10 Rotational Kinematics and Energy

Can you imagine life without rotating objects: vehicles without wheels, machinery without gears, carnivals without merry-go-rounds? The people on this roller coaster certainly know that rotational motion is very different from motion on a straight, linear stretch of track. In this chapter we show that the motion of rotating objects, such as a roller coaster executing a loop-the-loop, can be analyzed using many of the same methods that we applied earlier to linear motion.

It is certainly no exaggeration to say that rotation is a part of everyday life. After all, we live on a planet that rotates about its axis once a day and that revolves about the Sun once a year. The apparent motion of the Sun across the sky, for example, is actually the result of the Earth's rotational motion. In addition, engines that power cars and trucks have moving parts that rotate quite rapidly, as do CDs, CD-ROMs, and DVDs, not to mention the tumbling, rotating molecules in the air we breathe. Thus, a study of rotation yields results that apply to a great variety of natural phenomena.

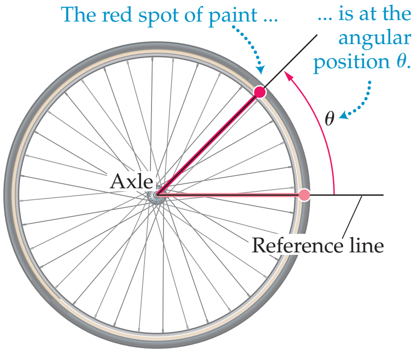
In this chapter, then, we study various aspects of rotational motion. As we do, we shall make extensive use of the close analogies that exist between rotational and linear motion. In fact, many of the results derived in earlier chapters can be applied to rotation by simply replacing linear quantities with their rotational counterparts.

10–1 Angular Position, Velocity, and Acceleration

To describe the motion of an object moving in a straight line, it is useful to establish a coordinate system with a definite origin and positive direction. In terms of this coordinate system we can measure the object's position, velocity, and acceleration.

Similarly, to describe rotational motion, we define "angular" quantities that are analogous to the linear position, velocity, and acceleration. These angular quantities form the basis of our study of rotation. We begin by defining the most basic angular quantity—the angular position.

Angular Position, θ



▲ **FIGURE 10–1 Angular position**
The angular position, θ, of a spot of paint on a bicycle wheel. The reference line, where $\theta = 0$, is drawn horizontal here but can be chosen in any direction.

Consider a bicycle wheel that is free to rotate about its axle, as shown in **Figure 10–1**. We say that the axle is the **axis of rotation** for the wheel. As the wheel rotates, each and every point on it moves in a circular path centered on the axis of rotation.

Now, suppose there is a small spot of red paint on the tire, and we want to describe the rotational motion of the spot. The **angular position** of the spot is defined to be the angle, θ, that a line from the axle to the spot makes with a reference line, as indicated in Figure 10–1.

Definition of Angular Position, θ

$$\theta = \text{angle measured from reference line} \tag{10–1}$$

SI unit: radian (rad), which is dimensionless

The reference line simply defines $\theta = 0$; it is analogous to the origin in a linear coordinate system. The reference line begins at the axis of rotation, and may be chosen to point in any direction—just as an origin may be placed anywhere along a coordinate axis. Once chosen, however, the reference line must be used consistently.

Note that the spot of paint in Figure 10–1 is rotated counterclockwise from the reference line by the angle θ. By convention, we say that this angle is positive. Similarly, clockwise rotations correspond to negative angles.

Sign Convention for Angular Position

By convention:

$\theta > 0$ counterclockwise rotation from reference line
$\theta < 0$ clockwise rotation from reference line

▲ Rotational motion is everywhere in our universe, on every scale of length and time. A galaxy like the one at left may take millions of years to complete a single rotation about its center, while the skater in the middle photo spins several times in a second. The bacterium at right moves in a corkscrew path by rapidly twirling its flagella (the fine projections at either end of the cell) like whips.

Now that we have established a reference line (for $\theta = 0$), and a positive direction of rotation (counterclockwise), we must choose units in which to measure angles. Common units are degrees (°) and revolutions (rev), where one revolution—that is, going completely around a circle—corresponds to 360°:

$$1 \text{ rev} = 360°$$

The most convenient units for scientific calculations, however, are radians. A **radian** (rad) is defined as follows:

> A radian is the angle for which the arc length on a circle of radius r is equal to the radius of the circle.

This definition is useful because it establishes a particularly simple relationship between an angle measured in radians and the corresponding arc length, as illustrated in **Figure 10–2**. For example, it follows from our definition that for an angle of one radian, the arc length s is equal to the radius: $s = r$. Similarly, an angle of two radians corresponds to an arc length of two radii, $s = 2r$, and so on. Thus, the arc length s for an arbitrary angle θ measured in radians is given by the following relation:

$$s = r\theta \qquad 10\text{–}2$$

This simple and straightforward relation does not hold for degrees or revolutions—additional conversion factors would be needed.

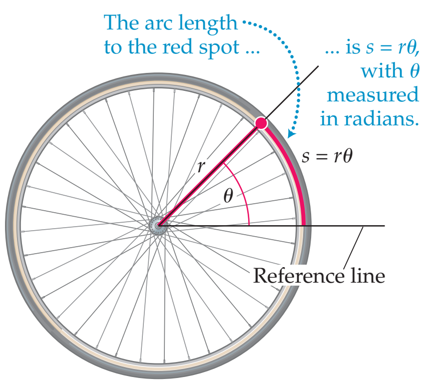
In one complete revolution, the arc length is the circumference of a circle, $C = 2\pi r$. Comparing with $s = r\theta$, we see that a complete revolution corresponds to 2π radians:

$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

Equivalently,

$$1 \text{ rad} = \frac{360°}{2\pi} = 57.3°$$

One final note on the units for angles: Radians, as well as degrees and revolutions, are dimensionless. In the relation $s = r\theta$, for example, the arc length and the radius both have SI units of meters. For the equation to be dimensionally consistent, it is necessary that θ have no dimensions. Still, if an angle θ is, let's say, three radians, we will write it as $\theta = 3$ rad to remind us of the angular units being used.



▲ **FIGURE 10–2 Arc length**
The arc length, s, from the reference line to the spot of paint is given by $s = r\theta$ if the angular position θ is measured in radians.

PROBLEM-SOLVING NOTE

Radians

Remember to measure angles in radians when using the relation $s = r\theta$.

Angular Velocity, ω

As the bicycle wheel in Figure 10–1 rotates, the angular position of the spot of red paint changes. This is illustrated in **Figure 10–3**. The **angular displacement** of the spot, $\Delta\theta$, is

$$\Delta\theta = \theta_f - \theta_i$$

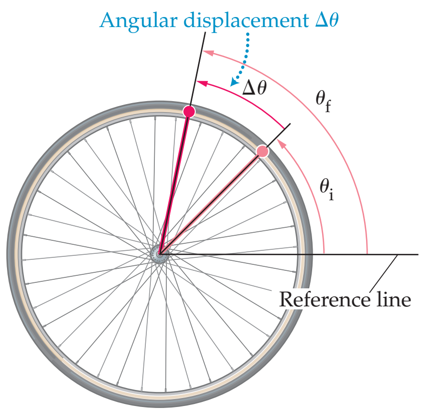
If we divide the angular displacement by the time, Δt, during which the displacement occurs, the result is the **average angular velocity,** ω_{av}.

Definition of Average Angular Velocity, ω_{av}

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad 10\text{–}3$$

SI unit: radian per second (rad/s) = s^{-1}

This is analogous to the definition of the average linear velocity $v_{av} = \Delta x/\Delta t$. Note that the units of linear velocity are m/s, whereas the units of angular velocity are rad/s.



▲ **FIGURE 10–3 Angular displacement**
As the wheel rotates, the spot of paint undergoes an angular displacement, $\Delta\theta = \theta_f - \theta_i$.

▲ Star trails provide a clear illustration of the relationship between angle, arc, and radius in circular motion. The stars, of course, do not actually move like this, but because of the Earth's rotation they appear to follow circular paths across the sky each night, with Polaris, the North Star, very near the axis of rotation. This photo was made by opening the camera shutter for an extended period of time. Notice that each star moves through the same angle in the course of the exposure. However, the farther a star is from the axis of rotation, the longer the arc it traces out in a given period of time. (Can you estimate the length of the exposure?)

In addition to the average angular velocity, we can define an **instantaneous angular velocity** as the limit of ω_{av} as the time interval, Δt, approaches zero. The instantaneous angular velocity, then, is

Definition of Instantaneous Angular Velocity, ω

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \qquad 10\text{–}4$$

SI unit: rad/s = s^{-1}

Generally, we shall refer to the instantaneous angular velocity simply as the angular velocity.

Note that we call ω the angular velocity, not the angular speed. The reason is that ω can be positive or negative, depending on the sense of rotation. For example, if the red paint spot rotates in the counterclockwise sense, the angular position, θ, increases. As a result, $\Delta\theta$ is positive and therefore, so is ω. Similarly, clockwise rotation corresponds to a negative $\Delta\theta$ and hence a negative ω.

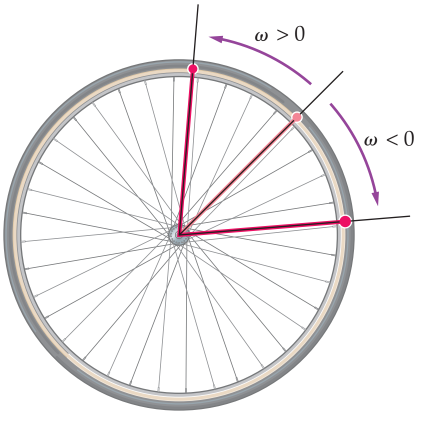
Sign Convention for Angular Velocity

By convention:

$\omega > 0$ counterclockwise rotation
$\omega < 0$ clockwise rotation

The sign convention for angular velocity is illustrated in **Figure 10–4**. In analogy with linear motion, the sign of ω indicates the *direction* of the angular velocity *vector*, as we shall see in detail in Chapter 11. Similarly, the magnitude of the angular velocity is the **angular speed,** just as in the one-dimensional case.

In Exercise 10–1 we utilize the definitions and conversion factors presented so far in this section.



▲ **FIGURE 10–4 Angular speed and velocity**
Counterclockwise rotation is defined to correspond to a positive angular velocity, ω. Similarly, clockwise rotation corresponds to a negative angular velocity. The magnitude of the angular velocity is referred to as the angular speed.

EXERCISE 10–1

(a) An old phonograph record rotates clockwise at $33\frac{1}{3}$ rpm (revolutions per minute). What is its angular velocity in rad/s? **(b)** If a CD rotates at 22.0 rad/s, what is its angular speed in rpm?

SOLUTION

a. Convert from rpm to rad/s, and note that clockwise rotation corresponds to a negative angular velocity:

$$\omega = -33\tfrac{1}{3}\text{ rpm} = \left(-33\tfrac{1}{3}\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = -3.49\text{ rad/s}$$

b. Converting angular speed from rad/s to rpm gives

$$\omega = \left(22.0\frac{\text{rad}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 210\frac{\text{rev}}{\text{min}} = 210\text{ rpm}$$

Note that the same symbol, ω, is used for both angular velocity and angular speed in Exercise 10–1. Which quantity is meant in a given situation will be clear from the context in which it is used.

As a simple application of angular velocity, consider the following question: An object rotates with a constant angular velocity, ω. How much time, T, is required for it to complete one full revolution?

To solve this problem, note that since ω is constant, the instantaneous angular velocity is equal to the average angular velocity. That is,

$$\omega = \omega_{av} = \frac{\Delta\theta}{\Delta t}$$

In one revolution, we know that $\Delta\theta = 2\pi$ and $\Delta t = T$. Therefore,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{T}$$

Finally, solving for T we find

$$T = \frac{2\pi}{\omega}$$

The time to complete one revolution, T, is referred to as the **period.**

Definition of Period, *T*

$$T = \frac{2\pi}{\omega} \qquad 10\text{–}5$$

SI unit: second, s

EXERCISE 10–2

Find the period of a record that is rotating at 45 rpm.

SOLUTION

To apply $T = 2\pi/\omega$ we must first express ω in terms of rad/s:

$$45 \text{ rpm} = \left(45\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 4.7 \text{ rad/s}$$

Now we can calculate the period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \text{ rad}}{4.7 \text{ rad/s}} = 1.3 \text{ s}$$

Angular Acceleration, α

If the angular velocity of the rotating bicycle wheel increases or decreases with time, we say that the wheel experiences an **angular acceleration,** α. The average angular acceleration is the change in angular velocity, $\Delta\omega$, in a given interval of time, Δt:

Definition of Average Angular Acceleration, α_{av}

$$\alpha_{av} = \frac{\Delta\omega}{\Delta t} \qquad 10\text{–}6$$

SI unit: radian per second per second $(\text{rad/s}^2) = \text{s}^{-2}$

Note that the SI units of α are rad/s^2, which, since rad is dimensionless, is simply s^{-2}.

As expected, the instantaneous angular acceleration is the limit of α_{av} as the time interval, Δt, approaches zero:

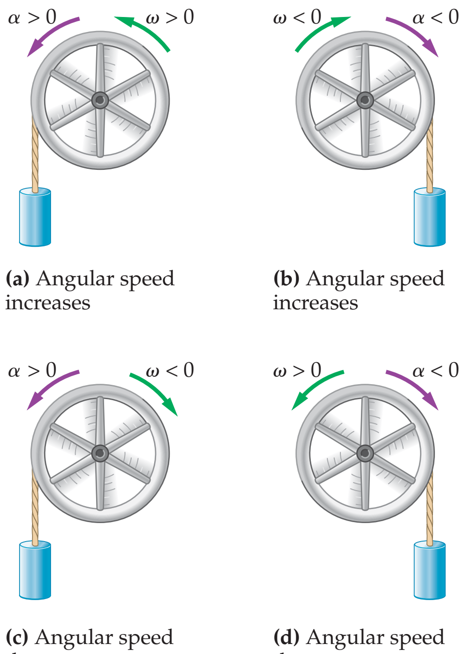
Definition of Instantaneous Angular Acceleration, α

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} \qquad 10\text{–}7$$

SI unit: $\text{rad/s}^2 = \text{s}^{-2}$

When referring to the instantaneous angular acceleration, we will usually just say angular acceleration.

The sign of the angular acceleration is determined by whether the change in angular velocity is positive or negative. For example, if ω is becoming more positive, so that ω_f is greater than ω_i, it follows that α is positive. Similarly, if ω is becoming more negative, so that ω_f is less than ω_i, it follows that α is negative. Therefore, if ω and α have the same sign, the speed of rotation is increasing. If ω and α have opposite signs, the speed of rotation is decreasing. This is illustrated in **Figure 10–5**.



▲ **FIGURE 10–5 Angular acceleration and angular speed**

When angular velocity and acceleration have the same sign, as in **(a)** and **(b)**, the angular speed increases. When angular velocity and angular acceleration have opposite signs, as in **(c)** and **(d)**, the angular speed decreases.

EXERCISE 10–3

As the wind dies, a windmill that was rotating at 2.1 rad/s begins to slow down with a constant angular acceleration of 0.45 rad/s^2. How long does it take for the windmill to come to a complete stop?

SOLUTION

If we choose the initial angular velocity to be positive, the angular acceleration is negative, corresponding to a deceleration. Hence, Equation 10–6 gives

$$\Delta t = \frac{\Delta\omega}{\alpha_{\text{av}}} = \frac{\omega_{\text{f}} - \omega_{\text{i}}}{\alpha} = \frac{0 - 2.1\ \text{rad/s}}{-0.45\ \text{rad/s}^2} = 4.7\ \text{s}$$

10–2 Rotational Kinematics

Just as the kinematics of Chapter 2 described linear motion, rotational kinematics describes rotational motion. In this section, as in Chapter 2, we concentrate on the important special case of constant acceleration.

As an example of a system with constant angular acceleration, consider the pulley shown in **Figure 10–6**. Wrapped around the circumference of the pulley is a string, with a mass attached to its free end. When the mass is released, the pulley begins to rotate—slowly at first, but then faster and faster. As we shall see in Chapter 11, the pulley is accelerating with constant angular acceleration.

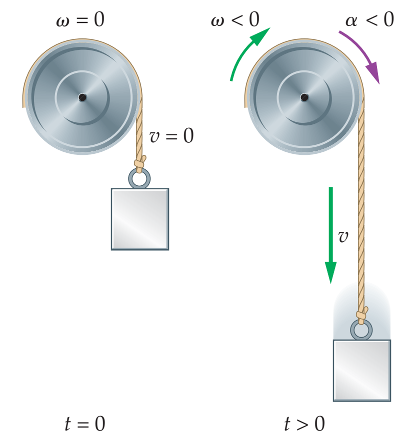


FIGURE 10–6 A pulley with constant angular acceleration

A mass is attached to a string wrapped around a pulley. As the mass falls, it causes the pulley to increase its angular speed with a constant angular acceleration.

Since α is constant, it follows that the average and instantaneous angular accelerations are equal. Hence,

$$\alpha = \alpha_{\text{av}} = \frac{\Delta\omega}{\Delta t}$$

Suppose the pulley starts with the initial angular velocity ω_0 at time $t = 0$, and that at the later time t its angular velocity is ω. Substituting these values into the preceding expression for α yields

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{t - 0} = \frac{\omega - \omega_0}{t}$$

Rearranging, we see that the angular velocity, ω, varies with time as follows:

$$\omega = \omega_0 + \alpha t \qquad \text{10–8}$$

EXERCISE 10–4

If the angular velocity of the pulley in Figure 10–6 is −8.4 rad/s at a given time, and its angular acceleration is −2.8 rad/s^2, what is the angular velocity of the pulley 1.5 s later?

SOLUTION

The angular velocity, ω, is found by applying Equation 10–8:

$$\omega = \omega_0 + \alpha t = -8.4\ \text{rad/s} + (-2.8\ \text{rad/s}^2)(1.5\ \text{s}) = -12.6\ \text{rad/s}$$

Note that the angular speed has increased, as expected, since ω and α have the same sign.

Note the close analogy between Equation 10–8 for angular velocity and the corresponding relation for linear velocity, Equation 2–7:

$$v = v_0 + at$$

Clearly, the equation for angular velocity can be obtained from our previous equation for linear velocity by replacing v with ω and replacing a with α. This type of analogy between linear and angular quantities can be most useful both in deriving angular equations—by starting with linear equations and using analogies—and in obtaining a better physical understanding of angular systems. Several linear-to-angular analogs are listed in the adjacent table.

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Using these analogies, we can rewrite all the kinematic equations in Chapter 2 in angular form. The following table gives both the linear kinematic equations and their angular counterparts.

Linear Equation (a = constant)		Angular Equation (α = constant)	
$v = v_0 + at$	2–7	$\omega = \omega_0 + \alpha t$	10–8
$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$	10–9
$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	10–10
$v^2 = v_0{}^2 + 2a(x - x_0)$	2–12	$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$	10–11

In solving kinematic problems involving rotation, we apply these angular equations in the same way that the linear equations were applied in Chapter 2. In a sense, then, this material is a review—since the mathematics is essentially the same. The only difference comes in the physical interpretation of the results. We will emphasize the rotational interpretations throughout the chapter.

PROBLEM-SOLVING NOTE

Rotational Kinematics

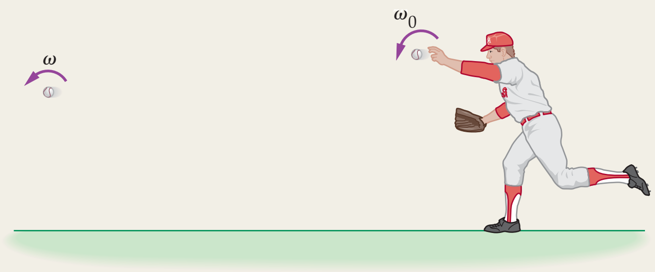
Using analogies between linear and angular quantities often helps when solving problems involving rotational kinematics.

EXAMPLE 10–1 THROWN FOR A CURVE

To throw a curve ball, a pitcher gives the ball an initial angular speed of 36.0 rad/s. When the catcher gloves the ball 0.595 s later, its angular speed has decreased (due to air resistance) to 34.2 rad/s. **(a)** What is the ball's angular acceleration, assuming it to be constant? **(b)** How many revolutions does the ball make before being caught?

PICTURE THE PROBLEM

We choose the ball's initial direction of rotation to be positive. As a result, the angular acceleration will be negative. We can also identify the initial angular velocity to be $\omega_0 = 36.0\ \text{rad/s}$, and the final angular velocity to be $\omega = 34.2\ \text{rad/s}$.



STRATEGY

The problem states that the angular acceleration of the ball is constant. It follows that Equations 10–8 to 10–11 apply to its rotation.

a. To relate angular velocity to time, we use $\omega = \omega_0 + \alpha t$. This can be solved for α.

b. To relate angle to time we use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$. The angular displacement of the ball is $\theta - \theta_0$.

SOLUTION

Part (a)

1. Solve $\omega = \omega_0 + \alpha t$ for the angular acceleration, α:

$$\omega = \omega_0 + \alpha t$$
$$\alpha = \frac{\omega - \omega_0}{t}$$

2. Substitute numerical values to find α:

$$\alpha = \frac{\omega - \omega_0}{t}$$
$$= \frac{34.2\ \text{rad/s} - 36.0\ \text{rad/s}}{0.595\ \text{s}} = -3.03\ \text{rad/s}^2$$

Part (b)

3. Use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ to calculate the angular displacement of the ball:

$$\theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2$$
$$= (36.0\ \text{rad/s})(0.595\ \text{s}) + \tfrac{1}{2}(-3.03\ \text{rad/s}^2)(0.595\ \text{s})^2$$
$$= 20.9\ \text{rad}$$

2. Convert the angular displacement to revolutions:

$$\theta - \theta_0 = 20.9\ \text{rad} = 20.9\ \text{rad}\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 3.33\ \text{rev}$$

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INSIGHT

The ball rotates through three-and-one-third revolutions during its time in flight.

An alternative method of solution is to use the kinematic relation given in Equation 10–9. This procedure yields $\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t = 20.9$ rad, in agreement with our previous result.

PRACTICE PROBLEM

(a) What is the angular velocity of the ball 0.500 s after it is thrown? **(b)** What is the ball's angular velocity after it completes its first full revolution? [**Answer:** (a) Use $\omega = \omega_0 + \alpha t$ to find $\omega = 34.5$ rad/s. (b) Use $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$ to find $\omega = 35.5$ rad/s.]

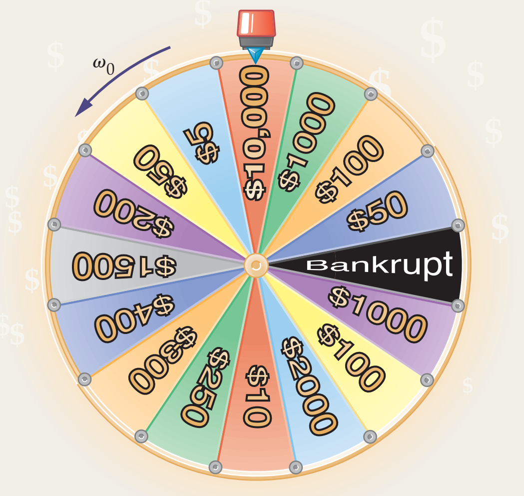
Some related homework problems: Problem 19, Problem 22

EXAMPLE 10–2 WHEEL OF MISFORTUNE

On a certain game show, contestants spin a wheel when it is their turn. One contestant gives the wheel an initial angular speed of 3.40 rad/s. It then rotates through one-and-one-quarter revolutions and comes to rest on the BANKRUPT space. **(a)** Find the angular acceleration of the wheel, assuming it to be constant. **(b)** How long does it take for the wheel to come to rest?

PICTURE THE PROBLEM

We choose the initial angular velocity to be positive, $\omega_0 = +3.40$ rad/s, and indicate it with a counterclockwise rotation in our sketch. Since the wheel slows to a stop, the angular acceleration must be negative; that is, in the clockwise direction. After a rotation of 1.25 rev the wheel will read BANKRUPT.



STRATEGY

As in Example 10–1, we can use the kinematic equations for constant angular acceleration, Equations 10–8 to 10–11.

a. To begin, we are given the initial angular velocity, $\omega_0 = +3.40$ rad/s, the final angular velocity, $\omega = 0$ (the wheel comes to rest), and the angular displacement, $\theta - \theta_0 = 1.25$ rev. We can find the angular acceleration using $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$.

b. Knowing the angular velocity and acceleration, we can find the time with $\omega = \omega_0 + \alpha t$.

SOLUTION

Part (a)

1. Solve $\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$ for the angular acceleration, α:

$$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$$
$$\alpha = \frac{\omega^2 - \omega_0{}^2}{2(\theta - \theta_0)}$$

2. Convert $\theta - \theta_0 = 1.25$ rev to radians:

$$\theta - \theta_0 = 1.25\text{ rev} = 1.25\text{ rev}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 7.85\text{ rad}$$

3. Substitute numerical values to find α:

$$\alpha = \frac{\omega^2 - \omega_0{}^2}{2(\theta - \theta_0)} = \frac{0 - (3.40\text{ rad/s})^2}{2(7.85\text{ rad})} = -0.736\text{ rad/s}^2$$

Part (b)

4. Solve $\omega = \omega_0 + \alpha t$ for the time, t:

$$\omega = \omega_0 + \alpha t$$
$$t = \frac{\omega - \omega_0}{\alpha}$$

5. Substitute numerical values to find t:

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 3.40\text{ rad/s}}{(-0.736\text{ rad/s}^2)} = 4.62\text{ s}$$

INSIGHT

Note that it was not necessary to define a reference line; that is, a direction for $\theta = 0$. All we need to know is the angular displacement, $\theta - \theta_0$, not the individual angles θ and θ_0. Finally, notice that we can also solve Equation 10–9 for the time in part (b), which yields $t = 2(\theta - \theta_0)/(\omega_0 + \omega) = 4.62$ s, as expected.

PRACTICE PROBLEM

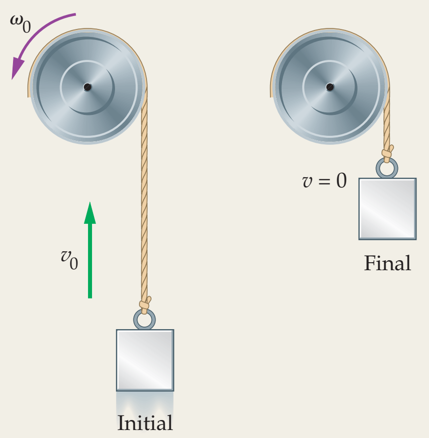
What is the angular speed of the wheel after one complete revolution? [**Answer:** $\omega = 1.52$ rad/s]

Some related homework problems: Problem 18, Problem 20

Finally, we consider a pulley that is rotating in such a way that initially it is lifting a mass with speed v. Gravity acting on the mass causes it and the pulley to slow and momentarily come to rest.

ACTIVE EXAMPLE 10–1 FIND THE TIME TO REST

A pulley rotating in the counterclockwise direction is attached to a mass suspended from a string. The mass causes the pulley's angular velocity to decrease with a constant angular acceleration $\alpha = -2.10\ \text{rad/s}^2$. **(a)** If the pulley's initial angular velocity is $\omega_0 = 5.40$ rad/s, how long does it take for the pulley to come to rest? **(b)** Through what angle does the pulley turn during this time?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. (a) Relate angular velocity to time:	$\omega = \omega_0 + \alpha t$
2. Solve for the time, t:	$t = (\omega - \omega_0)/\alpha$
3. Substitute numerical values:	$t = 2.57$ s
4. (b) Use $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ to solve for $\theta - \theta_0$:	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 = 6.94$ rad
5. Alternatively, use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$:	$\theta - \theta_0 = (\omega^2 - \omega_0^2)/2\alpha = 6.94$ rad

INSIGHT

After the pulley comes to rest, it immediately begins to rotate in the clockwise direction as the mass falls. The pulley's angular *acceleration* is constant—it has the same value before the pulley stops, when it stops, and after it begins rotating in the opposite direction. This is analogous to a projectile thrown straight upward, where the linear velocity starts out positive, goes to zero, then changes sign, all while the linear acceleration remains constant in the negative direction.

YOUR TURN

Find the angular displacement of the pulley at the time when its angular velocity is half its initial value.

(Answers to **Your Turn** *problems are given in the back of the book.)*

10–3 Connections Between Linear and Rotational Quantities

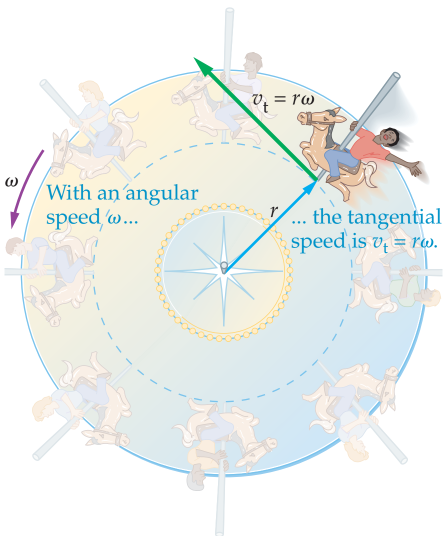
At a local county fair a child rides on a merry-go-round. The ride completes one circuit every $T = 7.50$ s. Therefore, the angular velocity of the child, from Equation 10–5, is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{7.50\ \text{s}} = 0.838\ \text{rad/s}$$

The path followed by the child is circular, with the center of the circle at the axis of rotation of the merry-go-round. In addition, at any instant of time the child is moving in a direction that is *tangential* to the circular path, as **Figure 10–7** shows. What is the tangential speed, v_t, of the child? In other words, what is the speed of the wind in the child's face?

We can find the child's tangential speed by dividing the circumference of the circular path, $2\pi r$, by the time required to complete one circuit, T. Thus,

$$v_t = \frac{2\pi r}{T} = r\left(\frac{2\pi}{T}\right)$$



▲ **FIGURE 10–7 Angular and linear speed**
Overhead view of a child riding on a merry-go-round. The child's path is a circle centered on the merry-go-round's axis of rotation. At any given time the child is moving tangential to the circular path with a speed $v_t = r\omega$.

Because $2\pi/T$ is simply ω, we can express the tangential speed as follows:

Tangential Speed of a Rotating Object

$$v_t = r\omega \qquad \text{10–12}$$

SI unit: m/s

Note that ω must be given in rad/s for this relation to be valid.

In the case of the merry-go-round, if the radius of the child's circular path is $r = 4.25$ m, the tangential speed is $v_t = r\omega = (4.25\text{ m})(0.838\text{ rad/s}) = 3.56\text{ m/s}$. When it is clear that we are referring to the tangential speed, we will often drop the subscript t, and simply write $v = r\omega$.

REAL-WORLD PHYSICS
The operation of a CD

An interesting application of the relation between linear and angular speeds is provided in the operation of a compact disk (CD). As you know, a CD is played by shining a laser beam onto the disk, and then converting the pattern of reflected light into a pattern of sound waves. For proper operation, however, the linear speed of the disk where the laser beam shines on it must be maintained at the constant value of 1.25 m/s. As the CD is played, the laser beam scans the disk in a spiral track from near the center outward to the rim. In order to maintain the required linear speed, the angular speed of the disk must decrease as the beam scans outward. The required angular speeds are determined in the following Exercise.

EXERCISE 10–5

Find the angular speed a CD must have to give a linear speed of 1.25 m/s when the laser beam shines on the disk **(a)** 2.50 cm and **(b)** 6.00 cm from its center.

SOLUTION

a. Using $v = 1.25$ m/s and $r = 0.0250$ m in Equation 10–12, we find

$$\omega = \frac{v}{r} = \frac{1.25\text{ m/s}}{0.0250\text{ m}} = 50.0\text{ rad/s} = 477\text{ rpm}$$

b. Similarly, with $r = 0.0600$ m we find

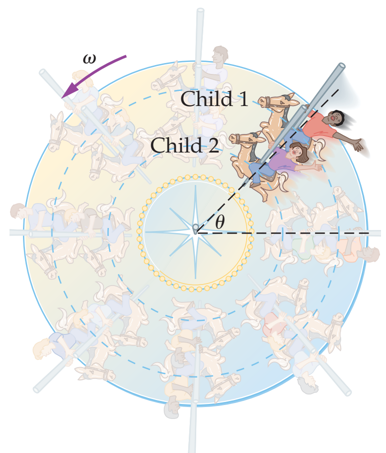
$$\omega = \frac{v}{r} = \frac{1.25\text{ m/s}}{0.0600\text{ m}} = 20.8\text{ rad/s} = 199\text{ rpm}$$

Thus, a CD slows from about 500 rpm to roughly 200 rpm as it plays.

How do the angular and tangential speeds of an object vary from one point to another? We explore this question in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 10–1 COMPARE THE SPEEDS

Two children ride on a merry-go-round, with child 1 at a greater distance from the axis of rotation than child 2. Is the angular speed of child 1 **(a)** greater than, **(b)** less than, or **(c)** the same as the angular speed of child 2?



REASONING AND DISCUSSION

At any given time, the angle θ for child 1 is the same as the angle for child 2, as shown. Therefore, when the angle for child 1 has gone through 2π, for example, so has the angle for child 2. As a result, they have the same angular speed. In fact, *each and every point on the merry-go-round has exactly the same angular speed.*

The tangential speeds are different, however. Child 1 has the greater tangential speed since he travels around a larger circle in the same time that child 2 travels around a smaller circle. This is in agreement with the relation $v = r\omega$, since the radius to child 1 is greater than the radius to child 2. That is, $v_1 = r_1\omega > v_2 = r_2\omega$.

ANSWER

(c) The angular speeds are the same.

◀ In the photo at left, two identical plastic letter "E"s have been placed on a rotating turntable at different distances from the axis of rotation. The stretching and blurring of the image of the outermost letter clearly show that it is moving faster than the letter closer to the axis. Similarly, the boy near the rim of this playground merry-go-round is moving faster than the girl near the hub.

Because the children on the merry-go-round move in a circular path, they experience a centripetal acceleration, a_{cp} (Section 6–5). The centripetal acceleration is always directed toward the axis of rotation and has a magnitude given by

$$a_{cp} = \frac{v^2}{r}$$

Note that the speed v in this expression is the tangential speed, $v = v_t = r\omega$, and therefore the centripetal acceleration in terms of ω is

$$a_{cp} = \frac{(r\omega)^2}{r}$$

Canceling one power of r, we have

Centripetal Acceleration of a Rotating Object

$$a_{cp} = r\omega^2 \qquad 10\text{–}13$$

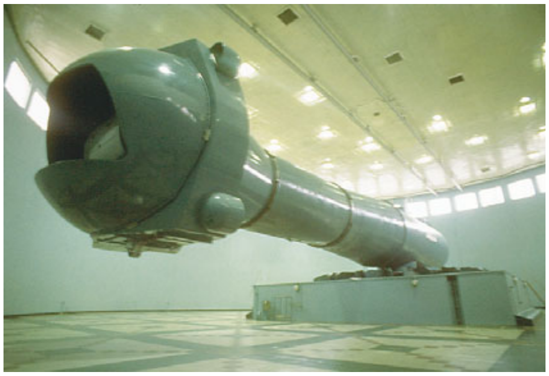
SI unit: m/s^2

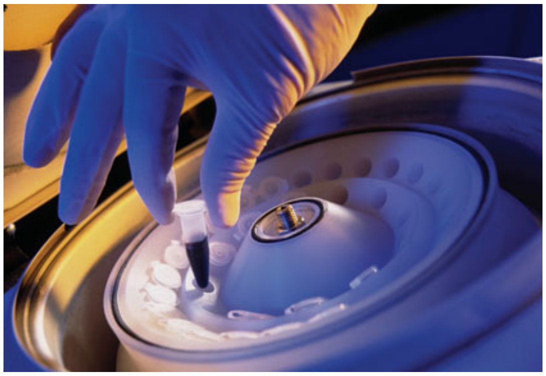
If the radius of a child's circular path on the merry-go-round is 4.25 m, and the angular speed of the ride is 0.838 rad/s, the centripetal acceleration of the child is $a_{cp} = r\omega^2 = (4.25\ m)(0.838\ rad/s)^2 = 2.98\ m/s^2$.

REAL-WORLD PHYSICS

The centrifuge

Though the centripetal acceleration of a merry-go-round is typically only a fraction of the acceleration of gravity, rotating devices referred to as **centrifuges** can produce centripetal accelerations many times greater than gravity. For example, the world's most powerful research centrifuge, operated by the U.S. Army Corps of Engineers, can subject 2.2-ton payloads to accelerations as high as $350g$ (350 times greater than the acceleration of gravity). This centrifuge is used to study earthquake engineering and dam erosion. The Air Force uses centrifuges to subject prospective jet pilots to the accelerations they will experience during rapid flight maneuvers, and in the future NASA may even use a human-powered centrifuge for gravity studies aboard the International Space Station.





▲ The large centrifuge shown at left, at the Gagarin Cosmonaut Training Center, is used to train Russian cosmonauts for space missions. This device, which rotates at 36 rpm, can produce a centripetal acceleration of over 290 m/s^2, 30 times the acceleration of gravity. The device at right is a microhematocrit centrifuge, used to separate blood cells from plasma. The volume of red blood cells in a given quantity of whole blood is a major factor in determining the oxygen-carrying capacity of the blood, an important clinical indicator.

The centrifuges most commonly encountered in everyday life are those found in virtually every medical laboratory in the world. These devices, which can produce centripetal accelerations in excess of 13,000g, are used to separate blood cells from blood plasma. They do this by speeding up the natural tendency of cells to settle out of plasma from days to minutes. The ratio of the packed cell volume to the total blood volume gives the *hematocrit value*, which is a useful clinical indicator of blood quality. In the next Example we consider the operation of a *microhematocrit centrifuge*, which measures the hematocrit value of a small (micro) sample of blood.

REAL-WORLD PHYSICS: BIO
The microhematocrit centrifuge

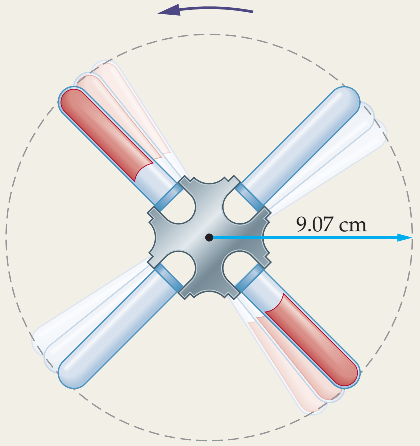
EXAMPLE 10–3 THE MICROHEMATOCRIT

REAL-WORLD PHYSICS: BIO

In a microhematocrit centrifuge, small samples of blood are placed in heparinized capillary tubes (heparin is an anticoagulant). The tubes are rotated at 11,500 rpm, with the bottoms of the tubes 9.07 cm from the axis of rotation. **(a)** Find the linear speed of the bottom of the tubes. **(b)** What is the centripetal acceleration at the bottom of the tubes?

PICTURE THE PROBLEM

Our sketch shows a top view of the centrifuge, with the capillary tubes rotating at 11,500 rpm. Notice that the bottoms of the tubes move in a circular path of radius 9.07 cm.



STRATEGY

a. Linear and angular speeds are related by $v = r\omega$. Once we convert the angular speed to rad/s we can use this relation to determine v.

b. The centripetal acceleration is $a_{cp} = r\omega^2$. Using ω from part (a) yields the desired result.

SOLUTION

Part (a)

1. Convert the angular speed, ω, to radians per second:

$$\omega = (11{,}500\ \text{rev/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 1.20 \times 10^3\ \text{rad/s}$$

2. Use $v = r\omega$ to calculate the linear speed:

$$v = r\omega = (0.0907\ \text{m})(1.20 \times 10^3\ \text{rad/s}) = 109\ \text{m/s}$$

Part (b)

3. Calculate the centripetal acceleration using $a_{cp} = r\omega^2$:

$$a_{cp} = r\omega^2 = (0.0907\ \text{m})(1.20 \times 10^3\ \text{rad/s})^2 = 131{,}000\ \text{m/s}^2$$

4. As a check, calculate the centripetal acceleration using $a_{cp} = v^2/r$:

$$a_{cp} = \frac{v^2}{r} = \frac{(109\ \text{m/s})^2}{0.0907\ \text{m}} = 131{,}000\ \text{m/s}^2$$

INSIGHT

Note that every point on a tube has the same angular speed. As a result, points near the top of a tube have smaller linear speeds and centripetal accelerations than do points near the bottom of a tube. In this case, the bottoms of the tubes experience a centripetal acceleration about 13,400 times greater than the acceleration of gravity on the surface of the Earth; that is, $a_{cp} = 131{,}000\ \text{m/s}^2 = 13{,}400g$.

PRACTICE PROBLEM

What angular speed must this centrifuge have if the centripetal acceleration at the bottom of the tubes is to be $98{,}100\ \text{m/s}^2$ ($\approx 10{,}000g$)? [**Answer:** $\omega = \sqrt{a_{cp}/r} = 1040\ \text{rad/s} = 9930\ \text{rpm}$]

Some related homework problems: Problem 34, Problem 37

If the angular speed of the merry-go-round in Conceptual Checkpoint 10–1 changes, the tangential speed of the children changes as well. It follows, then, that the children will experience a tangential acceleration, a_t. We can determine a_t by considering the relation $v_t = r\omega$. If ω changes by the amount $\Delta\omega$, with r remaining constant, the corresponding change in tangential speed is

$$\Delta v_t = r\Delta\omega$$

If this change in ω occurs in the time Δt, the tangential acceleration is

$$a_t = \frac{\Delta v_t}{\Delta t} = r\frac{\Delta\omega}{\Delta t}$$

Since $\Delta\omega/\Delta t$ is the angular acceleration, α, we find that

Tangential Acceleration of a Rotating Object

$$a_t = r\alpha \qquad \text{10–14}$$

SI unit: m/s^2

As with the tangential speed, we will often drop the subscript t in a_t when no confusion will arise.

In general, the children on the merry-go-round may experience both tangential and centripetal accelerations at the same time. Recall that a_t is due to a changing tangential speed, and that a_{cp} is caused by a changing direction of motion, even if the tangential speed remains constant. To summarize:

Tangential Versus Centripetal Acceleration

$a_t = r\alpha$	due to changing angular speed
$a_{cp} = r\omega^2$	due to changing direction of motion

As the names suggest, the tangential acceleration is always tangential to an object's path; the centripetal acceleration is always perpendicular to its path.

In cases in which both the centripetal and tangential accelerations are present, the total acceleration is the vector sum of the two, as indicated in **Figure 10–8**. Note that $\vec{\mathbf{a}}_t$ and $\vec{\mathbf{a}}_{cp}$ are at right angles to one another, and hence the magnitude of the total acceleration is given by the Pythagorean theorem:

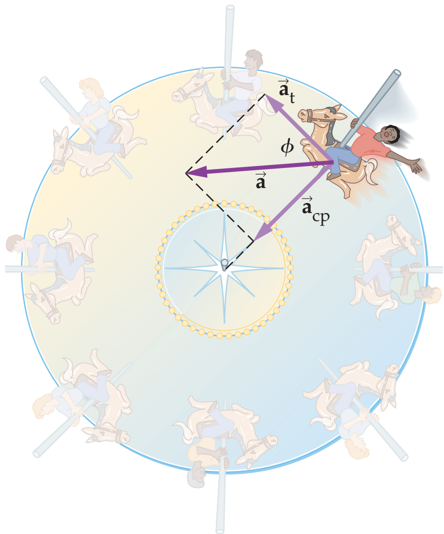
$$a = \sqrt{a_t^{\,2} + a_{cp}^{\,2}}$$

The direction of the total acceleration, measured relative to the tangential direction, is

$$\phi = \tan^{-1}\left(\frac{a_{cp}}{a_t}\right)$$

This angle is shown in Figure 10–8.

In the next Active Example, we consider an object that is rotating with a constant angular acceleration, α. In this case, the tangential acceleration, $a_t = r\alpha$, is constant in magnitude. On the other hand, the centripetal acceleration, $a_{cp} = r\omega^2$, changes with time since the angular speed changes.



▲ **FIGURE 10–8 Centripetal and tangential acceleration**

If the angular speed of the merry-go-round is increased, the child will experience two accelerations: (i) a tangential acceleration, $\vec{\mathbf{a}}_t$, and (ii) a centripetal acceleration, $\vec{\mathbf{a}}_{cp}$. The child's total acceleration, $\vec{\mathbf{a}}$, is the vector sum of $\vec{\mathbf{a}}_t$ and $\vec{\mathbf{a}}_{cp}$.

ACTIVE EXAMPLE 10–2 FIND THE ACCELERATION

Suppose the centrifuge in Example 10–3 is starting up with a constant angular acceleration of 95.0 rad/s^2. **(a)** What are the magnitudes of the centripetal, tangential, and total accelerations of the bottom of a tube when the angular speed is 8.00 rad/s? **(b)** What angle does the total acceleration make with the direction of motion?

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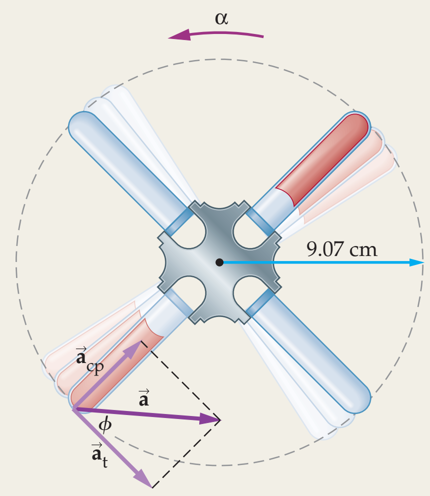

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SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Calculate the centripetal acceleration: $a_{cp} = r\omega^2 = 5.80\ \text{m/s}^2$
2. Calculate the tangential acceleration: $a_t = r\alpha = 8.62\ \text{m/s}^2$
3. Find the magnitude of the total acceleration: $a = \sqrt{a_{cp}^2 + a_t^2} = 10.4\ \text{m/s}^2$

Part (b)

4. Find the angle ϕ for the total acceleration: $\phi = \tan^{-1}(a_{cp}/a_t) = 33.9°$

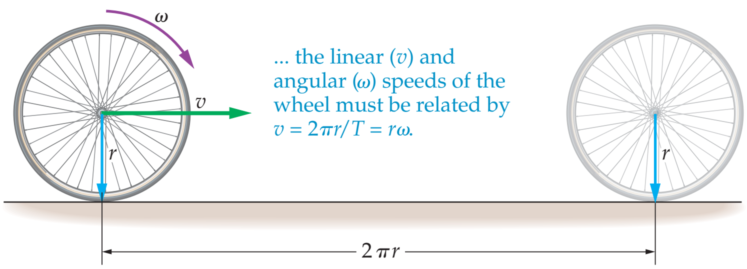
INSIGHT

Note that all points on a tube have the same angular speed. In addition, all points have the same angular acceleration. In contrast, different points have different centripetal and tangential accelerations, due to their dependence on the distance r from the axis of rotation.

YOUR TURN

Find the magnitude and direction of the total acceleration of a point *halfway* between the top and bottom of a tube.

(Answers to **Your Turn** *problems are given in the back of the book.)*



▶ **FIGURE 10–9 Rolling without slipping**
A wheel of radius r rolling without slipping. During one complete revolution, the center of the wheel moves forward through a distance $2\pi r$.

10–4 Rolling Motion

We began this chapter with a bicycle wheel rotating about its axle. In that case, the axle was at rest and every point on the wheel, such as the spot of red paint, moved in a circular path about the axle. We would like to consider a different situation now. Suppose the bicycle wheel is rolling freely, as indicated in **Figure 10–9**, with no slipping between the tire and the ground. The wheel still rotates about the axle, but the axle itself is moving in a straight line. As a result, the motion of the wheel is a combination of both rotational motion and linear (or **translational**) motion.

To see the connection between the wheel's rotational and translational motions, we show one full rotation of the wheel in Figure 10–9. During this rotation, the axle translates forward through a distance equal to the circumference of the wheel, $2\pi r$. Because the time required for one rotation is the period, T, the translational speed of the axle is

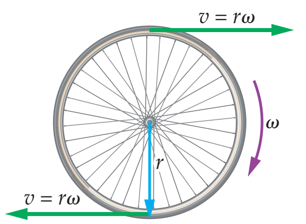
$$v = \frac{2\pi r}{T}$$

Recalling that $\omega = 2\pi/T$, we find

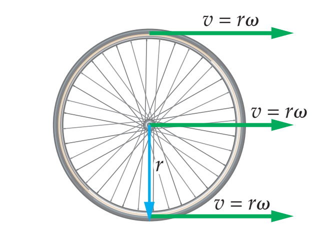
$$v = r\omega = v_t \qquad \text{10–15}$$

Hence, the translational speed of the axle is equal to the tangential speed of a point on the rim of a wheel spinning with angular speed ω.

A rolling object, then, combines rotational motion with angular speed ω, and translational motion with linear speed $v = r\omega$, where r is the radius of the object. Let's consider these two motions one at a time. First, in **Figure 10–10 (a)** we show pure rotational motion with angular speed ω. In this case, the axle is at rest, and points at the top and bottom of the wheel have tangential velocities that are equal in magnitude, $v = r\omega$, but point in opposite directions.



(a) Pure rotational motion



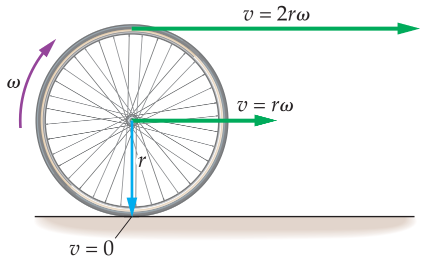
(b) Pure translational motion

▲ **FIGURE 10–10 Rotational and translational motions of a wheel**
(a) In pure rotational motion, the velocities at the top and bottom of the wheel are in opposite directions. **(b)** In pure translational motion, each point on the wheel moves with the same speed in the same direction.

Next, we consider translational motion with speed $v = r\omega$. This is illustrated in **Figure 10–10 (b)**, where we see that each point on the wheel moves in the same direction with the same speed. If this were the only motion the wheel had, it would be skidding across the ground, instead of rolling without slipping.

Finally, we combine these two motions by simply adding the velocity vectors in Figures 10–10 (a) and (b). The result is shown in **Figure 10–11**. At the top of the wheel the two velocity vectors are in the same direction, so they sum to give a speed of $2r\omega$. At the axle, the velocity vectors sum to give a speed $r\omega$. Finally, at the bottom of the wheel, the velocity vectors from rotation and translation have equal magnitude, but are in opposite directions. As a result, these velocities cancel, giving a speed of zero where the wheel is in contact with the ground.

The fact that the bottom of the wheel is instantaneously at rest, so that it is in static contact with the ground, is precisely what is meant by "rolling without slipping." Thus, a wheel that rolls without slipping is just like the situation when you are walking—even though your body as a whole moves forward, the soles of your shoes are momentarily at rest every time you place them on the ground. This point was discussed in detail in Conceptual Checkpoint 6–1.



▲ **FIGURE 10–11 Velocities in rolling motion**

In a wheel that rolls without slipping, the point in contact with the ground is instantaneously at rest. The center of the wheel moves forward with the speed $v = r\omega$, and the top of the wheel moves forward with twice that speed, $v = 2r\omega$.

EXERCISE 10–6

A car with tires of radius 32 cm drives on the highway at 55 mph. **(a)** What is the angular speed of the tires? **(b)** What is the linear speed of the tops of the tires?

SOLUTION

a. Using Equation 10–15 we find

$$\omega = \frac{v}{r} = \frac{(55\text{ mph})\left(\dfrac{0.447\text{ m/s}}{1\text{ mph}}\right)}{0.32\text{ m}} = 77\text{ rad/s}$$

This is about 12 revolutions per second.

b. The tops of the tires have a speed of $2v = 110$ mph.

▲ This photograph of a rolling wheel gives a visual indication of the speed of its various parts. The bottom of the wheel is at rest at any instant, so the image there is sharp. The top of the wheel has the greatest speed, and the image there shows the most blurring. (Compare Figure 10–11.)

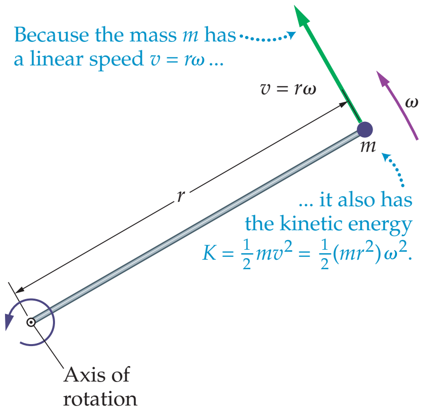
10–5 Rotational Kinetic Energy and the Moment of Inertia

An object in motion has kinetic energy, whether that motion is translational, rotational, or a combination of the two. In translational motion, for example, the kinetic energy of a mass m moving with a speed v is $K = \frac{1}{2}mv^2$. We cannot use this expression for a rotating object, however, because the speed v of each particle within a rotating object varies with its distance r from the axis of rotation, as we have seen in Equation 10–12. Thus, there is no unique value of v for an entire rotating object. On the other hand, there *is* a unique value of ω, the angular speed, that applies to all particles in the object.

To see how the kinetic energy of a rotating object depends on its angular speed, we start with a particularly simple system consisting of a rod of length r and negligible mass rotating about one end with an angular speed ω. Attached to the other end of the rod is a point mass m, as **Figure 10–12** shows. To find the kinetic energy of the mass, recall that its linear speed is $v = r\omega$ (Equation 10–12). Therefore, the translational kinetic energy of the mass m is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m(r\omega)^2 = \tfrac{1}{2}(mr^2)\omega^2 \qquad 10\text{–}16$$

Notice that the kinetic energy of the mass depends not only on the angular speed squared (analogous to the way the translational kinetic energy depends on the linear speed squared), but also on the radius squared—that is, the kinetic energy depends on the *distribution* of mass in the rotating object. To be specific, mass near the axis of rotation contributes little to the kinetic energy since its speed ($v = r\omega$) is small. On the other hand, the farther a mass is from the axis of rotation, the greater its speed v for a given angular velocity, and thus the greater its kinetic energy.



▲ **FIGURE 10–12 Kinetic energy of a rotating object**

As this rod rotates about the axis of rotation with an angular speed ω, the mass has a speed of $v = r\omega$. It follows that the kinetic energy of the mass is $K = \frac{1}{2}mv^2 = \frac{1}{2}(mr^2)\omega^2$.

You have probably noticed that the kinetic energy in Equation 10–16 is similar in form to the translational kinetic energy. Instead of $\frac{1}{2}(m)v^2$, we now have $\frac{1}{2}(mr^2)\omega^2$. Clearly, then, the quantity mr^2 plays the role of the mass for the rotating object. This "rotational mass" is given a special name in physics: the **moment of inertia,** I. Thus, in general, the kinetic energy of an object rotating with an angular speed ω can be written as:

Rotational Kinetic Energy

$$K = \frac{1}{2}I\omega^2 \qquad \text{10–17}$$

SI unit: J

The greater the moment of inertia—which some books call the rotational inertia—the greater an object's rotational kinetic energy. As we have just seen, in the special case of a point mass m a distance r from the axis of rotation, the moment of inertia is simply $I = mr^2$.

We now show how to find the moment of inertia for an object of arbitrary, fixed shape, as in **Figure 10–13**. Suppose, for example, that this object rotates about the axis indicated in the figure with an angular speed ω. To calculate the kinetic energy of the object, we first imagine dividing it into a collection of small mass elements, m_i. We then calculate the kinetic energy of each element and sum over all elements. This extends to a large number of mass elements what we did for the single mass m.

Following this plan, the total kinetic energy of an arbitrary rotating object is

$$K = \sum\left(\tfrac{1}{2}m_i v_i^2\right)$$

In this expression, m_i is the mass of one of the small mass elements and v_i is its speed. If m_i is at the radius r_i from the axis of rotation, as indicated in Figure 10–13, its speed is $v_i = r_i\omega$. Note that it is not necessary to write a separate angular speed, ω_i, for each element, because *all* mass elements of the object have exactly the same angular speed, ω. Therefore,

$$K = \sum\left(\tfrac{1}{2}m_i r_i^2\omega^2\right) = \tfrac{1}{2}\left(\sum m_i r_i^2\right)\omega^2$$

Now, in analogy with our results for the single mass, we can define the moment of inertia, I, as follows:

Definition of Moment of Inertia, *I*

$$I = \sum m_i r_i^2 \qquad \text{10–18}$$

SI unit: $kg \cdot m^2$

The precise value of I for a given object depends on its distribution of mass. A simple example of this dependence is given in the following Exercise.

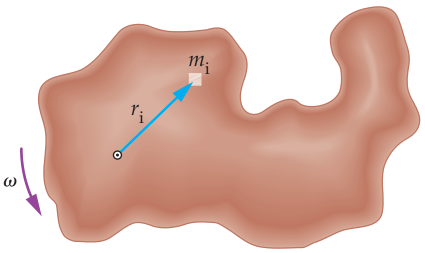


FIGURE 10–13 Kinetic energy of a rotating object of arbitrary shape

To calculate the kinetic energy of an object of arbitrary shape as it rotates about an axis with angular speed ω, imagine dividing it into small mass elements, m_i. The total kinetic energy of the object is the sum of the kinetic energies of all the mass elements.

EXERCISE 10–7

Use the general definition of the moment of inertia, as given in Equation 10–18, to find the moment of inertia for the dumbbell-shaped object shown in **Figure 10–14**. Note that the axis of rotation goes through the center of the object and points out of the page. In addition, assume that the masses may be treated as point masses.

SOLUTION

Referring to Figure 10–14, we see that $m_1 = m_2 = m$ and $r_1 = r_2 = r$. Therefore, the moment of inertia is

$$I = \sum m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 = mr^2 + mr^2 = 2mr^2$$

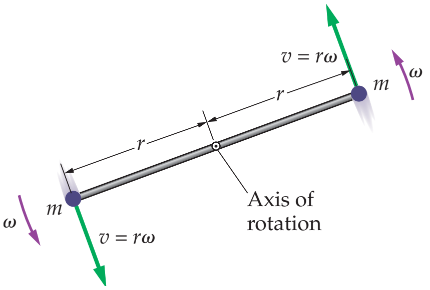


FIGURE 10–14 A dumbbell-shaped object rotating about its center

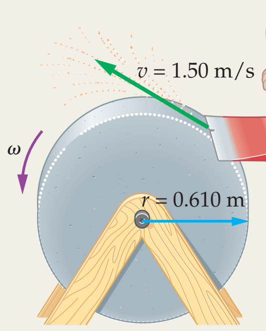
The connection between rotational kinetic energy and the moment of inertia is explored in more detail in the following Example.

EXAMPLE 10–4 NOSE TO THE GRINDSTONE

A grindstone with a radius of 0.610 m is being used to sharpen an ax. **(a)** If the linear speed of the stone relative to the ax is 1.50 m/s, and the stone's rotational kinetic energy is 13.0 J, what is its moment of inertia? **(b)** If the linear speed is doubled to 3.00 m/s, what is the corresponding kinetic energy of the grindstone?

PICTURE THE PROBLEM

Our sketch shows the grindstone spinning with an angular speed ω, which is not given in the problem statement. We do know, however, that the linear speed of the grindstone at its rim is $v = 1.50$ m/s and that its radius is $r = 0.610$ m. At this rate of rotation, the stone has a kinetic energy of 13.0 J.



STRATEGY

a. Recall that rotational kinetic energy and moment of inertia are related by $K = \frac{1}{2}I\omega^2$; thus $I = 2K/\omega^2$. We are not given ω, but we can find it from the connection between linear and angular speed, $v = r\omega$. Thus, we begin by finding ω. We then use ω, along with the kinetic energy K, to find I.

b. Find the new angular speed with $\omega = v/r$. Use I from part (a), along with $K = \frac{1}{2}I\omega^2$, to find the new kinetic energy.

SOLUTION

Part (a)

1. Find the angular speed of the grindstone: $\omega = \dfrac{v}{r} = \dfrac{1.50 \text{ m/s}}{0.610 \text{ m}} = 2.46 \text{ rad/s}$

2. Solve for the moment of inertia in terms of kinetic energy: $K = \frac{1}{2}I\omega^2 \quad \text{or} \quad I = \dfrac{2K}{\omega^2}$

3. Substitute numerical values for K and ω: $I = \dfrac{2K}{\omega^2} = \dfrac{2(13.0 \text{ J})}{(2.46 \text{ rad/s})^2} = 4.30 \text{ J}\cdot\text{s}^2 = 4.30 \text{ kg}\cdot\text{m}^2$

Part (b)

4. Find the angular speed of the grindstone corresponding to $v = 3.00$ m/s: $\omega = \dfrac{v}{r} = \dfrac{3.00 \text{ m/s}}{0.610 \text{ m}} = 4.92 \text{ rad/s}$

5. Determine the kinetic energy, K, using the moment of inertia, I, from part (a): $K = \frac{1}{2}I\omega^2 = \frac{1}{2}(4.30 \text{ kg}\cdot\text{m}^2)(4.92 \text{ rad/s})^2 = 52.0 \text{ J}$

INSIGHT

(a) We found I by relating it to the rotational kinetic energy of the grindstone. Later in this section we show how to calculate the moment of inertia of a disk directly, given its radius and mass. **(b)** Doubling the linear speed, v, results in a doubling of the angular speed, ω. The kinetic energy K depends on ω^2; therefore doubling ω increases K by a factor of 4, from 13.0 J to 4(13.0 J) = 52.0 J.

PRACTICE PROBLEM

When the ax is pressed firmly against the grindstone for sharpening, the angular speed of the grindstone decreases. If the rotational kinetic energy of the grindstone is cut in half to 6.50 J, what is its angular speed? [**Answer:** The moment of inertia is unchanged; it depends only on the size, shape, and mass of the grindstone. Hence, $\omega = \sqrt{2K/I} = 1.74$ rad/s, which is smaller than the original $\omega = 2.46$ rad/s by a factor of $\sqrt{2}$.]

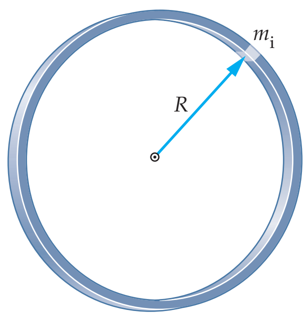
Some related homework problems: Problem 56, Problem 57

We return now to the dependence of the moment of inertia on the particular shape, or mass distribution, of an object. Suppose, for example, that a mass M is formed into the shape of a *hoop* of radius R. In addition, consider the case where the axis of rotation is perpendicular to the plane of the hoop and passes through its center, as shown in **Figure 10–15**. This is similar to a bicycle wheel rotating about its axle, if one ignores the spokes. In terms of small mass elements, we can write the moment of inertia as

$$I = \sum m_i r_i^2$$

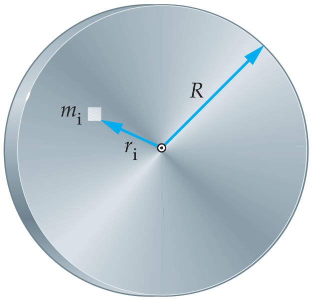
Each mass element of the hoop, however, is at the same radius R from the axis of rotation; that is, $r_i = R$. Hence, the moment of inertia in this case is

$$I = \sum m_i r_i^2 = \sum m_i R^2 = \left(\sum m_i\right) R^2$$



▲ FIGURE 10–15 The moment of inertia of a hoop

Consider a hoop of mass M and radius R. Each small mass element is at the same distance, R, from the center of the hoop. The moment of inertia in this case is $I = MR^2$.



▲ **FIGURE 10–16 The moment of inertia of a disk**

Consider a disk of mass M and radius R. Mass elements for the disk are at distances from the center ranging from 0 to R. The moment of inertia in this case is $I = \frac{1}{2}MR^2$.

Clearly, the sum of all the elementary masses is simply the total mass of the hoop, $\Sigma m_i = M$. Therefore, the moment of inertia of a hoop of mass M and radius R is

$$I = MR^2 \text{ (hoop)}$$

In contrast, if the same mass, M, is formed into a uniform *disk* of the same radius, R, the moment of inertia is different. To see this, note that it is no longer true that $r_i = R$ for all mass elements. In fact, most of the mass elements are closer to the axis of rotation than was the case for the hoop, as indicated in **Figure 10–16**. Thus, since the r_i are generally less than R, the moment of inertia will be smaller for the disk than for the hoop. A detailed calculation, summing over all mass elements, yields the following result:

$$I = \tfrac{1}{2}MR^2 \text{ (disk)}$$

As expected, I is less for the disk than for the hoop.

EXERCISE 10–8

If the grindstone in Example 10–4 is a uniform disk, what is its mass?

SOLUTION

Applying the preceding equation yields

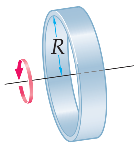
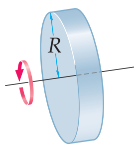
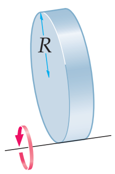
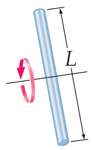
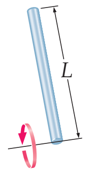
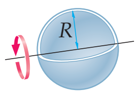
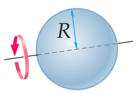
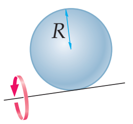
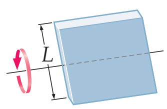
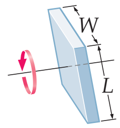
$$M = \frac{2I}{R^2} = \frac{2(4.30\ \text{kg}\cdot\text{m}^2)}{(0.610\ \text{m})^2} = 23.1\ \text{kg}$$

Thus, the grindstone has a weight of roughly 51 lb.

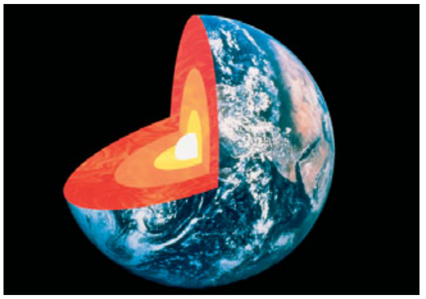
Table 10–1 collects moments of inertia for a variety of objects. Note that in all cases the moment of inertia is of the form $I = (\text{constant})MR^2$. It is only the constant in front of MR^2 that changes from one object to another.

Note also that objects of the same general shape but with different mass distributions—such as solid and hollow spheres—have different moments of inertia. In particular, a hollow sphere has a larger I than a solid sphere of the same mass, for the same reason that a hoop's moment of inertia is greater than a disk's—more of its mass is at a greater distance from the axis of rotation. Thus, I is a measure of both the shape *and* the mass distribution of an object.

TABLE 10–1 Moments of Inertia for Uniform, Rigid Objects of Various Shapes and Total Mass *M*

				
Hoop or cylindrical shell $I = MR^2$	Disk or solid cylinder $I = \frac{1}{2}MR^2$	Disk or solid cylinder (axis at rim) $I = \frac{3}{2}MR^2$	Long thin rod (axis through midpoint) $I = \frac{1}{12}ML^2$	Long thin rod (axis at one end) $I = \frac{1}{3}ML^2$
				
Hollow sphere $I = \frac{2}{3}MR^2$	Solid sphere $I = \frac{2}{5}MR^2$	Solid sphere (axis at rim) $I = \frac{7}{5}MR^2$	Solid plate (axis through center, in plane of plate) $I = \frac{1}{12}ML^2$	Solid plate (axis perpendicular to plane of plate) $I = \frac{1}{12}M(L^2 + W^2)$

Consider, for example, the moment of inertia of the Earth. If the Earth were a uniform sphere of mass M_E and radius R_E, its moment of inertia would be $\frac{2}{5}M_E R_E^2 = 0.4 M_E R_E^2$. In fact, the Earth's moment of inertia is only $0.331 M_E R_E^2$, considerably less than for a uniform sphere. This is due to the fact that the Earth is not homogeneous, but instead has a dense inner core surrounded by a less dense outer core and an even less dense mantle. The resulting concentration of mass near its axis of rotation gives the Earth a much smaller moment of inertia than it would have if its mass were uniformly distributed.



▲ The distribution of mass in the Earth is not uniform. Dense materials, like iron and nickel, have concentrated near the center, while less dense materials, like silicon and aluminum, have risen to the surface. This concentration of mass near the axis of rotation lowers the Earth's moment of inertia.

On the other hand, if the polar ice caps were to melt and release their water into the oceans, the Earth's moment of inertia would increase. This is because mass that had been near the axis of rotation (in the polar ice) would now be distributed more or less uniformly around the Earth (in the oceans). With more of the Earth's mass at greater distances from the axis of rotation, the moment of inertia would increase. If such an event were to occur, not only would the moment of inertia increase, but the length of the day would increase as well. We shall discuss the reasons for this in the next chapter.

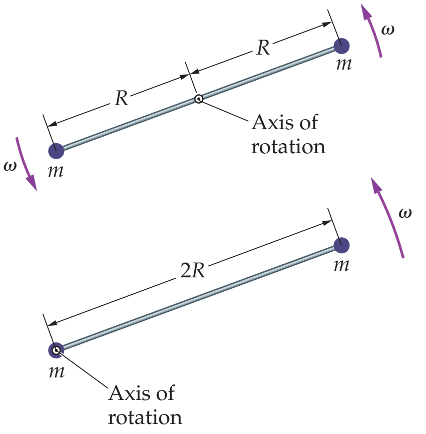
The moment of inertia of an object also depends on the location and orientation of the axis of rotation. If the axis of rotation is moved, all of the r_i change, leading to a different result for I. This is investigated for the dumbbell system in the following Conceptual Checkpoint.

REAL-WORLD PHYSICS

Moment of inertia of the Earth

CONCEPTUAL CHECKPOINT 10–2 COMPARE THE MOMENTS OF INERTIA

If the dumbbell-shaped object in Figure 10–14 is rotated about one end, is its moment of inertia **(a)** more than, **(b)** less than, or **(c)** the same as the moment of inertia about its center? As before, assume that the masses can be treated as point masses.



REASONING AND DISCUSSION

As we saw in Exercise 10–7, the moment of inertia about the center of the dumbbell is $I = 2mR^2$. When the axis is at one end, that mass is at the radius $r = 0$, and the other mass is at $r = 2R$. Therefore, the moment of inertia is

$$I = \sum m_i r_i^2 = m \cdot 0 + m(2R)^2 = 4mR^2$$

Thus, the moment of inertia doubles when the axis of rotation is moved from the center to one end.

The reason I increases is that the moment of inertia depends on the radius squared. Hence, even small increases in r can cause significant increases in I. By moving the axis to one end, the radius to the other mass is increased to its greatest possible value. As a result, I increases.

ANSWER

(a) The moment of inertia is greater about one end than about the center.

Finally, we summarize in the accompanying table the similarities between the translational kinetic energy, $K = \frac{1}{2}mv^2$, and the rotational kinetic energy, $K = \frac{1}{2}I\omega^2$. As expected, we see that the linear speed, v, has been replaced with the angular speed, ω. In addition, note that the mass m has been replaced with the moment of inertia I.

Linear Quantity	Angular Quantity
v	ω
m	I
$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$

As suggested by these analogies, the moment of inertia I plays the same role in rotational motion that mass plays in translational motion. For example, the larger I the more resistant an object is to any change in its angular velocity—an object with a large I is difficult to start rotating, and once it is rotating, it is difficult to stop. We shall see further applications of this analogy in the next chapter when we consider angular momentum.

10–6 Conservation of Energy

In this section, we consider the mechanical energy of objects that roll without slipping, and show how to apply energy conservation to such systems. In addition, we consider objects that rotate as a string or rope unwinds: for example, a pulley with a string wrapped around its circumference, or a yo-yo with a string wrapped around its axle. As long as the unwinding process and the rolling motion occur without slipping, the two situations are basically the same—at least as far as energy considerations are concerned.

To apply energy conservation to rolling objects, we first need to determine the kinetic energy of rolling motion. In Section 10–4 we saw that rolling motion is a combination of rotation and translation. It follows, then, that the kinetic energy of a rolling object is simply the sum of its translational kinetic energy, $\frac{1}{2}mv^2$, and its rotational kinetic energy, $\frac{1}{2}I\omega^2$:

PROBLEM-SOLVING NOTE

Energy Conservation with Rotational Motion

When applying energy conservation to a system with rotational motion, be sure to include the rotational kinetic energy, $\frac{1}{2}I\omega^2$.

Kinetic Energy of Rolling Motion

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \qquad \text{10–19}$$

Note that I in this expression is the moment of inertia about the center of the rolling object.

We can simplify the expression for the kinetic energy of a rolling object by using the fact that linear and angular speeds are related. In fact, recall that $v = r\omega$ (Equation 10–12), which can be rewritten as $\omega = v/r$. Substituting this into our expression for the rolling kinetic energy yields

Kinetic Energy of Rolling Motion: Alternative Form

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{r}\right)^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) \qquad \text{10–20}$$

Since $I = (\text{constant})mr^2$, the last term in Equation 10–20 is a constant that depends on the shape and mass distribution of the rolling object.

A special case of some interest is the point particle. In this case, by definition, all of the mass is at a single point. Therefore, $r = 0$, and hence $I = 0$. Substituting $I = 0$ in either Equation 10–19 or Equation 10–20 yields $K = \frac{1}{2}mv^2$, as expected.

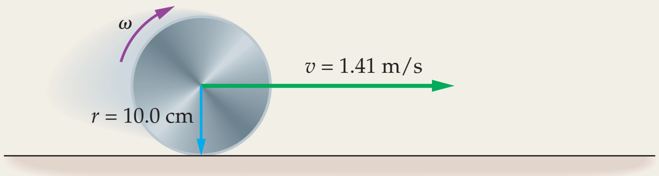
Next, we apply Equations 10–19 and 10–20 to a disk that rolls with no slipping.

EXAMPLE 10–5 LIKE A ROLLING DISK

A 1.20-kg disk with a radius of 10.0 cm rolls without slipping. If the linear speed of the disk is 1.41 m/s, find **(a)** the translational kinetic energy, **(b)** the rotational kinetic energy, and **(c)** the total kinetic energy of the disk.

PICTURE THE PROBLEM

Because the disk rolls without slipping, the angular speed and the linear speed are related by $v = r\omega$. Note that the linear speed is $v = 1.41$ m/s and the radius is $r = 10.0$ cm. Finally, we are given that the mass of the disk is 1.20 kg.



STRATEGY

We calculate each contribution to the kinetic energy separately. The linear kinetic energy, of course, is simply $\frac{1}{2}mv^2$. For the rotational kinetic energy, $\frac{1}{2}I\omega^2$, we must use the fact that the moment of inertia for a disk is $I = \frac{1}{2}mr^2$. Finally, since the disk rolls without slipping, its angular speed is $\omega = v/r$.

SOLUTION

Part (a)

1. Calculate the translational kinetic energy, $\frac{1}{2}mv^2$: $\quad \frac{1}{2}mv^2 = \frac{1}{2}(1.20\text{ kg})(1.41\text{ m/s})^2 = 1.19\text{ J}$

Part (b)

2. Calculate the rotational kinetic energy symbolically, using $I = \frac{1}{2}mr^2$ and $\omega = v/r$: $\quad \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)$

3. Substitute the numerical value for $\frac{1}{2}mv^2$ (the translational kinetic energy) obtained in Step 1: $\quad \frac{1}{2}I\omega^2 = \frac{1}{2}(1.19\text{ J}) = 0.595\text{ J}$

Part (c)

4. Sum the kinetic energies obtained in parts (a) and (b): $\quad K = 1.19\text{ J} + 0.595\text{ J} = 1.79\text{ J}$

5. Note that the same result is obtained using Equation 10–20:

$$K = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) = \tfrac{1}{2}mv^2\left(1 + \frac{1}{2}\right)$$
$$= \tfrac{3}{2}\left(\tfrac{1}{2}mv^2\right) = \tfrac{3}{2}(1.19\ \text{J}) = 1.79\ \text{J}$$

INSIGHT

The symbolic result in Step 2 shows that the rotational kinetic energy of a uniform disk rolling without slipping is precisely one-half the disk's translational kinetic energy. Thus, 2/3 of the disk's kinetic energy is translational, 1/3 rotational. This result is independent of the disk's radius, as we can see by the cancellation of the radius r in Step 2.

To understand this cancellation, note that a larger disk has a larger moment of inertia, since it has mass farther from the axis of rotation. On the other hand, the larger disk also has a smaller angular speed, since the angular speed is inversely proportional to the radius: $\omega = v/r$. These two effects cancel, giving the same rotational kinetic energy for uniform disks of any radius—provided their linear speed is the same.

PRACTICE PROBLEM

Repeat this problem for the case of a rolling, hollow sphere. [**Answer:** (a) 1.19 J, (b) 0.793 J, (c) 1.98 J]

Some related homework problems: Problem 58, Problem 62

CONCEPTUAL CHECKPOINT 10–3 COMPARE KINETIC ENERGIES

A solid sphere and a hollow sphere of the same mass and radius roll without slipping at the same speed. Is the kinetic energy of the solid sphere **(a)** more than, **(b)** less than, or **(c)** the same as the kinetic energy of the hollow sphere?

REASONING AND DISCUSSION

Both spheres have the same translational kinetic energy since they have the same mass and speed. The rotational kinetic energy, however, is proportional to the moment of inertia. Since the hollow sphere has the greater moment of inertia, it has the greater kinetic energy.

ANSWER

(b) The solid sphere has less kinetic energy than the hollow sphere.

Now that we can calculate the kinetic energy of rolling motion, we show how to apply it to energy conservation. For example, consider an object of mass m, radius r, and moment of inertia I at the top of a ramp, as shown in **Figure 10–17**. The object is released from rest and allowed to roll to the bottom, a vertical height h below the starting point. What is the object's speed on reaching the bottom?

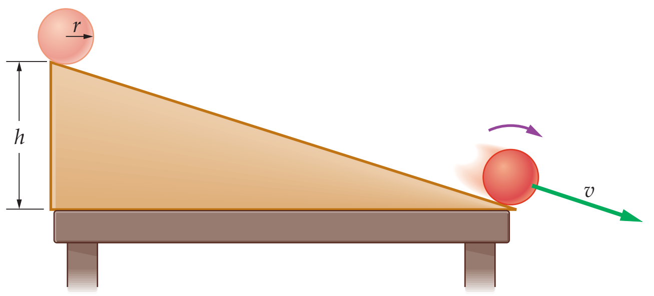


FIGURE 10–17 An object rolls down an incline

An object starts at rest at the top of an inclined plane and rolls without slipping to the bottom. The speed of the object at the bottom depends on its moment of inertia—a larger moment of inertia results in a lower speed.

The simplest way to solve this problem is to use energy conservation. To do so, we set the initial mechanical energy at the top (i) equal to the final mechanical energy at the bottom (f). That is,

$$K_i + U_i = K_f + U_f$$

Since we are dealing with rolling motion, the kinetic energy is

$$K = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

The potential energy is simply that due to the uniform gravitational field. Therefore,

$$U = mgy$$

With $y = h$ at the top of the ramp and the object starting at rest, we have

$$K_{\mathrm{i}} + U_{\mathrm{i}} = 0 + mgh = mgh$$

Similarly, with $y = 0$ at the bottom of the ramp and the object rolling with a speed v, we find

$$K_{\mathrm{f}} + U_{\mathrm{f}} = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) + 0 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

Setting the initial and final energies equal yields

$$mgh = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right)$$

Solving for v, we find

$$v = \sqrt{\frac{2gh}{1 + \dfrac{I}{mr^2}}}$$

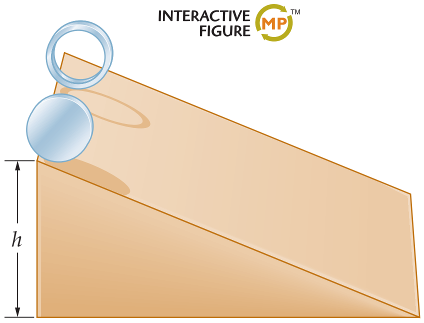
Let's quickly check one special case: namely, $I = 0$. With this substitution, we find

$$v = \sqrt{2gh}$$

This is the speed an object would have after falling straight down with no rotation through a distance h. Thus, setting $I = 0$ means there is no rotational kinetic energy, and hence the result is the same as for a point particle. As I becomes larger, the speed at the bottom of the ramp is smaller.

CONCEPTUAL CHECKPOINT 10–4 WHICH OBJECT WINS THE RACE?

A disk and a hoop of the same mass and radius are released at the same time at the top of an inclined plane. Does the disk reach the bottom of the plane **(a)** before, **(b)** after, or **(c)** at the same time as the hoop?



REASONING AND DISCUSSION

As we have just seen, the larger the moment of inertia, I, the smaller the speed, v. Hence the object with the larger moment of inertia (the hoop in this case) loses the race to the bottom, because its speed is less than the speed of the disk at any given height.

Another way to think about this is to recall that both objects have the same mechanical energy to begin with, namely, mgh. For the hoop, more of this initial potential energy goes into rotational kinetic energy, since the hoop has the larger moment of inertia; therefore, less energy is left for translational motion. As a result, the hoop moves more slowly and loses the race.

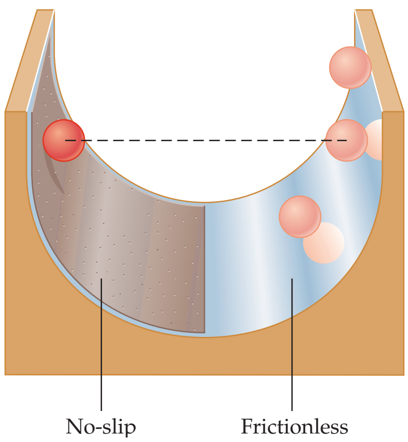
ANSWER

(a) The disk wins the race by reaching the bottom before the hoop.

In the next Conceptual Checkpoint, we consider the effects of a surface that changes from nonslip to frictionless.

CONCEPTUAL CHECKPOINT 10–5 COMPARE HEIGHTS

A ball is released from rest on a no-slip surface, as shown. After reaching its lowest point, the ball begins to rise again, this time on a frictionless surface. When the ball reaches its maximum height on the frictionless surface, is it **(a)** at a greater height, **(b)** at a lesser height, or **(c)** at the same height as when it was released?



REASONING AND DISCUSSION

As the ball descends on the no-slip surface, it begins to rotate, increasing its angular speed until it reaches the lowest point of the surface. When it begins to rise again, there is no friction to slow the rotational motion; thus, the ball continues to rotate with the same angular speed it had at its lowest point. Therefore, some of the ball's initial gravitational potential energy remains in the form of rotational kinetic energy. As a result, less energy is available to be converted back into gravitational potential energy, and the height is less.

ANSWER

(b) The height on the frictionless side is less.

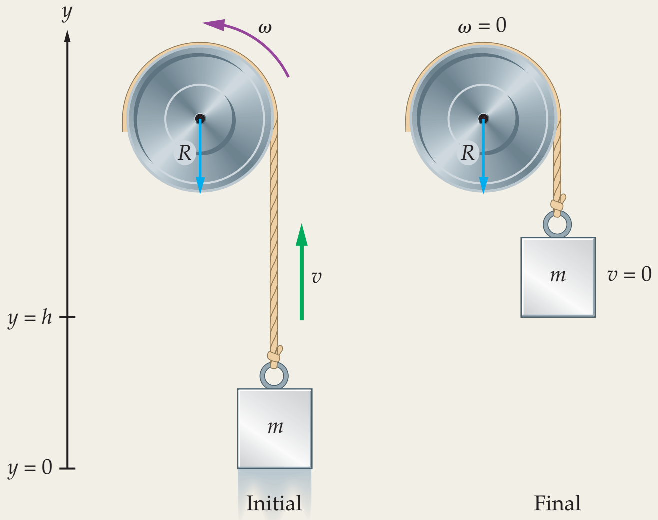
We can also apply energy conservation to the case of a pulley, or similar object, with a string that winds or unwinds without slipping. In such cases, the relation $v = r\omega$ is valid and we can follow the same methods applied to an object that rolls without slipping.

EXAMPLE 10–6 SPINNING WHEEL

A block of mass m is attached to a string that is wrapped around the circumference of a wheel of radius R and moment of inertia I. The wheel rotates freely about its axis and the string wraps around its circumference without slipping. Initially the wheel rotates with an angular speed ω, causing the block to rise with a linear speed v. To what height does the block rise before coming to rest? Give a symbolic answer.

PICTURE THE PROBLEM

Note in our sketch that we choose the origin of the y axis to be at the initial height of the block. The positive y direction, as usual, is chosen to be upward. When the block comes to rest, then, it is at the height $y = h > 0$, where h is to be determined from the initial speed of the block and the properties of the wheel.



STRATEGY

The problem statement gives two key pieces of information. First, the string wraps onto the disk without slipping; therefore, $v = R\omega$. Second, the wheel rotates freely, which means that the mechanical energy of the system is conserved. Thus, at the height h the initial kinetic energy of the system has been converted to gravitational potential energy. This condition can be used to find h.

Before we continue, note that the mechanical energy of the system includes the following contributions: (i) linear kinetic energy for the block, (ii) rotational kinetic energy for the wheel, and (iii) gravitational potential energy for the block. We do not include the gravitational potential energy of the wheel because its height does not change.

SOLUTION

1. Write an expression for the initial mechanical energy of the system, E_i, including all three contributions mentioned in the Strategy:

$$E_i = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{R}\right)^2 + 0$$

2. Write an expression for the final mechanical energy of the system, E_f:

$$E_f = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgy = 0 + 0 + mgh$$

3. Set the initial and final mechanical energies equal to one another, $E_i = E_f$:

$$E_i = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{R}\right)^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mR^2}\right) = E_f = mgh$$

4. Solve for the height, h:

$$h = \left(\frac{v^2}{2g}\right)\left(1 + \frac{I}{mR^2}\right)$$

INSIGHT

If the block were moving upward with speed v on its own—not attached to anything—it would rise to the height $h = v^2/2g$. We recover this result if $I = 0$, since in that case it is as if the wheel were not there. If the wheel is there, and I is nonzero, the block rises to a height that is *greater* than $v^2/2g$. The reason is that the wheel has kinetic energy, in addition to the kinetic energy of the block, and the sum of these kinetic energies must be converted to gravitational potential energy before the block and the wheel stop moving.

PRACTICE PROBLEM

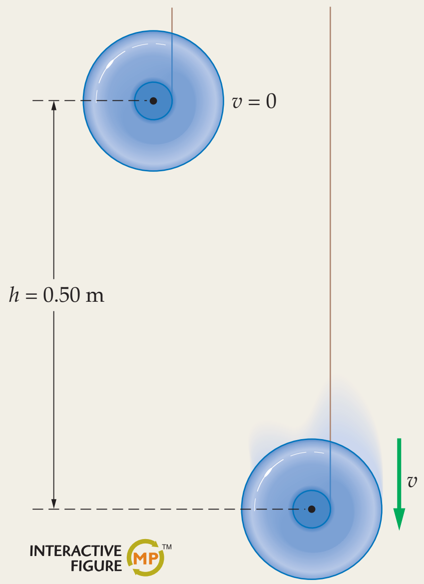
Suppose the wheel is a disk with a mass equal to the mass m of the block. Find an expression for the height h in this case. [**Answer:** The moment of inertia of the wheel is $I = \tfrac{1}{2}mR^2$. Therefore, $h = (3/2)(v^2/2g)$.]

Some related homework problems: Problem 66, Problem 70, Problem 73

The situation with a yo-yo is similar, as we see in the next Active Example.

ACTIVE EXAMPLE 10–3 FIND THE YO-YO'S SPEED

Yo-Yo man releases a yo-yo from rest and allows it to drop, as he keeps the top end of the string stationary. The mass of the yo-yo is 0.056 kg, its moment of inertia is $2.9 \times 10^{-5}\ \mathrm{kg \cdot m^2}$, and the radius, r, of the axle the string wraps around is 0.0064 m. What is the linear speed, v, of the yo-yo after it has dropped through a height $h = 0.50$ m?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial energy of the system: $E_i = mgh$
2. Write the final energy of the system: $E_f = \frac{1}{2}mv^2(1 + I/mr^2)$
3. Set $E_f = E_i$ and solve for v: $v = \sqrt{2gh/(1 + I/mr^2)}$
4. Substitute numerical values: $v = 0.85\ \mathrm{m/s}$

INSIGHT

The linear speed of the yo-yo is $v = r\omega$, where r is the radius of the axle from which the string unwraps without slipping. Therefore, the r in the term I/mr^2 is the radius of the axle. The outer radius of the yo-yo affects its moment of inertia, but since I is given to us in the problem statement, the outer radius is not pertinent.

YOUR TURN

If the yo-yo's moment of inertia is increased, does its final speed increase, decrease, or stay the same? Calculate the final speed for the case $I = 3.9 \times 10^{-5}\ \mathrm{kg \cdot m^2}$.

(Answers to **Your Turn** *problems are given in the back of the book.)*

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Our definitions of position, velocity, and acceleration from Chapter 2 are generalized in Section 10–1 to apply to rotational motion. We then use the kinematics of Chapters 2 and 4 in Section 10–2 to relate these quantities. The basic equations of motion are the same; only the names have been changed.

The kinetic energy, first defined in Chapter 7, plays a key role in defining the moment of inertia in Section 10–5.

Conservation of energy (Chapter 8) is just as important in rotational motion as it is in linear motion. We apply it to rotational motion in Section 10–6.

LOOKING AHEAD

In Chapter 11 we relate force to angular acceleration, in much the same way that force and acceleration are related in linear motion. This results in the concept of torque in Section 11–1.

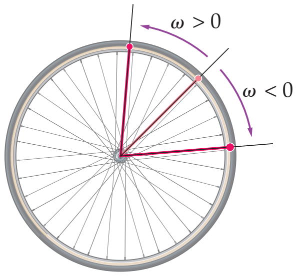
Just as linear speed is related to linear momentum (Chapter 9), angular speed is related to angular momentum. This is discussed in detail in Section 11–6.

Though a bit surprising at first, rotational motion is directly related to the motion of a pendulum swinging back and forth, and to the motion of a mass oscillating up and down on a spring. These connections are established in Section 13–3.

CHAPTER SUMMARY

10–1 ANGULAR POSITION, VELOCITY, AND ACCELERATION

To describe rotational motion, rotational analogues of position, velocity, and acceleration are defined.



Angular Position

Angular position, θ, is the angle measured from an arbitrary reference line:

$$\theta\ (\text{in radians}) = \text{arc length}/\text{radius} = s/r \qquad 10\text{–}2$$

Angular Velocity

Angular velocity, ω, is the rate of change of angular position. The average angular velocity is

$$\omega_{av} = \frac{\Delta\theta}{\Delta t} \qquad 10\text{–}3$$

The instantaneous angular velocity is the limit of ω_{av} as Δt approaches zero:

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \qquad 10\text{–}4$$

Angular Acceleration

Angular acceleration, α, is the rate of change of angular velocity. The average angular acceleration is

$$\alpha_{av} = \frac{\Delta \omega}{\Delta t} \qquad 10\text{–}6$$

The instantaneous angular acceleration is the limit of α_{av} as Δt approaches zero:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} \qquad 10\text{–}7$$

Period of Rotation

The period, T, is the time required to complete one full rotation. If the angular velocity is constant, T is related to ω as follows:

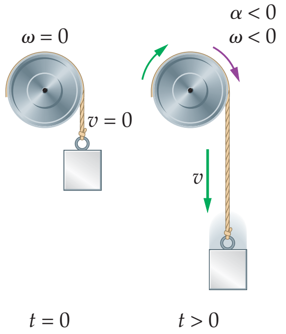
$$T = \frac{2\pi}{\omega} \qquad 10\text{–}5$$

Sign Convention

Counterclockwise rotations are positive; clockwise rotations are negative.

10–2 ROTATIONAL KINEMATICS

Rotational kinematics is the description of angular motion, in the same way that linear kinematics describes linear motion. In both cases, we assume constant acceleration.



Linear–Angular Analogues

Rotational kinematics is related to linear kinematics by the following linear–angular analogies:

Linear Quantity	Angular Quantity
x	θ
v	ω
a	α

Kinematic Equations (Constant Acceleration)

The equations of rotational kinematics are the same as the equations of linear kinematics, with the substitutions indicated by the linear–angular analogies:

Linear Equation		Angular Equation	
$v = v_0 + at$	2–7	$\omega = \omega_0 + \alpha t$	10–8
$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10	$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$	10–9
$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	10–10
$v^2 = v_0{}^2 + 2a(x - x_0)$	2–12	$\omega^2 = \omega_0{}^2 + 2\alpha(\theta - \theta_0)$	10–11

10–3 CONNECTIONS BETWEEN LINEAR AND ROTATIONAL QUANTITIES

A point on a rotating object follows a circular path. At any instant of time, the point is moving in a direction tangential to the circle, with a linear speed and acceleration. The linear speed and acceleration are related to the angular speed and acceleration.

Tangential Speed

The tangential speed, v_t, of a point on a rotating object is

$$v_t = r\omega \qquad 10\text{–}12$$

Centripetal Acceleration

The centripetal acceleration, a_{cp}, of a point on a rotating object is

$$a_{cp} = r\omega^2 \qquad 10\text{–}13$$

Centripetal acceleration is due to a change in direction of motion.

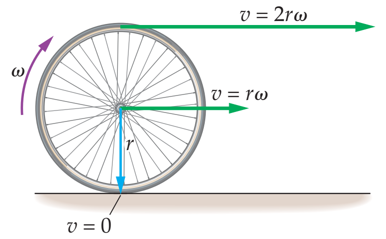
Tangential Acceleration
The tangential acceleration, a_t, of a point on a rotating object is

$$a_t = r\alpha \qquad 10\text{–}14$$

Tangential acceleration is due to a change in speed.

Total Acceleration
The total acceleration of a rotating object is the vector sum of its tangential and centripetal accelerations.

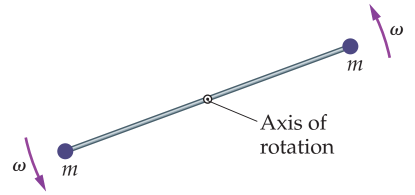
10–4 ROLLING MOTION



Rolling motion is a combination of translational and rotational motions. An object of radius r, rolling without slipping, translates with linear speed v and rotates with angular speed

$$\omega = v/r \qquad 10\text{–}15$$

10–5 ROTATIONAL KINETIC ENERGY AND THE MOMENT OF INERTIA



Rotating objects have kinetic energy, just as objects in linear motion have kinetic energy.

Rotational Kinetic Energy
The kinetic energy of a rotating object is

$$K = \tfrac{1}{2}I\omega^2 \qquad 10\text{–}17$$

The quantity I is the moment of inertia.

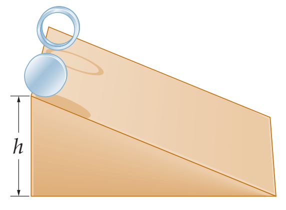
Moment of Inertia, Discrete Masses
The moment of inertia, I, of a collection of masses, m_i, at distances r_i from the axis of rotation is

$$I = \sum m_i r_i^2 \qquad 10\text{–}18$$

Moment of Inertia, Continuous Distribution of Mass
In a continuous object, the moment of inertia is calculated by dividing the object into a collection of small mass elements and summing $m_i r_i^2$ for each element. Results for a variety of continuous objects are collected in Table 10–1 on p. 314.

Linear–Angular Analogue
The moment of inertia is the rotational analogue to mass in linear systems. In particular, an object with a large moment of inertia is hard to start rotating and hard to stop rotating.

10–6 CONSERVATION OF ENERGY



Energy conservation can be applied to a variety of rotational systems in the same way that it is applied to translational systems.

Kinetic Energy of Rolling Motion
The kinetic energy of an object that rolls without slipping is

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 \qquad 10\text{–}19$$

Since rolling without slipping implies that $\omega = v/r$, the kinetic energy can be written as follows:

$$K = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\left(\frac{v}{r}\right)^2 = \tfrac{1}{2}mv^2\left(1 + \frac{I}{mr^2}\right) \qquad 10\text{–}20$$

Energy Conservation
Conservation of mechanical energy is a statement that the initial kinetic plus potential energy is equal to the final kinetic plus potential energy: $K_i + U_i = K_f + U_f$. By taking into account both rotational and translational kinetic energy, energy conservation can be applied in the same way as was done for linear systems.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Apply rotational kinematics with constant angular acceleration.	Rotational kinematics is completely analogous to the linear kinematics studied in Chapter 2. Angular problems are solved in the same way as the corresponding linear problems.	Example 10–1, Example 10–2 Active Example 10–1
Relate linear and angular motion.	Linear speed and angular speed are related by $v = r\omega$. Similarly, linear and angular accelerations are related by $a = r\alpha$. The centripetal acceleration of an object in circular motion is $a_{cp} = r\omega^2$.	Example 10–3 Active Example 10–2
Find the rotational kinetic energy of an object.	Rotational kinetic energy is given by $K = \frac{1}{2}I\omega^2$. The moment of inertia, I, plays the same role in rotational motion as the mass in linear motion.	Example 10–4, Example 10–5
Apply energy conservation to a rotational system.	To use energy conservation in a system with rotational motion, it is necessary to include the kinetic energy of rotation as one of the forms of energy.	Example 10–6 Active Example 10–3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A rigid object rotates about a fixed axis. Do all points on the object have the same angular speed? Do all points on the object have the same linear speed? Explain.
2. Can you drive your car in such a way that your tangential acceleration is zero while at the same time your centripetal acceleration is nonzero? Give an example if your answer is yes, state why not if your answer is no.
3. Can you drive your car in such a way that your tangential acceleration is nonzero while at the same time your centripetal acceleration is zero? Give an example if your answer is yes, state why not if your answer is no.
4. The fact that the Earth rotates gives people in New York a linear speed of about 750 mi/h. Where should you stand on the Earth to have the smallest possible linear speed?
5. At the local carnival you and a friend decide to take a ride on the Ferris wheel. As the wheel rotates with a constant angular speed, your friend poses the following questions: **(a)** Is my linear velocity constant? **(b)** Is my linear speed constant? **(c)** Is the magnitude of my centripetal acceleration constant? **(d)** Is the direction of my centripetal acceleration constant? What is your answer to each of these questions?
6. Why should changing the axis of rotation of an object change its moment of inertia, given that its shape and mass remain the same?
7. Give a common, everyday example for each of the following: **(a)** An object that has zero rotational kinetic energy but nonzero translational kinetic energy. **(b)** An object that has zero translational kinetic energy but nonzero rotational kinetic energy. **(c)** An object that has nonzero rotational and translational kinetic energies.
8. Two spheres have identical radii and masses. How might you tell which of these spheres is hollow and which is solid?
9. At the grocery store you pick up a can of beef broth and a can of chunky beef stew. The cans are identical in diameter and weight. Rolling both of them down the aisle with the same initial speed, you notice that the can of chunky stew rolls much farther than the can of broth. Why?
10. Suppose we change the race shown in Conceptual Checkpoint 10–4 so that a hoop of radius R and mass M races a hoop of radius R and mass $2M$. **(a)** Does the hoop with mass M finish before, after, or at the same time as the hoop with mass $2M$? Explain. **(b)** How would your answer to part (a) change if the hoops had different radii? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

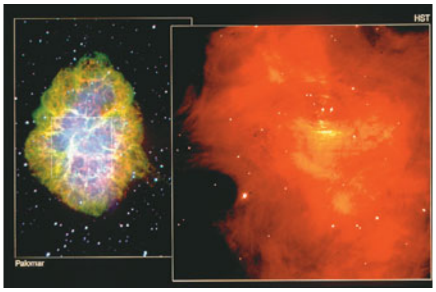
SECTION 10–1 ANGULAR POSITION, VELOCITY, AND ACCELERATION

1. • The following angles are given in degrees. Convert them to radians: 30°, 45°, 90°, 180°.
2. • The following angles are given in radians. Convert them to degrees: $\pi/6$, 0.70π, 1.5π, 5π.
3. • Find the angular speed of **(a)** the minute hand and **(b)** the hour hand of the famous clock in London, England, that rings the bell known as Big Ben.
4. • Express the angular velocity of the second hand on a clock in the following units: **(a)** rev/hr, **(b)** deg/min, and **(c)** rad/s.
5. • Rank the following in order of increasing angular speed: an automobile tire rotating at 2.00×10^3 deg/s, an electric drill rotating at 400.0 rev/min, and an airplane propeller rotating at 40.0 rad/s.
6. • A spot of paint on a bicycle tire moves in a circular path of radius 0.33 m. When the spot has traveled a linear distance of 1.95 m, through what angle has the tire rotated? Give your answer in radians.

7. • What is the angular speed (in rev/min) of the Earth as it orbits about the Sun?

8. • Find the angular speed of the Earth as it spins about its axis. Give your result in rad/s.

9. • **The Crab Nebula** One of the most studied objects in the night sky is the Crab nebula, the remains of a supernova explosion observed by the Chinese in 1054. In 1968 it was discovered that a pulsar—a rapidly rotating neutron star that emits a pulse of radio waves with each revolution—lies near the center of the Crab nebula. The period of this pulsar is 33 ms. What is the angular speed (in rad/s) of the Crab nebula pulsar?



The photo at left is a true-color visible light image of the Crab nebula. In the false-color breakout, the pulsar can be seen as the left member of the pair of stars just above the center of the frame. (Problems 9 and 106)

10. •• **IP** A 3.5-inch floppy disk in a computer rotates with a period of 2.00×10^{-1} s. What are **(a)** the angular speed of the disk and **(b)** the linear speed of a point on the rim of the disk? **(c)** Does a point near the center of the disk have an angular speed that is greater than, less than, or the same as the angular speed found in part (a)? Explain. (*Note:* A 3.5-inch floppy disk is 3.5 inches in diameter.)

11. •• The angle an airplane propeller makes with the horizontal as a function of time is given by $\theta = (125\ \text{rad/s})t + (42.5\ \text{rad/s}^2)t^2$. **(a)** Estimate the instantaneous angular velocity at $t = 0.00$ s by calculating the average angular velocity from $t = 0.00$ s to $t = 0.010$ s. **(b)** Estimate the instantaneous angular velocity at $t = 1.000$ s by calculating the average angular velocity from $t = 1.000$ s to $t = 1.010$ s. **(c)** Estimate the instantaneous angular velocity at $t = 2.000$ s by calculating the average angular velocity from $t = 2.000$ s to $t = 2.010$ s. **(d)** Based on your results from parts (a), (b), and (c), is the angular acceleration of the propeller positive, negative, or zero? Explain. **(e)** Calculate the average angular acceleration from $t = 0.00$ s to $t = 1.00$ s and from $t = 1.00$ s to $t = 2.00$ s.

SECTION 10–2 ROTATIONAL KINEMATICS

12. • **CE** An object at rest begins to rotate with a constant angular acceleration. If this object rotates through an angle θ in the time t, through what angle did it rotate in the time $t/2$?

13. • **CE** An object at rest begins to rotate with a constant angular acceleration. If the angular speed of the object is ω after the time t, what was its angular speed at the time $t/2$?

14. • In Active Example 10–1, how long does it take before the angular velocity of the pulley is equal to −5.0 rad/s?

15. • In Example 10–2, through what angle has the wheel turned when its angular speed is 2.45 rad/s?

16. • The angular speed of a propeller on a boat increases with constant acceleration from 12 rad/s to 26 rad/s in 2.5 revolutions. What is the acceleration of the propeller?

17. • The angular speed of a propeller on a boat increases with constant acceleration from 11 rad/s to 28 rad/s in 2.4 seconds. Through what angle did the propeller turn during this time?

18. •• After fixing a flat tire on a bicycle you give the wheel a spin. **(a)** If its initial angular speed was 6.35 rad/s and it rotated 14.2 revolutions before coming to rest, what was its average angular acceleration? **(b)** For what length of time did the wheel rotate?

19. •• **IP** A ceiling fan is rotating at 0.96 rev/s. When turned off, it slows uniformly to a stop in 2.4 min. **(a)** How many revolutions does the fan make in this time? **(b)** Using the result from part (a), find the number of revolutions the fan must make for its speed to decrease from 0.96 rev/s to 0.48 rev/s.

20. •• A discus thrower starts from rest and begins to rotate with a constant angular acceleration of 2.2 rad/s^2. **(a)** How many revolutions does it take for the discus thrower's angular speed to reach 6.3 rad/s? **(b)** How much time does this take?

21. •• **Half Time** At 3:00 the hour hand and the minute hand of a clock point in directions that are 90.0° apart. What is the first time after 3:00 that the angle between the two hands has decreased by half to 45.0°?

When the little hand is on the 3 and the big hand is on the 12 . . . . (Problem 21)

22. •• **BIO** A centrifuge is a common laboratory instrument that separates components of differing densities in solution. This is accomplished by spinning a sample around in a circle with a large angular speed. Suppose that after a centrifuge in a medical laboratory is turned off, it continues to rotate with a constant angular deceleration for 10.2 s before coming to rest. **(a)** If its initial angular speed was 3850 rpm, what is the magnitude of its angular deceleration? **(b)** How many revolutions did the centrifuge complete after being turned off?

23. •• **The Slowing Earth** The Earth's rate of rotation is constantly decreasing, causing the day to increase in duration. In the year 2006 the Earth took about 0.840 s longer to complete 365 revolutions than it did in the year 1906. What was the average angular acceleration of the Earth during this time? Give your answer in rad/s^2.

24. •• **IP** A compact disk (CD) speeds up uniformly from rest to 310 rpm in 3.3 s. **(a)** Describe a strategy that allows you to calculate the number of revolutions the CD makes in this time. **(b)** Use your strategy to find the number of revolutions.

25. •• When a carpenter shuts off his circular saw, the 10.0-inch-diameter blade slows from 4440 rpm to 0.00 rpm in 2.50 s. **(a)** What is the angular acceleration of the blade? **(b)** What is the distance traveled by a point on the rim of the blade during the deceleration? **(c)** What is the magnitude of the net displacement of a point on the rim of the blade during the deceleration?

