y + B_y$. If $\vec{\mathbf{C}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, then $C_x = A_x - B_x$ and $C_y = A_y - B_y$.

3–4 UNIT VECTORS

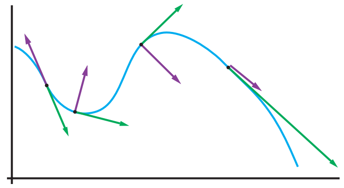


***x* Unit Vector**
Written $\hat{\mathbf{x}}$, the x unit vector is a dimensionless vector of unit length in the positive x direction.

***y* Unit Vector**
Written $\hat{\mathbf{y}}$, the y unit vector is a dimensionless vector of unit length in the positive y direction.

Vector Addition
$\vec{\mathbf{A}} + \vec{\mathbf{B}} = (A_x + B_x)\hat{\mathbf{x}} + (A_y + B_y)\hat{\mathbf{y}}$

3–5 POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS



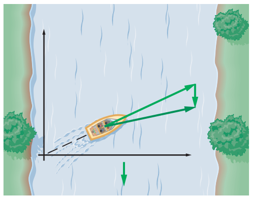
Position Vector
The position vector $\vec{\mathbf{r}}$ points from the origin to a particle's location.

Displacement Vector
The displacement vector $\Delta\vec{\mathbf{r}}$ is the change in position; $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i$.

Velocity Vector
The velocity vector $\vec{\mathbf{v}}$ points in the direction of motion and has a magnitude equal to the speed.

Acceleration Vector
The acceleration vector $\vec{\mathbf{a}}$ indicates how quickly and in what direction the velocity is changing. It need not point in the direction of motion.

3–6 RELATIVE MOTION



Velocity of Object 1 Relative to Object 3
$\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$, where object 2 can be anything.

Reversing the Subscripts on a Velocity
$\vec{\mathbf{v}}_{12} = -\vec{\mathbf{v}}_{21}$.

PROBLEM-SOLVING SUMMARY

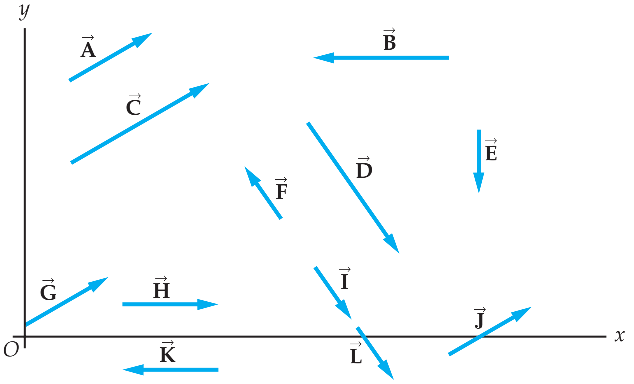
Type of Problem	Relevant Physical Concepts	Related Examples
Add or subtract vectors.	Resolve the vectors into x and y components, then add or subtract the components.	Active Example 3–1 Exercise 3–5
Calculate the average velocity.	Divide the displacement, $\Delta\vec{\mathbf{r}}$, by the elapsed time, Δt.	Exercise 3–6
Calculate the average acceleration.	Divide the change in velocity, $\Delta\vec{\mathbf{v}}$, by the elapsed time, Δt.	Active Example 3–2
Find the relative velocity of object 1 with respect to object 3.	Use $\vec{\mathbf{v}}_{13} = \vec{\mathbf{v}}_{12} + \vec{\mathbf{v}}_{23}$ with the appropriate choices for 1, 2, and 3.	Example 3–2 Exercise 3–8

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. For the following quantities, indicate which is a scalar and which is a vector: **(a)** the time it takes for you to run the 100-yard dash; **(b)** your displacement after running the 100-yard dash; **(c)** your average velocity while running; **(d)** your average speed while running.
2. Which, if any, of the vectors shown in **Figure 3–30** are equal?



▲ **FIGURE 3–30** Conceptual Question 2

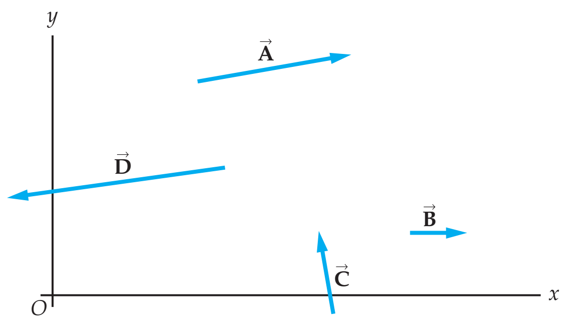
3. Given that $\vec{A} + \vec{B} = 0$, **(a)** how does the magnitude of $\vec{B}$ compare with the magnitude of $\vec{A}$? **(b)** How does the direction of $\vec{B}$ compare with the direction of $\vec{A}$?
4. Can a component of a vector be greater than the vector's magnitude?
5. Suppose that $\vec{A}$ and $\vec{B}$ have nonzero magnitude. Is it possible for $\vec{A} + \vec{B}$ to be zero?
6. Can a vector with zero magnitude have one or more components that are nonzero? Explain.
7. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A^2 + B^2 = C^2$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?
8. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A + B = C$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?
9. Given that $\vec{A} + \vec{B} = \vec{C}$, and that $A - B = C$, how are $\vec{A}$ and $\vec{B}$ oriented relative to one another?
10. Vector $\vec{A}$ has x and y components of equal magnitude. What can you say about the possible directions of $\vec{A}$?
11. The components of a vector $\vec{A}$ satisfy the relation $A_x = -A_y \neq 0$. What are the possible directions of $\vec{A}$?
12. Use a sketch to show that two vectors of unequal magnitude cannot add to zero, but that three vectors of unequal magnitude can.
13. Rain is falling vertically downward and you are running for shelter. To keep driest, should you hold your umbrella vertically, tilted forward, or tilted backward? Explain.
14. When sailing, the wind feels stronger when you sail upwind ("beating") than when you are sailing downwind ("running"). Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

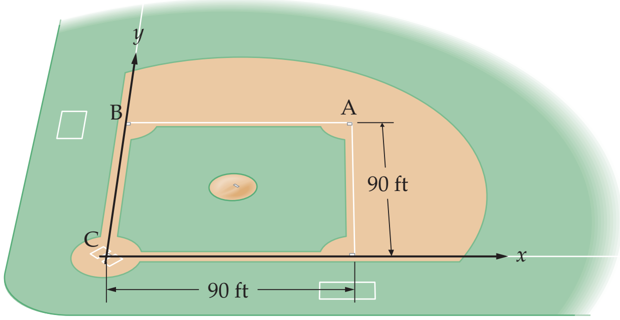
SECTION 3–2 THE COMPONENTS OF A VECTOR

1. • **CE** Suppose that each component of a certain vector is doubled. **(a)** By what multiplicative factor does the magnitude of the vector change? **(b)** By what multiplicative factor does the direction angle of the vector change?
2. • **CE** Rank the vectors in **Figure 3–31** in order of increasing magnitude.



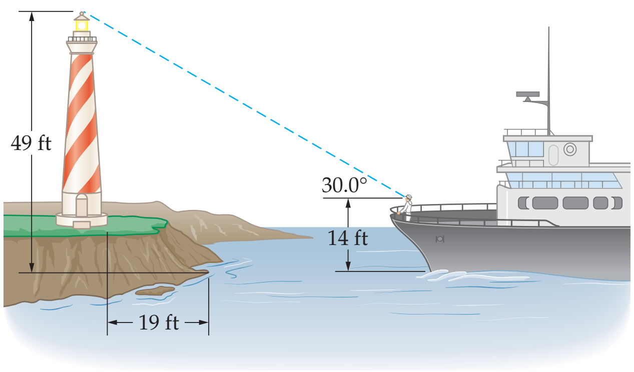
▲ **FIGURE 3–31** Problems 2, 3, and 4

3. • **CE** Rank the vectors in Figure 3–31 in order of increasing value of their x component.
4. • **CE** Rank the vectors in Figure 3–31 in order of increasing value of their y component.
5. • The press box at a baseball park is 32.0 ft above the ground. A reporter in the press box looks at an angle of 15.0° below the horizontal to see second base. What is the horizontal distance from the press box to second base?
6. • You are driving up a long, inclined road. After 1.2 miles you notice that signs along the roadside indicate that your elevation has increased by 530 ft. **(a)** What is the angle of the road above the horizontal? **(b)** How far do you have to drive to gain an additional 150 ft of elevation?
7. • **A One-Percent Grade** A road that rises 1 ft for every 100 ft traveled horizontally is said to have a 1% grade. Portions of the Lewiston grade, near Lewiston, Idaho, have a 6% grade. At what angle is this road inclined above the horizontal?
8. • Find the x and y components of a position vector $\vec{r}$ of magnitude $r = 75$ m, if its angle relative to the x axis is **(a)** 35.0° and **(b)** 65.0°.
9. • A baseball "diamond" **(Figure 3–32)** is a square with sides 90 ft in length. If the positive x axis points from home plate to first base, and the positive y axis points from home plate to third base, find the displacement vector of a base runner who has just hit **(a)** a double, **(b)** a triple, or **(c)** a home run.



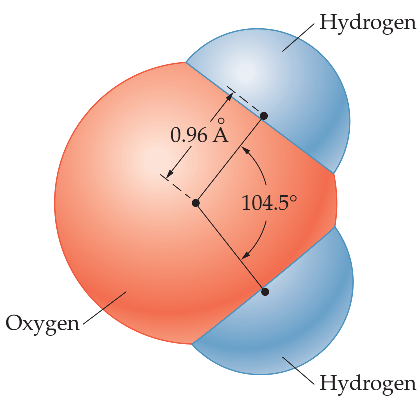
▲ **FIGURE 3–32** Problem 9

10. •• A lighthouse that rises 49 ft above the surface of the water sits on a rocky cliff that extends 19 ft from its base, as shown in **Figure 3–33**. A sailor on the deck of a ship sights the top of the lighthouse at an angle of 30.0° above the horizontal. If the sailor's eye level is 14 ft above the water, how far is the ship from the rocks?



▲ **FIGURE 3–33** Problem 10

11. •• H_2O A water molecule is shown schematically in **Figure 3–34**. The distance from the center of the oxygen atom to the center of a hydrogen atom is 0.96 Å, and the angle between the hydrogen atoms is 104.5°. Find the center-to-center distance between the hydrogen atoms. (1 Å $= 10^{-10}$ m.)



▲ **FIGURE 3–34** Problem 11

12. •• **IP** The x and y components of a vector $\vec{r}$ are $r_x = 14$ m and $r_y = -9.5$ m, respectively. Find **(a)** the direction and **(b)** the magnitude of the vector $\vec{r}$. **(c)** If both r_x and r_y are doubled, how do your answers to parts (a) and (b) change?

13. •• **IP The Longitude Problem** In 1755, John Harrison (1693–1776) completed his fourth precision chronometer, the H4, which eventually won the celebrated Longitude Prize. (For the human drama behind the Longitude Prize, see *Longitude*, by Dava Sobel.) When the minute hand of the H4 indicated 10 minutes past the hour, it extended 3.0 cm in the horizontal direction. **(a)** How long was the H4's minute hand? **(b)** At 10 minutes past the hour,

Not just a watch! The Harrison H4. (Problem 13)

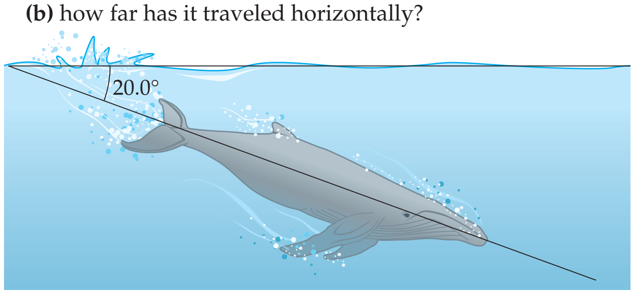
was the extension of the minute hand in the vertical direction more than, less than, or equal to 3.0 cm? Explain. **(c)** Calculate the vertical extension of the minute hand at 10 minutes past the hour.

14. •• You drive a car 680 ft to the east, then 340 ft to the north. **(a)** What is the magnitude of your displacement? **(b)** Using a sketch, estimate the direction of your displacement. **(c)** Verify your estimate in part (b) with a numerical calculation of the direction.

15. •• Vector $\vec{A}$ has a magnitude of 50 units and points in the positive x direction. A second vector, $\vec{B}$, has a magnitude of 120 units and points at an angle of 70° below the x axis. Which vector has **(a)** the greater x component, and **(b)** the greater y component?

16. •• A treasure map directs you to start at a palm tree and walk due north for 15.0 m. You are then to turn 90° and walk 22.0 m; then turn 90° again and walk 5.00 m. Give the distance from the palm tree, and the direction relative to north, for each of the four possible locations of the treasure.

17. •• A whale comes to the surface to breathe and then dives at an angle of 20.0° below the horizontal (**Figure 3–35**). If the whale continues in a straight line for 150 m, **(a)** how deep is it, and **(b)** how far has it traveled horizontally?



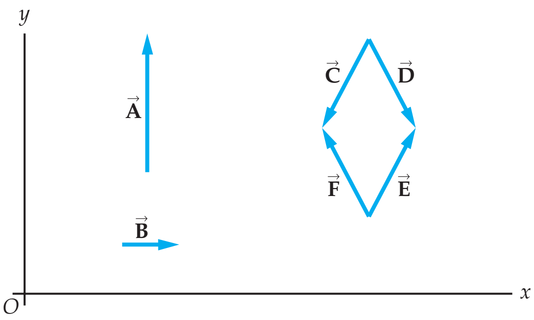
▲ **FIGURE 3–35** Problem 17

SECTION 3–3 ADDING AND SUBTRACTING VECTORS

18. • **CE** Consider the vectors $\vec{A}$ and $\vec{B}$ shown in **Figure 3–36**. Which of the other four vectors in the figure ($\vec{C}$, $\vec{D}$, $\vec{E}$, and $\vec{F}$) best represents the *direction* of **(a)** $\vec{A} + \vec{B}$, **(b)** $\vec{A} - \vec{B}$, and **(c)** $\vec{B} - \vec{A}$,?

19. • **CE** Refer to **Figure 3–36** for the following questions: **(a)** Is the magnitude of $\vec{A} + \vec{D}$ greater than, less than, or equal to the magnitude of $\vec{A} + \vec{E}$? **(b)** Is the magnitude of $\vec{A} + \vec{E}$ greater than, less than, or equal to the magnitude of $\vec{A} + \vec{F}$?

20. • A vector $\vec{A}$ has a magnitude of 40.0 m and points in a direction 20.0° below the positive x axis. A second vector, $\vec{B}$, has a magnitude of 75.0 m and points in a direction 50.0° above the positive x axis. **(a)** Sketch the vectors $\vec{A}$, $\vec{B}$, and $\vec{C} = \vec{A} + \vec{B}$. **(b)** Using the component method of vector addition, find the magnitude and direction of the vector $\vec{C}$.



▲ **FIGURE 3–36** Problems 18 and 19

21. • An air traffic controller observes two airplanes approaching the airport. The displacement from the control tower to plane 1 is given by the vector $\vec{A}$, which has a magnitude of 220 km and points in a direction 32° north of west. The displacement from the control tower to plane 2 is given by the vector $\vec{B}$, which has a magnitude of 140 km and points 65° east of north. **(a)** Sketch the vectors $\vec{A}$, $-\vec{B}$, and $\vec{D} = \vec{A} - \vec{B}$. Notice that $\vec{D}$ is the displacement from plane 2 to plane 1. **(b)** Find the magnitude and direction of the vector $\vec{D}$.

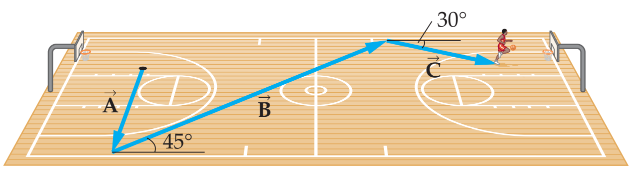
22. • The initial velocity of a car, $\vec{v}_i$, is 45 km/h in the positive x direction. The final velocity of the car, $\vec{v}_f$, is 66 km/h in a direction that points 75° above the positive x axis. **(a)** Sketch the vectors $-\vec{v}_i$, $\vec{v}_f$, and $\Delta\vec{v} = \vec{A}_f - \vec{A}_i$. **(b)** Find the magnitude and direction of the change in velocity, $\Delta\vec{v}$.

23. •• Vector $\vec{A}$ points in the positive x direction and has a magnitude of 75 m. The vector $\vec{C} = \vec{A} + \vec{B}$ points in the positive y direction and has a magnitude of 95 m. **(a)** Sketch $\vec{A}$, $\vec{B}$, and $\vec{C}$. **(b)** Estimate the magnitude and direction of the vector $\vec{B}$. **(c)** Verify your estimate in part (b) with a numerical calculation.

24. •• Vector $\vec{A}$ points in the negative x direction and has a magnitude of 22 units. The vector $\vec{B}$ points in the positive y direction. **(a)** Find the magnitude of $\vec{B}$ if $\vec{A} + \vec{B}$ has a magnitude of 37 units. **(b)** Sketch $\vec{A}$ and $\vec{B}$.

25. •• Vector $\vec{A}$ points in the negative y direction and has a magnitude of 5 units. Vector $\vec{B}$ has twice the magnitude and points in the positive x direction. Find the direction and magnitude of **(a)** $\vec{A} + \vec{B}$, **(b)** $\vec{A} - \vec{B}$, and **(c)** $\vec{B} - \vec{A}$.

26. •• A basketball player runs down the court, following the path indicated by the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in **Figure 3–37**. The magnitudes of these three vectors are $A = 10.0$ m, $B = 20.0$ m, and $C = 7.0$ m. Find the magnitude and direction of the net displacement of the player using **(a)** the graphical method and **(b)** the component method of vector addition. Compare your results.



▲ **FIGURE 3–37** Problem 26

SECTION 3–4 UNIT VECTORS

27. • A particle undergoes a displacement $\Delta\vec{r}$ of magnitude 54 m in a direction 42° below the x axis. Express $\Delta\vec{r}$ in terms of the unit vectors $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$.

28. • A vector has a magnitude of 3.50 m and points in a direction that is 145° counterclockwise from the x axis. Find the x and y components of this vector.

29. • A vector $\vec{A}$ has a length of 6.1 m and points in the negative x direction. Find **(a)** the x component and **(b)** the magnitude of the vector $-3.7\,\vec{A}$.

30. • The vector $-5.2\,\vec{A}$ has a magnitude of 34 m and points in the positive x direction. Find **(a)** the x component and **(b)** the magnitude of the vector $\vec{A}$.

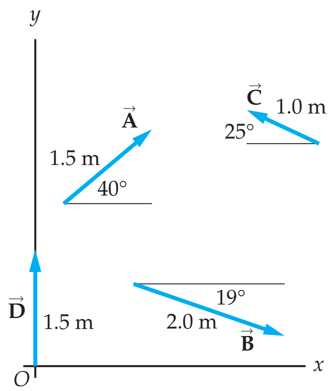
31. • Find the direction and magnitude of the vectors.
(a) $\vec{A} = (5.0\text{ m})\hat{\mathbf{x}} + (-2.0\text{ m})\hat{\mathbf{y}}$,
(b) $\vec{B} = (-2.0\text{ m})\hat{\mathbf{x}} + (5.0\text{ m})\hat{\mathbf{y}}$, and **(c)** $\vec{A} + \vec{B}$.

32. • Find the direction and magnitude of the vectors.
(a) $\vec{A} = (25\text{ m})\hat{\mathbf{x}} + (-12\text{ m})\hat{\mathbf{y}}$,
(b) $\vec{B} = (2.0\text{ m})\hat{\mathbf{x}} + (15\text{ m})\hat{\mathbf{y}}$, and **(c)** $\vec{A} + \vec{B}$.

33. • For the vectors given in Problem 32, express **(a)** $\vec{A} - \vec{B}$ and **(b)** $\vec{B} - \vec{A}$ in unit vector notation.

34. • Express each of the vectors in **Figure 3–38** in unit vector notation.

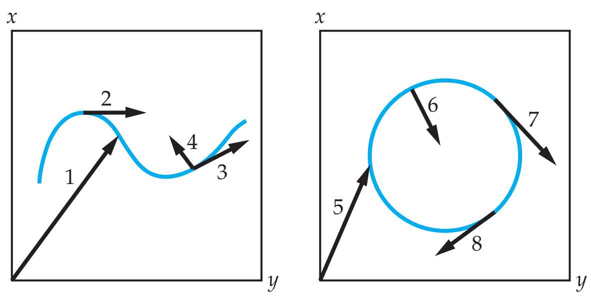
35. •• Referring to the vectors in Figure 3–38, express the sum $\vec{A} + \vec{B} + \vec{C}$ in unit vector notation.



▲ **FIGURE 3–38** Problems 34 and 35

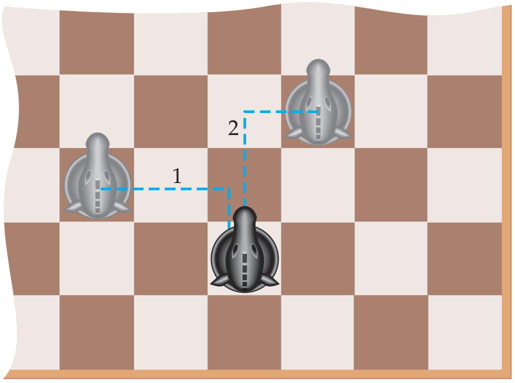
SECTION 3–5 POSITION, DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

36. • **CE** The blue curves shown in **Figure 3–39** display the constant-speed motion of two different particles in the x-y plane. For each of the eight vectors in Figure 3–39, state whether it is **(a)** a position vector, **(b)** a velocity vector, or **(c)** an acceleration vector for the particles.



▲ **FIGURE 3–39** Problem 36

37. • **IP Moving the Knight** Two of the allowed chess moves for a knight are shown in **Figure 3–40**. **(a)** Is the magnitude of displacement 1 greater than, less than, or equal to the magnitude of displacement 2? Explain. **(b)** Find the magnitude and direction of the knight's displacement for each of the two moves. Assume that the checkerboard squares are 3.5 cm on a side.



▲ **FIGURE 3–40** Problem 37

38. • **IP** In its daily prowl of the neighborhood, a cat makes a displacement of 120 m due north, followed by a 72-m displacement due west. **(a)** Find the magnitude and direction of the displacement required for the cat to return home. **(b)** If, instead, the cat had first prowled 72 m west and then 120 m north, how would this affect the displacement needed to bring it home? Explain.

39. • If the cat in Problem 38 takes 45 minutes to complete the 120-m displacement and 17 minutes to complete the 72-m displacement, what are the magnitude and direction of its average velocity during this 62-minute period of time?

40. • What are the direction and magnitude of your total displacement if you have traveled due west with a speed of 27 m/s for 125 s, then due south at 14 m/s for 66 s?

41. •• You drive a car 1500 ft to the east, then 2500 ft to the north. If the trip took 3.0 minutes, what were the direction and magnitude of your average velocity?

42. •• **IP** A jogger runs with a speed of 3.25 m/s in a direction 30.0° above the x axis. **(a)** Find the x and y components of the jogger's velocity. **(b)** How will the velocity components found in part (a) change if the jogger's speed is halved?

43. •• You throw a ball upward with an initial speed of 4.5 m/s. When it returns to your hand 0.92 s later, it has the same speed in the downward direction (assuming air resistance can be ignored). What was the average acceleration vector of the ball?

44. •• A skateboarder rolls from rest down an inclined ramp that is 15.0 m long and inclined above the horizontal at an angle of $\theta = 20.0°$. When she reaches the bottom of the ramp 3.00 s later her speed is 10.0 m/s. Show that the average acceleration of the skateboarder is $g \sin\theta$, where $g = 9.81\ \text{m/s}^2$.

45. •• Consider a skateboarder who starts from rest at the top of a ramp that is inclined at an angle of 17.5° to the horizontal. Assuming that the skateboarder's acceleration is $g \sin 17.5°$, find his speed when he reaches the bottom of the ramp in 3.25 s.

46. ••• **IP** **The Position of the Moon** Relative to the center of the Earth, the position of the Moon can be approximated by

$$\vec{\mathbf{r}} = (3.84 \times 10^8\ \text{m})\{\cos[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{x}} + \sin[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{y}}\}$$

where t is measured in seconds. **(a)** Find the magnitude and direction of the Moon's average velocity between $t = 0$ and $t = 7.38$ days. (This time is one-quarter of the 29.5 days it takes the Moon to complete one orbit.) **(b)** Is the instantaneous speed of the Moon greater than, less than, or the same as the average speed found in part (a)? Explain.

47. ••• **The Velocity of the Moon** The velocity of the Moon relative to the center of the Earth can be approximated by

$$\vec{\mathbf{v}} = (945\ \text{m/s})\{-\sin[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{x}} + \cos[(2.46 \times 10^{-6}\ \text{radians/s})t]\hat{\mathbf{y}}\}$$

where t is measured in seconds. To approximate the instantaneous acceleration of the Moon at $t = 0$, calculate the magnitude and direction of the average acceleration between the times **(a)** $t = 0$ and $t = 0.100$ days and **(b)** $t = 0$ and $t = 0.0100$ days. (The time required for the Moon to complete one orbit is 29.5 days.)

SECTION 3–6 RELATIVE MOTION

48. • **CE** The accompanying photo shows a KC-10A Extender using a boom to refuel an aircraft in flight. If the velocity of the KC-10A is 125 m/s due east relative to the ground, what is the velocity of the aircraft being refueled relative to **(a)** the ground, and **(b)** the KC-10A?

Air-to-air refueling. (Problem 48)

49. • As an airplane taxies on the runway with a speed of 16.5 m/s, a flight attendant walks toward the tail of the plane with a speed of 1.22 m/s. What is the flight attendant's speed relative to the ground?

50. • Referring to part (a) of Example 3–2, find the time it takes for the boat to reach the opposite shore if the river is 35 m wide.

51. •• As you hurry to catch your flight at the local airport, you encounter a moving walkway that is 85 m long and has a speed of 2.2 m/s relative to the ground. If it takes you 68 s to cover 85 m when walking on the ground, how long will it take you to cover the same distance on the walkway? Assume that you walk with the same speed on the walkway as you do on the ground.

52. •• In Problem 51, how long would it take you to cover the 85-m length of the walkway if, once you get on the walkway, you immediately turn around and start walking in the opposite direction with a speed of 1.3 m/s relative to the walkway?

53. •• **IP** The pilot of an airplane wishes to fly due north, but there is a 65-km/h wind blowing toward the east. **(a)** In what direction should the pilot head her plane if its speed relative to the air is 340 km/h? **(b)** Draw a vector diagram that illustrates your result in part (a). **(c)** If the pilot decreases the air speed of the plane, but still wants to head due north, should the angle found in part (a) be increased or decreased?

54. •• A passenger walks from one side of a ferry to the other as it approaches a dock. If the passenger's velocity is 1.50 m/s due north relative to the ferry, and 4.50 m/s at an angle of 30.0° west of north relative to the water, what are the direction and magnitude of the ferry's velocity relative to the water?

55. •• You are riding on a Jet Ski at an angle of 35° upstream on a river flowing with a speed of 2.8 m/s. If your velocity relative to the ground is 9.5 m/s at an angle of 20.0° upstream, what is the speed of the Jet Ski relative to the water? (*Note*: Angles are measured relative to the x axis shown in Example 3–2.)

56. •• **IP** In Problem 55, suppose the Jet Ski is moving at a speed of 12 m/s relative to the water. **(a)** At what angle must you point the Jet Ski if your velocity relative to the ground is to be perpendicular to the shore of the river? **(b)** If you increase the speed of the Jet Ski relative to the water, does the angle in part (a) increase, decrease, or stay the same? Explain. (*Note:* Angles are measured relative to the x axis shown in Example 3–2.)

57. ••• **IP** Two people take identical Jet Skis across a river, traveling at the same speed relative to the water. Jet Ski A heads directly across the river and is carried downstream by the current before reaching the opposite shore. Jet Ski B travels in a direction that is 35° upstream and arrives at the opposite shore directly across from the starting point. **(a)** Which Jet Ski reaches the opposite shore in the least amount of time? **(b)** Confirm your answer to part (a) by finding the ratio of the time it takes for the two Jet Skis to cross the river. (*Note:* Angles are measured relative to the x axis shown in Example 3–2.)

GENERAL PROBLEMS

58. • **CE Predict/Explain** Consider the vectors $\vec{\mathbf{A}} = (1.2\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (-3.4\text{ m})\hat{\mathbf{x}}$. **(a)** Is the magnitude of vector $\vec{\mathbf{A}}$ greater than, less than, or equal to the magnitude of vector $\vec{\mathbf{B}}$? **(b)** Choose the *best explanation* from among the following:
  I. The number 3.4 is greater than the number 1.2.
  II. The component of $\vec{\mathbf{B}}$ is negative.
  III. The vector $\vec{\mathbf{A}}$ points in the positive x direction.

59. • **CE Predict/Explain** Two vectors are defined as follows: $\vec{\mathbf{A}} = (-2.2\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (1.4\text{ m})\hat{\mathbf{y}}$. **(a)** Is the magnitude of $1.4\,\vec{\mathbf{A}}$ greater than, less than, or equal to the magnitude of $2.2\,\vec{\mathbf{B}}$? **(b)** Choose the *best explanation* from among the following:
  I. The vector $\vec{\mathbf{A}}$ has a negative component.
  II. A number and its negative have the same magnitude.
  III. The vectors $1.4\,\vec{\mathbf{A}}$ and $2.2\,\vec{\mathbf{B}}$ point in opposite directions.

60. • You slide a box up a loading ramp that is 10.0 ft long. At the top of the ramp the box has risen a height of 3.00 ft. What is the angle of the ramp above the horizontal?

61. • Find the direction and magnitude of the vector $2\vec{\mathbf{A}} + \vec{\mathbf{B}}$, where $\vec{\mathbf{A}} = (12.1\text{ m})\hat{\mathbf{x}}$ and $\vec{\mathbf{B}} = (-32.2\text{ m})\hat{\mathbf{y}}$.

62. •• **CE** The components of a vector $\vec{\mathbf{A}}$ satisfy $A_x < 0$ and $A_y < 0$. Is the direction angle of $\vec{\mathbf{A}}$ between 0° and 90°, between 90° and 180°, between 180° and 270°, or between 270° and 360°?

63. •• **CE** The components of a vector $\vec{\mathbf{B}}$ satisfy $B_x > 0$ and $B_y < 0$. Is the direction angle of $\vec{\mathbf{B}}$ between 0° and 90°, between 90° and 180°, between 180° and 270°, or between 270° and 360°?

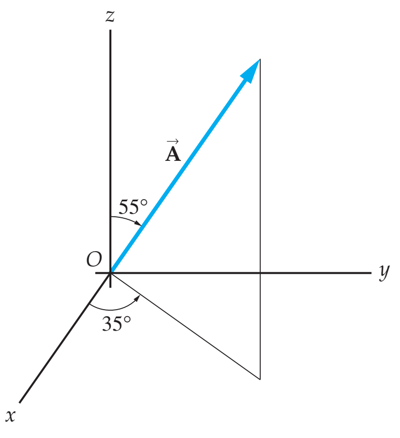
64. •• It is given that $\vec{\mathbf{A}} - \vec{\mathbf{B}} = (-51.4\text{ m})\hat{\mathbf{x}}$, $\vec{\mathbf{C}} = (62.2\text{ m})\hat{\mathbf{x}}$, and $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = (13.8\text{ m})\hat{\mathbf{x}}$. Find the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

65. •• **IP** Two students perform an experiment with a train and a ball. Michelle rides on a flatcar pulled at 8.35 m/s by a train on a straight, horizontal track; Gary stands at rest on the ground near the tracks. When Michelle throws the ball with an initial angle of 65.0° above the horizontal, from her point of view, Gary sees the ball rise straight up and back down above a fixed point on the ground. **(a)** Did Michelle throw the ball toward the front of the train or toward the rear of the train? Explain. **(b)** What was the initial speed of Michelle's throw? **(c)** What was the initial speed of the ball as seen by Gary?

66. •• An off-roader explores the open desert in her Hummer. First she drives 25° west of north with a speed of 6.5 km/h for 15 minutes, then due east with a speed of 12 km/h for 7.5 minutes. She completes the final leg of her trip in 22 minutes. What are the direction and speed of travel on the final leg? (Assume her speed is constant on each leg, and that she returns to her starting point at the end of the final leg.)

67. •• Find the x, y, and z components of the vector $\vec{\mathbf{A}}$ shown in **Figure 3–41**, given that $A = 65$ m.

68. •• A football is thrown horizontally with an initial velocity of $(16.6\text{ m/s})\hat{\mathbf{x}}$. Ignoring air resistance, the average acceleration of the football over any period of time is $(-9.81\text{ m/s}^2)\hat{\mathbf{y}}$. **(a)** Find the velocity vector of the ball 1.75 s after it is thrown. **(b)** Find the magnitude and direction of the velocity at this time.

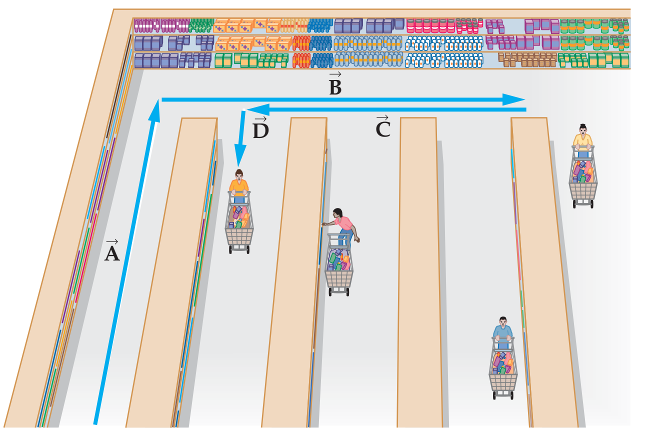


▲ **FIGURE 3–41** Problem 67

69. •• As a function of time, the velocity of the football described in Problem 68 can be written as $\vec{\mathbf{v}} = (16.6\text{ m/s})\hat{\mathbf{x}} - [(9.81\text{ m/s}^2)t]\hat{\mathbf{y}}$. Calculate the average acceleration vector of the football for the time periods **(a)** $t = 0$ to $t = 1.00$ s, **(b)** $t = 0$ to $t = 2.50$ s, and **(c)** $t = 0$ to $t = 5.00$ s. (If the acceleration of an object is constant, its average acceleration is the same for all time periods.)

70. •• Two airplanes taxi as they approach the terminal. Plane 1 taxies with a speed of 12 m/s due north. Plane 2 taxies with a speed of 7.5 m/s in a direction 20° north of west. **(a)** What are the direction and magnitude of the velocity of plane 1 relative to plane 2? **(b)** What are the direction and magnitude of the velocity of plane 2 relative to plane 1?

71. •• A shopper at the supermarket follows the path indicated by vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, $\vec{\mathbf{C}}$, and $\vec{\mathbf{D}}$ in **Figure 3–42**. Given that the



▲ **FIGURE 3–42** Problem 71

vectors have the magnitudes $A = 51$ ft, $B = 45$ ft, $C = 35$ ft, and $D = 13$ ft, find the total displacement of the shopper using **(a)** the graphical method and **(b)** the component method of vector addition. Give the direction of the displacement relative to the direction of vector $\vec{\mathbf{A}}$.

72. •• Initially, a particle is moving at 4.10 m/s at an angle of 33.5° above the horizontal. Two seconds later, its velocity is 6.05 m/s at an angle of 59.0° below the horizontal. What was the particle's average acceleration during these 2.00 seconds?

73. •• A passenger on a stopped bus notices that rain is falling vertically just outside the window. When the bus moves with constant velocity, the passenger observes that the falling raindrops are now making an angle of 15° with respect to the vertical. **(a)** What is the ratio of the speed of the raindrops to the speed of the bus? **(b)** Find the speed of the raindrops, given that the bus is moving with a speed of 18 m/s.

74. •• **A Big Clock** The clock that rings the bell known as Big Ben has an hour hand that is 9.0 feet long and a minute hand that is 14 feet long, where the distance is measured from the center of the clock to the tip of each hand. What is the tip-to-tip distance between these two hands when the clock reads 12 minutes after four o'clock?

75. •• **IP** Suppose we orient the x axis of a two-dimensional coordinate system along the beach at Waikiki. Waves approaching the beach have a velocity relative to the shore given by $\vec{\mathbf{v}}_{ws} = (1.3 \text{ m/s})\hat{\mathbf{y}}$. Surfers move more rapidly than the waves, but at an angle to the beach. The angle is chosen so that the surfers approach the shore with the same speed as the waves. **(a)** If a surfer has a speed of 7.2 m/s relative to the water, what is her direction of motion relative to the positive x axis? **(b)** What is the surfer's velocity relative to the wave? **(c)** If the surfer's speed is increased, will the angle in part (a) increase or decrease? Explain.

76. ••• **IP** Referring to Example 3–2, **(a)** what heading must the boat have if it is to land directly across the river from its starting point? **(b)** How much time is required for this trip if the river is 25.0 m wide? **(c)** Suppose the speed of the boat is increased, but it is still desired to land directly across from the starting point. Should the boat's heading be more upstream, more downstream, or the same as in part (a)? Explain.

77. ••• Vector $\vec{\mathbf{A}}$ points in the negative x direction. Vector $\vec{\mathbf{B}}$ points at an angle of 30.0° above the positive x axis. Vector $\vec{\mathbf{C}}$ has a magnitude of 15 m and points in a direction 40.0° below the positive x axis. Given that $\vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$, find the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

78. ••• As two boats approach the marina, the velocity of boat 1 relative to boat 2 is 2.15 m/s in a direction 47.0° east of north. If boat 1 has a velocity that is 0.775 m/s due north, what is the velocity (magnitude and direction) of boat 2?

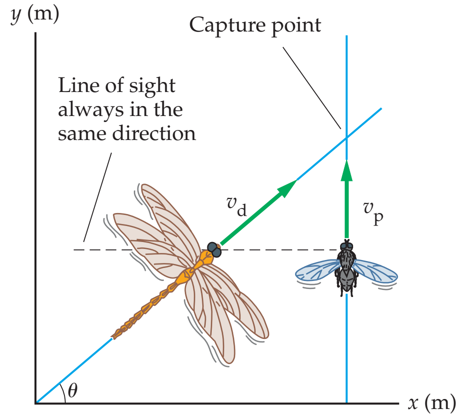
PASSAGE PROBLEMS

BIO Motion Camouflage in Dragonflies

Dragonflies, whose ancestors were once the size of hawks, have prowled the skies in search of small flying insects for over 250 million years. Faster and more maneuverable than any other insect, they even fold their front two legs in flight and tuck them behind their head to be as streamlined as possible. They also employ an intriguing stalking strategy known as "motion camouflage" to approach their prey almost undetected.

The basic idea of motion camouflage is for the dragonfly to move in such a way that the line of sight from the prey to the dragonfly is always in the same direction. Moving in this way, the dragonfly appears almost motionless to its prey, as if it were an object at infinity. Eventually the prey notices the dragonfly has grown in size and is therefore closer, but by that time it's too late for it to evade capture.

A typical capture scenario is shown in **Figure 3–43**, where the prey moves in the positive y direction with the constant speed $v_p = 0.750$ m/s, and the dragonfly moves at an angle $\theta = 48.5°$ to the x axis with the constant speed v_d. If the dragonfly chooses its speed correctly, the line of sight from the prey to the dragonfly will always be in the same direction—parallel to the x axis in this case.



▲ **FIGURE 3–43** Problems 79, 80, 81, and 82

79. • What speed must the dragonfly have if the line of sight, which is parallel to the x axis initially, is to remain parallel to the x axis?

A. 0.562 m/s **B.** 0.664 m/s
C. 1.00 m/s **D.** 1.13 m/s

80. • Suppose the dragonfly now approaches its prey along a path with $\theta > 48.5°$, but it still keeps the line of sight parallel to the x axis. Is the speed of the dragonfly in this new case greater than, less than, or equal to its speed in Problem 79?

81. • What is the correct "motion camouflage" speed of approach for a dragonfly pursuing its prey at the angle $\theta = 68.5°$?

A. 0.295 m/s **B.** 0.698 m/s
C. 0.806 m/s **D.** 2.05 m/s

82. •• If the dragonfly approaches its prey with a speed of 0.950 m/s, what angle θ is required to maintain a constant line of sight parallel to the x axis?

A. 37.9° **B.** 38.3°
C. 51.7° **D.** 52.1°

INTERACTIVE PROBLEMS

83. •• **IP** **Referring to Example 3–2** Suppose the speed of the boat relative to the water is 7.0 m/s. **(a)** At what angle to the x axis must the boat be headed if it is to land directly across the river from its starting position? **(b)** If the speed of the boat relative to the water is increased, will the angle needed to go directly across the river increase, decrease, or stay the same? Explain.

84. ••• **Referring to Example 3–2** Suppose the boat has a speed of 6.7 m/s relative to the water, and that the dock on the opposite shore of the river is at the location $x = 55$ m and $y = 28$ m relative to the starting point of the boat. **(a)** At what angle relative to the x axis must the boat be pointed in order to reach the other dock? **(b)** With the angle found in part (a), what is the speed of the boat relative to the ground?

4 Two-Dimensional Kinematics

When you hear the word "projectile," you probably think of an artillery shell or perhaps a home run into the upper deck. But as we'll see in this chapter, the term applies to any object moving under the influence of gravity alone. For example, each of these juggling balls undergoes projectile motion as it moves from one hand to the other. In this chapter we will explore the physical laws that govern such motion, and will learn—among other things—that these balls follow a parabolic path.

We now extend our study of kinematics to motion in two dimensions. This allows us to consider a much wider range of physical phenomena observed in everyday life. Of particular interest is **projectile motion,** the motion of objects that are initially launched—or "projected"—and that then continue moving under the influence of gravity alone. Examples of projectile motion include balls thrown from one person to another, water spraying from a hose, salmon leaping over rapids, and divers jumping from the cliffs of Acapulco.

The main idea of this chapter is quite simple: Horizontal and vertical motions are independent. That's it. For example, a ball thrown horizontally with a speed v continues to move with the same speed v in the horizontal direction, even as it falls with an increasing speed in the vertical direction. Similarly, the time of fall is the same whether a ball is dropped from rest straight down, or thrown horizontally. Simply put, each motion continues as if the other motion were not present.

This chapter develops and applies the idea of independence of motion to many common physical systems.

4–1 Motion in Two Dimensions

In this section we develop equations of motion to describe objects moving in two dimensions. First, we consider motion with constant velocity, determining x and y as functions of time. Next, we investigate motion with constant acceleration. We show that the one-dimensional kinematic equations of Chapter 2 can be extended in a straightforward way to apply to two dimensions.

Constant Velocity

To begin, consider the simple situation shown in **Figure 4–1**. A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26\ \text{m/s}$ in a direction 25° above the x axis. How far has the turtle moved in the x and y directions after 5.0 seconds?

First, note that the turtle moves in a straight line a distance

$$d = v_0 t = (0.26\ \text{m/s})(5.0\ \text{s}) = 1.3\ \text{m}$$

as indicated in **Figure 4–1(a)**. From the definitions of sine and cosine given in the previous chapter, we see that

$$x = d \cos 25° = 1.2\ \text{m}$$
$$y = d \sin 25° = 0.55\ \text{m}$$

An alternative way to approach this problem is to treat the x and y motions separately. First, we determine the speed of the turtle in each direction. Referring to **Figure 4–1(b)**, we see that the x component of velocity is

$$v_{0x} = v_0 \cos 25° = 0.24\ \text{m/s}$$

and the y component is

$$v_{0y} = v_0 \sin 25° = 0.11\ \text{m/s}$$

Next, we find the distance traveled by the turtle in the x and y directions by multiplying the speed in each direction by the time:

$$x = v_{0x} t = (0.24\ \text{m/s})(5.0\ \text{s}) = 1.2\ \text{m}$$

and

$$y = v_{0y} t = (0.11\ \text{m/s})(5.0\ \text{s}) = 0.55\ \text{m}$$

This is in agreement with our previous results. To summarize, we can think of the turtle's actual motion as a combination of separate x and y motions.

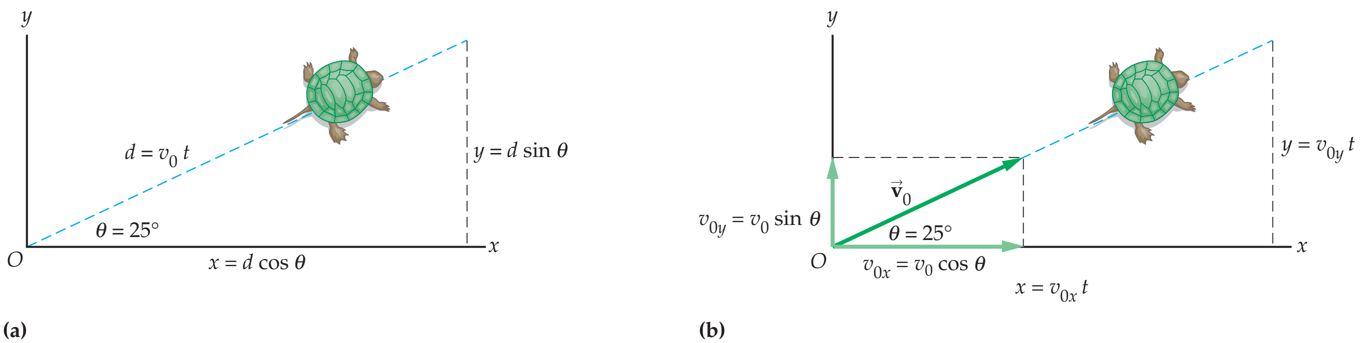
In general, the turtle might start at a position $x = x_0$ and $y = y_0$ at time $t = 0$. In this case, we have

$$x = x_0 + v_{0x} t \qquad \text{4–1}$$

and

$$y = y_0 + v_{0y} t \qquad \text{4–2}$$

as the x and y equations of motion.



▲ **FIGURE 4–1 Constant velocity**

A turtle walks from the origin with a speed of $v_0 = 0.26\ \text{m/s}$. **(a)** In a time t the turtle moves through a straight-line distance of $d = v_0 t$; thus the x and y displacements are $x = d \cos\theta$, $y = d \sin\theta$. **(b)** Equivalently, the turtle's x and y components of velocity are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$; hence $x = v_{0x} t$ and $y = v_{0y} t$.

Compare these equations with Equation 2–11, $x = x_0 + v_0t + \frac{1}{2}at^2$, which gives position as a function of time in one dimension. When acceleration is zero, as it is for the turtle, Equation 2–11 reduces to $x = x_0 + v_0t$. Replacing v_0 with the x component of the velocity, v_{0x}, yields Equation 4–1. Similarly, replacing each x in Equation 4–1 with y converts it to Equation 4–2, the y equation of motion.

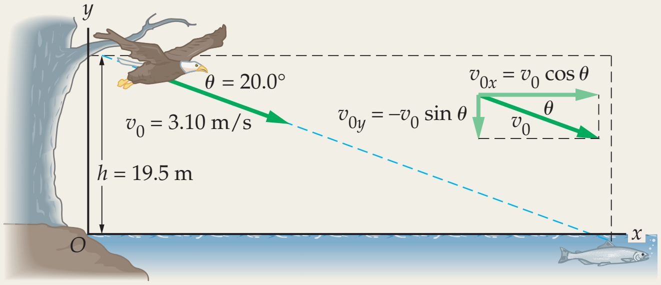
A situation illustrating the use of Equations 4–1 and 4–2 is given in Example 4–1.

EXAMPLE 4–1 THE EAGLE DESCENDS

An eagle perched on a tree limb 19.5 m above the water spots a fish swimming near the surface. The eagle pushes off from the branch and descends toward the water. By adjusting its body in flight, the eagle maintains a constant speed of 3.10 m/s at an angle of 20.0° below the horizontal. **(a)** How long does it take for the eagle to reach the water? **(b)** How far has the eagle traveled in the horizontal direction when it reaches the water?

PICTURE THE PROBLEM

We set up our coordinate system so that the eagle starts at $x_0 = 0$ and $y_0 = h = 19.5$ m. The water level is $y = 0$. As indicated in our sketch, $v_{0x} = v_0 \cos\theta$ and $v_{0y} = -v_0 \sin\theta$, where $v_0 = 3.10$ m/s and $\theta = 20.0°$. Notice that both components of the eagle's velocity are constant, and therefore the equations of motion given in Equations 4–1 and 4–2 apply.



STRATEGY

As usual in such problems, it is best to treat the eagle's flight as a combination of separate x and y motions. Since we are given the constant speed of the eagle, and the angle at which it descends, we can find the x and y components of its velocity. We then use the y equation of motion, $y = y_0 + v_{0y}t$, to find the time t when the eagle reaches the water. Finally, we use this value of t in the x equation of motion, $x = x_0 + v_{0x}t$, to find the horizontal distance the bird travels.

SOLUTION

Part (a)

1. Begin by determining v_{0x} and v_{0y}:

$$v_{0x} = v_0 \cos\theta = (3.10\ \text{m/s}) \cos 20.0° = 2.91\ \text{m/s}$$
$$v_{0y} = -v_0 \sin\theta = -(3.10\ \text{m/s}) \sin 20.0° = -1.06\ \text{m/s}$$

2. Now, set $y = 0$ in $y = y_0 + v_{0y}t$ and solve for t:

$$y = y_0 + v_{0y}t = h + v_{0y}t = 0$$
$$t = -\frac{h}{v_{0y}} = -\frac{19.5\ \text{m}}{(-1.06\ \text{m/s})} = 18.4\ \text{s}$$

Part (b)

3. Substitute $t = 18.4$ s into $x = x_0 + v_{0x}t$ to find x:

$$x = x_0 + v_{0x}t = 0 + (2.91\ \text{m/s})(18.4\ \text{s}) = 53.5\ \text{m}$$

INSIGHT

Notice how the two minus signs in Step 2 combine to give a positive time. One minus sign comes from setting $y = 0$, the other from the fact that v_{0y} is negative. No matter where we choose the origin, or what direction we choose to be positive, the time will always have the same value.

As mentioned in the problem statement, the eagle cannot travel in a straight line by simply dropping from the tree limb—it has to adjust its wings and tail to produce enough lift to balance the force of gravity. Airplanes do the same thing when they adjust their flight surfaces to make a smooth landing.

PRACTICE PROBLEM

What is the location of the eagle 2.00 s after it takes flight? [**Answer:** $x = 5.82$ m, $y = 17.4$ m]

Some related homework problems: Problem 2, Problem 3

Constant Acceleration

To study motion with constant acceleration in two dimensions we repeat what was done in one dimension in Chapter 2, but with separate equations for both x and y. For example, to obtain x as a function of time we start with $x = x_0 + v_0t + \frac{1}{2}at^2$ (Equation 2–11), and replace both v_0 and a with the corresponding x components, v_{0x} and a_x. This gives

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_xt^2 \qquad \text{4–3(a)}$$

To obtain y as a function of time, we write y in place of x in Equation 4–3(a):

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_yt^2 \qquad \text{4–3(b)}$$

These are the position-versus-time equations of motion for two dimensions. (In three dimensions we introduce a third coordinate direction and label it z. We would then simply replace x with z in Equation 4–3(a) to obtain z as a function of time.)

The same approach gives velocity as a function of time. Start with Equation 2–7, $v = v_0 + at$, and write it in terms of x and y components. This yields

$$v_x = v_{0x} + a_xt \qquad \text{4–4(a)}$$

$$v_y = v_{0y} + a_yt \qquad \text{4–4(b)}$$

Note that we simply repeat everything we did for one dimension, only now with separate equations for the x and y components.

Finally, we can write $v^2 = v_0^{\,2} + 2a\Delta x$ in terms of components as well:

$$v_x^{\,2} = v_{0x}^{\,2} + 2a_x\Delta x \qquad \text{4–5(a)}$$

$$v_y^{\,2} = v_{0y}^{\,2} + 2a_y\Delta y \qquad \text{4–5(b)}$$

The following table summarizes our results:

Table 4–1 Constant-Acceleration Equations of Motion

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$	$v_x = v_{0x} + a_xt$	$v_x^{\,2} = v_{0x}^{\,2} + 2a_x\Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$	$v_y = v_{0y} + a_yt$	$v_y^{\,2} = v_{0y}^{\,2} + 2a_y\Delta y$

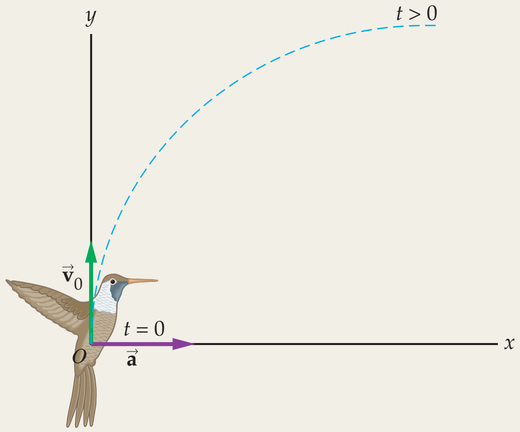
These are the fundamental equations that will be used to obtain *all* of the results presented throughout the rest of this chapter. Though it may appear sometimes that we are writing new sets of equations for different special cases, the equations aren't new—what we are actually doing is simply writing these equations again, but with specific values substituted for the constants that appear in them.

EXAMPLE 4–2 A HUMMER ACCELERATES

A hummingbird is flying in such a way that it is initially moving vertically with a speed of 4.6 m/s and accelerating horizontally at 11 m/s^2. Assuming the bird's acceleration remains constant for the time interval of interest, find **(a)** the horizontal and vertical distances through which it moves in 0.55 s and **(b)** its x and y velocity components at $t = 0.55$ s.

PICTURE THE PROBLEM

In our sketch we have placed the origin of a two-dimensional coordinate system at the location of the hummingbird at the initial time, $t = 0$. In addition, we have chosen the initial direction of motion to be in the positive y direction, and the direction of acceleration to be in the positive x direction. As a result, it follows that $x_0 = y_0 = 0$, $v_{0x} = 0$, $v_{0y} = 4.6$ m/s, $a_x = 11$ m/s^2, and $a_y = 0$. As the hummingbird moves upward, its x component of velocity increases, resulting in a curved path, as shown.



STRATEGY

(a) Since we want to relate position and time, we find the horizontal position of the hummingbird using $x = x_0 + v_{0x}t + \frac{1}{2}a_xt^2$, and the vertical position using $y = y_0 + v_{0y}t + \frac{1}{2}a_yt^2$. **(b)** The velocity components as a function of time can be found using $v_x = v_{0x} + a_xt$ and $v_y = v_{0y} + a_yt$.

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SOLUTION

Part (a)

1. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find x at $t = 0.55$ s: $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + 0 + \frac{1}{2}(11\ \text{m/s}^2)(0.55\ \text{s})^2 = 1.7\ \text{m}$

2. Use $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ to find y at $t = 0.55$ s: $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 0 + (4.6\ \text{m/s})(0.55\ \text{s}) + 0 = 2.5\ \text{m}$

Part (b)

3. Use $v_x = v_{0x} + a_x t$ to find v_x at $t = 0.55$ s: $v_x = v_{0x} + a_x t = 0 + (11\ \text{m/s}^2)(0.55\ \text{s}) = 6.1\ \text{m/s}$

4. Use $v_y = v_{0y} + a_y t$ to find v_y at $t = 0.55$ s: $v_y = v_{0y} + a_y t = 4.6\ \text{m/s} + (0)(0.55\ \text{s}) = 4.6\ \text{m/s}$

INSIGHT

In 0.55 s the hummingbird moves 1.7 m horizontally and 2.5 m vertically. The horizontal position of the bird will eventually increase more rapidly with time than the vertical position, due to the t^2 dependence of x as compared with the t dependence of y. This results in a curved, parabolic path for the hummingbird, as shown in our sketch. The bird's velocity at 0.55 s is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6.1\ \text{m/s})^2 + (4.6\ \text{m/s})^2} = 7.6\ \text{m/s}$ at an angle of $\theta = \tan^{-1}(v_y/v_x) = \tan^{-1}[(4.6\ \text{m/s})/(6.1\ \text{m/s})] = 37°$ above the x axis. It's clear the angle of flight must be less than 45° at this time, since the x component of velocity is greater than the y component.

PRACTICE PROBLEM

How much time is required for the hummingbird to move 2.0 m horizontally from its initial position? [**Answer:** $t = 0.60$ s]

Some related homework problems: Problem 4, Problem 5, Problem 62

4–2 Projectile Motion: Basic Equations

We now apply the independence of horizontal and vertical motions to projectiles. Just what do we mean by a projectile? Well, a **projectile** is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity. As you might expect, this covers a wide variety of physical systems.

In studying projectile motion we make the following assumptions:

- air resistance is ignored
- the acceleration due to gravity is constant, downward, and has a magnitude equal to $g = 9.81\ \text{m/s}^2$
- the Earth's rotation is ignored

Air resistance can be significant when a projectile moves with relatively high speed or if it encounters a strong wind. In many everyday situations, however, like tossing a ball to a friend or dropping a book, air resistance is relatively insignificant. As for the acceleration due to gravity, $g = 9.81\ \text{m/s}^2$, this value varies slightly from place to place on the Earth's surface and decreases with increasing altitude. In addition, the rotation of the Earth can be significant when considering projectiles that cover great distances. Little error is made in ignoring the variation of g or the rotation of the Earth, however, in the examples of projectile motion considered in this chapter.

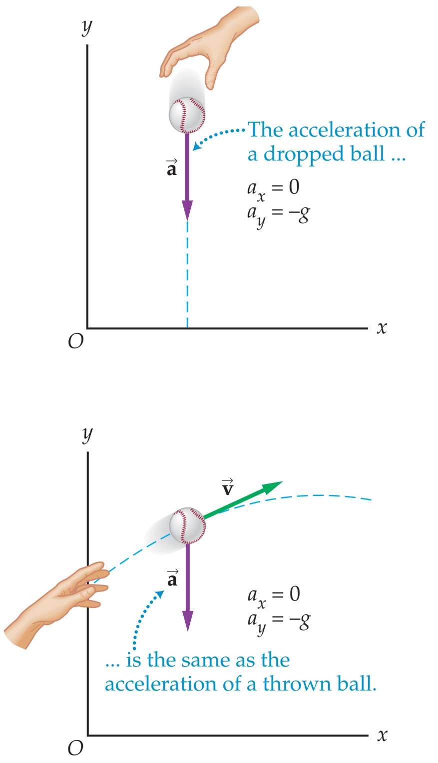
Let's incorporate these assumptions into the equations of motion given in the previous section. Suppose, as in Figure 4–2, that the x axis is horizontal and the y axis is vertical, with the positive direction upward. Since downward is the negative direction, it follows that

$$a_y = -9.81\ \text{m/s}^2 = -g$$

Gravity causes no acceleration in the x direction. Thus, the x component of acceleration is zero:

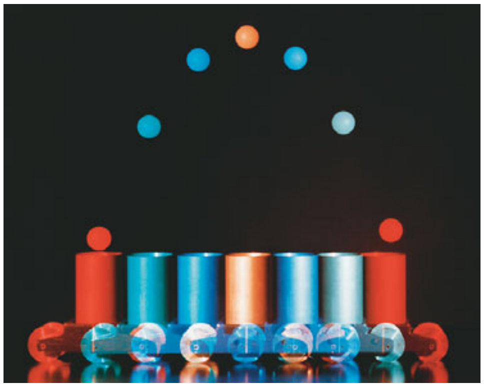
$$a_x = 0$$

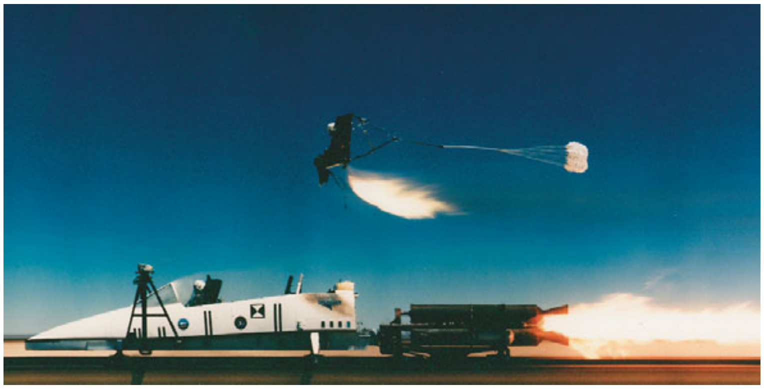
With these acceleration components substituted into the fundamental constant-acceleration equations of motion (Table 4–1) we find:



▲ FIGURE 4–2 Acceleration in free fall

All objects in free fall have acceleration components $a_x = 0$ and $a_y = -g$ when the coordinate system is chosen as shown here. This is true regardless of whether the object is dropped, thrown, kicked, or otherwise set into motion.





▲ In the multiple-exposure photo at left, a ball is projected upward from a moving cart. The ball retains its initial horizontal velocity; as a result, it follows a parabolic path and remains directly above the cart at all times. When the ball lands, it falls back into the cart, just as it would if the cart had been at rest. (In this sequence, the exposures were made at equal time intervals with light of different colors, making it easier to follow the relative motion of the ball and the cart.) In the photo at right, the pilot ejection seat of a jet fighter is being ground-tested. Here too the horizontal and vertical motions are independent; thus, the test dummy is still almost directly above the cockpit from which it was ejected. Note, however, that air resistance is beginning to reduce the dummy's horizontal velocity. Eventually, it will fall far behind the speeding plane.

Projectile Motion ($a_x = 0, a_y = -g$)

$$x = x_0 + v_{0x}t \qquad v_x = v_{0x} \qquad v_x^2 = v_{0x}^2$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad v_y = v_{0y} - gt \qquad v_y^2 = v_{0y}^2 - 2g\Delta y \qquad \text{4–6}$$

Note that in these expressions the positive y direction is upward and the quantity g is positive. All of our studies of *projectile motion* will use Equations 4–6 as our fundamental equations—again, special cases will simply correspond to substituting specific values for the constants.

A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Note that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second.

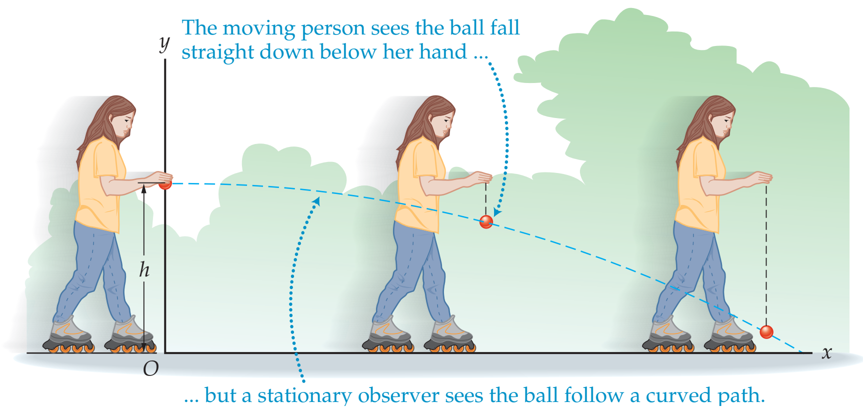
Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully. To you, its motion looks the same as before: It goes straight down, lands near your feet, bounces straight back up, and returns in about one second. This is illustrated in **Figure 4–3**. The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions were independent.

To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path is determined in the next section.

PROBLEM-SOLVING NOTE

Acceleration of a Projectile

When the x axis is chosen to be horizontal and the y axis points vertically upward, it follows that the acceleration of an ideal projectile is $a_x = 0$ and $a_y = -g$.



▲ **FIGURE 4–3 Independence of vertical and horizontal motions**

When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.

▲ This rollerblader may not be thinking about independence of motion, but the ball she released illustrates the concept perfectly; it continues to move horizontally with constant speed—even though she's no longer touching it—at the same time that it accelerates vertically downward.

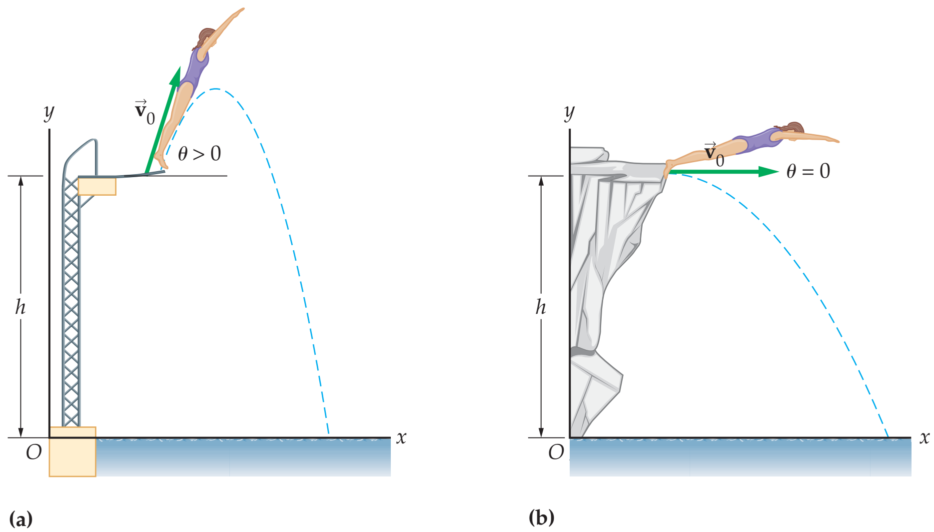


FIGURE 4–4 Launch angle of a projectile
(a) A projectile launched at an angle above the horizontal, $\theta > 0$. A launch below the horizontal would correspond to $\theta < 0$. **(b)** A projectile launched horizontally, $\theta = 0$. In this section we consider $\theta = 0$. The next section deals with $\theta \neq 0$.

4–3 Zero Launch Angle

A special case of some interest is a projectile launched horizontally, so that the angle between the initial velocity and the horizontal is $\theta = 0$. We devote this section to a brief look at this type of motion.

Equations of Motion

Suppose you are walking with a speed v_0 when you release a ball from a height h, as discussed in the previous section. If we choose ground level to be $y = 0$ and the release point to be directly above the origin, the initial position of the ball is given by

$$x_0 = 0$$

and

$$y_0 = h$$

This is illustrated in Figure 4–3.

The initial velocity is horizontal, corresponding to $\theta = 0$ in **Figure 4–4**. As a result, the x component of the initial velocity is simply the initial speed:

$$v_{0x} = v_0 \cos 0° = v_0$$

and the y component of the initial velocity is zero:

$$v_{0y} = v_0 \sin 0° = 0$$

Substituting these specific values into our fundamental equations for projectile motion (Equations 4–6) gives the following simplified results for zero launch angle ($\theta = 0$):

$$\begin{aligned} x &= v_0 t \qquad & v_x &= v_0 = \text{constant} \qquad & v_x^{\,2} &= v_0^{\,2} = \text{constant} \\ y &= h - \tfrac{1}{2} g t^2 \qquad & v_y &= -gt \qquad & v_y^{\,2} &= -2g\Delta y \end{aligned} \qquad 4\text{–}7$$

Note that the x component of velocity remains the same for all time and that the y component steadily decreases with time. As a result, x increases linearly with time, and y decreases with a t^2 dependence. Snapshots of this motion at equal time intervals are shown in **Figure 4–5**.

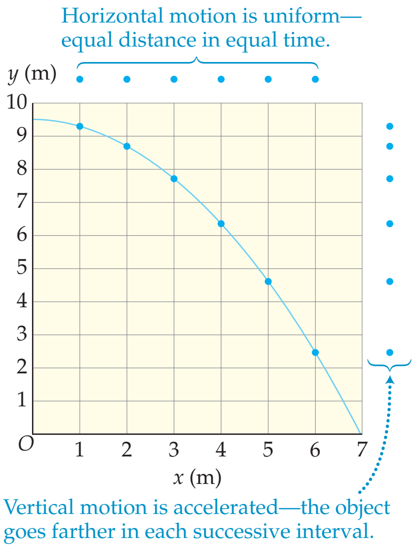


FIGURE 4–5 Trajectory of a projectile launched horizontally
In this plot, the projectile was launched from a height of 9.5 m with an initial speed of 5.0 m/s. The positions shown in the plot correspond to the times $t = 0.20$ s, 0.40 s, 0.60 s, ... Note the uniform motion in the x direction, and the accelerated motion in the y direction.

PROBLEM-SOLVING NOTE

Identify Initial Conditions

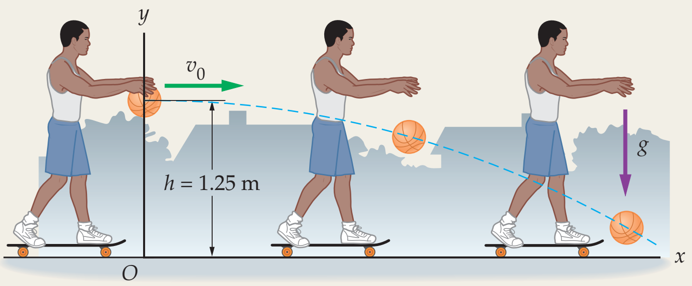
The launch point of a projectile determines x_0 and y_0. The initial velocity of a projectile determines v_{0x} and v_{0y}.

EXAMPLE 4–3 DROPPING A BALL

A person skateboarding with a constant speed of 1.30 m/s releases a ball from a height of 1.25 m above the ground. Given that $x_0 = 0$ and $y_0 = h = 1.25$ m, find x and y for **(a)** $t = 0.250$ s and **(b)** $t = 0.500$ s. **(c)** Find the velocity, speed, and direction of motion of the ball at $t = 0.500$ s.

PICTURE THE PROBLEM

The ball starts at $x_0 = 0$ and $y_0 = h = 1.25$ m. Its initial velocity is horizontal, therefore $v_{0x} = v_0 = 1.30$ m/s and $v_{0y} = 0$. In addition, it accelerates with the acceleration due to gravity in the negative y direction, $a_y = -g$, and moves with constant speed in the x direction, $a_x = 0$.



STRATEGY

The x and y positions are given by $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, respectively. We simply substitute time into these expressions. Similarly, the velocity components are $v_x = v_0$ and $v_y = -gt$.

SOLUTION

Part (a)

1. Substitute $t = 0.250$ s into the x and y equations of motion:

$$x = v_0 t = (1.30 \text{ m/s})(0.250 \text{ s}) = 0.325 \text{ m}$$
$$y = h - \tfrac{1}{2}gt^2$$
$$= 1.25 \text{ m} - \tfrac{1}{2}(9.81 \text{ m/s}^2)(0.250 \text{ s})^2 = 0.943 \text{ m}$$

Part (b)

2. Substitute $t = 0.500$ s into the x and y equations of motion: Note that the ball is only about an inch above the ground at this time:

$$x = v_0 t = (1.30 \text{ m/s})(0.500 \text{ s}) = 0.650 \text{ m}$$
$$y = h - \tfrac{1}{2}gt^2$$
$$= 1.25 \text{ m} - \tfrac{1}{2}(9.81 \text{ m/s}^2)(0.500 \text{ s})^2 = 0.0238 \text{ m}$$

Part (c)

3. First, calculate the x and y components of the velocity at $t = 0.500$ s using $v_x = v_0$ and $v_y = -gt$:

$$v_x = v_0 = 1.30 \text{ m/s}$$
$$v_y = -gt = -(9.81 \text{ m/s}^2)(0.500 \text{ s}) = -4.91 \text{ m/s}$$

4. Use these components to determine $\vec{\mathbf{v}}$, v, and θ:

$$\vec{\mathbf{v}} = (1.30 \text{ m/s})\hat{\mathbf{x}} + (-4.91 \text{ m/s})\hat{\mathbf{y}}$$
$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{(1.30 \text{ m/s})^2 + (-4.91 \text{ m/s})^2} = 5.08 \text{ m/s}$$
$$\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{(-4.91 \text{ m/s})}{1.30 \text{ m/s}} = -75.2°$$

INSIGHT

Note that the x position of the ball does not depend on the acceleration of gravity, g, and that its y position does not depend on the initial horizontal speed of the ball, v_0. For example, if the person is running when he drops the ball, the ball is moving faster in the horizontal direction, and it keeps up with the person when it is dropped. Its vertical motion doesn't change at all, however; it drops to the ground in exactly the same time and bounces back to the same height as before.

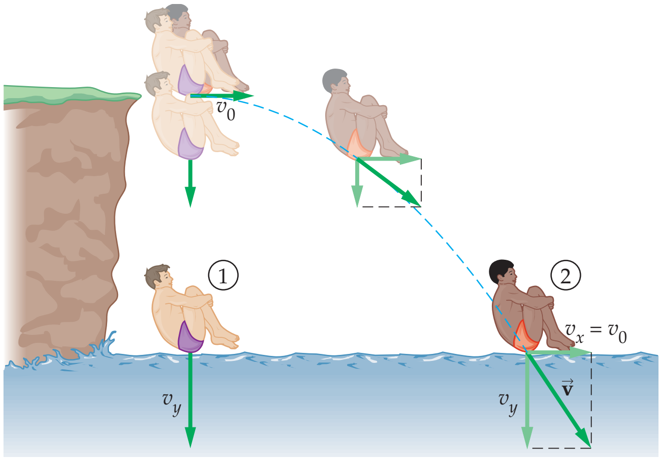
PRACTICE PROBLEM

How long does it take for the ball to land? [**Answer:** Referring to the results of part (b), it is clear that the time of landing is slightly greater than 0.500 s. Setting $y = 0$ gives a precise answer; $t = \sqrt{2h/g} = 0.505$ s.]

Some related homework problems: Problem 15, Problem 16, Problem 20

CONCEPTUAL CHECKPOINT 4–1 COMPARE SPLASHDOWN SPEEDS

Two youngsters dive off an overhang into a lake. Diver 1 drops straight down, diver 2 runs off the cliff with an initial horizontal speed v_0. Is the splashdown speed of diver 2 **(a)** greater than, **(b)** less than, or **(c)** equal to the splashdown speed of diver 1?



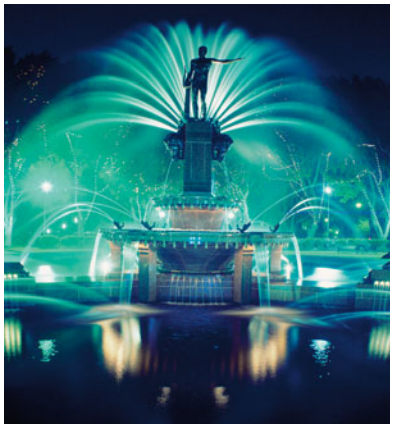
REASONING AND DISCUSSION

Note that neither diver has an initial y component of velocity, and that they both fall with the same vertical acceleration—the acceleration due to gravity. Therefore, the two divers fall for the same amount of time, and their y components of velocity are the same at splashdown. Since diver 2 also has a nonzero x component of velocity, unlike diver 1, the speed of diver 2 is greater.

ANSWER

(a) The speed of diver 2 is greater than that of diver 1.

REAL-WORLD PHYSICS
The parabolic trajectory of projectiles



▲ Lava bombs (top) and fountain jets (bottom) trace out parabolic paths, as is typical in projectile motion. The trajectories are only slightly altered by air resistance.

Parabolic Path

Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = x/v_0$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \tfrac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0^2}\right)x^2 \qquad 4\text{–}8$$

Note that y has the form

$$y = a + bx^2$$

where $a = h =$ constant and $b = -g/2v_0^2 =$ constant. This is the equation of a parabola that curves downward, a characteristic shape in projectile motion.

Landing Site

Where does a projectile land if it is launched horizontally with a speed v_0 from a height h?

The most direct way to answer this question is to set $y = 0$ in Equation 4–8, since $y = 0$ corresponds to ground level. This gives

$$0 = h - \left(\frac{g}{2v_0^2}\right)x^2$$

Solving for x yields the landing site:

$$x = v_0\sqrt{\frac{2h}{g}} \qquad 4\text{–}9$$

Note that we have chosen the positive sign for the square root since the projectile was launched in the positive x direction, and hence lands at a positive value of x.

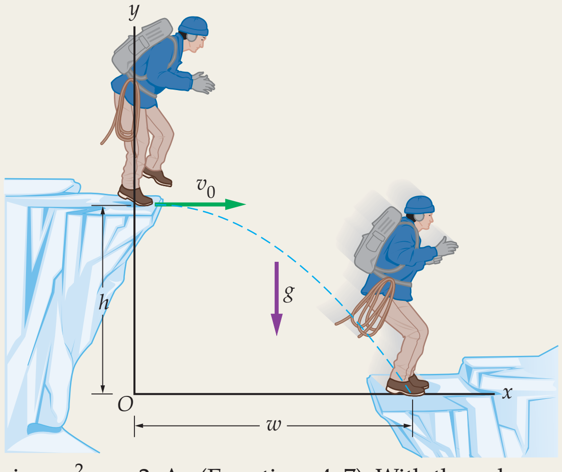
A useful alternative approach is to find the time of landing with the kinematic relations given in Equation 4–7, and then substitute this time into $x = v_0 t$. This approach is illustrated in the next Example.

EXAMPLE 4–4 JUMPING A CREVASSE

A mountain climber encounters a crevasse in an ice field. The opposite side of the crevasse is 2.75 m lower, and is separated horizontally by a distance of 4.10 m. To cross the crevasse, the climber gets a running start and jumps in the horizontal direction. **(a)** What is the minimum speed needed by the climber to safely cross the crevasse? If, instead, the climber's speed is 6.00 m/s, **(b)** where does the climber land, and **(c)** what is the climber's speed on landing?

PICTURE THE PROBLEM

The mountain climber jumps from $x_0 = 0$ and $y_0 = h = 2.75$ m. The landing site for part (a) is $x = w = 4.10$ m and $y = 0$. Note that the y position of the climber *decreases* by h, and therefore $\Delta y = -h = -2.75$ m. As for the initial velocity, we are given that $v_{0x} = v_0$ and $v_{0y} = 0$. Finally, with our choice of coordinates it follows that $a_x = 0$ and $a_y = -g$.



STRATEGY

We can model the climber as a projectile, and apply our equations for projectile motion with a horizontal launch.

a. From Equations 4–7 we have that $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$. Setting $y = 0$ determines the time of landing. Using this time in the x equation gives the horizontal landing position in terms of the initial speed.

b. We can now use the relation from part (a) to find x in terms of $v_0 = 6.00$ m/s.

c. We already know v_x, since it remains constant, and we can calculate v_y using $v_y^2 = -2g\Delta y$ (Equations 4–7). With the velocity components known, we can use the Pythagorean theorem to find the speed.

SOLUTION

Part (a)

1. Set $y = h - \frac{1}{2}gt^2$ equal to zero (landing condition) and solve for the corresponding time t:

$$y = h - \tfrac{1}{2}gt^2 = 0$$

$$t = \sqrt{\frac{2h}{g}}$$

2. Substitute this expression for t into the x equation of motion, $x = v_0 t$, and solve for the speed, v_0:

$$x = v_0 t = v_0\sqrt{\frac{2h}{g}} \quad \text{or} \quad v_0 = x\sqrt{\frac{g}{2h}}$$

3. Substitute numerical values in this expression:

$$v_0 = x\sqrt{\frac{g}{2h}} = (4.10\text{ m})\sqrt{\frac{9.81\text{ m/s}^2}{2(2.75\text{ m})}} = 5.48\text{ m/s}$$

Part (b)

4. Substitute $v_0 = 6.00$ m/s into the expression for x obtained in Step 2, $x = v_0\sqrt{2h/g}$:

$$x = v_0\sqrt{\frac{2h}{g}} = (6.00\text{ m/s})\sqrt{\frac{2(2.75\text{ m})}{9.81\text{ m/s}^2}} = 4.49\text{ m}$$

Part (c)

5. Use the fact that the x component of velocity does not change to determine v_x, and use $v_y^2 = -2g\Delta y$ to determine v_y. For v_y, note that we choose the minus sign for the square root because the climber is moving downward:

$$v_x = v_0 = 6.00\text{ m/s}$$

$$v_y = \pm\sqrt{-2g\Delta y}$$

$$= -\sqrt{-2(9.81\text{ m/s}^2)(-2.75\text{ m})} = -7.35\text{ m/s}$$

6. Use the Pythagorean theorem to determine the speed:

$$v = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(6.00\text{ m/s})^2 + (-7.35\text{ m/s})^2} = 9.49\text{ m/s}$$

INSIGHT

The minimum speed needed to safely cross the crevasse is 5.48 m/s. If the initial horizontal speed is 6.00 m/s, the climber will land 4.49 m − 4.10 m = 0.39 m beyond the edge of the crevasse with a speed of 9.49 m/s.

PRACTICE PROBLEM

(a) When the climber's speed is the minimum needed to cross the crevasse, $v_0 = 5.48$ m/s, how long is the climber in the air? **(b)** How long is the climber in the air when $v_0 = 6.00$ m/s? [**Answer: (a)** $t = x/v_0 = (4.10\text{ m})/(5.48\text{ m/s}) = 0.748$ s. **(b)** $t = x/v_0 = (4.49\text{ m})/(6.00\text{ m/s}) = 0.748$ s. The times are the same! The answer to both parts is simply the time needed to fall through a height h; $t = \sqrt{2h/g} = 0.748$ s.]

Some related homework problems: Problem 11, Problem 12, Problem 17

PROBLEM-SOLVING NOTE

Use Independence of Motion

Projectile problems can be solved by breaking the problem into its x and y components, and then solving for the motion of each component separately.

CONCEPTUAL CHECKPOINT 4–2 MINIMUM SPEED

If the height h is increased in the previous example but the width w remains the same, does the minimum speed needed to cross the crevasse **(a)** increase, **(b)** decrease, or **(c)** stay the same?

REASONING AND DISCUSSION

If the height is greater, the time of fall is also greater. Since the climber is in the air for a greater time, the horizontal distance covered for a given initial speed is also greater. Thus, if the width of the crevasse is the same, a lower initial speed allows for a safe crossing.

ANSWER

(b) The minimum speed decreases.

4–4 General Launch Angle

We now consider the more general case of a projectile launched at an arbitrary angle with respect to the horizontal. This means we can no longer use the simplifications associated with zero launch angle. As always, we return to our basic equations for projectile motion (Equations 4–6), and this time we simply let θ be nonzero.

Figure 4–6 (a) shows a projectile launched with an initial speed v_0 at an angle θ above the horizontal. Since the projectile starts at the origin, the initial x and y positions are zero:

$$x_0 = y_0 = 0$$

The components of the initial velocity are determined as indicated in **Figure 4–6 (b)**:

$$v_{0x} = v_0 \cos\theta$$

and

$$v_{0y} = v_0 \sin\theta$$

As a quick check, note that if $\theta = 0$, then $v_{0x} = v_0$ and $v_{0y} = 0$. Similarly, if $\theta = 90°$ we find $v_{0x} = 0$ and $v_{0y} = v_0$. These checks are depicted in **Figure 4–6 (c)**.

Substituting these results into the basic equations for projectile motion yields the following results for a general launch angle:

$$\begin{aligned} x &= (v_0 \cos\theta)t & v_x &= v_0 \cos\theta & v_x{}^2 &= v_0{}^2 \cos^2\theta \\ y &= (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 & v_y &= v_0 \sin\theta - gt & v_y{}^2 &= v_0{}^2 \sin^2\theta - 2g\Delta y \end{aligned} \qquad 4\text{–}10$$

Note that these equations, which are valid for any launch angle, reduce to the simpler Equations 4–7 when we set $\theta = 0$ and $y_0 = h$. In the next two Exercises, we use Equations 4–10 to calculate a projectile's position and velocity for three equally spaced times.

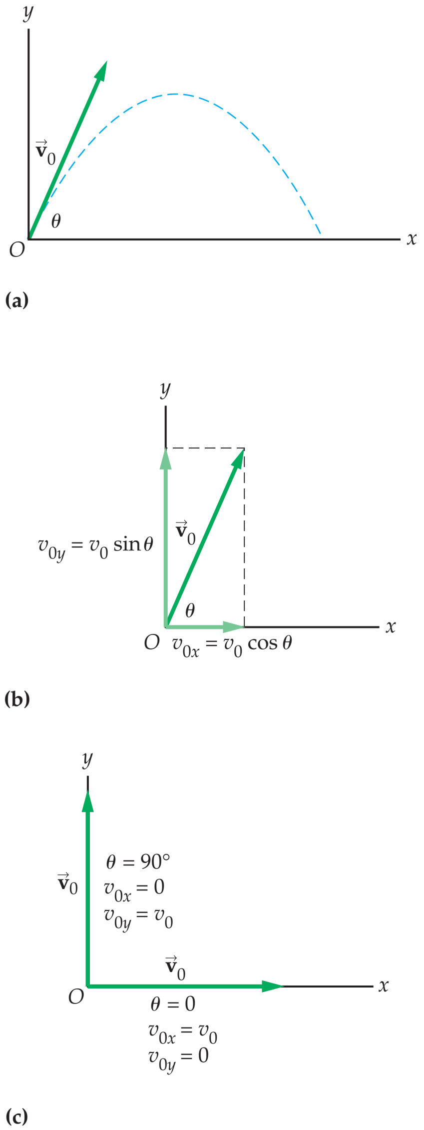


FIGURE 4–6 Projectile with an arbitrary launch angle

(a) A projectile launched from the origin at an angle θ above the horizontal. **(b)** The x and y components of the initial velocity. **(c)** Velocity components in the limits $\theta = 0$ and $\theta = 90°$.

EXERCISE 4–1

A projectile is launched from the origin with an initial speed of 20.0 m/s at an angle of 35.0° above the horizontal. Find the x and y positions of the projectile at times **(a)** $t = 0.500$ s, **(b)** $t = 1.00$ s, and **(c)** $t = 1.50$ s.

SOLUTION

a. $x = 8.19$ m, $y = 4.51$ m,

b. $x = 16.4$ m, $y = 6.57$ m,

c. $x = 24.6$ m, $y = 6.17$ m. Note that x increases steadily; y increases, then decreases.

EXERCISE 4–2

Referring to Exercise 4–1, find the velocity of the projectile at times **(a)** $t = 0.500$ s, **(b)** $t = 1.00$ s, and **(c)** $t = 1.50$ s.

SOLUTION

a. $\vec{\mathbf{v}} = (16.4\text{ m/s})\hat{\mathbf{x}} + (6.57\text{ m/s})\hat{\mathbf{y}}$,

b $\vec{\mathbf{v}} = (16.4\text{ m/s})\hat{\mathbf{x}} + (1.66\text{ m/s})\hat{\mathbf{y}}$,

c. $\vec{\mathbf{v}} = (16.4\text{ m/s})\hat{\mathbf{x}} + (-3.24\text{ m/s})\hat{\mathbf{y}}$.

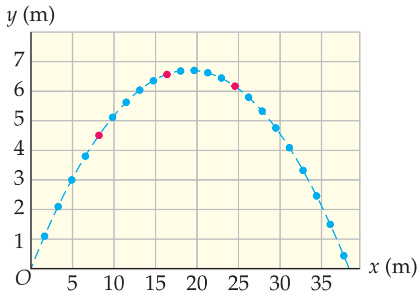


FIGURE 4–7 Snapshots of a trajectory
This plot shows a projectile launched from the origin with an initial speed of 20.0 m/s at an angle of 35.0° above the horizontal. The positions shown in the plot correspond to the times $t = 0.1$ s, 0.2 s, 0.3 s, ... Red dots mark the positions considered in Exercises 4–1 and 4–2.

Figure 4–7 shows the projectile referred to in the previous Exercises for a series of times spaced by 0.10 s. Note that the points in Figure 4–7 are not evenly spaced in terms of position, even though they are evenly spaced in time. In fact, the points bunch closer together at the top of the trajectory, showing that a comparatively large fraction of the flight time is spent near the highest point. This is why it seems that a basketball player soaring toward a slam dunk, or a ballerina performing a grand jeté, is "hanging" in air.



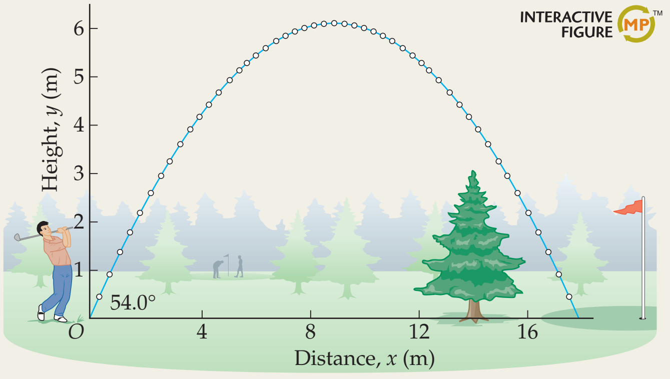
"Hanging" in air near the peak of a jump requires no special knack—in fact, it's an unavoidable consequence of the laws of physics. This phenomenon, which makes big leapers (such as deer and dancers) look particularly graceful, can also make life more dangerous for salmon fighting their way upstream to spawn.

EXAMPLE 4–5 A ROUGH SHOT

Chipping from the rough, a golfer sends the ball over a 3.00-m-high tree that is 14.0 m away. The ball lands at the same level from which it was struck after traveling a horizontal distance of 17.8 m—on the green, of course. **(a)** If the ball left the club 54.0° above the horizontal and landed on the green 2.24 s later, what was its initial speed? **(b)** How high was the ball when it passed over the tree?

PICTURE THE PROBLEM

Our sketch shows the ball taking flight from the origin, $x_0 = y_0 = 0$, with a launch angle of 54.0°, and arcing over the tree. The individual points along the parabolic trajectory correspond to equal time intervals.



STRATEGY

a. Since the projectile moves with constant speed in the x direction, the x component of velocity is simply horizontal distance divided by time. Knowing v_x and θ, we can find v_0 from $v_x = v_0 \cos\theta$.

b. We can use $x = (v_0 \cos\theta)t$ to find the time when the ball is at $x = 14.0$ m. Substituting this time into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ gives the height.

SOLUTION

Part (a)

1. Divide the horizontal distance, d, by the time of flight, t, to obtain v_x:

$$v_x = \frac{d}{t} = \frac{17.8\text{ m}}{2.24\text{ s}} = 7.95\text{ m/s}$$

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2. Use $v_x = v_0 \cos\theta$ to find v_0, the initial speed: $\quad v_x = v_0 \cos\theta \quad \text{or} \quad v_0 = \dfrac{v_x}{\cos\theta} = \dfrac{7.95\ \text{m/s}}{\cos 54.0°} = 13.5\ \text{m/s}$

Part (b)

3. Use $x = (v_0 \cos\theta)t$ to find the time when $x = 14.0$ m. Recall that $x_0 = 0$: $\quad x = (v_0 \cos\theta)t \quad \text{or} \quad t = \dfrac{x}{v_0 \cos\theta} = \dfrac{14.0\ \text{m}}{7.95\ \text{m/s}} = 1.76\ \text{s}$

4. Evaluate $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$ at the time found in Step 3. Recall that $y_0 = 0$:

$$\begin{aligned} y &= (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \\ &= [(13.5\ \text{m/s}) \sin 54.0°](1.76\ \text{s}) - \tfrac{1}{2}(9.81\ \text{m/s}^2)(1.76\ \text{s})^2 \\ &= 4.03\ \text{m} \end{aligned}$$

INSIGHT

The ball clears the top of the tree by 1.03 m and lands on the green 0.48 s later. When it lands, its speed (in the absence of air resistance) is again 13.5 m/s—the same as when it was launched. This result will be verified in the next section.

PRACTICE PROBLEM

What are the speed and direction of the ball when it passes over the tree? [**Answer:** To find the ball's speed and direction, note that $v_x = 7.95$ m/s and $v_y = v_0 \sin\theta - gt = -6.34$ m/s. It follows that $v = \sqrt{v_x^2 + v_y^2} = 10.2$ m/s and $\theta = \tan^{-1}(v_y/v_x) = -38.6°$.]

Some related homework problems: Problem 31, Problem 39

ACTIVE EXAMPLE 4–1 AN ELEVATED GREEN

A golfer hits a ball from the origin with an initial speed of 30.0 m/s at an angle of 50.0° above the horizontal. The ball lands on a green that is 5.00 m above the level where the ball was struck.

a. How long is the ball in the air?

b. How far has the ball traveled in the horizontal direction when it lands?

c. What are the speed and direction of motion of the ball just before it lands?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Let $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 5.00$ m and solve for t: $\quad t = 0.229\ \text{s}, 4.46\ \text{s}$

2. When $t = 0.229$ s, the ball is moving upward; when $t = 4.46$ s, the ball is on the way down. Choose the later time: $\quad t = 4.46\ \text{s}$

Part (b)

3. Substitute $t = 4.46$ s into $x = (v_0 \cos\theta)t$: $\quad x = 86.0\ \text{m}$

Part (c)

4. Use $v_x = v_0 \cos\theta$ to calculate v_x: $\quad v_x = 19.3\ \text{m/s}$

5. Substitute $t = 4.46$ s into $v_y = v_0 \sin\theta - gt$ to find v_y: $\quad v_y = -20.8\ \text{m/s}$

6. Calculate v and θ: $\quad v = 28.4\ \text{m/s}, \theta = -47.1°$

YOUR TURN

How long is the ball in the air if the green is 5.00 m *below* the level where the ball was struck?

(Answers to **Your Turn** *problems are given in the back of the book.)*

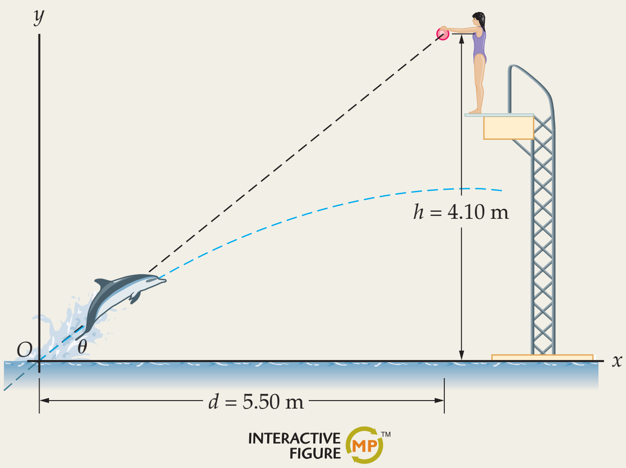
The next Example presents a classic situation in which two projectiles collide. One projectile is launched from the origin, and thus its equations of motion are given by Equations 4–10. The second projectile is simply dropped from a height, which is a special case of the equations of motion in Equations 4–7 with $v_0 = 0$.

EXAMPLE 4–6 A LEAP OF FAITH

A trained dolphin leaps from the water with an initial speed of 12.0 m/s. It jumps directly toward a ball held by the trainer a horizontal distance of 5.50 m away and a vertical distance of 4.10 m above the water. In the absence of gravity the dolphin would move in a straight line to the ball and catch it, but because of gravity the dolphin follows a parabolic path well below the ball's initial position, as shown in the sketch. If the trainer releases the ball the instant the dolphin leaves the water, show that the dolphin and the falling ball meet.

PICTURE THE PROBLEM

In our sketch we have the dolphin leaping from the water at the origin $x_0 = y_0 = 0$ with an angle above the horizontal given by $\theta = \tan^{-1}(h/d)$. The initial position of the ball is $x_0 = d = 5.50$ m and $y_0 = h = 4.10$ m, and its initial velocity is zero. The ball drops straight down with the acceleration of gravity, $a_y = -g$.



STRATEGY

We want to show that when the dolphin is at $x = d$, its height above the water is the same as the height of the ball above the water. To do this we first find the time when the dolphin is at $x = d$, then calculate y for the dolphin at this time. Next, we calculate y of the ball at the same time and then check to see if they are equal.

Since the ball drops from rest from a height h, its y equation of motion is $y = h - \frac{1}{2}gt^2$, as in Equations 4–7 in Section 4–3.

SOLUTION

1. Calculate the angle at which the dolphin leaves the water:

$$\theta = \tan^{-1}\left(\frac{h}{d}\right) = \tan^{-1}\left(\frac{4.10\text{ m}}{5.50\text{ m}}\right) = 36.7°$$

2. Use this angle and the initial speed to find the time t when the x position of the dolphin, x_d, is equal to 5.50 m. The x equation of motion is $x_d = (v_0 \cos\theta)t$:

$$x_d = (v_0 \cos\theta)t = [(12.0\text{ m/s})\cos 36.7°]t = (9.62\text{ m/s})t = 5.50\text{ m}$$

$$t = \frac{5.50\text{ m}}{9.62\text{ m/s}} = 0.572\text{ s}$$

3. Evaluate the y position of the dolphin, y_d, at $t = 0.572$ s. The y equation of motion is $y_d = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y_d = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2$$
$$= [(12.0\text{ m/s})\sin 36.7°](0.572\text{ s}) - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2$$
$$= 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

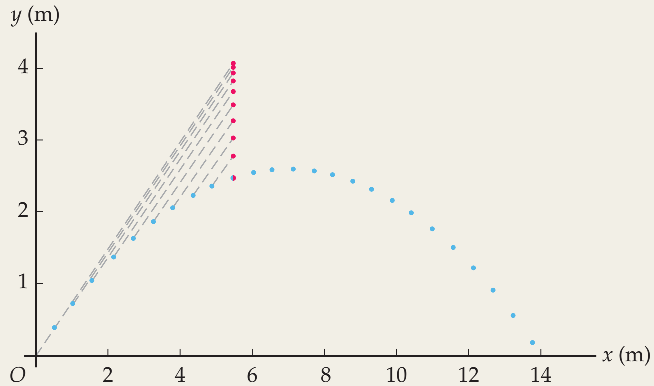
4. Finally, evaluate the y position of the ball, y_b, at $t = 0.572$ s. The ball's equation of motion is $y_b = h - \frac{1}{2}gt^2$:

$$y_b = h - \tfrac{1}{2}gt^2 = 4.10\text{ m} - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2$$
$$= 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

INSIGHT

In the absence of gravity, both the dolphin and the ball would be at $x = 5.50$ m and $y = 4.10$ m at $t = 0.572$ s. Because of gravity, however, the dolphin and the ball fall below their zero-gravity positions—and by the same amount, 1.60 m. In fact, from the point of view of the dolphin, the ball is always at the same angle of 36.7° above the horizontal until it is caught.

This is shown in the accompanying plot, where the red dots show the position of the ball at ten equally spaced times, and the blue dots show the position of the dolphin at the corresponding times. In addition, the dashed lines from the dolphin to the ball all make the same angle with the horizontal, 36.7°.



PRACTICE PROBLEM

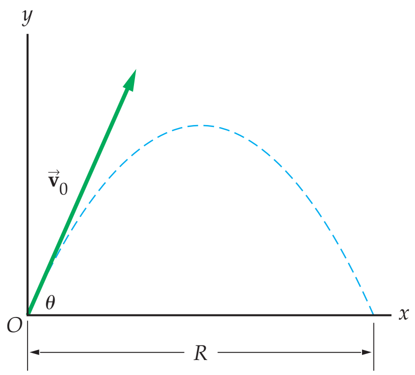
At what height does the dolphin catch the ball if it leaves the water with an initial speed of 8.00 m/s? [**Answer:** $y_d = y_b = 0.493$ m. If the dolphin's initial speed is less than 7.50 m/s, it reenters the water before catching the ball.]

Some related homework problems: Problem 31, Problem 40

4–5 Projectile Motion: Key Characteristics

We conclude this chapter with a brief look at some additional characteristics of projectile motion that are both interesting and useful. In all cases our results follow as a direct consequence of the fundamental kinematic equations (Equations 4–10) describing projectile motion.

Range



▲ **FIGURE 4–8 Range of a projectile**
The range R of a projectile is the horizontal distance it travels between its takeoff and landing positions.

The **range**, R, of a projectile is the horizontal distance it travels before landing. We consider the case shown in **Figure 4–8**, where the initial and final elevations are the same ($y = 0$). One way to obtain the range, then, is as follows: (i) Find the time when the projectile lands by setting $y = 0$ in the expression $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$; (ii) Substitute the time found in (i) into the x equation of motion.

Carrying out the first part of the calculation yields the following:

$$(v_0 \sin\theta)t - \tfrac{1}{2}gt^2 = 0 \quad \text{or} \quad (v_0 \sin\theta)t = \tfrac{1}{2}gt^2$$

Clearly, $t = 0$ is a solution to this equation—corresponding to the initial condition—but the solution we seek is a time that is greater than zero. We can find the desired time by dividing both sides of the equation by t. This gives

$$(v_0 \sin\theta) = \tfrac{1}{2}gt \quad \text{or} \quad t = \left(\frac{2v_0}{g}\right)\sin\theta \qquad \text{4–11}$$

This is the time when the projectile lands—also known as the time of flight.

Now, substitute this time into $x = (v_0 \cos\theta)t$ to find the value of x when the projectile lands:

$$x = (v_0\cos\theta)t = (v_0\cos\theta)\left(\frac{2v_0}{g}\right)\sin\theta = \left(\frac{2{v_0}^2}{g}\right)\sin\theta\cos\theta$$

This value of x is the range, R, thus

$$R = \left(\frac{2{v_0}^2}{g}\right)\sin\theta\cos\theta$$

Using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, as given in Appendix A, we can write this more compactly as follows:

$$R = \left(\frac{{v_0}^2}{g}\right)\sin 2\theta \quad \textit{(same initial and final elevation)} \qquad \text{4–12}$$

PROBLEM-SOLVING NOTE

Use the Same Math Regardless of the Initial Conditions

Once an object is launched, its trajectory follows the kinematic equations of motion, regardless of specific differences in the initial conditions. Thus, our equations of motion can be used to derive any desired characteristic of projectile motion, including range, symmetry, and maximum height.

ACTIVE EXAMPLE 4–2 FIND THE INITIAL SPEED

A football game begins with a kickoff in which the ball travels a horizontal distance of 45 yd and lands on the ground. If the ball was kicked at an angle of 40.0° above the horizontal, what was its initial speed?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 4–12 for the initial speed v_0:	$v_0 = \sqrt{gR/\sin 2\theta}$
2. Convert the range to meters:	$R = 41$ m
3. Substitute numerical values:	$v_0 = 20$ m/s

INSIGHT

Note that we choose the positive square root in Step 1 because we are interested only in the *speed* of the ball, which is always positive.

YOUR TURN

Suppose the initial speed of the ball is increased by 10%, to 22 m/s. By what percentage does the range increase?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note that R depends inversely on the acceleration of gravity, g—thus the smaller g, the larger the range. For example, a projectile launched on the Moon, where the acceleration of gravity is only about 1/6 that on Earth, travels about six times as far as it would on Earth. It was for this reason that astronaut Alan Shepard simply couldn't resist the temptation of bringing a golf club and ball with him on the third lunar landing mission in 1971. He ambled out onto the Fra Mauro Highlands and became the first person to hit a tee shot on the Moon. His distance was undoubtedly respectable—unfortunately, his ball landed in a sand trap.

REAL-WORLD PHYSICS

Golf on the Moon

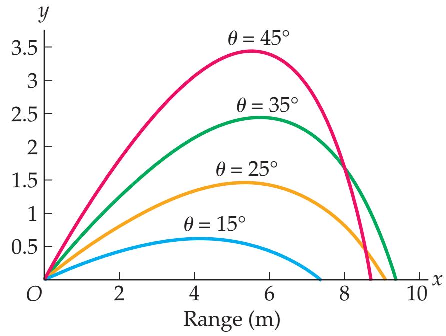
Now, what launch angle gives the greatest range? From Equation 4–12 we see that R varies with angle as $\sin 2\theta$; thus R is largest when $\sin 2\theta$ is largest—that is, when $\sin 2\theta = 1$. Since $\sin 90° = 1$, it follows that $\theta = 45°$ gives the maximum range. Thus

$$R_{\text{max}} = \frac{v_0^2}{g} \qquad \text{4–13}$$

As expected, the range (Equation 4–12) and maximum range (Equation 4–13) depend strongly on the initial speed of the projectile—they are both proportional to v_0^2.

Note that these results are specifically for the case where a projectile lands at the same level from which it was launched. If a projectile lands at a higher level, for example, the launch angle that gives maximum range is greater than 45°, and if it lands at a lower level, the angle for maximum range is less than 45°.

Finally, the range given here applies only to the ideal case of no air resistance. In cases where air resistance is significant, as in the flight of a rapidly moving golf ball, for example, the overall range of the ball is reduced. In addition, the maximum range occurs for a launch angle less than 45° (**Figure 4–9**). The reason is that with a smaller launch angle the golf ball is in the air for less time, giving air resistance less time to affect its flight.



▲ **FIGURE 4–9 Projectiles with air resistance**

Projectiles with the same initial speed but different launch angles showing the effects of air resistance. Notice that the maximum range occurs for a launch angle less than 45°, and that the projectiles return to the ground at a steeper angle than the launch angle.

Symmetry in Projectile Motion

There are many striking symmetries in projectile motion, beginning with the graceful symmetry of the parabola itself. As a first example, recall that earlier in this section, in Equation 4–11, we found the time when a projectile lands:

$$t = \left(\frac{2v_0}{g}\right)\sin\theta$$

Now, by symmetry, the time it takes a projectile to reach its highest point (in the absence of air resistance) should be just half this time. After all, the projectile moves in the x direction with constant speed, and the highest point—by symmetry—occurs at $x = \frac{1}{2}R$.

This all seems reasonable, but is there another way to check? Well, at the highest point the projectile is moving horizontally, thus its y component of velocity is zero. Let's find the time when $v_y = 0$ and compare with the time to land:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$

$$t = \left(\frac{v_0}{g}\right)\sin\theta \qquad \text{4–14}$$

As expected from symmetry, the time at the highest point is one-half the time at landing.

There is another interesting symmetry concerning speed. Recall that when a projectile is launched, its y component of velocity is $v_y = v_0 \sin\theta$. When the projectile lands, at time $t = (2v_0/g)\sin\theta$, its y component of velocity is

$$v_y = v_0 \sin\theta - gt = v_0 \sin\theta - g\left(\frac{2v_0}{g}\right)\sin\theta = -v_0 \sin\theta$$

▲ To be successful, a juggler must master the behavior of projectile motion. Physicist Richard Feynman shows that just knowing the appropriate equations is not enough; one must also practice. In this sense, learning to juggle is similar to learning to solve physics problems.

This is exactly the opposite of the y component of the velocity when it was launched. Since the x component of velocity is always the same, it follows that when the projectile lands, its speed, $v = \sqrt{v_x^2 + v_y^2}$, is the same as when it was launched—as one might expect from symmetry.

The velocities are different, however, since the direction of motion is different at launch and landing. Even so, there is still a symmetry—the initial velocity is *above* the horizontal by the angle θ; the landing velocity is *below* the horizontal by the same angle θ.

So far, these results have referred to launching and landing, which both occur at $y = 0$. The same symmetry extends to any level, though. That is, at a given height the speed of a projectile is the same on the way up as on the way down. In addition, the angle of the velocity above the horizontal on the way up is the same as the angle below the horizontal on the way down. This is illustrated in **Figure 4–10** and in the next Conceptual Checkpoint.

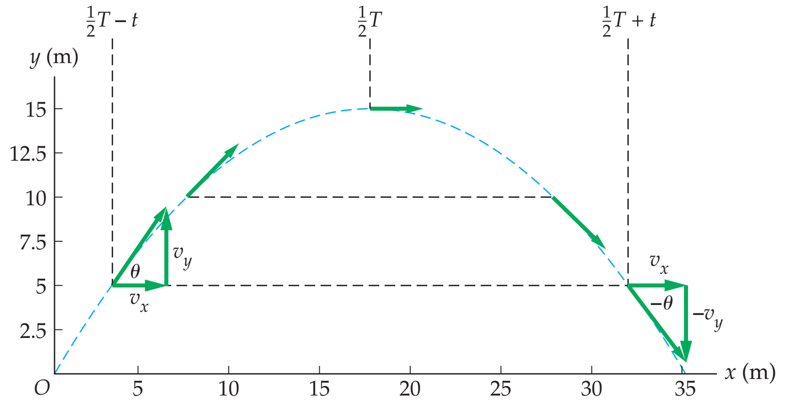
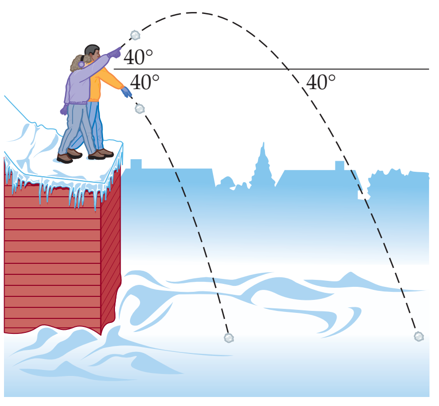


FIGURE 4–10 Velocity vectors for a projectile launched at the origin
At a given height the speed (length of velocity vector) is the same on the way up as on the way down. The direction of motion on the way up is above the horizontal by the same amount that it is below the horizontal on the way down. In this case, the total time of flight is T, and the greatest height is reached at the time $T/2$. Notice that the speed is the same at the time $(T/2) - t$ as it is at the time $(T/2) + t$.

CONCEPTUAL CHECKPOINT 4–3 COMPARE LANDING SPEEDS

You and a friend stand on a snow-covered roof. You both throw snowballs with the same initial speed, but in different directions. You throw your snowball downward, at 40° *below* the horizontal; your friend throws her snowball upward, at 40° *above* the horizontal. When the snowballs land on the ground, is the speed of your snowball **(a)** greater than, **(b)** less than, or **(c)** the same as the speed of your friend's snowball?



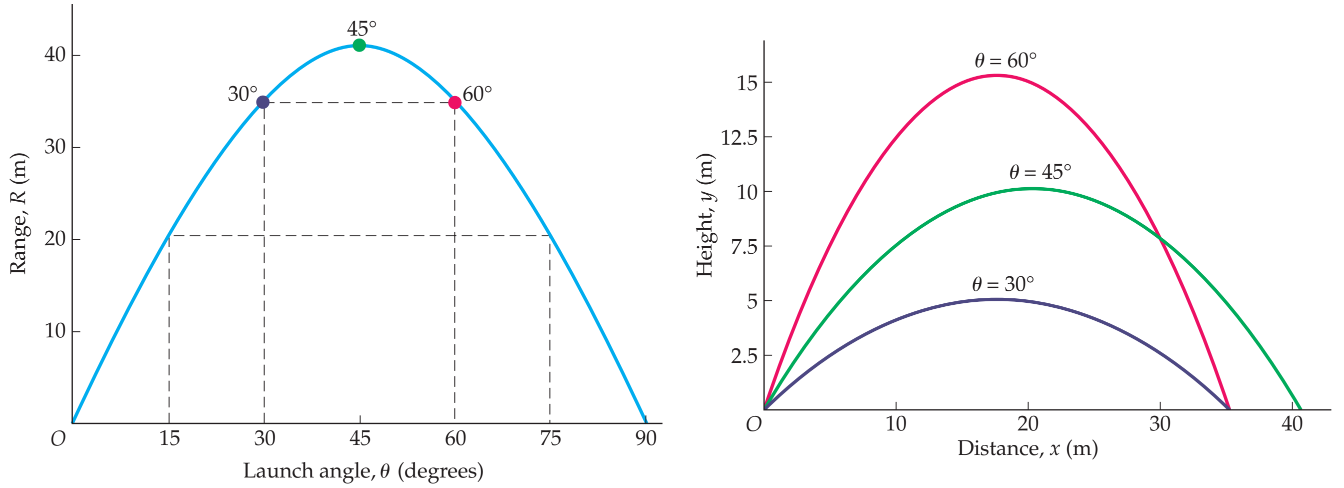
REASONING AND DISCUSSION

One consequence of symmetry in projectile motion is that when your friend's snowball returns to the level of the throw, its speed will be the same as the initial speed. In addition, it will be moving downward, at 40° below the horizontal. From that point on its motion is the same as that of your snowball; thus it lands with the same speed.

What if you throw your snowball horizontally? Or suppose you throw it straight down? In either case, the final speed is unchanged! In fact, for a given initial speed, the speed on landing simply doesn't depend on the direction in which you throw the ball. This is shown in Homework Problems 35 and 76. We return to this point in Chapter 8 when we discuss potential energy and energy conservation.

ANSWER

(c) The snowballs have the same speed.



(a) Launch angles that are greater or less than 45° by the same amount give the same range.

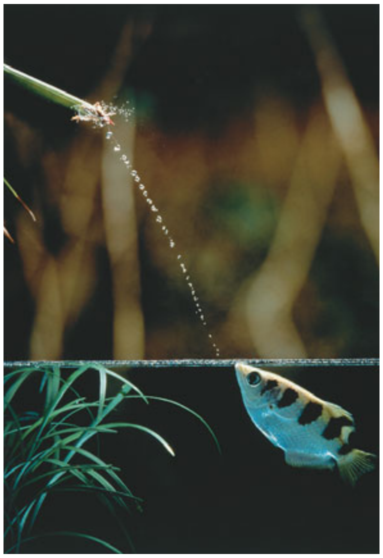
(b) Projectiles with $\theta = 30°$ and $\theta = 60°$ follow different paths but have the same range.

▲ **FIGURE 4–11 Range and launch angle in the absence of air resistance**

(a) A plot of range versus launch angle for a projectile launched with an initial speed of 20 m/s. Note that the maximum range occurs at $\theta = 45°$. Launch angles equally greater than or less than 45°, such as 30° and 60°, give the same range. **(b)** Trajectories of projectiles with initial speeds of 20 m/s and launch angles of 60°, 45°, and 30°. The projectiles with launch angles of 30° and 60° land at the same location.

As our final example of symmetry, consider the range R. A plot of R versus launch angle θ is shown in **Figure 4–11 (a)** for $v_0 = 20$ m/s. Note that in the absence of air resistance, R is greatest at $\theta = 45°$, as pointed out previously. In addition, we can see from the figure that the range for angles equally above or below 45° is the same. For example, if air resistance is negligible, the range for $\theta = 30°$ is the same as the range for $\theta = 60°$, as we can see in both parts (a) and (b) of Figure 4–11.

Symmetries such as these are just some of the many reasons why physicists find physics to be "beautiful" and "aesthetically pleasing." Discovering such patterns and symmetries in nature is really what physics is all about. A physicist does not consider the beauty of projectile motion to be diminished by analyzing it in detail. Just the opposite—detailed analysis reveals deeper, more subtle, and sometimes unexpected levels of beauty.



▲ An archerfish would have trouble procuring its lunch without an instinctive grasp of projectile motion.

Maximum Height

Let's follow up on an observation made earlier in this section, namely, that a projectile is at maximum height when its y component of velocity is zero. In fact, we will use this observation to determine the maximum height of an arbitrary projectile. This can be accomplished with the following two-step calculation: (i) Find the time when $v_y = 0$; (ii) Substitute this time into the y-versus-t equation of motion, $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$. This calculation is carried out in the next Example.

EXAMPLE 4–7 WHAT A SHOT!

The archerfish hunts by dislodging an unsuspecting insect from its resting place with a stream of water expelled from the fish's mouth. Suppose the archerfish squirts water with an initial speed of 2.30 m/s at an angle of 19.5° above the horizontal. When the stream of water reaches a beetle on a leaf at height h above the water's surface, it is moving horizontally.

a. How much time does the beetle have to react?

b. What is the height h of the beetle?

c. What is the horizontal distance d between the fish and the beetle when the water is launched?

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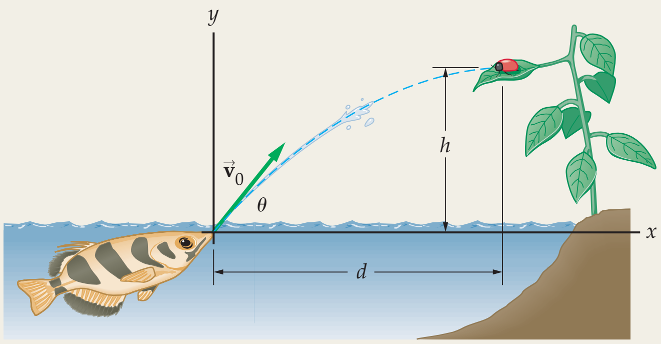
PICTURE THE PROBLEM

Our sketch shows the fish squirting water from the origin, $x_0 = y_0 = 0$, and the beetle at $x = d$, $y = h$. The stream of water starts off with a speed $v_0 = 2.30\ \mathrm{m/s}$ at an angle $\theta = 19.5°$ above the horizontal. Note that the water is moving horizontally when it reaches the beetle.



STRATEGY

a. Because the stream of water is moving horizontally when it reaches the beetle, it is at the top of its parabolic trajectory, as can be seen in Figure 4–10. This means that its y component of velocity is zero. Therefore, we can set $v_y = 0$ in $v_y = v_0 \sin\theta - gt$ and solve for the time t.

b. To find the maximum height of the stream of water, and of the beetle, we substitute the time found in part (a) into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$.

c. Similarly, we can find the horizontal distance d by substituting the time from part (a) into $x = (v_0 \cos\theta)t$.

SOLUTION

Part (a)

1. Set $v_y = v_0 \sin\theta - gt$ equal to zero and solve for the corresponding time t:

$$v_y = v_{0y} - gt = v_0 \sin\theta - gt = 0$$
$$t = \frac{v_0 \sin\theta}{g}$$

2. Substitute numerical values to determine the reaction time:

$$t = \frac{v_0 \sin\theta}{g} = \frac{(2.30\ \mathrm{m/s})\sin 19.5°}{9.81\ \mathrm{m/s^2}} = 0.0783\ \mathrm{s}$$

Part (b)

3. To calculate the height, we substitute $t = (v_0 \sin\theta)/g$ into $y = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y = (v_0 \sin\theta)\left(\frac{v_0 \sin\theta}{g}\right) - \frac{1}{2}g\left(\frac{v_0 \sin\theta}{g}\right)^2 = \frac{(v_0 \sin\theta)^2}{2g}$$

4. Substitute numerical values to find the height h:

$$h = \frac{(v_0 \sin\theta)^2}{2g} = \frac{[(2.30\ \mathrm{m/s})\sin 19.5°]^2}{2(9.81\ \mathrm{m/s^2})} = 0.0300\ \mathrm{m}$$

Part (c)

5. We can find the horizontal distance d using x as a function of time, $x = (v_0 \cos\theta)t$:

$$x = (v_0 \cos\theta)t$$
$$d = [(2.30\ \mathrm{m/s})\cos 19.5°](0.0783\ \mathrm{s}) = 0.170\ \mathrm{m}$$

INSIGHT

To hit the beetle, the fish aims 19.5° above the horizontal. For comparison, note that the straight-line angle to the beetle is $\tan^{-1}(0.0300/0.170) = 10.0°$. Therefore, the fish cannot aim directly at its prey if it wants a meal.

Finally, note that by working symbolically in Step 3 we have derived a general result for the maximum height of a projectile. In particular, we find $y_{max} = (v_0 \sin\theta)^2/2g$, a result that is valid for any launch speed and angle. As a check of our result, note that if we launch a projectile straight upward ($\theta = 90°$), the maximum height is $y_{max} = {v_0}^2/2g$. Comparing with the one-dimensional kinematics of Chapter 2, if an object is thrown straight upward with an initial speed v_0, and the object accelerates downward with the acceleration of gravity, $a = -g$, it comes to rest ($v = 0$) after covering a vertical distance Δy given by $0 = {v_0}^2 + 2(-g)\Delta y$. Solving for the distance yields $\Delta y = {v_0}^2/2g = y_{max}$. This is an example of the internal consistency that characterizes all of physics.

PRACTICE PROBLEM

How far does the stream of water go if it happens to miss the beetle? [**Answer:** By symmetry, the distance d is half the range. Thus the stream of water travels a distance $R = 2d = 0.340$ m.]

Some related homework problems: Problem 81, Problem 82

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

This chapter provides a number of opportunities to use the vector methods developed in Chapter 3. In Section 4–4, for example, we resolve a velocity vector into its x and y components, and then use the components in Equations 4–10.

The equations of one-dimensional kinematics derived in Chapter 2 are used again in this chapter, even though we are now studying kinematics in two dimensions. For example, the equations in Table 4–1 are the same as those used in Chapter 2, only now applied individually to the x and y directions.

LOOKING AHEAD

The basic idea behind projectile motion will be used again in Chapter 12, when we consider orbital motion. See, in particular, the illustration presented in Section 12–1.

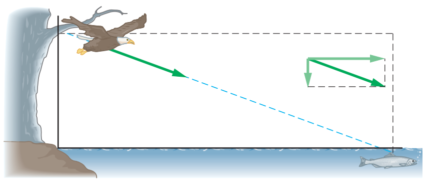
Two-dimensional kinematics comes up again when we study the motion of charged particles (like electrons) in electric fields. To see the connection, compare Figures 19–41 and 22–10 (a) with the person jumping a crevasse in Example 4–4. The same basic principles apply.

CHAPTER SUMMARY

4–1 MOTION IN TWO DIMENSIONS

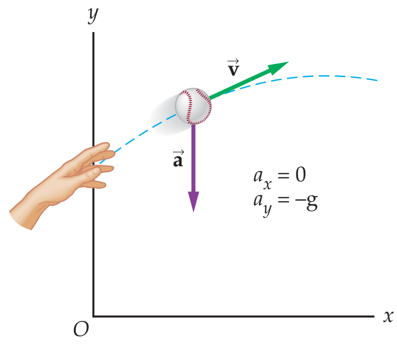
Independence of Motion

Components of motion in the x and y directions can be treated independently of one another. Thus, two-dimensional motion with constant acceleration is described by the same kinematic equations derived in Chapter 2, only now written in terms of x and y components.



4–2 PROJECTILE MOTION: BASIC EQUATIONS

Projectile motion refers to the path of an object after it is thrown, kicked, batted, or otherwise launched into the air. For the ideal case, we assume no air resistance and a constant downward acceleration of magnitude g.



Acceleration Components

In projectile motion, with the x axis horizontal and the y axis upward, the components of the acceleration of gravity are

$$a_x = 0$$
$$a_y = -g$$

x and y as Functions of Time

The x and y equations of motion are

$$x = x_0 + v_{0x}t$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad 4\text{–}6$$

v_x and v_y as Functions of Time

The velocity components vary with time as follows:

$$v_x = v_{0x}$$
$$v_y = v_{0y} - gt \qquad 4\text{–}6$$

v_x and v_y as Functions of Displacement

v_x and v_y vary with displacement as

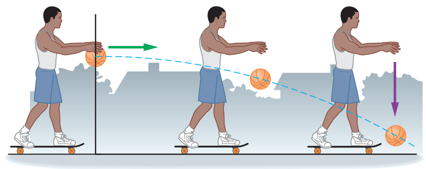
$$v_x^2 = v_{0x}^2$$
$$v_y^2 = v_{0y}^2 - 2g\Delta y \qquad 4\text{–}6$$

4–3 ZERO LAUNCH ANGLE

Equations of Motion

A projectile launched horizontally from $x_0 = 0, y_0 = h$ with an initial speed v_0 has the following equations of motion:

$$x = v_0 t \qquad v_x = v_0 \qquad v_x{}^2 = v_0{}^2$$
$$y = h - \tfrac{1}{2}gt^2 \qquad v_y = -gt \qquad v_y{}^2 = -2g\Delta y \qquad \text{4–7}$$



Parabolic Path

The path followed by a projectile launched horizontally with an initial speed v_0 is described by

$$y = h - \left(\frac{g}{2v_0{}^2}\right)x^2 \qquad \text{4–8}$$

This path is a parabola.

Landing Site

The landing site of a projectile launched horizontally is

$$x = v_0\sqrt{\frac{2h}{g}} \qquad \text{4–9}$$

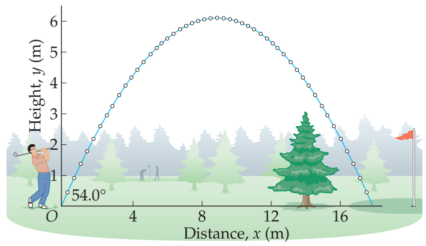
In this expression, v_0 is the initial speed and h is the initial height. Note that this result is simply the speed in the x direction multiplied by the time of fall.