26. •• **The World's Fastest Turbine** The drill used by most dentists today is powered by a small air turbine that can operate at angular speeds of 350,000 rpm. These drills, along with ultrasonic dental drills, are the fastest turbines in the world—far exceeding the angular speeds of jet engines. Suppose a drill starts at rest and comes up to operating speed in 2.1 s. **(a)** Find the angular acceleration produced by the drill, assuming it to be constant. **(b)** How many revolutions does the drill bit make as it comes up to speed?

An air-turbine dentist drill—faster than a jet engine. (Problem 26)

SECTION 10–3 CONNECTIONS BETWEEN LINEAR AND ROTATIONAL QUANTITIES

27. • **CE Predict/Explain** Two children, Jason and Betsy, ride on the same merry-go-round. Jason is a distance R from the axis of rotation; Betsy is a distance $2R$ from the axis. Is the rotational period of Jason greater than, less than, or equal to the rotational period of Betsy? **(b)** Choose the *best explanation* from among the following:
   I. The period is greater for Jason because he moves more slowly than Betsy.
   II. The period is greater for Betsy since she must go around a circle with a larger circumference.
   III. It takes the same amount of time for the merry-go-round to complete a revolution for all points on the merry-go-round.

28. • **CE** Referring to the previous problem, what are **(a)** the ratio of Jason's angular speed to Betsy's angular speed, **(b)** the ratio of Jason's linear speed to Betsy's linear speed, and **(c)** the ratio of Jason's centripetal acceleration to Betsy's centripetal acceleration?

29. • **CE Predict/Explain A Tall Building** The world's tallest building is the Taipei 101 Tower in Taiwan, which rises to a height of 508 m (1667 ft). **(a)** When standing on the top floor of the building, is your angular speed due to the Earth's rotation greater than, less than, or equal to your angular speed when you stand on the ground floor? **(b)** Choose the *best explanation* from among the following:
   I. The angular speed is the same at all distances from the axis of rotation.
   II. At the top of the building you are farther from the axis of rotation and hence you have a greater angular speed.
   III. You are spinning faster when you are closer to the axis of rotation.

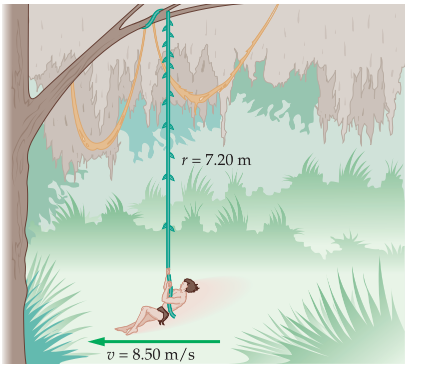
30. • The hour hand on a certain clock is 8.2 cm long. Find the tangential speed of the tip of this hand.

31. • Two children ride on the merry-go-round shown in Conceptual Checkpoint 10–1. Child 1 is 2.0 m from the axis of rotation, and child 2 is 1.5 m from the axis. If the merry-go-round completes one revolution every 4.5 s, find **(a)** the angular speed and **(b)** the linear speed of each child.

32. • The outer edge of a rotating Frisbee with a diameter of 29 cm has a linear speed of 3.7 m/s. What is the angular speed of the Frisbee?

33. • A carousel at the local carnival rotates once every 45 seconds. **(a)** What is the linear speed of an outer horse on the carousel, which is 2.75 m from the axis of rotation? **(b)** What is the linear speed of an inner horse that is 1.75 m from the axis of rotation?

34. •• **IP** Jeff of the Jungle swings on a vine that is 7.20 m long (**Figure 10–18**). At the bottom of the swing, just before hitting the tree, Jeff's linear speed is 8.50 m/s. **(a)** Find Jeff's angular speed at this time. **(b)** What centripetal acceleration does Jeff experience at the bottom of his swing? **(c)** What exerts the force that is responsible for Jeff's centripetal acceleration?



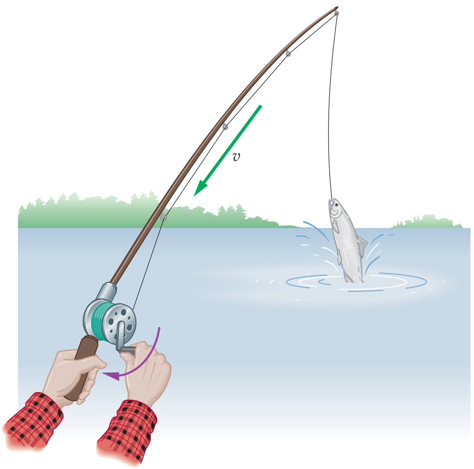
▲ **FIGURE 10–18** Problems 34 and 35

35. •• Suppose, in Problem 34, that at some point in his swing Jeff of the Jungle has an angular speed of 0.850 rad/s and an angular acceleration of 0.620 rad/s^2. Find the magnitude of his centripetal, tangential, and total accelerations, and the angle his total acceleration makes with respect to the tangential direction of motion.

36. •• A compact disk, which has a diameter of 12.0 cm, speeds up uniformly from 0.00 to 4.00 rev/s in 3.00 s. What is the tangential acceleration of a point on the outer rim of the disk at the moment when its angular speed is **(a)** 2.00 rev/s and **(b)** 3.00 rev/s?

37. •• **IP** When a compact disk with a 12.0-cm diameter is rotating at 5.05 rad/s, what are **(a)** the linear speed and **(b)** the centripetal acceleration of a point on its outer rim? **(c)** Consider a point on the CD that is halfway between its center and its outer rim. Without repeating all of the calculations required for parts (a) and (b), determine the linear speed and the centripetal acceleration of this point.

38. •• **IP** As Tony the fisherman reels in a "big one," he turns the spool on his fishing reel at the rate of 3.0 complete revolutions every second (**Figure 10–19**). **(a)** If the radius of the reel is 3.7 cm, what is the linear speed of the fishing line as it is reeled in? **(b)** How would your answer to part (a) change if the radius of the reel were doubled?



▲ **FIGURE 10–19** Problem 38

39. •• A Ferris wheel with a radius of 9.5 m rotates at a constant rate, completing one revolution every 36 s. Find the direction and magnitude of a passenger's acceleration when **(a)** at the top and **(b)** at the bottom of the wheel.

40. •• Suppose the Ferris wheel in the previous problem begins to decelerate at the rate of 0.22 $\mathrm{rad/s^2}$ when the passenger is at the top of the wheel. Find the direction and magnitude of the passenger's acceleration at that time.

41. •• **IP** A person swings a 0.52-kg tether ball tied to a 4.5-m rope in an approximately horizontal circle. **(a)** If the maximum tension the rope can withstand before breaking is 11 N, what is the maximum angular speed of the ball? **(b)** If the rope is shortened, does the maximum angular speed found in part (a) increase, decrease, or stay the same? Explain.

42. •• To polish a filling, a dentist attaches a sanding disk with a radius of 3.20 mm to the drill. **(a)** When the drill is operated at 2.15×10^4 rad/s, what is the tangential speed of the rim of the disk? **(b)** What period of rotation must the disk have if the tangential speed of its rim is to be 275 m/s?

43. •• In the previous problem, suppose the disk has an angular acceleration of 232 $\mathrm{rad/s^2}$ when its angular speed is 640 rad/s. Find both the tangential and centripetal accelerations of a point on the rim of the disk.

44. •• **The Bohr Atom** The Bohr model of the hydrogen atom pictures the electron as a tiny particle moving in a circular orbit about a stationary proton. In the lowest-energy orbit the distance from the proton to the electron is 5.29×10^{-11} m, and the linear speed of the electron is 2.18×10^6 m/s. **(a)** What is the angular speed of the electron? **(b)** How many orbits about the proton does it make each second? **(c)** What is the electron's centripetal acceleration?

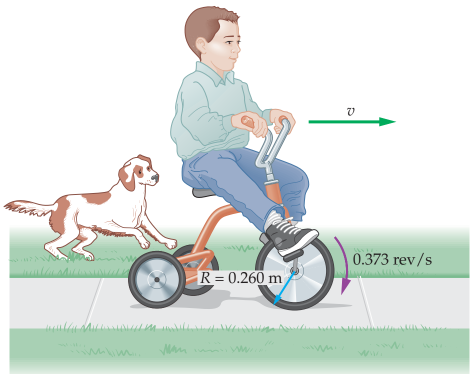
45. ••• A wheel of radius R starts from rest and accelerates with a constant angular acceleration α about a fixed axis. At what time t will the centripetal and tangential accelerations of a point on the rim have the same magnitude?

SECTION 10–4 ROLLING MOTION

46. • **CE** As you drive down the highway, the top of your tires are moving with a speed v. What is the reading on your speedometer?

47. •• The tires on a car have a radius of 31 cm. What is the angular speed of these tires when the car is driven at 15 m/s?

48. • A child pedals a tricycle, giving the driving wheel an angular speed of 0.373 rev/s (**Figure 10–20**). If the radius of the wheel is 0.260 m, what is the child's linear speed?



▲ **FIGURE 10–20** Problem 48

49. • A soccer ball, which has a circumference of 70.0 cm, rolls 14.0 yards in 3.35 s. What was the average angular speed of the ball during this time?

50. •• As you drive down the road at 17 m/s, you press on the gas pedal and speed up with a uniform acceleration of 1.12 $\mathrm{m/s^2}$ for 0.65 s. If the tires on your car have a radius of 33 cm, what is their angular displacement during this period of acceleration?

51. •• **IP** A bicycle coasts downhill and accelerates from rest to a linear speed of 8.90 m/s in 12.2 s. **(a)** If the bicycle's tires have a radius of 36.0 cm, what is their angular acceleration? **(b)** If the radius of the tires had been smaller, would their angular acceleration be greater than or less than the result found in part (a)?

SECTION 10–5 ROTATIONAL KINETIC ENERGY AND THE MOMENT OF INERTIA

52. • **CE Predict/Explain** The minute and hour hands of a clock have a common axis of rotation and equal mass. The minute hand is long, thin, and uniform; the hour hand is short, thick, and uniform. **(a)** Is the moment of inertia of the minute hand greater than, less than, or equal to the moment of inertia of the hour hand? **(b)** Choose the *best explanation* from among the following:
    - **I.** The hands have equal mass, and hence equal moments of inertia.
    - **II.** Having mass farther from the axis of rotation results in a greater moment of inertia.
    - **III.** The more compact hour hand concentrates its mass and has the greater moment of inertia.

53. • **CE Predict/Explain** Tons of dust and small particles rain down onto the Earth from space every day. As a result, does the Earth's moment of inertia increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
    - **I.** The dust adds mass to the Earth and increases its radius slightly.
    - **II.** As the dust moves closer to the axis of rotation, the moment of inertia decreases.
    - **III.** The moment of inertia is a conserved quantity and cannot change.

54. **CE** • **Predict/Explain** Suppose a bicycle wheel is rotated about an axis through its rim and parallel to its axle. **(a)** Is its moment of inertia about this axis greater than, less than, or equal to its moment of inertia about its axle? **(b)** Choose the *best explanation* from among the following:
    - **I.** The moment of inertia is greatest when an object is rotated about its center.
    - **II.** The mass and shape of the wheel remain the same.
    - **III.** Mass is farther from the axis when the wheel is rotated about the rim.

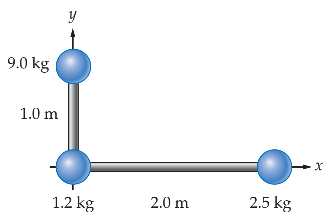
55. • The moment of inertia of a 0.98-kg bicycle wheel rotating about its center is 0.13 $\mathrm{kg \cdot m^2}$. What is the radius of this wheel, assuming the weight of the spokes can be ignored?

56. • What is the kinetic energy of the grindstone in Example 10–4 if it completes one revolution every 4.20 s?

57. • An electric fan spinning with an angular speed of 13 rad/s has a kinetic energy of 4.6 J. What is the moment of inertia of the fan?

58. • Repeat Example 10–5 for the case of a rolling hoop of the same mass and radius.

59. •• **CE** The L-shaped object in **Figure 10–21** can be rotated in one of the following three ways: case 1, rotation about the x axis; case 2, rotation about the y axis; and case 3, rotation about the



▲ **FIGURE 10–21** Problem 59

z axis (which passes through the origin perpendicular to the plane of the figure). Rank these three cases in order of increasing moment of inertia. Indicate ties where appropriate.

60. •• **IP** A 12-g CD with a radius of 6.0 cm rotates with an angular speed of 34 rad/s. **(a)** What is its kinetic energy? **(b)** What angular speed must the CD have if its kinetic energy is to be doubled?

61. •• When a pitcher throws a curve ball, the ball is given a fairly rapid spin. If a 0.15-kg baseball with a radius of 3.7 cm is thrown with a linear speed of 48 m/s and an angular speed of 42 rad/s, how much of its kinetic energy is translational and how much is rotational? Assume the ball is a uniform, solid sphere.

62. •• **IP** A basketball rolls along the floor with a constant linear speed v. **(a)** Find the fraction of its total kinetic energy that is in the form of rotational kinetic energy about the center of the ball. **(b)** If the linear speed of the ball is doubled to $2v$, does your answer to part (a) increase, decrease, or stay the same? Explain.

63. •• Find the rate at which the rotational kinetic energy of the Earth is decreasing. The Earth has a moment of inertia of $0.331 M_E R_E^2$, where $R_E = 6.38 \times 10^6$ m and $M_E = 5.97 \times 10^{24}$ kg, and its rotational period increases by 2.3 ms with each passing century. Give your answer in watts.

64. •• A lawn mower has a flat, rod-shaped steel blade that rotates about its center. The mass of the blade is 0.65 kg and its length is 0.55 m. **(a)** What is the rotational energy of the blade at its operating angular speed of 3500 rpm? **(b)** If all of the rotational kinetic energy of the blade could be converted to gravitational potential energy, to what height would the blade rise?

SECTION 10–6 CONSERVATION OF ENERGY

65. • **CE** Consider the physical situation shown in Conceptual Checkpoint 10–5. Suppose this time a ball is released from rest on the frictionless surface. When the ball comes to rest on the no-slip surface, is its height greater than, less than, or equal to the height from which it was released?

66. • Suppose the block in Example 10–6 has a mass of 2.1 kg and an initial upward speed of 0.33 m/s. Find the moment of inertia of the wheel if its radius is 8.0 cm and the block rises to a height of 7.4 cm before momentarily coming to rest.

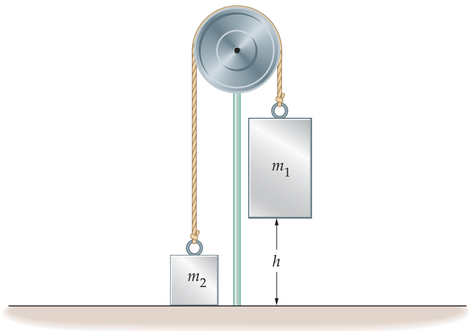
67. • Through what height must the yo-yo in Active Example 10–3 fall for its linear speed to be 0.65 m/s?

68. •• **CE** Suppose we change the race shown in Conceptual Checkpoint 10–4 to a race between three different disks. Let disk 1 have a mass M and a radius R, disk 2 have a mass M and a radius $2R$, and disk 3 have a mass $2M$ and a radius R. Rank the three disks in the order in which they finish the race. Indicate ties where appropriate.

69. •• Calculate the speeds of **(a)** the disk and **(b)** the hoop at the bottom of the inclined plane in Conceptual Checkpoint 10–4 if the height of the incline is 0.82 m.

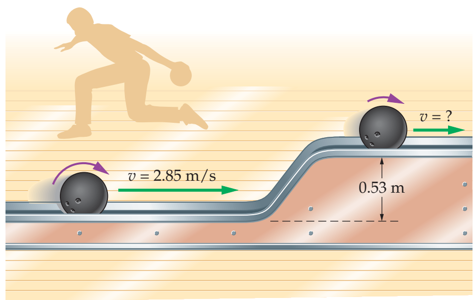
70. •• **IP Atwood's Machine** The two masses ($m_1 = 5.0$ kg and $m_2 = 3.0$ kg) in the Atwood's machine shown in **Figure 10–22** are released from rest, with m_1 at a height of 0.75 m above the floor. When m_1 hits the ground its speed is 1.8 m/s. Assuming that the pulley is a uniform disk with a radius of 12 cm, **(a)** outline a strategy that allows you to find the mass of the pulley. **(b)** Implement the strategy given in part (a) and determine the pulley's mass.

71. •• In Conceptual Checkpoint 10–5, assume the ball is a solid sphere of radius 2.9 cm and mass 0.14 kg. If the ball is released from rest at a height of 0.78 m above the bottom of the track on the no-slip side, **(a)** what is its angular speed when it is on the frictionless side of the track? **(b)** How high does the ball rise on the frictionless side?



▲ **FIGURE 10–22** Problem 70

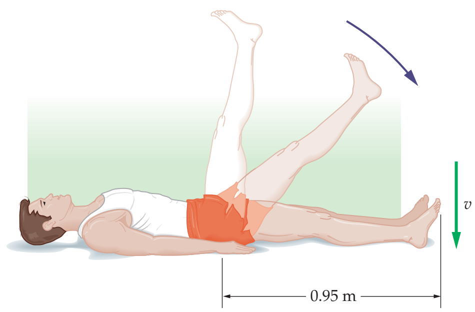
72. •• **IP** After you pick up a spare, your bowling ball rolls without slipping back toward the ball rack with a linear speed of 2.85 m/s (**Figure 10–23**). To reach the rack, the ball rolls up a ramp that rises through a vertical distance of 0.53 m. **(a)** What is the linear speed of the ball when it reaches the top of the ramp? **(b)** If the radius of the ball were increased, would the speed found in part (a) increase, decrease, or stay the same? Explain.



▲ **FIGURE 10–23** Problem 72

73. •• **IP** A 1.3-kg block is tied to a string that is wrapped around the rim of a pulley of radius 7.2 cm. The block is released from rest. **(a)** Assuming the pulley is a uniform disk with a mass of 0.31 kg, find the speed of the block after it has fallen through a height of 0.50 m. **(b)** If a small lead weight is attached near the rim of the pulley and this experiment is repeated, will the speed of the block increase, decrease, or stay the same? Explain.

74. •• After doing some exercises on the floor, you are lying on your back with one leg pointing straight up. If you allow your leg to fall freely until it hits the floor (**Figure 10–24**), what is the tangential speed of your foot just before it lands? Assume the leg can be treated as a uniform rod 0.95 m long that pivots freely about the hip.



▲ **FIGURE 10–24** Problem 74

75. ••• A 2.0-kg solid cylinder (radius = 0.10 m, length = 0.50 m) is released from rest at the top of a ramp and allowed to roll without slipping. The ramp is 0.75 m high and 5.0 m long. When the cylinder reaches the bottom of the ramp, what are

(a) its total kinetic energy, **(b)** its rotational kinetic energy, and **(c)** its translational kinetic energy?

76. ••• A 2.5-kg solid sphere (radius = 0.10 m) is released from rest at the top of a ramp and allowed to roll without slipping. The ramp is 0.75 m high and 5.6 m long. When the sphere reaches the bottom of the ramp, what are **(a)** its total kinetic energy, **(b)** its rotational kinetic energy, and **(c)** its translational kinetic energy?

GENERAL PROBLEMS

77. • **CE** When you stand on the observation deck of the Empire State Building in New York, is your linear speed due to the Earth's rotation greater than, less than, or the same as when you were waiting for the elevators on the ground floor?

78. • **CE Hard-Boiled Versus Raw Eggs** One way to tell whether an egg is raw or hard boiled—without cracking it open—is to place it on a kitchen counter and give it a spin. If you do this to two eggs, one raw the other hard boiled, you will find that one spins considerably longer than the other. Is the raw egg the one that spins a long time, or the one that stops spinning in a short time?

79. • **CE** When the Hoover Dam was completed and the reservoir behind it filled with water, did the moment of inertia of the Earth increase, decrease, or stay the same?

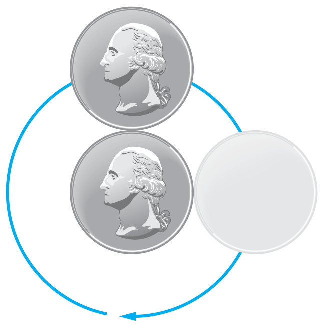
80. • **Weightless on the Equator** In Quito, Ecuador, near the equator, you weigh about half a pound less than in Barrow, Alaska, near the pole. Find the rotational period of the Earth that would make you feel weightless at the equator. (With this rotational period, your centripetal acceleration would be equal to the acceleration due to gravity, g.)

81. • A diver completes $2\frac{1}{2}$ somersaults during a 2.3-s dive. What was the diver's average angular speed during the dive?

82. • What linear speed must a 0.065-kg hula hoop have if its total kinetic energy is to be 0.12 J? Assume the hoop rolls on the ground without slipping.

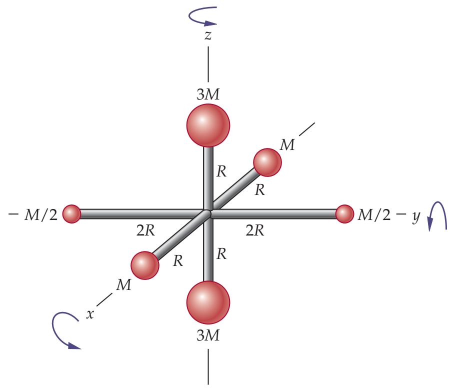
83. • **BIO Losing Consciousness** A pilot performing a horizontal turn will lose consciousness if she experiences a centripetal acceleration greater than 7.00 times the acceleration of gravity. What is the minimum radius turn she can make without losing consciousness if her plane is flying with a constant speed of 245 m/s?

84. •• **CE** Place two quarters on a table with their rims touching, as shown in **Figure 10–25**. While holding one quarter fixed, roll the other one—without slipping—around the circumference of the fixed quarter until it has completed one round trip. How many revolutions has the rolling quarter made about its center?



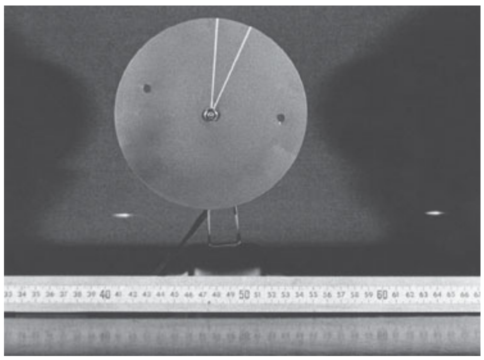
▲ **FIGURE 10–25** Problem 84

85. • **CE** The object shown in **Figure 10–26** can be rotated in three different ways: case 1, rotation about the x axis; case 2, rotation about the y axis; and case 3, rotation about the z axis. Rank these three cases in order of increasing moment of inertia. Indicate ties where appropriate.



▲ **FIGURE 10–26** Problem 85

86. •• The accompanying double-exposure photograph illustrates a method for determining the speed of a BB. The circular disk in the upper part of the photo rotates with a constant angular speed of 50.4 revolutions per second. A single white radial line drawn on the disk is seen in two locations in the double exposure. Below the disk are two bright images of a BB taken during the two exposures. Use the information given here and in the photo to estimate the speed of the BB.



Speeding BB and spinning wheel. (Problems 86 and 87)

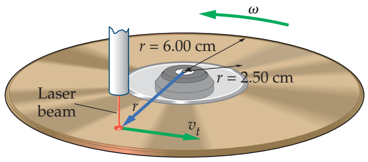
87. •• Referring to the previous problem, **(a)** estimate the linear speed of a point on the rim of the rotating disk. **(b)** By comparing the arc length between the two white lines to the distance covered by the BB, estimate the speed of the BB. **(c)** What radius must the disk have for the linear speed of a point on its rim to be the same as the speed of the BB? **(d)** Suppose a 1.0-g lump of putty is stuck to the rim of the disk. What centripetal force is required to hold the putty in place?

88. •• **IP When the Hands Align** A mathematically inclined friend e-mails you the following instructions: "Meet me in the cafeteria the first time after 2:00 P.M. today that the hands of a clock point in the same direction." **(a)** Is the desired meeting time before, after, or equal to 2:10 P.M.? Explain. **(b)** Is the desired meeting time before, after, or equal to 2:15 P.M.? Explain. **(c)** When should you meet your friend?

89. •• **IP** A diver runs horizontally off the end of a diving tower 3.0 m above the surface of the water with an initial speed of 2.6 m/s. During her fall she rotates with an average angular speed of 2.2 rad/s. **(a)** How many revolutions has she made when she hits the water? **(b)** How does your answer to part (a) depend on the diver's initial speed? Explain.

90. •• **IP** A potter's wheel of radius 6.8 cm rotates with a period of 0.52 s. What are **(a)** the linear speed and **(b)** the centripetal acceleration of a small lump of clay on the rim of the wheel? **(c)** How do your answers to parts (a) and (b) change if the period of rotation is doubled?

91. •• **IP** **Playing a CD** The record in an old-fashioned record player always rotates at the same angular speed. With CDs, the situation is different. For a CD to play properly, the point on the CD where the laser beam shines must have a linear speed $v_t = 1.25\ \mathrm{m/s}$, as indicated in **Figure 10–27**. **(a)** As the CD plays from the center outward, does its angular speed increase, decrease, or stay the same? Explain. **(b)** Find the angular speed of a CD when the laser beam is 2.50 cm from its center. **(c)** Repeat part (b) for the laser beam 6.00 cm from the center. **(d)** If the CD plays for 66.5 min, and the laser beam moves from 2.50 cm to 6.00 cm during this time, what is the CD's average angular acceleration?



▲ **FIGURE 10–27** Problem 91

92. •• **BIO** **Roller Pigeons** Pigeons are bred to display a number of interesting characteristics. One breed of pigeon, the "roller," is remarkable for the fact that it does a number of backward somersaults as it drops straight down toward the ground. Suppose a roller pigeon drops from rest and free falls downward for a distance of 14 m. If the pigeon somersaults at the rate of 12 rad/s, how many revolutions has it completed by the end of its fall?

93. •• As a marble with a diameter of 1.6 cm rolls down an incline, its center moves with a linear acceleration of $3.3\ \mathrm{m/s^2}$. **(a)** What is the angular acceleration of the marble? **(b)** What is the angular speed of the marble after it rolls for 1.5 s from rest?

94. •• A rubber ball with a radius of 3.2 cm rolls along the horizontal surface of a table with a constant linear speed v. When the ball rolls off the edge of the table, it falls 0.66 m to the floor below. If the ball completes 0.37 revolution during its fall, what was its linear speed, v?

95. •• A college campus features a large fountain surrounded by a circular pool. Two students start at the northernmost point of the pool and walk slowly around it in opposite directions. **(a)** If the angular speed of the student walking in the clockwise direction (as viewed from above) is 0.045 rad/s and the angular speed of the other student is 0.023 rad/s, how long does it take before they meet? **(b)** At what angle, measured clockwise from due north, do the students meet? **(c)** If the difference in linear speed between the students is 0.23 m/s, what is the radius of the fountain?

96. •• **IP** A yo-yo moves downward until it reaches the end of its string, where it "sleeps." As it sleeps—that is, spins in place—its angular speed decreases from 35 rad/s to 25 rad/s. During this time it completes 120 revolutions. **(a)** How long did it take for the yo-yo to slow from 35 rad/s to 25 rad/s? **(b)** How long does it take for the yo-yo to slow from 25 rad/s to 15 rad/s? Assume a constant angular acceleration as the yo-yo sleeps.

97. •• **IP** **(a)** An automobile with tires of radius 32 cm accelerates from 0 to 45 mph in 9.1 s. Find the angular acceleration of the tires. **(b)** How does your answer to part (a) change if the radius of the tires is halved?

98. •• **IP** In Problems 75 and 76 we considered a cylinder and a solid sphere, respectively, rolling down a ramp. **(a)** Which object do you expect to have the greater speed at the bottom of the ramp? **(b)** Verify your answer to part (a) by calculating the speed of the cylinder and of the sphere when they reach the bottom of the ramp.

99. •• A centrifuge (Problem 22) with an angular speed of 6050 rpm produces a maximum centripetal acceleration equal to $6840g$ (that is, 6840 times the acceleration of gravity). **(a)** What is the diameter of this centrifuge? **(b)** What force must the bottom of the sample holder exert on a 15.0-g sample under these conditions?

100. •• **A Yo-Yo with a Brain** Yomega ("The yo-yo with a brain") is constructed with a clever clutch mechanism in its axle that allows it to rotate freely and "sleep" when its angular speed is greater than a certain critical value. When the yo-yo's angular speed falls below this value, the clutch engages, causing the yo-yo to climb the string to the user's hand. If the moment of inertia of the yo-yo is $7.4 \times 10^{-5}\ \mathrm{kg \cdot m^2}$, its mass is 0.11 kg, and the string is 1.0 m long, what is the smallest angular speed that will allow the yo-yo to return to the user's hand?

A brain or just a clutch?
(Problem 100)

101. •• The rotor in a centrifuge has an initial angular speed of 430 rad/s. After 8.2 s of constant angular acceleration, its angular speed has increased to 550 rad/s. During this time, what were **(a)** the angular acceleration of the rotor and **(b)** the angle through which it turned?

102. •• **BIO** A honey bee has two pairs of wings that can beat 250 times a second. Estimate **(a)** the maximum angular speed of the wings and **(b)** the maximum linear speed of a wing tip.

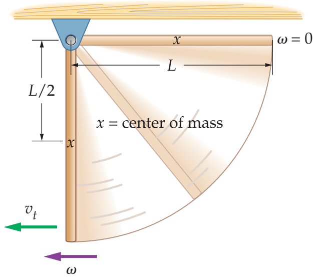
103. •• The Sun, with Earth in tow, orbits about the center of the Milky Way galaxy at a speed of 137 miles per second, completing one revolution every 240 million years. **(a)** Find the angular speed of the Sun relative to the center of the Milky Way. **(b)** Find the distance from the Sun to the center of the Milky Way.

104. •• A person walks into a room and switches on the ceiling fan. The fan accelerates with constant angular acceleration for 15 s until it reaches its operating angular speed of 1.9 rotations/s—after that its speed remains constant as long as the switch is "on." The person stays in the room for a short time; then, 5.5 minutes after turning the fan on, she switches it off again and leaves the room. The fan now decelerates with constant angular acceleration, taking 2.4 minutes to come to rest. What is the total number of revolutions made by the fan, from the time it was turned on until the time it stopped?

105. •• **BIO** **Preventing Bone Loss in Space** When astronauts return from prolonged space flights, they often suffer from bone loss, resulting in brittle bones that may take weeks for their bodies to rebuild. One solution may be to expose astronauts to periods of substantial "g forces" in a centrifuge carried aboard their spaceship. To test this approach, NASA conducted a study in which four people spent 22 hours each in a compartment attached to the end of a 28-foot arm that rotated with an angular speed of 10.0 rpm. **(a)** What centripetal acceleration did these volunteers experience? Express your answer in terms of g. **(b)** What was their linear speed?

106. ••• **Angular Acceleration of the Crab Nebula** The pulsar in the Crab nebula (Problem 9) was created by a supernova explosion that was observed on Earth in A.D. 1054. Its current period of rotation (33.0 ms) is observed to be increasing by 1.26×10^{-5} seconds per year. **(a)** What is the angular acceleration of the pulsar in $\mathrm{rad/s^2}$? **(b)** Assuming the angular acceleration of the pulsar to be constant, how many years will it take for the pulsar to slow to a stop? **(c)** Under the same assumption, what was the period of the pulsar when it was created?

107. ••• A thin, uniform rod of length L and mass M is pivoted about one end, as shown in **Figure 10–28**. The rod is released from rest in a horizontal position, and allowed to swing downward without friction or air resistance. When the rod is vertical, what are **(a)** its angular speed ω and **(b)** the tangential speed v_t of its free end?



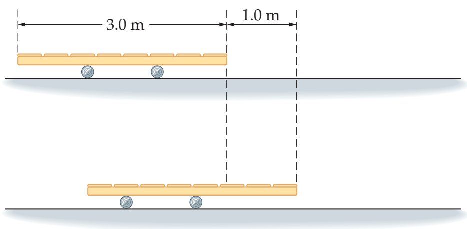
▲ **FIGURE 10–28** Problem 107

108. ••• **Center of Percussion** In the previous problem, suppose a small metal ball of mass $m = 2M$ is attached to the rod a distance d from the pivot. The rod and ball are released from rest in the horizontal position. **(a)** Show that when the rod reaches the vertical position, the speed of its tip is

$$v_t = \sqrt{3gL}\sqrt{\frac{1 + 4(d/L)}{1 + 6(d/L)^2}}$$

(b) At what finite value of d/L is the speed of the rod the same as it is for $d = 0$? (This value of d/L is the **center of percussion,** or "sweet spot," of the rod.)

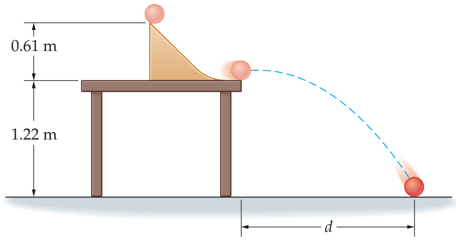
109. ••• A wooden plank rests on two soup cans laid on their sides. Each can has a diameter of 6.5 cm, and the plank is 3.0 m long. Initially, one can is placed 1.0 m inward from either end of the plank, as **Figure 10–29** shows. The plank is now pulled 1.0 m to the right, and the cans roll without slipping. **(a)** How far does the center of each can move? **(b)** How many rotations does each can make?



▲ **FIGURE 10–29** Problem 109

110. ••• A person rides on a 12-m-diameter Ferris wheel that rotates at the constant rate of 8.1 rpm. Calculate the magnitude and direction of the force that the seat exerts on a 65-kg person when he is **(a)** at the top of the wheel, **(b)** at the bottom of the wheel, and **(c)** halfway up the wheel.

111. ••• **IP** A solid sphere with a diameter of 0.17 m is released from rest; it then rolls without slipping down a ramp, dropping through a vertical height of 0.61 m. The ball leaves the bottom of the ramp, which is 1.22 m above the floor, moving horizontally (**Figure 10–30**). **(a)** Through what horizontal distance d does the ball move before landing? **(b)** How many revolutions does the ball make during its fall? **(c)** If the ramp were to be made frictionless, would the distance d increase, decrease, or stay the same? Explain.



▲ **FIGURE 10–30** Problem 111

PASSAGE PROBLEMS

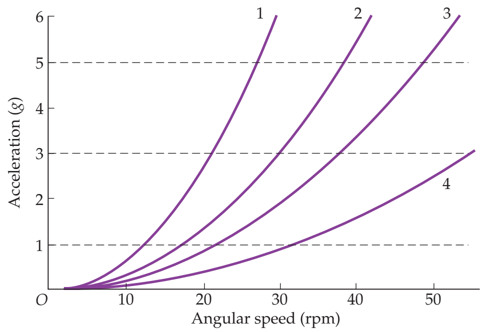
BIO Human-Powered Centrifuge

Space travel is fraught with hazards, not the least of which are the many side effects of prolonged weightlessness, including weakened muscles, bone loss, decreased coordination, and unsteady balance. If you are fortunate enough to go on a trip to Mars, which could take more than a year each way, you might be a bit "weak in the knees" by the time you arrive. This could lead to problems when you try to take your first "small step" on the surface.

To counteract these effects, NASA is looking into ways to provide astronauts with "portable gravity" on long space flights. One method under consideration is the human-powered centrifuge, which not only subjects the astronauts to artificial gravity, but also gives them aerobic exercise. The device is basically a rotating, circular platform on which two astronauts lie supine along a diameter, head-to-head at the center, with their feet at opposite rims, as shown in the accompanying photo. The radius of the platform in this test model is 6.25 ft. As one astronaut pedals to rotate the platform, the astronaut facing the other direction can exercise in the artificial gravity. Alternatively, a third astronaut on a stationary bicycle can provide the rotation for the other two.

Human-powered centrifuge, designed to give astronauts exercise and artificial gravity during long space flights.

Figure 10–31 shows the centripetal acceleration (in gs) produced by a rotating platform at four different radii. Notice that the acceleration increases as the square of the angular speed. Also indicated in Figure 10–31 are acceleration levels corresponding to 1, 3, and 5 gs. It is thought that enhanced gravitational effects may be desirable since the astronauts will experience the artificial gravity for only relatively brief periods of time during the flight.



▲ **FIGURE 10–31** Problems 112, 113, 114, and 115

112. • Rank the four curves shown in Figure 10–31 in order of increasing radius. Indicate ties where appropriate.

113. • What angular speed (in rpm) must the platform in this test model have to give a centripetal acceleration of 5.00 gs at the rim?

A. 5.07 rpm **B.** 26.1 rpm

C. 36.2 rpm **D.** 48.5 rpm

114. • Which of the curves shown in Figure 10–31 corresponds to the test model?

A. 1 **B.** 2

C. 3 **D.** 4

115. •• Estimate the radius corresponding to curve 4 in Figure 10–31.

A. 0.03 ft **B.** 0.3 ft

C. 3 ft **D.** 6 ft

INTERACTIVE PROBLEMS

116. •• **Referring to Conceptual Checkpoint 10–4** Suppose we race a disk and a hollow spherical shell, like a basketball. The spherical shell has a mass M and a radius R; the disk has a mass $2M$ and a radius $2R$. **(a)** Which object wins the race? If the two objects are released at rest, and the height of the ramp is $h = 0.75$ m, find the speed of **(b)** the disk and **(c)** the spherical shell when they reach the bottom of the ramp.

117. •• **Referring to Conceptual Checkpoint 10–4** Consider a race between the following three objects: object 1, a disk; object 2, a solid sphere; and object 3, a hollow spherical shell. All objects have the same mass and radius. **(a)** Rank the three objects in the order in which they finish the race. Indicate a tie where appropriate. **(b)** Rank the objects in order of increasing kinetic energy at the bottom of the ramp. Indicate a tie where appropriate.

118. •• **Referring to Active Example 10–3** **(a)** Suppose the radius of the axle the string wraps around is increased. Does the speed of the yo-yo after falling through a given height increase, decrease, or stay the same? **(b)** Find the speed of the yo-yo after falling from rest through a height $h = 0.50$ m if the radius of the axle is 0.0075 m. Everything else in Active Example 10–3 remains the same.

119. •• **Referring to Active Example 10–3** Suppose we use a new yo-yo that has the same mass as the original yo-yo and an axle of the same radius. The new yo-yo has a different mass distribution—most of its mass is concentrated near the rim. **(a)** Is the moment of inertia of the new yo-yo greater than, less than, or the same as that of the original yo-yo? **(b)** Find the moment of inertia of the new yo-yo if its speed after dropping from rest through a height $h = 0.50$ m is $v = 0.64$ m/s.

11 Rotational Dynamics and Static Equilibrium

Equilibrium, and the sense of serenity that comes with it, involves more than just forces that add to zero—it also depends on where the forces are applied. To keep from falling, for example, the forces exerted on this woman's hand and foot must add up to her total weight. But the total weight must be shared between her hand and foot in just the right way, or else her body will rotate and the pose will be lost. To ensure equilibrium, a new physical quantity—the torque—must also be zero. In this chapter we introduce the torque, and show that equilibrium occurs only when both the net force and the net torque are zero. We will also consider the consequences of nonzero torque.

In the previous chapter we learned how to describe uniformly accelerated rotational motion, but we did not discuss how a given angular acceleration is caused by a given force. The connection between forces and angular acceleration is the focus of this chapter.

We begin by defining a quantity that is the rotational equivalent of force. This quantity is called the *torque*. Although torque may not be as familiar a term as force, your muscles are exerting torques on your body at this very moment. In fact, every time you raise an arm, extend a finger, or stretch a leg, you exert torques to carry out these motions. Thus, our ability to move from place to place, or to hold our body still, is intimately related to our ability to exert precisely controlled torques on our limbs.

We also introduce the notion of *angular momentum* in this chapter and show that it is related to torque in essentially the same way that linear momentum is related to force. As a result, it follows that angular momentum is conserved when the net external torque acting on a system is zero. Thus, conservation of angular momentum joins conservation of energy and conservation of linear momentum as one of the fundamental principles on which all physics is based.

11–1 Torque

Suppose you want to loosen a nut by rotating it counterclockwise with a wrench. If you have ever used a wrench in this way, you probably know that the nut is more likely to turn if you apply your force as far from the nut as possible, as indicated in **Figure 11–1 (a)**. Applying a force near the nut would not be very effective—you could still get the nut to turn, but it would require considerably more effort! Similarly, it is much easier to open a revolving door if you push far from the axis of rotation, as indicated in **Figure 11–1 (b)**. Clearly, then, the tendency for a force to cause a rotation increases with the distance, r, from the axis of rotation to the force. As a result, it is useful to define a quantity called the **torque**, τ, that takes into account both the magnitude of the force, F, *and* the distance from the axis of rotation, r:

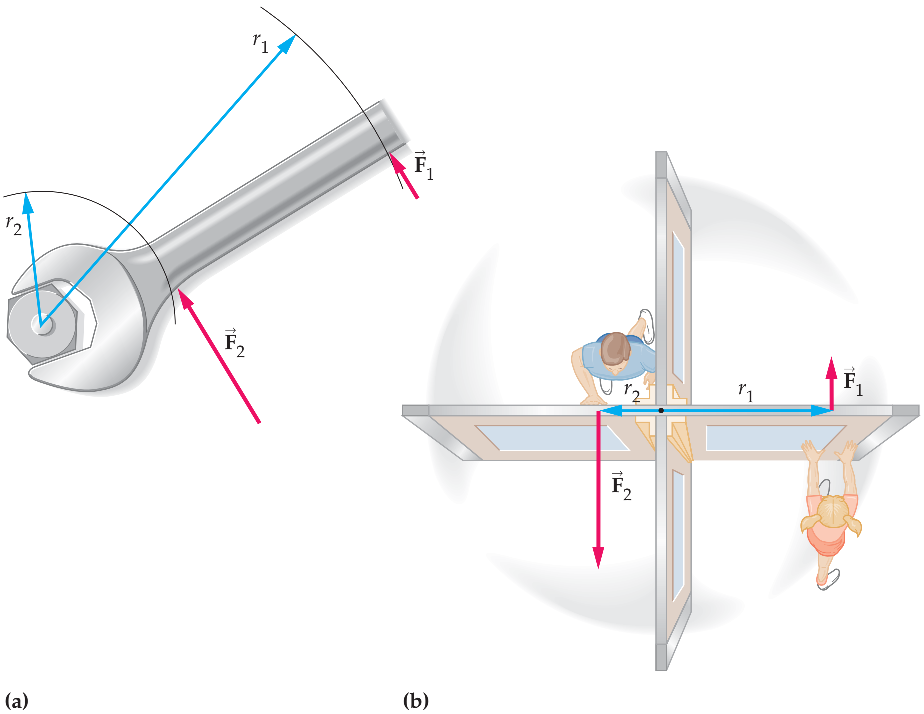
▲ The long handle of this wrench enables the user to produce a large torque without having to exert a very great force.

Definition of Torque, τ, for a Tangential Force

$$\tau = rF \qquad \text{11–1}$$

SI unit: N · m

Note that the torque increases with both the force and the distance.



◀ **FIGURE 11–1 Applying a torque**
(a) When a wrench is used to loosen a nut, less force is required if it is applied far from the nut. **(b)** Similarly, less force is required to open a revolving door if it is applied far from the axis of rotation.

Equation 11–1 is valid only when the applied force is *tangential* to a circle of radius r centered on the axis of rotation, as indicated in Figure 11–1. The more general case is considered later in this section. First, we use Equation 11–1 to determine how much force is needed to open a swinging door, depending on where we apply the force.

EXERCISE 11–1

To open the door in Figure 11–1 (b) a tangential force F is applied at a distance r from the axis of rotation. If the minimum torque required to open the door is 3.1 N · m, what force must be applied if r is **(a)** 0.94 m or **(b)** 0.35 m?

SOLUTION

(a) Setting $\tau = r_1F_1 = 3.1\ \text{N}\cdot\text{m}$, we find that the required force is

$$F_1 = \frac{\tau}{r_1} = \frac{3.1\ \text{N}\cdot\text{m}}{0.94\ \text{m}} = 3.3\ \text{N}$$

PROBLEM-SOLVING NOTE

The Units of Torque

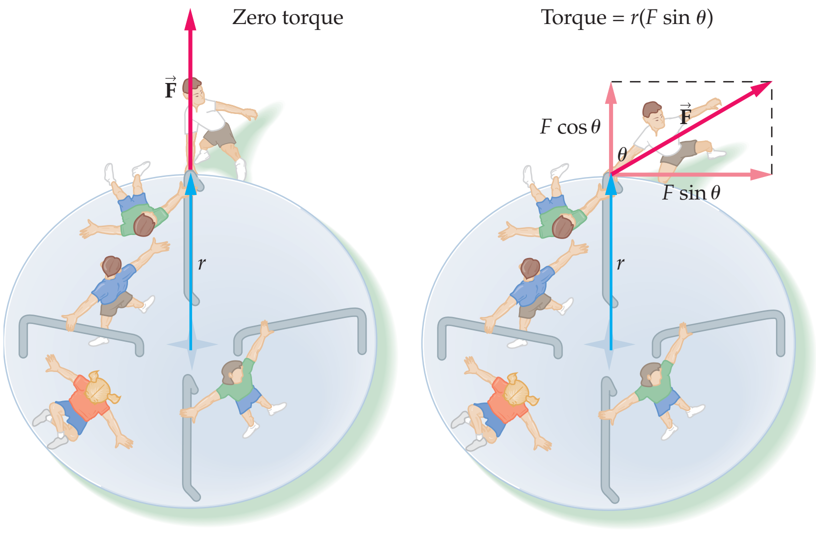
Note that the units of torque are N · m, the same as the units of work. Though their units are the same, torque, τ, and work, W, represent different physical quantities and should not be confused with one another.

(b) Repeat the calculation, this time with $r_2 = 0.35$ m:

$$F_2 = \frac{\tau}{r_2} = \frac{3.1\ \mathrm{N \cdot m}}{0.35\ \mathrm{m}} = 8.9\ \mathrm{N}$$

As expected, the required force is greater when it is applied closer to the hinges.

To this point we have considered tangential forces only. What happens if you exert a force in a direction that is not tangential? Suppose, for example, that you pull on a playground merry-go-round in a direction that is radial—that is, along a line that extends through the axis of rotation—as in **Figure 11–2 (a)**. In this case, your force has no tendency to cause a rotation. Instead, the axle of the merry-go-round simply exerts an equal and opposite force, and the merry-go-round remains at rest. Similarly, if you were to push or pull in a radial direction on a swinging door it would not rotate. We conclude that a *radial force produces zero torque*.



(a) A radial force produces zero torque

(b) Only the tangential component of force causes a torque

▶ **FIGURE 11–2 Only the tangential component of a force causes a torque**

(a) A radial force causes no rotation. In this case, the force $\vec{\mathbf{F}}$ is opposed by an equal and opposite force exerted by the axle of the merry-go-round. The merry-go-round does not rotate. **(b)** A force applied at an angle θ with respect to the radial direction. The radial component of this force, $F \cos \theta$, causes no rotation; the tangential component, $F \sin \theta$, can cause a rotation.

On the other hand, what if your force is at an angle θ relative to a radial line, as shown in **Figure 11–2 (b)**? To analyze this case, we first resolve the force vector $\vec{\mathbf{F}}$ into radial and tangential components. Referring to the figure, we see that the radial component has a magnitude of $F \cos \theta$, and the tangential component has a magnitude of $F \sin \theta$. Because it is the tangential component alone that causes rotation, we define the torque to have a magnitude of $r(F \sin \theta)$. That is,

General Definition of Torque, τ

$$\tau = r(F \sin \theta) \qquad 11\text{–}2$$

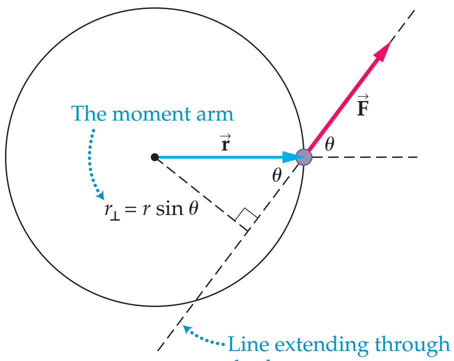
SI units: N · m

(More generally, the torque can be defined as the **cross product** between the vectors $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$; that is, $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$. The cross product is discussed in detail in Appendix A.)

As a quick check, note that a radial force corresponds to $\theta = 0$. In this case, $\tau = r(F \sin 0) = 0$, as expected. If the force is tangential, however, it follows that $\theta = \pi/2$. This gives $\tau = r(F \sin \pi/2) = rF$, in agreement with Equation 11–1.

An equivalent way to define the torque is in terms of the **moment arm**, $r_\perp$. The idea here is to extend a line through the force vector, as in **Figure 11–3**, and then draw a second line from the axis of rotation perpendicular to the line of the force. The perpendicular distance from the axis of rotation to the line of the force is defined to be $r_\perp$. From the figure, we see that

$$r_\perp = r \sin \theta$$



▲ **FIGURE 11–3 The moment arm**

To find the moment arm, $r_\perp$, for a given force, first extend a line through the force vector. This line is sometimes referred to as the "line of action." Next, drop a perpendicular line from the axis of rotation to the line of the force. The perpendicular distance is $r_\perp = r \sin \theta$.

In addition, we note that a simple rearrangement of the torque expression in Equation 11–2 yields

$$\tau = r(F \sin\theta) = (r \sin\theta)F$$

Thus, the torque can be written as the moment arm times the force:

$$\tau = r_{\perp}F \qquad 11\text{–}3$$

Just as a force applied to an object gives rise to a linear acceleration, a torque applied to an object gives rise to an angular acceleration. For example, if a torque acts on an object at rest, the object will begin to rotate; if a torque acts on a rotating object, the object's angular velocity will change. In fact, the greater the torque applied to an object, the greater its angular acceleration, as we shall see in the next section. For this reason, the sign of the torque is determined by the same convention used in Section 10–1 for angular acceleration:

Sign Convention for Torque

By convention, if a torque τ acts alone, then

$\tau > 0$ if the torque causes a counterclockwise angular acceleration
$\tau < 0$ if the torque causes a clockwise angular acceleration

In a system with more than one torque, the sign of each torque is determined by the type of angular acceleration *it alone* would produce. The net torque acting on the system, then, is the sum of each individual torque, taking into account the proper sign. This is illustrated in the following Example.

▲ The net torque on the wheel of this ship is the sum of the torques exerted by the two helmsmen. At the moment pictured, they are both exerting negative torques on the wheel, causing it to rotate in the clockwise direction. This will turn the boat to its left—or, in nautical terms, to port.

PROBLEM-SOLVING NOTE

The Sign of Torques

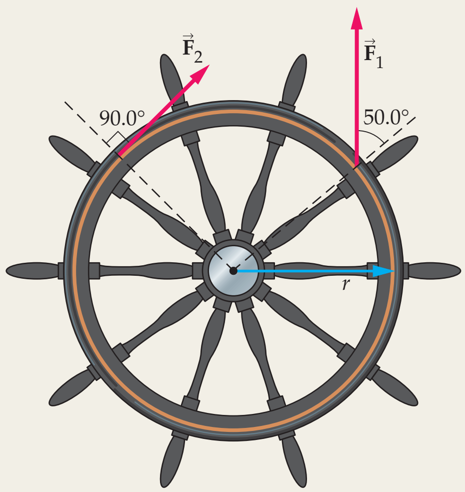
The sign of a torque is determined by the direction of rotation it would cause if it were the only torque acting in the system.

EXAMPLE 11–1 TORQUES TO THE LEFT AND TORQUES TO THE RIGHT

Two helmsmen, in disagreement about which way to turn a ship, exert the forces shown below on a ship's wheel. The wheel has a radius of 0.74 m, and the two forces have the magnitudes $F_1 = 72$ N and $F_2 = 58$ N. Find **(a)** the torque caused by $\vec{\mathbf{F}}_1$ and **(b)** the torque caused by $\vec{\mathbf{F}}_2$. **(c)** In which direction does the wheel turn as a result of these two forces?

PICTURE THE PROBLEM

Our sketch shows that both forces are applied at the distance $r = 0.74$ m from the axis of rotation. However, $F_1 = 72$ N is at an angle of 50.0° relative to the radial direction, whereas $F_2 = 58$ N is tangential, which means that its angle relative to the radial direction is 90.0°.



STRATEGY

For each force, we find the magnitude of the corresponding torque, using $\tau = rF\sin\theta$. As for the signs of the torques, we must consider the angular acceleration each force alone would cause. $\vec{\mathbf{F}}_1$ acting alone would cause the wheel to accelerate counterclockwise, hence its torque is positive. $\vec{\mathbf{F}}_2$ would accelerate the wheel clockwise if it acted alone, hence its torque is negative. If the sum of the two torques is positive, the wheel accelerates counterclockwise; if the sum of the two torques is negative, the wheel accelerates clockwise.

SOLUTION

Part (a)

1. Use Equation 11–2 to calculate the torque due to $\vec{\mathbf{F}}_1$. Recall that this torque is positive:

$$\tau_1 = rF_1 \sin 50.0° = (0.74\text{ m})(72\text{ N})\sin 50.0° = 41\text{ N}\cdot\text{m}$$

Part (b)

2. Similarly, calculate the torque due to $\vec{\mathbf{F}}_2$. Recall that this torque is negative:

$$\tau_2 = -rF_2 \sin 90.0° = -(0.74\text{ m})(58\text{ N}) = -43\text{ N}\cdot\text{m}$$

Part (c)

3. Sum the torques from parts (a) and (b) to find the net torque:

$$\tau_{\text{net}} = \tau_1 + \tau_2 = 41\text{ N}\cdot\text{m} - 43\text{ N}\cdot\text{m} = -2\text{ N}\cdot\text{m}$$

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INSIGHT

Because the net torque is negative, the wheel accelerates clockwise. Thus, even though $\vec{\mathbf{F}}_2$ is the smaller force, it has the greater effect in determining the wheel's direction of acceleration. This is because $\vec{\mathbf{F}}_2$ is applied tangentially, whereas $\vec{\mathbf{F}}_1$ is applied in a direction that is partially radial.

PRACTICE PROBLEM

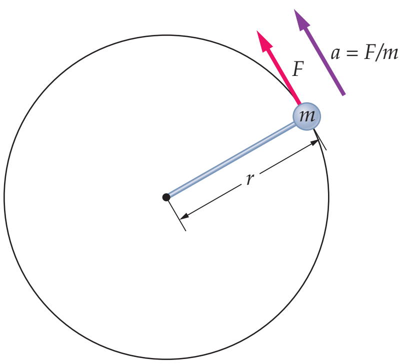
What magnitude of $\vec{\mathbf{F}}_2$ would yield zero net torque on the wheel? [**Answer:** $F_2 = 55$ N]

Some related homework problems: Problem 1, Problem 3

11–2 Torque and Angular Acceleration

In the previous section we indicated that a torque causes a change in the rotational motion of an object. To be more precise, a single torque, τ, acting on an object causes the object to have an angular acceleration, α. In this section we develop the specific relationship between τ and α.

Consider, for example, a small object of mass m connected to an axis of rotation by a light rod of length r, as in **Figure 11–4**. If a tangential force of magnitude F is applied to the mass, it will move with an acceleration given by Newton's second law:



▲ **FIGURE 11–4 Torque and angular acceleration**

A tangential force F applied to a mass m gives it a linear acceleration of magnitude $a = F/m$. The corresponding angular acceleration is $\alpha = \tau/I$, where $\tau = rF$ and $I = mr^2$.

$$a = \frac{F}{m}$$

From Equation 10–14, we know that the linear and angular accelerations are related by

$$\alpha = \frac{a}{r}$$

Combining these results yields the following expression for the angular acceleration:

$$\alpha = \frac{a}{r} = \frac{F}{mr}$$

Finally, multiplying both numerator and denominator by r gives

$$\alpha = \left(\frac{r}{r}\right)\frac{F}{mr} = \frac{rF}{mr^2}$$

Now this last result is rather interesting, since the numerator and denominator have simple interpretations. First, the numerator is the torque, $\tau = rF$, for the case of a tangential force (Equation 11–1). Second, the denominator is the moment of inertia of a single mass m rotating at a radius r; that is, $I = mr^2$. Therefore, we find that

$$\alpha = \frac{rF}{mr^2} = \frac{\tau}{I}$$

or, rewriting slightly,

$$\tau = I\alpha$$

Thus, once we calculate the torque, as described in the previous section, we can find the angular acceleration of a system using $\tau = I\alpha$. Notice that the angular acceleration is directly proportional to the torque, and inversely proportional to the moment of inertia—that is, a large moment of inertia means a small angular acceleration.

Now, the relationship $\tau = I\alpha$ was derived for the special case of a tangential force and a single mass rotating at a radius r. However, the result is completely general. For example, in a system with more than one torque, the relation $\tau = I\alpha$

is replaced with $\tau_{\text{net}} = \Sigma\tau = I\alpha$, where τ_{net} is the net torque acting on the system. This gives us the *rotational* version of Newton's second law:

Newton's Second Law for Rotational Motion

$$\sum \tau = I\alpha \qquad 11\text{–}4$$

If only a single torque acts on a system, we will simply write $\tau = I\alpha$.

EXERCISE 11–2

A light rope wrapped around a disk-shaped pulley is pulled tangentially with a force of 0.53 N. Find the angular acceleration of the pulley, given that its mass is 1.3 kg and its radius is 0.11 m.

SOLUTION

The torque applied to the disk is

$$\tau = rF = (0.11\text{ m})(0.53\text{ N}) = 5.8 \times 10^{-2}\text{ N}\cdot\text{m}$$

Since the pulley is a disk, its moment of inertia is given by

$$I = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(1.3\text{ kg})(0.11\text{ m})^2 = 7.9 \times 10^{-3}\text{ kg}\cdot\text{m}^2$$

Thus, the angular acceleration of the pulley is

$$\alpha = \frac{\tau}{I} = \frac{5.8 \times 10^{-2}\text{ N}\cdot\text{m}}{7.9 \times 10^{-3}\text{ kg}\cdot\text{m}^2} = 7.3\text{ rad/s}^2$$

It is easy to remember the rotational version of Newton's second law, $\Sigma\tau = I\alpha$, by using analogies between rotational and linear quantities. We have already seen that I is the analogue of m, and that α is the analogue of a. Similarly, τ, which causes an angular acceleration, is the analogue of F, which causes a linear acceleration. To summarize:

Linear Quantity	Angular Quantity
m	I
a	α
F	τ

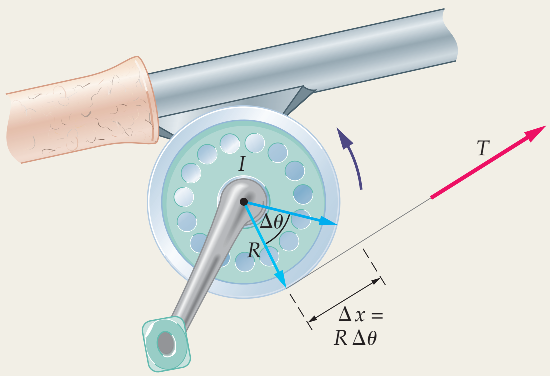
Thus, just as $\Sigma F = ma$ describes linear motion, $\Sigma\tau = I\alpha$ describes rotational motion.

EXAMPLE 11–2 A FISH TAKES THE LINE

A fisherman is dozing when a fish takes the line and pulls it with a tension T. The spool of the fishing reel is at rest initially and rotates without friction (since the fisherman left the drag off) as the fish pulls for a time t. If the radius of the spool is R, and its moment of inertia is I, find **(a)** the angular displacement of the spool, **(b)** the length of line pulled from the spool, and **(c)** the final angular speed of the spool.

PICTURE THE PROBLEM

Our sketch shows the fishing line being pulled tangentially from the spool with a tension T. Because the radius of the spool is R, the torque produced by the line is $\tau = RT$. Also note that as the spool rotates through an angle $\Delta\theta$, the line moves through a linear distance $\Delta x = R\Delta\theta$. Finally, the spool starts at rest, hence $\omega_0 = 0$.



STRATEGY

This is basically an angular kinematics problem, as in Chapter 10, but in this case we must first calculate the angular acceleration using $\alpha = \tau/I$. Once α is known, we can find the angular displacement, $\Delta\theta$, using $\theta = \theta_0 + \omega_0 t + \tfrac{1}{2}\alpha t^2$. Similarly, we can find the angular speed of the spool, ω, using $\omega = \omega_0 + \alpha t$.

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SOLUTION

1. Calculate the torque acting on the spool. Note that $\theta = 90°$, since the pull is tangential. The radius is $r = R$, and the force applied to the reel is the tension in the line, T:

$$\tau = rF \sin\theta = RT \sin 90° = RT$$

2. Using the result just obtained for the torque, find the angular acceleration of the reel:

$$\alpha = \frac{\tau}{I} = \frac{RT}{I}$$

Part (a)

3. Calculate the angular displacement $\Delta\theta = \theta - \theta_0$:

$$\Delta\theta = \theta - \theta_0 = \omega_0 t + \tfrac{1}{2}\alpha t^2 = \tfrac{1}{2}\alpha t^2 = \left(\frac{RT}{2I}\right)t^2$$

Part (b)

4. Calculate the length of line pulled from the spool with $\Delta x = R\,\Delta\theta$:

$$\Delta x = R\,\Delta\theta = \left(\frac{R^2T}{2I}\right)t^2$$

Part (c)

5. Use $\omega = \omega_0 + \alpha t$ to find the final angular speed:

$$\omega = \omega_0 + \alpha t = \left(\frac{RT}{I}\right)t$$

INSIGHT

Note that the final angular speed can also be obtained from the kinematic equation relating angular speed and angular distance; $\omega^2 = {\omega_0}^2 + 2\alpha\,\Delta\theta = 0 + 2(RT/I)(RT/2I)t^2 = (RT/I)^2\,t^2$.

This calculation also applies to other situations in which a "line" is pulled from a "reel." Examples include telephone line or sewing thread pulled from a spool.

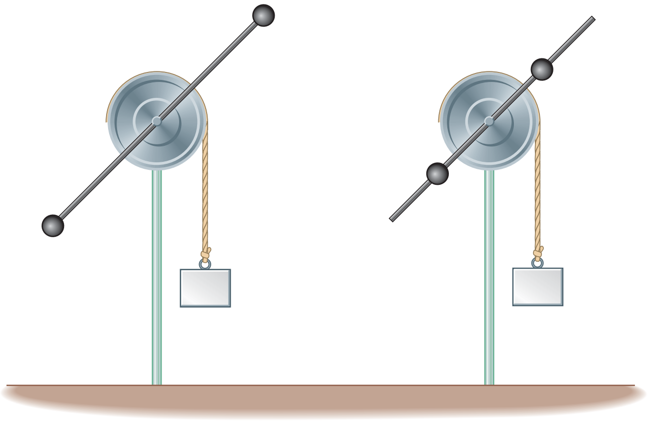
PRACTICE PROBLEM

How fast is the line moving at time t? [**Answer:** $v = R\omega = (R^2T/I)t$]

Some related homework problems: Problem 10, Problem 19

CONCEPTUAL CHECKPOINT 11–1 WHICH BLOCK LANDS FIRST?

The rotating systems shown below differ only in that the two spherical movable masses are positioned either far from the axis of rotation (left), or near the axis of rotation (right). If the hanging blocks are released simultaneously from rest, is it observed that **(a)** the block at left lands first, **(b)** the block at right lands first, or **(c)** both blocks land at the same time?



REASONING AND DISCUSSION

The net external torque, supplied by the hanging blocks, is the same for each of these systems. However, the moment of inertia of the system at right is less than that of the system at left because the movable masses are closer to the axis of rotation. Since the angular acceleration is inversely proportional to the moment of inertia ($\alpha = \tau_{net}/I$), the system at right has the greater angular acceleration, and it wins the race.

ANSWER

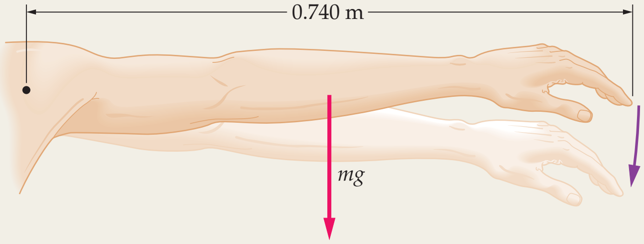
(b) The block at right lands first.

EXAMPLE 11–3 DROP IT

A person holds his outstretched arm at rest in a horizontal position. The mass of the arm is m and its length is 0.740 m. When the person releases his arm, allowing it to drop freely, it begins to rotate about the shoulder joint. Find **(a)** the initial angular acceleration of the arm, and **(b)** the initial linear acceleration of the man's hand. (*Hint:* In calculating the torque, assume the mass of the arm is concentrated at its midpoint. In calculating the angular acceleration, use the moment of inertia of a uniform rod of length L about one end; $I = \frac{1}{3}mL^2$.)

PICTURE THE PROBLEM

The arm is initially horizontal and at rest. When released, it rotates downward about the shoulder joint. The force of gravity, mg, acts at a distance of $(0.740\text{ m})/2 = 0.370\text{ m}$ from the shoulder.



STRATEGY

The angular acceleration, α, can be found using $\tau = I\alpha$. In this case, the initial torque is $\tau = mg(L/2)$, where $L = 0.740$ m, and the moment of inertia is $I = \frac{1}{3}mL^2$.

Once the initial angular acceleration is found, the corresponding linear acceleration is obtained from $a = r\alpha$.

SOLUTION

Part (a)

1. Use $\tau = I\alpha$ to find the angular acceleration, α:

$$\alpha = \frac{\tau}{I}$$

2. Write expressions for the initial torque, τ, and the moment of inertia, I:

$$\tau = mg\frac{L}{2}$$

$$I = \frac{1}{3}mL^2$$

3. Substitute τ and I into the expression for the angular acceleration. Note that the mass of the arm cancels:

$$\alpha = \frac{\tau}{I} = \frac{mgL/2}{mL^2/3} = \frac{3g}{2L}$$

4. Substitute numerical values:

$$\alpha = \frac{3g}{2L} = \frac{3(9.81\text{ m/s}^2)}{2(0.740\text{ m})} = 19.9\text{ rad/s}^2$$

Part (b)

5. Use $a = r\alpha$ to calculate the linear acceleration at the man's hand, a distance $r = L$ from the shoulder:

$$a = L\alpha = L\left(\frac{3g}{2L}\right) = \frac{3}{2}g = 14.7\text{ m/s}^2$$

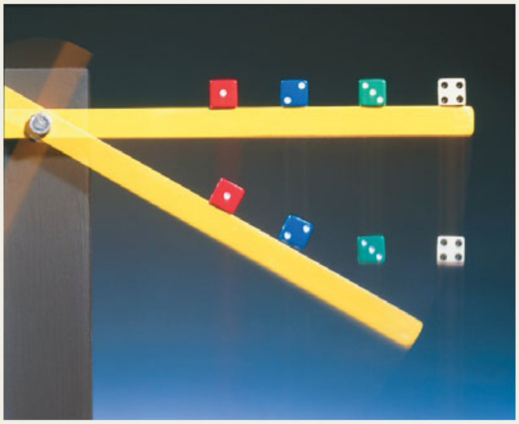
INSIGHT

Note that the linear acceleration of the hand is 1.50 times greater than the acceleration of gravity, regardless of the mass of the arm. This can be demonstrated with the following simple experiment: Hold your arm straight out with a pen resting on your hand. Now, relax your deltoid muscles, and let your arm rotate freely downward about your shoulder joint. Notice that as your arm falls downward, your hand moves more rapidly than the pen, which appears to "lift off" your hand. The pen drops with the acceleration of gravity, which is clearly less than the acceleration of the hand. This effect can be seen in the adjacent photo.

PRACTICE PROBLEM

At what distance from the shoulder is the initial linear acceleration of the arm equal to the acceleration of gravity?
[**Answer:** Set $a = r\alpha$ equal to g. This gives $r = 2L/3 = 0.493$ m.]

Some related homework problems: Problem 13, Problem 15



▲ As a rod of length L rotates freely about one end, points farther from the axle than $2L/3$ have an acceleration greater than g (see the Practice Problem for Example 11–3). Thus, the rod falls out from under the last two dice.

11–3 Zero Torque and Static Equilibrium

The parents of a young boy are supporting him on a long, lightweight plank, as illustrated in **Figure 11–5.** If the mass of the child is m, the upward forces exerted by the parents must sum to mg; that is,

$$F_1 + F_2 = mg$$

This condition ensures that the net force acting on the plank is zero. It *does not*, however, guarantee that the plank remains at rest.

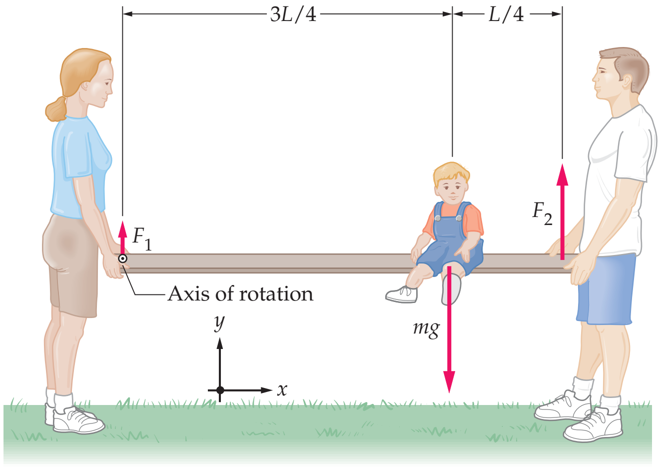


FIGURE 11–5 Forces required for static equilibrium

Two parents support a child on a lightweight plank of length L. For the calculation described in the text, we choose the axis of rotation to be the left end of the plank.

To see why, imagine for a moment that the parent on the right lets go of the plank and that the parent on the left increases her force until it is equal to the weight of the child. In this case, $F_1 = mg$ and $F_2 = 0$, which clearly satisfies the force equation we have just written. Since the right end of the plank is no longer supported, however, it drops toward the ground while the left end rises. In other words, the plank rotates in a clockwise sense. For the plank to remain completely at rest, with no translation or rotation, we must impose the following *two* conditions: First, the net force acting on the plank must be zero, so that there is no translational acceleration. Second, the net torque acting on the plank must also be zero, so that there is no rotational acceleration. If both of these conditions are met, an extended object, like the plank, will remain at rest if it starts at rest. To summarize:

Conditions for Static Equilibrium

For an extended object to be in static equilibrium, the following two conditions must be met:

(i) The net force acting on the object must be zero,

$$\sum F_x = 0, \qquad \sum F_y = 0 \tag{11–5}$$

(ii) The net torque acting on the object must be zero,

$$\sum \tau = 0 \tag{11–6}$$

Note that these two conditions are independent of one another; that is, satisfying one does *not* guarantee that the other is satisfied.

Let's apply these conditions to the plank that supports the child. First, we consider the forces acting on the plank, with upward chosen as the positive direction, as in Figure 11–5. Setting the net force equal to zero yields

$$F_1 + F_2 - mg = 0$$

Clearly, this agrees with the force equation we wrote down earlier.

Next, we apply the torque condition. To do so, we must first choose an axis of rotation. For example, we might take the left end of the plank to be the axis, as in Figure 11–5. With this choice, we see that the force F_1 exerts zero torque, since it acts directly through the axis of rotation. On the other hand, F_2 acts at the far end of the plank, a distance L from the axis. In addition, F_2 would cause a counterclockwise (positive) rotation if it acted alone, as we can see in Figure 11–5. Therefore, the torque due to F_2 is

$$\tau_2 = F_2 L$$

Finally, the weight of the child, mg, acts at a distance of $3L/4$ from the axis, and would cause a clockwise (negative) rotation if it acted alone. Hence, its torque is negative:

$$\tau_{mg} = -mg\left(\tfrac{3}{4}L\right)$$

Setting the net torque equal to zero, then, yields the following condition:

$$F_2 L - mg\left(\tfrac{3}{4}L\right) = 0$$

This torque condition, along with the force condition in $F_1 + F_2 - mg = 0$, can be used to determine the two unknowns, F_1 and F_2. For example, we can begin by canceling L in the torque equation to find F_2:

$$F_2 = \tfrac{3}{4}mg$$

Substituting this result into the force condition gives

$$F_1 + \tfrac{3}{4}mg - mg = 0$$

Therefore, F_1 is

$$F_1 = \tfrac{1}{4}mg$$

These two forces support the plank, *and* keep it from rotating. As one might expect, the force nearest the child is greatest.

Our choice of the left end of the plank as the axis of rotation was completely arbitrary. In fact, if an object is in static equilibrium, the net torque acting on it is zero, regardless of the location of the axis of rotation. Hence, we are free to choose an axis of rotation that is most convenient for a given problem. In general, it is useful to pick the axis to be at the location of one of the unknown forces. This eliminates that force from the torque condition, and simplifies the remaining algebra. We consider an alternative choice for the axis of rotation in the following Active Example.

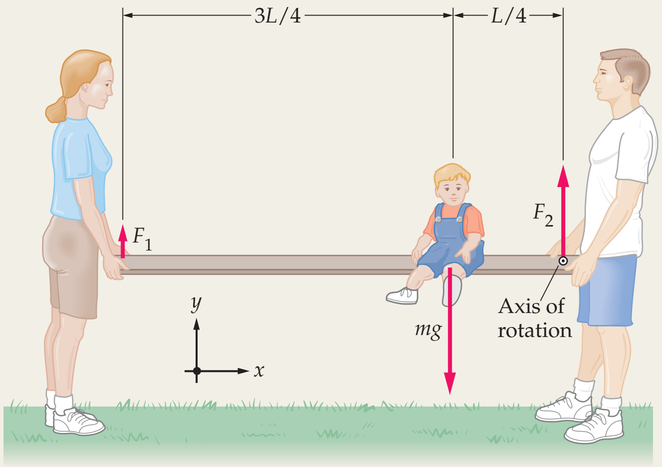
PROBLEM-SOLVING NOTE

Axis of Rotation

Any point in a system may be used as the axis of rotation when calculating torque. It is generally best, however, to choose an axis that gives zero torque for at least one of the unknown forces in the system. Such a choice simplifies the algebra needed to solve for the forces.

ACTIVE EXAMPLE 11–1 FIND THE FORCES: AXIS ON THE RIGHT

A child of mass m is supported on a light plank by his parents, who exert the forces F_1 and F_2 as indicated. Find the forces required to keep the plank in static equilibrium. Use the right end of the plank as the axis of rotation.



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SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the net force acting on the plank equal to zero: $F_1 + F_2 - mg = 0$
2. Set the net torque acting on the plank equal to zero: $-F_1(L) + mg\left(\frac{1}{4}L\right) = 0$
3. Note that the torque condition involves only one of the two unknowns, F_1. Use this condition to solve for F_1: $F_1 = \frac{1}{4}mg$
4. Substitute F_1 into the force condition to solve for F_2: $F_2 = mg - \frac{1}{4}mg = \frac{3}{4}mg$

INSIGHT

As expected, the results are identical to those obtained previously. Note that in this case the torque produced by the child would cause a counterclockwise rotation, hence it is positive. Thus, the magnitude *and* sign of the torque produced by a given force depend on the location chosen for the axis of rotation.

YOUR TURN

Suppose the child moves to a new position, with the result that the force exerted by the father is reduced to $0.60mg$. Did the child move to the left or to the right? How far did the child move?

(Answers to **Your Turn** *problems are given in the back of the book.)*

A third choice for the axis of rotation is considered in Problem 24. As expected, all three choices give the same results.

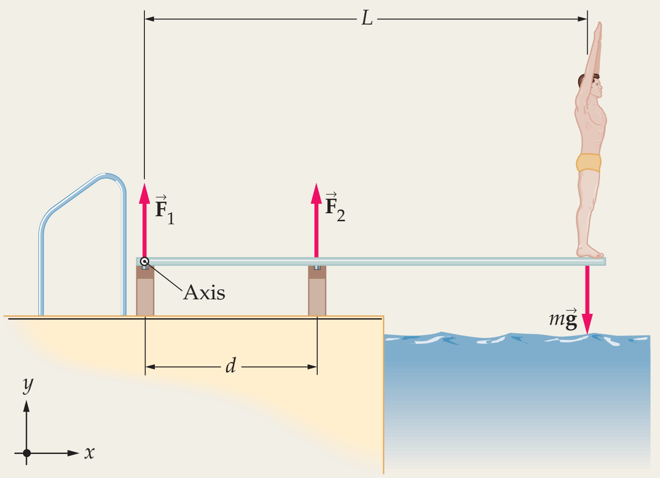
In the next Example, we show that the forces supporting a person or other object sometimes act in different directions. To emphasize the direction of the forces, we solve the Example in terms of the components of the relevant forces.

EXAMPLE 11–4 TAKING THE PLUNGE

A 5.00-m-long diving board of negligible mass is supported by two pillars. One pillar is at the left end of the diving board, as shown below; the other is 1.50 m away. Find the forces exerted by the pillars when a 90.0-kg diver stands at the far end of the board.

PICTURE THE PROBLEM

We choose upward to be the positive direction for the forces. When calculating torques, we use the left end of the diving board as the axis of rotation. Note that $\vec{\mathbf{F}}_2$ would cause a counterclockwise rotation if it acted alone, so its torque is positive. On the other hand, $m\vec{\mathbf{g}}$ would cause a clockwise rotation, so its torque is negative. Finally, $\vec{\mathbf{F}}_2$ acts at a distance d from the axis of rotation, and $m\vec{\mathbf{g}}$ acts at a distance L.



STRATEGY

As usual in static equilibrium problems, we use the conditions of (i) zero net force and (ii) zero net torque to determine the unknown forces, $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$. In this system all forces act in the positive or negative y direction; thus we need only set the net y component of force equal to zero.

SOLUTION

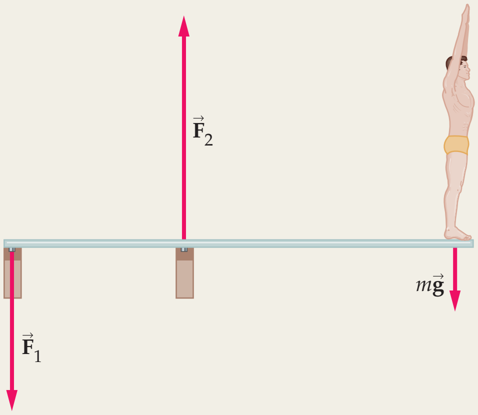
1. Set the net y component of force acting on the diving board equal to zero: $\sum F_y = F_{1,y} + F_{2,y} - mg = 0$
2. Calculate the torque due to each force, using the left end of the board as the axis of rotation. Note that each force is at right angles to the radius and that $\vec{\mathbf{F}}_1$ goes directly through the axis of rotation:
$$\tau_1 = F_{1,y}(0) = 0$$
$$\tau_2 = F_{2,y}(d)$$
$$\tau_3 = -mg(L)$$
3. Set the net torque acting on the diving board equal to zero: $\sum \tau = F_{1,y}(0) + F_{2,y}(d) - mg(L) = 0$
4. Solve the torque equation for the force $F_{2,y}$:
$$F_{2,y} = mg(L/d)$$
$$= (90.0\text{ kg})(9.81\text{ m/s}^2)(5.00\text{ m}/1.50\text{ m}) = 2940\text{ N}$$

5. Use the force equation to determine $F_{1,y}$:

$$F_{1,y} = mg - F_{2,y} = (90.0\ \text{kg})(9.81\ \text{m/s}^2) - 2940\ \text{N} = -2060\ \text{N}$$

INSIGHT

The first point to notice about our solution is that $F_{1,y}$ is negative, which means that $\vec{\mathbf{F}}_1$ is actually directed *downward*, as shown to the right. To see why, imagine for a moment that the board is no longer connected to the first pillar. In this case, the board would rotate clockwise about the second pillar, and the left end of the board would move upward. Thus, a downward force is required on the left end of the board to hold it in place.



The second point is that both pillars exert forces with magnitudes that are considerably larger than the diver's weight, $mg = 883$ N. In particular, the first pillar must pull downward with a force of $2.33mg$, while the second pillar pushes upward with a force of $2.33mg + mg = 3.33mg$. This is not unusual. In fact, it is common for the forces in a structure, such as a bridge, a building, or the human body, to be much greater than the weight it supports.

PRACTICE PROBLEM

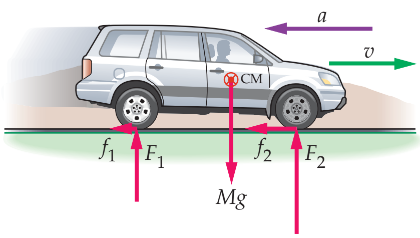
Find the forces exerted by the pillars when the diver is 1.00 m from the right end. [**Answer:** $F_{1,y} = -1470$ N, $F_{2,y} = 2350$ N]

Some related homework problems: Problem 26, Problem 32

To this point we have ignored the mass of the plank holding the child and the diving board holding the swimmer, since they were described as lightweight. If we want to consider the torque exerted by an extended object of finite mass, however, we can simply treat it as if all its mass were concentrated at its center of mass, as was done in similar situations in Section 9–7. We consider such a system in the next Active Example.

REAL-WORLD PHYSICS

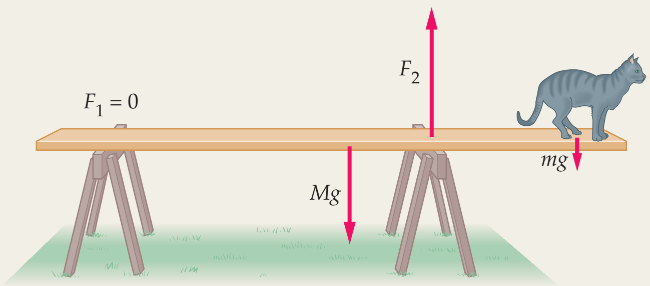
Applying the brakes



▲ As the brakes are applied on this SUV, rotational equilibrium demands that the normal forces exerted on the front tires be greater than the normal forces exerted on the rear tires—which is why braking cars are "nose down" during a rapid stop. For this reason, many cars use disk brakes for the front wheels and less powerful drum brakes for the rear wheels. As the disk brakes wear, they tend to coat the front wheels with dust from the brake pads, which give the front wheels a characteristic "dirty" look.

ACTIVE EXAMPLE 11–2 WALKING THE PLANK: FIND THE MASS

A cat walks along a uniform plank that is 4.00 m long and has a mass of 7.00 kg. The plank is supported by two sawhorses, one 0.440 m from the left end of the board and the other 1.50 m from its right end. When the cat reaches the right end, the plank just begins to tip. What is the mass of the cat?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Since the board is just beginning to tip, there is no weight on the left sawhorse: $F_1 = 0$

2. Calculate the torque about the right sawhorse: $Mg(0.500\ \text{m}) - mg(1.50\ \text{m}) = 0$

3. Solve the torque equation for the mass of the cat, m: $m = 0.333M = 2.33$ kg

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INSIGHT

Note that we did not include a torque for the left sawhorse, since F_1 is zero. As an exercise, you might try repeating the calculation with the axis of rotation at the left sawhorse, or at the center of mass of the plank.

YOUR TURN

Write both the zero force and zero torque conditions for the case where the axis of rotation is at the left sawhorse.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Forces with Both Vertical and Horizontal Components

Note that all of the previous examples have dealt with forces that point either directly upward or directly downward. We now consider a more general situation, where forces may have both vertical and horizontal components. For example, consider the wall-mounted lamp (sconce) shown in **Figure 11–6**. The sconce consists of a light curved rod that is bolted to the wall at its lower end. Suspended from the upper end of the rod, a horizontal distance H from the wall, is the lamp of mass m. The rod is also connected to the wall by a horizontal wire a vertical distance V above the bottom of the rod.

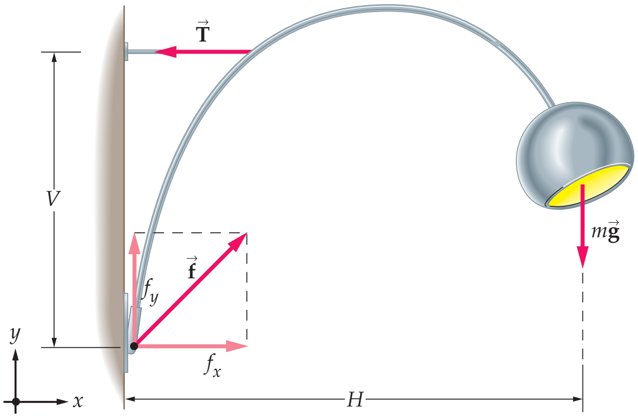


FIGURE 11–6 A lamp in static equilibrium

A wall-mounted lamp of mass m is suspended from a light curved rod. The bottom of the rod is bolted to the wall. The rod is also connected to the wall by a horizontal wire a vertical distance V above the bottom of the rod.

Now, suppose we are designing this sconce to be placed in the lobby of a building on campus. To ensure its structural stability, we would like to know the tension T the wire must exert and the vertical and horizontal components of the force $\vec{\mathbf{f}}$ that must be exerted by the bolt on the rod. This information will be important in deciding on the type of wire and bolt to be used in the construction.

To find these forces, we apply the same conditions as before: the net force and the net torque must be zero. In this case, however, forces may have both horizontal and vertical components. Thus, the condition of zero net force is really two separate conditions: (i) zero net force in the horizontal direction; and (ii) zero net force in the vertical direction. These two conditions plus (iii) zero net torque, allow for a full solution of the problem.

We begin with the torque condition. A convenient choice for the axis of rotation is the bottom end of the rod, since this eliminates one of the unknown forces ($\vec{\mathbf{f}}$). With this choice we can readily calculate the torques acting on the rod by using the moment arm expression for the torque, $\tau = r_{\perp}F$ (Equation 11–3). We find

$$\sum \tau = T(V) - mg(H) = 0$$

This relation can be solved immediately for the tension, giving

$$T = mg(H/V)$$

Note that the tension is increased if the wire is connected closer to the bottom of the rod; that is, if V is reduced.

Next, we apply the force conditions. First, we sum the y components of all the forces and set the sum equal to zero:

$$\sum F_y = f_y - mg = 0$$

Thus, the vertical component of the force exerted by the bolt simply supports the weight of the lamp:

$$f_y = mg$$

Finally, we sum the x components of the forces and set that sum equal to zero:

$$\sum F_x = f_x - T = 0$$

Clearly, the x component of the force exerted by the bolt is of the same magnitude as the tension, but it points in the opposite direction:

$$f_x = T = mg(H/V)$$

The bolt, then, pushes upward on the rod to support the lamp, and at the same time it pushes to the right to keep the rod from rotating.

For example, suppose the lamp in Figure 11–6 has a mass of 2.00 kg, and that $V = 12.0$ cm and $H = 15.0$ cm. In this case, we find the following forces:

$$T = mg(H/V) = (2.00 \text{ kg})(9.81 \text{ m/s}^2)(15.0 \text{ cm})/(12.0 \text{ cm}) = 24.5 \text{ N}$$
$$f_x = T = 24.5 \text{ N}$$
$$f_y = mg = (2.00 \text{ kg})(9.81 \text{ m/s}^2) = 19.6 \text{ N}$$

Note that f_x and T are greater than the weight, mg, of the lamp. Just as we found with the diving board in Example 11–4, the forces required of structural elements can be greater than the weight of the object to be supported—an important consideration when designing a structure like a bridge, an airplane, or a sconce. The same effect occurs in the human body. We find in Problem 25, for example, that the force exerted by the biceps to support a baseball in the hand is several times larger than the baseball's weight. Similar conclusions apply to muscles throughout the body.

In Example 11–5 we consider another system in which forces have both vertical and horizontal components.

▲ The chains that support this sign maintain it in a state of translational and rotational equilibrium. The forces in the chains are most easily analyzed by resolving them into vertical and horizontal components and applying the conditions for equilibrium. In particular, the net vertical force, the net horizontal force, and the net torque must all be zero.

REAL-WORLD PHYSICS

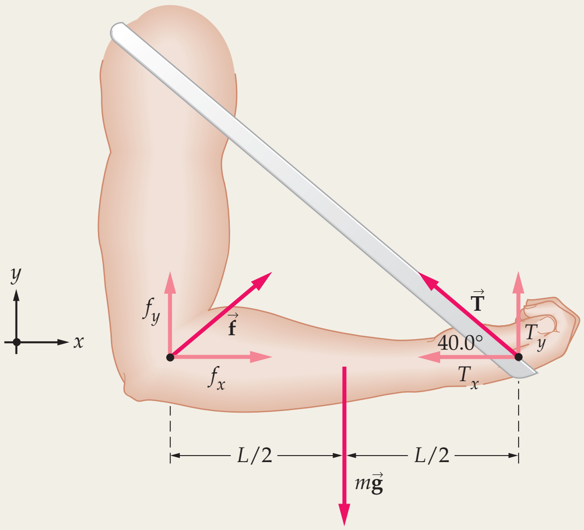
Forces required for structural stability

EXAMPLE 11–5 ARM IN A SLING

A hiker who has broken his forearm rigs a temporary sling using a cord stretching from his shoulder to his hand. The cord holds the forearm level and makes an angle of 40.0° with the horizontal where it attaches to the hand. Considering the forearm and hand to be uniform, with a total mass of 1.30 kg and a length of 0.300 m, find **(a)** the tension in the cord and **(b)** the horizontal and vertical components of the force, $\vec{\mathbf{f}}$, exerted by the humerus (the bone of the upper arm) on the radius and ulna (the bones of the forearm).

PICTURE THE PROBLEM

In our sketch, we use the typical conventions for the positive x and y directions. In addition, since the forearm and hand are assumed to be a uniform object, we indicate the weight mg as acting at its center. The length of the forearm and hand is $L = 0.300$ m. Finally, two other forces act on the forearm: (i) the tension in the cord, $\vec{\mathbf{T}}$, at an angle of 40.0° above the negative x axis, and (ii) the force $\vec{\mathbf{f}}$ exerted at the elbow joint.



STRATEGY

In this system there are three unknowns: T, f_x, and f_y. These unknowns can be determined using the following three conditions: (i) net torque equals zero; (ii) net x component of force equals zero; and (iii) net y component of force equals zero.

We start with the torque condition, using the elbow joint as the axis of rotation. As we shall see, this choice of axis eliminates f, and gives a direct solution for the tension T. Next, we use T and the two force conditions to determine f_x and f_y.

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SOLUTION

Part (a)

1. Calculate the torque about the elbow joint. Note that f causes zero torque, mg causes a negative torque, and the vertical component of T causes a positive torque. The horizontal component of T produces no torque, since it is on a line with the axis:

$$\sum \tau = (T \sin 40.0°)L - mg(L/2) = 0$$

2. Solve the torque condition for the tension, T:

$$T = \frac{mg}{2 \sin 40.0°} = \frac{(1.30\ \text{kg})(9.81\ \text{m/s}^2)}{2 \sin 40.0°} = 9.92\ \text{N}$$

Part (b)

3. Set the sum of the x components of force equal to zero, and solve for f_x:

$$\sum F_x = f_x - T \cos 40.0° = 0$$
$$f_x = T \cos 40.0° = (9.92\ \text{N}) \cos 40.0° = 7.60\ \text{N}$$

4. Set the sum of the y components of force equal to zero, and solve for f_y:

$$\sum F_y = f_y - mg + T \sin 40.0° = 0$$
$$f_y = mg - T \sin 40.0°$$
$$= (1.30\ \text{kg})(9.81\ \text{m/s}^2) - (9.92\ \text{N}) \sin 40.0° = 6.38\ \text{N}$$

INSIGHT

It is not necessary to determine T_x and T_y separately, since we know the direction of the cord. In particular, it is clear from our sketch that the components of $\vec{\mathbf{T}}$ are $T_x = -T \cos 40.0° = -7.60$ N and $T_y = T \sin 40.0° = 6.38$ N.

Did you notice that $\vec{\mathbf{f}}$ is at an angle of 40.0° with respect to the positive x axis, the same angle that $\vec{\mathbf{T}}$ makes with the negative x axis? The reason for this symmetry, of course, is that mg acts at the center of the forearm. If mg were to act closer to the elbow, for example, $\vec{\mathbf{f}}$ would make a larger angle with the horizontal, as we see in the following Practice Problem.

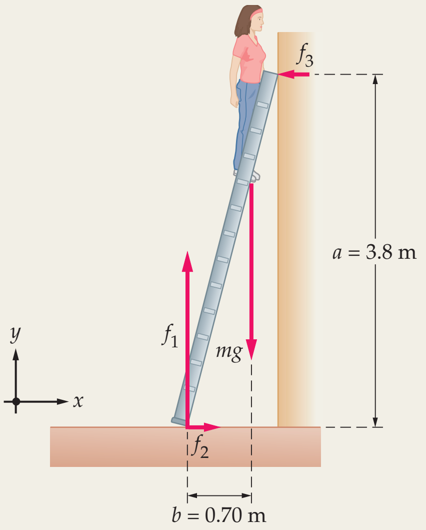
PRACTICE PROBLEM

Suppose the forearm and hand are nonuniform, and that the center of mass is located at a distance of $L/4$ from the elbow joint. What are T, f_x, and f_y in this case? [**Answer:** $T = 4.96$ N, $f_x = 3.80$ N, $f_y = 9.56$ N. In this case, $\vec{\mathbf{f}}$ makes an angle of 68.3° with the horizontal.]

Some related homework problems: Problem 33, Problem 94

ACTIVE EXAMPLE 11–3 DON'T WALK UNDER THE LADDER: FIND THE FORCES

An 85-kg person stands on a lightweight ladder, as shown. The floor is rough; hence, it exerts both a normal force, f_1, and a frictional force, f_2, on the ladder. The wall, on the other hand, is frictionless; it exerts only a normal force, f_3. Using the dimensions given in the figure, find the magnitudes of f_1, f_2, and f_3.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the net torque acting on the ladder equal to zero. Use the bottom of the ladder as the axis: $f_3(a) - mg(b) = 0$
2. Solve for f_3: $f_3 = mg(b/a) = 150$ N
3. Sum the x components of force and set equal to zero: $f_2 - f_3 = 0$
4. Solve for f_2: $f_2 = f_3 = 150$ N
5. Sum the y components of force and set equal to zero: $f_1 - mg = 0$
6. Solve for f_1: $f_1 = mg = 830$ N

INSIGHT

If the floor is quite smooth, the ladder might slip—it depends on whether the coefficient of static friction is great enough to provide the needed force $f_2 = 150$ N. In this case, the normal force exerted by the floor is $N = f_1 = 830$ N. Therefore, if the coefficient of static friction is greater than 0.18 [since $0.18(830\ \text{N}) = 150$ N], the ladder will stay put. Ladders often have rubberized pads on the bottom in order to increase the static friction, and hence increase the safety of the ladder.

YOUR TURN

Write both the zero force and zero torque conditions for the case where the axis of rotation is at the top of the ladder.

(Answers to **Your Turn** *problems are given in the back of the book.)*

11–4 Center of Mass and Balance

Suppose you decide to construct a mobile. To begin, you tie a thread to a light rod, as in **Figure 11–7**. Note that the rod extends a distance x_1 to the left of the thread and a distance x_2 to the right. At the left end of the rod you attach an object of mass m_1. What mass, m_2, should be attached to the right end if the rod is to be balanced?

From the discussions in the previous sections, it is clear that if the rod is to be in static equilibrium (balanced), the net torque acting on it must be zero. Taking the point where the thread is tied to the rod as the axis of rotation, this zero-torque condition can be written as:

$$m_1 g(x_1) - m_2 g(x_2) = 0$$

Canceling g and rearranging, we find

$$m_1 x_1 = m_2 x_2 \qquad \text{11–7}$$

This gives the following result for m_2:

$$m_2 = m_1(x_1/x_2)$$

For example, if $x_2 = 2x_1$, it follows that m_2 should be one-half of m_1.

Let's now consider a slightly different question: Where is the center of mass of m_1 and m_2? Choosing the origin of the x axis to be at the location of the thread, as indicated in Figure 11–7, we can use the definition of the center of mass, Equation 9–13, to find x_{cm}:

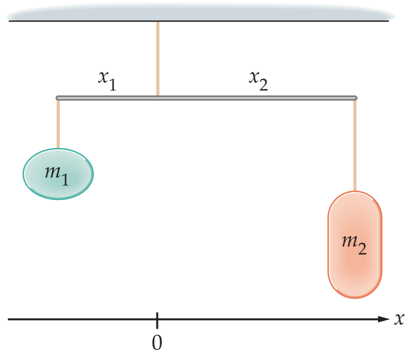
$$x_{\text{cm}} = \frac{m_1(-x_1) + m_2(x_2)}{m_1 + m_2} = -\left(\frac{m_1 x_1 - m_2 x_2}{m_1 + m_2}\right)$$

Referring to the zero-torque condition in Equation 11–7, we see that $m_1 x_1 - m_2 x_2 = 0$; hence the center of mass is at the origin:

$$x_{\text{cm}} = 0$$

This is precisely where the string is attached. We conclude, then, that the rod balances when the center of mass is directly below the point from which the rod is suspended. This is a general result.

Let's apply this result to the case of the mobile shown in the next Example.

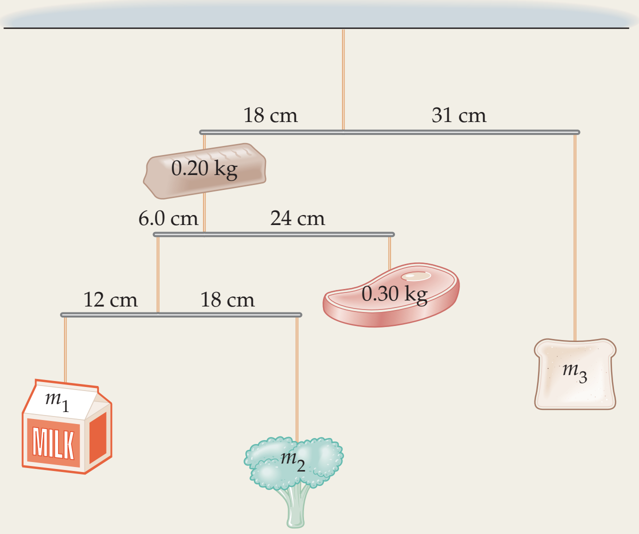


▲ **FIGURE 11–7 Zero torque and balance**
One section of a mobile. The rod is balanced when the net torque acting on it is zero. This is equivalent to having the center of mass directly under the suspension point.

EXAMPLE 11–6 A WELL-BALANCED MEAL

As a grade-school project, students construct a mobile representing some of the major food groups. Their completed artwork is shown below. Find the masses m_1, m_2, and m_3 that are required for a perfectly balanced mobile. Assume the strings and the horizontal rods have negligible mass.

PICTURE THE PROBLEM
The dimensions of the horizontal rods, and the values of the given masses, are indicated in our sketch. Note that each rod is balanced at its suspension point.



STRATEGY
We can find all three unknown masses by repeatedly applying the condition for balance, $m_1 x_1 = m_2 x_2$.

First, we apply the balance condition to m_1 and m_2, with the distances $x_1 = 12$ cm and $x_2 = 18$ cm. This gives a relation between m_1 and m_2.

To get a second relation between m_1 and m_2, we apply the balance condition again at the next higher level of the mobile. That is, the mass $(m_1 + m_2)$ at the distance 6.0 cm must balance the mass 0.30 kg at the distance 24 cm. These two conditions determine m_1 and m_2.

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To find m_3 we again apply the balance condition, this time with the mass $(m_1 + m_2 + 0.30\text{ kg} + 0.20\text{ kg})$ at the distance 18 cm, and the mass m_3 at the distance 31 cm.

SOLUTION

1. Apply the balance condition to m_1 and m_2:
$$m_1(12\text{ cm}) = m_2(18\text{ cm})$$
$$m_1 = (1.5)m_2$$
2. Apply the balance condition to the next level up in the mobile. Solve for the sum, $m_1 + m_2$:
$$(m_1 + m_2)(6.0\text{ cm}) = (0.30\text{ kg})(24\text{ cm})$$
$$m_1 + m_2 = \frac{(0.30\text{ kg})(24\text{ cm})}{6.0\text{ cm}} = 1.2\text{ kg}$$
3. Substitute $m_1 = (1.5)m_2$ into $m_1 + m_2 = 1.2\text{ kg}$ to find m_2:
$$(1.5)m_2 + m_2 = (2.5)m_2 = 1.2\text{ kg}$$
$$m_2 = 1.2\text{ kg}/2.5 = 0.48\text{ kg}$$
4. Use $m_1 = (1.5)m_2$ to find m_1:
$$m_1 = (1.5)m_2 = (1.5)0.48\text{ kg} = 0.72\text{ kg}$$
5. Apply the balance condition to the top level of the mobile:
$$(0.72\text{ kg} + 0.48\text{ kg} + 0.30\text{ kg} + 0.20\text{ kg})(18\text{ cm}) = m_3(31\text{ cm})$$
6. Solve for m_3:
$$m_3 = \frac{(1.70\text{ kg})(18\text{ cm})}{31\text{ cm}} = 0.99\text{ kg}$$

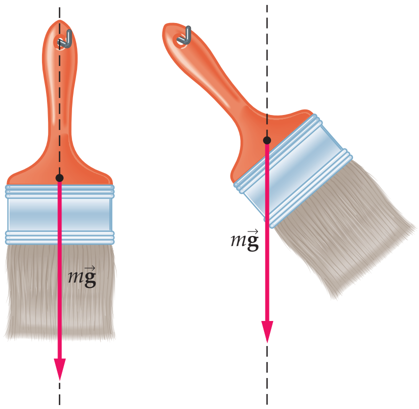
INSIGHT

With the values for m_1, m_2, and m_3 found above, the mobile balances at every level. In fact, the center of mass of the *entire* mobile is directly below the point where the uppermost string attaches to the ceiling.

PRACTICE PROBLEM

Find m_1, m_2, and m_3 if the 0.30-kg mass is replaced with a 0.40-kg mass. [**Answer:** $m_1 = 0.96\text{ kg}$, $m_2 = 0.64\text{ kg}$, $m_3 = 1.3\text{ kg}$]

Some related homework problems: Problem 43, Problem 45



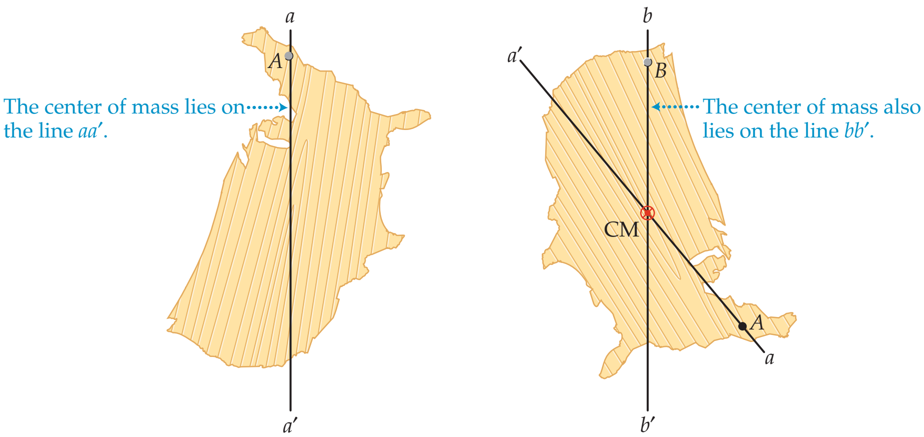
(a) Zero torque **(b)** Nonzero torque

▲ **FIGURE 11–8 Equilibrium of a suspended object**

(a) If an object's center of mass is directly below the suspension point, its weight creates zero torque and the object is in equilibrium. **(b)** When an object is rotated, so that the center of mass is no longer directly below the suspension point, the object's weight creates a torque. The torque tends to rotate the object to bring the center of mass under the suspension point.

In general, if you allow an arbitrarily shaped object to hang freely, its center of mass is directly below the suspension point. To see why, note that when the center of mass is directly below the suspension point, the torque due to gravity is zero, since the force of gravity extends right through the axis of rotation. This is shown in **Figure 11–8 (a)**. If the object is rotated slightly, as in **Figure 11–8 (b)**, the force of gravity is not in line with the axis of rotation—hence gravity produces a torque. This torque tends to rotate the object, bringing the center of mass back under the suspension point.

For example, suppose you cut a piece of wood into the shape of the continental United States, as shown in **Figure 11–9**, drill a small hole in it, and hang it from the



▲ **FIGURE 11–9 The geometric center of the United States**

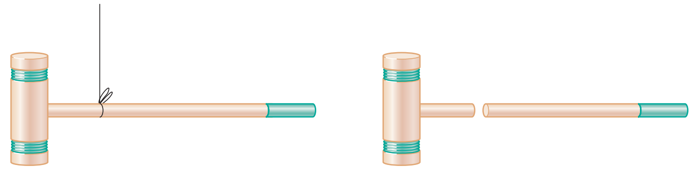
To find the center of mass of an irregularly shaped object, such as a wooden model of the continental United States, suspend it from two or more points. The center of mass lies on a vertical line extending downward from the suspension point. The intersection of these vertical lines gives the precise location of the center of mass.

point A. The result is that the center of mass lies somewhere on the line aa'. Similarly, if a second hole is drilled at point B, we find that the center of mass lies somewhere on the line bb'. The only point that is on both the line aa' *and* the line bb' is the point CM, near Smith Center, Kansas, which marks the location of the center of mass.

▲ In this scene from the movie *Mission Impossible*, Tom Cruise is attempting to download top-secret computer files without setting off the elaborate security system in the room. To accomplish this nearly impossible mission, he is suspended from the ceiling, since touching the floor would immediately give away his presence. To remain in equilibrium above the floor as he works, he must carefully adjust the position of his arms and legs to keep his center of mass directly below the suspension point.

CONCEPTUAL CHECKPOINT 11–2 COMPARE THE MASSES

A croquet mallet balances when suspended from its center of mass, as indicated in the drawing at left. If you cut the mallet in two at its center of mass, as in the drawing at right, how do the masses of the two pieces compare? **(a)** The masses are equal; **(b)** the piece with the head of the mallet has the greater mass; or **(c)** the piece with the head of the mallet has the smaller mass.



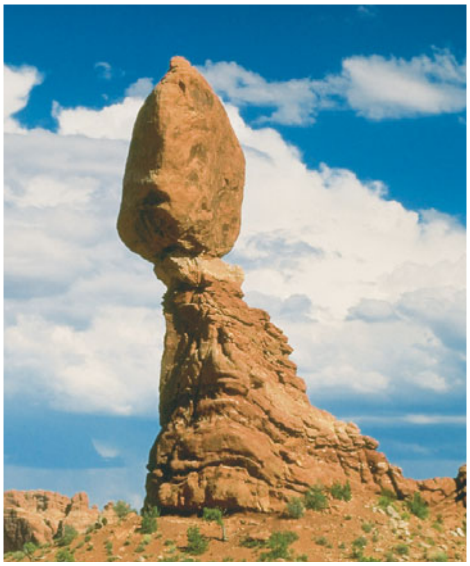
REASONING AND DISCUSSION

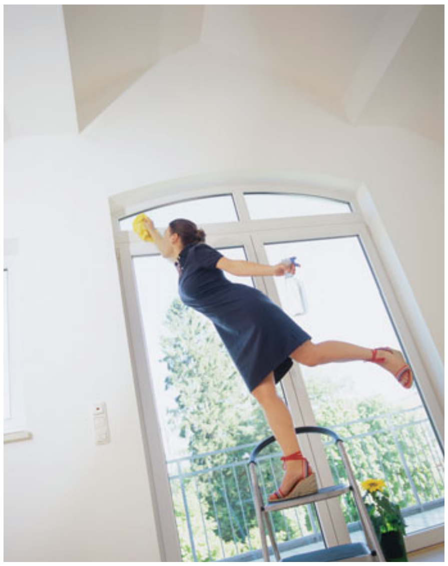
The mallet balances because the torques due to the two pieces are of equal magnitude. The piece with the head of the mallet extends a smaller distance from the point of suspension than does the other piece, hence its mass must be greater; that is, a large mass at a small distance creates the same torque as a small mass at a large distance.

ANSWER

(b) The piece with the head of the mallet has the greater mass.

Similar considerations apply to an object that is at rest on a surface, as opposed to being suspended from a point. In such a case, the object is in equilibrium as long as its center of mass is directly above the base on which it is supported. For example, when you stand upright with normal posture your feet provide a base of support, and your center of mass is above a point roughly halfway between your feet. If you lift your right foot from the floor—without changing your posture—you will begin to lose your balance and tip over. The reason is that your center of mass is no longer above the base of support, which is now your left foot. To balance on your left foot, you must lean slightly in that direction so as to position your center of mass directly above the foot. This principle applies to everything from a performer in a high-wire act to one of the "balancing rocks" that are a familiar sight in the desert Southwest. In Problem 44 we apply this condition for stability to a stack of books on the edge of a table.





◀ (Left) Although it looks precarious, this rock in Arches National Park, Utah, has probably been balancing above the desert for many thousands of years. It will remain secure on its perch as long as its center of mass lies above its base of support. (Right) Although her knowledge may be based more on practical experience than on physics, this woman knows exactly what she must do to keep from falling. By extending one leg backward as she leans forward, she keeps her center of mass safely positioned over the foot that supports her.

11–5 Dynamic Applications of Torque

In this section we focus on applications of Newton's second law for rotation. For example, consider a disk-shaped pulley of radius R and mass M with a string wrapped around its circumference, as in **Figure 11–10 (a)**. Hanging from the string is a mass m. When the mass is released, it accelerates downward and the pulley begins to rotate. If the pulley rotates without friction, and the string unwraps without slipping, what are the acceleration of the mass and the tension in the string?

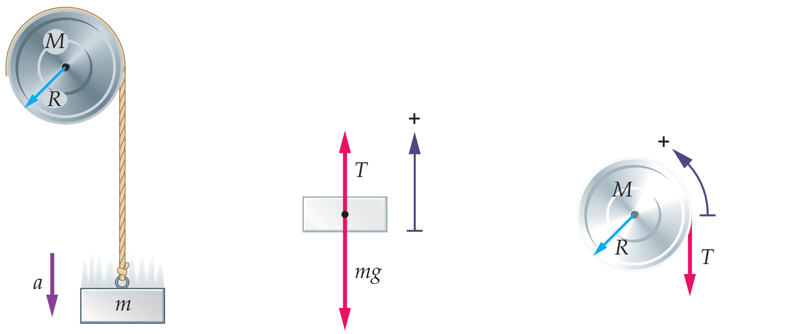


FIGURE 11–10 A mass suspended from a pulley

A mass m hangs from a string wrapped around the circumference of a disk-shaped pulley of radius R and mass M. When the mass is released, it accelerates downward. Positive directions of motion for the system are shown in parts (b) and (c). In part (c), the weight of the pulley acts downward at its center, and the axle exerts an upward force equal in magnitude to the weight of the pulley plus the tension in the string. Of the three forces acting on the pulley, only the tension in the string produces a torque about the axle.

At first it may seem that since the pulley rotates freely, the mass will simply fall with the acceleration of gravity. But remember, the pulley has a nonzero moment of inertia, $I > 0$, which means that it resists any change in its rotational motion. In order for the pulley to rotate, the string must pull downward on it. This means that the string also pulls upward on the mass m with a tension T. As a result, the net downward force on m is less than mg, and thus its acceleration is less than g.

To solve for the acceleration of the mass, we must apply Newton's second law to both the linear motion of the mass *and* the rotational motion of the pulley. The first step is to define a consistent choice of positive directions for the two motions. In Figure 11–10 (a) we note that when the pulley rotates counterclockwise, the mass moves upward. Thus, we choose counterclockwise to be positive for the pulley and upward to be positive for the mass.

With our positive directions established, we proceed to apply Newton's second law. Referring to the free-body diagram for the mass, shown in **Figure 11–10 (b)**, we see that

$$T - mg = ma \qquad 11\text{–}8$$

Similarly, the free-body diagram for the pulley is shown in **Figure 11–10 (c)**. Note that the tension in the string, T, exerts a tangential force on the pulley at a distance R from the axis of rotation. This produces a torque of magnitude TR. Since the tension tends to cause a clockwise rotation, it follows that the torque is negative; thus, $\tau = -TR$. As a result, Newton's second law for the pulley gives

$$-TR = I\alpha \qquad 11\text{–}9$$

Now, these two statements of Newton's second law are related by the fact that the string unwraps without slipping. As was discussed in Chapter 10, when a string unwraps without slipping, the angular and linear accelerations are related by

$$\alpha = \frac{a}{R}$$

Using this relation in Equation 11–9 we have

$$-TR = I\frac{a}{R}$$

or, dividing by R,

$$T = -I\frac{a}{R^2}$$

Substituting this result into Equation 11–8 yields

$$-I\frac{a}{R^2} - mg = ma$$

Finally, dividing by m and rearranging yields the acceleration, a:

$$a = -\frac{g}{\left(1 + \dfrac{I}{mR^2}\right)} \qquad \text{11–10}$$

Let's briefly check our solution for a. First, note that a is negative. This is to be expected, since the mass accelerates downward, which is the negative direction. Second, if the moment of inertia were zero, $I = 0$, or if the mass m were infinite, $m \to \infty$, the mass would fall with the acceleration of gravity, $a = -g$. When I is greater than zero and m is finite, however, the acceleration of the mass has a magnitude less than g. In fact, in the limit of an infinite moment of inertia, $I \to \infty$, the acceleration vanishes—the mass is simply unable to cause the pulley to rotate in this case.

The next Example presents another system in which Newton's laws are used to relate linear and rotational motions.

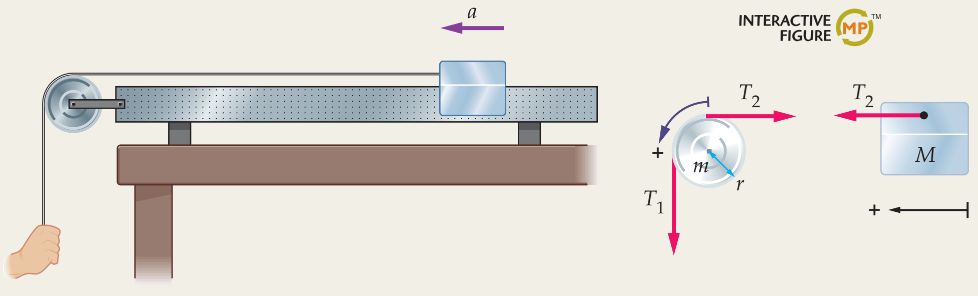
EXAMPLE 11–7 THE PULLEY MATTERS

A 0.31-kg cart on a horizontal air track is attached to a string. The string passes over a disk-shaped pulley of mass 0.080 kg and radius 0.012 m and is pulled vertically downward with a constant force of 1.1 N. Find **(a)** the tension in the string between the pulley and the cart and **(b)** the acceleration of the cart.

PICTURE THE PROBLEM

The system is shown below. We label the mass of the cart with M, the mass of the pulley with m, and the radius of the pulley with r. The applied downward force creates a tension $T_1 = 1.1$ N in the vertical portion of the string. The horizontal portion of the string, from the pulley to the cart, has a tension T_2. If the pulley had zero mass, these two tensions would be equal. In this case, however, T_2 will have a different value than T_1.

We also show the relevant forces acting on the pulley and the cart. The positive direction of rotation is counterclockwise, and the corresponding positive direction of motion for the cart is to the left.



Physical picture   Relevant forces

STRATEGY

The two unknowns, T_2 and a, can be found by applying Newton's second law to both the pulley and the cart. This gives two equations for two unknowns.

In applying Newton's second law to the pulley, note that since the pulley is a disk, it follows that $I = \frac{1}{2}mr^2$. Also, since the string is not said to slip as it rotates the pulley, we can assume that the angular and linear accelerations are related by $\alpha = a/r$.

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SOLUTION

Part (a)

1. Apply Newton's second law to the cart: $T_2 = Ma$
2. Apply Newton's second law to the pulley. Note that T_1 causes a positive torque, and T_2 causes a negative torque. In addition, use the relation $\alpha = a/r$: $\sum\tau = I\alpha$

$$rT_1 - rT_2 = \left(\tfrac{1}{2}mr^2\right)\left(\frac{a}{r}\right) = \tfrac{1}{2}mra$$

3. Use the cart equation, $T_2 = Ma$, to eliminate a in the pulley equation: $a = \dfrac{T_2}{M}$

$$rT_1 - rT_2 = \tfrac{1}{2}mr\left(\frac{T_2}{M}\right)$$

4. Cancel r and solve for T_2:

$$T_2 = \frac{T_1}{1 + m/2M} = \frac{1.1\text{ N}}{1 + 0.080\text{ kg}/[2(0.31\text{ kg})]} = 0.97\text{ N}$$

Part (b)

5. Use $T_2 = Ma$ to find the acceleration:

$$a = \frac{T_2}{M} = \frac{0.97\text{ N}}{0.31\text{ kg}} = 3.1\text{ m/s}^2$$

INSIGHT

Note that T_2 is less than T_1. As a result, the net torque acting on the pulley is in the counterclockwise direction, causing a rotation in that direction, as expected. If the mass of the pulley were zero $(m = 0)$, the two tensions would be equal, and the acceleration of the cart would be $T_1/M = 3.5\text{ m/s}^2$.

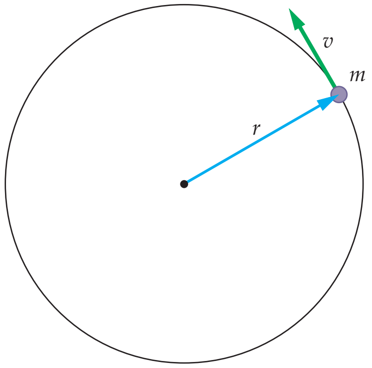
PRACTICE PROBLEM

What applied force is necessary to give the cart an acceleration of 2.2 m/s^2? [**Answer:** $T_1 = T_2(1 + m/2M) = (Ma)(1 + m/2M) = 0.77\text{ N}$]

Some related homework problems: Problem 49, Problem 50

11–6 Angular Momentum

When an object of mass m moves with a speed v in a straight line, we say that it has a linear momentum, $p = mv$. When the same object moves with an angular speed ω along the circumference of a circle of radius r, as in **Figure 11–11**, we say that it has an **angular momentum**, L. The magnitude of L is given by replacing m and v in the expression for p with their angular analogues I and ω (Section 10–5). Thus, we define the angular momentum as follows:



▲ **FIGURE 11–11 The angular momentum of circular motion**

A particle of mass m, moving in a circle of radius r with a speed v. This particle has an angular momentum of magnitude $L = rmv$.

Definition of the Angular Momentum, *L*

$$L = I\omega \qquad \text{11–11}$$

SI unit: $\text{kg}\cdot\text{m}^2/\text{s}$

This expression applies to any object undergoing angular motion, whether it is a point mass moving in a circle, as in Figure 11–11, or a rotating hoop, disk, or other object.

Returning for a moment to the case of a point mass m moving in a circle of radius r, recall that the moment of inertia in this case is $I = mr^2$ (Equation 10–18). In addition, the linear speed of the mass is $v = r\omega$ (Equation 10–12). Combining these results, we find

$$L = I\omega = (mr^2)(v/r) = rmv$$

Noting that mv is the linear momentum p, we find that the angular momentum of a point mass can be written in the following form:

$$L = rmv = rp \qquad \text{11–12}$$

It is important to recall that this expression applies specifically to a point particle moving along the circumference of a circle.

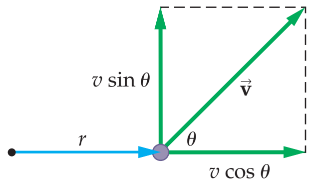
More generally, a point object may be moving at an angle θ with respect to a radial line, as indicated in **Figure 11–12 (a)**. In this case, it is only the tangential component of the momentum, $p \sin \theta = mv \sin \theta$, that contributes to the angular momentum, just as the tangential component of the force, $F \sin \theta$, is all that contributes to the torque. Thus, the magnitude of the angular momentum for a point particle is defined as:

Angular Momentum, *L*, for a Point Particle

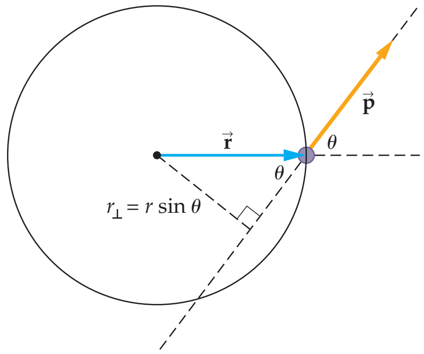
$$L = rp \sin \theta = rmv \sin \theta \qquad 11\text{–}13$$

SI unit: $\text{kg} \cdot \text{m}^2/\text{s}$

Note that if the particle moves in a circular path the angle θ is 90° and the angular momentum is $L = rmv$, in agreement with Equation 11–12. On the other hand, if the object moves radially, so that $\theta = 0$, the angular momentum is zero; $L = rmv \sin 0 = 0$.



(a)



(b)

▲ **FIGURE 11–12 The angular momentum of nontangential motion**

(a) When a particle moves at an angle θ with respect to the radial direction, only the tangential component of velocity, $v \sin \theta$, contributes to the angular momentum. In the case shown here, the particle's angular momentum has a magnitude given by $L = rmv \sin \theta$. **(b)** The angular momentum of an object can also be defined in terms of the moment arm, $r_\perp$. Since $r_\perp = r \sin \theta$, it follows that $L = rmv \sin \theta = r_\perp mv$. Note the similarity between this figure and Figure 11–3.

EXERCISE 11–3

Find the angular momentum of **(a)** a 0.13-kg Frisbee (considered to be a uniform disk of radius 7.5 cm) spinning with an angular speed of 1.15 rad/s, and **(b)** a 95-kg person running with a speed of 5.1 m/s on a circular track of radius 25 m.

SOLUTION

a. Recalling that $I = \frac{1}{2}mR^2$ for a uniform disk (Table 10–1), we have

$$L = I\omega$$
$$= \left(\tfrac{1}{2}mR^2\right)\omega = \tfrac{1}{2}(0.13\ \text{kg})(0.075\ \text{m})^2(1.15\ \text{rad/s}) = 4.2 \times 10^{-4}\ \text{kg} \cdot \text{m}^2/\text{s}$$

b. Treating the person as a particle of mass m, we find

$$L = rmv = (25\ \text{m})(95\ \text{kg})(5.1\ \text{m/s}) = 12{,}000\ \text{kg} \cdot \text{m}^2/\text{s}$$

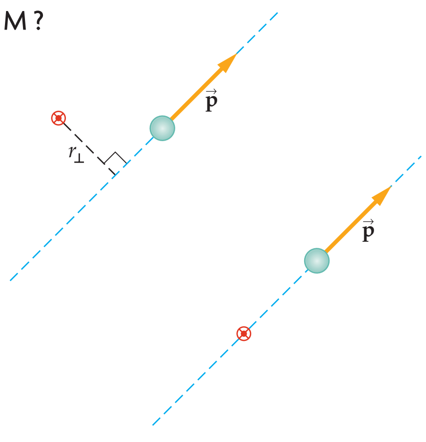
An alternative definition of the angular momentum uses the moment arm, $r_\perp$, as was done for the torque in Equation 11–3. To apply this definition, start by extending a line through the momentum vector, $\vec{p}$, as in **Figure 11–12 (b)**. Next, draw a line from the axis of rotation perpendicular to the line through $\vec{p}$. The perpendicular distance from the axis of rotation to the line of $\vec{p}$ is the moment arm. From the figure we see that $r_\perp = r \sin \theta$. Hence, from Equation 11–13, the angular momentum is

$$L = r_\perp p = r_\perp mv$$

If an object moves in a circle of radius r, the moment arm is $r_\perp = r$ and the angular momentum reduces to our earlier result, $L = rp$.

CONCEPTUAL CHECKPOINT 11–3 ANGULAR MOMENTUM?

Does an object moving in a straight line have nonzero angular momentum **(a)** always, **(b)** sometimes, or **(c)** never?

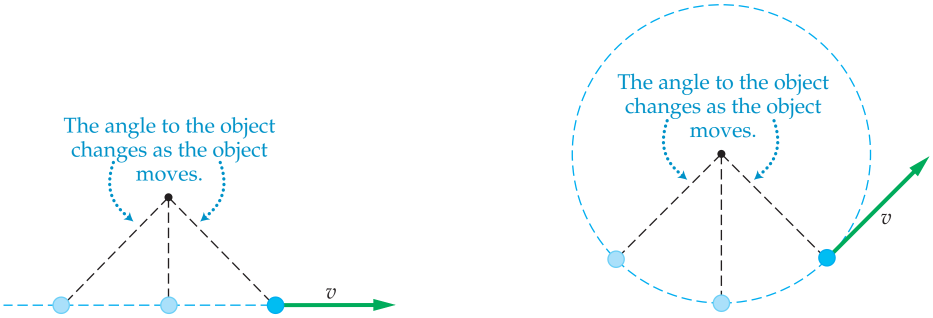


REASONING AND DISCUSSION

The answer is sometimes, because it depends on the choice of the axis of rotation. If the axis of rotation is not on the line drawn through the momentum vector, as in the left sketch at right, the moment arm is nonzero, and therefore $L = r_\perp p$ is also nonzero. If the axis of rotation is on the line of motion, as in the right sketch, the moment arm is zero; hence the linear momentum is radial and L vanishes.

ANSWER

(b) An object moving in a straight line may or may not have angular momentum, depending on the location of the axis of rotation.



(a) Linear motion **(b)** Circular motion

FIGURE 11–13 Angular momentum in linear and circular motion

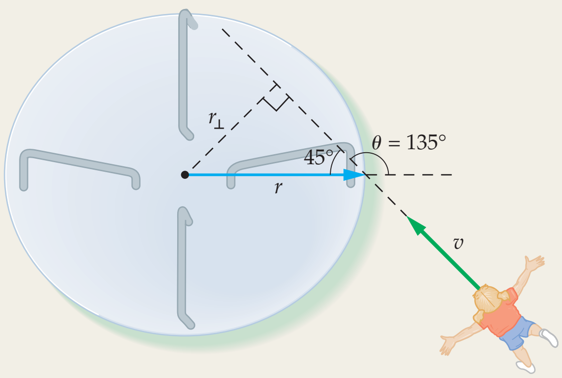
An object moving in **(a)** a straight line and **(b)** a circular path. In both cases, the angular position increases with time; hence, the angular momentum is positive.

Note that an object moving with a momentum p in a straight line that does not go through the axis of rotation has an *angular* position that changes with time. This is illustrated in **Figure 11–13 (a)**. It is for this reason that such an object is said to have an *angular* momentum.

The sign of L is determined by whether the angle to a given object is increasing or decreasing with time. For example, the object moving counterclockwise in a circular path in **Figure 11–13 (b)** has a positive angular momentum, since θ is increasing with time. Similarly, the object in Figure 11–13 (a) also has an angle θ that increases with time, hence its angular momentum is positive as well. On the other hand, if these objects were to have their direction of motion reversed, they would have angles that decrease with time and their angular momenta would be negative.

EXAMPLE 11–8 JUMP ON

Running with a speed of 4.10 m/s, a 21.2-kg child heads toward the rim of a merry-go-round. The radius of the merry-go-round is 2.00 m, and the child moves in the direction indicated. **(a)** What is the child's angular momentum with respect to the center of the merry-go-round? Use $L = rmv \sin\theta$. **(b)** What is the moment arm, $r_\perp$, in this case? **(c)** Find the angular momentum of the child with $L = r_\perp mv$.



PICTURE THE PROBLEM

Our sketch shows the child approaching the rim of the merry-go-round at an angle of 135° relative to the radial direction. Note that the line of motion of the child does not go through the axis of the merry-go-round. As a result, the child has a nonzero angular momentum with respect to that axis of rotation. We also indicate the moment arm, $r_\perp$, and the 45° angle that is opposite to it.

STRATEGY

a. The child's angular momentum can be found by applying $L = rmv \sin\theta$. In this case, we see from the sketch that $\theta = 135°$ and $r = 2.00$ m. The values of m and v are given in the problem statement.

b. and c. Our sketch shows that $r_\perp$ is the side of the right triangle opposite to the angle of 45°. It follows that $r_\perp = r \sin 45°$.

SOLUTION

Part (a)

1. Evaluate $L = rmv \sin\theta$:

$$L = rmv \sin\theta = (2.00\text{ m})(21.2\text{ kg})(4.10\text{ m/s}) \sin 135° = 123\text{ kg}\cdot\text{m}^2/\text{s}$$

Part (b)

2. Calculate the moment arm, $r_\perp$:

$$r_\perp = r \sin 45° = (2.00\text{ m}) \sin 45° = 1.41\text{ m}$$

Part (c)

3. Evaluate $L = r_\perp mv$:

$$L = r_\perp mv = (1.41\text{ m})(21.2\text{ kg})(4.10\text{ m/s}) = 123\text{ kg}\cdot\text{m}^2/\text{s}$$

INSIGHT

When the child lands on the merry-go-round, she will transfer angular momentum to it, causing the merry-go-round to rotate about its center. This will be discussed in more detail in the next section.

Notice that we use 45° in $r_\perp = r \sin 45°$ because we calculate the length of the opposite side of the right triangle indicated in our sketch. We could have used $r_\perp = r \sin 135°$ just as well, using the same angle as in $L = rmv \sin 135°$. The results are the same in either case, since $\sin 135° = \sin 45°$.

PRACTICE PROBLEM

For what angle relative to the radial line does the child have a maximum angular momentum? What is the angular momentum in this case? [**Answer:** $\theta = 90°$, for which $L = rmv = 174 \text{ kg}\cdot\text{m}^2/\text{s}$]

Some related homework problems: Problem 56, Problem 57, Problem 58

Next, we consider the rate of change of angular momentum with time. Since the moment of inertia is a constant—as long as the mass and shape of the object remain unchanged—the change in L in a time interval Δt is

$$\frac{\Delta L}{\Delta t} = I\frac{\Delta\omega}{\Delta t}$$

Recall, however, that $\Delta\omega/\Delta t$ is the angular acceleration, α. Therefore, we have

$$\frac{\Delta L}{\Delta t} = I\alpha$$

Since $I\alpha$ is the torque, it follows that Newton's second law for rotational motion can be written as

Newton's Second Law for Rotational Motion

$$\sum\tau = I\alpha = \frac{\Delta L}{\Delta t} \qquad \text{11–14}$$

Clearly, this is the rotational analogue of $\Sigma F_x = ma_x = \Delta p_x/\Delta t$. Just as force can be expressed as the change in *linear* momentum in a given time interval, the torque can be expressed as the change in *angular* momentum in a time interval.

EXERCISE 11–4

In a light wind, a windmill experiences a constant torque of $255 \text{ N}\cdot\text{m}$. If the windmill is initially at rest, what is its angular momentum 2.00 s later?

SOLUTION

Solve Equation 11–14 for the change in angular momentum due to a single torque τ:

$$\Delta L = L_\text{f} - L_\text{i} = \left(\sum\tau\right)\Delta t = \tau\,\Delta t$$

Since the initial angular momentum of the windmill is zero, its final angular momentum is

$$L_\text{f} = \tau\,\Delta t = (255 \text{ N}\cdot\text{m})(2.00 \text{ s}) = 510 \text{ kg}\cdot\text{m}^2/\text{s}$$

11–7 Conservation of Angular Momentum

When an ice skater goes into a spin and pulls her arms inward to speed up, she probably doesn't think about angular momentum. Neither does a diver who springs into the air and folds her body to speed her rotation. Most people, in fact, are not aware that the actions of these athletes are governed by the same basic laws of physics that cause a collapsing star to spin faster as it becomes a rapidly rotating pulsar. Yet in all these cases, as we shall see, **conservation of angular momentum** is at work.

▲ Once she has launched herself into space, this diver is essentially a projectile. However, the principle of conservation of angular momentum allows her to control the rotational part of her motion. By curling her body up into a tight "tuck," she decreases her moment of inertia, thereby increasing the speed of her spin. To slow down for an elegant entry into the water, she will extend her body, increasing her moment of inertia.

To see the origin of angular momentum conservation, consider an object with an initial angular momentum L_i acted on by a single torque τ. After a period of time, Δt, the object's angular momentum changes in accordance with Newton's second law:

$$\tau = \frac{\Delta L}{\Delta t}$$

Solving for ΔL, we find

$$\Delta L = L_\text{f} - L_\text{i} = \tau\,\Delta t$$

Thus, the final angular momentum of the object is

$$L_\text{f} = L_\text{i} + \tau\,\Delta t$$

If the torque acting on the object is zero, $\tau = 0$, it follows that the initial and final angular momenta are equal—that is, the angular momentum is conserved:

$$L_f = L_i \qquad (\text{if } \tau = 0)$$

Angular momentum is also conserved in systems acted on by more than one torque, provided that the *net external torque* is zero. The reason that internal torques can be ignored is that, just as internal forces come in equal and opposite pairs that cancel, so too do internal torques. As a result, the internal torques in a system sum to zero, and the net torque acting on it is simply the net external torque. Thus, for a general system, angular momentum is conserved if $\tau_{\text{net, ext}}$ is zero:

Conservation of Angular Momentum

$$L_f = L_i \quad (\text{if } \tau_{\text{net, ext}} = 0) \qquad \text{11–15}$$

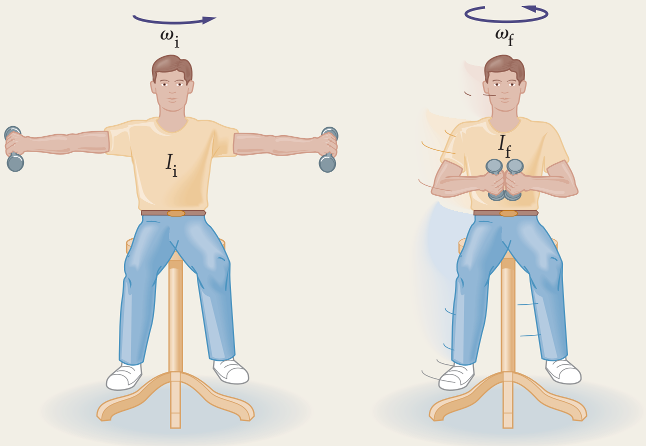
As an illustration of angular momentum conservation, we consider the case of a student rotating on a piano stool in the next Example. Notice how a change in moment of inertia results in a change in angular speed.

EXAMPLE 11–9 GOING FOR A SPIN

For a classroom demonstration, a student sits on a piano stool holding a sizable mass in each hand. Initially, the student holds his arms outstretched and spins about the axis of the stool with an angular speed of 3.72 rad/s. The moment of inertia in this case is 5.33 kg·m^2. While still spinning, the student pulls his arms in to his chest, reducing the moment of inertia to 1.60 kg·m^2. **(a)** What is the student's angular speed now? **(b)** Find the initial and final angular momenta of the student.

PICTURE THE PROBLEM

The initial and final configurations of the student are shown in our sketch. Clearly, the mass distribution in the final configuration, with the masses held closer to the axis of rotation, results in a smaller moment of inertia.



STRATEGY

Ignoring friction in the axis of the stool, since none was mentioned, we conclude that no external torques act on the system. As a result, the angular momentum is conserved. Therefore, setting the initial angular momentum, $L_i = I_i\omega_i$, equal to the final angular momentum, $L_f = I_f\omega_f$, yields the final angular speed.

SOLUTION

Part (a)

1. Apply angular momentum conservation to this system: $L_i = L_f$

   $I_i\omega_i = I_f\omega_f$

2. Solve for the final angular speed, ω_f: $\omega_f = \left(\dfrac{I_i}{I_f}\right)\omega_i$

3. Substitute numerical values: $\omega_f = \left(\dfrac{5.33 \text{ kg}\cdot\text{m}^2}{1.60 \text{ kg}\cdot\text{m}^2}\right)(3.72 \text{ rad/s}) = 12.4 \text{ rad/s}$

Part (b)

4. Use $L = I\omega$ to calculate the angular momentum. Substitute both initial and final values as a check:

   $L_i = I_i\omega_i = (5.33 \text{ kg}\cdot\text{m}^2)(3.72 \text{ rad/s}) = 19.8 \text{ kg}\cdot\text{m}^2/\text{s}$

   $L_f = I_f\omega_f = (1.60 \text{ kg}\cdot\text{m}^2)(12.4 \text{ rad/s}) = 19.8 \text{ kg}\cdot\text{m}^2/\text{s}$

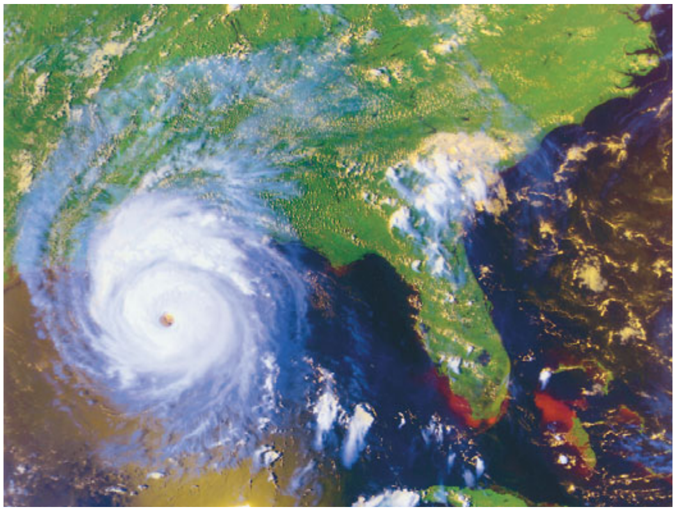
INSIGHT

Initially the student completes one revolution roughly every two seconds. After pulling the weights in, the student's rotation rate has increased to almost two revolutions a second—quite a dizzying pace. The same physics applies to a rotating diver or a spinning ice skater.

PRACTICE PROBLEM

What moment of inertia would be required to give a final spin rate of 10.0 rad/s? [**Answer:** $I_f = (\omega_i/\omega_f)I_i = 1.99 \text{ kg}\cdot\text{m}^2$]

Some related homework problems: Problem 65, Problem 67, Problem 74



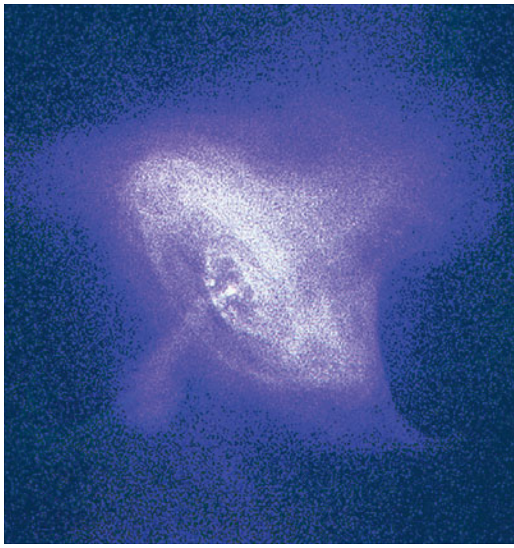
▲ This 1992 satellite photo of Hurricane Andrew (left), one of the most powerful hurricanes of recent decades, clearly suggests the rotating structure of the storm. The violence of the hurricane winds can be attributed in large part to conservation of angular momentum: as air is pushed inward toward the low pressure near the eye of the storm, its rotational velocity increases. The same principle, operating on a smaller scale, explains the tremendous destructive power of tornadoes. The tornado shown at right passed through downtown Miami on May 12, 1997.

REAL-WORLD PHYSICS

Hurricanes and tornadoes

An increasing angular speed, as experienced by the student in Example 11–9, can be observed in nature as well. For example, a hurricane draws circulating air in at ground level toward its "eye," where it then rises to an altitude of 10 miles or more. As air moves inward toward the axis of rotation, its angular speed increases, just as the masses held by the student speed up when they are pulled inward. For example, if the wind has a speed of only 3.0 mph at a distance of 300 miles from the center of the hurricane, it would speed up to 150 mph when it comes to within 6.0 miles of the center. Of course, this analysis ignores friction, which would certainly decrease the wind speed. Still, the basic principle—that a decreasing distance from the axis of rotation implies an increasing speed—applies to both the student and the hurricane. Similar behavior is observed in tornadoes and waterspouts.

Another example of conservation of angular momentum occurs in stellar explosions. On occasion a star will explode, sending a portion of its material out into space. After the explosion, the star collapses to a fraction of its original size, speeding up its rotation in the process. If the mass of the star is greater than 1.44 times the mass of the Sun, the collapse can continue until a *neutron star* is formed, with a radius of only about 10 to 20 km. Neutron stars have incredibly high densities; in fact, if you could bring a teaspoonful of neutron star material to the Earth, it would weigh about 100 million tons! In addition, neutron stars produce powerful beams of X-rays and other radiation that sweep across the sky like a gigantic lighthouse beam as the star rotates. On the Earth we see pulses of radiation from these rotating beams, one for each revolution of the star. These "pulsating stars," or *pulsars*, typically have periods ranging from about 2 ms to nearly 1 s. The Crab nebula (see Problems 9 and 106 in Chapter 10) is a famous example of such a system. The dependence of angular speed on radius for a collapsing star is considered in Active Example 11–4.



▲ Among the fastest rotating objects known in nature are pulsars: stars that have collapsed to a tiny fraction of their original size. Since all the angular momentum of a star must be conserved when it collapses, the dramatic decrease in radius is accompanied by a correspondingly great increase in rotational speed. The Crab nebula pulsar, the remains of a star whose explosion was observed on Earth nearly 1000 years ago, spins at about 30 rev/s. This X-ray photograph shows rings and jets of high-energy particles flying outward from the whirling neutron star at the center.

ACTIVE EXAMPLE 11–4 A STELLAR PERFORMANCE: FIND THE ANGULAR SPEED

A star of radius $R = 2.3 \times 10^8$ m rotates with an angular speed $\omega = 2.4 \times 10^{-6}$ rad/s. If this star collapses to a radius of 20.0 km, find its final angular speed. (Treat the star as if it were a uniform sphere, and assume that no mass is lost as the star collapses.)

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply conservation of angular momentum: $I_i\omega_i = I_f\omega_f$

CONTINUED ON NEXT PAGE

REAL-WORLD PHYSICS
Angular speed of a pulsar

CONTINUED FROM PREVIOUS PAGE

2. Write expressions for the initial and final moments of inertia: $I_\mathrm{i} = \frac{2}{5}MR_\mathrm{i}^2$ and $I_\mathrm{f} = \frac{2}{5}MR_\mathrm{f}^2$

3. Solve for the final angular speed: $\omega_\mathrm{f} = (I_\mathrm{i}/I_\mathrm{f})\omega_\mathrm{i} = (R_\mathrm{i}^2/R_\mathrm{f}^2)\omega_\mathrm{i}$

4. Substitute numerical values: $\omega_\mathrm{f} = 320\ \mathrm{rad/s}$

INSIGHT

The final angular speed corresponds to a period of about 20 ms, a typical period for pulsars. Since 320 rad/s is roughly 3000 rpm, it follows that a pulsar, which has the mass of a star, rotates as fast as the engine in a racing car.

YOUR TURN

At what radius will the star's period of rotation be equal to 15 ms?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note that if the student in Example 11–9 were to stretch his arms back out again, the resulting *increase* in the moment of inertia would cause a *decrease* in his angular speed. The same effect might apply to the Earth one day. For example, a melting of the polar ice caps would lead to an increase in the Earth's moment of inertia (as we saw in Chapter 10) and thus, by angular momentum conservation, the angular speed of the Earth would decrease. This would mean that more time would be required for the Earth to complete a revolution about its axis of rotation; that is, the day would lengthen.

Since angular momentum is conserved in the systems we have studied so far, it is natural to ask whether the energy is conserved as well. We consider this question in the next Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 11–4

COMPARE KINETIC ENERGIES

A skater pulls in her arms, decreasing her moment of inertia by a factor of two, and doubling her angular speed. Is her final kinetic energy **(a)** equal to, **(b)** greater than, or **(c)** less than her initial kinetic energy?

REASONING AND DISCUSSION

Let's calculate the initial and final kinetic energies, and compare them. The initial kinetic energy is

$$K_\mathrm{i} = \tfrac{1}{2}I_\mathrm{i}\omega_\mathrm{i}^2$$

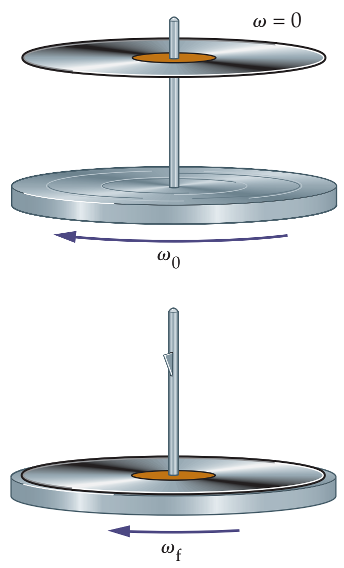
After pulling in her arms, the skater has half the moment of inertia and twice the angular speed. Hence, her final kinetic energy is

$$K_\mathrm{f} = \tfrac{1}{2}I_\mathrm{f}\omega_\mathrm{f}^2 = \tfrac{1}{2}(I_\mathrm{i}/2)(2\omega_\mathrm{i})^2 = 2\left(\tfrac{1}{2}I_\mathrm{i}\omega_\mathrm{i}^2\right) = 2K_\mathrm{i}$$

Thus, the fact that K depends on the square of ω leads to an increase in the kinetic energy. The source of this additional energy is the work done by the muscles in the skater's arms as she pulls them in to her body.

ANSWER

(b) The skater's kinetic energy increases.



▲ **FIGURE 11–14 A rotational collision**

A nonrotating record dropped onto a rotating turntable is an example of a "rotational collision." Since only internal forces are involved during the collision, the final angular momentum is equal to the initial angular momentum.

Rotational Collisions

In the not-too-distant past, a person would play music by placing a record on a rotating turntable. Suppose, for example, that a turntable with a moment of inertia I_t is rotating freely with an initial angular speed ω_0. A record, with a moment of inertia I_r and initially at rest, is dropped straight down onto the rotating turntable, as in **Figure 11–14**. When the record lands, frictional forces between it and the turntable cause the record to speed up and the turntable to slow down, until they both have the same angular speed. Since only internal forces are involved during this process, it follows that the system's angular momentum is conserved. We can think of this event, then, as a "rotational collision."

Before the collision, the angular momentum of the system is

$$L_\mathrm{i} = I_\mathrm{t}\omega_0$$

After the collision, when both the record and the turntable are rotating with the angular speed ω_f, the system's angular momentum is

$$L_\mathrm{f} = I_\mathrm{t}\omega_\mathrm{f} + I_\mathrm{r}\omega_\mathrm{f}$$

Setting $L_\mathrm{f} = L_\mathrm{i}$ yields the final angular speed:

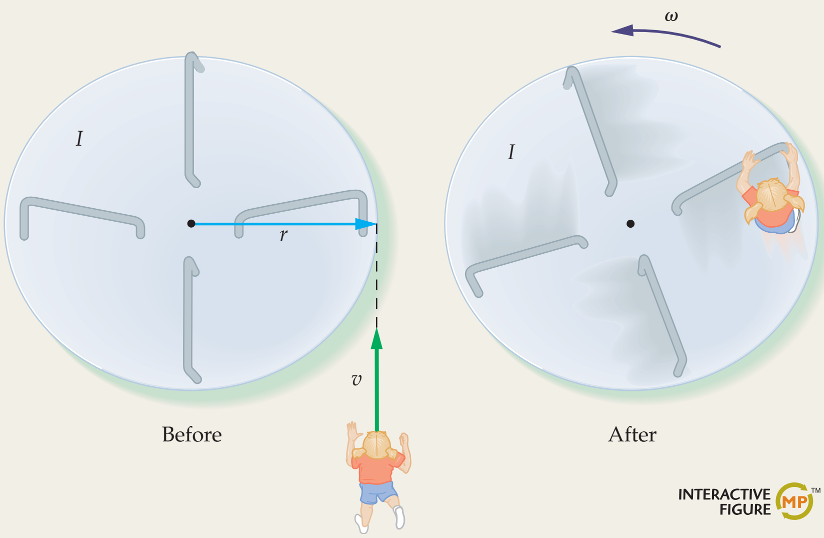
$$\omega_\mathrm{f} = \left(\frac{I_\mathrm{t}}{I_\mathrm{t} + I_\mathrm{r}}\right)\omega_0 \qquad \text{11–16}$$

Since this collision is completely inelastic, we expect the final kinetic energy to be less than the initial kinetic energy.

We conclude this section with a somewhat different example of a rotational collision. The physical principles involved are precisely the same, however.

ACTIVE EXAMPLE 11–5 CONSERVE ANGULAR MOMENTUM: FIND THE ANGULAR SPEED

A 34.0-kg child runs with a speed of 2.80 m/s tangential to the rim of a stationary merry-go-round. The merry-go-round has a moment of inertia of 512 kg · m^2 and a radius of 2.31 m. When the child jumps onto the merry-go-round, the entire system begins to rotate. What is the angular speed of the system?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the initial angular momentum of the child: $L_\mathrm{i} = rmv$
2. Write the final angular momentum of the system: $L_\mathrm{f} = (I + mr^2)\omega$
3. Set $L_\mathrm{f} = L_\mathrm{i}$ and solve for the angular speed: $\omega = rmv/(I + mr^2)$
4. Substitute numerical values: $\omega = 0.317$ rad/s

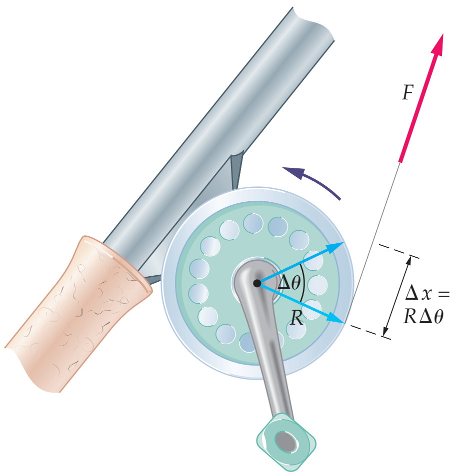
INSIGHT

If the moment of inertia of the merry-go-round had been zero, $I = 0$, the angular speed would be $\omega = v/r$. This means that the linear speed of the child, $r\omega = v$, is unchanged. If $I > 0$, however, the linear speed of the child is decreased. In this particular case, the child's linear speed after the collision is only $v = r\omega = 0.733$ m/s.

YOUR TURN

What initial speed does the child have if, after landing on the merry-go-round, it takes her 22.5 s to complete one revolution?

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 11–15 Rotational work**

A force F pulling a length of line Δx from a fishing reel does the work $W = F\,\Delta x$. In terms of torque and angular displacement, the work can be expressed as $W = \tau\,\Delta\theta$.

The initial and final kinetic energies of the system in Active Example 11–5 are considered in Problem 66.

11–8 Rotational Work and Power

Just as a force acting through a distance performs work on an object, so too does a torque acting through an angular displacement. To see this, consider again the fishing line pulled from a reel. If the line is pulled with a force F for a distance Δx, as in **Figure 11–15**, the work done on the reel is

$$W = F\Delta x$$

Now, since the line is unwinding without slipping, it follows that the linear displacement of the line, Δx, is related to the angular displacement of the reel, $\Delta\theta$, by the following relation:

$$\Delta x = R\Delta\theta$$

In this equation, R is the radius of the reel, and $\Delta\theta$ is measured in radians. Thus, the work can be written as

$$W = F\Delta x = FR\Delta\theta$$

Finally, the torque exerted on the reel by the line is $\tau = RF$, and hence the work done on the reel is simply torque times angular displacement:

Work Done by Torque

$$W = \tau\Delta\theta \qquad \text{11–17}$$

Note again the analogies between angular and linear quantities in $W = F\Delta x$ and $W = \tau\Delta\theta$. As usual, τ is the analogue of F, and θ is the analogue of x.

As we saw in Chapter 7, the net work done on an object is equal to the change in its kinetic energy. This is the work–energy theorem:

$$W = \Delta K = K_{\mathrm{f}} - K_{\mathrm{i}} \qquad \text{11–18}$$

The work-energy theorem applies regardless of whether the work is done by a force acting through a distance or a torque acting through an angle.

Similarly, power is the amount of work done in a given time, regardless whether the work is done by a force or a torque. In the case of a torque, we have $W = \tau\Delta\theta$, and hence

Power Produced by a Torque

$$P = \frac{W}{\Delta t} = \tau\frac{\Delta\theta}{\Delta t} = \tau\omega \qquad \text{11–19}$$

Again, the analogy is clear between $P = Fv$ for the linear case, and $P = \tau\omega$ for the rotational case.

EXERCISE 11–5

It takes a good deal of effort to make homemade ice cream. **(a)** If the torque required to turn the handle on an ice cream maker is 5.7 N·m, how much work is expended on each complete revolution of the handle? **(b)** How much power is required to turn the handle if each revolution is completed in 1.5 s?

SOLUTION

a. Applying Equation 11–17 yields

$$W = \tau\Delta\theta = (5.7\ \mathrm{N\cdot m})(2\pi\ \mathrm{rad}) = 36\ \mathrm{J}$$

b. Power is the work per time; that is,

$$P = W/\Delta t = (36\ \mathrm{J})/(1.5\ \mathrm{s}) = 24\ \mathrm{W}$$

Equivalently, the angular speed of the handle is $\omega = (2\pi)/T = (2\pi)/(1.5\ \mathrm{s}) = 4.2\ \mathrm{rad/s}$, and therefore Equation 11–19 yields $P = \tau\omega = (5.7\ \mathrm{N\cdot m})(4.2\ \mathrm{rad/s}) = 24\ \mathrm{W}$.

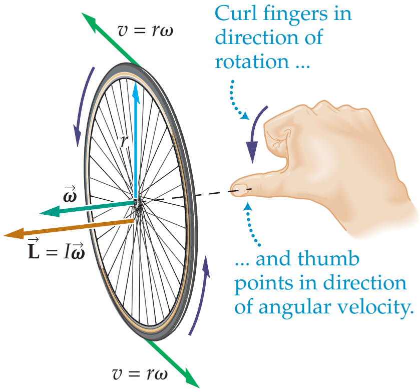
*11–9 The Vector Nature of Rotational Motion

We have mentioned many times that the angular velocity is a vector, and that we must be careful to use the proper sign for ω. But if the angular velocity is a vector, what is its direction?

To address this question, consider the rotating wheel shown in **Figure 11–16**. Each point on the rim of this wheel has a velocity vector pointing in a different direction in the plane of rotation. Since different parts of the wheel move in different directions, how can we assign a single direction to the angular velocity vector, $\vec{\boldsymbol{\omega}}$? The answer is that there is only one direction that remains fixed as the wheel rotates; the direction of the axis of rotation. By definition, then, the angular velocity vector, $\vec{\boldsymbol{\omega}}$, is taken to point along the axis of rotation.

Given that $\vec{\boldsymbol{\omega}}$ points along the axis of rotation, we must still decide whether it points to the left or to the right in Figure 11–16. The convention we use for assigning the direction of $\vec{\boldsymbol{\omega}}$ is referred to as the right-hand rule:

Right-Hand Rule for the Angular Velocity, $\vec{\boldsymbol{\omega}}$
Curl the fingers of the right hand in the direction of rotation.
The thumb now points in the direction of the angular velocity, $\vec{\boldsymbol{\omega}}$.



▲ **FIGURE 11–16 The right-hand rule for angular velocity**
The angular velocity, $\vec{\boldsymbol{\omega}}$, of a rotating wheel points along the axis of rotation. Its direction is given by the right-hand rule.

The right-hand rule for $\vec{\boldsymbol{\omega}}$ is illustrated in Figure 11–16.

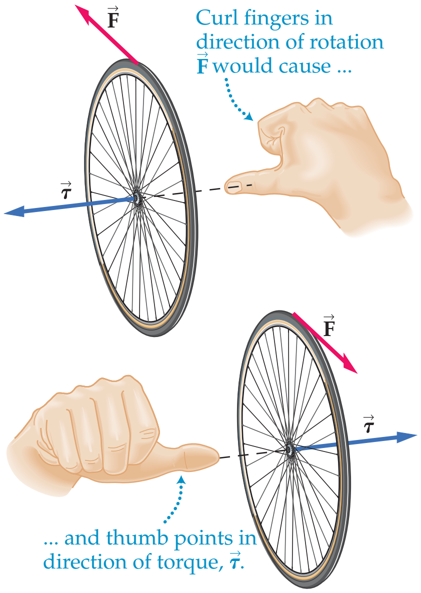
The same convention for direction applies to the angular momentum vector. First, recall that the angular momentum has a magnitude given by $L = I\omega$. Hence, we choose the direction of $\vec{\mathbf{L}}$ to be the same as the direction of $\vec{\boldsymbol{\omega}}$. That is

$$\vec{\mathbf{L}} = I\vec{\boldsymbol{\omega}} \qquad 11\text{–}20$$

The angular momentum vector is also illustrated in Figure 11–16.

Similarly, torque is a vector, and it too is defined to point along the axis of rotation. The right-hand rule for torque is similar to that for angular velocity:

Right-Hand Rule for Torque, $\vec{\boldsymbol{\tau}}$
Curl the fingers of the right hand in the direction of rotation that this torque would cause if it acted alone.
The thumb now points in the direction of the torque vector, $\vec{\boldsymbol{\tau}}$.



▲ **FIGURE 11–17 The right-hand rule for torque**
Examples of torque vectors obtained using the right-hand rule.

Examples of torque vectors are given in **Figure 11–17**.

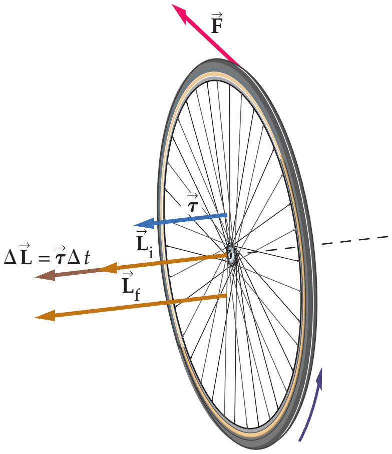
As an example of torque and angular momentum vectors, consider the spinning bicycle wheel shown in **Figure 11–18**. The angular momentum vector for the wheel points to the left, along the axis of rotation. If a person pushes on the rim of the wheel in the direction indicated, the resulting torque is also to the left, as shown in the figure. If this torque lasts for a time Δt, the angular momentum changes by the amount

$$\Delta\vec{\mathbf{L}} = \vec{\boldsymbol{\tau}}\,\Delta t$$

Adding $\Delta\vec{\mathbf{L}}$ to the original angular momentum $\vec{\mathbf{L}}_i$ yields the final angular momentum, $\vec{\mathbf{L}}_f$, shown in Figure 11–18. Since $\vec{\mathbf{L}}_f$ is in the same direction as $\vec{\mathbf{L}}_i$, but with a greater magnitude, it follows that the wheel is spinning in the same direction as

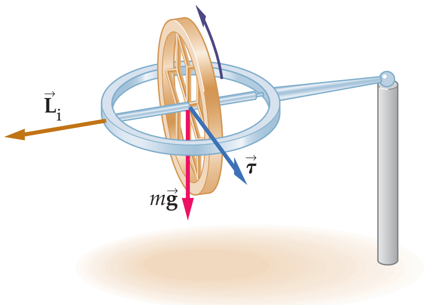
◀ Children have always been fascinated by tops—but not only children. The physicists in the photo at right, Wolfgang Pauli and Niels Bohr, seem as delighted by a spinning top as any child. Their contributions to modern physics, discussed in Chapter 30, helped to show that subatomic particles, the ultimate constituents of matter, have a property (now referred to as "spin") that is in some ways analogous to the rotation of a top or a gyroscope.

▲ The 1.5-inch fused quartz sphere shown here is no ordinary ball. In fact, it is the most perfect sphere ever manufactured. If the Earth were this smooth, the change in elevation from the deepest ocean trench to the highest mountain peak would be only 16 feet. Such precision is required because this sphere is designed to serve as the rotor for an extremely sensitive gyroscope. The device, a million times more sensitive than those used in the best inertial navigation systems, orbits the Earth as part of an experiment to test predictions of Einstein's theory of general relativity.



▲ **FIGURE 11–18 Torque and angular momentum vectors**

A tangential push on the spinning wheel in the direction shown causes a torque to the left. As a result, the angular momentum increases. Hence, the wheel spins faster, as expected.



▲ **FIGURE 11–19 The torque exerted on a gyroscope**

A spinning gyroscope has an initial angular momentum to the left. The torque due to gravity is out of the page.

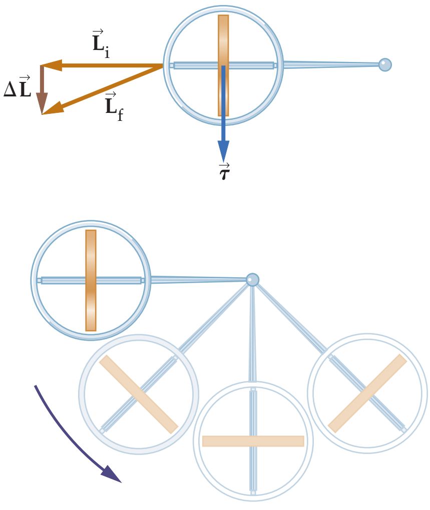
before, only faster. This is to be expected, considering the direction of the person's push on the wheel.

On the other hand, if a person pushes on the wheel in the opposite direction, the torque vector points to the right. As a result, $\Delta\vec{L}$ points to the right as well. When we add $\Delta\vec{L}$ and $\vec{L}_i$ to obtain the final angular momentum, $\vec{L}_f$, we find that it has the same direction as $\vec{L}_i$, but a smaller magnitude. Hence, we conclude that the wheel spins more slowly, as one would expect.

Finally, a case of considerable interest is when the torque and angular momentum vectors are at right angles to one another. The classic example of such a system is the **gyroscope**. To begin, consider a gyroscope whose axis of rotation is horizontal, as in **Figure 11–19**. If the gyroscope were to be released with no spin it would simply fall, rotating counterclockwise downward about its point of support. Curling the fingers of the right hand in the counterclockwise direction, we see that the thumb, and hence the torque due to gravity, points out of the page.

Next, imagine the gyroscope to be spinning rapidly—as would normally be the case—with its angular momentum pointing to the left in Figure 11–19. If the gyroscope is released now, it doesn't fall as before, even though the torque is the same. To see what happens instead, consider the change in angular momentum, $\Delta\vec{L}$, caused by the torque, $\vec{\tau}$, acting for a small interval of time. As shown in **Figure 11–20**, the small change, $\Delta\vec{L}$, is at right angles to $\vec{L}_i$; hence the final angular momentum, $\vec{L}_f$, is essentially the same length as $\vec{L}_i$, but pointing in a direction slightly out of the page. With each small interval of time, the angular momentum vector continues to change in direction so that, viewed from above as in Figure 11–20, the gyroscope as a whole rotates in a counterclockwise sense around its support point. This type of motion, where the axis of rotation changes direction with time, is referred to as **precession**.

Because of its spinning motion about its rotational axis, the Earth may be considered as one rather large gyroscope. Gravitational forces exerted on the Earth by the Sun and the Moon subject it to external torques that cause its rotational axis to precess. At the moment, the rotational axis of the Earth points toward Polaris, the "North Star," which remains almost fixed in position in time-lapse photographs while the other stars move in circular paths about it. In a few hundred years, however, Polaris will also move in a circular path in the sky because the Earth's axis of rotation will point in a different direction. After 26,000 years the Earth will complete one full cycle of precession, and Polaris will again be the pole star.



Top view

▲ **FIGURE 11–20 Precession of a gyroscope**

The gyroscope as viewed from above. In a time Δt the angular momentum changes by the amount $\Delta\vec{L} = \vec{\tau}\Delta t$. This causes the angular momentum vector, and hence the gyroscope as a whole, to rotate in a counterclockwise direction.

REAL-WORLD PHYSICS

Precession of the Earth

On a smaller scale, gyroscopes are used in the navigational systems of a variety of vehicles. In such applications, the rapidly spinning wheel of a gyroscope is mounted on nearly frictionless bearings so that it is practically free from external torques. If no torque acts on the gyroscope, its angular momentum vector remains unchanged both in magnitude and—here is the important point—in direction. With the axis of its gyroscope always pointing in the same, known direction, it is possible for a vehicle to maintain a desired direction of motion relative to the gyroscope's reference direction. On the Hubble Space Telescope, for example, six gyroscopes are used for pointing and stability, though it can operate with only three working gyroscopes if necessary.

REAL-WORLD PHYSICS
Gyroscopes in navigation and space

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of force (Chapters 5 and 6) is extended to torque, its rotational equivalent, in Section 11–1. We also apply Newton's laws to rotation in Section 11–6, just as for linear motion in Chapters 5 and 6.

The connection between rotational and linear quantities (Chapter 10) is used in Section 11–2 to relate torque to angular acceleration. In addition, we extend linear momentum (Chapter 9) to angular momentum in Sections 11–6 and 11–7.

Work and kinetic energy (Chapter 7) are applied to rotational systems in Section 11–8.

LOOKING AHEAD

Angular momentum and the conservation of angular momentum play important roles in the study of gravity. See, in particular, the discussion of Kepler's third law in Section 12–3.

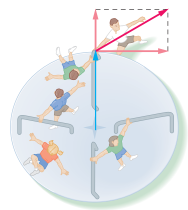
Torque arises in the discussion of magnetic fields and the forces they exert. See Section 22–5 in particular. The torques due to magnetic fields are also the key element in the operation of electric motors, as we see in Section 23–6.

Angular momentum is quantized (given discrete values) in the Bohr model of the hydrogen atom in Section 31–4.

CHAPTER SUMMARY

11–1 TORQUE

A force applied so as to cause an angular acceleration is said to exert a torque, τ.



Tangential Force
A force is tangential if it is tangent to a circle centered on the axis of rotation.

Torque Due to a Tangential Force
A tangential force F applied at a distance r from the axis of rotation produces a torque

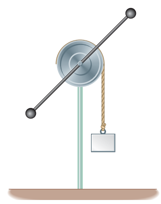
$$\tau = rF \qquad \text{11–1}$$

Torque for a General Force
A force exerted at an angle θ with respect to the radial direction, and applied at a distance r from the axis of rotation, produces the torque

$$\tau = rF\sin\theta \qquad \text{11–2}$$

11–2 TORQUE AND ANGULAR ACCELERATION

A single torque applied to an object gives it an angular acceleration.



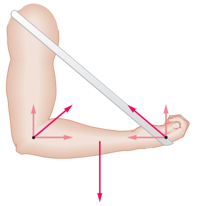
Newton's Second Law for Rotation
The connection between torque and angular acceleration is

$$\sum \tau = I\alpha \qquad \text{11–4}$$

In this expression, I is the moment of inertia about the axis of rotation and α is the angular acceleration about this axis.

Rotational/Translational Analogies
Torque is analogous to force, the moment of inertia is analogous to mass, and the angular acceleration is analogous to linear acceleration. Therefore, the rotational analogue of $F = ma$ is $\tau = I\alpha$.

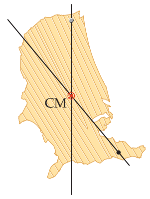
11–3 ZERO TORQUE AND STATIC EQUILIBRIUM



The conditions for an object to be in static equilibrium are that the total force and the total torque acting on the object must be zero:

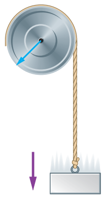
$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum \tau = 0$$

11–4 CENTER OF MASS AND BALANCE



An object balances when it is supported at its center of mass.

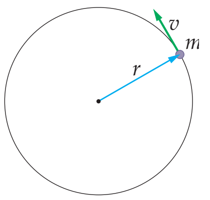
11–5 DYNAMIC APPLICATIONS OF TORQUE



Newton's second law can be applied to rotational systems in a way that is completely analogous to its application to linear systems.

Systems Involving Both Rotational and Linear Elements
In a system with both rotational and linear motions—such as a string passing over a pulley and attached to a mass—Newton's second law must be applied separately to the rotational and linear motions of the system. Connections between the two motions, such as $\alpha = a/r$, can be used to solve for all the accelerations in the system.

11–6 ANGULAR MOMENTUM



A moving object has angular momentum as long as its direction of motion does not extend through the axis of rotation.

Angular Momentum and Angular Speed
Angular momentum can be expressed in terms of angular speed and the moment of inertia as follows:

$$L = I\omega \qquad \text{11–11}$$

This is the rotational analogue of $p = mv$.

Tangential Motion
An object of mass m moving tangentially with a speed v at a distance r from the axis of rotation has an angular momentum, L, given by

$$L = rmv \qquad \text{11–12}$$

General Motion
If an object of mass m is a distance r from the axis of rotation and moves with a speed v at an angle θ with respect to the radial direction, its angular momentum is

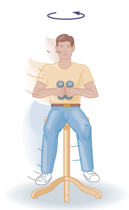
$$L = rmv \sin\theta \qquad \text{11–13}$$

Newton's Second Law
Newton's second law can be expressed in terms of the rate of change of the angular momentum:

$$\sum \tau = I\alpha = \frac{\Delta L}{\Delta t} \qquad \text{11–14}$$

This is the rotational analogue of $\Sigma F = \Delta p/\Delta t$.

11–7 CONSERVATION OF ANGULAR MOMENTUM



If the net external torque acting on a system is zero, its angular momentum is conserved:

$$L_f = L_i$$

Rotational Collisions
Systems in which two rotational objects come into contact can be thought of in terms of a "rotational collision." In such a case, the total angular momentum of the system is conserved.

11–8 ROTATIONAL WORK AND POWER

A torque acting through an angle does work, just as does a force acting through a distance.

Work Done by a Torque
A torque τ acting through an angle $\Delta\theta$ does a work W given by

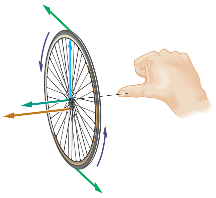
$$W = \tau\Delta\theta \qquad 11\text{–}17$$

Work–Energy Theorem
The work–energy theorem is

$$W = \Delta K = K_f - K_i \qquad 11\text{–}18$$

This theorem applies whether the work is done by a force or by a torque. In the linear case the kinetic energy is $\frac{1}{2}mv^2$; in the rotational case, the kinetic energy is $K = \frac{1}{2}I\omega^2$ (Equation 10–17).

*11–9 THE VECTOR NATURE OF ROTATIONAL MOTION



Rotational quantities have directions that point along the axis of rotation. The precise direction is given by the right-hand rule.

Right-Hand Rule
If the fingers of the *right hand* are curled in the direction of rotation, the thumb points in the direction of the rotational quantity in question. This rule applies to the angular velocity vector, $\vec{\omega}$, the angular acceleration vector, $\vec{\alpha}$, the angular momentum vector, $\vec{L}$, and the torque vector, $\vec{\tau}$.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the torque exerted on a system.	The torque exerted by a tangential force a distance r from the axis of rotation is $\tau = rF$. If the force is at an angle θ to the radial direction, the torque is $\tau = rF\sin\theta$.	Example 11–1
Determine the angular acceleration of a system.	First, calculate the torque exerted on the system. Next, find the angular acceleration using Newton's second law as applied to rotation, namely, $\tau = I\alpha$.	Examples 11–2, 11–3
Find the forces required for static equilibrium.	Static equilibrium requires that both the net force and the net torque acting on a system be zero.	Examples 11–4, 11–5, 11–6 Active Examples 11–1, 11–2, 11–3
Find the final angular momentum of a system.	A torque changes the angular momentum L of a system with time as follows: $\tau = \Delta L/\Delta t$. If no net torque acts on a system, its angular momentum is conserved.	Examples 11–8, 11–9 Active Examples 11–4, 11–5

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Two forces produce the same torque. Does it follow that they have the same magnitude? Explain.
2. A car pitches down in front when the brakes are applied sharply. Explain this observation in terms of torques.
3. A tightrope walker uses a long pole to aid in balancing. Why?
4. When a motorcycle accelerates rapidly from a stop it sometimes "pops a wheelie"; that is, its front wheel may lift off the ground. Explain this behavior in terms of torques.
5. Give an example of a system in which the net torque is zero but the net force is nonzero.
6. Give an example of a system in which the net force is zero but the net torque is nonzero.
7. Is the normal force exerted by the ground the same for all four tires on your car? Explain.
8. Give two everyday examples of objects that are not in static equilibrium.
9. Give two everyday examples of objects that are in static equilibrium.
10. Can an object have zero translational acceleration and, at the same time, have nonzero angular acceleration? If your answer is no, explain why not. If your answer is yes, give a specific example.
11. Stars form when a large rotating cloud of gas collapses. What happens to the angular speed of the gas cloud as it collapses?
12. What purpose does the tail rotor on a helicopter serve?

13. Is it possible to change the angular momentum of an object without changing its linear momentum? If your answer is no, explain why not. If your answer is yes, give a specific example.

14. Suppose a diver springs into the air with no initial angular velocity. Can the diver begin to rotate by folding into a tucked position? Explain.

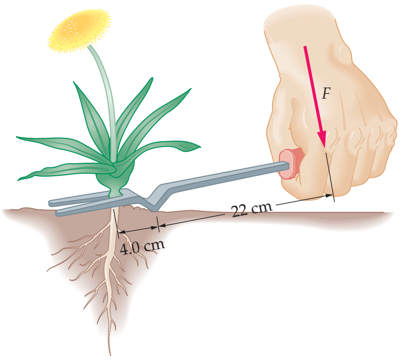
PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 11–1 TORQUE

1. • To tighten a spark plug, it is recommended that a torque of 15 N·m be applied. If a mechanic tightens the spark plug with a wrench that is 25 cm long, what is the minimum force necessary to create the desired torque?

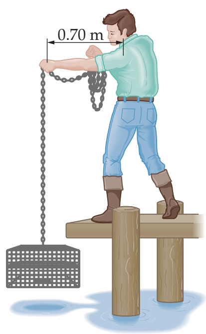
2. • **Pulling a Weed** The gardening tool shown in **Figure 11–21** is used to pull weeds. If a 1.23-N·m torque is required to pull a given weed, what force did the weed exert on the tool?



▲ **FIGURE 11–21** Problem 2

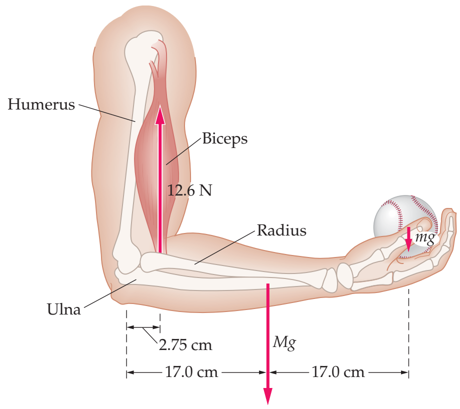
3. • A 1.61-kg bowling trophy is held at arm's length, a distance of 0.605 m from the shoulder joint. What torque does the trophy exert about the shoulder if the arm is **(a)** horizontal, or **(b)** at an angle of 22.5° below the horizontal?

4. • A person slowly lowers a 3.6-kg crab trap over the side of a dock, as shown in **Figure 11–22**. What torque does the trap exert about the person's shoulder?



▲ **FIGURE 11–22**
Problem 4

5. •• **IP BIO Force to Hold a Baseball** A person holds a 1.42-N baseball in his hand, a distance of 34.0 cm from the elbow joint, as shown in **Figure 11–23**. The biceps, attached at a distance of 2.75 cm from the elbow, exerts an upward force of 12.6 N on the forearm. Consider the forearm and hand to be a uniform rod with a mass of 1.20 kg. **(a)** Calculate the net torque acting on the forearm and hand. Use the elbow joint as the axis of rotation. **(b)** If the net torque obtained in part (a) is nonzero, in which direction will the forearm and hand rotate? **(c)** Would the torque exerted on the forearm by the biceps increase or decrease if the biceps were attached farther from the elbow joint?



▲ **FIGURE 11–23** Problems 5 and 19

6. •• At the local playground, a 16-kg child sits on the end of a horizontal teeter-totter, 1.5 m from the pivot point. On the other side of the pivot an adult pushes straight down on the teeter-totter with a force of 95 N. In which direction does the teeter-totter rotate if the adult applies the force at a distance of **(a)** 3.0 m, **(b)** 2.5 m, or **(c)** 2.0 m from the pivot?

SECTION 11–2 TORQUE AND ANGULAR ACCELERATION

7. • **CE Predict/Explain** Consider the pulley–block systems shown in Conceptual Checkpoint 11–1. **(a)** Is the tension in the string on the left-hand rotating system greater than, less than, or equal to the weight of the mass attached to that string? **(b)** Choose the *best explanation* from among the following:

I. The mass is in free fall once it is released.
II. The string rotates the pulley in addition to supporting the mass.
III. The mass accelerates downward.

8. • **CE Predict/Explain** Consider the pulley–block systems shown in Conceptual Checkpoint 11–1. **(a)** Is the tension in the string on the left-hand rotating system greater than, less than, or equal to the tension in the string on the right-hand rotating system? **(b)** Choose the *best explanation* from among the following:

I. The mass in the right-hand system has the greater downward acceleration.
II. The masses are equal.

III. The mass in the left-hand system has the greater downward acceleration.

9. • **CE** Suppose a torque rotates your body about one of three different axes of rotation: case A, an axis through your spine; case B, an axis through your hips; and case C, an axis through your ankles. Rank these three axes of rotation in increasing order of the angular acceleration produced by the torque. Indicate ties where appropriate.

10. • A torque of 0.97 N·m is applied to a bicycle wheel of radius 35 cm and mass 0.75 kg. Treating the wheel as a hoop, find its angular acceleration.

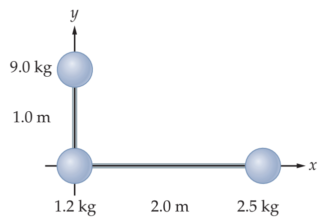
11. • When a ceiling fan rotating with an angular speed of 2.75 rad/s is turned off, a frictional torque of 0.120 N·m slows it to a stop in 22.5 s. What is the moment of inertia of the fan?

12. • When the play button is pressed, a CD accelerates uniformly from rest to 450 rev/min in 3.0 revolutions. If the CD has a radius of 6.0 cm and a mass of 17 g, what is the torque exerted on it?

13. •• A person holds a ladder horizontally at its center. Treating the ladder as a uniform rod of length 3.15 m and mass 8.42 kg, find the torque the person must exert on the ladder to give it an angular acceleration of 0.302 rad/s^2.

14. •• **IP** A wheel on a game show is given an initial angular speed of 1.22 rad/s. It comes to rest after rotating through 0.75 of a turn. **(a)** Find the average torque exerted on the wheel given that it is a disk of radius 0.71 m and mass 6.4 kg. **(b)** If the mass of the wheel is doubled and its radius is halved, will the angle through which it rotates before coming to rest increase, decrease, or stay the same? Explain. (Assume that the average torque exerted on the wheel is unchanged.)

15. •• The L-shaped object in **Figure 11–24** consists of three masses connected by light rods. What torque must be applied to this object to give it an angular acceleration of 1.20 rad/s^2 if it is rotated about **(a)** the x axis, **(b)** the y axis, or **(c)** the z axis (which is through the origin and perpendicular to the page)?

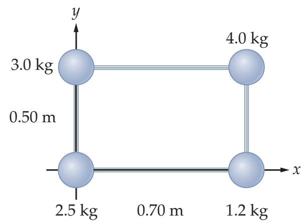


▲ **FIGURE 11–24** Problems 15, 16, and 82

16. •• **CE** The L-shaped object described in Problem 15 can be rotated in one of the following three ways: case A, about the x axis; case B, about the y axis; and case C, about the z axis (which passes through the origin perpendicular to the plane of the figure). If the same torque τ is applied in each of these cases, rank them in increasing order of the resulting angular acceleration. Indicate ties where appropriate.

17. •• **CE** A motorcycle accelerates from rest, and both the front and rear tires roll without slipping. **(a)** Is the force exerted by the ground on the rear tire in the forward or in the backward direction? Explain. **(b)** Is the force exerted by the ground on the front tire in the forward or in the backward direction? Explain. **(c)** If the moment of inertia of the front tire is increased, will the motorcycle's acceleration increase, decrease, or stay the same? Explain.

18. •• **IP** A torque of 13 N·m is applied to the rectangular object shown in **Figure 11–25**. The torque can act about the x axis, the y axis, or the z axis, which passes through the origin and points out of the page. **(a)** In which case does the object experience the greatest angular acceleration? The least angular acceleration? Explain. Find the angular acceleration when the torque acts about **(b)** the x axis, **(c)** the y axis, and **(d)** the z axis.



▲ **FIGURE 11–25** Problems 18 and 83

19. •• A fish takes the bait and pulls on the line with a force of 2.2 N. The fishing reel, which rotates without friction, is a cylinder of radius 0.055 m and mass 0.99 kg. **(a)** What is the angular acceleration of the fishing reel? **(b)** How much line does the fish pull from the reel in 0.25 s?

20. •• Repeat the previous problem, only now assume the reel has a friction clutch that exerts a restraining torque of 0.047 N·m.

SECTION 11–3 ZERO TORQUE AND STATIC EQUILIBRIUM

21. • **CE Predict/Explain** Suppose the person in Active Example 11–3 climbs higher on the ladder. **(a)** As a result, is the ladder more likely, less likely, or equally likely to slip? **(b)** Choose the *best explanation* from among the following:
   I. The forces are the same regardless of the person's position.
   II. The magnitude of f_2 must increase as the person moves upward.
   III. When the person is higher, the ladder presses down harder on the floor.

22. • A string that passes over a pulley has a 0.321-kg mass attached to one end and a 0.635-kg mass attached to the other end. The pulley, which is a disk of radius 9.40 cm, has friction in its axle. What is the magnitude of the frictional torque that must be exerted by the axle if the system is to be in static equilibrium?

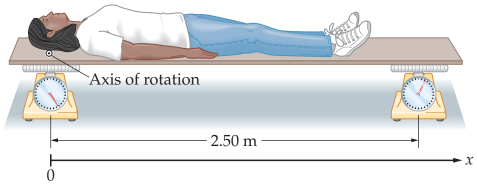
23. • To loosen the lid on a jar of jam 8.9 cm in diameter, a torque of 8.5 N·m must be applied to the circumference of the lid. If a jar wrench whose handle extends 15 cm from the center of the jar is attached to the lid, what is the minimum force required to open the jar?

24. • Consider the system in Active Example 11–1, this time with the axis of rotation at the location of the child. Write out both the condition for zero net force and the condition for zero net torque. Solve for the two forces.

25. •• **IP BIO** Referring to the person holding a baseball in Problem 5, suppose the biceps exert just enough upward force to keep the system in static equilibrium. **(a)** Is the force exerted by the biceps more than, less than, or equal to the combined weight of the forearm, hand, and baseball? Explain. **(b)** Determine the force exerted by the biceps.

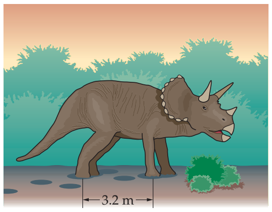
26. •• **IP BIO** **A Person's Center of Mass** To determine the location of her center of mass, a physics student lies on a lightweight plank supported by two scales 2.50 m apart, as

indicated in **Figure 11–26**. If the left scale reads 290 N, and the right scale reads 122 N, find **(a)** the student's mass and **(b)** the distance from the student's head to her center of mass.



▲ **FIGURE 11–26** Problem 26

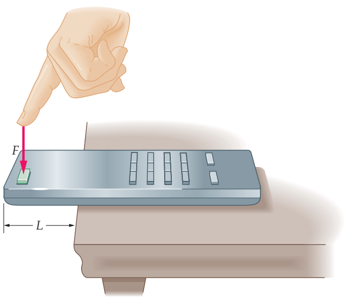
27. •• **Triceratops** A set of fossilized triceratops footprints discovered in Texas show that the front and rear feet were 3.2 m apart, as shown in **Figure 11–27**. The rear footprints were observed to be twice as deep as the front footprints. Assuming that the rear feet pressed down on the ground with twice the force exerted by the front feet, find the horizontal distance from the rear feet to the triceratops's center of mass.



▲ **FIGURE 11–27** Problem 27

28. •• **IP** A schoolyard teeter-totter with a total length of 5.2 m and a mass of 38 kg is pivoted at its center. A 19-kg child sits on one end of the teeter-totter. **(a)** Where should a parent push vertically downward with a force of 210 N in order to hold the teeter-totter level? **(b)** Where should the parent push with a force of 310 N? **(c)** How would your answers to parts (a) and (b) change if the mass of the teeter-totter were doubled? Explain.

29. •• A 0.122-kg remote control 23.0 cm long rests on a table, as shown in **Figure 11–28**, with a length L overhanging its edge. To operate the power button on this remote requires a force of 0.365 N. How far can the remote control extend beyond the edge of the table and still not tip over when you press the power button? Assume the mass of the remote is distributed uniformly, and that the power button is 1.41 cm from the overhanging end of the remote.



▲ **FIGURE 11–28** Problem 29

30. •• **IP** A 0.16-kg meterstick is held perpendicular to a vertical wall by a 2.5-m string going from the wall to the far end of the stick. **(a)** Find the tension in the string. **(b)** If a shorter string is used, will its tension be greater than, less than, or the same as that found in part (a)? **(c)** Find the tension in a 2.0-m string.

31. •• Repeat Example 11–4, this time with a uniform diving board that weighs 225 N.

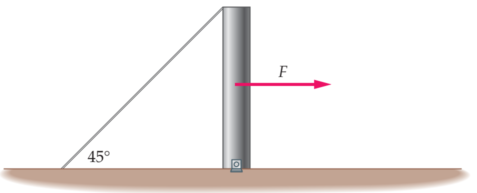
32. •• Babe Ruth steps to the plate and casually points to left center field to indicate the location of his next home run. The mighty Babe holds his bat across his shoulder, with one hand holding the small end of the bat. The bat is horizontal, and the distance from the small end of the bat to the shoulder is 22.5 cm. If the bat has a mass of 1.10 kg and has a center of mass that is 67.0 cm from the small end of the bat, find the magnitude and direction of the force exerted by **(a)** the hand and **(b)** the shoulder.

33. •• A uniform metal rod, with a mass of 3.7 kg and a length of 1.2 m, is attached to a wall by a hinge at its base. A horizontal wire bolted to the wall 0.51 m above the base of the rod holds the rod at an angle of 25° above the horizontal. The wire is attached to the top of the rod. **(a)** Find the tension in the wire. Find **(b)** the horizontal and **(c)** the vertical components of the force exerted on the rod by the hinge.

34. •• **IP** In the previous problem, suppose the wire is shortened, so that the rod now makes an angle of 35° with the horizontal. The wire is horizontal, as before. **(a)** Do you expect the tension in the wire to increase, decrease, or stay the same as a result of its new length? Explain. **(b)** Calculate the tension in the wire.

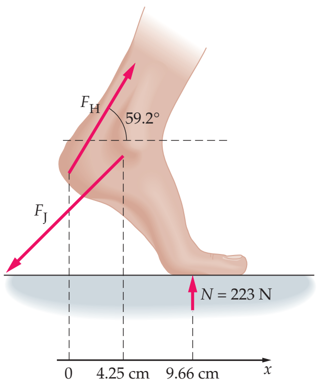
35. •• Repeat Active Example 11–3, this time with a uniform 7.2-kg ladder that is 4.0 m long.

36. •• A rigid, vertical rod of negligible mass is connected to the floor by a bolt through its lower end, as shown in **Figure 11–29**. The rod also has a wire connected between its top end and the floor. If a horizontal force F is applied at the midpoint of the rod, find **(a)** the tension in the wire, and **(b)** the horizontal and **(c)** the vertical components of force exerted by the bolt on the rod.



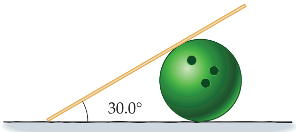
▲ **FIGURE 11–29** Problems 36, 111, and 112

37. ••• **BIO** **Forces in the Foot** **Figure 11–30** shows the forces acting on a sprinter's foot just before she takes off at the start of the race. Find the magnitude of the force exerted on the heel by the Achilles tendon, F_H, and the magnitude of the force exerted on the foot at the ankle joint, F_J.



▲ **FIGURE 11–30** Problem 37

38. ••• A stick with a mass of 0.214 kg and a length of 0.436 m rests in contact with a bowling ball and a rough floor, as shown in **Figure 11–31**. The bowling ball has a diameter of 21.6 cm, and the angle the stick makes with the horizontal is 30.0°. You may assume there is no friction between the stick and the bowling ball, though friction with the floor must be taken into account. **(a)** Find the magnitude of the force exerted on the stick by the bowling ball. **(b)** Find the horizontal component of the force exerted on the stick by the floor. **(c)** Repeat part (b) for the vertical component of the force.



▲ **FIGURE 11–31** Problem 38

39. ••• **IP** A uniform crate with a mass of 16.2 kg rests on a floor with a coefficient of static friction equal to 0.571. The crate is a uniform cube with sides 1.21 m in length. **(a)** What horizontal force applied to the top of the crate will initiate tipping? **(b)** If the horizontal force is applied halfway to the top of the crate, it will begin to slip before it tips. Explain.

40. ••• In the previous problem, **(a)** what is the minimum height where the force F can be applied so that the crate begins to tip before sliding? **(b)** What is the magnitude of the force in this case?

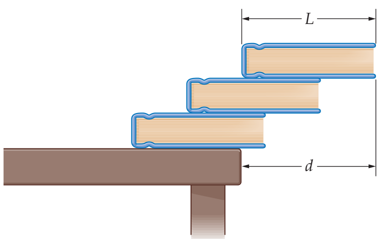
SECTION 11–4 CENTER OF MASS AND BALANCE

41. • A hand-held shopping basket 62.0 cm long has a 1.81-kg carton of milk at one end, and a 0.722-kg box of cereal at the other end. Where should a 1.80-kg container of orange juice be placed so that the basket balances at its center?

42. • If the cat in Active Example 11–2 has a mass of 2.8 kg, how close to the right end of the two-by-four can it walk before the board begins to tip?

43. •• **IP** A 0.34-kg meterstick balances at its center. If a necklace is suspended from one end of the stick, the balance point moves 9.5 cm toward that end. **(a)** Is the mass of the necklace more than, less than, or the same as that of the meterstick? Explain. **(b)** Find the mass of the necklace.

44. •• **Maximum Overhang** Three identical, uniform books of length L are stacked one on top the other. Find the maximum overhang distance d in **Figure 11–32** such that the books do not fall over.



▲ **FIGURE 11–32** Problems 44 and 107

45. •• A baseball bat balances 71.1 cm from one end. If a 0.560-kg glove is attached to that end, the balance point moves 24.7 cm toward the glove. Find the mass of the bat.

SECTION 11–5 DYNAMIC APPLICATIONS OF TORQUE

46. •• A 2.85-kg bucket is attached to a disk-shaped pulley of radius 0.121 m and mass 0.742 kg. If the bucket is allowed to fall, **(a)** what is its linear acceleration? **(b)** What is the angular acceleration of the pulley? **(c)** How far does the bucket drop in 1.50 s?

47. •• **IP** In the previous problem, **(a)** is the tension in the rope greater than, less than, or equal to the weight of the bucket? Explain. **(b)** Calculate the tension in the rope.

48. •• A child exerts a tangential 42.2-N force on the rim of a disk-shaped merry-go-round with a radius of 2.40 m. If the merry-go-round starts at rest and acquires an angular speed of 0.0860 rev/s in 3.50 s, what is its mass?

49. •• **IP** You pull downward with a force of 28 N on a rope that passes over a disk-shaped pulley of mass 1.2 kg and radius 0.075 m. The other end of the rope is attached to a 0.67-kg mass. **(a)** Is the tension in the rope the same on both sides of the pulley? If not, which side has the largest tension? **(b)** Find the tension in the rope on both sides of the pulley.

50. •• Referring to the previous problem, find the linear acceleration of the 0.67-kg mass.

51. ••• A uniform meterstick of mass M has an empty paint can of mass m hanging from one end. The meterstick and the can balance at a point 20.0 cm from the end of the stick where the can is attached. When the balanced stick–can system is suspended from a scale, the reading on the scale is 2.54 N. Find the mass of **(a)** the meterstick and **(b)** the paint can.

52. ••• **Atwood's Machine** An Atwood's machine consists of two masses, m_1 and m_2, connected by a string that passes over a pulley. If the pulley is a disk of radius R and mass M, find the acceleration of the masses.

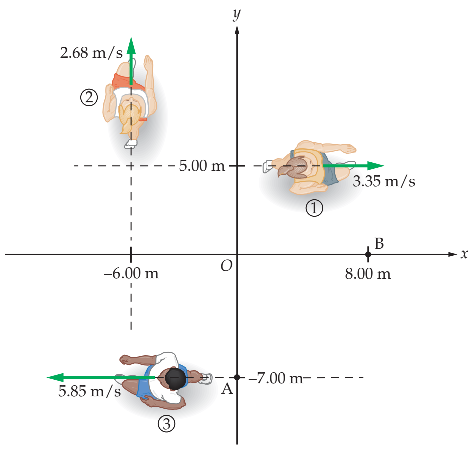
SECTION 11–6 ANGULAR MOMENTUM

53. • Calculate the angular momentum of the Earth about its own axis, due to its daily rotation. Assume that the Earth is a uniform sphere.

54. • A 0.015-kg record with a radius of 15 cm rotates with an angular speed of $33\frac{1}{3}$ rpm. Find the angular momentum of the record.

55. • In the previous problem, a 1.1-g fly lands on the rim of the record. What is the fly's angular momentum?

56. • Jogger 1 in **Figure 11–33** has a mass of 65.3 kg and runs in a straight line with a speed of 3.35 m/s. **(a)** What is the magnitude



▲ **FIGURE 11–33** Problems 56, 57, and 58 of the jogger's linear momentum? **(b)** What is the magnitude of the jogger's angular momentum with respect to the origin, O?

57. • Repeat the previous problem for the case of jogger 2, whose speed is 2.68 m/s and whose mass is 58.2 kg.

58. •• **IP** Suppose jogger 3 in Figure 11–33 has a mass of 62.2 kg and a speed of 5.85 m/s. **(a)** Is the magnitude of the jogger's angular momentum greater with respect to point A or point B? Explain. **(b)** Is the magnitude of the jogger's angular momentum with respect to point B greater than, less than, or the same as it is with respect to the origin, O? Explain. **(c)** Calculate the magnitude of the jogger's angular momentum with respect to points A, B, and O.

59. •• A torque of 0.12 N · m is applied to an egg beater. **(a)** If the egg beater starts at rest, what is its angular momentum after 0.65 s? **(b)** If the moment of inertia of the egg beater is 2.5×10^{-3} kg · m^2, what is its angular speed after 0.65 s?

60. •• A windmill has an initial angular momentum of 8500 kg · m^2/s. The wind picks up, and 5.86 s later the windmill's angular momentum is 9700 kg · m^2/s. What was the torque acting on the windmill, assuming it was constant during this time?

61. •• Two gerbils run in place with a linear speed of 0.55 m/s on an exercise wheel that is shaped like a hoop. Find the angular momentum of the system if each gerbil has a mass of 0.22 kg and the exercise wheel has a radius of 9.5 cm and a mass of 5.0 g.

SECTION 11–7 CONSERVATION OF ANGULAR MOMENTUM

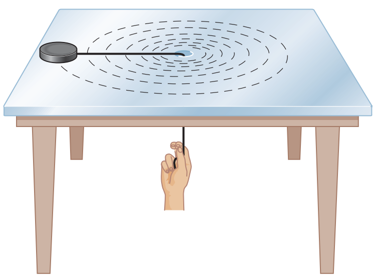
62. • **CE Predict/Explain** A student rotates on a frictionless piano stool with his arms outstretched, a heavy weight in each hand. Suddenly he lets go of the weights, and they fall to the floor. As a result, does the student's angular speed increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The loss of angular momentum when the weights are dropped causes the student to rotate more slowly.

II. The student's moment of inertia is decreased by dropping the weights.

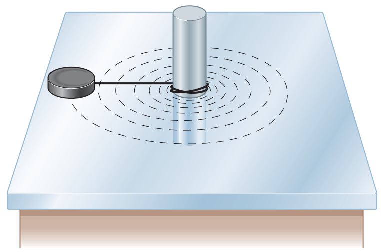
III. Dropping the weights exerts no torque on the student, but the floor exerts a torque on the weights when they land.

63. • **CE** A puck on a horizontal, frictionless surface is attached to a string that passes through a hole in the surface, as shown in **Figure 11–34**. As the puck rotates about the hole, the string is pulled downward, bringing the puck closer to the hole. During this process, do the puck's **(a)** linear speed, **(b)** angular speed, and **(c)** angular momentum increase, decrease, or stay the same?



▲ **FIGURE 11–34** Problems 63 and 93

64. • **CE** A puck on a horizontal, frictionless surface is attached to a string that wraps around a pole of finite radius, as shown in **Figure 11–35**. **(a)** As the puck moves along the spiral path, does its speed increase, decrease, or stay the same? Explain. **(b)** Does its angular momentum increase, decrease, or stay the same? Explain.



▲ **FIGURE 11–35** Problem 64

65. • As an ice skater begins a spin, his angular speed is 3.17 rad/s. After pulling in his arms, his angular speed increases to 5.46 rad/s. Find the ratio of the skater's final moment of inertia to his initial moment of inertia.

66. • Calculate both the initial and the final kinetic energies of the system described in Active Example 11–5.

67. • A diver tucks her body in midflight, decreasing her moment of inertia by a factor of two. By what factor does her angular speed change?

68. •• **IP** In the previous problem, **(a)** does the diver's kinetic energy increase, decrease, or stay the same? **(b)** Calculate the ratio of the final kinetic energy to the initial kinetic energy for the diver.

69. •• A disk-shaped merry-go-round of radius 2.63 m and mass 155 kg rotates freely with an angular speed of 0.641 rev/s. A 59.4-kg person running tangential to the rim of the merry-go-round at 3.41 m/s jumps onto its rim and holds on. Before jumping on the merry-go-round, the person was moving in the same direction as the merry-go-round's rim. What is the final angular speed of the merry-go-round?

70. •• **IP** In the previous problem, **(a)** does the kinetic energy of the system increase, decrease, or stay the same when the person jumps on the merry-go-round? **(b)** Calculate the initial and final kinetic energies for this system.

71. •• A student sits at rest on a piano stool that can rotate without friction. The moment of inertia of the student–stool system is 4.1 kg · m^2. A second student tosses a 1.5-kg mass with a speed of 2.7 m/s to the student on the stool, who catches it at a distance of 0.40 m from the axis of rotation. What is the resulting angular speed of the student and the stool?

72. •• **IP** Referring to the previous problem, **(a)** does the kinetic energy of the mass–student–stool system increase, decrease, or stay the same as the mass is caught? **(b)** Calculate the initial and final kinetic energies of the system.

73. •• **IP** A turntable with a moment of inertia of 5.4×10^{-3} kg · m^2 rotates freely with an angular speed of $33\frac{1}{3}$ rpm. Riding on the rim of the turntable, 15 cm from the center, is a cute, 32-g mouse. **(a)** If the mouse walks to the center of the turntable, will the turntable rotate faster, slower, or at the same rate? Explain. **(b)** Calculate the angular speed of the turntable when the mouse reaches the center.

74. •• A student on a piano stool rotates freely with an angular speed of 2.95 rev/s. The student holds a 1.25-kg mass in each outstretched arm, 0.759 m from the axis of rotation. The combined moment of inertia of the student and the stool, ignoring the two masses, is 5.43 kg · m^2, a value that remains constant. **(a)** As the student pulls his arms inward, his angular speed increases to 3.54 rev/s. How far are the masses from the axis of rotation at this time, considering the masses to be points? **(b)** Calculate the initial and final kinetic energies of the system.

75. ••• **Walking on a Merry-Go-Round** A child of mass m stands at rest near the rim of a stationary merry-go-round of radius R and moment of inertia I. The child now begins to walk around the circumference of the merry-go-round with a tangential speed v with respect to the merry-go-round's surface. **(a)** What is the child's speed with respect to the ground? Check your result in the limits **(b)** $I \to 0$ and **(c)** $I \to \infty$.

SECTION 11–8 ROTATIONAL WORK AND POWER

76. • **CE Predict/Explain** Two spheres of equal mass and radius are rolling across the floor with the same speed. Sphere 1 is a uniform solid; sphere 2 is hollow. Is the work required to stop sphere 1 greater than, less than, or equal to the work required to stop sphere 2? **(b)** Choose the *best explanation* from among the following:
   I. Sphere 2 has the greater moment of inertia and hence the greater rotational kinetic energy.
   II. The spheres have equal mass and speed; therefore, they have the same kinetic energy.
   III. The hollow sphere has less kinetic energy.

77. • How much work must be done to accelerate a baton from rest to an angular speed of 7.4 rad/s about its center? Consider the baton to be a uniform rod of length 0.53 m and mass 0.44 kg.

78. • Turning a doorknob through 0.25 of a revolution requires 0.14 J of work. What is the torque required to turn the doorknob?

79. • A person exerts a tangential force of 36.1 N on the rim of a disk-shaped merry-go-round of radius 2.74 m and mass 167 kg. If the merry-go-round starts at rest, what is its angular speed after the person has rotated it through an angle of 32.5°?

80. • To prepare homemade ice cream, a crank must be turned with a torque of 3.95 N·m. How much work is required for each complete turn of the crank?

81. • **Power of a Dental Drill** A popular make of dental drill can operate at a speed of 42,500 rpm while producing a torque of 3.68 oz·in. What is the power output of this drill? Give your answer in watts.

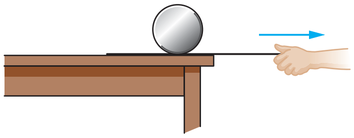
82. •• The L-shaped object in Figure 11–24 consists of three masses connected by light rods. Find the work that must be done on this object to accelerate it from rest to an angular speed of 2.35 rad/s about **(a)** the x axis, **(b)** the y axis, and **(c)** the z axis (which is through the origin and perpendicular to the page).

83. •• The rectangular object in Figure 11–25 consists of four masses connected by light rods. What power must be applied to this object to accelerate it from rest to an angular speed of 2.5 rad/s in 6.4 s about **(a)** the x axis, **(b)** the y axis, and **(c)** the z axis (which is through the origin and perpendicular to the page)?

84. •• **IP** A circular saw blade accelerates from rest to an angular speed of 3620 rpm in 6.30 revolutions. **(a)** Find the torque exerted on the saw blade, assuming it is a disk of radius 15.2 cm and mass 0.755 kg. **(b)** Is the angular speed of the saw blade after 3.15 revolutions greater than, less than, or equal to 1810 rpm? Explain. **(c)** Find the angular speed of the blade after 3.15 revolutions.

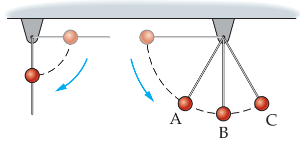
GENERAL PROBLEMS

85. • **CE** A uniform disk stands upright on its edge, and rests on a sheet of paper placed on a tabletop. If the paper is pulled horizontally to the right, as in **Figure 11–36**, **(a)** does the disk rotate clockwise or counterclockwise about its center? Explain. **(b)** Does the center of the disk move to the right, move to the left, or stay in the same location? Explain.



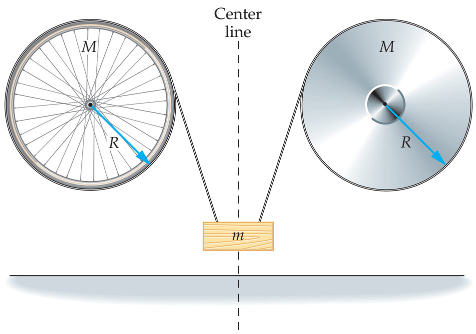
▲ **FIGURE 11–36** Problem 85

86. • **CE** Consider the two rotating systems shown in **Figure 11–37**, each consisting of a mass m attached to a rod of negligible mass pivoted at one end. On the left, the mass is attached at the midpoint of the rod; to the right, it is attached to the free end of the rod. The rods are released from rest in the horizontal position at the same time. When the rod to the left reaches the vertical position, is the rod to the right not yet vertical (location A), vertical (location B), or past vertical (location C)? Explain.



▲ **FIGURE 11–37** Problem 86

87. • **CE Predict/Explain** A disk and a hoop (bicycle wheel) of equal radius and mass each have a string wrapped around their circumferences. Hanging from the strings, halfway between the disk and the hoop, is a block of mass m, as shown in **Figure 11–38**. The disk and the hoop are free to rotate about their centers. When the block is allowed to fall, does it stay on the center line, move toward the right, or move toward the left? **(b)** Choose the *best explanation* from among the following:
   I. The disk is harder to rotate, and hence its angular acceleration is less than that of the wheel.
   II. The wheel has the greater moment of inertia and unwinds more slowly than the disk.
   III. The system is symmetric, with equal mass and radius on either side.



▲ **FIGURE 11–38** Problem 87

88. • **CE** A beetle sits at the rim of a turntable that is at rest but is free to rotate about a vertical axis. Suppose the beetle now begins to walk around the perimeter of the turntable. Does the beetle move forward, backward, or does it remain in the same location relative to the ground? Answer for two different cases, **(a)** the turntable is much more massive than the beetle and **(b)** the turntable is massless.

89. • **CE** A beetle sits near the rim of a turntable that is rotating without friction about a vertical axis. The beetle now begins to walk toward the center of the turntable. As a result, does the angular speed of the turntable increase, decrease, or stay the same? Explain.

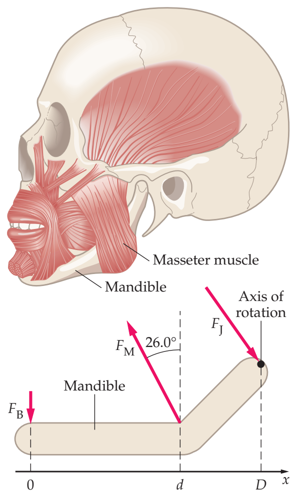
90. • **CE** Suppose the Earth were to magically expand, doubling its radius while keeping its mass the same. Would the length of the day increase, decrease, or stay the same? Explain.

91. • After getting a drink of water, a hamster jumps onto an exercise wheel for a run. A few seconds later the hamster is running in place with a speed of 1.3 m/s. Find the work done by the hamster to get the exercise wheel moving, assuming it is a hoop of radius 0.13 m and mass 6.5 g.

92. •• A 47.0-kg uniform rod 4.25 m long is attached to a wall with a hinge at one end. The rod is held in a horizontal position by a wire attached to its other end. The wire makes an angle of 30.0° with the horizontal, and is bolted to the wall directly above the hinge. If the wire can withstand a maximum tension of 1450 N before breaking, how far from the wall can a 68.0-kg person sit without breaking the wire?

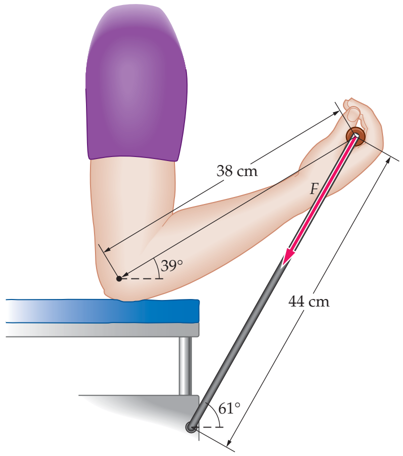
93. •• **IP** A puck attached to a string moves in a circular path on a frictionless surface, as shown in Figure 11–34. Initially, the speed of the puck is v and the radius of the circle is r. If the string passes through a hole in the surface, and is pulled downward until the radius of the circular path is $r/2$, **(a)** does the speed of the puck increase, decrease, or stay the same? **(b)** Calculate the final speed of the puck.

94. •• **BIO The Masseter Muscle** The masseter muscle, the principal muscle for chewing, is one of the strongest muscles for its size in the human body. It originates on the lower edge of the zygomatic arch (cheekbone) and inserts in the angle of the mandible. Referring to the lower diagram in **Figure 11–39**, where $d = 7.60$ cm and $D = 10.85$ cm, **(a)** find the torque produced about the axis of rotation by the masseter muscle. The force exerted by the masseter muscle is $F_M = 455$ N. **(b)** Find the biting force, F_B, exerted on the mandible by the upper teeth. Find **(c)** the horizontal and **(d)** the vertical component of the force F_J exerted on the mandible at the joint where it attaches to the skull. Assume that the mandible is in static equilibrium, and that upward is the positive vertical direction.



▲ **FIGURE 11–39** Problem 94

95. •• **Exercising the Biceps** You are designing exercise equipment to operate as shown in **Figure 11–40**, where a person pulls upward on an elastic cord. The cord behaves like an ideal spring and has an unstretched length of 31 cm. If you would like the torque about the elbow joint to be 81 N · m in the position shown, what force constant, k, is required for the cord?



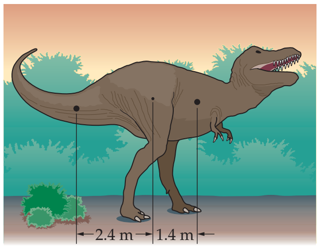
▲ **FIGURE 11–40** Problem 95

96. •• **Horsepower of a Car** Auto mechanics use the following formula to calculate the horsepower (HP) of a car engine:

$$\text{HP} = \text{Torque} \cdot \text{RPM}/C$$

In this expression, Torque is the torque produced by the engine in ft · lb, RPM is the angular speed of the engine in revolutions per minute, and C is a dimensionless constant. **(a)** Find the numerical value of C. **(b)** The Shelby Series 1 engine is advertised to generate 320 hp at 6500 rpm. What is the corresponding torque produced by this engine? Give your answer in ft · lb.

97. •• **Balancing a *T. rex*** Paleontologists believe that *Tyrannosaurus rex* stood and walked with its spine almost horizontal, as indicated in **Figure 11–41**, and that its tail was held off the ground to balance its upper torso about the hip joint. Given that the total mass of *T. rex* was 5400 kg, and that the placement of the center of mass of the tail and the upper torso was as shown in Figure 11–41, find the mass of the tail required for balance.

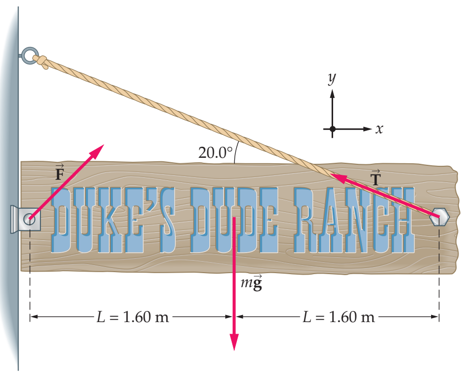


▲ **FIGURE 11–41** Problem 97

98. •• **IP** You hold a uniform, 28-g pen horizontal with your thumb pushing down on one end and your index finger pushing upward 3.5 cm from your thumb. The pen is 14 cm long. **(a)** Which of these two forces is greater in magnitude? **(b)** Find the two forces.

99. •• In Active Example 11–3, suppose the ladder is uniform, 4.0 m long, and weighs 60.0 N. Find the forces exerted on the ladder when the person is **(a)** halfway up the ladder and **(b)** three-fourths of the way up the ladder.