4–4 GENERAL LAUNCH ANGLE

Launch from the Origin

The equations of motion for a launch from the origin with an initial speed v_0 at an angle of θ with respect to the horizontal are

$$x = (v_0 \cos\theta)t \qquad v_x = v_0 \cos\theta \qquad v_x{}^2 = v_0{}^2\cos^2\theta$$
$$y = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 \qquad v_y = v_0 \sin\theta - gt \qquad v_y{}^2 = v_0{}^2\sin^2\theta - 2g\Delta y \qquad \text{4–10}$$

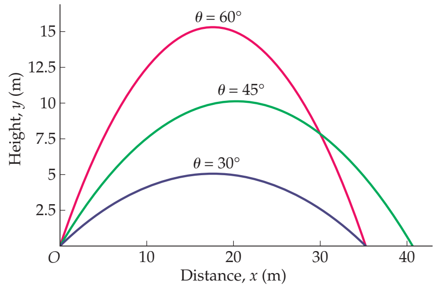


4–5 PROJECTILE MOTION: KEY CHARACTERISTICS

Range

The range of a projectile launched from the origin with an initial speed v_0 and a launch angle θ is

$$R = \left(\frac{v_0{}^2}{g}\right)\sin 2\theta \qquad \text{4–12}$$



This expression applies only to projectiles that land at the same level from which they were launched.

Symmetry

Projectile motion exhibits many symmetries. For example, the speed of a projectile depends only on its height and not on whether it is moving upward or downward.

Maximum Height

The maximum height of a projectile above its launch site is

$$y_{max} = \frac{v_0{}^2 \sin^2\theta}{2g}$$

In this equation, v_0 is the initial speed and θ is the launch angle.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Study two-dimensional motion with constant acceleration.	Motion in the x direction is independent of motion in the y direction. This is the basis for the equations of motion given in Table 4–1. Note that these equations are the same as the kinematic equations of Chapter 2, only written in terms of x and y components.	Examples 4–1, 4–2
Find the location and velocity of a projectile launched horizontally.	When a projectile is launched horizontally with a speed v_0 its initial velocity components are $v_{0x} = v_0$ and $v_{0y} = 0$. Make these substitutions in the equations of projectile motion given in Equations 4–6.	Examples 4–3, 4–4 Conceptual Checkpoints 4–1, 4–2
Find the location and velocity of a projectile launched with an arbitrary launch angle.	If a projectile is launched at an angle θ, its initial velocity components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. Make these substitutions in the equations of projectile motion given in Equations 4–6.	Examples 4–5, 4–6, 4–7 Active Examples 4–1, 4–2

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. What is the acceleration of a projectile when it reaches its highest point? What is its acceleration just before and just after reaching this point?
2. A projectile is launched with an initial speed of v_0 at an angle θ above the horizontal. It lands at the same level from which it was launched. What was its average velocity between launch and landing? Explain.
3. A projectile is launched from level ground. When it lands, its direction of motion has rotated clockwise through 60°. What was the launch angle? Explain.
4. In a game of baseball, a player hits a high fly ball to the outfield. **(a)** Is there a point during the flight of the ball where its velocity is parallel to its acceleration? **(b)** Is there a point where the ball's velocity is perpendicular to its acceleration? Explain in each case.
5. A projectile is launched with an initial velocity of $\vec{\mathbf{v}} = (4\text{ m/s})\hat{\mathbf{x}} + (3\text{ m/s})\hat{\mathbf{y}}$. What is the velocity of the projectile when it reaches its highest point? Explain.
6. A projectile is launched from a level surface with an initial velocity of $\vec{\mathbf{v}} = (2\text{ m/s})\hat{\mathbf{x}} + (4\text{ m/s})\hat{\mathbf{y}}$. What is the velocity of the projectile just before it lands? Explain.
7. Do projectiles for which air resistance is nonnegligible, such as a bullet fired from a rifle, have maximum range when the launch angle is greater than, less than, or equal to 45°? Explain.
8. Two projectiles are launched from the same point at the same angle above the horizontal. Projectile 1 reaches a maximum height twice that of projectile 2. What is the ratio of the initial speed of projectile 1 to the initial speed of projectile 2? Explain.
9. A child rides on a pony walking with constant velocity. The boy leans over to one side and a scoop of ice cream falls from his ice cream cone. Describe the path of the scoop of ice cream as seen by **(a)** the child and **(b)** his parents standing on the ground nearby.
10. Driving down the highway, you find yourself behind a heavily loaded tomato truck. You follow close behind the truck, keeping the same speed. Suddenly a tomato falls from the back of the truck. Will the tomato hit your car or land on the road, assuming you continue moving with the same speed and direction? Explain.
11. A projectile is launched from the origin of a coordinate system where the positive x axis points horizontally to the right and the positive y axis points vertically upward. What was the projectile's launch angle with respect to the x axis if, at its highest point, its direction of motion has rotated **(a)** clockwise through 50° or **(b)** counterclockwise through 30°? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

(Air resistance should be ignored in the problems for this chapter, unless specifically stated otherwise.)

SECTION 4–1 MOTION IN TWO DIMENSIONS

1. • **CE Predict/Explain** As you walk briskly down the street, you toss a small ball into the air. **(a)** If you want the ball to land in your hand when it comes back down, should you toss the ball straight upward, in a forward direction, or in a backward direction, relative to your body? **(b)** Choose the *best explanation* from among the following:
   - **I.** If the ball is thrown straight up you will leave it behind.
   - **II.** You have to throw the ball in the direction you are walking.
   - **III.** The ball moves in the forward direction with your walking speed at all times.

2. • A sailboat runs before the wind with a constant speed of 4.2 m/s in a direction 32° north of west. How far **(a)** west and **(b)** north has the sailboat traveled in 25 min?

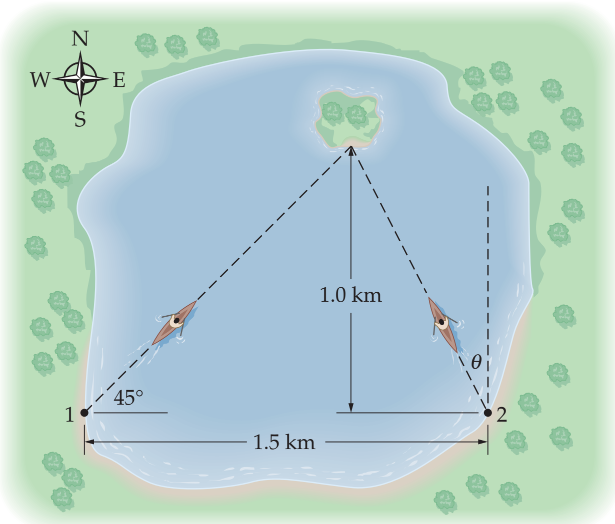
3. • As you walk to class with a constant speed of 1.75 m/s, you are moving in a direction that is 18.0° north of east. How much time does it take to change your displacement by **(a)** 20.0 m east or **(b)** 30.0 m north?

4. • Starting from rest, a car accelerates at $2.0\ \mathrm{m/s^2}$ up a hill that is inclined 5.5° above the horizontal. How far **(a)** horizontally and **(b)** vertically has the car traveled in 12 s?

5. •• **IP** A particle passes through the origin with a velocity of $(6.2\ \mathrm{m/s})\hat{\mathbf{y}}$. If the particle's acceleration is $(-4.4\ \mathrm{m/s^2})\hat{\mathbf{x}}$, **(a)** what are its x and y positions after 5.0 s? **(b)** What are v_x and v_y at this time? **(c)** Does the speed of this particle increase with time, decrease with time, or increase and then decrease? Explain.

6. •• An electron in a cathode-ray tube is traveling horizontally at $2.10 \times 10^9\ \mathrm{cm/s}$ when deflection plates give it an upward acceleration of $5.30 \times 10^{17}\ \mathrm{cm/s^2}$. **(a)** How long does it take for the electron to cover a horizontal distance of 6.20 cm? **(b)** What is its vertical displacement during this time?

7. •• Two canoeists start paddling at the same time and head toward a small island in a lake, as shown in **Figure 4–12**. Canoeist 1 paddles with a speed of 1.35 m/s at an angle of 45° north of east. Canoeist 2 starts on the opposite shore of the lake, a distance of 1.5 km due east of canoeist 1. **(a)** In what direction relative to north must canoeist 2 paddle to reach the island? **(b)** What speed must canoeist 2 have if the two canoes are to arrive at the island at the same time?



▲ **FIGURE 4–12** Problem 7

SECTION 4–3 ZERO LAUNCH ANGLE

8. • **CE Predict/Explain** Two divers run horizontally off the edge of a low cliff. Diver 2 runs with twice the speed of diver 1. **(a)** When the divers hit the water, is the horizontal distance covered by diver 2 twice as much, four times as much, or equal to the horizontal distance covered by diver 1? **(b)** Choose the *best explanation* from among the following:
   I. The drop time is the same for both divers.
   II. Drop distance depends on t^2.
   III. All divers in free fall cover the same distance.

9. • **CE Predict/Explain** Two youngsters dive off an overhang into a lake. Diver 1 drops straight down, and diver 2 runs off the cliff with an initial horizontal speed v_0. **(a)** Is the splashdown speed of diver 2 greater than, less than, or equal to the splashdown speed of diver 1? **(b)** Choose the *best explanation* from among the following:
   I. Both divers are in free fall, and hence they will have the same splashdown speed.
   II. The divers have the same vertical speed at splashdown, but diver 2 has the greater horizontal speed.
   III. The diver who drops straight down gains more speed than the one who moves horizontally.

10. • An archer shoots an arrow horizontally at a target 15 m away. The arrow is aimed directly at the center of the target, but it hits 52 cm lower. What was the initial speed of the arrow?

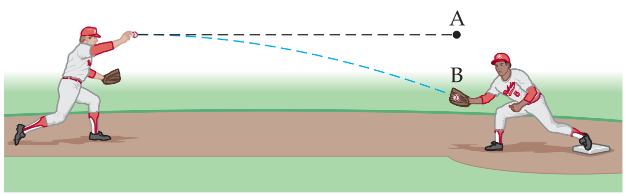
11. • **Victoria Falls** The great, gray-green, greasy Zambezi River flows over Victoria Falls in south central Africa. The falls are approximately 108 m high. If the river is flowing horizontally at 3.60 m/s just before going over the falls, what is the speed of the water when it hits the bottom? Assume the water is in free fall as it drops.

12. • A diver runs horizontally off the end of a diving board with an initial speed of 1.85 m/s. If the diving board is 3.00 m above the water, what is the diver's speed just before she enters the water?

13. • An astronaut on the planet Zircon tosses a rock horizontally with a speed of 6.95 m/s. The rock falls through a vertical distance of 1.40 m and lands a horizontal distance of 8.75 m from the astronaut. What is the acceleration of gravity on Zircon?

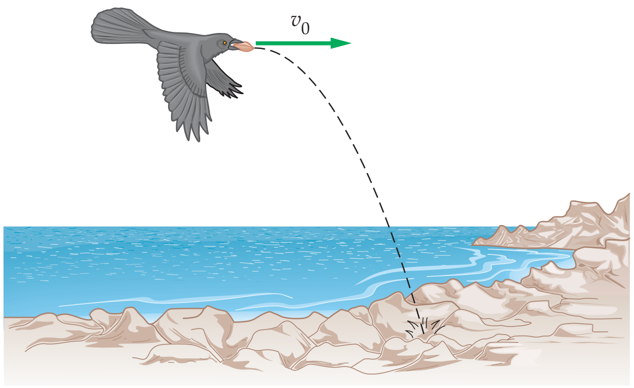
14. •• **IP Pitcher's Mounds** Pitcher's mounds are raised to compensate for the vertical drop of the ball as it travels a horizontal distance of 18 m to the catcher. **(a)** If a pitch is thrown horizontally with an initial speed of 32 m/s, how far does it drop by the time it reaches the catcher? **(b)** If the speed of the pitch is increased, does the drop distance increase, decrease, or stay the same? Explain. **(c)** If this baseball game were to be played on the Moon, would the drop distance increase, decrease, or stay the same? Explain.

15. •• Playing shortstop, you pick up a ground ball and throw it to second base. The ball is thrown horizontally, with a speed of 22 m/s, directly toward point A (**Figure 4–13**). When the ball reaches the second baseman 0.45 s later, it is caught at point B. **(a)** How far were you from the second baseman? **(b)** What is the distance of vertical drop, AB?



▲ **FIGURE 4–13** Problem 15

16. •• **IP** A crow is flying horizontally with a constant speed of 2.70 m/s when it releases a clam from its beak (**Figure 4–14**). The clam lands on the rocky beach 2.10 s later. Just before the clam lands, what is **(a)** its horizontal component of velocity, and **(b)** its vertical component of velocity? **(c)** How would your answers to parts (a) and (b) change if the speed of the crow were increased? Explain.

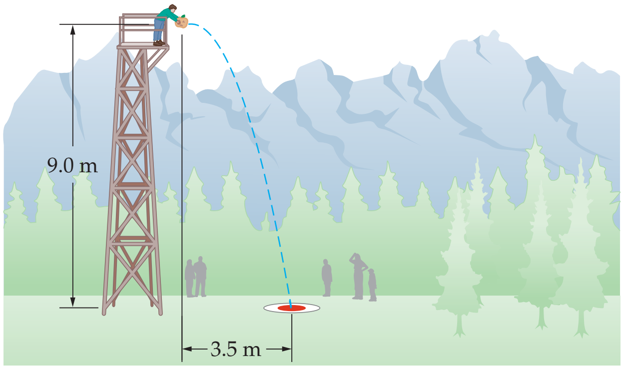


▲ **FIGURE 4–14** Problem 16

17. •• A mountain climber jumps a 2.8-m-wide crevasse by leaping horizontally with a speed of 7.8 m/s. **(a)** If the climber's direction of motion on landing is −45°, what is the height difference between the two sides of the crevasse? **(b)** Where does the climber land?

18. •• **IP** A white-crowned sparrow flying horizontally with a speed of 1.80 m/s folds its wings and begins to drop in free fall. **(a)** How far does the sparrow fall after traveling a horizontal distance of 0.500 m? **(b)** If the sparrow's initial speed is increased, does the distance of fall increase, decrease, or stay the same?

19. •• **Pumpkin Toss** In Denver, children bring their old jack-o-lanterns to the top of a tower and compete for accuracy in hitting a target on the ground (**Figure 4–15**). Suppose that the tower is 9.0 m high and that the bull's-eye is a horizontal distance of 3.5 m from the launch point. If the pumpkin is thrown horizontally, what is the launch speed needed to hit the bull's-eye?

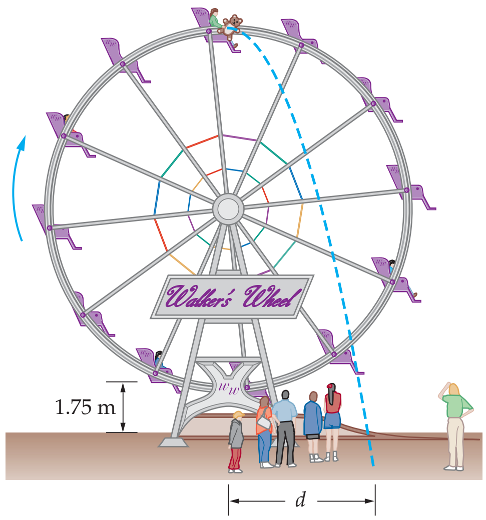


▲ **FIGURE 4–15** Problems 19 and 20

20. •• If, in the previous problem, a jack-o-lantern is given an initial horizontal speed of 3.3 m/s, what are the direction and magnitude of its velocity **(a)** 0.75 s later, and **(b)** just before it lands?

21. •• Fairgoers ride a Ferris wheel with a radius of 5.00 m (**Figure 4–16**). The wheel completes one revolution every 32.0 s. **(a)** What is the average speed of a rider on this Ferris wheel? **(b)** If a rider accidentally drops a stuffed animal at the top of the wheel, where does it land relative to the base of the ride? (Note: The bottom of the wheel is 1.75 m above the ground.)

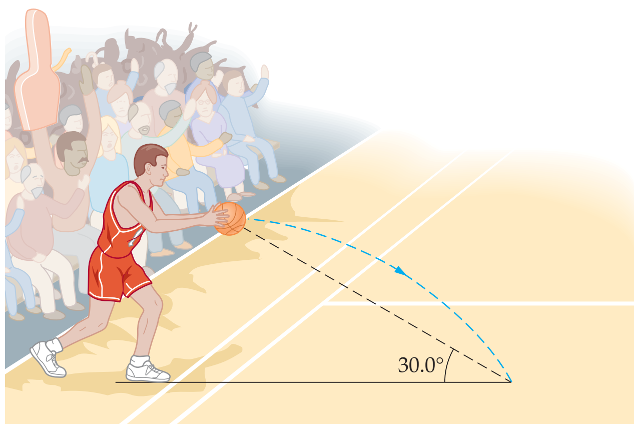
22. •• **IP** A swimmer runs horizontally off a diving board with a speed of 3.32 m/s and hits the water a horizontal distance of 1.78 m from the end of the board. **(a)** How high above the water was the diving board? **(b)** If the swimmer runs off the board with a reduced speed, does it take more, less, or the same time to reach the water?



▲ **FIGURE 4–16** Problems 21 and 42

23. •• **Baseball and the Washington Monument** On August 25, 1894, Chicago catcher William Schriver caught a baseball thrown from the top of the Washington Monument (555 ft, 898 steps). **(a)** If the ball was thrown horizontally with a speed of 5.00 m/s, where did it land? **(b)** What were the ball's speed and direction of motion when caught?

24. ••• A basketball is thrown horizontally with an initial speed of 4.20 m/s (**Figure 4–17**). A straight line drawn from the release point to the landing point makes an angle of 30.0° with the horizontal. What was the release height?



▲ **FIGURE 4–17** Problem 24

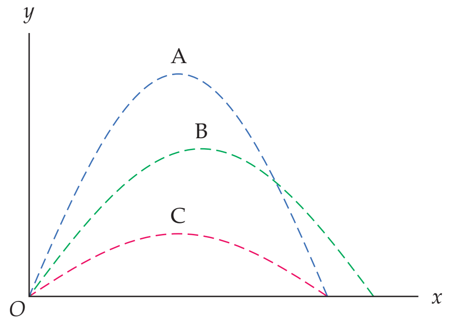
25. ••• **IP** A ball rolls off a table and falls 0.75 m to the floor, landing with a speed of 4.0 m/s. **(a)** What is the acceleration of the ball just before it strikes the ground? **(b)** What was the initial speed of the ball? **(c)** What initial speed must the ball have if it is to land with a speed of 5.0 m/s?

SECTION 4–4 GENERAL LAUNCH ANGLE

26. • **CE** A certain projectile is launched with an initial speed v_0. At its highest point its speed is $\frac{1}{2}v_0$. What was the launch angle of the projectile?

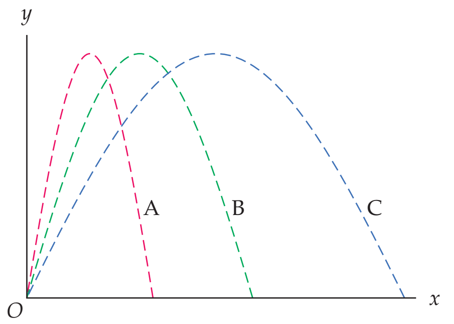
A. 30° **B.** 45° **C.** 60° **D.** 75°

27. • **CE** Three projectiles (A, B, and C) are launched with the same initial speed but with different launch angles, as shown in **Figure 4–18**. Rank the projectiles in order of increasing **(a)** horizontal component of initial velocity and **(b)** time of flight. Indicate ties where appropriate.



▲ **FIGURE 4–18** Problem 27

28. • **CE** Three projectiles (A, B, and C) are launched with different initial speeds so that they reach the same maximum height, as shown in **Figure 4–19**. Rank the projectiles in order of increasing **(a)** initial speed and **(b)** time of flight. Indicate ties where appropriate.



▲ **FIGURE 4–19** Problem 28

29. • A second baseman tosses the ball to the first baseman, who catches it at the same level from which it was thrown. The throw is made with an initial speed of 18.0 m/s at an angle of 37.5° above the horizontal. **(a)** What is the horizontal component of the ball's velocity just before it is caught? **(b)** How long is the ball in the air?

30. • Referring to the previous problem, what are the y component of the ball's velocity and its direction of motion just before it is caught?

31. • A cork shoots out of a champagne bottle at an angle of 35.0° above the horizontal. If the cork travels a horizontal distance of 1.30 m in 1.25 s, what was its initial speed?

32. • A soccer ball is kicked with a speed of 9.85 m/s at an angle of 35.0° above the horizontal. If the ball lands at the same level from which it was kicked, how long was it in the air?

33. •• In a game of basketball, a forward makes a bounce pass to the center. The ball is thrown with an initial speed of 4.3 m/s at an angle of 15° below the horizontal. It is released 0.80 m above the floor. What horizontal distance does the ball cover before bouncing?

34. •• Repeat the previous problem for a bounce pass in which the ball is thrown 15° *above* the horizontal.

35. •• **IP** Snowballs are thrown with a speed of 13 m/s from a roof 7.0 m above the ground. Snowball A is thrown straight downward; snowball B is thrown in a direction 25° above the horizontal. **(a)** Is the landing speed of snowball A greater than, less than, or the same as the landing speed of snowball B? Explain. **(b)** Verify your answer to part (a) by calculating the landing speed of both snowballs.

36. •• In the previous problem, find the direction of motion of the two snowballs just before they land.

37. •• A golfer gives a ball a maximum initial speed of 34.4 m/s. **(a)** What is the longest possible hole-in-one for this golfer? Neglect any distance the ball might roll on the green and assume that the tee and the green are at the same level. **(b)** What is the minimum speed of the ball during this hole-in-one shot?

38. •• What is the highest tree the ball in the previous problem could clear on its way to the longest possible hole-in-one?

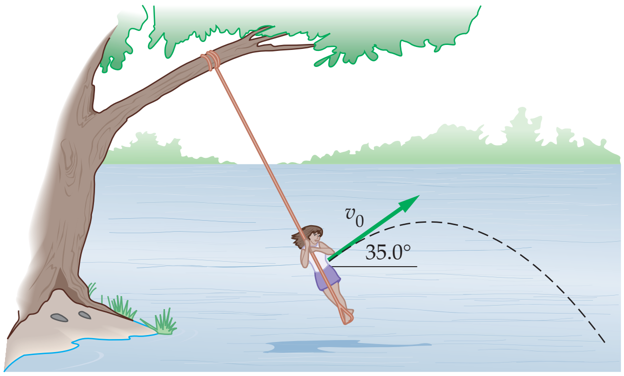
39. •• The "hang time" of a punt is measured to be 4.50 s. If the ball was kicked at an angle of 63.0° above the horizontal and was caught at the same level from which it was kicked, what was its initial speed?

40. •• In a friendly game of handball, you hit the ball essentially at ground level and send it toward the wall with a speed of 18 m/s at an angle of 32° above the horizontal. **(a)** How long does it take for the ball to reach the wall if it is 3.8 m away? **(b)** How high is the ball when it hits the wall?

41. •• **IP** In the previous problem, **(a)** what are the magnitude and direction of the ball's velocity when it strikes the wall? **(b)** Has the ball reached the highest point of its trajectory at this time? Explain.

42. •• A passenger on the Ferris wheel described in Problem 21 drops his keys when he is on the way up and at the 10 o'clock position. Where do the keys land relative to the base of the ride?

43. •• On a hot summer day, a young girl swings on a rope above the local swimming hole (**Figure 4–20**). When she lets go of the rope her initial velocity is 2.25 m/s at an angle of 35.0° above the horizontal. If she is in flight for 0.616 s, how high above the water was she when she let go of the rope?



▲ **FIGURE 4–20** Problem 43

44. •• A certain projectile is launched with an initial speed v_0. At its highest point its speed is $v_0/4$. What was the launch angle?

SECTION 4–5 PROJECTILE MOTION: KEY CHARACTERISTICS

45. • **Punkin Chunkin** In Sussex County, Delaware, a post-Halloween tradition is "Punkin Chunkin," in which contestants build cannons, catapults, trebuchets, and other devices to launch pumpkins and compete for the greatest distance. Though hard to believe, pumpkins have been projected a distance of 4086 feet in this contest. What is the minimum initial speed needed for such a shot?

46. • A dolphin jumps with an initial velocity of 12.0 m/s at an angle of 40.0° above the horizontal. The dolphin passes through the center of a hoop before returning to the water. If the dolphin is moving horizontally when it goes through the hoop, how high above the water is the center of the hoop?

47. • A player passes a basketball to another player who catches it at the same level from which it was thrown. The initial speed of the ball is 7.1 m/s, and it travels a distance of 4.6 m. What were **(a)** the initial direction of the ball and **(b)** its time of flight?

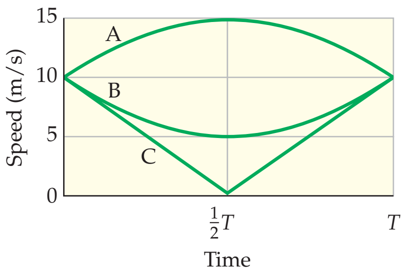
48. • A golf ball is struck with a five iron on level ground. It lands 92.2 m away 4.30 s later. What were **(a)** the direction and **(b)** the magnitude of the initial velocity?

49. • **A Record Toss** Babe Didrikson holds the world record for the longest baseball throw (296 ft) by a woman. For the following questions, assume that the ball was thrown at an angle of 45.0° above the horizontal, that it traveled a horizontal distance of 296 ft, and that it was caught at the same level from which it was thrown. **(a)** What was the ball's initial speed? **(b)** How long was the ball in the air?

50. • In the photograph to the left on page 87, suppose the cart that launches the ball is 11 cm high. Estimate **(a)** the launch speed of the ball and **(b)** the time interval between successive stroboscopic exposures.

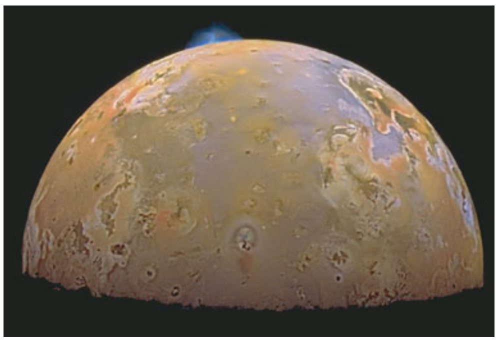
51. •• **CE Predict/Explain** You throw a ball into the air with an initial speed of 10 m/s at an angle of 60° above the horizontal. The ball returns to the level from which it was thrown in the time T. **(a)** Referring to **Figure 4–21**, which of the plots (A, B, or C) best represents the speed of the ball as a function of time? **(b)** Choose the *best explanation* from among the following:

I. Gravity causes the ball's speed to increase during its flight.
II. The ball has zero speed at its highest point.
III. The ball's speed decreases during its flight, but it doesn't go to zero.



▲ **FIGURE 4–21** Problem 51

52. •• **IP Volcanoes on Io** Astronomers have discovered several volcanoes on Io, a moon of Jupiter. One of them, named Loki, ejects lava to a maximum height of 2.00×10^5 m. **(a)** What is the initial speed of the lava? (The acceleration of gravity on Io is 1.80 m/s^2.) **(b)** If this volcano were on Earth, would the maximum height of the ejected lava be greater than, less than, or the same as on Io? Explain.



A volcano on Io, the innermost moon of Jupiter, displays the characteristic features of projectile motion. (Problem 52)

53. •• **IP** A soccer ball is kicked with an initial speed of 10.2 m/s in a direction 25.0° above the horizontal. Find the magnitude and direction of its velocity **(a)** 0.250 s and **(b)** 0.500 s after being kicked. **(c)** Is the ball at its greatest height before or after 0.500 s? Explain.

54. •• A second soccer ball is kicked with the same initial speed as in Problem 53. After 0.750 s it is at its highest point. What was its initial direction of motion?

55. •• **IP** A golfer tees off on level ground, giving the ball an initial speed of 46.5 m/s and an initial direction of 37.5° above the horizontal. **(a)** How far from the golfer does the ball land? **(b)** The next golfer in the group hits a ball with the same initial speed but at an angle above the horizontal that is greater than 45.0°. If the second ball travels the same horizontal distance as the first ball, what was its initial direction of motion? Explain.

56. •• **IP** One of the most popular events at Highland games is the hay toss, where competitors use a pitchfork to throw a bale of hay over a raised bar. Suppose the initial velocity of a bale of hay is $\vec{v} = (1.12\text{ m/s})\hat{x} + (8.85\text{ m/s})\hat{y}$. **(a)** After what minimum time is its speed equal to 5.00 m/s? **(b)** How long after the hay is tossed is it moving in a direction that is 45.0° below the horizontal? **(c)** If the bale of hay is tossed with the same initial speed, only this time straight upward, will its time in the air increase, decrease, or stay the same? Explain.

GENERAL PROBLEMS

57. • **CE** Child 1 throws a snowball horizontally from the top of a roof; child 2 throws a snowball straight down. Once in flight, is the acceleration of snowball 2 greater than, less than, or equal to the acceleration of snowball 1?

58. • **CE** The penguin to the left in the accompanying photo is about to land on an ice floe. Just before it lands, is its speed greater than, less than, or equal to its speed when it left the water?

This penguin behaves much like a projectile from the time it leaves the water until it touches down on the ice. (Problem 58)

59. • **CE Predict/Explain** A person flips a coin into the air and it lands on the ground a few feet away. **(a)** If the person were to perform an identical coin flip on an elevator rising with constant speed, would the coin's time of flight be greater than, less than, or equal to its time of flight when the person was at rest? **(b)** Choose the *best explanation* from among the following:

I. The floor of the elevator is moving upward, and hence it catches up with the coin in mid flight.

II. The coin has the same upward speed as the elevator when it is tossed, and the elevator's speed doesn't change during the coin's flight.
III. The coin starts off with a greater upward speed because of the elevator, and hence it reaches a greater height.

60. • **CE Predict/Explain** Suppose the elevator in the previous problem is rising with a constant upward acceleration, rather than constant velocity. **(a)** In this case, would the coin's time of flight be greater than, less than, or equal to its time of flight when the person was at rest? **(b)** Choose the *best explanation* from among the following:
I. The coin has the same acceleration once it is tossed, whether the elevator accelerates or not.
II. The elevator's upward speed increases during the coin's flight, and hence it catches up with the coin at a greater height than before.
III. The coin's downward acceleration is less than before because the elevator's upward acceleration partially cancels it.

61. • A train moving with constant velocity travels 170 m north in 12 s and an undetermined distance to the west. The speed of the train is 32 m/s. **(a)** Find the direction of the train's motion relative to north. **(b)** How far west has the train traveled in this time?

62. • Referring to Example 4–2, find **(a)** the x component and **(b)** the y component of the hummingbird's velocity at the time $t = 0.72$ s. **(c)** What is the bird's direction of travel at this time, relative to the positive x axis?

63. • A racket ball is struck in such a way that it leaves the racket with a speed of 4.87 m/s in the horizontal direction. When the ball hits the court, it is a horizontal distance of 1.95 m from the racket. Find the height of the racket ball when it left the racket.

64. •• **IP** A hot-air balloon rises from the ground with a velocity of $(2.00\ \mathrm{m/s})\hat{\mathbf{y}}$. A champagne bottle is opened to celebrate takeoff, expelling the cork horizontally with a velocity of $(5.00\ \mathrm{m/s})\hat{\mathbf{x}}$ relative to the balloon. When opened, the bottle is 6.00 m above the ground. **(a)** What is the initial velocity of the cork, as seen by an observer on the ground? Give your answer in terms of the x and y unit vectors. **(b)** What are the speed of the cork and its initial direction of motion as seen by the same observer? **(c)** Determine the maximum height above the ground attained by the cork. **(d)** How long does the cork remain in the air?

65. •• Repeat the previous problem, this time assuming that the balloon is *descending* with a speed of 2.00 m/s.

66. •• **IP** A soccer ball is kicked from the ground with an initial speed of 14.0 m/s. After 0.275 s its speed is 12.9 m/s. **(a)** Give a strategy that will allow you to calculate the ball's initial direction of motion. **(b)** Use your strategy to find the initial direction.

67. •• A particle leaves the origin with an initial velocity $\vec{\mathbf{v}} = (2.40\ \mathrm{m/s})\hat{\mathbf{x}}$, and moves with constant acceleration $\vec{\mathbf{a}} = (-1.90\ \mathrm{m/s^2})\hat{\mathbf{x}} + (3.20\ \mathrm{m/s^2})\hat{\mathbf{y}}$. **(a)** How far does the particle move in the x direction before turning around? **(b)** What is the particle's velocity at this time? **(c)** Plot the particle's position at $t = 0.500$ s, 1.00 s, 1.50 s, and 2.00 s. Use these results to sketch position versus time for the particle.

68. •• When the dried-up seed pod of a scotch broom plant bursts open, it shoots out a seed with an initial velocity of 2.62 m/s at an angle of 60.5° above the horizontal. If the seed pod is 0.455 m above the ground, **(a)** how long does it take for the seed to land? **(b)** What horizontal distance does it cover during its flight?

69. •• Referring to Problem 68, a second seed shoots out from the pod with the same speed but with a direction of motion 30.0° below the horizontal. **(a)** How long does it take for the second seed to land? **(b)** What horizontal distance does it cover during its flight?

70. •• A shot-putter throws the shot with an initial speed of 12.2 m/s from a height of 5.15 ft above the ground. What is the range of the shot if the launch angle is **(a)** 20.0°, **(b)** 30.0°, or **(c)** 40.0°?

71. •• **Pararescue Jumpers** Coast Guard pararescue jumpers are trained to leap from helicopters into the sea to save boaters in distress. The rescuers like to step off their helicopter when it is "ten and ten", which means that it is *ten* feet above the water and moving forward horizontally at *ten* knots. What are **(a)** the speed and **(b)** the direction of motion as a pararescuer enters the water following a ten and ten jump?

72. •• A ball thrown straight upward returns to its original level in 2.75 s. A second ball is thrown at an angle of 40.0° above the horizontal. What is the initial speed of the second ball if it also returns to its original level in 2.75 s?

73. •• **IP** To decide who pays for lunch, a passenger on a moving train tosses a coin straight upward with an initial speed of 4.38 m/s and catches it again when it returns to its initial level. From the point of view of the passenger, then, the coin's initial velocity is $(4.38\ \mathrm{m/s})\hat{\mathbf{y}}$. The train's velocity relative to the ground is $(12.1\ \mathrm{m/s})\hat{\mathbf{x}}$. **(a)** What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain. **(b)** Find the initial speed and direction of the coin as seen by an observer on the ground. **(c)** Use the expression for y_{max} derived in Example 4–7 to calculate the maximum height of the coin, as seen by an observer on the ground. **(d)** Calculate the maximum height of the coin from the point of view of the passenger, who sees only one-dimensional motion.

74. •• **IP** A cannon is placed at the bottom of a cliff 61.5 m high. If the cannon is fired straight upward, the cannonball just reaches the top of the cliff. **(a)** What is the initial speed of the cannonball? **(b)** Suppose a second cannon is placed at the top of the cliff. This cannon is fired horizontally, giving its cannonballs the same initial speed found in part (a). Show that the range of this cannon is the same as the maximum range of the cannon at the base of the cliff. (Assume the ground at the base of the cliff is level, though the result is valid even if the ground is not level.)

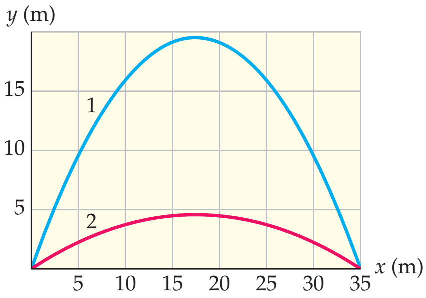
75. •• **Shot Put Record** The men's world record for the shot put, 23.12 m, was set by Randy Barnes of the United States on May 20, 1990. If the shot was launched from 6.00 ft above the ground at an initial angle of 42.0°, what was its initial speed?

76. •• Referring to Conceptual Checkpoint 4–3, suppose the two snowballs are thrown from an elevation of 15 m with an initial speed of 12 m/s. What is the speed of each ball when it is 5.0 m above the ground?

77. •• **IP** A hockey puck just clears the 2.00-m-high boards on its way out of the rink. The base of the boards is 20.2 m from the point where the puck is launched. **(a)** Given the launch angle of the puck, θ, outline a strategy that you can use to find its initial speed, v_0. **(b)** Use your strategy to find v_0 for $\theta = 15.0°$.

78. •• Referring to Active Example 4–2, suppose the ball is punted from an initial height of 0.750 m. What is the initial speed of the ball in this case?

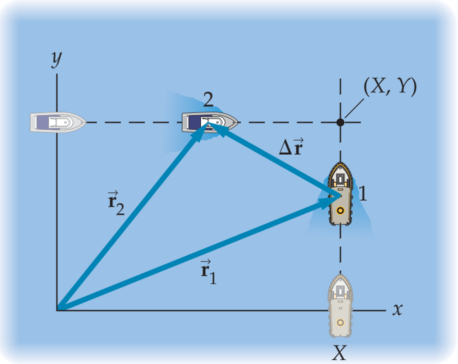
79. •• **A "Lob" Pass Versus a "Bullet"** A quarterback can throw a receiver a high, lazy "lob" pass or a low, quick "bullet" pass. These passes are indicated by curves 1 and 2, respectively, in **Figure 4–22**. **(a)** The lob pass is thrown with an initial speed of 21.5 m/s and its time of flight is 3.97 s. What is its launch angle?



▲ **FIGURE 4–22** Problem 79

(b) The bullet pass is thrown with a launch angle of 25.0°. What is the initial speed of this pass? **(c)** What is the time of flight of the bullet pass?

80\. ••• **Collision Course** A useful rule of thumb in boating is that if the heading from your boat to a second boat remains constant, the two boats are on a collision course. Consider the two boats shown in **Figure 4–23**. At time $t = 0$, boat 1 is at the location $(X, 0)$ and moving in the positive y direction; boat 2 is at $(0, Y)$ and moving in the positive x direction. The speed of boat 1 is v_1. **(a)** What speed must boat 2 have if the boats are to collide at the point (X, Y)? **(b)** Assuming boat 2 has the speed found in part (a), calculate the displacement from boat 1 to boat 2, $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$. **(c)** Use your results from part (b) to show that $(\Delta r)_y/(\Delta r)_x = -Y/X$, independent of time. This shows that $\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1$ maintains a constant direction until the collision, as specified in the rule of thumb.



▲ **FIGURE 4–23** Problem 80

81\. ••• As discussed in Example 4–7, the archerfish hunts by dislodging an unsuspecting insect from its resting place with a stream of water expelled from the fish's mouth. Suppose the archerfish squirts water with a speed of 2.15 m/s at an angle of 52.0° above the horizontal, and aims for a beetle on a leaf 3.00 cm above the water's surface. **(a)** At what horizontal distance from the beetle should the archerfish fire if it is to hit its target in the least time? **(b)** How much time will the beetle have to react?

82\. ••• **(a)** What is the greatest horizontal distance from which the archerfish can hit the beetle, assuming the same squirt speed and direction as in Problem 81? **(b)** How much time does the beetle have to react in this case?

83\. ••• Find the launch angle for which the range and maximum height of a projectile are the same.

84\. ••• A mountain climber jumps a crevasse of width W by leaping horizontally with speed v_0. **(a)** If the height difference between the two sides of the crevasse is h, what is the minimum value of v_0 for the climber to land safely on the other side? **(b)** In this case, what is the climber's direction of motion on landing?

85\. ••• Prove that the landing speed of a projectile is independent of launch angle for a given height of launch.

86\. •• **Maximum Height and Range** Prove that the maximum height of a projectile, H, divided by the range of the projectile, R, satisfies the relation $H/R = \frac{1}{4}\tan\theta$.

87\. •• **Landing on a Different Level** A projectile fired from $y = 0$ with initial speed v_0 and initial angle θ lands on a different level, $y = h$. Show that the time of flight of the projectile is

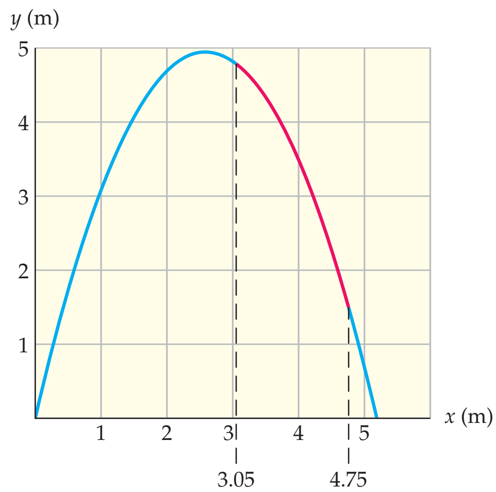
$$T = \tfrac{1}{2}T_0\left(1 + \sqrt{1 - \frac{h}{H}}\right)$$

where T_0 is the time of flight for $h = 0$ and H is the maximum height of the projectile.

88\. ••• A mountain climber jumps a crevasse by leaping horizontally with speed v_0. If the climber's direction of motion on landing is θ below the horizontal, what is the height difference h between the two sides of the crevasse?

89\. •••**IP** Referring to Problem 73, suppose the initial velocity of the coin tossed by the passenger is $\vec{\mathbf{v}} = (-2.25\text{ m/s})\hat{\mathbf{x}} + (4.38\text{ m/s})\hat{\mathbf{y}}$. The train's velocity relative to the ground is still $(12.1\text{ m/s})\hat{\mathbf{x}}$. **(a)** What is the minimum speed of the coin relative to the ground during its flight? At what point in the coin's flight does this minimum speed occur? Explain. **(b)** Find the initial speed and direction of the coin as seen by an observer on the ground. **(c)** Use the expression for y_{max} derived in Example 4–7 to calculate the maximum height of the coin, as seen by an observer on the ground. **(d)** Repeat part (c) from the point of view of the passenger. Verify that both observers calculate the same maximum height.

90\. ••• **Projectiles: Coming or Going?** Most projectiles continually move farther from the origin during their flight, but this is not the case if the launch angle is greater than $\cos^{-1}\left(\frac{1}{3}\right) = 70.5°$. For example, the projectile shown in **Figure 4–24** has a launch angle of 75.0° and an initial speed of 10.1 m/s. During the portion of its motion shown in red, it is moving closer to the origin—it is moving away on the blue portions. Calculate the distance from the origin to the projectile **(a)** at the start of the red portion, **(b)** at the end of the red portion, and **(c)** just before the projectile lands. Notice that the distance for part (b) is the smallest of the three.



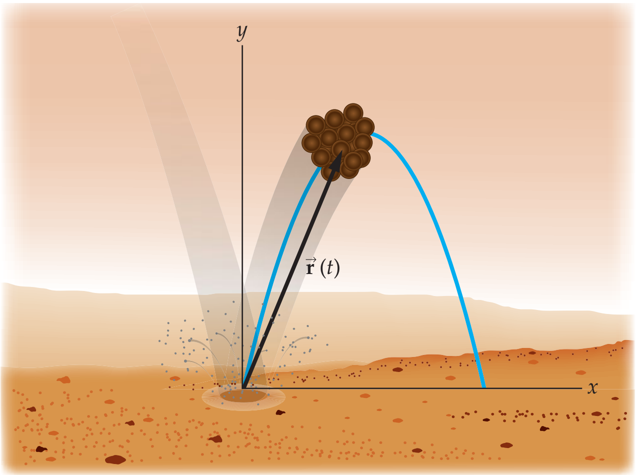
▲ **FIGURE 4–24** Problem 90

PASSAGE PROBLEMS

Landing Rovers on Mars

When the twin Mars exploration rovers, *Spirit* and *Opportunity*, set down on the surface of the red planet in January of 2004, their method of landing was both unique and elaborate. After initial braking with retro rockets, the rovers began their long descent through the thin Martian atmosphere on a parachute until they reached an altitude of about 16.7 m. At that point a system of four air bags with six lobes each were inflated, additional retro rocket blasts brought the craft to a virtual standstill, and the rovers detached from their parachutes. After a period of free fall to the surface, with an acceleration of 3.72 m/s^2, the rovers bounced about a dozen times before coming to rest. They then deflated their air bags, righted themselves, and began to explore the surface.

Figure 4–25 shows a rover with its surrounding cushion of air bags making its first contact with the Martian surface. After a typical first bounce the upward velocity of a rover would be 9.92 m/s at an angle of 75.0° above the horizontal. Assume this is the case for the problems that follow.



▲ **FIGURE 4–25** Problems 91, 92, 93, and 94

91. • What is the maximum height of a rover between its first and second bounces?

A. 2.58 m **B.** 4.68 m

C. 12.3 m **D.** 148 m

92. • How much time elapses between the first and second bounces?

A. 1.38 s **B.** 2.58 s

C. 5.15 s **D.** 5.33 s

93. • How far does a rover travel in the horizontal direction between its first and second bounces?

A. 13.2 m **B.** 49.4 m

C. 51.1 m **D.** 98.7 m

94. •• What is the average velocity of a rover between its first and second bounces?

A. 0

B. 2.57 m/s in the x direction

C. 9.92 m/s at 75.0° above the x axis

D. 9.58 m/s in the y direction

INTERACTIVE PROBLEMS

95. •• **Referring to Example 4–5 (a)** At what launch angle *greater* than 54.0° does the golf ball just barely miss the top of the tree in front of the green? Assume the ball has an initial speed of 13.5 m/s, and that the tree is 3.00 m high and is a horizontal distance of 14.0 m from the launch point. **(b)** Where does the ball land in the case described in part (a)? **(c)** At what launch angle *less* than 54.0° does the golf ball just barely miss the top of the tree in front of the green? **(d)** Where does the ball land in the case described in part (c)?

96. •• **Referring to Example 4–5** Suppose that the golf ball is launched with a speed of 15.0 m/s at an angle of 57.5° above the horizontal, and that it lands on a green 3.50 m above the level where it was struck. **(a)** What horizontal distance does the ball cover during its flight? **(b)** What increase in initial speed would be needed to increase the horizontal distance in part (a) by 7.50 m? Assume everything else remains the same.

97. •• **Referring to Example 4–6** Suppose the ball is dropped at the horizontal distance of 5.50 m, but from a new height of 5.00 m. The dolphin jumps with the same speed of 12.0 m/s. **(a)** What launch angle must the dolphin have if it is to catch the ball? **(b)** At what height does the dolphin catch the ball in this case? **(c)** What is the minimum initial speed the dolphin must have to catch the ball before it hits the water?

98. •• **IP Referring to Example 4–6** Suppose we change the dolphin's launch angle to 45.0°, but everything else remains the same. Thus, the horizontal distance to the ball is 5.50 m, the drop height is 4.10 m, and the dolphin's launch speed is 12.0 m/s. **(a)** What is the vertical distance between the dolphin and the ball when the dolphin reaches the horizontal position of the ball? We refer to this as the "miss distance." **(b)** If the dolphin's launch speed is reduced, will the miss distance increase, decrease, or stay the same? **(c)** Find the miss distance for a launch speed of 10.0 m/s.

5 Newton's Laws of Motion

Bobsledders know that a force is required to accelerate an object. In fact, the greater the force, the greater the acceleration. What they may not realize, however, is that forces always come in pairs that are equal in magnitude but opposite in direction. For example, when these athletes push on the bobsled, it pushes back on them with equal strength. All of these observations follow directly from Newton's three laws of motion, the subject of this chapter.

We are all subject to Newton's laws of motion, whether we know it or not. You can't move your body, drive a car, or toss a ball in a way that violates his rules. In short, our very existence is constrained and regulated by these three fundamental statements concerning matter and its motion.

Yet Newton's laws are surprisingly simple, especially when you consider that they apply equally well to galaxies, planets, comets, and yes, even apples falling from trees. In this chapter we present the three laws of Newton, and we show how they can be applied to everyday situations. Using them, we go beyond a simple description of motion, as in kinematics, to a study of the *causes* of motion, referred to as **dynamics.**

With the advent of Newtonian dynamics in 1687, science finally became quantitative and predictive. Edmund Halley, inspired by Newton's laws, used them to predict the return of the comet that today bears his name. In all of recorded history, no one had ever before predicted the appearance of a comet; in fact, they were generally regarded as supernatural apparitions. Though Halley didn't live to see his comet's return, his correct prediction illustrated the power of Newton's laws in a most dramatic and memorable way.

Today, we still recognize Newton's laws as the indispensable foundation for all of physics. It would be nice to say that these laws are the complete story when it comes to analyzing motion, but that is not the case. In the early part of the last century, physicists discovered that Newton's laws must be modified for objects moving at speeds near that of light and for objects comparable in size to atoms. In the world of everyday experience, however, Newton's laws still reign supreme.

5–1 Force and Mass

A **force,** simply put, is a push or a pull. When you push on a box to slide it across the floor, for example, or pull on the handle of a wagon to give a child a ride, you are exerting a force. Similarly, when you hold this book in your hand, you exert an upward force to oppose the downward pull of gravity. If you set the book on a table, the table exerts the same upward force you exerted a moment before. Forces are truly all around us.

Now, when you push or pull on something, there are two quantities that characterize the force you are exerting. The first is the strength or **magnitude** of your force; the second is the **direction** in which you are pushing or pulling. Because a force is determined by both a magnitude and a direction, it is a vector. We consider the vector properties of forces in more detail in Section 5–5.

In general, an object has several forces acting on it at any given time. In the previous example, a book at rest on a table experiences a downward force due to gravity and an upward force due to the table. If you push the book across the table, it also experiences a horizontal force due to your push. The total, or net, force exerted on the book is the vector sum of the individual forces acting on it.

After the net force acting on an object, the second key ingredient in Newton's laws is the **mass** of an object, which is a measure of how difficult it is to change its velocity—to start an object moving if it is at rest, to bring it to rest if it is moving, or to change its direction of motion. For example, if you throw a baseball or catch one thrown to you, the force required is not too great. But if you want to start a car moving or to stop one that is coming at you, the force involved is much greater. It follows that the mass of a car is greater than the mass of a baseball.

In agreement with everyday usage, mass can also be thought of as a measure of the quantity of matter in an object. Thus, it is clear that the mass of an automobile, for example, is much greater than the mass of a baseball, but much less than the mass of Earth. We measure mass in units of kilograms (kg), where one kilogram is defined as the mass of a standard cylinder of platinum-iridium, as discussed in Chapter 1. A list of typical masses is given in Table 5–1.

TABLE 5–1 Typical Masses in Kilograms (kg)

Earth	5.97×10^{24}
Space shuttle	2,000,000
Blue whale (largest animal on Earth)	178,000
Whale shark (largest fish)	18,000
Elephant (largest land animal)	5400
Automobile	1200
Human (adult)	70
Gallon of milk	3.6
Quart of milk	0.9
Baseball	0.145
Honeybee	0.00015
Bacterium	10^{-15}

These properties of force and mass are developed in detail in the next three sections.

5–2 Newton's First Law of Motion

If you've ever stood in line at an airport, pushing your bags forward a few feet at a time, you know that as soon as you stop pushing the bags, they stop moving. Observations such as this often lead to the erroneous conclusion that a force is required for an object to move. In fact, according to Newton's first law of motion, a force is required only to *change* an object's motion.

What is missing in this analysis is the force of friction between the bags and the floor. When you stop pushing the bags, it is not true that they stop moving because they no longer have a force acting on them. On the contrary, there is a rather large *frictional force* between the bags and the floor. It is this force that causes the bags to come to rest.

To see how motion is affected by reducing friction, imagine that you slide on dirt into second base during a baseball game. You won't slide very far before stopping. On the other hand, if you slide with the same initial speed on a sheet of ice—where the friction is much less than on a ball field—you slide considerably farther. If you could reduce the friction more, you would slide even farther.

In the classroom, air tracks allow us to observe motion with practically no friction. An example of such a device is shown in **Figure 5–1**. Note that air is blown through small holes in the track, creating a cushion of air for a small "cart" to ride on. A cart placed at rest on a level track remains at rest—unless you push on it to get it started.

Once set in motion, the cart glides along with constant velocity—constant speed in a straight line—until it hits a bumper at the end of the track. The bumper

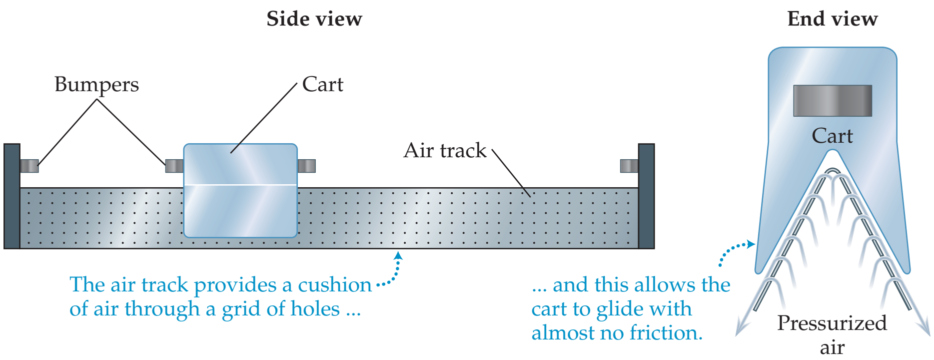


FIGURE 5–1 The air track
An air track provides a cushion of air on which a cart can ride with virtually no friction.

exerts a force on the cart, causing it to change its direction of motion. After bouncing off the bumper, the cart again moves with constant velocity. If the track could be extended to infinite length, and could be made perfectly frictionless, the cart would simply keep moving with constant velocity forever.

Newton's first law of motion summarizes these observations in the following statements:

Newton's First Law

An object at rest remains at rest as long as no net force acts on it.

An object moving with constant velocity continues to move with the same speed and in the same direction as long as no net force acts on it.

Notice the recurring phrase, "no net force," in these statements. It is important to realize that this can mean one of two things: (i) no force acts on the object; or (ii) forces act on the object, but they sum to zero. We shall see examples of the second possibility later in this chapter and again in the next chapter.

Newton's first law, which was first enunciated by Galileo, is also known as the **law of inertia,** which is appropriate since the literal meaning of the word *inertia* is "laziness." Speaking loosely, we can say that matter is "lazy," in that it won't change its motion unless forced to do so. For example, if an object is at rest, it won't start moving on its own. If an object is already moving with constant velocity, it won't alter its speed *or* direction, unless a force causes the change. We call this property of matter its inertia.

According to Newton's first law, being at rest and moving with constant velocity are actually equivalent. To see this, imagine two observers: one is in a train moving with constant velocity; the second is standing next to the tracks, at rest on the ground. The observer in the train places an ice cube on a dinner tray. From that person's point of view—that is, in that person's **frame of reference**—the ice cube has no net force acting on it and it is at rest on the tray. It obeys the first law. In the frame of reference of the observer on the ground, the ice cube has no net force on it and it moves with constant velocity. This also agrees with the first law. Thus Newton's first law holds for both observers: They both see an ice cube with zero net force moving with constant velocity—it's just that for the first observer the constant velocity happens to be zero.

In this example, we say that each observer is in an **inertial frame of reference;** that is, a frame of reference in which the law of inertia holds. In general, if one frame is an inertial frame of reference, then any frame of reference that moves with constant velocity relative to that frame is also an inertial frame of reference. Thus, if an object moves with constant velocity in one inertial frame, it is always possible to find another inertial frame in which the object is at rest. It is in this sense that there really isn't any difference between being at rest and moving with constant velocity. It's all relative—relative to the frame of reference the object is viewed from.

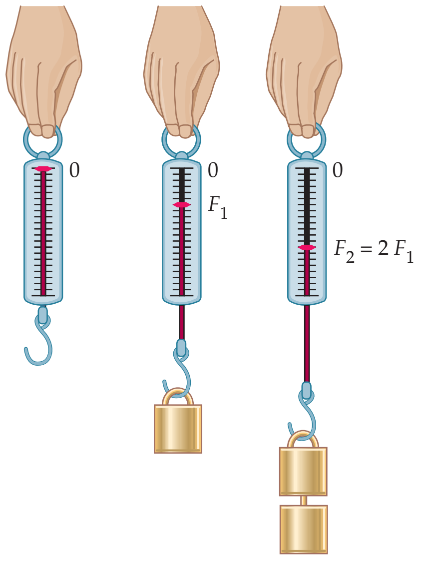
This gives us a more compact statement of the first law:

If the net force on an object is zero, its velocity is constant.

An air track provides a nearly frictionless environment for experiments involving linear motion.

As an example of a frame of reference that is not inertial, imagine that the train carrying the first observer suddenly comes to a halt. From the point of view of that observer, there is still no net force on the ice cube. However, because of the rapid braking, the ice cube flies off the tray. In fact, the ice cube simply continues to move forward with the same constant velocity while the *train* comes to rest. To the observer on the train, it appears that the ice cube has accelerated forward, even though no force acts on it, which is in violation of Newton's first law.

In general, any frame that accelerates relative to an inertial frame is a noninertial frame. The surface of the Earth accelerates slightly, due to its rotational and orbital motions, but since the acceleration is so small, it may be considered an excellent approximation to an inertial frame of reference. Unless specifically stated otherwise, we will always consider the surface of the Earth to be an inertial frame.



▲ **FIGURE 5–2 Calibrating a "force meter"** With two weights, the force exerted by the scale is twice the force exerted when only a single weight is attached.

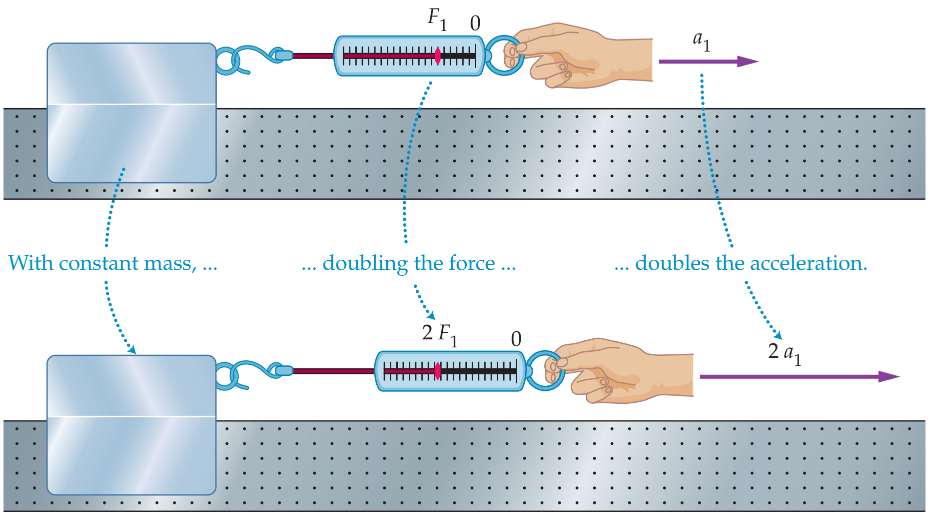
5–3 Newton's Second Law of Motion

To hold an object in your hand, you have to exert an upward force to oppose, or "balance," the force of gravity. If you suddenly remove your hand so that the only force acting on the object is gravity, it accelerates downward, as discussed in Chapter 2. This is one example of Newton's second law, which states, basically, that unbalanced forces cause accelerations.

To explore this in more detail, consider a spring scale of the type used to weigh fish. The scale gives a reading of the force, F, exerted by the spring contained within it. If we hang one weight from the scale, it gives a reading that we will call F_1. If two identical weights are attached, the scale reads $F_2 = 2F_1$, as indicated in **Figure 5–2**. With these two forces marked on the scale, we are ready to perform some force experiments.

First, attach the scale to an air-track cart, as in **Figure 5–3**. If we pull with a force F_1, we observe that the cart accelerates at the rate a_1. If we now pull with a force $F_2 = 2F_1$, the acceleration we observe is $a_2 = 2a_1$. Thus, the acceleration is proportional to the force—the greater the force, the greater the acceleration.

▶ **FIGURE 5–3 Acceleration is proportional to force** The spring calibrated in Figure 5–2 is used to accelerate a mass on a "frictionless" air track. If the force is doubled, the acceleration is also doubled.



Second, instead of doubling the force, let's double the mass of the cart by connecting two together, as in **Figure 5–4**. In this case, if we pull with a force F_1 we find an acceleration equal to $\frac{1}{2}a_1$. Thus, the acceleration is inversely proportional to mass—the greater the mass, the less the acceleration.

Combining these results, we find that in this simple case—with just one force in just one direction—the acceleration is given by

$$a = \frac{F}{m}$$

Rearranging the equation yields the form of Newton's law that is perhaps best known, $F = ma$.

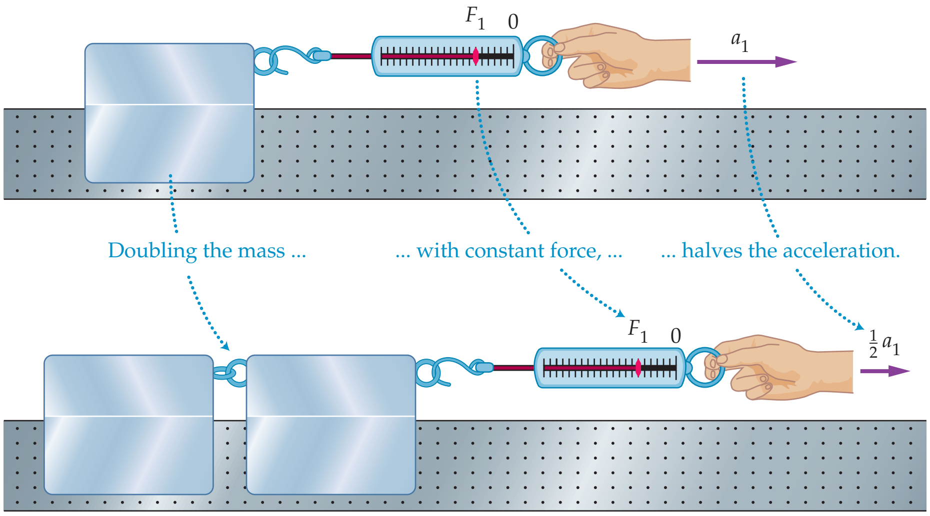


FIGURE 5–4 Acceleration is inversely proportional to mass
If the mass of an object is doubled but the force remains the same, the acceleration is halved.

In general, there may be several forces acting on a given mass, and these forces may be in different directions. Thus, we replace F with the sum of the force vectors acting on a mass:

$$\text{sum of force vectors} = \vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}}$$

The notation, $\Sigma\vec{\mathbf{F}}$, which uses the Greek letter sigma (Σ), is read "sum $\vec{\mathbf{F}}$." Recalling that acceleration is also a vector, we arrive at the formal statement of Newton's second law of motion:

Newton's Second Law

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} \quad \text{or} \quad \sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \tag{5–1}$$

In words:

If an object of mass m is acted on by a net force $\Sigma\vec{\mathbf{F}}$, it will experience an acceleration $\vec{\mathbf{a}}$ that is equal to the net force divided by the mass. Because the net force is a vector, the acceleration is also a vector. In fact, the direction of an object's acceleration is the *same* as the direction of the net force acting on it.

One should note that Newton's laws cannot be derived from anything more basic. In fact, this is what we mean by a law of nature. The validity of Newton's laws, and all other laws of nature, comes directly from comparisons with experiment.

In terms of vector components, an equivalent statement of the second law is:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \tag{5–2}$$

Note that Newton's second law holds independently for each coordinate direction. This component form of the second law is particularly useful when solving problems.

Let's pause for a moment to consider an important special case of the second law. Suppose an object has zero net force acting upon it. This may be because no forces act on it at all, or because it is acted on by forces whose vector sum is zero. In either case, we can state this mathematically as:

$$\sum \vec{\mathbf{F}} = 0$$

Now, according to Newton's second law, we conclude that the acceleration of this object must be zero:

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{0}{m} = 0$$

But if an object's acceleration is zero, its velocity must be constant. In other words, if the net force on an object is zero, the object moves with constant velocity. This is

▲ Even though the tugboat exerts a large force on this ship, the ship's acceleration is small. This is because the acceleration of an object is inversely proportional to its mass, and the mass of the ship is enormous. The force exerted on the unfortunate hockey player is much smaller. The resulting acceleration is much larger, however, due to the relatively small mass of the player compared to that of the ship.

Newton's first law. Thus we see that Newton's first and second laws are consistent with one another.

Forces are measured in units called, appropriately enough, the **newton (N).** In particular, one newton is defined as the force required to give one kilogram of mass an acceleration of 1 m/s^2. Thus,

$$1\ \text{N} = (1\ \text{kg})(1\ \text{m/s}^2) = 1\ \text{kg}\cdot\text{m/s}^2 \qquad 5\text{–}3$$

In everyday terms, a newton is roughly a quarter of a pound. Note that a force in newtons divided by a mass in kilograms has the units of acceleration:

$$\frac{1\ \text{N}}{1\ \text{kg}} = \frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{kg}} = 1\ \text{m/s}^2 \qquad 5\text{–}4$$

Other common units for force are presented in Table 5–2. Typical forces and their magnitudes in newtons are listed in Table 5–3.

TABLE 5–2 Units of Mass, Acceleration, and Force

System of units	Mass	Acceleration	Force
SI	kilogram (kg)	m/s^2	newton (N)
cgs	gram (g)	cm/s^2	dyne (dyn)
British	slug	ft/s^2	pound (lb)

(*Note:* 1 N $= 10^5$ dyne $= 0.225$ lb.)

TABLE 5–3 Typical Forces in Newtons (N)

Main engines of space shuttle	31,000,000
Pulling force of locomotive	250,000
Thrust of jet engine	75,000
Force to accelerate a car	7000
Weight of adult human	700
Weight of an apple	1
Weight of a rose	0.1
Weight of an ant	0.001

EXERCISE 5–1

The net force acting on a Jaguar XK8 has a magnitude of 6800 N. If the car's acceleration is 3.8 m/s^2, what is its mass?

SOLUTION

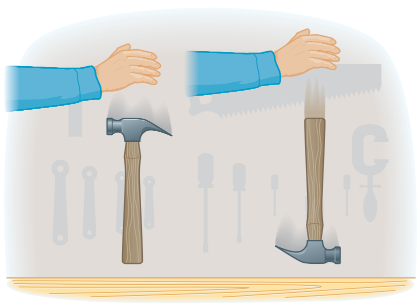
Since the net force and the acceleration are always in the same direction, we can replace the vectors in Equation 5–1 with magnitudes. Solving $\Sigma F = ma$ for the mass yields

$$m = \frac{\sum F}{a} = \frac{6800\ \text{N}}{3.8\ \text{m/s}^2} = 1800\ \text{kg}$$

The following Conceptual Checkpoint presents a situation in which both Newton's first and second laws play an important role.

CONCEPTUAL CHECKPOINT 5–1 TIGHTENING A HAMMER

The metal head of a hammer is loose. To tighten it, you drop the hammer down onto a table. Should you **(a)** drop the hammer with the handle end down, **(b)** drop the hammer with the head end down, or **(c)** do you get the same result either way?



REASONING AND DISCUSSION

It might seem that since the same hammer hits against the same table in either case, there shouldn't be a difference. Actually, there is.

In case (a) the handle of the hammer comes to rest when it hits the table, but the head continues downward until a force acts on it to bring it to rest. The force that acts on it is supplied by the handle, which results in the head being wedged more tightly onto the handle. Since the metal head is heavy, the force wedging it onto the handle is great. In case (b) the head of the hammer comes to rest, but the handle continues to move until a force brings it to rest. The handle is lighter than the head, however; thus the force acting on it is less, resulting in less tightening.

ANSWER

(a) Drop the hammer with the handle end down.

A similar effect occurs when you walk—with each step you take you tamp your head down onto your spine, as when dropping a hammer handle end down.

This causes you to grow shorter during the day! Try it. Measure your height first thing in the morning, then again before going to bed. If you're like many people, you'll find that you have shrunk by an inch or so during the day.

REAL-WORLD PHYSICS: BIO

How walking affects your height

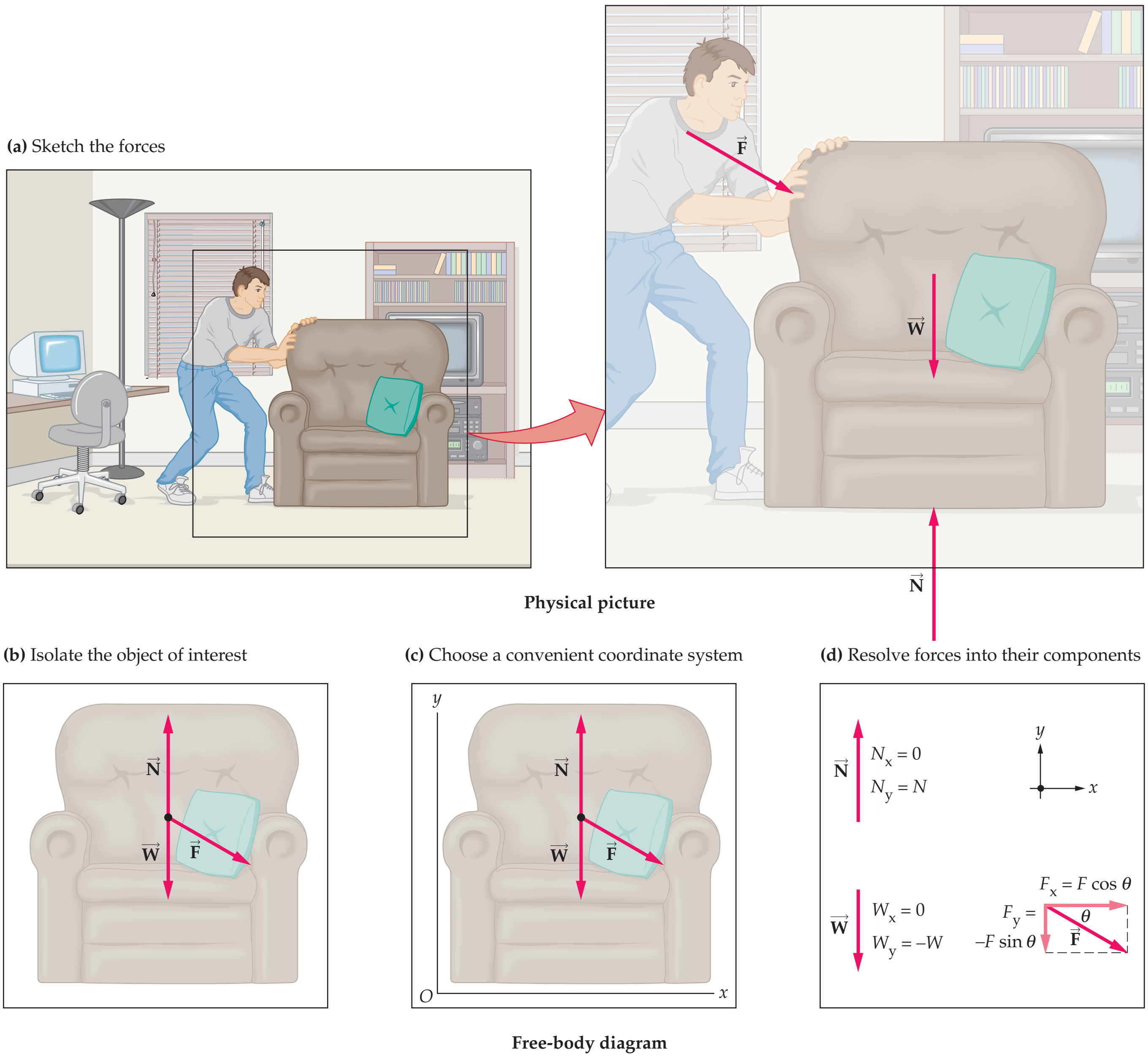
Free-Body Diagrams

When solving problems involving forces and Newton's laws, it is essential to begin by making a sketch that indicates *each and every external force* acting on a given object. This type of sketch is referred to as a **free-body diagram.** If we are concerned only with nonrotational motion, as is the case in this and the next chapter, we treat the object of interest as a point particle and apply each of the forces acting on the object to that point, as **Figure 5–5** shows. Once the forces are drawn, we choose a coordinate system and resolve each force into components. At this point, Newton's second law can be applied to each coordinate direction separately.

PROBLEM-SOLVING NOTE

External Forces

External forces acting on an object fall into two main classes: (i) Forces at the point of contact with another object, and (ii) forces exerted by an external agent, such as gravity.



▲ **FIGURE 5–5 Constructing and using a free-body diagram**

The four basic steps in constructing and using a free-body diagram are illustrated in these sketches. **(a)** Sketch all of the external forces acting on an object of interest. Note that only forces acting *on* the object are shown; none of the forces exerted *by* the object are included. **(b)** Isolate the object and treat it as a point particle. **(c)** Choose a convenient coordinate system. This will often mean aligning a coordinate axis to coincide with the direction of one or more forces in the system. **(d)** Resolve each of the forces into components using the coordinate system of part (c).

For example, in Figure 5–5 there are three external forces acting on the chair. One is the force $\vec{\mathbf{F}}$ exerted by the person. In addition, gravity exerts a downward force, $\vec{\mathbf{W}}$, which is simply the weight of the chair. Finally, the floor exerts an upward force on the chair that prevents it from falling toward the center of the Earth. This force is referred to as the *normal force*, $\vec{\mathbf{N}}$, because it is perpendicular (that is, normal) to the surface of the floor. We will consider the weight and the normal force in greater detail in Sections 5–6 and 5–7, respectively.

We can summarize the steps involved in constructing a free-body diagram as follows:

Sketch the Forces
Identify and sketch all of the external forces acting on an object. Sketching the forces roughly to scale will help in estimating the direction and magnitude of the net force.

Isolate the Object of Interest
Replace the object with a point particle of the same mass. Apply each of the forces acting on the object to that point.

Choose a Convenient Coordinate System
Any coordinate system will work; however, if the object moves in a known direction, it is often convenient to pick that direction for one of the coordinate axes. Otherwise, it is reasonable to choose a coordinate system that aligns with one or more of the forces acting on the object.

Resolve the Forces into Components
Determine the components of each force in the free-body diagram.

Apply Newton's Second Law to Each Coordinate Direction
Analyze motion in each coordinate direction using the component form of Newton's second law, as given in Equation 5–2.

PROBLEM-SOLVING NOTE

Picture the Problem

In problems involving Newton's laws, it is important to begin with a free-body diagram and to identify all the external forces that act on an object. Once these forces are identified and resolved into their components, Newton's laws can be applied in a straightforward way. It is crucial, however, that only external forces acting on the object be included, and that none of the external forces be omitted.

These basic steps are illustrated in Figure 5–5. Note that the figures in this chapter use the labels "Physical picture" to indicate a sketch of the physical situation and "Free-body diagram" to indicate a free-body sketch.

We start by applying this procedure to a simple one-dimensional example, saving two-dimensional systems for Section 5–5. Suppose, for instance, that you hold a book at rest in your hand. What is the magnitude of the upward force that your hand must exert to keep the book at rest? From everyday experience, we expect that the upward force must be equal in magnitude to the weight of the book, but let's see how this result can be obtained directly from Newton's second law.

We begin with a sketch of the physical situation, as shown in **Figure 5–6 (a)**. The corresponding free-body diagram, in **Figure 5–6 (b)**, shows just the book, represented by a point, and the forces acting on it. Note that two forces act on the book: (i) the downward force of gravity, $\vec{\mathbf{W}}$, and (ii) the upward force, $\vec{\mathbf{F}}$, exerted by your hand. Only the forces acting *on* the book are included in the free-body diagram.

Now that the free-body diagram is drawn, we indicate a coordinate system so that the forces can be resolved into components. In this case all the forces are vertical. Thus we draw a y axis in the vertical direction in Figure 5–6 (b). Note that we have chosen upward to be the positive direction. With this choice, the y components of the forces are $F_y = F$ and $W_y = -W$. It follows that

$$\sum F_y = F - W$$

Using the y component of the second law ($\Sigma F_y = ma_y$) we find

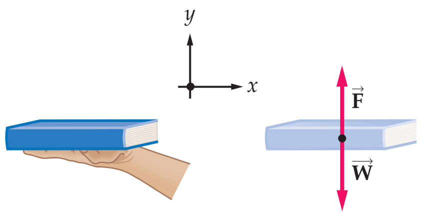
$$F - W = ma_y$$

Since the book remains at rest, its acceleration is zero. Thus, $a_y = 0$, which gives

$$F - W = ma_y = 0 \qquad \text{or} \qquad F = W$$

as expected.

Next, we consider a situation where the net force acting on an object is nonzero, meaning that its acceleration is also nonzero.



(a) Physical picture (b) Free-body diagram

▲ **FIGURE 5–6 A book supported in a person's hand**

(a) The physical situation. **(b)** The free-body diagram for the book, showing the two external forces acting on it. We also indicate our choice for a coordinate system.

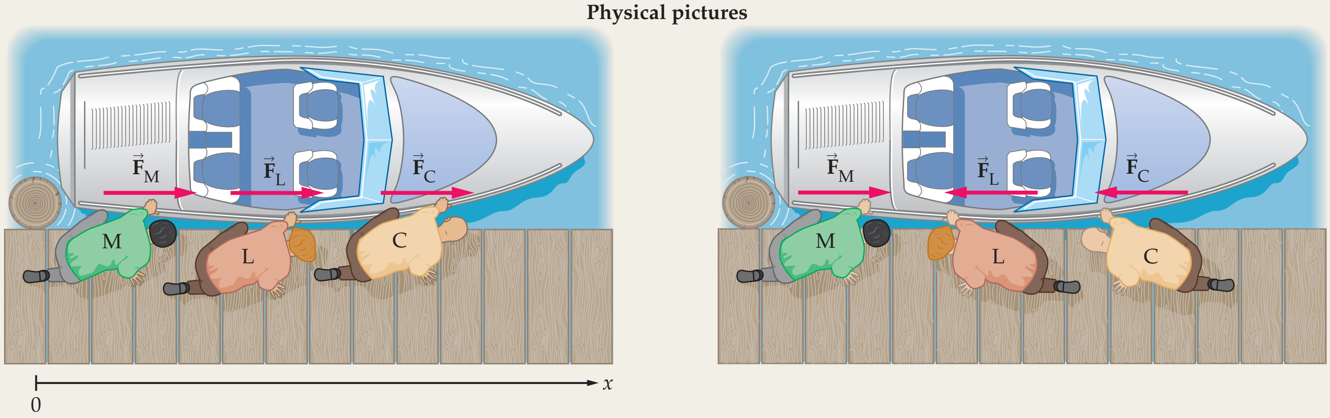
EXAMPLE 5–1 THREE FORCES

Moe, Larry, and Curly push on a 752-kg boat that floats next to a dock. They each exert an 80.5-N force parallel to the dock. **(a)** What is the acceleration of the boat if they all push in the same direction? Give both direction and magnitude. **(b)** What are the magnitude and direction of the boat's acceleration if Larry and Curly push in the opposite direction to Moe's push?

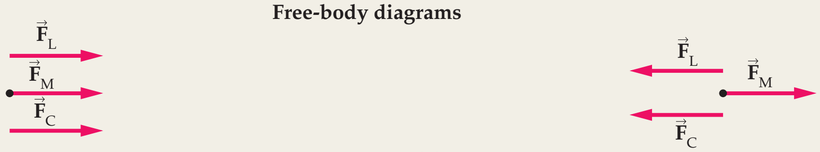
PICTURE THE PROBLEM

In our sketch we indicate the three relevant forces acting on the boat: $\vec{\mathbf{F}}_M$, $\vec{\mathbf{F}}_L$, and $\vec{\mathbf{F}}_C$. Note that we have chosen the positive x direction to the right, in the direction that all three push for part (a). Therefore, all three forces have a positive x component in part (a). In part (b), however, the forces exerted by Larry and Curly have negative x components.

Physical pictures



Free-body diagrams



(a) (b)

STRATEGY

Since we know the mass of the boat and the forces acting on it, we can find the acceleration using $\sum F_x = ma_x$. Even though this problem is one-dimensional, it is important to think of it in terms of vector components. For example, when we sum the x components of the forces, we are careful to use the appropriate signs—just as we always do when dealing with vectors.

SOLUTION

Part (a)

1. Write out the x component for each of the three forces: $F_{M,x} = F_{L,x} = F_{C,x} = 80.5\text{ N}$

2. Sum the x components of force and set equal to ma_x: $\sum F_x = F_{M,x} + F_{L,x} + F_{C,x} = 241.5\text{ N} = ma_x$

3. Divide by the mass to find a_x. Since a_x is positive, the acceleration is to the right, as expected:

$$a_x = \frac{\sum F_x}{m} = \frac{241.5\text{ N}}{752\text{ kg}} = 0.321\text{ m/s}^2$$

Part (b)

4. Again, start by writing the x component for each force:

$$F_{M,x} = 80.5\text{ N}$$
$$F_{L,x} = F_{C,x} = -80.5\text{ N}$$

5. Sum the x components of force and set equal to ma_x:

$$\sum F_x = F_{M,x} + F_{L,x} + F_{C,x}$$
$$= 80.5\text{ N} - 80.5\text{ N} - 80.5\text{ N} = -80.5\text{ N} = ma_x$$

6. Solve for a_x. In this case a_x is negative, indicating an acceleration to the left:

$$a_x = \frac{\sum F_x}{m} = \frac{-80.5\text{ N}}{752\text{ kg}} = -0.107\text{ m/s}^2$$

INSIGHT

The results of this Example are in agreement with everyday experience: three forces in the same direction cause more acceleration than three forces in opposing directions. The method of using vector components and being careful about their signs gives the expected results in a simple situation like this, and also works in more complicated situations where everyday experience may be of little help.

PRACTICE PROBLEM

If Moe, Larry, and Curly all push to the right with 85.0-N forces, and the boat accelerates at 0.530 m/s^2, what is its mass? **[Answer:** 481 kg]

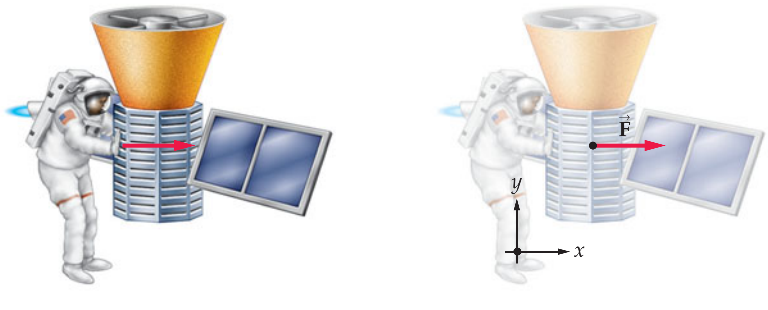
Some related homework problems: Problem 2, Problem 4

In some problems, we are given information that allows us to calculate an object's acceleration using the kinematic equations of Chapters 2 and 4. Once the acceleration is known, the second law can be used to find the net force that caused the acceleration.

REAL-WORLD PHYSICS
Astronaut jet packs

For example, suppose that an astronaut uses a jet pack to push a satellite toward the space shuttle. These jet packs, which are known to NASA as Manned Maneuvering Units, or MMUs, are basically small "one-person rockets" strapped to the back of an astronaut's spacesuit. An MMU contains pressurized nitrogen gas that can be released through varying combinations of 24 nozzles spaced around the unit, producing a force of about 10 pounds. The MMUs contain enough propellant for a six-hour EVA (extra-vehicular activity).

We show the physical situation in **Figure 5–7 (a)**, where an astronaut pushes on a 655-kg satellite. The corresponding free-body diagram for the satellite is shown in **Figure 5–7 (b)**. Note that we have chosen the x axis to point in the direction of the push. Now, if the satellite starts at rest and moves 0.675 m after 5.00 seconds of pushing, what is the force, F, exerted on it by the astronaut?



(a) Physical picture **(b) Free-body diagram**

FIGURE 5–7 An astronaut using a jet pack to push a satellite
(a) The physical situation. **(b)** The free-body diagram for the satellite. Only one force acts on the satellite, and it is in the positive x direction.

Clearly, we would like to use Newton's second law (basically, $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$) to find the force, but we know only the mass of the satellite, not its acceleration. We can find the acceleration, however, by assuming constant acceleration (after all, the force is constant) and using the kinematic equation relating position to time: $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$. We can choose the initial position of the satellite to be $x_0 = 0$, and we are given that it starts at rest, thus $v_{0x} = 0$. Hence,

$$x = \frac{1}{2}a_x t^2$$

Since we know the distance covered in a given time, we can solve for the acceleration:

$$a_x = \frac{2x}{t^2} = \frac{2(0.675\ \text{m})}{(5.00\ \text{s})^2} = 0.0540\ \text{m/s}^2$$

Now that kinematics has provided the acceleration, we use the x component of the second law to find the force. Only one force acts on the satellite, and its x component is F; thus,

$$\sum F_x = F = ma_x$$

$$F = ma_x = (655\ \text{kg})(0.0540\ \text{m/s}^2) = 35.4\ \text{N}$$

This force corresponds to a push of about 8 lb.

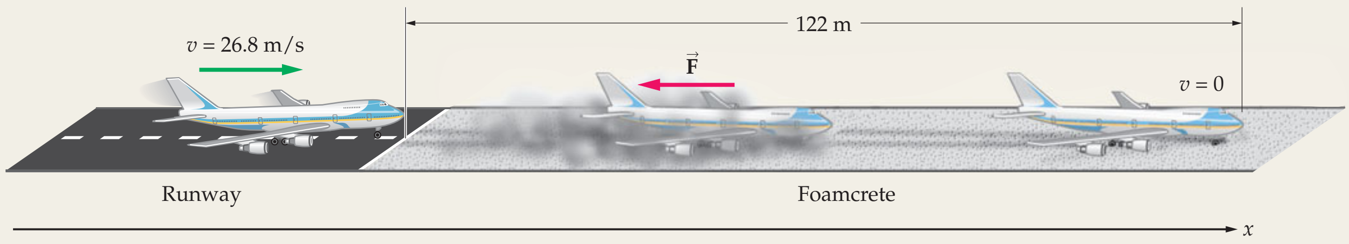
Another problem in which we use kinematics to find the acceleration is presented in the following Active Example.

▲ A technician inspects the landing gear of an airliner in a test of Foamcrete, a solid paving material that is just soft enough to collapse under the weight of an airliner. A plane that has run off the runway will slow safely to a stop as its wheels plow through the crumbling Foamcrete.

ACTIVE EXAMPLE 5–1 THE FORCE EXERTED BY FOAMCRETE

REAL-WORLD PHYSICS

Foamcrete is a substance designed to stop an airplane that has run off the end of a runway, without causing injury to passengers. It is solid enough to support a car, but crumbles under the weight of a large airplane. By crumbling, it slows the plane to a safe stop. For example, suppose a 747 jetliner with a mass of 1.75×10^5 kg and an initial speed of 26.8 m/s is slowed to a stop in 122 m. What is the magnitude of the average retarding force $\vec{\mathbf{F}}$ exerted by the Foamcrete on the plane?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use $v^2 = v_0{}^2 + 2a_x\,\Delta x$ to find the plane's average acceleration: $a_x = -2.94\ \mathrm{m/s^2}$
2. Sum the forces in the x direction. Let F represent the magnitude of the force $\vec{\mathbf{F}}$: $\sum F_x = -F$
3. Set the sum of forces equal to mass times acceleration: $-F = ma_x$
4. Solve for the magnitude of the average force, F: $F = -ma_x = 5.15 \times 10^5\ \mathrm{N}$

INSIGHT

Though the plane moves in the positive direction, its acceleration, and the net force exerted on it, are in the negative direction. As a result, the plane's speed decreases with time.

YOUR TURN

Find the plane's stopping distance if the magnitude of the average force exerted by the Foamcrete is doubled.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note again the care we take with the signs. The plane's acceleration is negative, hence the net force acting on it, $\vec{\mathbf{F}}$, is in the negative x direction. On the other hand, the magnitude of the force, F, is positive, as is always the case for magnitudes.

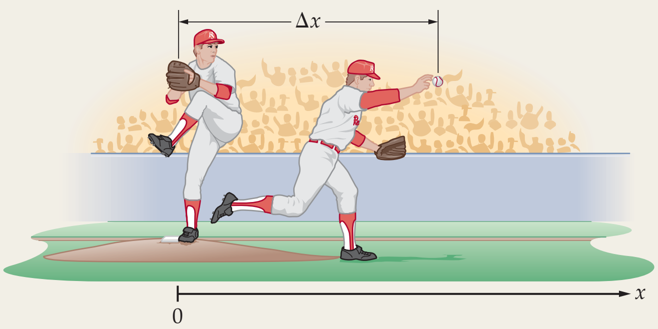
Finally, we end this section with an estimation problem.

EXAMPLE 5–2 PITCH MAN: ESTIMATE THE FORCE ON THE BALL

A pitcher throws a 0.15-kg baseball, accelerating it from rest to a speed of about 90 mi/h. Estimate the force exerted by the pitcher on the ball.

PICTURE THE PROBLEM

We choose the x axis to point in the direction of the pitch. Also indicated in the sketch is the distance over which the pitcher accelerates the ball, Δx. Since we are interested only in the pitch, and not in the subsequent motion of the ball, we ignore the effects of gravity.



STRATEGY

We know the mass, so we can find the force with $F_x = ma_x$ if we can estimate the acceleration. To find the acceleration, we start with the fact that $v_0 = 0$ and $v \approx 90$ mi/h. In addition, we can see from the sketch that a reasonable estimate for Δx is about 2.0 m. Combining these results with the kinematic equation $v^2 = v_0{}^2 + 2a_x\Delta x$ yields the acceleration, which we then use to find the force.

SOLUTION

1. Starting with the fact that 60 mi/h = 1 mi/min, perform a rough back-of-the-envelope conversion of 90 mi/h to meters per second: $v \approx 90\ \mathrm{mi/h} = \dfrac{1.5\ \mathrm{mi}}{\mathrm{min}} \approx \dfrac{2400\ \mathrm{m}}{60\ \mathrm{s}} = 40\ \mathrm{m/s}$
2. Solve $v^2 = v_0{}^2 + 2a_x\,\Delta x$ for the acceleration, a_x. Use the estimates $\Delta x \approx 2.0$ m and $v \approx 40$ m/s: $a_x = \dfrac{v^2 - v_0{}^2}{2\,\Delta x} \approx \dfrac{(40\ \mathrm{m/s})^2 - 0}{2(2.0\ \mathrm{m})} = 400\ \mathrm{m/s^2}$
3. Find the corresponding force with $F_x = ma_x$: $F_x = ma_x \approx (0.15\ \mathrm{kg})(400\ \mathrm{m/s^2}) = 60\ \mathrm{N} \approx 10\ \mathrm{lb}$

INSIGHT

On the one hand, this is a sizable force, especially when you consider that the ball itself weighs only about 1/3 lb. Thus, the pitcher exerts a force on the ball that is about 30 times greater than the force exerted by Earth's gravity. It follows that ignoring gravity during the pitch is a reasonable approximation.

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On the other hand, you might say that 10 lb isn't that much force for a person to exert. That's true, but this force is being exerted with an average speed of about 20 m/s, which means that the pitcher is actually generating about 1.5 horsepower—a sizable power output for a person. We will cover power in detail in Chapter 7, and relate it to human capabilities.

PRACTICE PROBLEM

What is the approximate speed of the pitch if the force exerted by the pitcher is $\frac{1}{2}(60\ \mathrm{N}) = 30\ \mathrm{N}$? **[Answer:** 30 m/s or 60 mi/h**]**

Some related homework problems: Problem 5, Problem 8

Another way to find the acceleration is to estimate the amount of time it takes to make the pitch. However, since the pitch is delivered so quickly—about 1/10 s—estimating the time would be more difficult than estimating the distance Δx.

5–4 Newton's Third Law of Motion

Nature never produces just one force at a time; *forces always come in pairs*. In addition, the forces in a pair, which always act on *different objects*, are equal in magnitude and opposite in direction. This is Newton's third law of motion.

Newton's Third Law

For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.

In a somewhat more specific form:

If object 1 exerts a force $\vec{\mathbf{F}}$ on object 2, then object 2 exerts a force $-\vec{\mathbf{F}}$ on object 1.

This law, more commonly known by its abbreviated form, "for every action there is an equal and opposite reaction," completes Newton's laws of motion.

Figure 5–8 illustrates some action-reaction pairs. Notice that there is always a reaction force, whether the action force pushes on something hard to move, like a refrigerator, or on something that moves with no friction, like an air-track cart. In some cases, the reaction force tends to be overlooked, as when the Earth exerts a *downward* gravitational force on the space shuttle, and the shuttle exerts an equal and opposite *upward* gravitational force on the Earth. Still, the reaction force always exists.

Another important aspect of the third law is that the action-reaction forces always act on *different* objects. This, again, is illustrated in Figure 5–8. Thus, in drawing a free-body diagram, only one of the action-reaction pair of forces would be drawn for a given object. The other force in the pair would appear in the free-body diagram of a different object. As a result, *the two forces do not cancel*.

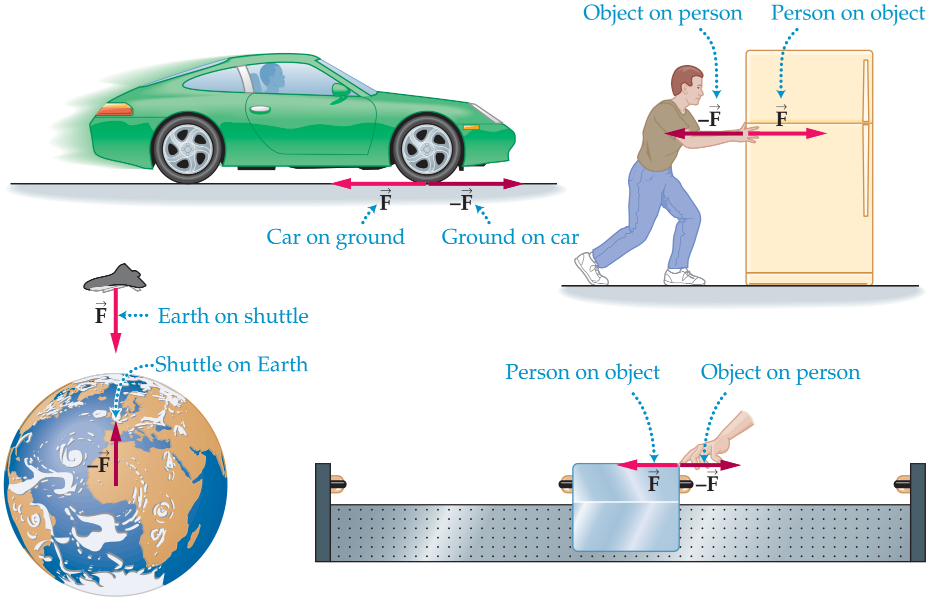


FIGURE 5–8 Examples of action-reaction force pairs

For example, consider a car accelerating from rest, as in Figure 5–8. As the car's engine turns the wheels, the tires exert a force on the road. By the third law, the road exerts an equal and opposite force on the car's tires. It is this second force—which acts on the car through its tires—that propels the car forward. The force exerted by the tires on the road does not accelerate the car.

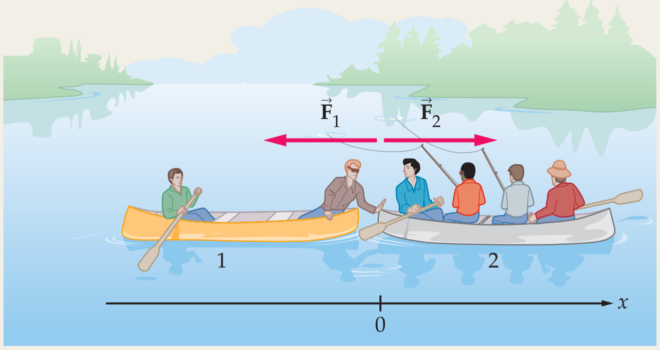
Since the action-reaction forces act on different objects, they generally produce different accelerations. This is the case in the next Example.

EXAMPLE 5–3 TIPPY CANOE

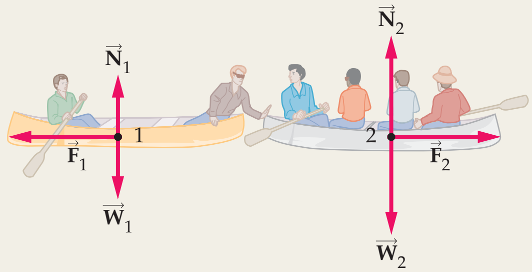
Two groups of canoeists meet in the middle of a lake. After a brief visit, a person in canoe 1 pushes on canoe 2 with a force of 46 N to separate the canoes. If the mass of canoe 1 and its occupants is $m_1 = 150$ kg, and the mass of canoe 2 and its occupants is $m_2 = 250$ kg, **(a)** find the acceleration the push gives to each canoe. **(b)** What is the separation of the canoes after 1.2 s of pushing?

PICTURE THE PROBLEM

We have chosen the positive x direction to point from canoe 1 to canoe 2. With this choice, the force exerted on canoe 2 is $\vec{\mathbf{F}}_2 = (+46\ \mathrm{N})\hat{\mathbf{x}}$. By Newton's third law, the force exerted on the person in canoe 1, and thus on canoe 1 itself if the person is firmly seated, is $\vec{\mathbf{F}}_1 = (-46\ \mathrm{N})\hat{\mathbf{x}}$. For convenience, we have placed the origin at the point where the canoes touch.



Physical picture



Free-body diagrams

STRATEGY

From Newton's third law, the force on canoe 1 is equal in magnitude to the force on canoe 2—the masses of the canoes are different, however, and therefore their accelerations are different as well. **(a)** We can find the acceleration of each canoe by solving $\sum F_x = ma_x$ for a_x. **(b)** The kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, can then be used to find the displacement of each canoe.

SOLUTION

Part (a)

1. Use Newton's second law to find the acceleration of canoe 2:
$$a_{2,x} = \frac{\sum F_{2,x}}{m_2} = \frac{46\ \mathrm{N}}{250\ \mathrm{kg}} = 0.18\ \mathrm{m/s^2}$$

2. Do the same calculation for canoe 1. Note that the acceleration of canoe 1 is in the negative direction:
$$a_{1,x} = \frac{\sum F_{1,x}}{m_1} = \frac{-46\ \mathrm{N}}{150\ \mathrm{kg}} = -0.31\ \mathrm{m/s^2}$$

Part (b)

3. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the position of canoe 2 at $t = 1.2$ s. From the problem statement, we know the canoes start at the origin $(x_0 = 0)$ and at rest $(v_{0x} = 0)$:
$$x_2 = \tfrac{1}{2}a_{2,x}t^2 = \tfrac{1}{2}(0.18\ \mathrm{m/s^2})(1.2\ \mathrm{s})^2 = 0.13\ \mathrm{m}$$

4. Repeat the calculation for canoe 1:
$$x_1 = \tfrac{1}{2}a_{1,x}t^2 = \tfrac{1}{2}(-0.31\ \mathrm{m/s^2})(1.2\ \mathrm{s})^2 = -0.22\ \mathrm{m}$$

5. Subtract the two positions to find the separation of the canoes:
$$x_2 - x_1 = 0.13\ \mathrm{m} - (-0.22\ \mathrm{m}) = 0.35\ \mathrm{m}$$

INSIGHT

The same magnitude of force acts on each canoe; hence the lighter one has the greater acceleration and the greater displacement. If the heavier canoe were replaced by a large ship of great mass, both vessels would still accelerate as a result of the push. However, the acceleration of the large ship would be so small as to be practically imperceptible. In this case, it would appear as if only the canoe moved, whereas, in fact, both vessels move.

PRACTICE PROBLEM

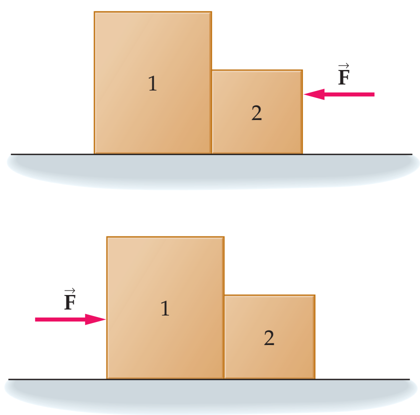
If the mass of canoe 2 is increased, does its acceleration increase, decrease, or stay the same? Check your answer by calculating the acceleration for the case where canoe 2 is replaced by a 25,000-kg ship. [**Answer:** The acceleration will decrease. In this case, $a = 0.0018\ \mathrm{m/s^2}$.]

Some related homework problems: Problem 18, Problem 19

When objects are touching one another, the action-reaction forces are often referred to as **contact forces.** The behavior of contact forces is explored in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 5–2 CONTACT FORCES

Two boxes—one large and heavy, the other small and light—rest on a smooth, level floor. You push with a force $\vec{\mathbf{F}}$ on either the small box or the large box. Is the contact force between the two boxes **(a)** the same in either case, **(b)** larger when you push on the large box, or **(c)** larger when you push on the small box?



REASONING AND DISCUSSION

Since the same force pushes on the boxes, you might think the force of contact is the same in both cases. It is not. What we can conclude, however, is that the boxes have the same acceleration in either case—the same net force acts on the same total mass, so the same acceleration, a, results.

To find the contact force between the boxes, we focus our attention on each box individually, and note that *Newton's second law must be satisfied for each of the boxes, just as it is for the entire two-box system*. For example, when the external force is applied to the small box, the only force acting on the *large* box (mass m_1) is the contact force; hence, the contact force must have a magnitude equal to m_1a. In the second case, the only force acting on the *small* box (mass m_2) is the contact force, and so the magnitude of the contact force is m_2a. Since m_1 is greater than m_2, it follows that the force of contact is larger when you push on the small box, m_1a, than when you push on the large box, m_2a.

To summarize, the contact force is larger when *it* must push the larger box.

ANSWER

(c) The contact force is larger when you push on the small box.

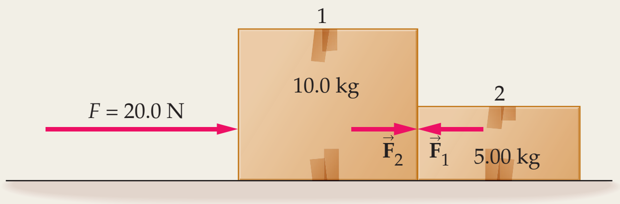
In the next Example, we calculate a numerical value for the contact force in a system similar to that described in Conceptual Checkpoint 5–2. We also show explicitly that Newton's third law is required for a full analysis of this system.

EXAMPLE 5–4 WHEN PUSH COMES TO SHOVE

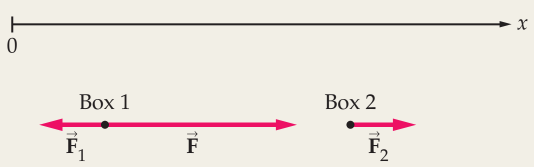
A box of mass $m_1 = 10.0$ kg rests on a smooth, horizontal floor next to a box of mass $m_2 = 5.00$ kg. If you push on box 1 with a horizontal force of magnitude $F = 20.0$ N, **(a)** what is the acceleration of the boxes? **(b)** What is the force of contact between the boxes?

PICTURE THE PROBLEM

We choose the x axis to be horizontal and pointing to the right. Thus, $\vec{\mathbf{F}} = (20.0\text{ N})\hat{\mathbf{x}}$. The contact forces are labeled as follows: $\vec{\mathbf{F}}_1$ is the contact force exerted on box 1; $\vec{\mathbf{F}}_2$ is the contact force exerted on box 2. By Newton's third law, the contact forces have the same magnitude, f, but point in opposite directions. With our coordinate system, we have $\vec{\mathbf{F}}_1 = -f\hat{\mathbf{x}}$ and $\vec{\mathbf{F}}_2 = f\hat{\mathbf{x}}$.



Physical picture



Free-body diagrams

INTERACTIVE FIGURE

STRATEGY

a. Since the two boxes are in contact, they have the same acceleration. We find this acceleration with Newton's second law; that is, we divide the net horizontal force by the total mass of the two boxes.

b. Now let's consider the system consisting solely of box 2. The mass in this case is 5.00 kg, and the only horizontal force acting on the system is $\vec{\mathbf{F}}_2$. Thus, we can find f, the magnitude of $\vec{\mathbf{F}}_2$, by requiring that box 2 have the acceleration found in part (a).

SOLUTION

Part (a)

1. Find the net horizontal force acting on the two boxes. Note that $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are equal in magnitude but opposite in direction. Hence, they sum to zero; $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$:

$$\sum_{\substack{\text{both}\\\text{boxes}}} F_x = F = 20.0\text{ N}$$

2. Divide the net force by the total mass, $m_1 + m_2$, to find the acceleration of the boxes:

$$a_x = \frac{\sum F_x}{m_1 + m_2} = \frac{20.0\text{ N}}{(10.0\text{ kg} + 5.00\text{ kg})} = \frac{20.0\text{ N}}{15.0\text{ kg}} = 1.33\text{ m/s}^2$$

Part (b)

3. Find the net horizontal force acting on box 2, and set it equal to the mass of box 2 times its acceleration:

$$\sum_{\text{box2}} F_x = F_{2,x} = f = m_2 a_x$$

4. Determine the magnitude of the contact force, f, by substituting numerical values for m_2 and a_x:

$$f = m_2 a_x = (5.00\text{ kg})(1.33\text{ m/s}^2) = 6.67\text{ N}$$

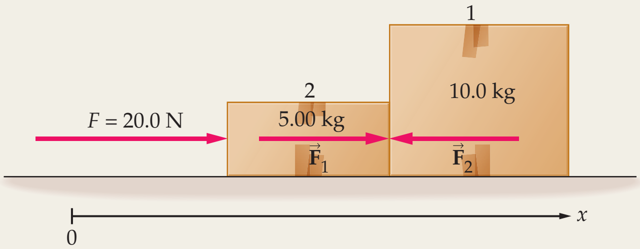
INSIGHT

Since the net horizontal force acting on box 1 is $F - f = 20.0\text{ N} - 6.67\text{ N} = 13.3\text{ N}$, it follows that its acceleration is $(13.3\text{ N})/(10.0\text{ kg}) = 1.33\text{ m/s}^2$. Thus, as expected, box 1 and box 2 have precisely the same acceleration.

If box 2 were not present, the 20.0-N force acting on box 1 would give it an acceleration of 2.00 m/s^2. As it is, the contact force between the boxes slows box 1 so that its acceleration is less than 2.00 m/s^2, and accelerates box 2 so that its acceleration is greater than zero. The precise value of the contact force is simply the value that gives both boxes the same acceleration.

PRACTICE PROBLEM

Suppose the relative positions of the boxes are reversed, so that F pushes on the small box, as shown here. Calculate the contact force for this case, and show that the force is greater than 6.67 N, as expected from Conceptual Checkpoint 5–2. [**Answer:** The contact force in this case is 13.3 N, double its previous value. This follows because the box being pushed has twice the mass of the box that was pushed originally.]



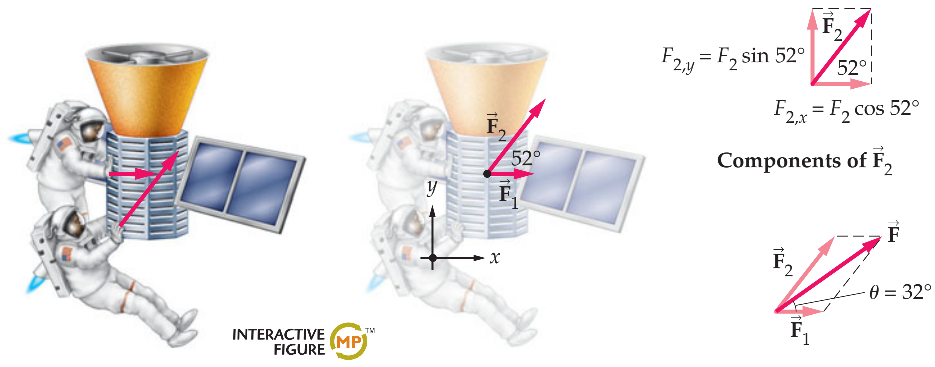
Some related homework problems: Problem 20, Problem 21

5–5 The Vector Nature of Forces: Forces in Two Dimensions

When we presented Newton's second law in Section 5–3, we said that an object's acceleration is equal to the net force acting on it divided by its mass. For example, if only a single force acts on an object, its acceleration is found to be in the same direction as the force. If more than one force acts on an object, experiments show that its acceleration is in the direction of the vector sum of the forces. Thus forces are indeed vectors, and they exhibit all the vector properties discussed in Chapter 3.

The mass of an object, on the other hand, is simply a positive number with no associated direction. It represents the amount of matter in an object.

As an example of the vector nature of forces, suppose two astronauts are using jet packs to push a 940-kg satellite toward the space shuttle, as shown in **Figure 5–9**. With the coordinate system indicated in the figure, astronaut 1 pushes in the positive x direction and astronaut 2 pushes in a direction 52° above the x axis.



(a) Physical picture **(b) Free-body diagram** **Total force**

FIGURE 5–9 Two astronauts pushing a satellite with forces that differ in magnitude and direction
The acceleration of the satellite can be found by calculating a_x and a_y separately, then combining these components to find a and θ.

If astronaut 1 pushes with a force of magnitude $F_1 = 26\text{ N}$ and astronaut 2 pushes with a force of magnitude $F_2 = 41\text{ N}$, what are the magnitude and direction of the satellite's acceleration?

The easiest way to solve a problem like this is to treat each coordinate direction independently of the other, just as we did many times when studying two-dimensional kinematics in Chapter 4. Thus, we first resolve each force into its x and y components. Referring to Figure 5–9, we see that for the x direction

$$F_{1,x} = F_1$$
$$F_{2,x} = F_2 \cos 52°$$

For the y direction

$$F_{1,y} = 0$$
$$F_{2,y} = F_2 \sin 52°$$

Next, we find the acceleration in the x direction by using the x component of Newton's second law:

$$\sum F_x = ma_x$$

Applied to this system, we have

$$\sum F_x = F_{1,x} + F_{2,x} = F_1 + F_2 \cos 52° = 26\text{ N} + (41\text{ N})\cos 52° = 51\text{ N} = ma_x$$

Solving for the acceleration yields

$$a_x = \frac{\sum F_x}{m} = \frac{51\text{ N}}{940\text{ kg}} = 0.054\text{ m/s}^2$$

Similarly, in the y direction we start with

$$\sum F_y = ma_y$$

This gives

$$\sum F_y = F_{1,y} + F_{2,y} = 0 + F_2 \sin 52° = (41\text{ N})\sin 52° = 32\text{ N} = ma_y$$

As a result, the y component of acceleration is:

$$a_y = \frac{\sum F_y}{m} = \frac{32\text{ N}}{940\text{ kg}} = 0.034\text{ m/s}^2$$

Thus, the satellite accelerates in both the x and the y directions. Its total acceleration has a magnitude of

$$a = \sqrt{a_x{}^2 + a_y{}^2} = \sqrt{(0.054\text{ m/s}^2)^2 + (0.034\text{ m/s}^2)^2} = 0.064\text{ m/s}^2$$

From Figure 5–9 we expect the total acceleration to be in a direction above the x axis but at an angle less than 52°. Straightforward calculation yields

$$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0.034\text{ m/s}^2}{0.054\text{ m/s}^2}\right) = \tan^{-1}(0.63) = 32°$$

This is the same direction as the total force in Figure 5–9, as expected.

The following Example and Active Example give further practice with resolving force vectors and using Newton's second law in component form.

PROBLEM-SOLVING NOTE

Component-by-Component Application of Newton's Laws

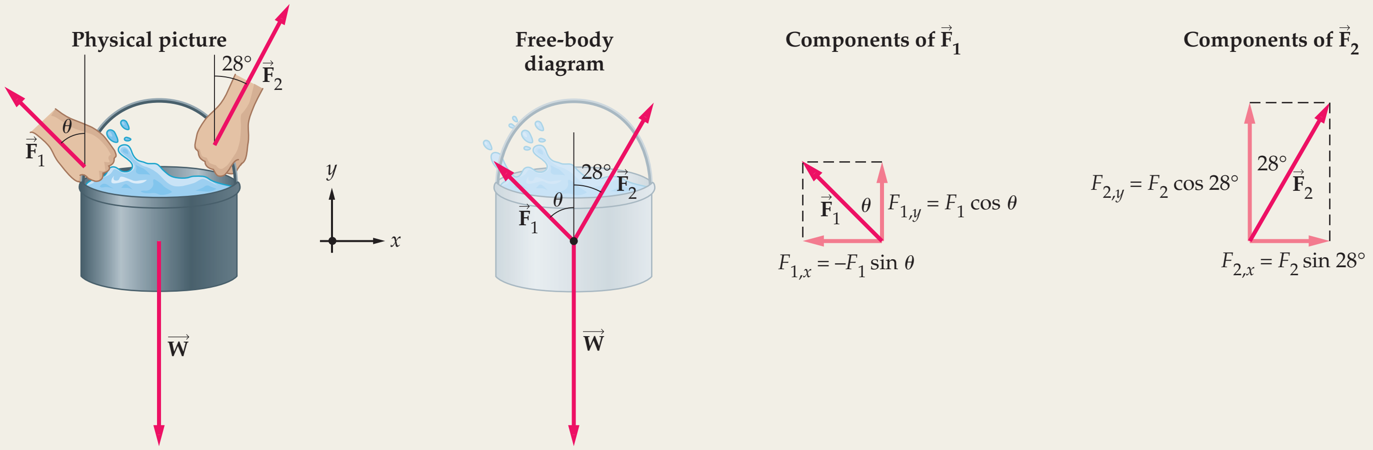
Newton's laws can be applied to each coordinate direction independently of the others. Therefore, when drawing a free-body diagram, be sure to include a coordinate system. Once the forces are resolved into their x and y components, the second law can be solved for each component separately. Working in a component-by-component fashion is the systematic way of using Newton's laws.

EXAMPLE 5–5 JACK AND JILL

Jack and Jill lift upward on a 1.30-kg pail of water, with Jack exerting a force $\vec{\mathbf{F}}_1$ of magnitude 7.0 N and Jill exerting a force $\vec{\mathbf{F}}_2$ of magnitude 11 N. Jill's force is exerted at an angle of 28° with the vertical, as shown below. **(a)** At what angle θ with respect to the vertical should Jack exert his force if the pail is to accelerate straight upward? **(b)** Determine the acceleration of the pail of water, given that its weight, $\vec{\mathbf{W}}$, has a magnitude of 12.8 N. (The simple connection between an object's mass and weight is presented in the next section.)

PICTURE THE PROBLEM

Our physical picture and free-body diagram show the pail and the three forces acting on it, as well as the angles relative to the vertical. In the panels at the right, we show the x and y components of the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. Notice, in particular, that $F_{1,x} = -F_1 \sin\theta$ and $F_{1,y} = F_1 \cos\theta$. Similarly, $F_{2,x} = F_2 \sin 28°$ and $F_{2,y} = F_2 \cos 28°$.



STRATEGY

a. We want the acceleration to be purely vertical. This means that the x component of acceleration must be zero, $a_x = 0$. For a_x to be zero it is necessary that the sum of forces in the x direction be zero, $\Sigma F_x = 0$. Since the x component of $\vec{\mathbf{F}}_1$ depends on the angle θ, the equation $\Sigma F_x = 0$ can be used to find θ.

b. Once the appropriate angle is found, we can use it to find the y component of $\vec{\mathbf{F}}_1$. Add this result to the y component of $\vec{\mathbf{F}}_2$. We're not done yet, though—to find the total y component of the force, $\Sigma \vec{\mathbf{F}}_y$, we must also add the weight of the pail, which points in the negative y direction. Finally, divide the total force by the mass of the pail, $m = 1.30$ kg, to obtain its acceleration, $a_y = (\Sigma F_y)/m$.

SOLUTION

Part (a)

1. Begin by writing out the x component of each force. Note that $\vec{\mathbf{W}}$ has no x component and that the x component of $\vec{\mathbf{F}}_1$ points in the negative x direction:

$$F_{1,x} = -F_1 \sin\theta \qquad F_{2,x} = F_2 \sin 28° \qquad W_x = 0$$

2. Sum the x components of force and set equal to zero. Note that θ is the only unknown in this equation:

$$\sum F_x = -F_1 \sin\theta + F_2 \sin 28° + 0 = ma_x = 0 \text{ or}$$
$$F_1 \sin\theta = F_2 \sin 28°$$

3. Solve for $\sin\theta$ and then for θ:

$$\sin\theta = \frac{F_2 \sin 28°}{F_1} = \frac{(11\text{ N}) \sin 28°}{7.0\text{ N}} = 0.74$$
$$\theta = \sin^{-1}(0.74) = 48°$$

Part (b)

4. First, determine the y component of each force. Note that $\vec{\mathbf{W}}$ points in the negative y direction and that the y components of both $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are positive:

$$F_{1,y} = F_1 \cos\theta = (7.0\text{ N}) \cos 48° = 4.7\text{ N}$$
$$F_{2,y} = F_2 \cos 28° = (11\text{ N}) \cos 28° = 9.7\text{ N}$$
$$W_y = -W = -12.8\text{ N}$$

5. Sum the y components of force and divide by the mass m to obtain the acceleration of the pail of water:

$$\sum F_y = F_1 \cos\theta + F_2 \cos 28° - W$$
$$= 4.7\text{ N} + 9.7\text{ N} - 12.8\text{ N} = 1.6\text{ N}$$
$$a_y = \left(\sum F_y\right)/m = (1.6\text{ N})/(1.3\text{ kg}) = 1.2\text{ m/s}^2$$

INSIGHT

Note that only the y components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ contribute to the vertical acceleration of the pail. The x components of the applied forces influence only the horizontal motion—they have no effect at all on the vertical acceleration of the pail. In this case the horizontal components of the applied forces cancel, and hence the pail moves straight upward with an acceleration of 1.2 m/s².

Finally, in the next section we shall see that the weight W of an object of mass m is $W = mg$. In this case, $W = (1.3\text{ kg})(9.81\text{ m/s}^2) = 12.8$ N.

PRACTICE PROBLEM

At what angle must Jack exert his force for the pail to accelerate straight upward if **(a)** $\vec{\mathbf{F}}_2$ is at an angle of 19° with the vertical or **(b)** $\vec{\mathbf{F}}_2$ is at an angle of 35° with the vertical? [**Answer:** **(a)** 31°, **(b)** 64°]

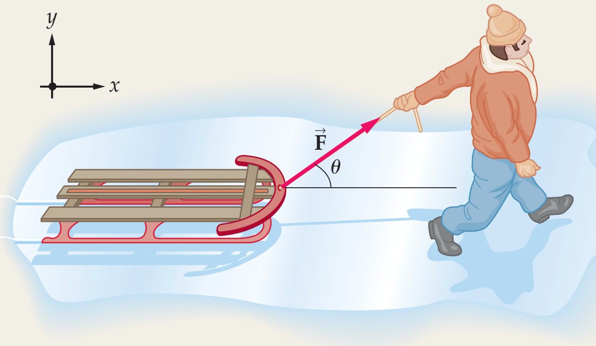
Some related homework problems: Problem 28, Problem 33

ACTIVE EXAMPLE 5–2 FIND THE SPEED OF THE SLED

A 4.60-kg sled is pulled across a smooth ice surface. The force acting on the sled is of magnitude 6.20 N and points in a direction 35.0° above the horizontal. If the sled starts at rest, how fast is it going after being pulled for 1.15 s?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the x component of $\vec{\mathbf{F}}$: $F_x = 5.08\text{ N}$
2. Apply Newton's second law to the x direction: $\sum F_x = F_x = ma_x$
3. Solve for the x component of acceleration: $a_x = 1.10\text{ m/s}^2$
4. Use $v_x = v_{0x} + a_x t$ to find the speed of the sled: $v_x = 1.27\text{ m/s}$



INSIGHT

Note that the y component of $\vec{\mathbf{F}}$ has no effect on the acceleration of the sled.

YOUR TURN

Suppose the angle of the force above the horizontal is decreased, and the sled is again pulled from rest for 1.15 s. **(a)** Is the final speed of the sled greater than, less than, or the same as before? Explain. **(b)** Find the final speed of the sled for the case $\theta = 25.0°$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

5–6 Weight

When you step onto a scale to weigh yourself, the scale gives a measurement of the pull of Earth's gravity. This is your weight, W. Similarly, the weight of any object on the Earth's surface is simply the gravitational force exerted on it by the Earth.

- The weight, W, of an object on the Earth's surface is the gravitational force exerted on it by the Earth.

As we know from everyday experience, the greater the mass of an object, the greater its weight. For example, if you put a brick on a scale and weigh it, you might get a reading of 9.0 N. If you put a second, identical brick on the scale—which doubles the mass—you will find a weight of 2(9.0 N) = 18 N. Clearly, there must be a simple connection between weight, W, and mass, m.

To see exactly what this connection is, consider taking one of the bricks just mentioned and letting it drop in free fall. As indicated in Figure 5–10, the only force acting on the brick is its weight, W, which is downward. If we choose upward to be the positive direction, we have

$$\sum F_y = -W$$

In addition, we know from Chapter 2 that the brick moves downward with an acceleration of $g = 9.81\text{ m/s}^2$ regardless of its mass. Thus,

$$a_y = -g$$

Using these results in Newton's second law

$$\sum F_y = ma_y$$

we find

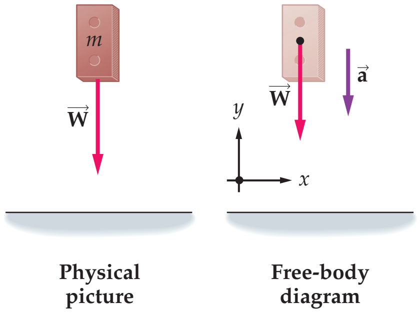
$$-W = -mg$$

Therefore, the weight of an object of mass m is $W = mg$:

Definition: Weight, *W*

$W = mg$ 5–5

SI unit: newton, N



▲ FIGURE 5–10 Weight and mass

A brick of mass m has only one force acting on it in free fall—its weight, $\vec{\mathbf{W}}$. The resulting acceleration has a magnitude $a = g$; hence $W = mg$.

Note that there is a clear distinction between weight and mass. Weight is a gravitational force, measured in newtons; mass is a measure of the inertia of an object, and it is given in kilograms. For example, if you were to travel to the Moon, your mass would not change—you would have the same amount of matter in you, regardless of your location. On the other hand, the gravitational force on the Moon's surface is less than the gravitational force on the Earth's surface. As a result, you would weigh less on the Moon than on the Earth, even though your mass is the same.

To be specific, on Earth an 81.0-kg person has a weight given by

$$W_{\text{Earth}} = mg_{\text{Earth}} = (81.0\text{ kg})(9.81\text{ m/s}^2) = 795\text{ N}$$

In contrast, the same person on the Moon, where the acceleration of gravity is 1.62 m/s^2, weighs only

$$W_{\text{Moon}} = mg_{\text{Moon}} = (81.0\text{ kg})(1.62\text{ m/s}^2) = 131\text{ N}$$

This is roughly one-sixth the weight on Earth. If, sometime in the future, there is a Lunar Olympics, the Moon's low gravity would be a boon for pole-vaulters, gymnasts, and others.

Finally, since weight is a force—which is a vector quantity—it has both a magnitude and a direction. Its magnitude, of course, is mg, and its direction is simply the direction of gravitational acceleration. Thus, if $\vec{\mathbf{g}}$ denotes a vector of magnitude g, pointing in the direction of free-fall acceleration, the weight of an object can be written in vector form as follows:

$$\vec{\mathbf{W}} = m\vec{\mathbf{g}}$$

We use the weight vector and its magnitude, mg, in the next Example.

▲ At the moment this picture was taken, the acceleration of both climbers was zero because the net force acting on them was zero. In particular, the upward forces exerted on the lower climber by the other climber and the ropes exactly cancel the downward force that gravity exerts on her.