100. •• When you arrive at Duke's Dude Ranch, you are greeted by the large wooden sign shown in **Figure 11–42**. The left end of the sign is held in place by a bolt, the right end is tied to a rope that makes an angle of 20.0° with the horizontal. If the sign is uniform, 3.20 m long, and has a mass of 16.0 kg, what are **(a)** the tension in the rope, and **(b)** the horizontal and vertical components of the force, $\vec{\mathbf{F}}$, exerted by the bolt?

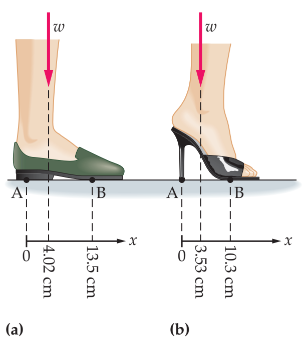


▲ **FIGURE 11–42** Problem 100

101. •• A 67.0-kg person stands on a lightweight diving board supported by two pillars, one at the end of the board, the other 1.10 m away. The pillar at the end of the board exerts a downward force of 828 N. **(a)** How far from that pillar is the person standing? **(b)** Find the force exerted by the second pillar.

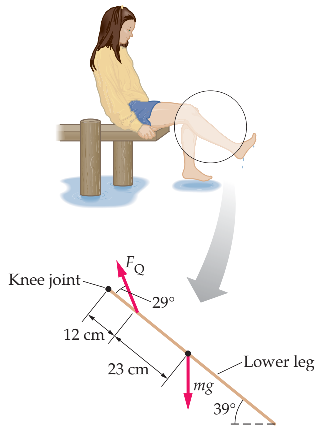
102. •• In Example 11–4, find $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ as a function of the distance, x, of the swimmer from the left end of the diving board. Assume that the diving board is uniform and has a mass of 85.0 kg.

103. •• **Flats Versus Heels** A woman might wear a pair of flat shoes to work during the day, as in **Figure 11–43 (a)**, but a pair of high heels, **Figure 11–43 (b)**, when going out for the evening. Assume that each foot supports half her weight, $w = W/2 = 279$ N, and that the forces exerted by the floor on her feet occur at the points A and B in both figures. Find the forces F_A (point A) and F_B (point B) for **(a)** flat shoes and **(b)** high heels. **(c)** How have the high heels changed the weight distribution between the woman's heels and toes?



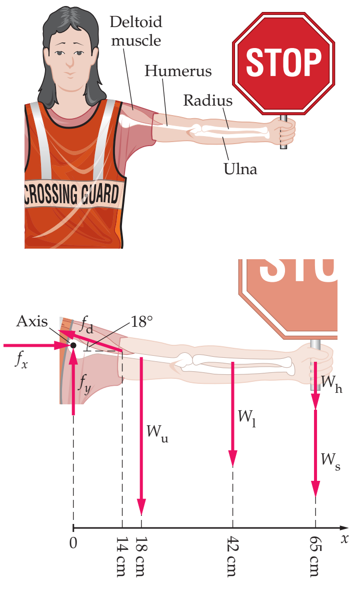
▲ **FIGURE 11–43** Problem 103

104. •• **BIO** A young girl sits at the edge of a dock by the bay, dipping her feet in the water. At the instant shown in **Figure 11–44**, she holds her lower leg stationary with her quadriceps muscle at an angle of 39° with respect to the horizontal. Use the information given in the figure, plus the fact that her lower leg has a mass of 3.4 kg, to determine the magnitude of the force, F_Q, exerted on the lower leg by the quadriceps.



▲ **FIGURE 11–44** Problem 104

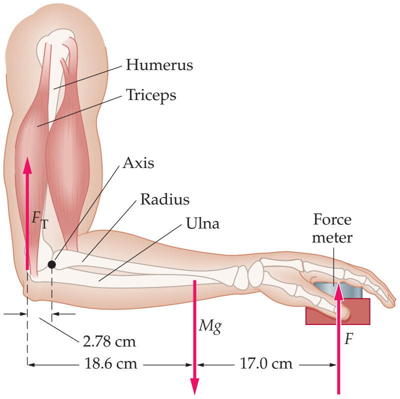
105. •• **BIO** **Deltoid Muscle** A crossing guard holds a STOP sign at arm's length, as shown in **Figure 11–45**. Her arm is horizontal, and we assume that the deltoid muscle is the only muscle supporting her arm. The weight of her upper arm is $W_u = 18$ N, the weight of her lower arm is $W_l = 11$ N, the weight of her hand is $W_h = 4.0$ N, and the weight of the sign is $W_s = 8.9$ N. The location where each of these forces acts on the arm is indicated in the figure. A force of magnitude f_d is exerted on the humerus by the deltoid, and the shoulder joint exerts a force on the humerus with horizontal and vertical components given by f_x, and f_y, respectively. **(a)** Is the magnitude of f_d greater than, less than, or equal to the magnitude of f_x? Explain. Find **(b)** f_d, **(c)** f_x, and **(d)** f_y. (The weights in Figure 11–45 are drawn to scale; the unknown forces are to be determined. If a force is found to be negative, its direction is opposite to that shown.)



Free-Body Diagram of the Arm

▲ **FIGURE 11–45** Problem 105

106. •• **BIO** **Triceps** To determine the force a person's triceps muscle can exert, a doctor uses the procedure shown in **Figure 11–46**, where the patient pushes down with the palm of his hand on a force meter. Given that the weight of the lower arm is $Mg = 15.6$ N, and that the force meter reads $F = 89.0$ N, what is the force F_T exerted vertically upward by the triceps?

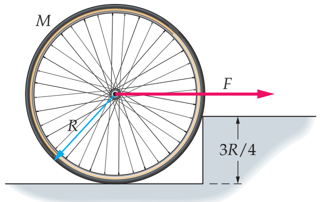


▲ **FIGURE 11–46** Problem 106

107. •• **IP** Suppose a fourth book, the same as the other three, is added to the stack of books shown in Figure 11–32. **(a)** What is the maximum overhang distance, d, in this case? **(b)** If the mass of each book is increased by the same amount, does your answer to part (a) increase, decrease, or stay the same? Explain.

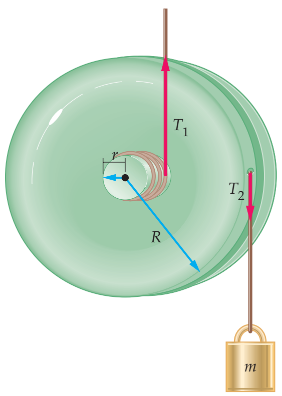
108. •• **IP** Suppose partial melting of the polar ice caps increases the moment of inertia of the Earth from $0.331\ M_E R_E^2$ to $0.332\ M_E R_E^2$. **(a)** Would the length of a day (the time required for the Earth to complete one revolution about its axis) increase or decrease? Explain. **(b)** Calculate the change in the length of a day. Give your answer in seconds.

109. ••• A bicycle wheel of radius R and mass M is at rest against a step of height $3R/4$, as illustrated in **Figure 11–47**. Find the minimum horizontal force F that must be applied to the axle to make the wheel start to rise up over the step.



▲ **FIGURE 11–47** Problem 109

110. ••• A 0.101-kg yo-yo has an outer radius R that is 5.60 times greater than the radius r of its axle. The yo-yo is in equilibrium if a mass m is suspended from its outer edge, as shown in **Figure 11–48**. Find the tension in the two strings, T_1 and T_2, and the mass m.



▲ **FIGURE 11–48** Problem 110

111. ••• In Problem 36, assume that the rod has a mass of M and that its bottom end simply rests on the floor, held in place by static friction. If the coefficient of static friction is μ_s, find the maximum force F that can be applied to the rod at its midpoint before it slips.

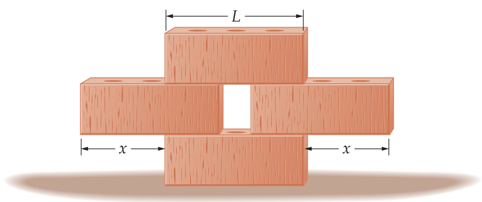
112. ••• In the previous problem, suppose the rod has a mass of 2.3 kg and the coefficient of static friction is 1/7. **(a)** Find the greatest force F that can be applied at the midpoint of the rod without causing it to slip. **(b)** Show that if F is applied 1/8 of the way down from the top of the rod, it will never slip at all, no matter how large the force F.

113. ••• A cylinder of mass m and radius r has a string wrapped around its circumference. The upper end of the string is held fixed, and the cylinder is allowed to fall. Show that its linear acceleration is $(2/3)g$.

114. ••• Repeat the previous problem, replacing the cylinder with a solid sphere. Show that its linear acceleration is $(5/7)g$.

115. ••• A mass M is attached to a rope that passes over a disk-shaped pulley of mass m and radius r. The mass hangs to the left side of the pulley. On the right side of the pulley, the rope is pulled downward with a force F. Find **(a)** the acceleration of the mass, **(b)** the tension in the rope on the left side of the pulley, and **(c)** the tension in the rope on the right side of the pulley. **(d)** Check your results in the limits $m \to 0$ and $m \to \infty$.

116. ••• **Bricks in Equilibrium** Consider a system of four uniform bricks of length L stacked as shown in **Figure 11–49**. What is the maximum distance, x, that the middle bricks can be displaced outward before they begin to tip?



▲ **FIGURE 11–49** Problem 116

PASSAGE PROBLEMS

BIO Correcting Torsiversion

Torsiversion is a medical condition in which a tooth is rotated away from its normal position about the long axis of the root. Studies show that about 2 percent of the population suffer from this condition to some degree. For those who do, the improper alignment of the tooth can lead to tooth-to-tooth collisions during eating, as well as other problems. Typical patients display a rotation ranging from 20° to 60°, with an average around 30°.

An example is shown in **Figure 11–50 (a)**, where the first premolar is not only displaced slightly from its proper location in the negative y direction, but also rotated clockwise from its normal orientation. To correct this condition, an orthodontist might use an archwire and a bracket to apply both a force and a torque to the tooth. In the simplest case, two forces are applied to the tooth in different locations, as indicated by F_1 and F_2 in Figure 11–50 (a). These two forces, if chosen properly, can reposition the tooth by exerting a net force in the positive y direction, and also reorient it by applying a torque in the counterclockwise direction.

In a typical case, it may be desired to have a net force in the positive y direction of 1.8 N. In addition, the distances in Figure 11–50 (a) can be taken to be $d = 3.2$ mm and $D = 4.5$ mm. Given these conditions, a range of torques is possible for various values of the y components of the forces, F_{1y} and F_{2y}. For example, **Figure 11–50 (b)** shows the values of F_{1y} and F_{2y} necessary to produce a given torque, where the torque is measured about the center of the tooth (which is also the origin of the coordinate system). Notice that the two forces always add to 1.8 N in the positive y direction, though one of the forces changes sign as the torque is increased.

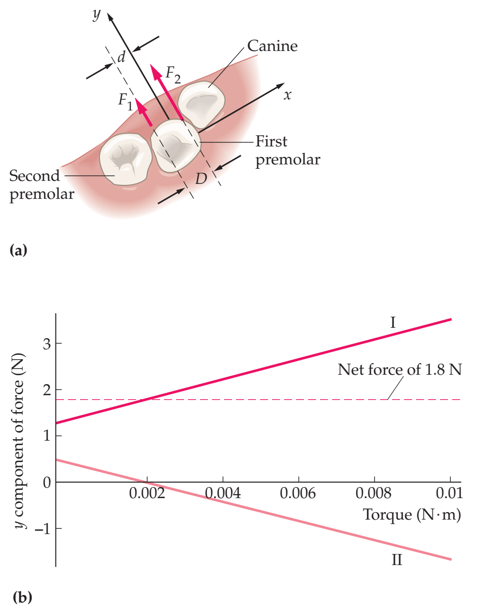


FIGURE 11–50 Problems 117, 118, 119, and 120

117. • The two, solid straight lines in Figure 11–50 (b) represent the two forces applied to the tooth. Which line corresponds to which force?

A. I = F_{1y}, II = F_{2y} **B.** I = F_{2y}, II = F_{1y}

118. • What is the value of the torque that corresponds to one of the forces being equal to zero?

A. 0.0023 N · m **B.** 0.0058 N · m

C. 0.0081 N · m **D.** 0.017 N · m

119. •• Find the values of F_{1y} and F_{2y} required to give zero net torque.

A. $F_{1y} = -1.2$ N, $F_{2y} = 3.0$ N **B.** $F_{1y} = 1.1$ N, $F_{2y} = 0.75$ N

C. $F_{1y} = -0.73$ N, $F_{2y} = 2.5$ N **D.** $F_{1y} = 0.52$ N, $F_{2y} = 1.3$ N

120. •• Find the values of F_{1y} and F_{2y} required to give a net torque of 0.0099 N · m. This is a torque that would be effective at rotating the tooth.

A. $F_{1y} = -1.7$ N, $F_{2y} = 3.5$ N **B.** $F_{1y} = -3.8$ N, $F_{2y} = 5.6$ N

C. $F_{1y} = -0.23$ N, $F_{2y} = 2.0$ N **D.** $F_{1y} = 4.0$ N, $F_{2y} = -2.2$ N

INTERACTIVE PROBLEMS

121. •• **Referring to Example 11–7** Suppose the mass of the pulley is doubled, to 0.160 kg, and that everything else in the system remains the same. **(a)** Do you expect the value of T_2 to increase, decrease, or stay the same? Explain. **(b)** Calculate the value of T_2 for this case.

122. •• **Referring to Example 11–7** Suppose the mass of the cart is doubled, to 0.62 kg, and that everything else in the system remains the same. **(a)** Do you expect the value of T_2 to increase, decrease, or stay the same? Explain. **(b)** Calculate the value of T_2 for this case.

123. •• **Referring to Active Example 11–5** Suppose the child runs with a different initial speed, but that everything else in the system remains the same. What initial speed does the child have if the angular speed of the system after the collision is 0.425 rad/s?

124. •• **Referring to Active Example 11–5** Suppose everything in the system is as described in Active Example 11–5 except that the child approaches the merry-go-round in a direction that is not tangential. Find the angle θ between the direction of motion and the outward radial direction (as in Example 11–8) that is required if the final angular speed of the system is to be 0.272 rad/s.

PHYSICS IN

PERSPECTIVE

Momentum: A Conserved Quantity

When objects interact, momentum may be conserved while mechanical energy is dissipated. Why? These pages explore momentum conservation and point out key differences between momentum and mechanical energy.

1 How do linear and angular momentum relate?

The equations of linear and angular momentum are analogous, and all the principles presented on these pages apply to angular as well as linear momentum.

	Definition	Newton's 2nd law
Linear momentum:	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$	$\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}} = \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t}$
Angular momentum:	$\vec{\mathbf{L}} = I\vec{\boldsymbol{\omega}}$	$\Sigma\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}} = \dfrac{\Delta\vec{\mathbf{L}}}{\Delta t}$

Analogous quantities

Linear	Angular
Acceleration $\vec{\mathbf{a}}$	Angular acceleration $\vec{\boldsymbol{\alpha}}$
Force $\vec{\mathbf{F}}$	Torque $\vec{\boldsymbol{\tau}}$
Velocity $\vec{\mathbf{v}}$	Angular velocity $\vec{\boldsymbol{\omega}}$
Mass m	Moment of inertia I

2 Why is momentum conserved?

Momentum conservation follows from Newton's laws.

Recall that the general form of Newton's second law relates force to momentum:

An object's change in momentum … … equals the net force acting on the object …

$$\Sigma\vec{\mathbf{F}} = \Delta\vec{\mathbf{p}}/\Delta t \quad \text{or} \quad \Delta\vec{\mathbf{p}} = (\Sigma\vec{\mathbf{F}})\Delta t$$

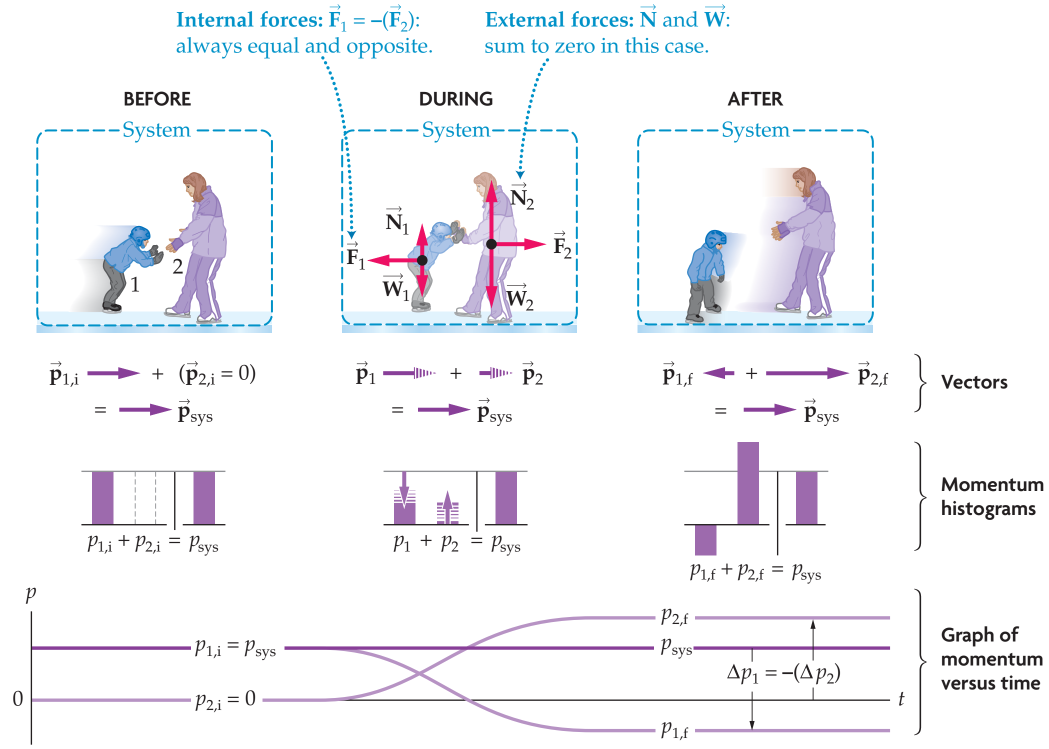
… multiplied by the time over which the force acts.

For an individual object, momentum is conserved (does not change) when the net force acting on the object is zero (that is, $\Delta\vec{\mathbf{p}} = 0$ when $\Sigma\vec{\mathbf{F}} = 0$).

For a system of objects, momentum conservation follows from Newton's third law:

- The momentum of a system of objects is the vector sum of the momenta of the individual objects.
- The forces between objects in the system (**internal forces**) cannot change the system's momentum because, by Newton's third law, the objects exert *equal but opposite forces* on each other, which cause *equal and opposite momentum changes.*
- **Thus, only external forces can change the momentum of a system.**

In the following interaction, the two skaters undergo equal and opposite momentum changes, whereas the system's momentum $\vec{\mathbf{p}}_{\text{sys}}$ is conserved.



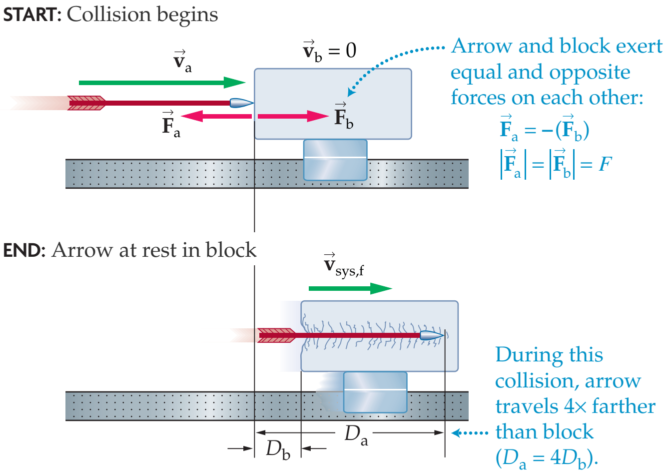
3 How can momentum be conserved when mechanical energy is not?

Force times time versus force times distance: Momentum change is due to *a force acting over a time* Δt, whereas changes in mechanical energy result from *a force acting over a distance* D (i.e., from work):

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}(\Delta t) \qquad \Delta E = W = F(D)$$

How do these relationships apply to the inelastic collision shown below?

Arrow shot into styrofoam block attached to air-track cart



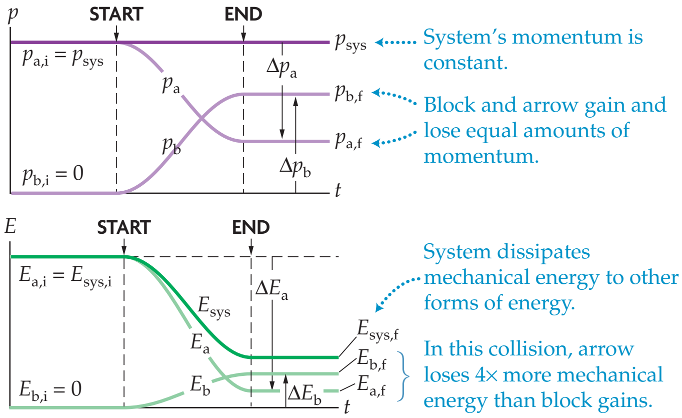
Momentum: The collision lasts the same time Δt for the arrow and block, so their mometum changes are equal and opposite:

$$\Delta\vec{\mathbf{p}}_a = \vec{\mathbf{F}}_a\Delta t = -(\vec{\mathbf{F}}_b)\Delta t = -\Delta\vec{\mathbf{p}}_b$$

Mechanical energy: The objects exert the same force magnitude F on each other, but the arrow travels farther during the collision because it penetrates the block: $D_a > D_b$. Thus, the arrow loses more mechanical energy than the block gains:

$$\Delta E_a = F_a(D_a) = -40\text{ J} \qquad \Delta E_b = F_b(D_b) = +10\text{ J}$$

Conclusion: The collision dissipates mechanical energy while conserving momentum, as the following graphs show:

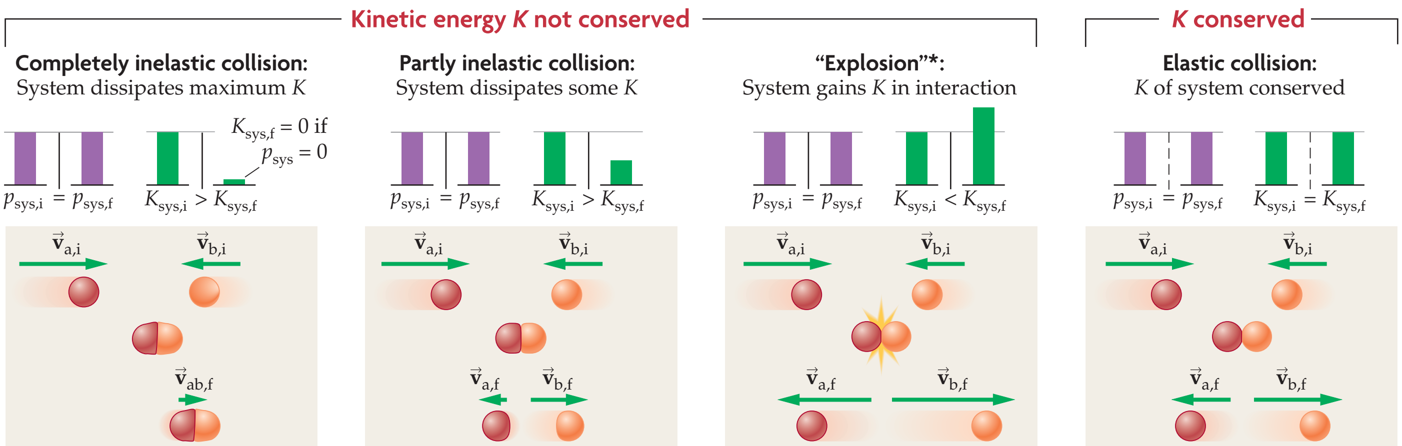


4 How does momentum conservation help us solve problems?

- You can use momentum conservation to analyze any interaction between objects for which the net external force acting on the system during the collision is zero (or is negligible compared to the internal forces).
- If the net external force is not negligible, you cannot use momentum conservation! This applies to the players at right, who push on the ground while colliding.
- For elastic collisions, you must use conservation of mechanical energy as well as conservation of momentum. (Section 9.6 solves these simultaneous equations for special cases.)

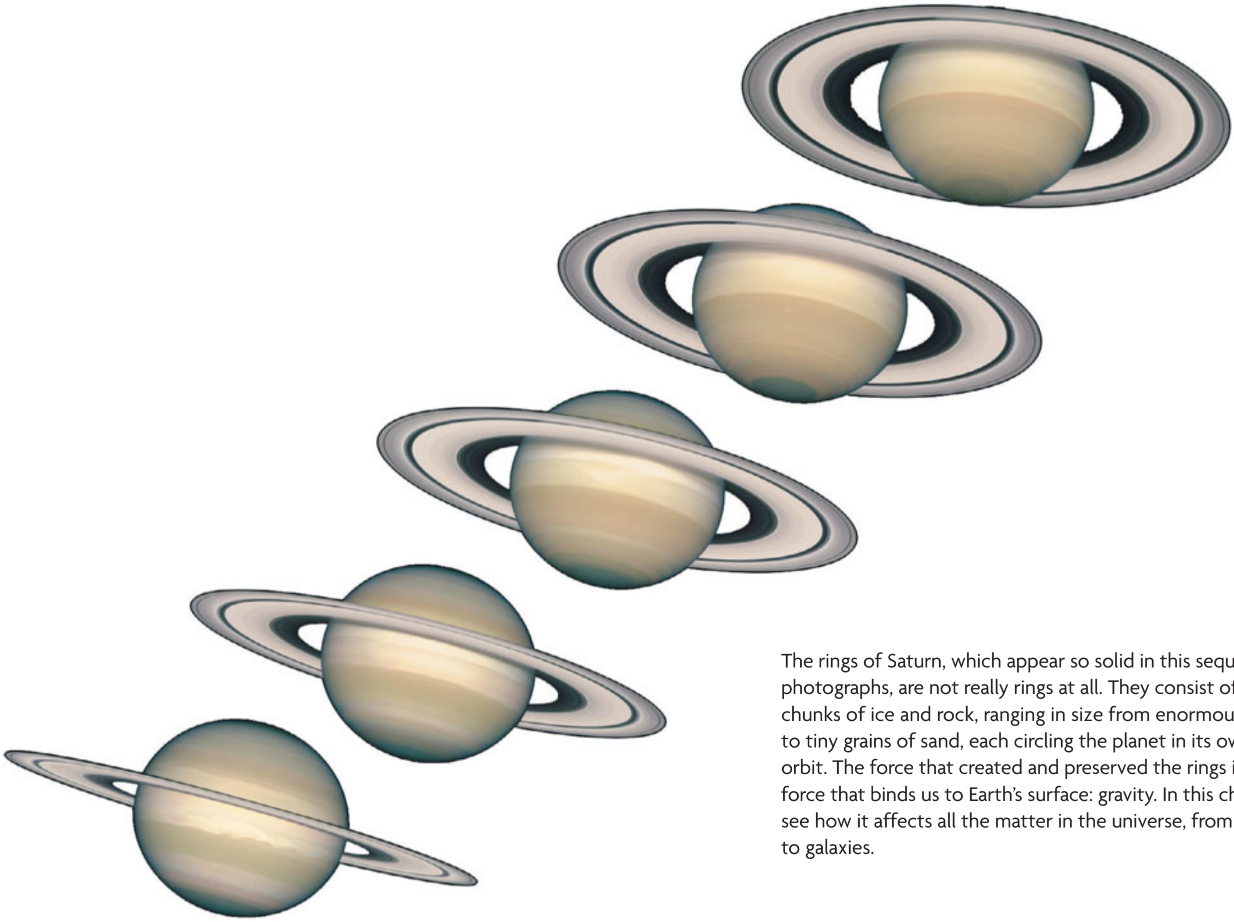
Collisions in which momentum is conserved can be categorized as follows, according to how kinetic energy changes or is conserved.

The external reaction forces of the earth on these players' feet cannot be ignored.



*Physicists use the term "explosion" to mean any interaction that adds kinetic energy to the system. Thus, the collision in Step 2 on the facing page is an explosion.

12 Gravity



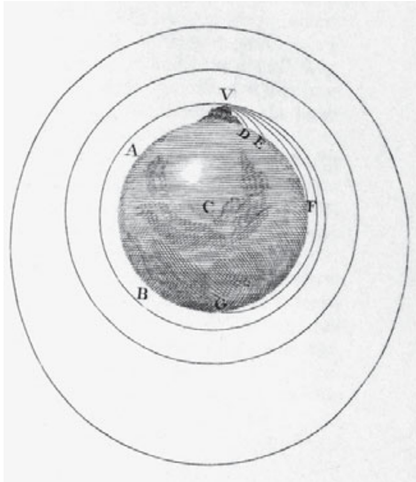
The rings of Saturn, which appear so solid in this sequence of photographs, are not really rings at all. They consist of countless chunks of ice and rock, ranging in size from enormous boulders to tiny grains of sand, each circling the planet in its own individual orbit. The force that created and preserved the rings is the same force that binds us to Earth's surface: gravity. In this chapter we'll see how it affects all the matter in the universe, from dust motes to galaxies.

The study of gravity has always been a central theme in physics, from Galileo's early experiments on free fall in the seventeenth century, to Einstein's general theory of relativity in the early years of the twentieth century, and Stephen Hawking's work on black holes in recent years. Perhaps the grandest milestone in this endeavor, however, was the discovery by Newton of the **universal law of gravitation**. With just one simple equation to describe the force of gravity, Newton was able to determine the orbits of planets, moons, and comets, and to explain such earthly phenomena as the tides and the fall of an apple.

Before Newton's work, it was generally thought that the heavens were quite separate from the Earth, and that they obeyed their own "heavenly" laws. Newton showed, on the contrary, that the same law of gravity that operates on the surface of the Earth applies to the Moon and to other astronomical objects. As a result of Newton's efforts, physics expanded its realm of applicability to natural phenomena throughout the universe.

So successful was Newton's law of gravitation that Edmond Halley (1656–1742) was able to use it to predict the return of the comet that today bears his name. Though he did not live to see its return in 1758, the fact that the comet did reappear when predicted was an event unprecedented in human history. Roughly a hundred years later, Newton's theory of gravity scored an even more impressive success. Astronomers observing the planet Uranus noticed small deviations in its orbit, which they thought might be due to the gravitational tug of a previously unknown planet. Using Newton's law to calculate the predicted position of the new planet—now called Neptune—it was found on the very first night of observations, September 23, 1846. The fact that Neptune was precisely where the law of gravitation said it should be still stands as one of the most astounding triumphs in the history of science.

Today, Newton's law of gravitation is used to determine the orbits that take spacecraft from the Earth to various destinations within our solar system and beyond. Appropriately enough, spacecraft were even sent to view Halley's comet at close range in 1986. In addition, the law allows us to calculate with pinpoint accuracy the time of solar eclipses and other astronomical events in the distant past and remote future. This incredibly powerful and precise law of nature is the subject of this chapter.



▲ In this illustration from his great work, the *Principia*, published in 1687, Newton presents a "thought experiment" to show the connection between free fall and orbital motion. Imagine throwing a projectile horizontally from the top of a mountain. The greater the initial speed of the projectile, the farther it travels in free fall before striking the ground. In the absence of air resistance, a great enough initial speed could result in the projectile circling the Earth and returning to its starting point. Thus, an object orbiting the Earth is actually in free fall—it simply has a large horizontal speed.

12–1 Newton's Law of Universal Gravitation

It's ironic, but the first fundamental force of nature to be recognized as such, **gravity**, is also the weakest of the fundamental forces. Still, it is the force most apparent to us in our everyday lives, and is the force responsible for the motion of the Moon, the Earth, and the planets. Yet the connection between falling objects on Earth and planets moving in their orbits was not known before Newton.

The flash of insight that came to Newton—whether it was due to seeing an apple fall to the ground or not—is simply this: The force causing an apple to accelerate downward is the same force causing the Moon to move in a circular path around the Earth. To put it another way, Newton was the first to realize that the Moon is *constantly falling* toward the Earth, though without ever getting closer to it, and that it falls for the same reason that an apple falls. This is illustrated in a classic drawing due to Newton, shown to the right.

To be specific, in the case of the apple the motion is linear as it accelerates downward toward the center of the Earth. In the case of the Moon the motion is circular with constant speed. As discussed in Section 6–5, an object in uniform circular motion accelerates toward the center of the circle. It follows, therefore, that the Moon *also* accelerates toward the center of the Earth. In fact, the force responsible for the Moon's centripetal acceleration is the Earth's gravitational attraction, the same force responsible for the fall of the apple.

To describe the force of gravity, Newton proposed the following simple law:

Newton's Law of Universal Gravitation

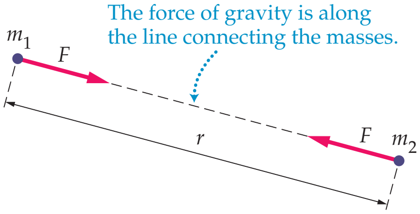
The force of gravity between any two point objects of mass m_1 and m_2 is attractive and of magnitude

$$F = G\frac{m_1 m_2}{r^2} \qquad 12\text{–}1$$

In this expression, r is the distance between the masses, and G is a constant referred to as the **universal gravitation constant**. Its value is

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2} \qquad 12\text{–}2$$

The force is directed along the line connecting the masses, as indicated in **Figure 12–1**.



▲ **FIGURE 12–1 Gravitational force between point masses**

Two point masses, m_1 and m_2, separated by a distance r exert equal and opposite attractive forces on one another. The magnitude of the forces, F, is given by Equation 12–1.

Note that each mass experiences a force of the same magnitude, $F = Gm_1m_2/r^2$, but acting in opposite directions. That is, the force of gravity between two objects forms an action-reaction pair.

According to Newton's law, all objects in the universe attract all other objects in the universe by way of the gravitational interaction. It is in this sense that the force law is termed "universal." Thus, the net gravitational force acting on you is due not only to the planet on which you stand, which is certainly responsible for the majority of the net force, but also to people nearby, planets, and even stars in far-off galaxies. In short, everything in the universe "feels" everything else, thanks to gravity.

The fact that G is such a small number means that the force of gravity between objects of human proportions is imperceptibly small. This is shown in the following Exercise.

EXERCISE 12–1

A man takes his dog for a walk on a deserted beach. Treating people and dogs as point objects for the moment, find the force of gravity between the 105-kg man and his 11.2-kg dog when they are separated by a distance of **(a)** 1.00 m and **(b)** 10.0 m.

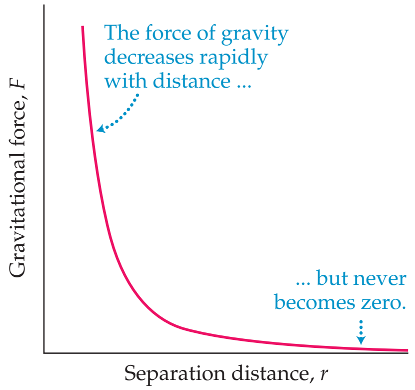
SOLUTION

a. Substituting numerical values into Equation 12–1 yields

$$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(105\ \mathrm{kg})(11.2\ \mathrm{kg})}{(1.00\ \mathrm{m})^2} = 7.84 \times 10^{-8}\ \mathrm{N}$$

b. Repeating the calculation for $r = 10.0$ m gives

$$F = G\frac{m_1 m_2}{r^2} = (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(105\ \mathrm{kg})(11.2\ \mathrm{kg})}{(1.00\ \mathrm{m})^2} = 7.84 \times 10^{-10}\ \mathrm{N}$$



▲ **FIGURE 12–2 Dependence of the gravitational force on separation distance, r**

The $1/r^2$ dependence of the gravitational force means that it decreases rapidly with distance. Still, it never completely vanishes. For this reason, we say that gravity is a force of infinite range; that is, every mass in the universe experiences a nonzero force from every other mass in the universe, no matter how far away.

The forces found in Exercise 12–1 are imperceptibly small. In comparison, the force exerted by the Earth on the man is 1030 N and the force exerted on the dog is 110 N—these forces are several orders of magnitude greater than the force between the man and the dog. In general, gravitational forces are significant only when large masses, such as the Earth or the Moon, are involved.

Exercise 12–1 also illustrates how rapidly the force of gravity decreases with distance. In particular, since F varies as $1/r^2$, it is said to have an **inverse square dependence** on distance. Thus, for example, an increase in distance by a factor of 10 results in a decrease in the force by a factor of $10^2 = 100$. A plot of the force of gravity versus distance is given in **Figure 12–2**. Note that even though the force diminishes rapidly with distance, it never completely vanishes; thus, we say that gravity is a force of infinite range.

Note also that the force of gravity between two masses depends on the product of the masses, m_1 times m_2. With this type of dependence, it follows that if either mass is doubled, the force of gravity is doubled as well. This would not be the case, for example, if the force of gravity depended on the *sum* of the masses, $m_1 + m_2$.

Finally, if a given mass is acted on by gravitational interactions with a number of other masses, the net force acting on it is simply the vector sum of each of the forces individually. This property of gravity is referred to as **superposition**. As an example, superposition implies that the net gravitational force exerted on you at this moment is the vector sum of the force exerted by the Earth, plus the force exerted by the Moon, plus the force exerted by the Sun, and so on. The following Example illustrates superposition.

PROBLEM-SOLVING NOTE

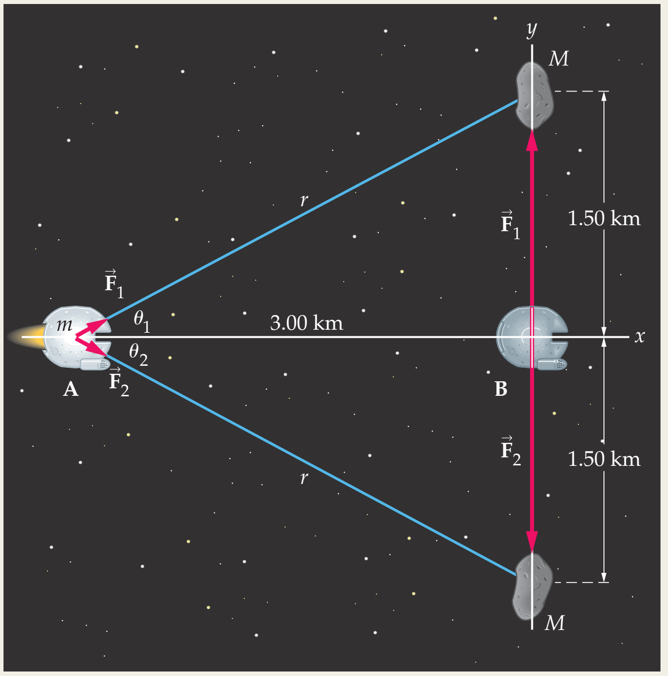
Net Gravitational Force

To find the net gravitational force acting on an object, you should (i) resolve each of the forces acting on the object into components and (ii) add the forces component by component.

EXAMPLE 12–1 HOW MUCH FORCE IS WITH YOU?

As part of a daring rescue attempt, the *Millennium Eagle* passes between a pair of twin asteroids, as shown. If the mass of the spaceship is 2.50×10^7 kg and the mass of each asteroid is 3.50×10^{11} kg, find the net gravitational force exerted on the *Millennium Eagle* **(a)** when it is at location A and **(b)** when it is at location B. Treat the spaceship and the asteroids as if they were point objects.

PICTURE THE PROBLEM
Our sketch shows the spaceship as it follows a path between the twin asteroids. The relevant distances and masses are indicated, as are the two points of interest, A and B. Note that at location A the force $\vec{\mathbf{F}}_1$ points above the x axis at the angle θ_1 (to be determined); the force $\vec{\mathbf{F}}_2$ points below the x axis at the angle $\theta_2 = -\theta_1$, as can be seen by symmetry. At location B, the two forces act in opposite directions.



STRATEGY
To find the net gravitational force exerted on the spaceship, we first determine the magnitude of the force exerted on it by each asteroid. This is done by using Equation 12–1 and the distances given in our sketch. Next, we resolve these forces into x and y components. Finally, we sum the force components to find the net force.

SOLUTION

Part (a)

1. Use the Pythagorean theorem to find the distance r from point A to each asteroid. Also, refer to the sketch to find the angle between $\vec{\mathbf{F}}_1$ and the x axis. The angle between $\vec{\mathbf{F}}_2$ and the x axis has the same magnitude but the opposite sign:

$$r = \sqrt{(3.00 \times 10^3\ \mathrm{m})^2 + (1.50 \times 10^3\ \mathrm{m})^2} = 3350\ \mathrm{m}$$
$$\theta_1 = \tan^{-1}\left(\frac{1.50 \times 10^3\ \mathrm{m}}{3.00 \times 10^3\ \mathrm{m}}\right) = \tan^{-1}(0.500) = 26.6°$$
$$\theta_2 = -\theta_1 = -26.6°$$

2. Use r and Equation 12–1 to calculate the magnitude of the forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ at point A:

$$F_1 = F_2 = G\frac{mM}{r^2}$$
$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(2.50 \times 10^7\ \mathrm{kg})(3.50 \times 10^{11}\ \mathrm{kg})}{(3350\ \mathrm{m})^2}$$
$$= 52.0\ \mathrm{N}$$

3. Use the values of θ_1 and θ_2 found in Step 1 to calculate the x and y components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$:

$$F_{1,x} = F_1 \cos\theta_1 = (52.0\ \mathrm{N})\cos 26.6° = 46.5\ \mathrm{N}$$
$$F_{1,y} = F_1 \sin\theta_1 = (52.0\ \mathrm{N})\sin 26.6° = 23.3\ \mathrm{N}$$
$$F_{2,x} = F_2 \cos\theta_2 = (52.0\ \mathrm{N})\cos(-26.6°) = 46.5\ \mathrm{N}$$
$$F_{2,y} = F_2 \sin\theta_2 = (52.0\ \mathrm{N})\sin(-26.6°) = -23.3\ \mathrm{N}$$

4. Add the components of $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ to find the components of the net force, $\vec{\mathbf{F}}$:

$$F_x = F_{1,x} + F_{2,x} = 93.0\ \mathrm{N}$$
$$F_y = F_{1,y} + F_{2,y} = 0$$

Part (b)

5. Use Equation 12–1 to find the magnitude of the forces exerted on the spaceship by the asteroids at location B:

$$F_1 = F_2 = G\frac{mM}{r^2}$$
$$= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(2.50 \times 10^7\ \mathrm{kg})(3.50 \times 10^{11}\ \mathrm{kg})}{(1.50 \times 10^3\ \mathrm{m})^2}$$
$$= 259\ \mathrm{N}$$

6. Use the fact that $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ have equal magnitudes and point in opposite directions to determine the net force, $\vec{\mathbf{F}}$, acting on the spaceship:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = 0$$

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INSIGHT

We find that the net force at location A is in the positive x direction, as one would expect by symmetry. At location B, where the force exerted by each asteroid is about 5 times greater than it is at location A, the *net* force is zero since the attractive forces exerted by the two asteroids are equal and opposite, and thus cancel. Note that the forces in our sketch have been drawn in correct proportion.

Rocket scientists often use the gravitational force between astronomical objects and spacecraft to accelerate the spacecraft and send them off to distant parts of the solar system. In fact, this gravitational attraction makes possible the "slingshot" effect illustrated in Figure 9–31.

PRACTICE PROBLEM

Find the net gravitational force acting on the spaceship when it is at the location $x = 5.00 \times 10^3$ m, $y = 0$. [**Answer:** 41.0 N in the negative x direction]

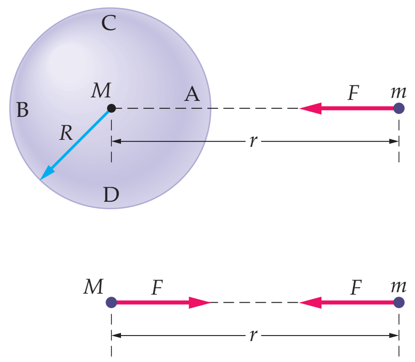
Some related homework problems: Problem 9, Problem 11, Problem 12

12–2 Gravitational Attraction of Spherical Bodies

Newton's law of gravity applies to point objects. How, then, do we calculate the force of gravity for an object of finite size? In general, the approach is to divide the finite object into a collection of small mass elements, then use superposition and the methods of calculus to determine the net gravitational force. For an arbitrary shape, this calculation can be quite difficult. For objects with a uniform spherical shape, however, the final result is remarkably simple, as was shown by Newton.

Uniform Sphere

Consider a uniform sphere of radius R and mass M, as in **Figure 12–3**. A point object of mass m is brought near the sphere, though still outside it at a distance r from its center. The object experiences a relatively strong attraction from mass near the point A, and a weaker attraction from mass near point B. In both cases the force is along the line connecting the mass m and the center of the sphere; that is, along the x axis. In addition, mass at the points C and D exert a net force that is also along the x axis—just as in the case of the twin asteroids in Example 12–1. Thus, the symmetry of the sphere guarantees that the net force it exerts on m is directed toward the sphere's center. The magnitude of the force exerted by the sphere must be calculated with the methods of calculus—which Newton invented and then applied to this problem. As a result of his calculations, Newton was able to show that **the net force exerted by the sphere on the mass m is the same as if all the mass of the sphere were concentrated at its center.** That is, the force between the mass m and the sphere of mass M has a magnitude that is simply



▲ **FIGURE 12–3 Gravitational force between a point mass and a sphere**
The force is the same as if all the mass of the sphere were concentrated at its center.

$$F = G\frac{mM}{r^2} \qquad 12\text{–}3$$

Let's apply this result to the case of a mass m on the surface of the Earth. If the mass of the Earth is M_E, and its radius is R_E, it follows that the force exerted on m by the Earth is

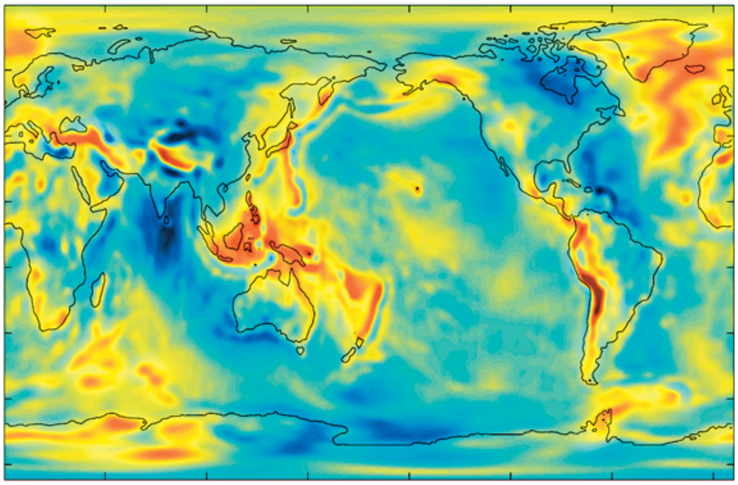
$$F = G\frac{mM_E}{{R_E}^2} = m\left(\frac{GM_E}{{R_E}^2}\right)$$

We also know, however, that the gravitational force experienced by a mass m on the Earth's surface is simply $F = mg$, where g is the acceleration due to gravity. Therefore, we see that

$$m\left(\frac{GM_E}{{R_E}^2}\right) = mg$$

or

$$g = \frac{GM_E}{{R_E}^2} = \frac{(6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2})(5.97 \times 10^{24}\,\mathrm{kg})}{(6.37 \times 10^6\,\mathrm{m})^2} = 9.81\,\mathrm{m/s^2} \qquad 12\text{–}4$$



This global model of the Earth's gravitational strength was constructed from a combination of surface gravity measurements and satellite tracking data. It shows how the acceleration of gravity varies from the value at an idealized "sea level" that takes into account the Earth's nonspherical shape. (The Earth is somewhat flattened at the poles—its radius is greatest at the equator.) Gravity is strongest in the red areas and weakest in the dark blue areas.

This result can be extended to objects above the Earth's surface, and hence farther from the center of the Earth, as we show in the next Example.

EXAMPLE 12–2 THE DEPENDENCE OF GRAVITY ON ALTITUDE

REAL-WORLD PHYSICS

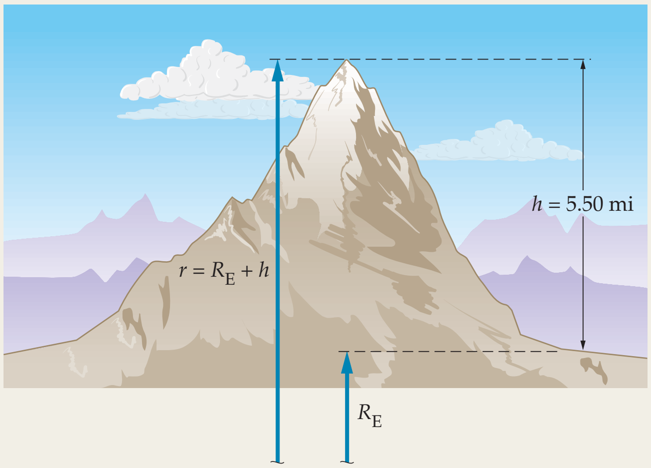
If you climb to the top of Mt. Everest, you will be about 5.50 mi above sea level. What is the acceleration due to gravity at this altitude?

PICTURE THE PROBLEM

At the top of the mountain, your distance from the center of the Earth is $r = R_E + h$, where $h = 5.50$ mi is the altitude.

STRATEGY

First, use $F = GmM_E/r^2$ to find the force due to gravity on the mountaintop. Then, set $F = mg_h$ to find the acceleration g_h at the height h.



SOLUTION

1. Calculate the force F due to gravity at a height h above the Earth's surface:

$$F = G\frac{mM_E}{(R_E + h)^2}$$

2. Set F equal to mg_h and solve for g_h:

$$F = G\frac{mM_E}{(R_E + h)^2} = mg_h$$

$$g_h = G\frac{M_E}{(R_E + h)^2}$$

3. Factor out R_E^2 from the denominator, and use the fact that $GM_E/R_E^2 = g$:

$$g_h = \left(\frac{GM_E}{R_E^2}\right)\frac{1}{\left(1 + \frac{h}{R_E}\right)^2} = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2}$$

4. Substitute numerical values, with $h = 5.50\text{ mi} = (5.50\text{ mi})(1609\text{ m/mi}) = 8850\text{ m}$, and $R_E = 6.37 \times 10^6$ m:

$$g_h = \frac{g}{\left(1 + \frac{h}{R_E}\right)^2} = \frac{9.81\text{ m/s}^2}{\left(1 + \frac{8850\text{ m}}{6.37 \times 10^6\text{ m}}\right)^2} = 9.78\text{ m/s}^2$$

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INSIGHT

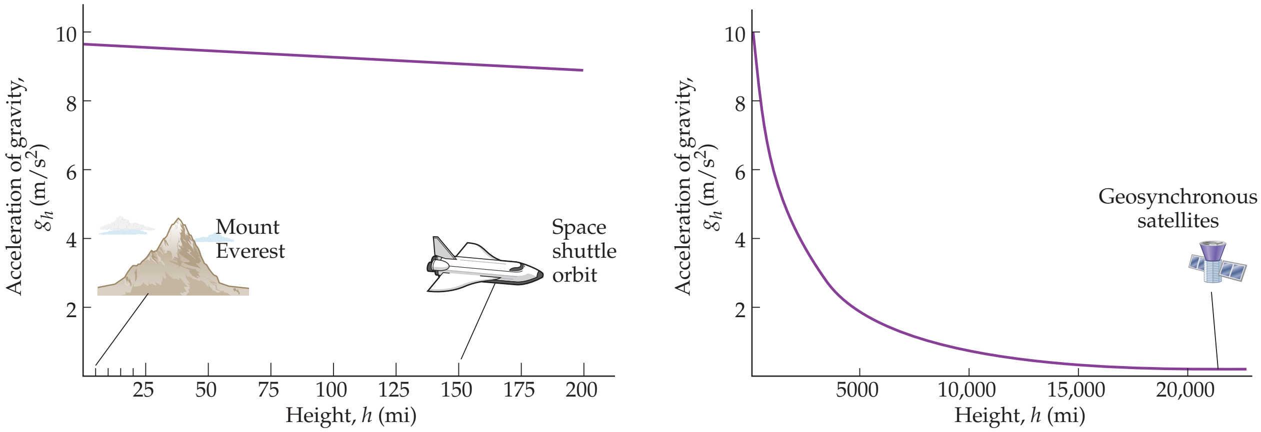
As expected, the acceleration due to gravity is less as one moves farther from the center of the Earth. Thus, if you were to climb to the top of Mt. Everest, you would lose weight—not only because of the physical exertion required for the climb, but also because of the reduced gravity. In particular, a person with a mass of 60 kg (about 130 lb) would lose about half a pound of weight just by standing on the summit of the mountain.

A plot of g_h as a function of h is shown in **Figure 12–4 (a)**. The plot indicates the altitude of Mt. Everest and the orbit of the space shuttle. **Figure 12–4 (b)** shows g_h out to the orbit of communications and weather satellites, which orbit at an altitude of roughly 22,300 mi.

PRACTICE PROBLEM

Find the acceleration due to gravity at the altitude of the space shuttle's orbit, 250 km above the Earth's surface. [**Answer:** $g_h = 9.08 \text{ m/s}^2$, a reduction of only 7.44% compared to the acceleration of gravity on the surface of the Earth.]

Some related homework problems: Problem 15, Problem 17



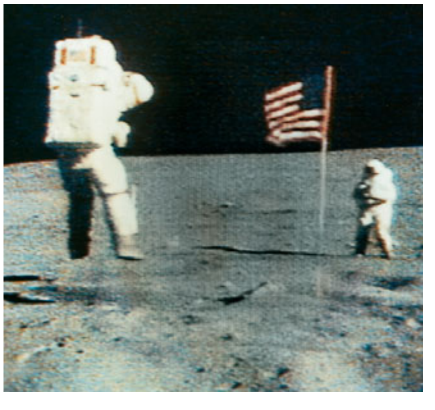
(a) Acceleration of gravity near the Earth's surface

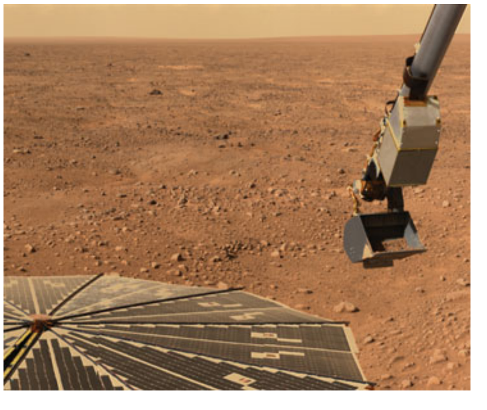
(b) Acceleration of gravity far from the Earth

FIGURE 12–4 ▲ **The acceleration due to gravity at a height *h* above the Earth's surface**

(a) In this plot, the peak of Mt. Everest is at about $h = 5.50$ mi, and the space shuttle orbit is at roughly $h = 150$ mi. **(b)** This shows the decrease in the acceleration of gravity from the surface of the Earth to an altitude of about 25,000 mi. The orbit of geosynchronous satellites—ones that orbit above a fixed point on the Earth—is at roughly $h = 22{,}300$ mi.

Equation 12–4 can be used to calculate the acceleration due to gravity on other objects in the solar system besides the Earth. For example, to calculate the acceleration due to gravity on the Moon, g_m, we simply use the mass and radius of the Moon in Equation 12–4. Once g_m is known, the weight of an object of mass m on the Moon is found by using $W_m = mg_m$.





▶ (Left) The weak lunar gravity permits astronauts, even encumbered by their massive space suits, to bound over the Moon's surface. The low gravitational pull, only about one-sixth that of Earth, is a consequence not only of the Moon's smaller size, but also of its lower average density. (Right) The force of gravity on the surface of Mars is only about 38% of its strength on Earth. This was an important factor in designing NASA's Phoenix Mars Lander, shown here lifting a scoop of dirt on its 16th Martian day after landing in May 2008.

EXERCISE 12–2

a. Find the acceleration due to gravity on the surface of the Moon.

b. The lunar rover had a mass of 225 kg. What was its weight on the Earth and on the Moon? (*Note:* The mass of the Moon is $M_m = 7.35 \times 10^{22}$ kg and its radius is $R_m = 1.74 \times 10^6$ m.)

SOLUTION

a. For the Moon, the acceleration due to gravity is

$$g_m = \frac{GM_m}{R_m^{\ 2}} = \frac{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(7.35 \times 10^{22}\ \text{kg})}{(1.74 \times 10^6\ \text{m})^2} = 1.62\ \text{m/s}^2$$

This is about one-sixth the acceleration due to gravity on the Earth.

b. On the Earth, the rover's weight was

$$W = mg = (225\ \text{kg})(9.81\ \text{m/s}^2) = 2210\ \text{N}$$

On the Moon, its weight was

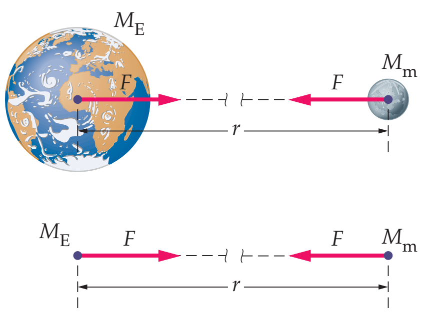
$$W_m = mg_m = (225\ \text{kg})(1.62\ \text{m/s}^2) = 365\ \text{N}$$

As expected, this is roughly one-sixth its Earth weight.

The replacement of a sphere with a point mass at its center can be applied to many physical systems. For example, the force of gravity between two spheres of finite size is the same as if *both* were replaced by point masses. Thus, the gravitational force between the Earth, with mass M_E, and the Moon, with mass M_m, is

$$F = G\frac{M_E M_m}{r^2}$$

The distance r in this expression is the center-to-center distance between the Earth and the Moon, as shown in **Figure 12–5**. It follows, then, that in many calculations involving the solar system, moons and planets can be treated as point objects.



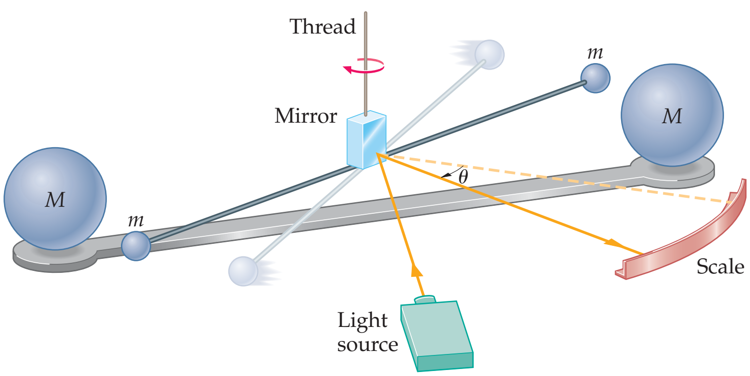
▲ FIGURE 12–5 Gravitational force between the Earth and the Moon

The force is the same as if both the Earth and the Moon were point masses. (The sizes of the Earth and Moon are in correct proportion in this figure, but the separation between the two should be much greater than that shown here. In reality, it is about 30 times the diameter of the Earth, and so would be about 2 ft on this scale.)

Weighing the Earth

The British physicist Henry Cavendish performed an experiment in 1798 that is often referred to as "weighing the Earth." What he did, in fact, was measure the value of the universal gravitation constant, G, that appears in Newton's law of gravity. As we have pointed out before, G is a very small number; hence a sensitive experiment is needed for its measurement. It is because of this experimental difficulty that G was not measured until more than 100 years after Newton published the law of gravitation.

In the Cavendish experiment, illustrated in **Figure 12–6**, two masses m are suspended from a thin thread. Near each suspended mass is a large stationary mass M, as shown. Each suspended mass is attracted by the force of gravity toward the large mass near it; hence the rod holding the suspended masses tends to rotate and twist the thread. The angle through which the thread twists can be measured by bouncing a beam of light from a mirror attached to the thread. If the force required to twist the thread through a given angle is known (from previous experiments), a measurement of the twist angle gives the magnitude of the force of gravity. Finally, knowing the masses m and M, and the distance between their centers, r, we can use Equation 12–1 to solve for G. Cavendish found $6.754 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$, in good agreement with the currently accepted value given in Equation 12–2.



◀ FIGURE 12–6 The Cavendish experiment

The gravitational attraction between the masses m and M causes the rod and the suspending thread to twist. Measurement of the twist angle allows for a direct measurement of the gravitational force.

To see why Cavendish is said to have weighed the Earth, recall that the force of gravity on the surface of the Earth, mg, can be written as follows:

$$mg = G\frac{mM_E}{{R_E}^2}$$

Canceling m and solving for M_E yields

$$M_E = \frac{g{R_E}^2}{G} \qquad \text{12–5}$$

Before the Cavendish experiment, the quantities g and R_E were known from direct measurement, but G had yet to be determined. When Cavendish measured G, he didn't actually "weigh" the Earth, of course. Instead, he calculated its mass, M_E.

EXERCISE 12–3

Use $M_E = g{R_E}^2/G$ to calculate the mass of the Earth.

SOLUTION

Substituting numerical values, we find

$$M_E = \frac{g{R_E}^2}{G} = \frac{(9.81\ \mathrm{m/s^2})(6.37 \times 10^6\ \mathrm{m})^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 5.97 \times 10^{24}\ \mathrm{kg}$$

As soon as Cavendish determined the mass of the Earth, geologists were able to use the result to calculate its average density; that is, its average mass per volume. Assuming a spherical Earth of radius R_E, its total volume is

$$V_E = \tfrac{4}{3}\pi {R_E}^3 = \tfrac{4}{3}\pi(6.37 \times 10^6\ \mathrm{m})^3 = 1.08 \times 10^{21}\ \mathrm{m^3}$$

Dividing this into the total mass yields the average density, ρ:

$$\rho = \frac{M_E}{V_E} = \frac{5.97 \times 10^{24}\ \mathrm{kg}}{1.08 \times 10^{21}\ \mathrm{m^3}} = 5530\ \mathrm{kg/m^3} = 5.53\ \mathrm{g/cm^3}$$

REAL-WORLD PHYSICS

The internal structure of the Earth and the Moon

This is an interesting result because typical rocks found near the surface of the Earth, such as granite, have a density of only about $3.00\ \mathrm{g/cm^3}$. We conclude, then, that the interior of the Earth must have a greater density than its surface. In fact, by analyzing the propagation of seismic waves around the world, we now know that the Earth has a rather complex interior structure, including a solid inner core with a density of about $15.0\ \mathrm{g/cm^3}$ (see Section 10–5).

A similar calculation for the Moon yields an average density of about $3.33\ \mathrm{g/cm^3}$, essentially the same as the density of the lunar rocks brought back during the Apollo program. Hence, it is likely that the Moon does not have an internal structure similar to that of the Earth.

Since G is a universal constant—with the same value everywhere in the universe—it can be used to calculate the mass of other bodies in the solar system as well. This is illustrated in the following Example.

EXAMPLE 12–3 MARS ATTRACTS!

After landing on Mars, an astronaut performs a simple experiment by dropping a rock. A quick calculation using the drop height and the time of fall yields a value of $3.73\ \mathrm{m/s^2}$ for the rock's acceleration. **(a)** Find the mass of Mars, given that its radius is $R_M = 3.39 \times 10^6$ m. **(b)** What is the acceleration of gravity due to Mars at a distance $2R_M$ from the center of the planet?

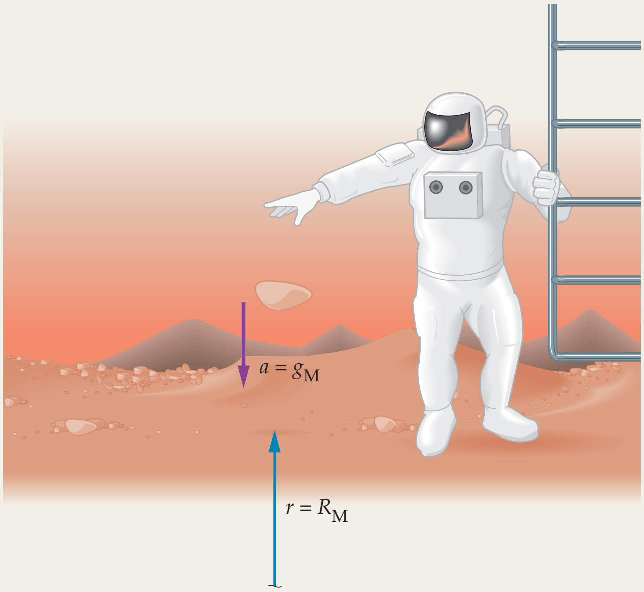
PICTURE THE PROBLEM

Our sketch shows an astronaut dropping a rock to the ground on the surface of Mars. If the acceleration of the rock is measured, we find $g_M = 3.73\ \mathrm{m/s^2}$, where the subscript M refers to Mars. In addition, we indicate the radius of Mars in our sketch, where $R_M = 3.39 \times 10^6$ m.

STRATEGY

a. Since the acceleration of gravity is g_M on the surface of Mars, it follows that the force of gravity on an object of mass m is $F = mg_M$. This force is also given by Newton's law of gravity—that is, $F = GmM_M/R_M^2$. Setting these expressions for the force equal to one another yields the mass of Mars, M_M.

b. Set $F = ma$ equal to $F = GmM_M/(2R_M)^2$ and solve for the acceleration, a.



SOLUTION

Part (a)

1. Set mg_M equal to GmM_M/R_M^2:
$$mg_M = G\frac{mM_M}{R_M^2}$$

2. Cancel m and solve for the mass of Mars:
$$M_M = \frac{g_M R_M^2}{G}$$

3. Substitute numerical values:
$$M_M = \frac{(3.73\ \mathrm{m/s^2})(3.39 \times 10^6\ \mathrm{m})^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 6.43 \times 10^{23}\ \mathrm{kg}$$

Part (b)

4. Apply Newton's law of gravity with $r = 2R_M$. Use the fact that $g_M = GM_M/R_M^2$ from Step 1 to simplify the calculation:
$$ma = G\frac{mM_M}{(2R_M)^2} \quad \text{or}$$
$$a = G\frac{M_M}{(2R_M)^2} = \tfrac{1}{4}\left(G\frac{M_M}{R_M^2}\right) = \tfrac{1}{4}(g_M) = \tfrac{1}{4}(3.73\ \mathrm{m/s^2}) = 0.933\ \mathrm{m/s^2}$$

INSIGHT

The important point here is that the universal gravitation constant, G, applies as well on Mars as on Earth, or any other object. Therefore, knowledge of the size and acceleration of gravity of an astronomical body is sufficient to determine its mass.

PRACTICE PROBLEM

If the radius of Mars were reduced to 3.00×10^6 m, with its mass remaining the same, would the acceleration of gravity on Mars increase, decrease, or stay the same? Check your answer by calculating the acceleration of gravity for this case. [**Answer:** The acceleration of gravity increases to $4.77\ \mathrm{m/s^2}$.]

Some related homework problems: Problem 20, Problem 21

12–3 Kepler's Laws of Orbital Motion

If you go outside each clear night and observe the position of Mars with respect to the stars, you will find that its apparent motion across the sky is rather complex. Instead of moving on a simple curved path, it occasionally reverses direction (this is known as *retrograde motion*). A few months later it reverses direction yet again and resumes its original direction of motion. Other planets exhibit similar odd behavior.

The Danish astronomer Tycho Brahe (1546–1601) followed the paths of the planets, and Mars in particular, for many years, even though the telescope had not yet been invented. He used, instead, an elaborate sighting device to plot the precise position of the planets. Brahe was joined in his work by Johannes Kepler (1571–1630) in 1600, and after Brahe's death, Kepler inherited his astronomical observations.

Kepler made good use of Brahe's life work, extracting from his carefully collected data the three laws of orbital motion we know today as Kepler's laws. These laws make it clear that the Sun and the planets do not orbit the Earth, as Ptolemy—the ancient Greek astronomer—claimed, but rather that the Earth, along with the other planets, orbit the Sun, as proposed by Copernicus (1473–1543).

Why the planets obey Kepler's laws no one knew—not even Kepler—until Newton considered the problem decades after Kepler's death. Newton was able to show that each of Kepler's laws follows as a direct consequence of the universal law of gravitation. In the remainder of this section we consider Kepler's three laws one at a time, and point out the connection between them and the law of gravitation.

Kepler's First Law

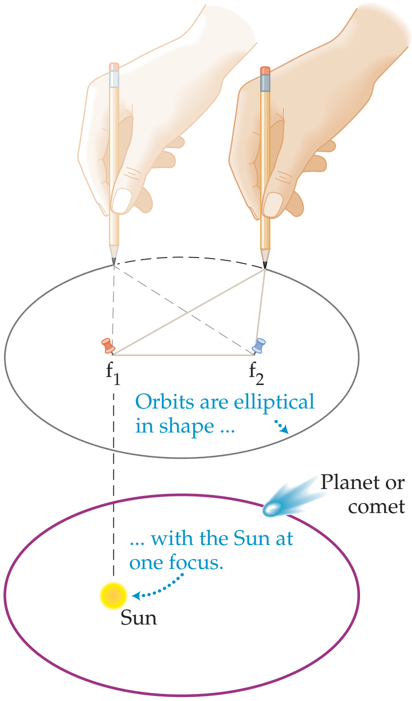
Kepler tried long and hard to find a circular orbit around the Sun that would match Brahe's observations of Mars. After all, up to that time everyone from Ptolemy to Copernicus believed that celestial objects moved in circular paths of one sort or another. Though the orbit of Mars was exasperatingly close to being circular, the small differences between a circular path and the experimental observations just could not be ignored. Eventually, after a great deal of hard work and disappointment over the loss of circular orbits, Kepler discovered that Mars followed an orbit that was elliptical rather than circular. The same applied to the other planets. This observation became Kepler's first law:

Planets follow elliptical orbits, with the Sun at one focus of the ellipse.

This is a fine example of the scientific method in action. Though Kepler expected and wanted to find circular orbits, he would not allow himself to ignore the data. If Brahe's observations had not been so accurate, Kepler probably would have chalked up the small differences between the data and a circular orbit to error. As it was, he had to discard a treasured—but incorrect—theory, and move on to an unexpected, but ultimately correct, view of nature.

Kepler's first law is illustrated in **Figure 12–7**, along with a definition of an ellipse in terms of its two foci. In the case where the two foci merge, as in **Figure 12–8**, the ellipse reduces to a circle. Thus, a circular orbit *is* allowed by Kepler's first law, but only as a special case.

Newton was able to show that, because the force of gravity decreases with distance as $1/r^2$, closed orbits must have the form of ellipses or circles, as stated in Kepler's first law. He also showed that orbits that are not closed—say the orbit of a comet that passes by the Sun once and then leaves the solar system—are either parabolic or hyperbolic.



▲ **FIGURE 12–7 Drawing an ellipse**

To draw an ellipse, put two tacks in a piece of cardboard. The tacks define the "foci" of the ellipse. Now connect a length of string to the two tacks, and use a pencil and the string to sketch out a smooth closed curve, as shown. This closed curve is an ellipse. In a planetary orbit a planet follows an elliptical path, with the Sun at one focus. Nothing is at the other focus.

Kepler's Second Law

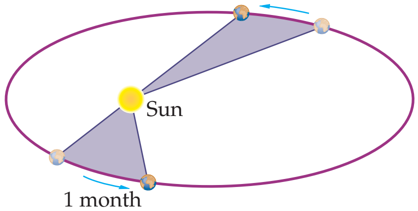
When Kepler plotted the position of a planet on its elliptical orbit, indicating at each position the time the planet was there, he made an interesting observation. First, draw a line from the Sun to a planet at a given time. Then a certain time later—perhaps a month—draw a line again from the Sun to the new position of the planet. The result is that the planet has "swept out" a wedge-shaped area, as indicated in **Figure 12–9 (a)**. If this procedure is repeated when the planet is on a different part of its orbit, another wedge-shaped area is generated. Kepler's observation was that the areas of these two wedges are equal:

As a planet moves in its orbit, it sweeps out an equal amount of area in an equal amount of time.

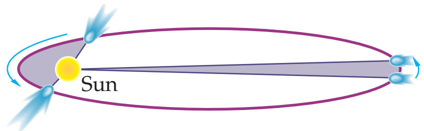
Kepler's second law follows from the fact that the force of gravity on a planet is directly toward the Sun. As a result, gravity exerts zero torque about the Sun,

▶ **FIGURE 12–8 The circle as a special case of the ellipse**

As the two foci of an ellipse approach one another, the ellipse becomes more circular. In the limit that the foci merge, the ellipse becomes a circle.



(a) Equal areas in equal times



(b) Equal areas in equal times for highly elliptical orbit

FIGURE 12–9 Kepler's second law
(a) The second law states that a planet sweeps out equal areas in equal times. **(b)** In a highly elliptical orbit, the long, thin area is equal to the broad, fan-shaped area.

which means that the angular momentum of a planet in its orbit must be conserved. As Newton showed, conservation of angular momentum is equivalent to the equal-area law stated by Kepler.

CONCEPTUAL CHECKPOINT 12–1 COMPARE SPEEDS

The Earth's orbit is slightly elliptical. In fact, the Earth is closer to the Sun during the northern hemisphere winter than it is during the summer. Is the speed of the Earth during winter **(a)** greater than, **(b)** less than, or **(c)** the same as its speed during summer?

REASONING AND DISCUSSION

According to Kepler's second law, the area swept out by the Earth per month is the same in winter as it is in summer. In winter, however, the radius from the Sun to the Earth is less than it is in summer. Therefore, if this smaller radius is to sweep out the same area, the Earth must move more rapidly.

ANSWER

(a) The speed of the Earth is greater during the winter.

Though we have stated the first two laws in terms of planets, they apply equally well to any object orbiting the Sun. For example, a comet might follow a highly elliptical orbit, as in **Figure 12–9 (b)**. When it is near the Sun, it moves very quickly, for the reason discussed in Conceptual Checkpoint 12–1, sweeping out a broad wedge-shaped area in a month's time. Later in its orbit, the comet is far from the Sun and moving slowly. In this case, the area it sweeps out in a month is a long, thin wedge. Still, the two wedges have equal areas.

Kepler's Third Law

Finally, Kepler studied the relation between the mean distance of a planet from the Sun, r, and its period—that is, the time, T, it takes for the planet to complete one orbit. **Figure 12–10** shows a plot of period versus distance for the planets of the solar system. Kepler tried to "fit" these results to a simple dependence between T and r. If he tried a linear fit—that is, T proportional to r (the bottom curve in Figure 12–10)—he found that the period did not increase rapidly enough with distance. On the other hand, if he tried T proportional to r^2 (the top curve in Figure 12–10), the period increased too rapidly. Splitting the difference, and trying T proportional to $r^{3/2}$, yields a good fit (the middle curve in Figure 12–10). This is Kepler's third law:

> The period, T, of a planet increases as its mean distance from the Sun, r, raised to the 3/2 power. That is,
>
> $$T = (\text{constant})r^{3/2} \qquad 12\text{–}6$$

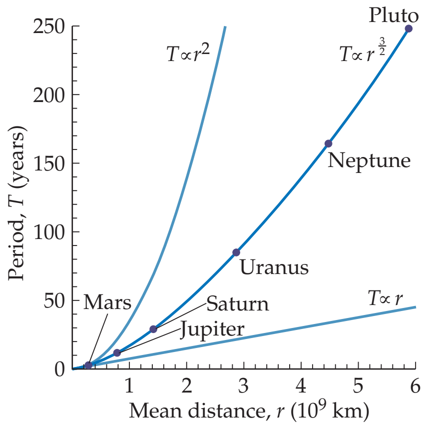


FIGURE 12–10 Kepler's third law and some near misses
These plots represent three possible mathematical relationships between period of revolution, T (in years), and mean distance from the Sun, r (in kilometers). The lower curve shows $T = (\text{constant})r$; the upper curve is $T = (\text{constant})r^2$. The middle curve, which fits the data, is $T = (\text{constant})r^{3/2}$. This is Kepler's third law.

It is straightforward to derive this result for the special case of a circular orbit. Consider, then, a planet orbiting the Sun at a distance r, as in **Figure 12–11**. Since the planet moves in a circular path, a centripetal force must act on it, as we saw in Section 6–5. In addition, this force must be directed toward the center of the circle; that is, toward the Sun. It is as if you were to swing a ball on the end of a string in a circle above your head, as in Figure 6–12 (p. 169). In order for the ball to move in a circular path, you have to exert a force on the ball toward the center of the circular path. This force is exerted through the string. In the case of a planet orbiting the Sun, the centripetal force is provided by the force of gravity between the Sun and the planet.

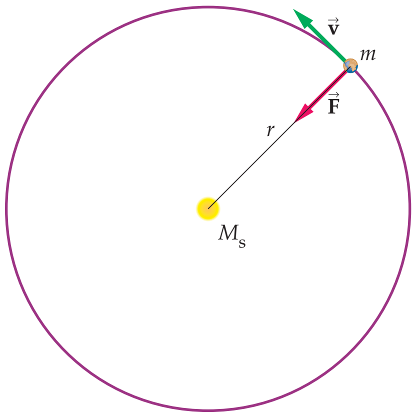


FIGURE 12–11 Centripetal force on a planet in orbit
As a planet revolves about the Sun in a circular orbit of radius r, the force of gravity between it and the Sun, $F = GM_s/r^2$, provides the required centripetal force.

If the planet has a mass m, and the Sun has a mass M_s, the force of gravity between them is

$$F = G\frac{mM_s}{r^2}$$

Now, this force creates the centripetal acceleration of the planet, a_{cp}, which, according to Equation 6–15, is

$$a_{cp} = \frac{v^2}{r}$$

Thus, the centripetal force necessary for the planet to orbit is ma_{cp}:

$$F = ma_{cp} = m\frac{v^2}{r}$$

Since the speed of the planet, v, is the circumference of the orbit, $2\pi r$, divided by the time to complete an orbit, T, we have

$$F = m\frac{v^2}{r} = m\frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 rm}{T^2}$$

Setting the centripetal force equal to the force of gravity yields

$$\frac{4\pi^2 rm}{T^2} = G\frac{mM_s}{r^2}$$

Eliminating m and rearranging, we find

$$T^2 = \frac{4\pi^2}{GM_s}r^3$$

or

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right)r^{3/2} = (\text{constant})r^{3/2} \qquad \text{12–7}$$

As predicted by Kepler, T is proportional to $r^{3/2}$.

Deriving Kepler's third law by using Newton's law of gravitation has allowed us to calculate the constant that multiplies $r^{3/2}$. Note that the constant depends on the mass of the Sun; that is, *T depends on the mass being orbited*. It does not depend on the mass of the planet orbiting the Sun, however, as long as the planet's mass is much less than the mass of the Sun. As a result, Equation 12–7 applies equally to all the planets.

PROBLEM-SOLVING NOTE

The Mass in Kepler's Third Law

When applying Kepler's third law, recall that the mass in Equation 12–7, M_s, refers to the mass of the object being orbited. Thus, the third law can be applied to satellites of any object, as long as M_s is replaced by the orbited mass.

This result can also be applied to the case of a moon or a satellite (an artificial moon) orbiting a planet. To do so, we simply note that it is the planet that is being orbited, not the Sun. Hence, to apply Equation 12–7, we just replace the mass of the Sun, M_s, with the mass of the appropriate planet.

As an example, let's calculate the mass of Jupiter. One of the four moons of Jupiter discovered by Galileo is Io, which completes one orbit every 42 h 27 min =

▶ Kepler's laws of orbital motion apply to planetary satellites as well as planets. Jupiter, the largest planet in the solar system, has at least 16 moons, all of which travel in elliptical orbits that obey Kepler's laws. (The moons in the photo at left, passing in front of Jupiter, are Io and Europa, two of the four largest Jovian satellites discovered by Galileo in 1609.) Even some asteroids have been found to have their own satellites. The large cratered object in the photo at right is 243 Ida, an asteroid some 56 km long; its miniature companion at the top of the photo is Dactyl, about 1.5 km in diameter. Like all gravitationally bound bodies, Ida and Dactyl orbit their common center of mass.

1.53×10^5 s. Given that the average distance from the center of Jupiter to Io is 4.22×10^8 m, we can find the mass of Jupiter as follows:

$$T = \left(\frac{2\pi}{\sqrt{GM_J}}\right) r^{3/2}$$

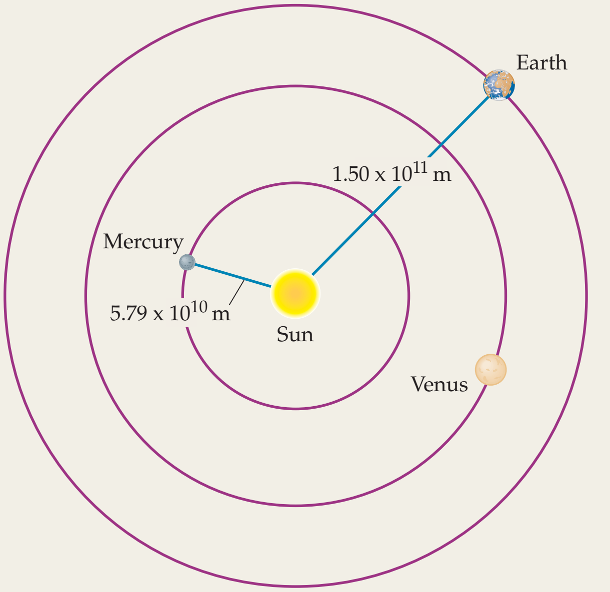
$$M_J = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2 (4.22 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1.53 \times 10^5 \text{ s})^2} = 1.90 \times 10^{27} \text{ kg}$$

EXAMPLE 12–4 THE SUN AND MERCURY

The Earth revolves around the Sun once a year at an average distance of 1.50×10^{11} m. **(a)** Use this information to calculate the mass of the Sun. **(b)** Find the period of revolution for the planet Mercury, whose average distance from the Sun is 5.79×10^{10} m.

PICTURE THE PROBLEM

Our sketch shows the orbits of Mercury, Venus, and the Earth in correct proportion. In addition, each of these orbits is slightly elliptical, though the deviation from circularity is too small for the eye to see. Finally, we indicate that the orbital radius for Mercury is 5.79×10^{10} m and the orbital radius for Earth is 1.50×10^{11} m.



STRATEGY

a. To find the mass of the Sun, we solve Equation 12–7 for M_s. Note that the period $T = 1$ yr must be converted to seconds before we evaluate the formula.

b. The period of Mercury is found by substituting $r = 5.79 \times 10^{10}$ m in Equation 12–7.

SOLUTION

Part (a)

1. Solve Equation 12–7 for the mass of the Sun:

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right) r^{3/2}$$

$$M_s = \frac{4\pi^2 r^3}{GT^2}$$

2. Calculate the period of the Earth in seconds:

$$T = 1 \text{ y}\left(\frac{365.24 \text{ days}}{1 \text{ y}}\right)\left(\frac{24 \text{ hr}}{1 \text{ day}}\right)\left(\frac{3600 \text{ s}}{1 \text{ hr}}\right) = 3.16 \times 10^7 \text{ s}$$

3. Substitute numerical values in the expression for the mass of the Sun obtained in Step 1:

$$M_s = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2 (1.50 \times 10^{11} \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(3.16 \times 10^7 \text{ s})^2}$$

$$= 2.00 \times 10^{30} \text{ kg}$$

Part (b)

4. Substitute $r = 5.79 \times 10^{10}$ m into Equation 12–7. In addition, use the mass of the Sun obtained in part (a):

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right) r^{3/2}$$

$$= \left(\frac{2\pi}{\sqrt{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(2.00 \times 10^{30} \text{ kg})}}\right) \times (5.79 \times 10^{10} \text{ m})^{3/2}$$

$$= 7.58 \times 10^6 \text{ s} = 0.240 \text{ y} = 87.7 \text{ days}$$

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INSIGHT

In part (a), notice that the mass of the Sun is almost a million times more than the mass of the Earth, as determined in Exercise 12–3. In fact, the Sun accounts for 99.9% of all the mass in the solar system.

In part (b) we see that Mercury, with its smaller orbital radius, has a shorter year than the Earth.

PRACTICE PROBLEM

Venus orbits the Sun with a period of 1.94×10^7 s. What is its average distance from the Sun? [**Answer:** $r = 1.08 \times 10^{11}$ m]

Some related homework problems: Problem 28, Problem 32

▲ Many weather and communications satellites are placed in geosynchronous orbits that allow them to remain "stationary" in the sky—that is, fixed over one point on the Earth's equator. Because the Earth rotates, the period of such a satellite must exactly match that of the Earth. The altitude needed for such an orbit is about 36,000 km (see Active Example 12–1). Other satellites, such as those used in the Global Positioning System (GPS), the Hubble Space Telescope, and the American space shuttles, operate at much lower altitudes—typically just a few hundred miles. The photo at left shows the communications satellite Intelsat VI just prior to its capture by astronauts of the space shuttle *Endeavour*. A launch failure had left the satellite stranded in low orbit. The astronauts snared the satellite (right) and fitted it with a new engine that boosted it to its geosynchronous orbit, where it is still in operation today.

REAL-WORLD PHYSICS

Geosynchronous satellites

A *geosynchronous satellite* is one that orbits above the equator with a period equal to one day. From the Earth, such a satellite appears to be in the same location in the sky at all times, making it particularly useful for applications such as communications and weather forecasting. From Kepler's third law, we know that a satellite has a period of one day only if its orbital radius has a particular value. We determine this value in the following Active Example.

ACTIVE EXAMPLE 12–1 FIND THE ALTITUDE OF A GEOSYNCHRONOUS SATELLITE

Find the altitude above the Earth's surface where a satellite orbits with a period of one day ($R_E = 6.37 \times 10^6$ m, $M_E = 5.97 \times 10^{24}$ kg, $T = 1$ day $= 8.64 \times 10^4$ s).

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Rewrite Equation 12–7, using the mass of the Earth in place of the mass of the Sun: $T = \left(2\pi/\sqrt{GM_E}\right)r^{3/2}$
2. Solve for the radius, r: $r = (T/2\pi)^{2/3}(GM_E)^{1/3}$
3. Substitute numerical values: $r = 4.22 \times 10^7$ m
4. Subtract the radius of the Earth to find the altitude: $r - R_E = 3.58 \times 10^7$ m

INSIGHT

Thus, *all* geosynchronous satellites orbit 3.58×10^7 m $\approx$ 22,300 mi above our heads.

YOUR TURN

Find the altitude above the surface of the Moon where a "lunasynchronous" satellite would orbit. [*Note:* The length of a lunar day is one month (27.332 days), which is why we see only one side of the Moon.]

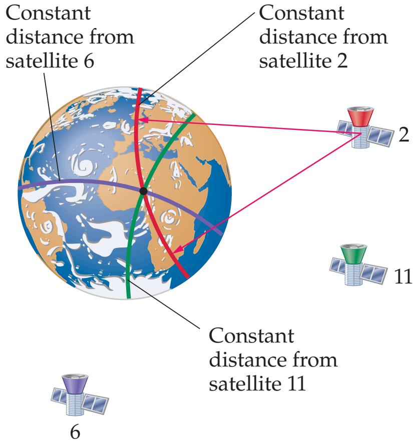
(Answers to **Your Turn** *problems are given in the back of the book.)*

Not all spacecraft are placed in geosynchronous orbits, however. The U.S. space shuttle, for example, orbits at an altitude of about 150 mi. At that altitude, it takes less than an hour and a half to complete one orbit. The International Space Station, operational although still under construction, orbits at a similar altitude.

REAL-WORLD PHYSICS

The Global Positioning System (GPS)

The 24 satellites of the Global Positioning System (GPS) are also in relatively low orbits. These satellites, which have an average altitude of 12,550 mi and orbit the Earth every 12 hours, are used to provide a precise determination of an observer's position anywhere on Earth. The operating principle of the GPS is illustrated in **Figure 12–12**. Imagine, for example, that satellite 2 emits a radio signal at a particular time (all GPS satellites carry atomic clocks on board). This signal travels away from the satellite with the speed of light (see Chapter 25) and is detected a short time later by an observer's GPS receiver. Multiplying the time delay by the speed of light gives the distance of the receiver from satellite 2. Thus, in our example, the observer must lie somewhere on the red circle in Figure 12–12. Similar time delay measurements for signals from satellite 11 show that the observer is also somewhere on the green circle; hence the observer is either at the point shown in Figure 12–12, or at the second intersection of the red and green circles on the other side of the planet. Measurements from satellite 6 can resolve the ambiguity and place the observer at the point shown in the figure. Measurements from additional satellites can even determine the observer's altitude. GPS receivers, which are used by hikers, boaters, and others who need to know their precise location, typically use signals from as many as 12 satellites. As currently operated, the GPS gives positions with a typical accuracy of 2 m to 10 m.



▲ **FIGURE 12–12 The Global Positioning System**

A system of 24 satellites in orbit about the Earth makes it possible to determine a person's location with great accuracy. Measuring the distance of a person from satellite 2 places the person somewhere on the red circle. Similar measurements using satellite 11 place the person's position somewhere on the green circle, and further measurements can pinpoint the person's location.

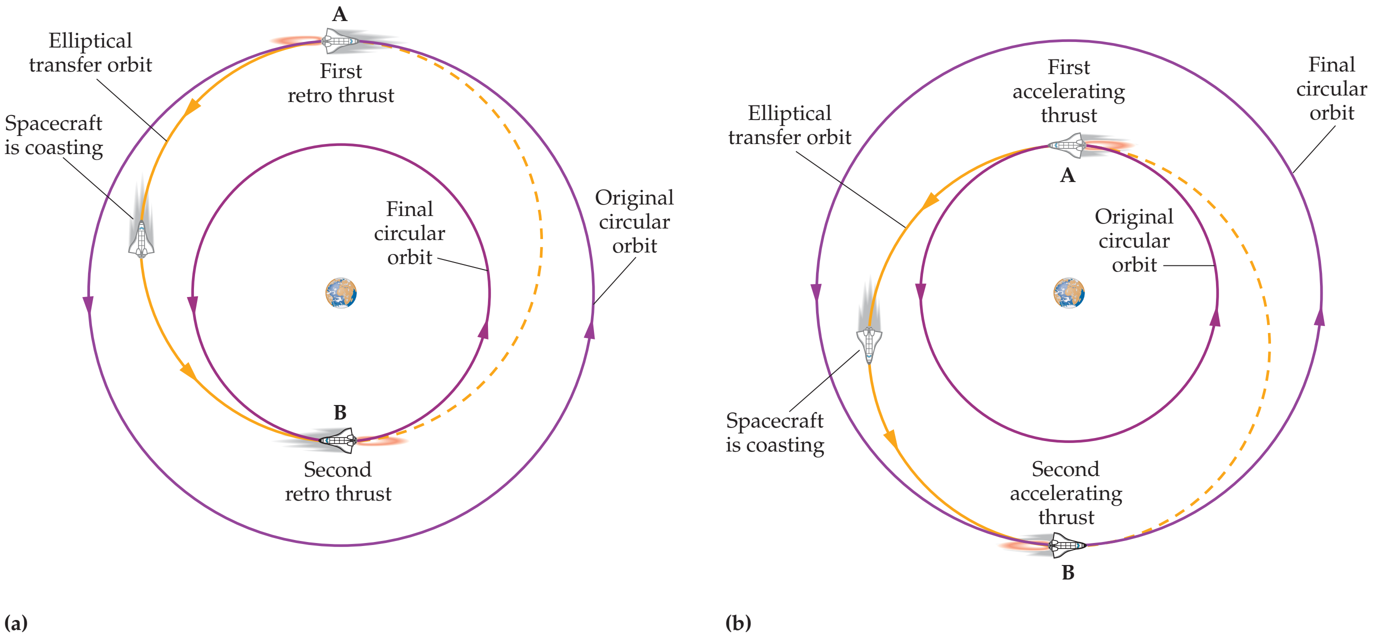
Orbital Maneuvers

We now show how Kepler's laws can give insight into maneuvering a satellite in orbit. Suppose, for example, that you are piloting a spacecraft in a circular orbit, and you would like to move to a lower circular orbit. As you might expect, you should begin by using your rockets to decrease your speed—that is, fire the rockets that point in the forward direction so that their thrust (Section 9–8) is opposite to your direction of motion. The result of firing the decelerating rockets at a given point A in your original orbit is shown in **Figure 12–13 (a)**. Note that your new orbit is not a circle, as desired, but rather an ellipse. To produce a circular orbit you can simply fire the decelerating rockets once again at point B, on the opposite side of the Earth from point A. The net result of these two firings is that you now move in a circular orbit of smaller radius.

REAL-WORLD PHYSICS

Maneuvering spacecraft

Similarly, to move to a larger orbit, you must fire your accelerating rockets twice. The first firing puts you into an elliptical orbit that moves farther from the Earth, as **Figure 12–13 (b)** shows. After the second firing you are again in a circular orbit. This simplest type of orbital transfer, requiring just two rocket burns, is referred to as a *Hohmann transfer*. The Hohmann transfer is the basic maneuver used to send spacecraft such as the Mars lander from Earth's orbit about the Sun to the orbit of Mars.



▲ **FIGURE 12–13 Orbital maneuvers**

(a) The radius of a satellite's orbit can be decreased by firing the decelerating rockets once at point A and again at point B. Between firings the satellite follows an elliptical orbit. The satellite speeds up as it falls inward toward the Earth during this maneuver. For this reason its final speed in the new circular orbit is greater than its speed in the original orbit, even though the decelerating rockets have slowed it down twice. **(b)** The radius of a satellite's orbit can be increased by firing the accelerating rockets once at point A and again at point B. Between firings the satellite follows an elliptical orbit. The satellite slows down as it moves farther from the Earth during this maneuver. For this reason its final speed in the new circular orbit is less than its speed in the original orbit, even though the accelerating rockets have sped it up twice.

CONCEPTUAL CHECKPOINT 12–2 WHICH ROCKETS TO USE?

As you pilot your spacecraft in a circular orbit about the Earth, you notice the space station you want to dock with several miles ahead in the same orbit. To catch up with the space station, should you **(a)** fire your accelerating rockets or **(b)** fire your decelerating rockets?

REASONING AND DISCUSSION

Since you want to catch up with something miles ahead, you must accelerate, right? Well, not in this case. Accelerating moves you into an elliptical orbit, as in Figure 12–13 (b), and with a second acceleration you can make your new orbit circular with a greater radius. Recall from Kepler's third law, however, that the larger the radius of an orbit the larger the period, as Equation 12–7 shows. Thus, on your new higher path you take longer to complete an orbit, so you fall farther behind the space station. The same is true even if you fire your rockets only once and stay on the elliptical orbit—it also has a longer period than the original orbit.

On the other hand, two decelerating burns will put you into a circular orbit of smaller radius, and thus smaller period. As a result, you complete an orbit in less time than before and catch up with the space station. After catching up, you can perform two accelerating burns to move you back into the original orbit to dock.

ANSWER

(b) You should fire your decelerating rockets.

12–4 Gravitational Potential Energy

In Chapter 8 we saw that the principle of conservation of energy can be used to solve a number of problems that would be difficult to handle with a straightforward application of Newton's laws of mechanics. Before we can apply energy conservation to astronomical situations, however, we must know the gravitational potential energy for a spherical object such as the Earth. Now you may be wondering, "Don't we already know the potential energy of gravity?" Well, in fact, in Chapter 8 we said that the gravitational potential energy a distance h above the Earth's surface is $U = mgh$. As was mentioned at the time, however, this result is valid only near the Earth's surface, where we can say that the acceleration of gravity, g, is approximately constant.

As the distance from the Earth increases we know that g decreases, as was shown in Example 12–2. It follows that mgh cannot be valid for arbitrary h. Indeed, it can be shown that the gravitational potential energy of a system consisting of a mass m a distance r from the center of the Earth is

$$U = -G\frac{mM_E}{r} \tag{12–8}$$

A plot of $U = -GmM_E/r$ is presented in **Figure 12–14**. Note that U approaches zero as r approaches infinity. This is a common convention in astronomical systems. In fact, since only *differences* in potential energy matter, as was mentioned in Chapter 8, the choice of the reference point ($U = 0$) is completely arbitrary. When we considered systems that were near the Earth's surface, it was natural to let $U = 0$ at ground level. When we consider, instead, distances of astronomical scale, it is generally more convenient to choose the potential energy to be zero when objects are separated by an infinite distance.

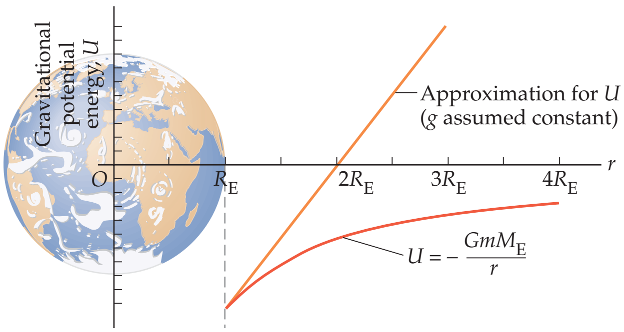


FIGURE 12–14 Gravitational potential energy as a function of the distance *r* from the center of the Earth
The lower curve in this plot shows the gravitational potential energy, $U = -GmM_E/r$, for r greater than R_E. Near the Earth's surface, U is approximately linear, corresponding to the result $U = mgh$ given in Chapter 8.

EXERCISE 12–4

Use Equation 12–8 to find the gravitational potential energy of a 12.0-kg meteorite when it is **(a)** one Earth radius above the surface of the Earth, and **(b)** on the surface of the Earth.

SOLUTION

a. In this case, the distance from the center of the Earth is $2R_E$, thus

$$U = -G\frac{mM_E}{2R_E}$$

$$= -(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})\frac{(12.0\,\mathrm{kg})(5.97 \times 10^{24}\,\mathrm{kg})}{2(6.37 \times 10^6\,\mathrm{m})} = -3.75 \times 10^8\,\mathrm{J}$$

b. Now, the distance from the center of the Earth is R_E, therefore

$$U = -\frac{GmM_E}{R_E}$$

$$= -(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})\frac{(12.0\,\mathrm{kg})(5.97 \times 10^{24}\,\mathrm{kg})}{6.37 \times 10^6\,\mathrm{m}} = -7.50 \times 10^8\,\mathrm{J}$$

Note that the potential energy in part (b) is twice what it was in part (a), since the distance from the center of the Earth to the meteorite has been halved.

At first glance, Equation 12–8 doesn't seem to bear any similarity to mgh, which we know to be valid near the surface of the Earth. Even so, there is a direct connection between these two expressions. Recall that when we say that the potential energy at a height h is mgh, what we mean is that when a mass m is raised from the ground to a height h, the potential energy of the system increases by the amount mgh. Let's calculate the corresponding difference in potential energy using Equation 12–8.

First, at a height h above the surface of the Earth we have $r = R_E + h$; hence the potential energy there is

$$U = -G\frac{mM_E}{R_E + h}$$

On the surface of the Earth, where $r = R_E$, we have

$$U = -G\frac{mM_E}{R_E}$$

The corresponding difference in potential energy is

$$\begin{aligned}\Delta U &= \left(-G\frac{mM_E}{R_E + h}\right) - \left(-G\frac{mM_E}{R_E}\right) \\ &= \left(-G\frac{mM_E}{R_E}\right)\left(\frac{1}{1 + h/R_E}\right) - \left(-G\frac{mM_E}{R_E}\right)\end{aligned}$$

If h is much smaller than the radius of the Earth, it follows that h/R_E is a small number. In this case, we can apply the useful approximation $1/(1 + x) \approx 1 - x$ [see Figure A–5 (b) in Appendix A] to write $1/(1 + h/R_E) \approx 1 - h/R_E$. As a result, we have

$$\Delta U = \left(-G\frac{mM_E}{R_E}\right)(1 - h/R_E) - \left(-G\frac{mM_E}{R_E}\right) = m\left[\frac{GM_E}{{R_E}^2}\right]h$$

The term in square brackets should look familiar—according to Equation 12–4 it is simply g. Hence, the increase in potential energy at the height h is

$$\Delta U = mgh$$

as expected.

The straight line in Figure 12–14 corresponds to the potential energy mgh. Near the Earth's surface, it is clear that mgh and $-GmM_E/r$ are in close agreement. For larger r, however, the fact that gravity is getting weaker means that the potential energy does not continue rising as rapidly as it would if gravity were of constant strength.

An important distinction between the potential energy, U, and the gravitational force, $\vec{\mathbf{F}}$, is that the force is a vector, whereas the potential energy is a scalar—that is, U is simply a number. As a result:

> The total gravitational potential energy of a system of objects is the sum of the gravitational potential energies of each pair of objects separately.

Since U is not a vector, there are no x or y components to consider, as would be the case with a vector. Finally, the potential energy given in Equation 12–8 applies to a mass m and the Earth, with mass M_E. More generally, if two point masses, m_1 and m_2, are separated by a distance r, their gravitational potential energy is

Gravitational Potential Energy, *U*

$$U = -G\frac{m_1 m_2}{r} \qquad 12\text{–}9$$

SI unit: joule, J

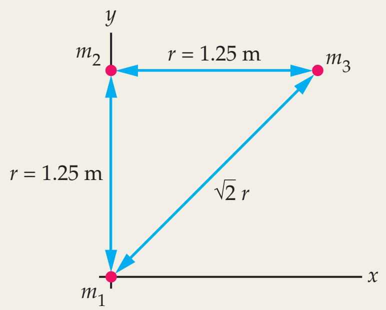
In the next Example we use this result, and the fact that U is a scalar, to find the total gravitational potential energy for a system of three point masses.

EXAMPLE 12–5 SIMPLE ADDITION

Three masses are positioned as follows: $m_1 = 2.5$ kg is at the origin; $m_2 = 0.75$ kg is at $x = 0$, $y = 1.25$ m; and $m_3 = 0.75$ kg is at $x = 1.25$ m and $y = 1.25$ m. Find the total gravitational potential energy of this system.

PICTURE THE PROBLEM

The masses and their positions are shown in our sketch. The horizontal and vertical distances are $r = 1.25$ m; the diagonal distance is $\sqrt{2}r$.



STRATEGY

The potential energy associated with each pair of masses is given by Equation 12–9. The total potential energy of the system is the sum of the potential energy for each of the three pairs of masses.

SOLUTION

1. Use Equation 12–9 to calculate the potential energy for masses 1 and 2:

$$U_{12} = -G\frac{m_1 m_2}{r_{12}}$$
$$= -(6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})\frac{(2.5\ \mathrm{kg})(0.75\ \mathrm{kg})}{(1.25\ \mathrm{m})}$$
$$= -1.0 \times 10^{-10}\ \mathrm{J}$$

2. Similarly, calculate the potential energy for masses 2 and 3:

$$U_{23} = -G\frac{m_2 m_3}{r_{23}}$$
$$= -(6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})\frac{(0.75\ \mathrm{kg})(0.75\ \mathrm{kg})}{(1.25\ \mathrm{m})}$$
$$= -3.0 \times 10^{-11}\ \mathrm{J}$$

3. Do the same calculation for masses 1 and 3:

$$U_{13} = -G\frac{m_1 m_3}{r_{13}}$$
$$= -(6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2})\frac{(2.5\ \mathrm{kg})(0.75\ \mathrm{kg})}{\sqrt{2}(1.25\ \mathrm{m})}$$
$$= -7.1 \times 10^{-11}\ \mathrm{J}$$

4. The total potential energy is the sum of the three contributions calculated above:

$$U_{\mathrm{total}} = U_{12} + U_{23} + U_{13}$$
$$= -1.0 \times 10^{-10}\ \mathrm{J} - 3.0 \times 10^{-11}\ \mathrm{J} - 7.1 \times 10^{-11}\ \mathrm{J}$$
$$= -2.0 \times 10^{-10}\ \mathrm{J}$$

INSIGHT

Note that the total gravitational potential energy of this system, $U_{\mathrm{total}} = -2.0 \times 10^{-10}$ J, is less than it would be if the separation of the masses were to approach infinity, in which case $U_{\mathrm{total}} = 0$. The implications of this change in potential energy, in terms of energy conservation, are considered in the next section.

PRACTICE PROBLEM

If the distance $r = 1.25$ m is reduced by a factor of two to $r = 0.625$ m, does the potential energy of the system increase, decrease, or stay the same? Verify your answer by calculating the potential energy in this case. [**Answer:** The potential energy decreases; that is, it becomes more negative. We find $U = 2(-2.0 \times 10^{-10}\ \mathrm{J})$.]

Some related homework problems: Problem 42, Problem 43

12–5 Energy Conservation

Now that we know the gravitational potential energy, U, at an arbitrary distance from a spherical object, we can apply energy conservation to astronomical situations in the same way we applied it to systems near the Earth's surface in Chapter 8. To be specific, the mechanical energy, E, of an object of mass m a distance r from the Earth is

$$E = K + U = \tfrac{1}{2}mv^2 - G\frac{mM_{\mathrm{E}}}{r} \qquad 12\text{–}10$$

Using energy conservation—that is, setting the initial mechanical energy equal to the final mechanical energy—we can answer questions such as the following: Suppose that an asteroid has zero speed infinitely far from the Earth. If this asteroid were to fall directly toward the Earth, what speed would it have when it strikes the Earth's surface?