EXAMPLE 5–6 WHERE'S THE FIRE?

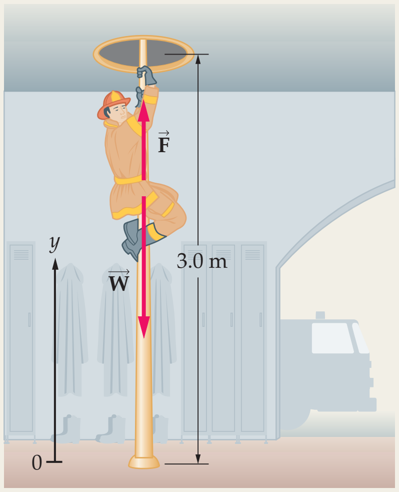
The fire alarm goes off, and a 97-kg fireman slides 3.0 m down a pole to the ground floor. Suppose the fireman starts from rest, slides with constant acceleration, and reaches the ground floor in 1.2 s. What was the upward force $\vec{\mathbf{F}}$ exerted by the pole on the fireman?

PICTURE THE PROBLEM

Our sketch shows the fireman sliding down the pole and the two forces acting on him: the upward force exerted by the pole, $\vec{\mathbf{F}}$, and the downward force of gravity, $\vec{\mathbf{W}}$. We choose the positive y direction to be upward, therefore $\vec{\mathbf{F}} = F\hat{\mathbf{y}}$ and $\vec{\mathbf{W}} = (-mg)\hat{\mathbf{y}}$. In addition, we choose $y = 0$ to be at ground level.

STRATEGY

The basic idea in approaching this problem is to apply Newton's second law to the y direction: $\sum F_y = ma_y$. The acceleration is not given directly, but we can find it using the kinematic equation $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$. Substituting the result for a_y into Newton's second law, along with $W_y = -W = -mg$, allows us to solve for the unknown force, $\vec{\mathbf{F}}$.



Physical picture

Free-body diagram

SOLUTION

1. Solve $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ for a_y, using the fact that $v_{0y} = 0$:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = y_0 + \tfrac{1}{2}a_y t^2$$

$$a_y = \frac{2(y - y_0)}{t^2}$$

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2. Substitute $y = 0$, $y_0 = 3.0$ m, and $t = 1.2$ s to find the acceleration: $a_y = \dfrac{2(0 - 3.0\text{ m})}{(1.2\text{ s})^2} = -4.2\text{ m/s}^2$

3. Sum the forces in the y direction: $\sum F_y = F - mg$

4. Set the sum of the forces equal to mass times acceleration: $F - mg = ma_y$

5. Solve for F, the y component of the force exerted by the pole. Use the result for F to write the force vector $\vec{\mathbf{F}}$:

$$F = mg + ma_y = m(g + a_y)$$
$$= (97\text{ kg})(9.81\text{ m/s}^2 - 4.2\text{ m/s}^2) = 540\text{ N}$$
$$\vec{\mathbf{F}} = (540\text{ N})\,\hat{\mathbf{y}}$$

INSIGHT

How is it that the pole exerts a force on the fireman? Well, by wrapping his arms and legs around the pole as he slides, the fireman exerts a downward force on the pole. By Newton's third law, the pole exerts an upward force of equal magnitude on the fireman. These forces are due to friction, which we shall study in detail in Chapter 6.

PRACTICE PROBLEM

What is the fireman's acceleration if the force exerted on him by the pole is 650 N? **[Answer:** $a_y = -3.1\text{ m/s}^2$]

Some related homework problems: Problem 36, Problem 40

Apparent Weight

We have all had the experience of riding in an elevator and feeling either heavy or light, depending on its motion. For example, when an elevator moving downward comes to rest by accelerating upward, we feel heavier. On the other hand, we feel lighter when an elevator moving upward comes to rest by accelerating downward. In short, the motion of an elevator can give rise to an **apparent weight** that differs from our true weight. Why?

The reason is that our sensation of weight in this case is due to the force exerted on our feet by the floor of the elevator. If this force is greater than our weight, mg, we feel heavy; if it is less than mg, we feel light.

As an example, imagine you are in an elevator that is moving with an upward acceleration a, as indicated in **Figure 5–11**. Two forces act on you: (i) your weight, W, acting downward; and (ii) the upward normal force exerted on your feet by the floor of the elevator. Let's call the second force W_a, since it represents your apparent weight—that is, W_a is the force that pushes upward on your feet and gives you the sensation of your "weight" pushing down on the floor. We can find W_a by applying Newton's second law to the vertical direction.

To be specific, the sum of the forces acting on you is

$$\sum F_y = W_a - W$$

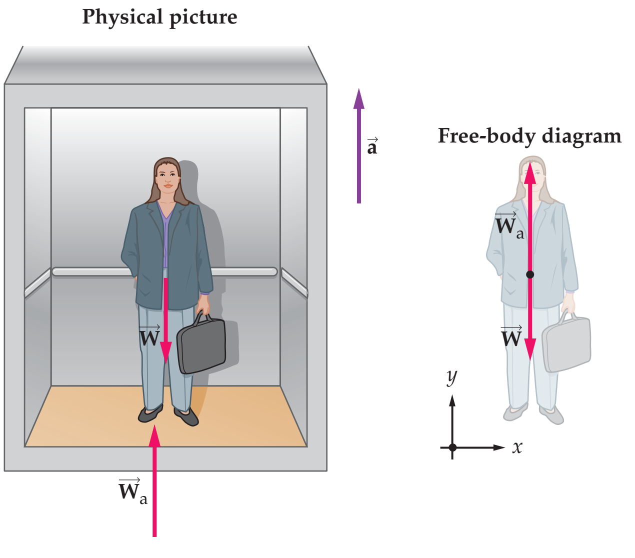


FIGURE 5–11 Apparent weight
A person rides in an elevator that is accelerating upward. Because the acceleration is upward, the net force must also be upward. As a result, the force exerted on the person by the floor of the elevator, $\vec{\mathbf{W}}_a$, must be greater than the person's weight, $\vec{\mathbf{W}}$. This means that the person feels heavier than normal.

By Newton's second law, this sum must equal ma_y. Since $a_y = a$, we find

$$W_a - W = ma$$

Solving for the apparent weight, W_a, yields

$$\begin{aligned} W_a &= W + ma \\ &= mg + ma = m(g + a) \end{aligned} \quad 5\text{–}6$$

Note that W_a is greater than your weight, mg, and hence you feel heavier. In fact, your apparent weight is precisely what it would be if you were suddenly "transported" to a planet where the acceleration of gravity is $g + a$ instead of g.

On the other hand, if the elevator accelerates downward, so that $a_y = -a$, your apparent weight is found by simply replacing a with $-a$ in Equation 5–6:

$$\begin{aligned} W_a &= W - ma \\ &= mg - ma = m(g - a) \end{aligned} \quad 5\text{–}7$$

In this case you feel lighter than usual.

We explore these results in the next Example, in which we consider weighing a fish on a scale. The reading on the scale is equal to the upward force it exerts on an object. Thus, the upward force exerted by the scale is the apparent weight, W_a.

EXAMPLE 5–7 HOW MUCH DOES THE SALMON WEIGH?

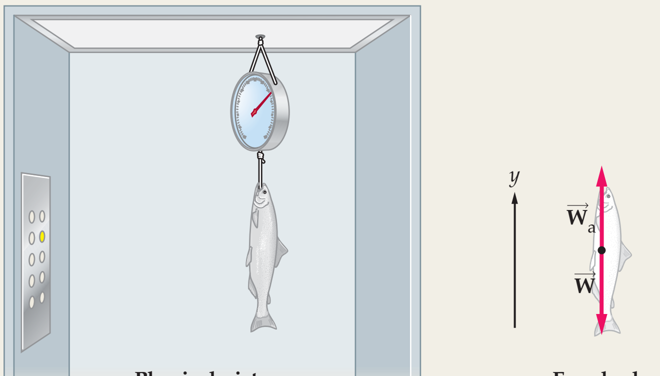
As part of an attempt to combine physics and biology in the same class, an instructor asks students to weigh a 5.0-kg salmon by hanging it from a fish scale attached to the ceiling of an elevator. What is the apparent weight of the salmon, $\vec{\mathbf{W}}_a$, if the elevator **(a)** is at rest, **(b)** moves with an upward acceleration of 2.5 m/s^2, or **(c)** moves with a downward acceleration of 3.2 m/s^2?

PICTURE THE PROBLEM

The free-body diagram for the salmon shows the weight of the salmon, $\vec{\mathbf{W}}$, and the force exerted by the scale, $\vec{\mathbf{W}}_a$. Note that upward is the positive direction. Therefore, the y component of $\vec{\mathbf{W}}$ is $-W = -mg$ and the y component of $\vec{\mathbf{W}}_a$ is W_a.

STRATEGY

We know the weight, $W = mg$, and the acceleration, a. To find the apparent weight, W_a, we use $\sum F_y = ma_y$. **(a)** Set $a_y = 0$. **(b)** Set $a_y = 2.5\ \mathrm{m/s^2}$. **(c)** Set $a_y = -3.2\ \mathrm{m/s^2}$.



Physical picture

Free-body diagram

SOLUTION

Part (a)

1. Sum the y component of the forces and set equal to mass times the y component of acceleration, with $a_y = 0$: $\sum F_y = W_a - W = ma_y = 0$
2. Solve for W_a, then write the vector $\vec{\mathbf{W}}$: $W_a = W = mg = (5.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) = 49\ \mathrm{N}$
   $\vec{\mathbf{W}}_a = (49\ \mathrm{N})\hat{\mathbf{y}}$

Part (b)

3. Again, sum the forces and set equal to mass times acceleration, this time with $a_y = a = 2.5\ \mathrm{m/s^2}$: $\sum F_y = W_a - W = ma_y = ma$
4. Solve for W_a, then write the vector $\vec{\mathbf{W}}_a$: $W_a = W + ma$
   $= mg + ma = 49\ \mathrm{N} + (5.0\ \mathrm{kg})(2.5\ \mathrm{m/s^2}) = 62\ \mathrm{N}$
   $\vec{\mathbf{W}}_a = (62\ \mathrm{N})\hat{\mathbf{y}}$

Part (c)

5. Finally, sum the forces and set equal to mass times acceleration, with $a_y = -a = -3.2\ \mathrm{m/s^2}$: $\sum F_y = W_a - W = ma_y = -ma$

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6. Solve for W_a, then write the vector $\vec{\mathbf{W}}_a$:

$$W_a = W - ma$$
$$= mg - ma = 49\ \mathrm{N} - (5.0\ \mathrm{kg})(3.2\ \mathrm{m/s^2}) = 33\ \mathrm{N}$$
$$\vec{\mathbf{W}}_a = (33\ \mathrm{N})\hat{\mathbf{y}}$$

INSIGHT

When the salmon is at rest, or moving with constant velocity, its acceleration is zero and the apparent weight is equal to the actual weight, mg. In part (b) the apparent weight is greater than the actual weight because the scale must exert an upward force capable not only of supporting the salmon, but of accelerating it upward as well. In part (c) the apparent weight is less than the actual weight. In this case the net force acting on the salmon is downward, and hence its acceleration is downward.

PRACTICE PROBLEM

What is the elevator's acceleration if the scale gives a reading of **(a)** 55 N or **(b)** 45 N? [**Answer:** **(a)** $a_y = 1.2\ \mathrm{m/s^2}$, **(b)** $a_y = -0.80\ \mathrm{m/s^2}$]

Some related homework problems: Problem 38, Problem 39

REAL-WORLD PHYSICS: BIO

Simulating weightlessness

▲ Astronaut candidates pose for a floating class picture during weightlessness training aboard the "vomit comet."

Let's return for a moment to Equation 5–7:

$$W_a = m(g - a)$$

This result indicates that a person feels lighter than normal when riding in an elevator with a downward acceleration a. In particular, if the elevator's downward acceleration is g—that is, if the elevator is in free fall—it follows that $W_a = m(g - g) = 0$. Thus, a person feels "weightless" (zero apparent weight) in a freely falling elevator!

NASA uses this effect when training astronauts. Trainees are sent aloft in a KC-135 airplane affectionately known as the "vomit comet" (since many trainees experience nausea along with the weightlessness). To generate an experience of weightlessness, the plane flies on a parabolic path—the same path followed by a projectile in free fall. Each round of weightlessness lasts about half a minute, after which the plane pulls up to regain altitude and start the cycle again. On a typical flight, trainees experience about 40 cycles of weightlessness. Many scenes in the movie *Apollo 13* were shot in 30-second takes aboard the vomit comet.

This idea of free-fall weightlessness applies to more than just the vomit comet. In fact, astronauts in orbit experience weightlessness for the same reason—they and their craft are actually in free fall. As we shall see in detail in Chapter 12 (Gravity), orbital motion is just a special case of free fall.

CONCEPTUAL CHECKPOINT 5–3 ELEVATOR RIDE

If you ride in an elevator moving upward with constant speed, is your apparent weight **(a)** the same as, **(b)** greater than, or **(c)** less than mg?

REASONING AND DISCUSSION

If the elevator is moving in a straight line with constant speed, its acceleration is zero. Now, if the acceleration is zero, the net force must also be zero. Hence, the upward force exerted by the floor of the elevator, W_a, must equal the downward force of gravity, mg. As a result, your apparent weight is equal to mg.

Note that this conclusion agrees with Equations 5–6 and 5–7, with $a = 0$.

ANSWER

(a) Your apparent weight is the same as mg.

5–7 Normal Forces

As you get ready for lunch, you take a can of soup from the cupboard and place it on the kitchen counter. The can is now at rest, which means that its acceleration is zero, so the net force acting on it is also zero. Thus, you know that the

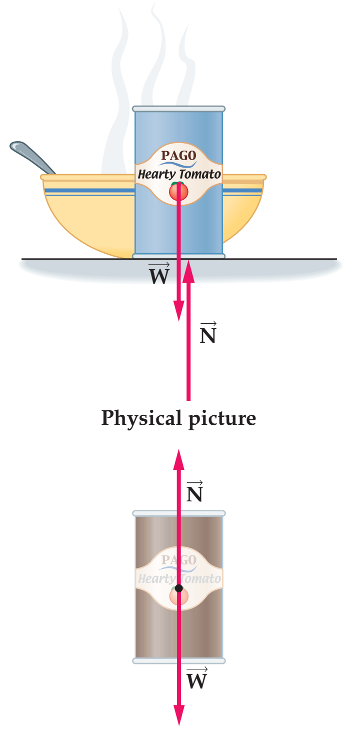


FIGURE 5–12 The normal force is perpendicular to a surface

A can of soup rests on a kitchen counter, which exerts a normal (perpendicular) force, $\vec{\mathbf{N}}$, to support it. In the special case shown here, the normal force is equal in magnitude to the weight, $W = mg$, and opposite in direction.

downward force of gravity is being opposed by an upward force exerted by the counter, as shown in **Figure 5–12**. As we have mentioned before, this force is referred to as the **normal force,** $\vec{\mathbf{N}}$. The reason the force is called normal is that it is *perpendicular to the surface*, and in mathematical terms, *normal* simply means perpendicular.

The origin of the normal force is the interaction between atoms in a solid that act to maintain its shape. When the can of soup is placed on the countertop, for example, it causes an imperceptibly small compression of the surface of the counter. This is similar to compressing a spring, and just like a spring, the countertop exerts a force to oppose the compression. Therefore, the greater the weight placed on the countertop, the greater the normal force it exerts to oppose being compressed.

In the example of the soup can and the countertop, the magnitude of the normal force is equal to the weight of the can. This is a special case, however. In general, the normal force may be greater than or less than the weight of an object.

To see how this can come about, consider pulling a 12.0-kg suitcase across a smooth floor by exerting a force, $\vec{\mathbf{F}}$, at an angle θ above the horizontal. The weight of the suitcase is $mg = (12.0\ \text{kg})(9.81\ \text{m/s}^2) = 118\ \text{N}$. The normal force will have a magnitude less than this, however, because the force $\vec{\mathbf{F}}$ has an upward component that supports part of the suitcase's weight. To be specific, suppose that $\vec{\mathbf{F}}$ has a magnitude of 45.0 N and that $\theta = 20.0°$. What is the normal force exerted by the floor on the suitcase?

The situation is illustrated in **Figure 5–13**, where we show the three forces acting on the suitcase: (i) the weight of the suitcase, $\vec{\mathbf{W}}$, (ii) the force $\vec{\mathbf{F}}$, and (iii) the normal force, $\vec{\mathbf{N}}$. We also indicate a typical coordinate system in the figure, with the x axis horizontal and the y axis vertical. Now, the key to solving a problem like this is to realize that since the suitcase does not move in the y direction, its y component of acceleration is zero; that is, $a_y = 0$. It follows, from Newton's second law, that the sum of the y components of force must also equal zero; that is, $\Sigma F_y = ma_y = 0$. Using this condition, we can solve for the one force that is unknown, $\vec{\mathbf{N}}$.

To find $\vec{\mathbf{N}}$, then, we start by writing out the y component of each force. For the weight we have $W_y = -mg = -118\ \text{N}$; for the applied force, $\vec{\mathbf{F}}$, the y component is $F_y = F \sin 20.0° = (45.0\ \text{N}) \sin 20.0° = 15.4\ \text{N}$; finally, the y component of the normal force is $N_y = N$. Setting the sum of the y components of force equal to zero yields

$$\sum F_y = W_y + F_y + N_y = -mg + F \sin 20.0° + N = 0$$

Solving for N gives

$$N = mg - F \sin 20.0° = 118\ \text{N} - 15.4\ \text{N} = 103\ \text{N}$$

In vector form,

$$\vec{\mathbf{N}} = N_y \hat{\mathbf{y}} = (103\ \text{N})\hat{\mathbf{y}}$$

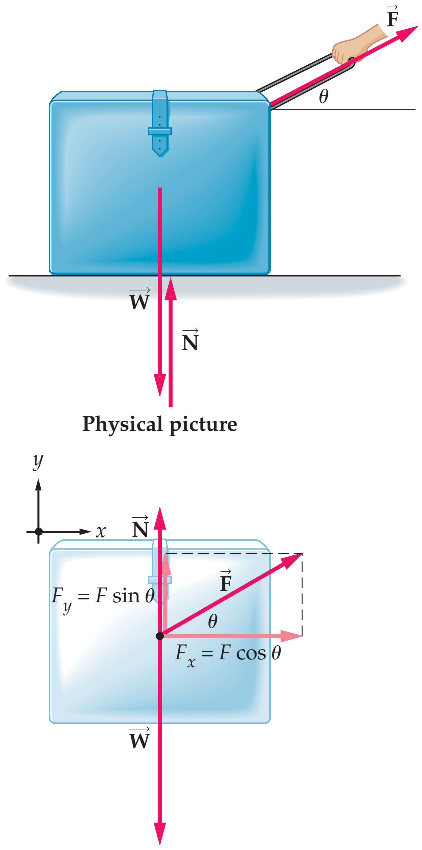


FIGURE 5–13 The normal force may differ from the weight

A suitcase is pulled across the floor by an applied force of magnitude F, directed at an angle θ above the horizontal. As a result of the upward component of $\vec{\mathbf{F}}$, the normal force $\vec{\mathbf{N}}$ will have a magnitude less than the weight of the suitcase.

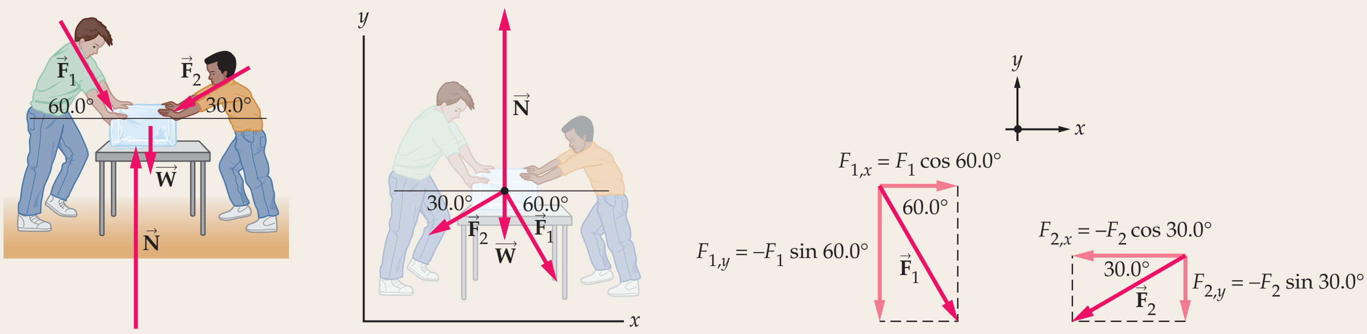
Thus, as mentioned, the normal force has a magnitude less than $mg = 118\text{ N}$ because the y component of $\vec{\mathbf{F}}$, $F_y = F \sin 20.0°$, supports part of the weight. In the following Example, however, the applied forces cause the normal force to be greater than the weight.

EXAMPLE 5–8 ICE BLOCK

A 6.0-kg block of ice is acted on by two forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$, as shown in the diagram. If the magnitudes of the forces are $F_1 = 13\text{ N}$ and $F_2 = 11\text{ N}$, find **(a)** the acceleration of the ice and **(b)** the normal force exerted on it by the table.

PICTURE THE PROBLEM

The sketch shows our choice of coordinate system, as well as all the forces acting on the block of ice. Note that $\vec{\mathbf{F}}_1$ has a positive x component and a negative y component; $\vec{\mathbf{F}}_2$ has negative x and y components. The weight and the normal force have only y components, therefore $W_x = 0$, $W_y = -W = -mg$, $N_x = 0$, and $N_y = N$.



Physical picture

Free-body diagram

Components of $\vec{\mathbf{F}}_1$ (enlarged)

Components of $\vec{\mathbf{F}}_2$ (enlarged)

STRATEGY

The basic idea in this problem is to apply Newton's second law to the x and y directions separately. **(a)** The block can accelerate only in the horizontal direction; thus we find the acceleration by solving $\Sigma F_x = ma_x$ for a_x. **(b)** There is no motion in the y direction, and therefore the acceleration in the y direction is zero. Hence, we can find the normal force $\vec{\mathbf{N}}$ by setting $\Sigma F_y = ma_y = 0$.

SOLUTION

Part (a)

1. Write out the x component of each force:

$$F_{1,x} = F_1 \cos 60.0° = (13\text{ N}) \cos 60.0° = 6.5\text{ N}$$
$$F_{2,x} = -F_2 \cos 30.0° = -(11\text{ N}) \cos 30.0° = -9.5\text{ N}$$
$$N_x = 0 \qquad W_x = 0$$

2. Sum the x components of force:

$$\sum F_x = F_{1,x} + F_{2,x} + N_x + W_x$$
$$= 6.5\text{ N} - 9.5\text{ N} + 0 + 0 = -3.0\text{ N}$$

3. Divide by the mass to obtain the acceleration:

$$a_x = \frac{\sum F_x}{m} = \frac{-3.0\text{ N}}{6.0\text{ kg}} = -0.50\text{ m/s}^2$$
$$\vec{\mathbf{a}} = (-0.50\text{ m/s}^2)\hat{\mathbf{x}}$$

Part (b)

4. Write out the y component of each force:

The only force we don't know is the normal. We represent its magnitude by N:

$$F_{1,y} = -F_1 \sin 60° = -(13\text{ N}) \sin 60.0° = -11\text{ N}$$
$$F_{2,y} = -F_2 \sin 30° = -(11\text{ N}) \sin 30.0° = -5.5\text{ N}$$
$$N_y = N \qquad W_y = -W = -mg$$

5. Sum the y components of force:

$$\sum F_y = F_{1,y} + F_{2,y} + N_y + W_y$$
$$= -11\text{ N} - 5.5\text{ N} + N - mg$$

6. Set this sum equal to 0 since the acceleration in the y direction is zero, and solve for N:

$$-11\text{ N} - 5.5\text{ N} + N - mg = 0$$
$$N = 11\text{ N} + 5.5\text{ N} + mg$$
$$= 11\text{ N} + 5.5\text{ N} + (6.0\text{ kg})(9.81\text{ m/s}^2) = 75\text{ N}$$

7. Finally, we write the normal force in vector form:

$$\vec{\mathbf{N}} = (75\text{ N})\hat{\mathbf{y}}$$

INSIGHT

The block accelerates to the left, even though the force acting to the right, $\vec{\mathbf{F}}_1$, has a greater magnitude than the force acting to the left, $\vec{\mathbf{F}}_2$. This is because $\vec{\mathbf{F}}_2$ has the greater x component. Also, note that the normal force is greater in magnitude than the weight, $mg = 59$ N.

In general, the normal force exerted by a surface is just as large as is necessary to prevent motion of an object into the surface. If the required force is larger than the material can provide, the surface will break.

PRACTICE PROBLEM

At what angle must $\vec{\mathbf{F}}_2$ be applied if the block of ice is to have zero acceleration? **[Answer:** $a_x = 0$ implies $F_1 \cos 60.0° = F_2 \cos\theta$. Thus, $\theta = 54°$.]

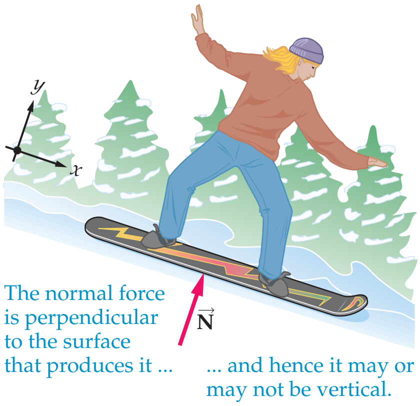
Some related homework problems: Problem 44, Problem 50

To this point, we have considered surfaces that are horizontal, in which case the normal force is vertical. When a surface is inclined, the normal force is still at right angles to the surface, even though it is no longer vertical. This is illustrated in **Figure 5–14**. (If friction is present, a surface may also exert a force that is parallel to its surface. This will be considered in detail in Chapter 6.)

When choosing a coordinate system for an inclined surface, it is generally best to have the x and y axes of the system parallel and perpendicular to the surface, respectively, as in **Figure 5–15**. One can imagine the coordinate system to be "bolted down" to the surface, so that when the surface is tilted the coordinate system tilts along with it.

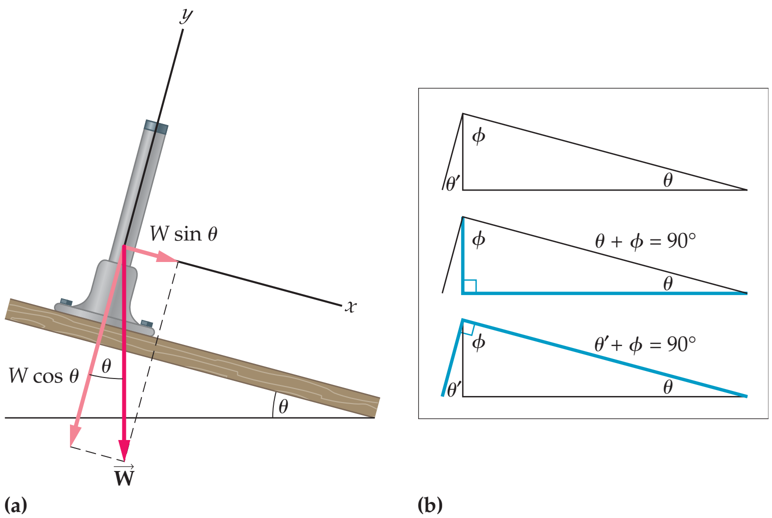
With this choice of coordinate system, there is no motion in the y direction, even on the inclined surface, and the normal force points in the positive y direction. Thus, the condition that determines the normal force is still $\Sigma F_y = ma_y = 0$, as before. In addition, if the object slides on the surface, its motion is purely in the x direction.

Finally, if the surface is inclined by an angle θ, note that the weight—which is still vertically downward—is at the same angle θ with respect to the negative



▲ FIGURE 5–14 An object on an inclined surface

The normal force $\vec{\mathbf{N}}$ is always at right angles to the surface; hence, it is not always in the vertical direction.



◀ FIGURE 5–15 Components of the weight on an inclined surface

Whenever a surface is tilted by an angle θ, the weight $\vec{\mathbf{W}}$ makes the same angle θ with respect to the negative y axis. This is proven in part (b), where we show that $\theta + \phi = 90°$, and that $\theta' + \phi = 90°$. From these results it follows that $\theta' = \theta$. The component of the weight perpendicular to the surface is $W_y = -W\cos\theta$; the component parallel to the surface is $W_x = W\sin\theta$.

y axis, as shown in Figure 5–15. As a result, the x and y components of the weight are

$$W_x = W \sin\theta = mg \sin\theta \qquad 5\text{–}8$$

and

$$W_y = -W \cos\theta = -mg \cos\theta \qquad 5\text{–}9$$

Let's quickly check some special cases of these results. First, if $\theta = 0$ the surface is horizontal, and we find $W_x = 0$, $W_y = -mg$, as expected. Second, if $\theta = 90°$ the surface is vertical; therefore, the weight is parallel to the surface, pointing in the positive x direction. In this case, $W_x = mg$ and $W_y = 0$.

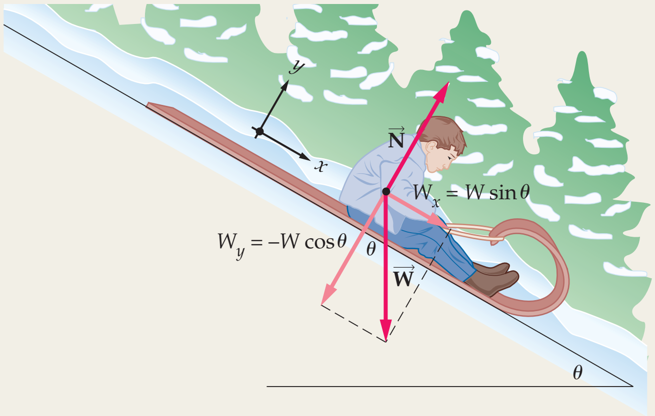
The next Example shows how to use the weight components to find the acceleration of an object on an inclined surface.

EXAMPLE 5–9 TOBOGGAN TO THE BOTTOM

A child of mass m rides on a toboggan down a slick, ice-covered hill inclined at an angle θ with respect to the horizontal. **(a)** What is the acceleration of the child? **(b)** What is the normal force exerted on the child by the toboggan?

PICTURE THE PROBLEM

We choose the x axis to be parallel to the slope, with the positive direction pointing downhill. Similarly, we choose the y axis to be perpendicular to the slope, pointing up and to the right. With these choices, the x component of $\vec{\mathbf{W}}$ is positive, $W_x = W \sin\theta$, and its y component is negative, $W_y = -W \cos\theta$. Finally, the x component of the normal force is zero, $N_x = 0$, and its y component is positive, $N_y = N$.



STRATEGY

Note that only two forces act on the child: (i) the weight, $\vec{\mathbf{W}}$, and (ii) the normal force, $\vec{\mathbf{N}}$. **(a)** We find the child's acceleration by solving $\sum F_x = ma_x$ for a_x. **(b)** Because there is no motion in the y direction, the y component of acceleration is zero. Therefore, we can find the normal force by setting $\sum F_y = ma_y = 0$.

SOLUTION

Part (a)

1. Write out the x components of the forces acting on the child:	$N_x = 0 \qquad W_x = W \sin\theta = mg \sin\theta$
2. Sum the x components of the forces and set equal to ma_x:	$\sum F_x = N_x + W_x = mg \sin\theta = ma_x$
3. Divide by the mass m to find the acceleration in the x direction:	$a_x = \dfrac{\sum F_x}{m} = \dfrac{mg \sin\theta}{m} = g \sin\theta$

Part (b)

4. Write out the y components of the forces acting on the child:	$N_y = N \qquad W_y = -W \cos\theta = -mg \cos\theta$
5. Sum the y components of the forces and set the sum equal to zero, since $a_y = 0$:	$\sum F_y = N_y + W_y = N - mg \cos\theta = ma_y = 0$
6. Solve for the magnitude of the normal force, N:	$N - mg \cos\theta = 0$ or $N = mg \cos\theta$
7. Write the normal force in vector form:	$\vec{\mathbf{N}} = (mg \cos\theta)\hat{\mathbf{y}}$

INSIGHT

Note that for θ between 0 and 90° the acceleration of the child is *less* than the acceleration of gravity. This is because only a *component* of the weight is causing the acceleration.

Let's check some special cases of our general result, $a_x = g \sin\theta$. First, let $\theta = 0$. In this case, we find zero acceleration; $a_x = g \sin 0 = 0$. This makes sense because with $\theta = 0$ the hill is actually level, and we don't expect an acceleration. Second, let $\theta = 90°$. In this case, the hill is vertical, and the toboggan should drop straight down in free fall. This also agrees with our general result; $a_x = g \sin 90° = g$.

PRACTICE PROBLEM

What is the child's acceleration if its mass is doubled to $2m$? [**Answer:** The acceleration is still $a_x = g \sin\theta$. As in free fall, the acceleration produced by gravity is independent of mass.]

Some related homework problems: Problem 45, Problem 49

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The fact that a constant force produces a constant acceleration gives special significance to the discussion of constant acceleration in Chapters 2 and 4.

All forces are vectors, and therefore the ability to use and manipulate vectors confidently is essential to a full and complete understanding of forces. Again, we see the importance of the vector material presented in Chapter 3.

As with two-dimensional kinematics in Chapter 4, where motion in the x and y directions were seen to be independent, the x and y components of force are independent as well. In particular, acceleration in the x direction depends only on the x component of force, and acceleration in the y direction depends only on the y component of force.

LOOKING AHEAD

Forces are a central theme throughout physics. In particular, we shall see in Chapters 7 and 8 that a force acting on an object over a distance changes its energy.

Another important application of forces is in the study of collisions. Central to this topic is the concept of momentum, a physical quantity that is changed when a force acts on an object over a period of time.

In this chapter we introduced the force law for gravity near the Earth's surface, $F = mg$. The more general law of gravity, valid at any location, is introduced in Chapter 12. Similarly, the force laws for electricity and magnetism are presented in Chapters 19 and 22, respectively.

CHAPTER SUMMARY

5–1 FORCE AND MASS

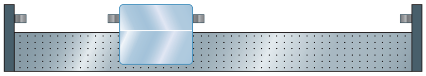
Force

A push or a pull.

Mass

A measure of the difficulty in accelerating an object. Equivalently, a measure of the quantity of matter in an object.

5–2 NEWTON'S FIRST LAW OF MOTION



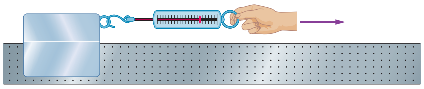
First Law (Law of Inertia)

If the net force on an object is zero, its velocity is constant.

Inertial Frame of Reference

Frame of reference in which the first law holds. All inertial frames of reference move with constant velocity relative to one another.

5–3 NEWTON'S SECOND LAW OF MOTION



Second Law

An object of mass m has an acceleration $\vec{\mathbf{a}}$ given by the net force $\Sigma\vec{\mathbf{F}}$ divided by m. That is

$$\vec{\mathbf{a}} = \sum \vec{\mathbf{F}}/m \qquad 5\text{–}1$$

Component Form

$$a_x = \sum F_x/m \qquad a_y = \sum F_y/m \qquad a_z = \sum F_z/m \qquad 5\text{–}2$$

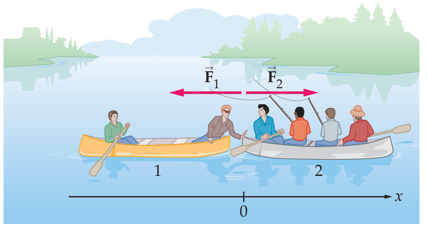
SI Unit: Newton (N)

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2 \qquad 5\text{–}3$$

Free-Body Diagram

A sketch showing all external forces acting on an object.

5–4 NEWTON'S THIRD LAW OF MOTION

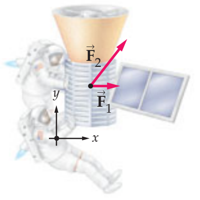


Physical picture

Third Law
For every force that acts on an object, there is a reaction force acting on a different object that is equal in magnitude and opposite in direction.

Contact Forces
Action-reaction pair of forces produced by physical contact of two objects.

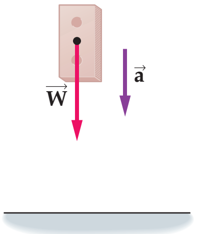
5–5 THE VECTOR NATURE OF FORCES: FORCES IN TWO DIMENSIONS



Forces are vectors.

Newton's second law can be applied to each component of force separately and independently.

5–6 WEIGHT



Gravitational force exerted *by the Earth on an object*.

On the surface of the Earth the weight, W, of an object of mass m has the magnitude

$$W = mg \quad 5\text{–}5$$

Apparent Weight
Force felt from contact with the floor or a scale in an accelerating system. For example, the sensation of feeling heavier or lighter in an accelerating elevator.

5–7 NORMAL FORCES

Force exerted by a surface that is *perpendicular* to the surface.

The normal force is equal to the weight of an object only in special cases. In general, the normal force is greater than or less than the object's weight.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Find the acceleration of an object.	Solve Newton's second law for each component of the acceleration; that is, $a_x = \Sigma F_x/m$ and $a_y = \Sigma F_y/m$.	Examples 5–1, 5–3, 5–4, 5–5, 5–8, 5–9 Active Examples 5–1, 5–2
Solve problems involving action-reaction forces.	Apply Newton's third law, being careful to note that the action-reaction forces act on different objects.	Examples 5–3, 5–4
Find the normal force exerted on an object.	Since there is no acceleration in the normal direction, set the sum of the normal components of force equal to zero.	Examples 5–8, 5–9

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Driving down the road, you hit the brakes suddenly. As a result, your body moves toward the front of the car. Explain, using Newton's laws.
2. You've probably seen pictures of someone pulling a tablecloth out from under glasses, plates, and silverware set out for a formal dinner. Perhaps you've even tried it yourself. Using Newton's laws of motion, explain how this stunt works.
3. As you read this, you are most likely sitting quietly in a chair. Can you conclude, therefore, that you are at rest? Explain.

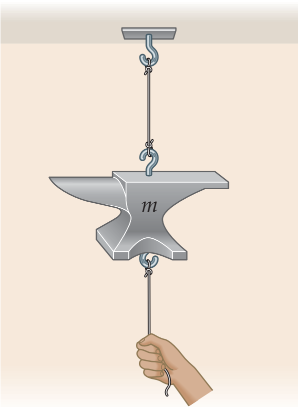
4. When a dog gets wet, it shakes its body from head to tail to shed the water. Explain, in terms of Newton's first law, why this works.

A dog uses the principle of inertia to shake water from its coat. (Conceptual Question 4)

5. A young girl slides down a rope. As she slides faster and faster she tightens her grip, increasing the force exerted on her by the rope. What happens when this force is equal in magnitude to her weight? Explain.

6. A drag-racing car accelerates forward because of the force exerted on it by the road. Why, then, does it need an engine? Explain.

7. A block of mass m hangs from a string attached to a ceiling, as shown in **Figure 5–16**. An identical string hangs down from the bottom of the block. Which string breaks if **(a)** the lower string is pulled with a slowly increasing force or **(b)** the lower string is jerked rapidly downward? Explain.



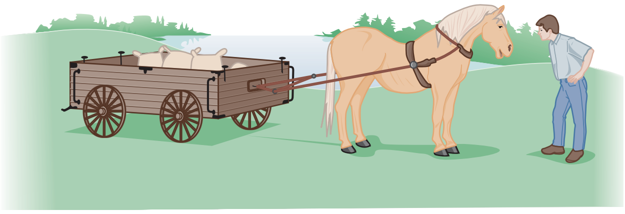
▲ **FIGURE 5–16** Conceptual Question 7

8. An astronaut on a space walk discovers that his jet pack no longer works, leaving him stranded 50 m from the spacecraft. If the jet pack is removable, explain how the astronaut can still use it to return to the ship.

9. Two untethered astronauts on a space walk decide to take a break and play catch with a baseball. Describe what happens as the game of catch progresses.

10. What are the action-reaction forces when a baseball bat hits a fast ball? What is the effect of each force?

11. In **Figure 5–17** Wilbur asks Mr. Ed, the talking horse, to pull a cart. Mr. Ed replies that he would like to, but the laws of nature just won't allow it. According to Newton's third law, he says, if he pulls on the wagon it pulls back on him with an equal force. Clearly, then, the net force is zero and the wagon will stay put. How should Wilbur answer the clever horse?



▲ **FIGURE 5–17** Conceptual Question 11

12. A whole brick has more mass than half a brick, thus the whole brick is harder to accelerate. Why doesn't a whole brick fall more slowly than half a brick? Explain.

13. The force exerted by gravity on a whole brick is greater than the force exerted by gravity on half a brick. Why, then, doesn't a whole brick fall faster than half a brick? Explain.

14. Is it possible for an object at rest to have only a single force acting on it? If your answer is yes, provide an example. If your answer is no, explain why not.

15. Is it possible for an object to be in motion and yet have zero net force acting on it? Explain.

16. A bird cage, with a parrot inside, hangs from a scale. The parrot decides to hop to a higher perch. What can you say about the reading on the scale **(a)** when the parrot jumps, **(b)** when the parrot is in the air, and **(c)** when the parrot lands on the second perch? Assume that the scale responds rapidly so that it gives an accurate reading at all times.

17. Suppose you jump from the cliffs of Acapulco and perform a perfect swan dive. As you fall, you exert an upward force on the Earth equal in magnitude to the downward force the Earth exerts on you. Why, then, does it seem that you are the one doing all the accelerating? Since the forces are the same, why aren't the accelerations?

18. A friend tells you that since his car is at rest, there are no forces acting on it. How would you reply?

19. Since all objects are "weightless" in orbit, how is it possible for an orbiting astronaut to tell if one object has more mass than another object? Explain.

20. To clean a rug, you can hang it from a clothesline and beat it with a tennis racket. Use Newton's laws to explain why beating the rug should have a cleansing effect.

21. If you step off a high board and drop to the water below, you plunge into the water without injury. On the other hand, if you were to drop the same distance onto solid ground, you might break a leg. Use Newton's laws to explain the difference.

22. A moving object is acted on by a net force. Give an example of a situation in which the object moves **(a)** in the same direction as the net force, **(b)** at right angles to the net force, or **(c)** in the opposite direction of the net force.

23. Is it possible for an object to be moving in one direction while the net force acting on it is in another direction? If your answer is yes, provide an example. If your answer is no, explain why not.

24. Since a bucket of water is "weightless" in space, would it hurt to kick the bucket? Explain.

25. In the movie *The Rocketeer*, a teenager discovers a jet-powered backpack in an old barn. The backpack allows him to fly at incredible speeds. In one scene, however, he uses the backpack to rapidly accelerate an old pickup truck that is being chased by "bad guys." He does this by bracing his arms against the cab of the pickup and firing the backpack, giving the truck the acceleration of a drag racer. Is the physics of this scene "Good," "Bad," or "Ugly?" Explain.

26. List three common objects that have a weight of approximately 1 N.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 5–3 NEWTON'S SECOND LAW OF MOTION

1. • **CE** An object of mass m is initially at rest. After a force of magnitude F acts on it for a time T, the object has a speed v. Suppose the mass of the object is doubled, and the magnitude of the force acting on it is quadrupled. In terms of T, how long does it take for the object to accelerate from rest to a speed v now?

2. • On a planet far, far away, an astronaut picks up a rock. The rock has a mass of 5.00 kg, and on this particular planet its weight is 40.0 N. If the astronaut exerts an upward force of 46.2 N on the rock, what is its acceleration?

3. • In a grocery store, you push a 12.3-kg shopping cart with a force of 10.1 N. If the cart starts at rest, how far does it move in 2.50 s?

4. • You are pulling your little sister on her sled across an icy (frictionless) surface. When you exert a constant horizontal force of 120 N, the sled has an acceleration of 2.5 m/s^2. If the sled has a mass of 7.4 kg, what is the mass of your little sister?

5. • A 0.53-kg billiard ball initially at rest is given a speed of 12 m/s during a time interval of 4.0 ms. What average force acted on the ball during this time?

6. • A 92-kg water skier floating in a lake is pulled from rest to a speed of 12 m/s in a distance of 25 m. What is the net force exerted on the skier, assuming his acceleration is constant?

7. •• **CE Predict/Explain** You drop two balls of equal diameter from the same height at the same time. Ball 1 is made of metal and has a greater mass than ball 2, which is made of wood. The upward force due to air resistance is the same for both balls. **(a)** Is the drop time of ball 1 greater than, less than, or equal to the drop time of ball 2? **(b)** Choose the *best explanation* from among the following:

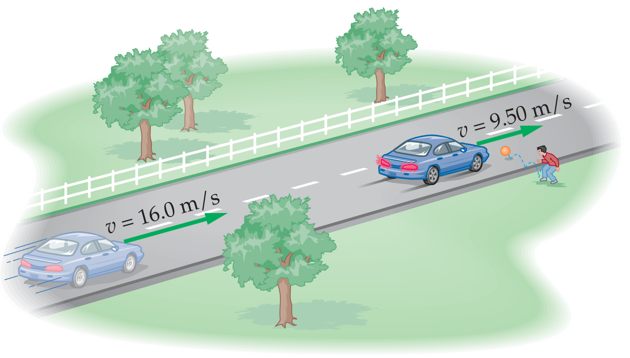
I. The acceleration of gravity is the same for all objects, regardless of mass.
II. The more massive ball is harder to accelerate.
III. Air resistance has less effect on the more massive ball.

8. •• **IP** A 42.0-kg parachutist is moving straight downward with a speed of 3.85 m/s. **(a)** If the parachutist comes to rest with constant acceleration over a distance of 0.750 m, what force does the ground exert on her? **(b)** If the parachutist comes to rest over a shorter distance, is the force exerted by the ground greater than, less than, or the same as in part (a)? Explain.

9. •• **IP** In baseball, a pitcher can accelerate a 0.15-kg ball from rest to 98 mi/h in a distance of 1.7 m. **(a)** What is the average force exerted on the ball during the pitch? **(b)** If the mass of the ball is increased, is the force required of the pitcher increased, decreased, or unchanged? Explain.

10. •• A major-league catcher gloves a 92-mi/h pitch and brings it to rest in 0.15 m. If the force exerted by the catcher is 803 N, what is the mass of the ball?

11. •• Driving home from school one day, you spot a ball rolling out into the street (**Figure 5–18**). You brake for 1.20 s, slowing your 950-kg car from 16.0 m/s to 9.50 m/s. **(a)** What was the average force exerted on your car during braking? **(b)** How far did you travel while braking?



▲ **FIGURE 5–18** Problem 11

12. •• **Stopping a 747** A 747 jetliner lands and begins to slow to a stop as it moves along the runway. If its mass is 3.50×10^5 kg, its speed is 27.0 m/s, and the net braking force is 4.30×10^5 N, **(a)** what is its speed 7.50 s later? **(b)** How far has it traveled in this time?

13. •• **IP** A drag racer crosses the finish line doing 202 mi/h and promptly deploys her drag chute (the small parachute used for braking). **(a)** What force must the drag chute exert on the 891-kg car to slow it to 45.0 mi/h in a distance of 185 m? **(b)** Describe the strategy you used to solve part (a).

SECTION 5–4 NEWTON'S THIRD LAW OF MOTION

14. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the magnitude of the force experienced by the car greater than, less than, or equal to the magnitude of the force experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. Action-reaction forces always have equal magnitude.
II. The truck has more mass, and hence the force exerted on it is greater.
III. The massive truck exerts a greater force on the lightweight car.

15. • **CE Predict/Explain** A small car collides with a large truck. **(a)** Is the acceleration experienced by the car greater than, less than, or equal to the acceleration experienced by the truck? **(b)** Choose the *best explanation* from among the following:

I. The truck exerts a larger force on the car, giving it the greater acceleration.
II. Both vehicles experience the same magnitude of force, therefore the lightweight car experiences the greater acceleration.
III. The greater force exerted on the truck gives it the greater acceleration.

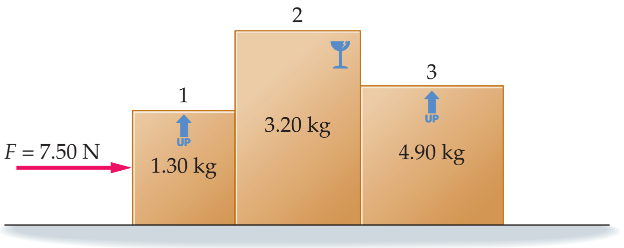
16. • You hold a brick at rest in your hand. **(a)** How many forces act on the brick? **(b)** Identify these forces. **(c)** Are these forces equal in magnitude and opposite in direction? **(d)** Are these forces an action-reaction pair? Explain.

17. • Referring to Problem 16, you are now accelerating the brick upward. **(a)** How many forces act on the brick in this case? **(b)** Identify these forces. **(c)** Are these forces equal in magnitude and opposite in direction? **(d)** Are these forces an action-reaction pair? Explain.

18. •• On vacation, your 1400-kg car pulls a 560-kg trailer away from a stoplight with an acceleration of $1.85\ \mathrm{m/s^2}$. **(a)** What is the net force exerted on the trailer? **(b)** What force does the trailer exert on the car? **(c)** What is the net force acting on the car?

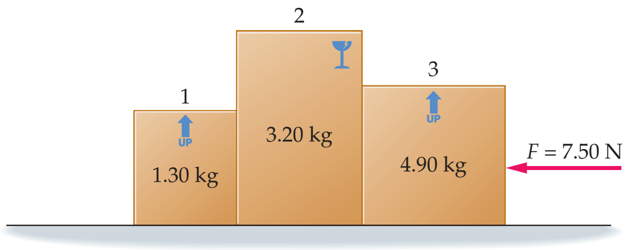
19. •• **IP** A 71-kg parent and a 19-kg child meet at the center of an ice rink. They place their hands together and push. **(a)** Is the force experienced by the child more than, less than, or the same as the force experienced by the parent? **(b)** Is the acceleration of the child more than, less than, or the same as the acceleration of the parent? Explain. **(c)** If the acceleration of the child is $2.6\ \mathrm{m/s^2}$ in magnitude, what is the magnitude of the parent's acceleration?

20. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in **Figure 5–19**. Find the magnitude of the contact force **(a)** between boxes 1 and 2, and **(b)** between boxes 2 and 3.



▲ **FIGURE 5–19** Problem 20

21. •• A force of magnitude 7.50 N pushes three boxes with masses $m_1 = 1.30$ kg, $m_2 = 3.20$ kg, and $m_3 = 4.90$ kg, as shown in **Figure 5–20**. Find the magnitude of the contact force **(a)** between boxes 1 and 2, and **(b)** between boxes 2 and 3.



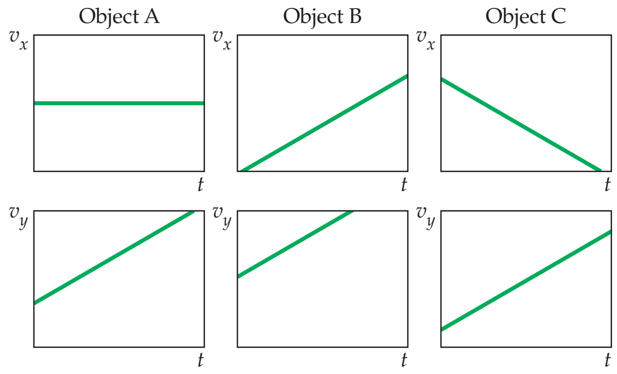
▲ **FIGURE 5–20** Problem 21

22. •• **IP** Two boxes sit side-by-side on a smooth horizontal surface. The lighter box has a mass of 5.2 kg; the heavier box has a mass of 7.4 kg. **(a)** Find the contact force between these boxes when a horizontal force of 5.0 N is applied to the light box. **(b)** If the 5.0-N force is applied to the heavy box instead, is the contact force between the boxes the same as, greater than, or less than the contact force in part (a)? Explain. **(c)** Verify your answer to part (b) by calculating the contact force in this case.

SECTION 5–5 THE VECTOR NATURE OF FORCES

23. • **CE** A skateboarder on a ramp is accelerated by a nonzero net force. For each of the following statements, state whether it is always true, never true, or sometimes true. **(a)** The skateboarder is moving in the direction of the net force. **(b)** The acceleration of the skateboarder is at right angles to the net force. **(c)** The acceleration of the skateboarder is in the same direction as the net force. **(d)** The skateboarder is instantaneously at rest.

24. • **CE** Three objects, A, B, and C, have x and y components of velocity that vary with time as shown in **Figure 5–21**. What is the direction of the net force acting on **(a)** object A, **(b)** object B, and **(c)** object C, as measured from the positive x axis? (All of the nonzero slopes have the same magnitude.)

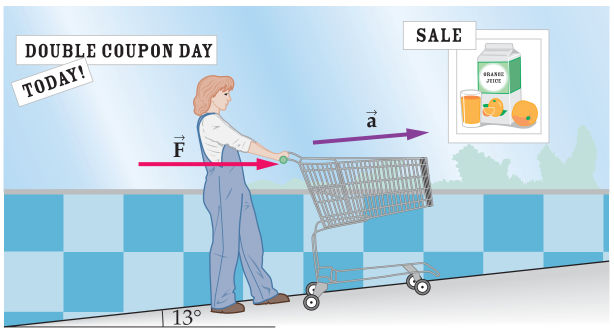


▲ **FIGURE 5–21** Problem 24

25. • A farm tractor tows a 3700-kg trailer up an 18° incline with a steady speed of 3.2 m/s. What force does the tractor exert on the trailer? (Ignore friction.)

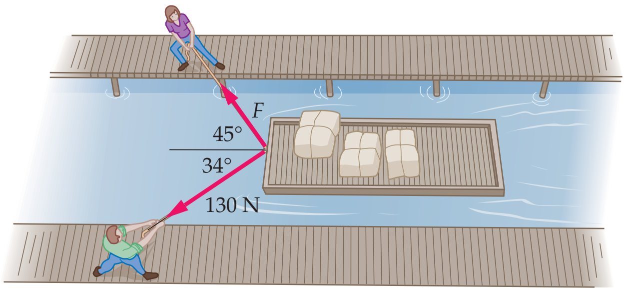
26. • A surfer "hangs ten," and accelerates down the sloping face of a wave. If the surfer's acceleration is $3.25\ \mathrm{m/s^2}$ and friction can be ignored, what is the angle at which the face of the wave is inclined above the horizontal?

27. • A shopper pushes a 7.5-kg shopping cart up a 13° incline, as shown in **Figure 5–22**. Find the magnitude of the horizontal force, $\vec{\mathbf{F}}$, needed to give the cart an acceleration of $1.41\ \mathrm{m/s^2}$.



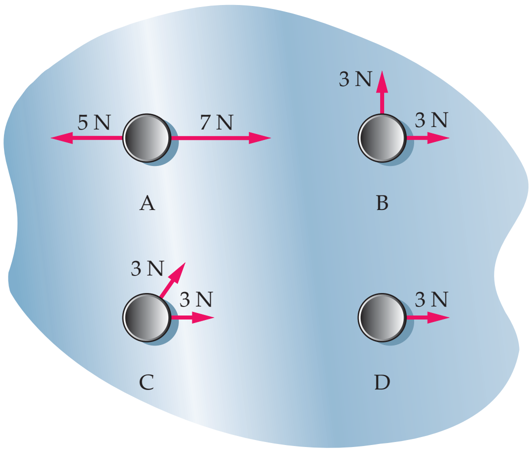
▲ **FIGURE 5–22** Problem 27

28. • Two crewmen pull a raft through a lock, as shown in **Figure 5–23**. One crewman pulls with a force of 130 N at an angle of 34° relative to the forward direction of the raft. The second crewman, on the opposite side of the lock, pulls at an angle of 45°. With what force should the second crewman pull so that the net force of the two crewmen is in the forward direction?



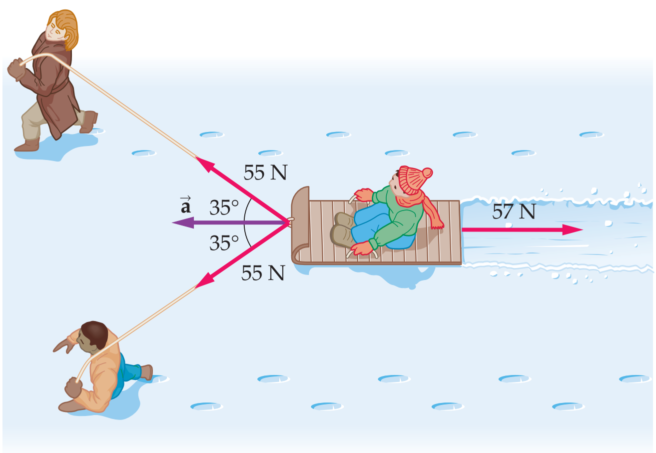
▲ **FIGURE 5–23** Problem 28

29. •• **CE** A hockey puck is acted on by one or more forces, as shown in **Figure 5–24**. Rank the four cases, A, B, C, and D, in order of the magnitude of the puck's acceleration, starting with the smallest. Indicate ties where appropriate.



▲ **FIGURE 5–24** Problem 29

30. •• To give a 19-kg child a ride, two teenagers pull on a 3.7-kg sled with ropes, as indicated in **Figure 5–25**. Both teenagers pull with a force of 55 N at an angle of 35° relative to the forward direction, which is the direction of motion. In addition, the snow exerts a retarding force on the sled that points opposite to the direction of motion, and has a magnitude of 57 N. Find the acceleration of the sled and child.



▲ **FIGURE 5–25** Problem 30

31. •• **IP** Before practicing his routine on the rings, a 67-kg gymnast stands motionless, with one hand grasping each ring and his feet touching the ground. Both arms slope upward at an angle of 24° above the horizontal. **(a)** If the force exerted by the rings on each arm has a magnitude of 290 N, and is directed along the length of the arm, what is the magnitude of the force exerted by the floor on his feet? **(b)** If the angle his arms make with the horizontal is greater that 24°, and everything else remains the same, is the force exerted by the floor on his feet greater than, less than, or the same as the value found in part (a)? Explain.

32. •• **IP** A 65-kg skier speeds down a trail, as shown in **Figure 5–26**. The surface is smooth and inclined at an angle of 22° with the horizontal. **(a)** Find the direction and magnitude of the net force acting on the skier. **(b)** Does the net force exerted on the skier increase, decrease, or stay the same as the slope becomes steeper? Explain.

▲ **FIGURE 5–26** Problems 32 and 45

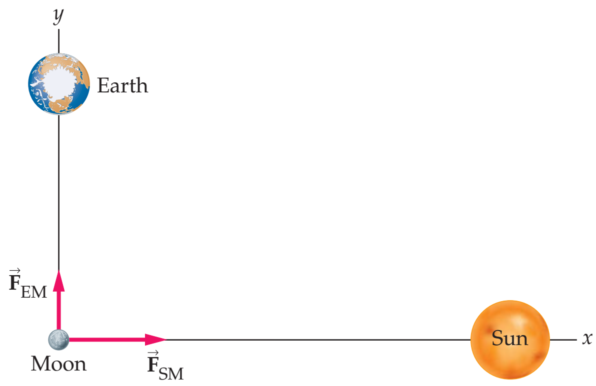
33. •• An object acted on by three forces moves with constant velocity. One force acting on the object is in the positive x direction and has a magnitude of 6.5 N; a second force has a magnitude of 4.4 N and points in the negative y direction. Find the direction and magnitude of the third force acting on the object.

34. •• A train is traveling up a 3.73° incline at a speed of 3.25 m/s when the last car breaks free and begins to coast without friction. **(a)** How long does it take for the last car to come to rest momentarily? **(b)** How far did the last car travel before momentarily coming to rest?

35. •• **The Force Exerted on the Moon** **Figure 5–27** shows the Earth, Moon, and Sun (not to scale) in their relative positions at the time when the Moon is in its third-quarter phase. Though few people realize it, the force exerted on the Moon by the Sun is actually greater than the force exerted on the Moon by the Earth. In fact, the force exerted on the Moon by the Sun has a magnitude of $F_{SM} = 4.34 \times 10^{20}$ N, whereas the force exerted by the Earth has a magnitude of only $F_{EM} = 1.98 \times 10^{20}$ N. These forces are indicated to scale in Figure 5–27. Find **(a)** the direction and **(b)** the magnitude of the net force acting on the Moon. **(c)** Given that the mass of the Moon is $M_M = 7.35 \times 10^{22}$ kg, find the magnitude of its acceleration at the time of the third-quarter phase.

SECTION 5–6 WEIGHT

36. • You pull upward on a stuffed suitcase with a force of 105 N, and it accelerates upward at 0.705 m/s^2. What are **(a)** the mass and **(b)** the weight of the suitcase?



▲ **FIGURE 5–27** Problem 35

37. • **BIO Brain Growth** A newborn baby's brain grows rapidly. In fact, it has been found to increase in mass by about 1.6 mg per minute. **(a)** How much does the brain's weight increase in one day? **(b)** How long does it take for the brain's weight to increase by 0.15 N?

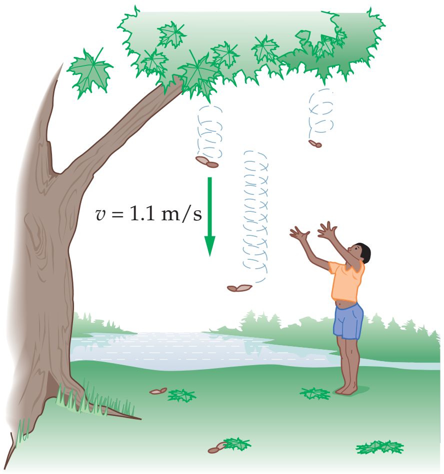
38. • Suppose a rocket launches with an acceleration of 30.5 m/s^2. What is the apparent weight of an 92-kg astronaut aboard this rocket?

39. • At the bow of a ship on a stormy sea, a crewman conducts an experiment by standing on a bathroom scale. In calm waters, the scale reads 182 lb. During the storm, the crewman finds a maximum reading of 225 lb and a minimum reading of 138 lb. Find **(a)** the maximum upward acceleration and **(b)** the maximum downward acceleration experienced by the crewman.

40. •• **IP** As part of a physics experiment, you stand on a bathroom scale in an elevator. Though your normal weight is 610 N, the scale at the moment reads 730 N. **(a)** Is the acceleration of the elevator upward, downward, or zero? Explain. **(b)** Calculate the magnitude of the elevator's acceleration. **(c)** What, if anything, can you say about the velocity of the elevator? Explain.

41. •• When you weigh yourself on good old *terra firma* (solid ground), your weight is 142 lb. In an elevator your apparent weight is 121 lb. What are the direction and magnitude of the elevator's acceleration?

42. •• **IP BIO Flight of the Samara** A 1.21-g samara—the winged fruit of a maple tree—falls toward the ground with a constant speed of 1.1 m/s (**Figure 5–28**). **(a)** What is the force of air resistance exerted on the samara? **(b)** If the constant speed of descent is greater than 1.1 m/s, is the force of air resistance greater than, less than, or the same as in part (a)? Explain.



▲ **FIGURE 5–28** Problem 42

43. ••• When you lift a bowling ball with a force of 82 N, the ball accelerates upward with an acceleration a. If you lift with a force of 92 N, the ball's acceleration is $2a$. Find **(a)** the weight of the bowling ball, and **(b)** the acceleration a.

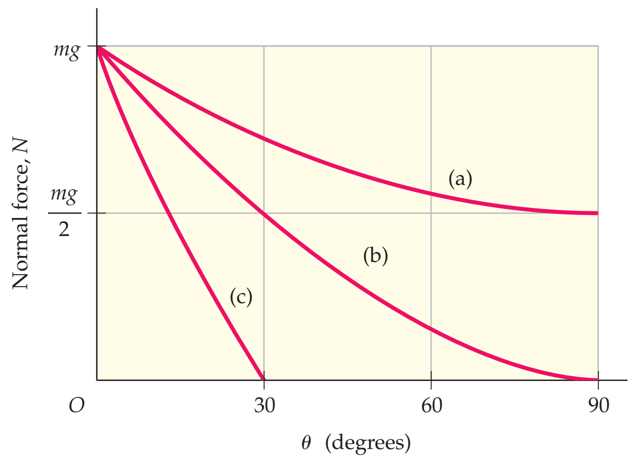
SECTION 5–7 NORMAL FORCES

44. • A 23-kg suitcase is being pulled with constant speed by a handle that is at an angle of 25° above the horizontal. If the normal force exerted on the suitcase is 180 N, what is the force F applied to the handle?

45. • **(a)** Draw a free-body diagram for the skier in Problem 32. **(b)** Determine the normal force acting on the skier.

46. • A 9.3-kg child sits in a 3.7-kg high chair. **(a)** Draw a free-body diagram for the child, and find the normal force exerted by the chair on the child. **(b)** Draw a free-body diagram for the chair, and find the normal force exerted by the floor on the chair.

47. •• **Figure 5–29** shows the normal force as a function of the angle θ for the suitcase shown in Figure 5–13. Determine the magnitude of the force $\vec{F}$ for each of the three curves shown in Figure 5–29. Give your answer in terms of the weight of the suitcase, mg.



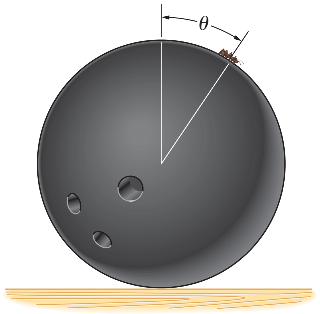
▲ **FIGURE 5–29** Problem 47

48. •• A 5.0-kg bag of potatoes sits on the bottom of a stationary shopping cart. **(a)** Sketch a free-body diagram for the bag of potatoes. **(b)** Now suppose the cart moves with a constant velocity. How does this affect your free-body diagram? Explain.

49. •• **IP** **(a)** Find the normal force exerted on a 2.9-kg book resting on a surface inclined at 36° above the horizontal. **(b)** If the angle of the incline is reduced, do you expect the normal force to increase, decrease, or stay the same? Explain.

50. •• **IP** A gardener mows a lawn with an old-fashioned push mower. The handle of the mower makes an angle of 35° with the surface of the lawn. **(a)** If a 219-N force is applied along the handle of the 19-kg mower, what is the normal force exerted by the lawn on the mower? **(b)** If the angle between the surface of the lawn and the handle of the mower is increased, does the normal force exerted by the lawn increase, decrease, or stay the same? Explain.

51. ••• An ant walks slowly away from the top of a bowling ball, as shown in **Figure 5–30**. If the ant starts to slip when the normal force on its feet drops below one-half its weight, at what angle θ does slipping begin?



▲ **FIGURE 5–30** Problem 51

GENERAL PROBLEMS

52. • **CE Predict/Explain** Riding in an elevator moving upward with constant speed, you begin a game of darts. **(a)** Do you have to aim your darts higher than, lower than, or the same as when you play darts on solid ground? **(b)** Choose the *best explanation* from among the following:
    - **I.** The elevator rises during the time it takes for the dart to travel to the dartboard.
    - **II.** The elevator moves with constant velocity. Therefore, Newton's laws apply within the elevator in the same way as on the ground.
    - **III.** You have to aim lower to compensate for the upward speed of the elevator.

53. • **CE Predict/Explain** Riding in an elevator moving with a constant upward acceleration, you begin a game of darts. **(a)** Do you have to aim your darts higher than, lower than, or the same as when you play darts on solid ground? **(b)** Choose the *best explanation* from among the following:
    - **I.** The elevator accelerates upward, giving its passengers a greater "effective" acceleration of gravity.
    - **II.** You have to aim lower to compensate for the upward acceleration of the elevator.
    - **III.** Since the elevator moves with a constant acceleration, Newton's laws apply within the elevator the same as on the ground.

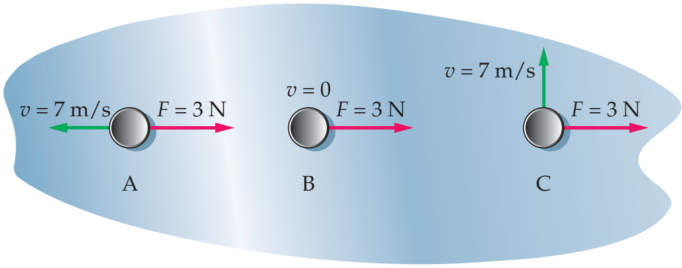
54. • **CE** Give the direction of the net force acting on each of the following objects. If the net force is zero, state "zero." **(a)** A car accelerating northward from a stoplight. **(b)** A car traveling southward and slowing down. **(c)** A car traveling westward with constant speed. **(d)** A skydiver parachuting downward with constant speed. **(e)** A baseball during its flight from pitcher to catcher (ignoring air resistance).

55. • **CE Predict/Explain** You jump out of an airplane and open your parachute after an extended period of free fall. **(a)** To decelerate your fall, must the force exerted on you by the parachute be greater than, less than, or equal to your weight? **(b)** Choose the *best explanation* from among the following:
    - **I.** Parachutes can only exert forces that are less than the weight of the skydiver.
    - **II.** The parachute exerts a force exactly equal to the skydiver's weight.
    - **III.** To decelerate after free fall, the net force acting on a skydiver must be upward.

56. • In a tennis serve, a 0.070-kg ball can be accelerated from rest to 36 m/s over a distance of 0.75 m. Find the magnitude of the average force exerted by the racket on the ball during the serve.

57. • A 51.5-kg swimmer with an initial speed of 1.25 m/s decides to coast until she comes to rest. If she slows with constant acceleration and stops after coasting 2.20 m, what was the force exerted on her by the water?

58. •• **CE** Each of the three identical hockey pucks shown in **Figure 5–31** is acted on by a 3-N force. Puck A moves with a speed of 7 m/s in a direction opposite to the force; puck B is instantaneously at rest; puck C moves with a speed of 7 m/s at right angles to the force. Rank the three pucks in order of the magnitude of their acceleration, starting with the smallest. Indicate ties with an equal sign.



▲ **FIGURE 5–31** Problem 58

59. •• **IP The VASIMR Rocket** NASA plans to use a new type of rocket, a Variable Specific Impulse Magnetoplasma Rocket (VASIMR), on future missions. A VASIMR can produce 1200 N of thrust (force) when in operation. If a VASIMR has a mass of 2.2×10^5 kg, **(a)** what acceleration will it experience? Assume that the only force acting on the rocket is its own thrust, and that the mass of the rocket is constant. **(b)** Over what distance must the rocket accelerate from rest to achieve a speed of 9500 m/s? **(c)** When the rocket has covered one-quarter the acceleration distance found in part (b), is its average speed 1/2, 1/3, or 1/4 its average speed during the final three-quarters of the acceleration distance? Explain.

60. •• An object of mass $m = 5.95$ kg has an acceleration $\vec{\mathbf{a}} = (1.17\text{ m/s}^2)\hat{\mathbf{x}} + (-0.664\text{ m/s}^2)\hat{\mathbf{y}}$. Three forces act on this object: $\vec{\mathbf{F}}_1$, $\vec{\mathbf{F}}_2$, and $\vec{\mathbf{F}}_3$. Given that $\vec{\mathbf{F}}_1 = (3.22\text{ N})\hat{\mathbf{x}}$ and $\vec{\mathbf{F}}_2 = (-1.55\text{ N})\hat{\mathbf{x}} + (2.05\text{ N})\hat{\mathbf{y}}$, find $\vec{\mathbf{F}}_3$.

61. •• At the local grocery store, you push a 14.5-kg shopping cart. You stop for a moment to add a bag of dog food to your cart. With a force of 12.0 N, you now accelerate the cart from rest through a distance of 2.29 m in 3.00 s. What was the mass of the dog food?

62. •• **IP BIO The Force of Running** Biomechanical research has shown that when a 67-kg person is running, the force exerted on each foot as it strikes the ground can be as great as 2300 N. **(a)** What is the ratio of the force exerted on the foot by the ground to the person's body weight? **(b)** If the only forces acting on the person are (i) the force exerted by the ground and (ii) the person's weight, what are the magnitude and direction of the person's acceleration? **(c)** If the acceleration found in part (b) acts for 10.0 ms, what is the resulting change in the vertical component of the person's velocity?

63. •• **IP BIO Grasshopper Liftoff** To become airborne, a 2.0-g grasshopper requires a takeoff speed of 2.7 m/s. It acquires this speed by extending its hind legs through a distance of 3.7 cm. **(a)** What is the average acceleration of the grasshopper during takeoff? **(b)** Find the magnitude of the average net force exerted

on the grasshopper by its hind legs during takeoff. **(c)** If the mass of the grasshopper increases, does the takeoff acceleration increase, decrease, or stay the same? **(d)** If the mass of the grasshopper increases, does the required takeoff force increase, decrease, or stay the same? Explain.

64. •• **Takeoff from an Aircraft Carrier** On an aircraft carrier, a jet can be catapulted from 0 to 155 mi/h in 2.00 s. If the average force exerted by the catapult is 9.35×10^5 N, what is the mass of the jet?

A jet takes off from the flight deck of an aircraft carrier. (Problem 64)

65. •• **IP** An archer shoots a 0.024-kg arrow at a target with a speed of 54 m/s. When it hits the target, it penetrates to a depth of 0.083 m. **(a)** What was the average force exerted by the target on the arrow? **(b)** If the mass of the arrow is doubled, and the force exerted by the target on the arrow remains the same, by what multiplicative factor does the penetration depth change? Explain.

66. •• An apple of mass $m = 0.13$ kg falls out of a tree from a height $h = 3.2$ m. **(a)** What is the magnitude of the force of gravity, mg, acting on the apple? **(b)** What is the apple's speed, v, just before it lands? **(c)** Show that the force of gravity times the height, mgh, is equal to $\frac{1}{2}mv^2$. (We shall investigate the significance of this result in Chapter 8.) Be sure to show that the dimensions are in agreement as well as the numerical values.

67. •• An apple of mass $m = 0.22$ kg falls from a tree and hits the ground with a speed of $v = 14$ m/s. **(a)** What is the magnitude of the force of gravity, mg, acting on the apple? **(b)** What is the time, t, required for the apple to reach the ground? **(c)** Show that the force of gravity times the time, mgt, is equal to mv. (We shall investigate the significance of this result in Chapter 9.) Be sure to show that the dimensions are in agreement as well as the numerical values.

68. •• **BIO The Fall of *T. rex*** Paleontologists estimate that if a *Tyrannosaurus rex* were to trip and fall, it would have experienced a force of approximately 260,000 N acting on its torso when it hit the ground. Assuming the torso has a mass of 3800 kg, **(a)** find the magnitude of the torso's upward acceleration as it comes to rest. (For comparison, humans lose consciousness with an acceleration of about $7g$.) **(b)** Assuming the torso is in free fall for a distance of 1.46 m as it falls to the ground, how much time is required for the torso to come to rest once it contacts the ground?

69. •• **Deep Space I** The NASA spacecraft *Deep Space I* was shut down on December 18, 2001, following a three-year journey to the asteroid Braille and the comet Borrelly. This spacecraft used a solar-powered ion engine to produce 0.064 ounces of thrust (force) by stripping electrons from neon atoms and accelerating the resulting ions to 70,000 mi/h. The thrust was only as much as the weight of a couple sheets of paper, but the engine operated continuously for 16,000 hours. As a result, the speed of the spacecraft increased by 7900 mi/h. What was the mass of *Deep Space I*? (Assume that the mass of the neon gas is negligible.)

70. •• Your groceries are in a bag with paper handles. The handles will tear off if a force greater than 51.5 N is applied to them. What is the greatest mass of groceries that can be lifted safely with this bag, given that the bag is raised **(a)** with constant speed, or **(b)** with an acceleration of 1.25 m/s^2?

71. •• **IP** While waiting at the airport for your flight to leave, you observe some of the jets as they take off. With your watch you find that it takes about 35 seconds for a plane to go from rest to takeoff speed. In addition, you estimate that the distance required is about 1.5 km. **(a)** If the mass of a jet is 1.70×10^5 kg, what force is needed for takeoff? **(b)** Describe the strategy you used to solve part (a).

72. •• **BIO Gecko Feet** Researchers have found that a gecko's foot is covered with hundreds of thousands of small hairs (*setae*) that allow it to walk up walls and even across ceilings. A single foot pad, which has an area of 1.0 cm^2, can attach to a wall or ceiling with a force of 11 N. **(a)** How many 250-g geckos could be suspended from the ceiling by a single foot pad? **(b)** Estimate the force per square centimeter that your body exerts on the soles of your shoes, and compare with the 11 N/cm^2 of the sticky gecko foot.

A Tokay gecko (*Gekko gecko*) shows off its famous feet. (Problem 72)

73. •• Two boxes are at rest on a smooth, horizontal surface. The boxes are in contact with one another. If box 1 is pushed with a force of magnitude $F = 12.00$ N, the contact force between the boxes is 8.50 N; if, instead, box 2 is pushed with the force F, the contact force is $12.00\text{ N} - 8.50\text{ N} = 3.50$ N. In either case, the boxes move together with an acceleration of 1.70 m/s^2. What is the mass of **(a)** box 1 and **(b)** box 2?

74. ••• **IP** Responding to an alarm, a 102-kg fireman slides down a pole to the ground floor, 3.3 m below. The fireman starts at rest and lands with a speed of 4.2 m/s. **(a)** Find the average force exerted on the fireman by the pole. **(b)** If the landing speed is half that in part (a), is the average force exerted on the fireman by the pole doubled? Explain. **(c)** Find the average force exerted on the fireman by the pole when the landing speed is 2.1 m/s.

75. ••• For a birthday gift, you and some friends take a hot-air balloon ride. One friend is late, so the balloon floats a couple of feet off the ground as you wait. Before this person arrives, the combined weight of the basket and people is 1220 kg, and the balloon is neutrally buoyant. When the late arrival climbs up into the basket, the balloon begins to accelerate downward at 0.56 m/s^2. What was the mass of the last person to climb aboard?

76. ••• A baseball of mass m and initial speed v strikes a catcher's mitt. If the mitt moves a distance Δx as it brings the ball to rest, what is the average force it exerts on the ball?

77. ••• When two people push in the same direction on an object of mass m they cause an acceleration of magnitude a_1. When the same people push in opposite directions, the acceleration of the object has a magnitude a_2. Determine the magnitude of the force exerted by each of the two people in terms of m, a_1, and a_2.

78. ••• An air-track cart of mass $m_1 = 0.14$ kg is moving with a speed $v_0 = 1.3$ m/s to the right when it collides with a cart of mass $m_2 = 0.25$ kg that is at rest. Each cart has a wad of putty on its bumper, and hence they stick together as a result of their collision. Suppose the average contact force between the carts is $F = 1.5$ N during the collision. **(a)** What is the acceleration of cart 1? Give direction and magnitude. **(b)** What is the acceleration of cart 2? Give direction and magnitude. **(c)** How long does it take for both carts to have the same speed? (Once the carts have the same speed the collision is over and the contact force vanishes.) **(d)** What is the final speed of the carts, v_f? **(e)** Show that $m_1 v_0$ is equal to $(m_1 + m_2)v_f$. (We shall investigate the significance of this result in Chapter 9.)

PASSAGE PROBLEMS

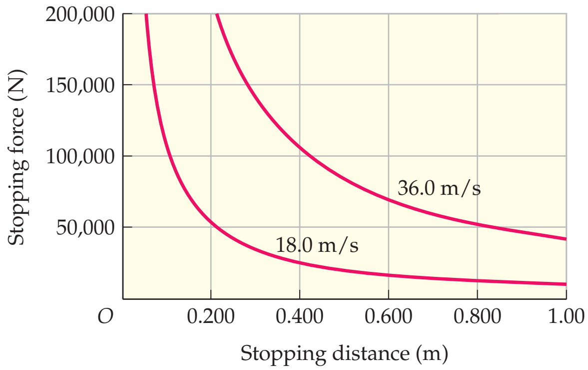
BIO Increasing Safety in a Collision

Safety experts say that an automobile accident is really a succession of three separate collisions. These can be described as follows: (1) the automobile collides with an obstacle and comes to rest; (2) people within the car continue to move forward until they collide with the interior of the car, or are brought to rest by a restraint system like a seatbelt or an air bag; (3) organs within the occupants' bodies continue to move forward until they collide with the body wall and are brought to rest. Not much can be done about the third collision, but the effects of the first two can be mitigated by increasing the distance over which the car and its occupants are brought to rest.

For example, the severity of the first collision is reduced by building collapsible "crumple zones" into the body of a car, and by placing compressible collision barriers near dangerous obstacles like bridge supports. The second collision is addressed primarily through the use of seatbelts and air bags. These devices reduce the force that acts on an occupant to survivable levels by increasing the distance over which he or she comes to rest. This is illustrated in Figure 5–32, where we see the force exerted on a 65.0-kg driver who slows from an initial speed of 18.0 m/s (lower curve) or 36.0 m/s (upper curve) to rest in a distance ranging from 5.00 cm to 1.00 m.

79. • The combination of "crumple zones" and air bags/seatbelts might increase the distance over which a person stops in a collision to as great as 1.00 m. What is the magnitude of the force exerted on a 65.0-kg driver who decelerates from 18.0 m/s to 0.00 m/s over a distance of 1.00 m?

A. 162 N	**B.** 585 N
C. 1.05×10^4 N	**D.** 2.11×10^4 N



▲ **FIGURE 5–32** Problems 79, 80, 81, and 82

80. • A driver who does not wear a seatbelt continues to move forward with a speed of 18.0 m/s (due to inertia) until something solid like the steering wheel is encountered. The driver now comes to rest in a much shorter distance—perhaps only a few centimeters. Find the magnitude of the net force acting on a 65.0-kg driver who is decelerated from 18.0 m/s to rest in 5.00 cm.

A. 3240 N	**B.** 1.17×10^4 N
C. 2.11×10^5 N	**D.** 4.21×10^5 N

81. • Suppose the initial speed of the driver is doubled to 36.0 m/s. If the driver still has a mass of 65.0 kg, and comes to rest in 1.00 m, what is the magnitude of the force exerted on the driver during this collision?

A. 648 N	**B.** 1170 N
C. 2.11×10^4 N	**D.** 4.21×10^4 N

82. • If both the speed and stopping distance of a driver are doubled, by what factor does the force exerted on the driver change?

A. 0.5	**B.** 1
C. 2	**D.** 4

INTERACTIVE PROBLEMS

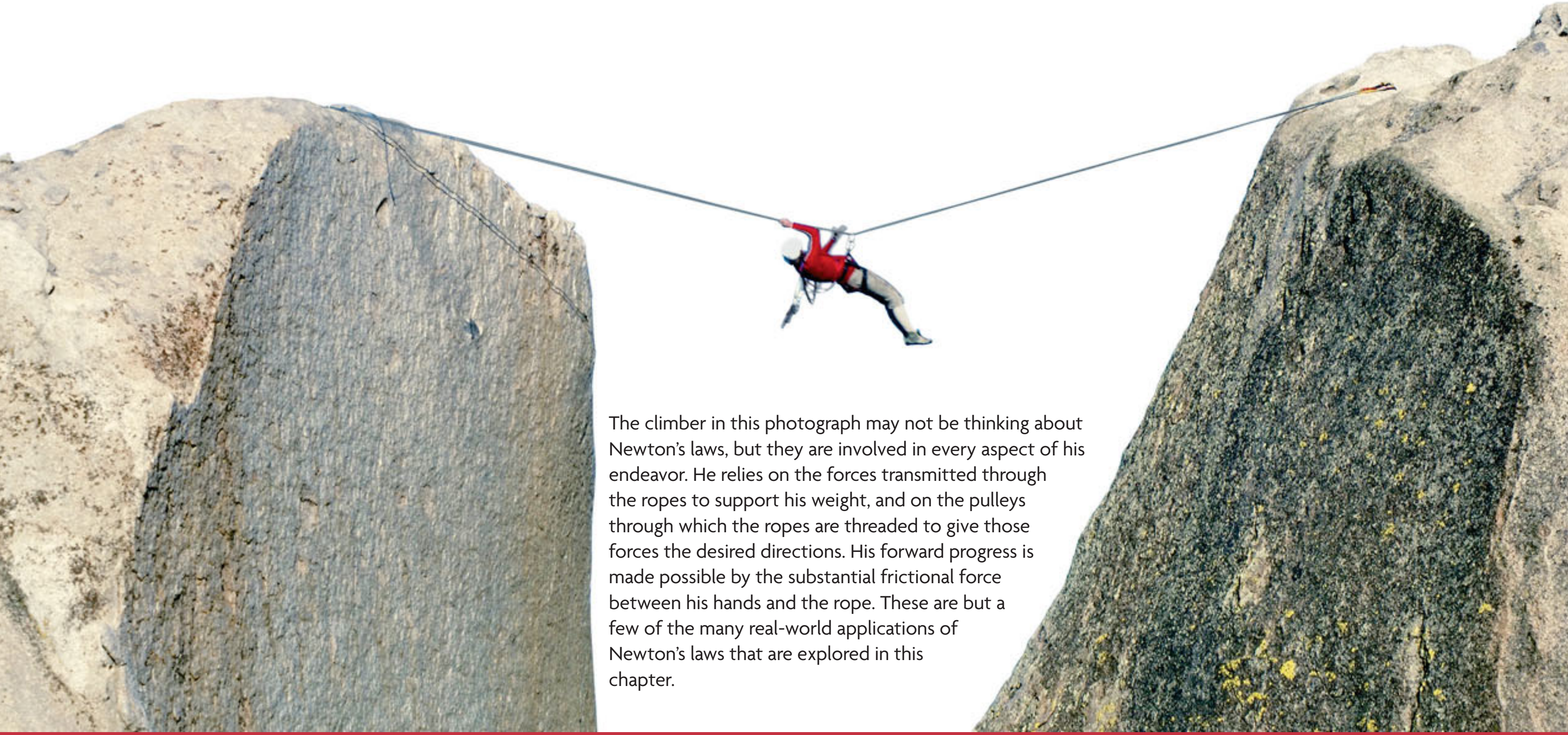
83. •• IP **Referring to Example 5–4** Suppose that we would like the contact force between the boxes to have a magnitude of 5.00 N, and that the only thing in the system we are allowed to change is the mass of box 2—the mass of box 1 is 10.0 kg and the applied force is 20.0 N. **(a)** Should the mass of box 2 be increased or decreased? Explain. **(b)** Find the mass of box 2 that results in a contact force of magnitude 5.00 N. **(c)** What is the acceleration of the boxes in this case?

84. •• **Referring to Example 5–4** Suppose the force of 20.0 N pushes on two boxes of unknown mass. We know, however, that the acceleration of the boxes is 1.20 m/s^2 and the contact force has a magnitude of 4.45 N. Find the mass of **(a)** box 1 and **(b)** box 2.

85. •• IP **Referring to Figure 5–9** Suppose the magnitude of $\vec{F}_2$ is increased from 41 N to 55 N, and that everything else in the system remains the same. **(a)** Do you expect the direction of the satellite's acceleration to be greater than, less than, or equal to 32°? Explain. Find **(b)** the direction and **(c)** the magnitude of the satellite's acceleration in this case.

86. •• IP **Referring to Figure 5–9** Suppose we would like the acceleration of the satellite to be at an angle of 25°, and that the only quantity we can change in the system is the magnitude of $\vec{F}_1$. **(a)** Should the magnitude of $\vec{F}_1$ be increased or decreased? Explain. **(b)** What is the magnitude of the satellite's acceleration in this case?

6 Applications of Newton's Laws



The climber in this photograph may not be thinking about Newton's laws, but they are involved in every aspect of his endeavor. He relies on the forces transmitted through the ropes to support his weight, and on the pulleys through which the ropes are threaded to give those forces the desired directions. His forward progress is made possible by the substantial frictional force between his hands and the rope. These are but a few of the many real-world applications of Newton's laws that are explored in this chapter.

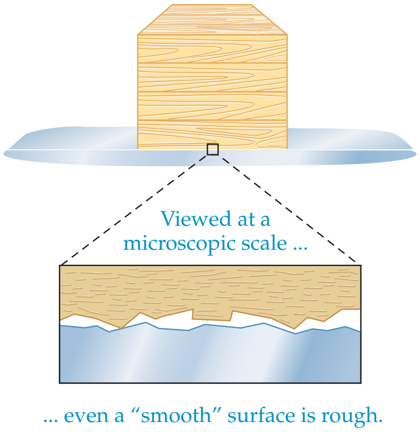
Newton's laws of motion can be applied to an immense variety of systems, a sampling of which was discussed in Chapter 5. In this chapter we extend our discussion of Newton's laws by introducing new types of forces and by considering new classes of systems.

For example, we begin by considering the forces due to friction between two surfaces. As we shall see, the force of friction is different depending on whether the surfaces are in static contact, or are moving relative to one another—an important consideration in antilock braking systems. And though friction may seem like something that should be eliminated, we show that it is actually essential to life as we know it.

Next, we investigate the forces exerted by strings and springs, and show how these forces can safely suspend a mountain climber over a chasm, or cushion the ride of a locomotive. Finally, we consider the key role that force plays in making circular motion possible.

6–1 Frictional Forces

In Chapter 5 we always assumed that surfaces were smooth and that objects could slide without resistance to their motion. No surface is perfectly smooth, however. When viewed on the atomic level, even the "smoothest" surface is actually rough and jagged, as indicated in **Figure 6–1**. To slide one such surface across another requires a force large enough to overcome the resistance of microscopic hills and valleys bumping together. This is the origin of the force we call **friction.**



▲ **FIGURE 6–1 The origin of friction**
Even "smooth" surfaces have irregularities when viewed at the microscopic level. This type of roughness contributes to friction.

We often think of friction as something that should be reduced, or even eliminated if possible. For example, roughly 20% of the gasoline you buy does nothing but overcome friction within your car's engine. Clearly, reducing that friction would be most desirable.

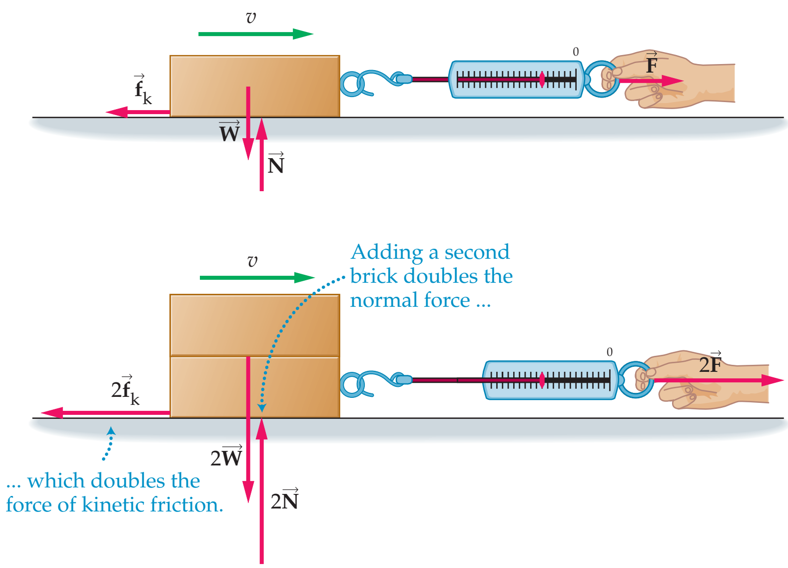
On the other hand, friction can be helpful—even indispensable—in other situations. Suppose, for example, that you are standing still and then decide to begin walking forward. The force that accelerates you is the force of friction between your shoes and the ground. We simply couldn't walk or run without friction—it's hard enough when friction is merely reduced, as on an icy sidewalk. Similarly, starting or stopping a car, or even turning a corner, all require friction. Friction is an important and common feature of everyday life.

Since friction is caused by the random, microscopic irregularities of a surface, and since it is greatly affected by other factors such as the presence of lubricants, there is no simple "law of nature" for friction. There are, however, some very useful rules of thumb that give us rather accurate, approximate results for calculating frictional forces. In what follows, we describe these rules of thumb for the two types of friction most commonly used in this text—kinetic friction and static friction.

Kinetic Friction

As its name implies, kinetic friction is the friction encountered when surfaces slide against one another with a finite relative speed. The force generated by this friction, which will be designated with the symbol f_k, acts to oppose the sliding motion at the point of contact between the surfaces.

A series of simple experiments illustrates the main characteristics of kinetic friction. First, imagine attaching a spring scale to a rough object, like a brick, and pulling it across a table, as shown in **Figure 6–2**. If the brick moves with constant velocity, Newton's second law tells us that the net force on the brick must be zero. Hence, the force read on the scale, F, has the same magnitude as the force of kinetic friction, f_k. Now, if we repeat the experiment, but this time put a second brick on top of the first, we find that the force needed to pull the brick with constant velocity is doubled, to $2F$.



▶ **FIGURE 6–2 The force of kinetic friction depends on the normal force**
In the top part of the figure, a force F is required to pull the brick with constant speed v. Thus the force of kinetic friction is $f_k = F$. In the bottom part of the figure, the normal force has been doubled, and so has the force of kinetic friction, to $f_k = 2F$.

From this experiment we see that when we double the normal force—by stacking up two bricks, for example—the force of kinetic friction is also doubled. In general, the force of kinetic friction is found to be proportional to the magnitude of the normal force, N. Stated mathematically, this observation can be written as follows:

$$f_k = \mu_k N \qquad 6\text{–}1$$

The constant of proportionality, μ_k (pronounced "mew sub k"), is referred to as the **coefficient of kinetic friction.** In Figure 6–2 the normal force is equal to the weight of the bricks, but this is a special case. The normal force is greater than the weight if someone pushes down on the bricks, and this would cause more friction, or less than the weight if the bricks are placed on an incline. The former case is considered in several homework problems, and the latter case is considered in Examples 6–2 and 6–3.

Since f_k and N are both forces, and hence have the same units, we see that μ_k is a dimensionless number. The coefficient of kinetic friction is always positive, and typical values range between 0 and 1, as indicated in Table 6–1. The interpretation of μ_k is simple: If $\mu_k = 0.1$, for example, the force of kinetic friction is one-tenth of the normal force. Simply put, the greater μ_k the greater the friction; the smaller μ_k the smaller the friction.

TABLE 6–1 Typical Coefficients of Friction

Materials	Kinetic, μ_k	Static, μ_s
Rubber on concrete (dry)	0.80	1–4
Steel on steel	0.57	0.74
Glass on glass	0.40	0.94
Wood on leather	0.40	0.50
Copper on steel	0.36	0.53
Rubber on concrete (wet)	0.25	0.30
Steel on ice	0.06	0.10
Waxed ski on snow	0.05	0.10
Teflon on Teflon	0.04	0.04
Synovial joints in humans	0.003	0.01

As we know from everyday experience, the force of kinetic friction tends to oppose motion, as shown in Figure 6–2. Thus, $f_k = \mu_k N$ is not a vector equation, because N is perpendicular to the direction of motion. When doing calculations with the force of kinetic friction, we use $f_k = \mu_k N$ to find its magnitude, and we draw its direction so that it is opposite to the direction of motion.

There are two more friction experiments of particular interest. First, suppose that when we pull a brick, we initially pull it at the speed v, then later at the speed $2v$. What forces do we measure? It turns out that the force of kinetic friction is approximately the same in each case—it certainly does not double when we double the speed. Second, let's try standing the brick on end, so that it has a smaller area in contact with the table. If this smaller area is half the previous area, is the force halved? No, the force remains essentially the same, regardless of the area of contact.

We summarize these observations with the following three rules of thumb for kinetic friction:

Rules of Thumb for Kinetic Friction

The force of kinetic friction between two surfaces is:

1. Proportional to the magnitude of the normal force, N, between the surfaces:

$$f_k = \mu_k N$$

2. Independent of the relative speed of the surfaces.
3. Independent of the area of contact between the surfaces.

▲ Friction plays an important role in almost everything we do. Sometimes it is desirable to reduce friction; in other cases we want as much friction as possible. For example, it is more fun to ride on a water slide (upper) if the friction is low. Similarly, an engine operates more efficiently when it is oiled. When running, however, we need friction to help us speed up, slow down, and make turns. The sole of this running shoe (lower), like a car tire, is designed to maximize friction.

Again, these rules are useful and fairly accurate, though they are still only approximate. For simplicity, when we do calculations involving kinetic friction in this text, we will use these rules as if they were exact.

Before we show how to use f_k in calculations, we should make a comment regarding rule 3. This rule often seems rather surprising and counterintuitive. How is it that a larger area of contact doesn't produce a larger force? One way to think about this is to consider that when the area of contact is large, the normal force is spread out over a large area, giving a small force per area, F/A. As a result, the microscopic hills and valleys are not pressed too deeply against one another. On the other hand, if the area is small, the normal force is concentrated in a small region, which presses the surfaces together more firmly, due to the large force per area. The net effect is roughly the same in either case.

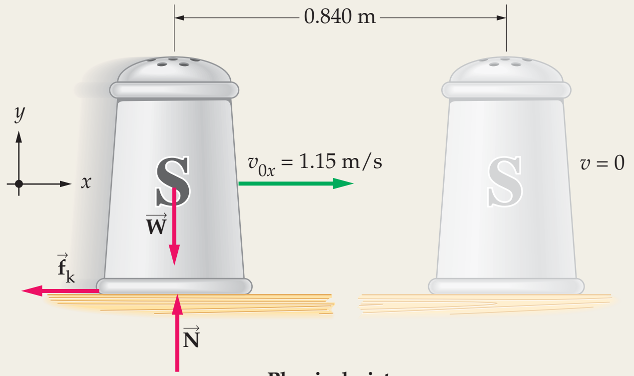
Now, let's consider a commonly encountered situation in which kinetic friction plays a decisive role.

EXAMPLE 6–1 PASS THE SALT—PLEASE

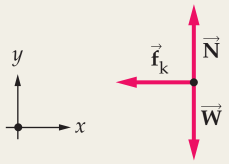
Someone at the other end of the table asks you to pass the salt. Feeling quite dashing, you slide the 50.0-g salt shaker in their direction, giving it an initial speed of 1.15 m/s. **(a)** If the shaker comes to rest with constant acceleration in 0.840 m, what is the coefficient of kinetic friction between the shaker and the table? **(b)** How much time is required for the shaker to come to rest if you slide it with an initial speed of 1.32 m/s?

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion, and the positive y direction to be upward. Two forces act in the y direction; the shaker's weight, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$, and the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$. Only one force acts in the x direction: the force of kinetic friction, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$. Note that the shaker moves through a distance of 0.840 m with an initial speed $v_{0x} = 1.15$ m/s.



Physical picture



Free-body diagram

STRATEGY

a. Since the frictional force has a magnitude of $f_k = \mu_k N$, it follows that $\mu_k = f_k/N$. Therefore, we need to find the magnitudes of the frictional force, f_k, and the normal force, N. To find f_k we set $\sum F_x = ma_x$, and find a_x with the kinematic equation $v_x^2 = v_{0x}^2 + 2a_x\Delta x$. To find N we set $a_y = 0$ (since there is no motion in the y direction) and solve for N using $\sum F_y = ma_y = 0$.

b. The coefficient of kinetic friction is independent of the sliding speed, and hence the acceleration of the shaker is also independent of the speed. As a result, we can use the acceleration from part (a) in the relation $v_x = v_{0x} + a_x t$ to find the sliding time.

SOLUTION

Part (a)

1. Set $\Sigma F_x = ma_x$ to find f_k in terms of a_x:
$$\sum F_x = -f_k = ma_x \quad \text{or} \quad f_k = -ma_x$$

2. Determine a_x by using the kinematic equation relating velocity to position, $v_x^2 = v_{0x}^2 + 2a_x\Delta x$:
$$v_x^2 = v_{0x}^2 + 2a_x\Delta x$$
$$a_x = \frac{v_x^2 - v_{0x}^2}{2\Delta x} = \frac{0 - (1.15\ \text{m/s})^2}{2(0.840\ \text{m})} = -0.787\ \text{m/s}^2$$

3. Set $\Sigma F_y = ma_y = 0$ to find the normal force, N:
$$\sum F_y = N + (-W) = ma_y = 0 \quad \text{or} \quad N = W = mg$$

4. Substitute $N = mg$ and $f_k = -ma_x$ (with $a_x = -0.787$ m/s^2) into $\mu_k = f_k/N$ to find μ_k:
$$\mu_k = \frac{f_k}{N} = \frac{-ma_x}{mg} = \frac{-a_x}{g} = \frac{-(-0.787\ \text{m/s}^2)}{9.81\ \text{m/s}^2} = 0.0802$$

Part (b)

5. Use $a_x = -0.787\ \text{m/s}^2$, $v_{0x} = 1.32\ \text{m/s}$, and $v_x = 0$ in $v_x = v_{0x} + a_x t$ to solve for the time, t:

$$v_x = v_{0x} + a_x t \quad \text{or}$$

$$t = \frac{v_x - v_{0x}}{a_x} = \frac{0 - (1.32\ \text{m/s})}{-0.787\ \text{m/s}^2} = 1.68\ \text{s}$$

INSIGHT

Note that m canceled in Step 4, so our result for the coefficient of friction is independent of the shaker's mass. For example, if we were to slide a shaker with twice the mass, but with the same initial speed, it would slide the same distance. It is unlikely this independence would have been apparent if we had worked the problem numerically rather than symbolically. Part (b) shows that the same comments apply to the sliding time—it too is independent of the shaker's mass.

PRACTICE PROBLEM

Given the same initial speed and a coefficient of kinetic friction equal to 0.120, what are **(a)** the acceleration of the shaker, and **(b)** the distance it slides? [**Answer: (a)** $a_x = -1.18\ \text{m/s}^2$, **(b)** 0.560 m]

Some related homework problems: Problem 3, Problem 18

In the next Example we consider a system that is inclined at an angle θ relative to the horizontal. As a result, the normal force responsible for the kinetic friction is less than the weight of the object. To be very clear about how we handle the force vectors in such a case, we begin by resolving each vector into its x and y components.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Incline

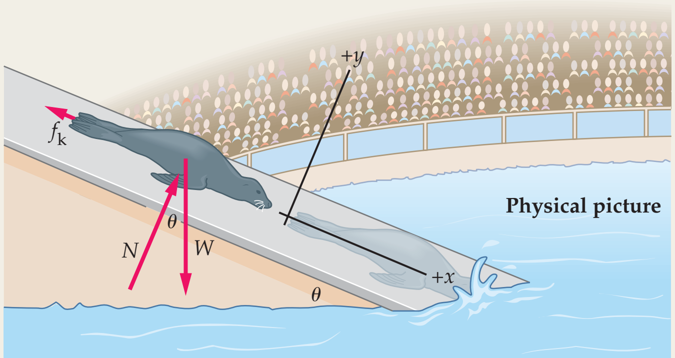
On an incline, align one axis (x) parallel to the surface, and the other axis (y) perpendicular to the surface. That way the motion is in the x direction. Since no motion occurs in the y direction, we know that $a_y = 0$.

EXAMPLE 6–2 MAKING A BIG SPLASH

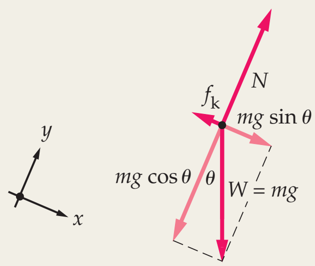
A trained sea lion slides from rest with constant acceleration down a 3.0-m-long ramp into a pool of water. If the ramp is inclined at an angle of 23° above the horizontal and the coefficient of kinetic friction between the sea lion and the ramp is 0.26, how long does it take for the sea lion to make a splash in the pool?

PICTURE THE PROBLEM

As is usual with inclined surfaces, we choose one axis to be parallel to the surface and the other to be perpendicular to it. In our sketch, the sea lion accelerates in the positive x direction ($a_x > 0$), having started from rest, $v_{0x} = 0$. We are free to choose the initial position of the sea lion to be $x_0 = 0$. There is no motion in the y direction, and therefore $a_y = 0$. Finally, we note from the free-body diagram that $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, $\vec{\mathbf{f}}_k = -\mu_k N\hat{\mathbf{x}}$, and $\vec{\mathbf{W}} = (mg \sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.



Physical picture



Free-body diagram

STRATEGY

We can use the kinematic equation relating position to time, $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to find the time of the sea lion's slide. It will be necessary, however, to first determine the acceleration of the sea lion in the x direction, a_x.

To find a_x we apply Newton's second law to the sea lion. First, we can find N by setting $\Sigma F_y = ma_y$ equal to zero (since $a_y = 0$). It is important to start by finding N because we need it to find the force of kinetic friction, $f_k = \mu_k N$. Using f_k in the sum of forces in the x direction, $\Sigma F_x = ma_x$, allows us to solve for a_x and, finally, for the time.

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SOLUTION

1. We begin by resolving each of the three force vectors into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{k,x} = -f_k = -\mu_k N \qquad f_{k,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$ to find N:

$$\sum F_y = N - mg\cos\theta = ma_y = 0$$

We see that N is less than the weight, mg:

$$N = mg\cos\theta$$

3. Next, set $\Sigma F_x = ma_x$:

$$\sum F_x = mg\sin\theta - \mu_k$$

Note that the mass cancels in this equation:

$$= mg\sin\theta - \mu_k mg\cos\theta = ma_x$$

4. Solve for the acceleration in the x direction, a_x:

$$a_x = g(\sin\theta - \mu_k\cos\theta)$$
$$= (9.81\ \mathrm{m/s^2})[\sin 23° - (0.26)\cos 23°]$$
$$= 1.5\ \mathrm{m/s^2}$$

5. Use $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$ to find the time when the sea lion reaches the bottom. We choose $x_0 = 0$, and we are given that $v_{0x} = 0$, hence we set $x = \frac{1}{2}a_x t^2 = 3.0$ m and solve for t:

$$x = \tfrac{1}{2}a_x t^2$$
$$t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2(3.0\ \mathrm{m})}{1.5\ \mathrm{m/s^2}}} = 2.0\ \mathrm{s}$$

INSIGHT

Note that we don't need the sea lion's mass to find the time. On the other hand, if we wanted the magnitude of the force of kinetic friction, $f_k = \mu_k N = \mu_k mg\cos\theta$, the mass would be needed.

It is useful to compare the sliding salt shaker in Example 6–1 with the sliding sea lion in this Example. In the case of the salt shaker, friction is the only force acting along the direction of motion (opposite to the direction of motion, in fact), and it brings the object to rest. Because of the slope on which the sea lion slides, however, it experiences both a component of its weight in the forward direction and the friction force opposite to the motion. Since the component of the weight is the larger of the two forces, the sea lion accelerates down the slope—friction only acts to slow its progress.

PRACTICE PROBLEM

How long would it take the sea lion to reach the water if there were no friction in this system? [**Answer:** 1.3 s]

Some related homework problems: Problem 11, Problem 72

Static Friction

Static friction tends to keep two surfaces from moving relative to one another. It, like kinetic friction, is due to the microscopic irregularities of surfaces that are in contact. In fact, static friction is typically stronger than kinetic friction because when surfaces are in static contact, their microscopic hills and valleys can nestle down deeply into one another, thus forming a strong connection between the surfaces that may even include molecular bonding. In kinetic friction, the surfaces bounce along relative to one another and don't become as firmly enmeshed.

As we did with kinetic friction, let's use the results of some simple experiments to determine the rules of thumb for static friction. We start with a brick at rest on a table, with no horizontal force pulling on it, as in **Figure 6–3**. Of course, in this case the force of static friction is zero; no force is needed to keep the brick from sliding.

Next, attach a spring scale to the brick and pull with a small force of magnitude F_1, a force small enough that the brick doesn't move. Since the brick is still at rest, it follows that the force of static friction, f_s, is equal in magnitude to the applied force; that is, $f_s = F_1$. Now, increase the applied force to a new value, F_2, which is still small enough that the brick stays at rest. In this case, the force of static friction has also increased so that $f_s = F_2$. If we continue increasing the applied force, we eventually reach a value beyond which the brick starts to move and kinetic friction takes over, as shown in the figure. Thus, there is an upper limit to the force that can be exerted by static friction, and we call this upper limit $f_{s,\max}$.

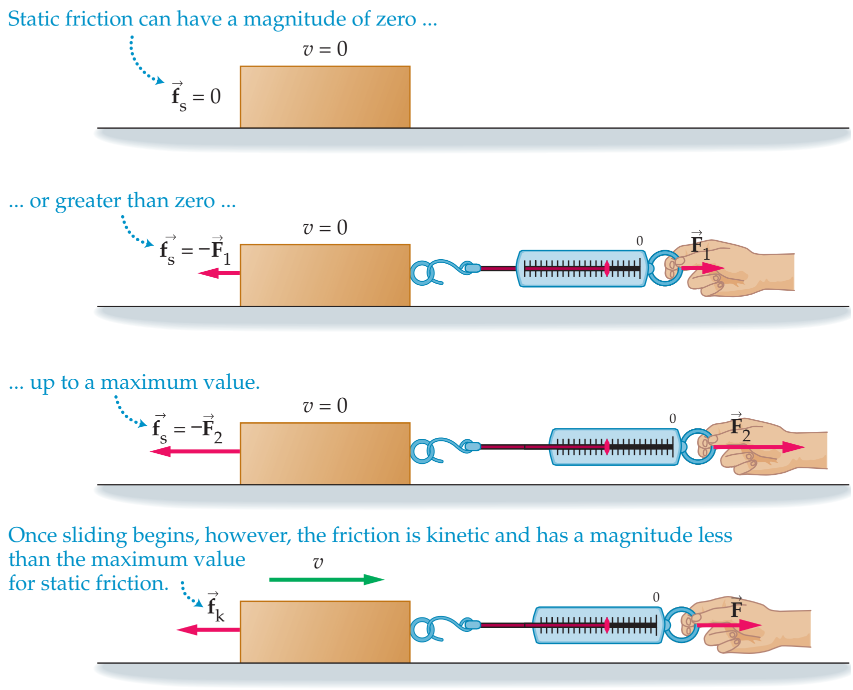


FIGURE 6–3 The maximum limit of static friction

As the force applied to an object increases, so does the force of static friction—up to a certain point. Beyond this maximum value, static friction can no longer hold the object, and it begins to slide. Now kinetic friction takes over.

To summarize, the force of static friction, f_s, can have any value between zero and $f_{s,max}$. This can be written mathematically as follows:

$$0 \leq f_s \leq f_{s,max} \qquad 6\text{–}2$$

Imagine repeating the experiment, only now with a second brick on top of the first. This doubles the normal force and it also doubles the maximum force of static friction. Thus, the maximum force is proportional to the magnitude of the normal force, or

$$f_{s,max} = \mu_s N \qquad 6\text{–}3$$

The constant of proportionality is called μ_s (pronounced "mew sub s"), the **coefficient of static friction.** Note that μ_s, like μ_k, is dimensionless. Typical values are given in Table 6–1. In most cases, μ_s is greater than μ_k, indicating that the force of static friction is greater than the force of kinetic friction, as mentioned. In fact, it is not uncommon for μ_s to be greater than 1, as in the case of rubber in contact with dry concrete.

Finally, two additional comments regarding the nature of static friction: (i) Experiments show that static friction, like kinetic friction, is independent of the area of contact. (ii) The force of static friction is not in the direction of the normal force, thus $f_{s,max} = \mu_s N$ is not a vector relation. The direction of f_s is parallel to the surface of contact, and opposite to the direction the object would move if there were no friction.

These observations are summarized in the following rules of thumb:

▲ The coefficient of static friction between two surfaces depends on many factors, including whether the surfaces are dry or wet. On the desert floor of Death Valley, California, occasional rains can reduce the friction between rocks and the sandy ground to such an extent that strong winds can move the rocks over considerable distances. This results in linear "rock trails," which record the direction of the winds at different times.

Rules of Thumb for Static Friction

The force of static friction between two surfaces has the following properties:

1. It takes on any value between zero and the maximum possible force of static friction, $f_{s,max} = \mu_s N$:

$$0 \leq f_s \leq \mu_s N$$

2. It is independent of the area of contact between the surfaces.
3. It is parallel to the surface of contact, and in the direction that opposes relative motion.

Next, we consider a practical method of determining the coefficient of static friction. As with the last Example, we begin by resolving all relevant force vectors into their x and y components.

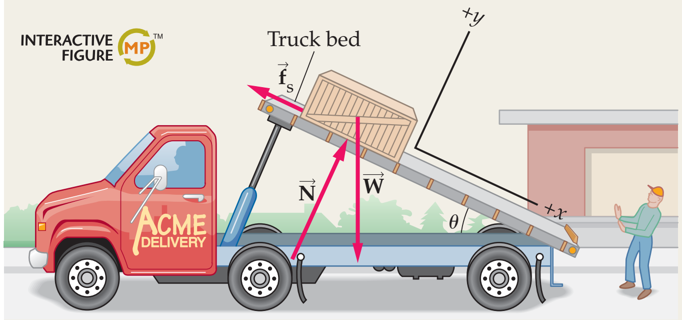
EXAMPLE 6–3 SLIGHTLY TILTED

A flatbed truck slowly tilts its bed upward to dispose of a 95.0-kg crate. For small angles of tilt the crate stays put, but when the tilt angle exceeds 23.2°, the crate begins to slide. What is the coefficient of static friction between the bed of the truck and the crate?

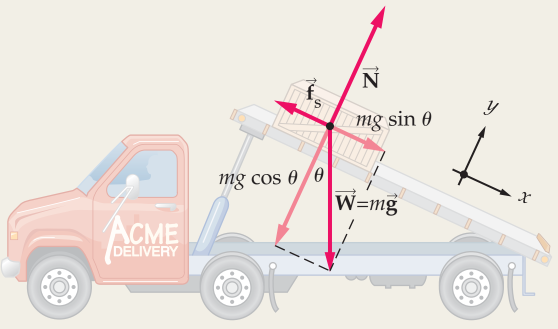
PICTURE THE PROBLEM

We align our coordinate system with the incline, and choose the positive x direction to point down the slope. Note that three forces act on the crate: the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$, the force of static friction, $\vec{\mathbf{f}}_s = -\mu_s N\hat{\mathbf{x}}$, and the weight, $\vec{\mathbf{W}} = (mg\sin\theta)\hat{\mathbf{x}} + (-mg\cos\theta)\hat{\mathbf{y}}$.

INTERACTIVE FIGURE



Physical picture



Free-body diagram

STRATEGY

When the crate is on the verge of slipping, but has not yet slipped, its acceleration is zero in both the x and y directions. In addition, "verge of slipping" means that the magnitude of the static friction is at its maximum value, $f_s = f_{s,max} = \mu_s N$. Thus, we set $\Sigma F_y = ma_y = 0$ to find N, then use $\Sigma F_x = ma_x = 0$ to find μ_s.

SOLUTION

1. Resolve the three force vectors acting on the crate into x and y components:

$$N_x = 0 \qquad N_y = N$$
$$f_{s,x} = -f_{s,max} = -\mu_s N \qquad f_{s,y} = 0$$
$$W_x = mg\sin\theta \qquad W_y = -mg\cos\theta$$

2. Set $\Sigma F_y = ma_y = 0$, since $a_y = 0$.

$$\sum F_y = N_y + f_{s,y} + W_y = N + 0 - mg\cos\theta = ma_y = 0$$

Solve for the normal force, N:

$$N = mg\cos\theta$$

3. Set $\Sigma F_x = ma_x = 0$, since the crate is at rest, and use the result for N obtained in Step 2:

$$\sum F_x = N_x + f_{s,x} + W_x = ma_x = 0$$
$$= 0 - \mu_s N + mg\sin\theta$$
$$= 0 - \mu_s mg\cos\theta + mg\sin\theta$$

4. Solve the expression for the coefficient of static friction, μ_s:

$$\mu_s mg\cos\theta = mg\sin\theta$$
$$\mu_s = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta = \tan 23.2° = 0.429$$

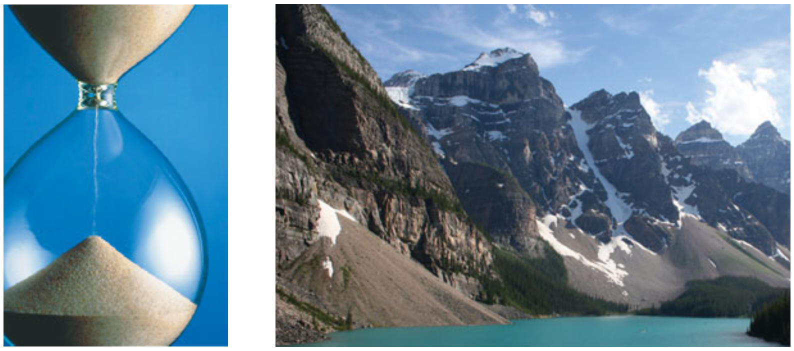
INSIGHT

In general, if an object is on the verge of slipping when the surface on which it rests is tilted at an angle θ_c, the coefficient of static friction between the object and the surface is $\mu_s = \tan\theta_c$. Note that this result is independent of the mass of the object. In particular, the critical angle for this crate is precisely the same whether it is filled with feathers or lead bricks.

PRACTICE PROBLEM

Find the magnitude of the force of static friction acting on the crate. [**Answer:** $f_{s,max} = \mu_s N = 367$ N]

Some related homework problems: Problem 12, Problem 82



The angle that the sloping sides of a sand pile (left) make with the horizontal is determined by the coefficient of static friction between grains of sand, in much the same way that static friction determines the angle at which the crate in Example 6–3 begins to slide. The same basic mechanism determines the angle of the cone-shaped mass of rock debris at the base of a cliff, known as a talus slope (right).

Recall that static friction can have magnitudes less than its maximum possible value. This point is emphasized in the following Active Example.

ACTIVE EXAMPLE 6–1 THE FORCE OF STATIC FRICTION

In the previous Example, what is the magnitude of the force of static friction acting on the crate when the truck bed is tilted at an angle of 20.0°?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Sum the x components of force acting on the crate: $\sum F_x = 0 - f_s + mg \sin\theta$
2. Set this sum equal to zero (since $a_x = 0$) and solve for the magnitude of the static friction force, f_s: $f_s = mg \sin\theta$
3. Substitute numerical values, including $\theta = 20.0°$: $f_s = 319\text{ N}$

INSIGHT

Notice that the force of static friction in this case has a magnitude (319 N) that is less than the value of 367 N found in the Practice Problem of Example 6–3, even though the coefficient of static friction is precisely the same.

YOUR TURN

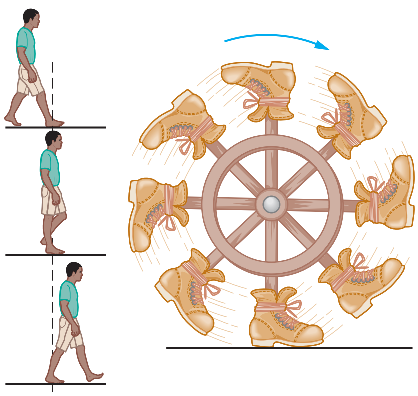
At what tilt angle will the force of static friction have a magnitude of 225 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Finally, friction often enters into problems dealing with vehicles with rolling wheels. In Conceptual Checkpoint 6–1, we consider which type of friction is appropriate in such cases.

CONCEPTUAL CHECKPOINT 6–1 FRICTION FOR ROLLING TIRES

A car drives with its tires rolling freely. Is the friction between the tires and the road **(a)** kinetic or **(b)** static?



REASONING AND DISCUSSION

A reasonable-sounding answer is that because the car is moving, the friction between its tires and the road must be kinetic friction—but this is not the case.

Actually, the friction is static because the bottom of the tire is in static contact with the road. To understand this, watch your feet as you walk. Even though you are moving, each foot is in static contact with the ground once you step down on it. Your foot doesn't move again until you lift it up and move it forward for the next step. A tire can be thought of as a succession of feet arranged in a circle, each of which is momentarily in static contact with the ground.

ANSWER

(b) The friction between the tires and the road is static friction.

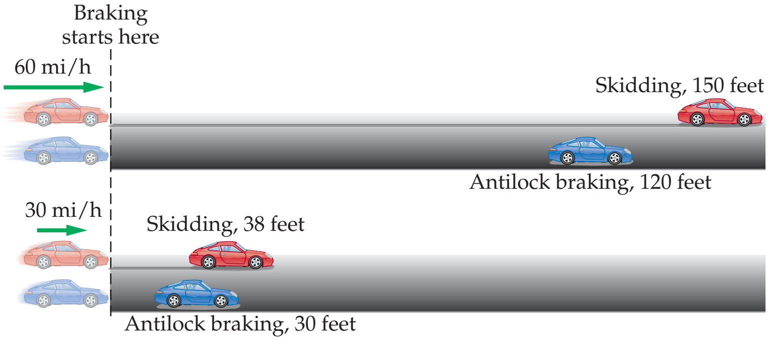
REAL-WORLD PHYSICS
Antilock braking systems

To summarize, if a car skids, the friction acting on it is kinetic; if its wheels are rolling, the friction is static. Since static friction is generally greater than kinetic friction, it follows that a car can be stopped in less distance if its wheels are rolling (static friction) than if its wheels are locked up (kinetic friction). This is the idea behind the antilock braking systems (ABS) that are available on many cars. When the brakes are applied in a car with ABS, an electronic rotation sensor at each wheel detects whether the wheel is about to start skidding. To prevent skidding, a small computer automatically begins to modulate the hydraulic pressure in the brake lines in short bursts, causing the brakes to release and then reapply in rapid succession. This allows the wheels to continue rotating, even in an emergency stop, and for static friction to determine the stopping distance. **Figure 6–4** shows a comparison of braking distances for cars with and without ABS. An added benefit of ABS is that a driver is better able to steer and control a braking car if its wheels are rotating.

(a) Front wheels locked; rear wheels free to turn

(b) Rear wheels locked; front wheels free to turn

▲ **Static Versus Kinetic Friction** Each of the two photos above shows five images of a toy car as it slides down an inclined surface. **(a)** In this photo the front wheels are locked, and skid on the surface, but the rear wheels roll without slipping. This means the front wheels experience kinetic friction and the rear wheels experience static friction. Because the force of kinetic friction is usually less than the force of static friction, the front wheels go down the incline first, pulling the rear wheels behind. **(b)** The situation is reversed in this photo, and the rear wheels are the ones that skid and experience a smaller frictional force. As a result, the rear wheels slide down the incline more quickly than the front wheels, causing the car to spin around. This change in behavior, which could be dangerous in a real-life situation, illustrates the significant differences between static and kinetic friction.



▲ **FIGURE 6–4 Stopping distance with and without ABS**
Antilock braking systems (ABS) allow a car to stop with static friction rather than kinetic friction—even in a case where a person slams on the brakes. As a result, the braking distance is reduced, due to the fact that μ_s is typically greater than μ_k. Professional drivers can beat the performance of ABS by carefully adjusting the force they apply to the brake pedal during a stop, but ABS provides essentially the same performance—within a few percent—for a person who simply pushes the brake pedal to the floor and holds it there.

6–2 Strings and Springs

A common way to exert a force on an object is to pull on it with a string, a rope, a cable, or a wire. Similarly, you can push or pull on an object if you attach it to a spring. In this section we discuss the basic features of strings and springs and how they transmit forces.

Strings and Tension

Imagine picking up a light string and holding it with one end in each hand. If you pull to the right with your right hand with a force T and to the left with your left hand with a force T, the string becomes taut. In such a case, we say that there is a **tension** T in the string. To be more specific, if your friend were to cut the string at some point, the tension T is the force pulling the ends apart, as illustrated in **Figure 6–5**—that is, T is the force your friend would have to exert with each hand to hold the cut ends together. Note that at any given point, the tension pulls equally to the right and to the left.

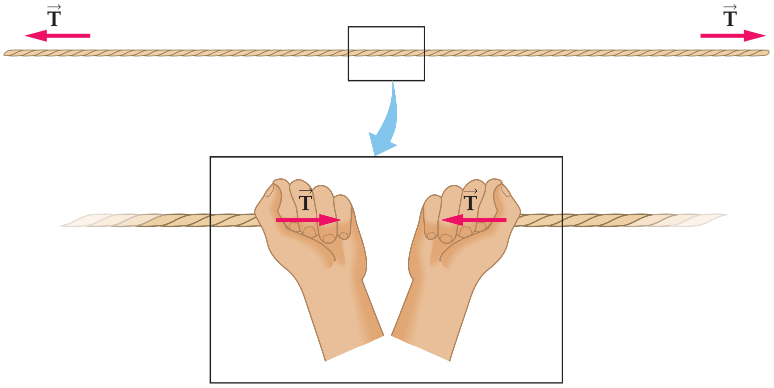


FIGURE 6–5 Tension in a string
A string, pulled from either end, has a tension, T. If the string were to be cut at any point, the force required to hold the ends together is T.

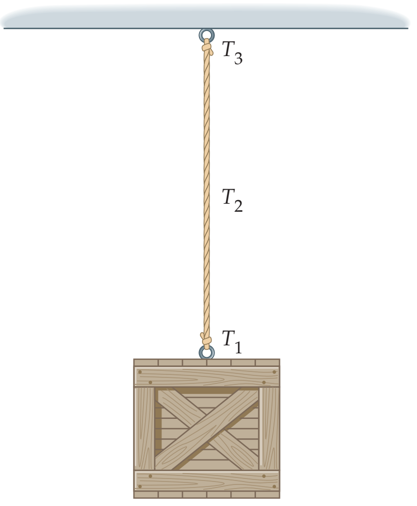


FIGURE 6–6 Tension in a heavy rope
Because of the weight of the rope, the tension is noticeably different at points 1, 2, and 3. As the rope becomes lighter, however, the difference in tension decreases. In the limit of a rope of zero mass, the tension is the same throughout the rope.

As an example, consider a rope that is attached to the ceiling at one end, and to a box with a weight of 105 N at the other end, as shown in **Figure 6–6**. In addition, suppose the rope is uniform, and that it has a total weight of 2.00 N. What is the tension in the rope (i) where it attaches to the box, (ii) at its midpoint, and (iii) where it attaches to the ceiling?

First, the rope holds the box at rest; thus, the tension where the rope attaches to the box is simply the weight of the box, $T_1 = 105$ N. At the midpoint of the rope, the tension supports the weight of the box, plus the weight of half the rope. Thus, $T_2 = 105\text{ N} + \frac{1}{2}(2.00\text{ N}) = 106$ N. Similarly, at the ceiling the tension supports the box plus all of the rope, giving a tension of $T_3 = 107$ N. Note that the tension pulls down on the ceiling but pulls up on the box.

From this discussion, we can see that the tension in the rope changes slightly from top to bottom because of the mass of the rope. If the rope had less mass, the difference in tension between its two ends would also be less. In particular, if the rope's mass were to be vanishingly small, the difference in tension would vanish as well. In this text, we will assume that all ropes, strings, wires, and so on are practically massless—unless specifically stated otherwise—and, hence, that the tension is the same throughout their length.

Pulleys are often used to redirect a force exerted by a string, as indicated in **Figure 6–7**. In the ideal case, a pulley has no mass and no friction in its bearings. Thus, *an ideal pulley simply changes the direction of the tension in a string, without changing its magnitude*. If a system contains more than one pulley, however, it is possible to arrange them in such a way as to "magnify a force," even if each pulley itself merely redirects the tension in a string. The traction device considered in the next Example shows one way this can be accomplished in a system that uses three ideal pulleys.

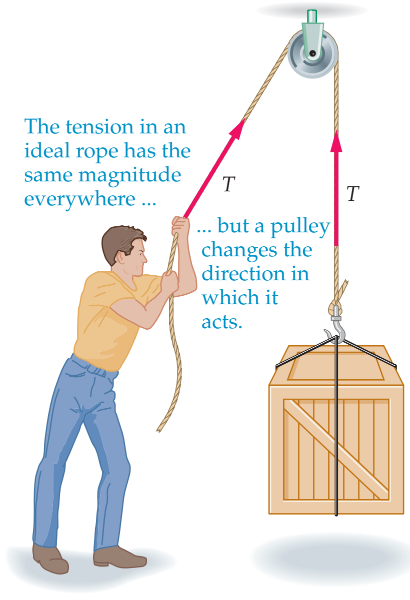


FIGURE 6–7 A pulley changes the direction of a tension

EXAMPLE 6–4 A BAD BREAK: SETTING A BROKEN LEG WITH TRACTION

A traction device employing three pulleys is applied to a broken leg, as shown in the sketch. The middle pulley is attached to the sole of the foot, and a mass m supplies the tension in the ropes. Find the value of the mass m if the force exerted on the sole of the foot by the middle pulley is to be 165 N.

CONTINUED ON NEXT PAGE

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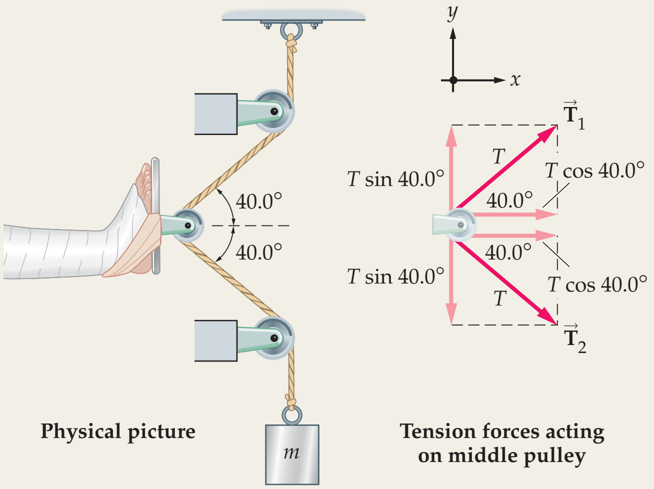
PICTURE THE PROBLEM

Our sketch shows the physical picture as well as the tension forces acting on the middle pulley. Notice that on the upper portion of the rope the tension is $\vec{\mathbf{T}}_1 = (T \cos 40.0°)\hat{\mathbf{x}} + (T \sin 40.0°)\hat{\mathbf{y}}$; on the lower portion it is $\vec{\mathbf{T}}_2 = (T \cos 40.0°)\hat{\mathbf{x}} + (-T \sin 40.0°)\hat{\mathbf{y}}$.



Physical picture

Tension forces acting on middle pulley

STRATEGY

We begin by noting that the rope supports the hanging mass m. As a result, the tension in the rope, T, must be equal in magnitude to the weight of the mass: $T = mg$.

Next, the pulleys simply change the direction of the tension without changing its magnitude. Therefore, the net force exerted on the sole of the foot is the sum of the tension T at 40.0° above the horizontal plus the tension T at 40.0° below the horizontal. We will calculate the net force component by component.

Once we calculate the net force acting on the foot, we set it equal to 165 N and solve for the tension T. Finally, we find the mass using the relation $T = mg$.

SOLUTION

1. First, consider the tension that acts upward and to the right on the middle pulley. Resolve this tension into x and y components:

$$T_{1,x} = T \cos 40.0° \qquad T_{1,y} = T \sin 40.0°$$

2. Next, consider the tension that acts downward and to the right on the middle pulley. Resolve this tension into x and y components. Note the minus sign in the y component:

$$T_{2,x} = T \cos 40.0° \qquad T_{2,y} = -T \sin 40.0°$$

3. Sum the x and y components of force acting on the middle pulley. We see that the net force acts only in the x direction, as one might expect from symmetry:

$$\sum F_x = T \cos 40.0° + T \cos 40.0° = 2T \cos 40.0°$$

$$\sum F_y = T \sin 40.0° - T \sin 40.0° = 0$$

4. Step 3 shows that the net force acting on the middle pulley is $2T \cos 40.0°$. Set this force equal to 165 N and solve for T:

$$2T \cos 40.0° = 165 \text{ N}$$

$$T = \frac{165 \text{ N}}{2 \cos 40.0°} = 108 \text{ N}$$

5. Solve for the mass, m, using $T = mg$:

$$T = mg$$

$$m = \frac{T}{g} = \frac{108 \text{ N}}{9.81 \text{ m/s}^2} = 11.0 \text{ kg}$$

INSIGHT

As pointed out earlier, this pulley arrangement "magnifies the force" in the sense that a 108-N weight attached to the rope produces a 165-N force exerted on the foot by the middle pulley. Note that the tension in the rope always has the same value—$T = 108$ N—as expected with ideal pulleys, but because of the arrangement of the pulleys the force applied to the foot by the rope is $2T \cos 40.0° > T$.

In addition, notice that the force exerted on the foot by the middle pulley produces an opposing force in the leg that acts in the direction of the head (a cephalad force), as desired to set a broken leg and keep it straight as it heals.

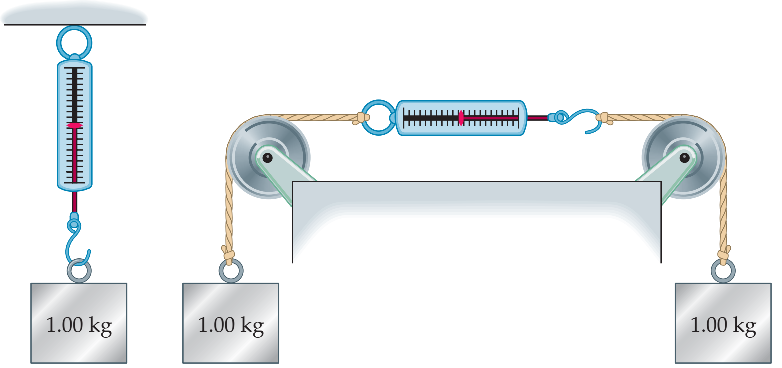
PRACTICE PROBLEM

(a) Would the required mass m increase or decrease if the angles in this device were changed from 40.0° to 30.0°? **(b)** Find the mass m for an angle of 30.0°. [**Answer: (a)** The required mass m would decrease. **(b)** 9.71 kg]

Some related homework problems: Problem 23, Problem 26, Problem 36

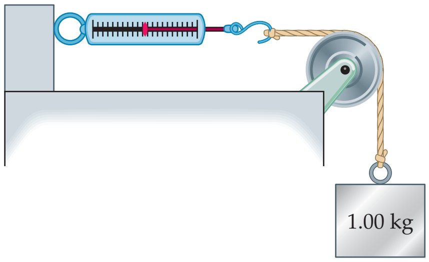
CONCEPTUAL CHECKPOINT 6–2 COMPARE THE READINGS ON THE SCALES

The scale at left reads 9.81 N. Is the reading of the scale at right **(a)** greater than 9.81 N, **(b)** equal to 9.81 N, or **(c)** less than 9.81 N?



REASONING AND DISCUSSION

Since a pulley simply changes the direction of the tension in a string without changing its magnitude, it is clear that the scale attached to the ceiling reads the same as the scale shown in the figure to the right.



There is no difference, however, between attaching the top end of the scale to something rigid and attaching it to another 1.00-kg hanging mass. In either case, the fact that the scale is at rest means that a force of 9.81 N must be exerted to the left on the top of the scale to balance the 9.81-N force exerted on the lower end of the scale. As a result, the two scales read the same.

ANSWER

(b) The reading of the scale at right is equal to 9.81 N.

Springs and Hooke's Law

Suppose you take a spring of length L, as shown in **Figure 6–8 (a)**, and attach it to a block. If you pull on the spring, causing it to stretch to a length $L + x$, the spring pulls on the block with a force of magnitude F. If you increase the length of the spring to $L + 2x$, the force exerted by the spring increases to $2F$. Similarly, if you compress the spring to a length $L - x$, the spring pushes on the block with a force of magnitude F, where F is the same force given previously. As you might expect, compression to a length $L - 2x$ results in a push of magnitude $2F$.

As a result of these experiments, we can say that a spring exerts a force that is proportional to the amount, x, by which it is stretched or compressed. Thus, if F is the magnitude of the spring force, we can say that

$$F = kx$$

In this expression, k is a constant of proportionality, referred to as the **force constant,** or, equivalently, as the spring constant. Since F has units of newtons and x has units of meters, it follows that k has units of newtons per meter, or N/m. The larger the value of k, the stiffer the spring.

To be more precise, consider the spring shown in **Figure 6–8 (b)**. Note that we have placed the origin of the x axis at the equilibrium length of the spring—that is, at the position of the spring when no force acts on it. Now, if we stretch the spring so that the end of the spring is at a positive value of x ($x > 0$), we find that the spring exerts a force of magnitude kx in the negative x direction. Thus, the spring force (which has only an x component) can be written as

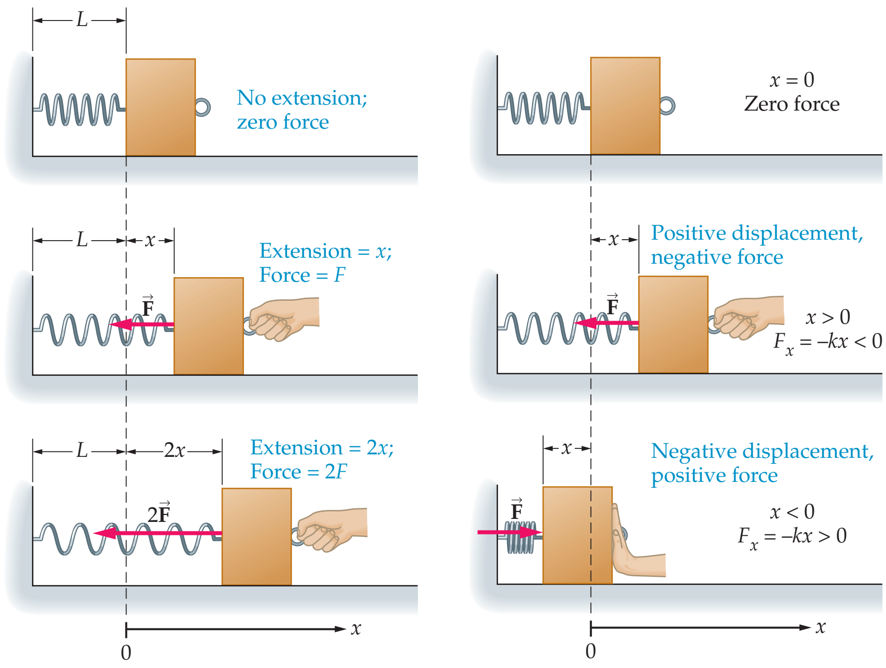
$$F_x = -kx$$

Similarly, consider compressing the spring so that its end is at a negative value of x ($x < 0$). In this case, the force exerted by the spring is of magnitude kx, and points in the positive x direction, as is shown in Figure 6–8 (b). Again, we can write the spring force as

$$F_x = -kx$$

FIGURE 6–8 The force exerted by a spring

When dealing with a spring, it is convenient to choose the origin at the equilibrium (zero force) position. In the cases shown here, the force is strictly in the x direction, and is given by $F_x = -kx$. Note that the minus sign means that the force is opposite to the displacement; that is, the force is restoring.



(a) Doubling the extension doubles the force.

(b) The spring force is opposite to the displacement from equilibrium.

To see that this is correct—that is, that F_x is positive in this case—recall that x is negative, which means that $(-x)$ is positive.

This result for the force of a spring is known as Hooke's law, after Robert Hooke (1635–1703). It is really just a good rule of thumb rather than a law of nature. Clearly, it can't work for any amount of stretching. For example, we know that if we stretch a spring far enough it will be permanently deformed, and will never return to its original length. Still, for small stretches or compressions, Hooke's law is quite accurate.

▲ Springs come in a variety of sizes and shapes. The large springs on a railroad car (top) are so stiff and heavy that you can't compress or stretch them by hand. Still, three of them are needed to smooth the ride of this car. In contrast, the delicate spiral spring inside a watch (bottom) flexes with even the slightest touch. It exerts enough force, however, to power the equally delicate mechanism of the watch.

Rules of Thumb for Springs (Hooke's Law)

A spring stretched or compressed by the amount x from its equilibrium length exerts a force whose x component is given by

$$F_x = -kx \text{ (gives magnitude and direction)} \qquad 6\text{–}4$$

If we are interested only in the magnitude of the force associated with a given stretch or compression, we use the somewhat simpler form of Hooke's law:

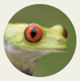
$$F = kx \text{ (gives magnitude only)} \qquad 6\text{–}5$$

In this text, we consider only **ideal springs**—that is, springs that are massless, and that are assumed to obey Hooke's law exactly.

Since the stretch of a spring and the force it exerts are proportional, we can now see how a spring scale operates. In particular, pulling on the two ends of a scale stretches the spring inside it by an amount proportional to the applied force. Once the scale is calibrated—by stretching the spring with a known, or reference, force—we can use it to measure other unknown forces.

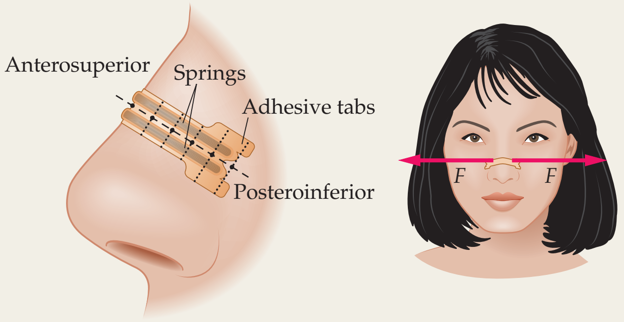
Finally, it is useful to note that Hooke's law, which we've introduced in the context of ideal springs, is particularly important in physics because it applies to so much more than just springs. For example, the forces that hold atoms together are often modeled by Hooke's law—that is, as "interatomic springs"—and these are the forces that are ultimately responsible for the normal force (Chapter 5), vibrations and oscillations (Chapter 13), wave motion (Chapter 14), and even the thermal expansion of solids (Chapter 16). And this just scratches the surface—Hooke's law comes up in one form or another in virtually every field of physics. In the following Active Example, we present a biomedical application of Hooke's law.

ACTIVE EXAMPLE 6–2 NASAL STRIPS



REAL-WORLD PHYSICS: BIO

An increasingly popular device for improving air flow through nasal passages is the nasal strip, which consists of two flat, polyester springs enclosed by an adhesive tape covering. Measurements show that a nasal strip can exert an outward force of 0.22 N on the nose, causing it to expand by 3.5 mm. **(a)** Treating the nose as an ideal spring, find its force constant in newtons per meter. **(b)** How much force would be required to expand the nose by 4.0 mm?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Solve the magnitude form of Hooke's law, $F = kx$, for the force constant, k: $k = F/x$
2. Substitute numerical values for F and x: $k = 62\ \mathrm{N/m}$

Part (b)

3. Use $F = kx$ to find the required force: $F = 0.25\ \mathrm{N}$

INSIGHT

Even though the human nose is certainly not an ideal spring, Hooke's law is still a useful way to model its behavior when dealing with forces and the stretches they cause.

YOUR TURN

Suppose a new nasal strip comes on the market that exerts an outward force of 0.32 N. What expansion of the nose will be caused by this strip?

(Answers to **Your Turn** *problems are given in the back of the book.)*

6–3 Translational Equilibrium

When we say that an object is in **translational equilibrium,** we mean that the net force acting on it is zero:

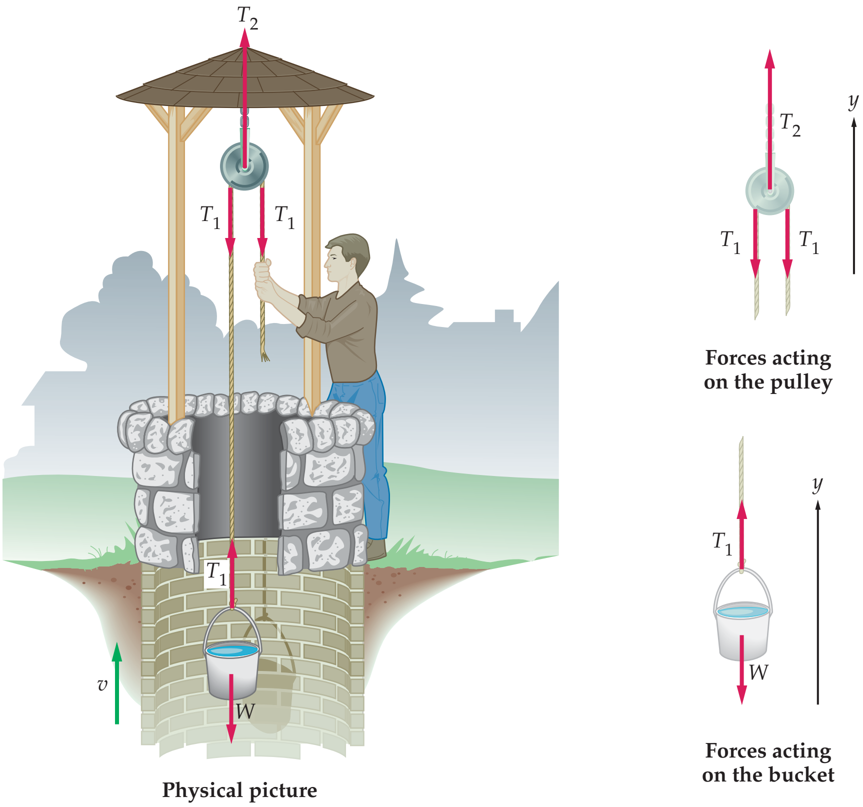
$$\sum \vec{\mathbf{F}} = 0 \qquad 6\text{–}6$$

From Newton's second law, this is equivalent to saying that the object's acceleration is zero. In two-dimensional systems, translational equilibrium implies two independent conditions: $\Sigma F_x = 0$ and $\Sigma F_y = 0$. In one dimension, only one of these conditions will apply.

Later, in Chapters 10 and 11, we will study objects that have both rotational and linear motions. In such cases, rotational equilibrium will be as important as translation equilibrium. For now, however, when we say equilibrium, we simply mean translational equilibrium.

As a first example, consider the one-dimensional situation illustrated in **Figure 6–9**. Here we see a person lifting a bucket of water from a well by pulling down on a rope that passes over a pulley. If the bucket's mass is m, and it is rising with constant speed v, what is the tension T_1 in the rope attached to the bucket? In addition, what is the tension T_2 in the chain that supports the pulley?

FIGURE 6–9 Raising a bucket
A person lifts a bucket of water from the bottom of a well with a constant speed, v. Because the speed is constant, the net force acting on the bucket must be zero.



To answer these questions, we first note that both the bucket and the pulley are in equilibrium; that is, they both have zero acceleration. As a result, the net force on each of them must be zero.

Let's start with the bucket. In Figure 6–9, we see that just two forces act on the bucket: (i) its weight $W = mg$ downward, and (ii) the tension in the rope, T_1 upward. If we take upward to be the positive direction, we can write $\Sigma F_y = 0$ for the bucket as follows:

$$T_1 - mg = 0$$

Therefore, the tension in the rope is $T_1 = mg$. Note that this is also the force the person must exert downward on the rope, as expected.

Next, we consider the pulley. In Figure 6–9, we see that three forces act on it: (i) the tension in the chain, T_2 upward, (ii) the tension in the part of the rope leading to the bucket, T_1 downward, and (iii) the tension in the part of the rope leading to the person, T_1 downward. Note that we don't include the weight of the pulley since we consider it to be ideal; that is, massless and frictionless. If we again take upward to be positive, the statement that the net force acting on the pulley is zero $(\Sigma F_y = 0)$ can be written

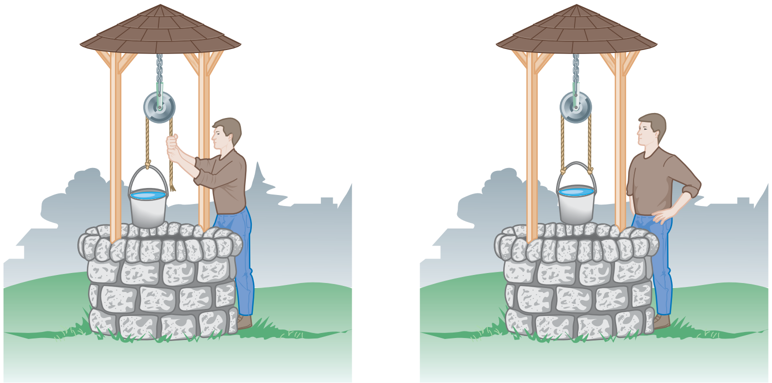
$$T_2 - T_1 - T_1 = 0$$

It follows that the tension in the chain is $T_2 = 2T_1 = 2mg$, twice the weight of the bucket of water!

In the next Conceptual Checkpoint we consider a slight variation of this situation.

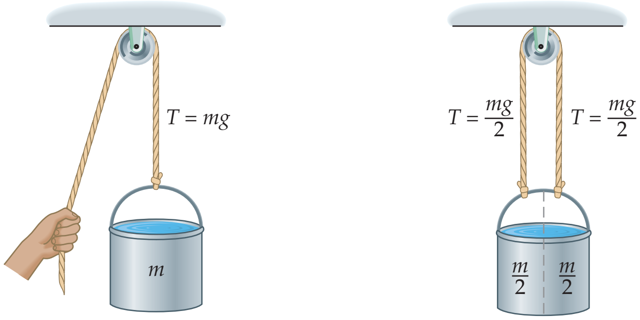
CONCEPTUAL CHECKPOINT 6–3 COMPARING TENSIONS

A person hoists a bucket of water from a well and holds the rope, keeping the bucket at rest, as at left. A short time later, the person ties the rope to the bucket so that the rope holds the bucket in place, as at right. In this case, is the tension in the rope **(a)** greater than, **(b)** less than, or **(c)** equal to the tension in the first case?



REASONING AND DISCUSSION

In the first case (left), the only upward force exerted on the bucket is the tension in the rope. Since the bucket is at rest, the tension must be equal in magnitude to the weight of the bucket. In the second case (right), the two ends of the rope exert equal upward forces on the bucket, hence the tension in the rope is only half the weight of the bucket. To see this more clearly, imagine cutting the bucket in half so that each end of the rope supports half the weight, as indicated in the accompanying diagram.



ANSWER

(b) The tension in the second case is less than in the first.

In the next two Examples, we consider two-dimensional systems in which forces act at various angles with respect to one another. Hence, our first step is to resolve the relevant vectors into their x and y components. Following that, we apply the conditions for translational equilibrium, $\Sigma F_x = 0$ and $\Sigma F_y = 0$.

EXAMPLE 6–5 SUSPENDED VEGETATION

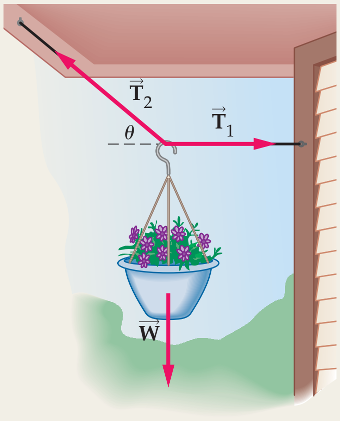
To hang a 6.20-kg pot of flowers, a gardener uses two wires—one attached horizontally to a wall, the other sloping upward at an angle of $\theta = 40.0°$ and attached to the ceiling. Find the tension in each wire.

PICTURE THE PROBLEM

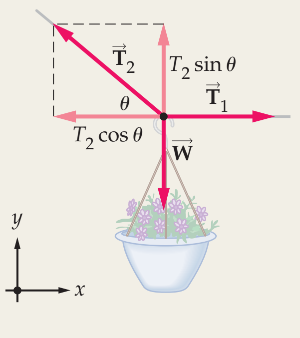
We choose a typical coordinate system, with the positive x direction to the right and the positive y direction upward. With this choice, tension 1 is in the positive x direction, $\vec{\mathbf{T}}_1 = T_1\hat{\mathbf{x}}$, the weight is in the negative y direction, $\vec{\mathbf{W}} = -mg\,\hat{\mathbf{y}}$, and tension 2 has a negative x component and a positive y component, $\vec{\mathbf{T}}_2 = (-T_2 \cos\theta)\hat{\mathbf{x}} + (T_2 \sin\theta)\hat{\mathbf{y}}$.

STRATEGY

The pot is at rest, and therefore the net force acting on it is zero. As a result, we can say that (i) $\Sigma F_x = 0$ and (ii) $\Sigma F_y = 0$. These two conditions allow us to determine the magnitude of the two tensions, T_1 and T_2.



Physical picture



Free-body diagram

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SOLUTION

1. First, resolve each of the forces acting on the pot into x and y components:

$$T_{1,x} = T_1 \qquad T_{1,y} = 0$$
$$T_{2,x} = -T_2 \cos\theta \qquad T_{2,y} = T_2 \sin\theta$$
$$W_x = 0 \qquad W_y = -mg$$

2. Now, set $\Sigma F_x = 0$. Note that this condition gives a relation between T_1 and T_2:

$$\sum F_x = T_{1,x} + T_{2,x} + W_x = T_1 + (-T_2\cos\theta) + 0 = 0$$
$$T_1 = T_2\cos\theta$$

3. Next, set $\Sigma F_y = 0$. This time, the resulting condition determines T_2 in terms of the weight, mg:

$$\sum F_y = T_{1,y} + T_{2,y} + W_y = 0 + T_2\sin\theta + (-mg) = 0$$
$$T_2\sin\theta = mg$$

4. Use the relation obtained in Step 3 to find T_2:

$$T_2 = \frac{mg}{\sin\theta} = \frac{(6.20\text{ kg})(9.81\text{ m/s}^2)}{\sin 40.0°} = 94.6\text{ N}$$

5. Finally, use the connection between the two tensions (obtained from $\Sigma F_x = 0$) to find T_1:

$$T_1 = T_2\cos\theta = (94.6\text{ N})\cos 40.0° = 72.5\text{ N}$$

INSIGHT

Notice that even though two wires suspend the pot, they both have tensions *greater* than the pot's weight, $mg = 60.8$ N. This is an important point for architects and engineers to consider when designing structures.

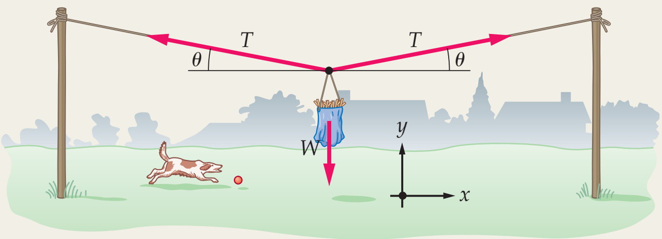
PRACTICE PROBLEM

Find T_1 and T_2 if the second wire slopes upward at the angle **(a)** $\theta = 20°$, **(b)** $\theta = 60.0°$, or **(c)** $\theta = 90.0°$. [**Answer: (a)** $T_1 = 167$ N, $T_2 = 178$ N **(b)** $T_1 = 35.1$ N, $T_2 = 70.2$ N **(c)** $T_1 = 0$, $T_2 = mg = 60.8$ N]

Some related homework problems: Problem 34, Problem 37

ACTIVE EXAMPLE 6–3 THE FORCES IN A LOW-TECH LAUNDRY

A 1.84-kg bag of clothespins hangs in the middle of a clothesline, causing it to sag by an angle $\theta = 3.50°$. Find the tension, T, in the clothesline.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the y component for each tension: $T_y = T\sin\theta$
2. Find the y component of the weight: $W_y = -mg$
3. Set $\Sigma F_y = 0$: $T\sin\theta + T\sin\theta - mg = 0$
4. Solve for T: $T = mg/(2\sin\theta) = 148$ N

INSIGHT

Note that we only considered the y components of force in our calculation. This is because forces in the x direction automatically balance, due to the symmetry of the system.

YOUR TURN

At what sag angle, θ, will the tension in the clothesline have a magnitude of 175 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

At 148 N, the tension in the clothesline is quite large, especially when you consider that the weight of the clothespin bag itself is only 18.1 N. The reason for such a large value is that the vertical component of the two tensions is $2T \sin \theta$, which, for $\theta = 3.50°$, is $(0.122)T$. If $(0.122)T$ is to equal the weight of the bag, it is clear that T must be roughly eight times the bag's weight.

If you and a friend were to pull on the two ends of the clothesline, in an attempt to straighten it out, you would find that no matter how hard you pulled, the line would still sag. You may be able to reduce θ to quite a small value, but as you do so the corresponding tension increases rapidly. In principle, it would take an infinite force to completely straighten the line and reduce θ to zero.

On the other hand, if θ were 90°, so that the two halves of the clothesline were vertical, the tension would be $T = mg/(2 \sin 90°) = mg/2$. In this case, each side of the line supports half the weight of the bag, as expected.

▲ Like the bag of clothespins in Active Example 6–3, this mountain climber is in static equilibrium. Since the ropes suspending the climber are nearly horizontal, the tension in them is significantly greater than the climber's weight.

6–4 Connected Objects

Interesting applications of Newton's laws arise when we consider accelerating objects that are tied together. Suppose, for example, that a force of magnitude F pulls two boxes—connected by a string—along a frictionless surface, as in **Figure 6–10**. In such a case, the string has a certain tension, T, and the two boxes have the same acceleration, a. Given the masses of the boxes and the applied force F, we would like to determine both the tension in the string and the acceleration of the boxes.

First, sketch the free-body diagram for each box. Box 1 has two horizontal forces acting on it: (i) the tension T to the left, and (ii) the force F to the right. Box 2 has only a single horizontal force, the tension T to the right. If we take the positive direction to be to the right, Newton's second law for the two boxes can be written as follows:

$$\begin{aligned} F - T &= m_1 a_1 = m_1 a \qquad &\text{box 1} \\ T &= m_2 a_2 = m_2 a \qquad &\text{box 2} \end{aligned} \qquad 6\text{–}7$$

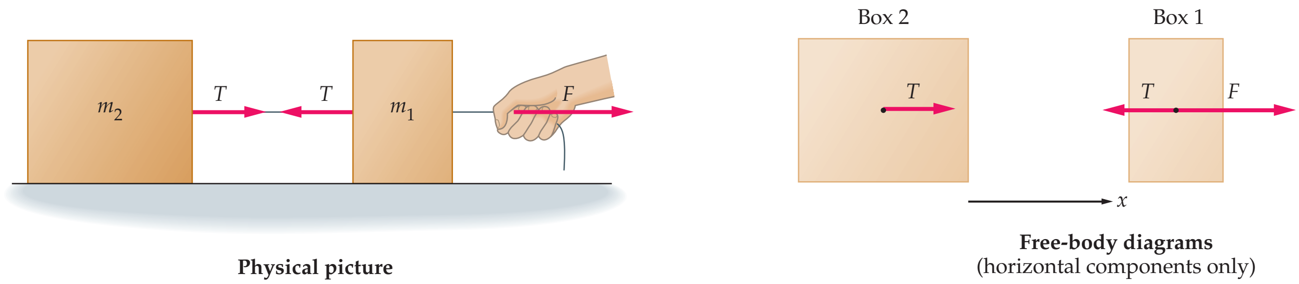
Since the boxes have the same acceleration, a, we have set $a_1 = a_2 = a$.

Next, we can eliminate the tension T by adding the two equations:

$$\begin{aligned} F - T &= m_1 a \\ T &= m_2 a \\ \hline F &= (m_1 + m_2)a \end{aligned}$$

With this result, it is straightforward to solve for the acceleration in terms of the applied force F:

$$a = \frac{F}{m_1 + m_2} \qquad 6\text{–}8$$



▲ **FIGURE 6–10 Two boxes connected by a string**
The string ensures that the two boxes have the same acceleration. This physical connection results in a mathematical connection, as shown in Equation 6–7. Note that in this case we treat each box as a separate system.

Finally, substitute this expression for a into either of the second-law equations to find the tension. The algebra is simpler if we use the equation for box 2. We find

$$T = m_2 a = \left(\frac{m_2}{m_1 + m_2}\right) F \tag{6–9}$$

It is left as an exercise to show that the equation for box 1 gives the same expression for T.

A second way to approach this problem is to treat both boxes together as a single system with a mass $m_1 + m_2$, as shown in **Figure 6–11**. The only *external* horizontal force acting on this system is the applied force F—the two tension forces are now *internal* to the system, and internal forces are not included when applying Newton's second law. As a result, the horizontal acceleration is simply $F/(m_1 + m_2)$, as given in Equation 6–8. This is certainly a quick way to find the acceleration a, but to find the tension T we must still use one of the relations given in Equations 6–7.

In general, we are always free to choose the "system" any way we like—we can choose any individual object, as when we considered box 1 and box 2 separately, or we can choose all the objects together. The important point is that Newton's second law is equally valid no matter what choice we make for the system, as long as we remember to include only forces *external* to *that system* in the corresponding free-body diagram.

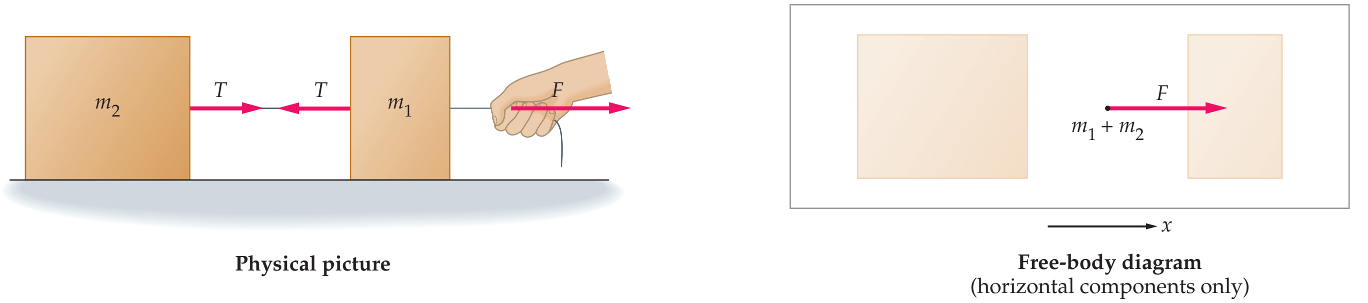
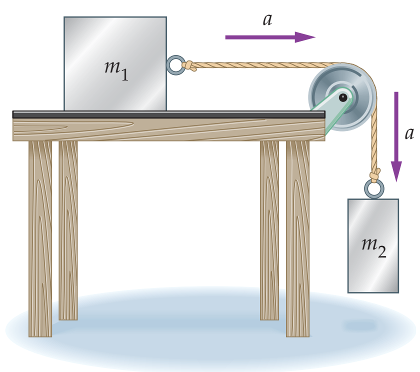


FIGURE 6–11 Two boxes, one system
In this case we consider the two boxes together as a single system of mass $m_1 + m_2$. The only external horizontal force acting on this system is $\vec{\mathbf{F}}$; hence the horizontal acceleration of the system is $a = F/(m_1 + m_2)$, in agreement with Equation 6–8.

CONCEPTUAL CHECKPOINT 6–4 TENSION IN THE STRING



Two masses, m_1 and m_2, are connected by a string that passes over a pulley. Mass m_1 slides without friction on a horizontal tabletop, and mass m_2 falls vertically downward. Both masses move with a constant acceleration of magnitude a. Is the tension in the string **(a)** greater than, **(b)** equal to, or **(c)** less than $m_2 g$?

REASONING AND DISCUSSION

First, note that m_2 accelerates downward, which means that the net force acting on it is downward. Only two forces act on m_2, however: the tension in the string (upward) and its weight (downward). Since the net force is downward, the tension in the string must be less than the weight, $m_2 g$.

A common misconception is that since m_2 has to pull m_1 behind it, the tension in the string must be greater than $m_2 g$. Certainly, attaching the string to m_1 has an effect on the tension. If the string were not attached, for example, its tension would be zero. Hence, m_2 pulling on m_1 increases the tension to a value greater than zero, though still less than $m_2 g$.

ANSWER

(c) The tension in the string is less than $m_2 g$.

In the next Example, we verify the qualitative conclusions given in the Conceptual Checkpoint with a detailed calculation. But first, a note about choosing a coordinate system for a problem such as this. Rather than apply the same coordinate system to both masses, it is useful to take into consideration the fact that a pulley simply changes the direction of the tension in a string. With this in mind, we choose a set of axes that "follow the motion" of the string, so that both masses accelerate in the positive x direction with accelerations of equal magnitude. Example 6–6 illustrates the use of this type of coordinate system.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Connected Objects

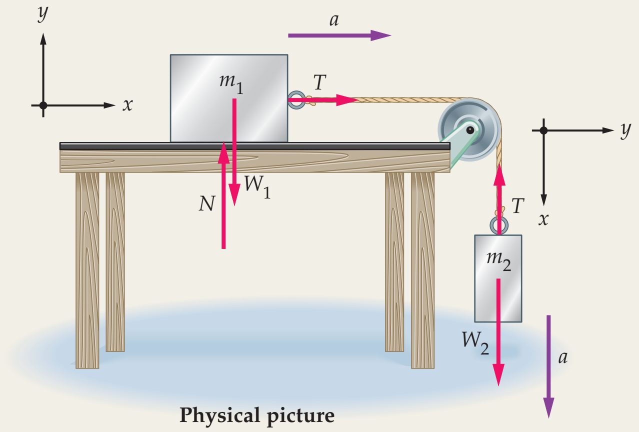
If two objects are connected by a string passing over a pulley, let the coordinate system follow the direction of the string. With this choice, both objects have accelerations of the same magnitude and in the same coordinate direction.

EXAMPLE 6–6 CONNECTED BLOCKS

A block of mass m_1 slides on a frictionless tabletop. It is connected to a string that passes over a pulley and suspends a mass m_2. Find **(a)** the acceleration of the masses and **(b)** the tension in the string.

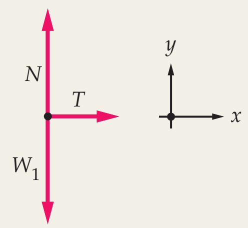
PICTURE THE PROBLEM

Our coordinate system follows the motion of the string so that both masses move in the positive x direction. Since the masses are connected, their accelerations have the same magnitude. Thus, $a_{1,x} = a_{2,x} = a$. In addition, note that the tension, $\vec{\mathbf{T}}$, is in the positive x direction for mass 1, but in the negative x direction for mass 2. Its magnitude, T, is the same for each mass, however. Finally, the weight of mass 2, W_2, acts in the positive x direction, whereas the weight of mass 1 is offset by the normal force, N.

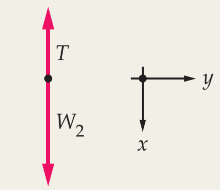


Physical picture

INTERACTIVE FIGURE



Free-body diagram for m_1



Free-body diagram for m_2

STRATEGY

Applying Newton's second law to the two masses yields the following relations: For mass 1, $\Sigma F_{1,x} = T = m_1 a_{1,x} = m_1 a$ and for mass 2, $\Sigma F_{2,x} = m_2 g - T = m_2 a_{2,x} = m_2 a$. These two equations can be solved for the two unknowns, a and T.

SOLUTION

Part (a)

1. First, write $\Sigma F_{1,x} = m_1 a$. Note that the only force acting on m_1 in the x direction is T:

$$\sum F_{1,x} = T = m_1 a$$
$$T = m_1 a$$

2. Next, write $\Sigma F_{2,x} = m_2 a$. In this case, two forces act in the x direction: $W_2 = m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$
$$m_2 g - T = m_2 a$$

3. Sum the two relations obtained to eliminate T:

$$\begin{aligned} T &= m_1 a \\ m_2 g - T &= m_2 a \\ \hline m_2 g &= (m_1 + m_2) a \end{aligned}$$

4. Solve for a:

$$a = \left(\frac{m_2}{m_1 + m_2}\right) g$$

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Part (b)

5. Substitute a into the first relation ($T = m_1 a$) to find T:

$$T = m_1 a = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g$$

INSIGHT

We could just as well have determined T using $m_2 g - T = m_2 a$, though the algebra is a bit messier. Also, note that $a = 0$ if $m_2 = 0$, and that $a = g$ if $m_1 = 0$, as expected. Similarly, $T = 0$ if either m_1 or m_2 is zero. This type of check, where you connect equations with physical situations, is one of the best ways to increase your understanding of physics.

PRACTICE PROBLEM

Find the tension for the case $m_1 = 1.50$ kg and $m_2 = 0.750$ kg, and compare the tension to $m_2 g$. [**Answer:** $a = 3.27\ \mathrm{m/s^2}$, $T = 4.91\ \mathrm{N} < m_2 g = 7.36\ \mathrm{N}$]

Some related homework problems: Problem 44, Problem 48

Conceptual Checkpoint 6–4 shows that the tension in the string should be less than $m_2 g$. Let's rewrite our solution for T to show that this is indeed the case. From Example 6–6 we have

$$T = \left(\frac{m_1 m_2}{m_1 + m_2}\right)g = \left(\frac{m_1}{m_1 + m_2}\right)m_2 g$$

Since the ratio $m_1/(m_1 + m_2)$ is always less than 1 (as long as m_2 is nonzero), it follows that $T < m_2 g$, as expected.

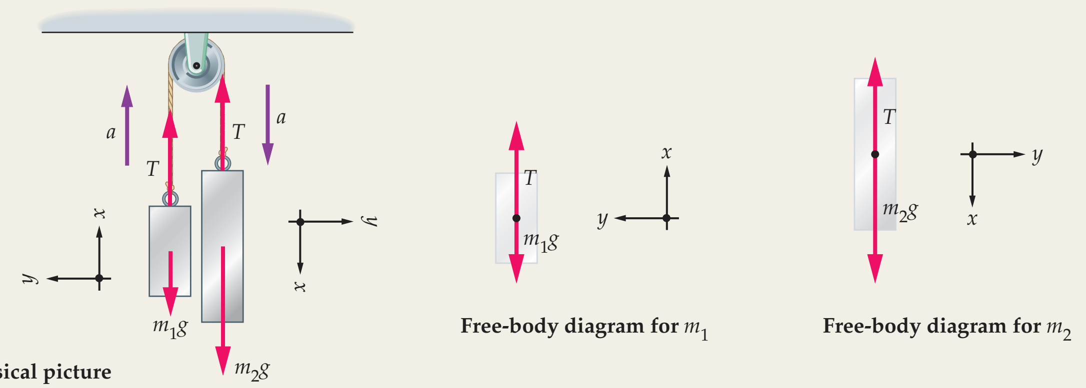
We conclude this section with a classic system that can be used to measure the acceleration of gravity. It is referred to as Atwood's machine, and it is basically two blocks of different mass connected by a string that passes over a pulley. The resulting acceleration of the blocks is related to the acceleration of gravity by a relatively simple expression, which we derive in the following Example.

EXAMPLE 6–7 ATWOOD'S MACHINE

Atwood's machine consists of two masses connected by a string that passes over a pulley, as shown below. Find the acceleration of the masses for general m_1 and m_2, and evaluate for the specific case $m_1 = 3.1$ kg, $m_2 = 4.4$ kg.

PICTURE THE PROBLEM

Our sketch shows Atwood's machine, along with our choice of coordinate directions for the two blocks. Note that both blocks accelerate in the positive x direction with accelerations of equal magnitude, a. From the free-body diagrams we can see that for mass 1 the weight is in the negative x direction and the tension is in the positive x direction. For mass 2, the tension is in the negative x direction and the weight is in the positive x direction. The tension has the same magnitude T for both masses, but their weights are different.



Physical picture **Free-body diagram for m_1** **Free-body diagram for m_2**

STRATEGY

To find the acceleration of the blocks, we follow the same strategy given in the previous Example. In particular, we start by applying Newton's second law to each block individually, using the fact that $a_{1,x} = a_{2,x} = a$. This gives two equations, both involving the tension T and the acceleration a. Eliminating T allows us to solve for the acceleration.

SOLUTION

1. Begin by writing out the expression $\Sigma F_{1,x} = m_1 a$. Note that two forces act in the x direction; T (positive direction) and $m_1 g$ (negative direction):

$$\sum F_{1,x} = T - m_1 g = m_1 a$$

2. Next, write out $\Sigma F_{2,x} = m_2 a$. The two forces acting in the x direction in this case are $m_2 g$ (positive direction) and T (negative direction):

$$\sum F_{2,x} = m_2 g - T = m_2 a$$

3. Sum the two relations obtained above to eliminate T:

$$\begin{aligned} T - m_1 g &= m_1 a \\ m_2 g - T &= m_2 a \\ \hline (m_2 - m_1)g &= (m_1 + m_2)a \end{aligned}$$

4. Solve for a:

$$a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$$

5. To evaluate the acceleration, substitute numerical values for the masses and for g:

$$\begin{aligned} a &= \left(\frac{m_2 - m_1}{m_1 + m_2}\right)g \\ &= \left(\frac{4.4\text{ kg} - 3.1\text{ kg}}{3.1\text{ kg} + 4.4\text{ kg}}\right)(9.81\text{ m/s}^2) = 1.7\text{ m/s}^2 \end{aligned}$$

INSIGHT

Since m_2 is greater than m_1, we find that the acceleration is positive, meaning that the masses accelerate in the positive x direction. On the other hand, if m_1 were greater than m_2, we would find that a is negative, indicating that the masses accelerate in the negative x direction. Finally, if $m_1 = m_2$ we have $a = 0$, as expected.

PRACTICE PROBLEM

If m_1 is increased by a small amount, does the acceleration of the blocks increase, decrease, or stay the same? Check your answer by evaluating the acceleration for $m_1 = 3.3$ kg. [**Answer:** If m_1 is increased only slightly, the acceleration will decrease. For $m_1 = 3.3$ kg, we find $a = 1.4\text{ m/s}^2$.]

Some related homework problems: Problem 48, Problem 50

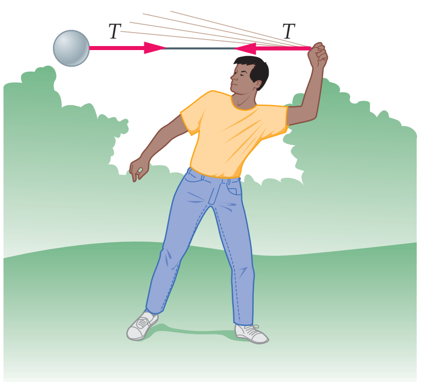
6–5 Circular Motion

According to Newton's second law, if no force acts on an object, it will move with constant speed in a constant direction. A force is required to change the speed, the direction, or both. For example, if you drive a car with constant speed on a circular track, the direction of the car's motion changes continuously. A force must act on the car to cause this change in direction. We would like to know two things about a force that causes circular motion: what is its direction, and what is its magnitude?

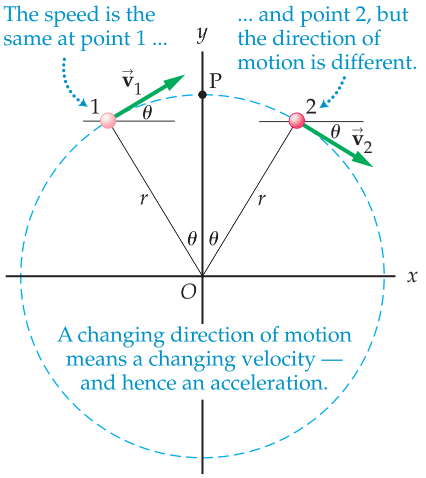
First, let's consider the direction of the force. Imagine swinging a ball tied to a string in a circle about your head, as shown in **Figure 6–12**. As you swing the ball, you feel a tension in the string pulling outward. Of course, on the other end of the string, where it attaches to the ball, the tension pulls inward, toward the center of the circle. Thus, the force the ball experiences is a force that is always directed toward the center of the circle. In summary,

> To make an object move in a circle with constant speed, a force must act on it that is directed toward the center of the circle.

Since the ball is acted on by a *force* toward the center of the circle, it follows that it must be *accelerating* toward the center of the circle. This might seem odd at



▲ **FIGURE 6–12 Swinging a ball in a circle** The tension in the string pulls outward on the person's hand and pulls inward on the ball.



▲ **FIGURE 6–13 A particle moving with constant speed in a circular path centered on the origin**

The speed of the particle is constant, but its velocity is constantly changing direction. Because the velocity changes, the particle is accelerating.

first: How can a ball that moves with constant speed have an acceleration? The answer is that acceleration is produced whenever the speed or direction of the velocity changes—and in circular motion, the direction changes continuously. The resulting center-directed acceleration is called **centripetal acceleration** (centripetal is from the Latin for "center seeking").

Let's calculate the magnitude of the centripetal acceleration, a_{cp}, for an object moving with a constant speed v in a circle of radius r. **Figure 6–13** shows the circular path of an object, with the center of the circle at the origin. To calculate the acceleration at the top of the circle, at point P, we first calculate the average acceleration from point 1 to point 2:

$$\vec{\mathbf{a}}_{av} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} \qquad \text{6–10}$$

The instantaneous acceleration at P is the limit of $\vec{\mathbf{a}}_{av}$ as points 1 and 2 move closer to P.

Referring to Figure 6–13, we see that $\vec{\mathbf{v}}_1$ is at an angle θ above the horizontal, and $\vec{\mathbf{v}}_2$ is at an angle θ below the horizontal. Both $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ have a magnitude v. Therefore, we can write the two velocities in vector form as follows:

$$\vec{\mathbf{v}}_1 = (v\cos\theta)\hat{\mathbf{x}} + (v\sin\theta)\hat{\mathbf{y}}$$
$$\vec{\mathbf{v}}_2 = (v\cos\theta)\hat{\mathbf{x}} + (-v\sin\theta)\hat{\mathbf{y}}$$

Substituting these results into $\vec{\mathbf{a}}_{av}$ gives

$$\vec{\mathbf{a}}_{av} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{\Delta t} = \frac{-2v\sin\theta}{\Delta t}\hat{\mathbf{y}} \qquad \text{6–11}$$

TABLE 6.2 $\frac{\sin\theta}{\theta}$ **for Values of** θ **Approaching Zero**

θ, radians	$\frac{\sin\theta}{\theta}$
1.00	0.841
0.500	0.959
0.250	0.990
0.125	0.997
0.0625	0.999

Note that $\vec{\mathbf{a}}_{av}$ points in the negative y direction—which, at point P, is toward the center of the circle.

To complete the calculation, we need Δt, the time it takes the object to go from point 1 to point 2. Since the object's speed is v, and the distance from point 1 to point 2 is $d = r(2\theta)$ where θ is measured in radians (see Appendix A, page A-2 for a discussion of radians and degrees), we find

$$\Delta t = \frac{d}{v} = \frac{2r\theta}{v} \qquad \text{6–12}$$

Combining this result for Δt with the previous result for $\vec{\mathbf{a}}_{av}$ gives

$$\vec{\mathbf{a}}_{av} = \frac{-2v\sin\theta}{(2r\theta/v)}\hat{\mathbf{y}} = -\frac{v^2}{r}\left(\frac{\sin\theta}{\theta}\right)\hat{\mathbf{y}} \qquad \text{6–13}$$

To find $\vec{\mathbf{a}}$ at point P, we let points 1 and 2 approach P, which means letting θ go to zero. Table 6–2 shows that as θ goes to zero $(\theta \to 0)$, the ratio $(\sin\theta)/\theta$ goes to 1:

$$\frac{\sin\theta}{\theta} \xrightarrow[\text{as } \theta \to 0]{} 1$$

Finally, then, the instantaneous acceleration at point P is

$$\vec{\mathbf{a}} = -\frac{v^2}{r}\hat{\mathbf{y}} = -a_{cp}\hat{\mathbf{y}} \qquad \text{6–14}$$

▲ The people enjoying this carnival ride are experiencing a centripetal acceleration of roughly 10 m/s^2 directed inward, toward the axis of rotation. The force needed to produce this acceleration, which keeps the riders moving in a circular path, is provided by the horizontal component of the tension in the chains.

As mentioned, the direction of the acceleration is toward the center of the circle, and now we see that its magnitude is

$$a_{cp} = \frac{v^2}{r} \qquad \text{6–15}$$

We can summarize these results as follows:

- When an object moves in a circle of radius r with constant speed v, its centripetal acceleration is $a_{cp} = v^2/r$.
- A force must be applied to an object to give it circular motion. For an object of mass m, the net force acting on it must have a magnitude given by

$$f_{cp} = ma_{cp} = m\frac{v^2}{r} \qquad 6\text{–}16$$

and must be directed toward the center of the circle.

Note that the **centripetal force,** f_{cp}, can be produced in any number of ways. For example, f_{cp} might be the tension in a string, as in the example with the ball, or it might be due to friction between tires and the road, as when a car turns a corner. In addition, f_{cp} could be the force of gravity causing a satellite, or the Moon, to orbit the Earth. Thus, f_{cp} is a force that must be present to cause circular motion, but the specific cause of f_{cp} varies from system to system.

We now show how these results for centripetal force and centripetal acceleration can be applied in practice.

PROBLEM-SOLVING NOTE

Choice of Coordinate System: Circular Motion

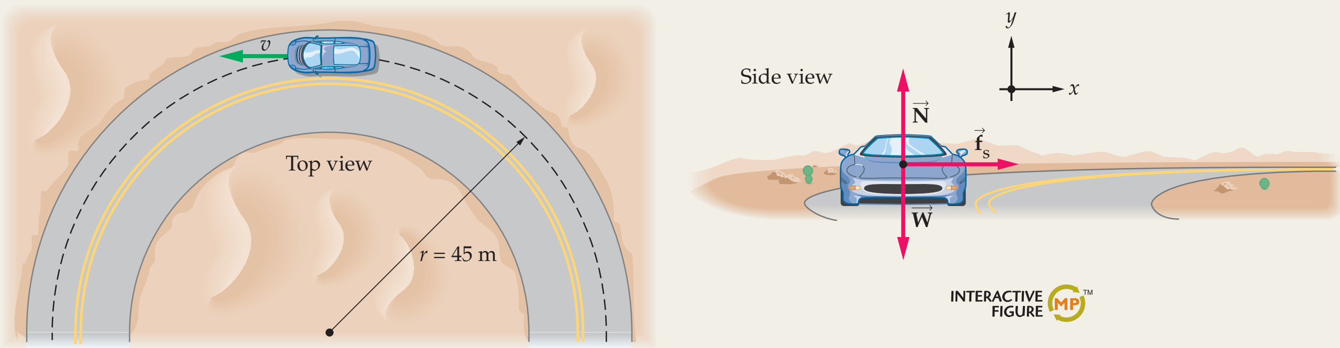
In circular motion, it is convenient to choose the coordinate system so that one axis points toward the center of the circle. Then, we know that the acceleration in that direction must be $a_{cp} = v^2/r$.

EXAMPLE 6–8 ROUNDING A CORNER

A 1200-kg car rounds a corner of radius $r = 45$ m. If the coefficient of static friction between the tires and the road is $\mu_s = 0.82$, what is the greatest speed the car can have in the corner without skidding?

PICTURE THE PROBLEM

In the first sketch we show a bird's-eye view of the car as it moves along its circular path. The next sketch shows the car moving directly toward the observer. Note that we have chosen the positive x direction to point toward the center of the circular path, and the positive y axis to point vertically upward. We also indicate the three forces acting on the car: gravity, $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$; the normal force, $\vec{\mathbf{N}} = N\hat{\mathbf{y}}$; and the force of static friction, $\vec{\mathbf{f}}_s = \mu_s N\hat{\mathbf{x}}$.



INTERACTIVE FIGURE

STRATEGY

In this system, the force of static friction provides the centripetal force required for the car to move in a circular path. That is why the force of friction is at right angles to the car's direction of motion; it is directed toward the center of the circle. In addition, the friction in this case is static because the car's tires are rolling without slipping—always making static contact with the ground. Finally, if the car moves faster, more centripetal force (i.e., more friction) is required. Thus, the greatest speed for the car corresponds to the maximum static friction, $f_s = \mu_s N$. Hence, if we set $\mu_s N$ equal to the centripetal force, $ma_{cp} = mv^2/r$, we can solve for v.

SOLUTION

1. Sum the x components of force to relate the force of static friction to the centripetal acceleration of the car: $\sum F_x = f_s = ma_x$

2. Since the car moves in a circular path, with the center of the circle in the x direction, it follows that $a_x = a_{cp} = v^2/r$. Make this substitution, along with $f_s = \mu_s N$ for the force of static friction: $\mu_s N = ma_{cp} = m\dfrac{v^2}{r}$

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3. Next, set the sum of the y components of force equal to zero, since $a_y = 0$: $\sum F_y = N - W = ma_y = 0$

4. Solve for the normal force: $N = W = mg$

5. Substitute the result $N = mg$ in Step 2 and solve for v. Notice that the mass of the car cancels:

$$\mu_s mg = m\frac{v^2}{r}$$

$$v = \sqrt{\mu_s rg}$$

6. Substitute numerical values to determine v: $v = \sqrt{(0.82)(45\text{ m})(9.81\text{ m/s}^2)} = 19\text{ m/s}$

INSIGHT

Note that the maximum speed is less if the radius is smaller (tighter corner) or if μ_s is smaller (slick road). The mass of the vehicle, however, is irrelevant. For example, the maximum speed is precisely the same for a motorcycle rounding this corner as it is for a large, heavily loaded truck.

PRACTICE PROBLEM

Suppose the situation described in this Example takes place on the Moon, where the acceleration of gravity is less than it is on Earth. If a lunar rover goes around this same corner, is its maximum speed greater than, less than, or the same as the result found in Step 4? To check your answer, find the maximum speed for a lunar rover when it rounds a corner with $r = 45\text{ m}$ and $\mu_s = 0.82$. (On the Moon, $g = 1.62\text{ m/s}^2$.) [**Answer:** The maximum speed will be less. On the Moon we find $v = 7.7\text{ m/s}$.]

Some related homework problems: Problem 55, Problem 57, Problem 61

REAL-WORLD PHYSICS

Skids and banked roadways

If you try to round a corner too rapidly, you may experience a skid; that is, your car may begin to slide sideways across the road. A common bit of road wisdom is that you should turn in the direction of the skid to regain control—which, to most people, sounds counterintuitive. The advice is sound, however. Suppose, for example, that you are turning to the left and begin to skid to the right. If you turn more sharply to the left to try to correct for the skid, you simply reduce the turning radius of your car, r. The result is that the centripetal acceleration, v^2/r, becomes larger, and an even larger force would be required from the road to make the turn. The tendency to skid would therefore be increased. On the other hand, if you turn slightly to the right when you start to skid, you *increase* your turning radius and the centripetal acceleration decreases. In this case your car may stop skidding, and you can then regain control of your vehicle.

You may also have noticed that many roads are tilted, or banked, when they round a corner. The same type of banking is observed on many automobile racetracks as well. Next time you drive around a banked curve, notice that the banking tilts you in toward the center of the circular path you are following. This is by

▲ The steeply banked track at the Talladega Speedway in Alabama (left) helps to keep the rapidly moving cars from skidding off along a tangential path. Even when there is no solid roadway, however, banking can still help—airplanes bank when making turns (center) to keep from "skidding" sideways. Banking is beneficial in another way as well. Occupants of cars on a banked roadway or of a banking airplane feel no sideways force when the banking angle is just right, so turns become a safer and more comfortable experience. For this reason, some trains use hydraulic suspension systems to bank when rounding corners (right), even though the tracks themselves are level.

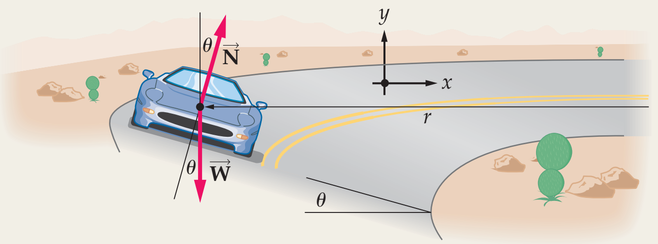
design. On a banked curve, the normal force exerted by the road contributes to the required centripetal force. If the tilt angle is just right, the normal force provides all of the centripetal force so that the car can negotiate the curve even if there is no friction between its tires and the road. The next Example determines the optimum banking angle for a given speed and given radius of turn.

EXAMPLE 6–9 BANK ON IT

If a roadway is banked at the proper angle, a car can round a corner without any assistance from friction between the tires and the road. Find the appropriate banking angle for a 900-kg car traveling at 20.5 m/s in a turn of radius 85.0 m.

PICTURE THE PROBLEM

Note that we choose the positive y axis to point vertically upward and the positive x direction to point toward the center of the circular path. Since $\vec{\mathbf{N}}$ is perpendicular to the banked roadway, it is at an angle θ to the y axis. Therefore, $\vec{\mathbf{N}} = (N \sin\theta)\hat{\mathbf{x}} + (N\cos\theta)\hat{\mathbf{y}}$ and $\vec{\mathbf{W}} = -W\hat{\mathbf{y}} = -mg\hat{\mathbf{y}}$.



STRATEGY

In order for the car to move in a circular path, there must be a force acting on it in the positive x direction. Since the weight $\vec{\mathbf{W}}$ has no x component, it follows that the normal force $\vec{\mathbf{N}}$ must supply the needed centripetal force. Thus, we find N by setting $\sum F_y = ma_y = 0$, since there is no motion in the y direction. Then we use N in $\sum F_x = ma_x = mv^2/r$ to find the angle θ.

SOLUTION

1. Start by determining N from the condition $\Sigma F_y = 0$:

$$\sum F_y = N\cos\theta - W = 0$$

$$N = \frac{W}{\cos\theta} = \frac{mg}{\cos\theta}$$

2. Next, set $\Sigma F_x = mv^2/r$:

$$\sum F_x = N\sin\theta$$

$$= ma_x = ma_{\text{cp}} = m\frac{v^2}{r}$$

3. Substitute $N = mg/\cos\theta$ (from $\Sigma F_y = 0$, Step 1) and solve for θ, using the fact that $\sin\theta/\cos\theta = \tan\theta$. Notice that, once again, the mass of the car cancels:

$$N\sin\theta = \frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{r}$$

$$\tan\theta = \frac{v^2}{gr} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$$

4. Substitute numerical values to determine θ:

$$\theta = \tan^{-1}\left[\frac{(20.5\ \text{m/s})^2}{(9.81\ \text{m/s}^2)(85.0\ \text{m})}\right] = 26.7°$$

INSIGHT

The symbolic result in Step 3 shows that the banking angle increases with increasing speed and decreasing radius of turn, as one would expect.

From the point of view of a passenger, the experience of rounding a properly banked corner is basically the same as riding on a level road—there are no “sideways forces” to make the turn uncomfortable. There is one small difference, however—the passenger feels heavier due to the increased normal force.

PRACTICE PROBLEM

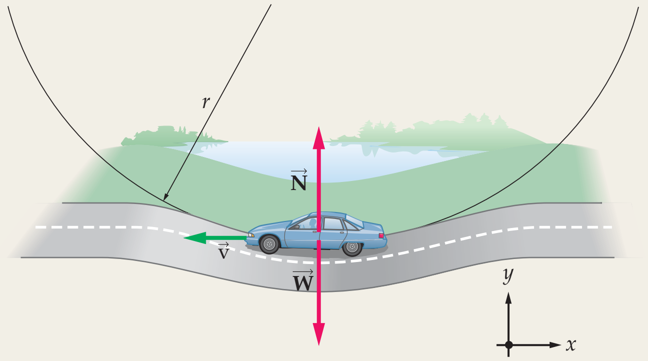
A turn of radius 65 m is banked at 30.0°. What speed should a car have in order to make the turn with no assistance from friction? [**Answer:** $v = 19\ \text{m/s}$]

Some related homework problems: Problem 58, Problem 107

If you’ve ever driven through a dip in the road, you know that you feel momentarily heavier near the bottom of the dip, just like a passenger in Example 6–9. This change in apparent weight is due to the approximately circular motion of the car, as we show next.

ACTIVE EXAMPLE 6–4 FIND THE NORMAL FORCE

While driving along a country lane with a constant speed of 17.0 m/s, you encounter a dip in the road. The dip can be approximated as a circular arc, with a radius of 65.0 m. What is the normal force exerted by a car seat on an 80.0-kg passenger when the car is at the bottom of the dip?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write $\Sigma F_y = ma_y$ for the passenger: $N - mg = ma_y$
2. Replace a_y with the centripetal acceleration: $a_y = v^2/r$
3. Solve for N: $N = mg + mv^2/r$
4. Substitute numerical values: $N = 1140\ \text{N}$

INSIGHT

At the bottom of the dip the normal force is greater than the weight of the passenger, since it must also supply the centripetal force. As a result, the passenger feels heavier than usual. In this case, the 80.0-kg passenger feels as if his mass has increased by 45%, to 116 kg!

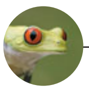
The same physics applies to a jet pilot who pulls a plane out of a high-speed dive. In that case, the magnitude of the effect can be much larger, resulting in a decrease of blood flow to the brain and eventually to loss of consciousness. Here's a case where basic physics really can be a matter of life and death.

YOUR TURN

At what speed will the magnitude of the normal force be equal to 1250 N?

(Answers to **Your Turn** *problems are given in the back of the book.)*

▲ A laboratory centrifuge of the kind commonly used to separate blood components.

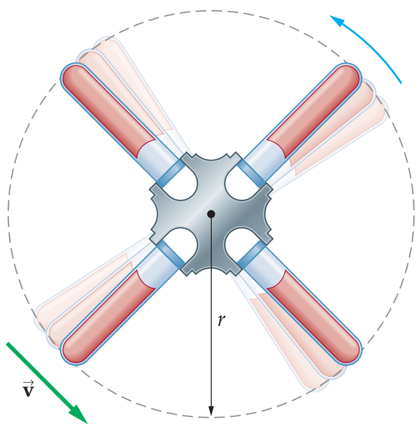


REAL-WORLD PHYSICS: BIO

Centrifuges and ultracentrifuges

A similar calculation can be applied to a car going over the top of a bump. In that case, circular motion results in a reduced apparent weight.

Finally, we determine the acceleration produced in a **centrifuge,** a common device in biological and medical laboratories that uses large centripetal accelerations to perform such tasks as separating red and white blood cells from serum. A simplified top view of a centrifuge is shown in **Figure 6–14**.



▲ **FIGURE 6–14 Simplified top view of a centrifuge in operation**

EXERCISE 6–1

The centrifuge in Figure 6–14 rotates at a rate that gives the bottom of the test tube a linear speed of 89.3 m/s. If the bottom of the test tube is 8.50 cm from the axis of rotation, what is the centripetal acceleration experienced there?

SOLUTION

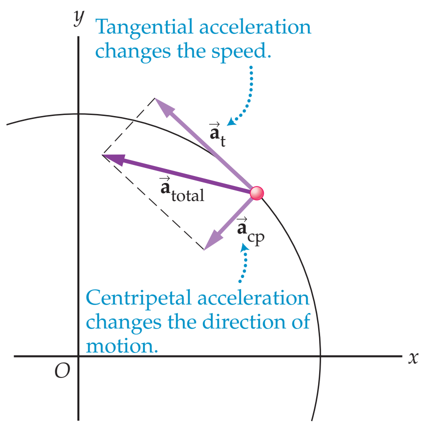
Applying the relation $a_{cp} = v^2/r$ yields

$$a_{cp} = \frac{v^2}{r} = \frac{(89.3\ \text{m/s})^2}{0.0850\ \text{m}} = 93{,}800\ \text{m/s}^2 = 9560g$$

In this expression, g is the acceleration of gravity, 9.81 m/s^2.

Thus, a centrifuge can produce centripetal accelerations that are many thousand times greater than the acceleration of gravity. In fact, devices referred to as **ultracentrifuges** can produce accelerations as great as 1 million g. Even in the relatively modest case considered in Exercise 6–1, the forces involved in a centrifuge can be quite significant. For example, if the contents of the test tube have a mass of 12.0 g, the centripetal force that must be exerted by the bottom of the tube is $(0.0120\ \text{kg})(9560\ g) = 1130\ \text{N}$, or about 250 lb!

Finally, an object moving in a circular path may increase or decrease its speed. In such a case, the object has both an acceleration tangential to its path that changes its speed, $\vec{\mathbf{a}}_t$, and a centripetal acceleration perpendicular to its path, $\vec{\mathbf{a}}_{cp}$, that changes its direction of motion. Such a situation is illustrated in **Figure 6–15**. The total acceleration of the object is the vector sum of $\vec{\mathbf{a}}_t$ and $\vec{\mathbf{a}}_{cp}$. We will explore this case more fully in Chapter 10.



▲ **FIGURE 6–15 A particle moving in a circular path with tangential acceleration**
In this case, the particle's speed is increasing at the rate given by a_t.

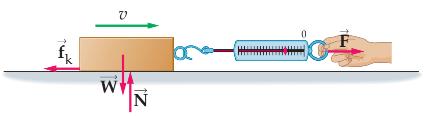
THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The equations of kinematics from Chapters 2 and 4 proved useful again in this chapter. See, in particular, Examples 6–1 and 6–2.	Our discussion of springs, and Hooke's law in particular, will be of importance when we consider oscillations in Chapter 13.
The discussion related to Figure 5–15 about angles on an inclined surface came into play when identifying the angles in Examples 6–2 and 6–9.	The basic ideas of translational equilibrium (Section 6–3) will be extended to more general objects in Chapter 11.
Our derivation of the direction and magnitude of centripetal acceleration (Section 6–5) made extensive use of our knowledge of vectors and how to resolve them into components.	Circular motion will come up again in a number of situations, but especially when we consider orbital motion in Chapter 12 and the Bohr model of the hydrogen atom in Chapter 31.

CHAPTER SUMMARY

6–1 FRICTIONAL FORCES

Frictional forces are due to the microscopic roughness of surfaces in contact. As a rule of thumb, friction is independent of the area of contact and independent of the relative speed of the surfaces.



Kinetic Friction

Friction experienced by surfaces that are in contact and moving relative to one another. The force of kinetic friction is given by

$$f_k = \mu_k N \qquad 6\text{–}1$$

In this expression, μ_k is the coefficient of kinetic friction and N is the magnitude of the normal force.

Static Friction

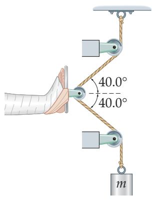
Friction experienced by surfaces that are in static contact. The maximum force of static friction is given by

$$f_{s,\max} = \mu_s N \qquad 6\text{–}3$$

In this expression, μ_s is the coefficient of static friction and N is the magnitude of the normal force. The force of static friction can have any magnitude between zero and its maximum value.

6–2 STRINGS AND SPRINGS

Strings and springs provide a common way of exerting forces on objects. Ideal strings and springs are massless.



Tension

The force transmitted through a string. The tension is the same throughout the length of an ideal string.

Hooke's Law

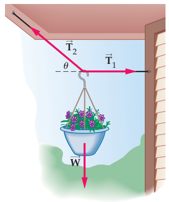
The force exerted by an ideal spring stretched by the amount x is

$$F_x = -kx \qquad 6\text{–}4$$

In words, the force exerted by a spring is proportional to the amount of stretch or compression, and is in the opposite direction.

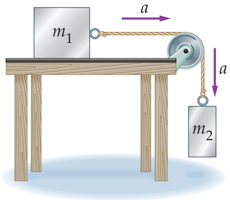
6–3 TRANSLATIONAL EQUILIBRIUM

An object is in translational equilibrium if the net force acting on it is zero. Equivalently, an object is in equilibrium if it has zero acceleration.



6–4 CONNECTED OBJECTS

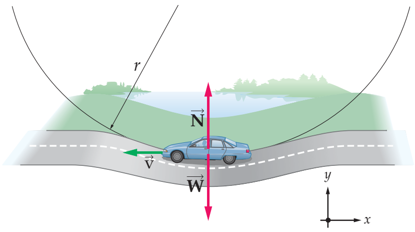
Connected objects are linked physically, and hence they are linked mathematically as well. For example, objects connected by strings have the same magnitude of acceleration.



6–5 CIRCULAR MOTION

An object moving with speed v in a circle of radius r has an acceleration of magnitude v^2/r directed toward the center of the circle: This is referred to as the centripetal acceleration, a_{cp}. If the object has a mass m, the force required for the circular motion is

$$f_{cp} = ma_{cp} = mv^2/r \qquad 6\text{–}16$$



PROBLEM-SOLVING SUMMARY

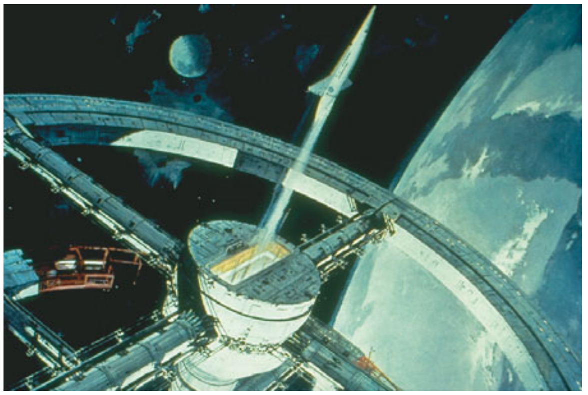
Type of Calculation	Relevant Physical Concepts	Related Examples
Find the acceleration when kinetic friction is present.	First, find the magnitude of the normal force, N. The corresponding kinetic friction has a magnitude of $f_k = \mu_k N$ and points opposite to the direction of motion. Include this force with the others when applying Newton's second law.	Examples 6–1, 6–2
Solve problems involving static friction.	Start by finding the magnitude of the normal force, N. The corresponding static friction has a magnitude between zero and $\mu_s N$. Its direction opposes motion.	Example 6–3 Active Example 6–1
Find the acceleration and the tension for masses connected by a string.	Apply Newton's second law to each mass separately. This generates two equations, which can be solved for the two unknowns, a and T.	Examples 6–6, 6–7
Solve problems involving circular motion.	Set up the coordinate system so that one axis points to the center of the circle. When applying Newton's second law to that direction, set the acceleration equal to $a_{cp} = v^2/r$.	Examples 6–8, 6–9 Active Example 6–4

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A clothesline always sags a little, even if nothing hangs from it. Explain.
2. In the *Jurassic Park* sequel, *The Lost World*, a man tries to keep a large vehicle from going over a cliff by connecting a cable from his Jeep to the vehicle. The man then puts the Jeep in gear and spins the rear wheels. Do you expect that spinning the tires will increase the force exerted by the Jeep on the vehicle? Why or why not?
3. When a traffic accident is investigated, it is common for the length of the skid marks to be measured. How could this information be used to estimate the initial speed of the vehicle that left the skid marks?
4. In a car with rear-wheel drive, the maximum acceleration is often less than the maximum deceleration. Why?
5. A train typically requires a much greater distance to come to rest, for a given initial speed, than does a car. Why?
6. Give some everyday examples of situations in which friction is beneficial.
7. At the local farm, you buy a flat of strawberries and place them on the backseat of the car. On the way home, you begin to brake as you approach a stop sign. At first the strawberries stay put, but as you brake a bit harder, they begin to slide off the seat. Explain.
8. It is possible to spin a bucket of water in a vertical circle and have none of the water spill when the bucket is upside down. How would you explain this to members of your family?
9. Water sprays off a rapidly turning bicycle wheel. Why?
10. Can an object be in equilibrium if it is moving? Explain.
11. In a dramatic circus act, a motorcyclist drives his bike around the inside of a vertical circle. How is this possible, considering that the motorcycle is upside down at the top of the circle?
12. The gravitational attraction of the Earth is only slightly less at the altitude of an orbiting spacecraft than it is on the Earth's surface. Why is it, then, that astronauts feel weightless?
13. A popular carnival ride has passengers stand with their backs against the inside wall of a cylinder. As the cylinder begins to spin, the passengers feel as if they are being pushed against the wall. Explain.
14. Referring to Question 13, after the cylinder reaches operating speed, the floor is lowered away, leaving the passengers "stuck" to the wall. Explain.
15. Your car is stuck on an icy side street. Some students on their way to class see your predicament and help out by sitting on the trunk of your car to increase its traction. Why does this help?
16. The parking brake on a car causes the rear wheels to lock up. What would be the likely consequence of applying the parking brake in a car that is in rapid motion? (*Note:* Do *not* try this at home.)
17. **BIO** The foot of your average gecko is covered with billions of tiny hair tips—called spatulae—that are made of keratin, the protein found in human hair. A subtle shift of the electron distribution in both the spatulae and the wall to which a gecko clings produces an adhesive force by means of the van der Waals interaction between molecules. Suppose a gecko uses its spatulae to cling to a vertical windowpane. If you were to describe this situation in terms of a coefficient of static friction, μ_s, what value would you assign to μ_s? Is this a sensible way to model the gecko's feat? Explain.
18. Discuss the physics involved in the spin cycle of a washing machine. In particular, how is circular motion related to the removal of water from the clothes?
19. The gas pedal and the brake pedal are capable of causing a car to accelerate. Can the steering wheel also produce an acceleration? Explain.
20. In the movie *2001: A Space Odyssey*, a rotating space station provides "artificial gravity" for its inhabitants. How does this work?



The rotating space station from the movie *2001: A Space Odyssey* (Conceptual Question 20)

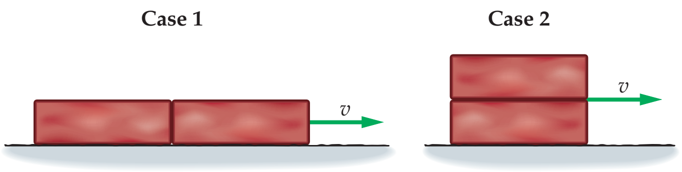
21. When rounding a corner on a bicycle or a motorcycle, the driver leans inward, toward the center of the circle. Why?
22. In *Robin Hood: Prince of Thieves*, starring Kevin Costner, Robin swings between trees on a vine that is on fire. At the lowest point of his swing, the vine burns through and Robin begins to fall. The next shot, from high up in the trees, shows Robin falling straight downward. Would you rate the physics of this scene "Good," "Bad," or "Ugly"? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 6–1 FRICTIONAL FORCES

1. • **CE Predict/Explain** You push two identical bricks across a tabletop with constant speed, v, as shown in **Figure 6–16**. In case 1, you place the bricks end to end; in case 2, you stack the bricks one on top of the other. **(a)** Is the force of kinetic friction in case 1 greater than, less than, or equal to the force of kinetic friction in case 2? **(b)** Choose the *best explanation* from among the following:



▲ **FIGURE 6–16** Problem 1

I. The normal force in case 2 is larger, and hence the bricks press down more firmly against the tabletop.
II. The normal force is the same in the two cases, and friction is independent of surface area.
III. Case 1 has more surface area in contact with the tabletop, and this leads to more friction.

2. • **CE Predict/Explain** Two drivers traveling side-by-side at the same speed suddenly see a deer in the road ahead of them and begin braking. Driver 1 stops by locking up his brakes and screeching to a halt; driver 2 stops by applying her brakes just to the verge of locking, so that the wheels continue to turn until her car comes to a complete stop. **(a)** All other factors being equal, is the stopping distance of driver 1 greater than, less than, or equal to the stopping distance of driver 2? **(b)** Choose the *best explanation* from among the following:
I. Locking up the brakes gives the greatest possible braking force.
II. The same tires on the same road result in the same force of friction.
III. Locked-up brakes lead to sliding (kinetic) friction, which is less than rolling (static) friction.

3. • A baseball player slides into third base with an initial speed of 4.0 m/s. If the coefficient of kinetic friction between the player and the ground is 0.46, how far does the player slide before coming to rest?

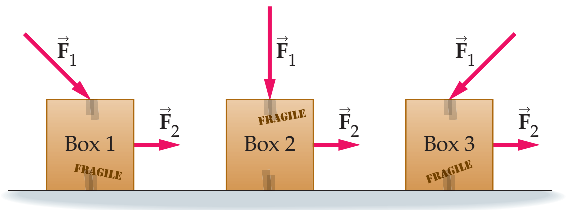
4. • A child goes down a playground slide with an acceleration of 1.26 m/s^2. Find the coefficient of kinetic friction between the child and the slide if the slide is inclined at an angle of 33.0° below the horizontal.

5. • Hopping into your Porsche, you floor it and accelerate at 12 m/s^2 without spinning the tires. Determine the minimum coefficient of static friction between the tires and the road needed to make this possible.

6. • When you push a 1.80-kg book resting on a tabletop, it takes 2.25 N to start the book sliding. Once it is sliding, however, it takes only 1.50 N to keep the book moving with constant speed. What are the coefficients of static and kinetic friction between the book and the tabletop?

7. • In Problem 6, what is the frictional force exerted on the book when you push on it with a force of 0.75 N?

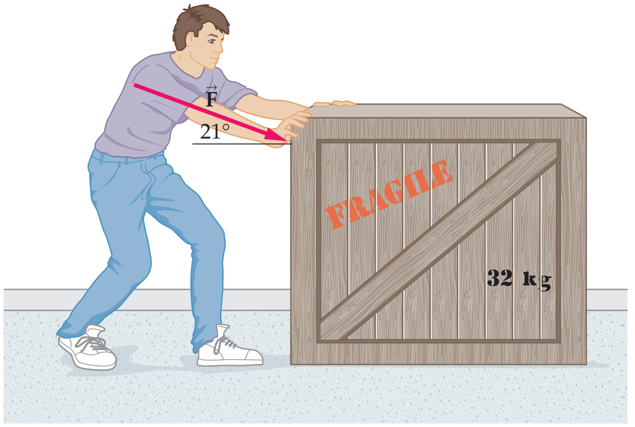
8. •• **CE** The three identical boxes shown in **Figure 6–17** remain at rest on a rough, horizontal surface, even though they are acted on by two different forces, $\vec{F}_1$ and $\vec{F}_2$. All of the forces labeled $\vec{F}_1$ have the same magnitude; all of the forces labeled $\vec{F}_2$ are identical to one another. Rank the boxes in order of increasing magnitude of the force static friction between them and the surface. Indicate ties where appropriate.



▲ **FIGURE 6–17** Problem 8

9. •• **IP** A tie of uniform width is laid out on a table, with a fraction of its length hanging over the edge. Initially, the tie is at rest. **(a)** If the fraction hanging from the table is increased, the tie eventually slides to the ground. Explain. **(b)** What is the coefficient of static friction between the tie and the table if the tie begins to slide when one-fourth of its length hangs over the edge?

10. •• To move a large crate across a rough floor, you push on it with a force F at an angle of 21° below the horizontal, as shown in **Figure 6–18**. Find the force necessary to start the crate moving, given that the mass of the crate is 32 kg and the coefficient of static friction between the crate and the floor is 0.57.



▲ **FIGURE 6–18** Problems 10, 11, and 106

11. •• In Problem 10, find the acceleration of the crate if the applied force is 330 N and the coefficient of kinetic friction is 0.45.

12. •• **IP** A 48-kg crate is placed on an inclined ramp. When the angle the ramp makes with the horizontal is increased to 26°, the crate begins to slide downward. **(a)** What is the coefficient of static friction between the crate and the ramp? **(b)** At what angle does the crate begin to slide if its mass is doubled?

13. •• **IP** A 97-kg sprinter wishes to accelerate from rest to a speed of 13 m/s in a distance of 22 m. **(a)** What coefficient of static friction is required between the sprinter's shoes and the track? **(b)** Explain the strategy used to find the answer to part (a).

14. •• **Coffee To Go** A person places a cup of coffee on the roof of her car while she dashes back into the house for a forgotten item. When she returns to the car, she hops in and takes off with the coffee cup still on the roof. **(a)** If the coefficient of static friction between the coffee cup and the roof of the car is 0.24, what is the maximum acceleration the car can have without causing the cup to slide? Ignore the effects of air resistance. **(b)** What is the smallest amount of time in which the person can accelerate the car from rest to 15 m/s and still keep the coffee cup on the roof?

15. •• **IP Force Times Distance I** At the local hockey rink, a puck with a mass of 0.12 kg is given an initial speed of $v = 5.3$ m/s. **(a)** If the coefficient of kinetic friction between the ice and the puck is 0.11, what distance d does the puck slide before coming to rest? **(b)** If the mass of the puck is doubled, does the frictional force F exerted on the puck increase, decrease, or stay the same? Explain. **(c)** Does the stopping distance of the puck increase, decrease, or stay the same when its mass is doubled? Explain. **(d)** For the situation considered in part (a), show that $Fd = \frac{1}{2}mv^2$.

(The significance of this result will be discussed in Chapter 7, where we will see that $\frac{1}{2}mv^2$ is the kinetic energy of an object.)

16. •• **IP** **Force Times Time** At the local hockey rink, a puck with a mass of 0.12 kg is given an initial speed of $v_0 = 6.7$ m/s. **(a)** If the coefficient of kinetic friction between the ice and the puck is 0.13, how much time t does it take for the puck to come to rest? **(b)** If the mass of the puck is doubled, does the frictional force F exerted on the puck increase, decrease, or stay the same? Explain. **(c)** Does the stopping time of the puck increase, decrease, or stay the same when its mass is doubled? Explain. **(d)** For the situation considered in part (a), show that $Ft = mv_0$. (The significance of this result will be discussed in Chapter 9, where we will see that mv is the momentum of an object.)

17. •• **Force Times Distance II** A block of mass $m = 1.95$ kg slides with an initial speed $v_i = 4.33$ m/s on a smooth, horizontal surface. The block now encounters a rough patch with a coefficient of kinetic friction given by $\mu_k = 0.260$. The rough patch extends for a distance $d = 0.125$ m, after which the surface is again frictionless. **(a)** What is the acceleration of the block when it is in the rough patch? **(b)** What is the final speed, v_f, of the block when it exits the rough patch? **(c)** Show that $-Fd = -(\mu_k mg)d = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$. (The significance of this result will be discussed in Chapter 7, where we will see that $\frac{1}{2}mv^2$ is the kinetic energy of an object.)

18. ••• **IP** The coefficient of kinetic friction between the tires of your car and the roadway is μ. **(a)** If your initial speed is v and you lock your tires during braking, how far do you skid? Give your answer in terms of v, μ, and m, the mass of your car. **(b)** If you double your speed, what happens to the stopping distance? **(c)** What is the stopping distance for a truck with twice the mass of your car, assuming the same initial speed and coefficient of kinetic friction?

SECTION 6–2 STRINGS AND SPRINGS

19. • **CE** A certain spring has a force constant k. **(a)** If this spring is cut in half, does the resulting half spring have a force constant that is greater than, less than, or equal to k? **(b)** If two of the original full-length springs are connected end to end, does the resulting double spring have a force constant that is greater than, less than, or equal to k?

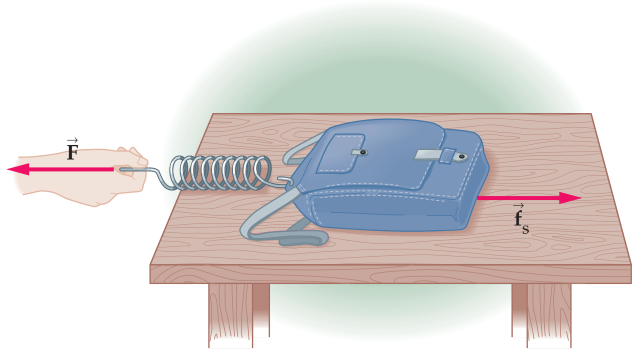
20. • Pulling up on a rope, you lift a 4.35-kg bucket of water from a well with an acceleration of 1.78 m/s^2. What is the tension in the rope?

21. • When a 9.09-kg mass is placed on top of a vertical spring, the spring compresses 4.18 cm. Find the force constant of the spring.

22. • A 110-kg box is loaded into the trunk of a car. If the height of the car's bumper decreases by 13 cm, what is the force constant of its rear suspension?

23. • A 50.0-kg person takes a nap in a backyard hammock. Both ropes supporting the hammock are at an angle of 15.0° above the horizontal. Find the tension in the ropes.

24. • **IP** A backpack full of books weighing 52.0 N rests on a table in a physics laboratory classroom. A spring with a force constant of 150 N/m is attached to the backpack and pulled horizontally, as indicated in **Figure 6–19**. **(a)** If the spring is pulled until it stretches 2.00 cm and the pack remains at rest, what is the force of friction exerted on the backpack by the table? **(b)** Does your answer to part (a) change if the mass of the backpack is doubled? Explain.



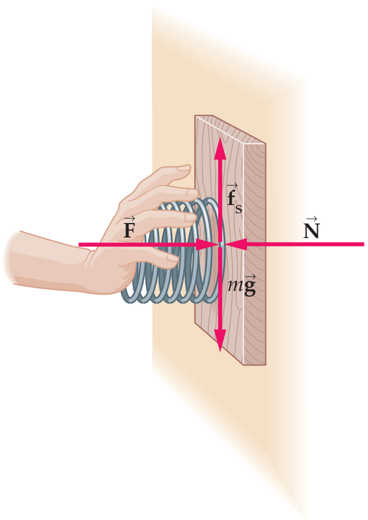
▲ **FIGURE 6–19** Problems 24 and 25

25. • If the 52.0-N backpack in Problem 24 begins to slide when the spring ($k = 150$ N/m) stretches by 2.50 cm, what is the coefficient of static friction between the backpack and the table?

26. •• **IP** The equilibrium length of a certain spring with a force constant of $k = 250$ N/m is 0.18 m. **(a)** What is the magnitude of the force that is required to hold this spring at twice its equilibrium length? **(b)** Is the magnitude of the force required to keep the spring compressed to half its equilibrium length greater than, less than, or equal to the force found in part (a)? Explain.

27. •• **IP** Illinois Jones is being pulled from a snake pit with a rope that breaks if the tension in it exceeds 755 N. **(a)** If Illinois Jones has a mass of 70.0 kg and the snake pit is 3.40 m deep, what is the minimum time that is required to pull our intrepid explorer from the pit? **(b)** Explain why the rope breaks if Jones is pulled from the pit in less time than that calculated in part (a).

28. •• **IP** A spring with a force constant of 120 N/m is used to push a 0.27-kg block of wood against a wall, as shown in **Figure 6–20**. **(a)** Find the minimum compression of the spring needed to keep the block from falling, given that the coefficient of static friction between the block and the wall is 0.46. **(b)** Does your answer to part (a) change if the mass of the block of wood is doubled? Explain.

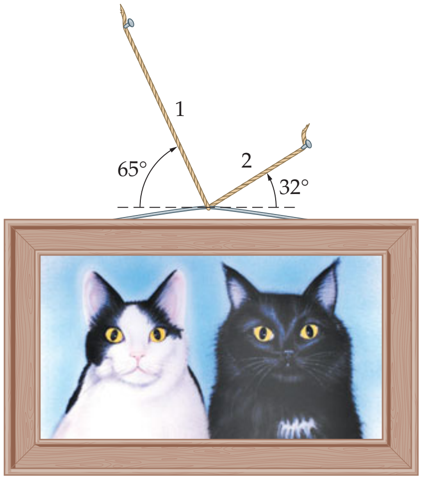


▲ **FIGURE 6–20** Problem 28

29. •• **IP** Your friend's 13.6-g graduation tassel hangs on a string from his rearview mirror. **(a)** When he accelerates from a stoplight, the tassel deflects backward toward the rear of the car. Explain. **(b)** If the tassel hangs at an angle of 6.44° relative to the vertical, what is the acceleration of the car?

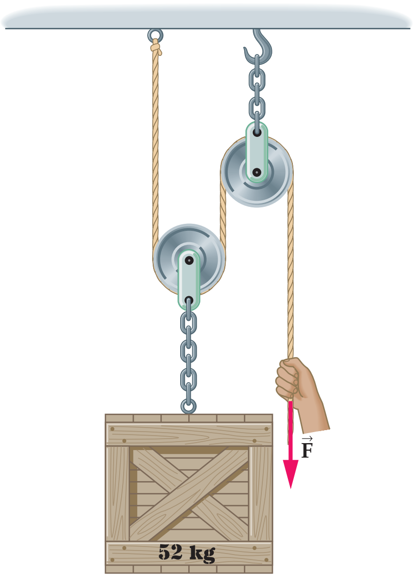
30. •• In Problem 29, **(a)** find the tension in the string holding the tassel. **(b)** At what angle to the vertical will the tension in the string be twice the weight of the tassel?

31. •• **IP** A picture hangs on the wall suspended by two strings, as shown in **Figure 6–21**. The tension in string 1 is 1.7 N. **(a)** Is the tension in string 2 greater than, less than, or equal to 1.7 N? Explain. **(b)** Verify your answer to part (a) by calculating the tension in string 2. **(c)** What is the weight of the picture?



▲ **FIGURE 6–21** Problems 31 and 83

32. •• **Mechanical Advantage** The pulley system shown in **Figure 6–22** is used to lift a 52-kg crate. Note that one chain connects the upper pulley to the ceiling and a second chain connects the lower pulley to the crate. Assuming the masses of the chains, pulleys, and ropes are negligible, determine **(a)** the force $\vec{F}$ required to lift the crate with constant speed, **(b)** the tension in the upper chain, and **(c)** the tension in the lower chain.



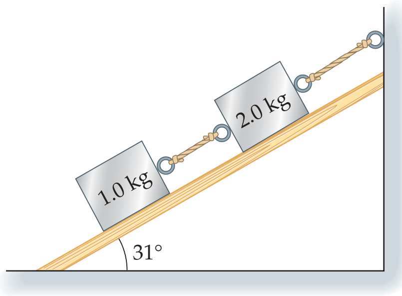
▲ **FIGURE 6–22** Problems 32 and 33

33. •• In Problem 32, determine **(a)** the force $\vec{F}$, **(b)** the tension in the upper chain, and **(c)** the tension in the lower chain, given that the crate is rising with an acceleration of 2.3 m/s^2.

SECTION 6–3 TRANSLATIONAL EQUILIBRIUM

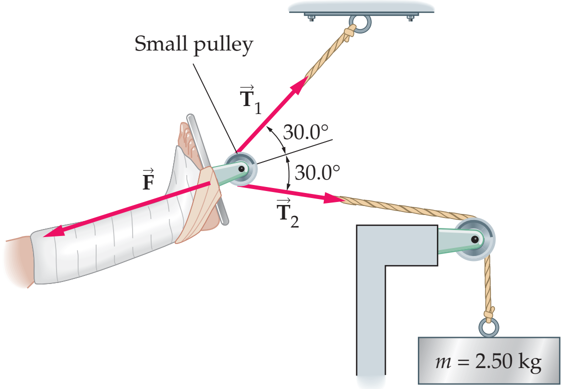
34. • Pulling the string on a bow back with a force of 28.7 lb, an archer prepares to shoot an arrow. If the archer pulls in the center of the string, and the angle between the two halves is 138°, what is the tension in the string?

35. • In **Figure 6–23** we see two blocks connected by a string and tied to a wall. The mass of the lower block is 1.0 kg; the mass of the upper block is 2.0 kg. Given that the angle of the incline is 31°, find the tensions in **(a)** the string connecting the two blocks and **(b)** the string that is tied to the wall.



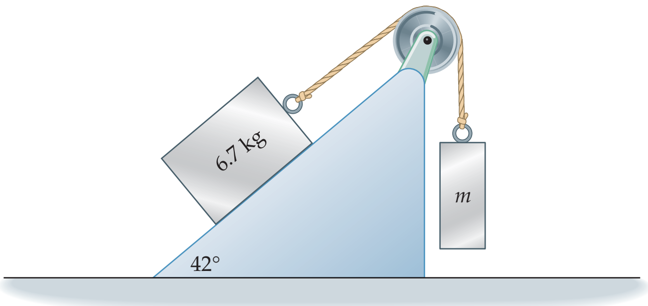
▲ **FIGURE 6–23** Problem 35

36. • **BIO Traction** After a skiing accident, your leg is in a cast and supported in a traction device, as shown in **Figure 6–24**. Find the magnitude of the force $\vec{F}$ exerted by the leg on the small pulley. (By Newton's third law, the small pulley exerts an equal and opposite force on the leg.) Let the mass m be 2.50 kg.



▲ **FIGURE 6–24** Problems 36 and 69

37. • Two blocks are connected by a string, as shown in **Figure 6–25**. The smooth inclined surface makes an angle of 42° with the horizontal, and the block on the incline has a mass of 6.7 kg. Find the mass of the hanging block that will cause the system to be in equilibrium. (The pulley is assumed to be ideal.)



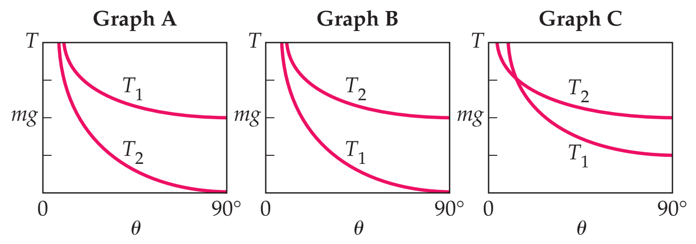
▲ **FIGURE 6–25** Problem 37

38. •• **CE Predict/Explain (a)** Referring to the hanging planter in Example 6–5, which of the three graphs (A, B, or C) in **Figure 6–26** shows an accurate plot of the tensions T_1 and T_2 as a function of the angle θ? **(b)** Choose the *best explanation* from among the following:

I. The two tensions must be equal at some angle between $\theta = 0$ and $\theta = 90°$.

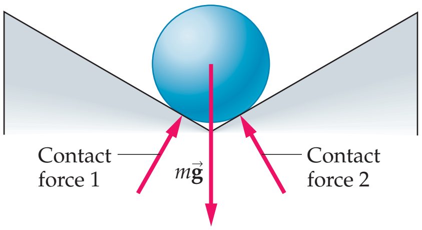
II. T_2 is greater than T_1 at all angles, and is equal to mg at $\theta = 90°$.

III. T_2 is less than T_1 at all angles, and is equal to 0 at $\theta = 90°$.



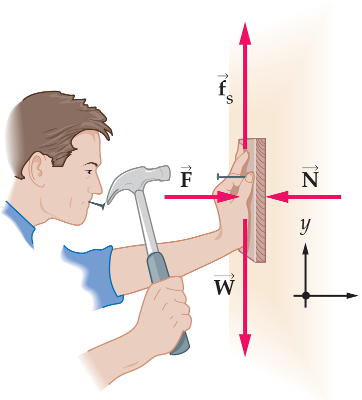
▲ **FIGURE 6–26** Problem 38

39. •• A 0.15-kg ball is placed in a shallow wedge with an opening angle of 120°, as shown in **Figure 6–27**. For each contact point between the wedge and the ball, determine the force exerted on the ball. Assume the system is frictionless.



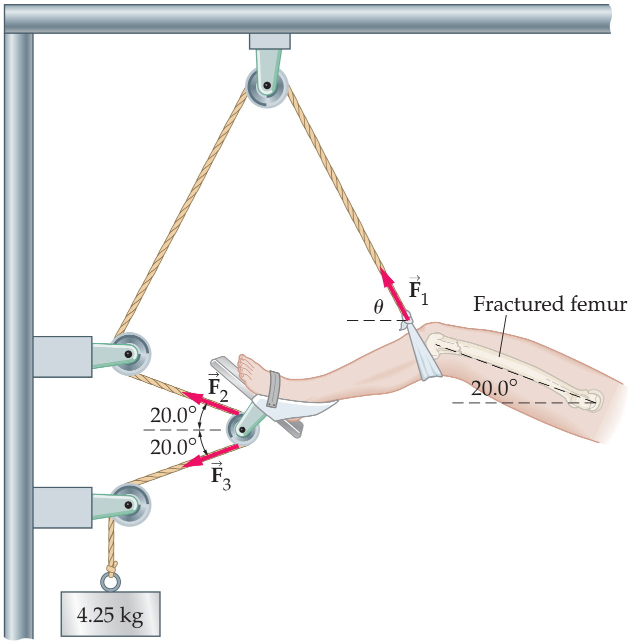
▲ **FIGURE 6–27** Problem 39

40. •• **IP** You want to nail a 1.6-kg board onto the wall of a barn. To position the board before nailing, you push it against the wall with a horizontal force $\vec{F}$ to keep it from sliding to the ground (**Figure 6–28**). **(a)** If the coefficient of static friction between the board and the wall is 0.79, what is the least force you can apply and still hold the board in place? **(b)** What happens to the force of static friction if you push against the wall with a force greater than that found in part (a)?



▲ **FIGURE 6–28** Problem 40

41. ••• **BIO The Russell Traction System** To immobilize a fractured femur (the thigh bone), doctors often utilize the Russell traction system illustrated in **Figure 6–29**. Notice that one force is applied directly to the knee, $\vec{F}_1$, while two other forces, $\vec{F}_2$ and $\vec{F}_3$, are applied to the foot. The latter two forces combine to give a force $\vec{F}_2 + \vec{F}_3$ that is transmitted through the lower leg to the knee. The result is that the knee experiences the total force $\vec{F}_{total} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$. The goal of this traction system is to have $\vec{F}_{total}$ directly in line with the fractured femur, at an angle of 20.0° above the horizontal. Find **(a)** the angle θ required to produce this alignment of $\vec{F}_{total}$ and **(b)** the magnitude of the force, $\vec{F}_{total}$ that is applied to the femur in this case. (Assume the pulleys are ideal.)



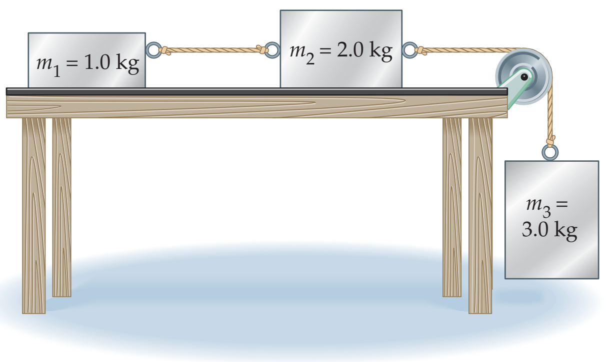
▲ **FIGURE 6–29** Problem 41

SECTION 6–4 CONNECTED OBJECTS

42. • **CE** In Example 6–6 (Connected Blocks), suppose m_1 and m_2 are both increased by a factor of 2. **(a)** Does the acceleration of the blocks increase, decrease, or stay the same? **(b)** Does the tension in the string increase, decrease, or stay the same?

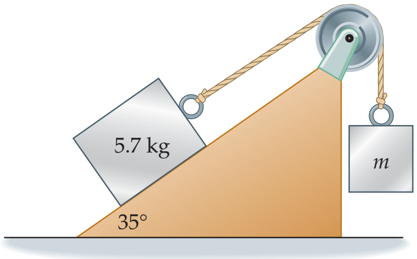
43. • **CE Predict/Explain** Suppose m_1 and m_2 in Example 6–7 (Atwood's Machine) are both increased by 1 kg. Does the acceleration of the blocks increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. The net force acting on the blocks is the same, but the total mass that must be accelerated is greater.
   II. The difference in the masses is the same, and this is what determines the net force on the system.
   III. The force exerted on each block is greater, leading to an increased acceleration.

44. • Find the acceleration of the masses shown in **Figure 6–30**, given that $m_1 = 1.0$ kg, $m_2 = 2.0$ kg, and $m_3 = 3.0$ kg. Assume the table is frictionless and the masses move freely.



▲ **FIGURE 6–30** Problems 44, 47, and 103

45. • Two blocks are connected by a string, as shown in **Figure 6–31**. The smooth inclined surface makes an angle of 35° with the horizontal, and the block on the incline has a mass of 5.7 kg. The mass of the hanging block is $m = 3.2$ kg. Find **(a)** the direction and **(b)** the magnitude of the hanging block's acceleration.

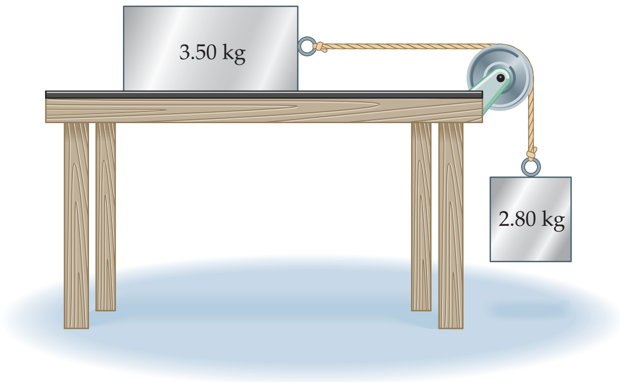


▲ **FIGURE 6–31** Problems 45 and 46

46. • Referring to Problem 45, find **(a)** the direction and **(b)** the magnitude of the hanging block's acceleration if its mass is $m = 4.2$ kg.

47. •• Referring to Figure 6–30, find the tension in the string connecting **(a)** m_1 and m_2 and **(b)** m_2 and m_3. Assume the table is frictionless and the masses move freely.

48. •• **IP** A 3.50-kg block on a smooth tabletop is attached by a string to a hanging block of mass 2.80 kg, as shown in **Figure 6–32**. The blocks are released from rest and allowed to move freely. **(a)** Is the tension in the string greater than, less than, or equal to the weight of the hanging mass? Find **(b)** the acceleration of the blocks and **(c)** the tension in the string.



▲ **FIGURE 6–32** Problem 48

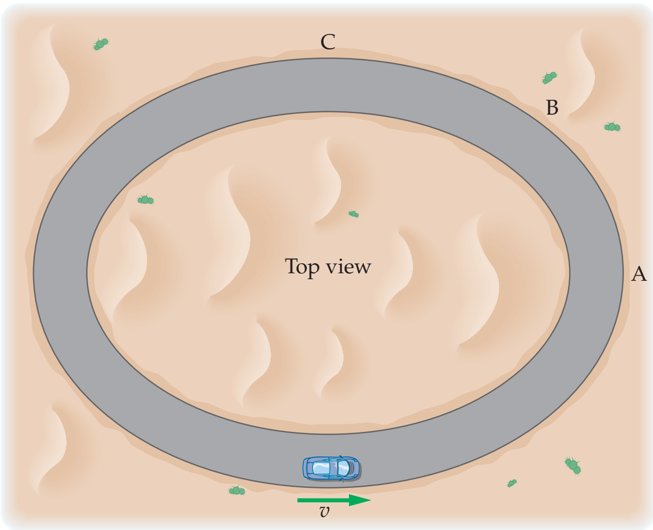
49. •• **IP** A 7.7-N force pulls horizontally on a 1.6-kg block that slides on a smooth horizontal surface. This block is connected by a horizontal string to a second block of mass $m_2 = 0.83$ kg on the same surface. **(a)** What is the acceleration of the blocks? **(b)** What is the tension in the string? **(c)** If the mass of block 1 is increased, does the tension in the string increase, decrease, or stay the same?

50. ••• **Buckets and a Pulley** Two buckets of sand hang from opposite ends of a rope that passes over an ideal pulley. One bucket is full and weighs 120 N; the other bucket is only partly filled and weighs 63 N. **(a)** Initially, you hold onto the lighter bucket to keep it from moving. What is the tension in the rope? **(b)** You release the lighter bucket and the heavier one descends. What is the tension in the rope now? **(c)** Eventually the heavier bucket lands and the two buckets come to rest. What is the tension in the rope now?

SECTION 6–5 CIRCULAR MOTION

51. • **CE** Suppose you stand on a bathroom scale and get a reading of 700 N. In principle, would the scale read more, less, or the same if the Earth did not rotate?

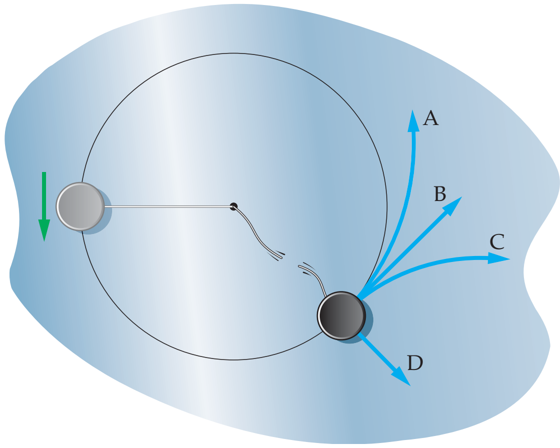
52. • **CE** A car drives with constant speed on an elliptical track, as shown in **Figure 6–33**. Rank the points A, B, and C in order of increasing likelihood that the car might skid. Indicate ties where appropriate.



▲ **FIGURE 6–33** Problem 52

53. • **CE** A car is driven with constant speed around a circular track. Answer each of the following questions with "Yes" or "No." **(a)** Is the car's velocity constant? **(b)** Is its speed constant? **(c)** Is the magnitude of its acceleration constant? **(d)** Is the direction of its acceleration constant?

54. • **CE** A puck attached to a string undergoes circular motion on an air table. If the string breaks at the point indicated in **Figure 6–34**, is the subsequent motion of the puck best described by path A, B, C, or D?



▲ **FIGURE 6–34** Problem 54

55. • When you take your 1300-kg car out for a spin, you go around a corner of radius 59 m with a speed of 16 m/s. The coefficient of static friction between the car and the road is 0.88. Assuming your car doesn't skid, what is the force exerted on it by static friction?

56. • Find the linear speed of the bottom of a test tube in a centrifuge if the centripetal acceleration there is 52,000 times the acceleration of gravity. The distance from the axis of rotation to the bottom of the test tube is 7.5 cm.

57. • **BIO A Human Centrifuge** To test the effects of high acceleration on the human body, the National Aeronautics and Space Administration (NASA) has constructed a large centrifuge at the Manned Spacecraft Center in Houston. In this device, astronauts are placed in a capsule that moves in a circular path with a radius of 15 m. If the astronauts in this centrifuge experience a centripetal acceleration 9.0 times that of gravity, what is the linear speed of the capsule?

58. • A car goes around a curve on a road that is banked at an angle of 33.5°. Even though the road is slick, the car will stay on the road without any friction between its tires and the road when its speed is 22.7 m/s. What is the radius of the curve?

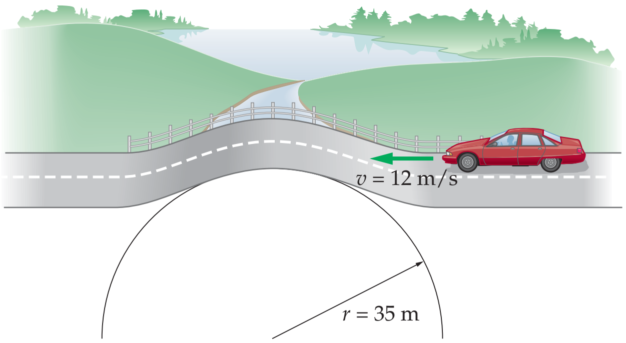
59. •• Jill of the Jungle swings on a vine 6.9 m long. What is the tension in the vine if Jill, whose mass is 63 kg, is moving at 2.4 m/s when the vine is vertical?

60. •• **IP** In Problem 59, **(a)** how does the tension in the vine change if Jill's speed is doubled? Explain. **(b)** How does the tension change if her mass is doubled instead? Explain.

61. •• **IP** **(a)** As you ride on a Ferris wheel, your apparent weight is different at the top than at the bottom. Explain. **(b)** Calculate your apparent weight at the top and bottom of a Ferris wheel, given that the radius of the wheel is 7.2 m, it completes one revolution every 28 s, and your mass is 55 kg.

A Ferris Wheel (Problems 61 and 84)

62. •• Driving in your car with a constant speed of 12 m/s, you encounter a bump in the road that has a circular cross section, as indicated in **Figure 6–35**. If the radius of curvature of the bump is 35 m, find the apparent weight of a 67-kg person in your car as you pass over the top of the bump.



▲ **FIGURE 6–35** Problems 62 and 63

63. •• Referring to Problem 62, at what speed must you go over the bump if people in your car are to feel "weightless"?

64. •• **IP** You swing a 4.6-kg bucket of water in a vertical circle of radius 1.3 m. **(a)** What speed must the bucket have if it is to complete the circle without spilling any water? **(b)** How does your answer depend on the mass of the bucket?

GENERAL PROBLEMS

65. • **CE** If you weigh yourself on a bathroom scale at the equator, is the reading you get greater than, less than, or equal to the reading you get if you weigh yourself at the North Pole?

66. • **CE** An object moves on a flat surface with an acceleration of constant magnitude. If the acceleration is always perpendicular to the object's direction of motion, **(a)** is the shape of the object's path circular, linear, or parabolic? **(b)** During its motion, does the object's velocity change in direction but not magnitude, change in magnitude but not direction, or change in both magnitude and direction? **(c)** Does its speed increase, decrease, or stay the same?

67. • **CE BIO** **Maneuvering a Jet** Humans lose consciousness if exposed to prolonged accelerations of more than about $7g$. This is of concern to jet fighter pilots, who may experience centripetal accelerations of this magnitude when making high-speed turns. Suppose we would like to decrease the centripetal acceleration of a jet. Rank the following changes in flight path in order of how effective they are in decreasing the centripetal acceleration, starting with the least effective: **A**, decrease the turning radius by a factor of two; **B**, decrease the speed by a factor of three; or **C**, increase the turning radius by a factor of four.

68. • **CE BIO** **Gravitropism** As plants grow, they tend to align their stems and roots along the direction of the gravitational field. This tendency, which is related to differential concentrations of plant hormones known as auxins, is referred to as *gravitropism*. As an illustration of gravitropism, experiments show that seedlings placed in pots on the rim of a rotating turntable do not grow in the vertical direction. Do you expect their stems to tilt inward—toward the axis of rotation—or outward—away from the axis of rotation?

69. • **BIO** A skateboard accident leaves your leg in a cast and supported by a traction device, as in Figure 6–24. Find the mass m that must be attached to the rope if the net force exerted by the small pulley on the foot is to have a magnitude of 37 N.

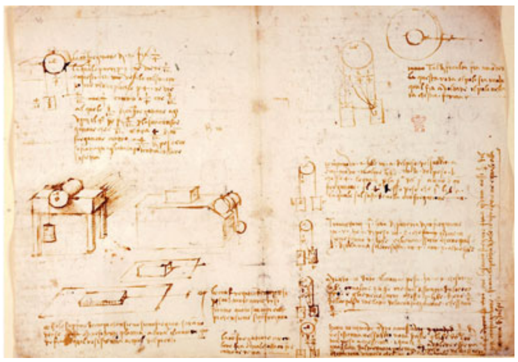
70. • Find the centripetal acceleration at the top of a test tube in a centrifuge, given that the top is 4.2 cm from the axis of rotation and that its linear speed is 77 m/s.

71. • Find the coefficient of kinetic friction between a 3.85-kg block and the horizontal surface on which it rests if an 850-N/m spring must be stretched by 6.20 cm to pull it with constant speed. Assume that the spring pulls in the horizontal direction.

72. • A child goes down a playground slide that is inclined at an angle of 26.5° below the horizontal. Find the acceleration of the child given that the coefficient of kinetic friction between the child and the slide is 0.315.

73. • When a block is placed on top of a vertical spring, the spring compresses 3.15 cm. Find the mass of the block, given that the force constant of the spring is 1750 N/m.

74. •• **The da Vinci Code** Leonardo da Vinci (1452–1519) is credited with being the first to perform quantitative experiments on friction, though his results weren't known until centuries later, due in part to the secret code (mirror writing) he used in his notebooks. Leonardo would place a block of wood on an inclined



Sketches from the notebooks of Leonardo da Vinci showing experiments he performed on friction (Problem 74) plane and measure the angle at which the block begins to slide. He reports that the coefficient of static friction was 0.25 in his experiments. At what angle did Leonardo's blocks begin to slide?

75. •• A force of 9.4 N pulls horizontally on a 1.1-kg block that slides on a rough, horizontal surface. This block is connected by a horizontal string to a second block of mass $m_2 = 1.92$ kg on the same surface. The coefficient of kinetic friction is $\mu_k = 0.24$ for both blocks. **(a)** What is the acceleration of the blocks? **(b)** What is the tension in the string?

76. •• You swing a 3.25-kg bucket of water in a vertical circle of radius 0.950 m. At the top of the circle the speed of the bucket is 3.23 m/s; at the bottom of the circle its speed is 6.91 m/s. Find the tension in the rope tied to the bucket at **(a)** the top and **(b)** the bottom of the circle.

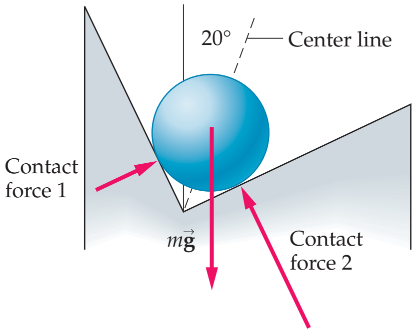
77. •• A 14-g coin slides upward on a surface that is inclined at an angle of 18° above the horizontal. The coefficient of kinetic friction between the coin and the surface is 0.23; the coefficient of static friction is 0.35. Find the magnitude and direction of the force of friction **(a)** when the coin is sliding and **(b)** after it comes to rest.

78. •• In Problem 77, the angle of the incline is increased to 25°. Find the magnitude and direction of the force of friction when the coin is **(a)** sliding upward initially and **(b)** sliding back downward later.

79. •• A physics textbook weighing 22 N rests on a table. The coefficient of static friction between the book and the table is $\mu_s = 0.60$; the coefficient of kinetic friction is $\mu_k = 0.40$. You push horizontally on the book with a force that gradually increases from 0 to 15 N, and then slowly decreases to 5.0 N, as indicated in the following table. For each value of the applied force given in the table, give the magnitude of the force of friction and state whether the book is accelerating, decelerating, at rest, or moving with constant speed.

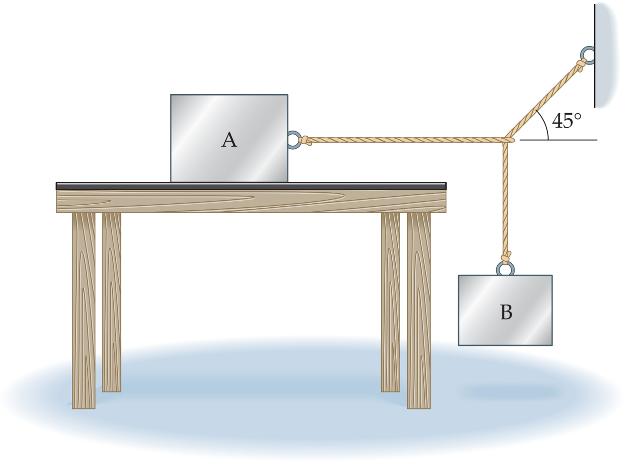
Applied force	Friction force	Motion
0		
5.0 N		
11 N		
15 N		
11 N		
8.0 N		
5.0 N		

80. •• A ball of mass m is placed in a wedge, as shown in **Figure 6–36**, in which the two walls meet at a right angle. Assuming the walls of the wedge are frictionless, determine the magnitude of **(a)** contact force 1 and **(b)** contact force 2.



▲ **FIGURE 6–36** Problem 80

81. •• **IP** The blocks shown in **Figure 6–37** are at rest. **(a)** Find the frictional force exerted on block A given that the mass of block A is 8.82 kg, the mass of block B is 2.33 kg, and the coefficient of static friction between block A and the surface on which it rests is 0.320. **(b)** If the mass of block A is doubled, does the frictional force exerted on it increase, decrease, or stay the same? Explain.



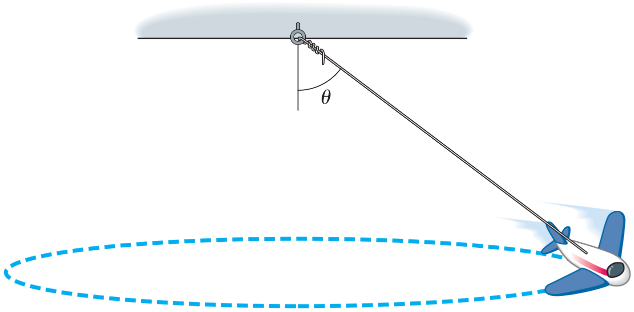
▲ **FIGURE 6–37** Problems 81 and 82

82. •• In part (a) of Problem 81, what is the maximum mass block B can have and the system still be in equilibrium?

83. •• **IP** A picture hangs on the wall suspended by two strings, as shown in Figure 6–21. The tension in string 2 is 1.7 N. **(a)** Is the tension in string 1 greater than, less than, or equal to 1.7 N? Explain. **(b)** Verify your answer to part (a) by calculating the tension in string 1. **(c)** What is the mass of the picture?