REAL-WORLD PHYSICS

The impact of meteorites

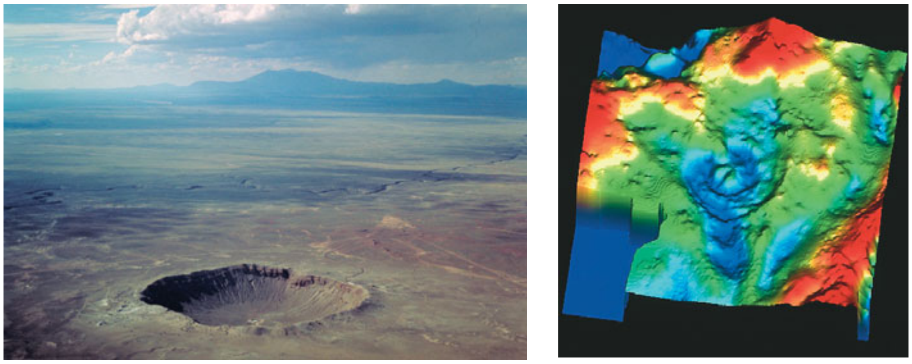
As you probably know, this is not an entirely academic question. Asteroids and comets, both large and small, have struck the Earth innumerable times during its history. In fact, a particularly large object appears to have struck the Earth on the Yucatan Peninsula in Mexico, near the town of Chicxulub, some 65 million years ago. Evidence suggests that this impact may have led to the mass extinctions of the Cretaceous period, during which the dinosaurs disappeared from the Earth. Unfortunately, such events are not limited to the distant past. For example, as recently as 50,000 years ago, an iron asteroid tens of meters in diameter and shining 10,000 times brighter than the Sun (from atmospheric heating) slammed into the ground near Winslow, Arizona, forming the 1.2-km-wide Barringer Meteor Crater. More recently yet, at sunrise on June 30, 1908, a relatively small stony asteroid streaked through the atmosphere and exploded at an altitude of several kilometers near the Tunguska River in Siberia. The energy released by the explosion was comparable to that of an H-bomb, and it flattened the forest for kilometers in all directions. One can only imagine the consequences if an event like this were to occur near a populated area. Finally, an uncomfortably close call occurred in the early evening of December 9, 1994, when an asteroid the size of a mountain passed the Earth at a distance only one-third the distance from the Earth to the Moon. Thus, though extremely unlikely, the scenarios depicted in movies such as *Armageddon* and *Deep Impact* are not completely unrealistic.

PROBLEM-SOLVING NOTE

Energy Conservation in Astronomical Systems

To apply conservation of energy to an object that moves far from the surface of a planet, one must use $U = -GmM/r$, where r is the distance from the center of the planet.

Returning to our original question, we can use energy conservation to determine the speed such an asteroid or comet might have when it hits the Earth. To begin, we assume the asteroid starts at rest, and hence its initial kinetic energy is zero, $K_i = 0$. In addition, the initial potential energy of the system, U_i, is also zero, since $U = -GmM_E/r$ approaches zero as r approaches infinity. As a result, the total initial mechanical energy of the asteroid–Earth system is zero: $E_i = K_i + U_i = 0$. Because gravity is a conservative force (as discussed in Section 8–1), the total mechanical energy remains constant as the asteroid falls toward the Earth. Thus, as the



▲ Bodies from space have struck the Earth countless times in the past and continue to do so on a regular basis. Most such objects are relatively small, ranging in size from grains of dust to fist-sized rocks, and burn up from friction as they pass through the atmosphere, creating the bright streaks that we know as meteors. But larger objects, including the occasional comet or asteroid, also cross our path from time to time, and some of these make it to the surface—often with very dramatic results. The crater above, in Arizona, must be of relatively recent origin (thousands rather than millions of years old), since erosion has not yet erased this scar on the Earth's surface.

The image at right is a false-color gravity anomaly map of the Chicxulub impact crater in Mexico. The object that struck here some 65 million years ago may have produced such far-reaching climatic disruption that the dinosaurs and many other species became extinct as a result. At the center of the crater the strength of gravity is lower than normal (blue) because of the presence of low-density rock: debris from the impact and sediments that have accumulated in the crater.

asteroid moves closer to the Earth and U becomes increasingly negative, the kinetic energy K must become increasingly positive so that their sum, $U + K$, is always zero.

We now set the initial energy equal to the final energy to determine the final speed, v_f. Recalling that the final distance r is the radius of the Earth, R_E, we have

$$E_i = E_f$$
$$0 = \tfrac{1}{2}mv_f^2 - G\frac{mM_E}{R_E}$$

Solving for the final speed yields

$$v_f = \sqrt{\frac{2GM_E}{R_E}} \qquad 12\text{–}11$$

Substituting numerical values into this expression gives

$$v_f = \sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{2(6.67 \times 10^{-11}\,\mathrm{N\cdot m^2/kg^2})(5.97 \times 10^{24}\,\mathrm{kg})}{6.37 \times 10^6\,\mathrm{m}}}$$
$$= 11{,}200\ \mathrm{m/s}\ (25{,}000\ \mathrm{mi/h}) \qquad 12\text{–}12$$

Thus, a typical asteroid hits the Earth moving at about 7.0 mi/s—about 16 times faster than a rifle bullet! Note that this result is independent of the asteroid's mass.

To help visualize energy conservation in this system, we plot the gravitational potential energy U in **Figure 12–15**. Also indicated in the plot is the total energy, $E = 0$. Since $U + K$ must always equal zero, the value of K goes up as the value of U goes down. This is illustrated graphically in the figure with the help of several histogram bars.

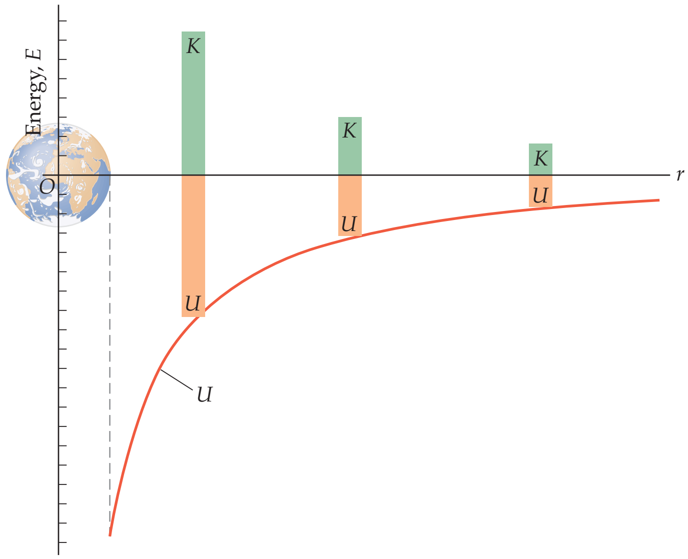


FIGURE 12–15 Potential and kinetic energies of an object falling toward Earth
As an object with zero total energy moves closer to the Earth, its gravitational potential energy, U, becomes increasingly negative. In order for the total energy to remain zero, $E = U + K = 0$, it is necessary for the kinetic energy to become increasingly positive.

Another way to think about this is to imagine a smooth wooden or plastic surface constructed to have the same shape as the plot of U shown in Figure 12–15. An object placed on this surface has a gravitational potential energy proportional to the height of the surface above a given reference level. Thus, if a small block is allowed to slide without friction on the surface, it will move downhill and speed up as it drops lower in elevation. That is, its kinetic energy will increase as the potential energy of the system decreases. This is completely analogous to the behavior of an asteroid as it "falls" toward the Earth.

A somewhat more elaborate plot showing the same physics is presented in **Figure 12–16**. The two-dimensional surface in this case represents the potential energy function U as one moves away from the Earth in any direction. In particular,

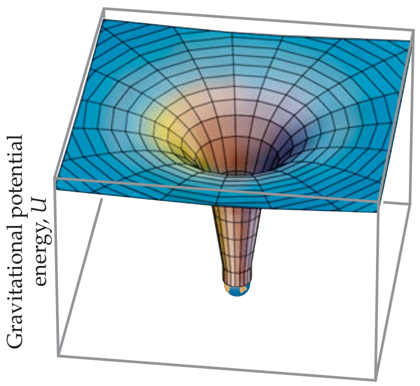


FIGURE 12–16 A gravitational potential "well"
The illustration is a three-dimensional plot of the gravitational potential energy near an object such as the Earth. An object approaching the Earth speeds up as it "falls" into the gravitational potential well.

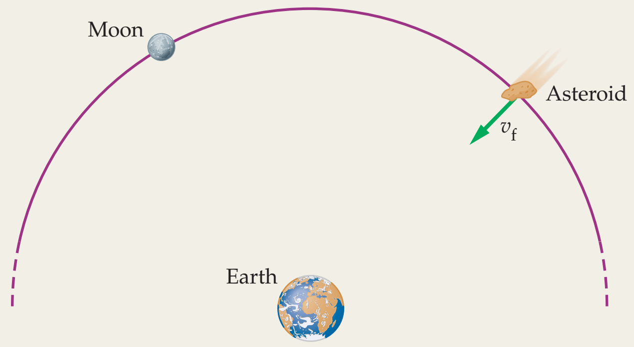
the dependence of U on distance r along any radial line in Figure 12–16 is the same as the shape of U versus r in Figure 12–15. Because the potential energy drops downward in such a plot, this type of situation is often referred to as a "potential well." If a marble is allowed to roll on such a surface, its motion is similar in many ways to the motion of an object near the Earth. In fact, if the marble is started with the right initial velocity, it will roll in a circular or elliptical "orbit" for a long time before falling into the center of the well. (Eventually, of course, the well does swallow up the marble. Though the retarding force of rolling friction is quite small, it still causes the marble to descend into a lower and lower orbit—just as air resistance causes a satellite to descend lower and lower into the Earth's atmosphere until it finally burns up.)

EXAMPLE 12–6 ARMAGEDDON RENDEZVOUS

In the movie *Armageddon*, a crew of hard-boiled oil drillers rendezvous with a menacing asteroid just as it passes the orbit of the Moon on its way toward Earth. Assuming the asteroid starts at rest infinitely far from the Earth, as in the previous discussion, find its speed when it passes the Moon's orbit. Assume the Moon orbits at a distance of $60R_E$ from the center of the Earth and that its gravitational influence may be neglected.

PICTURE THE PROBLEM

Our sketch shows the Earth, the Moon, and the asteroid. The initial position of the asteroid is at infinity, where its speed is zero. For the purposes of this problem, its final position is at the Moon's orbit, where its speed is v_f. At this point, the asteroid is heading directly for the Earth.



STRATEGY

The basic strategy is the same as that used to obtain the speed of an asteroid in Equation 12–12; namely, we set the initial energy equal to the final energy and solve for the final speed v_f. In this case, the final radius is $r = 60R_E$. As before, the initial energy is zero.

SOLUTION

1. Set the initial energy of the system equal to its final energy:

$$E_i = E_f$$

$$0 = \frac{1}{2}mv_f^2 - G\frac{mM_E}{60R_E}$$

2. Solve for the final speed, v_f:

$$v_f = \sqrt{\frac{2GM_E}{60R_E}} = \frac{1}{\sqrt{60}}\left(\sqrt{\frac{2GM_E}{R_E}}\right)$$

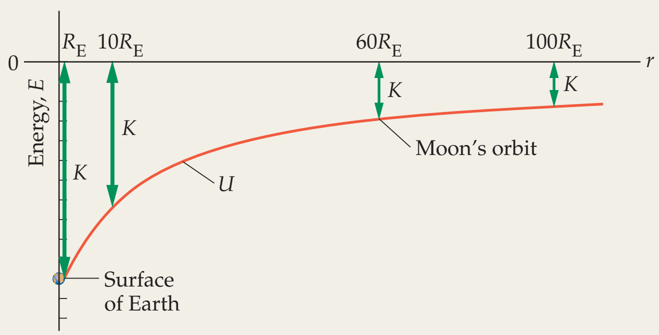
3. Substitute the numerical value given in Equation 12–12 for the quantity in parentheses:

$$v_f = \frac{1}{\sqrt{60}}(11{,}200 \text{ m/s}) = 1450 \text{ m/s} \sim 3200 \text{ mi/h}$$

INSIGHT

Note that the majority of the asteroid's increase in speed occurs after it passes the Moon. The reason for this can be seen in the accompanying plot of the gravitational potential energy, U.

Note that U drops downward more and more rapidly as the Earth is approached. Thus, while there is relatively little increase in K from infinite distance to $r = 60R_E$, there is a substantially larger increase in K from $r = 60R_E$ to $r = R_E$.



PRACTICE PROBLEM

At what distance from the center of the Earth is the asteroid's speed equal to 3535 m/s? [**Answer:** $r = 6.37 \times 10^7$ m $= 10R_E$]

Some related homework problems: Problem 50, Problem 52

ACTIVE EXAMPLE 12–2 FIND THE DISTANCE TO A SATELLITE

A satellite in an elliptical orbit has a speed of 9.00 km/s when it is at its closest approach to the Earth (perigee). The satellite is 7.00×10^6 m from the center of the Earth at this time. When the satellite is at its greatest distance from the center of the Earth (apogee), its speed is 3.66 km/s. How far is the satellite from the center of the Earth at apogee? ($R_E = 6.37 \times 10^6$ m, $M_E = 5.97 \times 10^{24}$ kg)

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the energy at perigee, E_1, equal to the energy at apogee, E_2:	$\frac{1}{2}mv_1^2 - GmM_E/r_1 = \frac{1}{2}mv_2^2 - GmM_E/r_2$
2. Solve for $1/r_2$:	$1/r_2 = 1/r_1 + (v_2^2 - v_1^2)/(2GM_E)$
3. Substitute numerical values:	$1/r_2 = 5.80 \times 10^{-8}\ \text{m}^{-1}$
4. Invert to obtain r_2:	$r_2 = 1.72 \times 10^7$ m

INSIGHT

In this case, apogee is about 2.5 times farther from the center of the Earth than perigee.

YOUR TURN

What is the speed of the satellite when it is 8.75×10^6 m from the center of the Earth?

(Answers to **Your Turn** *problems are given in the back of the book.)*

▲ Comet Hale-Bopp, one of the largest and brightest comets to visit our celestial neighborhood in recent decades, photographed in April 1997. While most of the planets and planetary satellites in the solar system have roughly circular orbits, the orbits of many comets are highly elliptical. In accordance with Kepler's second law, these objects spend most of their time moving slowly through cold, distant regions of the solar system (often far beyond the orbit of Pluto). Their visits to the inner solar system are infrequent and relatively brief.

Escape Speed

Resisting the pull of Earth's gravity has always held a fascination for the human species, from Daedalus and Icarus with their wings of feathers and wax, to Leonardo da Vinci and his flying machine, to the Montgolfier brothers and their hot-air balloons. In his 1865 novel, *From the Earth to the Moon*, Jules Verne imagined launching a spacecraft to the Moon by firing it straight upward from a cannon. Not a bad idea—if you could survive the initial blast. Today, rockets are fired into space using the same basic idea, though they smooth out the initial blast by burning their engines over a period of several minutes.

Suppose, then, that you would like to launch a rocket of mass m with an initial speed sufficient not only to reach the Moon, but to allow it to escape the Earth altogether. If we refer to this speed as the **escape speed** for the Earth, v_e, the initial energy of the rocket is

$$E_i = K_i + U_i = \frac{1}{2}mv_e^2 - G\frac{mM_E}{R_E}$$

If the rocket just barely escapes the Earth, its speed will decrease to zero as its distance from the Earth approaches infinity. Therefore, the rocket's final kinetic energy is zero, as is the potential energy of the system, since $U = -GmM_E/r$ goes to zero as $r \rightarrow \infty$. It follows that

$$E_f = K_f + U_f = 0 - 0 = 0$$

Equating these energies yields

$$\frac{1}{2}mv_e^2 - G\frac{mM_E}{R_E} = 0$$

Therefore, the escape speed from the Earth is

$$v_e = \sqrt{\frac{2GM_E}{R_E}} = 11{,}200\ \text{m/s} \approx 25{,}000\ \text{mi/h} \qquad 12\text{–}13$$

Note that the escape speed is precisely the same as the speed of the asteroid calculated in Equation 12–12. This is not surprising when you consider that an object launched from the Earth to infinity is just the reverse of an object falling from infinity to the Earth.

The result given in Equation 12–13 can be applied to other astronomical objects as well by simply replacing M_E and R_E with the appropriate mass and radius for that object.

EXERCISE 12–5

Calculate the escape speed for an object launched from the Moon.

SOLUTION

For the Moon we use $M_m = 7.35 \times 10^{22}$ kg and $R_m = 1.74 \times 10^6$ m. With these values, the escape speed is

$$v_e = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2(6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})(7.35 \times 10^{22}\ \mathrm{kg})}{1.74 \times 10^6\ \mathrm{m}}}$$
$$= 2370\ \mathrm{m/s}\ (5320\ \mathrm{mi/h})$$

The relatively low escape speed of the Moon means that it is much easier to launch a rocket into space from the Moon than from the Earth. For example, the tiny lunar module that blasted off from the Moon to return the astronauts to Earth could not have come close to escaping from the Earth.

REAL-WORLD PHYSICS
Planetary atmospheres

Similarly, the Moon's low escape speed is the reason it has no atmosphere. Even if you could magically supply the Moon with an atmosphere, it would soon evaporate into space because the individual molecules in the air move with speeds great enough to escape. On the Earth, however, where the escape speed is much higher, gravity can prevent the rapidly moving molecules from moving off into space. Even so, light molecules, like hydrogen and helium, move faster for a given temperature than the heavier molecules like nitrogen and oxygen, as we shall see in Chapter 17. For this reason, the Earth's atmosphere contains virtually no hydrogen or helium. (In fact, helium was first discovered on the Sun, as we point out in Chapter 31; hence its name, which derives from the Greek word for the Sun, "helios.") Since a stable atmosphere is a likely requirement for the development of life, it follows that the escape speed is an important quantity when considering the possibility of life on other planets.

CONCEPTUAL CHECKPOINT 12–3 COMPARE ESCAPE SPEEDS

Is the escape speed for a 10-N rocket **(a)** equal to, **(b)** less than, or **(c)** greater than the escape speed for a 10,000-N rocket?

REASONING AND DISCUSSION

The derivation of the escape speed in Equation 12–13 shows that the mass of the rocket, m, cancels. Hence, the escape speed is the same for all objects, regardless of their mass. On the other hand, the kinetic energy required to give the 10,000-N rocket the escape speed is 1000 times greater than the kinetic energy required for the 10-N rocket.

ANSWER

(a) Equal. The escape speed is independent of the mass that is escaping.

EXAMPLE 12–7 HALF ESCAPE

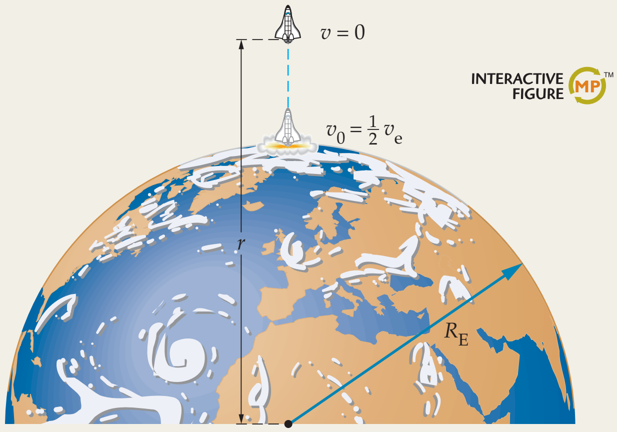
Suppose Jules Verne's cannon launches a rocket straight upward with an initial speed equal to one-half the escape speed. How far from the center of the Earth does this rocket travel before momentarily coming to rest? (Ignore air resistance in the Earth's atmosphere.)

PICTURE THE PROBLEM

Our sketch shows the rocket launched vertically from the Earth's surface with an initial speed equal to half the escape speed, $v_0 = \frac{1}{2}v_e$. The rocket moves radially away from the Earth until it comes to rest, $v = 0$, at a distance r from the center of the Earth.

STRATEGY

Since we ignore air resistance, the final energy of the rocket, E_f, must be equal to its initial energy, E_0. Setting these energies equal determines the point where the rocket comes to rest.



SOLUTION

1. The initial speed, v_0, is one-half the escape speed. Use Equation 12–13 to write an expression for v_0:

$$v_0 = \tfrac{1}{2}v_e = \tfrac{1}{2}\sqrt{\frac{2GM_E}{R_E}} = \sqrt{\frac{GM_E}{2R_E}}$$

2. Write out the initial energy of the rocket, E_0:

$$E_0 = K_0 + U_0 = \tfrac{1}{2}mv_0^2 - \frac{GmM_E}{R_E}$$

$$= \tfrac{1}{2}m\left(\sqrt{\frac{GM_E}{2R_E}}\right)^2 - \frac{GmM_E}{R_E} = -\tfrac{3}{4}\frac{GmM_E}{R_E}$$

3. Write out the final energy of the rocket. Note that the rocket is a distance r from the center of the Earth when it comes to rest:

$$E_f = K_f + U_f = 0 - \frac{GmM_E}{r} = -\frac{GmM_E}{r}$$

4. Equate the initial and final energies:

$$-\tfrac{3}{4}\frac{GmM_E}{R_E} = -\frac{GmM_E}{r}$$

5. Solve the relation for r:

$$r = \tfrac{4}{3}R_E$$

INSIGHT

An initial speed of v_e allows the rocket to go to infinity before stopping. If the rocket is launched with half that initial speed, however, it can only rise to a height of $4R_E/3 - R_E = R_E/3$ above the Earth's surface. Quite a dramatic difference.

PRACTICE PROBLEM

Find the rocket's maximum distance from the center of the Earth, r, if its launch speed is $3v_e/4$. **[Answer:** $r = 16R_E/7 = 2.29R_E$]

Some related homework problems: Problem 49, Problem 56

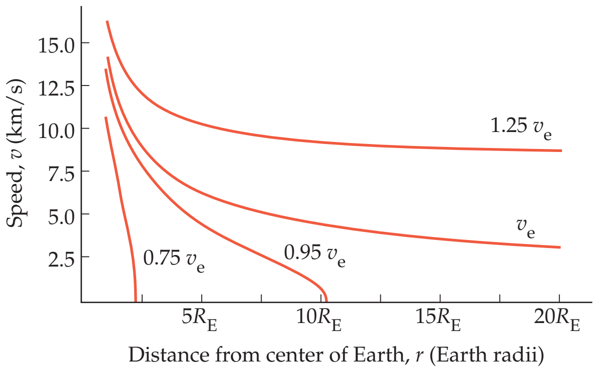


FIGURE 12–17 Speed of a rocket as a function of distance from the center of the Earth, *r*, for various vertical launch speeds The lower two curves show launch speeds that are less than the escape speed, v_e. In these cases the rocket comes to rest momentarily at a finite height above the Earth. The next higher curve shows the speed of a rocket launched with the escape speed, v_e. In this case, the rocket slows to zero speed as the distance approaches infinity. The top curve corresponds to a launch speed greater than the escape speed—this rocket has a finite speed even at infinite distance.

A plot of the speed of a rocket as a function of its distance from the center of the Earth is presented in **Figure 12–17** for a variety of initial speeds. Note that when the initial speed is less than the escape speed, the rocket comes to rest momentarily at a finite distance, r. In particular, if the launch speed is $0.75v_e$, as in the Practice Problem of Example 12–7, the rocket's maximum distance from the center of the Earth is $2.29R_E$.

Black Holes

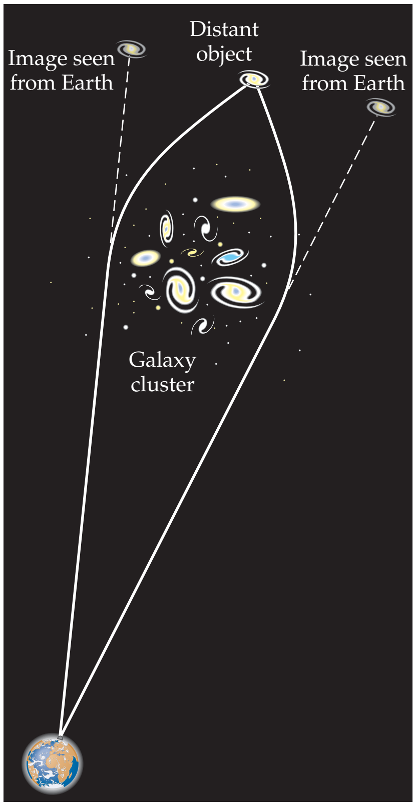
As we can see from Equation 12–13, the escape speed of an object increases with increasing mass and decreasing radius. Thus, for example, if a massive star were to collapse to a relatively small size, its escape speed would become very large. According to Einstein's theory of general relativity, the escape speed of a compressed, massive star could even exceed the speed of light. In this case nothing—not even light—could escape from the star. For this reason, such objects are referred to as *black holes*. Anything entering a black hole would be making a one-way trip to an unknown destiny.

Since black holes cannot be seen directly, our evidence for their existence is indirect. However, we can predict that as matter is drawn toward a black hole it should become heated to the point where it would emit strong beams of X-rays before disappearing from view. X-ray beams matching these predictions have in fact been observed. These observations, coupled with a variety of others, give astronomers confidence that massive black holes reside at the core of many galaxies—including our own!

REAL-WORLD PHYSICS

Black holes and gravitational lensing

Finally, just as a black hole can bend a beam of light back on itself and prevent it from escaping, any massive object can bend light—at least a little. For example, light from distant stars is deflected as it passes by the Sun by 1.75 seconds of an arc (the size of a quarter at a distance of 1.8 miles). Light passing by an entire galaxy of stars or cluster of galaxies can be bent by significant amounts, however, as **Figure 12–18** indicates. This effect is referred to as *gravitational lensing*, since the galaxies act much like the lenses we will study in Chapter 26. Because of gravitational lensing, the images of very distant galaxies or quasars in deep-space astronomical photographs sometimes appear in duplicate, in quadruplicate, or even spread out into circular arcs.



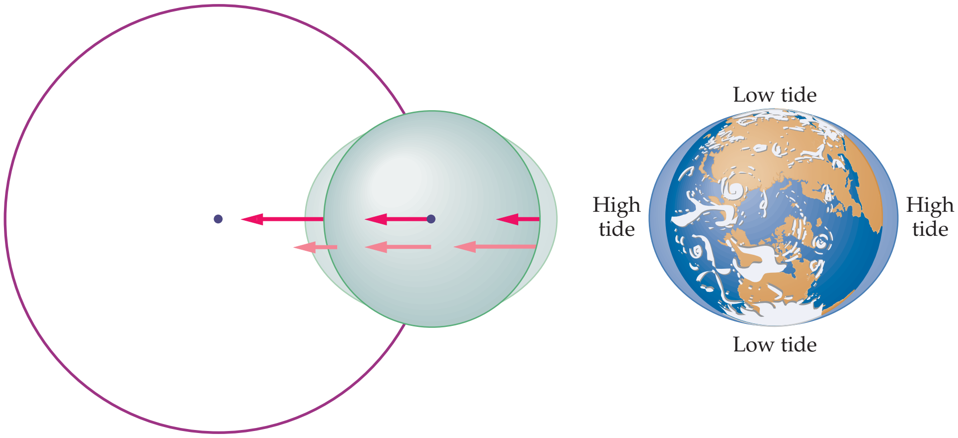
▲ **FIGURE 12–18 Gravitational lensing**
Astronomers often find that very distant objects seem to produce multiple images in their photographs. The cause is the gravitational attraction of intervening galaxies or clusters of galaxies, which are so massive that they can significantly bend the light from remote objects as it passes by them on its way to Earth.

*12–6 Tides

The reason for the ocean tides that rise and fall twice a day was a perplexing and enduring mystery until Newton introduced his law of universal gravitation. Even Galileo, who made so many advances in physics and astronomy, could not explain the tides. However, with the understanding that a force is required to cause an object to move in a circular path, and that the force of gravity becomes weaker with distance, it is possible to describe the tides in detail. In this section we show how it can be done. In addition, we extend the basic idea of tides to several related phenomena.

To begin, consider the idealized situation shown in **Figure 12–19 (a)**. Here we see an object of finite size (a moon or a planet, for example) orbiting a point mass. If all the mass of the object were concentrated at its center, the gravitational force exerted on it by the central mass would be precisely the amount needed to cause it to move in its circular path. Since the object is of finite size, however, the force exerted on various parts of it has different magnitudes. For example, points closer to the central mass experience a greater force than points farther away.

To see the effect of this variation in force, we use a dark red vector in Figure 12–19 (a) to indicate the force exerted by the central mass at three different points on the object. In addition, we use a light red vector to show the force that is required at each of these three points to cause a mass at that distance to orbit the central mass. Comparing these vectors, we see that the forces are identical at the center of the object—as expected. On the near side of the object, however, the force exerted by the central mass is larger than the force needed to hold the object in orbit, and on the far side the force due to the central mass is less than the force needed to hold the object in orbit. The result is that the near side of the object is pulled closer to the central mass and the far side tends to move farther from the central mass. This causes an egg-shaped deformation of the object, as indicated in Figure 12–19 (a).



(a) The mechanism responsible for tides

(b) Tidal deformations on Earth

▲ **FIGURE 12–19 The reason for two tides a day**

(a) Tides are caused by a disparity between the gravitational force exerted at various points on a finite-sized object (dark red arrows) and the centripetal force needed for circular motion (light red arrows). Note that the gravitational force decreases with distance, as expected. On the other hand, the centripetal force required to keep an object moving in a circular path *increases* with distance. On the near side, therefore, the gravitational force is stronger than required, and the object is stretched inward. On the far side, the gravitational force is weaker than required, and the object stretches outward. **(b)** On the Earth, the water in the oceans responds more to the deforming effects of tides than do the solid rocks of the land. The result is two high tides and two low tides daily on opposite sides of the Earth.

REAL-WORLD PHYSICS
Tides

Any two objects orbiting one another cause deformations of this type. For example, the Earth causes a deformation in the Moon, and the Moon causes a similar deformation in the Earth. In **Figure 12–19 (b)** we show the Earth and the waters of its ocean deformed into an egg shape. Since the waters in the oceans can flow, they deform much more than the underlying rocky surface of the Earth. As a result, the water level relative to the surface of the Earth is greater at the *tidal bulges* shown in the figure. As the Earth rotates about its axis, a person at a given location will observe two high tides and two low tides each day. This is the basic mechanism of the tides on Earth, but the actual situation is complicated by the shape of the coastline at different locations and by the additional tidal effects due to the Sun.

The Moon has no oceans, of course, but the tidal bulges produced in it by the Earth are the reason we see only one side of the Moon. Specifically, the Earth exerts gravitational forces on the tidal bulges of the Moon, causing them to point directly toward the Earth. If the Moon were to rotate slightly away from this alignment, the forces exerted by the Earth would cause a torque that would return the Moon to the original alignment. The net result is that the Moon's period of rotation about its axis is equal to its period of revolution about the Earth. This effect, known as **tidal locking**, is common among the various moons in the solar system.

REAL-WORLD PHYSICS
Tidal locking

A particularly interesting example of tidal locking is provided by Jupiter's moon Io, a site of intense volcanism (see the photo on p. 107). Io follows an elliptical orbit around Jupiter, and its tidal deformation is larger when it is closer to Jupiter than when it is farther away. As a result, each time Io orbits Jupiter it is squeezed into a greater deformation and then released. This continual flexing of

◀ Tides on Earth are caused chiefly by the Moon's gravitational pull, though at full and new moon, when the Moon and Sun are aligned, the Sun's gravity can enhance the effect. In some places on Earth, such as the Bay of Fundy between Maine and Nova Scotia, local topographic conditions produce abnormally large tides.

Io causes its internal temperature to rise, just as a rubber ball gets warmer if you squeeze and release it in your hand over and over. It is this mechanism that is largely responsible for Io's ongoing volcanic activity.

REAL-WORLD PHYSICS

Roche limit and Saturn's rings

In extreme cases, tidal deformation can become so large that an object is literally torn apart. Since tidal deformation increases as a moon moves closer to the planet it orbits, there is a limiting orbital radius—known as the **Roche limit**—inside of which this breakup occurs. A most spectacular example of this effect can be seen in the rings of Saturn, all of which exist well within the Roche limit. The small chunks of ice and other materials that make up the rings may be the remains of a moon that moved too close to Saturn and was destroyed by tidal forces. On the other hand, they may represent material that tidal forces prevented from aggregating to form a moon in the first place. In either case, this dramatic debris field will now never coalesce to form a moon—tidal effects will not allow such a process to occur. Similar remarks apply to the smaller, much fainter rings that spacecraft have observed around Jupiter, Uranus, and Neptune.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The general force of gravity, as presented in Equation 12–1, is a vector quantity. Therefore, vector calculations (Chapter 3) are important here. See, in particular, Example 12–1. We also use the connection between force and acceleration, $F = ma$ (Chapter 5), in Section 12–2.

The conservation of angular momentum (Chapter 11) plays a key role in gravity, leading to Kepler's second law in Section 12–3.

Just as the force of gravity is generalized in this chapter, so too is the gravitational potential energy. Thus, the expression $U = mgh$ (Chapter 8) is generalized to $U = -Gm_1m_2/r$ in Section 12–4. We then use this new form of the potential energy in situations involving energy conservation in Section 12–5.

LOOKING AHEAD

In Chapter 19 we shall see that the force between two electric charges, denoted q_1 and q_2, has exactly the same form as the general force of gravity between two masses, Equation 12–1. The electric force is referred to as Coulomb's law, and is presented in Equation 19–5.

The force between electric charges is conservative, and hence it leads to an electric potential energy that has the same form as the gravitational potential energy in Section 12–4. See Sections 20–1 and 20–2.

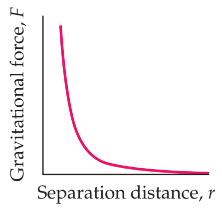
The analysis used to derive Kepler's third law in Section 12–3 is used again when we explore the Bohr model of the hydrogen atom in Chapter 31. The calculation is the same, but in hydrogen the Coulomb force between electric charges (Equation 19–5) is responsible for the orbital motion.

CHAPTER SUMMARY

12–1 NEWTON'S LAW OF UNIVERSAL GRAVITATION

The force of gravity between two point masses, m_1 and m_2, separated by a distance r is attractive and of magnitude

$$F = G\frac{m_1m_2}{r^2} \qquad 12\text{–}1$$



G is the universal gravitation constant:

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2} \qquad 12\text{–}2$$

Gravity exerts an action-reaction pair of forces on m_1 and m_2; that is, the force exerted by gravity on m_1 is equal in magnitude but opposite in direction to the force exerted on m_2.

Inverse Square Dependence

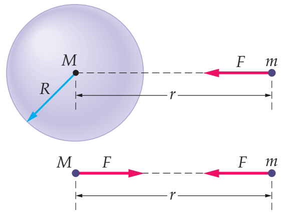
The force of gravity decreases with distance, r, as $1/r^2$. This is referred to as an inverse square dependence.

Superposition

If more than one mass exerts a gravitational force on a given object, the net force is simply the vector sum of each force individually.

12–2 GRAVITATIONAL ATTRACTION OF SPHERICAL BODIES

In calculating gravitational forces, spherical objects can be replaced by point masses.



Uniform Sphere
If a mass m is outside a uniform sphere of mass M, the gravitational force between m and the sphere is equivalent to the force exerted by a point mass M located at the center of the sphere.

Acceleration of Gravity
Replacing the Earth with a point mass at its center, we find that the acceleration of gravity on the surface of the Earth is

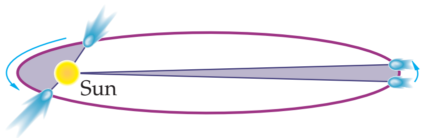
$$g = \frac{GM_E}{{R_E}^2} \qquad 12\text{–}4$$

Weighing the Earth
Cavendish was the first to determine the value of the universal gravitation constant G by direct experiment. Knowing G allows one to calculate the mass of the Earth:

$$M_E = \frac{g{R_E}^2}{G} \qquad 12\text{–}5$$

12–3 KEPLER'S LAWS OF ORBITAL MOTION

Kepler determined three laws that describe the motion of the planets in our solar system. Newton showed that Kepler's laws are a direct consequence of his law of universal gravitation.



Kepler's First Law
The orbits of the planets are ellipses, with the Sun at one focus.

Kepler's Second Law
Planets sweep out equal area in equal time.

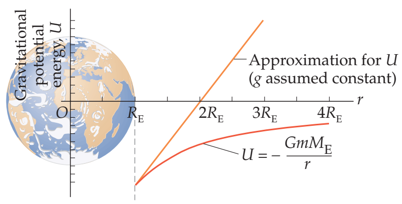
Kepler's Third Law
The period of a planet's orbit, T, is proportional to the 3/2 power of its average distance from the Sun, r:

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right)r^{3/2} = (\text{constant})r^{3/2} \qquad 12\text{–}7$$

12–4 GRAVITATIONAL POTENTIAL ENERGY

The gravitational potential energy, U, between two point masses m_1 and m_2 separated by a distance r is

$$U = -G\frac{m_1 m_2}{r} \qquad 12\text{–}9$$

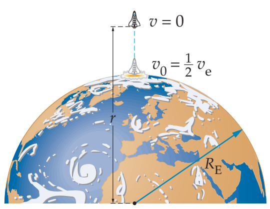


Zero Level
The zero level of the gravitational potential energy between two point masses is chosen to be at infinite separation of the two masses.

***U* Is a Scalar**
The gravitational potential energy, U, is a scalar. Therefore, the total potential energy for a group of objects is simply the numerical sum of the potential energy associated with each pair of masses.

12–5 ENERGY CONSERVATION

With the gravitational potential energy given in Section 12–4, energy conservation can be applied to astronomical situations.



Total Mechanical Energy
An object with mass m, speed v, and at a distance r from the center of the Earth has a total energy given by

$$E = K + U = \tfrac{1}{2}mv^2 - \frac{GmM_E}{r} \qquad 12\text{–}10$$

Escape Speed
An object launched from the surface of the Earth with the escape speed v_e can move infinitely far from the Earth. In the limit of infinite separation, the object slows to zero speed.

The escape speed for the Earth is given by

$$v_e = \sqrt{\frac{2GM_E}{R_E}} \qquad 12\text{–}13$$

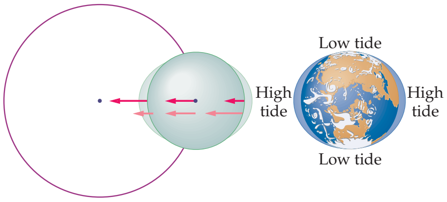
Its numerical value is 11,200 m/s = 25,000 mi/h. A similar expression can be applied to other astronomical bodies.

*12–6 TIDES

Tides result from the variation of the gravitational force from one side of an astronomical object to the other side.

Tidal Locking
Tidal locking occurs when one astronomical object always points its tidal bulge at the object it orbits.

Roche Limit
Tidal deformation increases as an astronomical object moves closer to the body it orbits. At the Roche limit, the tidal deformation is so great that it breaks the object into small pieces.



(a) (b)

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the force due to gravity.	The magnitude of the force is given by Newton's law of universal gravitation, $F = Gm_1m_2/r^2$. The direction of the force is attractive and along the line connecting m_1 and m_2. If more than one force is involved, the net force is the vector sum of the individual forces.	Examples 12–1, 12–2, 12–3
Relate the period of a planet to the radius of its orbit and the mass of the body it orbits.	Use Kepler's third law, $T = \left(2\pi/\sqrt{GM}\right)r^{3/2}$.	Example 12–4 Active Example 12–1
Determine the speed of an object at a particular location, given its initial speed and location.	Use energy conservation, with the gravitational potential energy given by $U = -Gm_1m_2/r$.	Examples 12–6, 12–7 Active Example 12–2

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

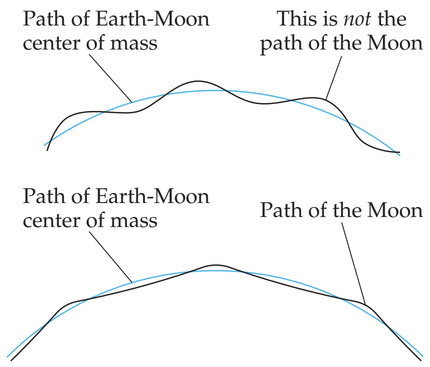
(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. It is often said that astronauts in orbit experience weightlessness because they are beyond the pull of Earth's gravity. Is this statement correct? Explain.
2. When a person passes you on the street, you do not feel a gravitational tug. Explain.
3. Two objects experience a gravitational attraction. Give a reason why the gravitational force between them does not depend on the sum of their masses.
4. Imagine bringing the tips of your index fingers together. Each finger contains a certain finite mass, and the distance between them goes to zero as they come into contact. From the force law $F = Gm_1m_2/r^2$ one might conclude that the attractive force between the fingers is infinite, and, therefore, that your fingers must remain forever stuck together. What is wrong with this argument?
5. Does the radius vector of Mars sweep out the same amount of area per time as that of the Earth? Why or why not?
6. When a communications satellite is placed in a geosynchronous orbit above the equator, it remains fixed over a given point on the ground. Is it possible to put a satellite into an orbit so that it remains fixed above the North Pole? Explain.
7. **The Mass of Pluto** On June 22, 1978, James Christy made the first observation of a moon orbiting Pluto. Until that time the mass of Pluto was not known, but with the discovery of its moon, Charon, its mass could be calculated with some accuracy. Explain.
8. Rockets are launched into space from Cape Canaveral in an easterly direction. Is there an advantage to launching to the east versus launching to the west? Explain.
9. One day in the future you may take a pleasure cruise to the Moon. While there you might climb a lunar mountain and throw a rock horizontally from its summit. If, in principle, you could throw the rock fast enough, it might end up hitting you in the back. Explain.
10. Apollo astronauts orbiting the Moon at low altitude noticed occasional changes in their orbit that they attributed to localized concentrations of mass below the lunar surface. Just what effect would such "mascons" have on their orbit?

11. If you light a candle on the space shuttle—which would not be a good idea—would it burn the same as on the Earth? Explain.

12. The force exerted by the Sun on the Moon is more than twice the force exerted by the Earth on the Moon. Should the Moon be thought of as orbiting the Earth or the Sun? Explain.

13. **The Path of the Moon** The Earth and Moon exert gravitational forces on one another as they orbit the Sun. As a result, the path they follow is not the simple circular orbit you would expect if either one orbited the Sun alone. Occasionally you will see a suggestion that the Moon follows a path like a sine wave centered on a circular path, as in the upper part of **Figure 12–20**. This is *incorrect*. The Moon's path is qualitatively like that shown in the lower part of Figure 12–20. Explain. (Refer to Conceptual Question 12.)



▲ **FIGURE 12–20** Conceptual Question 13

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 12–1 NEWTON'S LAW OF UNIVERSAL GRAVITATION

1. • **CE** System A has masses m and m separated by a distance r; system B has masses m and $2m$ separated by a distance $2r$; system C has masses $2m$ and $3m$ separated by a distance $2r$; and system D has masses $4m$ and $5m$ separated by a distance $3r$. Rank these systems in order of increasing gravitational force. Indicate ties where appropriate.

2. • In each hand you hold a 0.16-kg apple. What is the gravitational force exerted by each apple on the other when their separation is **(a)** 0.25 m and **(b)** 0.50 m?

3. • A 6.1-kg bowling ball and a 7.2-kg bowling ball rest on a rack 0.75 m apart. **(a)** What is the force of gravity exerted on each of the balls by the other ball? **(b)** At what separation is the force of gravity between the balls equal to 2.0×10^{-9} N?

4. • A communications satellite with a mass of 480 kg is in a circular orbit about the Earth. The radius of the orbit is 35,000 km as measured from the center of the Earth. Calculate **(a)** the weight of the satellite on the surface of the Earth and **(b)** the gravitational force exerted on the satellite by the Earth when it is in orbit.

5. • **The Attraction of Ceres** Ceres, the largest asteroid known, has a mass of roughly 8.7×10^{20} kg. If Ceres passes within 14,000 km of the spaceship in which you are traveling, what force does it exert on you? (Use an approximate value for your mass, and treat yourself and the asteroid as point objects.)

6. • In one hand you hold a 0.11-kg apple, in the other hand a 0.24-kg orange. The apple and orange are separated by 0.85 m. What is the magnitude of the force of gravity that **(a)** the orange exerts on the apple and **(b)** the apple exerts on the orange?

7. •• **IP** A spaceship of mass m travels from the Earth to the Moon along a line that passes through the center of the Earth and the center of the Moon. **(a)** At what distance from the center of the Earth is the force due to the Earth twice the magnitude of the force due to the Moon? **(b)** How does your answer to part (a) depend on the mass of the spaceship? Explain.

8. •• At new moon, the Earth, Moon, and Sun are in a line, as indicated in **Figure 12–21**. Find the direction and magnitude of the net gravitational force exerted on **(a)** the Earth, **(b)** the Moon, and **(c)** the Sun.

▲ **FIGURE 12–21** Problem 8

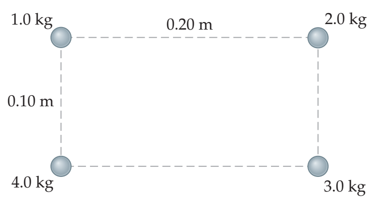
9. •• When the Earth, Moon, and Sun form a right triangle, with the Moon located at the right angle, as shown in **Figure 12–22**, the Moon is in its third-quarter phase. (The Earth is viewed here from above its North Pole.) Find the magnitude and direction of the net force exerted on the Moon. Give the direction relative to the line connecting the Moon and the Sun.

▲ **FIGURE 12–22** Problems 9, 10, and 73

10. •• Repeat the previous problem, this time finding the magnitude and direction of the net force acting on the Sun. Give the direction relative to the line connecting the Sun and the Moon.

11. •• **IP** Three 6.75-kg masses are at the corners of an equilateral triangle and located in space far from any other masses. **(a)** If the sides of the triangle are 1.25 m long, find the magnitude of the net force exerted on each of the three masses. **(b)** How does your answer to part (a) change if the sides of the triangle are doubled in length?

12. •• **IP** Four masses are positioned at the corners of a rectangle, as indicated in **Figure 12–23. (a)** Find the magnitude and direction of the net force acting on the 2.0-kg mass. **(b)** How do your answers to part (a) change (if at all) if all sides of the rectangle are doubled in length?



▲ **FIGURE 12–23** Problems 12 and 42

13. ••• Suppose that three astronomical objects (1, 2, and 3) are observed to lie on a line, and that the distance from object 1 to object 3 is D. Given that object 1 has four times the mass of object 3 and seven times the mass of object 2, find the distance between objects 1 and 2 for which the net force on object 2 is zero.

SECTION 12–2 GRAVITATIONAL ATTRACTION OF SPHERICAL BODIES

14. • Find the acceleration due to gravity on the surface of **(a)** Mercury and **(b)** Venus.

15. • At what altitude above the Earth's surface is the acceleration due to gravity equal to $g/2$?

16. • Two 6.7-kg bowling balls, each with a radius of 0.11 m, are in contact with one another. What is the gravitational attraction between the bowling balls?

17. • What is the acceleration due to Earth's gravity at a distance from the center of the Earth equal to the orbital radius of the Moon?

18. • **Gravity on Titan** Titan is the largest moon of Saturn and the only moon in the solar system known to have a substantial atmosphere. Find the acceleration due to gravity on Titan's surface, given that its mass is 1.35×10^{23} kg and its radius is 2570 km.

19. •• **IP** At a certain distance from the center of the Earth, a 4.6-kg object has a weight of 2.2 N. **(a)** Find this distance. **(b)** If the object is released at this location and allowed to fall toward the Earth, what is its initial acceleration? **(c)** If the object is now moved twice as far from the Earth, by what factor does its weight change? Explain. **(d)** By what factor does its initial acceleration change? Explain.

20. •• The acceleration due to gravity on the Moon's surface is known to be about one-sixth the acceleration due to gravity on the Earth. Given that the radius of the Moon is roughly one-quarter that of the Earth, find the mass of the Moon in terms of the mass of the Earth.

21. •• **IP An Extraterrestrial Volcano** Several volcanoes have been observed erupting on the surface of Jupiter's closest Galilean moon, Io. Suppose that material ejected from one of these volcanoes reaches a height of 5.00 km after being projected straight upward with an initial speed of 134 m/s. Given that the radius of Io is 1820 km, **(a)** outline a strategy that allows you to calculate the mass of Io. **(b)** Use your strategy to calculate Io's mass.

22. •• **IP Verne's Trip to the Moon** In his novel *From the Earth to the Moon*, Jules Verne imagined that astronauts inside a spaceship would walk on the floor of the cabin when the force exerted on the ship by the Earth was greater than the force exerted by the Moon. When the force exerted by the Moon was greater, he thought the astronauts would walk on the ceiling of the cabin. **(a)** At what distance from the center of the Earth would the forces exerted on the spaceship by the Earth and the Moon be equal? **(b)** Explain why Verne's description of gravitational effects is incorrect.

23. ••• Consider an asteroid with a radius of 19 km and a mass of 3.35×10^{15} kg. Assume the asteroid is roughly spherical. **(a)** What is the acceleration due to gravity on the surface of the asteroid? **(b)** Suppose the asteroid spins about an axis through its center, like the Earth, with a rotational period T. What is the smallest value T can have before loose rocks on the asteroid's equator begin to fly off the surface?

SECTION 12–3 KEPLER'S LAWS OF ORBITAL MOTION

24. • **CE Predict/Explain The Speed of the Earth** The orbital speed of the Earth is greatest around January 4 and least around July 4. **(a)** Is the distance from the Earth to the Sun on January 4 greater than, less than, or equal to its distance from the Sun on July 4? **(b)** Choose the *best explanation* from among the following:
    I. The Earth's orbit is circular, with equal distance from the Sun at all times.
    II. The Earth sweeps out equal area in equal time, thus it must be closer to the Sun when it is moving faster.
    III. The greater the speed of the Earth, the greater its distance from the Sun.

25. • **CE** A satellite orbits the Earth in a circular orbit of radius r. At some point its rocket engine is fired in such a way that its speed increases rapidly by a small amount. As a result, do the **(a)** apogee distance and **(b)** perigee distance increase, decrease, or stay the same?

26. • **CE** Repeat the previous problem, only this time with the rocket engine of the satellite fired in such a way as to slow the satellite.

27. • **CE Predict/Explain The Earth–Moon Distance Is Increasing** Laser reflectors left on the surface of the Moon by the Apollo astronauts show that the average distance from the Earth to the Moon is increasing at the rate of 3.8 cm per year. **(a)** As a result, will the length of the month increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:
    I. The greater the radius of an orbit, the greater the period, which implies a longer month.
    II. The length of the month will remain the same due to conservation of angular momentum.
    III. The speed of the Moon is greater with increasing radius; therefore, the length of the month will be less.

28. • **Apollo Missions** On Apollo missions to the Moon, the command module orbited at an altitude of 110 km above the lunar surface. How long did it take for the command module to complete one orbit?

29. • Find the orbital speed of a satellite in a geosynchronous circular orbit 3.58×10^{7} m above the surface of the Earth.

30. • **An Extrasolar Planet** In July of 1999 a planet was reported to be orbiting the Sun-like star Iota Horologii with a period of 320 days. Find the radius of the planet's orbit, assuming that Iota Horologii has the same mass as the Sun. (This planet is presumably similar to Jupiter, but it may have large, rocky moons that enjoy a relatively pleasant climate.)

31. • Phobos, one of the moons of Mars, orbits at a distance of 9378 km from the center of the red planet. What is the orbital period of Phobos?

32. • The largest moon in the solar system is Ganymede, a moon of Jupiter. Ganymede orbits at a distance of 1.07×10^{9} m from the center of Jupiter with an orbital period of about 6.18×10^{5} s. Using this information, find the mass of Jupiter.

33. •• **IP An Asteroid with Its Own Moon** The asteroid 243 Ida has its own small moon, Dactyl. (See the photo on p. 390) **(a)** Outline a strategy to find the mass of 243 Ida, given that the orbital radius of Dactyl is 89 km and its period is 19 hr. **(b)** Use your strategy to calculate the mass of 243 Ida.

34. •• **GPS Satellites** GPS (Global Positioning System) satellites orbit at an altitude of 2.0×10^7 m. Find **(a)** the orbital period, and **(b)** the orbital speed of such a satellite.

35. •• **IP** Two satellites orbit the Earth, with satellite 1 at a greater altitude than satellite 2. **(a)** Which satellite has the greater orbital speed? Explain. **(b)** Calculate the orbital speed of a satellite that orbits at an altitude of one Earth radius above the surface of the Earth. **(c)** Calculate the orbital speed of a satellite that orbits at an altitude of two Earth radii above the surface of the Earth.

36. •• **IP** Calculate the orbital periods of satellites that orbit **(a)** one Earth radius above the surface of the Earth and **(b)** two Earth radii above the surface of the Earth. **(c)** How do your answers to parts (a) and (b) depend on the mass of the satellites? Explain. **(d)** How do your answers to parts (a) and (b) depend on the mass of the Earth? Explain.

37. •• **IP** The Martian moon Deimos has an orbital period that is greater than the other Martian moon, Phobos. Both moons have approximately circular orbits. **(a)** Is Deimos closer to or farther from Mars than Phobos? Explain. **(b)** Calculate the distance from the center of Mars to Deimos given that its orbital period is 1.10×10^5 s.

38. ••• **Binary Stars** Centauri A and Centauri B are binary stars with a separation of 3.45×10^{12} m and an orbital period of 2.52×10^9 s. Assuming the two stars are equally massive (which is approximately the case), determine their mass.

39. ••• Find the speed of Centauri A and Centauri B, using the information given in the previous problem.

SECTION 12–4 GRAVITATIONAL POTENTIAL ENERGY

40. • **Sputnik** The first artificial satellite to orbit the Earth was Sputnik I, launched October 4, 1957. The mass of Sputnik I was 83.5 kg, and its distances from the center of the Earth at apogee and perigee were 7330 km and 6610 km, respectively. Find the difference in gravitational potential energy for Sputnik I as it moved from apogee to perigee.

41. •• **CE Predict/Explain** **(a)** Is the amount of energy required to get a spacecraft from the Earth to the Moon greater than, less than, or equal to the energy required to get the same spacecraft from the Moon to the Earth? **(b)** Choose the *best explanation* from among the following:
   I. The escape speed of the Moon is less than that of the Earth; therefore, less energy is required to leave the Moon.
   II. The situation is symmetric, and hence the same amount of energy is required to travel in either direction.
   III. It takes more energy to go from the Moon to the Earth because the Moon is orbiting the Earth.

42. •• **IP** Consider the four masses shown in Figure 12–23. **(a)** Find the total gravitational potential energy of this system. **(b)** How does your answer to part (a) change if all the masses in the system are doubled? **(c)** How does your answer to part (a) change if, instead, all the sides of the rectangle are halved in length?

43. •• Calculate the gravitational potential energy of a 8.8-kg mass **(a)** on the surface of the Earth and **(b)** at an altitude of 350 km. **(c)** Take the difference between the results for parts (b) and (a), and compare with mgh, where $h = 350$ km.

44. •• Two 0.59-kg basketballs, each with a radius of 12 cm, are just touching. How much energy is required to change the separation between the centers of the basketballs to **(a)** 1.0 m and **(b)** 10.0 m? (Ignore any other gravitational interactions.)

45. •• Find the minimum kinetic energy needed for a 39,000-kg rocket to escape **(a)** the Moon or **(b)** the Earth.

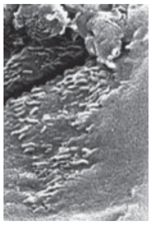
SECTION 12–5 ENERGY CONSERVATION

46. • **CE Predict/Explain** Suppose the Earth were to suddenly shrink to half its current diameter, with its mass remaining constant. **(a)** Would the escape speed of the Earth increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Since the radius of the Earth would be smaller, the escape speed would also be smaller.
   II. The Earth would have the same amount of mass, and hence its escape speed would be unchanged.
   III. The force of gravity would be much stronger on the surface of the compressed Earth, leading to a greater escape speed.

47. • **CE** Is the energy required to launch a rocket vertically to a height h greater than, less than, or equal to the energy required to put the same rocket into orbit at the height h? Explain.

48. • Suppose one of the Global Positioning System satellites has a speed of 4.46 km/s at perigee and a speed of 3.64 km/s at apogee. If the distance from the center of the Earth to the satellite at perigee is 2.00×10^4 km, what is the corresponding distance at apogee?

49. • **Meteorites from Mars** Several meteorites found in Antarctica are believed to have come from Mars, including the famous ALH84001 meteorite that some believe contains fossils of ancient life on Mars. Meteorites from Mars are thought to get to Earth by being blasted off the Martian surface when a large object (such as an asteroid or a comet) crashes into the planet. What speed must a rock have to escape Mars?



The meteorite ALH84001 (left), dislodged from the Martian surface by a tremendous impact, drifted through space for millions of years before falling to Earth in Antarctica about 13,000 years ago. The electron micrograph at right shows tubular structures within the meteorite; some scientists think they are traces of primitive, bacteria-like organisms that may have lived on Mars billions of years ago. (Problem 49)

50. • Referring to Example 12–1, if the *Millennium Eagle* is at rest at point A, what is its speed at point B?

51. • What is the launch speed of a projectile that rises vertically above the Earth to an altitude equal to one Earth radius before coming to rest momentarily?

52. • A projectile launched vertically from the surface of the Moon rises to an altitude of 365 km. What was the projectile's initial speed?

53. • Find the escape velocity for **(a)** Mercury and **(b)** Venus.

54. •• **IP** **Halley's Comet** Halley's comet, which passes around the Sun every 76 years, has an elliptical orbit. When closest to the Sun (perihelion) it is at a distance of 8.823×10^{10} m and moves with a speed of 54.6 km/s. The greatest distance between Halley's comet and the Sun (aphelion) is 6.152×10^{12} m. **(a)** Is the speed of Halley's comet greater than or less than 54.6 km/s

when it is at aphelion? Explain. **(b)** Calculate its speed at aphelion.

55. •• **The End of the Lunar Module** On Apollo Moon missions, the lunar module would blast off from the Moon's surface and dock with the command module in lunar orbit. After docking, the lunar module would be jettisoned and allowed to crash back onto the lunar surface. Seismometers placed on the Moon's surface by the astronauts would then pick up the resulting seismic waves. Find the impact speed of the lunar module, given that it is jettisoned from an orbit 110 km above the lunar surface moving with a speed of 1630 m/s.

56. •• If a projectile is launched vertically from the Earth with a speed equal to the escape speed, how high above the Earth's surface is it when its speed is half the escape speed?

57. •• Suppose a planet is discovered orbiting a distant star. If the mass of the planet is 10 times the mass of the Earth, and its radius is one-tenth the Earth's radius, how does the escape speed of this planet compare with that of the Earth?

58. •• A projectile is launched vertically from the surface of the Moon with an initial speed of 1050 m/s. At what altitude is the projectile's speed one-half its initial value?

59. •• To what radius would the Sun have to be contracted for its escape speed to equal the speed of light? (Black holes have escape speeds greater than the speed of light; hence we see no light from them.)

60. •• **IP** Two baseballs, each with a mass of 0.148 kg, are separated by a distance of 395 m in outer space, far from any other objects. **(a)** If the balls are released from rest, what speed do they have when their separation has decreased to 145 m? **(b)** Suppose the mass of the balls is doubled. Would the speed found in part (a) increase, decrease, or stay the same? Explain.

61. ••• On Earth, a person can jump vertically and rise to a height h. What is the radius of the largest spherical asteroid from which this person could escape by jumping straight upward? Assume that each cubic meter of the asteroid has a mass of 3500 kg.

*SECTION 12–6 TIDES

62. •• As will be shown in Problem 63, the magnitude of the tidal force exerted on an object of mass m and length a is approximately $4GmMa/r^3$. In this expression, M is the mass of the body causing the tidal force and r is the distance from the center of m to the center of M. Suppose you are 1 million miles away from a black hole whose mass is a million times that of the Sun. **(a)** Estimate the tidal force exerted on your body by the black hole. **(b)** At what distance will the tidal force be approximately 10 times greater than your weight?

63. ••• A dumbbell has a mass m on either end of a rod of length $2a$. The center of the dumbbell is a distance r from the center of the Earth, and the dumbbell is aligned radially. If $r \gg a$, show that the difference in the gravitational force exerted on the two masses by the Earth is approximately $4GmM_Ea/r^3$. (*Note:* The difference in force causes a tension in the rod connecting the masses. We refer to this as a *tidal force*.) [*Hint:* Use the fact that $1/(r-a)^2 - 1/(r+a)^2 \sim 4a/r^3$ for $r \gg a$.]

64. ••• Referring to the previous problem, suppose the rod connecting the two masses m is removed. In this case, the only force between the two masses is their mutual gravitational attraction. In addition, suppose the masses are spheres of radius a and mass $m = \frac{4}{3}\pi a^3 \rho$ that touch each other. (The Greek letter ρ stands for the density of the masses.) **(a)** Write an expression for the gravitational force between the masses m. **(b)** Find the distance from the center of the Earth, r, for which the gravitational force found in part (a) is equal to the tidal force found in Problem 63. This distance is known as the *Roche limit*. **(c)** Calculate the Roche limit for Saturn, assuming $\rho = 3330\ \text{kg/m}^3$. (The famous rings of Saturn are within the Roche limit for that planet. Thus, the innumerable small objects, composed mostly of ice, that make up the rings will never coalesce to form a moon.)

GENERAL PROBLEMS

65. • **CE** You weigh yourself on a scale inside an airplane flying due east above the equator. If the airplane now turns around and heads due west with the same speed, will the reading on the scale increase, decrease, or stay the same? Explain.

66. • **CE** Rank objects A, B, and C in **Figure 12–24** in order of increasing net gravitational force experienced by the object. Indicate ties where appropriate.

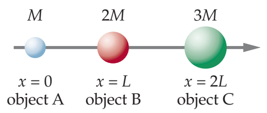


FIGURE 12–24
Problems 66 and 67

67. • **CE** Referring to Figure 12–24, rank objects A, B, and C in order of increasing initial acceleration each would experience if it alone were allowed to move. Indicate ties where appropriate.

68. • **CE** When the Moon is in its new-moon position (directly between the Earth and the Sun), does the net force exerted on it by the Sun and the Earth point toward the Sun, or point toward the Earth? Explain. (Refer to Conceptual Questions 12 and 13 as well as Figure 12–20.)

69. • **CE** A satellite goes through one complete orbit of the Earth. **(a)** Is the net work done on it by the Earth's gravitational force positive, negative, or zero? Explain. **(b)** Does your answer to part (a) depend on whether the orbit is circular or elliptical?

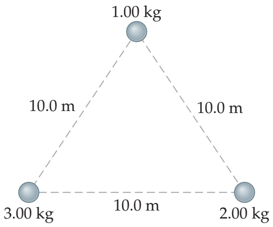
70. • **CE** **The Crash of Skylab** Skylab, the largest spacecraft ever to fall back to the Earth, met its fiery end on July 11, 1979, after flying directly over Everett, WA, on its last orbit. On the *CBS Evening News* the night before the crash, anchorman Walter Cronkite, in his rich baritone voice, made the following statement: "NASA says there is a little chance that Skylab will land in a populated area." After the commercial, he immediately corrected himself by saying, "I meant to say '*there is little chance*' Skylab will hit a populated area." In fact, it landed primarily in the Indian Ocean off the west coast of Australia, though several pieces were recovered near the town of Esperance, Australia, which later sent the U.S. State Department a $400 bill for littering. The cause of Skylab's crash was the friction it experienced in the upper reaches of the Earth's atmosphere. As the radius of Skylab's orbit decreased, did its speed increase, decrease, or stay the same? Explain.

71. • Consider a system consisting of three masses on the x axis. Mass $m_1 = 1.00$ kg is at $x = 1.00$ m; mass $m_2 = 2.00$ kg is at $x = 2.00$ m; and mass $m_3 = 3.00$ kg is at $x = 3.00$ m. What is the total gravitational potential energy of this system?

72. •• An astronaut exploring a distant solar system lands on an unnamed planet with a radius of 3860 km. When the astronaut jumps upward with an initial speed of 3.10 m/s, she rises to a height of 0.580 m. What is the mass of the planet?

73. •• **IP** When the Moon is in its third-quarter phase, the Earth, Moon, and Sun form a right triangle, as shown in Figure 12–22. Calculate the magnitude of the force exerted on the Moon by **(a)** the Earth and **(b)** the Sun. **(c)** Does it make more sense to think of the Moon as orbiting the Sun, with a small effect due to the Earth, or as orbiting the Earth, with a small effect due to the Sun?

74. •• An equilateral triangle 10.0 m on a side has a 1.00-kg mass at one corner, a 2.00-kg mass at another corner, and a 3.00-kg mass at the third corner (**Figure 12–25**). Find the magnitude and direction of the net force acting on the 1.00-kg mass.



▲ **FIGURE 12–25**
Problems 74 and 75

75. •• Suppose that each of the three masses in Figure 12–25 is replaced by a mass of 5.95 kg and radius 0.0714 m. If the balls are released from rest, what speed will they have when they collide at the center of the triangle? Ignore gravitational effects from any other objects.

76. •• **A Near Miss!** In the early morning hours of June 14, 2002, the Earth had a remarkably close encounter with an asteroid the size of a small city. The previously unknown asteroid, now designated 2002 MN, remained undetected until three days after it had passed the Earth. At its closest approach, the asteroid was 73,600 miles from the center of the Earth—about a third of the distance to the Moon. **(a)** Find the speed of the asteroid at closest approach, assuming its speed at infinite distance to be zero and considering only its interaction with the Earth. **(b)** Observations indicate the asteroid to have a diameter of about 2.0 km. Estimate the kinetic energy of the asteroid at closest approach, assuming it has an average density of 3.33 g/cm^3. (For comparison, a 1-megaton nuclear weapon releases about 5.6×10^{15} J of energy.)

77. •• **IP** Suppose a planet is discovered that has the same amount of mass in a given volume as the Earth, but has half its radius. **(a)** Is the acceleration due to gravity on this planet more than, less than, or the same as the acceleration due to gravity on the Earth? Explain. **(b)** Calculate the acceleration due to gravity on this planet.

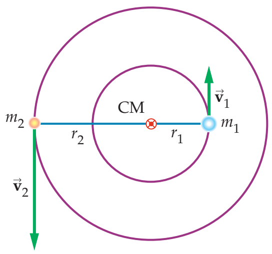
78. •• **IP** Suppose a planet is discovered that has the same total mass as the Earth, but half its radius. **(a)** Is the acceleration due to gravity on this planet more than, less than, or the same as the acceleration due to gravity on the Earth? Explain. **(b)** Calculate the acceleration due to gravity on this planet.

79. •• Show that the speed of a satellite in a circular orbit a height h above the surface of the Earth is

$$v = \sqrt{\frac{GM_E}{R_E + h}}$$

80. •• In a binary star system, two stars orbit about their common center of mass, as shown in **Figure 12–26**. If $r_2 = 2r_1$, what is the ratio of the masses m_2/m_1 of the two stars?

81. •• Find the orbital period of the binary star system described in the previous problem.



▲ **FIGURE 12–26**
Problems 80 and 81

82. •• Using the results from Problem 54, find the angular momentum of Halley's comet **(a)** at perihelion and **(b)** at aphelion. (Take the mass of Halley's comet to be 9.8×10^{14} kg.)

83. •• **Exploring Mars** In the not-too-distant future astronauts will travel to Mars to carry out scientific explorations. As part of their mission, it is likely that a "geosynchronous" satellite will be placed above a given point on the Martian equator to facilitate communications. At what altitude above the surface of Mars should such a satellite orbit? (*Note:* The Martian "day" is 24.6229 hours. Other relevant information can be found in Appendix C.)

84. •• **IP** A satellite is placed in Earth orbit 1000 miles higher than the altitude of a geosynchronous satellite. Referring to Active Example 12–1, we see that the altitude of the satellite is 23,300 mi. **(a)** Is the period of this satellite greater than or less than 24 hours? **(b)** As viewed from the surface of the Earth, does the satellite move eastward or westward? Explain. **(c)** Find the orbital period of this satellite.

85. •• Find the speed of the *Millennium Eagle* at point A in Example 12–1 if its speed at point B is 0.905 m/s.

86. •• Show that the force of gravity between the Moon and the Sun is always greater than the force of gravity between the Moon and the Earth.

87. •• The astronomical unit AU is defined as the mean distance from the Sun to the Earth ($1 \text{ AU} = 1.50 \times 10^{11}$ m). Apply Kepler's third law (Equation 12–7) to the solar system, and show that it can be written as

$$T = Cr^{3/2}$$

In this expression, the period T is measured in years, the distance r is measured in astronomical units, and the constant C has a magnitude that you must determine.

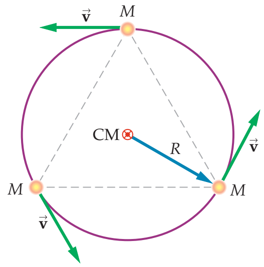
88. •• **(a)** Find the kinetic energy of a 1720-kg satellite in a circular orbit about the Earth, given that the radius of the orbit is 12,600 miles. **(b)** How much energy is required to move this satellite to a circular orbit with a radius of 25,200 miles?

89. •• **IP Space Shuttle Orbit** On a typical mission, the space shuttle ($m = 2.00 \times 10^6$ kg) orbits at an altitude of 250 km above the Earth's surface. **(a)** Does the orbital speed of the shuttle depend on its mass? Explain. **(b)** Find the speed of the shuttle in its orbit. **(c)** How long does it take for the shuttle to complete one orbit of the Earth?

90. ••• **IP** Consider an object of mass m orbiting the Earth at a radius r. **(a)** Find the speed of the object. **(b)** Show that the total mechanical energy of this object is equal to (-1) times its kinetic energy. **(c)** Does the result of part (b) apply to an object orbiting the Sun? Explain.

91. ••• In a binary star system two stars orbit about their common center of mass. Find the orbital period of such a system, given that the stars are separated by a distance d and have masses m and $2m$.

92. ••• Three identical stars, at the vertices of an equilateral triangle, orbit about their common center of mass (**Figure 12–27**). Find



▲ **FIGURE 12–27**
Problem 92

the period of this orbital motion in terms of the orbital radius, R, and the mass of each star, M.

93. ••• Find an expression for the kinetic energy of a satellite of mass m in an orbit of radius r about a planet of mass M.

94. ••• Referring to Example 12–1, find the x component of the net force acting on the *Millennium Eagle* as a function of x. Plot your result, showing both negative and positive values of x.

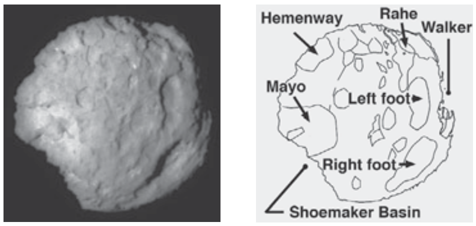
95. ••• A satellite orbits the Earth in an elliptical orbit. At perigee its distance from the center of the Earth is 22,500 km and its speed is 4280 m/s. At apogee its distance from the center of the Earth is 24,100 km and its speed is 3990 m/s. Using this information, calculate the mass of the Earth.

PASSAGE PROBLEMS

Exploring Comets with the *Stardust* Spacecraft

On February 7, 1999, NASA launched a spacecraft with the ambitious mission of making a close encounter with a comet, collecting samples from its tail, and returning the samples to Earth for analysis. This spacecraft, appropriately named *Stardust*, took almost five years to rendezvous with its objective—comet Wild 2 (pronounced "Vilt 2")—and another two years to return its samples. The reason for the long round trip is that the spacecraft had to make three orbits around the Sun, and also an Earth Gravity Assist (EGA) flyby, to increase its speed enough to put it in an orbit appropriate for the encounter.

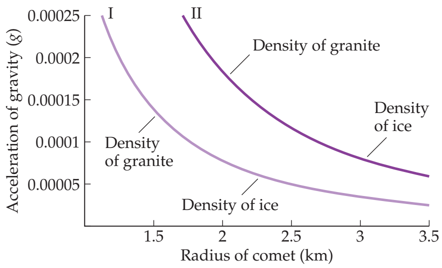
When *Stardust* finally reached comet Wild 2 on January 2, 2004, it flew within 147 miles of the comet's nucleus, snapping pictures and collecting tiny specks of dust in the glistening coma. The approach speed between the spacecraft and the comet at the encounter was a relatively "slow" 6200 m/s, so that dust particles could be collected safely without destroying the vehicle. Note that "slow" is put in quotation marks; after all, 6200 m/s is still about six times the speed of a rifle bullet!



Comet Wild 2 and some of its surface features, including the Walker basin, the site of unusual jets of outward-flowing dust and rocks.

The roughly spherical comet Wild 2 has a radius of 2.7 km, and the acceleration due to gravity on its surface is $0.00010g$. The two curves in **Figure 12–28** show the surface acceleration as a function of radius for a spherical comet with two different masses, one of which corresponds to comet Wild 2. Also indicated are radii at which these two hypothetical comets have densities equal to that of ice and granite.

The *Stardust* spacecraft is still in space; only its small return capsule came back to Earth. It has now been given a new assignment—to visit and photograph comet Tempel 1, the object of the Deep Impact collision on July 4, 2005. This mission, called New Exploration of Tempel 1 (NExT), is scheduled to make its close encounter with comet Tempel 1 on February 14, 2011.



▲ **FIGURE 12–28** Problems 96, 97, 98, and 99

96. • Which of the two curves in Figure 12–28 corresponds to comet Wild 2?

A. Curve I **B.** Curve II

97. • What is the mass of comet Wild 2?

A. 1.1×10^{8} kg **B.** 1.1×10^{12} kg

C. 1.1×10^{14} kg **D.** 1.1×10^{18} kg

98. • Find the speed needed to escape from the surface of comet Wild 2. (*Note:* It is easy for a person to jump upward with a speed of 3 m/s.)

A. 1.6 m/s **B.** 2.3 m/s

C. 72 m/s **D.** 230 m/s

99. •• Suppose comet Wild 2 had a small satellite in orbit around it, just as Dactyl orbits asteroid 243 Ida (see page 390). If this satellite were to orbit at twice the radius of the comet, what would be its period of revolution?

A. 0.93 h **B.** 2.9 h

C. 5.8 h **D.** 8.2 h

INTERACTIVE PROBLEMS

100. •• Find the orbital radius that corresponds to a "year" of 150 days.

101. •• **IP** Suppose the mass of the Sun is suddenly doubled, but the Earth's orbital radius remains the same. **(a)** Would the length of an Earth year increase, decrease, or stay the same? **(b)** Find the length of a year for the case of a Sun with twice the mass. **(c)** Suppose the Sun retains its present mass, but the mass of the Earth is doubled instead. Would the length of the year increase, decrease, or stay the same?

102. •• **IP Referring to Example 12–7 (a)** If the mass of the Earth were doubled, would the escape speed of a rocket increase, decrease, or stay the same? **(b)** Calculate the escape speed of a rocket for the case of an Earth with twice its present mass. **(c)** If the mass of the Earth retains its present value, but the mass of the rocket is doubled, does the escape speed increase, decrease, or stay the same?

103. •• **IP Referring to Example 12–7** Suppose the Earth is suddenly shrunk to half its present radius without losing any of its mass. **(a)** Would the escape speed of a rocket increase, decrease, or stay the same? **(b)** Find the escape speed for an Earth with half its present radius.

13 Oscillations About Equilibrium

In this era of atomic timekeepers and electronic digital readouts, a pendulum seems little more than a quaint reminder of the age of grandfather clocks. But pendulums played an important role in the development of physics, and analyzing the motion of a pendulum still provides insight into key physical principles. In this chapter we will explore the behavior of objects that swing, vibrate, or oscillate—and lay the foundations for understanding many natural phenomena, including sound.

In Chapter 11 we considered systems in static equilibrium. Such systems are seldom left undisturbed for very long, it seems, before they are displaced from equilibrium by a bump, a kick, or a nudge. When this happens to a system, it often results in **oscillations** back and forth from one side of the equilibrium position to the other.

The basic cause of oscillations is the fact that when an object is displaced from a position of stable equilibrium it experiences a *restoring force* that is directed back toward the equilibrium position. Thus, the restoring force accelerates the object in the direction of its initial, equilibrium position. When it reaches equilibrium the force acting on it is zero, but it doesn't come to rest. In moving back to equilibrium, it has gained speed and momentum, and hence its inertia carries it through the equilibrium position to the other side, where the restoring force is now in the opposite direction. The process repeats itself, leading to a series of oscillations.

Perhaps the most familiar oscillating system is the simple pendulum, like the ones that keep time in grandfather clocks. Modern digital wristwatches also use oscillators to keep time, but theirs are tiny quartz crystals. In fact, oscillating systems are found in nature over virtually all length scales, from water molecules that oscillate in a microwave oven, to planets that oscillate when struck by an asteroid, to the universe itself, which some think may oscillate in a series of “big bangs” followed by equally momentous “big crunches.”

13–1 Periodic Motion

▲ Periodic phenomena are found everywhere in nature, from the movements of the heavenly bodies to the vibration of individual atoms. The trace of an electrocardiogram (ECG or EKG), as shown here, records the rhythmic electrical activity that accompanies the beating of our hearts.

A motion that repeats itself over and over is referred to as **periodic motion.** The beating of your heart, the ticking of a clock, and the movement of a child on a swing are familiar examples. One of the key characteristics of a periodic system is the time required for the completion of one cycle of its repetitive motion. For example, the pendulum in a grandfather clock might take one second to swing from maximum displacement in one direction to maximum displacement in the opposite direction, or two seconds for a complete cycle of oscillation. In this case, we say that the **period**, T, of the pendulum is 2 s.

Definition of Period, *T*

T = time required for one cycle of a periodic motion

SI unit: seconds/cycle = s

Note that a cycle (that is, an oscillation) is dimensionless.

Closely related to the period is another common measure of periodic motion, the **frequency,** f. The frequency of an oscillation is simply the number of oscillations per unit of time. Thus, f tells us how frequently, or rapidly, an oscillation takes place—the higher the frequency, the more rapid the oscillations. By definition, the frequency is simply the inverse of the period, T:

Definition of Frequency, *f*

$$f = \frac{1}{T} \qquad \text{13–1}$$

SI unit: cycle/second = $1/\mathrm{s} = \mathrm{s}^{-1}$

Note that if the period of an oscillation, T, is very small, corresponding to rapid oscillations, the corresponding frequency, $f = 1/T$, will be large, as expected.

A special unit has been introduced for the measurement of frequency. It is the **hertz** (Hz), named for the German physicist Heinrich Hertz (1857–1894), in honor of his pioneering studies of radio waves. By definition, one Hz is one cycle per second:

$$1\ \mathrm{Hz} = 1\ \mathrm{cycle/second}$$

High frequencies are often measured in kilohertz (kHz), where 1 kHz = 10^3 Hz, or megahertz (MHz), where 1 MHz = 10^6 Hz.

EXERCISE 13–1

The processing “speed” of a computer refers to the number of binary operations it can perform in one second, so it is really a frequency. If the processor of a personal computer operates at 1.80 GHz, how much time is required for one processing cycle?

SOLUTION

The frequency of the computer’s processor is f = 1.80 GHz. Therefore, we can use Equation 13–1 to solve for the processing period:

$$T = \frac{1}{f} = \frac{1}{1.80\ \mathrm{GHz}} = \frac{1}{1.80 \times 10^9\ \mathrm{cycles/s}} = 5.56 \times 10^{-10}\ \mathrm{s}$$

The high frequency of the computer corresponds to a very small period—less than a billionth of a second to complete one operation. We next consider a situation in which the frequency is considerably smaller.

EXERCISE 13–2

A tennis ball is hit back and forth between two players warming up for a match. If it takes 2.31 s for the ball to go from one player to the other, what are the period and frequency of the ball's motion?

SOLUTION

The period of this motion is the time for the ball to complete one round trip from one player to the other and back. Therefore,

$$T = 2(2.31\ \text{s}) = 4.62\ \text{s}$$

$$f = \frac{1}{T} = \frac{1}{4.62\ \text{s}} = 0.216\ \text{Hz}$$

Notice that the period and frequency of periodic motion can vary over a remarkably large range. Table 13–1 gives a sampling of typical values of T and f.

TABLE 13–1 Typical Periods and Frequencies

System	Period (s)	Frequency (Hz)
Precession of the Earth	8.2×10^{11} (26,000 y)	1.2×10^{-12}
Hour hand of a clock	43,200 (12 h)	2.3×10^{-5}
Minute hand of a clock	3600	2.8×10^{-4}
Second hand of clock	60	0.017
Pendulum in grandfather clock	2.0	0.50
Human heartbeat	1.0	1.0
Lower range of human hearing	5.0×10^{-2}	20
Wing beat of housefly	5.0×10^{-3}	200
Upper range of human hearing	5.0×10^{-5}	20,000
Computer processor	5.6×10^{-10}	1.8×10^{9}

13–2 Simple Harmonic Motion

Periodic motion can take many forms, as illustrated by the tennis ball going back and forth between players in Exercise 13–2, or an expectant father pacing up and down in a hospital hallway. There is one type of periodic motion, however, that is of particular importance. It is referred to as **simple harmonic motion.**

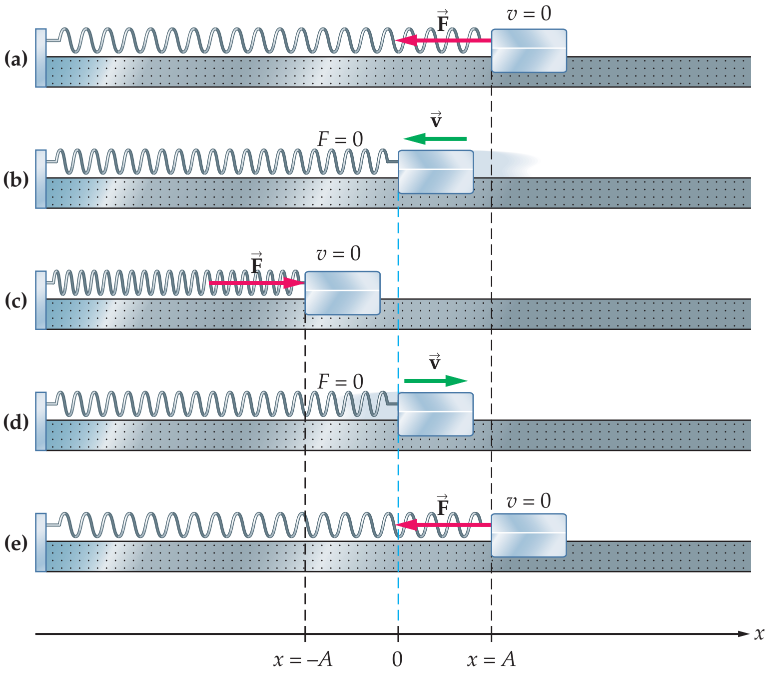
A classic example of simple harmonic motion is provided by the oscillations of a mass attached to a spring. (See Section 6–2 for a discussion of ideal springs and the forces they exert.) To be specific, consider an air-track cart of mass m attached to a spring of force constant k, as in **Figure 13–1**. When the spring is at its equilibrium length—neither stretched nor compressed—the cart is at the position $x = 0$, where it will remain at rest if left undisturbed. If the cart is displaced from equilibrium by a distance x, however, the spring exerts a restoring force given by Hooke's law, $F = -kx$. In words:

> A spring exerts a restoring force whose magnitude is proportional to the distance it is displaced from equilibrium.

This direct proportionality between distance from equilibrium and force is the key feature of a mass–spring system that leads to simple harmonic motion. As for the direction of the spring force:

> The force exerted by a spring is opposite in direction to its displacement from equilibrium; this accounts for the minus sign in $F = -kx$. In general, a restoring force is one that *always* points toward the equilibrium position.

Now, suppose the cart is released from rest at the location $x = A$. As indicated in Figure 13–1, the spring exerts a force on the cart to the left, causing the cart to accelerate toward the equilibrium position. When the cart reaches $x = 0$, the net



▶ FIGURE 13–1 A mass attached to a spring undergoes simple harmonic motion about $x = 0$

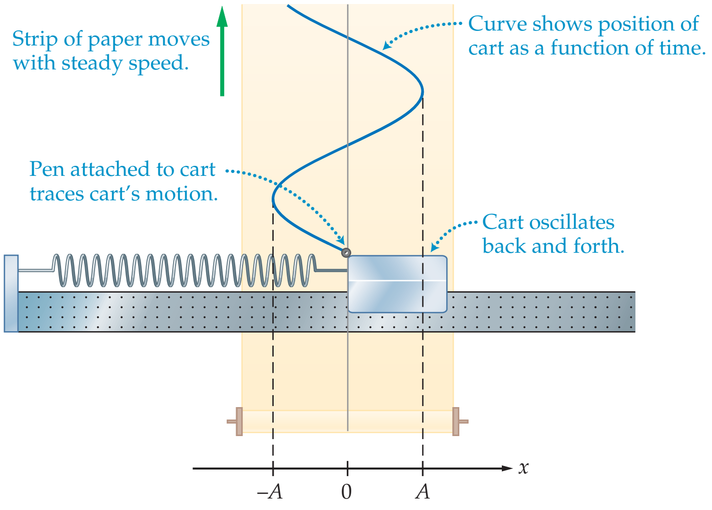
(a) The mass is at its maximum positive value of x. Its velocity is zero, and the force on it points to the left with maximum magnitude. **(b)** The mass is at the equilibrium position of the spring. Here the speed has its maximum value, and the force exerted by the spring is zero. **(c)** The mass is at its maximum displacement in the negative x direction. The velocity is zero here, and the force points to the right with maximum magnitude. **(d)** The mass is at the equilibrium position of the spring, with zero force acting on it and maximum speed. **(e)** The mass has completed one cycle of its oscillation about $x = 0$.

force acting on it is zero. Its speed is not zero at this point, however, and so it continues to move to the left. As the cart compresses the spring, it experiences a force to the right, causing it to decelerate and finally come to rest at $x = -A$. The spring continues to exert a force to the right; thus, the cart immediately begins to move to the right until it comes to rest again at $x = A$, completing one oscillation in the time T.

If a pen is attached to the cart, it can trace its motion on a strip of paper moving with constant speed, as indicated in **Figure 13–2**. On this "strip chart" we obtain a record of the cart's motion as a function of time. As we see in Figure 13–2, the motion of the cart looks like a sine or a cosine function.

Mathematical analysis, using the methods of calculus, shows that this is indeed the case; that is, the position of the cart as a function of time can be represented by a sine or a cosine function. The reason that either function works can be seen by considering **Figure 13–3**. If we take $t = 0$ to be at point 1, for example, the position as a function of time starts at zero, just like a sine function; if we choose $t = 0$ to be at point 2, however, the position versus time starts at its maximum value, just like a cosine function. It is really the same mathematical function, differing only in the choice of starting point.

Returning to Figure 13–1, note that the position of the mass oscillates between $x = +A$ and $x = -A$. Since A represents the extreme displacement of the cart on



▶ FIGURE 13–2 Displaying position versus time for simple harmonic motion

As an air-track cart oscillates about its equilibrium position, a pen attached to it traces its motion onto a moving sheet of paper. This produces a "strip chart," showing that the cart's motion has the shape of a sine or a cosine.

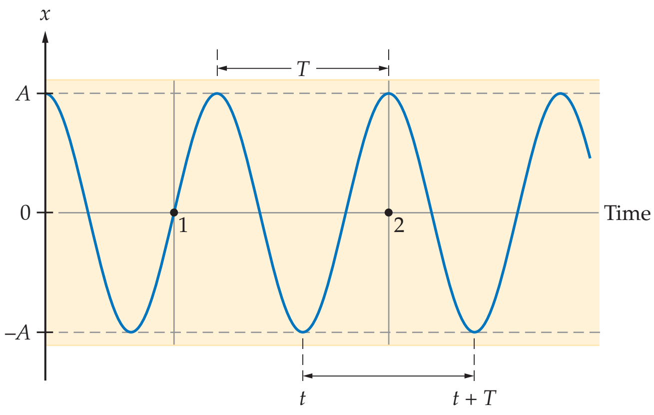


FIGURE 13–3 Simple harmonic motion as a sine or a cosine

The strip chart from Figure 13–2. The cart oscillates back and forth from $x = +A$ to $x = -A$, completing one cycle in the time T. The function traced by the pen can be represented as a sine function if $t = 0$ is taken to be at point 1, where the function is equal to zero. The function can be represented by a cosine if $t = 0$ is taken to be at point 2, where the function has its maximum value.

either side of equilibrium, we refer to it as the **amplitude** of the motion. It follows that the amplitude is one-half the total range of motion. In addition, recall that the cart's motion repeats with a period T. As a result, the position of the cart is the same at the time $t + T$ as it is at the time t, as shown in Figure 13–3. Combining all these observations results in the following mathematical description of position versus time:

Position Versus Time in Simple Harmonic Motion

$$x = A\cos\left(\frac{2\pi}{T}t\right) \qquad 13\text{–}2$$

SI unit: m

PROBLEM-SOLVING NOTE

Using Radians

When evaluating Equation 13–2, be sure you have your calculator set to "radians" mode.

This type of dependence on time—as a sine or a cosine—is characteristic of simple harmonic motion. With this particular choice, the position at $t = 0$ is $x = A\cos(0) = A$; thus, Equation 13–2 describes an object that has its maximum displacement at $t = 0$, as in Figure 13–3.

To see how Equation 13–2 works, recall that the cosine oscillates between +1 and −1. Therefore, $x = A\cos(2\pi t/T)$ will oscillate between $+A$ and $-A$, just as in the strip chart. Next, consider what happens if we replace the time t with the time $t + T$. This gives

$$x = A\cos\left(\frac{2\pi}{T}(t + T)\right)$$
$$= A\cos\left(\frac{2\pi}{T}t + \frac{2\pi}{T}T\right) = A\cos\left(\frac{2\pi}{T}t + 2\pi\right)$$

Finally, using the fact that $\cos(\theta + 2\pi) = \cos\theta$ for any angle θ, we can rewrite the last expression as follows:

$$x = A\cos\left(\frac{2\pi}{T}t\right)$$

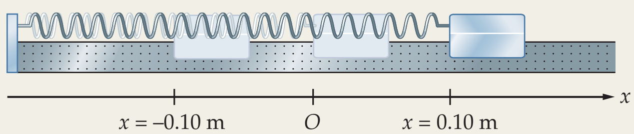
Therefore, as expected, the position at time $t + T$ is precisely the same as the position at time t.

EXAMPLE 13–1 SPRING TIME

An air-track cart attached to a spring completes one oscillation every 2.4 s. At $t = 0$ the cart is released from rest at a distance of 0.10 m from its equilibrium position. What is the position of the cart at **(a)** 0.30 s, **(b)** 0.60 s, **(c)** 2.7 s, and **(d)** 3.0 s?

PICTURE THE PROBLEM

In our sketch, we place the origin of the x-axis at the equilibrium position of the cart and the positive direction to point to the right. The cart is released from rest at $x = 0.10$ m, which means that its amplitude is $A = 0.10$ m. After it is released, the cart oscillates back and forth between $x = 0.10$ m and $x = -0.10$ m.



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STRATEGY

Note that the period of oscillation, $T = 2.4$ s, is given in the problem statement. Thus, we can find the position of the cart by evaluating $x = A\cos(2\pi t/T)$ at the desired times, using $A = 0.10$ m. (*Note*: Remember to have your calculator set to radian mode when evaluating these cosine functions.)

SOLUTION

Part (a)

1. Calculate x at the time $t = 0.30$ s:

$$x = A\cos\left(\frac{2\pi}{T}t\right) = (0.10\text{ m})\cos\left[\left(\frac{2\pi}{2.4\text{ s}}\right)(0.30\text{ s})\right]$$
$$= (0.10\text{ m})\cos(\pi/4) = 7.1\text{ cm}$$

Part (b)

2. Now, substitute $t = 0.60$ s:

$$x = A\cos\left(\frac{2\pi}{T}t\right) = (0.10\text{ m})\cos\left[\left(\frac{2\pi}{2.4\text{ s}}\right)(0.60\text{ s})\right]$$
$$= (0.10\text{ m})\cos(\pi/2) = 0$$

Part (c)

3. Repeat with $t = 2.7$ s:

$$x = A\cos\left(\frac{2\pi}{T}t\right) = (0.10\text{ m})\cos\left[\left(\frac{2\pi}{2.4\text{ s}}\right)(2.7\text{ s})\right]$$
$$= (0.10\text{ m})\cos(9\pi/4) = 7.1\text{ cm}$$

Part (d)

4. Repeat with $t = 3.0$ s:

$$x = A\cos\left(\frac{2\pi}{T}t\right) = (0.10\text{ m})\cos\left[\left(\frac{2\pi}{2.4\text{ s}}\right)(3.0\text{ s})\right]$$
$$= (0.10\text{ m})\cos(5\pi/2) = 0$$

INSIGHT

Note that the results for parts (c) and (d) are the same as for parts (a) and (b), respectively. This is because the times in (c) and (d) are greater than the corresponding times in (a) and (b) by one period; that is, $2.7\text{ s} = 0.30\text{ s} + 2.4\text{ s}$ and $3.0\text{ s} = 0.60\text{ s} + 2.4\text{ s}$.

PRACTICE PROBLEM

What is the first time the cart is at the position $x = -5.0$ cm? [**Answer:** $t = T/3 = 0.80$ s]

Some related homework problems: Problem 10, Problem 17, Problem 18

Though the motion of the air-track cart is strictly one-dimensional, it bears a close relationship to uniform circular motion. In the next section we explore this connection between simple harmonic motion and uniform circular motion in detail.

13–3 Connections Between Uniform Circular Motion and Simple Harmonic Motion

Imagine a turntable that rotates with a constant angular speed $\omega = 2\pi/T$, taking the time T to complete a revolution. At the rim of the turntable we place a small peg, as indicated in **Figure 13–4**. If we view the turntable from above, we see the peg undergoing uniform circular motion.

On the other hand, suppose we view the turntable from the side, so that the peg appears to move back and forth. Perhaps the easiest way to view this motion is to shine a light that casts a shadow of the peg on a screen, as shown in Figure 13–4. While the peg itself moves on a circular path, its shadow moves back and forth in a straight line.

To be specific, let the radius of the turntable be $r = A$, so that the shadow moves from $x = +A$ to $x = -A$. When the shadow is at $x = +A$, release a mass on a spring that is also at $x = +A$, so that the mass and the shadow start together. If we adjust the period of the mass so that it completes one oscillation in the same time T that the turntable completes one revolution, we find that the mass and the shadow move as one for all times. This is also illustrated in Figure 13–4. Since the mass undergoes simple harmonic motion, it follows that the shadow does so as well.

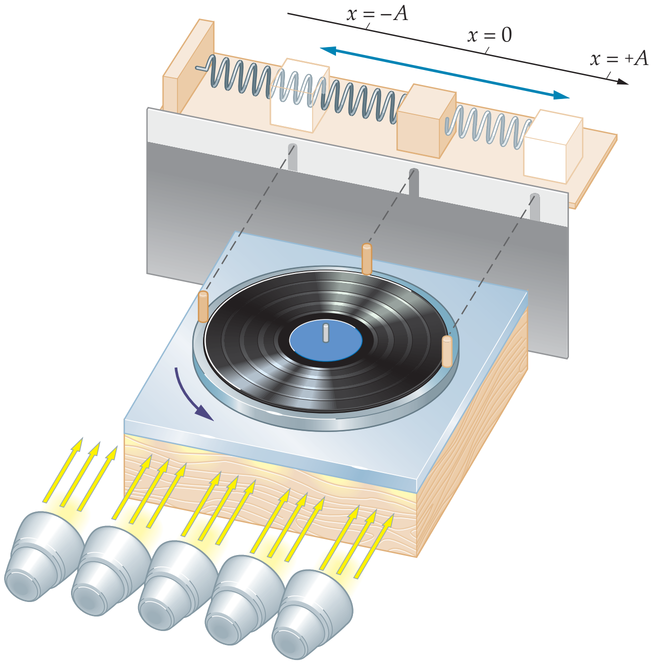


FIGURE 13–4 The relationship between uniform circular motion and simple harmonic motion

A peg is placed at the rim of a turntable that rotates with constant angular velocity. Viewed from above, the peg exhibits uniform circular motion. If the peg is viewed from the side, however, it appears to move back and forth in a straight line, as we can see by shining a light to cast a shadow of the peg onto a screen. The shadow moves with simple harmonic motion. If we compare this motion with the behavior of a mass on a spring, moving with the same period as the turntable and an amplitude of motion equal to the radius of the turntable, we find that the mass and the shadow of the peg move together in simple harmonic motion.

We now use this connection, plus our knowledge of circular motion, to obtain detailed results for the position, velocity, and acceleration of a particle undergoing simple harmonic motion.

Position

In **Figure 13–5** we show the peg at the angular position θ, where θ is measured relative to the x axis. If the peg starts at $\theta = 0$ at $t = 0$, and the turntable rotates with a constant angular speed ω, we know from Equation 10–10 that the angular position of the peg is simply

$$\theta = \omega t \qquad 13\text{–}3$$

That is, the angular position increases linearly with time.

Now, imagine drawing a radius vector of length A to the position of the peg, as indicated in Figure 13–5. When we project the shadow of the peg onto the screen, the shadow is at the location $x = A\cos\theta$, which is the x component of the radius vector. Therefore, the position of the shadow as a function of time is

Position of the Shadow as a Function of Time

$$x = A\cos\theta = A\cos(\omega t) = A\cos\left(\frac{2\pi}{T}t\right) \qquad 13\text{–}4$$

SI unit: m

Note that we have used Equation 13–3 to express θ in terms of the time, t. Clearly, Equations 13–4 and 13–2 are identical, so the shadow does indeed exhibit simple harmonic motion, just like a mass on a spring.

For notational simplicity, we will often write the position of a mass on a spring in the form $x = A\cos(\omega t)$, which is more compact than $x = A\cos(2\pi t/T)$. When referring to a rotating turntable, ω is called the angular speed; when referring to simple harmonic motion, or other periodic motion, we have a slightly different name for ω. In these situations, ω is called the **angular frequency:**

Definition of Angular Frequency, ω

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad 13\text{–}5$$

SI unit: $\text{rad/s} = \text{s}^{-1}$

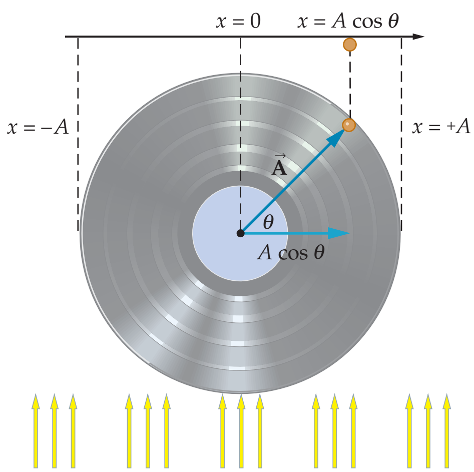


FIGURE 13–5 Position versus time in simple harmonic motion

A peg rotates on the rim of a turntable of radius A. When the peg is at the angular position θ, its shadow is at $x = A\cos\theta$. Note that $A\cos\theta$ is also the x component of the radius vector $\vec{\mathbf{A}}$ from the center of the turntable to the peg.

Velocity

We can find the velocity of the shadow in the same way that we determined its position; first find the velocity of the peg, then take its x component. The result of this calculation will be the velocity as a function of time for simple harmonic motion.

To begin, recall that the velocity of an object in uniform circular motion of radius r has a magnitude equal to

$$v = r\omega$$

In addition, the velocity is tangential to the object's circular path, as indicated in **Figure 13–6 (a)**. Therefore, referring to the figure, we see that when the peg is at the angular position θ, the velocity vector makes an angle θ with the vertical. As a result, the x component of the velocity is $-v \sin \theta$. Combining these results, we find that the velocity of the peg, along the x axis, is

$$v_x = -v \sin \theta = -r\omega \sin \theta$$

In what follows we shall drop the x subscript, since we know that the shadow and a mass on a spring move only along the x axis. Recalling that $r = A$ and $\theta = \omega t$, we have

Velocity in Simple Harmonic Motion

$$v = -A\omega \sin(\omega t) \qquad \text{13–6}$$

SI unit: m/s

We plot x and v for simple harmonic motion in **Figure 13–6 (b)**. Note that when the displacement from equilibrium is a maximum, the velocity is zero. This is to be expected, since at $x = +A$ and $x = -A$ the object is momentarily at rest as it turns around. Not surprisingly, these points are referred to as **turning points** of the motion.

On the other hand, the speed is a maximum when the displacement from equilibrium is zero. Similarly, a mass on a spring is moving with its greatest speed as it goes through $x = 0$. From the expression $v = -A\omega \sin(\omega t)$, and the fact that the largest value of $\sin \theta$ is 1, we see that the maximum speed of the mass is

$$v_{max} = A\omega \qquad \text{13–7}$$

After the mass passes $x = 0$ it begins either to compress or to stretch the spring, and hence it slows down.

Acceleration

The acceleration of an object in uniform circular motion has a magnitude given by

$$a_{cp} = r\omega^2$$

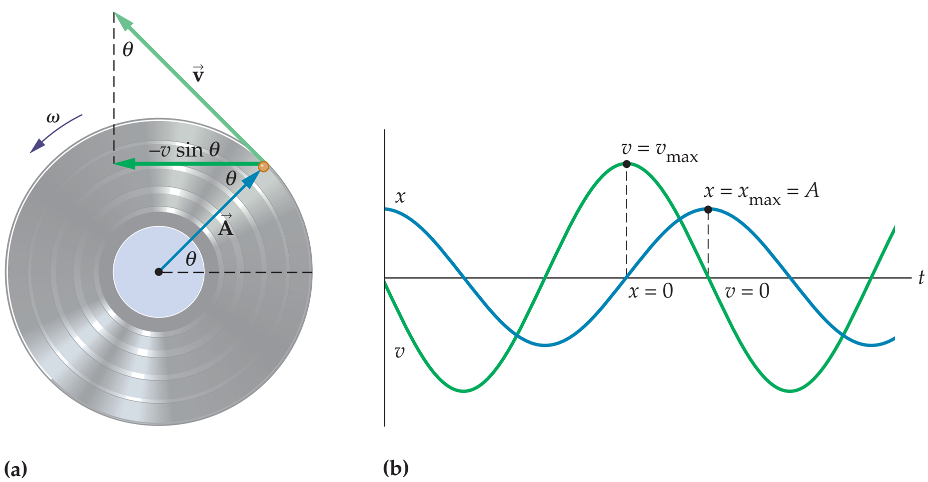


FIGURE 13–6 Velocity versus time in simple harmonic motion

(a) The velocity of a peg rotating on the rim of a turntable is tangential to its circular path. As a result, when the peg is at the angle θ, its velocity makes an angle of θ with the vertical. The x component of the velocity, then, is $-v \sin \theta$. **(b)** Position, x, and velocity, v, as a function of time for simple harmonic motion. The speed is greatest when the object passes through equilibrium, $x = 0$. On the other hand, the speed is zero when the position is greatest—that is, at the turning points. Finally, note that as x moves in the negative direction, the velocity is negative. Similar remarks apply to the positive direction.

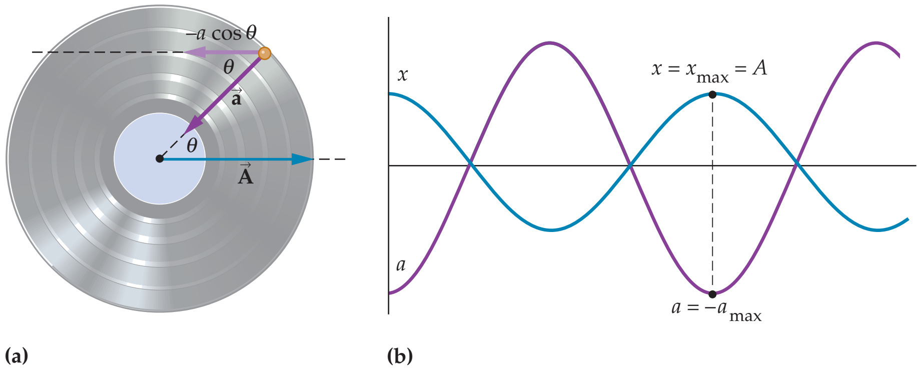


FIGURE 13–7 Acceleration versus time in simple harmonic motion

(a) The acceleration of a peg on the rim of a uniformly rotating turntable is directed toward the center of the turntable. Hence, when the peg is at the angle θ, the acceleration makes an angle θ with the horizontal. The x component of the acceleration is $-a\cos\theta$. **(b)** Position, x, and acceleration, a, as a function of time for simple harmonic motion. Note that when the position has its greatest positive value, the acceleration has its greatest negative value.

In addition, the direction of the acceleration is toward the center of the circular path, as indicated in **Figure 13–7 (a)**. Thus, when the angular position of the peg is θ, the acceleration vector is at an angle θ below the x axis, and its x component is $-a_{cp}\cos\theta$. Again setting $r = A$ and $\theta = \omega t$, we find

Acceleration in Simple Harmonic Motion

$$a = -A\omega^2\cos(\omega t) \qquad 13\text{–}8$$

SI unit: $\mathrm{m/s^2}$

The position and acceleration for simple harmonic motion are plotted in **Figure 13–7 (b)**. Note that the acceleration and position vary with time in the same way but with opposite signs. That is, when the position has its maximum *positive* value, the acceleration has its maximum *negative* value, and so on. After all, the restoring force of the spring is opposite to the position, hence the acceleration, $a = F/m$, must also be opposite to the position. In fact, comparing Equations 13–4 and 13–8 we see that the acceleration can be written as

$$a = -\omega^2 x$$

Finally, since the largest value of x is the amplitude A, we see that the maximum acceleration is of magnitude

$$a_{max} = A\omega^2 \qquad 13\text{–}9$$

We conclude this section with a few examples using position, velocity, and acceleration in simple harmonic motion.

PROBLEM-SOLVING NOTE

Be Sure to Use Radians

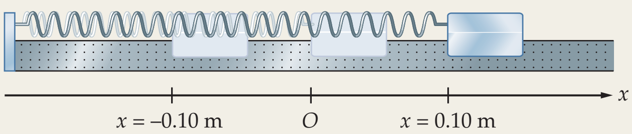
Note that Equations 13–4, 13–6, and 13–8 must all be evaluated in terms of radians.

EXAMPLE 13–2 VELOCITY AND ACCELERATION

As in Example 13–1, an air-track cart attached to a spring completes one oscillation every 2.4 s. At $t = 0$ the cart is released from rest with the spring stretched 0.10 m from its equilibrium position. What are the velocity and acceleration of the cart at **(a)** 0.30 s and **(b)** 0.60 s?