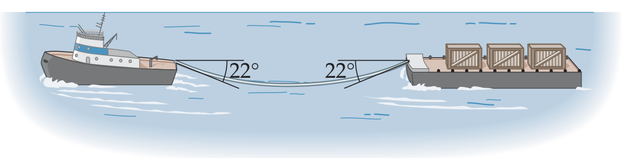
84. •• **IP** Referring to Problem 61, suppose the Ferris wheel rotates fast enough to make you feel "weightless" at the top. **(a)** How many seconds does it take to complete one revolution in this case? **(b)** How does your answer to part (a) depend on your mass? Explain. **(c)** What are the direction and magnitude of your acceleration when you are at the bottom of the wheel? Assume that its rotational speed has remained constant.

85. •• **A Conical Pendulum** A 0.075-kg toy airplane is tied to the ceiling with a string. When the airplane's motor is started, it moves with a constant speed of 1.21 m/s in a horizontal circle of radius 0.44 m, as illustrated in **Figure 6–38**. Find **(a)** the angle the string makes with the vertical and **(b)** the tension in the string.



▲ **FIGURE 6–38** Problem 85

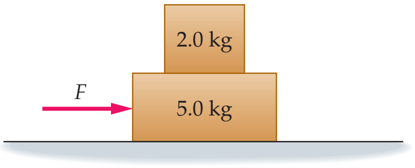
86. •• A tugboat tows a barge at constant speed with a 3500-kg cable, as shown in **Figure 6–39**. If the angle the cable makes with the hor-



▲ **FIGURE 6–39** Problem 86

izontal where it attaches to the barge and the tugboat is 22°, find the force the cable exerts on the barge in the forward direction.

87. •• **IP** Two blocks, stacked one on top of the other, can move without friction on the horizontal surface shown in **Figure 6–40**. The surface between the two blocks is rough, however, with a coefficient of static friction equal to 0.47. **(a)** If a horizontal force F is applied to the 5.0-kg bottom block, what is the maximum value F can have before the 2.0-kg top block begins to slip? **(b)** If the mass of the top block is increased, does the maximum value of F increase, decrease, or stay the same? Explain.



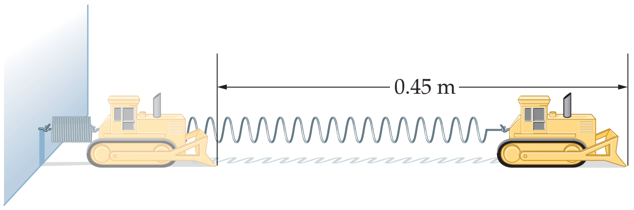
▲ **FIGURE 6–40** Problem 87

88. •• Find the coefficient of kinetic friction between a 4.7-kg block and the horizontal surface on which it rests if an 89-N/m spring must be stretched by 2.2 cm to pull the block with constant speed. Assume the spring pulls in a direction 13° above the horizontal.

89. •• **IP** In a daring rescue by helicopter, two men with a combined mass of 172 kg are lifted to safety. **(a)** If the helicopter lifts the men straight up with constant acceleration, is the tension in the rescue cable greater than, less than, or equal to the combined weight of the men? Explain. **(b)** Determine the tension in the cable if the men are lifted with a constant acceleration of 1.10 m/s^2.

90. •• At the airport, you pull a 18-kg suitcase across the floor with a strap that is at an angle of 45° above the horizontal. Find **(a)** the normal force and **(b)** the tension in the strap, given that the suitcase moves with constant speed and that the coefficient of kinetic friction between the suitcase and the floor is 0.38.

91. •• **IP** A light spring with a force constant of 13 N/m is connected to a wall and to a 1.2-kg toy bulldozer, as shown in **Figure 6–41**. When the electric motor in the bulldozer is turned on, it stretches the spring for a distance of 0.45 m before its tread begins to slip on the floor. **(a)** Which coefficient of friction (static or kinetic) can be determined from this information? Explain. **(b)** What is the numerical value of this coefficient of friction?

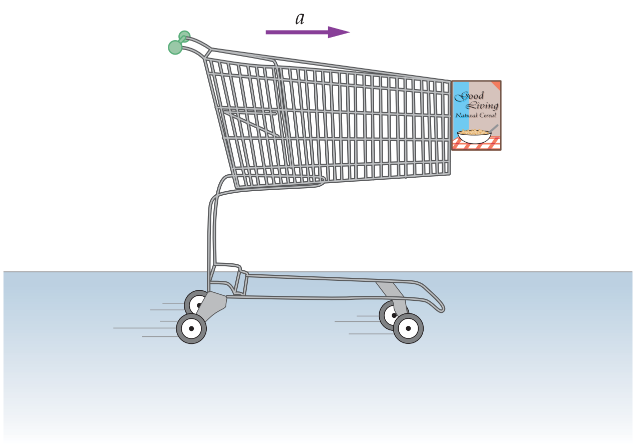


▲ **FIGURE 6–41** Problem 91

92. •• **IP** A 0.16-g spider hangs from the middle of the first thread of its future web. The thread makes an angle of 7.2° with the horizontal on both sides of the spider. **(a)** What is the tension in the thread? **(b)** If the angle made by the thread had been less than 7.2°, would its tension have been greater than, less than, or the same as in part (a)? Explain.

93. •• Find the acceleration the cart in **Figure 6–42** must have in order for the cereal box at the front of the cart not to fall. Assume that the coefficient of static friction between the cart and the box is 0.38.

94. •• **IP Playing a Violin** The tension in a violin string is 2.7 N. When pushed down against the neck of the violin, the string makes an angle of 4.1° with the horizontal. **(a)** With what force must you push down on the string to bring it into contact with the neck? **(b)** If the angle were less than 4.1°, would the required force be greater than, less than, or the same as in part (a)? Explain.



▲ **FIGURE 6–42** Problem 93

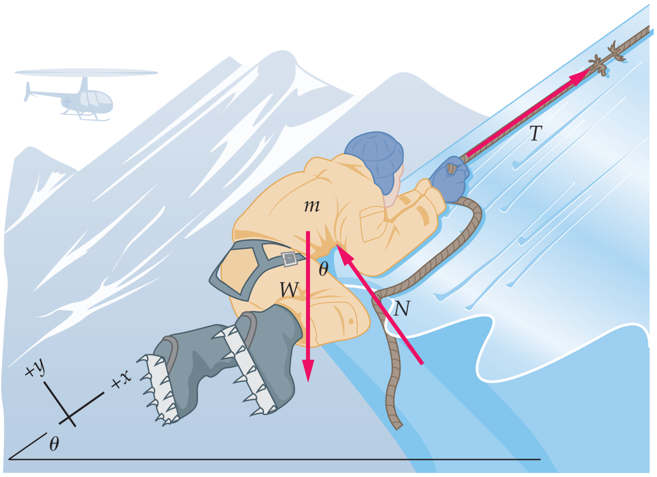
95. •• **IP** A pair of fuzzy dice hangs from a string attached to your rearview mirror. As you turn a corner with a radius of 98 m and a constant speed of 27 mi/h, what angle will the dice make with the vertical? Why is it unnecessary to give the mass of the dice?

96. •• Find the tension in each of the two ropes supporting a hammock if one is at an angle of 18° above the horizontal and the other is at an angle of 35° above the horizontal. The person sleeping in the hammock (unconcerned about tensions and ropes) has a mass of 68 kg.

97. •• As your plane circles an airport, it moves in a horizontal circle of radius 2300 m with a speed of 390 km/h. If the lift of the airplane's wings is perpendicular to the wings, at what angle should the plane be banked so that it doesn't tend to slip sideways?

98. •• **IP** A block with a mass of 3.1 kg is placed at rest on a surface inclined at an angle of 45° above the horizontal. The coefficient of static friction between the block and the surface is 0.50, and a force of magnitude F pushes upward on the block, parallel to the inclined surface. **(a)** The block will remain at rest only if F is greater than a minimum value, F_{min}, and less than a maximum value, F_{max}. Explain the reasons for this behavior. **(b)** Calculate F_{min}. **(c)** Calculate F_{max}.

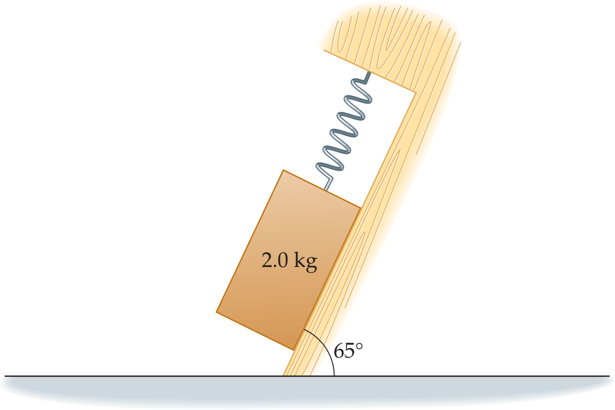
99. •• A mountain climber of mass m hangs onto a rope to keep from sliding down a smooth, ice-covered slope (**Figure 6–43**). Find a formula for the tension in the rope when the slope is inclined at an angle θ above the horizontal. Check your results in the limits $\theta = 0$ and $\theta = 90°$.



▲ **FIGURE 6–43** Problem 99

100. •• A child sits on a rotating merry-go-round, 2.3 m from its center. If the speed of the child is 2.2 m/s, what is the minimum coefficient of static friction between the child and the merry-go-round that will prevent the child from slipping?

101. ••• A 2.0-kg box rests on a plank that is inclined at an angle of 65° above the horizontal. The upper end of the box is attached to a spring with a force constant of 360 N/m, as shown in **Figure 6–44**. If the coefficient of static friction between the box and the plank is 0.22, what is the maximum amount the spring can be stretched and the box remain at rest?



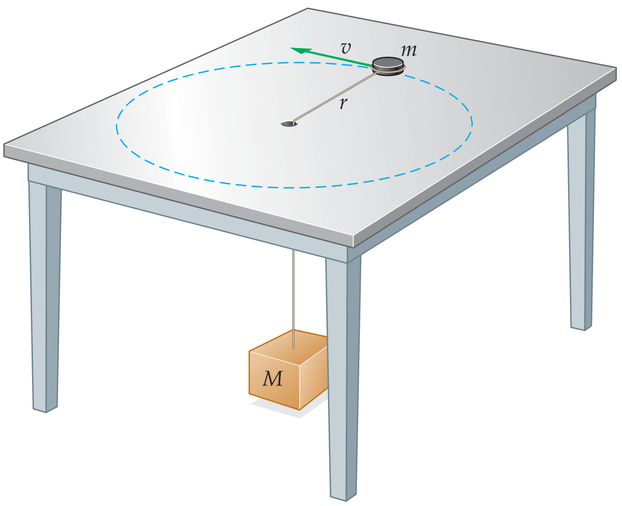
▲ **FIGURE 6–44** Problem 101

102. ••• A wood block of mass m rests on a larger wood block of mass M that rests on a wooden table. The coefficients of static and kinetic friction between all surfaces are μ_s and μ_k, respectively. What is the minimum horizontal force, F, applied to the lower block that will cause it to slide out from under the upper block?

103. ••• Find the tension in each of the two strings shown in Figure 6–30 for general values of the masses. Your answer should be in terms of m_1, m_2, m_3, and g.

104. ••• The coefficient of static friction between a rope and the table on which it rests is μ_s. Find the fraction of the rope that can hang over the edge of the table before it begins to slip.

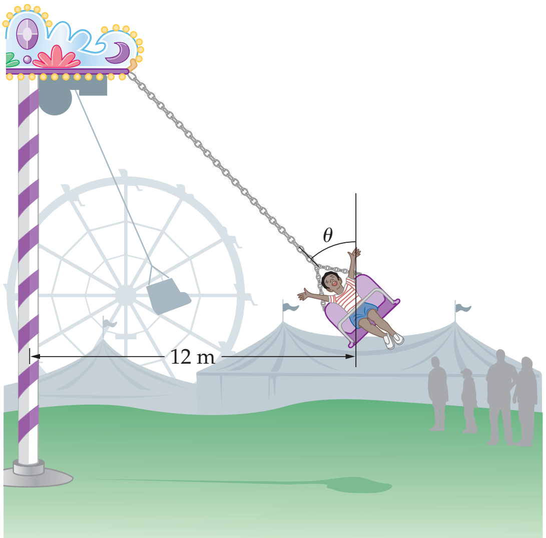
105. ••• A hockey puck of mass m is attached to a string that passes through a hole in the center of a table, as shown in **Figure 6–45**. The hockey puck moves in a circle of radius r. Tied to the other end of the string, and hanging vertically beneath the table, is a mass M. Assuming the tabletop is perfectly smooth, what speed must the hockey puck have if the mass M is to remain at rest?



▲ **FIGURE 6–45** Problem 105

106. ••• **The Force Needed to Move a Crate** To move a crate of mass m across a rough floor, you push down on it at an angle θ, as shown in Figure 6–18 for the special case of $\theta = 21°$. **(a)** Find the force necessary to start the crate moving as a function of θ, given that the coefficient of static friction between the crate and the floor is μ_s. **(b)** Show that it is impossible to move the crate, no matter how great the force, if the coefficient of static friction is greater than or equal to $1/\tan\theta$.

107. ••• **IP** A popular ride at amusement parks is illustrated in **Figure 6–46**. In this ride, people sit in a swing that is suspended from a rotating arm. Riders are at a distance of 12 m from the axis of rotation and move with a speed of 25 mi/h. **(a)** Find the centripetal acceleration of the riders. **(b)** Find the angle θ the supporting wires make with the vertical. **(c)** If you observe a ride like that in Figure 6–46, or as shown in the photo on page 170, you will notice that all the swings are at the same angle θ to the vertical, regardless of the weight of the rider. Explain.

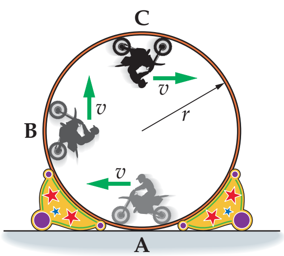


▲ **FIGURE 6–46** Problem 107

108. ••• **A Conveyor Belt** A box is placed on a conveyor belt that moves with a constant speed of 1.25 m/s. The coefficient of kinetic friction between the box and the belt is 0.780. **(a)** How much time does it take for the box to stop sliding relative to the belt? **(b)** How far does the box move in this time?

109. ••• You push a box along the floor against a constant force of friction. When you push with a horizontal force of 75 N, the acceleration of the box is 0.50 m/s^2; when you increase the force to 81 N, the acceleration is 0.75 m/s^2. Find **(a)** the mass of the box and **(b)** the coefficient of kinetic friction between the box and the floor.

110. ••• As part of a circus act, a person drives a motorcycle with constant speed v around the inside of a vertical track of radius r, as indicated in **Figure 6–47**. If the combined mass of the motorcycle and rider is m, find the normal force exerted on the motorcycle by the track at the points **(a)** A, **(b)** B, and **(c)** C.



▲ **FIGURE 6–47** Problem 110

PASSAGE PROBLEMS

BIO Nasal Strips

People in all walks of life use nasal strips, or external nasal dilator strips (ENDS), to alleviate a number of respiratory problems. First introduced to eliminate snoring, they are now finding use in a number of other areas. For example, dentists have found that nasal strips help patients breathe better during dental procedures, making the experience considerably more pleasant for both doctor and patient. Surprisingly, horse owners have also discovered the advantage of nasal strips, and have begun to apply large "horse-sized" strips to saddle horses—as well as racing thoroughbreds—to reduce fatigue and lung stress.

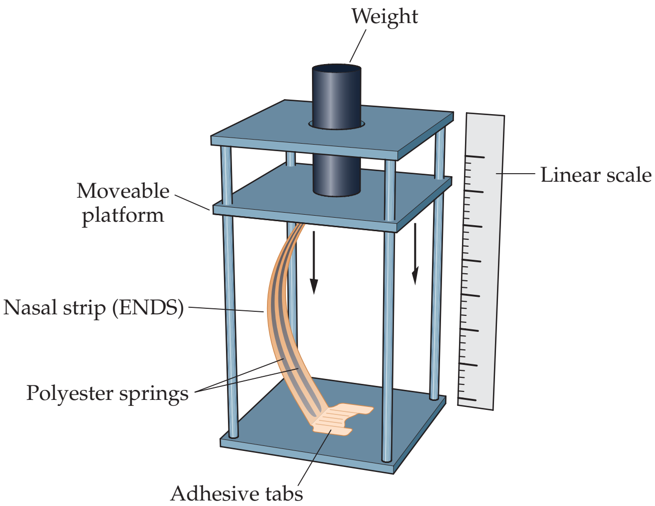
One of the great advantages of ENDS is that no drugs are involved; the strips are a purely mechanical device, consisting of two flat, polyester springs enclosed by an adhesive tape covering. When applied to the nose, they exert an outward force that enlarges the nasal passages and reduces the resistance to air flow (see the illustration in Active Example 6–2). The mechanism shown in **Figure 6–48 (a)** is used to measure the behavior of these strips. For example, if a 30-g weight is placed on the moveable platform (of negligible mass), the strip is found to compress from an initial length of 50 mm to a reduced length of 19 mm, as can be seen in **Figure 6–48 (b)**.

A thoroughbred racehorse with a nasal strip. Did it win by a nose?

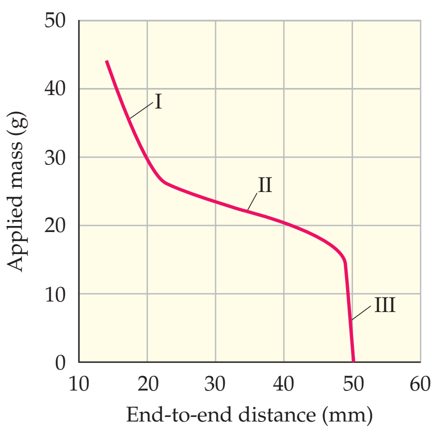
111. • On the straight-line segment I in Figure 6–48 (b) we see that increasing the applied mass from 26 g to 44 g results in a reduction of the end-to-end distance from 21 mm to 14 mm. What is the force constant in N/m on segment I?

A. 2.6 N/m **B.** 3.8 N/m

C. 9.8 N/m **D.** 25 N/m



(a)



(b)

▲ **FIGURE 6–48** Problems 111, 112, 113, and 114

112. • Is the force constant on segment II greater than, less than, or equal to the force constant on segment I?

113. • Which of the following is the best estimate for the force constant on segment II?

A. 0.83 N/m **B.** 1.3 N/m

C. 2.5 N/m **D.** 25 N/m

114. • Rank the straight segments I, II, and III in order of increasing "stiffness" of the nasal strip.

INTERACTIVE PROBLEMS

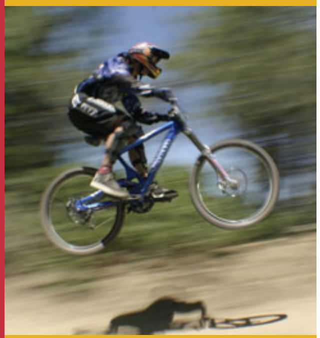
115. •• **IP Referring to Example 6–3** Suppose the coefficients of static and kinetic friction between the crate and the truck bed are 0.415 and 0.382, respectively. **(a)** Does the crate begin to slide at a tilt angle that is greater than, less than, or equal to 23.2°? **(b)** Verify your answer to part (a) by determining the angle at which the crate begins to slide. **(c)** Find the length of time it takes for the crate to slide a distance of 2.75 m when the tilt angle has the value found in part (b).

116. •• **IP Referring to Example 6–3** The crate begins to slide when the tilt angle is 17.5°. When the crate reaches the bottom of the flatbed, after sliding a distance of 2.75 m, its speed is 3.11 m/s. Find **(a)** the coefficient of static friction and **(b)** the coefficient of kinetic friction between the crate and the flatbed.

117. •• **Referring to Example 6–6** Suppose that the mass on the frictionless tabletop has the value $m_1 = 2.45$ kg. **(a)** Find the value of m_2 that gives an acceleration of 2.85 m/s^2. **(b)** What is the corresponding tension, T, in the string? **(c)** Calculate the ratio T/m_2g and show that it is less than 1, as expected.

118. •• **Referring to Example 6–8** **(a)** At what speed will the force of static friction exerted on the car by the road be equal to half the weight of the car? The mass of the car is $m = 1200$ kg, the radius of the corner is $r = 45$ m, and the coefficient of static friction between the tires and the road is $\mu_s = 0.82$. **(b)** Suppose that the mass of the car is now doubled, and that it moves with a speed that again makes the force of static friction equal to half the car's weight. Is this new speed greater than, less than, or equal to the speed in part (a)?

PHYSICS IN



PERSPECTIVE

Force, Acceleration, and Motion

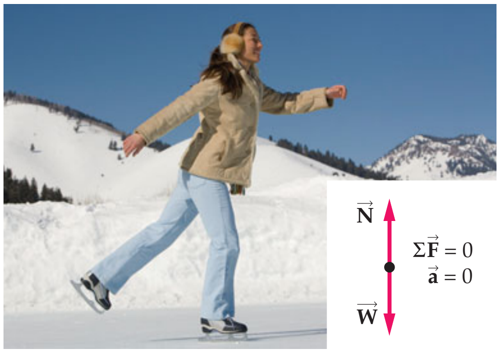
Motion does not require a force—but a *change* in motion does. On these pages we explore the connections between forces, as described in Newton's laws, and the types of motion we've studied in the first six chapters.

1 Objects that experience zero net force obey Newton's first law

If the net force $\vec{F}_{net} = \Sigma\vec{F}$ acting on an object is zero, the object's motion doesn't change—the object either remains at rest or continues to move with constant velocity, as Newton's first law states.

At rest

Motion at constant velocity



This behavior is consistent with Newton's second law for a net force of zero:

If the net force acting on an object is zero … $\Sigma\vec{F} = 0$, then $\vec{a} = \dfrac{\Sigma\vec{F}}{m} = 0$ … the object has zero acceleration.

2 All objects experience forces—the question is whether the object experiences a *net* force

All objects—moving or at rest—are acted on by forces. Even in outer space, objects experience gravitational and other forces. Therefore, the *net* force on the object is the quantity that matters.

Newton's first law seems at odds with our experience: If we stop exerting a force on a moving object, the object usually stops. But that is because we must counter friction and drag forces. In the photo, the net force *on the couch* is zero even though the person exerts a steady push.

3 Moving at constant velocity is equivalent to being at rest

When you sit in a jet flying in a straight line, you feel the same as when you are sitting at home, and objects around you behave the same.

From the point of view of physics, *there is no difference* between these situations; Newton's laws hold in both. We say that both represent *inertial frames of reference*.

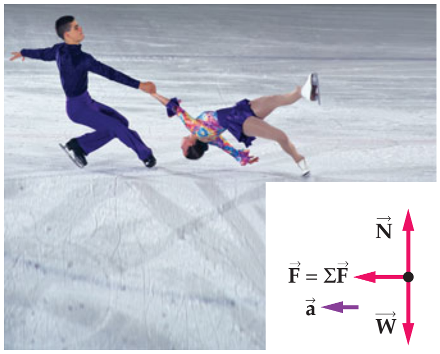
4 Objects that experience a nonzero net force obey Newton's second law

A nonzero net force accelerates an object—that is, causes its velocity to change in magnitude, direction, or both. We have studied the following three special types of accelerated motion:

Linear accelerated motion

Projectile motion

Circular motion



Accelerated motion obeys Newton's second law:

If a nonzero net force acts on an object … $\Sigma\vec{F} \neq 0,$ then $\vec{a} = \dfrac{\Sigma\vec{F}}{m} \neq 0$ … the object has an acceleration in the direction of the net force that is proportional to $\Sigma\vec{F}$ and inversely proportional to m.

5 The acceleration points in the direction of the net force

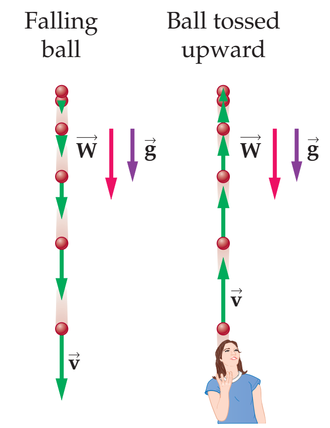
Linear accelerated motion
- Net force is parallel to motion.
- Velocity changes in magnitude but not in direction.

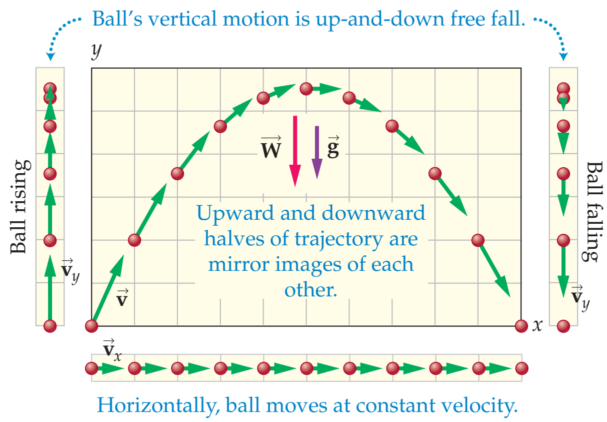
Special case: free fall
Constant downward acceleration $\vec{g}$

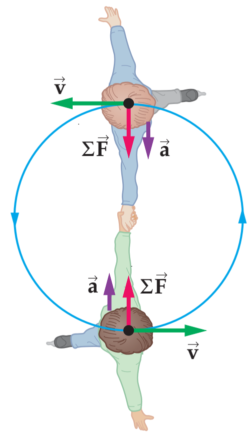
Parabolic motion
- Constant net force acts at angle to motion.
- Velocity changes in both magnitude and direction.

Special case: projectile motion
Constant downward acceleration $\vec{g}$

Circular motion (constant speed)
- Net force is constant in magnitude but always points toward the center of the circle. Thus, the net force is always at a right angle to the object's velocity.
- Velocity changes in direction but not in magnitude.

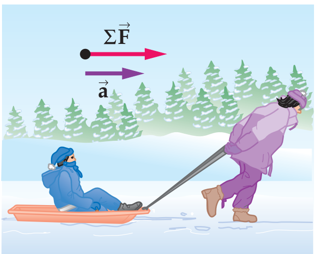


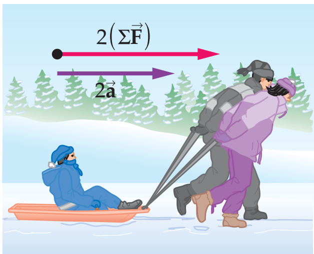




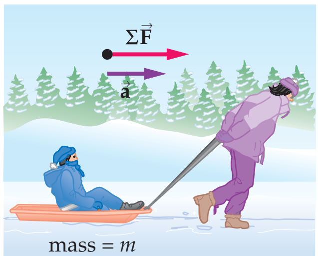
6 The acceleration magnitude is proportional to *F* and inversely proportional to *m*

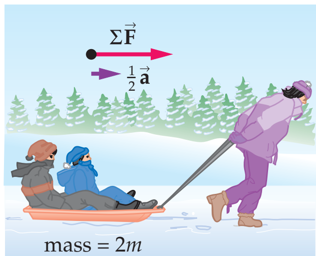
Doubling the net force acting on an object doubles the object's acceleration ($\vec{a} \propto \vec{F}$).





Doubling the object's mass m *halves* its acceleration ($\vec{a} \propto 1/m$).





7 Work and Kinetic Energy

We all know intuitively that motion, energy, and work are somehow related. For example, the chemical energy stored in this pitcher's muscles enables him to do work on a baseball. This means, basically, that he exerts a force on it over a distance. The work done on the ball appears as kinetic energy—the energy of motion—and when the ball is caught, its kinetic energy can in turn do work on the catcher. In this chapter we'll give precise definitions of the concepts of work, kinetic energy, and power, and explore the physical relationships among them.

The concept of force is one of the foundations of physics, as we have seen in the previous two chapters. Equally fundamental, though less obvious, is the idea that a force times the displacement through which it acts is also an important physical quantity. We refer to this quantity as the *work* done by a force.

Now, we all know what work means in everyday life: We get up in the morning and go to work, or we "work up a sweat" as we hike a mountain trail. Later in the day we eat lunch, which gives us the "energy" to continue working or to continue our hike. In this chapter we give a precise physical definition of work, and show how it is related to another important physical quantity—the energy of motion, or *kinetic energy*. When these concepts are extended in the next chapter, we are led to the rather sweeping observation that the total amount of energy in the universe remains constant at all times.

7–1 Work Done by a Constant Force

In this section we define work—in the physics sense of the word—and apply our definition to a variety of physical situations. We start with the simplest case; namely, the work done when force and displacement are in the same direction. Later in the section we generalize our definition to include cases where the force and displacement are in arbitrary directions. We conclude with a discussion of the work done on an object when it is acted on by more than one force.

Force in the Direction of Displacement

When we push a shopping cart in a store or pull a suitcase through an airport, we do work. The greater the force, the greater the work; the greater the distance, the greater the work. These simple ideas form the basis for our definition of work.

To be specific, suppose we push a box with a constant force $\vec{\mathbf{F}}$, as shown in **Figure 7–1**. If we move the box *in the direction of* $\vec{\mathbf{F}}$ through a displacement $\vec{\mathbf{d}}$, the **work** W we have done is Fd:

Definition of Work, *W*, When a Constant Force Is in the Direction of Displacement

$$W = Fd \qquad 7\text{–}1$$

SI unit: newton-meter $(\mathrm{N \cdot m})$ = joule, J

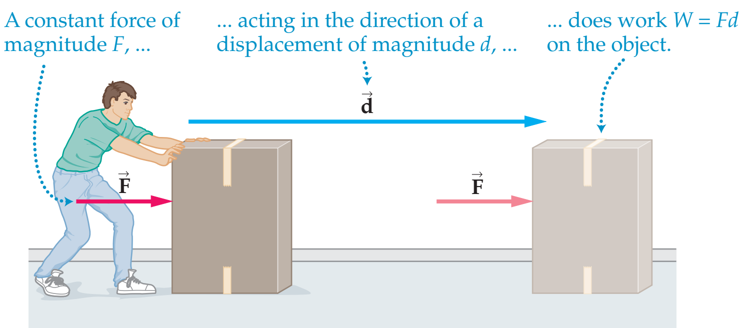


FIGURE 7–1 Work: constant force in the direction of motion

A constant force $\vec{\mathbf{F}}$ pushes a box through a displacement $\vec{\mathbf{d}}$. In this special case, where the force and displacement are in the *same* direction, the work done on the box by the force is $W = Fd$.

Note that work is the product of two magnitudes, and hence it is a scalar. In addition, notice that a small force acting over a large distance gives the same work as a large force acting over a small distance. For example, $W = (1\ \mathrm{N})(400\ \mathrm{m}) = (400\ \mathrm{N})(1\ \mathrm{m})$.

The dimensions of work are newtons (force) times meters (distance), or $\mathrm{N \cdot m}$. This combination of dimensions is called the **joule** (rhymes with "school," as commonly pronounced) in honor of James Prescott Joule (1818–1889), a dedicated physicist who is said to have conducted physics experiments even while on his honeymoon. We define a joule as follows:

Definition of the Joule, J

$$1\ \text{joule} = 1\ \mathrm{J} = 1\ \mathrm{N \cdot m} = 1(\mathrm{kg \cdot m/s^2}) \cdot \mathrm{m} = 1\ \mathrm{kg \cdot m^2/s^2} \qquad 7\text{–}2$$

To get a better feeling for work and the associated units, suppose you exert a force of 82.0 N on the box in Figure 7–1 and move it in the direction of the force through a distance of 3.00 m. The work you have done is

$$W = Fd = (82.0\ \mathrm{N})(3.00\ \mathrm{m}) = 246\ \mathrm{N \cdot m} = 246\ \mathrm{J}$$

Similarly, if you do 5.00 J of work to lift a book through a vertical distance of 0.750 m, the force you exerted on the book is

$$F = \frac{W}{d} = \frac{5.00\ \mathrm{J}}{0.750\ \mathrm{m}} = \frac{5.00\ \mathrm{N \cdot m}}{0.750\ \mathrm{m}} = 6.67\ \mathrm{N}$$

TABLE 7–1 Typical Values of Work

Activity	Equivalent work (J)
Annual U.S. energy use	8×10^{19}
Mt. St. Helens eruption	10^{18}
Burning one gallon of gas	10^{8}
Human food intake/day	10^{7}
Melting an ice cube	10^{4}
Lighting a 100-W bulb for 1 minute	6000
Heartbeat	0.5
Turning page of a book	10^{-3}
Hop of a flea	10^{-7}
Breaking a bond in DNA	10^{-20}

EXERCISE 7–1

One species of Darwin's finch, *Geospiza magnirostris*, can exert a force of 205 N with its beak as it cracks open a *Tribulus* seed case. If its beak moves through a distance of 0.40 cm during this operation, how much work does the finch do to get the seed?

SOLUTION

$W = Fd = (205\ \mathrm{N})(0.0040\ \mathrm{m}) = 0.82\ \mathrm{J}$

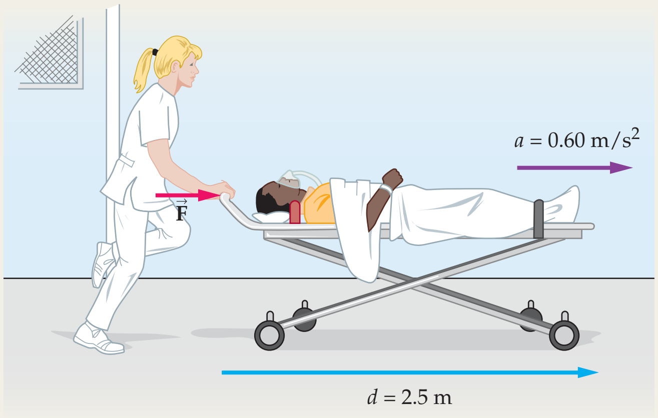
Just how much work is a joule, anyway? Well, you do one joule of work when you lift a gallon of milk through a height of about an inch, or lift an apple a meter. One joule of work lights a 100-watt lightbulb for 0.01 seconds or heats a glass of water 0.00125 degrees Celsius. Clearly, a joule is a modest amount of work in everyday terms. Additional examples of work are listed in Table 7–1.

EXAMPLE 7–1 HEADING FOR THE ER

An intern pushes a 72-kg patient on a 15-kg gurney, producing an acceleration of 0.60 m/s^2. **(a)** How much work does the intern do by pushing the patient and gurney through a distance of 2.5 m? Assume the gurney moves without friction. **(b)** How far must the intern push the gurney to do 140 J of work?

PICTURE THE PROBLEM

Our sketch shows the physical situation for this problem. Note that the force exerted by the intern is in the same direction as the displacement of the gurney; therefore, we know that $W = Fd$.



STRATEGY

We are not given the magnitude of the force F, so we cannot apply Equation 7–1 directly. However, we are given the mass and acceleration of the patient and gurney, from which we can calculate the force with $F = ma$. The work done by the intern is then $W = Fd$.

SOLUTION

Part (a)

1. First, find the force F exerted by the intern: $F = ma = (72\ \mathrm{kg} + 15\ \mathrm{kg})(0.60\ \mathrm{m/s^2}) = 52\ \mathrm{N}$

2. The work done by the intern, W, is the force times the distance: $W = Fd = (52\ \mathrm{N})(2.5\ \mathrm{m}) = 130\ \mathrm{J}$

Part (b)

3. Use $W = Fd$ to solve for the distance d: $W = Fd$ therefore $d = \dfrac{W}{F} = \dfrac{140\ \mathrm{J}}{52\ \mathrm{N}} = 2.7\ \mathrm{m}$

INSIGHT

You might wonder whether the work done by the intern depends on the speed of the gurney. The answer is no. The work done on an object, $W = Fd$, doesn't depend on whether the object moves through the distance d quickly or slowly. What does depend on the speed of the gurney is the *rate* at which work is done, as we discuss in detail in Section 7–4.

PRACTICE PROBLEM

If the total mass of the gurney plus patient is halved and the acceleration is doubled, does the work done by the intern increase, decrease, or remain the same? [**Answer:** The work remains the same.]

Some related homework problems: Problem 4, Problem 5

Before moving on, let's note an interesting point about our definition of work. It's clear from Equation 7–1 that *the work W is zero if the distance d is zero*—and this is true regardless of how great the force might be. For example, if you push against a solid wall you do no work on it, even though you may become tired from your efforts. Similarly, if you stand in one place holding a 50-pound suitcase in your hand, you do no work on the suitcase. The fact that we become tired when we push against a wall or hold a heavy object is due to the repeated contraction and expansion of individual cells within our muscles. Thus, even when we are "at rest," our muscles are doing mechanical work on the microscopic level.

The weightlifter at left does more work in raising 150 kilograms above her head than Atlas, who is supporting the entire world. Why?

Force at an Angle to the Displacement

In **Figure 7–2** we see a person pulling a suitcase on a level surface with a strap that makes an angle θ with the horizontal—in this case the force is at an angle to the direction of motion. How do we calculate the work now? Well, instead of force times distance, we say that work is the *component* of force in the *direction* of displacement times the magnitude of the displacement. In Figure 7–2, the component of force in the direction of the displacement is $F\cos\theta$ and the magnitude of the displacement is d. Therefore, the work is $F\cos\theta$ times d:

Definition of Work When the Angle Between a Constant Force and the Displacement Is θ

$$W = (F\cos\theta)d = Fd\cos\theta \qquad 7\text{–}3$$

SI unit: joule, J

Of course, in the case where the force is in the direction of motion, the angle θ is zero; then $W = Fd\cos 0^\circ = Fd \cdot 1 = Fd$, in agreement with Equation 7–1.

Equally interesting is a situation in which the force and the displacement are at right angles to one another. In this case $\theta = 90^\circ$ and the work done by the force F is zero; $W = Fd\cos 90^\circ = 0$.

This result leads naturally to an alternative way to think about the expression $W = Fd\cos\theta$. In **Figure 7–3** we show the displacement and the force for the suitcase in Figure 7–2. Notice that the displacement is equivalent to a displacement in the

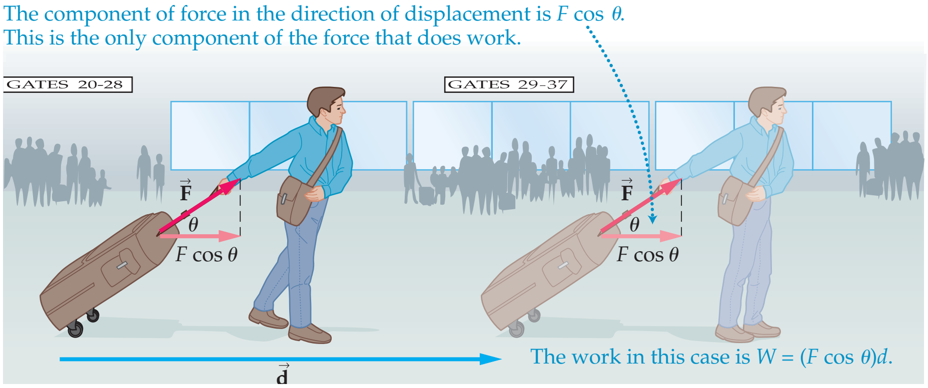
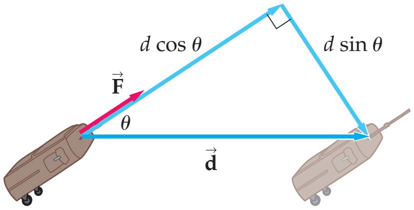


FIGURE 7–2 Work: force at an angle to the direction of motion

A person pulls a suitcase with a strap at an angle θ to the direction of motion. The component of force in the direction of motion is $F\cos\theta$, and the work done by the person is $W = (F\cos\theta)d$.



▲ **FIGURE 7–3 Force at an angle to direction of motion: another look**
The displacement of the suitcase in Figure 7–2 is equivalent to a displacement of magnitude $d\cos\theta$ in the direction of the force $\vec{\mathbf{F}}$, plus a displacement of magnitude $d\sin\theta$ perpendicular to the force. Only the displacement parallel to the force results in nonzero work, hence the total work done is $F(d\cos\theta)$ as expected.

direction of the force of magnitude $(d\cos\theta)$ *plus* a displacement at right angles to the force of magnitude $(d\sin\theta)$. Since the displacement at right angles to the force corresponds to zero work and the displacement in the direction of the force corresponds to a work $W = F(d\cos\theta)$, it follows that the work done in this case is $Fd\cos\theta$, as given in Equation 7–3. Thus, the work done by a force can be thought of in the following two *equivalent* ways:

(i) *Work is the component of force in the direction of the displacement times the magnitude of the displacement.*
(ii) *Work is the component of displacement in the direction of the force times the magnitude of the force.*

In either of these interpretations, the mathematical expression for work is exactly the same, $W = Fd\cos\theta$, where θ is the angle between the force vector and the displacement vector when they are placed tail-to-tail. This definition of θ is illustrated in Figure 7–3.

Finally, we can also express work as the **dot product** between the vectors $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$; that is, $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = Fd\cos\theta$. Note that the dot product, which is always a scalar, is simply the magnitude of one vector times the magnitude of the second vector times the cosine of the angle between them. We discuss the dot product in greater detail in Appendix A.

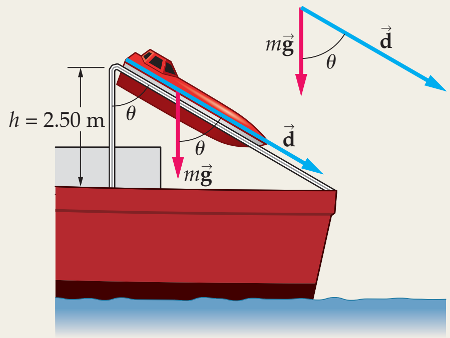
EXAMPLE 7–2 GRAVITY ESCAPE SYSTEM

REAL-WORLD PHYSICS

In a gravity escape system (GES), an enclosed lifeboat on a large ship is deployed by letting it slide down a ramp and then continuing in free fall to the water below. Suppose a 4970-kg lifeboat slides a distance of 5.00 m on a ramp, dropping through a vertical height of 2.50 m. How much work does gravity do on the boat?

PICTURE THE PROBLEM

From our sketch, we see that the force of gravity $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ are at an angle θ relative to one another when placed tail-to-tail, and that θ is also the angle the ramp makes with the vertical. In addition, we note that the vertical height of the ramp is $h = 2.50$ m and the length of the ramp is $d = 5.00$ m.



STRATEGY

By definition, the work done on the lifeboat by gravity is $W = Fd\cos\theta$, where $F = mg$, $d = 5.00$ m, and θ is the angle between $m\vec{\mathbf{g}}$ and $\vec{\mathbf{d}}$. We are not given θ in the problem statement, but from the right triangle that forms the ramp we see that $\cos\theta = h/d$. Once θ is determined from the geometry of our sketch, it is straightforward to calculate W.

SOLUTION

1. First, find the component of $\vec{\mathbf{F}} = m\vec{\mathbf{g}}$ in the direction of motion:

$$F\cos\theta = (mg)\left(\frac{h}{d}\right)$$
$$= (4970\ \text{kg})(9.81\ \text{m/s}^2)\left(\frac{2.50\ \text{m}}{5.00\ \text{m}}\right) = 24{,}400\ \text{N}$$

2. Multiply by distance to find the work:

$$W = (F\cos\theta)d = (24{,}400\ \text{N})(5.00\ \text{m}) = 122{,}000\ \text{J}$$

3. Alternatively, cancel d algebraically before substituting numerical values:

$$W = Fd\cos\theta = (mg)(d)\left(\frac{h}{d}\right)$$
$$= mgh = (4970\ \text{kg})(9.81\ \text{m/s}^2)(2.50\ \text{m}) = 122{,}000\ \text{J}$$

INSIGHT

The work is simply $W = mgh$, exactly the same as if the lifeboat had fallen straight down through the height h.

Notice that working the problem symbolically, as in Step 3, results in two distinct advantages. First, it makes for a simpler expression for the work. Second, and more importantly, it shows that the distance d cancels; hence the work depends on the height h but not on d. Such a result is not apparent when we work solely with numbers, as in Steps 1 and 2.

PRACTICE PROBLEM

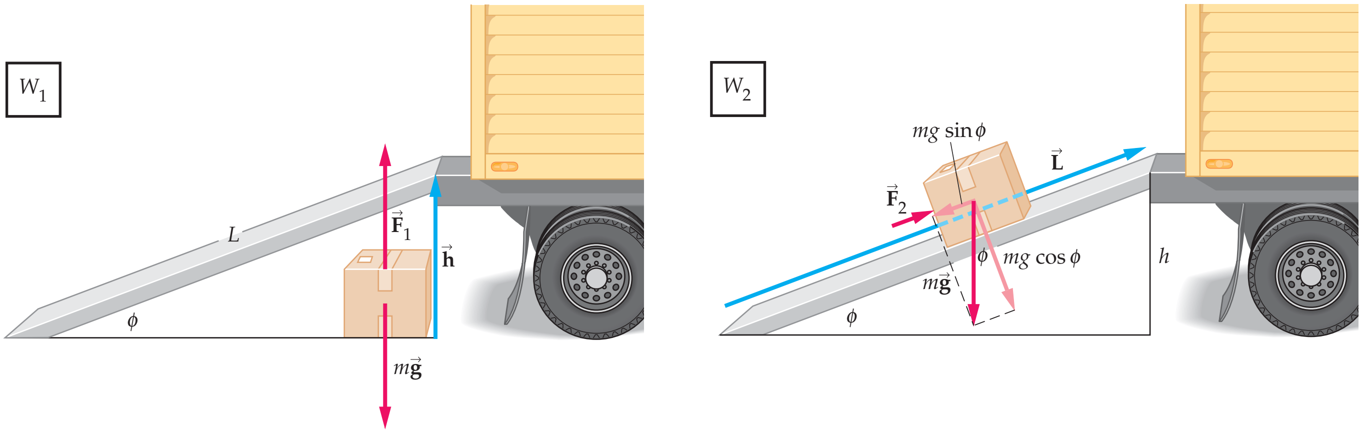
Suppose the lifeboat slides halfway to the water, gets stuck for a moment, and then starts up again and continues to the end of the ramp. What is the work done by gravity in this case? **[Answer:** The work done by gravity is exactly the same, $W = mgh$, independent of how the boat moves down the ramp.]

Some related homework problems: Problem 11, Problem 12

Next, we present a Conceptual Checkpoint that compares the work required to move an object along two different paths.

CONCEPTUAL CHECKPOINT 7–1 PATH DEPENDENCE OF WORK

You want to load a box into the back of a truck. One way is to lift it straight up through a height h, as shown, doing a work W_1. Alternatively, you can slide the box up a loading ramp a distance L, doing a work W_2. Assuming the box slides on the ramp without friction, which of the following is correct: **(a)** $W_1 < W_2$, **(b)** $W_1 = W_2$, **(c)** $W_1 > W_2$?



REASONING AND DISCUSSION

You might think that W_2 is less than W_1, since the force needed to slide the box up the ramp, F_2, is less than the force needed to lift it straight up. On the other hand, the distance up the ramp, L, is greater than the vertical distance, h, so perhaps W_2 should be greater than W_1. In fact, these two effects cancel exactly, giving $W_1 = W_2$.

To see this, we first calculate W_1. The force needed to lift the box with constant speed is $F_1 = mg$, and the height is h, therefore $W_1 = mgh$.

Next, the work to slide the box up the ramp with constant speed is $W_2 = F_2L$, where F_2 is the force required to push against the tangential component of gravity. In the figure we see that $F_2 = mg \sin \phi$. The figure also shows that $\sin \phi = h/L$; thus $W_2 = (mg \sin \phi)L = (mg)(h/L)L = mgh = W_1$.

Clearly, the ramp is a useful device—it reduces the *force* required to move the box upward from $F_1 = mg$ to $F_2 = mg(h/L)$. Even so, it doesn't decrease the amount of *work* we need to do. As we have seen, the reduced force on the ramp is offset by the increased distance.

ANSWER

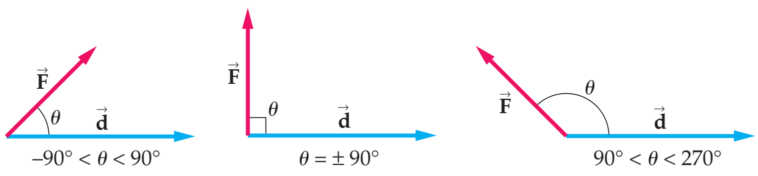
(b) $W_1 = W_2$

Negative Work and Total Work

Work depends on the angle between the force, $\vec{\mathbf{F}}$, and the displacement (or direction of motion), $\vec{\mathbf{d}}$. This dependence gives rise to three distinct possibilities, as shown in **Figure 7–4**:

(i) *Work is positive if the force has a component in the direction of motion* $(-90° < \theta < 90°)$.
(ii) *Work is zero if the force has no component in the direction of motion* $(\theta = \pm 90°)$.
(iii) *Work is negative if the force has a component opposite to the direction of motion* $(90° < \theta < 270°)$.

Thus, whenever we calculate work, we must be careful about its sign and not just assume it to be positive.



(a) Positive work **(b)** Zero work **(c)** Negative work

FIGURE 7–4 Positive, negative, and zero work
Work is positive when the force is in the same general direction as the displacement and is negative if the force is generally opposite to the displacement. Zero work is done if the force is at right angles to the displacement.

PROBLEM-SOLVING NOTE

Be Careful About the Angle θ

In calculating $W = Fd\cos\theta$ be sure that the angle you use in the cosine is the angle between the force and the displacement vectors when they are placed tail to tail. Sometimes θ may be used to label a different angle in a given problem. For example, θ is often used to label the angle of a slope, in which case it may have nothing to do with the angle between the force and the displacement. To summarize: Just because an angle is labeled θ doesn't mean it's automatically the correct angle to use in the work formula.

When more than one force acts on an object, the total work is the sum of the work done by each force separately. Thus, if force $\vec{\mathbf{F}}_1$ does work W_1, force $\vec{\mathbf{F}}_2$ does work W_2, and so on, the total work is

$$W_{\text{total}} = W_1 + W_2 + W_3 + \cdots = \sum W \qquad 7\text{–}4$$

Equivalently, the total work can be calculated by first performing a vector sum of all the forces acting on an object to obtain $\vec{\mathbf{F}}_{\text{total}}$ and then using our basic definition of work:

$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}\,d\cos\theta \qquad 7\text{–}5$$

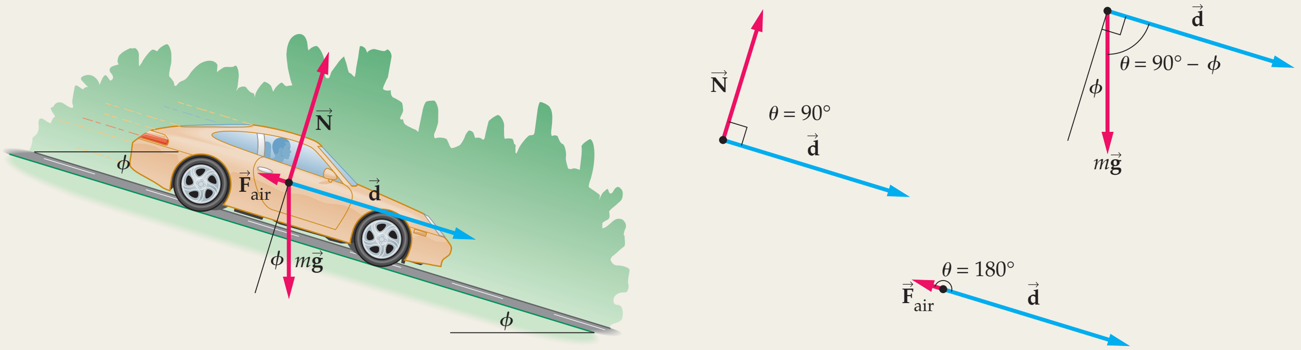
where θ is the angle between $\vec{\mathbf{F}}_{\text{total}}$ and the displacement $\vec{\mathbf{d}}$. In the next two Examples we calculate the total work in each of these ways.

EXAMPLE 7–3 A COASTING CAR I

A car of mass m coasts down a hill inclined at an angle ϕ below the horizontal. The car is acted on by three forces: (i) the normal force $\vec{\mathbf{N}}$ exerted by the road, (ii) a force due to air resistance, $\vec{\mathbf{F}}_{\text{air}}$, and (iii) the force of gravity, $m\vec{\mathbf{g}}$. Find the total work done on the car as it travels a distance d along the road.

PICTURE THE PROBLEM

Because ϕ is the angle the slope makes with the horizontal, it is also the angle between $m\vec{\mathbf{g}}$ and the downward normal direction, as was shown in Figure 5–15. It follows that the angle between $m\vec{\mathbf{g}}$ and the displacement $\vec{\mathbf{d}}$ is $\theta = 90° - \phi$. Our sketch also shows that the angle between $\vec{\mathbf{N}}$ and $\vec{\mathbf{d}}$ is $\theta = 90°$, and the angle between $\vec{\mathbf{F}}_{\text{air}}$ and $\vec{\mathbf{d}}$ is $\theta = 180°$.



STRATEGY

For each force we calculate the work using $W = Fd\cos\theta$, where θ is the angle between that particular force and the displacement $\vec{\mathbf{d}}$. The total work is the sum of the work done by each of the three forces.

SOLUTION

1. We start with the work done by the normal force, $\vec{\mathbf{N}}$. From the figure we see that $\theta = 90°$ for this force: $W_N = Nd\cos\theta = Nd\cos 90° = Nd(0) = 0$

2. For the force of air resistance, $\theta = 180°$: $W_{\text{air}} = F_{\text{air}}d\cos 180° = F_{\text{air}}d(-1) = -F_{\text{air}}d$

3. For gravity the angle θ is $\theta = 90° - \phi$, as indicated in the figure. Recall that $\cos(90° - \phi) = \sin\phi$ (see Appendix A): $W_{mg} = mgd\cos(90° - \phi) = mgd\sin\phi$

4. The total work is the sum of the individual works: $W_{\text{total}} = W_N + W_{\text{air}} + W_{mg} = 0 - F_{\text{air}}d + mgd\sin\phi$

INSIGHT

The normal force is perpendicular to the motion of the car, and thus does no work. Air resistance points in a direction that opposes the motion, so it does negative work. On the other hand, gravity has a component in the direction of motion; therefore, its work is positive. The physical significance of positive, negative, and zero work will be discussed in detail in the next section.

PRACTICE PROBLEM

Calculate the total work done on a 1550-kg car as it coasts 20.4 m down a hill with $\phi = 5.00°$. Let the force due to air resistance be 15.0 N. [**Answer:** $W_{\text{total}} = W_N + W_{\text{air}} + W_{mg} = 0 - F_{\text{air}}d + mgd\sin\phi = 0 - 306\text{ J} + 2.70 \times 10^4\text{ J} = 2.67 \times 10^4\text{ J}$]

Some related homework problems: Problem 15, Problem 81

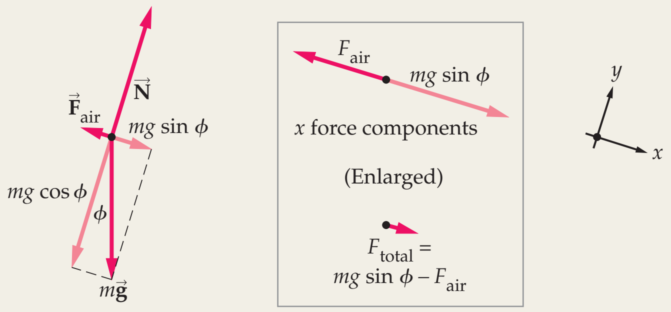
In the previous Example, we showed that the total work can be calculated by finding the work done by each force separately, and then summing the individual works. In the next Example, we take a different approach. We first sum the forces acting on the car to find F_{total}. Once the total force is determined, we calculate the total work using $W_{\text{total}} = F_{\text{total}} d \cos\theta$.

EXAMPLE 7–4 A COASTING CAR II

Consider the car described in Example 7–3. Calculate the total work done on the car using $W_{\text{total}} = F_{\text{total}} d \cos\theta$.

PICTURE THE PROBLEM

First, we choose the x axis to point down the slope, and the y axis to be at right angles to the slope. With this choice, there is no acceleration in the y direction, which means that the total force in that direction must be zero. As a result, the total force acting on the car is in the x direction. The magnitude of the total force is $mg \sin\phi - F_{\text{air}}$, as can be seen in our sketch.



STRATEGY

We begin by finding the x component of each force vector and then summing them to find the total force acting on the car. As can be seen from the figure, the total force points in the positive x direction; that is, in the same direction as the displacement. Therefore, the angle θ in $W = F_{\text{total}} d \cos\theta$ is zero.

SOLUTION

1. Referring to the figure above, we see that the *magnitude* of the total force is $mg \sin\phi$ minus F_{air}:

$$F_{\text{total}} = mg \sin\phi - F_{\text{air}}$$

2. The *direction* of $\vec{\mathbf{F}}_{\text{total}}$ is the same as the direction of $\vec{\mathbf{d}}$, thus $\theta = 0°$. We can now calculate W_{total}:

$$W_{\text{total}} = F_{\text{total}} d \cos\theta = (mg \sin\phi - F_{\text{air}}) d \cos 0° = mgd \sin\phi - F_{\text{air}} d$$

INSIGHT

Note that we were careful to calculate both the magnitude and the direction of the total force. The magnitude (which is always positive) gives F_{total} and the direction gives $\theta = 0°$, allowing us to use $W_{\text{total}} = F_{\text{total}} d \cos\theta$.

PRACTICE PROBLEM

Suppose the total work done on a 1620-kg car as it coasts 25.0 m down a hill with $\phi = 6.00°$ is $W_{\text{total}} = 3.75 \times 10^4$ J. Find the magnitude of the force due to air resistance. [**Answer:** $F_{\text{air}} d = -W_{\text{total}} + mgd \sin\phi = 4030$ J, thus $F_{\text{air}} = (4030 \text{ J})/d = 161$ N]

Some related homework problems: Problem 15, Problem 81

The full significance of positive versus negative work is seen in the next section, where we relate the work done on an object to the change in its speed.

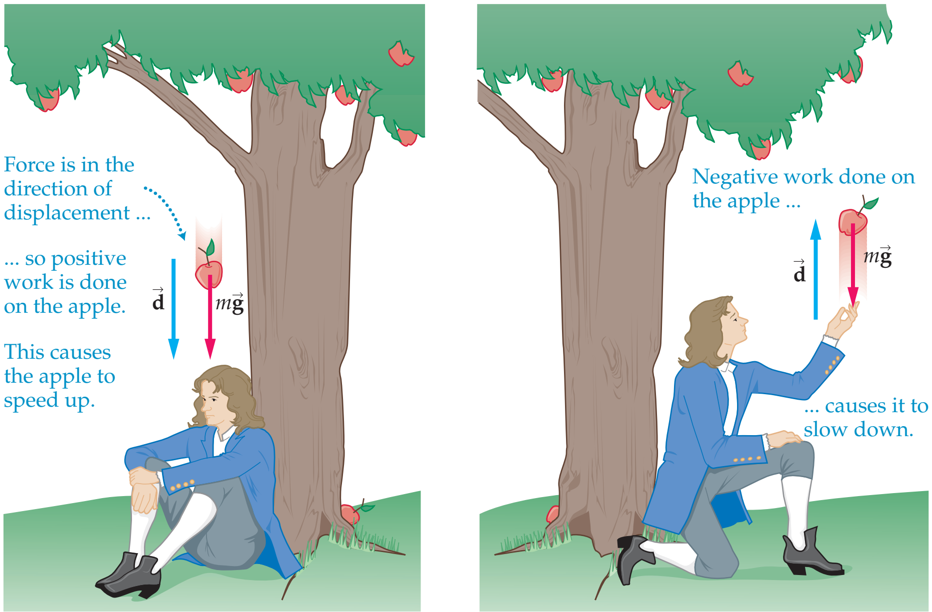
7–2 Kinetic Energy and the Work–Energy Theorem

Suppose you drop an apple. As it falls, gravity does positive work on it, as indicated in **Figure 7–5**, and its speed increases. If you toss the apple upward, gravity does negative work, and the apple slows down. In general, whenever the total work done on an object is positive, its speed increases; when the total work is negative, its speed decreases. In this section we derive an important result, the **work–energy theorem,** which makes this connection between work and change in speed precise.

To begin, consider an apple of mass m falling through the air, and suppose that two forces act on the apple—gravity, $m\vec{\mathbf{g}}$, and the average force of air resistance, $\vec{\mathbf{F}}_{\text{air}}$. The total force acting on the apple, $\vec{\mathbf{F}}_{\text{total}}$, gives the apple a constant downward acceleration of magnitude

$$a = F_{\text{total}}/m$$

Since the total force is downward and the motion is downward, the work done on the apple is positive.



Apple falling: $W > 0$, speed increases

Apple tossed upward: $W < 0$, speed decreases

FIGURE 7–5 Gravitational work
The work done by gravity on an apple that moves downward is positive. If the apple is in free fall, this positive work will result in an increase in speed. On the other hand, the work done by gravity on an apple that moves upward is negative. If the apple is in free fall, the negative work done by gravity will result in a decrease of speed.

Now, suppose the initial speed of the apple is v_i, and that after falling a distance d its speed increases to v_f. The apple falls with constant acceleration a, hence constant-acceleration kinematics (Equation 2–12) gives

$$v_f^2 = v_i^2 + 2ad$$

or, with a slight rearrangement,

$$2ad = v_f^2 - v_i^2$$

Next, substitute $a = F_{total}/m$ into this equation:

$$2\left(\frac{F_{total}}{m}\right)d = v_f^2 - v_i^2$$

Multiplying both sides by m and dividing by 2 yields

$$F_{total}d = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

where $F_{total}d$ is simply the total work done on the apple. Thus we find

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

showing that total work is directly related to change in speed, as just mentioned. Note that $W_{total} > 0$ means $v_f > v_i$, $W_{total} < 0$ means $v_f < v_i$, and $W_{total} = 0$ implies that $v_f = v_i$.

The quantity $\frac{1}{2}mv^2$ in the equation for W_{total} has a special significance in physics, as we shall see. We call it the **kinetic energy,** K:

Definition of Kinetic Energy, *K*

$$K = \tfrac{1}{2}mv^2 \qquad 7\text{–}6$$

SI unit: $kg \cdot m^2/s^2$ = joule, J

In general, the kinetic energy of an object is the energy due to its motion. We measure kinetic energy in joules, the same units as work, and both kinetic energy and work are scalars. Unlike work, however, kinetic energy is never negative. Instead, K is always greater than or equal to zero, independent of the direction of motion or the direction of any forces.

To get a feeling for typical values of kinetic energy, consider your kinetic energy when jogging. Assuming a mass of about 62 kg and a speed of 2.5 m/s, your kinetic energy is $K = \frac{1}{2}(62\text{ kg})(2.5\text{ m/s})^2 = 190$ J. Additional examples of kinetic energy are given in Table 7–2.

PROBLEM-SOLVING NOTE
Work Can Be Positive, Negative, or Zero

When you calculate work, be sure to keep track of whether it is positive or negative. The distinction is important, since positive work increases speed, whereas negative work decreases speed. Zero work, of course, has no effect on speed.

TABLE 7–2 Typical Kinetic Energies

Source	Approximate kinetic energy (J)
Jet aircraft at 500 mi/h	10^9
Car at 60 mi/h	10^6
Home-run baseball	10^3
Person at walking speed	50
Housefly in flight	10^{-3}

EXERCISE 7–2

A truck moving at 15 m/s has a kinetic energy of 4.2×10^5 J. **(a)** What is the mass of the truck? **(b)** By what multiplicative factor does the kinetic energy of the truck increase if its speed is doubled?

SOLUTION

(a) $K = \frac{1}{2}mv^2$; therefore $m = 2K/v^2 = 3700$ kg. **(b)** Kinetic energy depends on the speed squared, and hence doubling the speed increases the kinetic energy by a factor of four.

In terms of kinetic energy, the work–energy theorem can be stated as follows:

Work–Energy Theorem

The total work done on an object is equal to the change in its kinetic energy:

$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad 7\text{–}7$$

Thus, the work–energy theorem says that when a force acts on an object over a distance—doing work on it—the result is a change in the speed of the object, and hence a change in its energy of motion. Equation 7–7 is the quantitative expression of this connection.

Finally, though we have derived the work–energy theorem for a force that is constant in direction and magnitude, it is valid for any force, as can be shown using the methods of calculus. In fact, the work–energy theorem is completely general, making it one of the more important and fundamental results in physics. It is also a very handy tool for problem solving, as we shall see many times throughout this text.

PROBLEM-SOLVING NOTE

Starts from Rest Means $v_i = 0$

A problem statement that uses a phrase like "starts from rest" or "is raised from rest" is telling you that $v_i = 0$.

EXERCISE 7–3

How much work is required for a 74-kg sprinter to accelerate from rest to 2.2 m/s?

SOLUTION

Since $v_i = 0$, we have $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 = \frac{1}{2}(74 \text{ kg})(2.2 \text{ m/s})^2 = 180$ J.

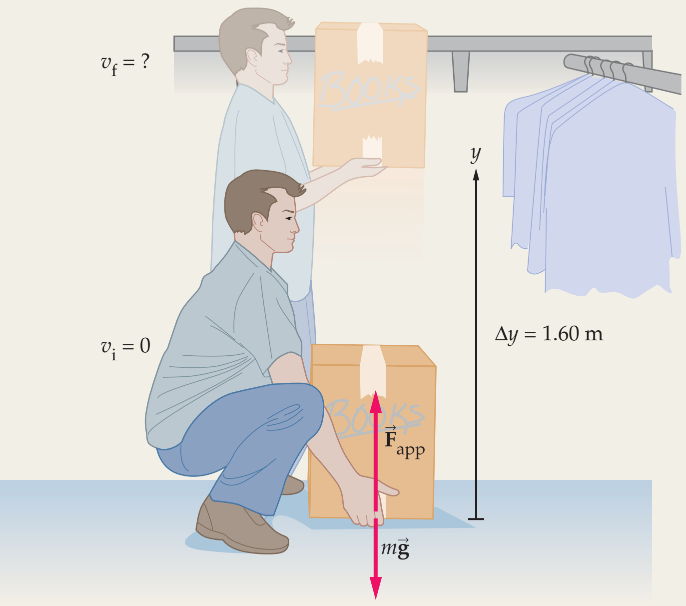
We now present a variety of Examples showing how the work–energy theorem is used in practical situations.

EXAMPLE 7–5 HIT THE BOOKS

A 4.10-kg box of books is lifted vertically from rest a distance of 1.60 m with a constant, upward applied force of 52.7 N. Find **(a)** the work done by the applied force, **(b)** the work done by gravity, and **(c)** the final speed of the box.

PICTURE THE PROBLEM

Our sketch shows that the direction of motion of the box is upward. In addition, we see that the applied force, $\vec{F}_{\text{app}}$, is upward and the force of gravity, $m\vec{g}$, is downward. Finally, the box is lifted from rest ($v_i = 0$) through a distance $\Delta y = 1.60$ m.



STRATEGY

The applied force is in the direction of motion, so the work it does, W_{app}, is positive. Gravity is opposite in direction to the motion; thus its work, W_g, is negative. The total work is the sum of W_{app} and W_g, and the final speed of the box is found by applying the work–energy theorem, $W_{\text{total}} = \Delta K$.

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SOLUTION

Part (a)

1. First we find the work done by the applied force. In this case, $\theta = 0°$ and the distance is $\Delta y = 1.60$ m:

$$W_{app} = F_{app} \cos 0° \, \Delta y = (52.7\text{ N})(1)(1.60\text{ m}) = 84.3\text{ J}$$

Part (b)

2. Next, we calculate the work done by gravity. The distance is $\Delta y = 1.60$ m, as before, but now $\theta = 180°$:

$$W_g = mg \cos 180° \, \Delta y$$
$$= (4.10\text{ kg})(9.81\text{ m/s}^2)(-1)(1.60\text{ m}) = -64.4\text{ J}$$

Part (c)

3. The total work done on the box, W_{total}, is the sum of W_{app} and W_g:

$$W_{total} = W_{app} + W_g = 84.3\text{ J} - 64.4\text{ J} = 19.9\text{ J}$$

4. To find the final speed, v_f, we apply the work–energy theorem. Recall that the box started at rest, thus $v_i = 0$:

$$W_{total} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2W_{total}}{m}} = \sqrt{\frac{2(19.9\text{ J})}{4.10\text{ kg}}} = 3.12\text{ m/s}$$

INSIGHT

As a check on our result, we can find v_f in a completely different way. First, calculate the acceleration of the box with the result $a = (F_{app} - mg)/m = 3.04\text{ m/s}^2$. Next, use this result in the kinematic equation $v^2 = v_0^2 + 2a\Delta y$. With $v_0 = 0$ and $\Delta y = 1.60$ m, we find $v = 3.12$ m/s, in agreement with the results using the work–energy theorem.

PRACTICE PROBLEM

If the box is lifted only a quarter of the distance, is the final speed 1/8, 1/4, or 1/2 of the value found in Step 4? Calculate v_f in this case as a check on your answer. [**Answer:** Since work depends linearly on Δy, and v_f depends on the square root of the work, it follows that the final speed is $\sqrt{1/4} = \frac{1}{2}$ the value in Step 4. Letting $\Delta y = (1.60\text{ m})/4 = 0.400$ m, we find $v_f = \frac{1}{2}(3.12\text{ m/s}) = 1.56$ m/s.]

Some related homework problems: Problem 19, Problem 24, Problem 25

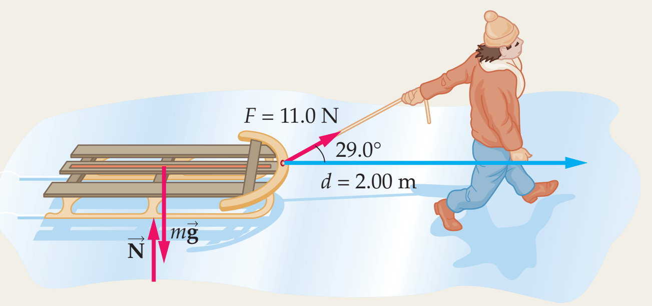
In the previous Example the initial speed was zero. This is not always the case, of course. The next Example illustrates how to use the work–energy theorem when the initial velocity is nonzero.

EXAMPLE 7–6 PULLING A SLED

A boy exerts a force of 11.0 N at 29.0° above the horizontal on a 6.40-kg sled. Find **(a)** the work done by the boy and **(b)** the final speed of the sled after it moves 2.00 m, assuming the sled starts with an initial speed of 0.500 m/s and slides horizontally without friction.

PICTURE THE PROBLEM

Our sketch shows the direction of motion and the directions of each of the forces. Note that the normal force and the force due to gravity are vertical, whereas the displacement is horizontal. The force exerted by the boy has both a vertical component, $F \sin\theta$, and a horizontal component, $F \cos\theta$.



STRATEGY

a. The forces $\vec{N}$ and $m\vec{g}$ do no work because they are at right angles to the horizontal displacement. The force exerted by the boy, however, has a horizontal component that does positive work on the sled. Therefore, the total work is simply the work done by the boy.

b. After calculating this work, we find v_f by applying the work–energy theorem with $v_i = 0.500$ m/s.

SOLUTION

Part (a)

1. The work done by the boy is $(F \cos\theta)d$, where $\theta = 29.0°$. This is also the total work done on the sled:

$$W_{boy} = (F \cos\theta)d$$
$$= (11.0\text{ N})(\cos 29.0°)(2.00\text{ m}) = 19.2\text{ J} = W_{total}$$

Part (b)

2. Use the work–energy theorem to solve for the final speed:

$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$\tfrac{1}{2}mv_f^2 = W_{\text{total}} + \tfrac{1}{2}mv_i^2$$

$$v_f = \sqrt{\frac{2W_{\text{total}}}{m} + v_i^2}$$

3. Substitute numerical values to get the final answer:

$$v_f = \sqrt{\frac{2(19.2\ \text{J})}{6.40\ \text{kg}} + (0.500\ \text{m/s})^2}$$

$$= 2.50\ \text{m/s}$$

INSIGHT

If the sled had started from rest, instead of with an initial speed of 0.500 m/s, would its final speed be 2.50 m/s − 0.500 m/s = 2.00 m/s?

No. If the initial speed is zero, then $v_f = \sqrt{\frac{2W_{\text{total}}}{m}} = \sqrt{\frac{2(19.2\ \text{J})}{6.40\ \text{kg}}} = 2.45\ \text{m/s}$. Why don't the speeds add and subtract in a straightforward way? The reason is that the work–energy theorem depends on the *square* of the speeds rather than on v_i and v_f directly.

PRACTICE PROBLEM

Suppose the sled starts with a speed of 0.500 m/s and has a final speed of 2.50 m/s after the boy pulls it through a distance of 3.00 m. What force did the boy exert on the sled? [**Answer:** $F = W_{\text{total}}/(d \cos\theta) = \Delta K/(d\cos\theta) = 7.32\ \text{N}$]

Some related homework problems: Problem 28, Problem 61

The final speeds in the previous Examples could have been found using Newton's laws and the constant-acceleration kinematics of Chapter 2, as indicated in the Insight following Example 7–5. The work–energy theorem provides an alternative method of calculation that is often much easier to apply than Newton's laws. We return to this point in Chapter 8.

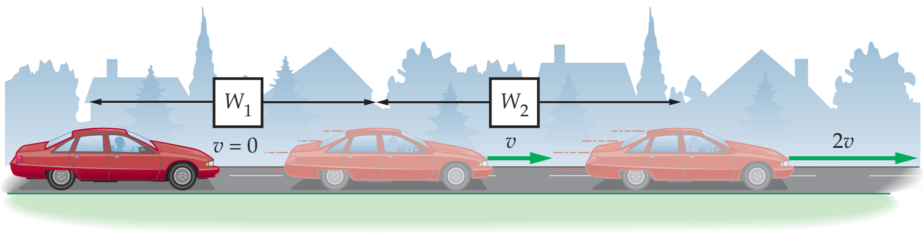
PROBLEM-SOLVING NOTE

Be Careful About Linear Reasoning

Though some relations are linear—if you *double* the mass, you *double* the kinetic energy—others are not. For example, if you *double* the speed, you *quadruple* the kinetic energy. Be careful not to jump to conclusions based on linear reasoning.

CONCEPTUAL CHECKPOINT 7–2 COMPARE THE WORK

To accelerate a certain car from rest to the speed v requires the work W_1. The work needed to accelerate the car from v to $2v$ is W_2. Which of the following is correct: **(a)** $W_2 = W_1$, **(b)** $W_2 = 2\,W_1$, **(c)** $W_2 = 3W_1$, **(d)** $W_2 = 4W_1$?



REASONING AND DISCUSSION

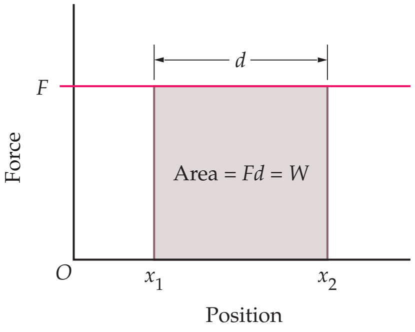
A common mistake is to reason that since we increase the speed by the same amount in each case, the work required is the same. It is not, and the reason is that work depends on the speed squared rather than on the speed itself.

To see how this works, first calculate W_1, the work needed to go from rest to a speed v.

From the work–energy theorem, with $v_i = 0$ and $v_f = v$, we find $W_1 = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv^2$. Similarly, the work needed to go from rest, $v_i = 0$, to a speed $v_f = 2v$, is simply $\frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4W_1$. Therefore, the work needed to increase the speed from v to $2v$ is the difference: $W_2 = 4W_1 - W_1 = 3W_1$.

ANSWER

(c) $W_2 = 3W_1$



▲ **FIGURE 7–6 Graphical representation of the work done by a constant force**

A constant force F acting through a distance d does a work $W = Fd$. Note that Fd is also equal to the shaded area between the force line and the x axis.

7–3 Work Done by a Variable Force

Thus far we have calculated work only for constant forces, yet most forces in nature vary with position. For example, the force exerted by a spring depends on how far the spring is stretched, and the force of gravity between planets depends on their separation. In this section we show how to calculate the work for a force that varies with position.

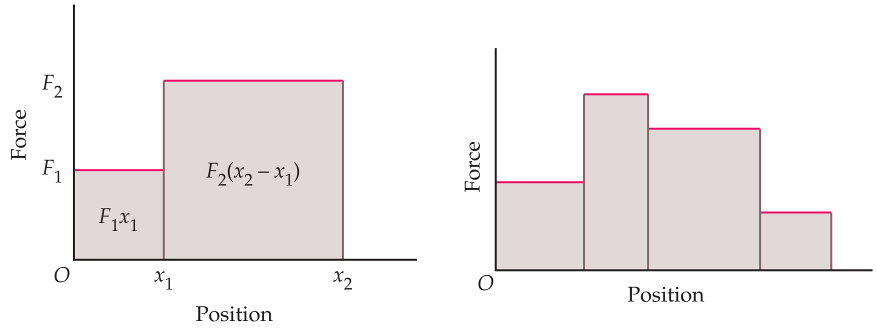
First, let's review briefly the case of a constant force, and develop a graphical interpretation of work. **Figure 7–6** shows a constant force plotted versus position, x. If the force acts in the positive x direction and moves an object a distance d, from x_1 to x_2, the work it does is $W = Fd = F(x_2 - x_1)$. Referring to the figure, we see that the work is equal to the shaded area[1] between the force line and the x axis.

Next, consider a force that has the value F_1 from $x = 0$ to $x = x_1$ and a different value F_2 from $x = x_1$ to $x = x_2$, as in **Figure 7–7 (a)**. The work in this case is the sum of the works done by F_1 and F_2. Therefore, $W = F_1x_1 + F_2(x_2 - x_1)$ which, again, is the area between the force lines and the x axis. Clearly, this type of calculation can be extended to a force with any number of different values, as indicated in **Figure 7–7 (b)**.

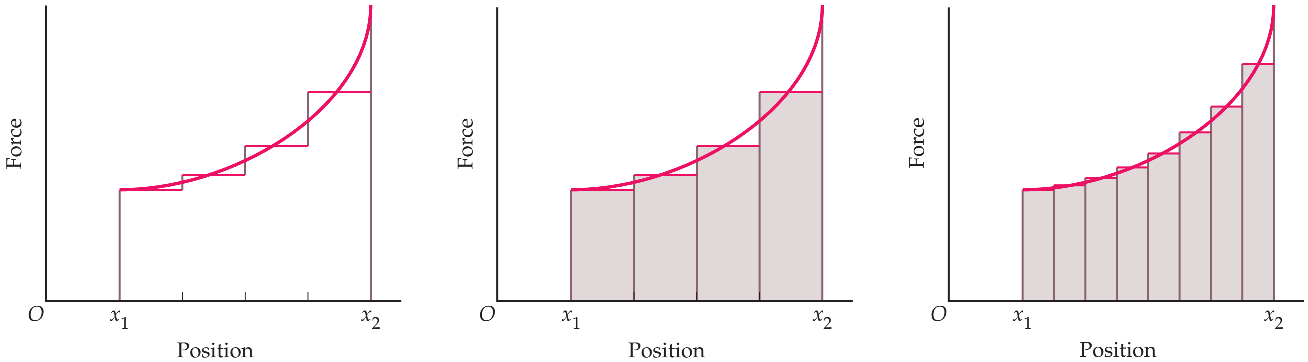
If a force varies continuously with position, we can approximate it with a series of constant values that follow the shape of the curve, as shown in **Figure 7–8 (a)**. It follows that the work done by the continuous force is approximately equal to the area of the corresponding rectangles, as **Figure 7–8 (b)** shows. The approximation can be made better by using more rectangles, as illustrated in **Figure 7–8 (c)**. In the

▶ **FIGURE 7–7 Work done by a nonconstant force**

(a) A force with a value F_1 from 0 to x_1 and a value F_2 from x_1 to x_2 does the work $W = F_1x_1 + F_2(x_2 - x_1)$. This is simply the area of the two shaded rectangles. **(b)** If a force takes on a number of different values, the work it does is still the total area between the force lines and the x axis, just as in part (a).



(a) **(b)**



(a) Approximating a continuous force

(b) Approximating the work done by a continuous force

(c) A better approximation

▲ **FIGURE 7–8 Work done by a continuously varying force**

(a) A continuously varying force can be approximated by a series of constant values that follow the shape of the curve. **(b)** The work done by the continuous force is approximately equal to the area of the small rectangles corresponding to the constant values of force shown in part (a). **(c)** In the limit of an infinite number of vanishingly small rectangles, we see that the work done by the force is equal to the area between the force curve and the x axis.

[1]Usually, area has the dimensions of (length) × (length), or length^2. In this case, however, the vertical axis is force and the horizontal axis is distance. As a result, the dimensions of area are (force) × (distance), which in SI units is $\mathrm{N \cdot m = J}$.

limit of an infinite number of vanishingly small rectangles, the area of the rectangles becomes identical to the area under the force curve. Hence this area is the work done by the continuous force. To summarize:

The work done by a force in moving an object from x_1 to x_2 is equal to the corresponding area between the force curve and the x axis.

A case of particular interest is that of a spring. Since the force exerted by a spring is given by $F_x = -kx$ (Section 6–2), it follows that the force we must exert to hold it at the position x is $+kx$. This is illustrated in **Figure 7–9**, where we also show that the corresponding force curve is a straight line extending from the origin. Therefore, the work we do in stretching a spring from $x = 0$ (equilibrium) to the general position x is the shaded, triangular area shown in **Figure 7–10**. This area is equal to $\frac{1}{2}$(base)(height), where in this case the base is x and the height is kx. As a result, the work is $\frac{1}{2}(x)(kx) = \frac{1}{2}kx^2$. Similar reasoning shows that the work needed to compress a spring a distance x is also $\frac{1}{2}kx^2$. Therefore,

Work to Stretch or Compress a Spring a Distance *x* from Equilibrium

$$W = \frac{1}{2}kx^2 \qquad 7\text{–}8$$

SI unit: joule, J

We can get a feeling for the amount of work required to compress a typical spring in the following Exercise.

EXERCISE 7–4

The spring in a pinball launcher has a force constant of 405 N/m. How much work is required to compress the spring a distance of 3.00 cm?

SOLUTION

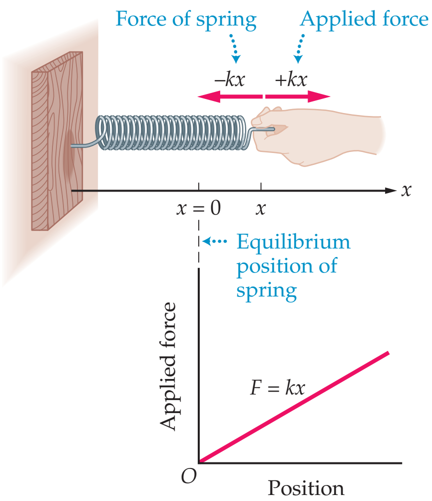
$W = \frac{1}{2}kx^2 = \frac{1}{2}(405\ \text{N/m})(0.0300\ \text{m})^2 = 0.182\ \text{J}$

Note that the work done in compressing or expanding a spring varies with the second power of x, the displacement from equilibrium. The consequences of this dependence are explored throughout the rest of this section.

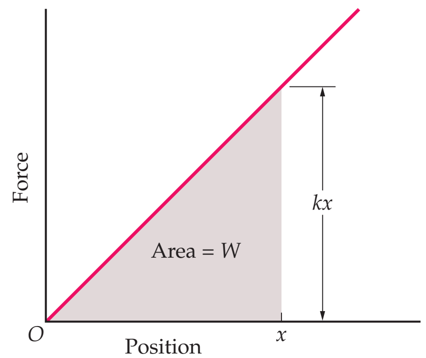
Before we consider a specific example, however, recall that the results for a spring apply to more than just the classic case of a helical coil of wire. In fact, any flexible structure satisfies the relations $F_x = -kx$ and $W = \frac{1}{2}kx^2$, given the appropriate value of the force constant, k, and small enough displacements, x. Several examples were mentioned in Section 6–2.

Here we consider an example from the field of nanotechnology; namely, the cantilevers used in **atomic-force microscopy** (AFM). As we show in Example 7–7, a typical atomic-force cantilever is basically a thin silicon bar about 250 μm in length, supported at one end like a diving board, with a sharp, hanging point at the other end. When the point is pulled across the surface of a material—like an old-fashioned phonograph needle in the groove of a record—individual atoms on the surface cause the point to move up and down, deflecting the cantilever. These deflections, which can be measured by reflecting a laser beam from the top of the cantilever, are then converted into an atomic-level picture of the surface, as shown in the accompanying photograph.

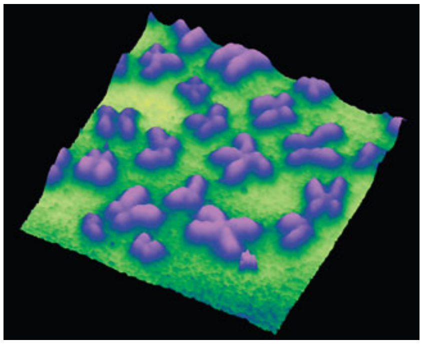
A typical force constant for an AFM cantilever is on the order of 1 N/m, much smaller than the 100–500 N/m force constant of a common lab spring. The implications of this are discussed in the following Example.



▲ **FIGURE 7–9 Stretching a spring**
The force we must exert on a spring to stretch it a distance x is $+kx$. Thus, applied force versus position for a spring is a straight line of slope k.



▲ **FIGURE 7–10 Work needed to stretch a spring a distance *x***
The work done is equal to the shaded area, which is a right triangle. The area of the triangle is $\frac{1}{2}(x)(kx) = \frac{1}{2}kx^2$.



▲ Human chromosomes, as imaged by an atomic-force microscope.

EXAMPLE 7–7 FLEXING AN AFM CANTILEVER

The work required to deflect a typical AFM cantilever by 0.10 nm is 1.2×10^{-20} J. **(a)** What is the force constant of the cantilever, treating it as an ideal spring? **(b)** How much work is required to increase the deflection of the cantilever from 0.10 nm to 0.20 nm?

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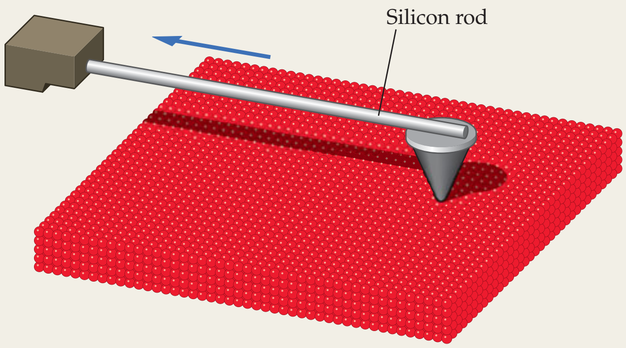
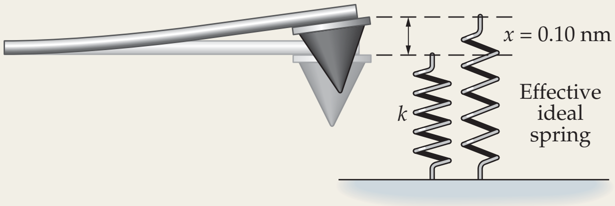
PICTURE THE PROBLEM

The sketch on the left shows the cantilever and its sharp point being dragged across the surface of a material. In the sketch to the right, we show an exaggerated view of the cantilever's deflection, and indicate that it is equivalent to the stretch of an "effective" ideal spring with a force constant k.





STRATEGY

a. Given that $W = 1.2 \times 10^{-20}$ J for a deflection of $x = 0.10$ nm, we can find the effective force constant k using $W = \frac{1}{2}kx^2$.

b. To find the work required to deflect from $x = 0.10$ nm to $x = 0.20$ nm, $W_{1\to2}$, we calculate the work to deflect from $x = 0$ to $x = 0.20$ nm, $W_{0\to2}$, and then subtract the work needed to deflect from $x = 0$ to $x = 0.10$ nm, $W_{0\to1}$. (Note that we *cannot* simply assume the work to go from $x = 0.10$ nm to $x = 0.20$ nm is the same as the work to go from $x = 0$ to $x = 0.10$ nm.)

SOLUTION

Part (a)

1. Solve $W = \frac{1}{2}kx^2$ for the force constant k:

$$k = \frac{2W}{x^2} = \frac{2(1.2 \times 10^{-20}\ \text{J})}{(0.10 \times 10^{-9}\ \text{m})^2} = 2.4\ \text{N/m}$$

Part (b)

2. First, calculate the work needed to deflect the cantilever from $x = 0$ to $x = 0.20$ nm:

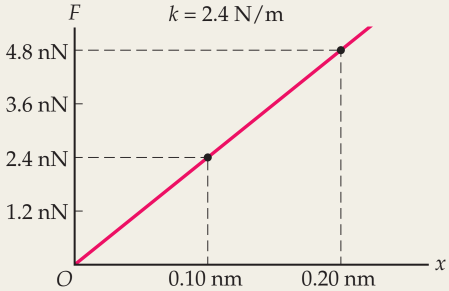
$$W_{0\to2} = \tfrac{1}{2}kx^2$$
$$= \tfrac{1}{2}(2.4\ \text{N/m})(0.2 \times 10^{-9}\ \text{m})^2 = 4.8 \times 10^{-20}\ \text{J}$$

3. Subtract from the above result the work to deflect from $x = 0$ to $x = 0.10$ nm, which the problem statement gives as 1.2×10^{-20} J:

$$W_{1\to2} = W_{0\to2} - W_{0\to1}$$
$$= 4.8 \times 10^{-20}\ \text{J} - 1.2 \times 10^{-20}\ \text{J} = 3.6 \times 10^{-20}\ \text{J}$$

INSIGHT

Our results show that more energy is needed to deflect the cantilever the second 0.10 nm than to deflect it the first 0.10 nm. Why? The reason is that the force of the cantilever increases with distance; thus, the average force over the second 0.10 nm is greater than the average force over the first 0.10 nm. In fact, we can see from the adjacent figure that the average force between 0.10 nm and 0.20 nm (3.6 nN) is three times the average force between 0 and 0.10 nm (1.2 nN). It follows, then, that the work required for the second 0.10 nm is three times the work required for the first 0.10 nm.



PRACTICE PROBLEM

A second cantilever has half the force constant of the cantilever in this Example. Is the work required to deflect the second cantilever by 0.20 nm greater than, less than, or equal to the work required to deflect the cantilever in this Example by 0.10 nm? [**Answer:** Halving the force constant halves the work, but doubling the deflection quadruples the work. The net effect is that the work increases by a factor of two, to 2.4×10^{-20} J.]

Some related homework problems: Problem 32, Problem 38

An equivalent way to calculate the work for a variable force is to multiply the average force, F_{av}, by the distance, d:

$$W = F_{av}d \tag{7–9}$$

For a spring that is stretched a distance x from equilibrium the force varies linearly from 0 to kx. Thus, the average force is $F_{av} = \frac{1}{2}kx$, as indicated in **Figure 7–11**. Therefore, the work is

$$W = \tfrac{1}{2}kx(x) = \tfrac{1}{2}kx^2$$

As expected, our result agrees with Equation 7–8.

Finally, when you stretch or compress a spring from its equilibrium position, the work you do is always positive. The work done *by* a spring, however, may be positive or negative, depending on the situation. For example, consider a block sliding to the right with an initial speed v_0 on a smooth, horizontal surface, as shown in **Figure 7–12 (a)**. When the block begins to compress the spring, as in **Figure 7–12 (b)**, the spring exerts a force on the block to the left—that is, opposite to the block's direction of motion. As a result, the spring does *negative* work on the block, which causes the block's speed to decrease. Eventually the negative work done by the spring, $W = -\frac{1}{2}kx^2$, is equal in magnitude to the initial kinetic energy of the block. At this point, **Figure 7–12 (c)**, the block comes to rest momentarily, and $W = \Delta K = K_f - K_i = 0 - K_i = -K_i = -\frac{1}{2}mv_0^2 = -\frac{1}{2}kx^2$. We apply this result in Active Example 7–1.

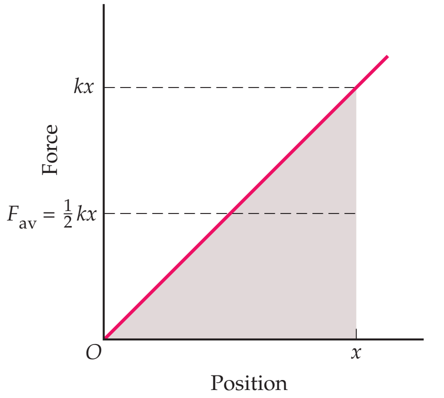


FIGURE 7–11 Work done in stretching a spring: average force

The average force of a spring from $x = 0$ to x is $F_{av} = \frac{1}{2}kx$, and the work done is $W = F_{av}d = \frac{1}{2}kx(x) = \frac{1}{2}kx^2$.

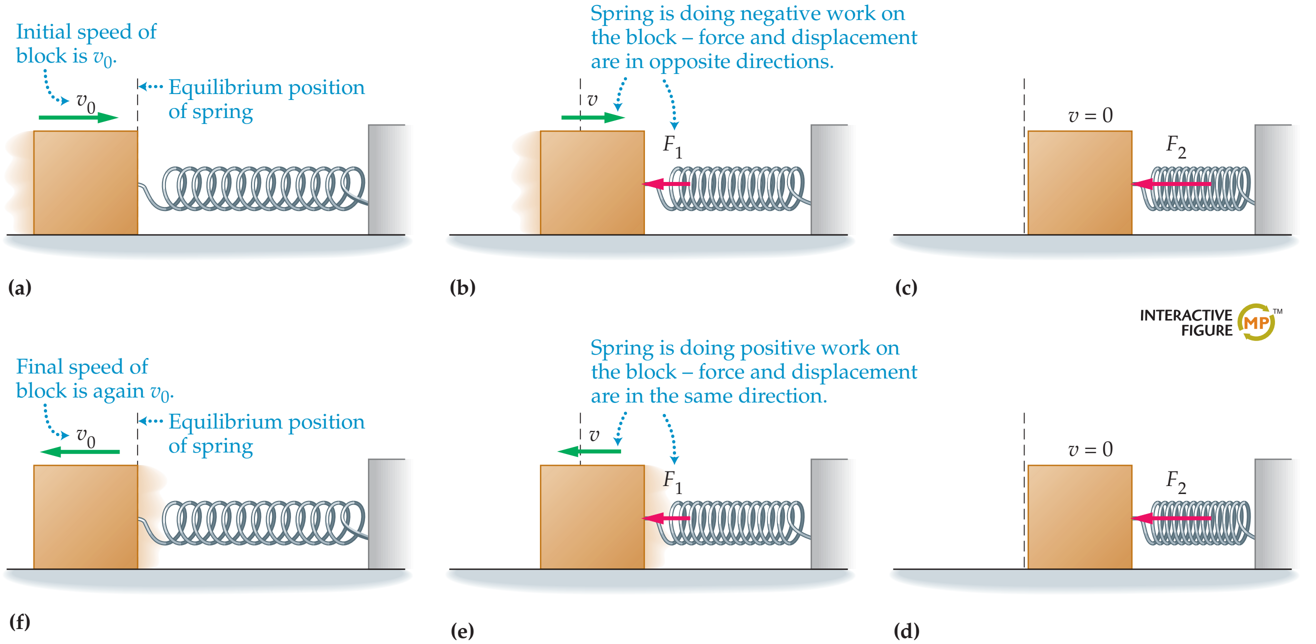


FIGURE 7–12 The work done by a spring can be positive or negative

(a) A block slides to the right on a frictionless surface with a speed v_0 until it encounters a spring. **(b)** The spring now exerts a force to the left—opposite to the block's motion—and hence it does negative work on the block. This causes the block's speed to decrease. **(c)** The negative work done by the spring eventually is equal in magnitude to the block's initial kinetic energy, at which point the block comes to rest momentarily. As the spring expands, **(d)** and **(e)**, it does positive work on the block and increases its speed. **(f)** When the block leaves the spring its speed is again equal to v_0.

ACTIVE EXAMPLE 7–1 A BLOCK COMPRESSES A SPRING

Suppose the block in Figure 7–12 (a) has a mass of 1.5 kg and moves with an initial speed of $v_0 = 2.2$ m/s. Find the compression of the spring, whose force constant is 475 N/m, when the block momentarily comes to rest.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the initial and final kinetic energies of the block: $K_i = 3.6$ J, $K_f = 0$
2. Calculate the change in kinetic energy of the block: $\Delta K = -3.6$ J
3. Set the negative work done by the spring equal to the change in kinetic energy of the block: $-\frac{1}{2}kx^2 = \Delta K = -3.6$ J
4. Solve for the compression, x, and substitute numerical values: $x = 0.12$ m

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INSIGHT

After the block comes to rest, the spring expands back to its equilibrium position, as shown in **Figures 7–12 (d)–(f)**. During this expansion the force exerted by the spring is in the same direction as the block's motion, and hence it does *positive* work in the amount $W = \frac{1}{2}kx^2$. As a result, the block leaves the spring with the same speed it had initially.

YOUR TURN

Find the compression of the spring for the case where the mass of the block is doubled to 3.0 kg.

(Answers to **Your Turn** *problems are given in the back of the book.)*

7–4 Power

Power is a measure of how *quickly* work is done. To be precise, suppose the work W is performed in the time t. The average power delivered during this time is defined as follows:

Definition of Average Power, *P*

$$P = \frac{W}{t} \qquad 7\text{–}10$$

SI unit: J/s = watt, W

For simplicity of notation we drop the usual subscript av for an average quantity and simply understand that the power P refers to an average power unless stated otherwise.

Note that the dimensions of power are joules (work) per second (time). We define one joule per second to be a watt (W), after James Watt (1736–1819), the Scottish engineer and inventor who played a key role in the development of practical steam engines:

$$1 \text{ watt} = 1 \text{ W} = 1 \text{ J/s} \qquad 7\text{–}11$$

Of course, the watt is the unit of power used to rate the output of lightbulbs. Another common unit of power is the horsepower (hp), which is used to rate the output of car engines. It is defined as follows:

$$1 \text{ horsepower} = 1 \text{ hp} = 746 \text{ W} \qquad 7\text{–}12$$

Though it sounds like a horse should be able to produce one horsepower, in fact, a horse can generate only about 2/3 hp for sustained periods. The reason for the discrepancy is that when James Watt defined the horsepower—as a way to characterize the output of his steam engines—he purposely chose a unit that was overly generous to the horse, so that potential investors couldn't complain he was overstating the capability of his engines.

To get a feel for the magnitude of the watt and the horsepower, consider the power you might generate when walking up a flight of stairs. Suppose, for example, that an 80.0-kg person walks up a flight of stairs in 20.0 s, and that the altitude gain is 12.0 ft (3.66 m). Referring to Example 7–2 and Conceptual Checkpoint 7–1, we find that the work done by the person is $W = mgh = (80.0 \text{ kg})(9.81 \text{ m/s}^2)(3.66 \text{ m}) = 2870 \text{ J}$. To find the power, we simply divide by the time: $P = W/t = (2870 \text{ J})/(20.0 \text{ s}) = 144 \text{ W} = 0.193 \text{ hp}$. Thus, a leisurely stroll up the stairs requires about 1/5 hp or 150 W. Similarly, the power produced by a sprinter bolting out of the starting blocks is about 1 hp, and the greatest power most people can produce for sustained periods of time is roughly 1/3 to 1/2 hp. Further examples of power are given in Table 7–3.

TABLE 7–3 Typical Values of Power

Source	Approximate power (W)
Hoover Dam	1.34×10^9
Car moving at 40 mi/h	7×10^4
Home stove	1.2×10^4
Sunlight falling on one square meter	1380
Refrigerator	615
Television	200
Person walking up stairs	150
Human brain	20

REAL-WORLD PHYSICS: BIO

Human power output and flight

Human-powered flight is a feat just barely within our capabilities, since the most efficient human-powered airplanes require a steady power output of about 1/3 hp. On August 23, 1977, the *Gossamer Condor*, designed by Paul MacCready and flown by Bryan Allen, became the first human-powered airplane to complete a prescribed one-mile, figure-eight course and claim the Kremer Prize of £50,000. Allen, an accomplished bicycle racer, used bicycle-like pedals to spin the pro-

▲ The *Gossamer Albatross* on its record-breaking flight across the English Channel in 1979. On two occasions the aircraft actually touched the surface of the water, but the pilot was able to maintain control and complete the 22.25-mile flight.

peller. Controlling the slow-moving craft while pedaling at full power was no easy task. Allen also piloted the *Gossamer Albatross*, which, in 1979, became the first (and so far the only) human-powered aircraft to fly across the English Channel. This 22.25-mile flight—from Folkestone, England, to Cap Gris-Nez, France—took 2 hours 49 minutes and required a total energy output roughly equivalent to climbing to the top of the Empire State Building 10 times.

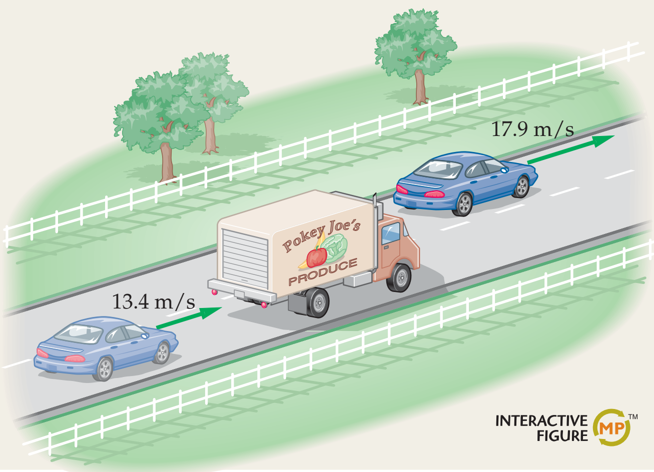
Power output is also an important factor in the performance of a car. For example, suppose it takes a certain amount of work, W, to accelerate a car from 0 to 60 mi/h. If the average power provided by the engine is P, then according to Equation 7–10 the amount of time required to reach 60 mi/h is $t = W/P$. Clearly, the greater the power P, the less the time required to accelerate. Thus, in a loose way of speaking, we can say that the power of a car is a measure of "how fast it can go fast."

EXAMPLE 7–8 PASSING FANCY

To pass a slow-moving truck, you want your fancy 1.30×10^3-kg car to accelerate from 13.4 m/s (30.0 mi/h) to 17.9 m/s (40.0 mi/h) in 3.00 s. What is the minimum power required for this pass?

PICTURE THE PROBLEM

Our sketch shows the car accelerating from an initial speed of $v_i = 13.4$ m/s to a final speed of $v_f = 17.9$ m/s. We assume the road is level, so that no work is done against gravity, and that friction and air resistance may be ignored.



STRATEGY

Power is work divided by time, and work is equal to the change in kinetic energy as the car accelerates. We can determine the change in kinetic energy from the given mass of the car and its initial and final speeds. With this information at hand, we can determine the power with the relation $P = W/t = \Delta K/t$.

SOLUTION

1. First, calculate the change in kinetic energy:

$$\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1.30 \times 10^3\ \text{kg})(17.9\ \text{m/s})^2 - \tfrac{1}{2}(1.30 \times 10^3\ \text{kg})(13.4\ \text{m/s})^2 = 9.16 \times 10^4\ \text{J}$$

2. Divide by time to find the minimum power. (The actual power would have to be greater to overcome frictional losses.):

$$P = \frac{W}{t} = \frac{\Delta K}{t} = \frac{9.16 \times 10^4\ \text{J}}{3.00\ \text{s}} = 3.05 \times 10^4\ \text{W} = 40.9\ \text{hp}$$

INSIGHT

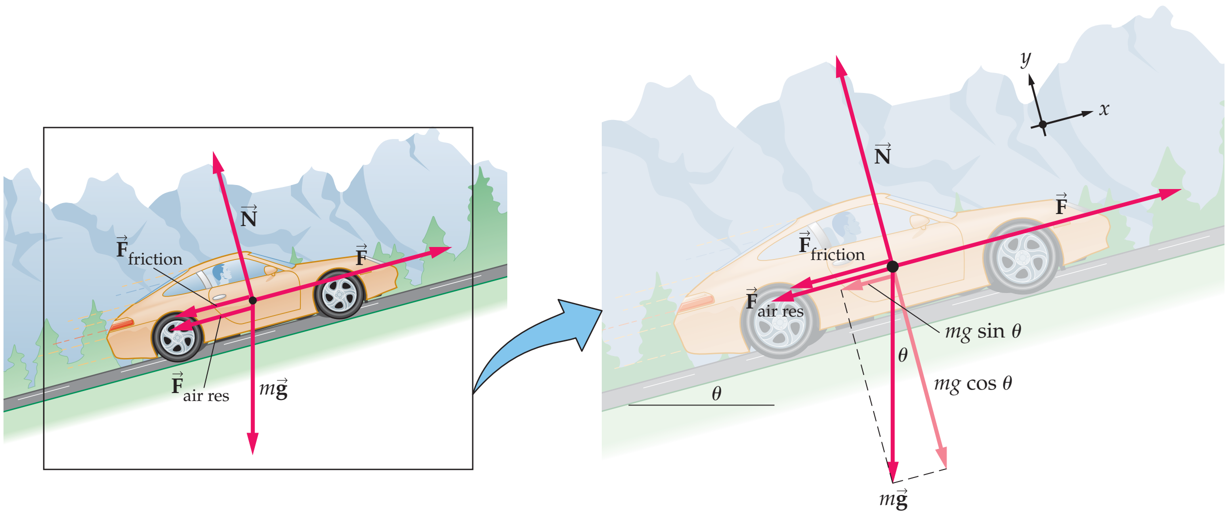
Suppose that your fancy car continues to produce the same 3.05×10^4 W of power as it accelerates from $v = 17.9$ m/s (40.0 mi/h) to $v = 22.4$ ms (50.0 mi/h). Is the time required more than, less than, or equal to 3.00 s? It will take more than 3.00 s. The reason is that ΔK is greater for a change in speed from 40.0 mi/h to 50.0 mi/h than for a change in speed from 30.0 mi/h to 40.0 mi/h, because K depends on speed squared. Since ΔK is greater, the time $t = \Delta K/P$ is also greater.

PRACTICE PROBLEM

Find the time required to accelerate from 40.0 mi/h to 50.0 mi/h with 3.05×10^4 W of power. [**Answer:** First, $\Delta K = 1.18 \times 10^5$ J. Second, $P = \Delta K/t$ can be solved for time to give $t = \Delta K/P$. Thus, $t = 3.87$ s.]

Some related homework problems: Problem 44, Problem 59

Finally, consider a system in which a car, or some other object, is moving with a constant speed v. For example, a car might be traveling uphill on a road inclined at an angle θ above the horizontal. To maintain a constant speed, the engine must exert a constant force F equal to the combined effects of friction, gravity, and air



▲ **FIGURE 7–13 Driving up a hill**
A car traveling uphill at constant speed requires a constant force, F, of magnitude $mg \sin\theta + F_{\text{air res}} + F_{\text{friction}}$, applied in the direction of motion.

resistance, as indicated in **Figure 7–13**. Now, as the car travels a distance d, the work done by the engine is $W = Fd$, and the power it delivers is

$$P = \frac{W}{t} = \frac{Fd}{t}$$

Since the car has a constant speed, $v = d/t$, it follows that

$$P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv \qquad \text{7–13}$$

Note that power is directly proportional to both the force and the speed. For example, suppose you push a heavy shopping cart with a force F. You produce twice as much power when you push at 2 m/s than when you push at 1 m/s, even though you are pushing no harder. It's just that the amount of work you do in a given time period is doubled.

ACTIVE EXAMPLE 7–2 FIND THE MAXIMUM SPEED

It takes a force of 1280 N to keep a 1500-kg car moving with constant speed up a slope of 5.00°. If the engine delivers 50.0 hp to the drive wheels, what is the maximum speed of the car?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Convert the power of 50.0 hp to watts: $P = 3.73 \times 10^4$ W
2. Solve Equation 7–13 for the speed v: $v = P/F$
3. Substitute numerical values for the power and force: $v = 29.1$ m/s

INSIGHT
Thus, the maximum speed of the car on this slope is approximately 65 mi/h.

YOUR TURN
How much power is required for a maximum speed of 32.0 m/s?

(Answers to **Your Turn** *problems are given in the back of the book.)*

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Even though work and kinetic energy are scalar quantities, the idea of vectors, and vector components in particular (Chapter 3), was used in the definition of work in Section 7–1.

The kinematic equations of motion for constant acceleration (Chapters 2 and 4) were used in the derivation of kinetic energy in Section 7–2. In particular, we used the relation between the speed of an object and the distance through which it accelerates.

The basic concepts of force, mass, and acceleration (Chapters 5 and 6) were used throughout this chapter. One particular force, the force exerted by a spring (Chapter 6), played a key role in Section 7–3.

LOOKING AHEAD

In Chapter 8 we introduce the concept of potential energy. The combination of kinetic and potential energy is referred to as the mechanical energy, which will play a central role in our discussion of the conservation of energy.

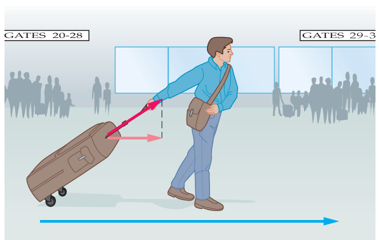
Collisions are studied in Chapter 9. As we shall see, the kinetic energy before and after a collision is an important characterizing feature. Look for the discussion of elastic versus inelastic collision in particular.

The concept of kinetic energy plays a significant role in many areas of physics. Look for it to reappear when we study rotational motion in Chapter 10, and in Section 10–5 in particular. Kinetic energy is also important when we study ideal gases in Chapter 17—in fact, Section 17–2 is titled Kinetic Theory.

CHAPTER SUMMARY

7–1 WORK DONE BY A CONSTANT FORCE

A force exerted through a distance performs mechanical work.



Force in Direction of Motion

In this, the simplest case, work is force times distance:

$$W = Fd \qquad 7\text{–}1$$

Force at an Angle θ to Motion

Work is the component of force in the direction of motion, $F\cos\theta$, times distance, d:

$$W = (F\cos\theta)d = Fd\cos\theta \qquad 7\text{–}3$$

Negative and Total Work

Work is negative if the force opposes the motion; that is, if $\theta > 90°$. If more than one force does work, the total work is the sum of the works done by each force separately:

$$W_{\text{total}} = W_1 + W_2 + W_3 + \cdots \qquad 7\text{–}4$$

Equivalently, sum the forces first to find F_{total}, then

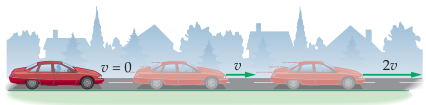
$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}\, d\cos\theta \qquad 7\text{–}5$$

Units

The SI unit of work and energy is the joule, J:

$$1\ \text{J} = 1\ \text{N}\cdot\text{m} \qquad 7\text{–}2$$

7–2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM



Total work is equal to the change in kinetic energy:

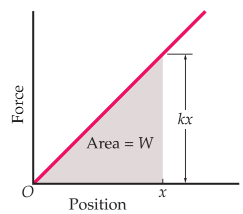
$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad 7\text{–}7$$

Note: To apply this theorem correctly, you must use the *total* work. Kinetic energy is one-half mass times speed squared:

$$K = \tfrac{1}{2}mv^2 \qquad 7\text{–}6$$

It follows that kinetic energy is always positive or zero.

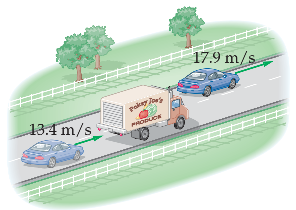
7–3 WORK DONE BY A VARIABLE FORCE



Work is equal to the area between the force curve and the displacement on the x axis. For the case of a spring force, the work to stretch or compress a distance x from equilibrium is

$$W = \tfrac{1}{2}kx^2 \qquad 7\text{–}8$$

7–4 POWER



Average power is work divided by the time required to do the work:

$$P = \frac{W}{t} \qquad 7\text{–}10$$

Equivalently, power is force times speed:

$$P = Fv \qquad 7\text{–}13$$

Units

The SI unit of power is the watt, W:

$$1\ \text{W} = 1\ \text{J/s} \qquad 7\text{–}11$$

$$746\ \text{W} = 1\ \text{hp} \qquad 7\text{–}12$$

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Find the work done by a constant force.	Work is defined as force times displacement, $W = Fd$, when F is in the direction of motion. Use $W = (F\cos\theta)d$ when there is an angle θ between the force and the direction of motion.	Examples 7–1 through 7–6
Calculate the change in speed.	The change in kinetic energy is given by the work–energy theorem, $W_{\text{total}} = \Delta K$. From this, the change in speed can be found by recalling that $K = \frac{1}{2}mv^2$. Be sure W_{total} is the total work and that it has the correct sign.	Examples 7–5, 7–6
Calculate the power.	Find the work done, then divide by time: $P = W/t$. Alternatively, find the force, then multiply by the speed: $P = Fv$.	Example 7–8 Active Example 7–2

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Is it possible to do work on an object that remains at rest?
2. A friend makes the statement, "Only the total force acting on an object can do work." Is this statement true or false? If it is true, state why; if it is false, give a counterexample.
3. A friend makes the statement, "A force that is always perpendicular to the velocity of a particle does no work on the particle." Is this statement true or false? If it is true, state why; if it is false, give a counterexample.
4. The net work done on a certain object is zero. What can you say about its speed?
5. To get out of bed in the morning, do you have to do work? Explain.
6. Give an example of a frictional force doing negative work.
7. Give an example of a frictional force doing positive work.
8. A ski boat moves with constant velocity. Is the net force acting on the boat doing work? Explain.
9. A package rests on the floor of an elevator that is rising with constant speed. The elevator exerts an upward normal force on the package, and hence does positive work on it. Why doesn't the kinetic energy of the package increase?
10. An object moves with constant velocity. Is it safe to conclude that no force acts on the object? Why, or why not?
11. Engine 1 does twice the work of engine 2. Is it correct to conclude that engine 1 produces twice as much power as engine 2? Explain.
12. Engine 1 produces twice the power of engine 2. Is it correct to conclude that engine 1 does twice as much work as engine 2? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

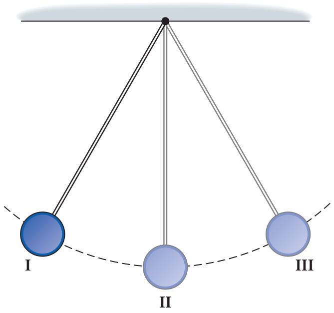
Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 7–1 WORK DONE BY A CONSTANT FORCE

1. • **CE** The International Space Station orbits the Earth in an approximately circular orbit at a height of $h = 375$ km above the Earth's surface. In one complete orbit, is the work done by the Earth on the space station positive, negative, or zero? Explain.

2. • **CE** A pendulum bob swings from point I to point II along the circular arc indicated in **Figure 7–14**. **(a)** Is the work done on the bob by gravity positive, negative, or zero? Explain. **(b)** Is the work done on the bob by the string positive, negative, or zero? Explain.

3. • **CE** A pendulum bob swings from point II to point III along the circular arc indicated in Figure 7–14. **(a)** Is the work done on



▲ **FIGURE 7–14** Problems 2 and 3

the bob by gravity positive, negative, or zero? Explain. **(b)** Is the work done on the bob by the string positive, negative, or zero? Explain.

4. • A farmhand pushes a 26-kg bale of hay 3.9 m across the floor of a barn. If she exerts a horizontal force of 88 N on the hay, how much work has she done?

5. • Children in a tree house lift a small dog in a basket 4.70 m up to their house. If it takes 201 J of work to do this, what is the combined mass of the dog and basket?

6. • Early one October, you go to a pumpkin patch to select your Halloween pumpkin. You lift the 3.2-kg pumpkin to a height of 1.2 m, then carry it 50.0 m (on level ground) to the check-out stand. **(a)** Calculate the work you do on the pumpkin as you lift it from the ground. **(b)** How much work do you do on the pumpkin as you carry it from the field?

7. • The coefficient of kinetic friction between a suitcase and the floor is 0.272. If the suitcase has a mass of 71.5 kg, how far can it be pushed across the level floor with 642 J of work?

8. •• You pick up a 3.4-kg can of paint from the ground and lift it to a height of 1.8 m. **(a)** How much work do you do on the can of paint? **(b)** You hold the can stationary for half a minute, waiting for a friend on a ladder to take it. How much work do you do during this time? **(c)** Your friend decides against the paint, so you lower it back to the ground. How much work do you do on the can as you lower it?

9. •• **IP** A tow rope, parallel to the water, pulls a water skier directly behind the boat with constant velocity for a distance of 65 m before the skier falls. The tension in the rope is 120 N. **(a)** Is the work done on the skier by the rope positive, negative, or zero? Explain. **(b)** Calculate the work done by the rope on the skier.

10. •• **IP** In the situation described in the previous problem, **(a)** is the work done on the boat by the rope positive, negative, or zero? Explain. **(b)** Calculate the work done by the rope on the boat.

11. •• A child pulls a friend in a little red wagon with constant speed. If the child pulls with a force of 16 N for 10.0 m, and the handle of the wagon is inclined at an angle of 25° above the horizontal, how much work does the child do on the wagon?

12. •• A 51-kg packing crate is pulled with constant speed across a rough floor with a rope that is at an angle of 43.5° above the horizontal. If the tension in the rope is 115 N, how much work is done on the crate to move it 8.0 m?

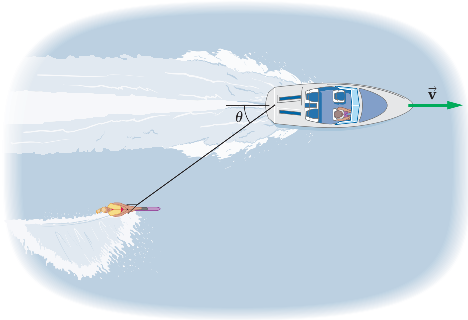
13. •• **IP** To clean a floor, a janitor pushes on a mop handle with a force of 50.0 N. **(a)** If the mop handle is at an angle of 55° above the horizontal, how much work is required to push the mop 0.50 m? **(b)** If the angle the mop handle makes with the horizontal is increased to 65°, does the work done by the janitor increase, decrease, or stay the same? Explain.

14. •• A small plane tows a glider at constant speed and altitude. If the plane does 2.00×10^5 J of work to tow the glider 145 m and the tension in the tow rope is 2560 N, what is the angle between the tow rope and the horizontal?

15. •• A young woman on a skateboard is pulled by a rope attached to a bicycle. The velocity of the skateboarder is $\vec{\mathbf{v}} = (4.1 \text{ m/s})\hat{\mathbf{x}}$ and the force exerted on her by the rope is $\vec{\mathbf{F}} = (17 \text{ N})\hat{\mathbf{x}} + (12 \text{ N})\hat{\mathbf{y}}$. **(a)** Find the work done on the skateboarder by the rope in 25 seconds. **(b)** Assuming the velocity of the bike is the same as that of the skateboarder, find the work the rope does on the bicycle in 25 seconds.

16. •• To keep her dog from running away while she talks to a friend, Susan pulls gently on the dog's leash with a constant force given by $\vec{\mathbf{F}} = (2.2 \text{ N})\hat{\mathbf{x}} + (1.1 \text{ N})\hat{\mathbf{y}}$. How much work does she do on the dog if its displacement is **(a)** $\vec{\mathbf{d}} = (0.25 \text{ m})\hat{\mathbf{x}}$, **(b)** $\vec{\mathbf{d}} = (0.25 \text{ m})\hat{\mathbf{y}}$, or **(c)** $\vec{\mathbf{d}} = (-0.50 \text{ m})\hat{\mathbf{x}} + (-0.25 \text{ m})\hat{\mathbf{y}}$?

17. •• Water skiers often ride to one side of the center line of a boat, as shown in Figure 7–15. In this case, the ski boat is traveling at 15 m/s and the tension in the rope is 75 N. If the boat does 3500 J of work on the skier in 50.0 m, what is the angle θ between the tow rope and the center line of the boat?



▲ **FIGURE 7–15** Problems 17 and 69

SECTION 7–2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM

18. • **CE** A pitcher throws a ball at 90 mi/h and the catcher stops it in her glove. **(a)** Is the work done on the ball by the pitcher positive, negative, or zero? Explain. **(b)** Is the work done on the ball by the catcher positive, negative, or zero? Explain.

19. • How much work is needed for a 73-kg runner to accelerate from rest to 7.7 m/s?

20. • **Skylab's Reentry** When Skylab reentered the Earth's atmosphere on July 11, 1979, it broke into a myriad of pieces. One of the largest fragments was a 1770-kg lead-lined film vault, and it landed with an estimated speed of 120 m/s. What was the kinetic energy of the film vault when it landed?

21. • **IP** A 9.50-g bullet has a speed of 1.30 km/s. **(a)** What is its kinetic energy in joules? **(b)** What is the bullet's kinetic energy if its speed is halved? **(c)** If its speed is doubled?

22. •• **CE Predict/Explain** The work W_0 accelerates a car from 0 to 50 km/h. **(a)** Is the work required to accelerate the car from 50 km/h to 150 km/h equal to $2W_0$, $3W_0$, $8W_0$, or $9W_0$? **(b)** Choose the *best explanation* from among the following:

I. The work to accelerate the car depends on the speed squared.
II. The final speed is three times the speed that was produced by the work W_0.
III. The increase in speed from 50 km/h to 150 km/h is twice the increase in speed from 0 to 50 km/h.

23. •• **CE** Jogger A has a mass m and a speed v, jogger B has a mass $m/2$ and a speed $3v$, jogger C has a mass $3m$ and a speed $v/2$, and jogger D has a mass $4m$ and a speed $v/2$. Rank the joggers in order of increasing kinetic energy. Indicate ties where appropriate.

24. •• **IP** A 0.14-kg pinecone falls 16 m to the ground, where it lands with a speed of 13 m/s. **(a)** With what speed would the pinecone have landed if there had been no air resistance? **(b)** Did air resistance do positive work, negative work, or zero work on the pinecone? Explain.

25. •• In the previous problem, **(a)** how much work was done on the pinecone by air resistance? **(b)** What was the average force of air resistance exerted on the pinecone?

26. •• At $t = 1.0$ s, a 0.40-kg object is falling with a speed of 6.0 m/s. At $t = 2.0$ s, it has a kinetic energy of 25 J. **(a)** What is the kinetic energy of the object at $t = 1.0$ s? **(b)** What is the speed of the object at $t = 2.0$ s? **(c)** How much work was done on the object between $t = 1.0$ s and $t = 2.0$ s?

27. •• After hitting a long fly ball that goes over the right fielder's head and lands in the outfield, the batter decides to keep going past second base and try for third base. The 62.0-kg player begins sliding 3.40 m from the base with a speed of 4.35 m/s. If the player comes to rest at third base, **(a)** how much work was done on the player by friction? **(b)** What was the coefficient of kinetic friction between the player and the ground?

28. •• **IP** A 1100-kg car coasts on a horizontal road with a speed of 19 m/s. After crossing an unpaved, sandy stretch of road 32 m long, its speed decreases to 12 m/s. **(a)** Was the net work done on the car positive, negative, or zero? Explain. **(b)** Find the magnitude of the average net force on the car in the sandy section.

29. •• **IP** **(a)** In the previous problem, the car's speed decreased by 7.0 m/s as it coasted across a sandy section of road 32 m long. If the sandy portion of the road had been only 16 m long, would the car's speed have decreased by 3.5 m/s, more than 3.5 m/s, or less than 3.5 m/s? Explain. **(b)** Calculate the change in speed in this case.

30. •• A 65-kg bicyclist rides his 8.8-kg bicycle with a speed of 14 m/s. **(a)** How much work must be done by the brakes to bring the bike and rider to a stop? **(b)** How far does the bicycle travel if it takes 4.0 s to come to rest? **(c)** What is the magnitude of the braking force?

SECTION 7–3 WORK DONE BY A VARIABLE FORCE

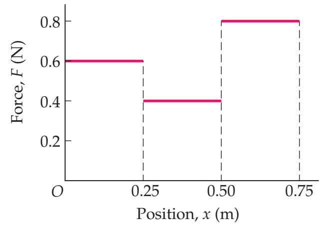
31. • **CE** A block of mass m and speed v collides with a spring, compressing it a distance Δx. What is the compression of the spring if the force constant of the spring is increased by a factor of four?

32. • A spring with a force constant of 3.5×10^4 N/m is initially at its equilibrium length. **(a)** How much work must you do to stretch the spring 0.050 m? **(b)** How much work must you do to compress it 0.050 m?

33. • A 1.2-kg block is held against a spring of force constant 1.0×10^4 N/m, compressing it a distance of 0.15 m. How fast is the block moving after it is released and the spring pushes it away?

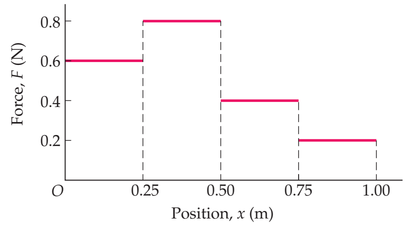
34. • Initially sliding with a speed of 2.2 m/s, a 1.8-kg block collides with a spring and compresses it 0.31 m before coming to rest. What is the force constant of the spring?

35. • The force shown in **Figure 7–16** moves an object from $x = 0$ to $x = 0.75$ m. **(a)** How much work is done by the force? **(b)** How much work is done by the force if the object moves from $x = 0.15$ m to $x = 0.60$ m?



▲ **FIGURE 7–16** Problem 35

36. • An object is acted on by the force shown in **Figure 7–17**. What is the final position of the object if its initial position is $x = 0.40$ m and the work done on it is equal to **(a)** 0.21 J, or **(b)** -0.19 J?



▲ **FIGURE 7–17** Problems 36 and 40

37. •• **CE** A block of mass m and speed v collides with a spring, compressing it a distance Δx. What is the compression of the spring if the mass of the block is halved and its speed is doubled?

38. •• To compress spring 1 by 0.20 m takes 150 J of work. Stretching spring 2 by 0.30 m requires 210 J of work. Which spring is stiffer?

39. •• **IP** It takes 180 J of work to compress a certain spring 0.15 m. **(a)** What is the force constant of this spring? **(b)** To compress the spring an additional 0.15 m, does it take 180 J, more than 180 J, or less than 180 J? Verify your answer with a calculation.

40. •• The force shown in Figure 7–17 acts on a 1.7-kg object whose initial speed is 0.44 m/s and initial position is $x = 0.27$ m. **(a)** Find the speed of the object when it is at the location $x = 0.99$ m. **(b)** At what location would the object's speed be 0.32 m/s?

41. ••• A block is acted on by a force that varies as $(2.0 \times 10^4\ \text{N/m})x$ for $0 \le x \le 0.21$ m, and then remains constant at 4200 N for larger x. How much work does the force do on the block in moving it **(a)** from $x = 0$ to $x = 0.30$ m, or **(b)** from $x = 0.10$ m to $x = 0.40$ m?

SECTION 7–4 POWER

42. • **CE** Force F_1 does 5 J of work in 10 seconds, force F_2 does 3 J of work in 5 seconds, force F_3 does 6 J of work in 18 seconds, and force F_4 does 25 J of work in 125 seconds. Rank these forces in order of increasing power they produce. Indicate ties where appropriate.

43. • **BIO** **Climbing the Empire State Building** A new record for running the stairs of the Empire State Building was set on February 3, 2003. The 86 flights, with a total of 1576 steps, was run in 9 minutes and 33 seconds. If the height gain of each step was 0.20 m, and the mass of the runner was 70.0 kg, what was his average power output during the climb? Give your answer in both watts and horsepower.

44. • How many joules of energy are in a kilowatt-hour?

45. • Calculate the power output of a 1.4-g fly as it walks straight up a windowpane at 2.3 cm/s.

46. • An ice cube is placed in a microwave oven. Suppose the oven delivers 105 W of power to the ice cube and that it takes 32,200 J to melt it. How long does it take for the ice cube to melt?

47. • You raise a bucket of water from the bottom of a deep well. If your power output is 108 W, and the mass of the bucket and the water in it is 5.00 kg, with what speed can you raise the bucket? Ignore the weight of the rope.

48. •• In order to keep a leaking ship from sinking, it is necessary to pump 12.0 lb of water each second from below deck up a height of 2.00 m and over the side. What is the minimum horsepower motor that can be used to save the ship?

49. •• **IP** A kayaker paddles with a power output of 50.0 W to maintain a steady speed of 1.50 m/s. **(a)** Calculate the resistive force exerted by the water on the kayak. **(b)** If the kayaker doubles her power output, and the resistive force due to the water remains the same, by what factor does the kayaker's speed change?

50. •• **BIO Human-Powered Flight** Human-powered aircraft require a pilot to pedal, as in a bicycle, and produce a sustained power output of about 0.30 hp. The *Gossamer Albatross* flew across the English Channel on June 12, 1979, in 2 h 49 min. **(a)** How much energy did the pilot expend during the flight? **(b)** How many Snickers candy bars (280 Cal per bar) would the pilot have to consume to be "fueled up" for the flight? [*Note:* The nutritional calorie, 1 Cal, is equivalent to 1000 calories (1000 cal) as defined in physics. In addition, the conversion factor between calories and joules is as follows: 1 Cal = 1000 cal = 1 kcal = 4186 J.]

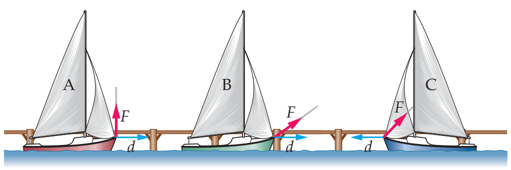
51. •• **IP** A grandfather clock is powered by the descent of a 4.35-kg weight. **(a)** If the weight descends through a distance of 0.760 m in 3.25 days, how much power does it deliver to the clock? **(b)** To increase the power delivered to the clock, should the time it takes for the mass to descend be increased or decreased? Explain.

52. •• **BIO The Power You Produce** Estimate the power you produce in running up a flight of stairs. Give your answer in horsepower.

53. ••• **IP** A certain car can accelerate from rest to the speed v in T seconds. If the power output of the car remains constant, **(a)** how long does it take for the car to accelerate from v to $2v$? **(b)** How fast is the car moving at $2T$ seconds after starting?

GENERAL PROBLEMS

54. • **CE** As the three small sailboats shown in **Figure 7–18** drift next to a dock, because of wind and water currents, students pull on a line attached to the bow and exert forces of equal magnitude F. Each boat drifts through the same distance d. Rank the three boats (A, B, and C) in order of increasing work done on the boat by the force F. Indicate ties where appropriate.



▲ **FIGURE 7–18** Problem 54

55. • **CE** A youngster rides on a skateboard with a speed of 2 m/s. After a force acts on the youngster, her speed is 3 m/s. Was the work done by the force positive, negative, or zero? Explain.

56. • **CE Predict/Explain** A car is accelerated by a constant force, F. The distance required to accelerate the car from rest to the speed v is Δx. **(a)** Is the distance required to accelerate the car from the speed v to the speed $2v$ equal to Δx, $2\Delta x$, $3\Delta x$, or $4\Delta x$? **(b)** Choose the *best explanation* from among the following:

I. The final speed is twice the initial speed.
II. The increase in speed is the same in each case.
III. Work is force times distance, and work depends on the speed squared.

57. • **CE** Car 1 has four times the mass of car 2, but they both have the same kinetic energy. If the speed of car 2 is v, is the speed of car 1 equal to $v/4$, $v/2$, $2v$, or $4v$? Explain.

58. • **BIO Muscle Cells** Biological muscle cells can be thought of as nanomotors that use the chemical energy of ATP to produce mechanical work. Measurements show that the active proteins within a muscle cell (such as myosin and actin) can produce a force of about 7.5 pN and displacements of 8.0 nm. How much work is done by such proteins?

59. • When you take a bite out of an apple, you do about 19 J of work. Estimate **(a)** the force and **(b)** the power produced by your jaw muscles during the bite.

60. • A Mountain bar has a mass of 0.045 kg and a calorie rating of 210 Cal. What speed would this candy bar have if its kinetic energy were equal to its metabolic energy? [See the note following Problem 50.]

61. • A small motor runs a lift that raises a load of bricks weighing 836 N to a height of 10.7 m in 23.2 s. Assuming that the bricks are lifted with constant speed, what is the minimum power the motor must produce?

62. • You push a 67-kg box across a floor where the coefficient of kinetic friction is $\mu_k = 0.55$. The force you exert is horizontal. **(a)** How much power is needed to push the box at a speed of 0.50 m/s? **(b)** How much work do you do if you push the box for 35 s?

63. • **BIO The Beating Heart** The average power output of the human heart is 1.33 watts. **(a)** How much energy does the heart produce in a day? **(b)** Compare the energy found in part (a) with the energy required to walk up a flight of stairs. Estimate the height a person could attain on a set of stairs using nothing more than the daily energy produced by the heart.

64. • **The Atmos Clock** The Atmos clock (the so-called perpetual motion clock) gets its name from the fact that it runs off pressure variations in the atmosphere, which drive a bellows containing a mixture of gas and liquid ethyl chloride. Because the power to drive these clocks is so limited, they must be very efficient. In fact, a single 60.0-W lightbulb could power 240 million Atmos clocks simultaneously. Find the amount of energy, in joules, required to run an Atmos clock for one day.

65. •• **CE** The work W_0 is required to accelerate a car from rest to the speed v_0. How much work is required to accelerate the car **(a)** from rest to the speed $v_0/2$ and **(b)** from $v_0/2$ to v_0?

66. •• **CE** A work W_0 is required to stretch a certain spring 2 cm from its equilibrium position. **(a)** How much work is required to stretch the spring 1 cm from equilibrium? **(b)** Suppose the spring is already stretched 2 cm from equilibrium. How much additional work is required to stretch it to 3 cm from equilibrium?

67. •• After a tornado, a 0.55-g straw was found embedded 2.3 cm into the trunk of a tree. If the average force exerted on the straw by the tree was 65 N, what was the speed of the straw when it hit the tree?

68. •• You throw a glove straight upward to celebrate a victory. Its initial kinetic energy is K and it reaches a maximum height h. What is the kinetic energy of the glove when it is at the height $h/2$?

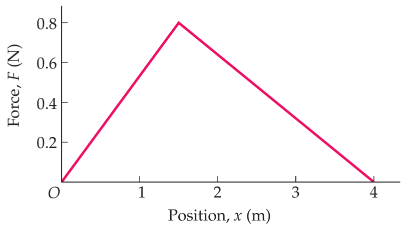
69. •• The water skier in Figure 7–15 is at an angle of 35° with respect to the center line of the boat, and is being pulled at a constant speed of 14 m/s. If the tension in the tow rope is 90.0 N, **(a)** how much work does the rope do on the skier in 10.0 s? **(b)** How much work does the resistive force of water do on the skier in the same time?

70. •• **IP** A sled with a mass of 5.80 kg is pulled along the ground through a displacement given by $\vec{\mathbf{d}} = (4.55\text{ m})\hat{\mathbf{x}}$. (Let the x axis be horizontal and the y axis be vertical.) **(a)** How much work is done on the sled when the force acting on it is $\vec{\mathbf{F}} = (2.89\text{ N})\hat{\mathbf{x}} + (0.131\text{ N})\hat{\mathbf{y}}$? **(b)** How much work is done on the sled when the force acting on it is $\vec{\mathbf{F}} = (2.89\text{ N})\hat{\mathbf{x}} + (0.231\text{ N})\hat{\mathbf{y}}$? **(c)** If the mass of the sled is increased, does the work done by the forces in parts (a) and (b) increase, decrease, or stay the same? Explain.

71. •• **IP** A 0.19-kg apple falls from a branch 3.5 m above the ground. **(a)** Does the power delivered to the apple by gravity increase, decrease, or stay the same during the time the apple falls to the ground? Explain. Find the power delivered by gravity to the apple when the apple is **(b)** 2.5 m and **(c)** 1.5 m above the ground.

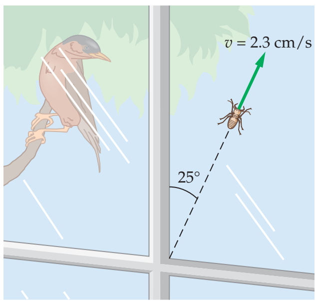
72. •• A juggling ball of mass m is thrown straight upward from an initial height h with an initial speed v_0. How much work has gravity done on the ball **(a)** when it reaches its greatest height, h_{max}, and **(b)** when it reaches ground level? **(c)** Find an expression for the kinetic energy of the ball as it lands.

73. •• The force shown in **Figure 7–19** acts on an object that moves along the x axis. How much work is done by the force as the object moves from **(a)** $x = 0$ to $x = 2.0$ m, **(b)** $x = 1.0$ m to $x = 4.0$ m, and **(c)** $x = 3.5$ m to $x = 1.2$ m?



▲ **FIGURE 7–19** Problem 73

74. •• Calculate the power output of a 1.8-g spider as it walks up a windowpane at 2.3 cm/s. The spider walks on a path that is at 25° to the vertical, as illustrated in **Figure 7–20**.



▲ **FIGURE 7–20** Problem 74

75. •• The motor of a ski boat produces a power of 36,600 W to maintain a constant speed of 14.0 m/s. To pull a water skier at the same constant speed, the motor must produce a power of 37,800 W. What is the tension in the rope pulling the skier?

76. •• **Cookie Power** To make a batch of cookies, you mix half a bag of chocolate chips into a bowl of cookie dough, exerting a 21-N force on the stirring spoon. Assume that your force is always in the direction of motion of the spoon. **(a)** What power is needed to move the spoon at a speed of 0.23 m/s? **(b)** How much work do you do if you stir the mixture for 1.5 min?

77. •• **IP** A pitcher accelerates a 0.14-kg hardball from rest to 42.5 m/s in 0.060 s. **(a)** How much work does the pitcher do on the ball? **(b)** What is the pitcher's power output during the pitch? **(c)** Suppose the ball reaches 42.5 m/s in less than 0.060 s. Is the power produced by the pitcher in this case more than, less than, or the same as the power found in part (b)? Explain.

78. •• **Catapult Launcher** A catapult launcher on an aircraft carrier accelerates a jet from rest to 72 m/s. The work done by the catapult during the launch is 7.6×10^7 J. **(a)** What is the mass of the jet? **(b)** If the jet is in contact with the catapult for 2.0 s, what is the power output of the catapult?

79. •• **BIO Brain Power** The human brain consumes about 22 W of power under normal conditions, though more power may be required during exams. **(a)** How long can one Snickers bar (see the note following Problem 50) power the normally functioning brain? **(b)** At what rate must you lift a 3.6-kg container of milk (one gallon) if the power output of your arm is to be 22 W? **(c)** How long does it take to lift the milk container through a distance of 1.0 m at this rate?

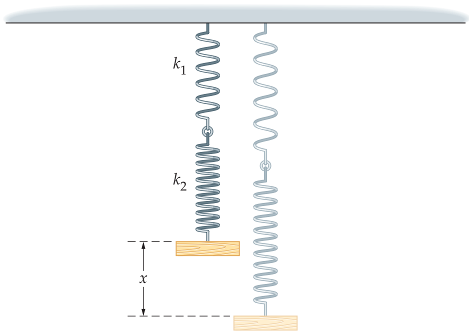
80. •• **IP** A 1300-kg car delivers a constant 49 hp to the drive wheels. We assume the car is traveling on a level road and that all frictional forces may be ignored. **(a)** What is the acceleration of this car when its speed is 14 m/s? **(b)** If the speed of the car is doubled, does its acceleration increase, decrease, or stay the same? Explain. **(c)** Calculate the car's acceleration when its speed is 28 m/s.

81. •• **Meteorite** On October 9, 1992, a 27-pound meteorite struck a car in Peekskill, NY, creating a dent about 22 cm deep. If the initial speed of the meteorite was 550 m/s, what was the average force exerted on the meteorite by the car?

An interplanetary fender bender (Problem 81)

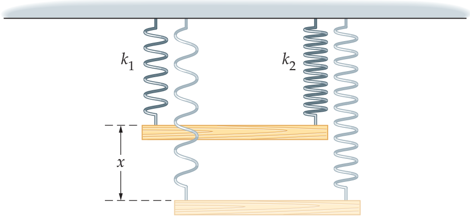
82. ••• **BIO Powering a Pigeon** A pigeon in flight experiences a force of air resistance given approximately by $F = bv^2$, where v is the flight speed and b is a constant. **(a)** What are the units of the constant b? **(b)** What is the largest possible speed of the pigeon if its maximum power output is P? **(c)** By what factor does the largest possible speed increase if the maximum power is doubled?

83. ••• **Springs in Series** Two springs, with force constants k_1 and k_2, are connected in series, as shown in **Figure 7–21**. How much work is required to stretch this system a distance x from the equilibrium position?



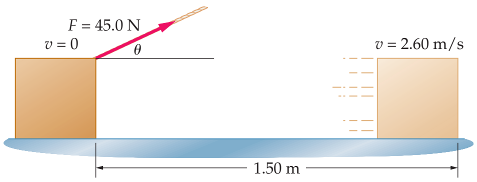
▲ **FIGURE 7–21** Problem 83

84. ••• **Springs in Parallel** Two springs, with force constants k_1 and k_2, are connected in parallel, as shown in **Figure 7–22**. How much work is required to stretch this system a distance x from the equilibrium position?



▲ **FIGURE 7–22** Problem 84

85. ••• A block rests on a horizontal frictionless surface. A string is attached to the block, and is pulled with a force of 45.0 N at an angle θ above the horizontal, as shown in **Figure 7–23**. After the block is pulled through a distance of 1.50 m, its speed is 2.60 m/s, and 50.0 J of work has been done on it. **(a)** What is the angle θ? **(b)** What is the mass of the block?



▲ **FIGURE 7–23** Problem 85

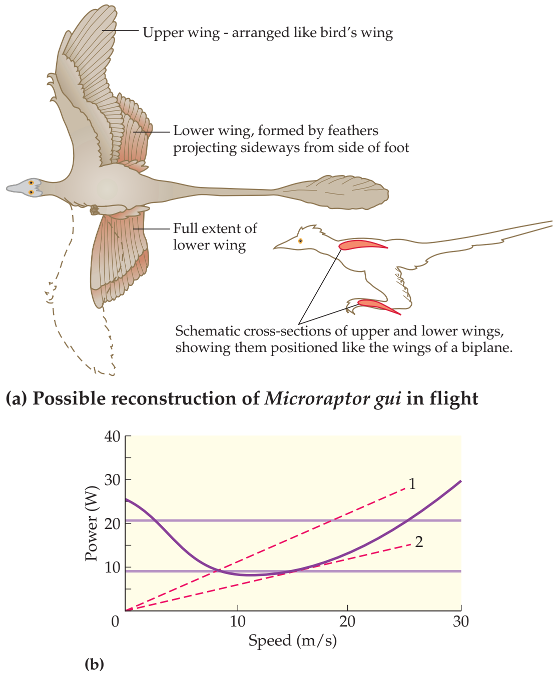
PASSAGE PROBLEMS

BIO *Microraptor gui*: The Biplane Dinosaur

The evolution of flight is a subject of intense interest in paleontology. Some subscribe to the "cursorial" (or ground-up) hypothesis, in which flight began with ground-dwelling animals running and jumping after prey. Others favor the "arboreal" (or trees-down) hypothesis, in which tree-dwelling animals, like modern-day flying squirrels, developed flight as an extension of gliding from tree to tree.

A recently discovered fossil from the Cretaceous period in China supports the arboreal hypothesis and adds a new element—it suggests that feathers on both the wings and the lower legs and feet allowed this dinosaur, *Microraptor gui*, to glide much like a biplane, as shown in **Figure 7–24 (a)**. Researchers have produced a detailed computer simulation of *Microraptor*, and with its help have obtained the power-versus-speed plot presented in **Figure 7–24 (b)**. This curve shows how much power is required for flight at speeds between 0 and 30 m/s. Notice that the power increases at high speeds, as expected, but is also high for low speeds, where the dinosaur is almost hovering. A minimum of 8.1 W is needed for flight at 10 m/s. The lower horizontal line shows the estimated 9.8-W power output of *Microraptor*, indicating the small range of speeds for which flight would be possible. The upper horizontal line shows the wider range of flight speeds that would be available if *Microraptor* were able to produce 20 W of power.

Also of interest are the two dashed, straight lines labeled 1 and 2. These lines represent constant ratios of power to speed; that is, a constant value for P/v. Referring to Equation 7–13, we see that $P/v = Fv/v = F$, so the lines 1 and 2 correspond to lines of constant force. Line 2 is interesting in that it has the smallest slope that still touches the power-versus-speed curve.



▲ **FIGURE 7–24** Problems 86, 87, 88, and 89

86. • Estimate the range of flight speeds for *Microraptor gui* if its power output is 9.8 W.

A. 0–7.7 m/s **B.** 7.7–15 m/s **C.** 15–30 m/s **D.** 0–15 m/s

87. • What approximate range of flight speeds would be possible if *Microraptor gui* could produce 20 W of power?

A. 0–25 m/s **B.** 25–30 m/s **C.** 2.5–25 m/s **D.** 0–2.5 m/s

88. •• How much energy would *Microraptor* have to expend to fly with a speed of 10 m/s for 1.0 minute?

A. 8.1 J **B.** 81 J **C.** 490 J **D.** 600 J

89. • Estimate the minimum force that *Microraptor* must exert to fly.

A. 0.65 N **B.** 1.3 N **C.** 1.0 N **D.** 10 N

INTERACTIVE PROBLEMS

90. •• **Referring to Figure 7–12** Suppose the block has a mass of 1.4 kg and an initial speed of 0.62 m/s. **(a)** What force constant must the spring have if the maximum compression is to be 2.4 cm? **(b)** If the spring has the force constant found in part (a), find the maximum compression if the mass of the block is doubled *and* its initial speed is halved.

91. •• **IP Referring to Figure 7–12** In the situation shown in Figure 7–12 (d), a spring with a force constant of 750 N/m is compressed by 4.1 cm. **(a)** If the speed of the block in Figure 7–12 (f) is 0.88 m/s, what is its mass? **(b)** If the mass of the block is doubled, is the final speed greater than, less than, or equal to 0.44 m/s? **(c)** Find the final speed for the case described in part (b).

92. •• **IP Referring to Example 7–8** Suppose the car has a mass of 1400 kg and delivers 48 hp to the wheels. **(a)** How long does it take for the car to increase its speed from 15 m/s to 25 m/s? **(b)** Would the time required to increase the speed from 5.0 m/s to 15 m/s be greater than, less than, or equal to the time found in part (a)? **(c)** Determine the time required to accelerate from 5.0 m/s to 15 m/s.

8 Potential Energy and Conservation of Energy

Probably everyone has seen a high jumper spring into the air, slow, hang motionless in midair for an instant, and then start to descend, picking up speed on the way. At the top of the trajectory, where has the kinetic energy gone? And how does it reappear as the jumper descends? As we answer these questions, we will find that there are other kinds of energy besides those considered in the last chapter.

One of the greatest accomplishments of physics is the concept of energy and its conservation. To realize, for example, that there is an important physical quantity that we can neither see nor touch is an impressive leap of the imagination. Even more astonishing, however, is the discovery that energy comes in a multitude of forms, and that the sum total of all these forms of energy is a constant. The universe, in short, has a certain amount of energy, and that energy simply ebbs and flows from one form to another, with the total amount remaining fixed.

In this chapter we focus on the conservation of energy, the first "conservation law" to be studied in this text. Though only a handful of conservation laws are known, they are all of central importance in physics. Not only do they give deep insight into the workings of nature, they are also practical tools in problem solving. As we shall see in this chapter, many problems that would be difficult to solve using Newton's laws can be solved with ease using the principle of energy conservation.

8–1 Conservative and Nonconservative Forces

In physics, we classify forces according to whether they are *conservative* or *nonconservative*. The key distinction is that when a **conservative force** acts, the work it does is stored in the form of energy that can be released at a later time. In this section, we sharpen this distinction and explore some examples of conservative and nonconservative forces.

Perhaps the simplest case of a conservative force is gravity. Imagine lifting a box of mass m from the floor to a height h, as in **Figure 8–1**. To lift the box with constant speed, the force you must exert against gravity is mg. Since the upward distance is h, the work you do on the box is $W = mgh$. If you now release the box and allow it to drop back to the floor, gravity does the same work, $W = mgh$, and in the process gives the box an equivalent amount of kinetic energy.

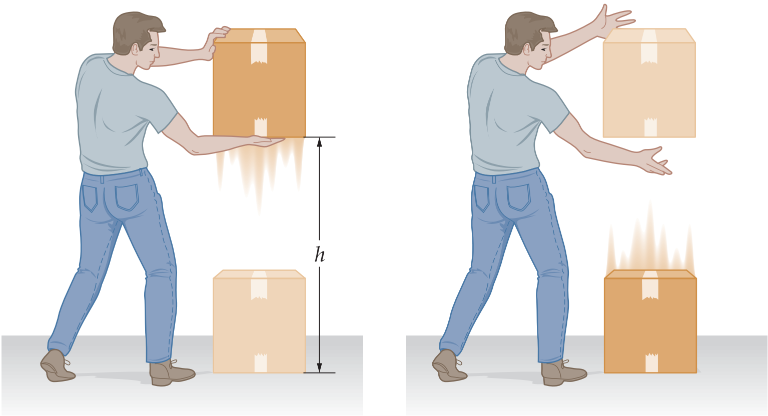


FIGURE 8–1 Work against gravity
Lifting a box against gravity with constant speed takes a work mgh. When the box is released, gravity does the same work on the box as it falls. Gravity is a conservative force.

Contrast this with the force of kinetic friction, which is nonconservative. To slide a box of mass m across the floor with constant speed, as shown in **Figure 8–2**, you must exert a force of magnitude $\mu_k N = \mu_k mg$. After sliding the box a distance d, the work you have done is $W = \mu_k mgd$. In this case, when you release the box it simply stays put—friction does no work on it after you let go. Thus, the work done by a **nonconservative force** cannot be recovered later as kinetic energy; instead, it is converted to other forms of energy, such as a slight warming of the floor and box in our example.

The differences between conservative and nonconservative forces are even more apparent if we consider moving an object around a closed path. Consider, for example, the path shown in **Figure 8–3**. If we move a box of mass m along this path, the total work done by gravity is the sum of the work done on each segment of the path; that is, $W_{\text{total}} = W_{\text{AB}} + W_{\text{BC}} + W_{\text{CD}} + W_{\text{DA}}$. The work done by gravity from A to B and from C to D is zero, since the force is at right angles to the displacement on these segments. Thus $W_{\text{AB}} = W_{\text{CD}} = 0$. On the segment from B to C, gravity does negative work (displacement and force are in opposite directions),

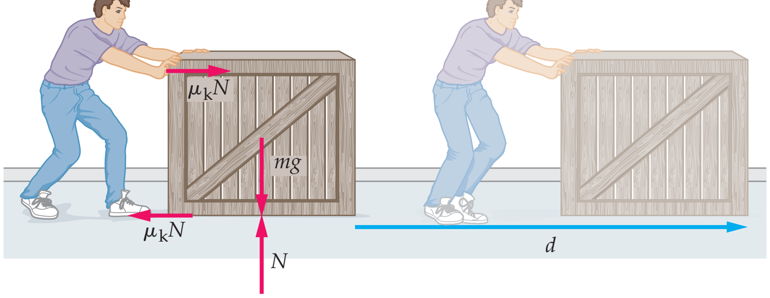


FIGURE 8–2 Work against friction
Pushing a box with constant speed against friction takes a work $\mu_k mgd$. When the box is released, it quickly comes to rest and friction does no further work. Friction is a nonconservative force.

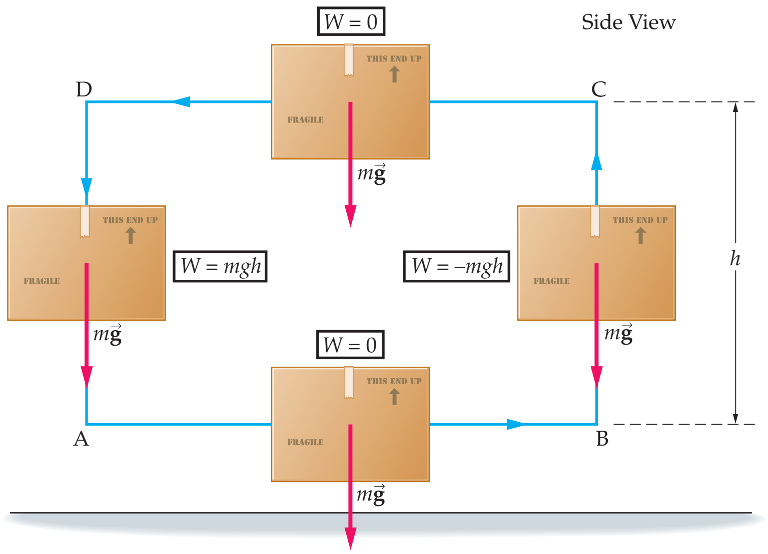


FIGURE 8–3 Work done by gravity on a closed path is zero
Gravity does no work on the two horizontal segments of the path. On the two vertical segments, the amounts of work done are equal in magnitude but opposite in sign. Therefore, the total work done by gravity on this—or any—closed path is zero.

but it does positive work from D to A (displacement and force are in the same direction). Hence, $W_{BC} = -mgh$ and $W_{DA} = mgh$. As a result, the total work done by gravity is zero:

$$W_{total} = 0 + (-mgh) + 0 + mgh = 0$$

With friction, the results are quite different. If we push the box around the closed horizontal path shown in **Figure 8–4**, the total work done by friction does not vanish. In fact, friction does the negative work $W = -f_k d = -\mu_k mgd$ on each segment. Therefore, the total work done by kinetic friction is

$$W_{total} = (-\mu_k mgd) + (-\mu_k mgd) + (-\mu_k mgd) + (-\mu_k mgd) = -4\,\mu_k mgd$$

These results lead to the following definition of a conservative force:

Conservative Force: Definition 1
A conservative force is a force that does zero total work on any closed path.

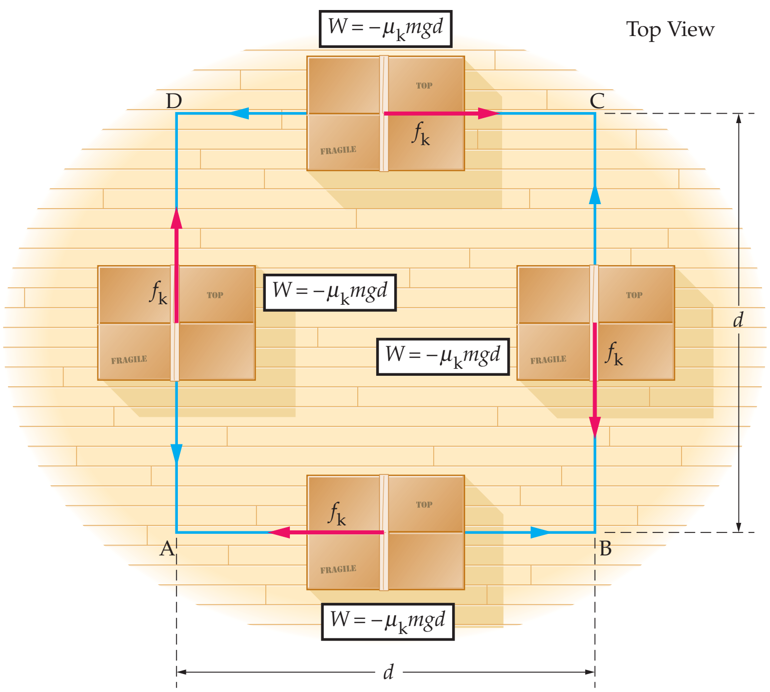
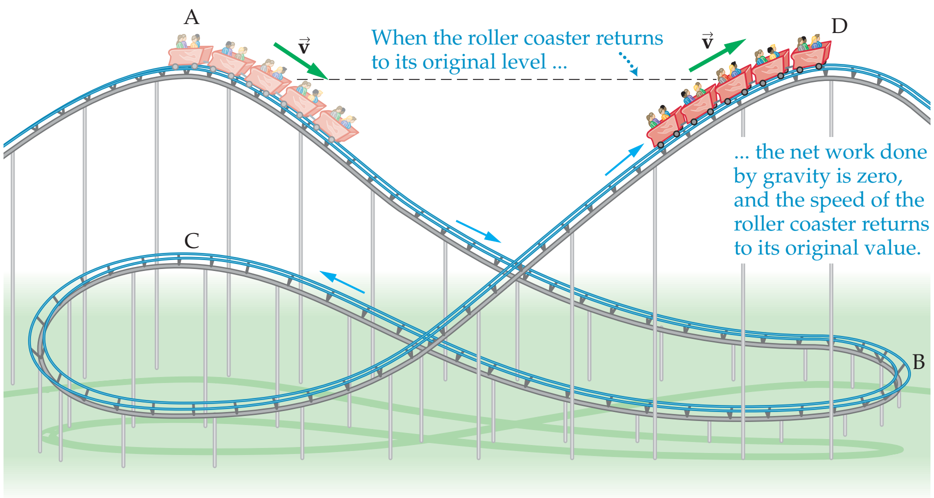


FIGURE 8–4 Work done by friction on a closed path is nonzero
The work done by friction when an object moves through a distance d is $-\mu_k mgd$. Thus, the total work done by friction on a closed path is nonzero. In this case, it is equal to $-4\,\mu_k mgd$.



▲ **FIGURE 8–5 Gravity is a conservative force**

If frictional forces can be ignored, a roller coaster car will have the same speed at points A and D, since they are at the same height. Hence, after any complete circuit of the track the speed of the car returns to its initial value. It follows that the change in kinetic energy is zero for a complete circuit, and, therefore, the work done by gravity is also zero.

A roller coaster provides a good illustration of this definition. If a car on a roller coaster has a speed v at point A in **Figure 8–5**, it speeds up as it drops to point B, slows down as it approaches point C, and so on. When the car returns to its original height, at point D, it will again have the speed v, as long as friction and other nonconservative forces can be neglected. Similarly, if the car completes a circuit of the track and returns to point A, it will again have the speed v. Hence, a car's kinetic energy is unchanged ($\Delta K = 0$) after *any* complete circuit of the track. From the work–energy theorem, $W_{total} = \Delta K$, it follows that the work done by gravity is zero for the closed path of the car, as expected for a conservative force.

This property of conservative forces has interesting consequences. For instance, consider the closed paths shown in **Figure 8–6**. On each of these paths, we know that the work done by a conservative force is zero. Thus, it follows from paths 1 and 2 that $W_{total} = W_1 + W_2 = 0$, or

$$W_2 = -W_1$$

Similarly, using paths 1 and 3 we have $W_{total} = W_1 + W_3 = 0$, or

$$W_3 = -W_1$$

As a result, we see that the work done on path 3 is the same as the work done on path 2:

$$W_3 = W_2$$

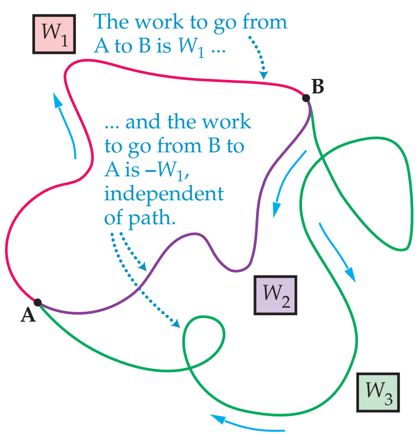
But paths 2 and 3 are arbitrary, as long as they start at point B and end at point A. This leads to an equivalent definition of a conservative force:

Conservative Force: Definition 2

If the work done by a force in going from an arbitrary point A to an arbitrary point B is *independent of the path* from A to B, the force is conservative.

This definition is given an explicit check in Example 8–1.

Table 8–1 summarizes the different kinds of conservative and nonconservative forces we have encountered thus far in this text.



▲ **FIGURE 8–6 The work done by a conservative force is independent of path**

Considering paths 1 and 2, we see that $W_1 + W_2 = 0$, or $W_2 = -W_1$. From paths 1 and 3, however, we see that $W_1 + W_3 = 0$, or $W_3 = -W_1$. It follows, then, that $W_3 = W_2$, since they are both equal to $-W_1$; hence the work done in going from A to B is independent of the path.

TABLE 8–1 Conservative and Nonconservative Forces

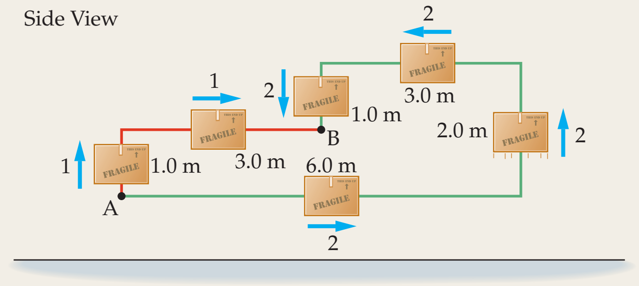
Force	Section
Conservative forces	
Gravity	5–6
Spring force	6–2
Nonconservative forces	
Friction	6–1
Tension in a rope, cable, etc.	6–2
Forces exerted by a motor	7–4
Forces exerted by muscles	5–3

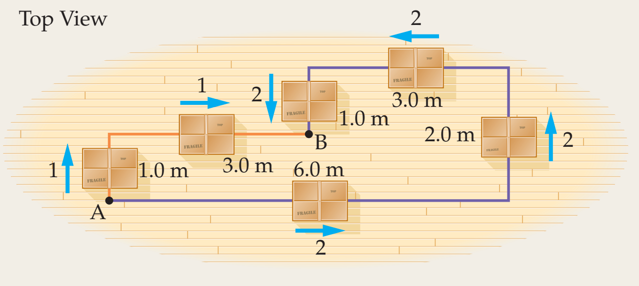
EXAMPLE 8–1 DIFFERENT PATHS, DIFFERENT FORCES

(a) A 4.57-kg box is moved with constant speed from A to B along the two paths shown at left below. Calculate the work done by gravity on each of these paths. **(b)** The same box is pushed across a floor from A to B along path 1 and path 2 at right below. If the coefficient of kinetic friction between the box and the surface is $\mu_k = 0.63$, how much work is done by friction along each path?

PICTURE THE PROBLEM

Part (a) of our sketch shows two different paths a box might be taken through in going from point A to point B. Path 1 is indicated by two red lines, indicating a vertical displacement of 1.0 m and a horizontal displacement of 3.0 m. Path 2, indicated in green, consists of two horizontal and two vertical displacements. In this case, we are interested in the work done by gravity. Part (b) shows the same basic paths—path 1 in orange and path 2 in purple—only this time on a rough floor. Here it is the force of kinetic friction that is of interest.





STRATEGY

To calculate the work for each path, we break it down into segments. Path 1 is made up of two segments, path 2 has four segments.

a. For gravity, the work is zero on horizontal segments. On vertical segments, the work done by gravity is positive when motion is downward and negative when motion is upward.

b. The work done by kinetic friction is negative on all segments of both paths.

SOLUTION

Part (a)

1. Using $W = Fd = mgy$, calculate the work done by gravity along the two segments of path 1:

$$W_1 = -(4.57\text{ kg})(9.81\text{ m/s}^2)(1.0\text{ m}) + 0 = -45\text{ J}$$

2. In the same way, calculate the work done by gravity along the four segments of path 2:

$$W_2 = 0 - (4.57\text{ kg})(9.81\text{ m/s}^2)(2.0\text{ m}) + 0 + (4.57\text{ kg})(9.81\text{ m/s}^2)(1.0\text{ m}) = -45\text{ J}$$

Part (b)

3. Using $F = \mu_k N$, calculate the work done by kinetic friction along the two segments of path 1:

$$W_1 = -(0.63)(4.57\text{ kg})(9.81\text{ m/s}^2)(1.0\text{ m}) - (0.63)(4.57\text{ kg})(9.81\text{ m/s}^2)(3.0\text{ m}) = -110\text{ J}$$

4. Similarly, calculate the work done by kinetic friction along the four segments of path 2:

$$W_2 = -(0.63)(4.57\text{ kg})(9.81\text{ m/s}^2)(6.0\text{ m}) - (0.63)(4.57\text{ kg})(9.81\text{ m/s}^2)(2.0\text{ m}) - (0.63)(4.57\text{ kg})(9.81\text{ m/s}^2)(3.0\text{ m}) - (0.63)(4.57\text{ kg})(9.81\text{ m/s}^2)(1.0\text{ m}) = -340\text{ J}$$

INSIGHT

As expected, the conservative force of gravity gives the same work in going from A to B, regardless of the path. The work done by kinetic friction, however, is greater on the path of greater length.

PRACTICE PROBLEM

The work done by gravity when the box is moved from point B to a point C is 140 J. Is point C above or below point B? What is the vertical distance between points B and C? [**Answer:** Point C is 3.1 m below point B.]

Some related homework problems: Problem 2, Problem 3

8–2 Potential Energy and the Work Done by Conservative Forces

Work must be done to lift a bowling ball from the floor to a shelf. Once on the shelf, the bowling ball has zero kinetic energy, just as it did on the floor. Even so, the work done in lifting the ball has not been lost. If the ball is allowed to fall from the shelf, gravity does the same amount of work on it as you did to lift it in the first place. As a result, the work you did is "recovered" in the form of kinetic energy. Thus we say that when the ball is lifted to a new position, there is an increase in **potential energy,** U, and that this potential energy can be converted to kinetic energy when the ball falls.

▲ Because gravity is a conservative force, the work done against gravity in lifting these logs (left) can, in principle, all be recovered. If the logs are released, for example, they will acquire an amount of kinetic energy exactly equal to the work done to lift them and to the gravitational potential energy that they gained in being lifted. Friction, by contrast, is a nonconservative force. Some of the work done by this spinning grindstone (right) goes into removing material from the object being ground, while the rest is transformed into sound energy and (especially) heat. Most of this work can never be recovered as kinetic energy.

In a sense, potential energy is a storage system for energy. When we increase the separation between the ball and the ground, the work we do is stored in the form of an increased potential energy. Not only that, but the storage system is perfect, in the sense that the energy is never lost, as long as the separation remains the same. The ball can rest on the shelf for a million years, and still, when it falls, it gains the same amount of kinetic energy.

Work done against friction, however, is not "stored" as potential energy. Instead, it is dissipated into other forms of energy such as heat or sound. The same is true of other nonconservative forces. Only conservative forces have the potential-energy storage system.

Before proceeding, we should point out an interesting difference between kinetic and potential energy. Kinetic energy is given by the expression $K = \frac{1}{2}mv^2$, no matter what force might be involved. On the other hand, each different conservative force has a different expression for its potential energy. To see how this comes about, we turn now to a precise definition of potential energy.

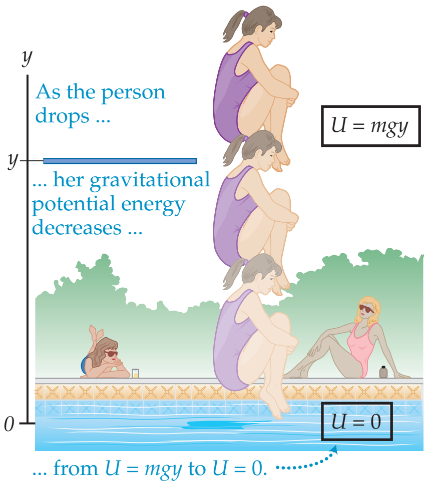
Potential Energy, *U*

When a conservative force does an amount of work W_c (the subscript c stands for conservative), the corresponding potential energy U is changed according to the following definition:

Definition of Potential Energy, *U*

$$W_c = U_i - U_f = -(U_f - U_i) = -\Delta U \qquad 8\text{–}1$$

SI unit: joule, J



▲ **FIGURE 8–7 Gravitational potential energy**

A person drops from a diving board into a swimming pool. The diving board is at the height y, and the surface of the water is at $y = 0$. We choose the gravitational potential energy to be zero at $y = 0$; hence, the potential energy is mgy at the diving board.

PROBLEM-SOLVING NOTE

Zero of Potential Energy

When working potential energy problems it is important to make a definite choice for the location where the potential energy is to be set equal to zero. Any location can be chosen, but once the choice is made, it must be used consistently.

▲ The $U = 0$ level for the gravitational potential energy of this system can be assigned to the point where the diver starts his dive, to the water level, or to any other level. Regardless of the choice, however, his kinetic energy when he strikes the water will be exactly equal to the difference in gravitational potential energy between his launch and splashdown points.

In words, the work done by a conservative force is equal to the negative of the change in potential energy. For example, when an object falls, gravity does *positive* work on it and its potential energy *decreases*. Similarly, when an object is lifted, gravity does *negative* work and the potential energy *increases*.

Note that since work is a scalar with units of joules, the same is true of potential energy. In addition, our definition determines only the *difference* in potential energy between two points, not the actual value of the potential energy. Hence, we are free to choose the place where the potential energy is zero ($U = 0$) in much the same way we are free to choose the location of the origin in a coordinate system.

Gravity

Let's apply our definition of potential energy to the force of gravity near the Earth's surface. Suppose a person of mass m drops a distance y from a diving board into a pool, as shown in **Figure 8–7**. As the person drops, gravity does the work

$$W_c = mgy$$

Applying the definition given in Equation 8–1, the corresponding change in potential energy is

$$-\Delta U = U_i - U_f = W_c = mgy$$

In this expression, U_i is the potential energy when the diver is on the board, and U_f is the potential energy when the diver enters the water. Rearranging slightly, we have

$$U_i = mgy + U_f \qquad \text{8–2}$$

Note that U_i is greater than U_f.

As mentioned above, we are free to choose $U = 0$ anywhere we like; *only the difference* in U is important. For example, if you slip and fall to the ground, you hit with the same thud whether you fall in Denver (altitude 1 mile) or in Honolulu (at sea level). It's the difference in height that matters, not the height itself. (The acceleration of gravity does vary slightly with altitude, as we shall see, but the difference is small enough to be unimportant in this case.) The only point to be careful about when choosing a location for $U = 0$ is to be consistent with the choice once it is made.

In general, we choose $U = 0$ in a convenient location. In Figure 8–7, a reasonable place for $U = 0$ is the surface of the water, where $y = 0$; that is, $U_f = 0$. Then, Equation 8–2 becomes $U_i = mgy$. If we omit the subscript on U_i, letting U stand for the potential energy at the arbitrary height y, we have

Gravitational Potential Energy (Near Earth's Surface)

$$U = mgy \qquad \text{8–3}$$

Note that the gravitational potential energy depends only on the height, y, and is independent of horizontal position.

EXERCISE 8–1

Find the gravitational potential energy of a system consisting of a 65-kg person on a 3.0-m-high diving board. Let $U = 0$ be at water level.

SOLUTION

Substituting $m = 65$ kg and $y = 3.0$ m in Equation 8–3 yields

$$U = mgy = (65\text{ kg})(9.81\text{ m/s}^2)(3.0\text{ m}) = 1900\text{ J}$$

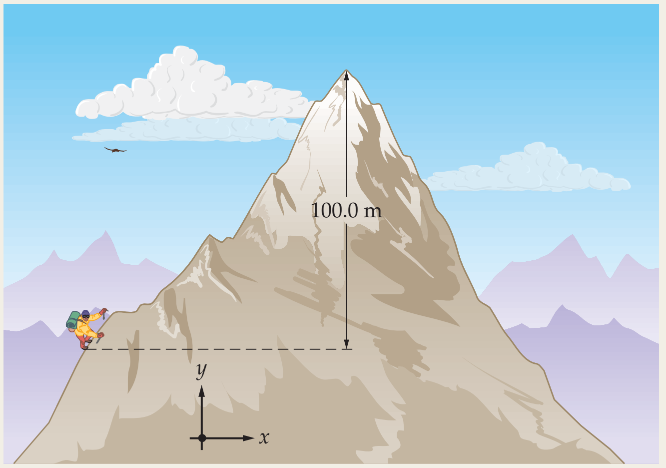
The next Example considers the change in gravitational potential energy of a mountain climber, given different choices for the location of $U = 0$.

EXAMPLE 8–2 PIKES PEAK OR BUST

An 82.0-kg mountain climber is in the final stage of the ascent of 4301-m-high Pikes Peak. What is the change in gravitational potential energy as the climber gains the last 100.0 m of altitude? Let $U = 0$ be **(a)** at sea level or **(b)** at the top of the peak.

PICTURE THE PROBLEM

Our sketch shows the mountain climber and the last 100.0 m of altitude to be climbed. We choose a typical coordinate system, with the positive y axis upward and the positive x axis to the right.



STRATEGY

The gravitational potential energy of the Earth–climber system depends only on the height y; the path followed in gaining the last 100.0 m of altitude is unimportant. The change in potential energy is $\Delta U = U_f - U_i = mgy_f - mgy_i$, where y_f is the altitude of the peak and y_i is 100.0 m less than y_f.

SOLUTION

Part (a)

1. Calculate ΔU with $y_f = 4301$ m and $y_i = 4201$ m:

$$\begin{aligned}\Delta U &= mgy_f - mgy_i \\ &= (82.0\text{ kg})(9.81\text{ m/s}^2)(4301\text{ m}) \\ &\quad - (82.0\text{ kg})(9.81\text{ m/s}^2)(4201\text{ m}) = 80{,}400\text{ J}\end{aligned}$$

Part (b)

2. Calculate ΔU with $y_f = 0$ and $y_i = -100.0$ m:

$$\begin{aligned}\Delta U &= mgy_f - mgy_i \\ &= (82.0\text{ kg})(9.81\text{ m/s}^2)(0) \\ &\quad - (82.0\text{ kg})(9.81\text{ m/s}^2)(-100.0\text{ m}) = 80{,}400\text{ J}\end{aligned}$$

INSIGHT

As expected, the *change* in gravitational potential energy does not depend on where we choose $U = 0$. Nor does it depend on the path taken between the initial and final points.

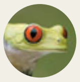
PRACTICE PROBLEM

Find the altitude of the climber for which the gravitational potential energy of the Earth–climber system is 1.00×10^5 J less than it is when the climber is at the summit. [**Answer:** 4180 m]

Some related homework problems: Problem 10, Problem 17

A single item of food can be converted into a surprisingly large amount of potential energy. This is shown for the case of a candy bar in Example 8–3.

EXAMPLE 8–3 CONVERTING FOOD ENERGY TO MECHANICAL ENERGY



REAL-WORLD PHYSICS: BIO

A candy bar called the Mountain Bar has a calorie content of 212 Cal = 212 kcal, which is equivalent to an energy of 8.87×10^5 J. If an 81.0-kg mountain climber eats a Mountain Bar and magically converts it all to potential energy, what gain of altitude would be possible?

PICTURE THE PROBLEM

We show the mountain climber eating the candy bar at a given level on the mountain, which we can take to be $y = 0$. The altitude gain, then, corresponds to $y = h$.

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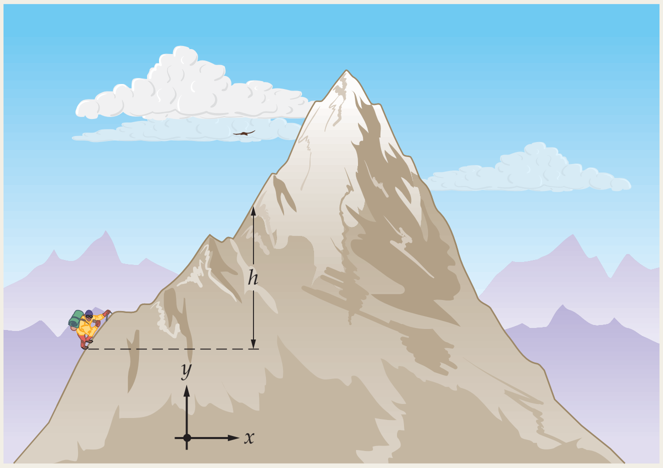
STRATEGY

The initial gravitational potential energy of the Earth–climber system is $U = 0$; the final potential energy is $U = mgh$. To find the altitude gain, set $U = mgh$ equal to the energy provided by the candy bar, 8.87×10^5 J, and solve for h.



SOLUTION

1. Solve $U = mgh$ for h:

$$U = mgh$$
$$h = \frac{U}{mg}$$

2. Substitute numerical values, with $U = 8.87 \times 10^5$ J:

$$h = \frac{U}{mg} = \frac{8.87 \times 10^5\ \text{J}}{(81.0\ \text{kg})(9.81\ \text{m/s}^2)} = 1120\ \text{m}$$

INSIGHT

This is more than two-thirds of a mile in elevation. Even if we take into account the fact that metabolic efficiency is only about 25%, the height would still be 280 m, or nearly two-tenths of a mile. It's remarkable just how much our bodies can do with so little.

PRACTICE PROBLEM

If the mass of the mountain climber is increased—by adding more items to the backpack, for example—does the possible elevation gain increase, decrease, or stay the same? Calculate the elevation gain for a climber with a mass of 91.0 kg. [**Answer:** The altitude gain will decrease. For $m = 91.0$ kg we find $h = 994$ m.]

Some related homework problems: Problem 10, Problem 17

▲ Because springs, and bungee cords, exert conservative forces, they can serve as energy storage devices. In this case, the stretched bungee cord is beginning to give up the energy it has stored, and to convert that potential energy into kinetic energy as the jumper is pulled rapidly skyward.

We have been careful *not* to say that the potential energy of the mountain climber—or any object—increases when its height increases. The reason is that the potential energy is a property of an entire system, not of its individual parts. The correct statement is that if an object is lifted, the potential energy of the Earth–object system is increased.

Springs

Consider a spring that is stretched from its equilibrium position a distance x. According to Equation 7–8, the work required to cause this stretch is $W = \frac{1}{2}kx^2$. Therefore, if the spring is released—and allowed to move from the stretched position back to the equilibrium position—it will do the same work, $\frac{1}{2}kx^2$. From our definition of potential energy, then, we see that

$$W_c = \tfrac{1}{2}kx^2 = U_i - U_f \qquad 8\text{–}4$$

Note that in this case U_f is the potential energy when the spring is at $x = 0$ (equilibrium position), and U_i is the potential energy when the spring is stretched by the amount x.

A convenient choice for $U = 0$ is the equilibrium position of the spring. With this choice we have $U_f = 0$, and Equation 8–4 becomes $U_i = \frac{1}{2}kx^2$. Omitting the subscript i, so that U represents the potential energy of the spring for an arbitrary amount of stretch x, we have

Potential Energy of a Spring

$$U = \tfrac{1}{2}kx^2 \qquad 8\text{–}5$$

Since U depends on x^2, which is positive even if x is negative, the potential energy of a spring is always greater than or equal to zero. Thus, a spring's potential energy increases whenever it is displaced from equilibrium.

EXERCISE 8–2

Find the potential energy of a spring with force constant $k = 680\ \text{N/m}$ if it is **(a)** stretched by 5.00 cm or **(b)** compressed by 7.00 cm.

SOLUTION

Substituting $x = 0.0500\ \text{m}$ and $x = -0.0700\ \text{m}$ in Equation 8–5 yields

a. $U = \frac{1}{2}(680\ \text{N/m})(0.0500\ \text{m})^2 = 0.850\ \text{J}$

b. $U = \frac{1}{2}(680\ \text{N/m})(-0.0700\ \text{m})^2 = 1.67\ \text{J}$

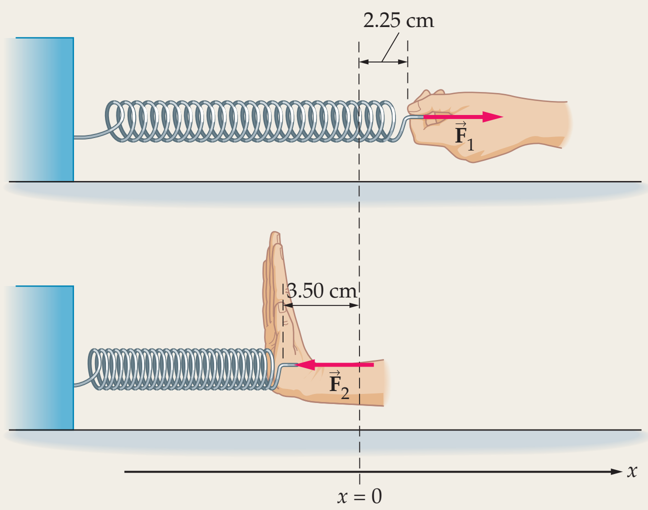
Finally, comparing Equation 8–3 to Equation 8–5, we see that the potential energies for gravity and for a spring are given by different expressions. As mentioned, each conservative force has its own potential energy.

EXAMPLE 8–4 COMPRESSED ENERGY AND THE JUMP OF A FLEA

When a force of 120.0 N is applied to a certain spring, it causes a stretch of 2.25 cm. What is the potential energy of this spring when it is **(a)** compressed by 3.50 cm or **(b)** expanded by 7.00 cm?

PICTURE THE PROBLEM

The top sketch shows the spring stretched 2.25 cm by the force $F_1 = 120.0\ \text{N}$. The lower sketch shows the same spring compressed by a second force, F_2, which causes a compression of 3.50 cm. An expansion of the spring by 7.00 cm would look similar to the top sketch.



STRATEGY

From the first piece of information—a certain force causes a certain stretch—we can calculate the force constant using $F = kx$. Once we know k, we find the potential energy for either a compression or an expansion with $U = \frac{1}{2}kx^2$.

SOLUTION

1. Solve $F = kx$ for the spring constant, k:

$$F = kx$$

$$k = \frac{F}{x} = \frac{120.0\ \text{N}}{0.0225\ \text{m}} = 5330\ \text{N/m}$$

Part (a)

2. Substitute $k = 5330\ \text{N/m}$ and $x = -0.0350\ \text{m}$ into the potential energy expression, $U = \frac{1}{2}kx^2$:

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(5330\ \text{N/m})(-0.0350\ \text{m})^2 = 3.26\ \text{J}$$

Part (b)

3. Substitute $k = 5330\ \text{N/m}$ and $x = 0.0700\ \text{m}$ into the potential energy expression, $U = \frac{1}{2}kx^2$:

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(5330\ \text{N/m})(0.0700\ \text{m})^2 = 13.1\ \text{J}$$

INSIGHT

Though this Example deals with ideal springs, the same basic physics applies to many other real-world situations. A case in point is the jump of a flea, in which a flea can propel itself up to 100 times its body length. The physics behind this feat is the slow

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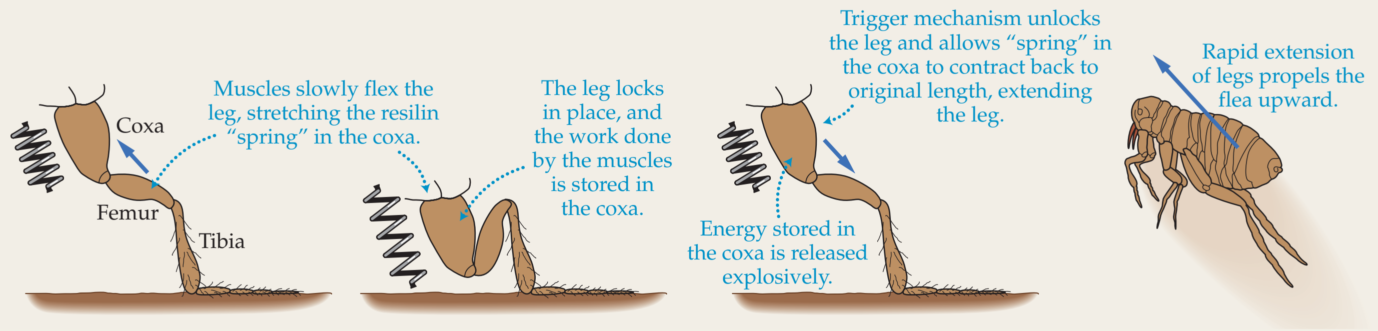
accumulation of energy in a "springy" strip of resilin in the coxa of the leg, as shown in the accompanying sketches, and the sudden release of this energy at a later time. Specifically, as the flea's muscles flex the leg, the resilin strip in the coxa is stretched, storing the work done by the muscles in the form of potential energy, $U = \frac{1}{2}kx^2$. Later, when a trigger mechanism unlocks the flexed leg, the energy stored in the resilin is released explosively—rapidly extending the leg and propelling the flea upward. See Problem 89 for a calculation using the force constant of resilin.



PRACTICE PROBLEM

What stretch is necessary for the spring in this Example to have a potential energy of 5.00 J? [**Answer:** 4.33 cm]

Some related homework problems: Problem 12, Problem 16

The jump of a flea is similar in many respects to the operation of a bow and arrow. In the latter case, the work done in slowly pulling the string back is stored in the flex of the bow. The string is held in place while aim is taken, and then released to allow the bow to return to its original shape. This propels the arrow forward with great speed—a speed many times faster than could be obtained by simply throwing the arrow with the same arm muscles that pulled back on the string. In fact, if the string returns to its original position in 1/1000th the time it took to pull the string back, the power it delivers to the arrow is magnified by a factor of 1000. Similarly, the spring-loaded jump of the flea gives it a much greater takeoff speed than if it relied solely on muscle power. An analogous process occurs in the flash unit of a camera, as we shall see in Chapter 20.

8–3 Conservation of Mechanical Energy

In this section, we show how potential energy can be used as a powerful tool in solving a variety of problems and in gaining greater insight into the workings of physical systems. To do so, we begin by defining the **mechanical energy,** E, as the sum of the potential and kinetic energies of an object:

$$E = U + K \qquad 8\text{–}6$$

The significance of mechanical energy is that it is **conserved** in systems involving only conservative forces. By conserved, we mean that its value never changes; that is, $E =$ constant. (In situations where nonconservative forces are involved, the mechanical energy can change, as when friction causes warming by converting mechanical energy to thermal energy. When *all* possible forms of energy are considered, energy is always found to be conserved.)

To show that E is conserved for conservative forces, we start with the work–energy theorem from Chapter 7:

$$W_{\text{total}} = \Delta K = K_f - K_i$$

Suppose for a moment that the system has only a single force and that the force is conservative. If this is the case, then the total work, W_{total}, is the work done by the conservative force, W_c:

$$W_{\text{total}} = W_c$$

From the definition of potential energy, we know that $W_c = -\Delta U = U_i - U_f$. Combining these results, we have

$$W_{total} = W_c$$

$$K_f - K_i = U_i - U_f$$

With a slight rearrangement we find

$$U_f + K_f = U_i + K_i$$

or

$$E_f = E_i$$

Since the initial and final points can be chosen arbitrarily, it follows that E is conserved:

$$E = \text{constant}$$

If the system has more than one conservative force, the only change to these results is to replace U with the sum of potential energies of all the forces.

To summarize:

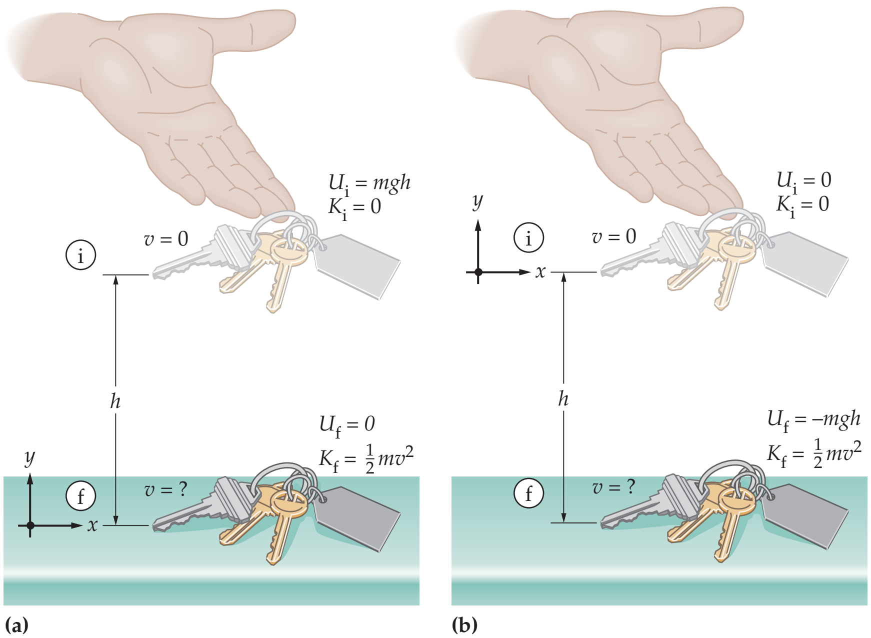
Conservation of Mechanical Energy
In systems with conservative forces only, the mechanical energy E is conserved; that is, $E = U + K = \text{constant}$.

In terms of physical systems, conservation of mechanical energy means that energy can be converted between potential and kinetic forms, but that the sum remains the same. As an example, in the roller coaster shown in Figure 8–5, the gravitational potential energy decreases as the car approaches point B; as it does, the car's kinetic energy increases by the same amount. From a practical point of view, conservation of mechanical energy means that many physics problems can be solved by what amounts to simple bookkeeping.

For example, consider a key chain of mass m that is dropped to the floor from a height h, as illustrated in **Figure 8–8**. The question is, how fast are the keys moving just before they land? We know how to solve this problem using Newton's laws and kinematics, but now let's see how energy conservation can be used instead.

First, note that the only force acting on the keys is gravity—ignoring air resistance, of course—and that gravity is a conservative force. As a result, we can say

▲ A roller coaster (top) illustrates the conservation of mechanical energy. With every descent, gravitational potential energy is converted into kinetic energy; with every rise, kinetic energy is converted back into gravitational potential energy. If friction is neglected, the total mechanical energy of the car remains constant. The same principle is exploited at a pumped-storage facility, such as this one at the Mormon Flat Dam in Phoenix, Arizona (bottom). When surplus electrical power is available, it is used to pump water uphill into the reservoir. This process, in effect, stores electrical energy as gravitational potential energy. When power demand is high, the stored water is allowed to flow back downhill through the electrical generators in the dam, converting the gravitational energy to kinetic energy and the kinetic energy to electrical energy.



◀ **FIGURE 8–8 Solving a kinematics problem using conservation of energy**
(a) A set of keys falls to the floor. Ignoring frictional forces, we know that the mechanical energy at points i and f must be equal; $E_i = E_f$. Using this condition, we can find the speed of the keys just before they land. **(b)** The same physical situation as in part (a), except this time we have chosen $y = 0$ to be at the point where the keys are dropped. As before, we set $E_i = E_f$ to find the speed of the keys just before they land. The result is the same.

PROBLEM-SOLVING NOTE

Conservative Systems

A convenient approach to problems involving energy conservation is to first sketch the system, and then label the initial and final points with i and f, respectively. To apply energy conservation, write out the energy at these two points and set $E_i = E_f$.

that $E = U + K$ is constant during the entire time the keys are falling. To solve the problem, then, we pick two points on the motion of the keys, say i and f in Figure 8–8, and we set the mechanical energy equal at these points:

$$E_i = E_f \tag{8–7}$$

Writing this out in terms of potential and kinetic energies, we have

$$U_i + K_i = U_f + K_f \tag{8–8}$$

This one equation—which is nothing but bookkeeping—can be used to solve for the one unknown, the final speed.

To be specific, in Figure 8–8 (a) we choose $y = 0$ at ground level, which means that $U_i = mgh$. In addition, the fact that the keys are released from rest means that $K_i = 0$. Similarly, at point f—just before hitting the ground—the energy is all kinetic, and the potential energy is zero; that is, $U_f = 0$, $K_f = \frac{1}{2}mv^2$. Substituting these values into Equation 8–8, we find

$$mgh + 0 = 0 + \tfrac{1}{2}mv^2$$

Canceling m and solving for v yields the same result we get with kinematics:

$$v = \sqrt{2gh}$$

Suppose, instead, that we had chosen $y = 0$ to be at the release point of the keys, as in Figure 8–8 (b), so that the keys land at $y = -h$. Now, when the keys are released, we have $U_i = 0$ and $K_i = 0$, and when they land $U_f = -mgh$ and $K_f = \frac{1}{2}mv^2$. Substituting these results in $U_i + K_i = U_f + K_f$ yields

$$0 + 0 = -mgh + \tfrac{1}{2}mv^2$$

Solving for v gives the same result:

$$v = \sqrt{2gh}$$

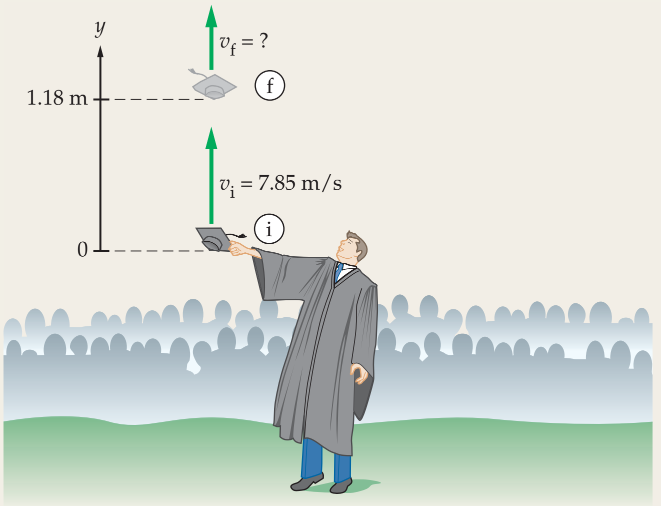
Thus, as expected, changing the zero level has no effect on the physical results.

EXAMPLE 8–5 GRADUATION FLING

At the end of a graduation ceremony, graduates fling their caps into the air. Suppose a 0.120-kg cap is thrown straight upward with an initial speed of 7.85 m/s, and that frictional forces can be ignored. **(a)** Use kinematics to find the speed of the cap when it is 1.18 m above the release point. **(b)** Show that the mechanical energy at the release point is the same as the mechanical energy 1.18 m above the release point.

PICTURE THE PROBLEM

In our sketch we choose $y = 0$ to be at the level where the cap is released with an initial speed of 7.85 m/s. In addition, note that we designate the release point as i (initial) and the point at which $y = 1.18$ m as f (final). It is the speed at point f that we wish to find.



STRATEGY

a. The cap is in free fall, which justifies the use of constant-acceleration kinematics. Since we want to relate velocity to position, we use $v_y{}^2 = v_{0y}{}^2 + 2a_y\Delta y$ (Section 2–5). In this case, $v_{0y} = 7.85$ m/s, $\Delta y = 1.18$ m, and $a_y = -g$. Substituting these values gives v_y.

b. At each point we simply calculate $E = U + K$, with $U = mgy$ and $K = \frac{1}{2}mv^2$.

SOLUTION

Part (a)

1. Use kinematics to solve for v_y:

$$v_y^2 = v_{0y}^2 + 2a_y\,\Delta y$$
$$v_y = \pm\sqrt{v_{0y}^2 + 2a_y\,\Delta y}$$

2. Substitute $v_{0y} = 7.85\ \mathrm{m/s}$, $\Delta y = 1.18\ \mathrm{m}$, and $a_y = -g$ to find v_y. Choose the plus sign, since we are interested only in the speed:

$$v_y = \sqrt{v_{0y}^2 + 2a_y\,\Delta y}$$
$$= \sqrt{(7.85\ \mathrm{m/s})^2 + 2(-9.81\ \mathrm{m/s^2})(1.18\ \mathrm{m})} = 6.20\ \mathrm{m/s}$$

Part (b)

3. Calculate E_i. At this point $y_\mathrm{i} = 0$ and $v_\mathrm{i} = 7.85\ \mathrm{m/s}$:

$$E_\mathrm{i} = U_\mathrm{i} + K_\mathrm{i} = mgy_\mathrm{i} + \tfrac{1}{2}mv_\mathrm{i}^2$$
$$= 0 + \tfrac{1}{2}(0.120\ \mathrm{kg})(7.85\ \mathrm{m/s})^2 = 3.70\ \mathrm{J}$$

4. Calculate E_f. At this point $y_\mathrm{f} = 1.18\ \mathrm{m}$ and $v_\mathrm{f} = 6.20\ \mathrm{m/s}$:

$$E_\mathrm{f} = U_\mathrm{f} + K_\mathrm{f} = mgy_\mathrm{f} + \tfrac{1}{2}mv_\mathrm{f}^2$$
$$= (0.120\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(1.18\ \mathrm{m}) + \tfrac{1}{2}(0.120\ \mathrm{kg})(6.20\ \mathrm{m/s})^2$$
$$= 1.39\ \mathrm{J} + 2.31\ \mathrm{J} = 3.70\ \mathrm{J}$$

INSIGHT

As expected, E_f is equal to E_i. In the remaining Examples in this section we turn this process around; we start with $E_\mathrm{f} = E_\mathrm{i}$, and use this relation to find a final speed or a final height. As we shall see, this procedure of using energy conservation is a more powerful approach—it actually makes the calculations simpler.

PRACTICE PROBLEM

Use energy conservation to find the height at which the speed of the cap is 5.00 m/s. [**Answer:** 1.87 m]

Some related homework problems: Problem 30, Problem 31, and Problem 33

An interesting extension of this Example is shown in **Figure 8–9**. In this case, we are given that the speed of the cap is v_i at the height y_i, and we would like to know its speed v_f when it is at the height y_f.

To find v_f, we apply energy conservation to the points i and f:

$$U_\mathrm{i} + K_\mathrm{i} = U_\mathrm{f} + K_\mathrm{f}$$

Writing out U and K specifically for these two points yields the following:

$$mgy_\mathrm{i} + \tfrac{1}{2}mv_\mathrm{i}^2 = mgy_\mathrm{f} + \tfrac{1}{2}mv_\mathrm{f}^2$$

As before, we cancel m and solve for the unknown speed, v_f:

$$v_\mathrm{f} = \sqrt{v_\mathrm{i}^2 + 2g(y_\mathrm{i} - y_\mathrm{f})}$$

This result is in agreement with the kinematic equation, $v_y^2 = v_{0y}^2 + 2a_y\,\Delta y$.

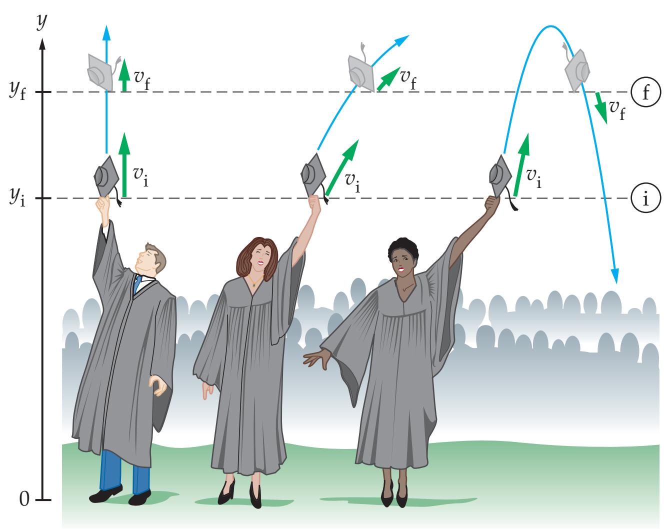


FIGURE 8–9 Speed is independent of path

If the speed of the cap is v_i at the height y_i, its speed is v_f at the height y_f, independent of the path between the two heights. This assumes, of course, that frictional forces can be neglected.

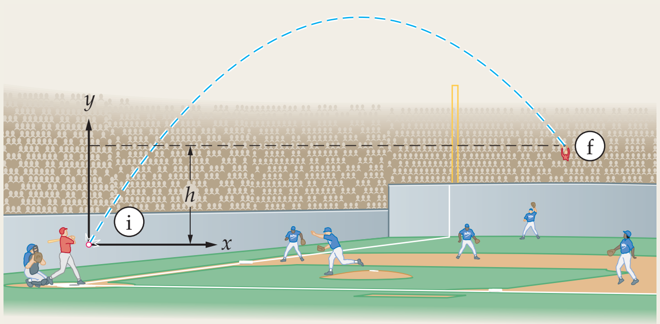
Note that v_f depends only on y_i and y_f, not on the path connecting them. This is because conservative forces such as gravity do work that is path-independent. What this means physically is that the cap has the same speed v_f at the height y_f, whether it goes straight upward or follows some other trajectory, as in Figure 8–9. All that matters is the height difference.

EXAMPLE 8–6 CATCHING A HOME RUN

In the bottom of the ninth inning, a player hits a 0.15-kg baseball over the outfield fence. The ball leaves the bat with a speed of 36 m/s, and a fan in the bleachers catches it 7.2 m above the point where it was hit. Assuming frictional forces can be ignored, find **(a)** the kinetic energy of the ball when it is caught and **(b)** its speed when caught.

PICTURE THE PROBLEM

Our sketch shows the ball's trajectory. We label the hit point i and the catch point f. At point i we choose $y_i = 0$; at point f, then, $y_f = h = 7.2$ m. In addition, we are given that $v_i = 36$ m/s; v_f is to be determined.



STRATEGY

a. Because frictional forces can be ignored, it follows that the initial mechanical energy is equal to the final mechanical energy; that is, $U_i + K_i = U_f + K_f$. Use this relation to find K_f.

b. Once K_f is determined, use $K_f = \frac{1}{2}mv_f^2$ to find v_f.

SOLUTION

Part (a)

1. Begin by writing U and K for point i:

$$U_i = 0$$
$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(0.15\text{ kg})(36\text{ m/s})^2 = 97\text{ J}$$

2. Next, write U and K for point f:

$$U_f = mgh = (0.15\text{ kg})(9.81\text{ m/s}^2)(7.2\text{ m}) = 11\text{ J}$$
$$K_f = \tfrac{1}{2}mv_f^2$$

3. Set the total mechanical energy at point i, $E_i = U_i + K_i$, equal to the total mechanical energy at point f, $E_f = U_f + K_f$, and solve for K_f:

$$U_i + K_i = U_f + K_f$$
$$0 + 97\text{ J} = 11\text{ J} + K_f$$
$$K_f = 97\text{ J} - 11\text{ J} = 86\text{ J}$$

Part (b)

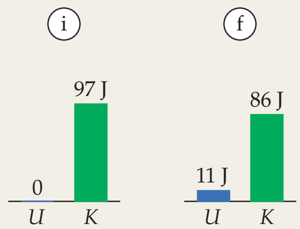
4. Use $K_f = \frac{1}{2}mv_f^2$ to find v_f:

$$K_f = \tfrac{1}{2}mv_f^2$$
$$v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(86\text{ J})}{0.15\text{ kg}}} = 34\text{ m/s}$$

INSIGHT

To find the ball's speed when it was caught, we need to know the height of point f, but we don't need to know any details about the ball's trajectory. For example, it is not necessary to know the angle at which the ball leaves the bat or its maximum height.

The histograms to the right show the values of U and K at the points i and f. Notice that the energy of the system is mostly kinetic at the time the ball is caught.



PRACTICE PROBLEM

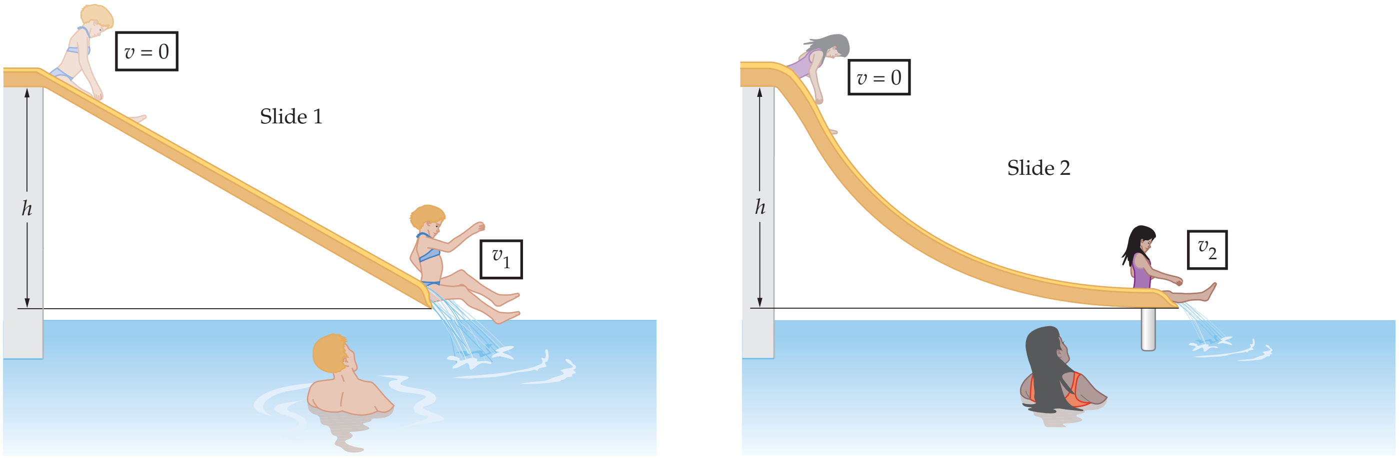
If the mass of the ball were increased, would the catch speed be greater than, less than, or the same as the value we just found? [**Answer:** The same. U and K depend on mass in the same way, hence the mass cancels.]

Some related homework problems: Problem 29, Problem 30

The connection between height difference and speeds is explored further in the following Conceptual Checkpoint and Example.

CONCEPTUAL CHECKPOINT 8–1 COMPARE THE FINAL SPEEDS

Swimmers at a water park can enter a pool using one of two frictionless slides of equal height. Slide 1 approaches the water with a uniform slope; slide 2 dips rapidly at first, then levels out. Is the speed v_2 at the bottom of slide 2 **(a)** greater than, **(b)** less than, or **(c)** the same as the speed v_1 at the bottom of slide 1?



REASONING AND DISCUSSION

In both cases, the same amount of potential energy, mgh, is converted to kinetic energy. Since the conversion of gravitational potential energy to kinetic energy is the *only* energy transaction taking place, it follows that the speed is the same for each slide.

Interestingly, although the final speeds are the same, the time required to reach the water is less for slide 2. The reason is that swimmer 2 reaches a high speed early and maintains it, whereas the speed of swimmer 1 increases slowly and steadily.

ANSWER

(c) The speeds are the same.

EXAMPLE 8–7 SKATEBOARD EXIT RAMP

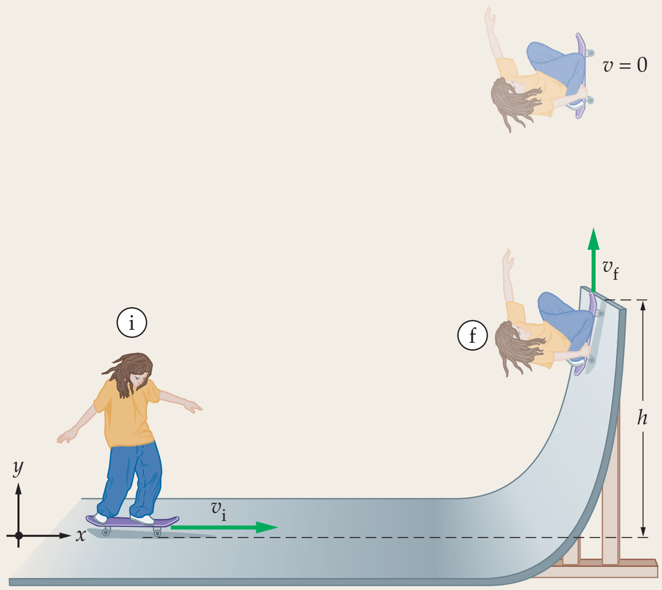
A 55-kg skateboarder enters a ramp moving horizontally with a speed of 6.5 m/s and leaves the ramp moving vertically with a speed of 4.1 m/s. Find the height of the ramp, assuming no energy loss to frictional forces.

PICTURE THE PROBLEM

We choose $y = 0$ to be the level of the bottom of the ramp, thus the gravitational potential energy is zero there. Point i indicates the skateboarder entering the ramp with a speed of 6.5 m/s; point f is the top of the ramp, where the speed is 4.1 m/s.

STRATEGY

To find h, simply set the initial energy, $E_i = U_i + K_i$, equal to the final energy, $E_f = U_f + K_f$.



SOLUTION

1. Write expressions for U_i and K_i: $U_i = mg \cdot 0 = 0$ $K_i = \frac{1}{2}mv_i^2$

2. Write expressions for U_f and K_f: $U_f = mgh$ $K_f = \frac{1}{2}mv_f^2$

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3. Set the total mechanical energy at point i, $E_i = U_i + K_i$, equal to the total mechanical energy at point f, $E_f = U_f + K_f$:

$$U_i + K_i = U_f + K_f$$
$$0 + \tfrac{1}{2}mv_i^2 = mgh + \tfrac{1}{2}mv_f^2$$

4. Solve for h. Note that m cancels:

$$mgh = \tfrac{1}{2}mv_i^2 - \tfrac{1}{2}mv_f^2$$
$$h = \frac{v_i^2 - v_f^2}{2g}$$

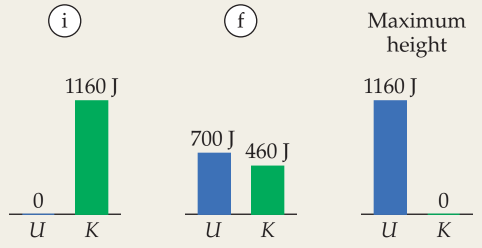
5. Substitute numerical values:

$$h = \frac{(6.5\ \mathrm{m/s})^2 - (4.1\ \mathrm{m/s})^2}{2(9.81\ \mathrm{m/s^2})} = 1.3\ \mathrm{m}$$

INSIGHT

Note that our value for h is independent of the shape of the ramp—it is equally valid for one with the shape shown here, or one that simply inclines upward at a constant angle. In addition, the height does not depend on the person's mass, as we see in Step 4.

The histograms to the right show U and K to scale at the points i and f, as well as at the maximum height where $K = 0$.



PRACTICE PROBLEM

What is the skateboarder's maximum height above the bottom of the ramp? [**Answer:** 2.2 m]

Some related homework problems: Problem 29, Problem 33

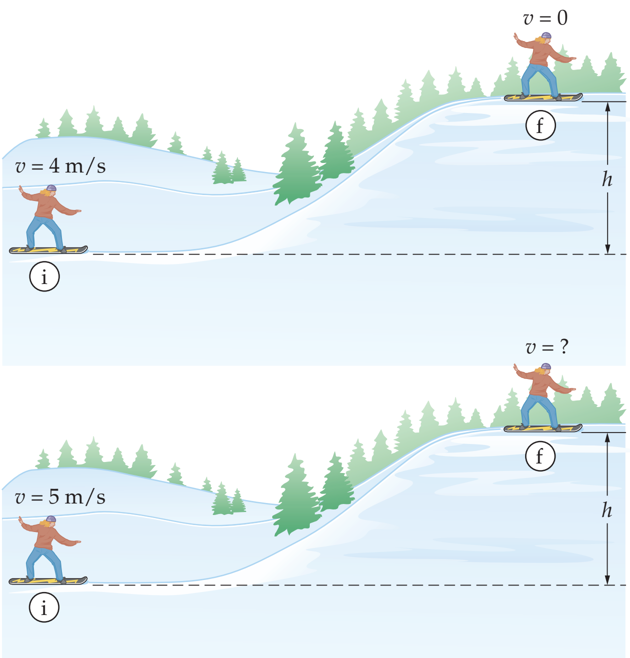
▲ Does the shape of the slide matter? (See Conceptual Checkpoint 8–1.)

It is interesting to express the equation in Step 3 from Example 8–7 in words. First, the left side of the equation is the initial kinetic energy of the skateboarder, $\tfrac{1}{2}mv_i^2$. This is the initial energy content of the system. At point f the system still has the same amount of energy, only now part of it, mgh, is in the form of gravitational potential energy. The remainder is the final kinetic energy, $\tfrac{1}{2}mv_f^2$.

Conceptual Checkpoint 8–2 considers the effect of a slight change in the initial speed of an object.

CONCEPTUAL CHECKPOINT 8–2 WHAT IS THE FINAL SPEED?

A snowboarder coasts on a smooth track that rises from one level to another. If the snowboarder's initial speed is 4 m/s, the snowboarder just makes it to the upper level and comes to rest. With a slightly greater initial speed of 5 m/s, the snowboarder is still moving to the right on the upper level. Is the snowboarder's final speed in this case **(a)** 1 m/s, **(b)** 2 m/s, or **(c)** 3 m/s?



REASONING AND DISCUSSION

A plausible-sounding answer is that since the initial speed is greater by 1 m/s in the second case, the final speed should be greater by 1 m/s as well. Therefore, the answer should be 0 + 1 m/s = 1 m/s. This is incorrect, however.

As surprising as it may seem, an increase in the initial speed from 4 m/s to 5 m/s results in an increase in the final speed from 0 to 3 m/s. This is due to the fact that kinetic energy depends on v^2 rather than v; thus, it is the difference in v^2 that counts. In this case, the initial value of v^2 increases from 16 m²/s² to 25 m²/s², for a total increase of 25 m²/s² − 16 m²/s² = 9 m²/s². The final value of v^2 must increase by the same amount, 9 m²/s² = (3 m/s)². As a result, the final speed is 3 m/s.

ANSWER

(c) The final speed of the snowboarder in the second case is 3 m/s.

Let's check the results of the previous Conceptual Checkpoint with a specific numerical example. Suppose the snowboarder has a mass of 74.0 kg. It follows that in the first case the initial kinetic energy is $K_i = \frac{1}{2}(74.0\text{ kg})(4.00\text{ m/s})^2 = 592\text{ J}$. At the top of the hill all of this kinetic energy is converted to gravitational potential energy, mgh.

In the second case, the initial speed of the snowboarder is 5.00 m/s; thus, the initial kinetic energy is $K_i = \frac{1}{2}(74.0\text{ kg})(5.00\text{ m/s})^2 = 925\text{ J}$. When the snowboarder reaches the top of the hill, 592 J of this kinetic energy is converted to gravitational potential energy, leaving the snowboarder with a final kinetic energy of 925 J − 592 J = 333 J. The corresponding speed is given by

$$\tfrac{1}{2}mv^2 = 333\text{ J}$$

$$v = \sqrt{\frac{2(333\text{ J})}{m}} = \sqrt{\frac{2(333\text{ J})}{74.0\text{ kg}}} = \sqrt{9.00\text{ m}^2/\text{s}^2} = 3.00\text{ m/s}$$

Thus, as expected, the snowboarder in the second case has a final speed of 3.00 m/s.

We conclude this section with two Examples involving springs.

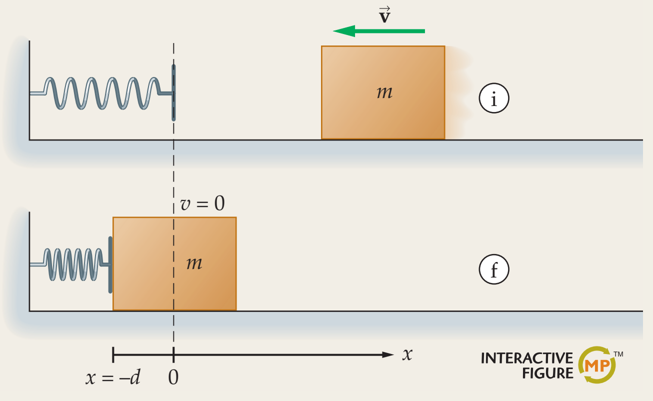
EXAMPLE 8–8 SPRING TIME

A 1.70-kg block slides on a horizontal, frictionless surface until it encounters a spring with a force constant of 955 N/m. The block comes to rest after compressing the spring a distance of 4.60 cm. Find the initial speed of the block. (Ignore air resistance and any energy lost when the block initially contacts the spring.)

PICTURE THE PROBLEM

Point i refers to times before the block makes contact with the spring, which means the block has a speed v and the end of the spring is at $x = 0$. Point f refers to the time when the block has come to rest, and the spring is compressed to $x = -d = -4.60$ cm.

We can choose the center of the block to be the $y = 0$ level. With this choice, the gravitational potential energy of the system is zero at all times.



STRATEGY

Set E_i equal to E_f to find the one unknown, v. Note that the initial energy, E_i, is the kinetic energy of the block before it reaches the spring. The final energy, E_f, is the potential energy of the compressed spring.

SOLUTION

1. Write expressions for U_i and K_i. For U, we consider only the potential energy of the spring, $U = \frac{1}{2}kx^2$: $\quad U_i = \frac{1}{2}k \cdot 0^2 = 0 \qquad K_i = \frac{1}{2}mv^2$
2. Do the same for U_f and K_f: $\quad U_f = \frac{1}{2}k(-d)^2 = \frac{1}{2}kd^2 \qquad K_f = \frac{1}{2}m \cdot 0^2 = 0$

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3. Set the initial mechanical energy, $E_i = U_i + K_i$, equal to the final mechanical energy, $E_f = U_f + K_f$, and solve for v:

$$U_i + K_i = U_f + K_f$$
$$0 + \tfrac{1}{2}mv^2 = \tfrac{1}{2}kd^2 + 0$$
$$v = d\sqrt{\frac{k}{m}}$$

4. Substitute numerical values:

$$v = d\sqrt{\frac{k}{m}} = (0.0460\ \text{m})\sqrt{\frac{955\ \text{N/m}}{1.70\ \text{kg}}} = 1.09\ \text{m/s}$$

INSIGHT

After the block comes to rest, the spring expands again, converting its potential energy back into the kinetic energy of the block. When the block leaves the spring, moving to the right, its speed is once again 1.09 m/s.

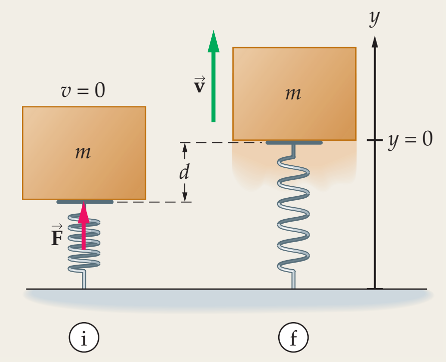
PRACTICE PROBLEM

What is the compression distance, d, if the block's initial speed is 0.500 m/s? [**Answer:** 2.11 cm]

Some related homework problems: Problem 32, Problem 34

ACTIVE EXAMPLE 8–1 FIND THE SPEED OF THE BLOCK

Suppose the spring and block in Example 8–8 are oriented vertically, as shown here. Initially, the spring is compressed 4.60 cm and the block is at rest. When the block is released, it accelerates upward. Find the speed of the block when the spring has returned to its equilibrium position.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write an expression for the initial mechanical energy E_i: $\quad E_i = U_i + K_i = -mgd + \tfrac{1}{2}kd^2 + 0$

2. Write an expression for the final mechanical energy E_f: $\quad E_f = U_f + K_f = 0 + 0 + \tfrac{1}{2}mv^2$

3. Set E_i equal to E_f and solve for v:

$$-mgd + \tfrac{1}{2}kd^2 = \tfrac{1}{2}mv^2$$
$$v = \sqrt{kd^2/m - 2gd}$$

4. Substitute numerical values: $\quad v = 0.535\ \text{m/s}$

INSIGHT

In this system, part of the initial potential energy of the spring $(\tfrac{1}{2}kd^2)$ goes into increasing the gravitational potential energy of the block (mgd). The remainder of the initial energy, $\tfrac{1}{2}kd^2 - mgd$, is converted into the block's kinetic energy.

YOUR TURN

What is the speed of the block when the spring is only halfway back to its equilibrium position?

(Answers to **Your Turn** *problems are given in the back of the book.)*

8–4 Work Done by Nonconservative Forces

Nonconservative forces change the amount of mechanical energy in a system. They might decrease the mechanical energy by converting it to thermal energy, or increase it by converting muscular work to kinetic or potential energy. In some systems, both types of processes occur at the same time.

To see the connection between the work done by a nonconservative force, W_{nc}, and the mechanical energy, E, we return once more to the work–energy theorem, which says that the *total* work is equal to the change in kinetic energy:

$$W_{total} = \Delta K$$

Suppose, for instance, that a system has one conservative and one nonconservative force. In this case, the total work is the sum of the conservative work W_c and the nonconservative work W_{nc}:

$$W_{total} = W_c + W_{nc}$$

Recalling that conservative work is related to the change in potential energy by the definition given in Equation 8–1, $W_c = -\Delta U$, we have

$$W_{total} = -\Delta U + W_{nc} = \Delta K$$

Solving this relation for the nonconservative work yields

$$W_{nc} = \Delta U + \Delta K$$

Finally, since the total mechanical energy is $E = U + K$, it follows that the *change* in mechanical energy is $\Delta E = \Delta U + \Delta K$. As a result, the nonconservative work is simply the change in mechanical energy:

$$W_{nc} = \Delta E = E_f - E_i \qquad 8\text{–}9$$

If more than one nonconservative force acts, we simply add the nonconservative work done by each such force to obtain W_{nc}.

At this point it may be useful to collect the three "working relationships" that have been introduced in the last two chapters:

$$\begin{aligned} W_{total} &= \Delta K \\ W_c &= -\Delta U \\ W_{nc} &= \Delta E \end{aligned} \qquad 8\text{–}10$$

Note that positive nonconservative work increases the total mechanical energy of a system, while negative nonconservative work decreases the mechanical energy—and converts it to other forms. In the next Example, for instance, part of the initial mechanical energy of a leaf is converted to heat and other forms of energy by air resistance as it falls to the ground.

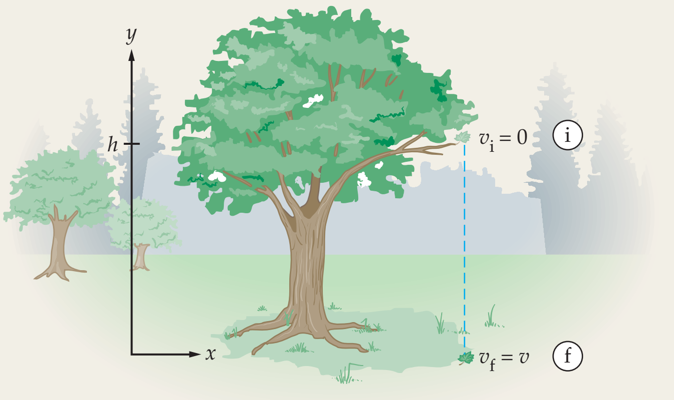
PROBLEM-SOLVING NOTE

Nonconservative Systems

Start by sketching the system and labeling the initial and final points with i and f, respectively. The initial and final mechanical energies are related to the nonconservative work by $W_{nc} = E_f - E_i$.

EXAMPLE 8–9 A LEAF FALLS IN THE FOREST: FIND THE NONCONSERVATIVE WORK

Deep in the forest, a 17.0-g leaf falls from a tree and drops straight to the ground. If its initial height was 5.30 m and its speed on landing was 1.3 m/s, how much nonconservative work was done on the leaf?



PICTURE THE PROBLEM

The leaf drops from rest at a height $y = h = 5.30$ m and lands with a speed $v = 1.3$ m/s at $y = 0$. These two points are labeled i and f, respectively.

STRATEGY

To begin, calculate the initial mechanical energy, E_i, and the final mechanical energy, E_f. Once these energies have been determined, the nonconservative work is $W_{nc} = \Delta E = E_f - E_i$.

SOLUTION

1. Evaluate U_i, K_i, and E_i:

$$U_i = mgh = (0.0170\text{ kg})(9.81\text{ m/s}^2)(5.30\text{ m}) = 0.884\text{ J}$$
$$K_i = \tfrac{1}{2}m\cdot 0^2 = 0$$
$$E_i = U_i + K_i = 0.884\text{ J}$$

2. Next, evaluate U_f, K_f, and E_f.

$$U_f = mg\cdot 0 = 0$$
$$K_f = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.0170\text{ kg})(1.3\text{ m/s})^2 = 0.014\text{ J}$$
$$E_f = U_f + K_f = 0.014\text{ J}$$

3. Use $W_{nc} = \Delta E$ to find the nonconservative work:

$$W_{nc} = \Delta E = E_f - E_i = 0.014\text{ J} - 0.884\text{ J} = -0.870\text{ J}$$

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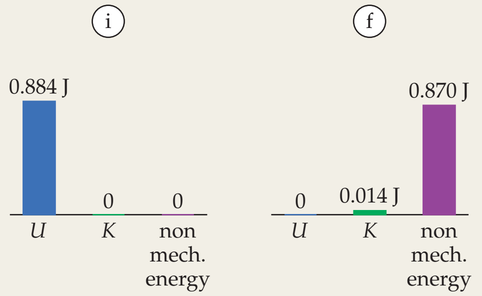
INSIGHT

Note that most of the initial mechanical energy is dissipated as the leaf falls. This is indicated in the histograms to the right. The small amount that remains (only about 1.6%) appears as the kinetic energy of the leaf just before it lands. If a cherry had fallen from the tree, it would have struck the ground with a considerably greater speed—perhaps five times the speed of the leaf. In that case, the percentage of the initial potential energy remaining as kinetic energy would have been $5^2 = 25$ times greater than the percentage retained by the leaf.



PRACTICE PROBLEM

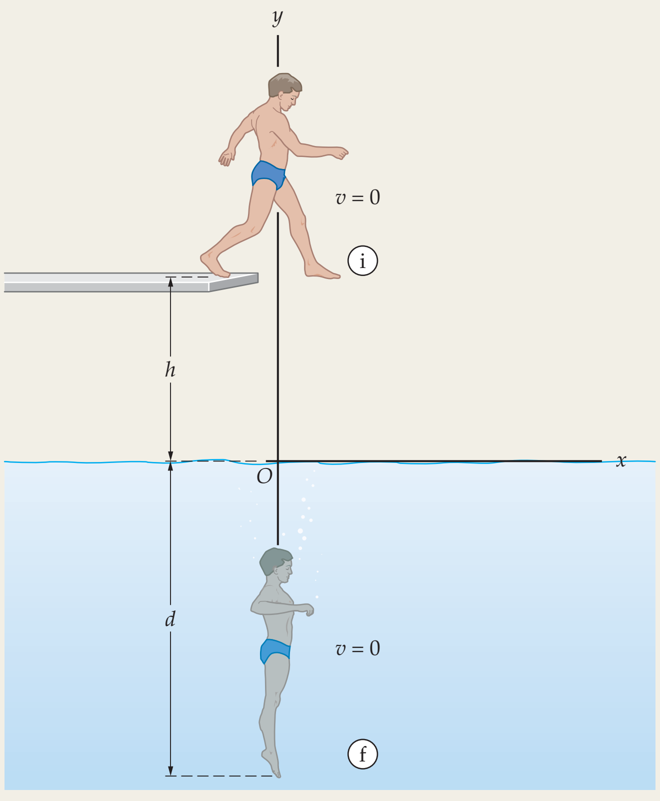
What was the average nonconservative force exerted on the leaf as it fell? [**Answer:** $W_{nc} = -Fh$, $F = -W_{nc}/h = 0.16$ N, upward]

Some related homework problems: Problem 42, Problem 43

In the following Active Example, we use a knowledge of the nonconservative work to find the depth at which a diver comes to rest.

ACTIVE EXAMPLE 8–2 FIND THE DIVER'S DEPTH

A 95.0-kg diver steps off a diving board and drops into the water 3.00 m below. At some depth d below the water's surface, the diver comes to rest. If the nonconservative work done on the diver is $W_{nc} = -5120$ J, what is the depth, d?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial mechanical energy, E_i: $E_i = mgh + 0 = mgh$
2. Write the final mechanical energy, E_f: $E_f = mg(-d) + 0 = -mgd$

3. Set W_{nc} equal to ΔE:	$W_{nc} = \Delta E = E_f - E_i = -mgd - mgh$
4. Solve for d:	$d = -(W_{nc} + mgh)/mg$
5. Substitute numerical values:	$d = 2.49$ m

INSIGHT

Another way to write Step 3 is $E_f = E_i + W_{nc}$. In words, this equation says that the final mechanical energy is the initial mechanical energy plus the nonconservative work done on the system. In this case, $W_{nc} < 0$; hence the final mechanical energy is less than the initial mechanical energy.

YOUR TURN

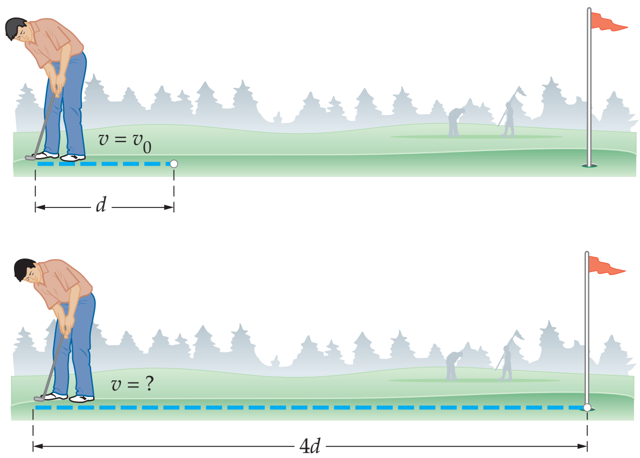
Suppose the diver descends to a depth of 3.50 m. How much nonconservative work is done in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

We now present a Conceptual Checkpoint that further examines the relationship between nonconservative work and distance.

CONCEPTUAL CHECKPOINT 8–3 JUDGING A PUTT

A golfer badly misjudges a putt, sending the ball only one-quarter of the distance to the hole. The original putt gave the ball an initial speed of v_0. If the force of resistance due to the grass is constant, would an initial speed of **(a)** $2v_0$, **(b)** $3v_0$, or **(c)** $4v_0$ be needed to get the ball to the hole from its original position?



REASONING AND DISCUSSION

In the original putt, the ball started with a kinetic energy of $\frac{1}{2}mv_0^2$ and came to rest in the distance d. The kinetic energy was dissipated by the nonconservative force due to grass resistance, F, which does the work $W_{nc} = -Fd$. Since the change in mechanical energy is $\Delta E = 0 - \frac{1}{2}mv_0^2 = -\frac{1}{2}mv_0^2$, it follows from $W_{nc} = \Delta E$ that $Fd = \frac{1}{2}mv_0^2$. Therefore, to go four times the distance, $4d$, we need to give the ball four times as much kinetic energy. Noting that kinetic energy is proportional to v^2, we see that the initial speed need only be doubled.

ANSWER

(a) The initial speed should be doubled to $2v_0$.

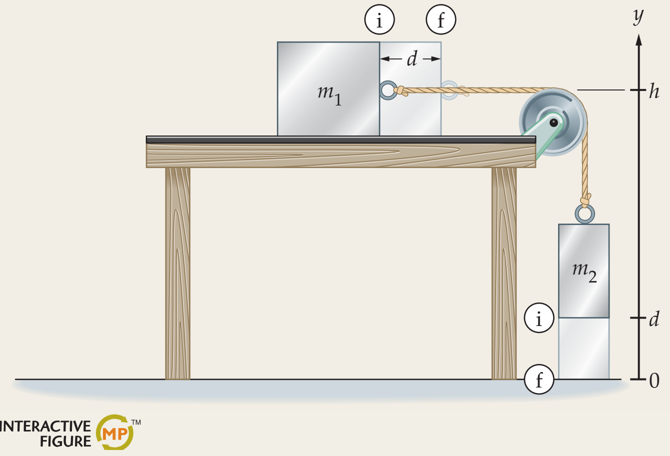
A common example of a nonconservative force is kinetic friction. In the next Example, we show how to include the effects of friction in a system that also includes kinetic energy and gravitational potential energy.

EXAMPLE 8–10 LANDING WITH A THUD

A block of mass $m_1 = 2.40$ kg is connected to a second block of mass $m_2 = 1.80$ kg, as shown here. When the blocks are released from rest, they move through a distance $d = 0.500$ m, at which point m_2 hits the floor. Given that the coefficient of kinetic friction between m_1 and the horizontal surface is $\mu_k = 0.450$, find the speed of the blocks just before m_2 lands.

PICTURE THE PROBLEM

We choose $y = 0$ to be at floor level; therefore, the gravitational potential energy of m_2 is zero when it lands. The potential energy of m_1 doesn't change during this process; it is always m_1gh. Thus, it isn't necessary to know the value of h. Note that we label the beginning and ending points with i and f, respectively.



STRATEGY

Since a nonconservative force (friction) is doing work in this system, we use $W_{nc} = \Delta E = E_f - E_i$. Thus, we must calculate not only the mechanical energies, E_i and E_f, but also the nonconservative work, W_{nc}. Note that E_f can be written in terms of the unknown speed of the blocks just before m_2 lands. Therefore, we can set W_{nc} equal to ΔE and solve for the final speed.

SOLUTION

1. Evaluate U_i, K_i, and E_i. Be sure to include contributions from both masses:

$$U_i = m_1gh + m_2gd$$
$$K_i = \tfrac{1}{2}m_1 \cdot 0^2 + \tfrac{1}{2}m_2 \cdot 0^2 = 0$$
$$E_i = U_i + K_i = m_1gh + m_2gd$$

2. Next, evaluate U_f, K_f, and E_f. Note that E_f depends on the unknown speed, v:

$$U_f = m_1gh + 0$$
$$K_f = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$
$$E_f = U_f + K_f = m_1gh + \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2$$

3. Calculate the nonconservative work, W_{nc}. Recall that the force of friction is $f_k = \mu_k N = \mu_k m_1 g$, and that it points opposite to the displacement of distance d:

$$W_{nc} = -f_k d = -\mu_k m_1 g d$$

4. Set W_{nc} equal to $\Delta E = E_f - E_i$. Notice that m_1gh cancels because it occurs in both E_i and E_f.

$$W_{nc} = E_f - E_i$$
$$-\mu_k m_1 g d = \tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 - m_2gd$$

5. Solve for v:

$$v = \sqrt{\frac{2(m_2 - \mu_k m_1)gd}{m_1 + m_2}}$$

6. Substitute numerical values:

$$v = \sqrt{\frac{2[1.80\text{ kg} - (0.450)(2.40\text{ kg})](9.81\text{ m/s}^2)(0.500\text{ m})}{1.80\text{ kg} + 2.40\text{ kg}}}$$
$$= 1.30\text{ m/s}$$

INSIGHT

Note that Step 4 can be rearranged as follows: $\tfrac{1}{2}m_1v^2 + \tfrac{1}{2}m_2v^2 = m_2gd - \mu_k m_1 g d$. Translating this to words, we can say that the final kinetic energy of the blocks is equal to the initial gravitational potential energy of m_2, minus the energy dissipated by friction.

PRACTICE PROBLEM

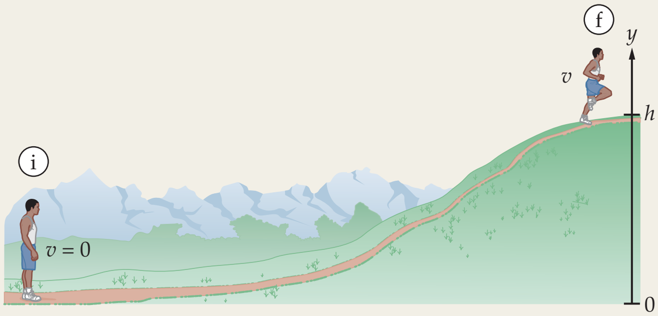
Find the coefficient of kinetic friction if the final speed of the blocks is 0.950 m/s. [**Answer:** $\mu_k = 0.589$]

Some related homework problems: Problem 46, Problem 51, Problem 66, Problem 106

Finally, we present an Active Example for the common situation of a system in which two different nonconservative forces do work.

ACTIVE EXAMPLE 8–3 MARATHON MAN: FIND THE HEIGHT OF THE HILL

An 80.0-kg jogger starts from rest and runs uphill into a stiff breeze. At the top of the hill the jogger has done the work $W_{nc1} = +1.80 \times 10^4$ J, air resistance has done the work $W_{nc2} = -4420$ J, and the jogger's speed is 3.50 m/s. Find the height of the hill.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial mechanical energy, E_i:	$E_i = U_i + K_i = 0 + 0 = 0$
2. Write the final mechanical energy, E_f:	$E_f = U_f + K_f = mgh + \frac{1}{2}mv^2$
3. Set W_{nc} equal to ΔE:	$W_{nc} = \Delta E = mgh + \frac{1}{2}mv^2$
4. Use $W_{nc} = \Delta E$ to solve for h:	$h = (W_{nc} - \frac{1}{2}mv^2)/mg$
5. Calculate the total nonconservative work:	$W_{nc} = W_{nc1} + W_{nc2} = 13{,}600$ J
6. Substitute numerical values to determine h:	$h = 16.7$ m

INSIGHT

As usual when dealing with energy calculations, our final result is independent of the shape of the hill.

YOUR TURN

Suppose the jogger's mass had been 90.0 kg rather than 80.0 kg. What would be the height of the hill in this case?

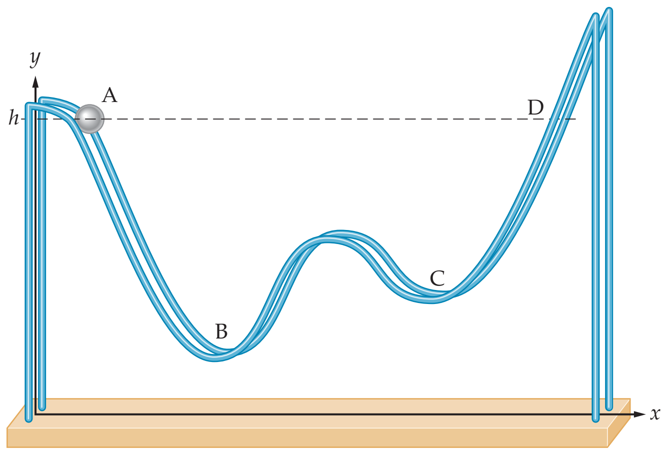
(Answers to **Your Turn** *problems are given in the back of the book.)*

▲ Highways that descend steeply are often provided with escape ramps that enable truck drivers whose brakes fail to bring their rigs to a safe stop. These ramps provide a perfect illustration of the conservation of energy. From a physics point of view, the driver's problem is to get rid of an enormous amount of kinetic energy in the safest possible way. The ramps run uphill, so some of the kinetic energy is simply converted back into gravitational potential energy (just as in a roller coaster). In addition, the ramps are typically surfaced with sand or gravel, allowing much of the initial kinetic energy to be dissipated by friction into other forms of energy, such as sound and heat.

8–5 Potential Energy Curves and Equipotentials

Figure 8–10 shows a metal ball rolling on a roller coaster–like track. Initially the ball is at rest at point A. Since the height at A is $y = h$, the ball's initial mechanical energy is $E_0 = mgh$. If friction and other nonconservative forces can be ignored, the ball's mechanical energy remains fixed at E_0 throughout its motion. Thus,

$$E = U + K = E_0$$



◀ **FIGURE 8–10 A ball rolling on a frictionless track**

The ball starts at A, where $y = h$, with zero speed. Its greatest speed occurs at B. At D, where $y = h$ again, its speed returns to zero.

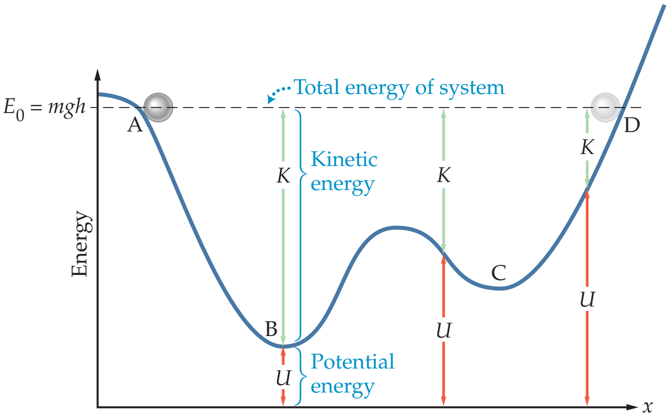


FIGURE 8–11 Gravitational potential energy versus position for the track shown in Figure 8–10

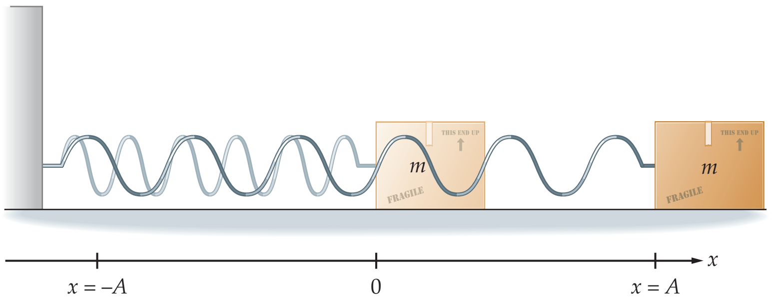
The shape of the potential energy curve is exactly the same as the shape of the track. In this case, the total mechanical energy is fixed at its initial value, $E_0 = U + K = mgh$. Because the height of the curve is U, by definition, it follows that K is the distance from the curve up to the dashed line at $E_0 = mgh$. Note that K is largest at B. In addition, K vanishes at A and D, which are turning points of the motion.

As the ball moves, its potential energy falls and rises in the same way as the track. After all, the gravitational potential energy, $U = mgy$, is directly proportional to the height of the track, y. In a sense, then, the track itself represents a graph of the corresponding potential energy.

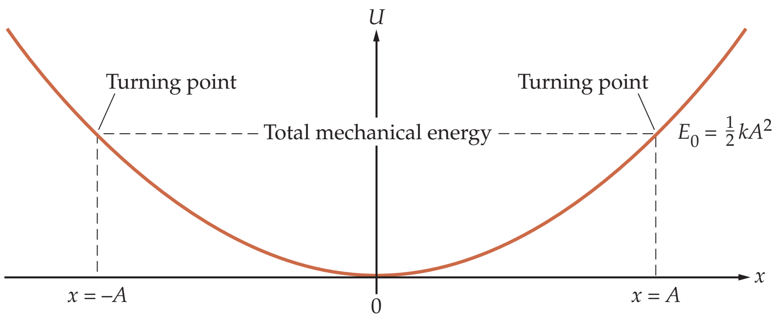
This is shown explicitly in **Figure 8–11**, where we plot energy on the vertical axis and x on the horizontal axis. The potential energy U looks just like the track in Figure 8–10. In addition, we plot a horizontal line at the value E_0, indicating the constant energy of the ball. Since the potential energy plus the kinetic energy must always add up to E_0, it follows that K is the amount of energy from the potential energy curve up to the horizontal line at E_0. This is also shown in Figure 8–11.

Examining an energy plot like Figure 8–11 gives a great deal of information about the motion of an object. For example, at point B the potential energy has its lowest value, and thus the kinetic energy is greatest there. At point C the potential energy has increased, indicating a corresponding decrease in kinetic energy. As the ball continues to the right, the potential energy increases until, at point D, it is again equal to the total energy, E_0. At this point the kinetic energy is zero, and the ball comes to rest momentarily. It then "turns around" and begins to move to the left, eventually returning to point A where it again stops, changes direction, and begins a new cycle. Points A and D, then, are referred to as **turning points** of the motion.

Turning points are also seen in the motion of a mass on a spring, as indicated in **Figure 8–12**. Figure 8–12 (a) shows a mass pulled to the position $x = A$, and released from rest; Figure 8–12 (b) shows the potential energy of the system, $U = \frac{1}{2}kx^2$.



(a)



(b)

FIGURE 8–12 A mass on a spring

(a) A spring is stretched by an amount A, giving it a potential energy of $U = \frac{1}{2}kA^2$. **(b)** The potential energy curve, $U = \frac{1}{2}kx^2$, for the spring in (a). Because the mass starts at rest, its initial mechanical energy is $E_0 = \frac{1}{2}kA^2$. The mass oscillates between $x = A$ and $x = -A$.

Starting the system this way gives it an initial energy $E_0 = \frac{1}{2}kA^2$, shown by the horizontal line in Figure 8–12 (b). As the mass moves to the left, its speed increases, reaching a maximum where the potential energy is lowest, at $x = 0$. If no nonconservative forces act, the mass continues to $x = -A$, where it stops momentarily before returning to $x = A$. This type of **oscillatory motion** will be studied in detail in Chapter 13.

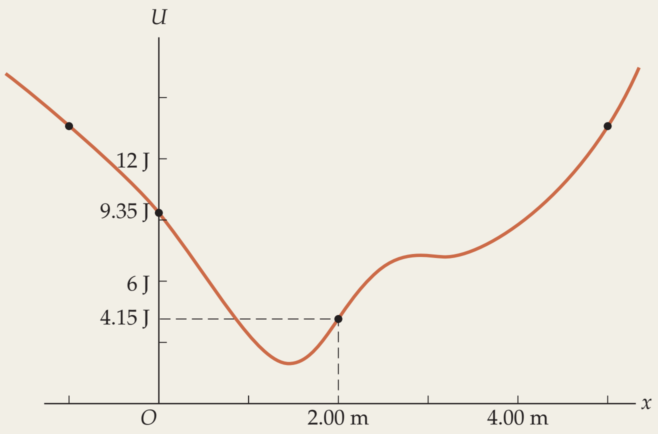
The next Example uses a potential-energy curve to find the speed of an object at a given value of x.

EXAMPLE 8–11 A POTENTIAL PROBLEM

A 1.60-kg object in a conservative system moves along the x axis, where the potential energy is as shown. A physical example would be a bead sliding on a wire with the shape of the potential energy curve. If the object's speed at $x = 0$ is 2.30 m/s, what is its speed at $x = 2.00$ m?

PICTURE THE PROBLEM

The plot shows U as a function of x. The values of U at $x = 0$ and $x = 2.00$ m are 9.35 J and 4.15 J, respectively. It follows that the object's speed at $x = 2.00$ m will be greater than its speed at $x = 0$.



STRATEGY

Since mechanical energy is conserved, we know that the total energy at $x = 0$ $(U_i + K_i)$ is equal to the total energy at $x = 2.00$ m $(U_f + K_f)$.

The problem statement gives U_i, and since we also know the speed at $x = 0$ we can use $K = \frac{1}{2}mv^2$ to calculate the corresponding kinetic energy, K_i. At $x = 2.00$ m we know the potential energy, U_f; hence we can use $U_i + K_i = U_f + K_f$ to solve for K_f. Once the final kinetic energy is known, it is possible to solve for the final speed by once again using $K = \frac{1}{2}mv^2$.

SOLUTION

1. Evaluate U_i, K_i, and E_i at $x = 0$:

$$U_i = 9.35\text{ J}$$
$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(1.60\text{ kg})(2.30\text{ m/s})^2 = 4.23\text{ J}$$
$$E_i = U_i + K_i = 9.35\text{ J} + 4.23\text{ J} = 13.58\text{ J}$$

2. Write expressions for U_f, K_f, and E_f at $x = 2.00$ m:

$$U_f = 4.15\text{ J}$$
$$K_f = \tfrac{1}{2}mv_f^2$$
$$E_f = U_f + K_f = 4.15\text{ J} + \tfrac{1}{2}mv_f^2$$

3. Set E_f equal to E_i and solve for v_f:

$$4.15\text{ J} + \tfrac{1}{2}mv_f^2 = 13.58\text{ J}$$

Solve for v_f:

$$v_f = \sqrt{\frac{2(13.58\text{ J} - 4.15\text{ J})}{m}}$$

4. Substitute the numerical value of the object's mass:

$$v_f = \sqrt{\frac{2(13.58\text{ J} - 4.15\text{ J})}{1.60\text{ kg}}} = 3.43\text{ m/s}$$

INSIGHT

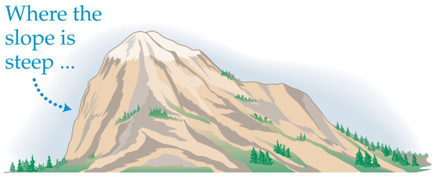
As we see in Step 1, the total mechanical energy of the system is 13.58 J. This means that turning points for this object occur at values of x where $U = 13.58$ J.