PICTURE THE PROBLEM

Once again, we place the origin of the x axis at the equilibrium position of the cart, with the positive direction pointing to the right. In addition, the cart is released from rest at $x = 0.10$ m, which means that it will have zero speed at $x = 0.10$ m and $x = -0.10$ m. Its speed will be a maximum at $x = 0$, however, which is also the point where the acceleration is zero.



STRATEGY

After calculating the angular frequency, $\omega = 2\pi/T$, we simply substitute $t = 0.30$ s and $t = 0.60$ s into $v = -A\omega\sin(\omega t)$ and $a = -A\omega^2\cos(\omega t)$. Remember to set your calculators to radian mode.

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SOLUTION

1. Calculate the angular frequency for this motion:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{(2.4\ \mathrm{s})} = 2.6\ \mathrm{rad/s}$$

Part (a)

2. Calculate v at the time $t = 0.30$ s. Express ω in terms of π—that is, $\omega = 2\pi/(2.4\ \mathrm{s})$. This isn't necessary, but it makes it easier to evaluate the sine and cosine functions. (See the discussion following the Example for additional details):

$$v = -A\omega \sin(\omega t)$$
$$= -(0.10\ \mathrm{m})(2.6\ \mathrm{rad/s}) \sin\left[\left(\frac{2\pi}{2.4\ \mathrm{s}}\right)(0.30\ \mathrm{s})\right]$$
$$= -(26\ \mathrm{cm/s}) \sin(\pi/4) = -18\ \mathrm{cm/s}$$

3. Similarly, calculate a at $t = 0.30$ s:

$$a = -A\omega^2 \cos(\omega t)$$
$$= -(0.10\ \mathrm{m})(2.6\ \mathrm{rad/s})^2 \cos\left[\left(\frac{2\pi}{2.4\ \mathrm{s}}\right)(0.30\ \mathrm{s})\right]$$
$$= -(68\ \mathrm{cm/s^2}) \cos(\pi/4) = -48\ \mathrm{cm/s^2}$$

Part (b)

4. Calculate v at the time $t = 0.60$ s:

$$v = -A\omega \sin(\omega t)$$
$$= -(0.10\ \mathrm{m})(2.6\ \mathrm{rad/s}) \sin\left[\left(\frac{2\pi}{2.4\ \mathrm{s}}\right)(0.60\ \mathrm{s})\right]$$
$$= -(26\ \mathrm{cm/s}) \sin(\pi/2) = -26\ \mathrm{cm/s}$$

5. Similarly, calculate a at $t = 0.60$ s:

$$a = -A\omega^2 \cos(\omega t)$$
$$= -(0.10\ \mathrm{m})(2.6\ \mathrm{rad/s})^2 \cos\left[\left(\frac{2\pi}{2.4\ \mathrm{s}}\right)(0.60\ \mathrm{s})\right]$$
$$= -(68\ \mathrm{cm/s^2}) \cos(\pi/2) = 0$$

INSIGHT

Note that the cart speeds up from $t = 0.30$ s to $t = 0.60$ s; in fact, the maximum speed of the cart, $v_{\mathrm{max}} = A\omega = 26$ cm/s, occurs at $t = 0.60\ \mathrm{s} = T/4$. Referring to Example 13–1, we see that this is precisely the time when the cart is at the equilibrium position, $x = 0$. As expected, the acceleration is zero at this time.

PRACTICE PROBLEM

What is the first time the velocity of the cart is +26 cm/s? [**Answer:** $v = +26$ cm/s at $t = 3T/4 = 1.8$ s]

Some related homework problems: Problem 23, Problem 24

PROBLEM-SOLVING NOTE

Expressing Time in Terms of the Period

When evaluating the expression $x = A\cos(2\pi t/T)$ at the time t, it is often helpful to express t in terms of the period T.

In problems like the preceding Example, it is often useful to express the time t in terms of the period, T. For example, if the period is $T = 2.4$ s it follows that $t = 0.60$ s is $T/4$. Thus, the angular frequency times the time is

$$\omega t = \left(\frac{2\pi}{T}\right)\left(\frac{T}{4}\right) = \pi/2$$

This result was used in Steps 4 and 5 in Example 13–2. Using $\omega t = \pi/2$ we find that the position of the cart is

$$x = A\cos(\omega t) = A\cos(\pi/2) = 0$$

Similarly, the velocity of the cart is

$$v = -A\omega\sin(\omega t) = -A\omega\sin(\pi/2) = -A\omega$$

and its acceleration is

$$a = -A\omega^2\cos(\omega t) = -A\omega^2\cos(\pi/2) = 0$$

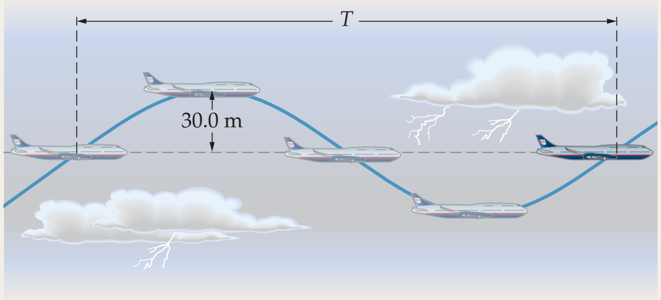
When expressed in this way, it is clear why x and a are zero and why v has its maximum negative value.

EXAMPLE 13–3 TURBULENCE!

On December 29, 1997, a United Airlines flight from Tokyo to Honolulu was hit with severe turbulence 31 minutes after takeoff. Data from the airplane's "black box" indicated the 747 moved up and down with an amplitude of 30.0 m and a maximum acceleration of $1.8g$. Treating the up-and-down motion of the plane as simple harmonic, find **(a)** the time required for one complete oscillation and **(b)** the plane's maximum vertical speed.

PICTURE THE PROBLEM

Our sketch shows the 747 airliner moving up and down with an amplitude of $A = 30.0$ m relative to its normal horizontal flight path. The period of motion, T, is the time required for one complete cycle of this up-and-down motion.



INTERACTIVE FIGURE

STRATEGY

We are given the maximum acceleration and the amplitude of motion. With these quantities, and the basic equations of simple harmonic motion, we can determine the other characteristics of the motion.

a. We know that the maximum acceleration of simple harmonic motion is $a_{max} = A\omega^2$ (Equation 13–9). This relation can be solved for ω in terms of the known quantities a_{max} and A. We rearrange $\omega = 2\pi/T$ to solve for the period of motion, T.

b. The maximum vertical speed is found using $v_{max} = A\omega$ (Equation 13–7).

SOLUTION

Part (a)

1. Relate a_{max} to the angular frequency, ω: $a_{max} = A\omega^2$
2. Solve for ω, and express in terms of T: $\omega = \sqrt{a_{max}/A} = 2\pi/T$
3. Solve for T and substitute numerical values for g and A:

$$T = \frac{2\pi}{\sqrt{a_{max}/A}} = \frac{2\pi}{\sqrt{1.8g/A}} = \frac{2\pi}{\sqrt{1.8(9.81\text{ m/s}^2)/(30.0\text{ m})}} = 8.2\text{ s}$$

Part (b)

4. Calculate the maximum vertical speed using $v_{max} = A\omega = 2\pi A/T$:

$$v_{max} = A\omega = \frac{2\pi A}{T} = \frac{2\pi(30.0\text{ m})}{8.2\text{ s}} = 23\text{ m/s}$$

INSIGHT

We don't expect the up-and-down motion of the plane to be exactly simple harmonic—after all, it's unlikely the plane's path has precisely the shape of a cosine function. Still, the approximation is reasonable. In fact, many systems are well approximated by simple harmonic motion, and hence the equations derived in this section are used widely in physics.

Notice that the maximum vertical speed of the passengers (23 m/s) is roughly 50 mi/h, and that this speed is first in the upward direction, and then 4.1 s later in the downward direction.

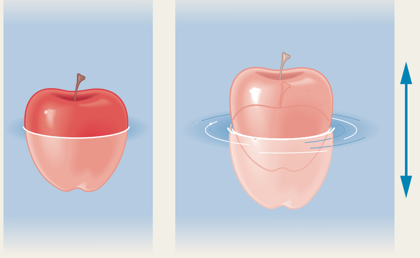
PRACTICE PROBLEM

What amplitude of motion would result in a maximum acceleration of $0.50g$, everything else remaining the same? [**Answer:** $A = 0.50g/\omega^2 = 0.50gT^2/4\pi^2 = 8.4$ m]

Some related homework problems: Problem 23, Problem 25

ACTIVE EXAMPLE 13–1 BOBBING FOR APPLES: FIND THE POSITION, VELOCITY, AND ACCELERATION

A Red Delicious apple floats in a barrel of water. If you lift the apple 2.00 cm above its floating level and release it, it bobs up and down with a period of $T = 0.750$ s. Assuming the motion is simple harmonic, find the position, velocity, and acceleration of the apple at the times **(a)** $T/4$ and **(b)** $T/2$.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Identify the amplitude of motion: $A = 2.00$ cm
2. Calculate the angular frequency: $\omega = 2\pi/T = 8.38$ rad/s

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Part (a)

3. Evaluate $x = A\cos(\omega t)$ at $t = T/4$: $x = A\cos(\pi/2) = 0$

4. Evaluate $v = -A\omega\sin(\omega t)$ at $t = T/4$: $v = -A\omega\sin(\pi/2) = -A\omega$ $= -16.8\text{ cm/s}$

5. Evaluate $a = -A\omega^2\cos(\omega t)$ at $t = T/4$: $a = -A\omega^2\cos(\pi/2) = 0$

Part (b)

6. Evaluate $x = A\cos(\omega t)$ at $t = T/2$: $x = A\cos(\pi) = -A = -2.00\text{ cm}$

7. Evaluate $v = -A\omega\sin(\omega t)$ at $t = T/2$: $v = -A\omega\sin(\pi) = 0$

8. Evaluate $a = -A\omega^2\cos(\omega t)$ at $t = T/2$: $a = -A\omega^2\cos(\pi) = A\omega^2 = 140\text{ cm/s}^2$

INSIGHT

The acceleration in part (b) may seem rather large, but remember that $140\text{ cm/s}^2 = 1.40\text{ m/s}^2$, so the acceleration is only a fraction of the acceleration of gravity.

YOUR TURN

The maximum kinetic energy of this bobbing apple is 0.00388 J. What is its mass?

(Answers to **Your Turn** *problems are given in the back of the book.)*

13–4 The Period of a Mass on a Spring

In this section we show how the period of a mass on a spring is related to the mass, m, the force constant of the spring, k, and the amplitude of motion, A. As a first step, note that the net force acting on the mass at the position x is

$$F = -kx$$

Now, since $F = ma$, it follows that

$$ma = -kx$$

Substituting the time dependence of x and a, as given in Equations 13–4 and 13–8 in the previous section, we find

$$m[-A\omega^2\cos(\omega t)] = -k[A\cos(\omega t)]$$

Canceling $-A\cos(\omega t)$ from each side of the equation yields

$$\omega^2 = k/m$$

or

$$\omega = \sqrt{\frac{k}{m}} \qquad 13\text{–}10$$

Finally, noting that $\omega = 2\pi/T$, it follows that the period of a mass on a spring is

Period of a Mass on a Spring

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad 13\text{–}11$$

SI unit: s

EXERCISE 13–3

When a 0.22-kg air-track cart is attached to a spring, it oscillates with a period of 0.84 s. What is the force constant for this spring?

SOLUTION

From Equation 13–11 we find

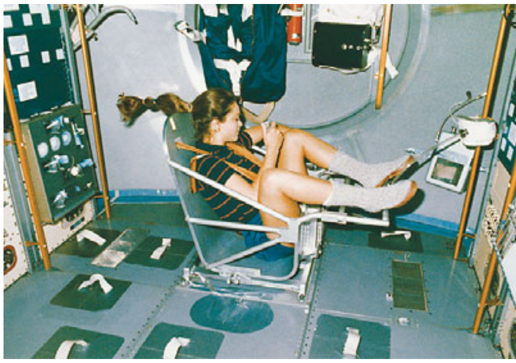
$$k = 4\pi^2 m/T^2 = 12\text{ N/m}$$

As one might expect, the period increases with the mass and decreases with the spring's force constant. For example, a larger mass has greater inertia, and hence it takes longer for the mass to move back and forth through an oscillation. On the other hand, a larger value of the force constant, k, indicates a stiffer spring. Clearly, a mass on a stiff spring completes an oscillation in less time than one on a soft, squishy spring.

The relationship between mass and period given in Equation 13–11 is used by NASA to measure the mass of astronauts in orbit. Recall that astronauts are in free fall as they orbit, as was discussed in Chapter 12, and therefore they are "weightless." As a result, they cannot simply step onto a bathroom scale to determine their mass. Thus, NASA has developed a device, known as the Body Mass Measurement Device (BMMD), to get around this problem. The BMMD is basically a spring attached to a chair, into which an astronaut is strapped. As the astronaut oscillates back and forth, the period of oscillation is measured. Knowing the force constant k and the period of oscillation T, the astronaut's mass can be determined from Equation 13–11. The result is simply $m = kT^2/4\pi^2$. See Problem 76 for an application of this result.

Note that the period given in Equation 13–11 is independent of the amplitude, A, which canceled in the derivation of this equation. This might seem counterintuitive at first: Shouldn't it take more time for a mass to cover the greater distance implied by a larger amplitude? While it is true that a mass will cover a greater distance when the amplitude is increased, it is also true that a larger amplitude implies a larger force exerted by the spring. With a greater force acting on it, the mass moves more rapidly; in fact, the speed of the mass is increased just enough that it covers the greater distance in precisely the same time.

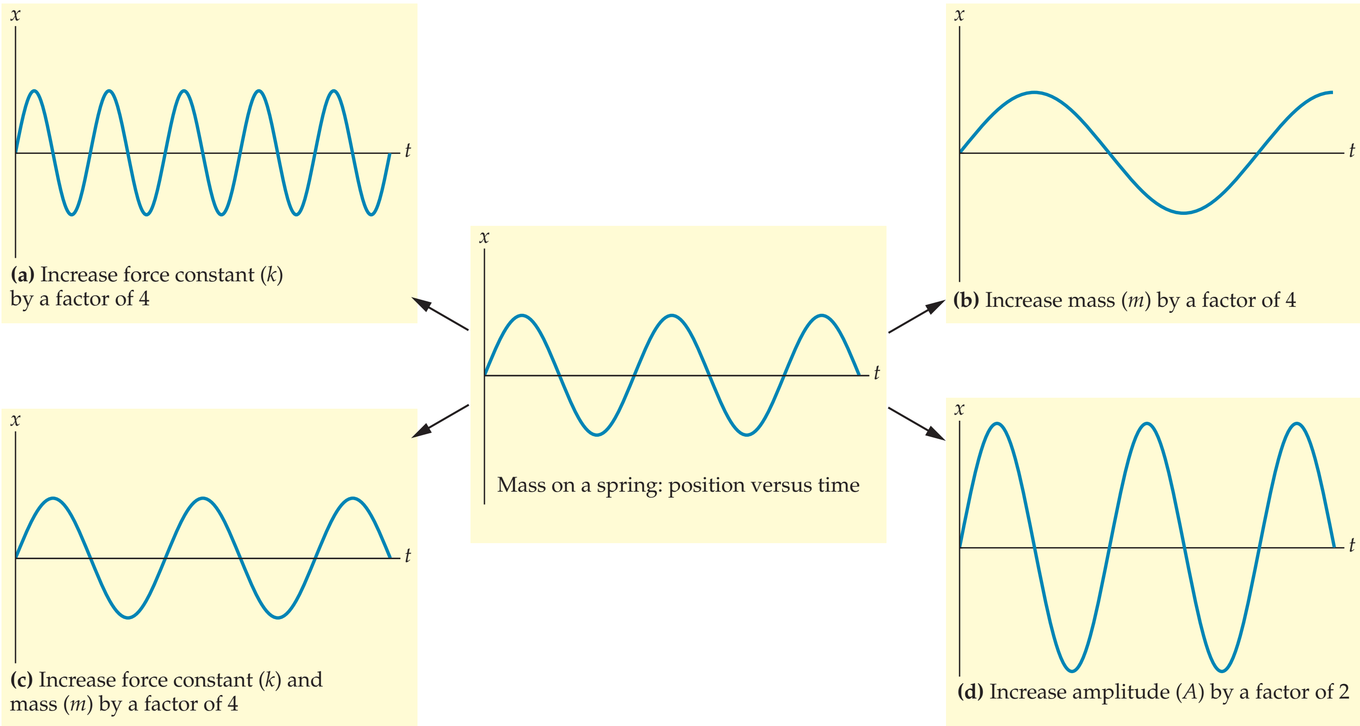
These relationships between the motion of a mass on a spring and the mass, the force constant, and the amplitude are summarized in **Figure 13–8**.



▲ Because astronauts are in free fall when orbiting the Earth, they behave as if they were "weightless." It is nevertheless possible to determine the mass of an astronaut by exploiting the properties of oscillatory motion. The chair into which astronaut Tamara Jernigan is strapped is attached to a spring. If the force constant of the spring is known, her mass can be determined simply by measuring the period with which she rocks back and forth. This instrument is known as a Body Mass Measurement Device (BMMD).

REAL-WORLD PHYSICS

Measuring the mass of a "weightless" astronaut



▲ **FIGURE 13–8 Factors affecting the motion of a mass on a spring**

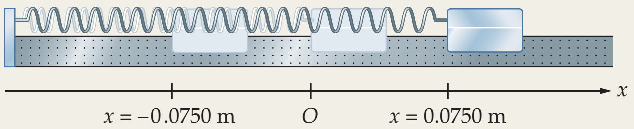
The motion of a mass on a spring is determined by the force constant of the spring, k, the mass, m, and the amplitude, A. **(a)** Increasing the force constant causes the mass to oscillate with a greater frequency. **(b)** Increasing the mass lowers the frequency of oscillation. **(c)** If the force constant and the mass are both increased by the same factor, the effects described in parts (a) and (b) cancel, resulting in no change in the motion. **(d)** An increase in amplitude has no effect on the oscillation frequency. However, it will increase the maximum speed and maximum acceleration of the mass.

EXAMPLE 13–4 SPRING INTO MOTION

A 0.120-kg mass attached to a spring oscillates with an amplitude of 0.0750 m and a maximum speed of 0.524 m/s. Find **(a)** the force constant and **(b)** the period of motion.

PICTURE THE PROBLEM

Our sketch shows a mass oscillating about the equilibrium position of a spring, which we place at $x = 0$. The amplitude of the oscillations is 0.0750 m, and therefore the mass moves back and forth between $x = 0.0750$ m and $x = -0.0750$ m. The maximum speed of the mass, which occurs at $x = 0$, is $v_{max} = 0.524$ m/s.



STRATEGY

a. To find the force constant, we first use the maximum speed, $v_{max} = A\omega$ (Equation 13–7), to determine the angular frequency ω. Once we know ω, we can obtain the force constant with $\omega = \sqrt{k/m}$ (Equation 13–10).

b. We can find the period from the angular frequency, using $\omega = 2\pi/T$. Alternatively, we can use the force constant and the mass in $T = 2\pi\sqrt{m/k}$ (Equation 13–11).

SOLUTION

Part (a)

1. Calculate the angular frequency in terms of the maximum speed:

$$v_{max} = A\omega$$

$$\omega = \frac{v_{max}}{A} = \frac{0.524\text{ m/s}}{0.0750\text{ m}} = 6.99\text{ rad/s}$$

2. Solve $\omega = \sqrt{k/m}$ for the force constant:

$$\omega = \sqrt{k/m}$$

$$k = m\omega^2 = (0.120\text{ kg})(6.99\text{ rad/s})^2 = 5.86\text{ N/m}$$

Part (b)

3. Use $\omega = 2\pi/T$ to find the period:

$$\omega = 2\pi/T$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6.99\text{ rad/s}} = 0.899\text{ s}$$

4. Use $T = 2\pi\sqrt{m/k}$ to find the period:

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\frac{0.120\text{ kg}}{5.86\text{ N/m}}} = 0.899\text{ s}$$

INSIGHT

What would happen if we were to attach a larger mass to this same spring and release it with the same amplitude? The answer is that the period would increase, the angular frequency would decrease, and hence the maximum speed would also decrease.

PRACTICE PROBLEM

What is the maximum acceleration of the mass described in this Example? [**Answer:** $a_{max} = A\omega^2 = 3.66\text{ m/s}^2$]

Some related homework problems: Problem 36, Problem 38

ACTIVE EXAMPLE 13–2 MASS ON A SPRING: FIND THE FORCE CONSTANT AND THE MASS

When a 0.420-kg mass is attached to a spring, it oscillates with a period of 0.350 s. If, instead, a different mass, m_2, is attached to the same spring, it oscillates with a period of 0.700 s. Find **(a)** the force constant of the spring and **(b)** the mass m_2.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Let the initial mass and period be m_1 and T_1, respectively: $m_1 = 0.420$ kg, $T_1 = 0.350$ s

2. Use Equation 13–11 to write an expression for T_1: $T_1 = 2\pi\sqrt{m_1/k}$

3. Solve this expression for the force constant, k: $k = 4\pi^2 m_1/T_1^{\,2} = 135$ N/m

Part (b)

4. Write an expression for T_2: $T_2 = 2\pi\sqrt{m_2/k}$

5. Solve for m_2: $m_2 = kT_2^{\,2}/4\pi^2 = 1.68\ \text{kg}$

INSIGHT

In general, to double the period of a mass on a given spring—as in this case where it went from 0.350 s to 0.700 s—the mass must be increased by a factor of 4, in accordance with Equation 13–11. Therefore, an alternative way to find the second mass is $m_2 = 4(0.420\ \text{kg}) = 1.68\ \text{kg}$, in agreement with our result in Step 5.

YOUR TURN

The maximum speed of the second mass is 0.787 m/s. What is its amplitude of motion?

(Answers to **Your Turn** *problems are given in the back of the book.)*

A Vertical Spring

To this point we have considered only springs that are horizontal, and that are therefore unstretched at their equilibrium position. In many cases, however, we may wish to consider a vertical spring, as in **Figure 13–9**.

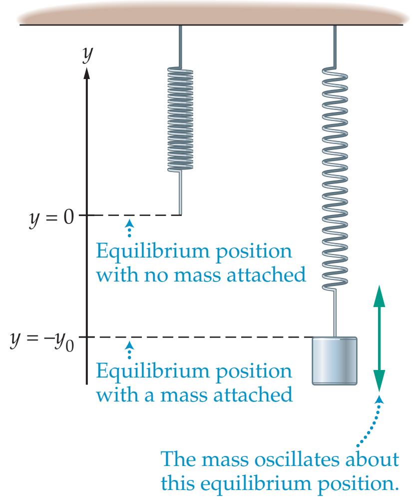
Now, when a mass m is attached to a vertical spring, it causes the spring to stretch. In fact, the vertical spring is in equilibrium when it exerts an upward force equal to the weight of the mass. That is, the spring stretches by an amount y_0 given by

$$ky_0 = mg$$

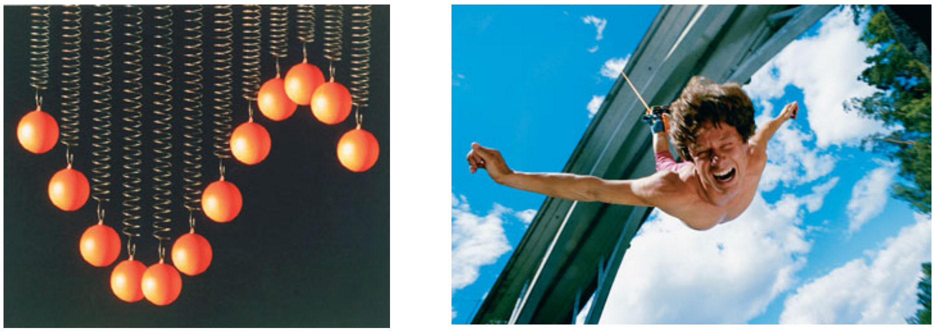
or

$$y_0 = mg/k \qquad 13\text{–}12$$

Thus, a mass on a vertical spring oscillates about the equilibrium point $y = -y_0$. In all other respects the oscillations are the same as for a horizontal spring. In particular, the motion is simple harmonic, and the period is given by Equation 13–11.



▲ **FIGURE 13–9 A mass on a vertical spring**
A mass stretches a vertical spring from its initial equilibrium at $y = 0$ to a new equilibrium at $y = -y_0 = -mg/k$. The mass executes simple harmonic motion about this new equilibrium.



▲ A ball attached to a vertical spring oscillates up and down with simple harmonic motion. If successive images taken at equal time intervals are displaced laterally, as in the sequence of photos at left, the ball appears to trace out a sinusoidal pattern (compare with Figure 13–2). The bungee jumper at right will oscillate in a similar fashion, though friction and air resistance will reduce the amplitude of his bounces.

EXAMPLE 13–5 IT'S A STRETCH

A 0.260-kg mass is attached to a vertical spring. When the mass is put into motion, its period is 1.12 s. **(a)** How much does the mass stretch the spring when it is at rest in its equilibrium position? **(b)** Suppose this experiment is repeated on a planet where the acceleration due to gravity is twice what it is on Earth. By what multiplicative factors do the period and equilibrium stretch change?

CONTINUED ON NEXT PAGE

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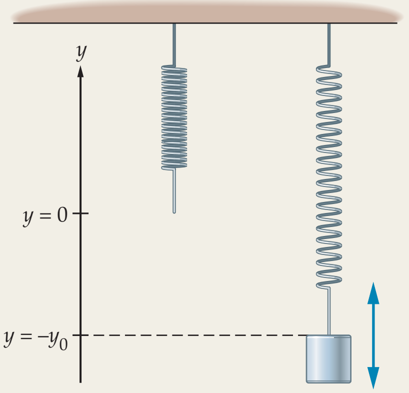
PICTURE THE PROBLEM

We choose the vertical axis to have its origin at the unstretched position of the spring. Once the mass is attached, the spring stretches to the position $y = -y_0$. The mass oscillates about this point with a period of 1.12 s.



STRATEGY

a. In order to find the stretch of the spring, $y_0 = mg/k$, we need to know the force constant, k. We can find k from the period of oscillation—that is, from $T = 2\pi\sqrt{m/k}$.

b. Replace g with $2g$ in both $T = 2\pi\sqrt{m/k}$ and $y_0 = mg/k$. Note that g does not occur in the expression for the period T.

SOLUTION

Part (a)

1. Use the period $T = 2\pi\sqrt{m/k}$ to solve for the force constant:

$$T = 2\pi\sqrt{m/k}$$

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.260\text{ kg})}{(1.12\text{ s})^2} = 8.18\text{ kg/s}^2 = 8.18\text{ N/m}$$

2. Set the magnitude of the spring force, ky_0, equal to mg to solve for y_0:

$$ky_0 = mg$$

$$y_0 = \frac{mg}{k} = \frac{(0.260\text{ kg})(9.81\text{ m/s}^2)}{8.18\text{ N/m}} = 0.312\text{ m}$$

Part (b)

3. Use $2g$ in place of g in the expressions for the period T and the magnitude of the equilibrium stretch, y_0:

$$T = 2\pi\sqrt{m/k} \xrightarrow[g\to 2g]{} T$$

$$y_0 = mg/k \xrightarrow[g\to 2g]{} 2y_0$$

INSIGHT

We see that doubling the force of gravity has no effect on the period (changes it by a factor of 1), but doubles the equilibrium stretch. Therefore, one would observe the same period of oscillation on the Moon or Mars—or even in orbit, as in the case of the Body Mass Measurement Device mentioned earlier in this section.

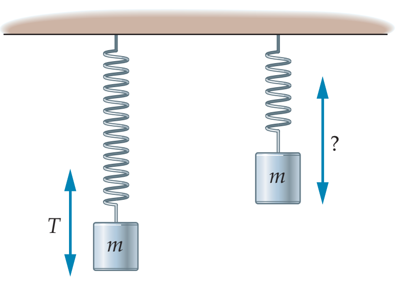
PRACTICE PROBLEM

A 0.170-kg mass stretches a vertical spring 0.250 m when at rest. What is its period when set into vertical motion? [**Answer:** $T = 1.00$ s]

Some related homework problems: Problem 39, Problem 42

CONCEPTUAL CHECKPOINT 13–1 COMPARE PERIODS

When a mass m is attached to a vertical spring with a force constant k, it oscillates with a period T. If the spring is cut in half and the same mass is attached to it, is the period of oscillation **(a)** greater than, **(b)** less than, or **(c)** equal to T?



REASONING AND DISCUSSION

The downward force exerted by the mass is of magnitude mg, as is the upward force exerted by the spring. Note that *each coil* of the spring experiences the *same force*, just as each point in a string experiences the same tension. Therefore, each coil elongates by the same amount, regardless of how many coils there are in a given spring. It follows, then, that the total elongation of the spring with half the number of coils is half the total elongation of the longer spring. Since half the elongation for the same applied force means a greater force constant, the half-spring has a larger value of k—it is stiffer. As a result, its period of oscillation is less than the period of the full spring.

ANSWER

(b) The period for the half-spring is less than for the full spring.

13–5 Energy Conservation in Oscillatory Motion

In an ideal system with no friction or other nonconservative forces, the total energy is conserved. For example, the total energy E of a mass on a horizontal spring is the sum of its kinetic energy, $K = \frac{1}{2}mv^2$, and its potential energy, $U = \frac{1}{2}kx^2$. Therefore,

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad 13\text{–}13$$

Since E remains the same throughout the motion, it follows that there is a continual tradeoff between kinetic and potential energy.

This energy tradeoff is illustrated in **Figure 13–10**, where the horizontal line represents the total energy of the system, E, and the parabolic curve is the spring's potential energy, U. At any given value of x, the sum of U and K must equal E; therefore, since U is the amount of energy from the axis to the parabola, K is the amount of energy from the parabola to the horizontal line. We can see, then, that the kinetic energy vanishes at the turning points, $x = +A$ and $x = -A$, as expected. On the other hand, the kinetic energy is greatest at $x = 0$ where the potential energy vanishes.

Since the mass oscillates back and forth with time, the kinetic and potential energies also change with time. For example, the potential energy is

$$U = \frac{1}{2}kx^2$$

Letting $x = A\cos(\omega t)$, we have

$$U = \frac{1}{2}kA^2\cos^2(\omega t) \qquad 13\text{–}14$$

Clearly, the maximum value of U is

$$U_{max} = \frac{1}{2}kA^2$$

From Figure 13–10, we see that the maximum value of U is simply the total energy of the system, E. Therefore

$$E = U_{max} = \frac{1}{2}kA^2 \qquad 13\text{–}15$$

This result leads to the following conclusion:

> In simple harmonic motion, the total energy is proportional to the square of the amplitude of motion.

Similarly, the kinetic energy of the mass is

$$K = \frac{1}{2}mv^2$$

Letting $v = -A\omega\sin(\omega t)$ yields

$$K = \frac{1}{2}mA^2\omega^2\sin^2(\omega t)$$

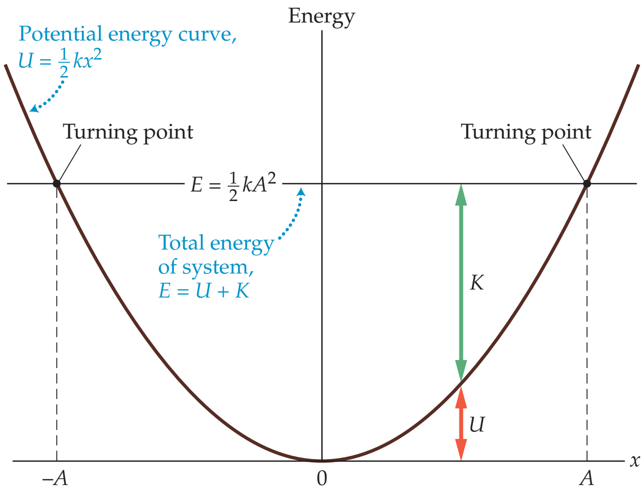
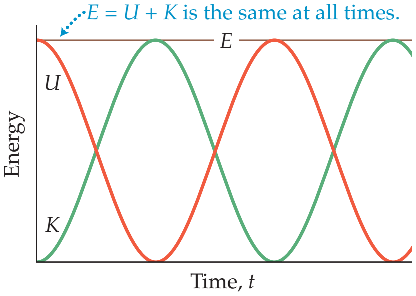


FIGURE 13–10 Energy as a function of position in simple harmonic motion
The parabola represents the potential energy of the spring, $U = \frac{1}{2}kx^2$. The horizontal line shows the total energy of the system, $E = U + K$, which is constant. It follows that the distance from the parabola to the total-energy line is the kinetic energy, K. Note that the kinetic energy vanishes at the turning points, $x = A$ and $x = -A$. At these points the energy is purely potential, and thus the total energy of the system is $E = \frac{1}{2}kA^2$.



▲ **FIGURE 13–11 Energy as a function of time in simple harmonic motion**

The sum of the potential energy, U, and the kinetic energy, K, is equal to the (constant) total energy E at all times. Note that when one energy (U or K) has its maximum value, the other energy is zero.

REAL-WORLD PHYSICS

Maximum Potential and Kinetic Energy

The maximum potential energy of a mass–spring system is the same as the maximum kinetic energy of the mass. When the system has its maximum potential energy, the kinetic energy of the mass is zero; when the mass has its maximum kinetic energy, the potential energy of the system is zero.

It follows that the maximum kinetic energy is

$$K_{max} = \tfrac{1}{2}mA^2\omega^2 \qquad 13\text{–}16$$

As noted, the maximum kinetic energy occurs when the potential energy is zero; hence the maximum kinetic energy must equal the total energy, E. That is,

$$E = U + K = U_{max} + 0 = 0 + K_{max}$$

At first glance, Equation 13–16 doesn't seem to be the same as Equation 13–15. However, if we recall that $\omega^2 = k/m$ (Equation 13–10), we see that

$$K_{max} = \tfrac{1}{2}mA^2\omega^2 = \tfrac{1}{2}mA^2(k/m) = \tfrac{1}{2}kA^2$$

Therefore, $K_{max} = U_{max} = E$, as expected, and the kinetic energy as a function of time is

$$K = \tfrac{1}{2}kA^2\sin^2(\omega t) \qquad 13\text{–}17$$

K and U are plotted as functions of time in **Figure 13–11**. The horizontal line at the top is E, the sum of U and K at all times. This shows quite graphically the back-and-forth tradeoff of energy between kinetic and potential. Mathematically, we can see that the total energy E is constant as follows:

$$E = U + K = \tfrac{1}{2}kA^2\cos^2(\omega t) + \tfrac{1}{2}kA^2\sin^2(\omega t)$$
$$= \tfrac{1}{2}kA^2[\cos^2(\omega t) + \sin^2(\omega t)] = \tfrac{1}{2}kA^2$$

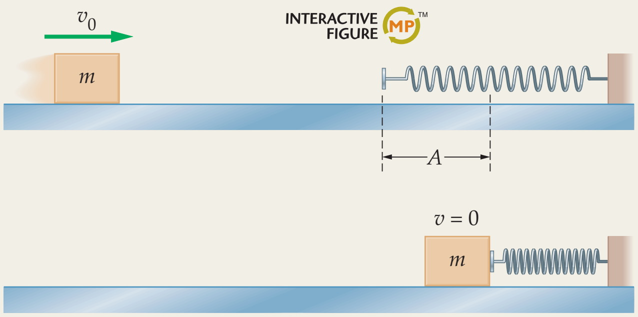
The last step follows from the fact that $\cos^2\theta + \sin^2\theta = 1$ for all θ.

EXAMPLE 13–6 STOP THE BLOCK

A 0.980-kg block slides on a frictionless, horizontal surface with a speed of 1.32 m/s. The block encounters an unstretched spring with a force constant of 245 N/m, as shown in the sketch. **(a)** How far is the spring compressed before the block comes to rest? **(b)** How long is the block in contact with the spring before it comes to rest?

PICTURE THE PROBLEM

As our sketch shows, the initial energy of the system is entirely kinetic; namely, the kinetic energy of the block with mass $m = 0.980$ kg and speed $v_0 = 1.32$ m/s. When the block momentarily comes to rest after compressing the spring by the amount A, its kinetic energy has been converted into the potential energy of the spring.



STRATEGY

a. We can find the compression, A, by using energy conservation. We set the initial kinetic energy of the block, $\tfrac{1}{2}mv_0^{\,2}$, equal to the spring potential energy, $\tfrac{1}{2}kA^2$, and solve for A.

b. If the mass were attached to the spring, it would complete one oscillation in the time $T = 2\pi\sqrt{m/k}$. In moving from the equilibrium position of the spring to maximum compression, the mass has undergone one-quarter of a cycle; thus the time is $T/4$.

SOLUTION

Part (a)

1. Set the initial kinetic energy of the block equal to the spring potential energy: $\tfrac{1}{2}mv_0^{\,2} = \tfrac{1}{2}kA^2$

2. Solve for A, the maximum compression: $A = v_0\sqrt{m/k} = (1.32\text{ m/s})\sqrt{\dfrac{0.980\text{ kg}}{245\text{ N/m}}} = 0.0835\text{ m}$

Part (b)

3. Calculate the period of one oscillation: $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{\dfrac{0.980\text{ kg}}{245\text{ N/m}}} = 0.397\text{ s}$

4. Divide T by four, since the block has been in contact with the spring for one-quarter of an oscillation: $t = \tfrac{1}{4}T = \tfrac{1}{4}(0.397\text{ s}) = 0.0993\text{ s}$

INSIGHT

If the horizontal surface had been rough, some of the block's initial kinetic energy would have been converted to thermal energy. In this case, the maximum compression of the spring would be less than that just calculated.

PRACTICE PROBLEM

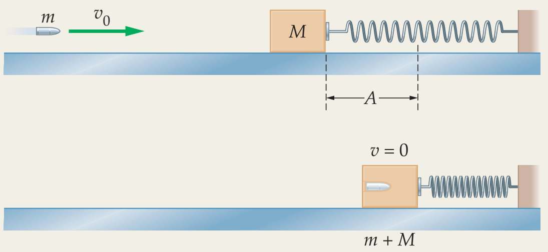
If the initial speed of the mass in this Example is increased, does the time required to bring it to rest increase, decrease, or stay the same? Check your answer by calculating the time for an initial speed of 1.50 m/s. [**Answer:** Increasing v increases the amplitude, A. The period is independent of amplitude, however. Thus, the time is the same, $t = 0.0993$ s.]

Some related homework problems: Problem 54, Problem 89

In the following Active Example, we consider a bullet striking a block that is attached to a spring. As the bullet embeds itself in the block, the completely inelastic bullet–block collision dissipates some of the initial kinetic energy into thermal energy. Only the kinetic energy remaining after the collision is available for compressing the spring.

ACTIVE EXAMPLE 13–3 BULLET-BLOCK COLLISION: FIND THE COMPRESSION AND COMPRESSION TIME

A bullet of mass m embeds itself in a block of mass M, which is attached to a spring of force constant k. If the initial speed of the bullet is v_0, find **(a)** the maximum compression of the spring and **(b)** the time for the bullet–block system to come to rest. (See Example 9–5 for a similar system involving a pendulum.)



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Use momentum conservation to find the final speed, v, of the bullet–block system: $mv_0 = (m + M)v$, $v = mv_0/(m + M)$
2. Find the kinetic energy of the bullet–block system after the collision: $\frac{1}{2}(m + M)v^2 = \frac{1}{2}m^2v_0^2/(m + M)$
3. Set this kinetic energy equal to $\frac{1}{2}kA^2$ to find the maximum compression of the spring: $A = mv_0/\sqrt{k(m + M)}$

Part (b)

4. As in the previous Example, the time to come to rest is one-quarter of a period: $t = T/4 = \frac{1}{2}\pi\sqrt{(m + M)/k}$

INSIGHT

Note that the maximum compression depends directly on the initial speed of the bullet. Thus, a measurement of the compression could be used to determine the bullet's speed. On the other hand, the time for the bullet–block system to come to rest is independent of the bullet's speed.

YOUR TURN

Suppose the force constant of the spring is quadrupled. By what factor does the amplitude of compression, A, change? By what factor does the time to come to rest, t, change?

(Answers to **Your Turn** *problems are given in the back of the book.)*

13–6 The Pendulum

One Sunday in 1583, as Galileo Galilei attended services in a cathedral in Pisa, Italy, he suddenly realized something interesting about the chandeliers hanging from the ceiling. Air currents circulating through the cathedral had set them in motion with small oscillations, and Galileo noticed that chandeliers of equal length oscillated with equal periods, even if their amplitudes were different. Indeed, as any given chandelier oscillated with decreasing amplitude its period remained constant. He verified this observation by timing the oscillations with his pulse!

Galileo was much struck by this observation, and after rushing home, he experimented with pendula constructed from different lengths of string and different weights. Continuing to use his pulse as a stopwatch, he observed that the period of a pendulum varies with its length, but is independent of the weight attached to the string. Thus, in one exhilarating afternoon, the young medical student discovered the key characteristics of a pendulum and launched himself on a new career in science. Later, he would go on to construct the first crude pendulum clock and a medical device, known as the pulsilogium, to measure a patient's pulse rate.

REAL-WORLD PHYSICS
The pendulum clock and pulsilogium

In modern terms, we would say that the chandeliers observed by Galileo were undergoing simple harmonic motion, as expected for small oscillations. As we know, the period of simple harmonic motion is independent of amplitude. The fact that the period is also independent of the mass is a special property of the pendulum, as we shall see next.

The Simple Pendulum

A simple pendulum consists of a mass m suspended by a light string or rod of length L. The pendulum has a stable equilibrium when the mass is directly below the suspension point, and oscillates about this position if displaced from it.

To understand the behavior of the pendulum, let's begin by considering the potential energy of the system. As shown in **Figure 13–12**, when the pendulum is at an angle θ with respect to the vertical, the mass m is above its lowest point by a vertical height $L(1 - \cos\theta)$. If we let the potential energy be zero at $\theta = 0$, the potential energy for general θ is

$$U = mgL(1 - \cos\theta) \qquad \text{13–18}$$

This function is plotted in **Figure 13–13**.

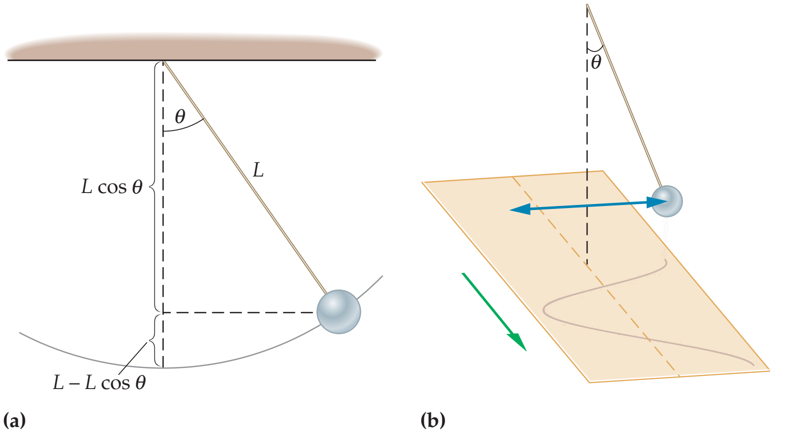


FIGURE 13–12 Motion of a pendulum
(a) As a pendulum swings away from its equilibrium position, it rises a vertical distance $L - L\cos\theta = L(1 - \cos\theta)$. **(b)** As a pendulum bob swings back and forth, it leaks a trail of sand onto a moving sheet of paper, creating a "strip chart" similar to that produced by a mass on a spring in Figure 13–2. In particular, the angle of the pendulum with the vertical varies with time like a sine or a cosine.

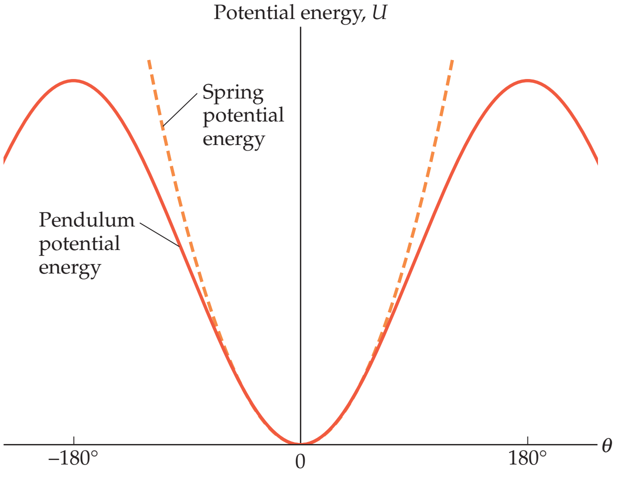


FIGURE 13–13 The potential energy of a simple pendulum
As a simple pendulum swings away from the vertical by an angle θ, its potential energy increases, as indicated by the solid curve. Near $\theta = 0$ the potential energy of the pendulum is essentially the same as that of a mass on a spring (dashed curve). Therefore, when a pendulum oscillates with small displacements from the vertical, it exhibits simple harmonic motion—the same as a mass on a spring.

Note that the stable equilibrium of the pendulum corresponds to a minimum of the potential energy, as expected. Near this minimum, the shape of the potential energy curve is approximately the same as for a mass on a spring, as indicated in Figure 13–13. As a result, when a pendulum oscillates with small displacements from the vertical, its motion is virtually the same as the motion of a mass on a spring; that is, the pendulum exhibits simple harmonic motion.

Next, we consider the forces acting on the mass m. In **Figure 13–14** we show the force of gravity, $m\vec{\mathbf{g}}$, and the tension force in the supporting string, $\vec{\mathbf{T}}$. The tension acts in the radial direction and supplies the force needed to keep the mass moving along its circular path. The net tangential force acting on m, then, is simply the tangential component of its weight:

$$F = mg \sin\theta$$

The direction of the net tangential force is always toward the equilibrium point. Thus F is a restoring force, as expected.

Now, for small angles θ (measured in *radians*), the sine of θ is approximately equal to the angle itself. That is,

$$\sin\theta \approx \theta$$

This is illustrated in **Figure 13–15**. Notice that there is little difference between $\sin\theta$ and θ for angles smaller than about $\pi/8$ rad $= 22.5$ degrees. In addition, we see from Figure 13–14 that the arc length displacement of the mass from equilibrium is

$$s = L\theta$$

Equivalently,

$$\theta = s/L$$

Therefore, if the mass m is displaced from equilibrium by a small arc length s, the force it experiences is restoring and of magnitude

$$F = mg\sin\theta \approx mg\theta = (mg/L)s \qquad \text{13–19}$$

Note that the restoring force is proportional to the displacement, just as expected for simple harmonic motion.

Let's compare the pendulum to a mass on a spring. In the latter case, the restoring force has a magnitude given by

$$F = kx$$

The restoring force acting on the pendulum has precisely the same form, if we let $x = s$ and

$$k = mg/L$$

Therefore, the period of a pendulum is simply the period of a mass on a spring, $T = 2\pi\sqrt{m/k}$, with k replaced by mg/L:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(mg/L)}}$$

Canceling the mass m, we find

Period of a Pendulum (small amplitude)

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad \text{13–20}$$

SI unit: s

This is the classic formula for the period of a pendulum. Note that T depends on the length of the pendulum, L, and on the acceleration of gravity, g. It is independent, however, of the mass m and the amplitude A, as noted by Galileo.

The mass does not appear in the expression for the period of a pendulum, for the same reason that different masses free fall with the same acceleration. In particular, a large mass tends to move more slowly because of its large inertia; on the

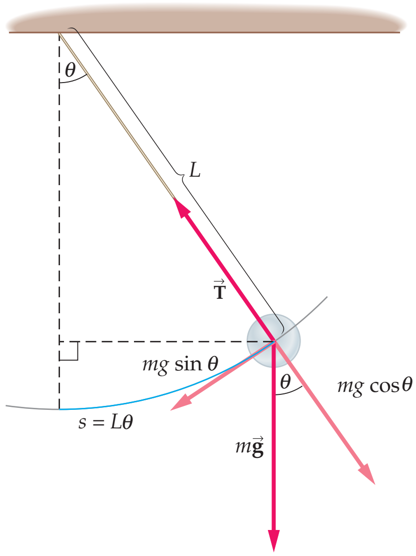


FIGURE 13–14 Forces acting on a pendulum bob
When the bob is displaced by an angle θ from the vertical, the restoring force is the tangential component of the weight, $mg\sin\theta$.

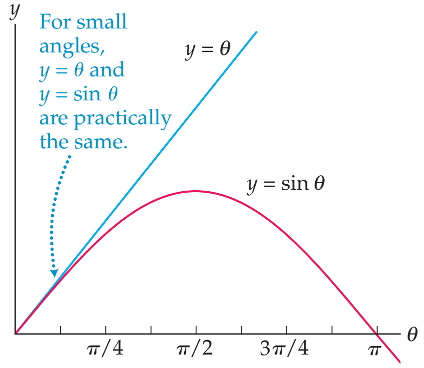


FIGURE 13–15 Relationship between sin θ and θ
For small angles measured in radians, $\sin\theta$ is approximately equal to θ. Thus, when considering small oscillations of a pendulum, we can replace $\sin\theta$ with θ. (See also Appendix A.)

other hand, the larger a mass the greater the gravitational force acting on it. These two effects cancel in free fall, as well as in a pendulum.

EXERCISE 13–4

The pendulum in a grandfather clock is designed to take one second to swing in each direction; that is, 2.00 seconds for a complete period. Find the length of a pendulum with a period of 2.00 seconds.

SOLUTION

Solve Equation 13–20 for L and substitute numerical values:

REAL-WORLD PHYSICS
Adjusting a grandfather clock

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.81\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2}{4\pi^2} = 0.994\ \mathrm{m}$$

CONCEPTUAL CHECKPOINT 13–2 RAISE OR LOWER THE WEIGHT?

If you look carefully at a grandfather clock, you will notice that the weight at the bottom of the pendulum can be moved up or down by turning a small screw. Suppose you have a grandfather clock at home that runs slow. Should you turn the adjusting screw so as to **(a)** raise the weight or **(b)** lower the weight?

REASONING AND DISCUSSION

To make the clock run faster, we want it to go from *tick* to *tick* more rapidly; in other words, we want the period of the pendulum to be decreased. From Equation 13–20 we can see that shortening the pendulum—that is, decreasing L—decreases the period. Hence, the weight should be raised, which effectively shortens the pendulum.

ANSWER

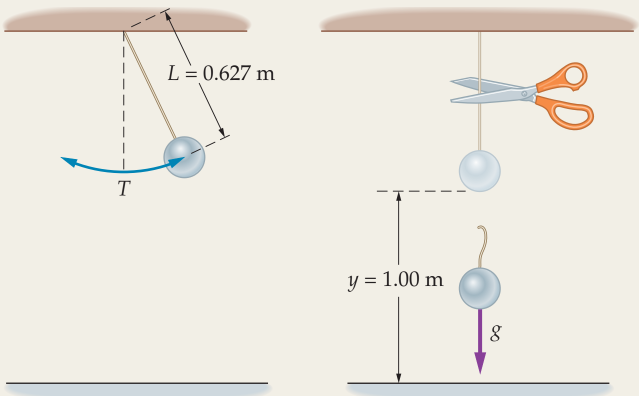
(a) The weight should be raised. This shortens the period and makes the clock run faster.

EXAMPLE 13–7 DROP TIME

A pendulum is constructed from a string 0.627 m long attached to a mass of 0.250 kg. When set in motion, the pendulum completes one oscillation every 1.59 s. If the pendulum is held at rest and the string is cut, how long will it take for the mass to fall through a distance of 1.00 m?

PICTURE THE PROBLEM

Note that the pendulum has a length $L = 0.627$ m and a period of oscillation $T = 1.59$ s. When the pendulum is held at rest in a vertical position, a pair of scissors is used to cut the string. The mass then falls straight downward with an acceleration g through a distance $y = 1.00$ m.



STRATEGY

At first it might seem that the period of oscillation and the time of fall are unrelated. Recall, however, that the period of a pendulum depends on both its length *and* the acceleration of gravity *g at the location of the pendulum*.

To solve this problem, then, we first use the period T to find the acceleration of gravity g. Once g is known, the time of fall is a straightforward kinematics problem (see Example 2–10).

SOLUTION

1. Use the formula for the period of a pendulum to solve for the acceleration of gravity:
$$T = 2\pi\sqrt{L/g} \quad \text{or} \quad g = \frac{4\pi^2 L}{T^2}$$
2. Substitute numerical values to find g:
$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(0.627\ \mathrm{m})}{(1.59\ \mathrm{s})^2} = 9.79\ \mathrm{m/s^2}$$
3. Use kinematics to solve for the time to drop from rest through a distance y:
$$y = \tfrac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{2y/g}$$
4. Substitute $y = 1.00$ m and the value of g just found to find the time:
$$t = \sqrt{2y/g} = \sqrt{\frac{2(1.00\ \mathrm{m})}{9.79\ \mathrm{m/s^2}}} = 0.452\ \mathrm{s}$$

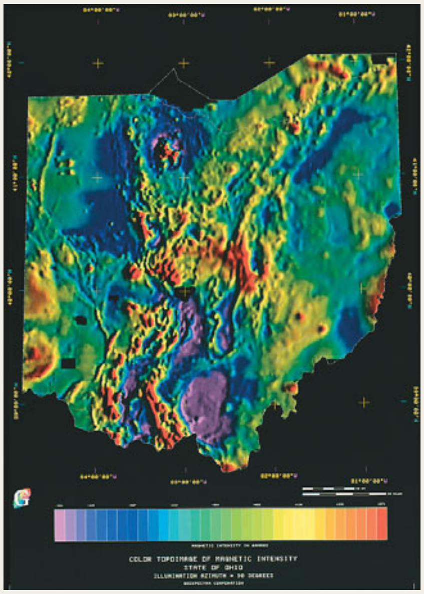
INSIGHT

The fact that the acceleration of gravity g varies from place to place on the Earth was mentioned in Chapter 2. In fact, "gravity maps," such as the one shown in the photo at the right, are valuable tools for geologists attempting to understand the underground properties of a given region. One instrument geologists use to make gravity maps is basically a very precise pendulum whose period can be accurately measured. Slight changes in the period from one location to another, or from one elevation to another, correspond to slight changes in g. More common in recent decades is an electronic "gravimeter" that uses a mass on a spring and relates the force of gravity to the stretch of the spring. These gravimeters can measure g with an accuracy of one part in 1000 million.

PRACTICE PROBLEM

If a mass falls 1.00 m in a time of 0.451 s, what are **(a)** the acceleration of gravity and **(b)** the period of a pendulum of length 0.500 m at that location? [**Answer:** (a) 9.83 m/s^2, (b) 1.42 s]

Some related homework problems: Problem 60, Problem 62



▲ A map of gravitational strength for the state of Ohio. The purple areas are those where the gravitational field is weakest. Areas where the field is strongest (red) represent regions where denser rocks lie near the surface.

*The Physical Pendulum

In the ideal version of a simple pendulum, a bob of mass m swings back and forth a distance L from the suspension point. All of the pendulum's mass is assumed to be concentrated in the bob, which is treated as a point mass. On the other hand, a **physical pendulum** is one in which the mass is not concentrated at a point, but instead is distributed over a finite volume. Examples are shown in **Figure 13–16**. Detailed mathematical analysis shows that if the moment of inertia (Chapter 10) of a physical pendulum about its axis of rotation is I, and the distance from the axis to the center of mass is ℓ, the period of the pendulum is given by the following:

Period of a Physical Pendulum

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(\sqrt{\frac{I}{m\ell^2}}\right) \qquad \text{13–21}$$

SI unit: s

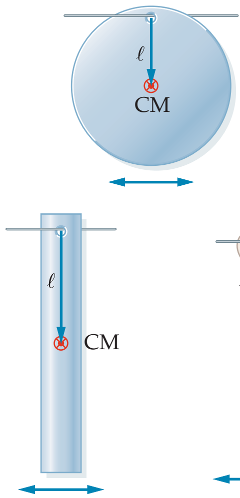
Note that the first part of the expression, $2\pi\sqrt{\ell/g}$, is the period of a simple pendulum with all its mass concentrated at the center of mass. The second factor, $\sqrt{I/m\ell^2}$, is a correction that takes into account the size and shape of the physical pendulum. Thus, writing the period in this form, rather than canceling one power of ℓ, makes for a convenient comparison with the simple pendulum.

To see how Equation 13–21 works in practice, we first apply it to a simple pendulum of mass m and length L. In this case, the moment of inertia is $I = mL^2$, and the distance to the center of mass is $\ell = L$. As a result, the period is

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(\sqrt{\frac{I}{m\ell^2}}\right) = 2\pi\sqrt{\frac{L}{g}}\left(\sqrt{\frac{mL^2}{mL^2}}\right) = 2\pi\sqrt{\frac{L}{g}}$$

Thus, as expected, Equation 13–21 also applies to a simple pendulum.

Next, we apply Equation 13–21 to a nontrivial physical pendulum; namely, your leg. When you walk, your leg rotates about the hip joint much like a uniform rod pivoted about one end, as indicated in **Figure 13–17**. Thus, if we approximate your leg as a uniform rod of length L, its period can be found using



▲ **FIGURE 13–16 Examples of physical pendula**

In each case, an object of definite size and shape oscillates about a given pivot point. The period of oscillation depends in detail on the location of the pivot point as well as on the distance ℓ from it to the center of mass, CM.

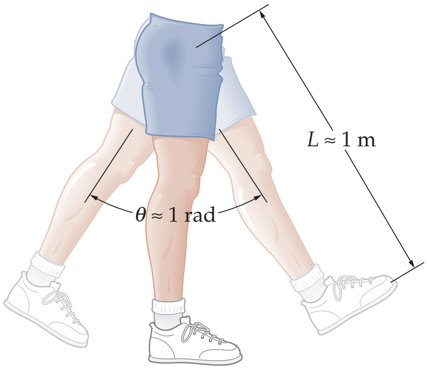


FIGURE 13–17 The leg as a physical pendulum

As a person walks, each leg swings much like a physical pendulum. A reasonable approximation is to treat the leg as a uniform rod about 1 m in length.

Equation 13–21. Recall from Chapter 10 (see Table 10–1) that the moment of inertia of a rod of length L about one end is

$$I = \tfrac{1}{3}mL^2$$

Similarly, the center of mass of your leg is essentially at the center of your leg; thus,

$$\ell = \tfrac{1}{2}L$$

Combining these results, we find that the period of your leg is roughly

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(\sqrt{\frac{I}{m\ell^2}}\right) = 2\pi\sqrt{\frac{\frac{1}{2}L}{g}}\left(\sqrt{\frac{\frac{1}{3}mL^2}{m\left(\frac{1}{2}L\right)^2}}\right) = 2\pi\sqrt{\frac{L}{g}}\left(\sqrt{\frac{2}{3}}\right)$$

Given that a typical human leg is about a meter long ($L = 1.0$ m), we find that its period of oscillation about the hip is approximately $T = 1.6$ s.

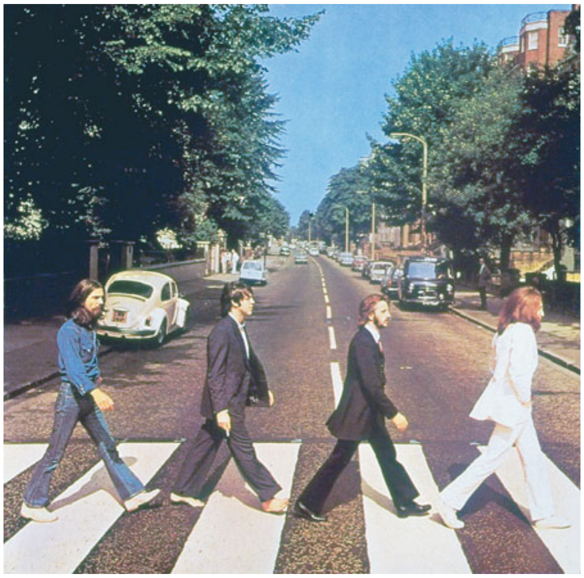
REAL-WORLD PHYSICS: BIO

Walking speed

The significance of this result is that the natural walking pace of humans and other animals is largely controlled by the swinging motion of their legs as physical pendula. In fact, a great deal of research in animal locomotion has focused on precisely this type of analysis. In the case just considered, suppose that the leg in Figure 13–17 swings through an angle of roughly 1.0 radian, as the foot moves from behind the hip to in front of the hip for the next step. The arc length through which the foot moves is $s = r\theta \approx (1.0\text{ m})(1.0\text{ rad}) = 1.0$ m. Since it takes half a period ($T/2 = 0.80$ s) for the foot to move from behind the hip to in front of it, the average speed of the foot is roughly

$$v = \frac{d}{t} \approx \frac{1.0\text{ m}}{0.80\text{ s}} \approx 1.3\text{ m/s}$$

This is the typical speed of a person taking a brisk walk.



When animals walk, the swinging movement of their legs can be approximated fairly well by treating the legs as physical pendula. Such analysis has proved useful in analyzing the gaits of various creatures, from Beatles to elephants.

PROBLEM-SOLVING NOTE

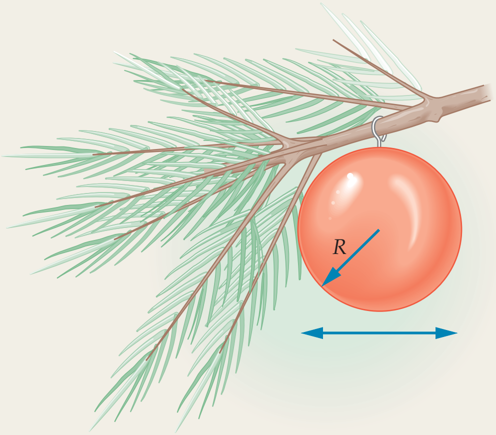
The Period of a Physical Pendulum

When finding the period of a physical pendulum, recall that I is the moment of inertia about the pivot point and ℓ is the distance from the pivot point to the center of mass.

Returning to Equation 13–21, we can see that the smaller the moment of inertia I, the smaller the period. After all, an object with a small moment of inertia rotates quickly and easily. It therefore completes an oscillation in less time than an object with a larger moment of inertia. In the case of the human leg, the fact that its mass is distributed uniformly along its length—rather than concentrated at the far end—means that its moment of inertia and period are less than for a simple pendulum 1 m long.

ACTIVE EXAMPLE 13–4 FIND THE PERIOD OF OSCILLATION

A Christmas ornament is made from a hollow glass sphere of mass M and radius R. The ornament is suspended from a small hook near its surface. If the ornament is nudged slightly, what is its period of oscillation? (*Note*: The moment of inertia about the pivot point is $I = \frac{5}{3}MR^2$.)



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Identify the distance from the axis to the center of mass: $\ell = R$
2. Substitute I and ℓ into Equation 13–21: $T = 2\pi\sqrt{\frac{R}{g}}\left(\sqrt{\frac{5}{3}}\right)$

INSIGHT

The period is greater than that of a simple pendulum of length R.

YOUR TURN

Would you expect the period for a solid spherical ornament to be greater than, less than, or the same as for a hollow spherical ornament? Verify your conclusion by calculating T for a solid spherical ornament. Refer to Table 10–1 for the appropriate moment of inertia.

(Answers to **Your Turn** *problems are given in the back of the book.)*

13–7 Damped Oscillations

To this point we have restricted our considerations to oscillating systems in which no mechanical energy is gained or lost. In most physical systems, however, there is some loss of mechanical energy to friction, air resistance, or other nonconservative forces. As the mechanical energy of a system decreases, its amplitude of oscillation decreases as well, as expected from Equation 13–15. This type of motion is referred to as a **damped oscillation.**

In a typical situation, an oscillating mass may lose its mechanical energy to a force such as air resistance that is proportional to the speed of the mass and opposite in direction. The force in such a case can be written as

$$\vec{\mathbf{F}} = -b\vec{\mathbf{v}}$$

The constant b is referred to as the **damping constant**; it is a measure of the strength of the damping force, and its SI units are kg/s.

If the damping constant is small, the system will continue to oscillate, but with a continuously decreasing amplitude. This type of motion, referred to as **underdamped,** is illustrated in **Figure 13–18 (a)**. In such cases, the amplitude decreases exponentially with time. Thus, if A_0 is the initial amplitude of an oscillating mass m, the amplitude at the time t is

$$A = A_0 e^{-bt/2m}$$

The exponential dependence of the amplitude is indicated by the dashed curve in the figure.

As the damping is increased, a point is reached where the system no longer oscillates, but instead simply relaxes back to the equilibrium position, as shown in **Figure 13–18 (b)**. A system with this type of behavior is said to be **critically damped.** If the damping is increased beyond this point, the system is said to be **overdamped.** In this case, the system still returns to equilibrium without oscillating, but the time required is greater. This is also illustrated in the figure.

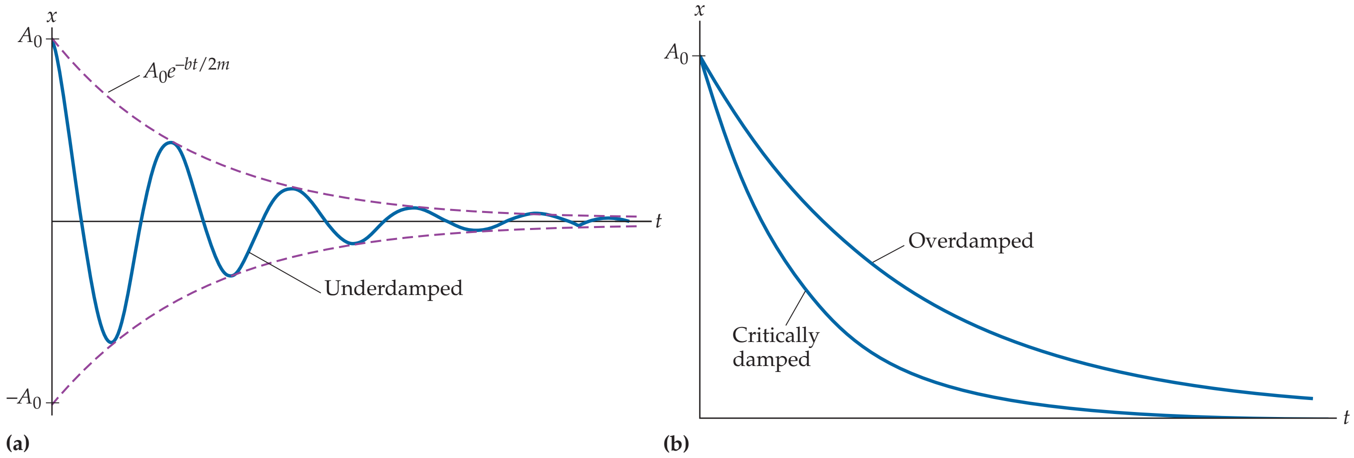


FIGURE 13–18 Damped oscillations

(a) In underdamped oscillations, the position continues to oscillate as a function of time, but the amplitude of oscillation decreases exponentially. **(b)** In critically damped and overdamped motion, no oscillations occur. Instead, an object simply settles back to its equilibrium position without overshooting. Equilibrium is reached most rapidly in the critically damped case.

Some mechanical systems are designed to be near the condition for critical damping. If such a system is displaced from equilibrium, it will return to equilibrium, without oscillating, in the shortest possible time. For example, shock absorbers are designed so that a car that has just hit a bump will return to equilibrium quickly, without a lot of up-and-down oscillations.

13–8 Driven Oscillations and Resonance

In the previous section we considered the effects of removing energy from an oscillating system. It is also possible, however, to increase the energy of a system, or to replace the energy lost to various forms of friction. This can be done by applying an external force that does positive work.

Suppose, for example, that you hold the end of a string from which a small weight is suspended, as in Figure 13–19. If the weight is set in motion and you hold your hand still, it will soon stop oscillating. If you move your hand back and forth in a horizontal direction, however, you can keep the weight oscillating indefinitely. The motion of your hand is said to be "driving" the weight, leading to **driven oscillations.**

The response of the weight in this example depends on the frequency of your hand's back-and-forth motion, as you can readily verify for yourself. For instance, if you move your hand very slowly, the weight will simply track the motion of your hand. Similarly, if you oscillate your hand very rapidly, the weight will exhibit only small oscillations. Oscillating your hand at an intermediate frequency, however, can result in large amplitude oscillations for the weight.

Just what is an appropriate intermediate frequency? Well, to achieve a large response, your hand should drive the weight at the frequency at which it oscillates when not being driven. This is referred to as the **natural frequency,** f_0, of the system. For example, the natural frequency of a pendulum of length L is simply the inverse of its period:

$$f_0 = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Similarly, the natural frequency of a mass on a spring is

$$f_0 = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

In general, driving any system at a frequency near its natural frequency results in large oscillations.

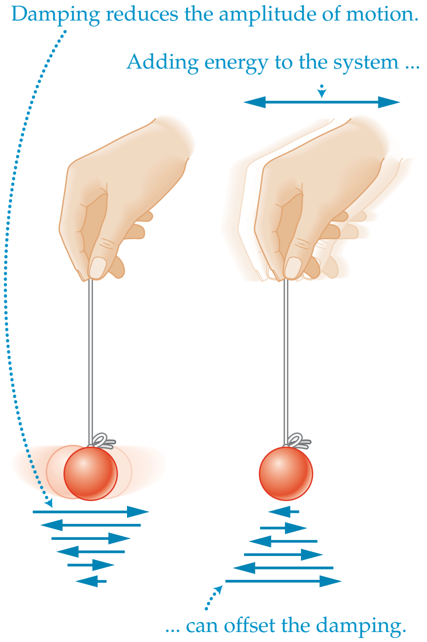
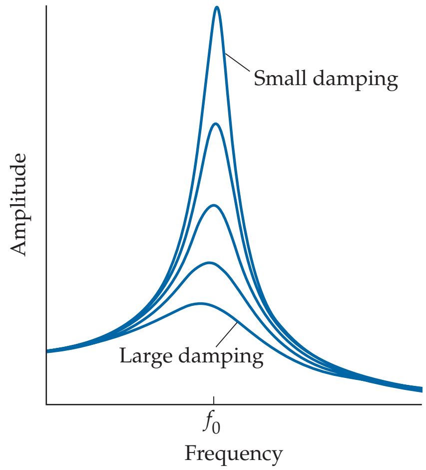


FIGURE 13–19 Driven oscillations

If the support point of a pendulum is held still, its oscillations quickly die away due to damping. If the support point is oscillated back and forth, however, the pendulum will continue swinging. This is called "driving" the pendulum. If the driving frequency is close to the natural frequency of the pendulum—the frequency at which it oscillates when the support point is held still—its amplitude of motion can become quite large.

As an example, **Figure 13–20** shows a plot of amplitude, A, versus driving frequency, f, for a mass on a spring. Note the large amplitude for frequencies near f_0. This type of large response, due to frequency matching, is known as **resonance,** and the curves shown in Figure 13–20 are referred to as **resonance curves.** The five curves shown in Figure 13–20 correspond to different amounts of damping, as indicated. As we can see, systems with small damping have a high, narrow peak on their resonance curve. This means that resonance is a large effect in these systems, and that it is very sensitive to frequency. On the other hand, systems with large damping have resonance peaks that are broad and low.



▲ **FIGURE 13–20 Resonance curves for various amounts of damping**
When the damping is small, the amplitude of oscillation can become very large for frequencies close to the natural frequency, f_0. When the damping is large, the amplitude has only a low, broad peak near the natural frequency.

Resonance plays an important role in a variety of physical systems, from a pendulum to atoms in a laser to a tuner in a radio or TV. As we shall see in Chapter 24, for example, adjusting the tuning knob in a radio changes the resonance frequency of the electric circuit in the tuner. When its resonance frequency matches the frequency being broadcast by a station (101 MHz, perhaps), that station is picked up. To change stations, we simply change the resonance frequency of the tuner to the frequency of another station. A good tuner will have little damping, so stations that are even slightly "off resonance" will have small response, and hence will not be heard.

Mechanical examples of resonance are all around us as well. In fact, you might want to try the following experiment the next time you notice a spider in its web. Move close to the web and hum, starting with a low pitch. Slowly increase the pitch of your humming, and soon you will notice the spider react excitedly. You have hit the resonance frequency of its web, causing it to vibrate and making the spider think he has snagged a lunch. If you continue to increase your humming frequency, the spider quiets down again, because you have gone past the resonance. Each individual spider web you encounter will resonate at a specific, and different, frequency.

REAL-WORLD PHYSICS
Radio tuners and spiderwebs

Man-made structures can show resonance effects as well. One of the most dramatic and famous examples is the collapse of Washington's Tacoma Narrows bridge in 1940. High winds through the narrows had often set the bridge into a gentle swaying motion, resulting in its being known by the affectionate nickname "Galloping Girdie." During one particular wind storm, however, the bridge experienced a resonance-like effect, and the amplitude of its swaying motion began to increase. Alarmed officials closed the bridge to traffic, and a short time later the swaying motion became so great that the bridge broke apart and fell into the waters below. Needless to say, bridges built since that time have been designed to prevent such catastrophic oscillations.

REAL-WORLD PHYSICS
Resonance: bridges

▲ Anyone who has ever pushed a child on a swing (left) knows that the timing of the pushes is critical. If they are synchronized with the natural frequency of the swing, the amplitude can increase rapidly. This phenomenon of resonance can have dangerous consequences. In 1940, the Tacoma Narrows bridge (right), completed only four months earlier, collapsed when high winds set the bridge swaying at one of its resonant frequencies.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

Uniform circular motion (Chapter 10) is used to better understand simple harmonic motion in Section 13–3.

Newton's second law, $F = ma$ (Chapter 5), is used in the discussion of a mass on a spring in Section 13–4. We also use the force law for a spring, $F = kx$ (Chapter 6), in that section.

The concepts of kinetic energy (Chapter 7) and potential energy (Chapter 8) play key roles in understanding oscillatory motion. In particular, we apply energy conservation to oscillations in Section 13–5.

LOOKING AHEAD

Frequency, f, and period, T, play key roles in the study of mechanical waves and sound in Chapter 14. They reappear when we study electromagnetic waves in Chapter 25.

Though a mass on a spring may seem far removed from an electric circuit, we show in Chapter 24 that there is in fact a deep connection. In particular, the motion of a mass on a spring is directly analogous to the current in a specific type of circuit referred to as an *RLC* circuit.

In Chapter 30 we study the beginnings of modern physics, and blackbody radiation in particular. As we shall see, the frequency f of light is directly related to the energy of a "particle of light," referred to as a photon.

CHAPTER SUMMARY

13–1 PERIODIC MOTION

Periodic motion repeats after a definite length of time.

Period

The period, T, is the time required for a motion to repeat:

$$T = \text{time required for one cycle of a periodic motion}$$

Frequency

Frequency, f, is the number of oscillations per unit time.

Equivalently, f is the inverse of the period:

$$f = \frac{1}{T} \qquad 13\text{–}1$$

Angular Frequency

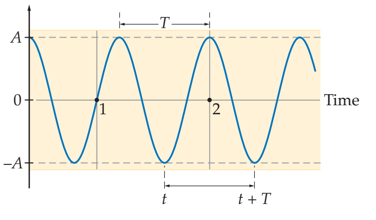
Angular frequency, ω, is 2π times the frequency:

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad 13\text{–}5$$

Rapid motion corresponds to a short period and a large frequency.

13–2 SIMPLE HARMONIC MOTION

A particular type of periodic motion is simple harmonic motion. A classic example of simple harmonic motion is the oscillation of a mass attached to a spring.



Restoring Force

Simple harmonic motion occurs when the restoring force is proportional to the displacement from equilibrium.

Amplitude

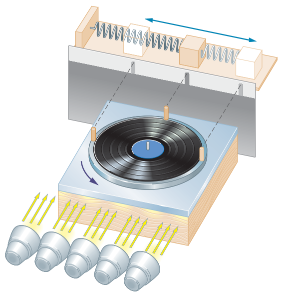
The maximum displacement from equilibrium is referred to as the amplitude, A.

Position Versus Time

The position, x, of an object undergoing simple harmonic motion varies with time as $A\cos(\omega t)$:

$$x = A\cos\left(\frac{2\pi}{T}t\right) = A\cos(\omega t) \qquad 13\text{–}2,\ 13\text{–}4$$

13–3 CONNECTIONS BETWEEN UNIFORM CIRCULAR MOTION AND SIMPLE HARMONIC MOTION



A close relationship exists between uniform circular motion and simple harmonic motion. In particular, circular motion viewed from the side—by projecting a shadow on a screen, for example—is simple harmonic.

Velocity

The velocity as a function of time in simple harmonic motion is

$$v = -A\omega \sin(\omega t) \qquad 13\text{–}6$$

Acceleration

The acceleration as a function of time in simple harmonic motion is

$$a = -A\omega^2 \cos(\omega t) \qquad 13\text{–}8$$

Maximum Speed and Acceleration

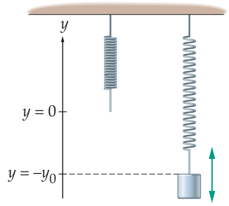
The maximum speed of an object in simple harmonic motion is

$$v_{\text{max}} = A\omega \qquad 13\text{–}7$$

Its maximum acceleration has a magnitude of

$$a_{\text{max}} = A\omega^2 \qquad 13\text{–}9$$

13–4 THE PERIOD OF A MASS ON A SPRING



An important special case of simple harmonic motion is a mass on a spring.

Period

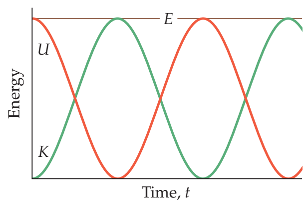
The period of a mass m attached to a spring of force constant k is

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad 13\text{–}11$$

Vertical Spring

A mass attached to a vertical spring causes it to stretch to a new equilibrium position. Oscillations about this new equilibrium are simple harmonic, with a period given by Equation 13–11.

13–5 ENERGY CONSERVATION IN OSCILLATORY MOTION



In an ideal oscillatory system, the total energy remains constant. This means that the kinetic and potential energies of the system vary with time in such a way that their sum remains fixed.

Total Energy

The total energy in simple harmonic motion is proportional to the amplitude, A, squared. For a mass on a spring, the total energy, E, is

$$E = \tfrac{1}{2}kA^2 \qquad 13\text{–}15$$

Potential Energy as a Function of Time

For a mass on a spring, the potential energy varies with time as follows:

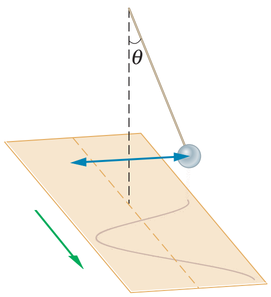
$$U = \tfrac{1}{2}kA^2\cos^2(\omega t) \qquad 13\text{–}14$$

Kinetic Energy as a Function of Time

Similarly, the kinetic energy as a function of time is

$$K = \tfrac{1}{2}kA^2\sin^2(\omega t) \qquad 13\text{–}17$$

13–6 THE PENDULUM



A pendulum oscillating with small amplitude also exhibits simple harmonic motion.

Simple Pendulum

A simple, or ideal, pendulum is one in which all the mass is concentrated at a single point a distance L from the suspension point.

Period of a Simple Pendulum

The period of a simple pendulum of length L is

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad 13\text{–}20$$

***Physical Pendulum**

In a physical pendulum, the mass is distributed throughout a finite volume. Thus, a physical pendulum has a definite shape, whereas a simple pendulum is characterized by a point mass.

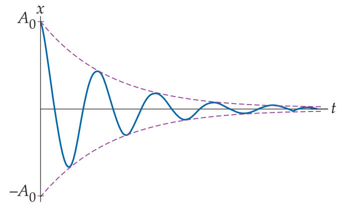
***Period of a Physical Pendulum**

The period of a physical pendulum is

$$T = 2\pi\sqrt{\frac{\ell}{g}}\left(\sqrt{\frac{I}{m\ell^2}}\right) \qquad 13\text{–}21$$

In this expression, I is the moment of inertia about the pivot point, and ℓ is the distance from the pivot point to the center of mass.

13–7 DAMPED OSCILLATIONS



Systems in which mechanical energy is lost to other forms, such as heat or sound, eventually come to rest at the equilibrium position. How they move as they come to rest depends on the amount of damping.

Underdamping

In an underdamped case, a system of mass m and damping constant b continues to oscillate as its amplitude steadily decreases with time. The decrease in amplitude is exponential:

$$A = A_0 e^{-bt/2m}$$

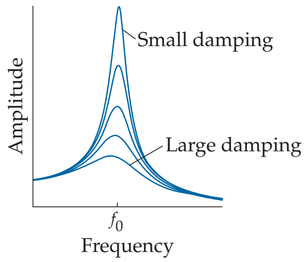
Critical Damping

In a system with critical damping, no oscillations occur. The system simply relaxes back to equilibrium in the least possible time.

Overdamping

An overdamped system also relaxes back to equilibrium with no oscillations. The relaxation occurs more slowly in this case than in critical damping.

13–8 DRIVEN OSCILLATIONS AND RESONANCE



If an oscillating system is driven by an external force, it is possible for energy to be added to the system. This added energy may simply replace energy lost to friction, or, in the case of resonance, it may result in oscillations of large amplitude and energy.

Natural Frequency

The natural frequency of an oscillating system is the frequency at which it oscillates when free from external disturbances.

Resonance

Resonance is the response of an oscillating system to a driving force of the appropriate frequency.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the position, velocity, and acceleration as a function of time for an object undergoing simple harmonic motion.	In simple harmonic motion, the position, velocity, and acceleration are all sinusoidal functions of time. In particular, $x = A\cos(\omega t)$, $v = -A\omega\sin(\omega t)$, and $a = -A\omega^2\cos(\omega t)$, where $\omega = 2\pi/T$.	Examples 13–1, 13–2 Active Example 13–1
Calculate the maximum speed and acceleration for simple harmonic motion.	The maximum speed is $v_{max} = A\omega$, and the maximum acceleration is $a_{max} = A\omega^2$.	Examples 13–3, 13–4
Relate the period of a mass on a spring to the mass and the force constant.	The period of a mass m attached to a spring of force constant k is $T = 2\pi\sqrt{m/k}$.	Examples 13–4, 13–5, 13–6 Active Examples 13–2, 13–3
Relate the period of a pendulum to its length and to the acceleration of gravity.	The period of a simple pendulum of length L is $T = 2\pi\sqrt{L/g}$. Note that the period is independent of the mass, but does depend on the acceleration of gravity.	Example 13–7
Find the period of a physical pendulum.	Identify the moment of inertia, I, about the pivot point and the distance from the pivot point to the center of mass, ℓ. Then use $T = 2\pi\sqrt{\ell/g}\left(\sqrt{I/m\ell^2}\right)$.	Active Example 13–4

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A basketball player dribbles a ball with a steady period of T seconds. Is the motion of the ball periodic? Is it simple harmonic? Explain.
2. A person rides on a Ferris wheel that rotates with constant angular speed. If the Sun is directly overhead, does the person's shadow on the ground undergo periodic motion? Does it undergo simple harmonic motion? Explain.
3. An air-track cart bounces back and forth between the two ends of an air track. Is this motion periodic? Is it simple harmonic? Explain.
4. If a mass m and a mass $2m$ oscillate on identical springs with identical amplitudes, they both have the same maximum kinetic energy. How can this be? Shouldn't the larger mass have more kinetic energy? Explain.
5. An object oscillating with simple harmonic motion completes a cycle in a time T. If the object's amplitude is doubled, the time required for one cycle is still T, even though the object covers twice the distance. How can this be? Explain.
6. The position of an object undergoing simple harmonic motion is given by $x = A\cos(Bt)$. Explain the physical significance of the constants A and B. What is the frequency of this object's motion?
7. The velocity of an object undergoing simple harmonic motion is given by $v = -C\sin(Dt)$. Explain the physical significance of the constants C and D. What are the amplitude and period of this object's motion?
8. The pendulum bob in Figure 13–12 leaks sand onto the strip chart. What effect does this loss of sand have on the period of the pendulum? Explain.
9. Soldiers on the march are often ordered to break cadence in their step when crossing a bridge. Why is this a good idea?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated conceptual/quantitative problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 13–1 PERIODIC MOTION

1. • A small cart on a 5.0-m-long air track moves with a speed of 0.85 m/s. Bumpers at either end of the track cause the cart to reverse direction and maintain the same speed. Find the period and frequency of this motion.

2. • A person in a rocking chair completes 12 cycles in 21 s. What are the period and frequency of the rocking?

3. • While fishing for catfish, a fisherman suddenly notices that the bobber (a floating device) attached to his line is bobbing up and down with a frequency of 2.6 Hz. What is the period of the bobber's motion?

4. • If you dribble a basketball with a frequency of 1.77 Hz, how long does it take for you to complete 12 dribbles?

5. • You take your pulse and observe 74 heartbeats in a minute. What are the period and frequency of your heartbeat?

6. •• **IP** **(a)** Your heart beats with a frequency of 1.45 Hz. How many beats occur in a minute? **(b)** If the frequency of your heartbeat increases, will the number of beats in a minute increase, decrease, or stay the same? **(c)** How many beats occur in a minute if the frequency increases to 1.55 Hz?

7. •• You rev your car's engine to 2700 rpm (rev/min). **(a)** What are the period and frequency of the engine? **(b)** If you change the period of the engine to 0.044 s, how many rpms is it doing?

SECTION 13–2 SIMPLE HARMONIC MOTION

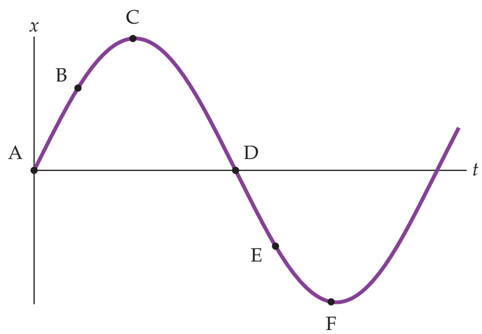
8. • **CE** A mass moves back and forth in simple harmonic motion with amplitude A and period T. **(a)** In terms of A, through what distance does the mass move in the time T? **(b)** Through what distance does it move in the time $5T/2$?

9. • **CE** A mass moves back and forth in simple harmonic motion with amplitude A and period T. **(a)** In terms of T, how long does it take for the mass to move through a total distance of $2A$? **(b)** How long does it take for the mass to move through a total distance of $3A$?

10. • The position of a mass oscillating on a spring is given by $x = (3.2\text{ cm})\cos[2\pi t/(0.58\text{ s})]$. **(a)** What is the period of this motion? **(b)** What is the first time the mass is at the position $x = 0$?

11. • The position of a mass oscillating on a spring is given by $x = (7.8\text{ cm})\cos[2\pi t/(0.68\text{ s})]$. **(a)** What is the frequency of this motion? **(b)** When is the mass first at the position $x = -7.8$ cm?

12. •• **CE** A position-versus-time plot for an object undergoing simple harmonic motion is given in **Figure 13–21**. Rank the six points indicated in the figure in order of increasing **(a)** speed, **(b)** velocity, and **(c)** acceleration. Indicate ties where necessary.



▲ **FIGURE 13–21** Problem 12

13. •• **CE** A mass on a spring oscillates with simple harmonic motion of amplitude A about the equilibrium position $x = 0$. Its maximum speed is v_{max} and its maximum acceleration is a_{max}. **(a)** What is the speed of the mass at $x = 0$? **(b)** What is the acceleration of the mass at $x = 0$? **(c)** What is the speed of the mass at $x = A$? **(d)** What is the acceleration of the mass at $x = A$?

14. •• A mass oscillates on a spring with a period of 0.73 s and an amplitude of 5.4 cm. Write an equation giving x as a function of time, assuming the mass starts at $x = A$ at time $t = 0$.

15. •• **IP Molecular Oscillations** An atom in a molecule oscillates about its equilibrium position with a frequency of 2.00×10^{14} Hz and a maximum displacement of 3.50 nm. **(a)** Write an expression giving x as a function of time for this atom, assuming that $x = A$ at $t = 0$. **(b)** If, instead, we assume that $x = 0$ at $t = 0$, would your expression for position versus time use a sine function or a cosine function? Explain.

16. •• A mass oscillates on a spring with a period T and an amplitude 0.48 cm. The mass is at the equilibrium position $x = 0$ at $t = 0$, and is moving in the positive direction. Where is the mass at the times **(a)** $t = T/8$, **(b)** $t = T/4$, **(c)** $t = T/2$ and **(d)** $t = 3T/4$? **(e)** Plot your results for parts (a) through (d) with the vertical axis representing position and the horizontal axis representing time.

17. •• The position of a mass on a spring is given by $x = (6.5 \text{ cm}) \cos[2\pi t/(0.88 \text{ s})]$. **(a)** What is the period, T, of this motion? **(b)** Where is the mass at $t = 0.25$ s? **(c)** Show that the mass is at the same location at $0.25 \text{ s} + T$ seconds as it is at 0.25 s.

18. •• **IP** A mass attached to a spring oscillates with a period of 3.35 s. **(a)** If the mass starts from rest at $x = 0.0440$ m and time $t = 0$, where is it at time $t = 6.37$ s? **(b)** Is the mass moving in the positive or negative x direction at $t = 6.37$ s? Explain.

19. ••• An object moves with simple harmonic motion of period T and amplitude A. During one complete cycle, for what length of time is the position of the object greater than $A/2$?

20. ••• An object moves with simple harmonic motion of period T and amplitude A. During one complete cycle, for what length of time is the speed of the object greater than $v_{max}/2$?

21. ••• An object executing simple harmonic motion has a maximum speed v_{max} and a maximum acceleration a_{max}. Find (a) the amplitude and (b) the period of this motion. Express your answers in terms of v_{max} and a_{max}.

SECTION 13–3 CONNECTIONS BETWEEN UNIFORM CIRCULAR MOTION AND SIMPLE HARMONIC MOTION

22. • A ball rolls on a circular track of radius 0.62 m with a constant angular speed of 1.3 rad/s in the counterclockwise direction. If the angular position of the ball at $t = 0$ is $\theta = 0$, find the x component of the ball's position at the times 2.5 s, 5.0 s, and 7.5 s. Let $\theta = 0$ correspond to the positive x direction.

23. •• An object executing simple harmonic motion has a maximum speed of 4.3 m/s and a maximum acceleration of 0.65 m/s^2. Find **(a)** the amplitude and **(b)** the period of this motion.

24. • A child rocks back and forth on a porch swing with an amplitude of 0.204 m and a period of 2.80 s. Assuming the motion is approximately simple harmonic, find the child's maximum speed.

25. •• **IP** A 30.0-g goldfinch lands on a slender branch, where it oscillates up and down with simple harmonic motion of amplitude 0.0335 m and period 1.65 s. **(a)** What is the maximum acceleration of the finch? Express your answer as a fraction of the acceleration of gravity, g. **(b)** What is the maximum speed of the goldfinch? **(c)** At the time when the goldfinch experiences its maximum acceleration, is its speed a maximum or a minimum? Explain.

26. •• **BIO Tuning Forks in Neurology** Tuning forks are used in the diagnosis of nervous afflictions known as large-fiber polyneuropathies, which are often manifested in the form of reduced sensitivity to vibrations. Disorders that can result in this type of pathology include diabetes and nerve damage from exposure to heavy metals. The tuning fork in **Figure 13–22** has a frequency of 128 Hz. If the tips of the fork move with an amplitude of 1.25 mm, find **(a)** their maximum speed and **(b)** their maximum acceleration. Give your answer to part (b) as a multiple of g.

▲ **FIGURE 13–22** A Buck neurological hammer with tuning fork and Wartenburg pinwheel. (Problem 26)

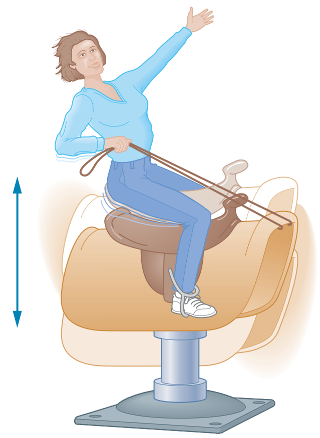
27. •• A vibrating structural beam in a spacecraft can cause problems if the frequency of vibration is fairly high. Even if the amplitude of vibration is only a fraction of a millimeter, the acceleration of the beam can be several times greater than the acceleration due to gravity. As an example, find the maximum acceleration of a beam that vibrates with an amplitude of 0.25 mm at the rate of 110 vibrations per second. Give your answer as a multiple of g.

28. •• A peg on a turntable moves with a constant tangential speed of 0.77 m/s in a circle of radius 0.23 m. The peg casts a shadow on a wall. Find the following quantities related to the motion of the shadow: **(a)** the period, **(b)** the amplitude, **(c)** the maximum speed, and **(d)** the maximum magnitude of the acceleration.

29. •• The pistons in an internal combustion engine undergo a motion that is approximately simple harmonic. If the amplitude of motion is 3.5 cm, and the engine runs at 1700 rev/min, find **(a)** the maximum acceleration of the pistons and **(b)** their maximum speed.

30. •• A 0.84-kg air cart is attached to a spring and allowed to oscillate. If the displacement of the air cart from equilibrium is $x = (10.0 \text{ cm}) \cos[(2.00 \text{ s}^{-1})t + \pi]$, find **(a)** the maximum kinetic energy of the cart and **(b)** the maximum force exerted on it by the spring.

31. •• **IP** A person rides on a mechanical bucking horse (see **Figure 13–23**) that oscillates up and down with simple harmonic motion. The period of the bucking is 0.74 s and the amplitude is slowly increasing. At a certain amplitude the rider must hang on to prevent separating from the mechanical horse. **(a)** Give a strategy that will allow you to calculate this amplitude. **(b)** Carry out your strategy and find the desired amplitude.



▲ **FIGURE 13–23** Problem 31

SECTION 13–4 THE PERIOD OF A MASS ON A SPRING

32. • **CE Predict/Explain** If a mass m is attached to a given spring, its period of oscillation is T. If two such springs are connected end to end and the same mass m is attached, **(a)** is the resulting period of oscillation greater than, less than, or equal to T? **(b)** Choose the *best explanation* from among the following:

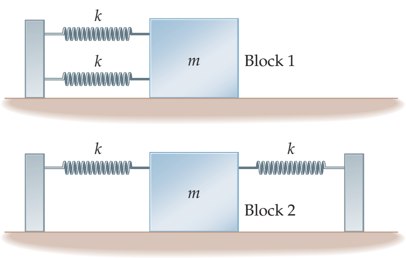
I. Connecting two springs together makes the spring stiffer, which means that less time is required for an oscillation.
II. The period of oscillation does not depend on the length of a spring, only on its force constant and the mass attached to it.
III. The longer spring stretches more easily, and hence takes longer to complete an oscillation.

33. • **CE Predict/Explain** An old car with worn-out shock absorbers oscillates with a given frequency when it hits a speed bump. If the driver adds a couple of passengers to the car and hits another speed bump, **(a)** is the car's frequency of oscillation greater than, less than, or equal to what it was before? **(b)** Choose the *best explanation* from among the following:

I. Increasing the mass on a spring increases its period, and hence decreases its frequency.
II. The frequency depends on the force constant of the spring but is independent of the mass.
III. Adding mass makes the spring oscillate more rapidly, which increases the frequency.

34. • **CE Predict/Explain** The two blocks in **Figure 13–24** have the same mass, m. All the springs have the same force constant, k, and are at their equilibrium length. When the blocks are set into oscillation, **(a)** is the period of block 1 greater than, less than, or equal to the period of block 2? **(b)** Choose the *best explanation* from among the following:

I. Springs in parallel are stiffer than springs in series; therefore the period of block 1 is smaller than the period of block 2.
II. The two blocks experience the same restoring force for a given displacement from equilibrium, and hence they have equal periods of oscillation.
III. The force of the two springs on block 2 partially cancel one another, leading to a longer period of oscillation.



▲ **FIGURE 13–24** Problems 34 and 38

35. • Show that the units of the quantity $\sqrt{k/m}$ are s^{-1}.

36. • A 0.46-kg mass attached to a spring undergoes simple harmonic motion with a period of 0.77 s. What is the force constant of the spring?

37. •• **CE** System A consists of a mass m attached to a spring with a force constant k; system B has a mass $2m$ attached to a spring with a force constant k; system C has a mass $3m$ attached to a spring with a force constant $6k$; and system D has a mass m attached to a spring with a force constant $4k$. Rank these systems in order of increasing period of oscillation.

38. •• Find the periods of block 1 and block 2 in Figure 13–24, given that $k = 49.2$ N/m and $m = 1.25$ kg.

39. •• When a 0.50-kg mass is attached to a vertical spring, the spring stretches by 15 cm. How much mass must be attached to the spring to result in a 0.75-s period of oscillation?

40. •• A spring with a force constant of 69 N/m is attached to a 0.57-kg mass. Assuming that the amplitude of motion is 3.1 cm, determine the following quantities for this system: **(a)** ω, **(b)** v_{max}, **(c)** T.

41. •• Two people with a combined mass of 125 kg hop into an old car with worn-out shock absorbers. This causes the springs to compress by 8.00 cm. When the car hits a bump in the road, it oscillates up and down with a period of 1.65 s. Find **(a)** the total load supported by the springs and **(b)** the mass of the car.

42. •• A 0.85-kg mass attached to a vertical spring of force constant 150 N/m oscillates with a maximum speed of 0.35 m/s. Find the following quantities related to the motion of the mass: **(a)** the period, **(b)** the amplitude, **(c)** the maximum magnitude of the acceleration.

43. •• When a 0.213-kg mass is attached to a vertical spring, it causes the spring to stretch a distance d. If the mass is now displaced slightly from equilibrium, it is found to make 102 oscillations in 56.7 s. Find the stretch distance, d.

44. •• **IP** The springs of a 511-kg motorcycle have an effective force constant of 9130 N/m. **(a)** If a person sits on the motorcycle, does its period of oscillation increase, decrease, or stay the same? **(b)** By what percent and in what direction does the period of oscillation change when a 112-kg person rides the motorcycle?

45. ••• **IP** If a mass m is attached to a given spring, its period of oscillation is T. If two such springs are connected end to end, and the same mass m is attached, **(a)** is its period greater than, less than, or the same as with a single spring? **(b)** Verify your answer to part (a) by calculating the new period, T', in terms of the old period T.

SECTION 13–5 ENERGY CONSERVATION IN OSCILLATORY MOTION

46. • How much work is required to stretch a spring 0.133 m if its force constant is 9.17 N/m?

47. • A 0.321-kg mass is attached to a spring with a force constant of 13.3 N/m. If the mass is displaced 0.256 m from equilibrium and released, what is its speed when it is 0.128 m from equilibrium?

48. • Find the total mechanical energy of the system described in the previous problem.

49. •• A 1.8-kg mass attached to a spring oscillates with an amplitude of 7.1 cm and a frequency of 2.6 Hz. What is its energy of motion?

50. •• **IP** A 0.40-kg mass is attached to a spring with a force constant of 26 N/m and released from rest a distance of 3.2 cm from the equilibrium position of the spring. **(a)** Give a strategy that allows you to find the speed of the mass when it is halfway to the equilibrium position. **(b)** Use your strategy to find this speed.

51. •• **(a)** What is the maximum speed of the mass in the previous problem? **(b)** How far is the mass from the equilibrium position when its speed is half the maximum speed?

52. •• A bunch of grapes is placed in a spring scale at a supermarket. The grapes oscillate up and down with a period of 0.48 s, and the spring in the scale has a force constant of 650 N/m. What are **(a)** the mass and **(b)** the weight of the grapes?

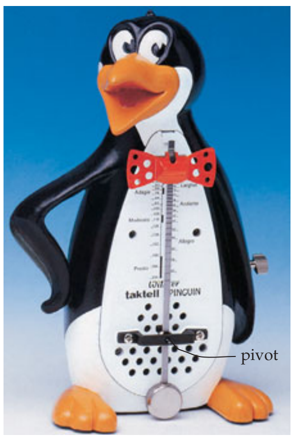
53. •• What is the maximum speed of the grapes in the previous problem if their amplitude of oscillation is 2.3 cm?

54. •• **IP** A 0.505-kg block slides on a frictionless horizontal surface with a speed of 1.18 m/s. The block encounters an unstretched spring and compresses it 23.2 cm before coming to rest. **(a)** What is the force constant of this spring? **(b)** For what length of time is the block in contact with the spring before it comes to rest? **(c)** If the force constant of the spring is increased, does the time required to stop the block increase, decrease, or stay the same? Explain.

55. •• A 2.25-g bullet embeds itself in a 1.50-kg block, which is attached to a spring of force constant 785 N/m. If the maximum compression of the spring is 5.88 cm, find **(a)** the initial speed of the bullet and **(b)** the time for the bullet–block system to come to rest.

SECTION 13–6 THE PENDULUM

56. • **CE** Metronomes, such as the penguin shown in the photo, are useful devices for music students. If it is desired to have the metronome tick with a greater frequency, should the penguin's bow tie be moved upward or downward?



How do you like my tie? (Problem 56)

57. • **CE Predict/Explain** A grandfather clock keeps correct time at sea level. If the clock is taken to the top of a nearby mountain, **(a)** would you expect it to keep correct time, run slow, or run fast? **(b)** Choose the *best explanation* from among the following:

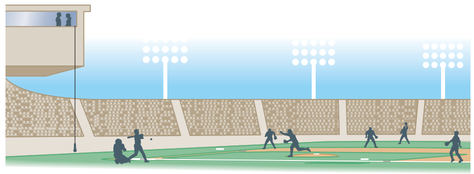
I. Gravity is weaker at the top of the mountain, leading to a greater period of oscillation.

II. The length of the pendulum is unchanged, and therefore its period remains the same.

III. The extra gravity from the mountain causes the period to decrease.

58. • **CE** A pendulum of length L has a period T. How long must the pendulum be if its period is to be $2T$?

59. • An observant fan at a baseball game notices that the radio commentators have lowered a microphone from their booth to just a few inches above the ground, as shown in **Figure 13–25**. The microphone is used to pick up sound from the field and from the fans. The fan also notices that the microphone is slowly swinging back and forth like a simple pendulum. Using her digital watch, she finds that 10 complete oscillations take 60.0 s. How high above the field is the radio booth? (Assume the microphone and its cord can be treated as a simple pendulum.)



▲ **FIGURE 13–25** Problem 59

60. • A simple pendulum of length 2.5 m makes 5.0 complete swings in 16 s. What is the acceleration of gravity at the location of the pendulum?

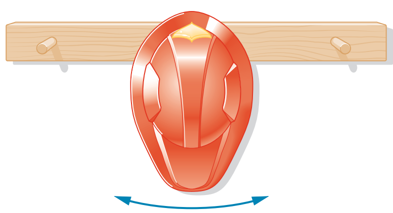
61. • **United Nations Pendulum** A large pendulum with a 200-lb gold-plated bob 12 inches in diameter is on display in the lobby of the United Nations building. The pendulum has a length of 75 ft. It is used to show the rotation of the Earth—for this reason it is referred to as a Foucault pendulum. What is the least amount of time it takes for the bob to swing from a position of maximum displacement to the equilibrium position of the pendulum? (Assume that the acceleration due to gravity is $g = 9.81\ \text{m/s}^2$ at the UN building.)

62. • Find the length of a simple pendulum that has a period of 1.00 s. Assume that the acceleration of gravity is $g = 9.81\ \text{m/s}^2$.

63. •• **IP** If the pendulum in the previous problem were to be taken to the Moon, where the acceleration of gravity is $g/6$, **(a)** would its period increase, decrease, or stay the same? **(b)** Check your result in part (a) by calculating the period of the pendulum on the Moon.

***64.** •• A hula hoop hangs from a peg. Find the period of the hoop as it gently rocks back and forth on the peg. (For a hoop with axis at the rim $I = 2mR^2$, where R is the radius of the hoop.)

***65.** •• A fireman tosses his 0.98-kg hat onto a peg, where it oscillates as a physical pendulum (**Figure 13–26**). If the center of mass of the hat is 8.4 cm from the pivot point, and its period of oscillation is 0.73 s, what is the moment of inertia of the hat about the pivot point?



▲ **FIGURE 13–26** Problem 65

***66.** •• **IP** Consider a meterstick that oscillates back and forth about a pivot point at one of its ends. **(a)** Is the period of a simple pendulum of length $L = 1.00$ m greater than, less than, or the same as the period of the meterstick? Explain. **(b)** Find the length L of a simple pendulum that has a period equal to the period of the meterstick.

*67. •• On the construction site for a new skyscraper, a uniform beam of steel is suspended from one end. If the beam swings back and forth with a period of 2.00 s, what is its length?

*68. •• **BIO** **(a)** Find the period of a child's leg as it swings about the hip joint. Assume the leg is 0.55 m long and can be treated as a uniform rod. **(b)** Estimate the child's walking speed.

69. ••• Suspended from the ceiling of an elevator is a simple pendulum of length L. What is the period of this pendulum if the elevator **(a)** accelerates upward with an acceleration a, or **(b)** accelerates downward with an acceleration whose magnitude is greater than zero but less than g? Give your answer in terms of L, g, and a.

GENERAL PROBLEMS

70. • **CE** An object undergoes simple harmonic motion with a period T. In the time $3T/2$ the object moves through a total distance of $12D$. In terms of D, what is the object's amplitude of motion?

71. • **CE** A mass on a string moves with simple harmonic motion. If the period of motion is doubled, with the force constant and the amplitude remaining the same, by what multiplicative factor do the following quantities change: **(a)** angular frequency, **(b)** frequency, **(c)** maximum speed, **(d)** maximum acceleration, **(e)** total mechanical energy?

72. • **CE** If the amplitude of a simple harmonic oscillator is doubled, by what multiplicative factor do the following quantities change: **(a)** angular frequency, **(b)** frequency, **(c)** period, **(d)** maximum speed, **(e)** maximum acceleration, **(f)** total mechanical energy?