PRACTICE PROBLEM

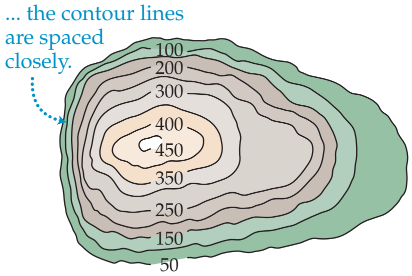
Using the graph provided, estimate the location of the turning points for this object. [**Answer:** $x = -1.00$ m and $x = 5.00$ m]

Some related homework problems: Problem 57, Problem 58

Oscillatory motion between turning points is also observed in molecules. If the energy of oscillation is relatively small, as is usual at room temperature, the atoms in a molecule simply vibrate back and forth—like masses connected by a spring. As long as no energy is gained or lost, the molecular oscillations continue unchanged. On the other hand, if the energy of the molecule is increased by



Side view of mountain



Contour map of mountain (from above)

▲ **FIGURE 8–13 A contour map**

A small mountain (top, in side view) is very steep on the left, more gently sloping on the right. A contour map of this mountain (bottom) shows a series of equal-altitude contour lines from 50 ft to 450 ft. Notice that the contour lines are packed close together where the terrain is steep, but are widely spaced where it is more level.

heating, or some other mechanism, the molecule will eventually dissociate—fly apart—as the atoms move to infinite separation.

In some cases, a two-dimensional plot of potential energy contours is useful. For instance, **Figure 8–13** shows a contour map of a hill. Each contour corresponds to a given altitude and, hence, to a given value of the gravitational potential energy. In general, lines corresponding to constant values of potential energy are called **equipotentials.** Since the altitude changes by equal amounts from one contour to the next, it follows that when gravitational equipotentials are packed close together, the corresponding terrain is steep. On the other hand, when the equipotentials are widely spaced, the ground is nearly flat, since a large horizontal distance is required for a given change in altitude. We shall see similar plots with similar interpretations when we study electric potential energy in Chapter 21.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of work, first introduced in Chapter 7 as force times distance, is used again in this chapter. For example, we use work in Section 8–1 to illustrate the difference between conservative and nonconservative forces.

Work (Chapter 7) is used again in Section 8–2 to introduce the concept of potential energy, U, and to define its change.

LOOKING AHEAD

Conservation of energy is one of the key elements in the study of elastic collisions. See, in particular, Section 9–6 and Example 9–7.

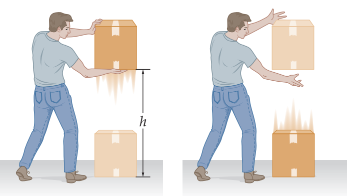
The wide-ranging importance of energy conservation is illustrated by its use in the following disparate topics: rotational motion (Section 10–6), gravitation (Section 12–5), oscillatory motion (Section 13–5), fluid dynamics (Section 15–7), and phase changes (Section 17–6).

CHAPTER SUMMARY

8–1 CONSERVATIVE AND NONCONSERVATIVE FORCES

Conservative forces conserve the mechanical energy of a system. Thus, in a conservative system the total mechanical energy remains constant.

Nonconservative forces convert mechanical energy into other forms of energy, or convert other forms of energy into mechanical energy.



Conservative Force, Definition
A conservative force does zero total work on any closed path. In addition, the work done by a conservative force in going from point A to point B is *independent of the path* from A to B.

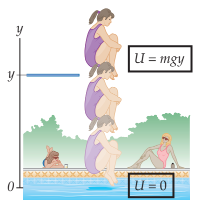
Examples of Conservative Forces
Gravity, spring.

Nonconservative Force, Definition
The work done by a nonconservative force on a closed path is nonzero. The work is also path-dependent.

Examples of Nonconservative Forces
Friction, air resistance, tension in ropes and cables, forces exerted by muscles and motors.

8–2 POTENTIAL ENERGY AND THE WORK DONE BY CONSERVATIVE FORCES

Potential energy, U, can "store" energy in a system. Energy in the form of potential energy can be converted to kinetic or other forms of energy.



Potential Energy, Definition
The work done by a conservative force is the negative of the change in potential energy:

$$W_c = -\Delta U = U_i - U_f \qquad 8\text{–}1$$

Zero Level
Any location can be chosen for $U = 0$. Once the choice is made, however, it must be used consistently.

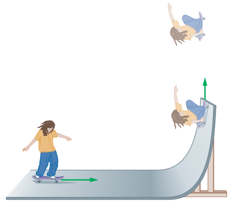
Gravity
Choosing $y = 0$ to be the zero level near Earth's surface,

$$U = mgy. \quad 8\text{–}3$$

Spring
Choosing $x = 0$ (the equilibrium position) to be the zero level,

$$U = \tfrac{1}{2}kx^2. \quad 8\text{–}5$$

8–3 CONSERVATION OF MECHANICAL ENERGY

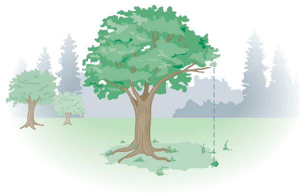


Mechanical energy, E, is conserved in systems with conservative forces only.

Mechanical Energy, Definition
Mechanical energy is the sum of the potential and kinetic energies of a system:

$$E = U + K \quad 8\text{–}6$$

8–4 WORK DONE BY NONCONSERVATIVE FORCES

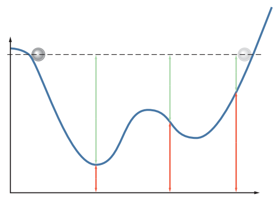


Nonconservative forces can change the mechanical energy of a system.

Change in Mechanical Energy
The work done by a nonconservative force is equal to the change in the mechanical energy of a system:

$$W_{nc} = \Delta E = E_f - E_i \quad 8\text{–}9$$

8–5 POTENTIAL ENERGY CURVES AND EQUIPOTENTIALS



A potential energy curve plots U as a function of position.

An equipotential plot shows contours corresponding to constant values of U.

Turning Points
Turning points occur where an object stops momentarily before reversing direction. At turning points the kinetic energy is zero.

Oscillatory Motion
An object moving back and forth between two turning points is said to have oscillatory motion.

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Calculate the gravitational or spring potential energy.	The potential energy for gravity is $U = mgy$; the potential energy for a spring is $U = \frac{1}{2}kx^2$.	Examples 8–2, 8–3, 8–4
Apply energy conservation in a system involving gravity.	Choose a horizontal level for $y = 0$, then use $U = mgy$.	Examples 8–6, 8–7 Active Example 8–1
Apply energy conservation in a system involving a spring.	Use $U = \frac{1}{2}kx^2$, where x measures the expansion or compression of the spring from its equilibrium position.	Example 8–8, Active Example 8–1
Find the nonconservative work done on a system.	Calculate the initial energy, E_i, and the final energy, E_f. Then use $W_{nc} = \Delta E = E_f - E_i$.	Examples 8–9, 8–10, Active Examples 8–2, 8–3

CONCEPTUAL QUESTIONS

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Is it possible for the kinetic energy of an object to be negative? Is it possible for the gravitational potential energy of an object to be negative? Explain.

2. An avalanche occurs when a mass of snow slides down a steep mountain slope. Discuss the energy conversions responsible for water vapor rising to form clouds, falling as snow on a mountain, and then sliding down a slope as an avalanche.

3. If the stretch of a spring is doubled, the force it exerts is also doubled. By what factor does the spring's potential energy increase?
4. When a mass is placed on top of a vertical spring, the spring compresses and the mass moves downward. Analyze this system in terms of its mechanical energy.
5. If a spring is stretched so far that it is permanently deformed, its force is no longer conservative. Why?
6. An object is thrown upward to a person on a roof. At what point is the object's kinetic energy at maximum? At what point is the potential energy of the system at maximum? At what locations do these energies have their minimum values?
7. It is a law of nature that the total energy of the universe is conserved. What do politicians mean, then, when they urge "energy conservation"?
8. Discuss the various energy conversions that occur when a person performs a pole vault. Include as many conversions as you can, and consider times before, during, and after the actual vault itself.

How many energy conversions can you identify? (Conceptual Question 8)

9. Discuss the nature of the work done by the equipment shown in this photo. What types of forces are involved?

Conservative or nonconservative? (Conceptual Question 9)

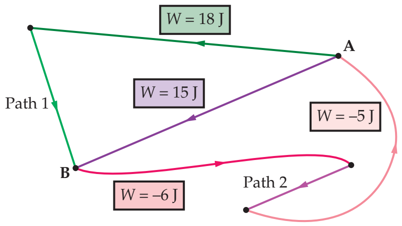
10. A toy frog consists of a suction cup and a spring. When the suction cup is pressed against a smooth surface, the frog is held down. When the suction cup lets go, the frog leaps into the air. Discuss the behavior of the frog in terms of energy conversions.
11. If the force on an object is zero, does that mean the potential energy of the system is zero? If the potential energy of a system is zero, is the force zero?
12. When a ball is thrown upward, its mechanical energy, $E = mgy + \frac{1}{2}mv^2$, is constant with time if air resistance can be ignored. How does E vary with time if air resistance cannot be ignored?
13. When a ball is thrown upward, it spends the same amount of time on the way up as on the way down—as long as air resistance can be ignored. If air resistance is taken into account, is the time on the way down the same as, greater than, or less than the time on the way up? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* ***(a)*** *your prediction of a physical outcome, and* ***(b)*** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 8–1 CONSERVATIVE AND NONCONSERVATIVE FORCES

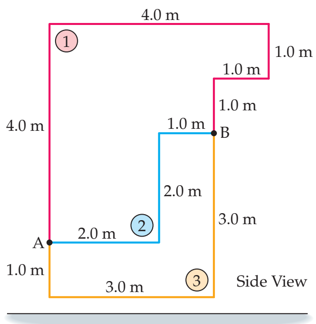
1. • **CE** The work done by a conservative force is indicated in **Figure 8–14** for a variety of different paths connecting the points A and B. What is the work done by this force **(a)** on path 1 and **(b)** on path 2?



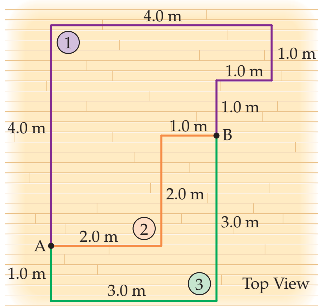
▲ **FIGURE 8–14** Problem 1

2. • Calculate the work done by gravity as a 3.2-kg object is moved from point A to point B in **Figure 8–15** along paths 1, 2, and 3.

3. • Calculate the work done by friction as a 3.7-kg box is slid along a floor from point A to point B in **Figure 8–16** along paths 1, 2, and 3. Assume that the coefficient of kinetic friction between the box and the floor is 0.26.

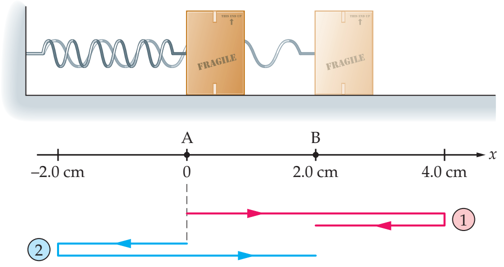


▲ **FIGURE 8–15** Problem 2



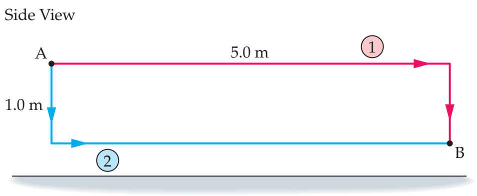
▲ **FIGURE 8–16** Problem 3

4. • **IP** A 4.1-kg block is attached to a spring with a force constant of 550 N/m, as shown in **Figure 8–17**. **(a)** Find the work done by the spring on the block as the block moves from A to B along paths 1 and 2. **(b)** How do your results depend on the mass of the block? Specifically, if you increase the mass, does the work done by the spring increase, decrease, or stay the same? (Assume the system is frictionless.)



▲ **FIGURE 8–17** Problems 4 and 6

5. • **IP** **(a)** Calculate the work done by gravity as a 5.2-kg object is moved from A to B in **Figure 8–18** along paths 1 and 2. **(b)** How do your results depend on the mass of the block? Specifically, if you increase the mass, does the work done by gravity increase, decrease, or stay the same?



▲ **FIGURE 8–18** Problem 5

6. •• In the system shown in Figure 8–17, suppose the block has a mass of 2.7 kg, the spring has a force constant of 480 N/m, and the coefficient of kinetic friction between the block and the floor is 0.16. **(a)** Find the work done on the block by the spring and by friction as the block is moved from point A to point B along path 2. **(b)** Find the work done on the block by the spring and by friction if the block is moved directly from point A to point B.

SECTION 8–2 POTENTIAL ENERGY AND THE WORK DONE BY CONSERVATIVE FORCES

7. • **CE Predict/Explain** Ball 1 is thrown to the ground with an initial downward speed; ball 2 is dropped to the ground from rest. Assuming the balls have the same mass and are released from the same height, is the change in gravitational potential energy of ball 1 greater than, less than, or equal to the change in gravitational potential energy of ball 2? **(b)** Choose the *best explanation* from among the following:
   - **I.** Ball 1 has the greater total energy, and therefore more energy can go into gravitational potential energy.
   - **II.** The gravitational potential energy depends only on the mass of the ball and the drop height.
   - **III.** All of the initial energy of ball 2 is gravitational potential energy.

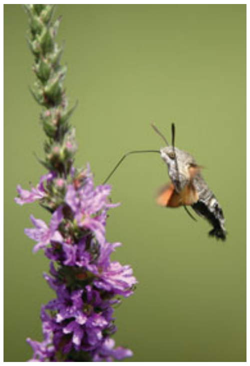
8. • **CE** A mass is attached to the bottom of a vertical spring. This causes the spring to stretch and the mass to move downward. **(a)** Does the potential energy of the spring increase, decrease, or stay the same during this process? Explain. **(b)** Does the gravitational potential energy of the Earth–mass system increase, decrease, or stay the same during this process? Explain.

9. • As an Acapulco cliff diver drops to the water from a height of 46 m, his gravitational potential energy decreases by 25,000 J. What is the diver's weight in newtons?

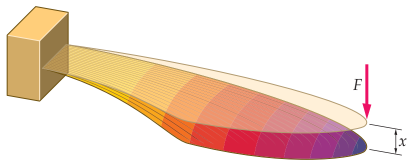
10. • Find the gravitational potential energy of an 88-kg person standing atop Mt. Everest at an altitude of 8848 m. Use sea level as the location for $y = 0$.

11. • **Jeopardy!** Contestants on the game show *Jeopardy!* depress spring-loaded buttons to "buzz in" and provide the question corresponding to the revealed answer. The force constant on these buttons is about 130 N/m. Estimate the amount of energy it takes—at a minimum—to buzz in.

12. •• **BIO The Wing of the Hawkmoth** Experiments performed on the wing of a hawkmoth (*Manduca sexta*) show that it deflects by a distance of $x = 4.8$ mm when a force of magnitude $F = 3.0$ mN is applied at the tip, as indicated in **Figure 8–19**. Treating the wing as an ideal spring, find **(a)** the force constant of the wing and **(b)** the energy stored in the wing when it is deflected. **(c)** What force must be applied to the tip of the wing to store twice the energy found in part (b)?



Hummingbird hawkmoth (*Manduca sexta*).
(Problem 12)



▲ **FIGURE 8–19** Problem 12

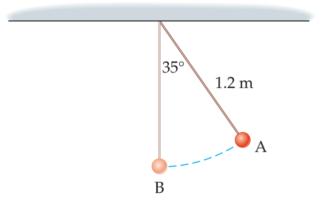
13. •• **IP** A vertical spring stores 0.962 J in spring potential energy when a 3.5-kg mass is suspended from it. **(a)** By what multiplicative factor does the spring potential energy change if the mass attached to the spring is doubled? **(b)** Verify your answer to part (a) by calculating the spring potential energy when a 7.0-kg mass is attached to the spring.

14. •• Pushing on the pump of a soap dispenser compresses a small spring. When the spring is compressed 0.50 cm, its potential energy is 0.0025 J. **(a)** What is the force constant of the spring? **(b)** What compression is required for the spring potential energy to equal 0.0084 J?

15. •• A force of 4.1 N is required to stretch a certain spring by 1.4 cm. **(a)** How far must this spring be stretched for its potential energy to be 0.020 J? **(b)** How much stretch is required for the spring potential energy to be 0.080 J?

16. •• **IP** The work required to stretch a certain spring from an elongation of 4.00 cm to an elongation of 5.00 cm is 30.5 J. **(a)** Is the work required to increase the elongation of the spring from 5.00 cm to 6.00 cm greater than, less than, or equal to 30.5 J? Explain. **(b)** Verify your answer to part (a) by calculating the required work.

17. •• A 0.33-kg pendulum bob is attached to a string 1.2 m long. What is the change in the gravitational potential energy of the system as the bob swings from point A to point B in **Figure 8–20**?



▲ **FIGURE 8–20** Problems 17, 34, 35, and 74

SECTION 8–3 CONSERVATION OF MECHANICAL ENERGY

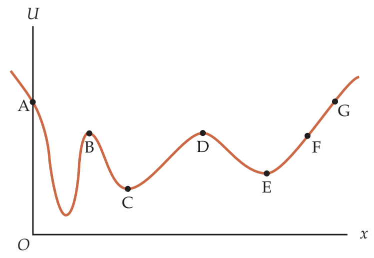
18. • **CE Predict/Explain** You throw a ball upward and let it fall to the ground. Your friend drops an identical ball straight down to the ground from the same height. Is the change in kinetic energy of your ball greater than, less than, or equal to the change in kinetic energy of your friend's ball? **(b)** Choose the *best explanation* from among the following:
    I. Your friend's ball converts all its initial energy into kinetic energy.
    II. Your ball is in the air longer, which results in a greater change in kinetic energy.
    III. The change in gravitational potential energy is the same for each ball, which means the change in kinetic energy must be the same also.

19. • **CE** Suppose the situation described in Conceptual Checkpoint 8–2 is repeated on the fictional planet Epsilon, where the acceleration due to gravity is less than it is on the Earth. **(a)** Would the height of a hill on Epsilon that causes a reduction in speed from 4 m/s to 0 be greater than, less than, or equal to the height of the corresponding hill on Earth? Explain. **(b)** Consider the hill on Epsilon discussed in part (a). If the initial speed at the bottom of the hill is 5 m/s, will the final speed at the top of the hill be greater than, less than, or equal to 3 m/s? Explain.

20. • **CE Predict/Explain** When a ball of mass m is dropped from rest from a height h, its kinetic energy just before landing is K. Now, suppose a second ball of mass $4m$ is dropped from rest from a height $h/4$. **(a)** Just before ball 2 lands, is its kinetic energy $4K$, $2K$, K, $K/2$, or $K/4$? **(b)** Choose the *best explanation* from among the following:
    I. The two balls have the same initial energy.
    II. The more massive ball will have the greater kinetic energy.
    III. The reduced drop height results in a reduced kinetic energy.

21. • **CE Predict/Explain** When a ball of mass m is dropped from rest from a height h, its speed just before landing is v. Now, suppose a second ball of mass $4m$ is dropped from rest from a height $h/4$. **(a)** Just before ball 2 lands, is its speed $4v$, $2v$, v, $v/2$, or $v/4$? **(b)** Choose the *best explanation* from among the following:
    I. The factors of 4 cancel; therefore, the landing speed is the same.
    II. The two balls land with the same kinetic energy; therefore, the ball of mass $4m$ has the speed $v/2$.
    III. Reducing the height by a factor of 4 reduces the speed by a factor of 4.

22. • **CE** For an object moving along the x axis, the potential energy of the frictionless system is shown in **Figure 8–21**. Suppose the object is released from rest at the point A. Rank the other points in the figure in increasing order of the object's speed. Indicate ties where appropriate.



▲ **FIGURE 8–21** Problems 22 and 23

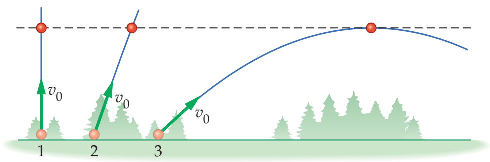
23. • **CE** Referring to Problem 22, suppose the object is released from rest at a point halfway between the points F and G. Rank the other points in the figure in increasing order of the object's speed, if the object can reach that point. Indicate ties where appropriate.

24. • At an amusement park, a swimmer uses a water slide to enter the main pool. If the swimmer starts at rest, slides without friction, and descends through a vertical height of 2.31 m, what is her speed at the bottom of the slide?

25. • In the previous problem, find the swimmer's speed at the bottom of the slide if she starts with an initial speed of 0.840 m/s.

26. • **IP** A player passes a 0.600-kg basketball downcourt for a fast break. The ball leaves the player's hands with a speed of 8.30 m/s and slows down to 7.10 m/s at its highest point. **(a)** Ignoring air resistance, how high above the release point is the ball when it is at its maximum height? **(b)** How would doubling the ball's mass affect the result in part (a)? Explain.

27. •• **CE** Three balls are thrown upward with the same initial speed v_0, but at different angles relative to the horizontal, as shown in **Figure 8–22**. Ignoring air resistance, indicate which of the following statements is correct: At the dashed level, **(A)** ball 3 has the lowest speed; **(B)** ball 1 has the lowest speed; **(C)** all three balls have the same speed; **(D)** the speed of the balls depends on their mass.



▲ **FIGURE 8–22** Problem 27

28. •• **IP** In a tennis match, a player wins a point by hitting the ball sharply to the ground on the opponent's side of the net. **(a)** If the ball bounces upward from the ground with a speed of 16 m/s, and is caught by a fan in the stands with a speed of 12 m/s, how high above the court is the fan? Ignore air resistance. **(b)** Explain why it is not necessary to know the mass of the tennis ball.

29. •• A 0.21-kg apple falls from a tree to the ground, 4.0 m below. Ignoring air resistance, determine the apple's kinetic energy, K, the gravitational potential energy of the system, U, and the total mechanical energy of the system, E, when the apple's height above the ground is **(a)** 4.0 m, **(b)** 3.0 m, **(c)** 2.0 m, **(d)** 1.0 m, and **(e)** 0 m. Take ground level to be $y = 0$.

30. •• **IP** A 2.9-kg block slides with a speed of 1.6 m/s on a frictionless horizontal surface until it encounters a spring. **(a)** If the block compresses the spring 4.8 cm before coming to rest, what is the force constant of the spring? **(b)** What initial speed should the block have to compress the spring by 1.2 cm?

31. •• A 0.26-kg rock is thrown vertically upward from the top of a cliff that is 32 m high. When it hits the ground at the base of the cliff, the rock has a speed of 29 m/s. Assuming that air resistance can be ignored, find **(a)** the initial speed of the rock and **(b)** the greatest height of the rock as measured from the base of the cliff.

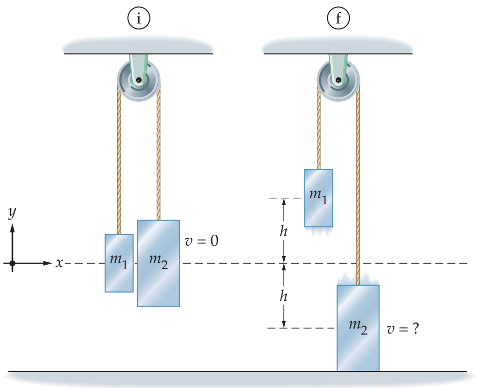
32. •• A 1.40-kg block slides with a speed of 0.950 m/s on a frictionless horizontal surface until it encounters a spring with a force constant of 734 N/m. The block comes to rest after compressing the spring 4.15 cm. Find the spring potential energy, U, the kinetic energy of the block, K, and the total mechanical energy of the system, E, for compressions of **(a)** 0 cm, **(b)** 1.00 cm, **(c)** 2.00 cm, **(d)** 3.00 cm, and **(e)** 4.00 cm.

33. •• A 5.76-kg rock is dropped and allowed to fall freely. Find the initial kinetic energy, the final kinetic energy, and the change in kinetic energy for **(a)** the first 2.00 m of fall and **(b)** the second 2.00 m of fall.

34. •• **IP** Suppose the pendulum bob in Figure 8–20 has a mass of 0.33 kg and is moving to the right at point B with a speed of 2.4 m/s. Air resistance is negligible. **(a)** What is the change in the system's gravitational potential energy when the bob reaches point A? **(b)** What is the speed of the bob at point A? **(c)** If the mass of the bob is increased, does your answer to part (a) increase, decrease, or stay the same? Explain. **(d)** If the mass of the bob is increased, does your answer to part (b) increase, decrease, or stay the same? Explain.

35. •• **IP** In the previous problem, **(a)** what is the bob's kinetic energy at point B? **(b)** At some point the bob will come to rest momentarily. Without doing an additional calculation, determine the change in the system's gravitational potential energy between point B and the point where the bob comes to rest. **(c)** Find the maximum angle the string makes with the vertical as the bob swings back and forth. Ignore air resistance.

36. ••• The two masses in the Atwood's machine shown in **Figure 8–23** are initially at rest at the same height. After they are released, the large mass, m_2, falls through a height h and hits the floor, and the small mass, m_1, rises through a height h. **(a)** Find the speed of the masses just before m_2 lands, giving your answer in terms of m_1, m_2, g, and h. Assume the ropes and pulley have negligible mass and that friction can be ignored. **(b)** Evaluate your answer to part (a) for the case $h = 1.2$ m, $m_1 = 3.7$ kg, and $m_2 = 4.1$ kg.



▲ **FIGURE 8–23** Problems 36, 37, 82, and 99

37. ••• In the previous problem, suppose the masses have an initial speed of 0.20 m/s, and that m_2 is moving upward. How high does m_2 rise above its initial position before momentarily coming to rest, given that $m_1 = 3.7$ kg and $m_2 = 4.1$ kg?

SECTION 8–4 WORK DONE BY NONCONSERVATIVE FORCES

38. • **CE** You coast up a hill on your bicycle with decreasing speed. Your friend pedals up the hill with constant speed. **(a)** Ignoring friction, does the mechanical energy of the you–bike–Earth system increase, decrease, or stay the same? Explain. **(b)** Does the mechanical energy of the friend–bike–Earth system increase, decrease, or stay the same? Explain.

39. • **CE Predict/Explain** On reentry, the space shuttle's protective heat tiles become extremely hot. **(a)** Is the mechanical energy of the shuttle–Earth system when the shuttle lands greater than, less than, or the same as when it is in orbit? **(b)** Choose the *best explanation* from among the following:
    I. Dropping out of orbit increases the mechanical energy of the shuttle.
    II. Gravity is a conservative force.
    III. A portion of the mechanical energy has been converted to heat energy.

40. • Catching a wave, a 77-kg surfer starts with a speed of 1.3 m/s, drops through a height of 1.65 m, and ends with a speed of 8.2 m/s. How much nonconservative work was done on the surfer?

41. • At a playground, a 19-kg child plays on a slide that drops through a height of 2.3 m. The child starts at rest at the top of the slide. On the way down, the slide does a nonconservative work of −361 J on the child. What is the child's speed at the bottom of the slide?

42. • Starting at rest at the edge of a swimming pool, a 72.0-kg athlete swims along the surface of the water and reaches a speed of 1.20 m/s by doing the work $W_{nc1} = +161$ J. Find the nonconservative work, W_{nc2}, done by the water on the athlete.

43. • A 17,000-kg airplane lands with a speed of 82 m/s on a stationary aircraft carrier deck that is 115 m long. Find the work done by nonconservative forces in stopping the plane.

44. • **IP** The driver of a 1300-kg car moving at 17 m/s brakes quickly to 11 m/s when he spots a local garage sale. **(a)** Find the change in the car's kinetic energy. **(b)** Explain where the "missing" kinetic energy has gone.

45. •• **CE** You ride your bicycle down a hill, maintaining a constant speed the entire time. **(a)** As you ride, does the gravitational potential energy of the you–bike–Earth system increase, decrease, or stay the same? Explain. **(b)** Does the kinetic energy of you and your bike increase, decrease, or stay the same? Explain. **(c)** Does the mechanical energy of the you–bike–Earth system increase, decrease, or stay the same? Explain.

46. •• Suppose the system in Example 8–10 starts with m_2 moving downward with a speed of 1.3 m/s. What speed do the masses have just before m_2 lands?

47. •• A 42.0-kg seal at an amusement park slides from rest down a ramp into the pool below. The top of the ramp is 1.75 m higher than the surface of the water, and the ramp is inclined at an angle of 35.0° above the horizontal. If the seal reaches the water with a speed of 4.40 m/s, what are **(a)** the work done by kinetic friction and **(b)** the coefficient of kinetic friction between the seal and the ramp?

48. •• A 1.9-kg rock is released from rest at the surface of a pond 1.8 m deep. As the rock falls, a constant upward force of 4.6 N is exerted on it by water resistance. Calculate the nonconservative work, W_{nc}, done by water resistance on the rock, the gravitational potential energy of the system, U, the kinetic energy of the rock, K, and the total mechanical energy of the system, E, when the depth of the rock below the water's surface is **(a)** 0 m, **(b)** 0.50 m, and **(c)** 1.0 m. Let $y = 0$ be at the bottom of the pond.

49. •• A 1250-kg car drives up a hill that is 16.2 m high. During the drive, two nonconservative forces do work on the car: (i) the force of friction, and (ii) the force generated by the car's engine. The work done by friction is -3.11×10^5 J; the work done by the engine is $+6.44 \times 10^5$ J. Find the change in the car's kinetic energy from the bottom of the hill to the top of the hill.

50. •• **IP** An 81.0-kg in-line skater does +3420 J of nonconservative work by pushing against the ground with his skates. In addition, friction does −715 J of nonconservative work on the skater. The skater's initial and final speeds are 2.50 m/s and 1.22 m/s, respectively. **(a)** Has the skater gone uphill, downhill, or remained at the same level? Explain. **(b)** Calculate the change in height of the skater.

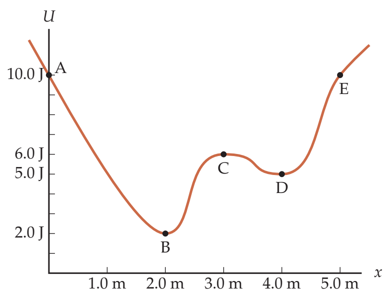
51. •• In Example 8–10, suppose the two masses start from rest and are moving with a speed of 2.05 m/s just before m_2 hits the floor. **(a)** If the coefficient of kinetic friction is $\mu_k = 0.350$, what is the distance of travel, d, for the masses? **(b)** How much conservative work was done on this system? **(c)** How much nonconservative work was done on this system? **(d)** Verify the three work relations given in Equations 8–10.

52. •• **IP** A 15,800-kg truck is moving at 12.0 m/s when it starts down a 6.00° incline in the Canadian Rockies. At the start of the descent the driver notices that the altitude is 1630 m. When she reaches an altitude of 1440 m, her speed is 29.0 m/s. Find the change in **(a)** the gravitational potential energy of the system and **(b)** the truck's kinetic energy. **(c)** Is the total mechanical energy of the system conserved? Explain.

53. ••• A 1.80-kg block slides on a rough horizontal surface. The block hits a spring with a speed of 2.00 m/s and compresses it a distance of 11.0 cm before coming to rest. If the coefficient of kinetic friction between the block and the surface is $\mu_k = 0.560$, what is the force constant of the spring?

SECTION 8–5 POTENTIAL ENERGY CURVES AND EQUIPOTENTIALS

54. • **Figure 8–24** shows a potential energy curve as a function of x. In qualitative terms, describe the subsequent motion of an object that starts at rest at point A.



▲ **FIGURE 8–24** Problems 54, 55, 56, and 59

55. • An object moves along the x axis, subject to the potential energy shown in Figure 8–24. The object has a mass of 1.1 kg and starts at rest at point A. **(a)** What is the object's speed at point B? **(b)** At point C? **(c)** At point D? **(d)** What are the turning points for this object?

56. • A 1.34-kg object moves along the x axis, subject to the potential energy shown in Figure 8–24. If the object's speed at point C is 1.25 m/s, what are the approximate locations of its turning points?

57. • A 23-kg child swings back and forth on a swing suspended by 2.5-m-long ropes. Plot the gravitational potential energy of this system as a function of the angle the ropes make with the vertical, assuming the potential energy is zero when the ropes are vertical. Consider angles up to 90° on either side of the vertical.

58. •• Find the turning-point angles in the previous problem if the child has a speed of 0.89 m/s when the ropes are vertical. Indicate the turning points on a plot of the system's potential energy.

59. •• The potential energy of a particle moving along the x axis is shown in Figure 8–24. When the particle is at $x = 1.0$ m it has 3.6 J of kinetic energy. Give approximate answers to the following questions. **(a)** What is the total mechanical energy of the system? **(b)** What is the smallest value of x the particle can reach? **(c)** What is the largest value of x the particle can reach?

60. •• A block of mass $m = 0.95$ kg is connected to a spring of force constant $k = 775$ N/m on a smooth, horizontal surface. **(a)** Plot the potential energy of the spring from $x = -5.00$ cm to $x = 5.00$ cm. **(b)** Determine the turning points of the block if its speed at $x = 0$ is 1.3 m/s.

61. •• A ball of mass $m = 0.75$ kg is thrown straight upward with an initial speed of 8.9 m/s. **(a)** Plot the gravitational potential energy of the block from its launch height, $y = 0$, to the height $y = 5.0$ m. Let $U = 0$ correspond to $y = 0$. **(b)** Determine the turning point (maximum height) of this mass.

62. ••• Two blocks, each of mass m, are connected on a frictionless horizontal table by a spring of force constant k and equilibrium length L. Find the maximum and minimum separation between the two blocks in terms of their maximum speed, v_{max}, relative to the table. (The two blocks always move in opposite directions as they oscillate back and forth about a fixed position.)

GENERAL PROBLEMS

63. • **CE** You and a friend both solve a problem involving a skier going down a slope. When comparing solutions, you notice that your choice for the $y = 0$ level is different than the $y = 0$ level chosen by your friend. Will your answers agree or disagree on the following quantities: **(a)** the skier's potential energy; **(b)** the skier's change in potential energy; **(c)** the skier's kinetic energy?

64. • **CE** A particle moves under the influence of a conservative force. At point A the particle has a kinetic energy of 12 J; at point B the particle is momentarily at rest, and the potential energy of the system is 25 J; at point C the potential energy of the system is 5 J. **(a)** What is the potential energy of the system when the particle is at point A? **(b)** What is the kinetic energy of the particle at point C?

65. • **CE** A leaf falls to the ground with constant speed. Is $U_i + K_i$ for this system greater than, less than, or the same as $U_f + K_f$ for this system? Explain.

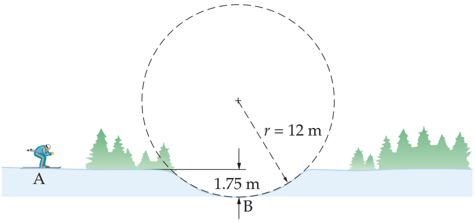
66. • **CE** Consider the two-block system shown in Example 8–10. **(a)** As block 2 descends through the distance d, does its mechanical energy increase, decrease, or stay the same? Explain. **(b)** Is the nonconservative work done on block 2 by the tension in the rope positive, negative, or zero? Explain.

67. •• **CE** Taking a leap of faith, a bungee jumper steps off a platform and falls until the cord brings her to rest. Suppose you analyze this system by choosing $y = 0$ at the platform level, and your friend chooses $y = 0$ at ground level. **(a)** Is the jumper's initial potential energy in your calculation greater than, less than, or equal to the same quantity in your friend's calculation? Explain. **(b)** Is the change in the jumper's potential energy in your calculation greater than, less than, or equal to the same quantity in your friend's calculation? Explain.

68. •• **IP** A sled slides without friction down a small, ice-covered hill. If the sled starts from rest at the top of the hill, its speed at the bottom is 7.50 m/s. **(a)** On a second run, the sled starts with a speed of 1.50 m/s at the top. When it reaches the bottom of the hill, is its speed 9.00 m/s, more than 9.00 m/s, or less than 9.00 m/s? Explain. **(b)** Find the speed of the sled at the bottom of the hill after the second run.

69. •• In the previous problem, what is the height of the hill?

70. •• A 68-kg skier encounters a dip in the snow's surface that has a circular cross section with a radius of curvature of 12 m. If the skier's speed at point A in **Figure 8–25** is 8.0 m/s, what is the normal force exerted by the snow on the skier at point B? Ignore frictional forces.



▲ **FIGURE 8–25** Problem 70

71. •• **Running Shoes** The soles of a popular make of running shoe have a force constant of 2.0×10^5 N/m. Treat the soles as ideal springs for the following questions. **(a)** If a 62-kg person stands in a pair of these shoes, with her weight distributed equally on both feet, how much does she compress the soles? **(b)** How much energy is stored in the soles of her shoes when she's standing?

72. •• **Nasal Strips** The force required to flex a nasal strip and apply it to the nose is 0.25 N; the energy stored in the strip when flexed is 0.0022 J. Assume the strip to be an ideal spring for the following calculations. Find **(a)** the distance through which the strip is flexed and **(b)** the force constant of the strip.

73. •• **IP** A pendulum bob with a mass of 0.13 kg is attached to a string with a length of 0.95 m. We choose the potential energy to be zero when the string makes an angle of 90° with the vertical. **(a)** Find the potential energy of this system when the string makes an angle of 45° with the vertical. **(b)** Is the magnitude of the change in potential energy from an angle of 90° to 45° greater than, less than, or the same as the magnitude of the change from 45° to 0°? Explain. **(c)** Calculate the potential energy of the system when the string is vertical.

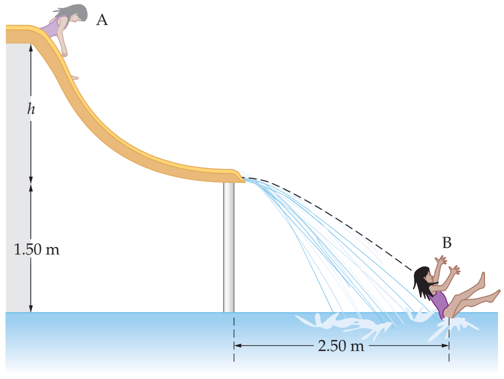
74. •• Suppose the pendulum bob in Figure 8–20 has a mass of 0.25 kg. **(a)** How much work does gravity do on the bob as it moves from point A to point B? **(b)** From point B to point A? **(c)** How much work does the string do on the bob as it moves from point A to point B? **(d)** From point B to point A?

75. •• An 1865-kg airplane starts at rest on an airport runway at sea level. **(a)** What is the change in mechanical energy of the airplane if it climbs to a cruising altitude of 2420 m and maintains a constant speed of 96.5 m/s? **(b)** What cruising speed would the plane need at this altitude if its increase in kinetic energy is to be equal to its increase in potential energy?

76. •• **IP** At the local playground a child on a swing has a speed of 2.02 m/s when the swing is at its lowest point. **(a)** To what maximum vertical height does the child rise, assuming he sits still and "coasts"? Ignore air resistance. **(b)** How do your results change if the initial speed of the child is halved?

77. •• The water slide shown in **Figure 8–26** ends at a height of 1.50 m above the pool. If the person starts from rest at point A and lands in the water at point B, what is the height h of the water slide? (Assume the water slide is frictionless.)

78. •• If the height of the water slide in Figure 8–26 is $h = 3.2$ m, and the person's initial speed at point A is 0.54 m/s, what is the new horizontal distance between the base of the slide and the splashdown point of the person?

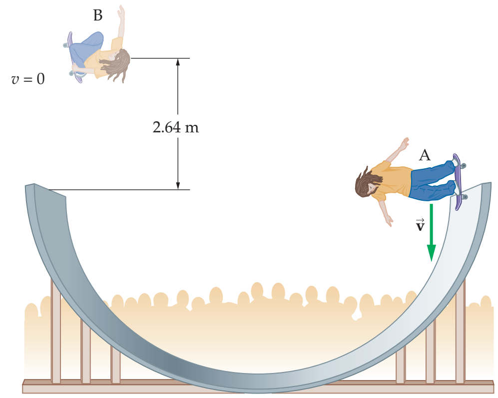


▲ **FIGURE 8–26** Problems 77 and 78

79. •• **IP** A person is to be released from rest on a swing pulled away from the vertical by an angle of 20.0°. The two frayed ropes of the swing are 2.75 m long, and will break if the tension in either of them exceeds 355 N. **(a)** What is the maximum weight the person can have and not break the ropes? **(b)** If the person is released at an angle greater than 20.0°, does the maximum weight increase, decrease, or stay the same? Explain.

80. •• **IP** A car is coasting without friction toward a hill of height h and radius of curvature r. **(a)** What initial speed, v_0, will result in the car's wheels just losing contact with the roadway as the car crests the hill? **(b)** What happens if the initial speed of the car is greater than the value found in part (a)?

81. •• A skateboarder starts at point A in **Figure 8–27** and rises to a height of 2.64 m above the top of the ramp at point B. What was the skateboarder's initial speed at point A?



▲ **FIGURE 8–27** Problem 81

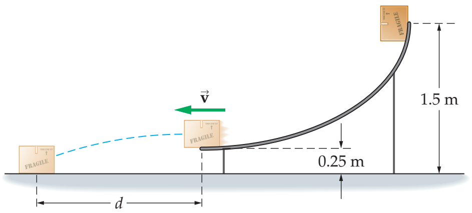
82. •• In the Atwood's machine of Problem 36, the mass m_2 remains at rest once it hits the floor, but the mass m_1 continues moving upward. How much higher does m_1 go after m_2 has landed? Give your answer for the case $h = 1.2$ m, $m_1 = 3.7$ kg, and $m_2 = 4.1$ kg.

83. •• An 8.70-kg block slides with an initial speed of 1.56 m/s up a ramp inclined at an angle of 28.4° with the horizontal. The coefficient of kinetic friction between the block and the ramp is 0.62. Use energy conservation to find the distance the block slides before coming to rest.

84. •• Repeat the previous problem for the case of an 8.70-kg block sliding down the ramp, with an initial speed of 1.56 m/s.

85. •• Jeff of the Jungle swings on a 7.6-m vine that initially makes an angle of 37° with the vertical. If Jeff starts at rest and has a mass of 78 kg, what is the tension in the vine at the lowest point of the swing?

86. •• A 1.9-kg block slides down a frictionless ramp, as shown in **Figure 8–28**. The top of the ramp is 1.5 m above the ground; the bottom of the ramp is 0.25 m above the ground. The block leaves the ramp moving horizontally, and lands a horizontal distance d away. Find the distance d.



▲ **FIGURE 8–28** Problems 86 and 87

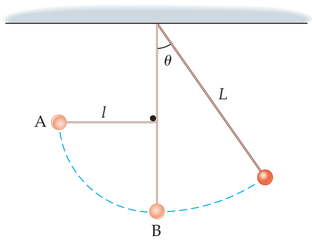
87. •• Suppose the ramp in Figure 8–28 is not frictionless. Find the distance d for the case in which friction on the ramp does −9.7 J of work on the block before it becomes airborne.

88. •• **BIO Compressing the Ground** A running track at Harvard University uses a surface with a force constant of 2.5×10^5 N/m. This surface is compressed slightly every time a runner's foot lands on it. The force exerted by the foot, according to the Saucony shoe company, has a magnitude of 2700 N for a typical runner. Treating the track's surface as an ideal spring, find **(a)** the amount of compression caused by a foot hitting the track and **(b)** the energy stored briefly in the track every time a foot lands.

89. •• **BIO A Flea's Jump** The resilin in the upper leg (coxa) of a flea has a force constant of about 26 N/m, and when the flea cocks its jumping legs, the resilin in each leg is stretched by approximately 0.10 mm. Given that the flea has a mass of 0.50 mg, and that two legs are used in a jump, estimate the maximum height a flea can attain by using the energy stored in the resilin. (Assume the resilin to be an ideal spring.)

90. ••• **IP** A trapeze artist of mass m swings on a rope of length L. Initially, the trapeze artist is at rest and the rope makes an angle θ with the vertical. **(a)** Find the tension in the rope when it is vertical. **(b)** Explain why your result for part (a) depends on L in the way it does.

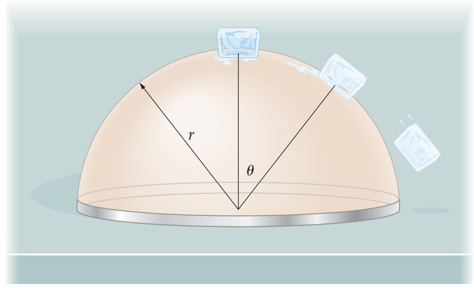
91. ••• **IP Tension at the Bottom** A ball of mass m is attached to a string of length L and released from rest at the point A in **Figure 8–29**. **(a)** Show that the tension in the string when the ball reaches point B is $3mg$, independent of the length l. **(b)** Give a detailed physical explanation for the fact that the tension at point B is independent of the length l.



▲ **FIGURE 8–29** Problems 91 and 92

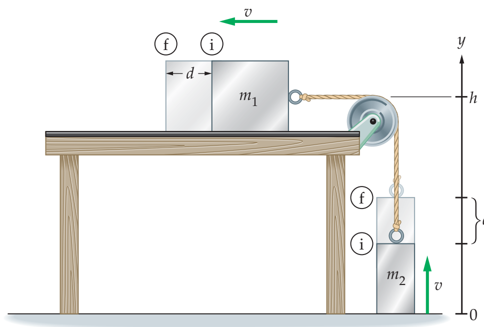
92. ••• **IP** In Figure 8–29, suppose that $L = 0.652$ m and $l = 0.325$ m. **(a)** Find the maximum angle the string makes with the vertical when the mass is released from rest at point A and swings as far to the right as it can. **(b)** At the point found in part (a), find the height of the mass above point B. Explain the physical significance of your result. **(c)** Give the angle of part (a) as a general expression in terms of L and l.

93. ••• An ice cube is placed on top of an overturned spherical bowl of radius r, as indicated in **Figure 8–30**. If the ice cube slides downward from rest at the top of the bowl, at what angle θ does it separate from the bowl? In other words, at what angle does the normal force between the ice cube and the bowl go to zero?



▲ **FIGURE 8–30** Problem 93

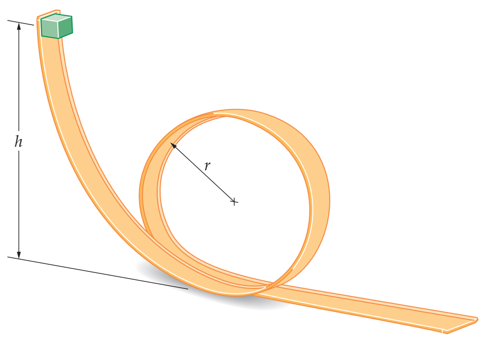
94. ••• **IP** The two blocks shown in **Figure 8–31** are moving with an initial speed v. **(a)** If the system is frictionless, find the distance d the blocks travel before coming to rest. (Let $U = 0$ correspond to the initial position of block 2.) **(b)** Is the work done on block 2 by the rope positive, negative, or zero? Explain. **(c)** Calculate the work done on block 2 by the rope.



▲ **FIGURE 8–31** Problems 94 and 95

95. ••• **IP** Consider the system shown in Figure 8–31. **(a)** What initial speed v is required if the blocks $m_1 = 2.4$ kg and $m_2 = 1.1$ kg are to travel a distance $d = 6.5$ cm before coming to rest? Assume the coefficient of kinetic friction between m_1 and the tabletop is $\mu_k = 0.25$. **(b)** Is the work done on m_2 by the rope positive, negative, or zero? Explain. **(c)** Calculate the work done on m_2 by the rope.

96. ••• **IP Loop-the-Loop** **(a)** A block of mass m slides from rest on a frictionless loop-the-loop track, as shown in **Figure 8–32**. What is the minimum release height, h, required for the block to maintain contact with the track at all times? Give your answer in terms of the radius of the loop, r. **(b)** Explain why the release height obtained in part (a) is independent of the block's mass.



▲ **FIGURE 8–32** Problem 96

97. ••• **Figure 8–33** shows a 1.75-kg block at rest on a ramp of height h. When the block is released, it slides without friction to the bottom of the ramp, and then continues across a surface that is frictionless except for a rough patch of width 10.0 cm that has a coefficient of kinetic friction $\mu_k = 0.640$. Find h such that the block's speed after crossing the rough patch is 3.50 m/s.

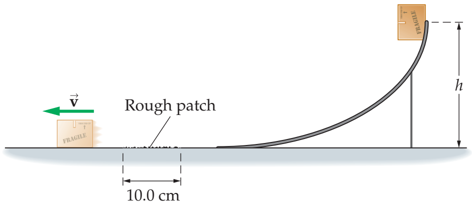


FIGURE 8–33 Problem 97

98. ••• In **Figure 8–34** a 1.2-kg block is held at rest against a spring with a force constant $k = 730$ N/m. Initially, the spring is compressed a distance d. When the block is released, it slides across a surface that is frictionless except for a rough patch of width 5.0 cm that has a coefficient of kinetic friction $\mu_k = 0.44$. Find d such that the block's speed after crossing the rough patch is 2.3 m/s.

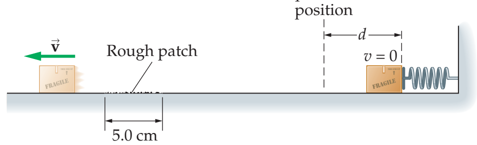


FIGURE 8–34 Problem 98

99. ••• **IP Using Work and Energy to Calculate Tension** Consider the Atwood's machine shown in Figure 8–23, with $h = 1.2$ m, $m_1 = 3.7$ kg, and $m_2 = 4.1$ kg. In this problem, we show how to calculate the tension in the rope using energy and work, rather than Newton's laws. **(a)** Is the change in mechanical energy for block 2 as it drops through the height h positive, negative, or zero? Explain. **(b)** Use energy conservation applied to the entire system to calculate the change in mechanical energy for block 2 as it drops through the height h. **(c)** Use your answer to part (b), and the known drop height, to find the magnitude of the tension in the rope.

PASSAGE PROBLEMS

BIO The Flight of the Dragonflies

Of all the animals you're likely to see on a summer's day, the most ancient is the dragonfly. In fact, the fossil record for dragonflies extends back over 250 million years, more than twice as long as for birds. Ancient dragonflies could be as large as a hawk, and were surely buzzing around the heads of both *T. Rex* and *Triceratops*.

Dragonflies belong to the order Odonata ("toothed jaws") and the suborder Anisoptera ("different wings"), a reference to the fact that their hindwings are wider front-to-back than their forewings. (Damselflies, in contrast, have forewings and hindwings that are the same.) Although ancient in their lineage, dragonflies are the fastest flying and most acrobatic of all insects; some of their maneuvers subject them to accelerations as great as $20g$.

The properties of dragonfly wings, and how they account for such speed and mobility, have been of great interest to biologists. **Figure 8–35 (a)** shows an experimental setup designed to measure the force constant of Plexiglas models of wings, which are used in wind tunnel tests. A downward force is applied to the model wing at the tip (1 for hindwing, 2 for forewing) or at two-thirds the distance to the tip (3 for hindwing, 4 for forewing). As the force is varied in magnitude, the resulting deflection of the wing is measured. The results are shown in **Figure 8–35 (b)**. Notice that significant differences are seen between the hindwings and forewings, as one might expect from their different shapes.

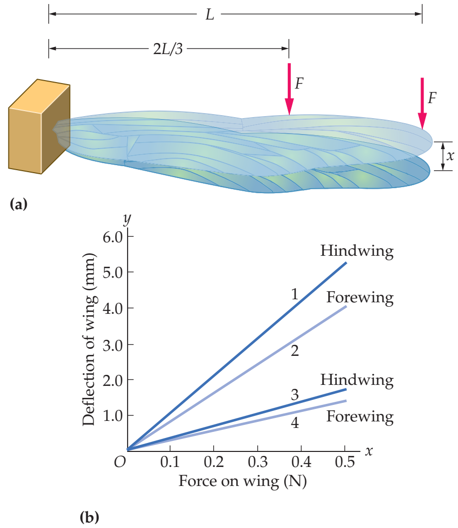


FIGURE 8–35 Problems 100, 101, 102, and 103

100. • Treating the model wing as an ideal spring, what is the force constant of the hindwing when a force is applied to its tip?

A. 94 N/m **B.** 130 N/m **C.** 290 N/m **D.** 330 N/m

101. • What is the force constant of the hindwing when a force is applied at two-thirds the distance from the base of the wing to the tip?

A. 94 N/m **B.** 130 N/m

C. 290 N/m **D.** 330 N/m

102. • Which of the wings is "stiffer"?

A. The hindwing. **B.** The forewing.

C. Depends on where the force is applied.

D. They are equally "stiff."

103. •• How much energy is stored in the forewing when a force at the tip deflects it by 3.5 mm?

A. 0.766 mJ **B.** 49.0 mJ **C.** 0.219 J **D.** 1.70 kJ

INTERACTIVE PROBLEMS

104. •• **IP Referring to Example 8–8** Consider a spring with a force constant of 955 N/m. **(a)** Suppose the mass of the block is 1.70 kg, but its initial speed can be varied. What initial speed is required to give a maximum spring compression of 4.00 cm? **(b)** Suppose the initial speed of the block is 1.09 m/s, but its mass can be varied. What mass is required to give a maximum spring compression of 4.00 cm?

105. •• **Referring to Example 8–8** Suppose the block is released from rest with the spring compressed 5.00 cm. The mass of the block is 1.70 kg and the force constant of the spring is 955 N/m. **(a)** What is the speed of the block when the spring expands to a compression of only 2.50 cm? **(b)** What is the speed of the block after it leaves the spring?

106. •• **Referring to Example 8–10** Suppose we would like the landing speed of block 2 to be increased to 1.50 m/s. **(a)** Should the coefficient of kinetic friction between block 1 and the tabletop be increased or decreased? **(b)** Find the required coefficient of kinetic friction for a landing speed of 1.50 m/s. Note that $m_1 = 2.40$ kg, $m_2 = 1.80$ kg, and $d = 0.500$ m.

PHYSICS IN

PERSPECTIVE

Energy: A Breakthrough in Physics

The concept of energy is a surprisingly recent addition to physics—in fact, Galileo and Newton knew nothing about it. Energy was difficult to discover because it can't be seen or touched, and because it takes so many different forms. Nevertheless, energy is central to our modern world.

1 Energy takes multiple forms

This scene shows a few of the myriad energy transformations that make life possible. Throughout the universe, energy continually changes form and moves from system to system.

Energy from the Sun travels to the Earth as sunlight.

Plants store solar energy in the chemical bonds of food molecules.

Animals convert food energy to many forms. Ultimately, most of it leaves the body as work done on the environment and as heat.

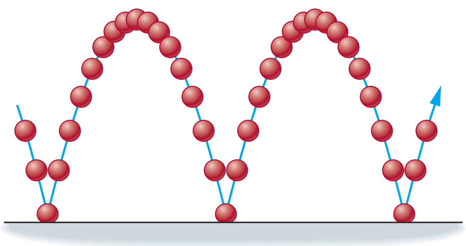
A leaping dog is a projectile, interconverting gravitational potential energy and kinetic energy. A thrown frisbee adds a new twist, trading part of its kinetic energy for lift, which does work to keep the frisbee airborne.

2 Energy is always conserved; mechanical energy is sometimes conserved

No energy is lost or gained during energy transfers and transformations. Therefore, the total energy of the universe is conserved (stays the same). This is a fundamental law of physics.

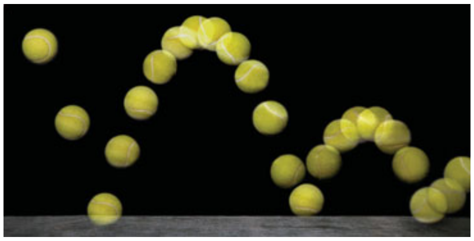
Although *energy* is always conserved, *mechanical energy* (kinetic and potential energy) is conserved only by conservative forces such as gravity. If nonconservative forces such as friction do work in a system, some mechanical energy is dissipated to other forms of energy.

Ideal bouncing ball: Energy and mechanical energy both conserved



If only gravity and spring forces did work on a bouncing ball, the ball would bounce to the same height forever—potential and kinetic energy would interconvert with no loss.

Real bouncing ball: Mechanical energy dissipated (but energy conserved)



In a real ball, air drag and friction within the ball gradually dissipate mechanical energy to thermal energy and other forms, so the ball loses height with each bounce.

3 ... But what *is* energy?

Energy is a very abstract concept—you can define it as *the scalar quantity that is conserved during energy transformations.* This seems hard to grasp until you realize that *money* is just as abstract and works in much the same way. Money can take many forms—cash, a checking account, a savings account—while its amount doesn't change. You can think of potential energy as money in the bank and of kinetic energy as cash.

The fact that energy is so abstract explains why this fundamental concept didn't become part of physics until about 200 years ago, 120 years *after* Newton formulated his laws.

4 Work done by conservative forces conserves mechanical energy, whereas work done by nonconservative forces dissipates it

Let us use the two cases shown below to explore the equations that relate work to mechanical energy within a given system:

$$W_{tot} = \Delta K \qquad W_c = -(\Delta U) \qquad W_{nc} = \Delta E_{mech}$$

For clarity, on these pages we denote mechanical energy by E_{mech} rather than the usual E.

- $W_{tot} = \Delta K$: This is the work–energy theorem.
- $W_c = -(\Delta U)$: Within a given system, only conservative forces can change the potential energy. *Positive* W_c increases the system's kinetic energy at the expense of potential energy—that is why the ΔU term has a minus sign. (*Negative* W_c results in an *increase* in potential energy.)
- $W_{nc} = \Delta E_{mech}$: Since nonconservative forces in a system act to dissipate mechanical energy to other forms of energy, the work done by these forces equals the change in mechanical energy.

Symbols used on this page:

E_{mech}, $E_{nonmech}$: Mechanical and nonmechanical energy, respectively

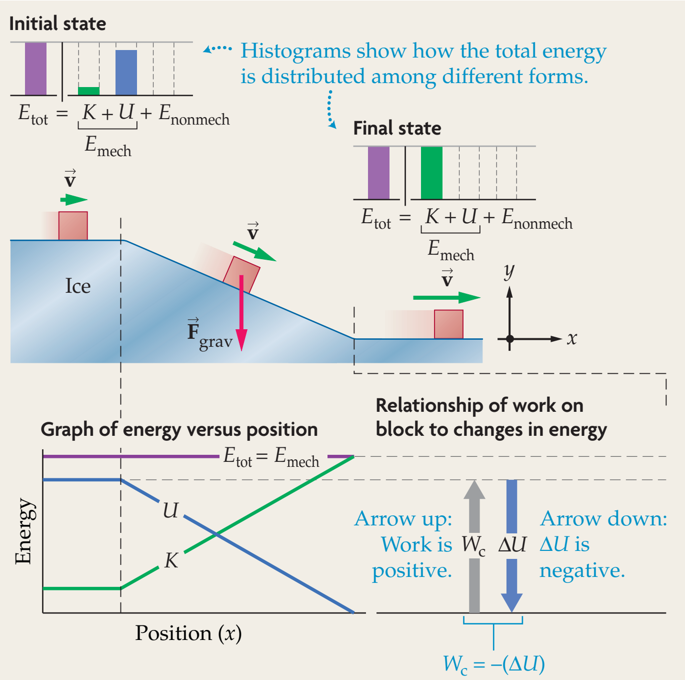
E_{tot}: Total energy (mechanical plus nonmechanical)

W_c, W_{nc}: Work done by conservative and nonconservative forces, respectively

W_{tot}: Total work ($W_c + W_{nc}$)

Case 1: Block slides down slippery slope

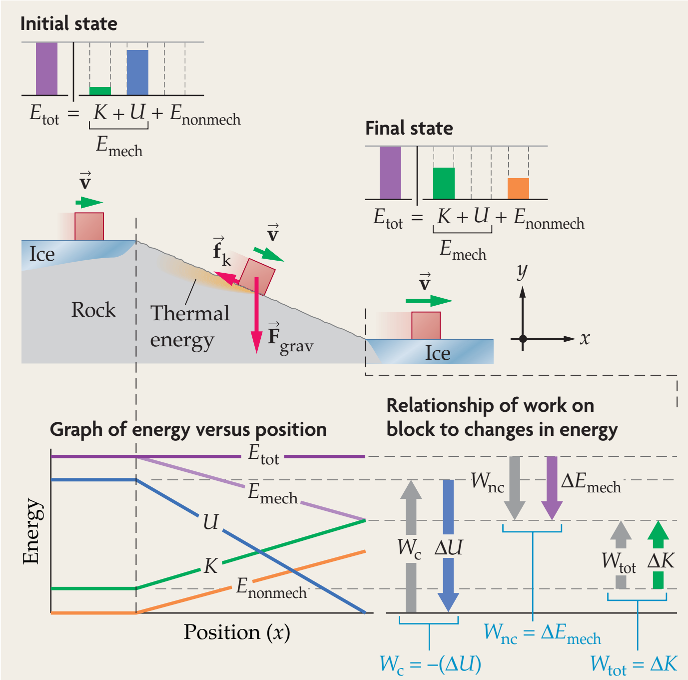
Only the conservative force of gravity does work on the block



- Initially the system has both kinetic and potential energy.
- The block slides down because the conservative force of gravity does positive work on it, changing the system's potential energy to kinetic energy. (We know the work is positive because the block speeds up.)
- In the final state, the system has only kinetic energy. Mechanical energy has been conserved.

Case 2: Block slides down slope with friction

In addition to gravity, nonconservative friction does work on the block



- Initially the system has both kinetic and potential energy.
- The block slides down because gravity does positive work on it, changing the system's potential energy to kinetic energy. At the same time, friction does negative work on the block, dissipating some of its kinetic energy to nonmechanical energy.
- The system's final energy is partly kinetic and partly nonmechanical. Energy has been conserved, but not mechanical energy.

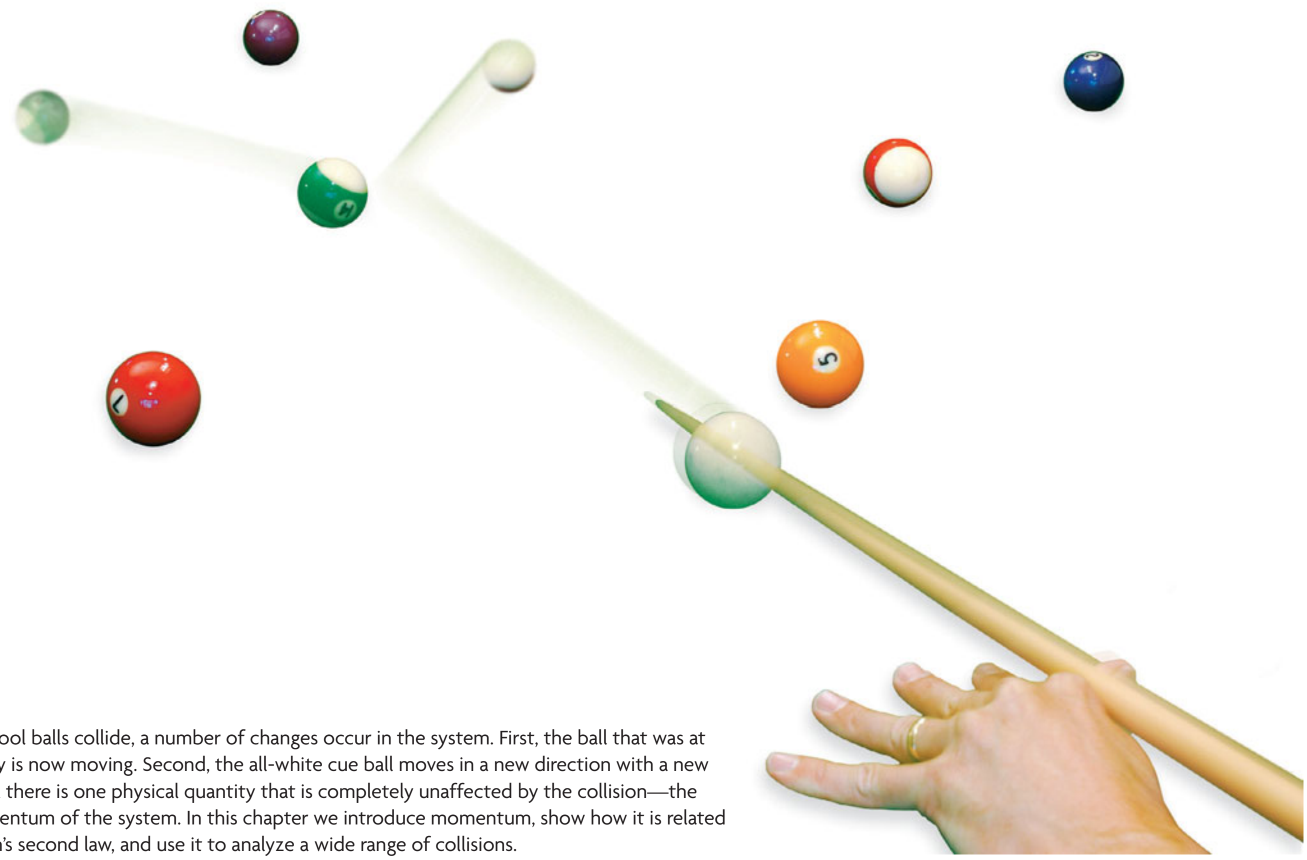
5 Conservation of mechanical energy is useful for solving problems

If nonconservative forces do no work in a system (or if the work they do is negligible compared to that done by conservative forces), you can use energy conservation to predict the system's behavior.

In the case of the water slides at right, it would be nearly impossible to predict a person's final speed using Newton's laws—you would need to know the net force acting at each position along the slide.

To apply energy conservation, all you need to know are the person's initial and final heights.

9 Linear Momentum and Collisions



As these pool balls collide, a number of changes occur in the system. First, the ball that was at rest initially is now moving. Second, the all-white cue ball moves in a new direction with a new speed. Still there is one physical quantity that is completely unaffected by the collision—the total momentum of the system. In this chapter we introduce momentum, show how it is related to Newton's second law, and use it to analyze a wide range of collisions.

Conservation laws play a central role in physics. In this chapter we introduce the concept of *momentum* and show that it, like energy, is a conserved quantity. Nothing we can do—in fact, nothing that can occur in nature—can change the total energy or the total momentum of the universe.

As with conservation of energy, we shall see that the conservation of momentum provides a powerful way of approaching a variety of problems that would be extremely difficult to solve using Newton's laws directly. In particular, problems involving the collision of two or more objects—such as a baseball bat striking a ball or one car bumping into another at an intersection—are especially well suited to a momentum approach. Finally, we introduce the concept of the *center of mass* and show that it allows us to extend many of the results that have been obtained for point particles to systems involving more realistic objects.

9–1 Linear Momentum

Imagine for a moment that you are sitting at rest on a skateboard that can roll without friction on a smooth surface. If you catch a heavy, slow-moving ball tossed to you by a friend, you begin to move. If, on the other hand, your friend tosses you a light, yet fast-moving ball, the net effect may be the same—that is, catching a lightweight ball moving fast enough will cause you to move with the same speed as when you caught the heavy ball.

In physics, the previous observations are made precise by defining a quantity called the **linear momentum**, $\vec{\mathbf{p}}$, which is defined as the product of the mass m and velocity $\vec{\mathbf{v}}$ of an object:

Definition of Linear Momentum, $\vec{\mathbf{p}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \tag{9–1}$$

SI unit: $\mathrm{kg \cdot m/s}$

In our example, if the heavy ball has twice the mass of the light ball but the light ball has twice the speed of the heavy ball, the momenta of the two balls are equal in magnitude. We can see from Equation 9–1 that the units of linear momentum are simply the units of mass times the units of velocity: $\mathrm{kg \cdot m/s}$. There is no special shorthand name given to this combination of units.

It is important to note that a constant *linear* momentum $\vec{\mathbf{p}}$ is the momentum of an object of mass m that is *moving in a straight line* with a velocity $\vec{\mathbf{v}}$. In Chapter 11 we introduce a similar quantity to describe the momentum of an object that rotates. This momentum will be referred to as the *angular momentum*. In general, when we simply say momentum, we are referring to the linear momentum $\vec{\mathbf{p}}$. We will always specify angular momentum when referring to the momentum associated with rotation.

Because the velocity $\vec{\mathbf{v}}$ is a vector with both a magnitude and a direction, so too is the momentum, $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$. The next Exercise gives some feeling for the *magnitude* of the momentum, $p = mv$, for everyday objects.

EXERCISE 9–1

(a) A 1180-kg car drives along a city street at 30.0 miles per hour (13.4 m/s). What is the magnitude of the car's momentum? **(b)** A major-league pitcher can give a 0.142-kg baseball a speed of 101 mi/h (45.1 m/s). Find the magnitude of the baseball's momentum.

SOLUTION

a. Using $p = mv$, we find

$$p_c = m_c v_c = (1180\ \mathrm{kg})(13.4\ \mathrm{m/s}) = 15{,}800\ \mathrm{kg \cdot m/s}$$

b. Similarly,

$$p_b = m_b v_b = (0.142\ \mathrm{kg})(45.1\ \mathrm{m/s}) = 6.40\ \mathrm{kg \cdot m/s}$$

As an illustration of the vector nature of momentum, consider the situations shown in **Figures 9–1 (a)** and **(b)**. In Figure 9–1 (a), a 0.10-kg beanbag bear is dropped to the floor, where it hits with a speed of 4.0 m/s and sticks. In Figure 9–1 (b) a 0.10-kg rubber ball also hits the floor with a speed of 4.0 m/s, but in this case the ball bounces upward off the floor. Assuming an ideal rubber ball, its initial upward speed is 4.0 m/s. Now the question in each case is, "What is the change in momentum?"

To approach the problem systematically, we introduce a coordinate system as shown in Figure 9–1. With this choice, we can see that neither object has momentum in the x direction; thus we need only consider the y component of momentum, p_y. The problem, therefore, is one-dimensional; still, we must be careful about the sign of p_y.

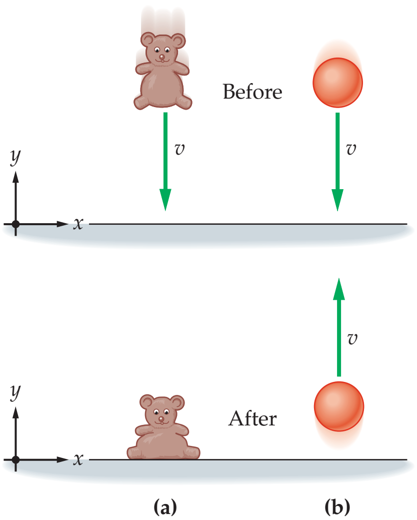


FIGURE 9–1 Change in momentum
A beanbag bear and a rubber ball, with the same mass m and the same downward speed v, hit the floor. **(a)** The beanbag bear comes to rest on hitting the floor. Its change in momentum is mv upward. **(b)** The rubber ball bounces upward with a speed v. Its change in momentum is $2mv$ upward.

We begin with the beanbag. Just before hitting the floor, it moves downward (that is, in the negative y direction) with a speed of $v = 4.0\ \mathrm{m/s}$. Letting m stand for the mass of the beanbag, we find that the initial momentum is

$$p_{y,\mathrm{i}} = m(-v)$$

After landing on the floor, the beanbag is at rest; hence, its final momentum is zero:

$$p_{y,\mathrm{f}} = m(0) = 0$$

Therefore the change in momentum is

$$\begin{aligned}\Delta p_y &= p_{y,\mathrm{f}} - p_{y,\mathrm{i}} = 0 - m(-v) = mv \\ &= (0.10\ \mathrm{kg})(4.0\ \mathrm{m/s}) = 0.40\ \mathrm{kg \cdot m/s}\end{aligned}$$

Note that the change in momentum is positive—that is, in the upward direction. This makes sense because, before the bag landed, it had a negative (downward) momentum in the y direction. In order to increase the momentum from a negative value to zero, it is necessary to add a positive (upward) momentum.

Next, consider the rubber ball in Figure 9–1 (b). Before bouncing, its momentum is

$$p_{y,\mathrm{i}} = m(-v)$$

the same as for the beanbag. After bouncing, when the ball is moving in the upward (positive) direction, its momentum is

$$p_{y,\mathrm{f}} = mv$$

PROBLEM-SOLVING NOTE

Coordinate Systems

Be sure to draw a coordinate system for momentum problems, even if the problem is only one-dimensional. It is important to use the coordinate system to assign the correct sign to velocities and momenta in the system.

As a result, the change in momentum for the rubber ball is

$$\begin{aligned}\Delta p_y &= p_{y,\mathrm{f}} - p_{y,\mathrm{i}} = mv - m(-v) = 2mv \\ &= 2(0.10\ \mathrm{kg})(4.0\ \mathrm{m/s}) = 0.80\ \mathrm{kg \cdot m/s}\end{aligned}$$

This is *twice* the change in momentum of the beanbag! The reason is that in this case, the momentum in the y direction must first be increased from $-mv$ to 0, then increased again from 0 to mv. For the beanbag, the change was merely from $-mv$ to 0.

Note how important it is to be careful about the vector nature of the momentum and to use the correct sign for p_y. Otherwise, we might have concluded—erroneously—that the rubber ball had zero change in momentum, since the *magnitude* of its momentum was unchanged by the bounce. In fact, its momentum does change due to the change in its *direction* of motion.

One additional point: Since momentum is a vector, the total momentum of a system of objects is the *vector* sum of the momenta of all the objects. That is,

$$\vec{\mathbf{p}}_{\mathrm{total}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 + \cdots \qquad 9\text{–}2$$

This is illustrated for the case of three objects in the following Example.

EXAMPLE 9–1 DUCK, DUCK, GOOSE: ADDING MOMENTA

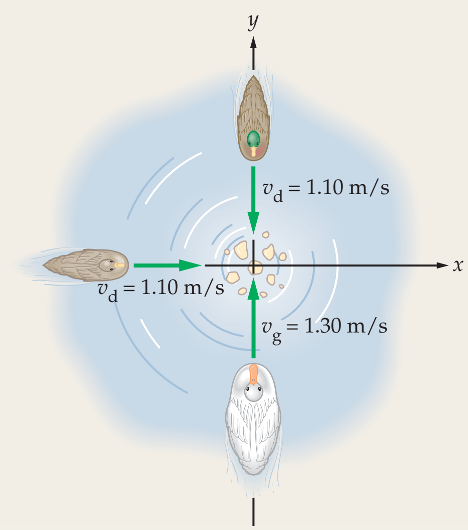
At a city park, a person throws some bread into a duck pond. Two 4.00-kg ducks and a 9.00-kg goose paddle rapidly toward the bread, as shown in our sketch. If the ducks swim at 1.10 m/s, and the goose swims with a speed of 1.30 m/s, find the magnitude and direction of the total momentum of the three birds.

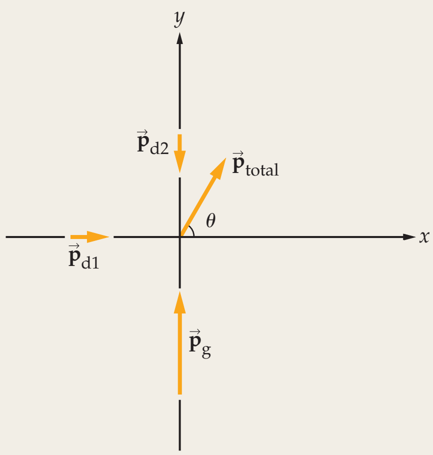
PICTURE THE PROBLEM

In our sketch we place the origin where the bread floats on the water. Note that duck 1 swims in the positive x direction, duck 2 swims in the negative y direction, and the goose swims in the positive y direction. Therefore, $\vec{\mathbf{p}}_{\mathrm{d1}} = m_{\mathrm{d}}v_{\mathrm{d}}\hat{\mathbf{x}}$, $\vec{\mathbf{p}}_{\mathrm{d2}} = -m_{\mathrm{d}}v_{\mathrm{d}}\hat{\mathbf{y}}$, and $\vec{\mathbf{p}}_{\mathrm{g}} = m_{\mathrm{g}}v_{\mathrm{g}}\hat{\mathbf{y}}$, where $v_{\mathrm{d}} = 1.10\ \mathrm{m/s}$, $m_{\mathrm{d}} = 4.00\ \mathrm{kg}$, $v_{\mathrm{g}} = 1.30\ \mathrm{m/s}$, and $m_{\mathrm{g}} = 9.00\ \mathrm{kg}$. The total momentum, $\vec{\mathbf{p}}_{\mathrm{total}}$, points at an angle θ relative to the positive x axis.

STRATEGY

Write the momentum of each bird as a vector, using unit vectors in the x and y directions. Next, sum these vectors component by component to find the total momentum. Finally, use the components of the total momentum to calculate its magnitude and direction.





SOLUTION

1. Use x and y unit vectors to express the momentum of each bird in vector form:

$$\vec{p}_{d1} = m_d v_d \hat{x} = (4.00\text{ kg})(1.10\text{ m/s})\hat{x} = (4.40\text{ kg}\cdot\text{m/s})\hat{x}$$

$$\vec{p}_{d2} = -m_d v_d \hat{y} = -(4.00\text{ kg})(1.10\text{ m/s})\hat{y} = -(4.40\text{ kg}\cdot\text{m/s})\hat{y}$$

$$\vec{p}_g = m_g v_g \hat{y} = (9.00\text{ kg})(1.30\text{ m/s})\hat{y} = (11.7\text{ kg}\cdot\text{m/s})\hat{y}$$

2. Sum the momentum vectors to obtain the total momentum:

$$\begin{aligned}\vec{p}_{total} &= \vec{p}_{d1} + \vec{p}_{d2} + \vec{p}_g \\ &= (4.40\text{ kg}\cdot\text{m/s})\hat{x} + [-4.40\text{ kg}\cdot\text{m/s} + 11.7\text{ kg}\cdot\text{m/s}]\hat{y} \\ &= (4.40\text{ kg}\cdot\text{m/s})\hat{x} + (7.30\text{ kg}\cdot\text{m/s})\hat{y}\end{aligned}$$

3. Calculate the magnitude of the total momentum:

$$\begin{aligned}p_{total} &= \sqrt{p_{total,x}^2 + p_{total,y}^2} \\ &= \sqrt{(4.40\text{ kg}\cdot\text{m/s})^2 + (7.30\text{ kg}\cdot\text{m/s})^2} \\ &= 8.52\text{ kg}\cdot\text{m/s}\end{aligned}$$

4. Calculate the direction of the total momentum:

$$\theta = \tan^{-1}\left(\frac{p_{total,y}}{p_{total,x}}\right) = \tan^{-1}\left(\frac{7.30\text{ kg}\cdot\text{m/s}}{4.40\text{ kg}\cdot\text{m/s}}\right) = 58.9°$$

INSIGHT

Note that the momentum of each bird depends only on its mass and velocity; it is independent of the bird's location. In addition, we observe that the magnitude of the total momentum is less than the sum of the magnitudes of each bird's momentum individually. This is generally the case when dealing with vector addition—the only exception is when all vectors point in the same direction.

PRACTICE PROBLEM

Should the speed of the goose be increased or decreased if the total momentum of the three birds is to point in the positive x direction? Verify your answer by calculating the required speed. [**Answer:** The goose's speed must be decreased. Setting the momentum of the goose equal to minus the momentum of duck 2 yields $v_g = 0.489$ m/s.]

Some related homework problems: Problem 1, Problem 2, Problem 3

9–2 Momentum and Newton's Second Law

In Chapter 5 we introduced Newton's second law:

$$\sum \vec{F} = m\vec{a}$$

As mentioned, this expression is valid only for objects that have constant mass. The more general law, which holds even if the mass changes, is expressed in terms of momentum. In fact, Newton's original statement of the second law was in just this form:

Newton's Second Law

$$\sum \vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t} \qquad 9\text{–}3$$

That is, the net force acting on an object is equal to the change in its momentum divided by the time interval during which the change occurs—in other words, the net force is the rate of change of momentum with time.

To show the connection between these two statements of the second law, consider the change in momentum, $\Delta \vec{\mathbf{p}}$. Since $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, we have

$$\Delta \vec{\mathbf{p}} = \vec{\mathbf{p}}_{\mathrm{f}} - \vec{\mathbf{p}}_{\mathrm{i}} = m_{\mathrm{f}}\vec{\mathbf{v}}_{\mathrm{f}} - m_{\mathrm{i}}\vec{\mathbf{v}}_{\mathrm{i}}$$

However, if the mass is constant, so that $m_{\mathrm{f}} = m_{\mathrm{i}} = m$, it follows that the change in momentum is simply m times $\Delta \vec{\mathbf{v}}$:

$$\Delta \vec{\mathbf{p}} = m_{\mathrm{f}}\vec{\mathbf{v}}_{\mathrm{f}} - m_{\mathrm{i}}\vec{\mathbf{v}}_{\mathrm{i}} = m(\vec{\mathbf{v}}_{\mathrm{f}} - \vec{\mathbf{v}}_{\mathrm{i}}) = m\Delta \vec{\mathbf{v}}$$

As a result, Newton's second law, for objects of constant mass, can be written as follows:

$$\sum \vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t} = m\frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

Finally, recall that acceleration is the rate of change of velocity with time:

$$\vec{\mathbf{a}} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

Therefore, we can write Equation 9–3 as

$$\sum \vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t} = m\vec{\mathbf{a}} \qquad 9\text{–}4$$

Hence, the two statements are equivalent if the mass is constant.

It should be noted, however, that $\Sigma \vec{\mathbf{F}} = \Delta \vec{\mathbf{p}}/\Delta t$ is the general form of Newton's second law, and that it is valid no matter how the mass may vary. In the remainder of this chapter we use this form of the second law to investigate the connections between forces and changes in momentum.

9–3 Impulse

The pitcher delivers a fastball, the batter takes a swing, and with a crack of the bat the ball that was approaching home plate at 95.0 mi/h is now heading toward the pitcher at 115 mi/h. In the language of physics, we say that the bat has delivered an **impulse,** $\vec{\mathbf{I}}$, to the ball.

During the brief time the ball and bat are in contact—perhaps as little as a thousandth of a second—the force between them rises rapidly to a large value, as shown in **Figure 9–2**, then falls back to zero as the ball takes flight. It would be almost impossible, of course, to describe every detail of the way the force varies with time. Instead, we focus on the average force exerted by the bat, $\vec{\mathbf{F}}_{\mathrm{av}}$, which is also shown in Figure 9–2. The impulse, then, is defined to be $\vec{\mathbf{F}}_{\mathrm{av}}$ times the length of time, Δt, that the ball and bat are in contact, which is simply the area under the force-versus-time curve:

Definition of Impulse, $\vec{\mathbf{I}}$

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\mathrm{av}}\,\Delta t \qquad 9\text{–}5$$

SI unit: $\mathrm{N \cdot s = kg \cdot m/s}$

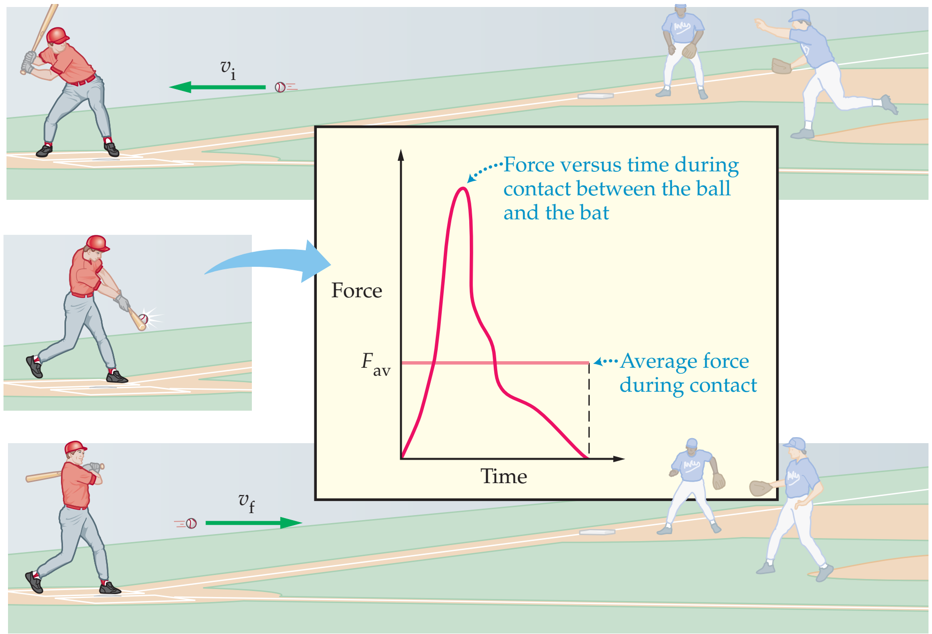


FIGURE 9–2 The average force during a collision

The force between two objects that collide, as when a bat hits a baseball, rises rapidly to very large values, then drops again to zero in a matter of milliseconds. Rather than try to describe the complex behavior of the force, we focus on its average value, F_{av}. Note that the area under the F_{av} rectangle is the same as the area under the actual force curve.

Note that impulse is a vector and that it points in the same direction as the average force. In addition, its units are $\mathrm{N \cdot s} = (\mathrm{kg \cdot m/s^2}) \cdot \mathrm{s} = \mathrm{kg \cdot m/s}$, the same as the units of momentum.

It is no accident that impulse and momentum have the same units. In fact, rearranging Newton's second law, Equation 9–3, we see that the average force times Δt is simply the change in momentum of the ball due to the bat:

$$\vec{\mathbf{F}}_{av} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$$

$$\vec{\mathbf{F}}_{av}\,\Delta t = \Delta\vec{\mathbf{p}}$$

Hence, in general, impulse is just the change in momentum:

Momentum–Impulse Theorem

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{av}\,\Delta t = \Delta\vec{\mathbf{p}} \qquad 9\text{–}6$$

For instance, if we know the impulse delivered to an object—that is, its change in momentum—and the time interval during which the change occurs, we can find the average force that caused the impulse.

As an example, let's calculate the impulse given to the baseball considered at the beginning of this section, as well as the average force between the ball and the bat. First, set up a coordinate system with the positive x axis pointing from home plate toward the pitcher's mound, as indicated in **Figure 9–3**. If the ball's mass is 0.145 kg, its initial momentum—which is in the negative x direction—is

$$\vec{\mathbf{p}}_i = -mv_i\hat{\mathbf{x}} = -(0.145\ \mathrm{kg})(95.0\ \mathrm{mi/h})\left(\frac{0.447\ \mathrm{m/s}}{1\ \mathrm{mi/h}}\right)\hat{\mathbf{x}} = -(6.16\ \mathrm{kg \cdot m/s})\hat{\mathbf{x}}$$

Immediately after the hit, the ball's final momentum is in the positive x direction:

$$\vec{\mathbf{p}}_f = mv_f\hat{\mathbf{x}} = (0.145\ \mathrm{kg})(115\ \mathrm{mi/h})\left(\frac{0.447\ \mathrm{m/s}}{1\ \mathrm{mi/h}}\right)\hat{\mathbf{x}} = (7.45\ \mathrm{kg \cdot m/s})\hat{\mathbf{x}}$$

The impulse, then, is

$$\vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = [7.45\ \mathrm{kg \cdot m/s} - (-6.16\ \mathrm{kg \cdot m/s})]\hat{\mathbf{x}} = (13.6\ \mathrm{kg \cdot m/s})\hat{\mathbf{x}}$$

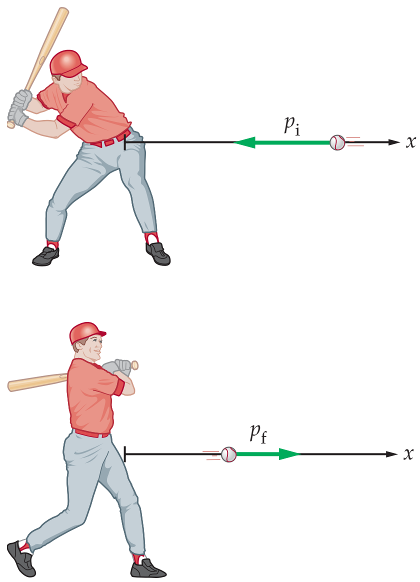


FIGURE 9–3 Hitting a baseball

A batter hits a ball, sending it back toward the pitcher's mound. The impulse delivered to the ball by the bat changes the ball's momentum from $-p_i\hat{\mathbf{x}}$ to $p_f\hat{\mathbf{x}}$.

REAL-WORLD PHYSICS

The force between a ball and a bat

▲ When a softball is hit by a bat (top), an enormous force (thousands of newtons) acts for a very short period of time—perhaps only a few ms. During this time, the ball is dramatically deformed by the impact. To keep the same thing from happening to a pole vaulter, who must fall nearly 20 feet after clearing the bar (bottom), a deeply padded landing area is provided. The change in the pole vaulter's momentum as he is brought to a stop, $mv = F\Delta t$, is the same whether he lands on a mat or on concrete. However, the padding is very yielding, greatly prolonging the time Δt during which he is in contact with the mat. The corresponding force on the vaulter is thus markedly decreased.

If the ball and bat are in contact for 1.20 ms $= 1.20 \times 10^{-3}$ s, a typical time, the average force is

$$\vec{\mathbf{F}}_{\text{av}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{\vec{\mathbf{I}}}{\Delta t} = \frac{(13.6\ \text{kg}\cdot\text{m/s})\hat{\mathbf{x}}}{1.20 \times 10^{-3}\ \text{s}} = (1.13 \times 10^{4}\ \text{N})\hat{\mathbf{x}}$$

Note that the average force is in the positive x direction; that is, toward the pitcher, as expected. In addition, the magnitude of the average force is remarkably large. In everyday units, the force between the ball and the bat is more than 2500 pounds! This explains why the ball is observed in high-speed photographs to deform significantly during a hit—the force is so large that, for an instant, it partially flattens the ball. Finally, notice that the weight of the ball, which is only about 0.3 lb, is completely negligible compared to the forces involved during the hit.

In problems that are strictly one-dimensional, we can drop the vector notation when dealing with impulse. However, we must still be careful about the signs of the various quantities in the system. This is illustrated in the following Active Example.

ACTIVE EXAMPLE 9–1 FIND THE FINAL SPEED OF THE BALL

A 0.144-kg baseball is moving toward home plate with a speed of 43.0 m/s when it is bunted (hit softly). The bat exerts an average force of 6.50×10^{3} N on the ball for 1.30 ms. The average force is directed toward the pitcher, which we take to be the positive x direction. What is the final speed of the ball?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Relate change in momentum to impulse (Equation 9–5): $\Delta p = p_{\text{f}} - p_{\text{i}} = I = F_{\text{av}}\,\Delta t$
2. Solve for the final momentum: $p_{\text{f}} = F_{\text{av}}\,\Delta t + p_{\text{i}}$
3. Calculate the initial momentum: $p_{\text{i}} = -6.19\ \text{kg}\cdot\text{m/s}$
4. Calculate the impulse: $I = F_{\text{av}}\,\Delta t = 8.45\ \text{kg}\cdot\text{m/s}$
5. Use these results to find the final momentum: $p_{\text{f}} = 2.26\ \text{kg}\cdot\text{m/s}$
6. Divide by the mass to find the final velocity: $v_{\text{f}} = p_{\text{f}}/m = 15.7\ \text{m/s}$

INSIGHT

With our choice of coordinate system, we see that the initial momentum of the ball was in the negative x direction. The impulse applied to the ball, however, resulted in a final momentum (and velocity) in the positive x direction.

YOUR TURN

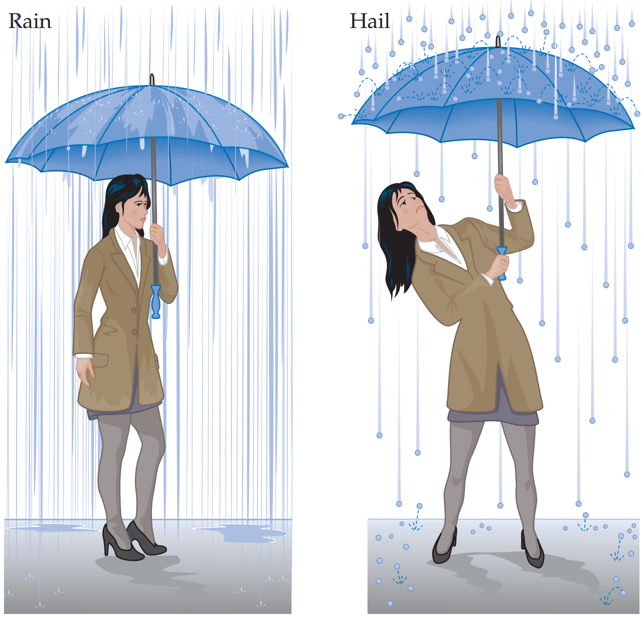
Suppose the bat is in contact with the ball for 2.60 ms rather than 1.30 ms. What is the final speed of the ball in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

We saw in Section 9–1 that the change in momentum is different for an object that hits something and sticks compared with an object that hits and bounces off. This means that the impulse, and hence the force, is different in the two cases. We explore this in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 9–1 RAIN VERSUS HAIL

A person stands under an umbrella during a rain shower. A few minutes later the raindrops turn to hail—though the number of "drops" hitting the umbrella per time and their speed remain the same. Is the force required to hold the umbrella in the hail **(a)** the same as, **(b)** more than, or **(c)** less than the force required in the rain?



▲ Most bats can take off simply by dropping from their perch on a branch or the ceiling of a cave, but vampire bats like this one must leap from the ground to become airborne. They do so by rocking forward onto their front limbs and then pushing off, using the extremely strong pectoral muscles that are also their main source of power in flight. Pushing downward on the ground, a bat experiences an upward reaction force exerted on it by the ground, with a corresponding impulse sufficient to propel it upward a considerable distance. In fact, a vampire bat can launch itself 1 m or more into the air in a mere 30 ms.

REASONING AND DISCUSSION

When raindrops strike the umbrella, they tend to splatter and run off; when hailstones hit the umbrella, they bounce back upward. As a result, the change in momentum is greater for the hail—just as the change in momentum is greater for a rubber ball bouncing off the floor than it is for a beanbag landing on the floor. Hence, the impulse and the force are greater with hail.

ANSWER

(b) The force is greater in the hail.

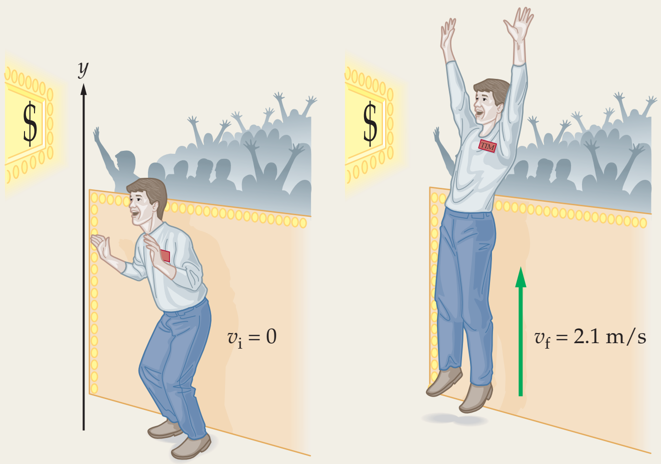
We conclude this section with an additional calculation involving impulse.

EXAMPLE 9–2 JUMPING FOR JOY

After winning a prize on a game show, a 72-kg contestant jumps for joy. **(a)** If the jump results in an upward speed of 2.1 m/s, what is the impulse experienced by the contestant? **(b)** Before the jump, the floor exerts an upward force of mg on the contestant. What additional average upward force does the floor exert if the contestant pushes down on it for 0.36 s during the jump?

PICTURE THE PROBLEM

Our sketch shows that the contestant's motion is purely one-dimensional, with a final speed of 2.1 m/s in the positive vertical direction. Note that we have chosen the positive y direction to be upward, therefore $\vec{\mathbf{v}}_i = 0$ and $\vec{\mathbf{v}}_f = (2.1\ \mathrm{m/s})\hat{\mathbf{y}}$.



STRATEGY

a. From the momentum–impulse theorem, we know that impulse is equal to the change in momentum. We are given the initial and final velocities of the contestant, and his mass as well; hence the change in momentum, $\Delta\vec{\mathbf{p}}$, can be calculated using the definition of momentum, $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$.

b. The average value of the additional force exerted on the contestant by the floor is $\Delta\vec{\mathbf{p}}/\Delta t$, where Δt is given as 0.36 s and $\Delta\vec{\mathbf{p}}$ is calculated in part (a).

SOLUTION

Part (a)

1. Write an expression for the impulse, noting that $\vec{\mathbf{v}}_i = 0$: $\quad \vec{\mathbf{I}} = \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = m\vec{\mathbf{v}}_f$

2. Substitute numerical values: $\quad \vec{\mathbf{I}} = m\vec{\mathbf{v}}_f = (72\ \mathrm{kg})(2.1\ \mathrm{m/s})\hat{\mathbf{y}} = (150\ \mathrm{kg\cdot m/s})\hat{\mathbf{y}}$

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Part (b)

3. Express the average force in terms of the impulse $\vec{\mathbf{I}}$ and the time interval Δt:

$$\vec{\mathbf{F}}_{\mathrm{av}} = \frac{\vec{\mathbf{I}}}{\Delta t} = \frac{(150\ \mathrm{kg \cdot m/s})\hat{\mathbf{y}}}{0.36\ \mathrm{s}} = (420\ \mathrm{kg \cdot m/s^2})\hat{\mathbf{y}} = (420\ \mathrm{N})\hat{\mathbf{y}}$$

INSIGHT

The magnitude of the additional average force exerted by the floor is rather large; in fact, 420 N is approximately 95 lb, or about 60% of the contestant's weight of 160 lb. Thus, the total upward force exerted by the floor is $mg + 420\ \mathrm{N} = 710\ \mathrm{N} + 420\ \mathrm{N}$, which is about 250 lb. The contestant, of course, exerts the same force downward. Fortunately, the contestant only needs to exert that force for a third of a second.

When the contestant lands, an impulse is required to bring him to rest. If he lands with stiff legs, the impulse occurs in a short time, resulting in a large force delivered to the knees—with possible harmful effects. If he bends his legs on landing, on the other hand, the time duration is significantly increased, and the force applied to the contestant is correspondingly reduced.

PRACTICE PROBLEM

Suppose the contestant lands with a speed of 2.1 m/s and comes to rest in 0.25 s. What is the magnitude of the average force exerted by the floor during landing? [**Answer:** $mg + 600\ \mathrm{N} \sim 290\ \mathrm{lb}$]

Some related homework problems: Problem 13, Problem 14

9–4 Conservation of Linear Momentum

In this section we turn to perhaps the most significant aspect of linear momentum—the fact that it is a conserved quantity. In this respect, it plays a fundamental role in physics similar to that of energy. We shall also see that momentum conservation leads to calculational simplifications, making it of great practical significance.

First, recall that the net force acting on an object is equal to the rate of change of its momentum

$$\sum \vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$$

Rearranging this expression, we find that the change in momentum during a time interval Δt is

$$\Delta \vec{\mathbf{p}} = \left(\sum \vec{\mathbf{F}}\right)\Delta t \qquad 9\text{–}7$$

Clearly, then, if the net force acting on an object is zero,

$$\sum \vec{\mathbf{F}} = 0$$

its change in momentum is also zero:

$$\Delta \vec{\mathbf{p}} = \left(\sum \vec{\mathbf{F}}\right)\Delta t = 0$$

Writing the change of momentum in terms of its initial and final values, we have

$$\Delta \vec{\mathbf{p}} = \vec{\mathbf{p}}_{\mathrm{f}} - \vec{\mathbf{p}}_{\mathrm{i}} = 0$$

or

$$\vec{\mathbf{p}}_{\mathrm{f}} = \vec{\mathbf{p}}_{\mathrm{i}} \qquad 9\text{–}8$$

Since the momentum does not change in this case, we say that it is **conserved.** To summarize:

Conservation of Momentum

If the net force acting on an object is zero, its momentum is conserved; that is,

$$\vec{\mathbf{p}}_{\mathrm{f}} = \vec{\mathbf{p}}_{\mathrm{i}}$$

Note that in some cases the force may be zero in one direction and nonzero in another. For example, an object in free fall has a nonzero y component of force, $F_y \neq 0$, but no force in the x direction, $F_x = 0$. As a result, the object's y component of momentum changes with time while its x component of momentum remains constant. Therefore, in applying momentum conservation, we must remember that both the force and the momentum are vector quantities and that the momentum conservation principle applies separately to each coordinate direction.

Thus far, our discussion has referred to the forces acting on a single object. Next, we consider a system composed of more than one object.

Internal Versus External Forces

The net force acting on a system of objects is the sum of forces applied from outside the system (external forces, $\vec{\mathbf{F}}_{\text{ext}}$) and forces acting between objects within the system (internal forces, $\vec{\mathbf{F}}_{\text{int}}$). Thus, we can write

$$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}} = \sum \vec{\mathbf{F}}_{\text{ext}} + \sum \vec{\mathbf{F}}_{\text{int}}$$

As we shall see, internal and external forces play very *different* roles in terms of how they affect the momentum of a system.

To illustrate the distinction, consider the case of two canoes floating at rest next to one another on a lake, as described in Example 5–3 and shown in **Figure 9–4**. In this case, let's consider the "system" to be the two canoes and the people inside them. When a person in canoe 1 pushes on canoe 2, a force $\vec{\mathbf{F}}_2$ is exerted on canoe 2. By Newton's third law, an equal and opposite force, $\vec{\mathbf{F}}_1 = -\vec{\mathbf{F}}_2$, is exerted on the person in canoe 1. Note that $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are internal forces, since they act between objects in the system. In addition, note that they sum to zero:

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (-\vec{\mathbf{F}}_2) + \vec{\mathbf{F}}_2 = 0$$

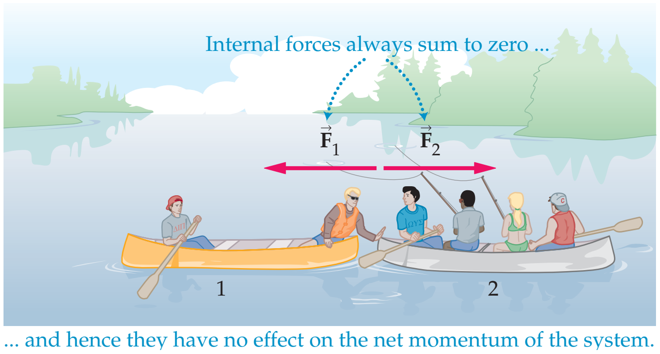


FIGURE 9–4 Separating two canoes
A system comprised of two canoes and their occupants. The forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are internal to the system. They sum to zero.

This is a special case, of course, but it demonstrates the following general principles:

- Internal forces, like all forces, always occur in action-reaction pairs.
- Because the forces in action-reaction pairs are equal and opposite—due to Newton's third law—internal forces must *always* sum to zero:

$$\sum \vec{\mathbf{F}}_{\text{int}} = 0$$

The fact that internal forces always cancel means that the net force acting on a system of objects is simply the sum of the *external* forces acting on it:

$$\vec{\mathbf{F}}_{\text{net}} = \sum \vec{\mathbf{F}}_{\text{ext}} + \sum \vec{\mathbf{F}}_{\text{int}} = \sum \vec{\mathbf{F}}_{\text{ext}}$$

The external forces, on the other hand, may or may not sum to zero—it all depends on the particular situation. For example, if the system consists of the two canoes in Figure 9–4, the external forces are the weights of the people and the canoes acting downward, and the upward, normal force exerted by the water to keep the canoes afloat. These forces sum to zero, and there is no acceleration in the vertical direction. In the next few sections we consider a variety of systems in which the external forces either sum to zero, or are so small that they can be ignored. Later, in Section 9–7, we consider situations where the external forces do not sum to zero and hence must be taken into account.

▲ If the astronaut in this photo pushes on the satellite, the satellite exerts an equal but opposite force on him, in accordance with Newton's third law. If we are calculating the change in the astronaut's momentum, we must take this force into account. However, if we define the system to be the astronaut *and* the satellite, the forces between them are internal to the system. Whatever effect they may have on the astronaut or the satellite individually, they do not affect the momentum of the system as a whole. Therefore, whether a particular force counts as internal or external depends entirely on where we draw the boundaries of the system.

Finally, how do external and internal forces affect the momentum of a system? To see the connection, first note that Newton's second law gives the change in the net momentum for a given time interval Δt:

$$\Delta\vec{\mathbf{p}}_{\text{net}} = \vec{\mathbf{F}}_{\text{net}}\,\Delta t$$

Because the internal forces cancel, however, the change in the net momentum is directly related to the net *external* force:

$$\Delta\vec{\mathbf{p}}_{\text{net}} = \left(\sum\vec{\mathbf{F}}_{\text{ext}}\right)\Delta t \qquad 9\text{–}9$$

Therefore, the key distinction between internal and external forces is the following:

Conservation of Momentum for a System of Objects

- *Internal* forces have absolutely no effect on the net momentum of a system.
- If the *net external* force acting on a system is zero, its net momentum is conserved. That is,

$$\vec{\mathbf{p}}_{1,\text{f}} + \vec{\mathbf{p}}_{2,\text{f}} + \vec{\mathbf{p}}_{3,\text{f}} + \cdots = \vec{\mathbf{p}}_{1,\text{i}} + \vec{\mathbf{p}}_{2,\text{i}} + \vec{\mathbf{p}}_{3,\text{i}} + \cdots$$

It is important to note that these statements apply only to the *net* momentum of a system, not to the momentum of each individual object. For example, suppose a system consists of two objects, 1 and 2, and that the net external force acting on the system is zero. As a result, the net momentum must remain constant:

$$\vec{\mathbf{p}}_{\text{net}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$$

This does not mean, however, that $\vec{\mathbf{p}}_1$ is constant or that $\vec{\mathbf{p}}_2$ is constant. All we can say is that the *sum* of $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_2$ does not change.

As a specific example, consider the case of the two canoes floating on a lake, as described previously. Initially the momentum of the system is zero, because the canoes are at rest. After a person pushes the canoes apart, they are both moving, and hence both have nonzero momentum. Thus, the momentum of each canoe has changed. On the other hand, because the net external force acting on the system is zero, the sum of the canoes' momenta must still vanish. We show this in the next Example.

PROBLEM-SOLVING NOTE

Internal Versus External Forces

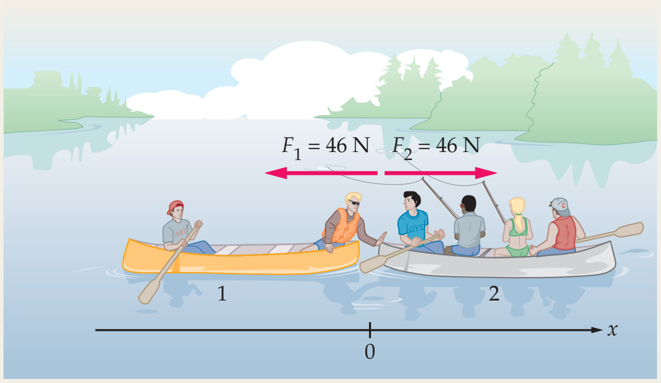
It is important to keep in mind that internal forces cannot change the momentum of a system—only a net external force can do that.

EXAMPLE 9–3 TIPPY CANOE: COMPARING VELOCITY AND MOMENTUM

Two groups of canoeists meet in the middle of a lake. After a brief visit, a person in canoe 1 pushes on canoe 2 with a force of 46 N to separate the canoes. If the mass of canoe 1 and its occupants is 130 kg, and the mass of canoe 2 and its occupants is 250 kg, find the momentum of each canoe after 1.20 s of pushing.

PICTURE THE PROBLEM

We choose the positive x direction to point from canoe 1 to canoe 2. With this choice, the force exerted on canoe 2 is $\vec{\mathbf{F}}_2 = (46\text{ N})\hat{\mathbf{x}}$ and the force exerted on canoe 1 is $\vec{\mathbf{F}}_1 = (-46\text{ N})\hat{\mathbf{x}}$.



STRATEGY

First, we find the acceleration of each canoe using $a_x = F_x/m$. Next, we use $v_x = v_{0x} + a_x t$ to find the velocity at time t. Note that the canoes start at rest, hence $v_{0x} = 0$. Finally, the momentum can be calculated using $p_x = mv_x$.

SOLUTION

1. Use Newton's second law to find the acceleration of canoe 2:
$$a_{2,x} = \frac{\sum F_{2,x}}{m_2} = \frac{46\text{ N}}{250\text{ kg}} = 0.18\text{ m/s}^2$$
2. Do the same calculation for canoe 1. Note that the acceleration of canoe 1 is in the negative direction:
$$a_{1,x} = \frac{\sum F_{1,x}}{m_1} = \frac{-46\text{ N}}{130\text{ kg}} = -0.35\text{ m/s}^2$$
3. Calculate the velocity of each canoe at $t = 1.20$ s:
$$v_{1,x} = a_{1,x}t = (-0.35\text{ m/s}^2)(1.20\text{ s}) = -0.42\text{ m/s}$$
$$v_{2,x} = a_{2,x}t = (0.18\text{ m/s}^2)(1.20\text{ s}) = 0.22\text{ m/s}$$
4. Calculate the momentum of each canoe at $t = 1.20$ s:
$$p_{1,x} = m_1v_{1,x} = (130\text{ kg})(-0.42\text{ m/s}) = -55\text{ kg}\cdot\text{m/s}$$
$$p_{2,x} = m_2v_{2,x} = (250\text{ kg})(0.22\text{ m/s}) = 55\text{ kg}\cdot\text{m/s}$$

INSIGHT

Note that the sum of the momenta of the two canoes is zero. This is just what one would expect: The canoes start at rest with zero momentum, there is zero net external force acting on the system, hence the final momentum must also be zero. The final velocities *do not* add to zero; it is momentum ($m\vec{\mathbf{v}}$) that is conserved, not velocity ($\vec{\mathbf{v}}$).

Finally, we solved this problem using one-dimensional kinematics so that we could clearly see the distinction between velocity and momentum. An alternative way to calculate the final momentum of each canoe is to use $\Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_\text{f} - \vec{\mathbf{p}}_\text{i} = \vec{\mathbf{F}}\Delta t$. For canoe 1 we have $\vec{\mathbf{p}}_{1,\text{f}} = \vec{\mathbf{F}}_1\,\Delta t + \vec{\mathbf{p}}_{1,\text{i}} = (-46\text{ N})\hat{\mathbf{x}}(1.20\text{ s}) + 0 = (-55\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}}$, in agreement with our results above. A similar calculation yields $\vec{\mathbf{p}}_{2,\text{f}} = (55\text{ kg}\cdot\text{m/s})\hat{\mathbf{x}}$ for canoe 2.

PRACTICE PROBLEM

What are the final momenta if the canoes are pushed apart with a force of 56 N? [**Answer:** $p_{1,x} = -67\text{ kg}\cdot\text{m/s}$, $p_{2,x} = 67\text{ kg}\cdot\text{m/s}$]

Some related homework problems: Problem 21, Problem 22

In a situation like that described in Example 9–3, the person in canoe 1 pushes canoe 2 away. At the same time, canoe 1 begins to move in the opposite direction. This is referred to as **recoil.** It is essentially the same as the recoil one experiences when firing a gun or when turning on a strong stream of water.

A particularly interesting example of recoil involves the human body. Perhaps you have noticed, when resting quietly in a rocking or reclining chair, that the chair wobbles back and forth slightly about once a second. The reason for this movement is that each time your heart pumps blood in one direction (from the atria to the ventricles, then to the aorta and pulmonary arteries, and so on) your body recoils in the opposite direction. Because the recoil depends on the force exerted by your heart on the blood and the volume of blood expelled from the heart with each beat, it is possible to gain valuable medical information regarding the health of your heart by analyzing the recoil it produces.

REAL-WORLD PHYSICS: BIO
The ballistocardiograph

The medical instrument that employs the physical principle of recoil is called the *ballistocardiograph*. It is a completely noninvasive technology that simply requires the patient to sit comfortably in a chair fitted with sensitive force sensors under the seat and behind the back. Sophisticated bathroom scales also utilize this technology. A ballistocardiographic (BCG) scale detects the recoil vibrations of the body as a person stands on the scale. This allows the BCG scale to display not only the person's body weight but his or her heart rate as well.

REAL-WORLD PHYSICS: BIO
Heartbeat detectors

A more dramatic application of heartbeat recoil is currently being used at the Riverbend Maximum Security Institution in Tennessee. The only successful breakout from this prison occurred when four inmates hid in a secret compartment in a delivery truck that was leaving the facility. The institution now uses a heartbeat recoil detector that would have foiled this escape. Vehicles leaving the prison must stop at a checkpoint where a small motion detector is attached to it with a suction cup. Any persons hidden in the vehicle will reveal their presence by the very beating of their hearts. These heartbeat detectors have proved to be 100 percent effective, even though the recoil of the heart may displace a large truck by only a few millionths of an inch. Similar systems are being used at other high-security installations and border crossings.

CONCEPTUAL CHECKPOINT 9–2 MOMENTUM VERSUS KINETIC ENERGY

In Example 9–3, the final momentum of the system (consisting of the two canoes and their occupants) is equal to the initial momentum of the system. Is the final kinetic energy **(a)** equal to, **(b)** less than, or **(c)** greater than the initial kinetic energy?

REASONING AND DISCUSSION

The final momentum of the two canoes is zero because one canoe has a positive momentum and the other has a negative momentum of the same magnitude. The two momenta, then, sum to zero. Kinetic energy, which is $\frac{1}{2}mv^2$, cannot be negative; hence no such cancellation is possible. Both canoes have positive kinetic energies, and therefore, the final kinetic energy is greater than the initial kinetic energy, which is zero.

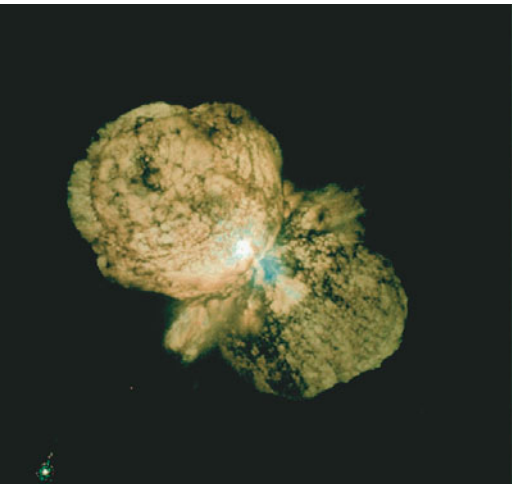
Where does the increase in kinetic energy come from? It comes from the muscular work done by the person who pushes the canoes apart.

ANSWER

(c) K_f is greater than K_i.

REAL-WORLD PHYSICS

Stellar explosions



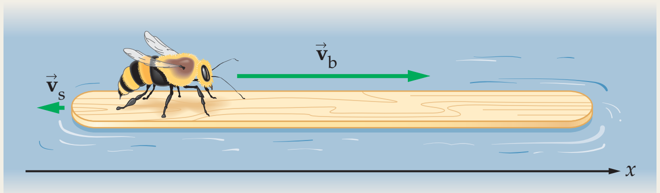
▲ This Hubble Space Telescope photograph shows the aftermath of a violent explosion of the star Eta Carinae. The explosion, which was observed on Earth in 1841 and briefly made Eta Carinae the second brightest star in the sky, produced two bright lobes of matter spewing outward in opposite directions. In this photograph, these lobes have expanded to about the size of our solar system. The momentum of the star before the explosion must be the same as the total momentum of the star and the bright lobes after the explosion. Since the lobes are roughly symmetric and move in opposite directions, their net momentum is essentially zero. Thus, we conclude that the momentum of the star itself was virtually unchanged by the explosion.

A special case of some interest is the universe. Since there is nothing external to the universe—by definition—it follows that the net external force acting on it is zero. Therefore, its net momentum is conserved. No matter what happens—a comet collides with the Earth, a star explodes and becomes a supernova, a black hole swallows part of a galaxy—the total momentum of the universe simply cannot change. A particularly vivid illustration of momentum conservation in our own galaxy is provided by the exploding star Eta Carinae. As can be seen in the Hubble Space Telescope photograph, jets of material are moving away from the star in opposite directions, just like the canoes moving apart from one another in Example 9–3.

Conservation of momentum also applies to the more everyday situation described in the next Active Example.

ACTIVE EXAMPLE 9–2 FIND THE VELOCITY OF THE BEE

A honeybee with a mass of 0.150 g lands on one end of a floating 4.75-g popsicle stick. After sitting at rest for a moment, it runs toward the other end with a velocity $\vec{\mathbf{v}}_b$ relative to the still water. The stick moves in the opposite direction with a speed of 0.120 cm/s. What is the velocity of the bee? (Let the direction of the bee's motion be the positive x direction.)



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the total momentum of the system equal to zero: $\vec{\mathbf{p}}_b + \vec{\mathbf{p}}_s = 0$
2. Solve for the momentum of the bee: $\vec{\mathbf{p}}_b = -\vec{\mathbf{p}}_s = m_b v_b \hat{\mathbf{x}}$
3. Calculate the momentum of the stick: $\vec{\mathbf{p}}_s = -m_s v_s \hat{\mathbf{x}} = (-0.570 \text{ g} \cdot \text{cm/s})\hat{\mathbf{x}}$
4. Calculate the momentum of the bee: $\vec{\mathbf{p}}_b = m_b v_b \hat{\mathbf{x}} = -\vec{\mathbf{p}}_s = (0.570 \text{ g} \cdot \text{cm/s})\hat{\mathbf{x}}$
5. Divide by the bee's mass to find its velocity: $\vec{\mathbf{v}}_b = \vec{\mathbf{p}}_b / m_b = (3.80 \text{ cm/s})\hat{\mathbf{x}}$

INSIGHT

Because only internal forces are at work while the bee walks on the stick, the system's total momentum must remain zero.

YOUR TURN

Suppose the mass of the popsicle stick is 9.50 g rather than 4.75 g. What is the bee's velocity in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

9–5 Inelastic Collisions

We now turn our attention to **collisions.** By a collision we mean a situation in which two objects strike one another, and in which the net external force is either zero or negligibly small. For example, if two train cars roll along on a level track and hit one another, this is a collision. In this case, the net external force—the weight downward and the normal force exerted by the tracks upward—is zero. As a result, the momentum of the two-car system is conserved.

Another example of a collision is a baseball being struck by a bat. In this case, the external forces are not zero because the weight of the ball is not balanced by any other force. However, as we have seen in Section 9–3, the forces exerted during the hit are much larger than the weight of the ball or the bat. Hence, to a good approximation, we may neglect the external forces (the weight of the ball and bat) in this case, and say that the momentum of the ball–bat system is conserved.

Now it may seem surprising at first, but the fact that the momentum of a system is conserved during a collision does not necessarily mean that the system's kinetic energy is conserved. In fact most, or even all, of a system's kinetic energy may be converted to other forms during a collision while, at the same time, not one bit of momentum is lost. This shall be explored in detail in this section.

In general, collisions are categorized according to what happens to the kinetic energy of the system. There are two possibilities. After a collision, the final kinetic energy, K_f, is either equal to the initial kinetic energy, K_i, or it is not. If $K_f = K_i$, the collision is said to be **elastic.** We shall consider elastic collisions in the next section.

On the other hand, the kinetic energy may change during a collision. Usually it decreases due to losses associated with sound, heat, and deformation. Sometimes it increases, if the collision sets off an explosion, for instance. In any event, collisions in which the kinetic energy is not conserved are referred to as **inelastic:**

Inelastic Collisions

In an inelastic collision, the momentum of a system is conserved,

$$\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i$$

but its kinetic energy is not,

$$K_f \neq K_i$$

In both elastic and inelastic collisions, momentum is conserved. The same is not true of kinetic energy, however. In the largely inelastic collision at left, much of the hockey players' initial kinetic energy is transformed into work: rearranging the players' anatomies and shattering the glass of the rink. In the highly elastic collision at right, the ball rebounds with very little diminution of its kinetic energy (though a little energy is lost as sound and heat).

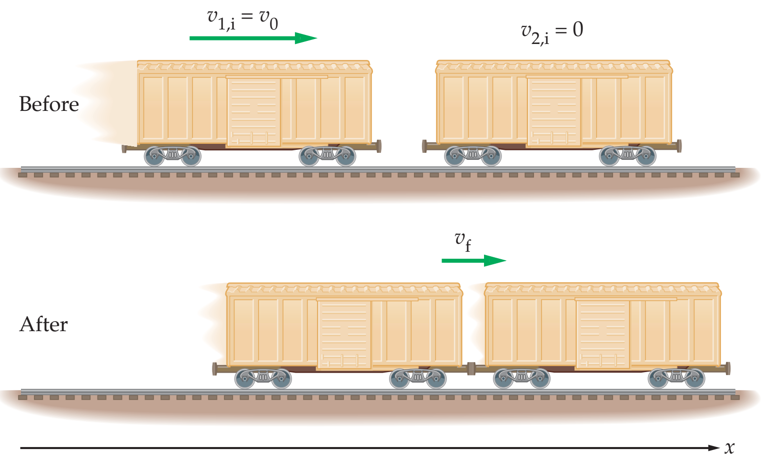
Finally, in the special case where objects stick together after the collision, we say that the collision is **completely inelastic.**

Completely Inelastic Collisions
When objects stick together after colliding, the collision is completely inelastic.

In a completely inelastic collision, the maximum amount of kinetic energy is lost. If the total momentum of the system is zero, this means that all of the kinetic energy is lost. For systems with nonzero total momentum, however, some kinetic energy will remain after the collision—still, the amount lost is the maximum permitted by momentum conservation.

Inelastic Collisions in One Dimension

Consider a system of two identical train cars of mass m on a smooth, level track. One car is at rest initially while the other moves toward it with a speed v_0, as shown in Figure 9–5. When the cars collide, the coupling mechanism latches, causing the cars to stick together and move as a unit. What is the speed of the cars after the collision?



▶ **FIGURE 9–5 Railroad cars collide and stick together**
A moving train car collides with an identical car that is stationary. After the collision, the cars stick together and move with the same speed.

To answer this question, we begin by considering the general case that applies to any completely inelastic collision, and then we look at the specific case of the two train cars. In general, suppose that two masses, m_1 and m_2, have initial velocities $v_{1,i}$ and $v_{2,i}$ respectively. The initial momentum of the system is

$$p_i = m_1v_{1,i} + m_2v_{2,i}$$

After the collision, the objects move together with a common velocity v_f. Therefore, the final momentum is

$$p_f = (m_1 + m_2)v_f$$

Equating the initial and final momenta yields $m_1v_{1,i} + m_2v_{2,i} = (m_1 + m_2)v_f$, or

$$v_f = \frac{m_1v_{1,i} + m_2v_{2,i}}{m_1 + m_2} \qquad 9\text{–}10$$

We can apply this general result to the case of the two railroad cars by noting that $m_1 = m_2 = m$, $v_{1,i} = v_0$, and $v_{2,i} = 0$. Thus, the final velocity is

$$v_f = \frac{mv_0 + m \cdot 0}{m + m} = \frac{m}{2m}v_0 = \frac{1}{2}v_0 \qquad 9\text{–}11$$

As you might have guessed, the final speed is one-half the initial speed.

EXERCISE 9–2

A 1200-kg car moving at 2.5 m/s is struck in the rear by a 2600-kg truck moving at 6.2 m/s. If the vehicles stick together after the collision, what is their speed immediately after colliding? (Assume that external forces may be ignored.)

SOLUTION

Applying Equation 9–10 with $m_1 = 1200$ kg, $v_{1,i} = 2.5$ m/s, $m_2 = 2600$ kg, and $v_{2,i} = 6.2$ m/s yields $v_f = 5.0$ m/s.

During the collision of the railroad cars, some of the initial kinetic energy is converted to other forms. Some propagates away as sound, some is converted to heat, some creates permanent deformations in the metal of the latching mechanism. The precise amount of kinetic energy that is lost is addressed in the following Conceptual Checkpoint.

PROBLEM-SOLVING NOTE

Momentum Versus Energy Conservation

Be sure to distinguish between momentum conservation and energy conservation. A common error is to assume that kinetic energy is conserved just because the momentum is conserved.

CONCEPTUAL CHECKPOINT 9–3 HOW MUCH KINETIC ENERGY IS LOST?