73. • **CE** A mass m is suspended from the ceiling of an elevator by a spring of force constant k. When the elevator is at rest, the period of the mass is T. Does the period increase, decrease, or remain the same when the elevator **(a)** moves upward with constant speed or **(b)** moves upward with constant acceleration?

74. • **CE** A pendulum of length L is suspended from the ceiling of an elevator. When the elevator is at rest, the period of the pendulum is T. Does the period increase, decrease, or remain the same when the elevator **(a)** moves upward with constant speed or **(b)** moves upward with constant acceleration?

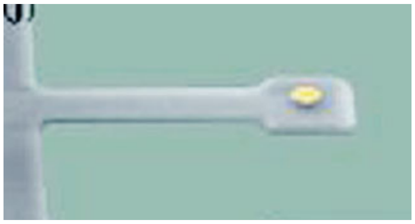
75. • A 1.8-kg mass is attached to a spring with a force constant of 59 N/m. If the mass is released with a speed of 0.25 m/s at a distance of 8.4 cm from the equilibrium position of the spring, what is its speed when it is halfway to the equilibrium position?

76. • **BIO** **Measuring an Astronaut's Mass** An astronaut uses a Body Mass Measurement Device (BMMD) to determine her mass. What is the astronaut's mass, given that the force constant of the BMMD is 2600 N/m and the period of oscillation is 0.85 s? (See the discussion on page 427 for more details on the BMMD.)

77. • A typical atom in a solid might oscillate with a frequency of 10^{12} Hz and an amplitude of 0.10 angstrom (10^{-11} m). Find the maximum acceleration of the atom and compare it with the acceleration of gravity.

78. • **Sunspot Observations** Sunspots vary in number as a function of time, exhibiting an approximately 11-year cycle. Galileo made the first European observations of sunspots in 1610, and daily observations were begun in Zurich in 1749. At the present time we are well into the 23rd observed cycle. What is the frequency of the sunspot cycle? Give your answer in Hz.

79. • **BIO** **Weighing a Bacterium** Scientists are using tiny, nanoscale cantilevers 4 micrometers long and 500 nanometers wide—essentially miniature diving boards—as a sensitive way to measure mass. The cantilevers oscillate up and down with a frequency that depends on the mass placed near the tip, and a laser beam is used to measure the frequency. A single *E. coli* bacterium was measured to have a mass of 665 femtograms = 6.65×10^{-16} kg with this device, as the cantilever oscillated with a frequency of 14.5 MHz. Treating the cantilever as an ideal, massless spring, find its effective force constant.



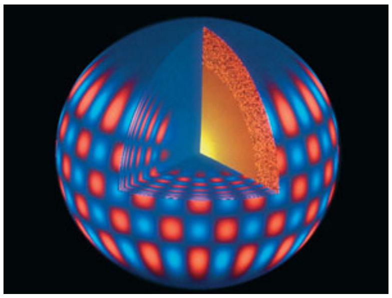
A silicon and silicon nitride cantilever with a 50-nanometer gold dot near its tip. (Problem 79)

80. •• **CE** An object undergoing simple harmonic motion with a period T is at the position $x = 0$ at the time $t = 0$. At the time $t = 0.25T$ the position of the object is positive. State whether x is positive, negative, or zero at the following times: **(a)** $t = 1.5T$, **(b)** $t = 2T$, **(c)** $t = 2.25T$, and **(d)** $t = 6.75T$.

81. •• The maximum speed of a 3.1-kg mass attached to a spring is 0.68 m/s, and the maximum force exerted on the mass is 11 N. **(a)** What is the amplitude of motion for this mass? **(b)** What is the force constant of the spring? **(c)** What is the frequency of this system?

82. •• The acceleration of a block attached to a spring is given by $a = -(0.302\ \mathrm{m/s^2})\cos([2.41\ \mathrm{rad/s}]t)$. **(a)** What is the frequency of the block's motion? **(b)** What is the maximum speed of the block? **(c)** What is the amplitude of the block's motion?

83. •• **Helioseismology** In 1962, physicists at Cal Tech discovered that the surface of the Sun vibrates due to the violent nuclear reactions that roil within its core. This has led to a new field of solar science known as helioseismology. A typical vibration of the Sun is shown in **Figure 13–27**; it has a period of 5.7 minutes. The blue patches in Figure 13–27 are moving outward; the red patches are moving inward. **(a)** Find the angular frequency of this vibration. **(b)** The maximum speed at which a patch of the surface moves during a vibration is 4.5 m/s. What is the amplitude of the vibration, assuming it to be simple harmonic motion?



▲ **FIGURE 13–27** A typical vibration pattern of the Sun. (Problem 83)

84. •• **IP** A 9.50-g bullet, moving horizontally with an initial speed v_0, embeds itself in a 1.45-kg pendulum bob that is initially at rest. The length of the pendulum is $L = 0.745$ m. After the collision, the pendulum swings to one side and comes to rest when it has gained a vertical height of 12.4 cm. **(a)** Is the kinetic energy of the bullet–bob system immediately after the collision greater than, less than, or the same as the kinetic energy of the system just before the collision? Explain. **(b)** Find the initial speed of the bullet. **(c)** How long does it take for the bullet–bob system to come to rest for the first time?

85. •• **BIO Spiderweb Oscillations** A 1.44-g spider oscillates on its web, which has a damping constant of 3.30×10^{-5} kg/s. How long does it take for the spider's amplitude of oscillation to decrease by 10.0 percent?

86. •• An object undergoes simple harmonic motion with a period T and amplitude A. In terms of T, how long does it take the object to travel from $x = A$ to $x = A/2$?

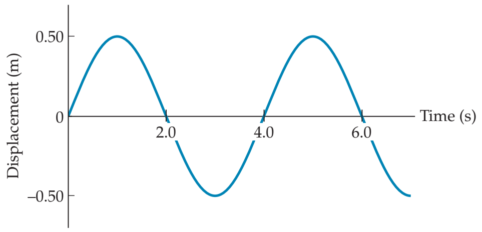
87. •• Find the period of oscillation of a disk of mass 0.32 kg and radius 0.15 m if it is pivoted about a small hole drilled near its rim.

88. •• Calculate the ratio of the kinetic energy to the potential energy of a simple harmonic oscillator when its displacement is half its amplitude.

89. •• A 0.363-kg mass slides on a frictionless floor with a speed of 1.24 m/s. The mass strikes and compresses a spring with a force constant of 44.5 N/m. **(a)** How far does the mass travel after contacting the spring before it comes to rest? **(b)** How long does it take for the spring to stop the mass?

90. •• A large rectangular barge floating on a lake oscillates up and down with a period of 4.5 s. Find the damping constant for the barge, given that its mass is 2.44×10^5 kg and that its amplitude of oscillation decreases by a factor of 2.0 in 5.0 minutes.

91. •• **IP** **Figure 13–28** shows a displacement-versus-time graph of the periodic motion of a 3.8-kg mass on a spring. **(a)** Referring to the figure, do you expect the maximum speed of the mass to be greater than, less than, or equal to 0.50 m/s? Explain. **(b)** Calculate the maximum speed of the mass. **(c)** How much energy is stored in this system?

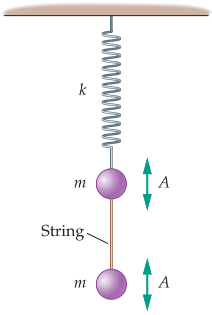


▲ **FIGURE 13–28** Problems 91 and 92

92. •• **IP** A 3.8-kg mass on a spring oscillates as shown in the displacement-versus-time graph in Figure 13–28. **(a)** Referring to the graph, at what times between $t = 0$ and $t = 6.0$ s does the mass experience a force of maximum magnitude? Explain. **(b)** Calculate the magnitude of the maximum force exerted on the mass. **(c)** At what times shown in the graph does the mass experience zero force? Explain. **(d)** How much force is exerted on the mass at the time $t = 0.50$ s?

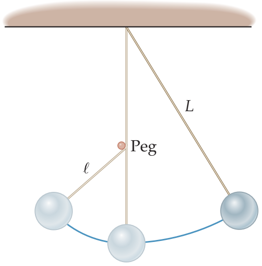
93. •• A 0.45-kg crow lands on a slender branch and bobs up and down with a period of 1.5 s. An eagle flies up to the same branch, scaring the crow away, and lands. The eagle now bobs up and down with a period of 4.8 s. Treating the branch as an ideal spring, find **(a)** the effective force constant of the branch and **(b)** the mass of the eagle.

94. ••• A mass m is connected to the bottom of a vertical spring whose force constant is k. Attached to the bottom of the mass is a string that is connected to a second mass m, as shown in **Figure 13–29**. Both masses are undergoing simple harmonic vertical motion of amplitude A. At the instant when the acceleration of the masses is a maximum in the upward direction the string breaks, allowing the lower mass to drop to the floor. Find the resulting amplitude of motion of the remaining mass.



▲ **FIGURE 13–29** Problem 94

95. ••• **IP** Consider the pendulum shown in **Figure 13–30**. Note that the pendulum's string is stopped by a peg when the bob swings to the left, but moves freely when the bob swings to the right. **(a)** Is the period of this pendulum greater than, less than, or the same as the period of the same pendulum without the peg? **(b)** Calculate the period of this pendulum in terms of L and ℓ. **(c)** Evaluate your result for $L = 1.0$ m and $\ell = 0.25$ m.



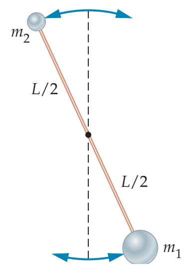
▲ **FIGURE 13–30** Problem 95

96. ••• When a mass m is attached to a vertical spring with a force constant k, it stretches the spring by the amount L. Calculate **(a)** the period of this mass and **(b)** the period of a simple pendulum of length L.

97. ••• An object undergoes simple harmonic motion of amplitude A and angular frequency ω about the equilibrium point $x = 0$. Use energy conservation to show that the speed of the object at the general position x is given by the following expression:

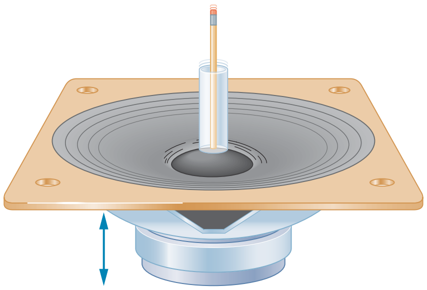
$$v = \omega\sqrt{A^2 - x^2}$$

***98.** ••• A physical pendulum consists of a light rod of length L suspended in the middle. A large mass m_1 is attached to one end of the rod, and a lighter mass m_2 is attached to the other end, as illustrated in **Figure 13–31**. Find the period of oscillation for this pendulum.



▲ **FIGURE 13–31** Problem 98

99. ••• **IP** A vertical hollow tube is connected to a speaker, which vibrates vertically with simple harmonic motion (**Figure 13–32**). The speaker operates with constant amplitude, A, but variable frequency, f. A slender pencil is placed inside the tube. **(a)** At low frequencies the pencil stays in contact with the speaker at all times; at higher frequencies the pencil begins to rattle. Explain the reason for this behavior. **(b)** Find an expression for the frequency at which rattling begins.



▲ **FIGURE 13–32** Problem 99

PASSAGE PROBLEMS

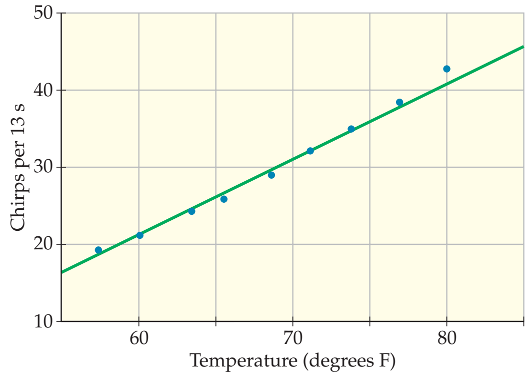
BIO A Cricket Thermometer, by Jiminy

Insects are ectothermic, which means their body temperature is largely determined by the temperature of their surroundings. This can have a number of interesting consequences. For example, the wing coloration in some butterfly species is determined by the ambient temperature, as is the body color of several species of dragonfly. In addition, the wing beat frequency of beetles taking flight varies with temperature due to changes in the resonant frequency of their thorax.

The origin of such temperature effects can be traced back to the fact that molecules have higher speeds and greater energy as temperature is increased (see Chapters 16 and 17). Thus, for example, molecules that collide and react as part of the metabolic process will do so more rapidly when the reactions are occurring at a higher temperature. As a result, development rates, heart rates, wing beats, and other processes all occur more rapidly.

One of the most interesting thermal effects is the temperature dependence of chirp rate in certain insects. This behavior has been observed in cone-headed grasshoppers, as well as several types of cricket. A particularly accurate connection between chirp rate and temperature is found in the snowy tree cricket (*Oecanthus fultoni* Walker), which chirps at a rate that follows the expression $N = T - 39$, where N is the number of chirps in 13 seconds, and T is the numerical value of the temperature in degrees Fahrenheit. This formula, which is known as Dolbear's law, is plotted in **Figure 13–33** (green line) along with data points (blue dots) for the snowy tree cricket.

The snowy tree cricket.



▲ **FIGURE 13–33** Problems 100, 101, 102, and 103

100. • If the temperature is increased by 10 degrees Fahrenheit, how many additional chirps are heard in a 13-s interval?

A. 5 **B.** 10

C. 13 **D.** 39

101. • What is the temperature in degrees Fahrenheit if a cricket is observed to give 35 chirps in 13 s?

A. 13 °F **B.** 35 °F

C. 74 °F **D.** 90 °F

102. • What is the frequency of the cricket's chirping (in Hz) when the temperature is 68 °F?

A. 0.45 Hz **B.** 2.2 Hz

C. 5.2 Hz **D.** 29 Hz

103. •• Suppose the temperature decreases uniformly from 75 °F to 63 °F in 12 minutes. How many chirps does the cricket produce during this time?

A. 28 **B.** 1700

C. 3800 **D.** 22,000

INTERACTIVE PROBLEMS

104. •• **IP Referring to Example 13–3** Suppose we can change the plane's period of oscillation, while keeping its amplitude of motion equal to 30.0 m. **(a)** If we want to reduce the maximum acceleration of the plane, should we increase or decrease the period? Explain. **(b)** Find the period that results in a maximum acceleration of $1.0g$.

105. •• **IP Referring to Example 13–6** Suppose the force constant of the spring is doubled, but the mass and speed of the block are still 0.980 kg and 1.32 m/s, respectively. **(a)** By what multiplicative factor do you expect the maximum compression of the spring to change? Explain. **(b)** Find the new maximum compression of the spring. **(c)** Find the time required for the mass to come to rest after contacting the spring.

106. •• **IP Referring to Example 13–6** **(a)** If the block's initial speed is increased, does the total time the block is in contact with the spring increase, decrease, or stay the same? **(b)** Find the total time of contact for $v_0 = 1.65$ m/s, $m = 0.980$ kg, and $k = 245$ N/m.

14 Waves and Sound

Have you ever wondered why a grand piano has this somewhat peculiar shape? It's not just tradition—there's also a physical reason, having to do with the way vibrating strings produce sound. But to understand this and other aspects of sound, it is first necessary to learn about waves in general—for sound, as we shall see, is merely a particular kind of wave, though one that has a special importance in our lives.

In the last chapter, we studied the behavior of an oscillator. Here, we consider the behavior of a series of oscillators that are connected to one another. Connecting oscillators leads to an assortment of new phenomena, including waves on a string, water waves, and sound. In this chapter, we focus our attention on the behavior of such waves, and in particular on the way they propagate, their speed of propagation, and their interactions with one another. Later, in Chapter 25, we shall see that light is also a type of wave, and that it displays many of the same phenomena exhibited by the waves considered in this chapter.

14–1 Types of Waves

Consider a group of swings in a playground swing set. We know that each swing by itself behaves like a simple pendulum; that is, like an oscillator. Now, let's connect the swings to one another. To be specific, suppose we tie a rope from the seat of the first swing to its neighbor, and then another rope from the second swing to the third swing, and so on. When the swings are at rest—in equilibrium— the connecting ropes have no effect. If you now sit in the first swing and begin oscillating—thus "disturbing" the equilibrium—the connecting ropes cause the other swings along the line to start oscillating as well. You have created a traveling disturbance.

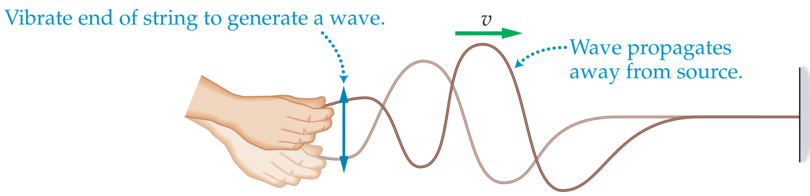
In general, a disturbance that propagates from one place to another is referred to as a **wave.** Waves propagate with well-defined speeds determined by the properties of the material through which they travel. In addition, waves carry energy. For example, part of the energy you put into sound waves when you speak is carried to the ears of others, where some of the sound energy is converted into electrical energy carried by nerve impulses to the brain which, in turn, creates the sensation of hearing.

It is important to distinguish between the motion of the wave itself and the motion of the individual particles that make up the wave. Common examples include the waves that propagate through a field of wheat. The individual wheat stalks sway back and forth as a wave passes, but they do not change their location. Similarly, a "wave" at a ball game may propagate around the stadium more quickly than a person can run, but the individual people making up the wave simply stand and sit in one place. From these simple examples it is clear that waves can come in a variety of types. We discuss some of the more common types in this section. In addition, we show how the speed of a wave is related to some of its basic properties.

▲ A wave can be viewed as a disturbance that propagates through space. Although the wave itself moves steadily in one direction, the particles that create the wave do not share in this motion. Instead, they oscillate back and forth about their equilibrium positions. The water in an ocean wave, for example, moves mainly up and down—as it passes, you bob up and down with it rather than being carried onto the shore. Similarly, the people in a human "wave" at a ballpark simply stand or raise their arms in place—they do not travel around the stadium.

Transverse Waves

Perhaps the easiest type of wave to visualize is a wave on a string, as illustrated in **Figure 14–1**. To generate such a wave, start by tying one end of a long string or rope to a wall. Pull on the free end with your hand, producing a tension in the string, and then move your hand up and down. As you do so, a wave will travel along the string toward the wall. In fact, if your hand moves up and down with simple harmonic motion, the wave on the string will have the shape of a sine or a cosine; we refer to such a wave as a **harmonic wave.**

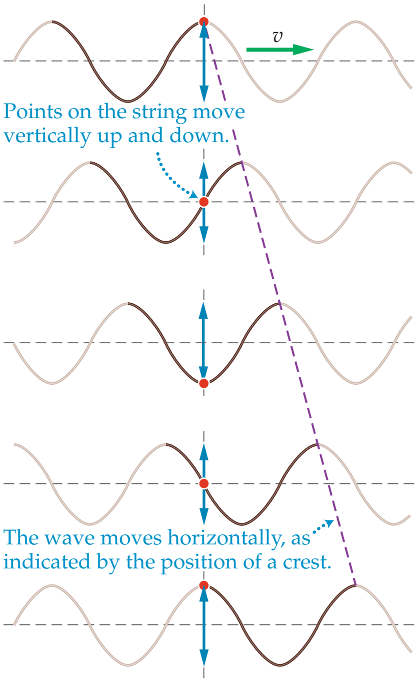


◀ **FIGURE 14–1 A wave on a string**
Vibrating one end of a string with an up-and-down motion generates a wave that travels away from its point of origin.

Note that the wave travels in the horizontal direction, even though your hand oscillates vertically about one spot. In fact, if you look at any point on the string, it too moves vertically up and down, with no horizontal motion at all. This is shown in **Figure 14–2**, where we see the location of an individual point on a string as a wave travels past. Notice, in particular, that the displacement of particles in a string is at right angles to the direction of propagation of the wave. A wave with this property is called a **transverse wave:**

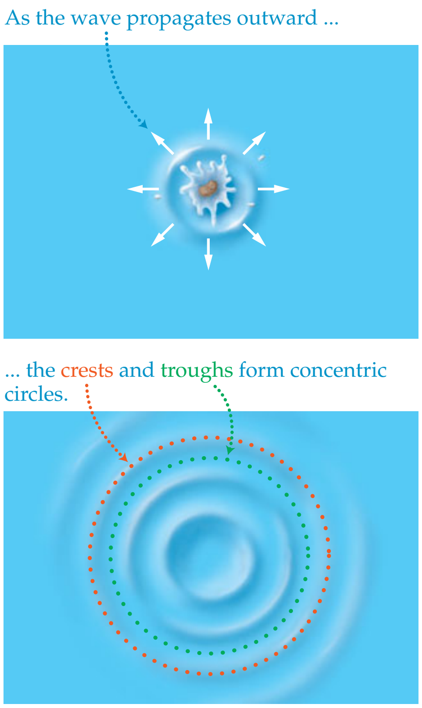
> In a transverse wave, the displacement of individual particles is at *right angles* to the direction of propagation of the wave.

Other examples of transverse waves include light and radio waves. These will be discussed in detail in Chapter 25.



▲ **FIGURE 14–2 The motion of a wave on a string**

As a wave on a string moves horizontally, all points on the string vibrate in the vertical direction, as indicated by the blue arrow.



▲ **FIGURE 14–4 Water waves from a disturbance**

An isolated disturbance in a pool of water, caused by a pebble dropped into the water, creates waves that propagate symmetrically away from the disturbance. The crests and troughs form concentric circles on the surface of the water as they move outward.

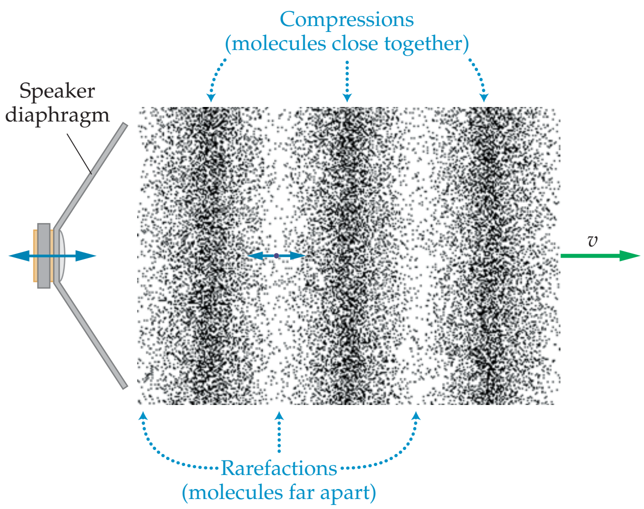
Longitudinal Waves

Longitudinal waves differ from transverse waves in the way that particles in the wave move. In particular, a longitudinal wave is defined as follows:

> In a longitudinal wave, the displacement of individual particles is parallel to the direction of propagation of the wave.

The classic example of a longitudinal wave is sound. When you speak, for example, the vibrations in your vocal cords create a series of compressions and expansions (rarefactions) in the air. The same kind of situation occurs with a loudspeaker, as illustrated in **Figure 14–3**. Here we see a speaker diaphragm vibrating horizontally with simple harmonic motion. As it moves to the right it compresses the air momentarily; as it moves to the left it rarefies the air. A series of compressions and rarefactions then travel horizontally away from the loudspeaker with the speed of sound.

Figure 14–3 also indicates the motion of an individual particle in the air as a sound wave passes. Note that the particle moves back and forth horizontally; that is, in the same direction as the propagation of the wave. The particle does not travel with the wave—each individual particle simply oscillates about a given position in space.



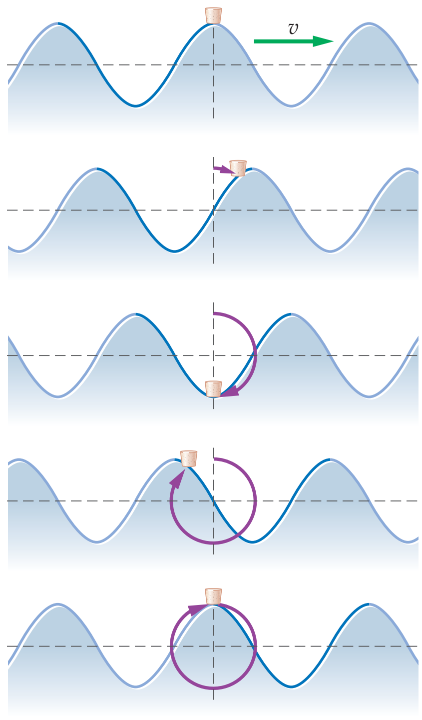
▲ **FIGURE 14–3 Sound produced by a speaker**

As the diaphragm of a speaker vibrates back and forth, it alternately compresses and rarefies the surrounding air. These regions of high and low density propagate away from the speaker with the speed of sound. Individual particles in the air oscillate back and forth about a given position, as indicated by the blue arrow.

Water Waves

If a pebble is dropped into a pool of water, a series of concentric waves move away from the drop point. This is illustrated in **Figure 14–4**. To visualize the movement of the water as a wave travels by, place a small piece of cork into the water. As a wave passes, the motion of the cork will trace out the motion of the water itself, as indicated in **Figure 14–5**.

Notice that the cork moves in a roughly circular path, returning to approximately its starting point. Thus, each element of water moves both vertically and horizontally as the wave propagates by in the horizontal direction. In this sense, a water wave is a combination of both transverse and longitudinal waves. This makes the water wave more difficult to analyze. Hence, most of our results will refer to the simpler cases of purely transverse and purely longitudinal waves.



▶ **FIGURE 14–5 The motion of a water wave**

As a water wave passes a given point, a molecule (or a small piece of cork) moves in a roughly circular path. This means that the water molecules move both vertically and horizontally. In this sense, the water wave has characteristics of both transverse and longitudinal waves.

Wavelength, Frequency, and Speed

A simple wave can be thought of as a regular, rhythmic disturbance that propagates from one point to another, repeating itself both in *space* and in *time*. We now show that the repeat length and the repeat time of a wave are directly related to its speed of propagation.

We begin by considering the snapshots of a wave shown in **Figure 14–6**. Points on the wave corresponding to maximum upward displacement are referred to as **crests;** points corresponding to maximum downward displacement are called **troughs.** The distance from one crest to the next, or from one trough to the next, is the repeat length—or **wavelength,** λ—of the wave.

Definition of Wavelength, λ

λ = distance over which a wave repeats

SI unit: m

Similarly, the repeat time—or **period,** T—of a wave is the time required for one wavelength to pass a given point, as illustrated in Figure 14–6. Closely related to the period of a wave is its **frequency,** f, which, as with oscillations, is defined by the relation $f = 1/T$.

Combining these observations, we see that a wave travels a distance λ in the time T. Applying the definition of speed—distance divided by time—it follows that the speed of a wave is

Speed of a Wave

$$v = \frac{\text{distance}}{\text{time}} = \frac{\lambda}{T} = \lambda f \qquad \text{14–1}$$

SI unit: m/s

This result applies to all waves.

EXERCISE 14–1

Sound waves travel in air with a speed of 343 m/s. The lowest frequency sound we can hear is 20.0 Hz; the highest frequency is 20.0 kHz. Find the wavelength of sound for frequencies of 20.0 Hz and 20.0 kHz.

SOLUTION

Solve Equation 14–1 for λ:

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{20.0\ \text{s}^{-1}} = 17.2\ \text{m}$$

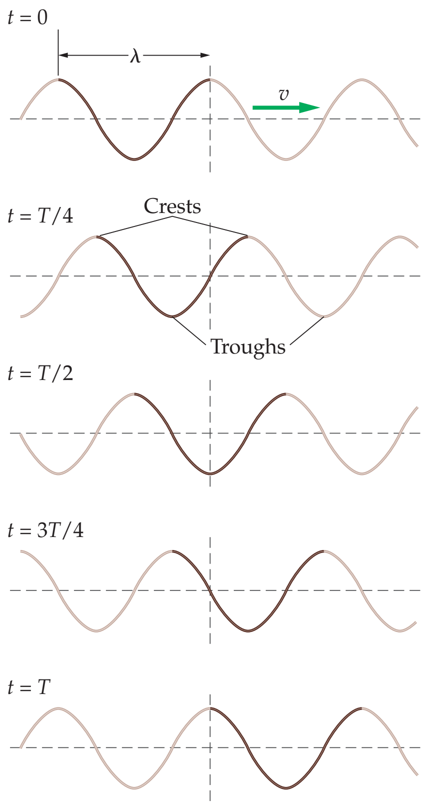
$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{20{,}000\ \text{s}^{-1}} = 1.72\ \text{cm}$$

14–2 Waves on a String

In this section we consider some of the basic properties of waves traveling on a string, a rope, a wire, or any similar linear medium.

The Speed of a Wave on a String

The speed of a wave is determined by the properties of the medium through which it propagates. In the case of a string of length L, there are two basic



▲ **FIGURE 14–6 The speed of a wave**

A wave repeats over a distance equal to the wavelength, λ. The time necessary for a wave to move one wavelength is the period, T. Thus, the speed of a wave is $v = \lambda/T = \lambda f$.

characteristics that determine the speed of a wave: (i) the tension in the string, and (ii) the mass of the string.

Let's begin with the tension, which is the force F transmitted through the string (we will use F for the tension rather than T, to avoid confusion between the tension and the period). Clearly, there must be a tension in a string in order for it to propagate a wave. Imagine, for example, that a string lies on a smooth floor with both ends free. If you take one end into your hand and shake it, the portions of the string near your hand will oscillate slightly, but no wave will travel to the other end of the string. If someone else takes hold of the other end of the string and pulls enough to set up a tension, then any movement you make on your end will propagate to the other end. In fact, if the tension is increased—so that the string becomes less slack—waves will travel through the string more rapidly.

Next, we consider the mass m of the string. A heavy string responds slowly to a given disturbance because of its inertia. Thus, if you try sending a wave through a kite string or a large rope, both under the same tension, you will find that the wave in the rope travels more slowly. In general, the heavier a rope or string the slower the speed of waves in it. Of course, the total mass of a string doesn't really matter; a longer string has more mass, but its other properties are basically the same. What is important is the mass of the string per length. We give this quantity the label μ:

Definition of Mass per Length, μ

μ = mass per length = m/L

SI unit: kg/m

To summarize, we expect the speed v to increase with the tension F and decrease with the mass per length, μ. Assuming these are the only factors determining the speed of a wave on a string, we can obtain the dependence of v on F and μ using dimensional analysis (see Chapter 1, Section 3). First, we identify the dimensions of v, F, and μ:

$$[v] = \mathrm{m/s}$$
$$[F] = \mathrm{N} = \mathrm{kg \cdot m/s^2}$$
$$[\mu] = \mathrm{kg/m}$$

Next, we seek a combination of F and μ that has the dimensions of v; namely, m/s. Suppose, for example, that v depends on F to the power a and μ to the power b. Then, we have

$$v = F^a \mu^b$$

In terms of dimensions, this equation is

$$\mathrm{m/s} = (\mathrm{kg \cdot m/s^2})^a (\mathrm{kg/m})^b = \mathrm{kg}^{a+b}\mathrm{m}^{a-b}\mathrm{s}^{-2a}$$

Comparing dimensions, we see that kg does not appear on the left side of the equation; therefore, we conclude that $a + b = 0$ so that kg does not appear on the right side of the equation. Hence, $a = -b$. Looking at the time dimension, s, we see that on the left we have s^{-1}; thus on the right side we must have $-2a = -1$, or $a = \frac{1}{2}$. It follows that $b = -a = -\frac{1}{2}$. This gives the following result:

Speed of a Wave on a String, v

$$v = \sqrt{\frac{F}{\mu}} \qquad 14\text{–}2$$

SI unit: m/s

As expected, the speed increases with F and decreases with μ.

Dimensional analysis does not guarantee that this is the complete, final result; there could be a dimensionless factor like $\frac{1}{2}$ or 2π left unaccounted for. It turns out, however, that a complete analysis based on Newton's laws gives precisely the same result.

EXERCISE 14–2

A 5.0-m length of rope, with a mass of 0.52 kg, is pulled taut with a tension of 46 N. Find the speed of waves on the rope.

SOLUTION

First, calculate the mass per length, μ:

$$\mu = m/L = (0.52\text{ kg})/(5.0\text{ m}) = 0.10\text{ kg/m}$$

Now, substitute μ and F into Equation 14–2:

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{46\text{ N}}{0.10\text{ kg/m}}} = 21\text{ m/s}$$

PROBLEM-SOLVING NOTE

Mass Versus Mass-per-Length

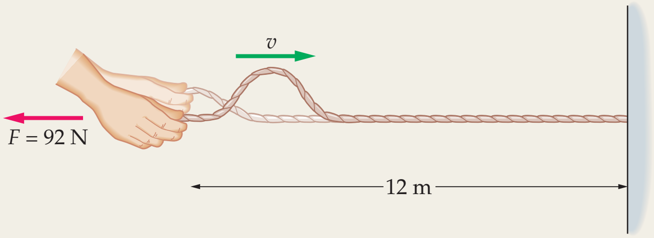
To find the mass of a string, multiply its mass per length, μ, by its length L. That is, $m = \mu L$.

EXAMPLE 14–1 A WAVE ON A ROPE

A 12-m rope is pulled tight with a tension of 92 N. When one end of the rope is given a "thunk" it takes 0.45 s for the disturbance to propagate to the other end. What is the mass of the rope?

PICTURE THE PROBLEM

Our sketch shows a wave pulse traveling with a speed v from one end of the rope to the other, a distance of 12 m. The tension in the rope is 92 N, and the travel time of the pulse is 0.45 s.



STRATEGY

We know that the speed of waves (disturbances) on a rope is determined by the tension and the mass per length. Thus, we first calculate the speed of the wave with the information given in the problem statement. Next, we solve for the mass per length, then multiply by the length to get the mass.

SOLUTION

1. Calculate the speed of the wave: $v = \dfrac{d}{t} = \dfrac{12\text{ m}}{0.45\text{ s}} = 27\text{ m/s}$
2. Use $v = \sqrt{F/\mu}$ to solve for the mass per length: $\mu = F/v^2$
3. Substitute numerical values for F and v: $\mu = \dfrac{F}{v^2} = \dfrac{92\text{ N}}{(27\text{ m/s})^2} = 0.13\text{ kg/m}$
4. Multiply μ by $L = 12$ m to find the mass: $m = \mu L = (0.13\text{ kg/m})(12\text{ m}) = 1.6\text{ kg}$

INSIGHT

Note that the speed of a wave on this rope (about 60 mi/h) is comparable to the speed of a car on a highway. This speed could be increased even further by pulling harder on the rope, thus increasing its tension.

PRACTICE PROBLEM

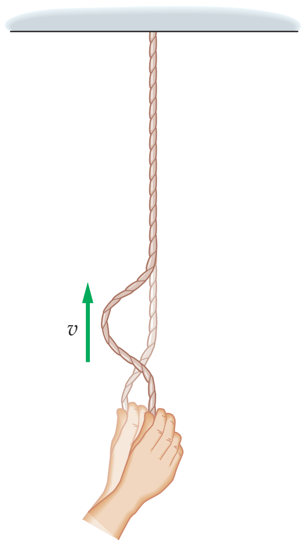
If the tension in this rope is doubled, how long will it take for the thunk to travel from one end to the other? [**Answer:** In this case the wave speed is $v = 38$ m/s; hence the time is $t = 0.32$ s.]

Some related homework problems: Problem 14, Problem 15, Problem 16

In the following Conceptual Checkpoint, we consider the speed of a wave on a vertical rope of finite mass.

CONCEPTUAL CHECKPOINT 14–1 SPEED OF A WAVE

A rope of length L and mass M hangs from a ceiling. If the bottom of the rope is given a gentle wiggle, a wave will travel to the top of the rope. As the wave travels upward does its speed **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

The speed of the wave is determined by the tension in the rope and its mass per length. The mass per length is the same from bottom to top, but not the tension. In particular, the tension at any point in the rope is equal to the weight of rope below that point. Thus, the tension increases from almost zero near the bottom to essentially Mg near the top. Since the tension increases with height, so too does the speed, according to Equation 14–2.

ANSWER

(a) The speed increases.

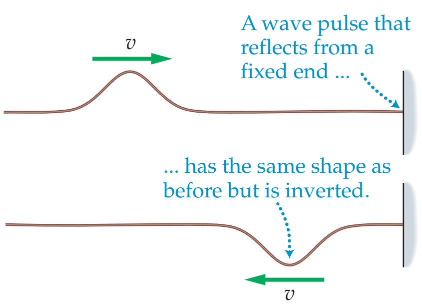


FIGURE 14–7 A reflected wave pulse: fixed end

A wave pulse on a string is inverted when it reflects from an end that is tied down.

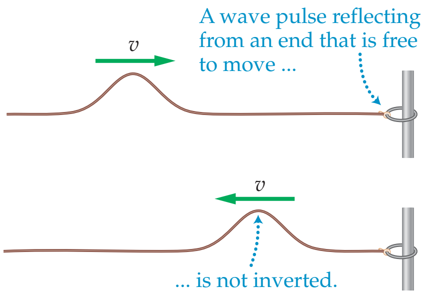


FIGURE 14–8 A reflected wave pulse: free end

A wave pulse on a string whose end is free to move is reflected without inversion.

Reflections

Thus far we have discussed only the situation in which a wave travels along a string; but at some point the wave must reach the end of the string. What happens then? Clearly, we expect the wave to be reflected, but the precise way in which the reflection occurs needs to be considered.

Suppose, for example, that the far end of a string is anchored firmly into a wall, as shown in **Figure 14–7**. If you give a flick to your end of the string, you set up a wave "pulse" that travels toward the far end. When it reaches the end, it exerts an upward force on the wall, trying to pull it up into the pulse. Since the end is tied down, however, the wall exerts an equal and opposite downward force to keep the end at rest. Thus, the wall exerts a downward force on the string that is just the *opposite* of the upward force you exerted when you created the pulse. As a result, the reflection is an inverted, or upside-down, pulse, as indicated in Figure 14–7. We shall encounter this same type of inversion under reflection when we consider the reflection of light in Chapter 28.

Another way to tie off the end of the string is shown in **Figure 14–8**. In this case, the string is tied to a small ring that slides vertically with little friction on a vertical pole. In this way, the string still has a tension in it, since it pulls on the ring, but it is also free to move up and down.

Consider a pulse moving along such a string, as in Figure 14–8. When the pulse reaches the end, it lifts the ring upward and then lowers it back down. In fact, the pulse flicks the far end of the string in the *same* way that you flicked it when you created the pulse. Therefore, the far end of the string simply creates a new pulse, identical to the first, only traveling in the opposite direction. This is illustrated in the figure.

Thus, when waves reflect, they may or may not be inverted, depending on how the reflection occurs.

*14–3 Harmonic Wave Functions

If a wave is generated by oscillating one end of a string with simple harmonic motion, the waves will have the shape of a sine or a cosine. This is shown in **Figure 14–9**, where the y direction denotes the vertical displacement of the string, and $y = 0$ corresponds to the flat string with no wave present. In what follows, we consider the mathematical formula that describes y as a function of time, t, and position, x, for such a harmonic wave.

First, note that the harmonic wave in Figure 14–9 repeats when x increases by an amount equal to the wavelength, λ. Thus, the dependence of the wave on x must be of the form

$$y(x) = A\cos\left(\frac{2\pi}{\lambda}x\right) \qquad 14\text{–}3$$

To see that this is the correct dependence, note that replacing x with $x + \lambda$ gives the same value for y:

$$y(x+\lambda) = A\cos\left[\frac{2\pi}{\lambda}(x+\lambda)\right] = A\cos\left(\frac{2\pi}{\lambda}x + 2\pi\right) = A\cos\left(\frac{2\pi}{\lambda}x\right) = y(x)$$

It follows that Equation 14–3 describes a vertical displacement that repeats with a wavelength λ, as desired for a wave.

This is only part of the "wave function," however, since we have not yet described the way the wave changes with time. This is illustrated in Figure 14–9, where we see a harmonic wave at time $t = 0$, $t = T/4$, $t = T/2$, $t = 3T/4$, and $t = T$. Note that the peak in the wave that was originally at $x = 0$ at $t = 0$ moves to $x = \lambda/4$, $x = \lambda/2$, $x = 3\lambda/4$, and $x = \lambda$ for the times just given. Thus, the position x of this peak can be written as follows:

$$x = \lambda\frac{t}{T}$$

Equivalently, we can say that the peak that was at $x = 0$ is now at the location given by

$$x - \lambda\frac{t}{T} = 0$$

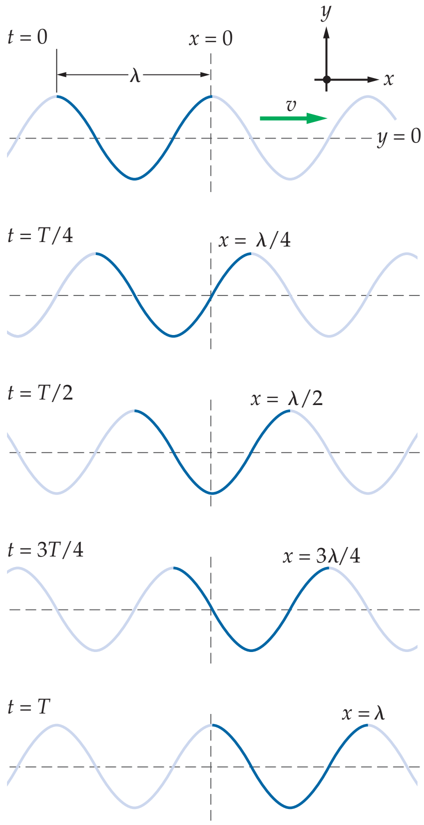
Similarly, the peak that was originally at $x = \lambda$ at $t = 0$ is at the following position at the general time t:

$$x - \lambda\frac{t}{T} = \lambda$$

In general, if the position of a given point on a wave at $t = 0$ is $x(0)$, and its position at the time t is $x(t)$, the relation between these positions is $x(t) - \lambda t/T = x(0)$. Therefore, to take into account the time dependence of a wave, we replace $x = x(0)$ in Equation 14–3 with $x(0) = x - \lambda t/T$. This yields the harmonic wave function:

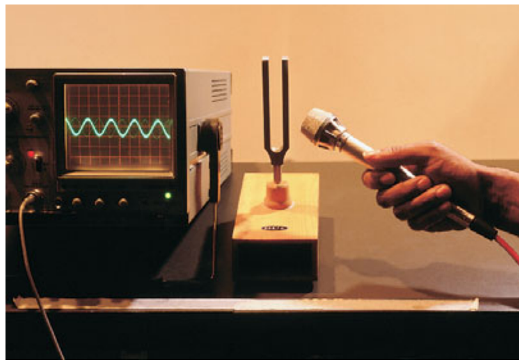
$$y(x,t) = A\cos\left[\frac{2\pi}{\lambda}\left(x - \lambda\frac{t}{T}\right)\right] = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \qquad 14\text{–}4$$

Note that the wave function, $y(x, t)$, depends on both time and position, and that the wave repeats whenever position increases by the wavelength, λ, or time increases by the period, T.



▲ **FIGURE 14–9 A harmonic wave moving to the right**

As a wave moves, the peak that was at $x = 0$ at time $t = 0$ moves to the position $x = \lambda t/T$ at the time t.

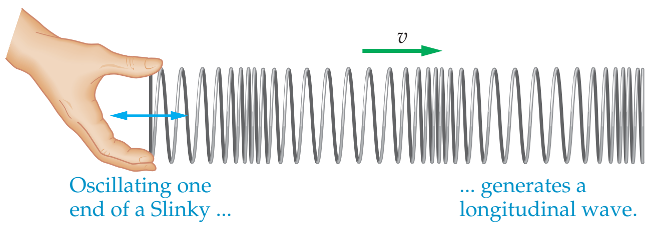


▲ An oscilloscope connected to a microphone can be used to display the wave form of a pure tone, created here by a tuning fork. The trace on the screen shows that the wave form is sinusoidal.

14–4 Sound Waves

The first thing we do when we come into this world is make a sound. It is many years later before we realize that sound is a wave propagating through the air at a speed of about 770 mi/h. More years are required to gain an understanding of the physics of a sound wave.

A useful mechanical model of a sound wave is provided by a Slinky. If we oscillate one end of a Slinky back and forth horizontally, as in **Figure 14–10**, we send out a longitudinal wave that also travels in the horizontal direction. The wave



◀ **FIGURE 14–10 A wave on a Slinky**

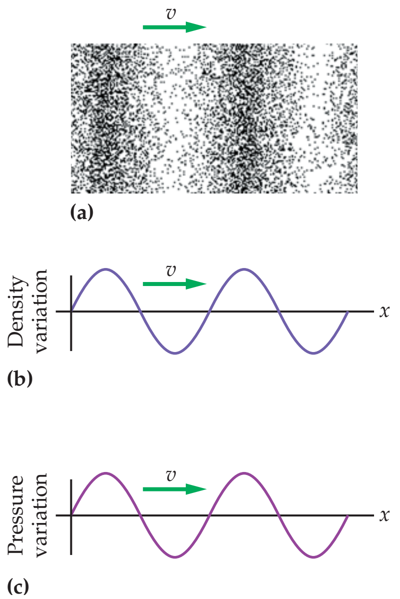
If one end of a Slinky is oscillated back and forth, a series of longitudinal waves are produced. These Slinky waves are analogous to sound waves.

consists of regions where the coils of the Slinky are compressed alternating with regions where the coils are more widely spaced.

In close analogy with the Slinky model, a speaker produces sound waves by oscillating a diaphragm back and forth horizontally, as we saw in Figure 14–3. Just as with the Slinky, a wave travels away from the source horizontally. The wave consists of compressed regions alternating with rarefied regions.

At first glance, the sound wave seems very different from the wave on a string. In particular, the sound wave doesn't seem to have the nice, sinusoidal shape of a wave. Certainly, Figure 14–3 gives no hint of such a wavelike shape.

If we plot the appropriate quantities, however, the classic wave shape emerges. For example, in **Figure 14–11 (a)** we plot the rarefactions and compressions of a typical sound wave, while in **Figure 14–11 (b)** we plot the fluctuations in the density of the air versus x. Clearly, the density oscillates in a wavelike fashion. Similarly, **Figure 14–11 (c)** shows a plot of the fluctuations in the pressure of the air as a function of x. In regions where the density is high, the pressure is also high; and where the density is low, the pressure is low. Thus, pressure versus position again shows that a sound wave has the usual wavelike properties.



▲ **FIGURE 14–11 Wave properties of sound**
A sound wave moving through the air **(a)** produces a wavelike disturbance in the **(b)** density and **(c)** pressure of the air.

Just like the speed of a wave on a string, the speed of sound is determined by the properties of the medium through which it propagates. In air, under normal atmospheric pressure and temperature, the speed of sound is approximately the following:

Speed of Sound in Air (at room temperature, 20 °C)

$v = 343\ \mathrm{m/s} \approx 770\ \mathrm{mi/h}$

SI unit: m/s

When we refer to the speed of sound in this text we will always assume the value is 343 m/s, unless stated specifically otherwise.

As we shall see in Chapter 17, where we study the kinetic theory of gases, the speed of sound in air is directly related to the speed of the molecules themselves. Did you know, for example, that the air molecules colliding with your body at this moment have speeds that are essentially the speed of sound? As the air is heated the molecules will move faster, and hence the speed of sound also increases with temperature.

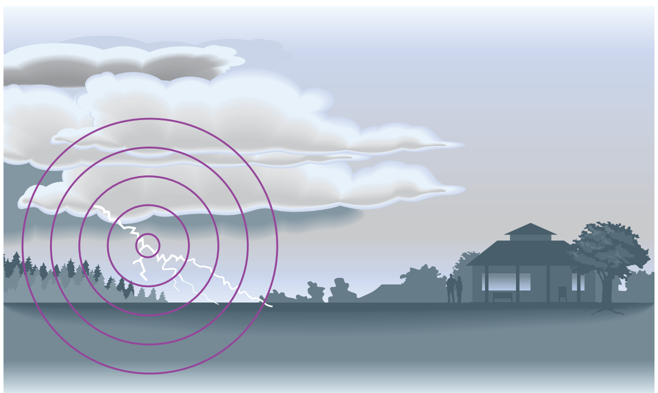
In a solid, the speed of sound is determined in part by how stiff the material is. The stiffer the material, the faster the sound wave, just as having more tension in a string causes a faster wave. Thus the speed of sound in plastic is rather high (2680 m/s), and in steel it is greater still (5960 m/s). Both speeds are much higher than the speed in air, which is certainly a "squishy" material in comparison. Table 14–1 gives a sampling of sound speed in a range of different materials.

TABLE 14–1 Speed of Sound in Various Materials

Material	Speed (m/s)
Aluminum	6420
Granite	6000
Steel	5960
Pyrex glass	5640
Copper	5010
Plastic	2680
Fresh water (20 °C)	1482
Fresh water (0 °C)	1402
Hydrogen (0 °C)	1284
Helium (0 °C)	965
Air (20 °C)	343
Air (0 °C)	331

CONCEPTUAL CHECKPOINT 14–2 HOW FAR TO THE LIGHTNING?

Five seconds after a brilliant flash of lightning, thunder shakes the house. Was the lightning **(a)** about a mile away, **(b)** much closer than a mile, or **(c)** much farther away than a mile?



REASONING AND DISCUSSION

As mentioned, the speed of sound is 343 m/s, which is just over 1000 ft/s. Thus, in five seconds sound travels slightly more than one mile. This gives rise to the following popular rule of thumb: The distance to a lightning strike (in miles) is the time for the thunder to arrive (in seconds) divided by 5.

Notice that we have neglected the travel time of light in our discussion. This is because light propagates with such a high speed (approximately 186,000 mi/s) that its travel time is about a million times less than that of sound.

ANSWER

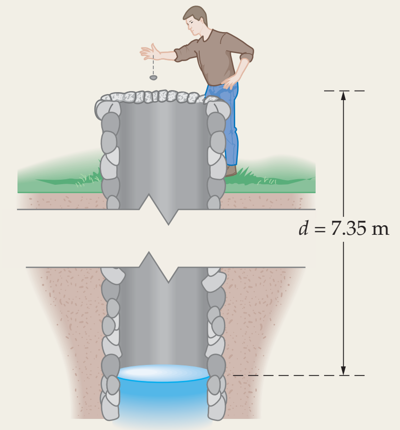
(a) The lightning was about a mile away.

EXAMPLE 14–2 WISHING WELL

You drop a stone from rest into a well that is 7.35 m deep. How long does it take before you hear the splash?

PICTURE THE PROBLEM

Our sketch shows the well into which the stone is dropped. Notice that the depth of the well is $d = 7.35$ m. After the stone falls a distance d, the sound from the splash rises the same distance d before it is heard.



STRATEGY

The time until the splash is heard is the sum of (i) the time, t_1, for the stone to drop a distance d, and (ii) the time, t_2, for sound to travel a distance d.

For the time of drop, we use one-dimensional kinematics with an initial velocity $v = 0$, since the stone is dropped from rest, and an acceleration g. Therefore, the relationship between distance and time for the stone is $d = \frac{1}{2}gt_1^2$, with $g = 9.81 \text{ m/s}^2$.

For the sound wave, we use $d = vt_2$, with $v = 343$ m/s.

SOLUTION

1. Calculate the time for the stone to drop:

$$d = \frac{1}{2}gt_1^2$$

$$t_1 = \sqrt{\frac{2d}{g}} = \sqrt{\frac{2(7.35 \text{ m})}{9.81 \text{ m/s}^2}} = 1.22 \text{ s}$$

2. Calculate the time for sound to travel a distance d:

$$d = vt_2$$

$$t_2 = \frac{d}{v} = \frac{7.35 \text{ m}}{343 \text{ m/s}} = 0.0214 \text{ s}$$

3. Sum the times found above:

$$t = t_1 + t_2 = 1.22 \text{ s} + 0.0214 \text{ s} = 1.24 \text{ s}$$

INSIGHT

Note that the time of travel for the sound is quite small, only a couple hundredths of a second. It is still nonzero, however, and must be taken into account to obtain the correct total time.

In addition, notice that we use the same speed for a sound wave whether it is traveling horizontally, vertically upward, or vertically downward—its speed is independent of its direction of motion. As a result, the waves emanating from a source of sound propagate outward in a spherical pattern, with the wave crests forming concentric spheres around the source.

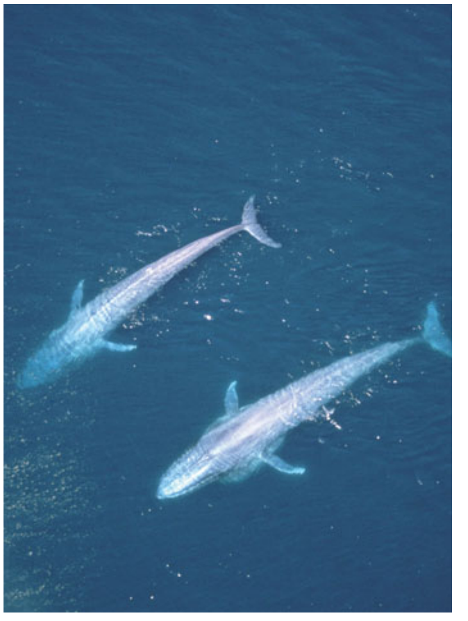
PRACTICE PROBLEM

You drop a stone into a well and hear the splash 1.47 s later. How deep is the well? [**Answer:** 10.2 m]

Some related homework problems: Problem 30, Problem 31

The Frequency of a Sound Wave

When we hear a sound, its frequency makes a great impression on us; in fact, the frequency determines the **pitch** of a sound. For example, the keys on a piano produce sound with frequencies ranging from 55 Hz for the key farthest to the left to 4187 Hz for the rightmost key. Similarly, as you hum a song you change the shape and size of your vocal chords slightly to change the frequency of the sound you produce.



▶ Many animal species use sound waves with frequencies that are too high (ultrasonic) or too low (infrasonic) for human ears to detect. Bats, for example, navigate in the dark and locate their prey by means of a system of biological sonar. They emit a continuous stream of ultrasonic sounds and detect the echoes from objects around them. Blue whales, by contrast, communicate over long distances by means of infrasonic sounds.

The frequency range of human hearing extends well beyond the range of a piano, however. As a rule of thumb, humans can hear sounds between 20 Hz on the low-frequency end and 20,000 Hz on the high-frequency end. Sounds with frequencies above this range are referred to as **ultrasonic,** while those with frequencies lower than 20 Hz are classified as **infrasonic.** Though we are unable to hear ultrasound and infrasound, these frequencies occur commonly in nature, and are used in many technological applications as well.

REAL-WORLD PHYSICS

Ultrasonic sounds in nature

For example, bats and dolphins produce ultrasound almost continuously as they go about their daily lives. By listening to the echoes of their calls—that is, by using *echolocation*—they are able to navigate about their environment and detect their prey. As a defense mechanism, some of the insects that are preyed upon by bats have the ability to hear the ultrasonic frequency of a hunting bat and take evasive action. For instance, the praying mantis has a specialized ultrasound receptor on its abdomen that allows it to take cover in response to an approaching bat. More dramatically, certain moths fold their wings in flight and drop into a precipitous dive toward the ground when they hear a bat on the prowl.

REAL-WORLD PHYSICS: BIO

Medical applications of ultrasound: ultrasonic scans

Medical applications of ultrasound are also common. Perhaps the most familiar is the ultrasound scan that is used to image a fetus in the womb. By sending bursts of ultrasound into the body and measuring the time delay of the resulting echoes—the technological equivalent of echolocation—it is possible to map out the location of structures that lie hidden beneath the skin. In addition to imaging the interior of a body, ultrasound can also produce changes within the body that would otherwise require surgery. For example, in a technique called *shock wave lithotripsy* (SWL), an intense beam of ultrasound is concentrated onto a kidney stone that must be removed. After being hit with as many as 1000 to 3000 pulses of sound (at 23 joules per pulse), the stone is fractured into small pieces that the body can then eliminate on its own.

REAL-WORLD PHYSICS: BIO

Medical applications of ultrasound: shock wave lithotripsy

As for infrasound, it has been discovered in recent years that elephants can communicate with one another using sounds with frequencies as low as 15 Hz. In fact, it may be that *most* elephant communication is infrasonic. These sounds, which humans feel as vibration rather than hear as sound, can carry over an area of about thirty square kilometers on the dry African savanna. And elephants are not alone in this ability. Whales, such as the blue and the finback, produce powerful infrasonic calls as well. Since sound generally travels farther in water than in air, the whale calls can be heard by others of their species over distances of thousands of kilometers.

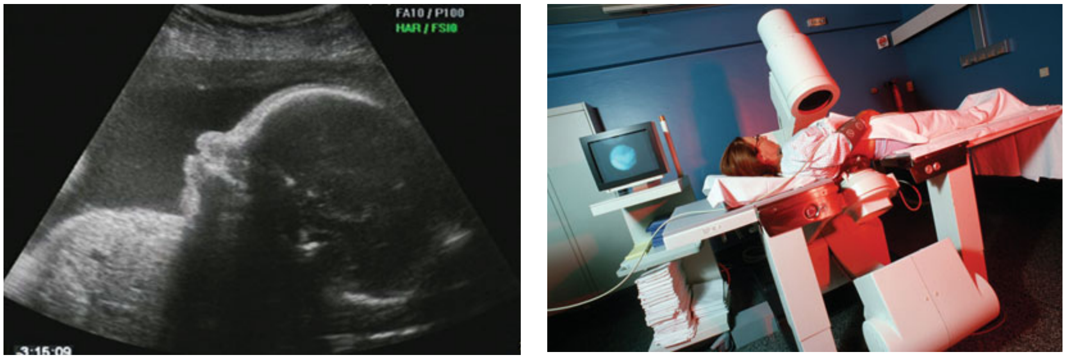
REAL-WORLD PHYSICS

Infrasonic communication among animals

One final example of infrasound is related to a dramatic event that occurred in southern New Mexico about a decade ago. At 12:47 in the afternoon of October 10, 1997, a meteor shining as bright as the full Moon streaked across the sky for a few brief moments. The event was observed not just visually, however, but with

REAL-WORLD PHYSICS

Infrasound produced by meteors



▲ Ultrasound is used in medicine both as an imaging medium and for therapeutic purposes. Ultrasound scans, or sonograms, are created by beaming ultrasonic pulses into the body and measuring the time required for the echoes to return. This technique is commonly used to evaluate heart function (echocardiograms) and to visualize the fetus in the uterus, as shown above (left). In shock wave lithotripsy (right), pulses of high-frequency sound waves are used to shatter kidney stones into fragments that can be passed in the urine.

infrasound as well. An array of special microphones at the Los Alamos National Laboratory—originally designed to listen for clandestine nuclear weapons tests—heard the infrasonic boom created by the meteor. By tracking the sonic signals of such meteors it may be possible to recover fragments that manage to reach the ground. The Los Alamos detector is in constant operation, and it detects about ten rather large objects (2 m or more in diameter) entering the Earth's atmosphere each year.

It should be noted, in light of the wide range of frequencies observed in sound, that the speed of sound is the same for all frequencies. Thus, in the relation

$$v = \lambda f$$

the speed v remains fixed. For example, if the frequency of a wave is doubled, its wavelength is halved, so that the speed v stays the same. The fact that different frequencies travel with the same speed is evident when we listen to an orchestra in a large room. Different instruments are producing sounds of different frequencies, but we hear the sounds at the same time. Otherwise, listening to music from a distance would be quite a different and inharmonious experience.

14–5 Sound Intensity

The noise made by a jackhammer is much louder than the song of a sparrow. On this we can all agree. But how do we express such an observation physically? What physical quantity determines the loudness of a sound? We address these questions in this section, and we also present a quantitative scale by which loudness may be measured.

Intensity

The loudness of a sound is determined by its **intensity;** that is, by the amount of energy that passes through a given area in a given time. This is illustrated in Figure 14–12. If the energy E passes through the area A in the time t, the intensity, I, of the wave carrying the energy is

$$I = \frac{E}{At}$$

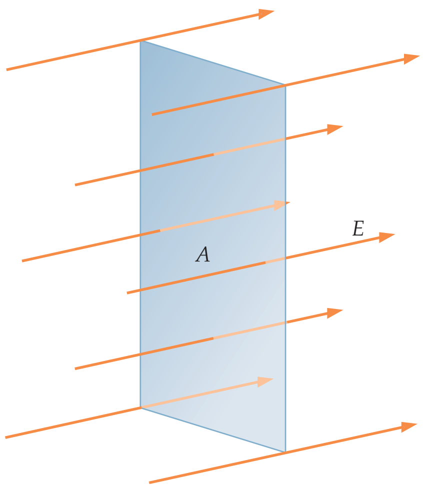
Recalling that power is energy per time, $P = E/t$, we can express the intensity as follows:

Definition of Intensity, *I*

$$I = \frac{P}{A} \qquad 14\text{–}5$$

SI unit: W/m^2

The units are those of power (watts, W) divided by area (meters squared, m^2).



▲ **FIGURE 14–12 Intensity of a wave**
If a wave carries an energy E through an area A in the time t, the corresponding intensity is $I = E/At = P/A$, where $P = E/t$ is the power.

TABLE 14–2 Sound Intensities (W/m^2)

Loudest sound produced in laboratory	10^9
Saturn V rocket at 50 m	10^8
Rupture of the eardrum	10^4
Jet engine at 50 m	10
Threshold of pain	1
Rock concert	10^{-1}
Jackhammer at 1 m	10^{-3}
Heavy street traffic	10^{-5}
Conversation at 1 m	10^{-6}
Classroom	10^{-7}
Whisper at 1 m	10^{-10}
Normal breathing	10^{-11}
Threshold of hearing	10^{-12}

Though we have introduced the concept of intensity in terms of sound, it applies to all types of waves. For example, the intensity of light from the Sun as it reaches the Earth's upper atmosphere is about 1380 W/m^2. If this intensity could be heard as sound, it would be painfully loud—roughly the equivalent of four jet airplanes taking off simultaneously. By comparison, the intensity of microwaves in a microwave oven is even greater, about 6000 W/m^2, whereas the intensity of a whisper is an incredibly tiny 10^{-10} W/m^2. A selection of representative intensities is given in Table 14–2.

EXERCISE 14–3

A loudspeaker puts out 0.15 W of sound through a square area 2.0 m on each side. What is the intensity of this sound?

SOLUTION

Applying Equation 14–5, with $A = (2.0\text{ m})^2$, we find

$$I = \frac{P}{A} = \frac{0.15\text{ W}}{(2.0\text{ m})^2} = 0.038\text{ W/m}^2$$

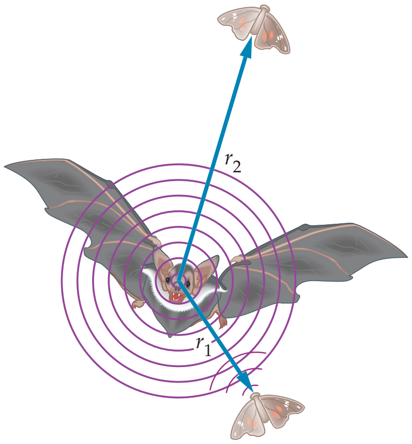


FIGURE 14–13 Echolocation

Two moths, at distances r_1 and r_2, hear the sonar signals sent out by a bat. The intensity of the signal decreases with the square of the distance from the bat. The bat, in turn, hears the echoes sent back by the moths. It can then use the direction and intensity of the returning echoes to locate its prey.

When we listen to a source of sound, such as a person speaking or a radio playing a song, we notice that the loudness of the sound decreases as we move away from the source. This means that the intensity of the sound is also decreasing. The reason for this reduction in intensity is simply that the energy emitted per time by the source spreads out over a larger area—just as spreading a certain amount of jam over a larger piece of bread reduces the intensity of the taste.

In **Figure 14–13** we show a source of sound (a bat) and two observers (moths) listening at the distances r_1 and r_2. Notice that the waves emanating from the bat propagate outward spherically, with the wave crests forming a series of concentric spheres. Assuming no reflections of sound, and a power output by the bat equal to P, the intensity detected by the first moth is

$$I_1 = \frac{P}{4\pi r_1^2}$$

In writing this expression, we have used the fact that the area of a sphere of radius r is $A = 4\pi r^2$. Similarly, the second moth hears the same sound with an intensity of

$$I_2 = \frac{P}{4\pi r_2^2}$$

The power P is the same in each case—it simply represents the amount of sound emitted by the bat. Solving for the intensity at moth 2 in terms of the intensity at moth 1 we find

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 \qquad \text{14–6}$$

In words, the intensity falls off with the square of the distance; doubling the distance reduces the intensity by a factor of 4.

To summarize, the intensity a distance r from a point source of power P is

Intensity with Distance from a Point Source

$$I = \frac{P}{4\pi r^2} \qquad \text{14–7}$$

SI unit: W/m^2

This result assumes that no sound is reflected—which could increase the amount of energy passing through a given area—that no sound is absorbed, and that the sound propagates outward spherically. These assumptions are applied in the next Example.

PROBLEM-SOLVING NOTE

Intensity Variation with Distance

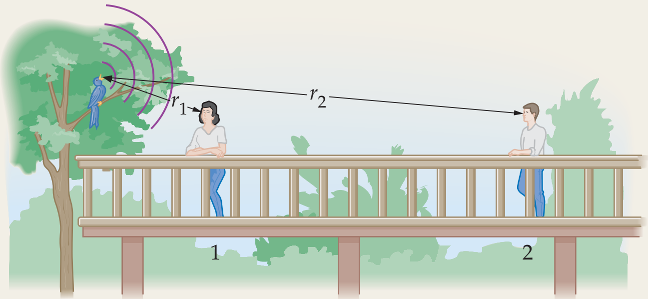
Suppose the intensity of a point source is I_1 at a distance r_1. This is enough information to find its intensity at any other distance. For example, to find the intensity I_2 at a distance r_2 we use the relation $I_2 = (r_1/r_2)^2 I_1$.

EXAMPLE 14–3 THE POWER OF SONG

Two people relaxing on a deck listen to a songbird sing. One person, only 1.00 m from the bird, hears the sound with an intensity of $2.80 \times 10^{-6}\ \mathrm{W/m^2}$. **(a)** What intensity is heard by the second person, who is 4.25 m from the bird? Assume that no reflected sound is heard by either person. **(b)** What is the power output of the bird's song?

PICTURE THE PROBLEM

Our sketch shows the two observers, one at a distance of $r_1 = 1.00$ m from the bird, the other at a distance of $r_2 = 4.25$ m. The sound emitted by the bird is assumed to spread out spherically, with no reflections.



STRATEGY

a. The two intensities are related by Equation 14–6, with $r_1 = 1.00$ m and $r_2 = 4.25$ m.

b. The power output can be obtained from the definition of intensity, $I = P/A$. We can calculate P for either observer, noting that $A = 4\pi r^2$.

SOLUTION

Part (a)

1. Substitute numerical values into Equation 14–6:

$$I_2 = \left(\frac{r_1}{r_2}\right)^2 I_1 = \left(\frac{1.00\ \mathrm{m}}{4.25\ \mathrm{m}}\right)^2 (2.80 \times 10^{-6}\ \mathrm{W/m^2})$$
$$= 1.55 \times 10^{-7}\ \mathrm{W/m^2}$$

Part (b)

2. Solve $I = P/A$ for the power, P, using data for observer 1:

$$I_1 = P/A_1$$
$$P = I_1 A_1 = (2.80 \times 10^{-6}\ \mathrm{W/m^2})[4\pi(1.00\ \mathrm{m})^2]$$
$$= 3.52 \times 10^{-5}\ \mathrm{W}$$

3. As a check, repeat the calculation for observer 2:

$$I_2 = P/A_2$$
$$P = I_2 A_2 = (1.55 \times 10^{-7}\ \mathrm{W/m^2})[4\pi(4.25\ \mathrm{m})^2]$$
$$= 3.52 \times 10^{-5}\ \mathrm{W}$$

INSIGHT

The intensity at observer 1 is $4.25^2 = 18.1$ times the intensity at observer 2. Even so, the bird only *seems* to be about 2.5 times louder to observer 1. The connection between intensity and perceived (subjective) loudness is discussed in detail later in this section.

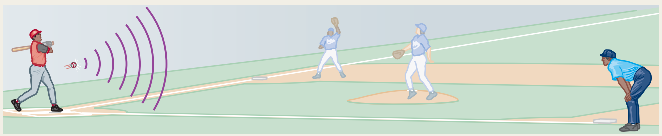
PRACTICE PROBLEM

If the intensity at observer 2 were $7.40 \times 10^{-7}\ \mathrm{W/m^2}$, how far would he be from the bird? [**Answer:** $r_2 = 1.95$ m]

Some related homework problems: Problem 36, Problem 41

ACTIVE EXAMPLE 14–1 THE BIG HIT: FIND THE INTENSITY

Ken Griffey, Jr., connects with a fast ball and sends it out of the park. A fan in the outfield bleachers, 140 m away, hears the hit with an intensity of $3.80 \times 10^{-7}\ \mathrm{W/m^2}$. Assuming no reflected sounds, what is the intensity heard by the first-base umpire, 90 ft (27.4 m) away from home plate?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Label the data given in the problem. Let the umpire be observer 1 and the fan be observer 2: $r_1 = 27.4$ m; $r_2 = 140$ m; $I_2 = 3.80 \times 10^{-7}\ \mathrm{W/m^2}$
2. Solve Equation 14–6 for I_1: $I_1 = (r_2/r_1)^2 I_2$
3. Substitute numerical values: $I_1 = 9.9 \times 10^{-6}\ \mathrm{W/m^2}$

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INSIGHT

For the fan, the sound from the hit is somewhat less intense than normal conversation. For the umpire it is comparable to the sound of a busy street.

YOUR TURN

Find the distance at which the sound of the hit has the intensity of a whisper. Refer to Table 14–2 for the necessary information.

(Answers to **Your Turn** *problems are given in the back of the book.)*

REAL-WORLD PHYSICS: BIO

Human perception of sound intensity

Human Perception of Sound

Hearing, like most of our senses, is incredibly versatile and sensitive. We can detect sounds that are about a million times fainter than a typical conversation, and listen to sounds that are a million times louder before experiencing pain. In addition, we are able to hear sounds over a wide range of frequencies, from 20 Hz to 20,000 Hz.

When detecting the faintest of sounds, our hearing is more sensitive than one would ever guess. For example, a faint sound, with an intensity of about 10^{-11} W/m^2, causes a displacement of molecules in the air of about 10^{-10} m. This displacement is roughly the diameter of an atom!

Equally interesting is the way we perceive the loudness of a sound. As an example, suppose you hear a sound of intensity I_1. Next, you listen to a second sound of intensity I_2, and this sound seems to be "twice as loud" as the first. If the two intensities are measured, it turns out that I_2 is about 10 times I_1. Similarly, a third sound, twice as loud as I_2, has an intensity I_3 that is 10 times greater than I_2. Thus, $I_2 = 10I_1$ and $I_3 = 10I_2 = 100I_1$.

Our perception of sound, then, is such that uniform increases in loudness correspond to intensities that increase by multiplicative factors. For this reason, a convenient scale to measure loudness depends on the logarithm of intensity, as we discuss next.

PROBLEM-SOLVING NOTE

Intensity Versus Intensity Level

When reading a problem statement, be sure to note carefully whether it refers to the intensity, I, or to the intensity level, β. These two quantities have similar names but completely different meanings and units, as indicated in the following table:

Physical quantity	Physical meaning	Units
Intensity, I	Energy per time per area	W/m^2
Intensity level, β	A measure of relative loudness	dB

Intensity Level and Decibels

In the study of sound, loudness is measured by the **intensity level** of a wave. Designated by the symbol β, the intensity level is defined as follows:

Definition of Intensity Level, β

$$\beta = (10\text{ dB})\log(I/I_0) \qquad 14\text{–}8$$

SI unit: decibel (dB), which is dimensionless

In this expression, log indicates the logarithm to the base 10, and I_0 is the intensity of the faintest sounds that can be heard. Experiments show this lowest detectable intensity to be

$$I_0 = 10^{-12}\text{ W/m}^2$$

Note that β is dimensionless; the only dimensions that enter into the definition are those of intensity, and they cancel in the logarithm. Still, just as with radians, it is convenient to label the values of intensity level with a name. The name we use—the bel—honors the work of Alexander Graham Bell (1847–1922), the inventor of the telephone. Since the bel is a fairly large unit, it is more common to measure β in units that are one-tenth of a bel. This unit is referred to as the **decibel,** and its abbreviation is dB.

To get a feeling for the decibel scale, let's start with the faintest sounds. If a sound has an intensity $I = I_0$, the corresponding intensity level is

$$\beta = (10\text{ dB})\log(I_0/I_0) = 10\log(1) = 0$$

Increasing the intensity by a factor of 10 makes the sound seem twice as loud. In terms of decibels, we have

$$\beta = (10 \text{ dB}) \log(10I_0/I_0) = (10 \text{ dB}) \log(10) = 10 \text{ dB}$$

Going up in intensity by another factor of 10 doubles the loudness of the sound again, and yields

$$\beta = (10 \text{ dB}) \log(100I_0/I_0) = (10 \text{ dB}) \log(100) = 20 \text{ dB}$$

Thus, *the loudness of a sound doubles with each increase in intensity level of 10 dB*. The *smallest* increase in intensity level that can be detected by the human ear is about 1 dB.

The intensity of a number of independent sound sources is simply the sum of the individual intensities. We use this fact in the following Example.

PROBLEM-SOLVING NOTE

Calculating the Intensity Level

When determining the intensity level β, be sure to use the base 10 logarithm (log), as opposed to the "natural," or base e, logarithm (ln).

EXAMPLE 14–4 PASS THE PACIFIER

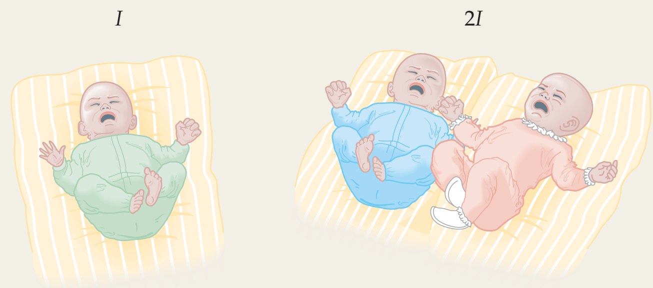
A crying child emits sound with an intensity of 8.0×10^{-6} W/m^2. Find **(a)** the intensity level in decibels for the child's sounds, and **(b)** the intensity level for this child and its twin, both crying with identical intensities.

PICTURE THE PROBLEM

We consider the crying sounds of either one or two children. Each child emits sound with an intensity $I = 8.0 \times 10^{-6}$ W/m^2. If two children are crying together, the total intensity of their sound is $2I$.

STRATEGY

The intensity level, β, is obtained by applying Equation 14–8.



(a) (b)

SOLUTION

Part (a)

1. Calculate β for $I = 8.0 \times 10^{-6}$ W/m^2:

$$\beta = (10 \text{ dB}) \log(I/I_0)$$
$$= (10 \text{ dB}) \log\left(\frac{8.0 \times 10^{-6} \text{ W/m}^2}{10^{-12} \text{ W/m}^2}\right) = (10 \text{ dB}) \log(8.0 \times 10^6)$$
$$= (10 \text{ dB}) \log(8.0) + (10 \text{ dB}) \log(10^6) = 69 \text{ dB}$$

Part (b)

2. Repeat the calculation with I replaced by $2I$:

$$\beta = (10 \text{ dB}) \log(2I/I_0)$$
$$= (10 \text{ dB}) \log(2) + (10 \text{ dB}) \log(I/I_0)$$
$$= 3.0 \text{ dB} + 69 \text{ dB} = 72 \text{ dB}$$

INSIGHT

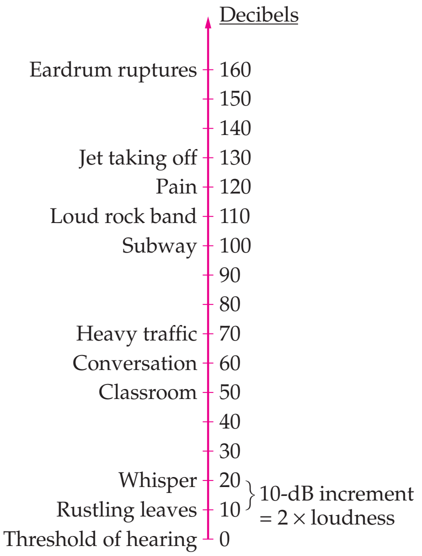
Note that the intensity level is increased by $(10 \text{ dB}) \log(2) = 3$ dB. This is a general rule: When the intensity is doubled, the intensity level, β, increases by 3 dB. Similarly, when the intensity is halved, β decreases by 3 dB.

PRACTICE PROBLEM

What is the intensity level of four identically crying quadruplets? [**Answer:** $\beta = 75$ dB]

Some related homework problems: Problem 38, Problem 39

Even though a change of 3 dB is relatively small—after all, a change of 10 dB is required to make a sound seem twice as loud—it still requires changing the intensity by a factor of two. For example, suppose a large nursery in a hospital has so many crying babies that the intensity level is 6 dB above the safe value, as determined by OSHA (Occupational Safety and Health Administration). To reduce



▲ **FIGURE 14–14 Representative intensity levels for common sounds**

the level by 6 dB it would be necessary to remove three-quarters of the children, leaving only one-quarter the original number. To our ears, however, the nursery will *sound* only 40 percent quieter!

Figure 14–14 shows the decibel scale with representative values indicated for a variety of common sounds.

14–6 The Doppler Effect

One of the most common physical phenomena involving sound is the change in pitch of a train whistle or a car horn as the vehicle moves past us. This change in pitch, due to the relative motion between a source of sound and the receiver, is called the **Doppler effect,** after the Austrian physicist Christian Doppler (1803–1853). If you listen carefully to the Doppler effect, you will notice that the pitch increases when the observer and the source are moving closer together, and decreases when the observer and source are separating.

One of the more fascinating aspects of the Doppler effect is the fact that it applies to all wave phenomena, not just to sound. In particular, the frequency of light is also Doppler-shifted when there is relative motion between the source and receiver. For light, this change in frequency means a change in color. In fact, most distant galaxies are observed to be red-shifted in the color of their light, which means they are moving away from the Earth. Some galaxies, however, are moving toward us, and their light shows a blue shift.

In the remainder of this section, we focus on the Doppler effect in sound waves. We show that the effect is different depending on whether the observer or the source is moving. Finally, both observer and source may be in motion, and we present results for such cases as well.

Moving Observer

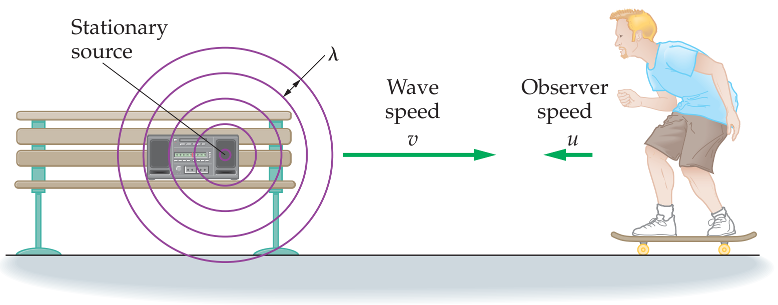
In **Figure 14–15** we see a stationary source of sound in still air. The radiated sound is represented by the circular patterns of compressions moving away from the source with a speed v. The distance between the compressions is the wavelength, λ, and the frequency of the sound is f. As for any wave, these quantities are related by

$$v = \lambda f$$

For an observer moving toward the source with a speed u, as in Figure 14–15, the sound *appears* to have a higher speed, $v + u$ (though, of course, the speed of sound relative to the air is always the same). As a result, more compressions move past the observer in a given time than if the observer had been at rest. To the observer, then, the sound has a frequency, f', that is higher than the frequency of the source, f.

We can find the frequency f' by first noting that the wavelength of the sound does not change—it is still λ. The speed, however, has increased to $v' = v + u$. Thus, we can solve $v' = \lambda f'$ for the frequency, which yields

$$f' = \frac{v'}{\lambda} = \frac{v + u}{\lambda}$$



▶ **FIGURE 14–15 The Doppler effect: a moving observer**

Sound waves from a stationary source form concentric circles moving outward with a speed v. To the observer, who moves toward the source with a speed u, the waves are moving with a speed $v + u$.

Finally, we recall from Equation 14–1 that $\lambda = v/f$, and hence

$$f' = \frac{v + u}{(v/f)} = \left(\frac{v + u}{v}\right)f = (1 + u/v)f$$

Note that f' is greater than f. This is the Doppler effect.

If the observer had been moving away from the source with a speed u, the sound would *appear* to the observer to have the reduced speed $v' = v - u$. Repeating the calculation just given, we find that

$$f' = \frac{v'}{\lambda} = \frac{v - u}{\lambda} = (1 - u/v)f$$

In this case the Doppler effect results in f' being less than f.

Combining these results, we have

Doppler Effect for Moving Observer

$$f' = (1 \pm u/v)f \qquad 14\text{–}9$$

SI unit: $1/\text{s} = \text{s}^{-1}$

In this expression u and v are speeds, and hence are always positive. The appropriate signs are obtained by using the *plus* sign when the observer moves *toward* the source, and the *minus* sign when the observer moves *away from* the source.

PROBLEM-SOLVING NOTE

Using the Correct Sign for the Doppler Effect

When an observer approaches a source, the frequency heard by the observer is greater than the frequency of the source; that is, $f' > f$. This means that we must use the plus sign in $f' = (1 \pm u/v)f$. Similarly, we must use the minus sign when the observer moves away from the source.

EXAMPLE 14–5 A MOVING OBSERVER

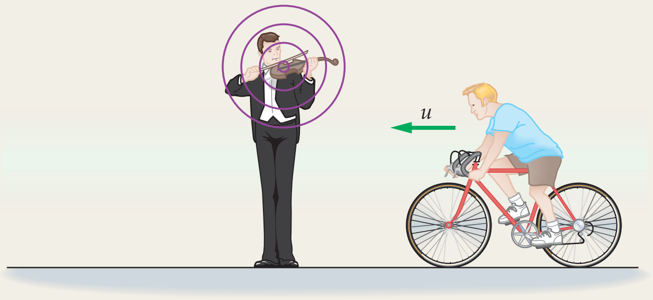
A street musician sounds the A string of his violin, producing a tone of 440 Hz. What frequency does a bicyclist hear as he **(a)** approaches and **(b)** recedes from the musician with a speed of 11.0 m/s?

PICTURE THE PROBLEM

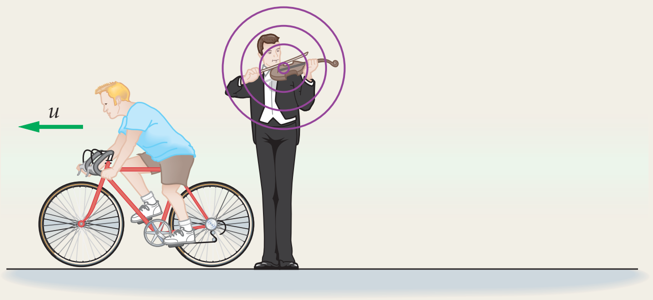
The sketch shows a stationary source of sound and a moving observer. In part (a) the observer approaches the source with a speed $u = 11.0$ m/s; in part (b) the observer has passed the source and is moving away with the same speed.

STRATEGY

The frequency heard by the observer is given by Equation 14–9, with the plus sign for part (a) and the minus sign for part (b).



(a)



(b)

SOLUTION

Part (a)

1. Apply Equation 14–9 with the plus sign and $u = 11.0$ m/s: $f' = (1 + u/v)f = \left(1 + \frac{11.0\text{ m/s}}{343\text{ m/s}}\right)(440\text{ Hz}) = 454\text{ Hz}$

Part (b)

2. Now use the minus sign in Equation 14–9: $f' = (1 - u/v)f = \left(1 - \frac{11.0\text{ m/s}}{343\text{ m/s}}\right)(440\text{ Hz}) = 426\text{ Hz}$

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INSIGHT

As the bicyclist passes the musician, the observed frequency of sound decreases, giving a "wow" effect. The difference in frequency is about 1 semitone, the frequency difference between adjacent notes on the piano. See Table 14–3 on p. 481 for semitones in the vicinity of middle C.

PRACTICE PROBLEM

If the bicyclist hears a frequency of 451 Hz when approaching the musician, what is his speed? [**Answer:** $u = 8.58\ \mathrm{m/s}$]

Some related homework problems: Problem 47, Problem 50

Moving Source

With a stationary observer and a moving source, the Doppler effect is not due to the sound wave appearing to have a higher or lower speed, as when the observer moves. On the contrary, the speed of a wave is determined by the properties of the medium through which it propagates. Thus, once the source emits a sound wave, it travels through the medium with its characteristic speed v regardless of what the source is doing.

By way of analogy, consider a water wave. The speed of such waves is the same whether they are created by a rock dropped into the water or by a stick moved rapidly through the water. To take an extreme case, the waves coming to the beach from a slow-moving tugboat move with the same speed as the waves produced by a 100-mph speed boat. The same is true of sound waves.

Consider, then, a source moving toward an observer with a speed u, as shown in **Figure 14–16**. If the frequency of the source is f, it emits one compression every T seconds, where $T = 1/f$. Therefore, during one cycle of the wave a compression travels a distance vT while the source moves a distance uT. As a result, the next compression is emitted a distance $vT - uT$ behind the previous compression, as illustrated in **Figure 14–17**. This means that the wavelength in the forward direction is

$$\lambda' = vT - uT = (v - u)T$$

Now, the speed of the wave is still v, as mentioned, hence

$$v = \lambda' f'$$

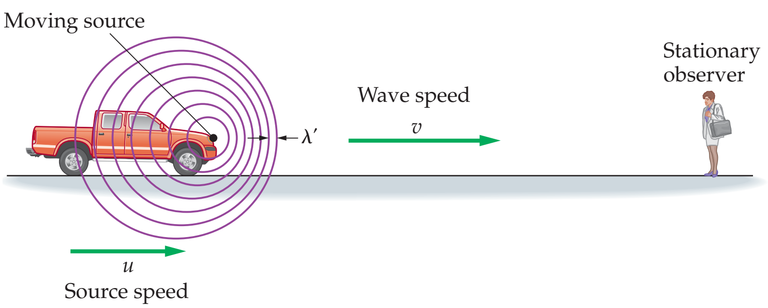


FIGURE 14–16 The Doppler effect: a moving source
Sound waves from a moving source are bunched up in the forward direction, causing a shorter wavelength and a higher frequency.

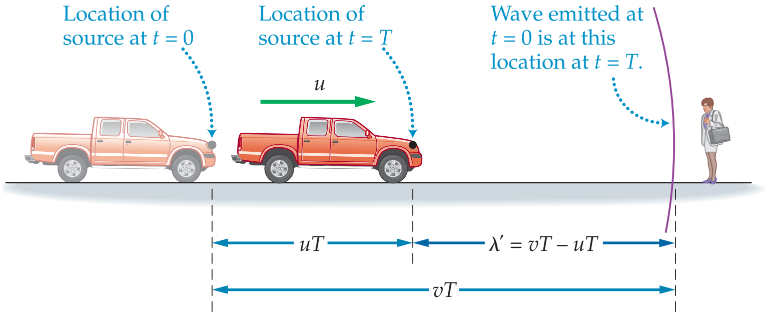


FIGURE 14–17 The Doppler-shifted wavelength
During one period, T, the wave emitted at $t = 0$ moves through a distance vT. In the same time, the source moves toward the observer through the distance uT. At the time $t = T$ the next wave is emitted from the source; hence, the distance between the waves (the wavelength) is $\lambda' = vT - uT$.

Solving for the new frequency, f', we find

$$f' = \frac{v}{\lambda'} = \frac{v}{(v - u)T}$$

Finally, recalling that $T = 1/f$, we have

$$f' = \frac{v}{(v - u)(1/f)} = \frac{v}{v - u}f = \left(\frac{1}{1 - u/v}\right)f$$

Note that f' is greater than f, as expected.

In the reverse direction, the wavelength is increased by the amount uT. Thus,

$$\lambda' = vT + uT = (v + u)T$$

It follows that the Doppler-shifted frequency is

$$f' = \frac{v}{(v + u)T} = \left(\frac{1}{1 + u/v}\right)f$$

This is less than the source frequency, f.

Finally, we can combine these results to yield

Doppler Effect for Moving Source

$$f' = \left(\frac{1}{1 \mp u/v}\right)f \qquad 14\text{–}10$$

SI unit: $1/s = s^{-1}$

As before, u and v are positive quantities. The *minus* sign is used when the source moves *toward* the observer, and the *plus* sign when the source moves *away from* the observer.

PROBLEM-SOLVING NOTE

Using Correct Signs

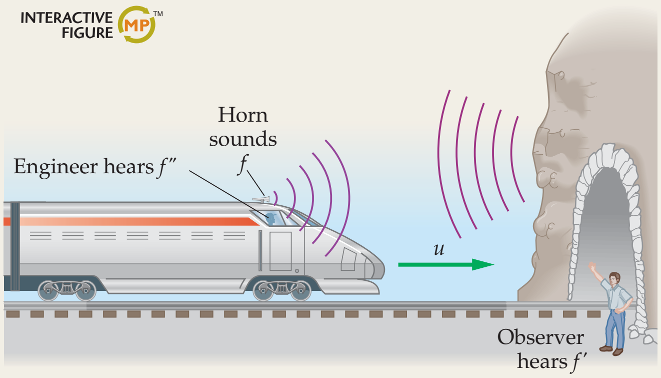
When a source approaches an observer, the frequency heard by the observer is greater than the frequency of the source; that is, $f' > f$. This means that we must use the minus sign in Equation 14–10, $f' = f/(1 - u/v)$, since this makes the denominator less than one. Similarly, use the plus sign when the source moves away from the observer.

EXAMPLE 14–6 WHISTLE STOP

A train sounds its whistle as it approaches a tunnel in a cliff. The whistle produces a tone of 650.0 Hz, and the train travels with a speed of 21.2 m/s. **(a)** Find the frequency heard by an observer standing near the tunnel entrance. **(b)** The sound from the whistle reflects from the cliff back to the engineer in the train. What frequency does the engineer hear?

PICTURE THE PROBLEM

The train moves with a speed $u = 21.2$ m/s and emits sound with a frequency $f = 650.0$ Hz. The observer near the tunnel hears the Doppler-shifted frequency f', and the engineer on the train hears the reflected sound at an even higher frequency f''.



STRATEGY

Two Doppler shifts are involved in this problem. The first is due to the motion of the train toward the cliff. This shift causes the observer at the cliff to hear sound with a higher frequency f', given by Equation 14–10 with the minus sign. The reflected sound has the same frequency, f'.

The second shift is due to the engineer moving toward the reflected sound. Thus, the engineer hears a frequency f'' that is greater than f'. We find f'' using Equation 14–9 with the plus sign.

SOLUTION

Part (a)

1. Use Equation 14–10, with the minus sign, to Doppler shift from f to f'.

$$f' = \left(\frac{1}{1 - u/v}\right)f = \left(\frac{1}{1 - \frac{21.2\text{ m/s}}{343\text{ m/s}}}\right)(650.0\text{ Hz})$$

$$= \left(\frac{1}{1 - 0.0618}\right)(650.0\text{ Hz}) = 693\text{ Hz}$$

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Part (b)

2. Now use Equation 14–9, with the plus sign, to Doppler shift from f' to f''.

$$f'' = (1 + u/v)f' = \left(1 + \frac{21.2\text{ m/s}}{343\text{ m/s}}\right)(693\text{ Hz})$$
$$= (1 + 0.0618)(693\text{ Hz}) = 736\text{ Hz}$$

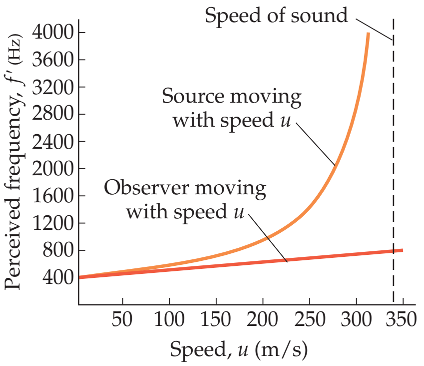
INSIGHT

Note that the reflected sound has the same frequency f' heard by the stationary observer, since all the cliff does is reverse the direction of motion of the sound heard at the cliff. Therefore, the cliff acts as a stationary source of sound at the frequency f'. The engineer's motion toward this stationary source results in the Doppler shift from f' to f''.

PRACTICE PROBLEM

If the stationary observer hears a frequency of 700.0 Hz, what are **(a)** the speed of the train and **(b)** the frequency heard by the engineer? [**Answer:** (a) $u = 24.5$ m/s, (b) $f'' = 750$ Hz]

Some related homework problems: Problem 45, Problem 103



▲ **FIGURE 14–18 Doppler-shifted frequency versus speed for a 400-Hz sound source**

The upper curve corresponds to a moving source, the lower curve to a moving observer. Notice that while the two cases give similar results for low speed, the high-speed behavior is quite different. In fact, the Doppler frequency for the moving source grows without limit for speeds near the speed of sound, while the Doppler frequency for the moving observer is relatively small. If a source moves faster than the speed of sound, the sound it produces is perceived not as a pure tone, but as a shock wave, commonly referred to as a sonic boom.

Now that we have obtained the Doppler effect for both moving observers and moving sources, it is interesting to compare the results. **Figure 14–18** shows the Doppler-shifted frequency versus speed for a 400-Hz source of sound. The upper curve corresponds to a moving source, the lower curve to a moving observer. Notice that while the two cases give similar results for low speed, the high-speed behavior is quite different. In fact, the Doppler frequency for the moving source grows without limit for speeds near the speed of sound, while the Doppler frequency for the moving observer is relatively small.

These results can be understood both in terms of mathematics—by simply comparing Equations 14–9 and 14–10—and physically. In physical terms, recall that a moving observer encounters wave crests separated by the wavelength, as indicated in Figure 14–15. Doubling the speed of the observer simply reduces the time required to move from one crest to the next by a factor of 2, which doubles the frequency. Thus, in general, the frequency is proportional to the speed, as we see in the lower curve in Figure 14–18. In contrast, when the source moves, as in Figure 14–16, the wave crests become "bunched up" in the forward direction, since the source is almost keeping up with the propagating waves. As the speed of the source approaches the speed of sound, the separation between crests approaches zero. Consequently, the frequency with which the crests pass a stationary observer approaches infinity, as indicated by the upper curve in Figure 14–18.

General Case

The results derived earlier in this section can be combined to give the Doppler effect for situations in which both observer and source move. Letting u_s be the speed of the source, and u_o be the speed of the observer, we have

Doppler Effect for Moving Source and Observer

$$f' = \left(\frac{1 \pm u_o/v}{1 \mp u_s/v}\right)f \qquad \text{14–11}$$

SI unit: $1/\text{s} = \text{s}^{-1}$

As in the previous cases, u_s, u_o, and v are positive quantities. In the numerator, the plus sign corresponds to the case in which the observer moves in the direction of the source, whereas the minus sign indicates motion in the opposite direction. In the denominator, the minus sign corresponds to the case in which the source moves in the direction of the observer, whereas the plus sign indicates motion in the opposite direction.

EXERCISE 14–4

A car moving at 18 m/s sounds its 550-Hz horn. A bicyclist, traveling with a speed of 7.2 m/s, moves toward the approaching car. What frequency is heard by the bicyclist?

SOLUTION

We use Equation 14–11 with $u_s = 18\text{ m/s}$ and $u_o = 7.2\text{ m/s}$. Since the source and observer are approaching, we use the plus sign in the numerator and the minus sign in the denominator:

$$f' = \left(\frac{1 + u_o/v}{1 - u_s/v}\right)f = \left(\frac{1 + 7.2/343}{1 - 18/343}\right)(550\text{ Hz}) = 590\text{ Hz}$$

REAL-WORLD PHYSICS

Radar guns

The Doppler effect is used in an amazing variety of technological applications. Perhaps one of the most familiar of these is the "radar gun" which is used to measure the speed of a pitched baseball or a car breaking the speed limit. Though the radar gun uses radio waves rather than sound waves, the basic physical principle is the same—by measuring the Doppler-shifted frequency of waves reflected from an object, it is possible to determine its speed. Doppler radar, used in weather forecasting, applies this same technology to tracking the motion of precipitation caused by storm clouds.

REAL-WORLD PHYSICS: BIO

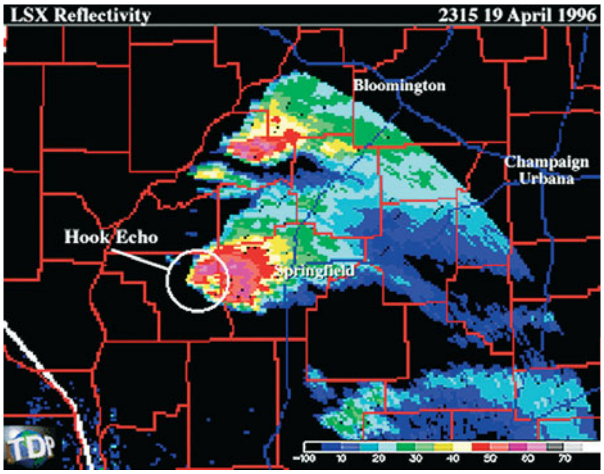
Measuring the speed of blood flow

In medicine, the Doppler shift is used to measure the speed of blood flow in an artery or in the heart itself. In this application, a beam of ultrasound is directed toward an artery in a patient. Some of the sound is reflected back by red blood cells moving through the artery. The reflected sound is detected, and its frequency is used to determine the speed of blood flow. If this information is color coded, with different colors indicating different speeds and directions of flow, an impressive image of blood flow can be constructed.

REAL-WORLD PHYSICS

Red shift of distant galaxies

Finally, the Doppler effect applies to the light of distant galaxies as well. For example, if a galaxy moves away from us—as most do—the light we observe from that galaxy has a lower frequency than if the galaxy were at rest relative to our galaxy, the Milky Way. Since red light has the lowest frequency of visible light, we refer to this reduction in frequency as a "red shift." Thus, by measuring the red shift of a galaxy, we can determine its speed relative to us. Such measurements show that the more distant a galaxy, the greater its speed relative to us—a result known as Hubble's law. This is just what one would expect from an explosion, or



▲ Many familiar and not-so-familiar devices utilize the Doppler effect. Doppler radar, now widely used at airports and for weather forecasting, makes it possible to determine the speed and direction of winds in a distant storm by measuring the Doppler shift they produce—winds shift the radar frequency upward if they blow toward the source and downward if they blow away from the source. The image at left is the Doppler radar scan of a severe thunderstorm that struck the town of Ogden, Illinois, on April 19, 1996. Reddish colors indicate winds blowing toward the radar station, bluish colors indicate winds blowing away. The hook-shaped echo marked on the image is characteristic of tornadoes in the making. In the photo at right, a medical technician uses a Doppler blood flow meter instead of a stethoscope while measuring the blood pressure of a patient.

"Big Bang," in which rapidly moving pieces travel farther in a given amount of time. Thus the Doppler effect, and red-shift measurements, provide strong evidence for the Big Bang and an expanding universe.

14–7 Superposition and Interference

So far we have considered only a single wave at a time. In this section we turn our attention to the way waves combine when more than one is present. As we shall see, the behavior of waves is quite simple in this respect.

Superposition

The combination of two or more waves to form a resultant wave is referred to as **superposition.** When waves are of small amplitude, they superpose in the simplest of ways—they just add. For example, consider two waves on a string, as in **Figure 14–19**, described by the wave functions y_1 and y_2. If these two waves are present on the same string at the same time, the result is a wave given by

$$y = y_1 + y_2$$

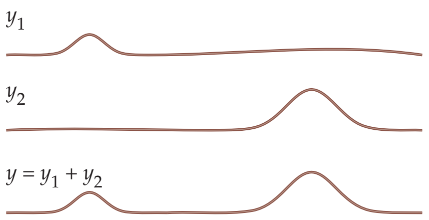
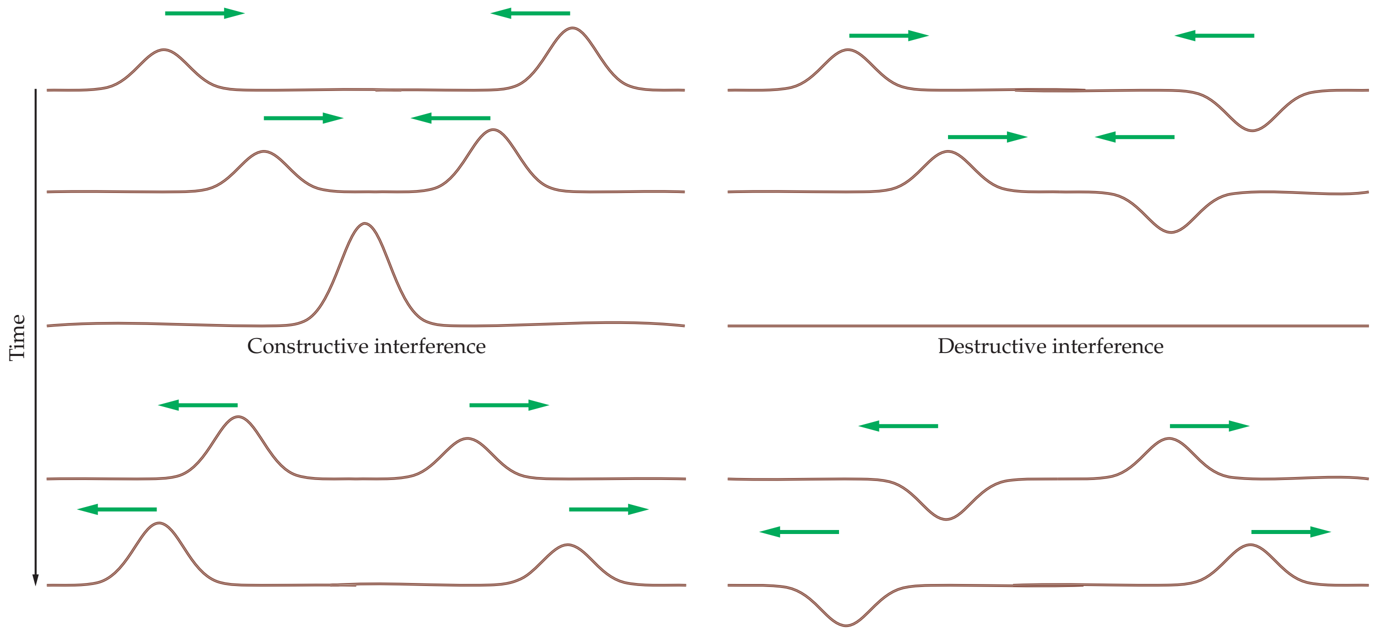


FIGURE 14–19 Wave superposition
Waves of small amplitude superpose (that is, combine) by simple addition.

To see how superposition works as a function of time, consider a string with two wave pulses on it, one traveling in each direction as shown in **Figure 14–20 (a)**. When the pulses arrive in the same region, they add, as stated. This is illustrated in Figure 14–20 (a). The question is, "What do the pulses look like after they have passed through one another? Does their interaction change them in any way?"

The answer is that the waves are completely unaffected by their interaction. This is also shown in Figure 14–20 (a). After the wave pulses pass through one another they continue on as if nothing had happened. It is like listening to an orchestra, where many different instruments are playing simultaneously, and their sounds are combining throughout the concert hall. Even so, you can still hear individual instruments, each making its own sound as if the others were not present.



(a) Two waves combine constructively **(b)** Two waves combine destructively

FIGURE 14–20 Interference
Wave pulses superpose as they pass through one another. Afterward, the pulses continue on unchanged. In **(a),** the pulses combine to give a larger amplitude. This is an example of *constructive interference*. When a positive pulse superposes with a negative pulse **(b),** the result is *destructive interference*. In this case, with symmetrical pulses, there is one moment of complete cancellation.

CONCEPTUAL CHECKPOINT 14–3 AMPLITUDE OF A RESULTANT WAVE

Since waves add, does the resultant wave y always have a greater amplitude than the individual waves y_1 and y_2?

REASONING AND DISCUSSION

The wave y is the sum of y_1 and y_2, but remember that y_1 and y_2 are sometimes positive and sometimes negative. Thus, if y_1 is positive at a given time, for example, and y_2 is negative, the sum $y_1 + y_2$ can be zero or even negative. For example, if y_1 and y_2 both have the amplitude A, the amplitude of y can take any value from 0 to $2A$.

ANSWER

No. The amplitude of y can be greater than, less than, or equal to the amplitudes of y_1 and y_2.

Interference

As simple as the principle of superposition is, it still leads to interesting consequences. For example, consider the wave pulses on a string shown in Figure 14–20 (a). When they combine, the resulting pulse has an amplitude equal to the sum of the amplitudes of the individual pulses. This is referred to as **constructive interference.**

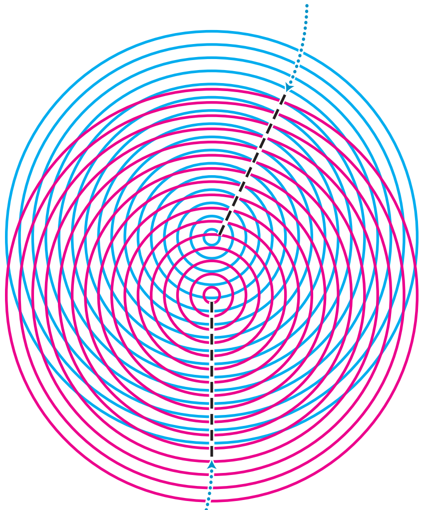
On the other hand, two pulses like those in **Figure 14–20 (b)** may combine. When this happens, the positive displacement of one wave adds to the negative displacement of the other to create a net displacement of zero. That is, the pulses momentarily cancel one another. This is **destructive interference.**

It is important to note that the waves don't simply disappear when they experience destructive interference. For example, in Figure 14–20 (b) the wave pulses continue on unchanged after they interact. This makes sense from an energy point of view—after all, each wave carries energy, hence the waves, along with their energy, cannot simply vanish. In fact, when the string is flat in Figure 14–20 (b) it has its greatest transverse speed—just like a swing has its highest speed when it is in its equilibrium position. Therefore, the energy of the wave is still present at this instant of time—it is just in the form of the kinetic energy of the string.