A railroad car of mass m and speed v collides and sticks to an identical railroad car that is initially at rest. After the collision, is the kinetic energy of the system **(a)** 1/2, **(b)** 1/3, or **(c)** 1/4 of its initial kinetic energy?

REASONING AND DISCUSSION

Before the collision, the kinetic energy of the system is

$$K_i = \frac{1}{2}mv^2$$

After the collision, the mass doubles and the speed is halved. Hence, the final kinetic energy is

$$K_f = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) = \frac{1}{2}K_i$$

Therefore, one-half of the initial kinetic energy is converted to other forms of energy. An equivalent way to arrive at this conclusion is to express the kinetic energy in terms of the momentum, $p = mv$:

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{m^2v^2}{m}\right) = \frac{p^2}{2m}$$

Since the momentum is the same before and after the collision, the fact that the mass doubles means the kinetic energy is halved.

ANSWER

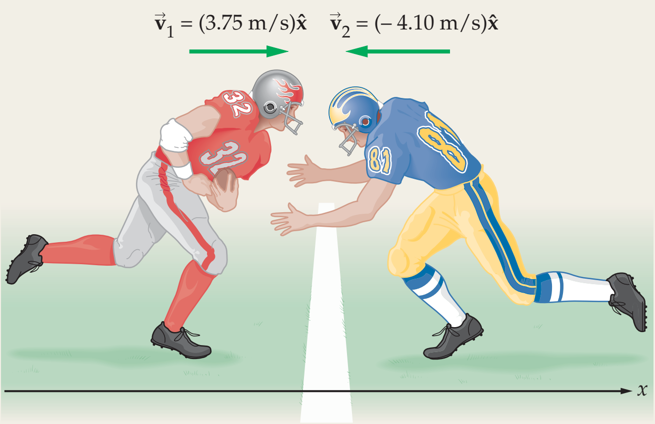
(a) The final kinetic energy is one-half the initial kinetic energy.

Note that we know the precise amount of kinetic energy that was lost, even though we don't know just how much went into sound, how much went into heat, and so on. It is not necessary to know all of those details to determine how much kinetic energy was lost.

We also know how much momentum was lost—none.

EXAMPLE 9–4 GOAL-LINE STAND

On a touchdown attempt, a 95.0-kg running back runs toward the end zone at 3.75 m/s. A 111-kg linebacker moving at 4.10 m/s meets the runner in a head-on collision. If the two players stick together, **(a)** what is their velocity immediately after the collision? **(b)** What are the initial and final kinetic energies of the system?



PICTURE THE PROBLEM

In our sketch, we let subscript 1 refer to the red-and-gray running back, who carries the ball, and subscript 2 refer to the blue-and-gold linebacker, who will make the tackle. The direction of the running back's initial motion is taken to be in the positive x direction. Therefore, the initial velocities of the players are $\vec{v}_1 = (3.75 \text{ m/s})\hat{x}$ and $\vec{v}_2 = (-4.10 \text{ m/s})\hat{x}$.

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STRATEGY

a. The final velocity can be found by applying momentum conservation to the system consisting of the two players. Initially, the players have momenta in opposite directions. After the collision, the players move together with a combined mass $m_1 + m_2$ and a velocity $\vec{\mathbf{v}}_f$.

b. The kinetic energies can be found by applying $\frac{1}{2}mv^2$ to the players individually to obtain the initial kinetic energy, and then to their combined motion for the final kinetic energy.

SOLUTION

Part (a)

1. Set the initial momentum equal to the final momentum:

$$m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = (m_1 + m_2)\vec{\mathbf{v}}_f$$

2. Solve for the final velocity and substitute numerical values, being careful to use the appropriate signs:

$$\vec{\mathbf{v}}_f = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$
$$= \frac{(95.0\text{ kg})(3.75\text{ m/s})\hat{\mathbf{x}} + (111\text{ kg})(-4.10\text{ m/s})\hat{\mathbf{x}}}{95.0\text{ kg} + 111\text{ kg}}$$
$$= (-0.480\text{ m/s})\hat{\mathbf{x}}$$

Part (b)

3. Calculate the initial kinetic energy of the two players:

$$K_i = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2$$
$$= \tfrac{1}{2}(95.0\text{ kg})(3.75\text{ m/s})^2 + \tfrac{1}{2}(111\text{ kg})(-4.10\text{ m/s})^2$$
$$= 1600\text{ J}$$

4. Calculate the final kinetic energy of the players, noting that they both move with the same velocity after the collision:

$$K_f = \tfrac{1}{2}(m_1 + m_2)v_f^2$$
$$= \tfrac{1}{2}(95.0\text{ kg} + 111\text{ kg})(-0.480\text{ m/s})^2 = 23.7\text{ J}$$

INSIGHT

After the collision, the two players are moving in the negative direction; that is, away from the end zone. This is because the linebacker had more negative momentum than the running back had positive momentum. As for the kinetic energy, of the original 1600 J, only 23.7 J is left after the collision. This means that over 98% of the original kinetic energy is converted to other forms. Even so, *none* of the momentum is lost.

PRACTICE PROBLEM

If the final speed of the two players is to be zero, should the speed of the running back be increased or decreased? Check your answer by calculating the required speed for the running back. [**Answer:** The running back's speed should be increased to 4.79 m/s.]

Some related homework problems: Problem 28, Problem 35

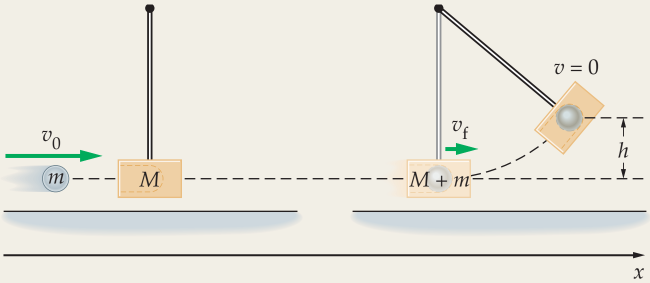
EXAMPLE 9–5 BALLISTIC PENDULUM

REAL-WORLD PHYSICS

In a ballistic pendulum, an object of mass m is fired with an initial speed v_0 at the bob of a pendulum. The bob has a mass M, and is suspended by a rod of negligible mass. After the collision, the object and the bob stick together and swing through an arc, eventually gaining a height h. Find the height h in terms of m, M, v_0, and g.

PICTURE THE PROBLEM

Our sketch shows the physical setup of a ballistic pendulum. Initially, only the object of mass m is moving, and it moves in the positive x direction with a speed v_0. Immediately after the collision, the bob and object move together with a new speed, v_f, which is determined by momentum conservation. Finally, the pendulum continues to swing to the right until its speed decreases to zero and it comes to rest at the height h.



INTERACTIVE FIGURE

STRATEGY

There are two distinct physical processes at work in the ballistic pendulum. The first is a completely inelastic collision between the bob and the object. Momentum is conserved during this collision, but kinetic energy is not. After the collision, the remaining kinetic energy is converted into gravitational potential energy, which determines how high the bob and object will rise.

SOLUTION

1. Set the momentum just before the bob–object collision equal to the momentum just after the collision. Let v_f be the speed just after the collision:
$$mv_0 = (M + m)v_f$$

2. Solve for the speed just after the collision, v_f:
$$v_f = \left(\frac{m}{M + m}\right)v_0$$

3. Calculate the kinetic energy just after the collision:
$$K_f = \frac{1}{2}(M + m)v_f^2 = \frac{1}{2}(M + m)\left(\frac{m}{M + m}\right)^2 v_0^2$$
$$= \frac{1}{2}mv_0^2\left(\frac{m}{M + m}\right)$$

4. Set the kinetic energy after the collision equal to the gravitational potential energy at the height h:
$$\frac{1}{2}mv_0^2\left(\frac{m}{M + m}\right) = (M + m)gh$$

5. Solve for the height, h:
$$h = \left(\frac{m}{M + m}\right)^2\left(\frac{v_0^2}{2g}\right)$$

INSIGHT

A ballistic pendulum is often used to measure the speed of a rapidly moving object, such as a bullet. If a bullet were shot straight up, it would rise to the height $v_0^2/2g$, which can be thousands of feet. On the other hand, if a bullet of mass m is fired into a ballistic pendulum, in which M is much greater than m, the bullet reaches only a small fraction of this height. Thus, the ballistic pendulum makes for a more convenient and practical measurement.

PRACTICE PROBLEM

A 7.00-g bullet is fired into a ballistic pendulum whose bob has a mass of 0.950 kg. If the bob rises to a height of 0.220 m, what was the initial speed of the bullet? [**Answer:** $v_0 = 284$ m/s. If this bullet were fired straight up, it would rise 4.11 km ≈ 13,000 ft in the absence of air resistance.]

Some related homework problems: Problem 32, Problem 33

Inelastic Collisions in Two Dimensions

Next we consider collisions in two dimensions, where we must conserve the momentum component by component. To do this, we set up a coordinate system and resolve the initial momentum into x and y components. Next, we demand that the final momentum have precisely the same x and y components as the initial momentum. That is,

$$p_{x,i} = p_{x,f}$$

and

$$p_{y,i} = p_{y,f}$$

The following Example shows how to carry out such a calculation in a practical situation.

PROBLEM-SOLVING NOTE

Sketch the System Before and After the Collision

In problems involving collisions, it is useful to draw the system before and after the collision. Be sure to label the relevant masses, velocities, and angles.

EXAMPLE 9–6 BAD INTERSECTION: ANALYZING A TRAFFIC ACCIDENT

REAL-WORLD PHYSICS

A car with a mass of 950 kg and a speed of 16 m/s approaches an intersection, as shown on the next page. A 1300-kg minivan traveling at 21 m/s is heading for the same intersection. The car and minivan collide and stick together. Find the speed and direction of the wrecked vehicles just after the collision, assuming external forces can be ignored.

PICTURE THE PROBLEM

In our sketch, we align the x and y axes with the crossing streets. With this choice, $\vec{\mathbf{v}}_1$ (the car's velocity) is in the positive x direction, and $\vec{\mathbf{v}}_2$ (the minivan's velocity) is in the positive y direction. In addition, the problem statement indicates that

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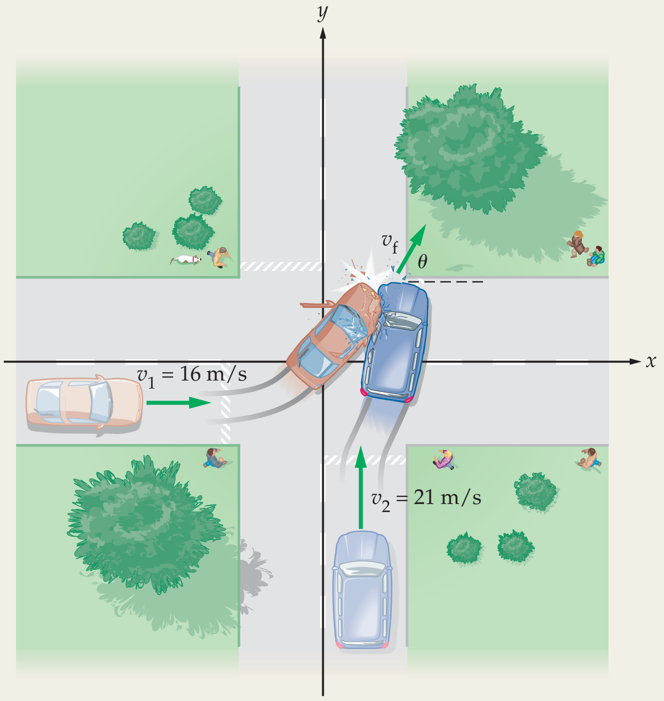
$m_1 = 950$ kg and $m_2 = 1300$ kg. After the collision, the two vehicles move together (as a unit) with a speed v_f in a direction θ with respect to the positive x axis.



STRATEGY

Because external forces can be ignored, the total momentum of the system must be conserved during the collision. This is really two conditions: (i) the x component of momentum is conserved, and (ii) the y component of momentum is conserved. These two conditions determine the two unknowns: the final speed, v_f, and the final direction, θ.

SOLUTION

1. Set the initial x component of momentum equal to the final x component of momentum:

$$m_1 v_1 = (m_1 + m_2) v_f \cos\theta$$

2. Do the same for the y component of momentum:

$$m_2 v_2 = (m_1 + m_2) v_f \sin\theta$$

3. Divide the y momentum equation by the x momentum equation. This eliminates v_f, giving an equation involving θ alone:

$$\frac{m_2 v_2}{m_1 v_1} = \frac{(m_1 + m_2) v_f \sin\theta}{(m_1 + m_2) v_f \cos\theta} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

4. Solve for θ:

$$\theta = \tan^{-1}\left(\frac{m_2 v_2}{m_1 v_1}\right) = \tan^{-1}\left[\frac{(1300\text{ kg})(21\text{ m/s})}{(950\text{ kg})(16\text{ m/s})}\right]$$
$$= \tan^{-1}(1.8) = 61°$$

5. The final speed can be found using either the x or the y momentum equation. Here we use the x equation:

$$v_f = \frac{m_1 v_1}{(m_1 + m_2)\cos\theta}$$
$$= \frac{(950\text{ kg})(16\text{ m/s})}{(950\text{ kg} + 1300\text{ kg})\cos 61°} = 14\text{ m/s}$$

INSIGHT

As a check, you should verify that the y momentum equation gives the same value for v_f.

When a collision occurs in the real world, a traffic-accident investigation team will measure skid marks at the scene of the crash and use this information—along with some basic physics—to determine the initial speeds and directions of the vehicles. This information is often presented in court, where it can lead to a clear identification of the driver at fault.

PRACTICE PROBLEM

Suppose the speed and direction immediately after the collision are known to be $v_f = 12.5$ m/s and $\theta = 42°$, respectively. Find the initial speed of each car. [**Answer:** $v_1 = 22$ m/s, $v_2 = 14$ m/s]

Some related homework problems: Problem 29, Problem 30

9–6 Elastic Collisions

In this section we consider collisions in which both momentum and kinetic energy are conserved. As mentioned in the previous section, such collisions are referred to as elastic:

Elastic Collisions

In an elastic collision, momentum and kinetic energy are conserved. That is,

$$\vec{\mathbf{p}}_{\mathrm{f}} = \vec{\mathbf{p}}_{\mathrm{i}}$$

and

$$K_{\mathrm{f}} = K_{\mathrm{i}}$$

Most collisions in everyday life are rather poor approximations to being elastic—usually there is a significant amount of energy converted to other forms. However, the collision of objects that bounce off one another with little deformation—like billiard balls, for example—provides a reasonably good approximation to an elastic collision. In the subatomic world, on the other hand, elastic collisions are common. Elastic collisions, then, are not merely an ideal that is approached but never attained—they are constantly taking place in nature.

Elastic Collisions in One Dimension

Consider a head-on collision of two carts on an air track, as pictured in **Figure 9–6**. The carts are provided with bumpers that give an elastic bounce when the carts collide. Let's suppose that initially cart 1 is moving to the right with a speed v_0 toward cart 2, which is at rest. If the masses of the carts are m_1 and m_2, respectively, then momentum conservation can be written as follows:

$$m_1 v_0 = m_1 v_{1,\mathrm{f}} + m_2 v_{2,\mathrm{f}}$$

In this expression, $v_{1,\mathrm{f}}$ and $v_{2,\mathrm{f}}$ are the final velocities of the two carts. Note that we say velocities, not speeds, since it is possible for cart 1 to reverse direction, in which case $v_{1,\mathrm{f}}$ would be negative.

Next, the fact that this is an elastic collision means the final velocities must also satisfy energy conservation:

$$\frac{1}{2}m_1 v_0^{\,2} = \frac{1}{2}m_1 v_{1,\mathrm{f}}^{\,2} + \frac{1}{2}m_2 v_{2,\mathrm{f}}^{\,2}$$

Thus, we now have two equations for the two unknowns, $v_{1,\mathrm{f}}$ and $v_{2,\mathrm{f}}$. Straightforward—though messy—algebra yields the following results:

$$\begin{aligned} v_{1,\mathrm{f}} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_0 \\ v_{2,\mathrm{f}} &= \left(\frac{2m_1}{m_1 + m_2}\right) v_0 \end{aligned} \qquad \text{9–12}$$

Note that the final velocity of cart 1 can be positive, negative, or zero, depending on whether m_1 is greater than, less than, or equal to m_2, respectively. The final velocity of cart 2, however, is always positive.

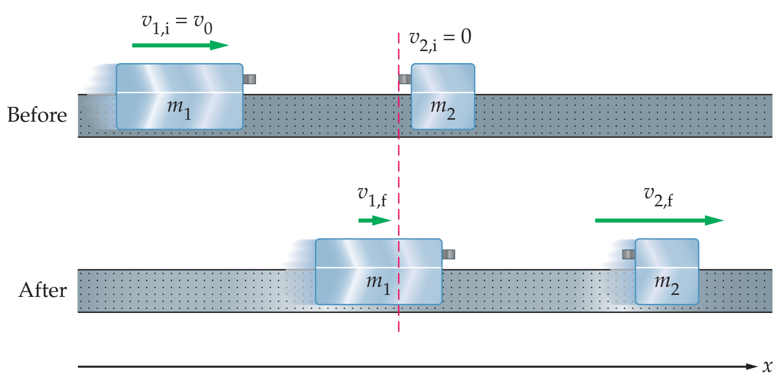


FIGURE 9–6 An elastic collision between two air carts

In the case pictured, $v_{1,\mathrm{f}}$ is to the right (positive), which means that m_1 is greater than m_2. In fact, we have chosen $m_1 = 2m_2$ for this plot; therefore, $v_{1,\mathrm{f}} = v_0/3$ and $v_{2,\mathrm{f}} = 4v_0/3$ as given by Equations 9–12. If m_1 were less than m_2, cart 1 would bounce back toward the left, meaning that $v_{1,\mathrm{f}}$ would be negative.

EXERCISE 9–3

At an amusement park, a 96.0-kg bumper car moving with a speed of 1.24 m/s bounces elastically off a 135-kg bumper car at rest. Find the final velocities of the cars.

SOLUTION

Using Equations 9–12, we find the final velocities to be $v_{1,\mathrm{f}} = -0.209\ \mathrm{m/s}$ and $v_{2,\mathrm{f}} = 1.03\ \mathrm{m/s}$. Note that the direction of travel of car 1 has been reversed.

Let's check a few special cases of our results. First, consider the case where the two carts have equal masses, $m_1 = m_2 = m$. Substituting into Equations 9–12, we find

$$v_{1,\mathrm{f}} = \left(\frac{m - m}{m + m}\right)v_0 = 0$$

and

$$v_{2,\mathrm{f}} = \left(\frac{2m}{m + m}\right)v_0 = v_0$$

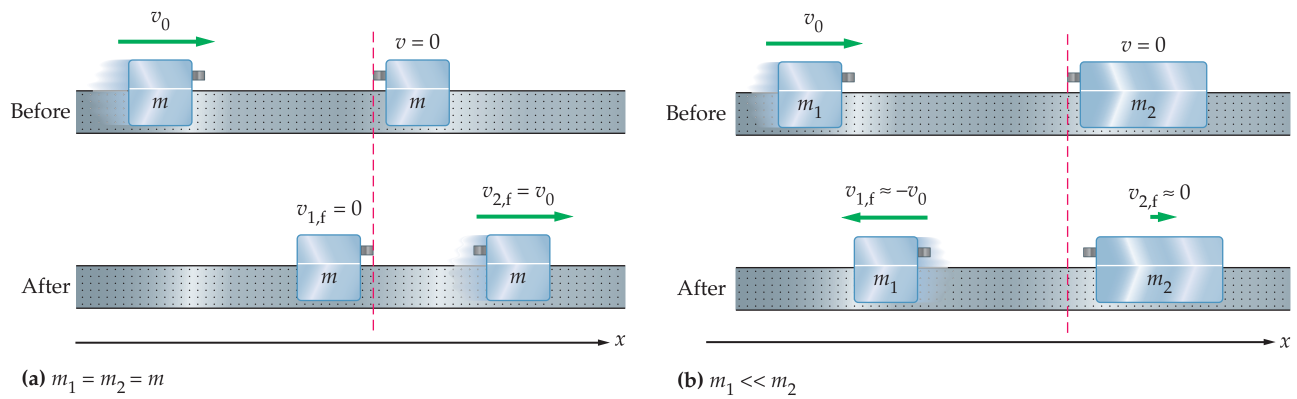
Thus, after the collision, the cart that was moving with velocity v_0 is now at rest, and the cart that was at rest is now moving with velocity v_0. In effect, the carts have "exchanged" velocities. This case is illustrated in **Figure 9–7 (a)**.

Next, suppose that m_2 is much greater than m_1, or, equivalently, that m_1 approaches zero. Returning to Equations 9–12, and setting $m_1 = 0$, we find

$$v_{1,\mathrm{f}} = \left(\frac{0 - m_2}{0 + m_2}\right)v_0 = \left(\frac{-m_2}{m_2}\right)v_0 = -v_0$$

and

$$v_{2,\mathrm{f}} = \frac{2 \cdot 0}{0 + m_2}v_0 = 0$$



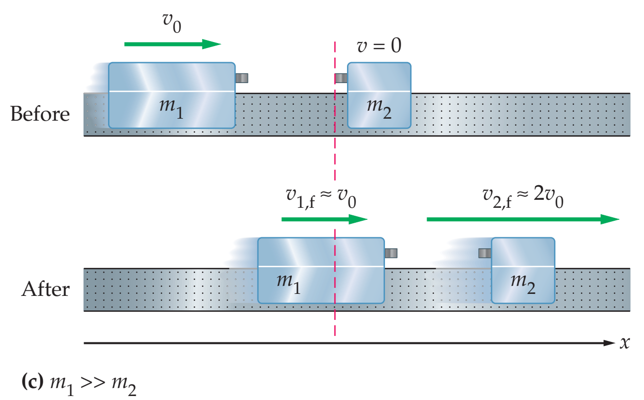


FIGURE 9–7 Elastic collisions between air carts of various masses

(a) Carts of equal mass exchange velocities when they collide. **(b)** When a light cart collides with a stationary heavy cart, its direction of motion is reversed. Its speed is practically unchanged. **(c)** When a heavy cart collides with a stationary light cart, it continues to move in the same direction with essentially the same speed. The light cart moves off with a speed that is roughly twice the initial speed of the heavy cart.

Physically, we interpret these results as follows: A very light cart collides with a heavy cart that is at rest. The heavy cart hardly budges, but the light cart is reflected, heading *backward* (remember the minus sign in $-v_0$) with the same speed it had initially. For example, if you throw a ball against a wall, the wall is the very heavy object and the ball is the light object. The ball bounces back with the same speed it had initially (assuming an ideal elastic collision). We show a case in which m_1 is much less than m_2 in **Figure 9–7 (b)**.

Finally, what happens when m_1 is much greater than m_2? To check this limit we can set m_2 equal to zero. We consider the results in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 9–4 SPEED AFTER A COLLISION

A hoverfly is happily maintaining a fixed position about 10 ft above the ground when an elephant charges out of the bush and collides with it. The fly bounces elastically off the forehead of the elephant. If the initial speed of the elephant is v_0, is the speed of the fly after the collision equal to **(a)** v_0, **(b)** $1.5v_0$, or **(c)** $2v_0$?

REASONING AND DISCUSSION

We can use Equations 9–12 to find the final speeds of the fly and the elephant. First, let m_1 be the mass of the elephant, and m_2 be the mass of the fly. Clearly, m_2 is vanishingly small compared with m_1, hence we can evaluate Equations 9–12 in the limit $m_2 \to 0$. This yields

$$v_{1,\mathrm{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_0 \xrightarrow[m_2 \to 0]{} \left(\frac{m_1}{m_1}\right)v_0 = v_0$$

and

$$v_{2,\mathrm{f}} = \left(\frac{2m_1}{m_1 + m_2}\right)v_0 \xrightarrow[m_2 \to 0]{} \left(\frac{2m_1}{m_1}\right)v_0 = 2v_0$$

As expected, the speed of the elephant is unaffected. The fly, however, rebounds with twice the speed of the elephant. **Figure 9–7 (c)** illustrates this case with air carts.

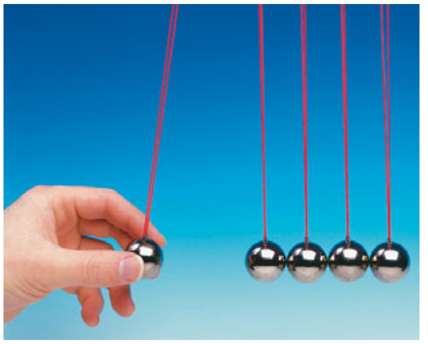
ANSWER

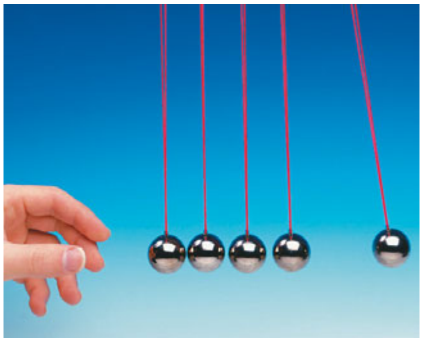
(c) The speed of the fly is $2v_0$.

Note that after the collision the fly is separating from the elephant with the speed $2v_0 - v_0 = v_0$. Before the collision the elephant was approaching the fly with the same speed, v_0. This is a special case of the following general result:

> The speed of separation after a head-on elastic collision is always equal to the speed of approach before the collision.

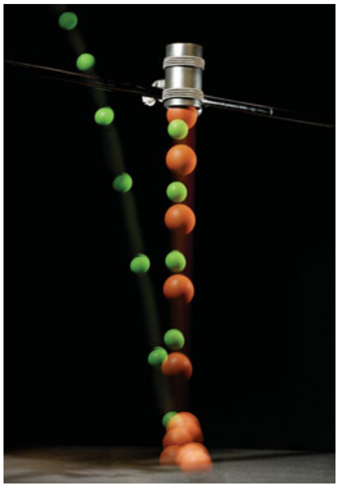
The proof of this statement is the subject of Problem 91.





▲ The apparatus shown here illustrates some of the basic features of elastic collisions between objects of equal mass. The device consists of five identical metal balls suspended by strings. When the end ball is pulled out to the side and then released so as to fall back and strike the second ball, it creates a rapid succession of elastic collisions among the balls. In each collision, one ball comes to rest while the next one begins to move with the original speed, just as with the air carts in Figure 9–7 (a). When the collisions reach the other end of the apparatus, the last ball swings out to the same height from which the first ball was released.

If two balls are pulled out and released, two balls swing out at the other side, and so on. To see why this must be so, imagine that the two balls swing in with a speed v and a single ball swings out at the other side with a speed v'. What value must v' have (a) to conserve momentum, and (b) to conserve kinetic energy? Since the required speed is $v' = 2v$ for (a) and $v' = \sqrt{2}v$ for (b), it follows that it is not possible to conserve both momentum and kinetic energy with two balls swinging in and one ball swinging out.



◀ **Momentum Transfer and Height Amplification**
In a collision between two objects of different mass, like the small and large balls in this photo, a significant amount of momentum can be transferred from the large object to the small object. Even though the total momentum is conserved, the small object can be given a speed that is significantly larger than any of the initial speeds. This is illustrated in the photo by the height to which the small ball bounces. A similar process occurs in the collapse of a star during a supernova explosion. The resulting collision can send jets of material racing away from the supernova at nearly the speed of light, just like the small ball that takes off with such a large speed in this collision.

Elastic Collisions in Two Dimensions

In a two-dimensional elastic collision, if we are given the final speed and direction of one of the objects, we can find the speed and direction of the other object using energy conservation and momentum conservation. For example, consider the collision of two 7.00-kg curling stones, as depicted in **Figure 9–8**. One stone is at rest initially, the other approaches with a speed $v_{1,\mathrm{i}} = 1.50\ \mathrm{m/s}$. The collision is not head-on, and after the collision, stone 1 moves with a speed of $v_{1,\mathrm{f}} = 0.610\ \mathrm{m/s}$ in a direction 66.0° away from the initial line of motion. What are the speed and direction of stone 2?

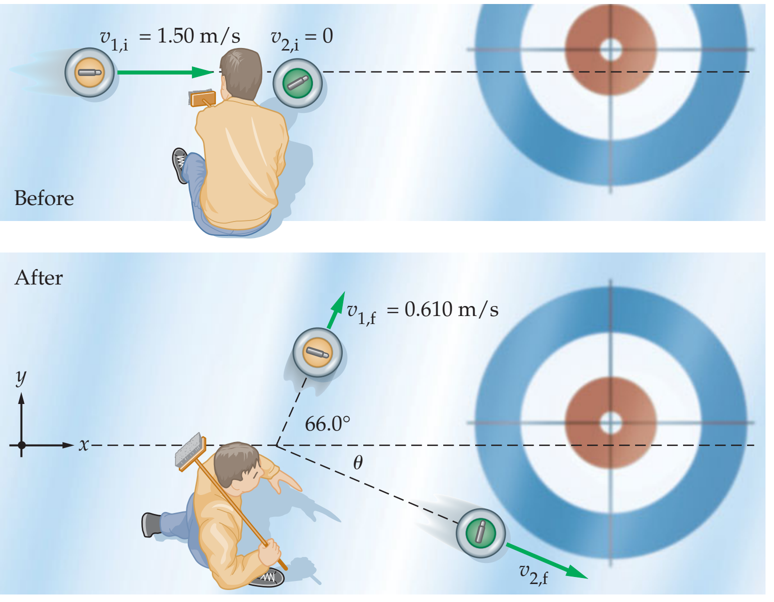


FIGURE 9–8 Two curling stones undergo an elastic collision
The speed of curling stone 2 after this collision can be determined using energy conservation; its direction of motion can be found using momentum conservation in either the x or the y direction.

PROBLEM-SOLVING NOTE
Kinetic Energy in Elastic Collisions
Remember that in elastic collisions, by definition, the kinetic energy is conserved.

First, let's find the speed of stone 2. The easiest way to do this is to simply require that the final kinetic energy be equal to the initial kinetic energy. Initially, the kinetic energy is

$$K_\mathrm{i} = \tfrac{1}{2}m_1 v_{1,\mathrm{i}}^2 = \tfrac{1}{2}(7.00\ \mathrm{kg})(1.50\ \mathrm{m/s})^2 = 7.88\ \mathrm{J}$$

After the collision stone 1 has a speed of 0.610 m/s and stone 2 has the speed $v_{2,\mathrm{f}}$. Hence, the final kinetic energy is

$$K_\mathrm{f} = \tfrac{1}{2}m_1 v_{1,\mathrm{f}}^2 + \tfrac{1}{2}m_2 v_{2,\mathrm{f}}^2 = \tfrac{1}{2}(7.00\ \mathrm{kg})(0.610\ \mathrm{m/s})^2 + \tfrac{1}{2}m_2 v_{2,\mathrm{f}}^2$$
$$= 1.30\ \mathrm{J} + \tfrac{1}{2}m_2 v_{2,\mathrm{f}}^2 = K_\mathrm{i}$$

Solving for the speed of stone 2, we find

$$v_{2,\mathrm{f}} = 1.37\ \mathrm{m/s}$$

Next, we can find the direction of motion of stone 2 by requiring that the momentum be conserved. For example, initially there is no momentum in the y direction. This must be true after the collision as well. Hence, we have the following condition:

$$0 = m_1 v_{1,\mathrm{f}} \sin 66.0° - m_2 v_{2,\mathrm{f}} \sin\theta$$

Solving for the angle θ we find

$$\theta = 24.0°$$

As a final check, compare the initial and final x components of momentum. Initially, we have

$$p_{x,\mathrm{i}} = m_1 v_{1,\mathrm{i}} = (7.00\ \mathrm{kg})(1.50\ \mathrm{m/s}) = 10.5\ \mathrm{kg \cdot m/s}$$

Following the collision, the x component of momentum is

$$\begin{aligned} p_{x,\mathrm{f}} &= m_1 v_{1,\mathrm{f}} \cos 66.0^\circ + m_2 v_{2,\mathrm{f}} \cos 24.0^\circ \\ &= (7.00\ \mathrm{kg})(0.610\ \mathrm{m/s}) \cos 66.0^\circ + (7.00\ \mathrm{kg})(1.37\ \mathrm{m/s}) \cos 24.0^\circ \\ &= 10.5\ \mathrm{kg \cdot m/s} \end{aligned}$$

As expected, the momentum is unchanged.

EXAMPLE 9–7 TWO FRUITS IN TWO DIMENSIONS: ANALYZING AN ELASTIC COLLISION

Two astronauts on opposite ends of a spaceship are comparing lunches. One has an apple, the other has an orange. They decide to trade. Astronaut 1 tosses the 0.130-kg apple toward astronaut 2 with a speed of 1.11 m/s. The 0.160-kg orange is tossed from astronaut 2 to astronaut 1 with a speed of 1.21 m/s. Unfortunately, the fruits collide, sending the orange off with a speed of 1.16 m/s at an angle of 42.0° with respect to its original direction of motion. Find the final speed and direction of the apple, assuming an elastic collision. Give the apple's direction relative to its original direction of motion.

PICTURE THE PROBLEM

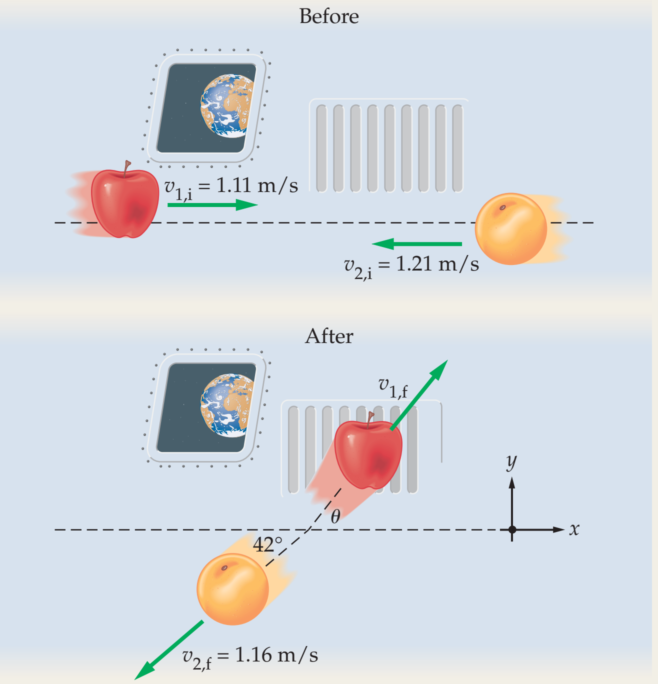
In our sketch we refer to the apple as object 1 and to the orange as object 2. We also choose the positive x direction to be in the initial direction of motion of the apple. We shall describe the "Before" and "After" sketches separately:

BEFORE

Initially, the apple moves in the positive x direction with a speed of 1.11 m/s, and the orange moves in the negative x direction with a speed of 1.21 m/s. There is no momentum in the y direction before the collision.

AFTER

After the collision, the orange moves with a speed of 1.16 m/s in a direction 42° below the negative x axis. As a result, the orange now has momentum in the negative y direction. To cancel this y momentum, the apple must move in a direction that is above the positive x axis, as indicated in the sketch.



STRATEGY

As described in the text, we first find the speed of the apple by demanding that the initial and final kinetic energies be the same. Next, we find the angle θ by conserving momentum in either the x or the y direction—the results are the same whichever direction is chosen.

SOLUTION

1. Calculate the initial kinetic energy of the system:

$$\begin{aligned} K_\mathrm{i} &= \tfrac{1}{2} m_1 v_{1,\mathrm{i}}^2 + \tfrac{1}{2} m_2 v_{2,\mathrm{i}}^2 \\ &= \tfrac{1}{2}(0.130\ \mathrm{kg})(1.11\ \mathrm{m/s})^2 + \tfrac{1}{2}(0.160\ \mathrm{kg})(1.21\ \mathrm{m/s})^2 \\ &= 0.197\ \mathrm{J} \end{aligned}$$

2. Calculate the final kinetic energy of the system in terms of $v_{1,\mathrm{f}}$:

$$\begin{aligned} K_\mathrm{f} &= \tfrac{1}{2} m_1 v_{1,\mathrm{f}}^2 + \tfrac{1}{2} m_2 v_{2,\mathrm{f}}^2 \\ &= \tfrac{1}{2}(0.130\ \mathrm{kg}) v_{1,\mathrm{f}}^2 + \tfrac{1}{2}(0.160\ \mathrm{kg})(1.16\ \mathrm{m/s})^2 \\ &= \tfrac{1}{2}(0.130\ \mathrm{kg}) v_{1,\mathrm{f}}^2 + 0.108\ \mathrm{J} \end{aligned}$$

3. Set $K_\mathrm{f} = K_\mathrm{i}$ to find $v_{1,\mathrm{f}}$:

$$v_{1,\mathrm{f}} = \sqrt{\frac{2(0.197\ \mathrm{J} - 0.108\ \mathrm{J})}{0.130\ \mathrm{kg}}} = 1.17\ \mathrm{m/s}$$

4. Set the final y component of momentum equal to zero to determine the angle, θ:

$$0 = m_1 v_{1,\mathrm{f}} \sin\theta - m_2 v_{2,\mathrm{f}} \sin 42.0^\circ$$

Solve for $\sin\theta$:

$$\sin\theta = \frac{m_2 v_{2,\mathrm{f}} \sin 42.0^\circ}{m_1 v_{1,\mathrm{f}}}$$

5. Substitute numerical values:

$$\sin\theta = \frac{(0.160\ \mathrm{kg})(1.16\ \mathrm{m/s}) \sin 42.0^\circ}{(0.130\ \mathrm{kg})(1.17\ \mathrm{m/s})} = 0.817$$

$$\theta = \sin^{-1}(0.817) = 54.8^\circ$$

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INSIGHT

The x momentum equation gives the same value for θ, as expected.

PRACTICE PROBLEM

Suppose that after the collision the apple moves in the positive y direction with a speed of 1.27 m/s. What are the final speed and direction of the orange in this case? [**Answer:** The orange moves with a speed of 1.07 m/s in a direction of 74.7° below the negative x axis.]

Some related homework problems: Problem 41, Problem 94

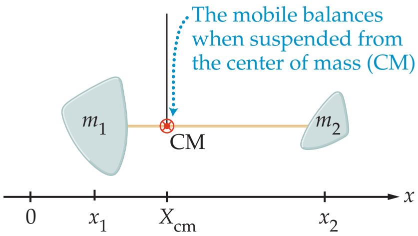


FIGURE 9–9 Balancing a mobile

Consider a portion of a mobile with masses m_1 and m_2 at the locations x_1 and x_2, respectively. The object balances when a string is attached at the center of mass. Since the center of mass is closer to m_1 than to m_2, it follows that m_1 is greater than m_2.

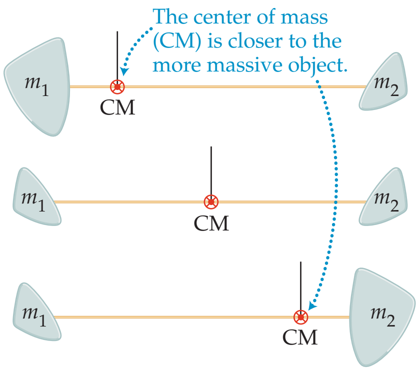
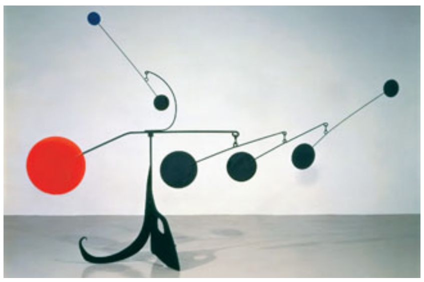


FIGURE 9–10 The center of mass of two objects

The center of mass is closest to the larger mass, or equidistant between the masses if they are equal.



Mobiles like *Myxomatose* by Alexander Calder illustrate the concept of center of mass with artistic flair. Each arm of the mobile is in balance because it is suspended at its center of mass.

9–7 Center of Mass

In this section we introduce the concept of the center of mass. We begin by defining its location for a given system of masses. Next we consider the motion of the center of mass and show how it is related to the net external force acting on the system. As we shall see, the center of mass plays a key role in the analysis of collisions.

Location of the Center of Mass

There is one point in any system of objects that has special significance—the **center of mass (CM).** One of the reasons the center of mass is so special is the fact that, in many ways, a system behaves as if all of its mass were concentrated there. As a result, a system can be balanced at its center of mass:

> The center of mass of a system of masses is the point where the system can be balanced in a uniform gravitational field.

For example, suppose you are making a mobile. At one stage in its construction, you want to balance a light rod with objects of mass m_1 and m_2 connected to either end, as indicated in **Figure 9–9**. To make the rod balance, you should attach a string to the center of mass of the system, just as if all its mass were concentrated at that point. In a sense, you can think of the center of mass as the "average" location of the system's mass.

To be more specific, suppose the two objects connected to the rod have the same mass. In this case the center of mass is at the midpoint of the rod, since this is where it balances. On the other hand, if one object has more mass than the other, the center of mass is closer to the heavier object, as indicated in **Figure 9–10**. In general, if a mass m_1 is on the x axis at the position x_1, and a mass m_2 is at the position x_2, as in Figure 9–9, the location of the center of mass, X_{cm}, is defined as the "weighted" average of the two positions:

Center of Mass for Two Objects

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1x_1 + m_2x_2}{M} \qquad \text{9–13}$$

Note that we have used $M = m_1 + m_2$ for the total mass of the two objects, and that the two positions, x_1 and x_2, are multiplied—or weighted—by their respective masses.

To see that this definition of X_{cm} agrees with our expectations, consider first the case where the masses are equal: $m_1 = m_2 = m$. In this case, $M = m_1 + m_2 = 2m$, and $X_{cm} = (mx_1 + mx_2)/2m = \frac{1}{2}(x_1 + x_2)$. Thus, as expected, if two masses are equal, their center of mass is halfway between them. On the other hand, if m_1 is significantly greater than m_2, it follows that $M = m_1 + m_2 \sim m_1$ and $m_1x_1 + m_2x_2 \sim m_1x_1$, since m_2 can be ignored in comparison to m_1. As a result, we find that $X_{cm} \sim m_1x_1/m_1 = x_1$; that is, the center of mass is essentially at the location of the extremely heavy mass, m_1. In general, as one mass becomes larger than the other, the center of mass moves closer to the larger mass.

EXERCISE 9–4

Suppose the masses in Figure 9–9 are separated by 0.500 m, and that $m_1 = 0.260$ kg and $m_2 = 0.170$ kg. What is the distance from m_1 to the center of mass of the system?

SOLUTION

Letting $x_1 = 0$ and $x_2 = 0.500$ m in Figure 9–9, we have

$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(0.260\text{ kg})\cdot 0 + (0.170\text{ kg})(0.500\text{ m})}{0.260\text{ kg} + 0.170\text{ kg}} = 0.198\text{ m}$$

Thus, the center of mass is closer to m_1 (the larger mass) than to m_2.

To extend the definition of X_{cm} to more general situations, first consider a system that contains many objects, not just two. In that case, X_{cm} is the sum of m times x for each object, divided by the total mass of the system, M. If, in addition, the objects in the system are not in a line, but are distributed in two dimensions, the center of mass will have both an x coordinate, X_{cm}, and a y coordinate, Y_{cm}. As one would expect, Y_{cm} is simply the sum of m times y for each object, divided by M. Thus, the x coordinate of the center of mass is

X Coordinate of the Center of Mass

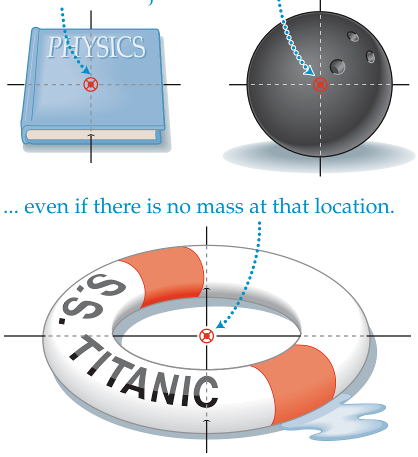
$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum mx}{M} \qquad 9\text{–}14$$

Similarly, the y coordinate of the center of mass is

Y Coordinate of the Center of Mass

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum my}{M} \qquad 9\text{–}15$$

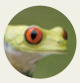
In systems with a continuous, uniform distribution of mass, the center of mass is at the geometric center of the object, as illustrated in **Figure 9–11**. Note that it is common for the center of mass to be located in a position where no mass exists, as in a life preserver, where the center of mass is precisely in the center of the hole.



▲ **FIGURE 9–11 Locating the center of mass**

In an object of continuous, uniform mass distribution, the center of mass is located at the geometric center of the object. In some cases, this means that the center of mass is not located within the object.

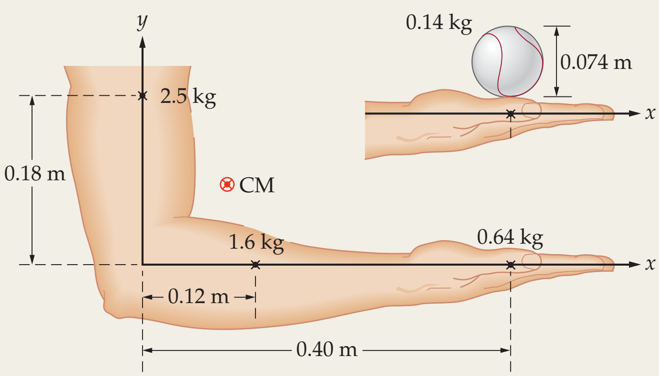
EXAMPLE 9–8 CENTER OF MASS OF THE ARM



A person's arm is held with the upper arm vertical, the lower arm and hand horizontal. **(a)** Find the center of mass of the arm in this configuration, given the following data: The upper arm has a mass of 2.5 kg and a center of mass 0.18 m above the elbow; the lower arm has a mass of 1.6 kg and a center of mass 0.12 m to the right of the elbow; the hand has a mass of 0.64 kg and a center of mass 0.40 m to the right of the elbow. **(b)** A 0.14-kg baseball is placed on the palm of the hand. If the diameter of the ball is 7.4 cm, find the center of mass of the arm–ball system.

PICTURE THE PROBLEM

We place the origin at the elbow, with the x and y axes pointing along the lower and upper arms, respectively. The center of mass of each of the three parts of the arm is indicated by an x; the center of mass of the entire arm is at the point labeled CM. The inset shows the baseball on the palm of the hand.



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STRATEGY

a. Using the information given in the problem statement, we can treat the arm as a system of three point masses placed as follows: 2.5 kg at (0, 0.18 m); 1.6 kg at (0.12 m, 0); 0.64 kg at (0.40 m, 0). We substitute these masses and locations into Equations 9–14 and 9–15 to find the x and y coordinates of the center of mass, respectively.

b. Treat the center of mass found in part (a) as a point particle with a mass 2.5 kg + 1.6 kg + 0.64 kg = 4.7 kg at the location (X_{cm}, Y_{cm}). The baseball can be treated as a point particle of mass 0.14 kg at the location (0.40 m, (0.074)/2 m).

SOLUTION

Part (a)

1. Calculate the x coordinate of the center of mass:

$$X_{cm} = \frac{(2.5\text{ kg})(0) + (1.6\text{ kg})(0.12\text{ m}) + (0.64\text{ kg})(0.40\text{ m})}{2.5\text{ kg} + 1.6\text{ kg} + 0.64\text{ kg}} = 0.095\text{ m}$$

2. Do the same calculation for the y coordinate of the center of mass:

$$Y_{cm} = \frac{(2.5\text{ kg})(0.18\text{ m}) + (1.6\text{ kg})(0) + (0.64\text{ kg})(0)}{2.5\text{ kg} + 1.6\text{ kg} + 0.64\text{ kg}} = 0.095\text{ m}$$

Part (b)

3. Calculate the new x coordinate of the center of mass:

$$X_{cm} = \frac{(4.7\text{ kg})(0.095\text{ m}) + (0.14\text{ kg})(0.40\text{ m})}{4.7\text{ kg} + 0.14\text{ kg}} = 0.10\text{ m}$$

4. Calculate the new y coordinate of the center of mass:

$$Y_{cm} = \frac{(4.7\text{ kg})(0.095\text{ m}) + (0.14\text{ kg})(0.037\text{ m})}{4.7\text{ kg} + 0.14\text{ kg}} = 0.093\text{ m}$$

INSIGHT

As is often the case, the center of mass of an arm held in this position is in a location where no mass exists—you might say the center of mass is having an out-of-body experience. This effect can sometimes be put to good use, as when the center of mass of a high jumper passes beneath the horizontal bar while the body passes above it. See Conceptual Question 18 for a photo of this technique in action, in the famous "Fosbury flop."

PRACTICE PROBLEM

Suppose the mass of the baseball is increased to 0.25 kg. **(a)** Does X_{cm} increase, decrease, or stay the same? **(b)** Does Y_{cm} increase, decrease, or stay the same? **(c)** Check your answers to parts (a) and (b) by finding the center of mass of the arm–ball system in this case. [**Answer:** (a) increases; (b) decreases; (c) $X_{cm} = 0.11$ m, $Y_{cm} = 0.092$ m]

Some related homework problems: Problem 51, Problem 53

Motion of the Center of Mass

Another reason the center of mass is of such importance is that its motion often displays a remarkable simplicity when compared with the motion of other parts of a system. To analyze this motion, we consider both the velocity and the acceleration of the center of mass. Each of these quantities is defined in complete analogy with the definition of the center of mass itself.

For example, to find the velocity of the center of mass, we first multiply the mass of each object in a system, m, by its velocity, $\vec{\mathbf{v}}$, to give $m_1\vec{\mathbf{v}}_1$, $m_2\vec{\mathbf{v}}_2$, and so on. Next, we add all these products together, $m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \cdots$, and divide by the total mass, $M = m_1 + m_2 + \cdots$. The result, by definition, is the velocity of the center of mass, $\vec{\mathbf{V}}_{cm}$:

Velocity of the Center of Mass

$$\vec{\mathbf{V}}_{cm} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m\vec{\mathbf{v}}}{M} \qquad 9\text{–}16$$

Comparing with Equation 9–14, we see that $\vec{\mathbf{V}}_{cm}$ is the same as X_{cm} with each position x replaced with a velocity vector $\vec{\mathbf{v}}$. In addition, note that the total mass of the system, M, times the velocity of the center of mass, $\vec{\mathbf{V}}_{\text{cm}}$, is simply the total momentum of the system:

$$M\vec{\mathbf{V}}_{\text{cm}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \cdots = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \cdots = \vec{\mathbf{p}}_{\text{total}}$$

To gain more information on how the center of mass moves, we next consider its acceleration, $\vec{\mathbf{A}}_{\text{cm}}$. As expected by analogy with $\vec{\mathbf{V}}_{\text{cm}}$, the acceleration of the center of mass is defined as follows:

Acceleration of the Center of Mass

$$\vec{\mathbf{A}}_{\text{cm}} = \frac{m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m\vec{\mathbf{a}}}{M} \qquad \text{9–17}$$

Note that the vector $\vec{\mathbf{A}}_{\text{cm}}$ contains terms like $m_1\vec{\mathbf{a}}_1$, $m_2\vec{\mathbf{a}}_2$, and so on, for each object in the system. From Newton's second law, however, we know that $m_1\vec{\mathbf{a}}_1$ is simply $\vec{\mathbf{F}}_1$, the net force acting on mass 1. The same conclusion applies to each of the masses. Therefore, we find that the total mass of the system, M, times the acceleration of the center of mass, $\vec{\mathbf{A}}_{\text{cm}}$, is simply the total force acting on the system:

$$M\vec{\mathbf{A}}_{\text{cm}} = m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + \cdots = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \cdots = \vec{\mathbf{F}}_{\text{total}}$$

Recall, however, that the total force acting on a system is the same as the net external force, $\vec{\mathbf{F}}_{\text{net,ext}}$, since the internal forces cancel. Therefore, $M\vec{\mathbf{A}}_{\text{cm}}$ is the net external force acting on the system:

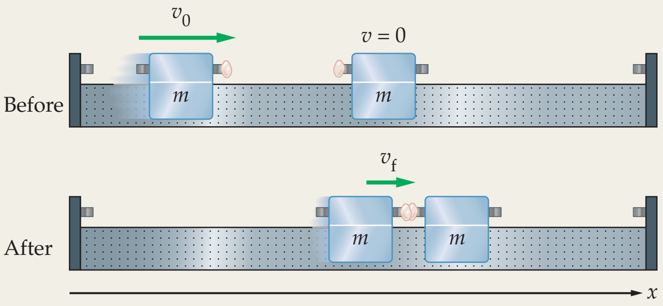
Newton's Second Law for a System of Particles

$$M\vec{\mathbf{A}}_{\text{cm}} = \vec{\mathbf{F}}_{\text{net,ext}} \qquad \text{9–18}$$

Zero Net External Force For systems in which $\vec{\mathbf{F}}_{\text{net,ext}}$ is zero, it follows that the acceleration of the center of mass is zero. Hence, if the center of mass is initially at rest, it remains at rest. Similarly, if the center of mass is moving initially, it continues to move with the same velocity. For example, in a collision between two air-track carts, the velocity of each cart changes as a result of the collision. The velocity of the center of mass of the two carts, however, is the same before and after the collision. We explore cases in which $\vec{\mathbf{F}}_{\text{net,ext}} = 0$ in the following Example and Active Example.

EXAMPLE 9–9 CRASH OF THE AIR CARTS

An air cart of mass m and speed v_0 moves toward a second, identical air cart that is at rest. When the carts collide they stick together and move as one. Find the velocity of the center of mass of this system **(a)** before and **(b)** after the carts collide.



INTERACTIVE FIGURE

PICTURE THE PROBLEM

We choose the positive x direction to be the direction of motion of the incoming cart, whose initial speed is v_0. Note that the carts have wads of putty on their bumpers; this ensures that they stick together when they collide and thereafter move as a unit. Their final speed is v_f.

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STRATEGY

a. We can find the velocity of the center of mass by applying Equation 9–16 to the case of just two masses; $\vec{\mathbf{V}}_{\text{cm}} = (m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2)/M$. In this case, $\vec{\mathbf{v}}_1 = v_0\hat{\mathbf{x}}$, $\vec{\mathbf{v}}_2 = 0$, and $m_1 = m_2 = m$.

b. After the collision the two masses have the same velocity, $\vec{\mathbf{v}}_{\text{f}} = v_{\text{f}}\hat{\mathbf{x}}$, which is given by momentum conservation (Equations 9–10 and 9–11). Hence, $\vec{\mathbf{V}}_{\text{cm}} = (m_1\vec{\mathbf{v}}_{\text{f}} + m_2\vec{\mathbf{v}}_{\text{f}})/M$.

SOLUTION

Part (a)

1. Use $\vec{\mathbf{V}}_{\text{cm}} = (m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2)/M$ to find the velocity of the center of mass before the collision:

$$\vec{\mathbf{V}}_{\text{cm}} = \frac{(m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2)}{m_1 + m_2} = \frac{(mv_0\hat{\mathbf{x}} + m\cdot 0)}{m + m} = \tfrac{1}{2}v_0\hat{\mathbf{x}}$$

Part (b)

2. Use momentum conservation in the x direction to find the speed of the carts after the collision:

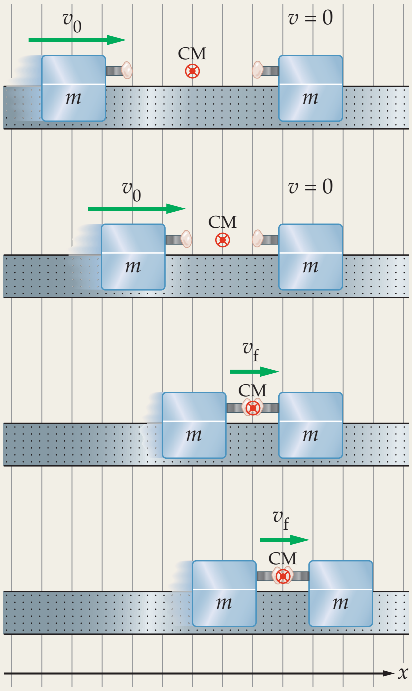
$$mv_0 = mv_{\text{f}} + mv_{\text{f}}$$
$$v_{\text{f}} = \tfrac{1}{2}v_0$$

3. Calculate the velocity of the center of mass of the two carts after the collision:

$$\vec{\mathbf{V}}_{\text{cm}} = \frac{(m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2)}{m_1 + m_2} = \frac{(mv_{\text{f}}\hat{\mathbf{x}} + mv_{\text{f}}\hat{\mathbf{x}})}{m + m} = v_{\text{f}}\hat{\mathbf{x}} = \tfrac{1}{2}v_0\hat{\mathbf{x}}$$

INSIGHT

As expected, the velocity of each cart changes when they collide. On the other hand, the velocity of the center of mass is completely unaffected by the collision. This is illustrated to the right, where we show a sequence of equal-time snapshots of the system just before and just after the collision. First, we note that the incoming cart moves two distance units for every time interval until it collides with the second cart. From that point on, the two carts are locked together, and move one distance unit per time interval. In contrast, the center of mass (CM), which is centered between the two equal-mass carts, progresses uniformly throughout the sequence, advancing one unit of distance for each time interval.



PRACTICE PROBLEM

If the mass of the cart that is moving initially is doubled to $2m$, does the velocity of the center of mass increase, decrease, or stay the same? Verify your answer by calculating the velocity of the center of mass in this case. [**Answer:** The velocity of the center of mass increases. We find that $\vec{\mathbf{V}}_{\text{cm}} = (2v_0/3)\hat{\mathbf{x}}$, both before and after the collision.]

Some related homework problems: Problem 54, Problem 57, Problem 81

ACTIVE EXAMPLE 9–3 FIND THE VELOCITY OF THE CENTER OF MASS

In Active Example 9–2 we found that as a 0.150-g bee runs with a speed of 3.80 cm/s in one direction, the 4.75-g popsicle stick on which it floats moves with a speed of 0.120 cm/s in the opposite direction. Find the velocity of the center of mass of the bee and the stick.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the velocity of the bee: $\vec{\mathbf{v}}_{\text{b}} = (3.80\text{ cm/s})\hat{\mathbf{x}}$

2. Write the velocity of the stick: $\vec{\mathbf{v}}_{\text{s}} = (-0.120\text{ cm/s})\hat{\mathbf{x}}$

3. Use these velocities to calculate $\vec{\mathbf{V}}_{\text{cm}}$: $\vec{\mathbf{V}}_{\text{cm}} = (m_{\text{b}}\vec{\mathbf{v}}_{\text{b}} + m_{\text{s}}\vec{\mathbf{v}}_{\text{s}})/(m_{\text{b}} + m_{\text{s}}) = 0$

INSIGHT

$\vec{\mathbf{V}}_{\text{cm}}$ is zero, and hence the center of mass stays at rest as the bee and the stick move. This is as expected, since the net external force is zero for this system, and the bee and stick started at rest initially.

YOUR TURN

If the bee increases its speed, will the velocity of the center of mass be nonzero?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Nonzero Net External Force Recall that Newton's second law, as expressed in Equation 9–18, states that the acceleration of the center of mass is related to the net external force as follows:

$$M\vec{\mathbf{A}}_{\text{cm}} = \vec{\mathbf{F}}_{\text{net,ext}}$$

This is completely analogous to the relationship between the acceleration of an object of mass m and the net force $\vec{\mathbf{F}}_{\text{net}}$ applied to it:

$$m\vec{\mathbf{a}} = \vec{\mathbf{F}}_{\text{net}}$$

Therefore, when $\vec{\mathbf{F}}_{\text{net,ext}}$ is nonzero, we can conclude the following:

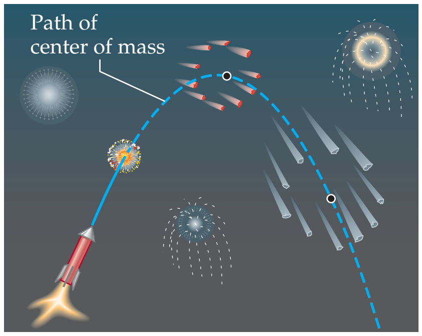
> The center of mass of a system accelerates precisely as if it were a point particle of mass M acted on by the force $\vec{\mathbf{F}}_{\text{net,ext}}$.

For this reason, the motion of the center of mass can be quite simple compared to the motion of its constituent parts. For example, a hammer tossed into the air with a rotation is shown in **Figure 9–12**. The motion of one part of the hammer, the tip of the handle, let's say, follows a complicated path in space. On the other hand, the path of the center of mass is a simple parabola, precisely the same path that a point mass would follow.

Similarly, consider a fireworks rocket launched into the sky, as illustrated in **Figure 9–13**. The center of mass of the rocket follows a parabolic path, ignoring air resistance. At some point in its path it explodes into numerous individual pieces. The explosion is due to internal forces, however, which must therefore sum to zero. Hence, the net external force acting on the pieces of the rocket is the same before, during, and after the explosion. As a result, the center of mass has a constant downward acceleration and continues to follow the original parabolic path. It is only when an additional external force acts on the system, as when one of the pieces of the rocket hits the ground, that the path of the center of mass changes.

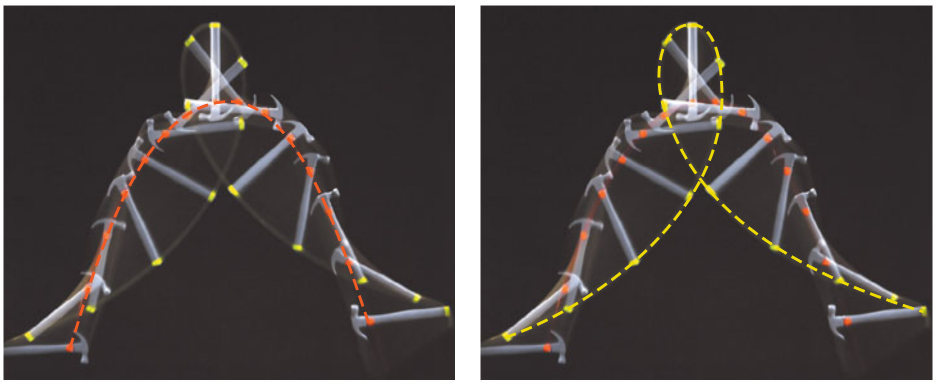
REAL-WORLD PHYSICS

An exploding rocket



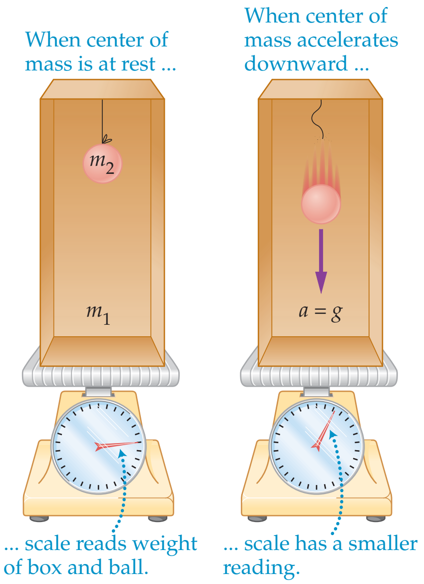
▲ **FIGURE 9–13 Center of mass of an exploding rocket**

A fireworks rocket follows a parabolic path, ignoring air resistance, until it explodes. After exploding, its center of mass continues on the same parabolic path until some of the fragments start to land.



▲ **FIGURE 9–12 Simple Motion of the Center of Mass**

As this hammer flies through the air, its motion is quite complex. Some parts of the hammer follow wild trajectories with strange loops and turns. There is one point on the hammer, however, that travels on a smooth, simple parabolic path—the center of mass. The center of mass (red path on the left) travels as if all the mass of the hammer were concentrated there; other points (yellow path on the right) follow complex paths that depend on the detailed shape and rotation of the hammer.



▲ **FIGURE 9–14 Weight and acceleration of the center of mass**

A box with a ball suspended from a string is weighed on a scale. The scale reads the weight of the box and the ball. When the string breaks and the ball falls with the acceleration of gravity, the scale reads only the weight of the box.

To see how to apply $M\vec{\mathbf{A}}_{cm} = \vec{\mathbf{F}}_{net,ext}$, consider the system shown in **Figure 9–14**. Here we see a box of mass m_1, inside of which is a ball of mass m_2 suspended from a light string. The entire system rests on a scale reading its weight. The scale exerts an upward force on the box of magnitude F_s. Initially, of course, $F_s = (m_1 + m_2)g$.

Now, suppose the string breaks, allowing the ball to fall with constant acceleration g toward the bottom of the box. What is the reading on the scale while the ball falls? We can guess that the answer should be simply $m_1 g$, the weight of the box alone, but let's analyze the problem from the point of view of the center of mass.

Taking upward as the positive direction, the net external force acting on the box and the ball is

$$F_{net,ext} = F_s - m_1 g - m_2 g$$

The acceleration of the center of mass is

$$A_{cm} = \frac{m_1 \cdot 0 - m_2 g}{M} = -\frac{m_2}{M} g$$

Setting $MA_{cm} = F_{net,ext}$ yields

$$MA_{cm} = M\left(-\frac{m_2}{M}\right)g = -m_2 g = F_{net,ext} = F_s - m_1 g - m_2 g$$

Finally, canceling the term $-m_2 g$ and solving for the weight read by the scale, F_s, we find, as expected, that

$$F_s = m_1 g$$

*9–8 Systems with Changing Mass: Rocket Propulsion

We close this chapter by considering systems in which the mass can change. A rocket, for example, changes its mass as its engines operate because it ejects part of the fuel as it burns. The burning process is produced by internal forces, hence the total momentum of the rocket and its fuel remains constant.

Consider, then, a rocket in outer space, far from any large, massive objects. When the rocket's engine is fired, it expels a certain mass of fuel out the back with a speed v. If the mass of the ejected fuel is Δm, then the momentum of the ejected fuel has a magnitude equal to $(\Delta m)v$. Since the total momentum of the system must still be zero, the rocket acquires an equivalent amount of momentum in the forward direction. Hence, the momentum increase of the rocket is

$$\Delta p = (\Delta m)v$$

If the mass of fuel Δm is ejected in the time Δt, the force exerted on the rocket is the change in its momentum divided by the time interval (Equation 9–3); that is,

$$F = \frac{\Delta p}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)v$$

The force exerted on the rocket by the ejected fuel is referred to as the **thrust.** Thus, the thrust of a rocket is

Thrust

$$\text{thrust} = \left(\frac{\Delta m}{\Delta t}\right)v \qquad 9\text{–}19$$

SI unit: newton, N

By $\Delta m/\Delta t$, we simply mean the amount of mass per time coming out of the rocket. For example, on the Saturn V rocket, the one used on the manned missions to the Moon, the main engines eject fuel at the rate of 13,800 kg/s with a speed of 2440 m/s. As a result, the thrust produced by these engines is

$$\text{thrust} = \left(\frac{\Delta m}{\Delta t}\right)v = (13{,}800\ \text{kg/s})(2440\ \text{m/s}) = 33.7 \times 10^6\ \text{N}$$

Since this is about 7.60 million pounds, and the weight of the rocket at liftoff is only 6.30 million pounds $= 28.0 \times 10^6$ N, the thrust is sufficient to launch the rocket and give it an upward acceleration. In fact, the initial net force acting on the rocket is

$$F_{\text{net}} = \text{thrust} - mg = 33.7 \times 10^6\ \text{N} - 28.0 \times 10^6\ \text{N} = 5.7 \times 10^6\ \text{N}$$

The rocket's initial weight is $W = mg = 28.0 \times 10^6$ N, and hence its initial mass is $m = W/g = 2.85 \times 10^6$ kg. Therefore, the rocket lifts off with an upward acceleration of

$$a = \frac{F_{\text{net}}}{m} = \frac{5.7 \times 10^6\ \text{N}}{2.85 \times 10^6\ \text{kg}} = 2.0\ \text{m/s}^2 \approx 0.20g$$

This is a rather gentle acceleration. The gentleness lasts only a matter of seconds, however, since the decreasing mass of the rocket results in an increasing acceleration.

REAL-WORLD PHYSICS

Saturn V rocket

EXERCISE 9–5

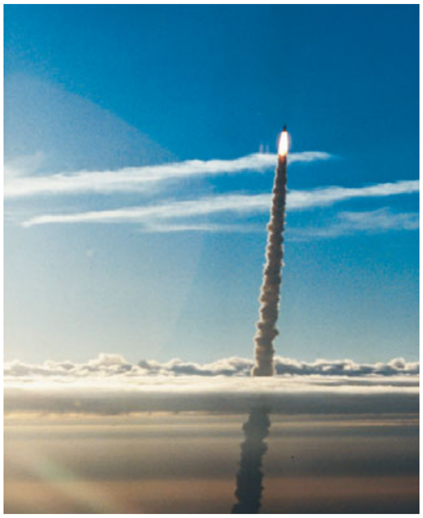
The ascent stage of the lunar lander was designed to produce 15,500 N of thrust at liftoff. If the speed of the ejected fuel is 2500 m/s, what is the rate at which the fuel must be burned?

SOLUTION

The rate of fuel consumption is

$$\frac{\Delta m}{\Delta t} = \frac{\text{thrust}}{v} = \frac{15{,}500\ \text{N}}{2500\ \text{m/s}} = 6.2\ \text{kg/s}$$

A common question regarding rockets is: "How can a rocket accelerate in outer space when it has nothing to push against?" The answer is that rockets, in effect, push against their own fuel. The situation is similar to firing a gun. When a bullet is ejected by the internal combustion of the gunpowder, the person firing the gun feels a recoil. If the person were in space, or standing on a frictionless surface, the recoil would give him or her a speed in the direction opposite to the bullet. The burning of a rocket engine provides a continuous recoil, almost as if the rocket were firing a steady stream of bullets out the back.



▲ A rocket (top) makes use of the principle of conservation of momentum: mass (the products of explosive burning of fuel) is ejected at high speed in one direction, causing the rocket to move in the opposite direction. The same method of propulsion has evolved in octopi (bottom) and some other animals. When danger threatens and a quick escape is needed, powerful muscles contract to create a jet of water that propels the animal to safety.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

We see in Section 9–1 that momentum is a vector quantity. Thus, the vector tools introduced in Chapter 3 again become important. In particular, we use vector components in our analysis of momentum conservation in Sections 9–5 and 9–6.

The connection between force (Chapter 5) and momentum is developed in Section 9–2. Newton's second law is central to impulse (Section 9–3), and Newton's third law is the key to conservation of momentum (Section 9–4).

Kinetic energy (Chapter 7) plays a key role in analyzing collisions, leading to the distinction between elastic and inelastic collisions. Potential energy (Chapter 8) enters into our analysis of the ballistic pendulum in Example 9–5.

In this chapter we see that force times the time over which it acts is related to a change in energy; in Chapters 7 and 8 we saw that force times the distance over which it acts leads to a change in energy.

LOOKING AHEAD

The concept of momentum is used again in Chapter 11, when we study the dynamics of rotational motion. In particular, we introduce angular momentum in Section 11–6 as an extension of the linear momentum introduced in this chapter.

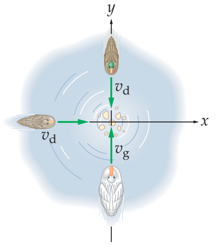
Angular momentum is used in our analysis of planetary orbits, and especially in the discussion of Kepler's second law in Section 12–3.

The idea of angular momentum having only certain allowed values is one of the key assumptions of the Bohr model of the hydrogen atom, as we show in Section 31–3.

Linear momentum plays an important role in quantum physics. For example, the momentum of a particle is related to its de Broglie wavelength (Section 30–5) and to the uncertainty principle (Section 30–6).

CHAPTER SUMMARY

9–1 LINEAR MOMENTUM



The linear momentum of an object of mass m moving with velocity $\vec{\mathbf{v}}$ is

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} \qquad 9\text{–}1$$

Momentum Is a Vector
Linear momentum is a vector, pointing in the same direction as the velocity vector, $\vec{\mathbf{v}}$.

Momentum of a System of Objects
In a system of several objects, the total linear momentum is the vector sum of the individual momenta:

$$\vec{\mathbf{p}}_{\text{total}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 + \vec{\mathbf{p}}_3 + \cdots \qquad 9\text{–}2$$

9–2 MOMENTUM AND NEWTON'S SECOND LAW

In terms of momentum, Newton's second law is

$$\sum \vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} \qquad 9\text{–}3$$

That is, the net force acting on an object is equal to the rate of change of its momentum.

Constant Mass
For cases in which the mass is constant, Newton's second law reduces to the familiar form

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} \qquad 9\text{–}4$$

9–3 IMPULSE

The impulse delivered to an object by an average force $\vec{\mathbf{F}}_{\text{av}}$ acting for a time Δt is

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}\,\Delta t \qquad 9\text{–}5$$

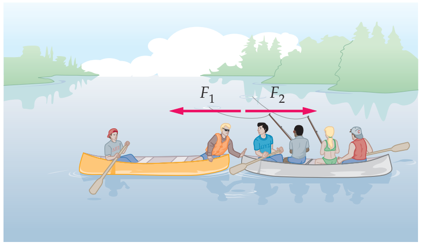
Impulse Is a Vector
Impulse is a vector, proportional to the force vector.

Impulse and Momentum
By Newton's second law, the impulse delivered to an object is equal to the change in its momentum:

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}\,\Delta t = \Delta\vec{\mathbf{p}} \qquad 9\text{–}6$$

Magnitude of the Impulse and Force
Since an impulse is often delivered in a very short time interval, the average force can be large.

9–4 CONSERVATION OF LINEAR MOMENTUM

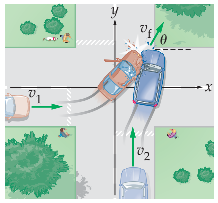


The momentum of an object is conserved (remains constant) if the net force acting on it is zero.

Internal/External Forces
In a system of objects, internal forces always sum to zero. The net force acting on a system of objects, then, is the sum of the external forces.

Conservation of Momentum in a System
In a system of objects, the net momentum is conserved if the net external force acting on the system is zero.

9–5 INELASTIC COLLISIONS



In collisions, we assume that external forces either sum to zero or are small enough to be ignored. Hence, momentum is conserved in all collisions.

Inelastic Collision
In an inelastic collision, the final kinetic energy is different from the initial kinetic energy. The kinetic energy is usually less after a collision, but it can also be more than the initial kinetic energy.

Completely Inelastic Collision
A collision in which objects hit and stick together is referred to as completely inelastic.

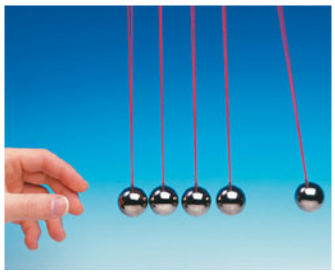
Collisions in One Dimension
A one-dimensional collision occurs along a line, which we can choose to be the x axis. After the collision, the x component of momentum is equal to the x component of momentum before the collision; that is, the x component of momentum is conserved.

If two objects, of mass m_1 and m_2 and with initial velocities $v_{1,i}$ and $v_{2,i}$, collide and stick, the final velocity is

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} \qquad \text{9–10}$$

Collisions in Two Dimensions
In a two-dimensional collision, there are two separate momentum relations to be satisfied: (i) the x component of momentum is conserved, and (ii) the y component of momentum is conserved.

9–6 ELASTIC COLLISIONS



In collisions, we assume that external forces either sum to zero or are small enough to be ignored. Hence, momentum is conserved in all collisions.

Elastic Collision
In an elastic collision, the final kinetic energy is equal to the initial kinetic energy.

Collisions in One Dimension
In an elastic collision in one dimension where mass m_1 is moving with an initial velocity v_0, and mass m_2 is initially at rest, the velocities of the masses after the collision are:

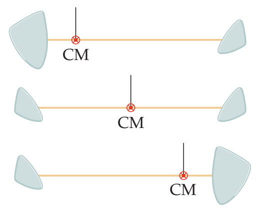
$$v_{1,f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_0$$

and 9–12

$$v_{2,f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_0$$

Collisions in Two Dimensions
In elastic collisions in two dimensions, three separate conditions are satisfied: (i) kinetic energy is conserved, (ii) the x component of momentum is conserved, and (iii) the y component of momentum is conserved.

9–7 CENTER OF MASS



The location of the center of mass of a two-dimensional system of objects is defined as follows:

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum mx}{M} \quad 9\text{–}14$$

and

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum my}{M} \quad 9\text{–}15$$

Motion of the Center of Mass

The velocity of the center of mass is

$$\vec{\mathbf{V}}_{cm} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m\vec{\mathbf{v}}}{M} \quad 9\text{–}16$$

Note that $M\vec{\mathbf{V}}_{cm} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \cdots = \vec{\mathbf{p}}_{total}$. If a system's momentum is conserved, its center of mass has constant velocity. Similarly, the acceleration of the center of mass is

$$\vec{\mathbf{A}}_{cm} = \frac{m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + \cdots}{m_1 + m_2 + \cdots} = \frac{\sum m\vec{\mathbf{a}}}{M} \quad 9\text{–}17$$

Note that $M\vec{\mathbf{A}}_{cm} = m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + \cdots =$ (net external force). That is,

$$M\vec{\mathbf{A}}_{cm} = \vec{\mathbf{F}}_{net,ext} \quad 9\text{–}18$$

The center of mass accelerates as if the net external force acted on a single object of mass $M = m_1 + m_2 + \cdots$.

*9–8 SYSTEMS WITH CHANGING MASS: ROCKET PROPULSION

The mass of a rocket changes because its engines expel fuel when they are fired. If fuel is expelled with the speed v and at the rate $\Delta m/\Delta t$, the thrust experienced by the rocket is

$$\text{thrust} = \left(\frac{\Delta m}{\Delta t}\right)v \quad 9\text{–}19$$

PROBLEM-SOLVING SUMMARY

Type of Calculation	Relevant Physical Concepts	Related Examples
Calculate the momentum of a system.	Each object in a system has a momentum of magnitude mv that points in the direction of its velocity vector. The total momentum is the vector sum of the individual momenta.	Example 9–1
Relate force and time to the impulse.	The impulse acting on a system is the average force, F_{av}, times the time interval, Δt.	Example 9–2 Active Example 9–1
Apply momentum conservation.	Momentum is conserved when the net external force acting on a system is zero.	Examples 9–3, 9–4, 9–5, 9–6, 9–7 Active Example 9–2
Find the center of mass.	The location of the center of mass is given by Equations 9–14 and 9–15.	Example 9–8
Determine the motion of the center of mass.	The center of mass moves the same as if it were a point particle of mass M (the total mass of the system) acted on by the net external force, $\vec{\mathbf{F}}_{net,ext}$.	Example 9–9 Active Example 9–3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. If you drop your keys, their momentum increases as they fall. Why is the momentum of the keys not conserved? Does this mean that the momentum of the universe increases as the keys fall? Explain.
2. By what factor does an object's kinetic energy change if its speed is doubled? By what factor does its momentum change?
3. A system of particles is known to have zero kinetic energy. What can you say about the momentum of the system?
4. A system of particles is known to have zero momentum. Does it follow that the kinetic energy of the system is also zero? Explain.
5. On a calm day you connect an electric fan to a battery on your sailboat and generate a breeze. Can the wind produced by the fan be used to power the sailboat? Explain.
6. In the previous question, can you use the wind generated by the fan to move a boat that has no sail? Explain why or why not.
7. Crash statistics show that it is safer to be riding in a heavy car in an accident than in a light car. Explain in terms of physical principles.
8. **(a)** As you approach a stoplight, you apply the brakes and bring your car to rest. What happened to your car's initial momentum? **(b)** When the light turns green, you accelerate until you reach cruising speed. What force was responsible for increasing your car's momentum?
9. An object at rest on a frictionless surface is struck by a second object. Is it possible for both objects to be at rest after the collision? Explain.
10. In the previous question, is it possible for one of the two objects to be at rest after the collision? Explain.
11. **(a)** Can two objects on a horizontal frictionless surface have a collision in which all the initial kinetic energy of the system is lost? Explain, and give a specific example if your answer is yes. **(b)** Can two such objects have a collision in which all the initial momentum of the system is lost? Explain, and give a specific example if your answer is yes.
12. Two cars collide at an intersection. If the cars do not stick together, can we conclude that their collision was elastic? Explain.
13. At the instant a bullet is fired from a gun, the bullet and the gun have equal and opposite momenta. Which object—the bullet or the gun—has the greater kinetic energy? Explain. How does your answer apply to the observation that it is safe to hold a gun while it is fired, whereas the bullet is deadly?
14. An hourglass is turned over, and the sand is allowed to pour from the upper half of the glass to the lower half. If the hourglass is resting on a scale, and the total mass of the hourglass and sand is M, describe the reading on the scale as the sand runs to the bottom.
15. In the classic movie *The Spirit of St. Louis*, Jimmy Stewart portrays Charles Lindbergh on his history-making transatlantic flight. Lindbergh is concerned about the weight of his fuel-laden airplane. As he flies over Newfoundland he notices a fly on the dashboard. Speaking to the fly, he wonders aloud, "Does the plane weigh less if you fly inside it as it's flying? Now that's an interesting question." What do you think?
16. A tall, slender drinking glass with a thin base is initially empty. **(a)** Where is the center of mass of the glass? **(b)** Suppose the glass is now filled slowly with water until it is completely full. Describe the position and motion of the center of mass during the filling process.
17. Lifting one foot into the air, you balance on the other foot. What can you say about the location of your center of mass?
18. In the "Fosbury flop" method of high jumping, named for the track and field star Dick Fosbury, an athlete's center of mass may pass under the bar while the athlete's body passes over the bar. Explain how this is possible.

The "Fosbury flop." (Conceptual Question 18)

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 9–1 LINEAR MOMENTUM

1. • Referring to Exercise 9–1, what speed must the baseball have if its momentum is to be equal in magnitude to that of the car? Give your result in miles per hour.
2. • Find the total momentum of the birds in Example 9–1 if the goose reverses direction.
3. •• A 26.2-kg dog is running northward at 2.70 m/s, while a 5.30-kg cat is running eastward at 3.04 m/s. Their 74.0-kg owner has the same momentum as the two pets taken together. Find the direction and magnitude of the owner's velocity.
4. •• **IP** Two air-track carts move toward one another on an air track. Cart 1 has a mass of 0.35 kg and a speed of 1.2 m/s. Cart 2 has a mass of 0.61 kg. **(a)** What speed must cart 2 have if the total momentum of the system is to be zero? **(b)** Since the momentum of the system is zero, does it follow that the kinetic energy of the system is also zero? **(c)** Verify your answer to part (b) by calculating the system's kinetic energy.
5. •• A 0.150-kg baseball is dropped from rest. If the magnitude of the baseball's momentum is 0.780 kg · m/s just before it lands on the ground, from what height was it dropped?
6. •• **IP** A 285-g ball falls vertically downward, hitting the floor with a speed of 2.5 m/s and rebounding upward with a speed of 2.0 m/s. **(a)** Find the magnitude of the change in the ball's momentum. **(b)** Find the change in the magnitude of the ball's momentum. **(c)** Which of the two quantities calculated in parts (a) and (b) is more directly related to the net force acting on the ball during its collision with the floor? Explain.

7. ••• Object 1 has a mass m_1 and a velocity $\vec{\mathbf{v}}_1 = (2.80 \text{ m/s})\hat{\mathbf{x}}$. Object 2 has a mass m_2 and a velocity $\vec{\mathbf{v}}_2 = (3.10 \text{ m/s})\hat{\mathbf{y}}$. The total momentum of these two objects has a magnitude of 17.6 kg · m/s and points in a direction 66.5° above the positive x axis. Find m_1 and m_2.

SECTION 9–3 IMPULSE

8. • **CE** Your car rolls slowly in a parking lot and bangs into the metal base of a light pole. In terms of safety, is it better for your collision with the light pole to be elastic, inelastic, or is the safety risk the same for either case? Explain.

9. • **CE Predict/Explain** A net force of 200 N acts on a 100-kg boulder, and a force of the same magnitude acts on a 100-g pebble. **(a)** Is the change of the boulder's momentum in one second greater than, less than, or equal to the change of the pebble's momentum in the same time period? **(b)** Choose the *best explanation* from among the following:
 I. The large mass of the boulder gives it the greater momentum.
 II. The force causes a much greater speed in the 100-g pebble, resulting in more momentum.
 III. Equal force means equal change in momentum for a given time.

10. • **CE Predict/Explain** Referring to the previous question, **(a)** is the change in the boulder's speed in one second greater than, less than, or equal to the change in speed of the pebble in the same time period? **(b)** Choose the *best explanation* from among the following:
 I. The large mass of the boulder results in a small acceleration.
 II. The same force results in the same change in speed for a given time.
 III. Once the boulder gets moving it is harder to stop than the pebble.

11. • **CE Predict/Explain** A friend tosses a ball of mass m to you with a speed v. When you catch the ball, you feel a noticeable sting in your hand, due to the force required to stop the ball. **(a)** If you now catch a second ball, with a mass $2m$ and speed $v/2$, is the sting you feel greater than, less than, or equal to the sting you felt when you caught the first ball? The time required to stop the two balls is the same. **(b)** Choose the *best explanation* from among the following:
 I. The second ball has less kinetic energy, since kinetic energy depends on v^2, and hence it produces less sting.
 II. The two balls have the same momentum, and hence they produce the same sting.
 III. The second ball has more mass, and hence it produces the greater sting.

12. • **CE** Force A has a magnitude F and acts for the time Δt, force B has a magnitude $2F$ and acts for the time $\Delta t/3$, force C has a magnitude $5F$ and acts for the time $\Delta t/10$, and force D has a magnitude $10F$ and acts for the time $\Delta t/100$. Rank these forces in order of increasing impulse. Indicate ties where appropriate.

13. • Find the magnitude of the impulse delivered to a soccer ball when a player kicks it with a force of 1250 N. Assume that the player's foot is in contact with the ball for 5.95×10^{-3} s.

14. • In a typical golf swing, the club is in contact with the ball for about 0.0010 s. If the 45-g ball acquires a speed of 67 m/s, estimate the magnitude of the force exerted by the club on the ball.

15. • A 0.50-kg croquet ball is initially at rest on the grass. When the ball is struck by a mallet, the average force exerted on it is 230 N. If the ball's speed after being struck is 3.2 m/s, how long was the mallet in contact with the ball?

16. • When spiking a volleyball, a player changes the velocity of the ball from 4.2 m/s to −24 m/s along a certain direction. If the impulse delivered to the ball by the player is −9.3 kg · m/s, what is the mass of the volleyball?

17. •• **IP** A 15.0-g marble is dropped from rest onto the floor 1.44 m below. **(a)** If the marble bounces straight upward to a height of 0.640 m, what are the magnitude and direction of the impulse delivered to the marble by the floor? **(b)** If the marble had bounced to a greater height, would the impulse delivered to it have been greater or less than the impulse found in part (a)? Explain.

18. •• To make a bounce pass, a player throws a 0.60-kg basketball toward the floor. The ball hits the floor with a speed of 5.4 m/s at an angle of 65° to the vertical. If the ball rebounds with the same speed and angle, what was the impulse delivered to it by the floor?

19. •• **IP** A 0.14-kg baseball moves toward home plate with a velocity $\vec{\mathbf{v}}_i = (-36 \text{ m/s})\hat{\mathbf{x}}$. After striking the bat, the ball moves vertically upward with a velocity $\vec{\mathbf{v}}_f = (18 \text{ m/s})\hat{\mathbf{y}}$. **(a)** Find the direction and magnitude of the impulse delivered to the ball by the bat. Assume that the ball and bat are in contact for 1.5 ms. **(b)** How would your answer to part (a) change if the mass of the ball were doubled? **(c)** How would your answer to part (a) change if the mass of the bat were doubled instead?

20. •• A player bounces a 0.43-kg soccer ball off her head, changing the velocity of the ball from $\vec{\mathbf{v}}_i = (8.8 \text{ m/s})\hat{\mathbf{x}} + (-2.3 \text{ m/s})\hat{\mathbf{y}}$ to $\vec{\mathbf{v}}_f = (5.2 \text{ m/s})\hat{\mathbf{x}} + (3.7 \text{ m/s})\hat{\mathbf{y}}$. If the ball is in contact with the player's head for 6.7 ms, what are **(a)** the direction and **(b)** the magnitude of the impulse delivered to the ball?

SECTION 9–4 CONSERVATION OF LINEAR MOMENTUM

21. • In a situation similar to Example 9–3, suppose the speeds of the two canoes after they are pushed apart are 0.58 m/s for canoe 1 and 0.42 m/s for canoe 2. If the mass of canoe 1 is 320 kg, what is the mass of canoe 2?

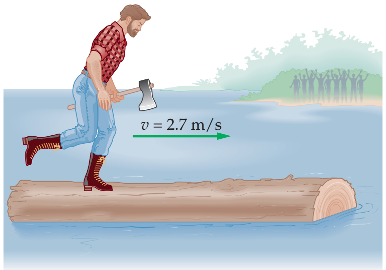
22. • Two ice skaters stand at rest in the center of an ice rink. When they push off against one another the 45-kg skater acquires a speed of 0.62 m/s. If the speed of the other skater is 0.89 m/s, what is this skater's mass?

23. • Suppose the bee in Active Example 9–2 has a mass of 0.175 g. If the bee walks with a speed of 1.41 cm/s relative to the still water, what is the speed of the 4.75-g stick relative to the water?

24. •• An object initially at rest breaks into two pieces as the result of an explosion. One piece has twice the kinetic energy of the other piece. What is the ratio of the masses of the two pieces? Which piece has the larger mass?

25. •• A 92-kg astronaut and a 1200-kg satellite are at rest relative to the space shuttle. The astronaut pushes on the satellite, giving it a speed of 0.14 m/s directly away from the shuttle. Seven and a half seconds later the astronaut comes into contact with the shuttle. What was the initial distance from the shuttle to the astronaut?

26. •• **IP** An 85-kg lumberjack stands at one end of a 380-kg floating log, as shown in **Figure 9–15**. Both the log and the lumberjack are at rest initially. **(a)** If the lumberjack now trots toward the other end of the log with a speed of 2.7 m/s relative to the log, what is the lumberjack's speed relative to the shore? Ignore friction between the log and the water. **(b)** If the mass of the log had been greater, would the lumberjack's speed relative to the shore be greater than, less than, or the same as in part (a)? Explain. **(c)** Check your answer to part (b) by calculating the lumberjack's speed relative to the shore for the case of a 450-kg log.



▲ **FIGURE 9–15** Problem 26

27. ••• A plate drops onto a smooth floor and shatters into three pieces of equal mass. Two of the pieces go off with equal speeds v at right angles to one another. Find the speed and direction of the third piece.

SECTION 9–5 INELASTIC COLLISIONS

28. • A cart of mass m moves with a speed v on a frictionless air track and collides with an identical cart that is stationary. If the two carts stick together after the collision, what is the final kinetic energy of the system?

29. • Suppose the car in Example 9–6 has an initial speed of 20.0 m/s and that the direction of the wreckage after the collision is 40.0° above the x axis. Find the initial speed of the minivan and the final speed of the wreckage.

30. • Two 72.0-kg hockey players skating at 5.45 m/s collide and stick together. If the angle between their initial directions was 115°, what is their speed after the collision?

31. •• **IP** **(a)** Referring to Exercise 9–2, is the final kinetic energy of the car and truck together greater than, less than, or equal to the sum of the initial kinetic energies of the car and truck separately? Explain. **(b)** Verify your answer to part (a) by calculating the initial and final kinetic energies of the system.

32. •• **IP** A bullet with a mass of 4.0 g and a speed of 650 m/s is fired at a block of wood with a mass of 0.095 kg. The block rests on a frictionless surface, and is thin enough that the bullet passes completely through it. Immediately after the bullet exits the block, the speed of the block is 23 m/s. **(a)** What is the speed of the bullet when it exits the block? **(b)** Is the final kinetic energy of this system equal to, less than, or greater than the initial kinetic energy? Explain. **(c)** Verify your answer to part (b) by calculating the initial and final kinetic energies of the system.

33. •• **IP** A 0.420-kg block of wood hangs from the ceiling by a string, and a 0.0750-kg wad of putty is thrown straight upward, striking the bottom of the block with a speed of 5.74 m/s. The wad of putty sticks to the block. **(a)** Is the mechanical energy of this system conserved? **(b)** How high does the putty–block system rise above the original position of the block?

34. •• A 0.430-kg block is attached to a horizontal spring that is at its equilibrium length, and whose force constant is 20.0 N/m. The block rests on a frictionless surface. A 0.0500-kg wad of putty is thrown horizontally at the block, hitting it with a speed of 2.30 m/s and sticking. How far does the putty–block system compress the spring?

35. ••• Two objects moving with a speed v travel in opposite directions in a straight line. The objects stick together when they collide, and move with a speed of $v/4$ after the collision. **(a)** What is the ratio of the final kinetic energy of the system to the initial kinetic energy? **(b)** What is the ratio of the mass of the more massive object to the mass of the less massive object?

SECTION 9–6 ELASTIC COLLISIONS

36. • The collision between a hammer and a nail can be considered to be approximately elastic. Calculate the kinetic energy acquired by a 12-g nail when it is struck by a 550-g hammer moving with an initial speed of 4.5 m/s.

37. • A 732-kg car stopped at an intersection is rear-ended by a 1720-kg truck moving with a speed of 15.5 m/s. If the car was in neutral and its brakes were off, so that the collision is approximately elastic, find the final speed of both vehicles after the collision.

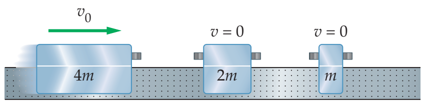
38. • **CE** Suppose you throw a rubber ball at an elephant that is charging directly at you (not a good idea). When the ball bounces back toward you, is its speed greater than, less than, or equal to the speed with which you threw it? Explain.

39. •• **IP** A charging bull elephant with a mass of 5240 kg comes directly toward you with a speed of 4.55 m/s. You toss a 0.150-kg rubber ball at the elephant with a speed of 7.81 m/s. **(a)** When the ball bounces back toward you, what is its speed? **(b)** How do you account for the fact that the ball's kinetic energy has increased?

40. •• **Moderating a Neutron** In a nuclear reactor, neutrons released by nuclear fission must be slowed down before they can trigger additional reactions in other nuclei. To see what sort of material is most effective in slowing (or moderating) a neutron, calculate the ratio of a neutron's final kinetic energy to its initial kinetic energy, K_f/K_i, for a head-on elastic collision with each of the following stationary target particles. (*Note:* The mass of a neutron is $m = 1.009$ u, where the atomic mass unit, u, is defined as follows: $1\ \text{u} = 1.66 \times 10^{-27}$ kg.) **(a)** An electron ($M = 5.49 \times 10^{-4}$ u). **(b)** A proton ($M = 1.007$ u). **(c)** The nucleus of a lead atom ($M = 207.2$ u).

41. •• In the apple-orange collision in Example 9–7, suppose the final velocity of the orange is 1.03 m/s in the negative y direction. What are the final speed and direction of the apple in this case?

42. •• The three air carts shown in **Figure 9–16** have masses, reading from left to right, of $4m$, $2m$, and m, respectively. The most massive cart has an initial speed of v_0; the other two carts are at rest initially. All carts are equipped with spring bumpers that give elastic collisions. **(a)** Find the final speed of each cart. **(b)** Verify that the final kinetic energy of the system is equal to the initial kinetic energy. (Assume the air track is long enough to accommodate all collisions.)



▲ **FIGURE 9–16** Problem 42

43. •• In this problem we show that when one ball is pulled to the left in the photo on page 275, only a single ball recoils to the right—under ideal elastic-collision conditions. To begin, suppose that each ball has a mass m, and that the ball coming in from the left strikes the other balls with a speed v_0. Now, consider the hypothetical case of two balls recoiling to the right. Determine the speed the two recoiling balls must have in order to satisfy **(a)** momentum conservation and **(b)** energy conservation. Since these speeds are not the same, it follows that momentum and energy cannot be conserved simultaneously with a recoil of two balls.

SECTION 9–7 CENTER OF MASS

44. • **CE Predict/Explain** A stalactite in a cave has drops of water falling from it to the cave floor below. The drops are equally

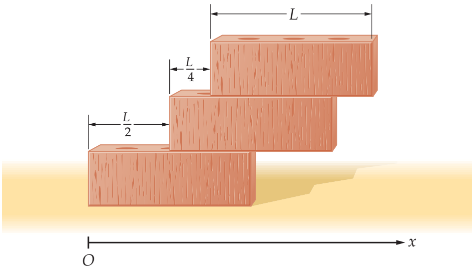
spaced in time and come in rapid succession, so that at any given moment there are many drops in midair. **(a)** Is the center of mass of the midair drops higher than, lower than, or equal to the halfway distance between the tip of the stalactite and the cave floor? **(b)** Choose the *best explanation* from among the following:

I. The drops bunch up as they near the floor of the cave.

II. The drops are equally spaced as they fall, since they are released at equal times.

III. Though equally spaced in time, the drops are closer together higher up.

45. • Find the x coordinate of the center of mass of the bricks shown in **Figure 9–17**.



▲ **FIGURE 9–17** Problem 45

46. • You are holding a shopping basket at the grocery store with two 0.56-kg cartons of cereal at the left end of the basket. The basket is 0.71 m long. Where should you place a 1.8-kg half gallon of milk, relative to the left end of the basket, so that the center of mass of your groceries is at the center of the basket?

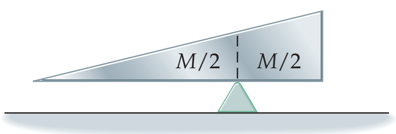
47. • **Earth–Moon Center of Mass** The Earth has a mass of 5.98×10^{24} kg, the Moon has a mass of 7.35×10^{22} kg, and their center-to-center distance is 3.85×10^{8} m. How far from the center of the Earth is the Earth–Moon center of mass? Is the Earth–Moon center of mass above or below the surface of the Earth? By what distance? (As the Earth and Moon orbit one another, their centers orbit about their common center of mass.)

48. •• **CE Predict/Explain** A piece of sheet metal of mass M is cut into the shape of a right triangle, as shown in **Figure 9–18**. A vertical dashed line is drawn on the sheet at the point where the mass to the left of the line ($M/2$) is equal to the mass to the right of the line (also $M/2$). The sheet is now placed on a fulcrum just under the dashed line and released from rest. **(a)** Does the metal sheet remain level, tip to the left, or tip to the right? **(b)** Choose the *best explanation* from among the following:

I. Equal mass on either side will keep the metal sheet level.

II. The metal sheet extends for a greater distance to the left, which shifts the center of mass to the left of the dashed line.

III. The center of mass is to the right of the dashed line because the metal sheet is thicker there.

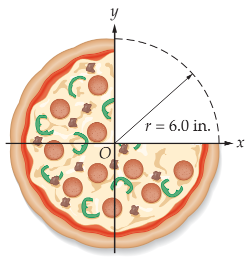


▲ **FIGURE 9–18** Problem 48

49. •• **CE** A pencil standing upright on its eraser end falls over and lands on a table. As the pencil falls, its eraser does not slip. The following questions refer to the contact force exerted on the pencil by the table. Let the positive x direction be in the direction the pencil falls, and the positive y direction be vertically upward. **(a)** During the pencil's fall, is the x component of the contact force positive, negative, or zero? Explain. **(b)** Is the y component of the contact force greater than, less than, or equal to the weight of the pencil? Explain.

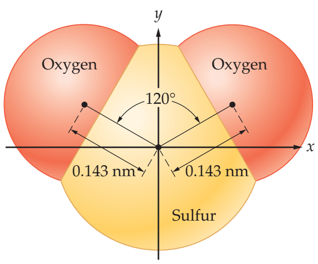
50. •• A cardboard box is in the shape of a cube with each side of length L. If the top of the box is missing, where is the center of mass of the open box? Give your answer relative to the geometric center of the box.

51. •• The location of the center of mass of the partially eaten, 12-inch-diameter pizza shown in **Figure 9–19** is $X_{cm} = -1.4$ in. and $Y_{cm} = -1.4$ in. Assuming each quadrant of the pizza to be the same, find the center of mass of the uneaten pizza above the x axis (that is, the portion of the pizza in the second quadrant).



▲ **FIGURE 9–19** Problem 51

52. •• **The Center of Mass of Sulfur Dioxide** Sulfur dioxide (SO_2) consists of two oxygen atoms (each of mass 16 u, where u is defined in Problem 40) and a single sulfur atom (of mass 32 u). The center-to-center distance between the sulfur atom and either of the oxygen atoms is 0.143 nm, and the angle formed by the three atoms is 120°, as shown in **Figure 9–20**. Find the x and y coordinates of the center of mass of this molecule.



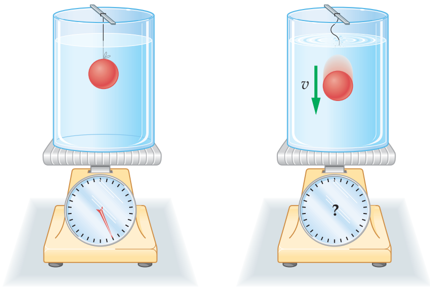
▲ **FIGURE 9–20** Problem 52

53. •• **IP** Three uniform metersticks, each of mass m, are placed on the floor as follows: stick 1 lies along the y axis from $y = 0$ to $y = 1.0$ m, stick 2 lies along the x axis from $x = 0$ to $x = 1.0$ m, stick 3 lies along the x axis from $x = 1.0$ m to $x = 2.0$ m. **(a)** Find the location of the center of mass of the metersticks. **(b)** How would the location of the center of mass be affected if the mass of the metersticks were doubled?

54. •• A 0.726-kg rope 2.00 meters long lies on a floor. You grasp one end of the rope and begin lifting it upward with a constant speed of 0.710 m/s. Find the position and velocity of the rope's center of mass from the time you begin lifting the rope to the time the last piece of rope lifts off the floor. Plot your results. (Assume the rope occupies negligible volume directly below the point where it is being lifted.)

55. •• Repeat the previous problem, this time lowering the rope onto a floor instead of lifting it.

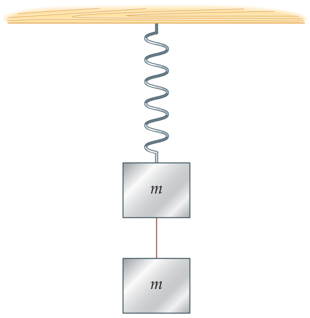
56. •• Consider the system shown in **Figure 9–21**. Assume that after the string breaks the ball falls through the liquid with constant speed. If the mass of the bucket *and* the liquid is 1.20 kg, and the



▲ **FIGURE 9–21** Problems 56 and 79

mass of the ball is 0.150 kg, what is the reading on the scale **(a)** before and **(b)** after the string breaks?

57. ••• A metal block of mass m is attached to the ceiling by a spring. Connected to the bottom of this block is a string that supports a second block of the same mass m, as shown in **Figure 9–22**. The string connecting the two blocks is now cut. **(a)** What is the net force acting on the two-block system immediately after the string is cut? **(b)** What is the acceleration of the center of mass of the two-block system immediately after the string is cut?



▲ **FIGURE 9–22** Problem 57

*SECTION 9–8 SYSTEMS WITH CHANGING MASS: ROCKET PROPULSION

58. • **Helicopter Thrust** During a rescue operation, a 5300-kg helicopter hovers above a fixed point. The helicopter blades send air downward with a speed of 62 m/s. What mass of air must pass through the blades every second to produce enough thrust for the helicopter to hover?

The powerful downdraft from this helicopter's blades creates a circular wave pattern in the water below. The thrust resulting from this downdraft is sufficient to support the weight of the helicopter. (Problem 58)

59. • **Rocks for a Rocket Engine** A child sits in a wagon with a pile of 0.65-kg rocks. If she can throw each rock with a speed of 11 m/s relative to the ground, causing the wagon to move, how many rocks must she throw per minute to maintain a constant average speed against a 3.4-N force of friction?

60. • A 57.8-kg person holding two 0.880-kg bricks stands on a 2.10-kg skateboard. Initially, the skateboard and the person are at rest. The person now throws the two bricks at the same time so that their speed relative to the person is 17.0 m/s. What is the recoil speed of the person and the skateboard relative to the ground, assuming the skateboard moves without friction?

61. •• In the previous problem, calculate the final speed of the person and the skateboard relative to the ground if the person throws the bricks one at a time. Assume that each brick is thrown with a speed of 17.0 m/s relative to the person.

62. •• A 0.540-kg bucket rests on a scale. Into this bucket you pour sand at the constant rate of 56.0 g/s. If the sand lands in the bucket with a speed of 3.20 m/s, **(a)** what is the reading of the scale when there is 0.750 kg of sand in the bucket? **(b)** What is the weight of the bucket and the 0.750 kg of sand?

63. •• **IP** Holding a long rope by its upper end, you lower it onto a scale. The rope has a mass of 0.13 kg per meter of length, and is lowered onto the scale at the constant rate of 1.4 m/s. **(a)** Calculate the thrust exerted by the rope as it lands on the scale. **(b)** At the instant when the amount of rope at rest on the scale has a weight of 2.5 N, does the scale read 2.5 N, more than 2.5 N, or less than 2.5 N? Explain. **(c)** Check your answer to part (b) by calculating the reading on the scale at this time.

GENERAL PROBLEMS

64. • **CE** Object A has a mass m, object B has a mass $2m$, and object C has a mass $m/2$. Rank these objects in order of increasing kinetic energy, given that they all have the same momentum. Indicate ties where appropriate.

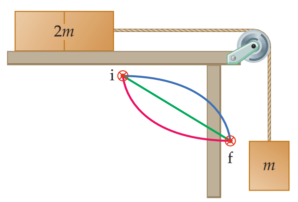
65. • **CE** Object A has a mass m, object B has a mass $4m$, and object C has a mass $m/4$. Rank these objects in order of increasing momentum, given that they all have the same kinetic energy. Indicate ties where appropriate.

66. • **CE Predict/Explain** A block of wood is struck by a bullet. **(a)** Is the block more likely to be knocked over if the bullet is metal and embeds itself in the wood, or if the bullet is rubber and bounces off the wood? **(b)** Choose the *best explanation* from among the following:
   I. The change in momentum when a bullet rebounds is larger than when it is brought to rest.
   II. The metal bullet does more damage to the block.
   III. Since the rubber bullet bounces off, it has little effect.

67. • **CE** A juggler performs a series of tricks with three bowling balls while standing on a bathroom scale. Is the average reading of the scale greater than, less than, or equal to the weight of the juggler plus the weight of the three balls? Explain.

68. • A 72.5-kg tourist climbs the stairs to the top of the Washington Monument, which is 555 ft high. How far does the Earth move in the opposite direction as the tourist climbs?

69. •• **CE Predict/Explain** **Figure 9–23** shows a block of mass $2m$ at rest on a horizontal, frictionless table. Attached to this block by a string that passes over a pulley is a second block, with a mass m.



▲ **FIGURE 9–23** Problem 69

The initial position of the center of mass of the blocks is indicated by the point i. The blocks are now released and allowed to accelerate; a short time later their center of mass is at the point f. **(a)** Did the center of mass follow the red path, the green path, or the blue path? **(b)** Choose the *best explanation* from among the following:

I. The center of mass must always be closer to the $2m$ block than to the m block.

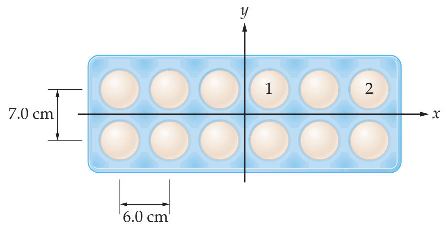
II. The center of mass starts at rest, and moves in a straight line in the direction of the net force.

III. The masses are accelerating, which implies parabolic motion.

70. •• A car moving with an initial speed v collides with a second stationary car that is one-half as massive. After the collision the first car moves in the same direction as before with a speed $v/3$. **(a)** Find the final speed of the second car. **(b)** Is this collision elastic or inelastic?

71. •• A 1.35-kg block of wood sits at the edge of a table, 0.782 m above the floor. A 0.0105-kg bullet moving horizontally with a speed of 715 m/s embeds itself within the block. What horizontal distance does the block cover before hitting the ground?

72. •• **IP** The carton of eggs shown in **Figure 9–24** is filled with a dozen eggs, each of mass m. Initially, the center of mass of the eggs is at the center of the carton. **(a)** Does the location of the center of mass of the eggs change more if egg 1 is removed or if egg 2 is removed? Explain. **(b)** Find the center of mass of the eggs when egg 1 is removed. **(c)** Find the center of mass of the eggs if egg 2 is removed instead.



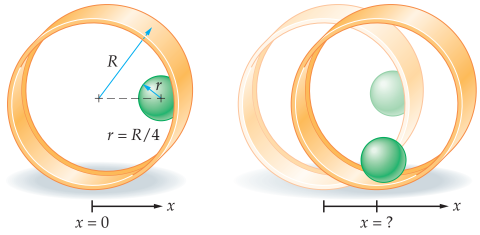
▲ **FIGURE 9–24** Problem 72

73. •• **The Force of a Storm** During a severe storm in Palm Beach, FL, on January 2, 1999, 31 inches of rain fell in a period of nine hours. Assuming that the raindrops hit the ground with a speed of 10 m/s, estimate the average upward force exerted by one square meter of ground to stop the falling raindrops during the storm. (*Note:* One cubic meter of water has a mass of 1000 kg.)

74. •• An apple that weighs 2.7 N falls vertically downward from rest for 1.4 s. **(a)** What is the change in the apple's momentum per second? **(b)** What is the total change in its momentum during the 1.4-second fall?

75. •• To balance a 35.5-kg automobile tire and wheel, a mechanic must place a 50.2-g lead weight 25.0 cm from the center of the wheel. When the wheel is balanced, its center of mass is exactly at the center of the wheel. How far from the center of the wheel was its center of mass before the lead weight was added?

76. •• A hoop of mass M and radius R rests on a smooth, level surface. The inside of the hoop has ridges on either side, so that it forms a track on which a ball can roll, as indicated in **Figure 9–25**. If a ball of mass $2M$ and radius $r = R/4$ is released as shown, the system rocks back and forth until it comes to rest with the ball at the bottom of the hoop. When the ball comes to rest, what is the x coordinate of its center?



▲ **FIGURE 9–25** Problem 76

77. •• **IP** A 63-kg canoeist stands in the middle of her 22-kg canoe. The canoe is 3.0 m long, and the end that is closest to land is 2.5 m from the shore. The canoeist now walks toward the shore until she comes to the end of the canoe. **(a)** When the canoeist stops at the end of her canoe, is her distance from the shore equal to, greater than, or less than 2.5 m? Explain. **(b)** Verify your answer to part (a) by calculating the distance from the canoeist to shore.

78. •• In the previous problem, suppose the canoeist is 3.4 m from shore when she reaches the end of her canoe. What is the canoe's mass?

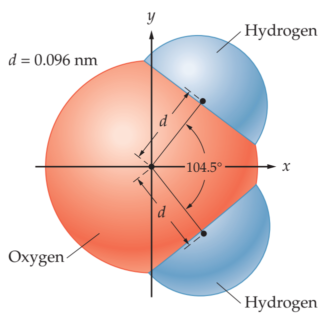
79. •• Referring to Problem 56, find the reading on the scale **(a)** before and **(b)** after the string breaks, assuming the ball falls through the liquid with an acceleration equal to $0.250g$.

80. •• A young hockey player stands at rest on the ice holding a 1.3-kg helmet. The player tosses the helmet with a speed of 6.5 m/s in a direction 11° above the horizontal, and recoils with a speed of 0.25 m/s. Find the mass of the hockey player.

81. •• Suppose the air carts in Example 9–9 are both moving to the right initially. The cart to the left has a mass m and an initial speed v_0; the cart to the right has an initial speed $v_0/2$. If the center of mass of this system moves to the right with a speed $2v_0/3$, what is the mass of the cart on the right?

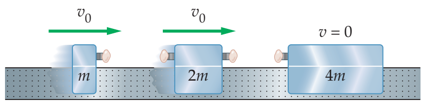
82. •• A long, uniform rope with a mass of 0.135 kg per meter lies on the ground. You grab one end of the rope and lift it at the constant rate of 1.13 m/s. Calculate the upward force you must exert at the moment when the top end of the rope is 0.525 m above the ground.

83. •• **The Center of Mass of Water** Find the center of mass of a water molecule, referring to **Figure 9–26** for the relevant angles and distances. The mass of a hydrogen atom is 1.0 u, and the mass of an oxygen atom is 16 u, where u is the atomic mass unit (see Problem 40). Use the center of the oxygen atom as the origin of your coordinate system.



▲ **FIGURE 9–26** Problem 83

84. •• The three air carts shown in **Figure 9–27** have masses, reading from left to right, of m, $2m$, and $4m$, respectively. Initially, the cart on the right is at rest, whereas the other two carts are moving to the right with a speed v_0. All carts are equipped with putty bumpers that give completely inelastic collisions. **(a)** Find



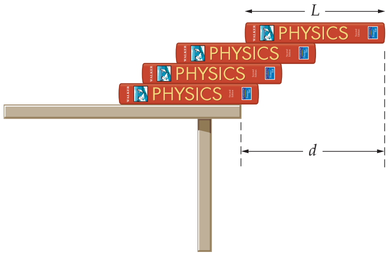
▲ **FIGURE 9–27** Problem 84

the final speed of the carts. **(b)** Calculate the ratio of the final kinetic energy of the system to the initial kinetic energy.

85. •• **IP** A fireworks rocket is launched vertically into the night sky with an initial speed of 44.2 m/s. The rocket coasts after being launched, then explodes and breaks into two pieces of equal mass 2.50 s later. **(a)** If each piece follows a trajectory that is initially at 45.0° to the vertical, what was their speed immediately after the explosion? **(b)** What is the velocity of the rocket's center of mass before and after the explosion? **(c)** What is the acceleration of the rocket's center of mass before and after the explosion?

86. •• **IP** The total momentum of two cars approaching an intersection is $\vec{\mathbf{p}}_{\text{total}} = (15{,}000\ \text{kg}\cdot\text{m/s})\hat{\mathbf{x}} + (2100\ \text{kg}\cdot\text{m/s})\hat{\mathbf{y}}$. **(a)** If the momentum of car 1 is $\vec{\mathbf{p}}_1 = (11{,}000\ \text{kg}\cdot\text{m/s})\hat{\mathbf{x}} + (-370\ \text{kg}\cdot\text{m/s})\hat{\mathbf{y}}$, what is the momentum of car 2? **(b)** Does your answer to part (a) depend on which car is closer to the intersection? Explain.

87. •• **Unlimited Overhang** Four identical textbooks, each of length L, are stacked near the edge of a table, as shown in **Figure 9–28**. The books are stacked in such a way that the distance they overhang the edge of the table, d, is maximized. Find the maximum overhang distance d in terms of L. In particular, show that $d > L$; that is, the top book is completely to the right of the table edge. (In principle, the overhang distance d can be made as large as desired simply by increasing the number of books in the stack.)



▲ **FIGURE 9–28** Problem 87

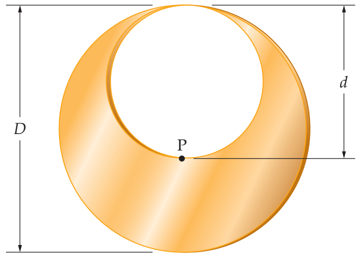
88. ••• Consider a one-dimensional, head-on elastic collision. One object has a mass m_1 and an initial velocity v_1; the other has a mass m_2 and an initial velocity v_2. Use momentum conservation and energy conservation to show that the final velocities of the two masses are

$$v_{1,\text{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)v_2$$

$$v_{2,\text{f}} = \left(\frac{2m_1}{m_1 + m_2}\right)v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)v_2$$

89. ••• Two air carts of mass $m_1 = 0.84$ kg and $m_2 = 0.42$ kg are placed on a frictionless track. Cart 1 is at rest initially, and has a spring bumper with a force constant of 690 N/m. Cart 2 has a flat metal surface for a bumper, and moves toward the bumper of the stationary cart with an initial speed $v = 0.68$ m/s. **(a)** What is the speed of the two carts at the moment when their speeds are equal? **(b)** How much energy is stored in the spring bumper when the carts have the same speed? **(c)** What is the final speed of the carts after the collision?

90. ••• **Golden Earrings and the Golden Ratio** A popular earring design features a circular piece of gold of diameter D with a circular cutout of diameter d, as shown in **Figure 9–29**. If this earring is to balance at the point P, show that the diameters must satisfy the condition $D = \phi d$, where $\phi = (1 + \sqrt{5})/2 = 1.61803\ldots$ is the famous "golden ratio."



▲ **FIGURE 9–29** Problem 90

91. ••• Two objects with masses m_1 and m_2 and initial velocities $v_{1,\text{i}}$ and $v_{2,\text{i}}$ move along a straight line and collide elastically. Assuming that the objects move along the same straight line after the collision, show that their relative velocities are unchanged; that is, show that $v_{1,\text{i}} - v_{2,\text{i}} = v_{2,\text{f}} - v_{1,\text{f}}$. (You can use the results given in Problem 88.)

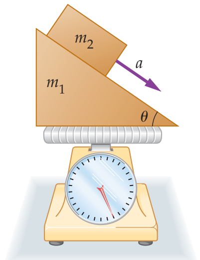
92. ••• **Amplified Rebound Height** Two small rubber balls are dropped from rest at a height h above a hard floor. When the balls are released, the lighter ball (with mass m) is directly above the heavier ball (with mass M). Assume the heavier ball reaches the floor first and bounces elastically; thus, when the balls collide, the ball of mass M is moving upward with a speed v and the ball of mass m is moving downward with essentially the same speed. In terms of h, find the height to which the ball of mass m rises after the collision. (Use the results given in Problem 88, and assume the balls collide at ground level.)

93. ••• On a cold winter morning, a child sits on a sled resting on smooth ice. When the 9.75-kg sled is pulled with a horizontal force of 40.0 N, it begins to move with an acceleration of 2.32 m/s^2. The 21.0-kg child accelerates too, but with a smaller acceleration than that of the sled. Thus, the child moves forward relative to the ice, but slides backward relative to the sled. Find the acceleration of the child relative to the ice.

94. ••• An object of mass m undergoes an elastic collision with an identical object that is at rest. The collision is not head-on. Show that the angle between the velocities of the two objects after the collision is 90°.

95. ••• **IP Weighing a Block on an Incline** A wedge of mass m_1 is firmly attached to the top of a scale, as shown in **Figure 9–30**. The inclined surface of the wedge makes an angle θ with the horizontal. Now, a block of mass m_2 is placed on the inclined surface of the wedge and allowed to accelerate without friction down the slope. **(a)** Show that the reading on the scale while the block slides is

$$(m_1 + m_2 \cos^2\theta)g$$



▲ **FIGURE 9–30** Problem 95

(b) Explain why the reading on the scale is less than $(m_1 + m_2)g$. **(c)** Show that the expression in part (a) gives the expected results for $\theta = 0$ and $\theta = 90°$.

96. ••• **IP** A uniform rope of length L and mass M rests on a table. **(a)** If you lift one end of the rope upward with a constant *speed*, v, show that the rope's center of mass moves upward with constant *acceleration*. **(b)** Next, suppose you hold the rope suspended in air, with its lower end just touching the table. If you now lower the rope with a constant speed, v, onto the table, is the acceleration of the rope's center of mass upward or downward? Explain your answer. **(c)** Find the magnitude and direction of the acceleration of the rope's center of mass for the case described in part (b). Compare with part (a).

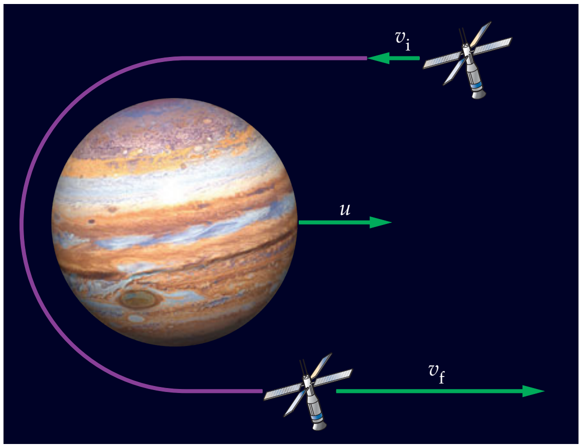
PASSAGE PROBLEMS

Navigating in Space: The Gravitational Slingshot

Many spacecraft navigate through space these days by using the "gravitational slingshot" effect, in which a close encounter with a planet results in a significant increase in magnitude and change in direction of the spacecraft's velocity. In fact, a spacecraft can attain a much greater speed with such a maneuver than it could produce with its own rockets.

The first use of this effect was on February 5, 1974, as the Mariner 10 probe—the first spacecraft to explore Mercury—made a close flyby of the planet Venus on the way to its final destination. More recently, the Cassini probe to Saturn, which was launched on October 15, 1997, and arrived at Saturn on July 1, 2004, made two close passes by Venus, followed by a flyby of Earth and a flyby of Jupiter.

A simplified version of the slingshot maneuver is illustrated in **Figure 9–31**, where we see a spacecraft moving to the left with an initial speed v_i, a planet moving to the right with a speed u, and the same spacecraft moving to the right with a final speed v_f after the encounter. The interaction can be thought of as an elastic collision in one dimension—as if the planet and spacecraft were two air carts on an air track. Both energy and momentum are conserved in this interaction, and hence the following simple condition is satisfied (see Problem 91): The relative speed of approach is equal to the relative speed of departure. This condition, plus the fact that the speed of the massive planet is essentially unchanged, can be used to determine the final speed of the spacecraft.



▲ **FIGURE 9–31** Problems 97, 98, 99, and 100

97. • From the perspective of an observer on the planet, what is the spacecraft's speed of approach?

A. $v_i + u$ **B.** $v_i - u$
C. $u - v_i$ **D.** $v_f - u$

98. • From the perspective of an observer on the planet, what is the spacecraft's speed of departure?

A. $v_f + u$ **B.** $v_f - u$
C. $u - v_f$ **D.** $v_i - u$

99. •• Set the speed of departure from Problem 98 equal to the speed of approach from Problem 97. Solving this relation for the final speed, v_f, yields:

A. $v_f = v_i + u$ **B.** $v_f = v_i - u$
C. $v_f = v_i + 2u$ **D.** $v_f = v_i - 2u$

100. •• Consider the special case in which $v_i = u$. By what factor does the kinetic energy of the spacecraft increase as a result of the encounter?

A. 4 **B.** 8
C. 9 **D.** 16

INTERACTIVE PROBLEMS

101. •• **Referring to Example 9–5** Suppose a bullet of mass $m = 6.75$ g is fired into a ballistic pendulum whose bob has a mass of $M = 0.675$ kg. **(a)** If the bob rises to a height of 0.128 m, what was the initial speed of the bullet? **(b)** What was the speed of the bullet–bob combination immediately after the collision takes place?

102. •• **Referring to Example 9–5** A bullet with a mass $m = 8.10$ g and an initial speed $v_0 = 320$ m/s is fired into a ballistic pendulum. What mass must the bob have if the bullet–bob combination is to rise to a maximum height of 0.125 m after the collision?

103. •• **Referring to Example 9–9** Suppose that cart 1 has a mass of 3.00 kg and an initial speed of 0.250 m/s. Cart 2 has a mass of 1.00 kg and is at rest initially. **(a)** What is the final speed of the carts? **(b)** How much kinetic energy is lost as a result of the collision?

104. •• **Referring to Example 9–9** Suppose the two carts have equal masses and are both moving to the right before the collision. The initial speed of cart 1 (on the left) is v_0 and the initial speed of cart 2 (on the right) is $v_0/2$. **(a)** What is the speed of the center of mass of this system? **(b)** What percentage of the initial kinetic energy is lost as a result of the collision? **(c)** Suppose the collision is elastic. What are the final speeds of the two carts in this case?