It should also be noted that interference is not limited to waves on a string; all waves exhibit interference effects. In fact, you could say that interference is one of the key characteristics that define waves. In general, when waves combine, they form **interference patterns** that include regions of both constructive and destructive interference. An example is shown in **Figure 14–21**, where two circular waves are interfering. Note the regions of constructive interference separated by regions of destructive interference.

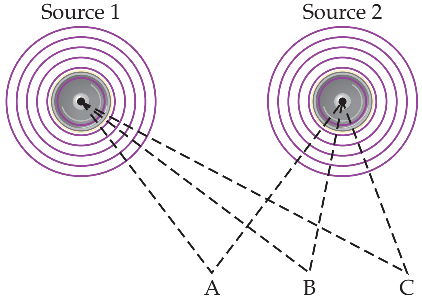
To understand the formation of such patterns, consider a system of two identical sources, as in **Figure 14–22**. Each source sends out waves consisting of alternating crests and troughs. We set up the system so that when one source emits a crest, the other emits a crest as well. Sources that are synchronized like this are said to be **in phase.**

Now, at a point like A, the distance to each source is the same. Thus, if the wave from one source produces a crest at point A, so too does the wave from the other source. As a result, with crest combining with crest, the interference at A is constructive.



▲ **FIGURE 14–21 Interference of circular waves**

Interference pattern formed by the superposition of two sets of circular waves. The light radial "rays" are regions where crest meets crest and trough meets trough (constructive interference). The dark areas in between the light rays are regions where the crest of one wave overlaps the trough of another wave (destructive interference).



▶ **FIGURE 14–22 Interference with two sources**

Suppose the two sources emit waves in phase. At point A the distance to each source is the same, and, hence, crest meets crest. The result is constructive interference. At B the distance from source 1 is greater than that from source 2 by half a wavelength. The result is crest meeting trough and destructive interference. Finally, at C the distance from source 1 is one wavelength greater than the distance from source 2. Hence, we find constructive interference at C. If the sources had been opposite in phase, then A and C would be points of destructive interference, and B would be a point of constructive interference.

Next consider point B. At this location the wave from source 1 must travel a greater distance than the wave from source 2. If the extra distance is half a wavelength, it follows that when the wave from source 2 produces a crest at B the wave from source 1 produces a trough. As a result, the waves combine to give destructive interference at B. At point C, on the other hand, the distance from source 1 is one wavelength greater than the distance from source 2. Hence the waves are in phase again at C, with crest meeting crest for constructive interference.

In general, then, we can say that constructive and destructive interference occur under the following conditions for two sources that are in phase:

> Constructive interference occurs when the path length from the two sources differs by $0, \lambda, 2\lambda, 3\lambda, \ldots$.
>
> Destructive interference occurs when the path length from the two sources differs by $\lambda/2, 3\lambda/2, 5\lambda/2, \ldots$.

Other path length differences result in intermediate degrees of interference, between the extremes of destructive and constructive interference.

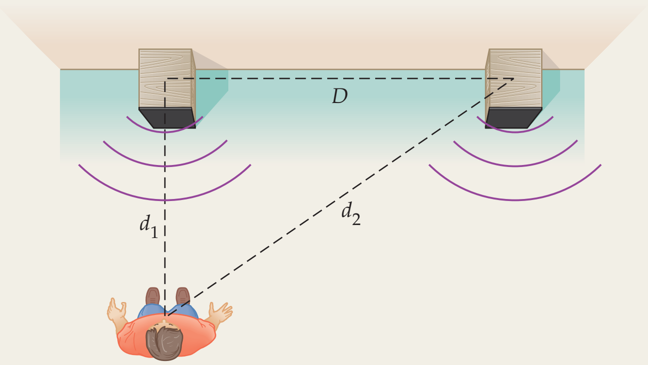
A specific example of interference patterns is provided by sound, using speakers that emit sound in phase with the same frequency. This situation is analogous to the two sources in Figure 14–22. As a result, constructive and destructive interference is to be expected, depending on the path length from each speaker. This is illustrated in the next Example.

EXAMPLE 14–7 SOUND OFF

Two speakers separated by a distance of 4.30 m emit sound of frequency 221 Hz. The speakers are in phase with one another. A person listens from a location 2.80 m directly in front of one of the speakers. Does the person hear constructive or destructive interference?

PICTURE THE PROBLEM

Our sketch shows the two speakers emitting sound in phase at the frequency $f = 221$ Hz. The speakers are separated by the distance $D = 4.30$ m, and the observer is a distance $d_1 = 2.80$ m directly in front of one of the speakers.



STRATEGY

The type of interference depends on whether the difference in path length, $d_2 - d_1$, is one or more wavelengths or an odd multiple of half a wavelength. Thus, we begin by calculating the wavelength, λ. Next, we find d_2, and compare the difference in path length to λ.

SOLUTION

1. Calculate the wavelength of this sound, using $v = \lambda f$. As usual, let $v = 343$ m/s be the speed of sound:
$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{221 \text{ Hz}} = 1.55 \text{ m}$$
2. Find the path length d_2:
$$d_2 = \sqrt{D^2 + d_1{}^2} = \sqrt{(4.30 \text{ m})^2 + (2.80 \text{ m})^2} = 5.13 \text{ m}$$
3. Determine the difference in path length, $d_2 - d_1$:
$$d_2 - d_1 = 5.13 \text{ m} - 2.80 \text{ m} = 2.33 \text{ m}$$
4. Divide λ into $d_2 - d_1$ to find the number of wavelengths that fit into the path difference:
$$\frac{d_2 - d_1}{\lambda} = \frac{2.33 \text{ m}}{1.55 \text{ m}} = 1.50$$

INSIGHT

Since the path difference is $3\lambda/2$, we expect destructive interference. In the ideal case, the person would hear no sound. As a practical matter, some sound will be reflected from objects in the vicinity, resulting in a finite sound intensity.

PRACTICE PROBLEM

We know that 221 Hz gives destructive interference. What is the lowest frequency that gives constructive interference for the case described in this Example? [**Answer:** Set $\lambda = d_2 - d_1 = 2.33$ m. This gives $f = 147$ Hz.]

Some related homework problems: Problem 61, Problem 63

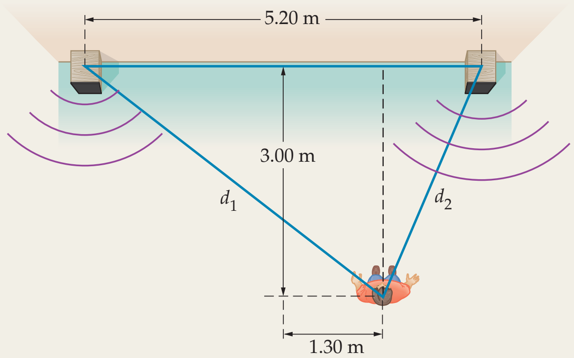
It is possible to connect a speaker with its wires reversed, which can result in a set of speakers that have **opposite phase.** In this case, as one speaker emits a compression the other sends out a rarefaction. When you set up a stereo system, it is important to be sure the wires are connected in a consistent fashion so that your speakers will be in phase.

REAL-WORLD PHYSICS
Connecting speakers in phase

If the two speakers in Figure 14–22 have opposite phase, for example, the conditions for constructive and destructive interference are changed, as are the interference patterns. For example, at point A, where the distances from the two speakers are the same, the wave from one speaker is a compression when the wave from the other speaker is a rarefaction. Thus, point A is now a point of destructive interference rather than constructive interference. In general, then, the conditions for constructive and destructive interference are simply reversed—a path difference of $0, \lambda, 2\lambda, \ldots$ results in destructive interference, a path difference of $\lambda/2, 3\lambda/2, \ldots$ results in constructive interference.

ACTIVE EXAMPLE 14–2 OPPOSITE PHASE INTERFERENCE

The speakers shown to the right have opposite phase. They are separated by a distance of 5.20 m and emit sound with a frequency of 104 Hz. A person stands 3.00 m in front of the speakers and 1.30 m to one side of the center line between them. What type of interference occurs at the person's location?



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the wavelength:	$\lambda = 3.30$ m
2. Find the path length d_1:	$d_1 = 4.92$ m
3. Find the path length d_2:	$d_2 = 3.27$ m
4. Calculate the path length difference, $d_1 - d_2$:	$d_1 - d_2 = 1.65$ m
5. Divide the path length difference by the wavelength:	$(d_1 - d_2)/\lambda = 0.500$

INSIGHT

The path difference is half a wavelength, and the speakers have opposite phase. As a result, the person experiences constructive interference.

YOUR TURN

Find the next higher frequency for which constructive interference occurs at the person's location.

(Answers to **Your Turn** *problems are given in the back of the book.)*

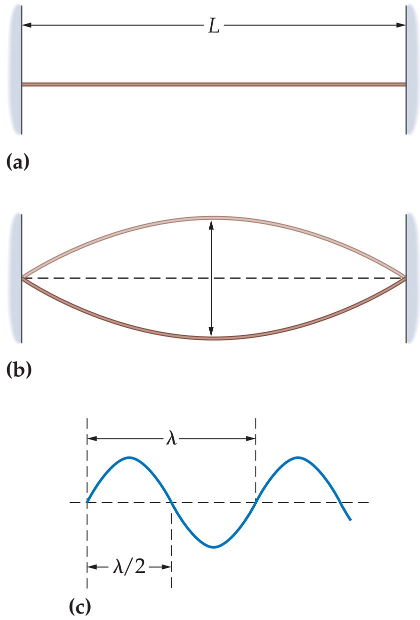
Destructive interference can be used to reduce the intensity of noise in a variety of situations, such as a factory, a busy office, or even the cabin of an airplane. The process, referred to as Active Noise Reduction (ANR), begins with a microphone that picks up the noise to be reduced. The signal from the microphone is then reversed in phase and sent to a speaker. As a result, the speaker emits sound that is opposite in phase to the incoming noise—in effect, the speaker produces "anti-noise." In this way, the noise is *actively* canceled by destructive interference, rather than simply reduced by absorption. The effect when wearing a pair of ANR headphones can be as much as a 30-dB reduction in the intensity level of noise.

REAL-WORLD PHYSICS
Active noise reduction

14–8 Standing Waves

If you have ever plucked a guitar string, or blown across the mouth of a pop bottle to create a tone, you have generated **standing waves.** In general, a standing wave is one that oscillates with time, but remains fixed in its location. It is in this sense that the wave is said to be "standing."

In some respects, a standing wave can be considered as resulting from constructive interference of a wave with itself. As one might expect, then, standing waves occur only if specific conditions are satisfied. We explore these conditions in this section for two cases: (i) waves on a string and (ii) sound waves in a hollow, cylindrical structure.



▲ **FIGURE 14–23 A standing wave**
(a) A string is tied down at both ends. **(b)** If the string is plucked in the middle, a standing wave results. This is the fundamental mode of oscillation of the string. **(c)** The fundamental consists of one-half (1/2) a wavelength between the two ends of the string. Hence, its wavelength is $2L$.

Waves on a String

We begin by considering a string of length L that is tied down at both ends, as in **Figure 14–23 (a)**. If you pluck this string in the middle it vibrates as shown in **Figure 14–23 (b)**. This is referred to as the **fundamental mode** of vibration for this string or, also, as the **first harmonic.** Clearly, the string assumes a wavelike shape, but because of the boundary conditions—the ends tied down—the wave stays in place.

As is clear from **Figure 14–23 (c)**, the fundamental mode corresponds to half a wavelength of a usual wave on a string. One can think of the fundamental as being formed by this wave reflecting back and forth between the walls holding the string. If the frequency is just right, the reflections combine to give constructive interference and the fundamental is formed; if the frequency differs from the fundamental frequency, the reflections result in destructive interference and a standing wave does not result.

We can find the frequency of the fundamental, or first harmonic, as follows: First use the fact that the wavelength of the first harmonic is twice the distance between the walls. Thus,

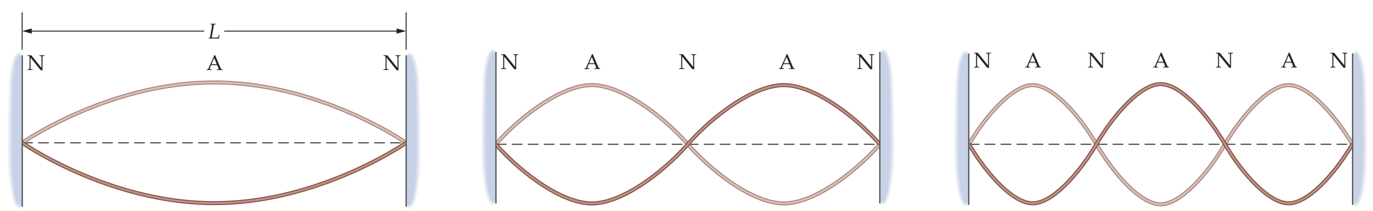
$$\lambda_1 = 2L$$

If the speed of waves on the string is v, it follows that the frequency of the first harmonic, f_1, is determined by $v = \lambda_1 f_1 = (2L)f_1$. Therefore,

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

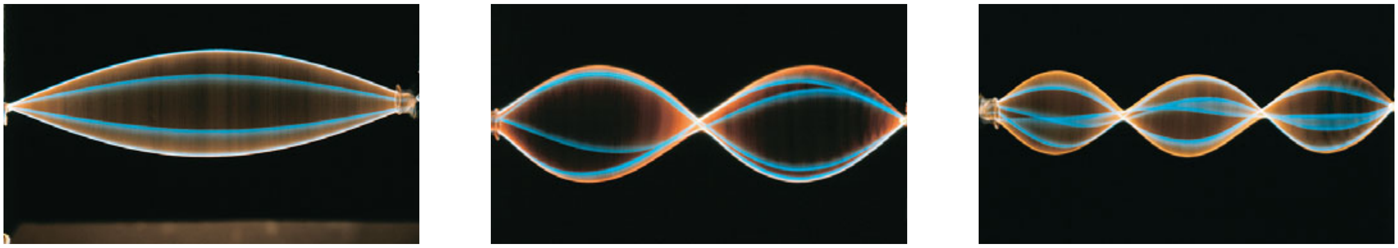
Note that the frequency of the first harmonic increases with the speed of the waves, and decreases as the string is lengthened.

The first harmonic is not the only standing wave that can exist on a string, however. In fact, there are an infinite number of standing wave modes—or **harmonics**—for any given string, with frequencies that are integer multiples of the first harmonic. To find the higher harmonics, note that the two ends of the string must remain fixed. Points on a standing wave that stay fixed are referred to as **nodes.** Halfway between any two nodes is a point on the wave that has a maximum displacement, as indicated in **Figure 14–24**. Such a point is called an **antinode.** Referring to Figure 14–24 (a), then, we see that the first harmonic consists of two nodes (N) and one antinode (A); the sequence is N-A-N.



(a) First harmonic (fundamental) **(b) Second harmonic** **(c) Third harmonic**

▲ **FIGURE 14–24 Harmonics**
The first three harmonics for a string tied down at both ends. Note that an extra half wavelength is added to go from one harmonic to the next. **(a)** $\lambda/2 = L$, $\lambda = 2L$; **(b)** $\lambda = L$; **(c)** $3\lambda/2 = L$, $\lambda = 2L/3$.



▲ The string in these multiflash photographs vibrates in one of three different standing wave patterns, each with its own characteristic frequency. The lowest frequency standing wave—the fundamental, or first harmonic—is shown in the photograph at left. In this case, there are only two nodes, one at each end of the string where it is tied down. If the length of the string is L, we see that the wavelength of the fundamental is twice this length, or $\lambda_1 = 2L$. Thus, if waves have a speed v on this string, their frequency is $f_1 = v/2L$. Higher harmonics are produced by adding one node at a time to the standing wave pattern. The second harmonic, shown in the middle photograph, has a node at either end and one in the middle. In this case the wavelength is $\lambda_2 = L$ and the frequency is $f_2 = v/L = 2f_1$. The photograph at right shows the third harmonic, where $\lambda_3 = 2L/3$ and $f_3 = v/(2L/3) = 3v/2L = 3f_1$. In general, the nth harmonic on a string tied down at both ends is $f_n = nf_1$.

The *second* harmonic can be constructed by including one more half wavelength in the standing wave, as in Figure 14–24 (b). This mode has the sequence N-A-N-A-N, and has one complete wavelength between the walls; that is, $\lambda_2 = L$. Therefore, the frequency of the second harmonic, f_2, is

$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$$

Similarly, the *third* harmonic again includes one more half wavelength, as in Figure 14–24 (c). Now there are one-and-a-half wavelengths in the length L, and hence $(3/2)\lambda_3 = L$, or $\lambda_3 = 2L/3$. The corresponding third-harmonic frequency, f_3, is

$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{2}{3}L} = 3\frac{v}{2L} = 3f_1$$

Note that the frequencies of the harmonics are increasing in integer steps; that is, each harmonic has a frequency that is an integer multiple of the first-harmonic frequency. Clearly, then, the sequence of standing waves is characterized by the following:

Standing Waves on a String

First harmonic (fundamental) frequency and wavelength:

$$f_1 = \frac{v}{2L} \qquad \lambda_1 = 2L \qquad \text{14–12}$$

Frequency and wavelength of the nth harmonic, with $n = 1, 2, 3, \ldots$:

$$f_n = nf_1 = n\frac{v}{2L} \qquad \lambda_n = \lambda_1/n = 2L/n \qquad \text{14–13}$$

▲ The photos above show a time sequence (from left to right) as a square metal plate that is initially at rest is vibrated vertically about its center. Initially the plate is covered with a uniform coating of salt crystals. As the plate is vibrated, however, a standing wave develops. The salt makes the wave pattern visible by collecting at the nodes, where the plate is at rest. Clearly, standing waves on a two-dimensional plate can be much more complex than the standing waves on a string.

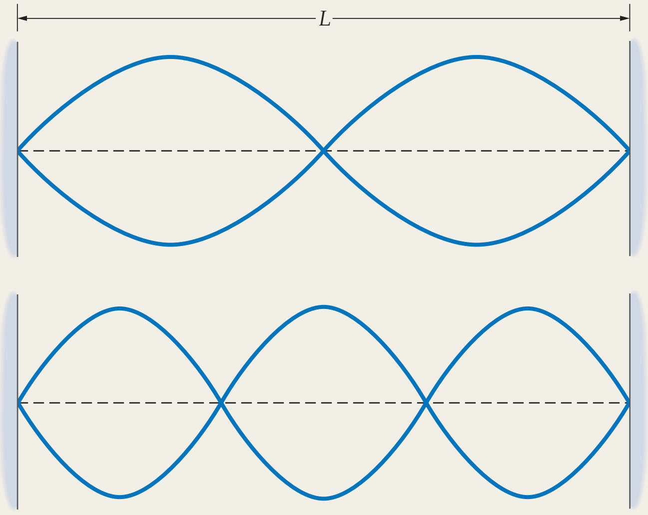
Note that the difference in frequency between any two successive harmonics is equal to the first-harmonic frequency, f_1, and that n represents the number of half wavelengths in the standing wave.

EXAMPLE 14–8 IT'S FUNDAMENTAL

One of the harmonics on a string 1.30 m long has a frequency of 15.60 Hz. The next higher harmonic has a frequency of 23.40 Hz. Find **(a)** the fundamental frequency, and **(b)** the speed of waves on this string.

PICTURE THE PROBLEM

The problem statement does not tell us directly which two harmonics have the given frequencies. We do know, however, that they are *successive* harmonics of the string. For example, if one harmonic has one node between the two ends of the string, the next harmonic has two nodes. Our sketch illustrates this case, which turns out to be appropriate for this problem.



STRATEGY

a. We know from Equation 14–13 that the frequencies of successive harmonics increase by f_1. That is, $f_2 = f_1 + f_1 = 2f_1$, $f_3 = f_2 + f_1 = 3f_1$, $f_4 = f_3 + f_1 = 4f_1$, ... Therefore, we can find the fundamental frequency, f_1, by taking the difference between the given frequencies.

b. Once the fundamental frequency is determined, we can find the speed of waves in the string from the relation $f_1 = v/2L$.

SOLUTION

Part (a)

1. The fundamental frequency is the difference between the two given frequencies: $f_1 = 23.40\text{ Hz} - 15.60\text{ Hz} = 7.80\text{ Hz}$

Part (b)

2. Solve $f_1 = v/2L$ for the speed, v:
$$f_1 = v/2L$$
$$v = 2Lf_1$$

3. Substitute numerical values: $v = 2Lf_1 = 2(1.30\text{ m})(7.80\text{ Hz}) = 20.3\text{ m/s}$

INSIGHT

Now that we know the fundamental frequency, we can identify the harmonics given in the problem statement. First, $15.6\text{ Hz} = 2(7.80\text{ Hz})$, so this is the second harmonic. The next mode, $23.4\text{ Hz} = 3(7.80\text{ Hz})$, is the third harmonic, as expected.

PRACTICE PROBLEM

Suppose the tension in this string is increased until the speed of the waves is 22.0 m/s. What are the frequencies of the first three harmonics in this case? [**Answer:** $f_1 = 8.46$ Hz, $f_2 = 16.9$ Hz, $f_3 = 25.4$ Hz]

Some related homework problems: Problem 72, Problem 73

When a guitar string is plucked or a piano string is struck, it vibrates primarily in its fundamental mode, with smaller contributions coming from the higher harmonics. It follows that notes of different pitch can be produced by using strings of different length. Recalling that the fundamental frequency for a string of length L is $f_1 = v/2L$, we see that long strings produce low frequencies and short strings produce high frequencies—all other variables remaining the same.

REAL-WORLD PHYSICS

The shape of a piano

This fact accounts for the general shape of a piano. Note that the strings shorten toward the right side of the piano, where the notes are of higher frequency. Similarly, a double bass is a larger instrument with longer strings than a violin, as one would expect by the different frequencies the instruments produce. To tune a stringed instrument, the tension in the strings is adjusted—since changing the length of the instrument is impractical. This in turn varies the speed v of waves on the string, and hence the fundamental frequency $f_1 = v/2L$ can be adjusted as desired.

The human ear responds to frequency in a rather interesting and unexpected way. In particular, frequencies that seem to increase by the same amount are in fact increasing by the same multiplicative factor. For example, if three frequencies, f_1, f_2, and f_3, sound equally spaced to our ears, you might think that f_2 is greater than f_1 by a certain amount, x, and that f_3 is greater than f_2 by the same amount. Mathematically, we would write this as $f_2 = f_1 + x$ and $f_3 = f_2 + x = f_1 + 2x$. In fact, when we measure the frequencies and compare, we find that f_2 is greater than f_1 by a multiplicative factor x, and that f_3 is greater than f_2 by the same factor; that is $f_2 = xf_1$ and $f_3 = xf_2 = x^2 f_1$.

REAL-WORLD PHYSICS: BIO

Human perception of pitch

For instance, middle C on the piano has a frequency of 261.7 Hz. If we move up one octave to the next C, the frequency is 523.3 Hz; going up one more octave, the next C is 1047 Hz. Note that with each octave the frequency doubles; that is, it goes up by a multiplicative factor of 2. Since there are 12 semitones in one octave of the chromatic scale, the frequency increase from one semitone to the next is $(2)^{1/12}$. The frequencies for a full chromatic octave are given in Table 14–3.

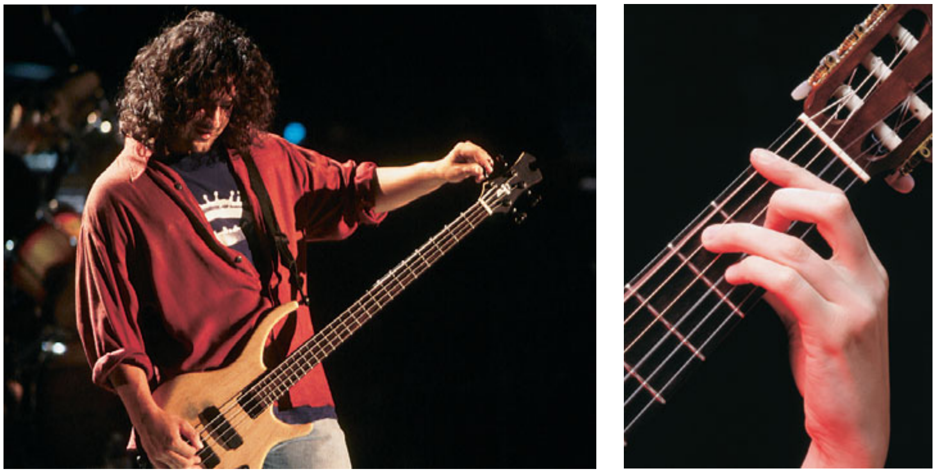
TABLE 14–3 Chromatic Musical Scale

Note	Frequency (Hz)
Middle C	261.7
C♯ (C-sharp) D♭ (D-flat)	277.2
D	293.7
D♯, E♭	311.2
E	329.7
F	349.2
F♯, G♭	370.0
G	392.0
G♯, A♭	415.3
A	440.0
A♯, B♭	466.2
B	493.9
C	523.3

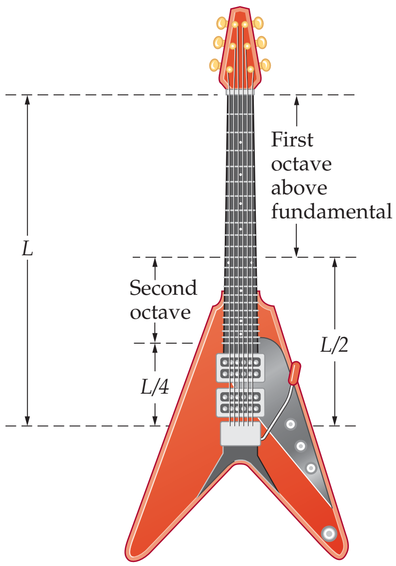
On a guitar two full octaves and more can be produced on a single string by pressing the string down against frets to effectively change its length. Notice that the separation between frets is not uniform. In particular, suppose the unfretted string has a fundamental frequency of 250 Hz. Since one octave up on the scale would be twice the frequency, 500 Hz, the length of the string must be halved to produce that note. To go to the next octave, and double the frequency again to 1000 Hz, the string must be shortened by a factor of two again, to one quarter its original length. This is illustrated in **Figure 14–25**. Since the distance between successive octaves is decreasing—in this case from $L/2$ to $L/4$—it follows that the spacing between frets must decrease as one goes to higher notes. As a result, the frets on a guitar are always more closely spaced as one moves toward the base of the neck.

REAL-WORLD PHYSICS

Frets on a guitar



▲ Three factors determine the pitch of a vibrating string: mass per unit length, μ; tension, F; and length, L. In an instrument such as a guitar, the first of these factors is fixed once the strings are put on. (Note in the photos that the strings vary in thickness; other things being equal, the heavier the string, the lower the pitch.) The second factor, the tension, can be varied by means of pegs that the player uses to tune the instrument (left), adjusting the pitch of each "open" string to its correct value. The third factor, the length of the string, is the only one that the performer controls while playing. Pressing a string against one of the frets (right), changes its effective length—the length of string that is free to vibrate—and thus the note that is produced.

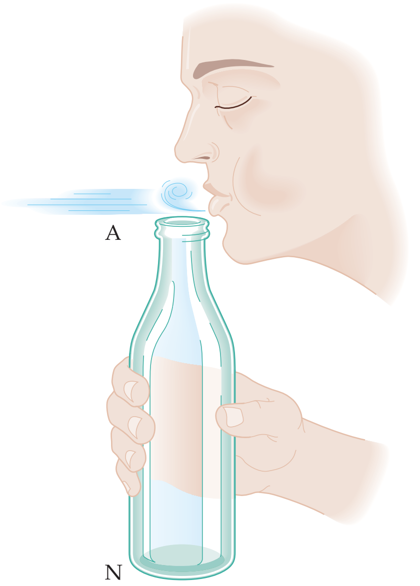


▲ **FIGURE 14–25 Frets on a guitar**

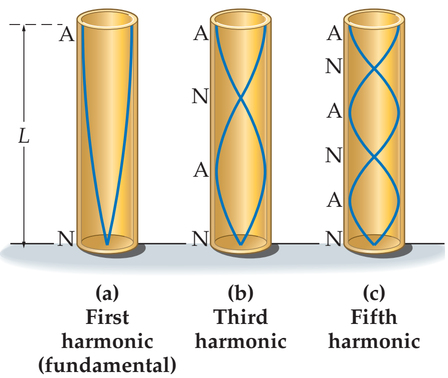
To go up one octave from the fundamental, the effective length of a guitar string must be halved. To increase one more octave, it is necessary to halve the length of the string again. Thus, the distance between frets is not uniform; they are more closely spaced near the base of the neck.

Vibrating Columns of Air

If you blow across the open end of a pop bottle, as in **Figure 14–26**, you hear a tone of a certain frequency. If you pour some water into the bottle and repeat the experiment, the sound you hear has a higher frequency. In both cases you have excited the fundamental mode of the column of air within the bottle. When water was added to the bottle, however, the column of air was shortened, leading to a higher frequency—in the same way that a shortened string has a higher frequency.



▲ **FIGURE 14–26 Exciting a standing wave**
When air is blown across the open top of a soda pop bottle, the turbulent air flow can cause an audible standing wave. The standing wave will have an antinode, A, at the top (where the air is moving) and a node, N, at the bottom (where the air cannot move).



▲ **FIGURE 14–27 Standing waves in a pipe that is open at one end**
The first three harmonics for waves in a column of air of length L that is open at one end: **(a)** $\lambda/4 = L$, $\lambda = 4L$; **(b)** $3\lambda/4 = L$, $\lambda = 4L/3$; **(c)** $5\lambda/4 = L$, $\lambda = 4L/5$.

Let's examine the situation more carefully. When you blow across the opening in the bottle, the result is a swirling movement of air that excites rarefactions and compressions, as illustrated in the figure. For this reason, the opening is an antinode (A) for sound waves. On the other hand, the bottom of the bottle is closed, preventing movement of the air; hence, it must be a node (N). Any standing wave in the bottle must have a node at the bottom and an antinode at the top.

The lowest frequency standing wave that is consistent with these conditions is shown in **Figure 14–27 (a)**. If we plot the density variation of the air for this wave, we see that one-quarter of a wavelength fits into the column of air in the bottle. Thus, if the length of the bottle is L, the first harmonic (fundamental) has a wavelength satisfying the following:

$$\tfrac{1}{4}\lambda = L$$
$$\lambda = 4L$$

The first-harmonic frequency, f_1, is given by

$$v = \lambda f_1$$

Solving for f_1 we find

$$f_1 = \frac{v}{\lambda} = \frac{v}{4L}$$

This is half the corresponding fundamental frequency for a wave on a string.

The next harmonic is produced by adding half a wavelength, just as in the case of the string. Thus, if the fundamental is represented by N-A, the second harmonic can be written as N-A-N-A. Since the distance from a node to an antinode is a quarter of a wavelength, we see that three-quarters (3/4) of a wavelength fits into the bottle for this mode. This is shown in **Figure 14–27 (b)**. Therefore, $3\lambda/4 = L$, and, hence,

$$\lambda = \tfrac{4}{3}L$$

As a result, the frequency is

$$\frac{v}{\lambda} = \frac{v}{\frac{4}{3}L} = 3\frac{v}{4L} = 3f_1$$

Notice that this is the *third* harmonic of the pipe, since its frequency is three times f_1.

Similarly, the next-higher harmonic is represented by N-A-N-A-N-A, as indicated in **Figure 14–27 (c)**. In this case, $5\lambda/4 = L$, and the frequency is

$$\frac{v}{\lambda} = \frac{v}{\frac{4}{5}L} = 5\frac{v}{4L} = 5f_1$$

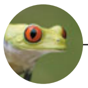
This is the *fifth* harmonic of the pipe.

Clearly, the progression of harmonics for a column of air that is closed at one end and open at the other end is described by the following frequencies and wavelengths:

Standing Waves in a Column of Air Closed at One End

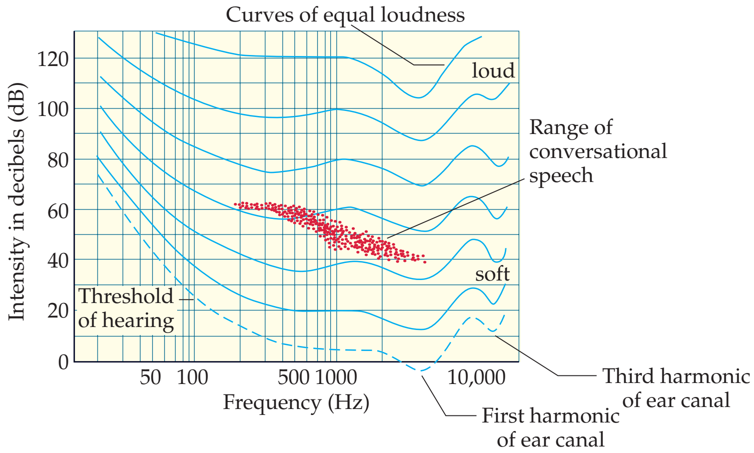
$$f_1 = \frac{v}{4L}$$
$$f_n = nf_1 = n\frac{v}{4L} \qquad n = 1, 3, 5, \ldots \qquad \text{14–14}$$
$$\lambda_n = \lambda_1/n = 4L/n$$

Note that only the odd harmonics are present in this case, as opposed to waves on a string, in which all integer harmonics occur.



REAL-WORLD PHYSICS: BIO
Human hearing and the ear canal

The human ear canal is an example of a column of air that is closed at one end (the eardrum) and open at the other end. Standing waves in the ear canal can lead to an increased sensitivity of hearing. This is illustrated in **Figure 14–28**, which shows "curves of equal loudness" as a function of frequency. Where these curves dip downward, sounds of lower intensity seem just as loud as sounds of higher intensity at other frequencies. The two prominent dips near 3500 Hz and 11,000 Hz are due to standing waves in the ear canal corresponding to Figures 14–27 (a) and (b), respectively.



▲ **FIGURE 14–28 Human response to sound**

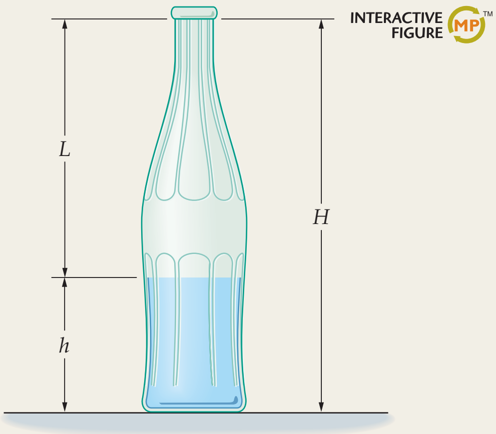
The human ear is more sensitive to some frequencies of sound than to others. For example, every point on a "curve of equal loudness" seems just as loud to us as any other point, even though the corresponding physical intensities may be quite different. To illustrate, note that the threshold of hearing is not equal to 0 dB for all frequencies. In fact, it is approximately 25 dB at 100 Hz, about 5 dB at 1000 Hz, and is even slightly negative near 3500 Hz. Thus, regions where the curves dip downward correspond to increased sensitivity of the ear—in fact, near 3500 Hz we can hear sounds that are a thousand times less intense than sounds at 100 Hz. The two most prominent dips occur near 3500 Hz and 11,000 Hz, corresponding to standing waves in the ear canal analogous to those shown in Figure 14–27 (a) and (b), respectively. (See Problem 71.)

EXAMPLE 14–9 POP MUSIC

An empty soda pop bottle is to be used as a musical instrument in a band. In order to be tuned properly, the fundamental frequency of the bottle must be 440.0 Hz. **(a)** If the bottle is 26.0 cm tall, how high should it be filled with water to produce the desired frequency? Treat the bottle as a pipe that is closed at one end (the surface of the water) and open at the other end. **(b)** What is the frequency of the next higher harmonic for this bottle?

PICTURE THE PROBLEM

In our sketch, we label the height of the bottle with $H = 26.0$ cm, and the unknown height of water with h. Clearly, then, the length of the vibrating column of air is $L = H - h$.



STRATEGY

a. Given the frequency of the fundamental ($f_1 = 440.0$ Hz) and the speed of sound in air ($v = 343$ m/s), we can use $f_1 = v/4L$ to solve for the length L of the air column. The height of water is then $h = H - L$.

b. The next higher harmonic for a pipe open at one end is the third harmonic ($n = 3$ in Equation 14–14). Thus, the next higher harmonic frequency for this bottle is $f_3 = 3f_1$.

SOLUTION

Part (a)

1. Solve $f_1 = v/4L$ for the length L: $\quad f_1 = v/4L \quad$ or $\quad L = v/4f_1$

2. Substitute numerical values: $\quad L = \dfrac{v}{4f_1} = \dfrac{343 \text{ m/s}}{4(440.0 \text{ Hz})} = 0.195 \text{ m}$

3. Use $h = H - L$ to find the height of the water: $\quad h = H - L = 0.260 \text{ m} - 0.195 \text{ m} = 0.065 \text{ m} = 6.5 \text{ cm}$

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Part (b)

4. Calculate the frequency of the third harmonic (the next highest) with $f_3 = 3f_1$:

$$f_3 = 3f_1 = 3(440.0\text{ Hz}) = 1320\text{ Hz}$$

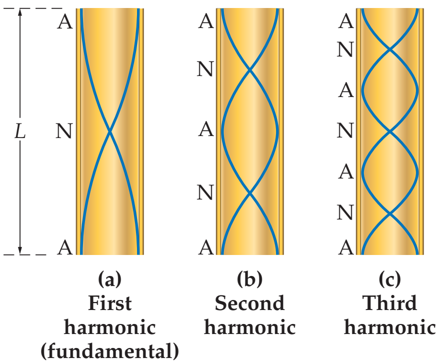
INSIGHT

If more water is added to the bottle, the air column will shorten and the fundamental frequency will become higher than 440.0 Hz. All higher harmonics would be increased in frequency as well.

PRACTICE PROBLEM

Calculate the fundamental frequency if the water level is increased to 7.00 cm. [**Answer:** $f_1 = 451$ Hz]

Some related homework problems: Problem 68, Problem 75



▲ **FIGURE 14–29 Standing waves in a pipe that is open at both ends**

The first three harmonics for waves in a column of air of length L that is open at both ends: **(a)** $\lambda/2 = L$, $\lambda = 2L$; **(b)** $\lambda = L$; **(c)** $3\lambda/2 = L$, $\lambda = 2L/3$.

It is also possible to excite standing waves in columns of air that are open at both ends, as illustrated in **Figure 14–29**. In this case there is an antinode at each end of the column. Hence, the first harmonic, or fundamental, is A-N-A, as shown in Figure 14–29 (a). Note that half a wavelength fits into the pipe, thus

$$f_1 = \frac{v}{2L}$$

This is the same as the corresponding result for a wave on a string.

The next harmonic is A-N-A-N-A, which fits one complete wavelength in the pipe. This harmonic is shown in Figure 14–29 (b), and has the frequency

$$f_2 = \frac{v}{L} = 2f_1$$

This is the *second* harmonic of the pipe. The rest of the harmonics continue in exactly the same manner as for waves on a string, with all integer harmonics present. Thus, the frequencies and wavelengths in a column of air open at both ends are as follows:

Standing Waves in a Column of Air Open at Both Ends

$$f_1 = \frac{v}{2L}$$

$$f_n = nf_1 = n\frac{v}{2L} \quad n = 1, 2, 3, \ldots \qquad \text{14–15}$$

$$\lambda_n = \lambda_1/n = 2L/n$$

CONCEPTUAL CHECKPOINT 14–4 TALKING WITH HELIUM

If you fill your lungs with helium and speak, you sound something like Donald Duck. From this observation, we can conclude that the speed of sound in helium must be **(a)** less than, **(b)** the same as, or **(c)** greater than the speed of sound in air.

REASONING AND DISCUSSION

When we speak with helium, our words are higher pitched. Looking at Equation 14–15, we see that for the frequency to increase, while the length of the vocal chords remains the same, the speed of sound must be higher.

ANSWER

(c) The speed of sound is greater in helium than in air.

REAL-WORLD PHYSICS

Organ pipes

A pipe organ uses a variety of pipes of different length, with some being open at both ends, others open at one end only. When a key is pressed on the console of the organ, air is forced through a given pipe. By accurately adjusting the length of the pipe it can be given the desired tone. In addition, since open and closed pipes have different harmonic frequencies, they sound distinctly different to the ear, even if they have the same fundamental frequency. Thus, by judiciously choosing both the length and the type of a pipe, an organ can be given a range of different sounds, allowing it to mimic a trumpet, a trombone, a clarinet, and so on.

Standing waves have also been observed in the Sun. Like an enormous, low-frequency musical instrument, the Sun vibrates once roughly every five minutes, a result of the roiling nuclear reactions that take place within its core. One of the goals of SOHO, the Solar and Heliospheric Observatory, is to study these solar vibrations in detail. By observing the variety of standing waves produced in the Sun, we can learn more about its internal structure and dynamics.

▲ Blowing across the mouth of a bottle (Figure 14–26) sets the air column within the bottle vibrating, producing a tone. This principle is put to use in the pipe organ. A large organ may have hundreds of pipes of different lengths, some open at both ends and some at only one, affording the performer great control over the tonal quality of the sound produced, as well as its pitch.

14–9 Beats

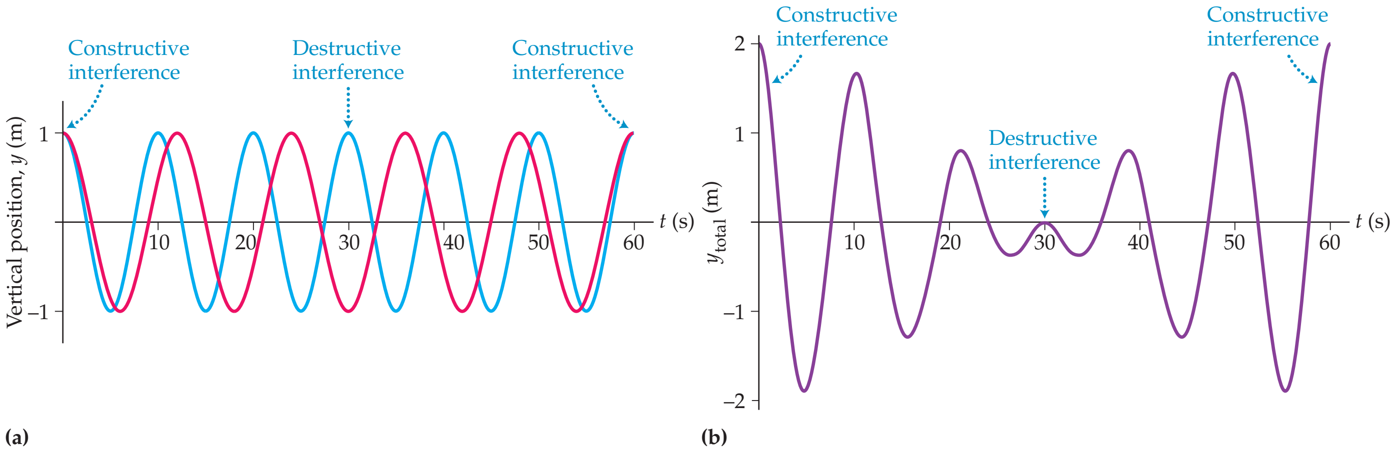
An interference pattern, such as that shown in Figure 14–21, is a snapshot at a given time, showing locations where constructive and destructive interference occur. It is an interference pattern in space. **Beats,** on the other hand, can be thought of as an interference pattern in time.

To be specific, imagine plucking two guitar strings that have slightly different frequencies. If you listen carefully, you notice that the sound produced by the strings is not constant in time. In fact, the intensity increases and decreases with a definite period. These fluctuations in intensity are the beats, and the frequency of successive maximum intensities is the **beat frequency.**

As an example, suppose two waves, with frequencies $f_1 = 1/T_1$ and $f_2 = 1/T_2$, interfere at a given, fixed location. At this location, each wave moves up and down with simple harmonic motion, as described by Equation 13–2. Applying this result to the vertical position, y, of each wave yields the following:

$$y_1 = A\cos\left(\frac{2\pi}{T_1}t\right) = A\cos(2\pi f_1 t)$$
$$y_2 = A\cos\left(\frac{2\pi}{T_2}t\right) = A\cos(2\pi f_2 t) \qquad 14\text{–}16$$

These equations are plotted in **Figure 14–30 (a)**, with $A = 1$ m, and their superposition, $y_{total} = y_1 + y_2$, is shown in **Figure 14–30 (b)**.



▲ **FIGURE 14–30 Interference of two waves with slightly different frequencies**
(a) A plot of the two waves, y_1 (blue) and y_2 (red), given in Equations 14–16. **(b)** The resultant wave y_{total} for the two waves shown in part (a). Note the alternately constructive and destructive interference leading to beats.

Note that at the time $t = 0$, both y_1 and y_2 are equal to A; thus their superposition gives $2A$. Since the waves have different frequencies, however, they do not stay in phase. At a later time, t_1, we find that $y_1 = A$ and $y_2 = -A$; their superposition gives zero at this time. At a still later time, $t_2 = 2t_1$, the waves are again in phase and add to give $2A$. Thus, a person listening to these two waves hears a sound whose amplitude and loudness vary with time; that is, the person hears beats.

Superposing these waves mathematically, we find

$$\begin{aligned} y_{total} &= y_1 + y_2 \\ &= A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t) \\ &= 2A\cos\left(2\pi \frac{f_1 - f_2}{2} t\right)\cos\left(2\pi \frac{f_1 + f_2}{2} t\right) \end{aligned} \qquad 14\text{–}17$$

The final step in the expression follows from the trigonometric identities given in Appendix A. The first part of y_{total} is

$$2A\cos\left(2\pi \frac{f_1 - f_2}{2} t\right)$$

This gives the slowly varying amplitude of the beats, as indicated in **Figure 14–31**. Since a loud sound is heard whenever this term is equal to $-2A$ or $2A$—which happens twice during any given oscillation—the beat frequency is

Definition of Beat Frequency

$$f_{beat} = |f_1 - f_2| \qquad 14\text{–}18$$

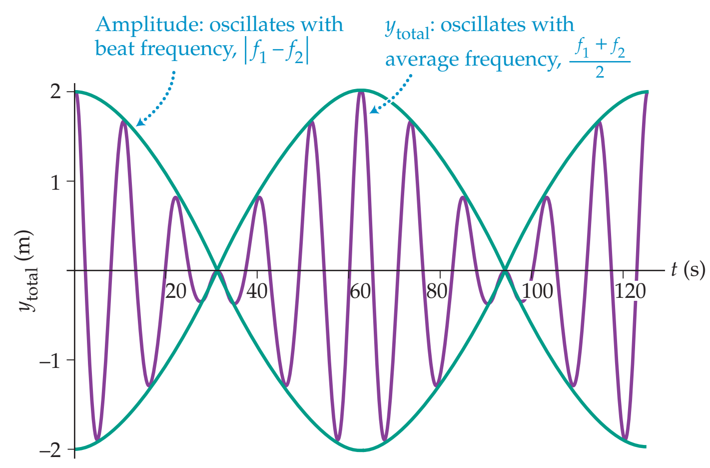
SI unit: $1/s = s^{-1}$

Finally, the rapid oscillations within each beat are due to the second part of y_{total}:

$$\cos\left(2\pi \frac{f_1 + f_2}{2} t\right)$$

Note that these oscillations have a frequency that is the average of the two input frequencies.

These results apply to any type of wave. In particular, if two sound waves produce beats, your ear will hear the average frequency with a loudness that varies with the beat frequency. For example, suppose the two guitar strings mentioned at the beginning of this section have the frequencies 438 Hz and 442 Hz. If you sound them simultaneously, you will hear the average frequency, 440 Hz, increasing



▲ **FIGURE 14–31 Beats**

Beats can be understood as oscillations at the average frequency, modulated by a slowly varying amplitude.

and decreasing in loudness with a beat frequency of 4 Hz. This means that you will hear maximum loudness 4 times a second. If the frequencies are brought closer together, the beat frequency will be less and fewer maxima will be heard each second.

Clearly, beats can be used to tune a musical instrument to a desired frequency. To tune a guitar string to 440 Hz, for example, the string can be played simultaneously with a 440-Hz tuning fork. Listening to the beats, the tension in the string can be increased or decreased until the beat frequency becomes vanishingly small. This technique applies only to frequencies that are reasonably close to begin with, since the maximum beat frequencies the ear can detect are about 15 to 20 Hz.

PROBLEM-SOLVING NOTE

Calculating the Beat Frequency

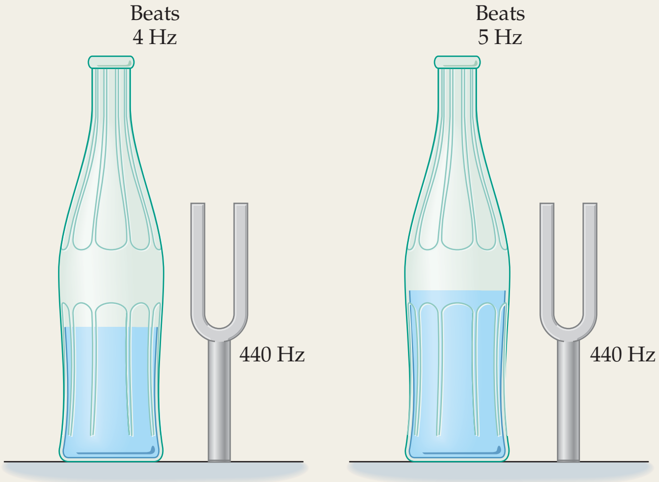
The beat frequency of two waves is the *magnitude* of the difference in their frequencies. Thus, the beat frequency is always positive.

EXAMPLE 14–10 GETTING A TUNE-UP

An experimental way to tune the soda pop bottle in Example 14–9 is to compare its frequency with that of a 440-Hz tuning fork. Initially, a beat frequency of 4 Hz is heard. As a small amount of water is added to that already present, the beat frequency increases steadily to 5 Hz. What were the initial and final frequencies of the bottle?

PICTURE THE PROBLEM

Our sketch shows the before and after situations for this problem. With the low water level the beat frequency is 4 Hz, with the higher level it is 5 Hz.



STRATEGY/SOLUTION

The fact that the initial beat frequency is 4 Hz means the initial frequency of the bottle is either 436 Hz or 444 Hz.

As water is added, we know from Example 14–9 that the bottle's frequency will increase. We also know that the new beat frequency is 5 Hz, and hence the final frequency is either 435 Hz or 445 Hz. Only 445 Hz satisfies the condition that the frequency must have increased.

Therefore, the initial frequency is 444 Hz, and the final frequency is 445 Hz.

INSIGHT

In this case, the initial frequency was too high. To tune the bottle properly, it is necessary to lower the water level.

PRACTICE PROBLEM

Suppose the initial beat frequency was 4 Hz and that adding a small amount of water caused the beat frequency to decrease steadily to 2 Hz. What were the initial and final frequencies in this case? [**Answer:** Initial frequency, 436 Hz; final frequency, 438 Hz]

Some related homework problems: Problem 82, Problem 84

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of frequency, first introduced in terms of oscillations (Chapter 13), is applied here to waves and sound in Sections 14–1 and 14–4, to the Doppler effect in 14–6, to standing waves in 14–8, and to beats in 14–9.

We use power (Chapter 7) in our definition of the intensity of a wave in Section 14–5.

Basic concepts from kinematics (Chapter 2) are used to derive the Doppler effect in Section 14–6.

LOOKING AHEAD

We next encounter waves when we study electricity and magnetism. In fact, we shall see in Chapter 25 that light is an electromagnetic wave, with both the electric and magnetic fields propagating much like a wave on a string.

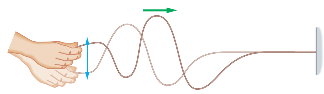
We return to the wave properties of light in Chapter 28, where we see that superposition and interference play a similar role for light as they do for sound.

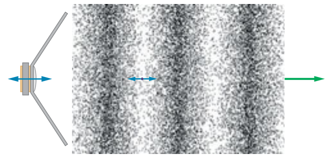
Another type of wave behavior is known as diffraction. This concept is also developed in Chapter 28.

CHAPTER SUMMARY

14–1 TYPES OF WAVES

A wave is a propagating disturbance.





Transverse Waves and Longitudinal Waves

In a transverse wave individual particles move at right angles to the direction of wave propagation. In a longitudinal wave individual particles move in the same direction as the wave propagation.

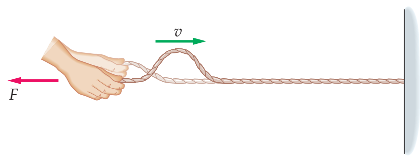
Wavelength, Frequency, and Speed

The wavelength, λ, frequency, f, and speed, v, of a wave are related by

$$v = \lambda f \qquad 14\text{–}1$$

14–2 WAVES ON A STRING

Transverse waves can propagate on a string held taut with a tension force, F.



Mass per Length

The mass per length of a string is $\mu = m/L$.

Speed of a Wave on a String

The speed of a wave on a string with a tension force F and a mass per length μ is

$$v = \sqrt{\frac{F}{\mu}} \qquad 14\text{–}2$$

Reflections

If the end of a string is fixed, the reflection of a wave is inverted. If the end of a string is free to move transversely, waves are reflected with no inversion.

*14–3 HARMONIC WAVE FUNCTIONS

A harmonic wave has the shape of a sine or a cosine.

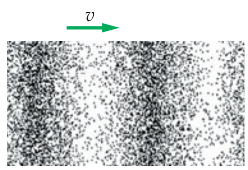
Wave Function

A harmonic wave of wavelength λ and period T is described by the following expression:

$$y(x, t) = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi}{T}t\right) \qquad 14\text{–}4$$

14–4 SOUND WAVES

A sound wave is a longitudinal wave of compressions and rarefactions that can travel through the air, as well as through other gases, liquids, and solids.



Speed of Sound

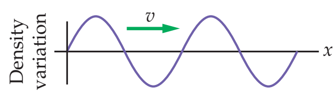
The speed of sound in air, under typical conditions, is $v = 343$ m/s.

Frequency of Sound

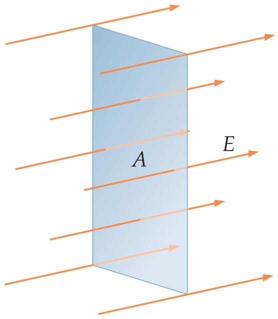
The frequency of sound determines its pitch. High-pitched sounds have high frequencies; low-pitched sounds have low frequencies.

Human Hearing Range

Human hearing extends from 20 Hz to 20,000 Hz.



14–5 SOUND INTENSITY



The loudness of a sound is determined by its intensity.

Intensity

Intensity, I, is a measure of the amount of energy per time that passes through a given area. Since energy per time is power, P, the intensity of a wave is

$$I = \frac{P}{A} \qquad 14\text{–}5$$

Point Source

If a point source emits sound with a power P, and there are no reflections, the intensity a distance r from the source is

$$I = \frac{P}{4\pi r^2} \qquad 14\text{–}7$$

Human Perception of Loudness

The intensity of a sound must be increased by a factor of 10 in order for it to seem twice as loud to our ears.

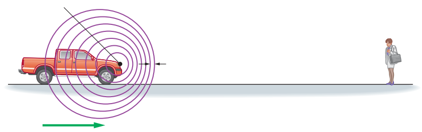
Intensity Level and Decibels

The intensity level, β, of a sound gives an indication of how loud it sounds to our ears. The intensity level is defined as follows:

$$\beta = 10 \log(I/I_0) \qquad 14\text{–}8$$

The value of β is given in decibels.

14–6 THE DOPPLER EFFECT



The change in frequency due to relative motion between a source and a receiver is called the Doppler effect.

Moving Observer

Suppose an observer is moving with a speed u relative to a stationary source. If the frequency of the source is f, and the speed of the waves is v, the frequency f' detected by the observer is

$$f' = (1 \pm u/v)f \qquad 14\text{–}9$$

The plus sign applies to the observer approaching the source, and the minus sign to the observer receding from the source.

Moving Source

If the source is moving with a speed u and the observer is at rest, the observed frequency is

$$f' = \left(\frac{1}{1 \mp u/v}\right)f \qquad 14\text{–}10$$

The minus sign applies to the source approaching the observer, and the plus sign to the source receding from the observer.

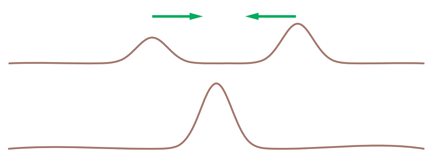
General Case

If the observer moves with a speed u_o and the source moves with a speed u_s, the Doppler effect gives

$$f' = \left(\frac{1 \pm u_o/v}{1 \mp u_s/v}\right)f \qquad 14\text{–}11$$

The meaning of the plus and minus signs is the same as for the moving-observer and moving-source cases given above.

14–7 SUPERPOSITION AND INTERFERENCE



Waves can combine to give a variety of effects.

Superposition

When two or more waves occupy the same location at the same time they simply add, $y_{total} = y_1 + y_2$.

Constructive Interference

Waves that add to give a larger amplitude exhibit constructive interference.

Destructive Interference

Waves that add to give a smaller amplitude exhibit destructive interference.

Interference Patterns

Waves that overlap can create patterns of constructive and destructive interference. These are referred to as interference patterns.

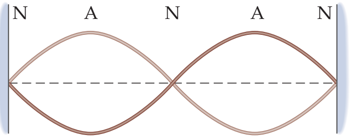
In Phase/Opposite Phase

Two sources are in phase if they both emit crests at the same time.

Sources have opposite phase if one emits a crest at the same time the other emits a trough.

14–8 STANDING WAVES

Standing waves oscillate in a fixed location.



Waves on a String

The fundamental, or first harmonic, corresponds to half a wavelength fitting into the length of the string. The fundamental for waves of speed v on a string of length L is

$$f_1 = \frac{v}{2L} \qquad \lambda_1 = 2L \tag{14–12}$$

The higher harmonics, with $n = 1, 2, 3, \ldots$, are described by

$$f_n = nf_1 = n(v/2L) \qquad \lambda_n = \lambda_1/n = 2L/n \tag{14–13}$$

Vibrating Columns of Air

The harmonics for a column of air closed at one end are

$$f_n = nf_1 = n(v/4L) \quad n = 1, 3, 5, \ldots \qquad \lambda_n = \lambda_1/n = 4L/n \tag{14–14}$$

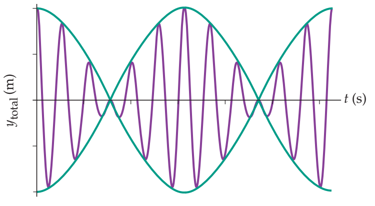
The harmonics for a column of air open at both ends are

$$f_n = nf_1 = n(v/2L) \quad n = 1, 2, 3, \ldots \qquad \lambda_n = \lambda_1/n = 2L/n \tag{14–15}$$

In both of these expressions the speed of sound is v and the length of the column is L.

14–9 BEATS

Beats occur when waves of slightly different frequencies interfere.



They can be thought of as interference patterns in time. To the ear, beats are perceived as an alternating loudness and softness to the sound.

Beat Frequency

If waves of frequencies f_1 and f_2 interfere, the beat frequency is

$$f_{beat} = |f_1 - f_2| \tag{14–18}$$

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the speed of a wave on a string, or relate the speed of a wave to the mass of a string.	The speed of a wave on a string is related to the tension in the string, F, and the string's mass per length, $\mu = m/L$, by the expression $v = \sqrt{F/\mu}$.	Example 14–1
Relate the intensity of a sound wave to its intensity level.	The intensity level of a sound wave, β, depends on the logarithm of the wave's intensity, I. The relation between β and I is $\beta = 10 \log(I/I_0)$, where $I_0 = 10^{-12}\ \mathrm{W/m^2}$.	Example 14–4
Calculate the Doppler shift for a moving source or observer.	If an observer and a source of sound with frequency f approach one another, the frequency heard by the observer is greater than f. If the source and observer recede from one another, the frequency heard by the observer is less than f.	Examples 14–5, 14–6
Calculate the beat frequency.	The beat frequency produced when sounds of frequency f_1 and f_2 are heard simultaneously is the magnitude of the difference in frequencies: $f_{beat} = \|f_1 - f_2\|$.	Example 14–10

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A long nail has been driven halfway into the side of a barn. How should you hit the nail with a hammer to generate a longitudinal wave? How should you hit it to generate a transverse wave?
2. What type of wave is exhibited by "amber waves of grain"?
3. At a ball game, a "wave" circulating through the stands can be an exciting event. What type of wave (longitudinal or transverse) are we talking about? Is it possible to change the type of wave? Explain how people might move their bodies to accomplish this.
4. In a classic TV commercial, a group of cats feed from bowls of cat food that are lined up side by side. Initially there is one cat for each bowl. When an additional cat is added to the scene, it runs to a bowl at the end of the line and begins to eat. The cat that was there originally moves to the next bowl, displacing that cat, which moves to the next bowl, and so on down the line. What type of wave have the cats created? Explain.
5. Describe how the sound of a symphony played by an orchestra would be altered if the speed of sound depended on the frequency of sound.
6. A "radar gun" is often used to measure the speed of a major league pitch by reflecting a beam of radio waves off a moving ball. Describe how the Doppler effect can give the speed of the ball from a measurement of the frequency of the reflected beam.
7. When you drive a nail into a piece of wood, you hear a tone with each blow of the hammer. In fact, the tone increases in pitch as the nail is driven farther into the wood. Explain.
8. Explain the function of the sliding part of a trombone.
9. When you tune a violin string, what causes its frequency to change?
10. On a guitar, some strings are single wires, others are wrapped with another wire to increase the mass per length. Which type of string would you expect to be used for a low-frequency note? Explain.
11. As a string oscillates in its fundamental mode, there are times when it is completely flat. Is the energy of oscillation zero at these times? Explain.
12. On a rainy day, while driving your car, you notice that your windshield wipers are moving in synchrony with the wiper blades of the car in front of you. After several cycles, however your wipers and the wipers of the other car are moving opposite to one another. A short time later the wipers are synchronous again. What wave phenomena do the wipers illustrate? Explain.
13. To play a C major chord on the piano, you hit the C, E, and G keys simultaneously. When you do so, you hear no beats. Why? (Refer to Table 14–3.)

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 14–1 TYPES OF WAVES

1. • A wave travels along a stretched horizontal rope. The vertical distance from crest to trough for this wave is 13 cm and the horizontal distance from crest to trough is 28 cm. What are **(a)** the wavelength and **(b)** the amplitude of this wave?

2. • A surfer floating beyond the breakers notes 14 waves per minute passing her position. If the wavelength of these waves is 34 m, what is their speed?

3. • The speed of surface waves in water decreases as the water becomes shallower. Suppose waves travel across the surface of a lake with a speed of 2.0 m/s and a wavelength of 1.5 m. When these waves move into a shallower part of the lake, their speed decreases to 1.6 m/s, though their frequency remains the same. Find the wavelength of the waves in the shallower water.

4. • **Tsunami** A tsunami traveling across deep water can have a speed of 750 km/h and a wavelength of 310 km. What is the frequency of such a wave?

5. •• **IP** A 4.5-Hz wave with an amplitude of 12 cm and a wavelength of 27 cm travels along a stretched horizontal string. **(a)** How far does a given peak on the wave travel in a time interval of 0.50 s? **(b)** How far does a knot on the string travel in the same time interval? **(c)** How would your answers to parts (a) and (b) change if the amplitude of the wave were halved? Explain.

6. •• **Deepwater Waves** The speed of a deepwater wave with a wavelength λ is given approximately by $v = \sqrt{g\lambda/2\pi}$. Find the speed and frequency of a deepwater wave with a wavelength of 4.5 m.

7. •• **Shallow-Water Waves** In shallow water of depth d the speed of waves is approximately $v = \sqrt{gd}$. Find the speed and frequency of a wave with wavelength 0.75 cm in water that is 2.6 cm deep.

SECTION 14–2 WAVES ON A STRING

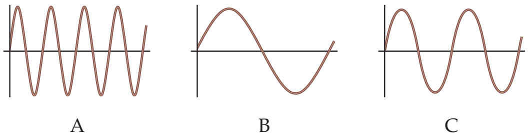
8. • **CE** Consider a wave on a string with constant tension. If the frequency of the wave is doubled, by what multiplicative factor does **(a)** the speed and **(b)** the wavelength change?

9. • **CE** Suppose you would like to double the speed of a wave on a string. By what multiplicative factor must you increase the tension?

10. • **CE Predict/Explain** Two strings are made of the same material and have equal tensions. String 1 is thick; string 2 is thin. **(a)** Is the speed of waves on string 1 greater than, less than, or equal to the speed of waves on string 2? **(b)** Choose the *best explanation* from among the following:
    I. Since the strings are made of the same material, the wave speeds will also be the same.
    II. A thick string implies a large mass per length and a slow wave speed.
    III. A thick string has a greater force constant, and therefore a greater wave speed.

11. • **CE Predict/Explain** Two strings are made of the same material and have waves of equal speed. String 1 is thick; string 2 is thin. **(a)** Is the tension in string 1 greater than, less than, or equal to the tension in string 2? **(b)** Choose the *best explanation* from among the following:
   I. String 1 must have a greater tension to compensate for its greater mass per length.
   II. String 2 will have a greater tension because it is thinner than string 1.
   III. Equal wave speeds implies equal tensions.

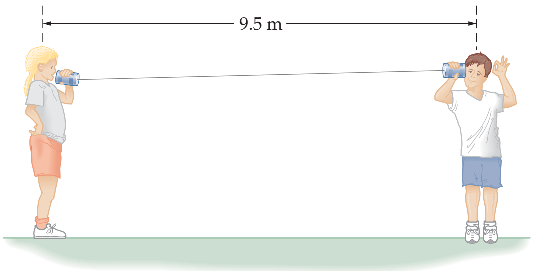
12. • **CE** The three waves, A, B and C, shown in **Figure 14–32** propagate on strings with equal tensions and equal mass per length. Rank the waves in order of increasing **(a)** frequency, **(b)** wavelength, and **(c)** speed. Indicate ties where appropriate.



▲ **FIGURE 14–32** Problem 12

13. • Waves on a particular string travel with a speed of 16 m/s. By what factor should the tension in this string be changed to produce waves with a speed of 32 m/s?

14. •• A brother and sister try to communicate with a string tied between two tin cans (**Figure 14–33**). If the string is 9.5 m long, has a mass of 32 g, and is pulled taut with a tension of 8.6 N, how much time does it take for a wave to travel from one end of the string to the other?



▲ **FIGURE 14–33** Problems 14 and 15

15. •• **IP** **(a)** Suppose the tension is increased in the previous problem. Does a wave take more, less, or the same time to travel from one end to the other? Calculate the time of travel for tensions of **(b)** 9.0 N and **(c)** 10.0 N.

16. •• **IP** A 5.2-m wire with a mass of 87 g is attached to the mast of a sailboat. If the wire is given a "thunk" at one end, it takes 0.094 s for the resulting wave to reach the other end. **(a)** What is the tension in the wire? **(b)** Would the tension found in part (a) be larger or smaller if the mass of the wire is greater than 87 g? **(c)** Calculate the tension for a 97-g wire.

17. •• Two steel guitar strings have the same length. String A has a diameter of 0.50 mm and is under 410.0 N of tension. String B has a diameter of 1.0 mm and is under a tension of 820.0 N. Find the ratio of the wave speeds, v_A/v_B, in these two strings.

18. ••• Use dimensional analysis to show how the speed v of a wave on a string of circular cross section depends on the tension in the string, T, the radius of the string, R, and its mass per volume, ρ.

*SECTION 14–3 HARMONIC WAVE FUNCTIONS

19. • Write an expression for a harmonic wave with an amplitude of 0.16 m, a wavelength of 2.1 m, and a period of 1.8 s. The wave is transverse, travels to the right, and has a displacement of 0.16 m at $t = 0$ and $x = 0$.

20. • Write an expression for a transverse harmonic wave that has a wavelength of 2.6 m and propagates to the right with a speed of 14.3 m/s. The amplitude of the wave is 0.11 m, and its displacement at $t = 0$ and $x = 0$ is 0.11 m.

21. •• **CE** The vertical displacement of a wave on a string is described by the equation $y(x, t) = A\sin(Bx - Ct)$, in which A, B, and C are positive constants. **(a)** Does this wave propagate in the positive or negative x direction? **(b)** What is the wavelength of this wave? **(c)** What is the frequency of this wave? **(d)** What is the smallest positive value of x where the displacement of this wave is zero at $t = 0$?

22. •• **CE** The vertical displacement of a wave on a string is described by the equation $y(x, t) = A\sin(Bx + Ct)$, in which A, B, and C are positive constants. **(a)** Does this wave propagate in the positive or negative x direction? **(b)** What is the physical meaning of the constant A? **(c)** What is the speed of this wave? **(d)** What is the smallest positive time t for which the wave has zero displacement at the point $x = 0$?

23. •• **IP** A wave on a string is described by the following equation:

$$y = (15\text{ cm})\cos\left(\frac{\pi}{5.0\text{ cm}}x - \frac{\pi}{12\text{ s}}t\right)$$

(a) What is the amplitude of this wave? **(b)** What is its wavelength? **(c)** What is its period? **(d)** What is its speed? **(e)** In which direction does the wave travel?

24. •• Consider the wave function given in the previous problem. Sketch this wave from $x = 0$ to $x = 10$ cm for the following times: **(a)** $t = 0$; **(b)** $t = 3.0$ s; **(c)** $t = 6.0$ s. **(d)** What is the least amount of time required for a given point on this wave to move from $y = 0$ to $y = 15$ cm? Verify your answer by referring to the sketches for parts (a), (b), and (c).

25. •• **IP** Four waves are described by the following equations, in which all distances are measured in centimeters and all times are measured in seconds:

$$y_A = 10\cos(3x - 4t)$$
$$y_B = 10\cos(5x + 4t)$$
$$y_C = 20\cos(-10x + 60t)$$
$$y_D = 20\cos(-4x - 20t)$$

(a) Which of these waves travel in the $+x$ direction? **(b)** Which of these waves travel in the $-x$ direction? **(c)** Which wave has the highest frequency? **(d)** Which wave has the greatest wavelength? **(e)** Which wave has the greatest speed?

SECTION 14–4 SOUND WAVES

26. • At Zion National Park a loud shout produces an echo 1.80 s later from a colorful sandstone cliff. How far away is the cliff?

27. • **BIO Dolphin Ultrasound** Dolphins of the open ocean are classified as Type II Odontocetes (toothed whales). These animals use ultrasonic "clicks" with a frequency of about 55 kHz to navigate and find prey. **(a)** Suppose a dolphin sends out a series of clicks that are reflected back from the bottom of the ocean 75 m below. How much time elapses before the dolphin hears the echoes of the clicks? (The speed of sound in seawater is approximately 1530 m/s.) **(b)** What is the wavelength of 55-kHz sound in the ocean?

28. • The lowest note on a piano is A, four octaves below the A given in Table 14–3. The highest note on a piano is C, four octaves above middle C. Find the frequencies and wavelengths (in air) of these notes.

29. •• **IP** A sound wave in air has a frequency of 425 Hz. **(a)** What is its wavelength? **(b)** If the frequency of the sound is increased, does its wavelength increase, decrease, or stay the same? Explain. **(c)** Calculate the wavelength for a sound wave with a frequency of 475 Hz.

30. •• **IP** When you drop a rock into a well, you hear the splash 1.5 seconds later. **(a)** How deep is the well? **(b)** If the depth of the well were doubled, would the time required to hear the splash be greater than, less than, or equal to 3.0 seconds? Explain.

31. •• A rock is thrown downward into a well that is 8.85 m deep. If the splash is heard 1.20 seconds later, what was the initial speed of the rock?

SECTION 14–5 SOUND INTENSITY

32. • **CE** If the distance to a point source of sound is doubled, by what multiplicative factor does the intensity change?

33. • The intensity level of sound in a truck is 92 dB. What is the intensity of this sound?

34. • The distance to a point source is decreased by a factor of three. **(a)** By what multiplicative factor does the intensity increase? **(b)** By what additive amount does the intensity level increase?

35. • Sound 1 has an intensity of 38.0 W/m^2. Sound 2 has an intensity level that is 2.5 dB greater than the intensity level of sound 1. What is the intensity of sound 2?

36. •• A bird-watcher is hoping to add the white-throated sparrow to her "life list" of species. How far could she be from the bird described in Example 14–3 and still be able to hear it? Assume no reflections or absorption of the sparrow's sound.

37. •• Residents of Hawaii are warned of the approach of a tsunami by sirens mounted on the tops of towers. Suppose a siren produces a sound that has an intensity level of 120 dB at a distance of 2.0 m. Treating the siren as a point source of sound, and ignoring reflections and absorption, find the intensity level heard by an observer at a distance of **(a)** 12 m and **(b)** 21 m from the siren. **(c)** How far away can the siren be heard?

38. •• In a pig-calling contest, a caller produces a sound with an intensity level of 110 dB. How many such callers would be required to reach the pain level of 120 dB?

39. •• **IP** Twenty violins playing simultaneously with the same intensity combine to give an intensity level of 82.5 dB. **(a)** What is the intensity level of each violin? **(b)** If the number of violins is increased to 40, will the combined intensity level be more than, less than, or equal to 165 dB? Explain.

40. •• **BIO The Human Eardrum** The radius of a typical human eardrum is about 4.0 mm. Find the energy per second received by an eardrum when it listens to sound that is **(a)** at the threshold of hearing and **(b)** at the threshold of pain.

41. ••• A point source of sound that emits uniformly in all directions is located in the middle of a large, open field. The intensity at Brittany's location directly north of the source is twice that at Phillip's position due east of the source. What is the distance between Brittany and Phillip if Brittany is 12.5 m from the source?

SECTION 14–6 THE DOPPLER EFFECT

42. • **CE Predict/Explain** A horn produces sound with frequency f_0. Let the frequency you hear when you are at rest and the horn moves toward you with a speed u be f_1; let the frequency you hear when the horn is at rest and you move toward it with a speed u be f_2. **(a)** Is f_1 greater than, less than, or equal to f_2? **(b)** Choose the *best explanation* from among the following:
   I. A moving observer encounters wave crests more often than a stationary observer, leading to a higher frequency.
   II. The relative speeds are the same in either case. Therefore, the frequencies will be the same as well.
   III. A moving source causes the wave crests to "bunch up," leading to a higher frequency than for a moving observer.

43. • **CE** You are heading toward an island in your speedboat when you see a friend standing on shore at the base of a cliff. You sound the boat's horn to get your friend's attention. Let the wavelength of the sound produced by the horn be λ_1, the wavelength as heard by your friend be λ_2, and the wavelength of the echo as heard on the boat be λ_3. Rank these wavelengths in order of increasing length. Indicate ties where appropriate.

44. • A person with perfect pitch sits on a bus bench listening to the 450-Hz horn of an approaching car. If the person detects a frequency of 470 Hz, how fast is the car moving?

45. • A train moving with a speed of 31.8 m/s sounds a 136-Hz horn. What frequency is heard by an observer standing near the tracks as the train approaches?

46. • In the previous problem, suppose the stationary observer sounds a horn that is identical to the one on the train. What frequency is heard from this horn by a passenger in the train?

47. • **BIO** A bat moving with a speed of 3.25 m/s and emitting sound of 35.0 kHz approaches a moth at rest on a tree trunk. **(a)** What frequency is heard by the moth? **(b)** If the speed of the bat is increased, is the frequency heard by the moth higher or lower? **(c)** Calculate the frequency heard by the moth when the speed of the bat is 4.25 m/s.

48. • A motorcycle and a police car are moving toward one another. The police car emits sound with a frequency of 502 Hz and has a speed of 27.0 m/s. The motorcycle has a speed of 13.0 m/s. What frequency does the motorcyclist hear?

49. • In the previous problem, suppose that the motorcycle and the police car are moving in the same direction, with the motorcycle in the lead. What frequency does the motorcyclist hear in this case?

50. •• Hearing the siren of an approaching fire truck, you pull over to the side of the road and stop. As the truck approaches, you hear a tone of 460 Hz; as the truck recedes, you hear a tone of 410 Hz. How much time will it take for the truck to get from your position to the fire 5.0 km away, assuming it maintains a constant speed?

51. •• With what speed must you approach a source of sound to observe a 15% change in frequency?

52. •• **IP** A particular jet engine produces a tone of 495 Hz. Suppose that one jet is at rest on the tarmac while a second identical jet flies overhead at 82.5% of the speed of sound. The pilot of each jet listens to the sound produced by the engine of the other jet. **(a)** Which pilot hears a greater Doppler shift? Explain. **(b)** Calculate the frequency heard by the pilot in the moving jet. **(c)** Calculate the frequency heard by the pilot in the stationary jet.

53. •• **IP** Two bicycles approach one another, each traveling with a speed of 8.50 m/s. **(a)** If bicyclist A beeps a 315-Hz horn, what frequency is heard by bicyclist B? **(b)** Which of the following would cause the greater increase in the frequency heard by bicyclist B: (i) bicyclist A speeds up by 1.50 m/s, or (ii) bicyclist B speeds up by 1.50 m/s? Explain.

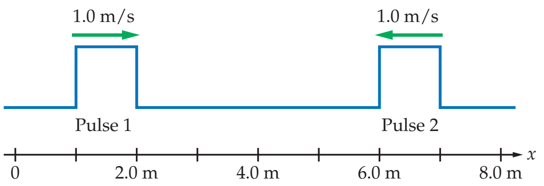
54. •• A train on one track moves in the same direction as a second train on the adjacent track. The first train, which is ahead of the second train and moves with a speed of 36.8 m/s, blows a horn whose frequency is 124 Hz. If the frequency heard on the second train is 135 Hz, what is its speed?

55. •• Two cars traveling with the same speed move directly away from one another. One car sounds a horn whose frequency is 205 Hz and a person in the other car hears a frequency of 192 Hz. What is the speed of the cars?

56. ••• **The Bullet Train** The Shinkansen, the Japanese "bullet" train, runs at high speed from Tokyo to Nagoya. Riding on the Shinkansen, you notice that the frequency of a crossing signal changes markedly as you pass the crossing. As you approach the crossing, the frequency you hear is f; as you recede from the crossing the frequency you hear is $2f/3$. What is the speed of the train?

SECTION 14–7 SUPERPOSITION AND INTERFERENCE

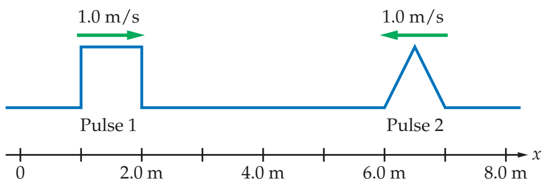
57. • Two wave pulses on a string approach one another at the time $t = 0$, as shown in **Figure 14–34**. Each pulse moves with a speed of 1.0 m/s. Make a careful sketch of the resultant wave at the times $t = 1.0$ s, 2.0 s, 2.5 s, 3.0 s, and 4.0 s, assuming that the superposition principle holds for these waves.



▲ **FIGURE 14–34** Problems 57 and 58

58. • Suppose pulse 2 in Problem 57 is inverted, so that it is a downward deflection of the string rather than an upward deflection. Repeat Problem 57 in this case.

59. • Two wave pulses on a string approach one another at the time $t = 0$, as shown in **Figure 14–35**. Each pulse moves with a speed of 1.0 m/s. Make a careful sketch of the resultant wave at the times $t = 1.0$ s, 2.0 s, 2.5 s, 3.0 s, and 4.0 s, assuming that the superposition principle holds for these waves.



▲ **FIGURE 14–35** Problems 59 and 60

60. • Suppose pulse 2 in Problem 59 is inverted, so that it is a downward deflection of the string rather than an upward deflection. Repeat Problem 59 in this case.

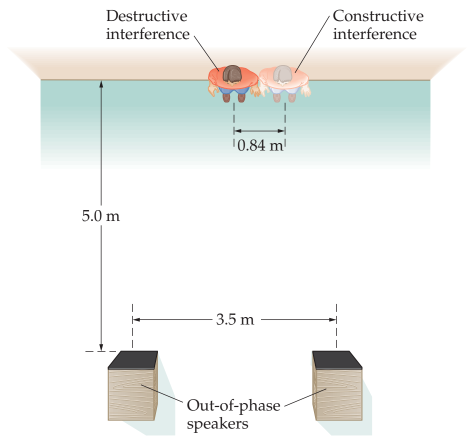
61. •• A pair of in-phase stereo speakers is placed side by side, 0.85 m apart. You stand directly in front of one of the speakers, 1.1 m from the speaker. What is the lowest frequency that will produce constructive interference at your location?

62. •• **IP** Two violinists, one directly behind the other, play for a listener directly in front of them. Both violinists sound concert A (440 Hz). **(a)** What is the smallest separation between the violinists that will produce destructive interference for the listener? **(b)** Does this smallest separation increase or decrease if the violinists produce a note with a higher frequency? **(c)** Repeat part (a) for violinists who produce sounds of 540 Hz.

63. •• Two loudspeakers are placed at either end of a gymnasium, both pointing toward the center of the gym and equidistant from it. The speakers emit 266-Hz sound that is in phase. An observer at the center of the gym experiences constructive interference. How far toward either speaker must the observer walk to first experience destructive interference?

64. •• **IP** **(a)** In the previous problem, does the required distance increase, decrease, or stay the same if the frequency of the speakers is lowered? **(b)** Calculate the distance to the first position of destructive interference if the frequency emitted by the speakers is lowered to 238 Hz.

65. •• Two speakers with opposite phase are positioned 3.5 m apart, both pointing toward a wall 5.0 m in front of them (**Figure 14–36**). An observer standing against the wall midway between the speakers hears destructive interference. If the observer hears constructive interference after moving 0.84 m to one side along the wall, what is the frequency of the sound emitted by the speakers?



▲ **FIGURE 14–36** Problem 65

66. •• Suppose, in Example 14–7, that the speakers have opposite phase. What is the lowest frequency that gives destructive interference in this case?

SECTION 14–8 STANDING WAVES

67. • **CE Predict/Explain** When you blow across the opening of a partially filled 2-L soda pop bottle you hear a tone. **(a)** If you take a sip of the pop and blow across the opening again, does the tone you hear have a higher frequency, a lower frequency, or the same frequency as before? **(b)** Choose the *best explanation* from among the following:

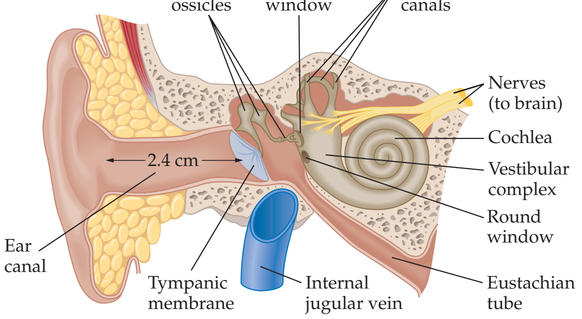
I. The same pop bottle will give the same frequency regardless of the amount of pop it contains.
II. The greater distance from the top of the bottle to the level of the pop results in a higher frequency.
III. A lower level of pop results in a longer column of air, and hence a lower frequency.

68. • An organ pipe that is open at both ends is 3.5 m long. What is its fundamental frequency?

69. • A string 1.5 m long with a mass of 2.6 g is stretched between two fixed points with a tension of 93 N. Find the frequency of the fundamental on this string.

70. •• **CE** A string is tied down at both ends. Some of the standing waves on this string have the following frequencies: 100 Hz, 200 Hz, 250 Hz, and 300 Hz. It is also known that there are no standing waves with frequencies between 250 Hz and 300 Hz. **(a)** What is the fundamental frequency of this string? **(b)** What is the frequency of the third harmonic?

71. •• **IP BIO Standing Waves in the Human Ear** The human ear canal is much like an organ pipe that is closed at one end (at the tympanic membrane or eardrum) and open at the other (**Figure 14–37**). A typical ear canal has a length of about 2.4 cm. **(a)** What are the fundamental frequency and wavelength of the ear canal? **(b)** Find the frequency and wavelength of the ear canal's third harmonic. (Recall that the third harmonic in this case is the standing wave with the second-lowest frequency.) **(c)** Suppose a person has an ear canal that is shorter than 2.4 cm. Is the fundamental frequency of that person's ear canal greater than, less than, or the same as the value found in part (a)? Explain. [Note that the frequencies found in parts (a) and (b) correspond closely to the frequencies of enhanced sensitivity in Figure 14–28.]



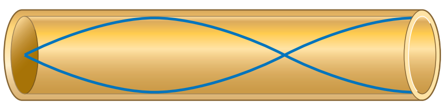
▲ **FIGURE 14–37** Problem 71

72. •• A guitar string 66 cm long vibrates with a standing wave that has three antinodes. **(a)** Which harmonic is this? **(b)** What is the wavelength of this wave?

73. •• **IP** A 12.5-g clothesline is stretched with a tension of 22.1 N between two poles 7.66 m apart. What is the frequency of **(a)** the fundamental and **(b)** the second harmonic? **(c)** If the tension in the clothesline is increased, do the frequencies in parts (a) and (b) increase, decrease, or stay the same? Explain.

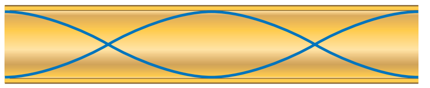
74. •• **IP** **(a)** In the previous problem, will the frequencies increase, decrease, or stay the same if a more massive rope is used? **(b)** Repeat Problem 73 for a clothesline with a mass of 15.0 g.

75. •• The organ pipe in **Figure 14–38** is 2.75 m long. **(a)** What is the frequency of the standing wave shown in the pipe? **(b)** What is the fundamental frequency of this pipe?



▲ **FIGURE 14–38** Problem 75

76. •• The frequency of the standing wave shown in **Figure 14–39** is 202 Hz. **(a)** What is the fundamental frequency of this pipe? **(b)** What is the length of the pipe?



▲ **FIGURE 14–39** Problem 76

77. ••• An organ pipe open at both ends has a harmonic with a frequency of 440 Hz. The next higher harmonic in the pipe has a frequency of 495 Hz. Find **(a)** the frequency of the fundamental and **(b)** the length of the pipe.

SECTION 14–9 BEATS

78. • **CE** When guitar strings A and B are plucked at the same time, a beat frequency of 2 Hz is heard. If string A is tightened, the beat frequency increases to 3 Hz. Which of the two strings had the lower frequency initially?

79. • **CE Predict/Explain** **(a)** Is the beat frequency produced when a 245-Hz tone and a 240-Hz tone are played together greater than, less than, or equal to the beat frequency produced when a 140-Hz tone and a 145-Hz tone are played together? **(b)** Choose the *best explanation* from among the following:
   I. The beat frequency is determined by the difference in frequencies and is independent of their actual values.
   II. The higher frequencies will produce a higher beat frequency.
   III. The percentage change in frequency for 240 and 245 Hz is less than for 140 and 145 Hz, resulting in a lower beat frequency.

80. • Two tuning forks have frequencies of 278 Hz and 292 Hz. What is the beat frequency if both tuning forks are sounded simultaneously?

81. • **Tuning a Piano** To tune middle C on a piano, a tuner hits the key and at the same time sounds a 261-Hz tuning fork. If the tuner hears 3 beats per second, what are the possible frequencies of the piano key?

82. • Two musicians are comparing their clarinets. The first clarinet produces a tone that is known to be 441 Hz. When the two clarinets play together they produce eight beats every 2.00 seconds. If the second clarinet produces a higher pitched tone than the first clarinet, what is the second clarinet's frequency?

83. •• **IP** Two strings that are fixed at each end are identical, except that one is 0.560 cm longer than the other. Waves on these strings propagate with a speed of 34.2 m/s, and the fundamental frequency of the shorter string is 212 Hz. **(a)** What beat frequency is produced if each string is vibrating with its fundamental frequency? **(b)** Does the beat frequency in part (a) increase or decrease if the longer string is increased in length? **(c)** Repeat part (a), assuming that the longer string is 0.761 cm longer than the shorter string.

84. •• **IP** A tuning fork with a frequency of 320.0 Hz and a tuning fork of unknown frequency produce beats with a frequency of 4.5 Hz. If the frequency of the 320.0-Hz fork is lowered slightly by placing a bit of putty on one of its tines, the new beat frequency is 7.5 Hz. **(a)** Which tuning fork has the lower frequency? Explain. **(b)** What is the final frequency of the 320.0-Hz tuning fork? **(c)** What is the frequency of the other tuning fork?

85. •• Identical cellos are being tested. One is producing a fundamental frequency of 130.9 Hz on a string that is 1.25 m long and has a mass of 109 g. On the second cello the same string is

fingered to reduce the length that can vibrate. If the beat frequency produced by these two strings is 4.33 Hz, what is the vibrating length of the second string?

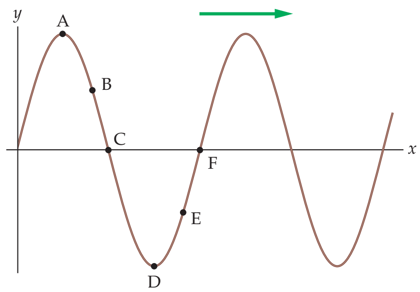
86. ••• A friend in another city tells you that she has two organ pipes of different lengths, one open at both ends, the other open at one end only. In addition, she has determined that the beat frequency caused by the second-lowest frequency of each pipe is equal to the beat frequency caused by the third-lowest frequency of each pipe. Her challenge to you is to calculate the length of the organ pipe that is open at both ends, given that the length of the other pipe is 1.00 m.

GENERAL PROBLEMS

87. • **CE** A harmonic wave travels along a string. **(a)** At a point where the displacement of the string is greatest, is the kinetic energy of the string a maximum or a minimum? Explain. **(b)** At a point where the displacement of the string is zero, is the kinetic energy of the string a maximum or a minimum? Explain.

88. • **CE** A harmonic wave travels along a string. **(a)** At a point where the displacement of the string is greatest, is the potential energy of the string a maximum or a minimum? Explain. **(b)** At a point where the displacement of the string is zero, is the potential energy of the string a maximum or a minimum? Explain.

89. • **CE** **Figure 14–40** shows a wave on a string moving to the right. For each of the points indicated on the string, A–F, state whether it is (**I**, moving upward; **II**, moving downward; or **III**, instantaneously at rest) at the instant pictured.



▲ **FIGURE 14–40** Problem 89

90. • **CE** You stand near the tracks as a train approaches with constant speed. The train is operating its horn continuously, and you listen carefully to the sound it makes. For each of the following properties of the sound, state whether it increases, decreases, or stays the same as the train gets closer: **(a)** the intensity; **(b)** the frequency; **(c)** the wavelength; **(d)** the speed.

91. • Sitting peacefully in your living room one stormy day, you see a flash of lightning through the windows. Eight and a half seconds later thunder shakes the house. Estimate the distance from your house to the bolt of lightning.

92. • The fundamental of an organ pipe that is closed at one end and open at the other end is 261.6 Hz (middle C). The second harmonic of an organ pipe that is open at both ends has the same frequency. What are the lengths of these two pipes?

93. • **The Loudest Animal** The loudest sound produced by a living organism on Earth is made by the bowhead whale (*Balaena mysticetus*). These whales can produce a sound that, if heard in air at a distance of 3.00 m, would have an intensity level of 127 dB. This is roughly the equivalent of 5000 trumpeting elephants. How far away can you be from a 127-dB sound and still just barely hear it?(Assume a point source, and ignore reflections and absorption.)

94. • **Hearing a Good Hit** Physicist Robert Adair, once appointed the "official physicist to the National League" by the commissioner of baseball, believes that the "crack of the bat" can tell an outfielder how well the ball has been hit. According to Adair, a good hit makes a sound of 510 Hz, while a poor hit produces a sound of 170 Hz. What is the difference in wavelength of these sounds?

95. • A standing wave of 603 Hz is produced on a string that is 1.33 m long and fixed on both ends. If the speed of waves on this string is 402 m/s, how many antinodes are there in the standing wave?

96. • **BIO** **Measuring Hearing Loss** To determine the amount of temporary hearing loss loud music can cause in humans, researchers studied a group of 20 adult females who were exposed to 110-dB music for 60 minutes. Eleven of the 20 subjects showed a 20.0-dB reduction in hearing sensitivity at 4000 Hz. What is the intensity corresponding to the threshold of hearing for these subjects?

97. •• **BIO** **Hearing a Pin Drop** The ability to hear a "pin drop" is the sign of sensitive hearing. Suppose a 0.55-g pin is dropped from a height of 28 cm, and that the pin emits sound for 1.5 s when it lands. Assuming all of the mechanical energy of the pin is converted to sound energy, and that the sound radiates uniformly in all directions, find the maximum distance from which a person can hear the pin drop. (This is the ideal maximum distance, but atmospheric absorption and other factors will make the actual maximum distance considerably smaller.)

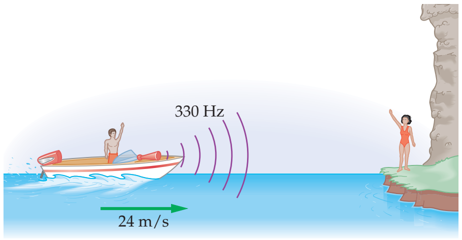
98. •• A machine shop has 120 equally noisy machines that together produce an intensity level of 92 dB. If the intensity level must be reduced to 82 dB, how many machines must be turned off?

99. •• **IP** When you blow across the top of a soda pop bottle you hear a fundamental frequency of 206 Hz. Suppose the bottle is now filled with helium. **(a)** Does the fundamental frequency increase, decrease, or stay the same? Explain. **(b)** Find the new fundamental frequency. (Assume that the speed of sound in helium is three times that in air.)

100. •• **Speed of a Tsunami** Tsunamis can have wavelengths between 100 and 400 km. Since this is much greater than the average depth of the oceans (about 4.3 km), the ocean can be considered as shallow water for these waves. Using the speed of waves in shallow water of depth d given in Problem 7, find the typical speed for a tsunami. (*Note*: In the open ocean, tsunamis generally have an amplitude of less than a meter, allowing them to pass ships unnoticed. As they approach shore, however, the water depth decreases and the waves slow down. This can result in an increase of amplitude to as much as 37 m or more.)

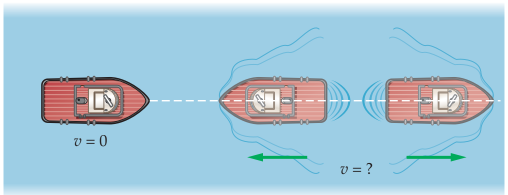
101. •• Two trains with 124-Hz horns approach one another. The slower of the two trains has a speed of 26 m/s. What is the speed of the fast train if an observer standing near the tracks between the trains hears a beat frequency of 4.4 Hz?

102. •• **IP** Jim is speeding toward James Island with a speed of 24 m/s when he sees Betsy standing on shore at the base of a cliff (**Figure 14–41**). Jim sounds his 330-Hz horn. **(a)** What frequency does Betsy hear? **(b)** Jim can hear the echo of his horn reflected back to him by the cliff. Is the frequency of this echo greater than or equal to the frequency heard by Betsy? Explain. **(c)** Calculate the frequency Jim hears in the echo from the cliff.



▲ **FIGURE 14–41** Problem 102

103. •• Two ships in a heavy fog are blowing their horns, both of which produce sound with a frequency of 175.0 Hz (**Figure 14–42**). One ship is at rest; the other moves on a straight line that passes through the one at rest. If people on the stationary ship hear a beat frequency of 3.5 Hz, what are the two possible speeds and directions of motion of the moving ship?



▲ **FIGURE 14–42** Problem 103

104. •• **BIO Cracking Your Knuckles** When you "crack" a knuckle, you cause the knuckle cavity to widen rapidly. This, in turn, allows the synovial fluid to expand into a larger volume. If this expansion is sufficiently rapid, it causes a gas bubble to form in the fluid in a process known as *cavitation*. This is the mechanism responsible for the cracking sound. (Cavitation can also cause pits in rapidly rotating ship's propellers.) If a "crack" produces a sound with an intensity level of 57 dB at your ear, which is 18 cm from the knuckle, how far from your knuckle can the "crack" be heard? Assume the sound propagates uniformly in all directions, with no reflections or absorption.

105. •• A steel guitar string has a tension T, length L, and diameter D. Give the multiplicative factor by which the fundamental frequency of the string changes under the following conditions: **(a)** The tension in the string is increased by a factor of 4. The diameter is D and the length is L. **(b)** The diameter of the string is increased by a factor of 3. The tension is T and the length is L. **(c)** The length of the string is halved. The tension is T and the diameter is D.

106. •• A Slinky has a mass of 0.28 kg and negligible length. When it is stretched 1.5 m, it is found that transverse waves travel the length of the Slinky in 0.75 s. **(a)** What is the force constant, k, of the Slinky? **(b)** If the Slinky is stretched farther, will the time required for a wave to travel the length of the Slinky increase, decrease, or stay the same? Explain. **(c)** If the Slinky is stretched 3.0 m, how much time does it take a wave to travel the length of the Slinky? (The Slinky stretches like an ideal spring, and propagates transverse waves like a rope with variable tension.)

107. •• **IP BIO OSHA Noise Standards** OSHA, the Occupational Safety and Health Administration, has established standards for workplace exposure to noise. According to OSHA's Hearing Conservation Standard, the permissible noise exposure per day is 95.0 dB for 4 hours or 105 dB for 1 hour. Assuming the eardrum is 9.5 mm in diameter, find the energy absorbed by the eardrum **(a)** with 95.0 dB for 4 hours and **(b)** with 105 dB for 1 hour. **(c)** Is OSHA's safety standard simply a measure of the amount of energy absorbed by the ear? Explain.

108. •• **IP Thundersticks at Ball Games** "Thundersticks" are a popular noisemaking device at many sporting events. A typical thunderstick is a hollow plastic tube about 82 cm long and 8.5 cm in diameter. When two thundersticks are hit sharply together, they produce a copious amount of noise. **(a)** Which dimension, the length or diameter, is more important in determining the frequency of the sound emitted by the thundersticks? Explain. **(b)** Estimate the characteristic frequency of the thunderstick's sound. **(c)** Suppose a single pair of thundersticks produces sound with an intensity level of 95 dB. What is the intensity level of 1200 pairs of thundersticks clapping simultaneously?

109. •• An organ pipe 2.5 m long is open at one end and closed at the other end. What is the linear distance between a node and the adjacent antinode for the third harmonic in this pipe?

110. •• Two identical strings with the same tension vibrate at 631 Hz. If the tension in one of the strings is increased by 2.25%, what is the resulting beat frequency?

111. •• **The Sound of a Black Hole** Astronomers using the Chandra X-ray Observatory have discovered that the Perseus Black Hole, some 250 million light years away, produces sound waves in the gaseous halo that surrounds it. The frequency of this sound is the same as the frequency of the 59th B-flat below the B-flat given in Table 14–3. How long does it take for this sound wave to complete one cycle? Give your answer in years.

112. •• **BIO The Love Song of the Midshipman Fish** When the plainfin midshipman fish (*Porichthys notatus*) migrates from deep Pacific waters to the west coast of North America each summer, the males begin to sing their "love song," which some describe as sounding like a low-pitched motorboat. Houseboat residents and shore dwellers are kept awake for nights on end by the amorous fish. The love song consists of a single note, the second A flat below middle C. **(a)** If the speed of sound in seawater is 1531 m/s, what is the wavelength of the midshipman's song? **(b)** What is the wavelength of the sound after it emerges into the air? (Information on the musical scale is given in Table 14–3.)

113. ••• **IP** A rope of length L and mass M hangs vertically from a ceiling. The tension in the rope is only that due to its own weight. **(a)** Suppose a wave starts near the bottom of the rope and propagates upward. Does the speed of the wave increase, decrease, or stay the same as it moves up the rope? Explain. **(b)** Show that the speed of waves a height y above the bottom of the rope is $v = \sqrt{gy}$.

114. ••• Experiments on water waves show that the speed of waves in shallow water is independent of their wavelength (see Problem 7). Using this observation and dimensional analysis, determine how the speed v of shallow-water waves depends on the depth of the water, d, the mass per volume of water, ρ, and the acceleration of gravity, g.

115. ••• A deepwater wave of wavelength λ has a speed given approximately by $v = \sqrt{g\lambda/2\pi}$. Find an expression for the period of a deepwater wave in terms of its wavelength. (Note the similarity of your result to the period of a pendulum.)

116. ••• **Beats and Standing Waves** In Problem 63, suppose the observer walks toward one speaker with a speed of 1.35 m/s. **(a)** What frequency does the observer hear from each speaker? **(b)** What beat frequency does the observer hear? **(c)** How far must the observer walk to go from one point of constructive interference to the next? **(d)** How many times per second does the observer hear maximum loudness from the speakers? Compare your result with the beat frequency from part (b).

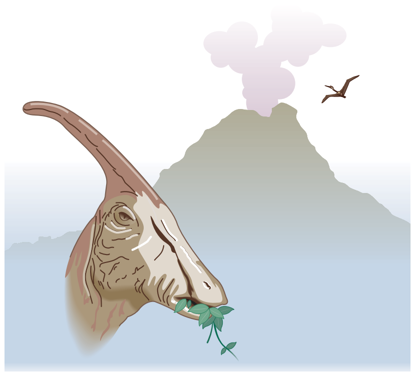
PASSAGE PROBLEMS

BIO The Sound of a Dinosaur

Modern-day animals make extensive use of sounds in their interactions with others. Some sounds are meant primarily for members of the same species, like the cooing calls of a pair of doves, the long-range infrasound communication between elephants, or the songs of the hump-backed whale. Other sounds may be used as a threat to other species, such as the rattle of a rattlesnake or the roar of a lion.

There is little doubt that extinct animals used sounds in much the same ways. But how can we ever hear the call of a long-vanished animal like a dinosaur when sounds don't fossilize? In some cases, basic physics may have the answer.

Consider, for example, the long-crested, duck-billed dinosaur *Parasaurolophus walkeri*, which roamed the Earth 75 million years ago. This dinosaur possessed the largest crest of any duck bill—so long, in fact, that there was a notch in *P. walkeri*'s spine to make room for the crest when its head was tilted backward. Many paleontologists believe the air passages in the dinosaur's crest acted like bent organ pipes open at both ends, and that they produced sounds *P. walkeri* used to communicate with others of its kind. As air was forced through the passages, the predominant sound they produced would be the fundamental standing wave, with a small admixture of higher harmonics as well. The frequencies of these standing waves can be determined with basic physical principles. **Figure 14–43** presents a plot of the lowest ten harmonics of a pipe that is open at both ends as a function of the length of the pipe.



The long crest of *Parasaurolophus walkeri* played a key role in its communications with others.

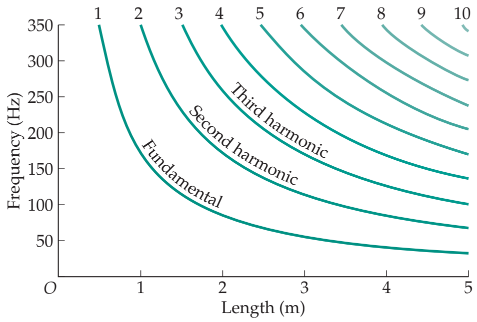


FIGURE 14–43 Standing wave frequencies as a function of length for a pipe open at both ends. The first ten harmonics ($n = 1$–10) are shown. (Problems 117, 118, 119, and 120)

117. • Suppose the air passages in a certain *P. walkeri* crest produce a bent tube 2.7 m long. What is the fundamental frequency of this tube, assuming the bend has no effect on the frequency? (For comparison, a typical human hearing range is 20 Hz to 20 kHz.)

A. 0.0039 Hz **B.** 32 Hz

C. 64 Hz **D.** 130 Hz

118. • Paleontologists believe the crest of a female *P. walkeri* was probably shorter than the crest of a male. If this was the case, would the fundamental frequency of a female be greater than, less than, or equal to the fundamental frequency of a male?

119. • Suppose the fundamental frequency of a particular female was 74 Hz. What was the length of the air passages in this female's crest?

A. 1.2 m **B.** 2.3 m

C. 2.7 m **D.** 4.6 m

120. • As a young *P. walkeri* matured, the air passages in its crest might increase in length from 1.5 m to 2.7 m, causing a decrease in the standing wave frequencies. Referring to Figure 14–43, do you expect the change in the fundamental frequency to be greater than, less than, or equal to the change in the second harmonic frequency?

INTERACTIVE PROBLEMS

121. •• **IP Referring to Example 14–6** Suppose the engineer adjusts the speed of the train until the sound he hears reflected from the cliff is 775 Hz. The train's whistle still produces a tone of 650.0 Hz. **(a)** Is the new speed of the train greater than, less than, or equal to 21.2 m/s? Explain. **(b)** Find the new speed of the train.

122. •• **Referring to Example 14–6** Suppose the train is backing away from the cliff with a speed of 18.5 m/s and is sounding its 650.0-Hz whistle. **(a)** What is the frequency heard by the observer standing near the tunnel entrance? **(b)** What is the frequency heard by the engineer?

123. •• **IP Referring to Example 14–9** Suppose we add more water to the soda pop bottle. **(a)** Does the fundamental frequency increase, decrease, or stay the same? Explain. **(b)** Find the fundamental frequency if the height of water in the bottle is increased to 7.5 cm. The height of the bottle is still 26.0 cm.

124. •• **IP Referring to Example 14–9** The speed of sound increases slightly with temperature. **(a)** Does the fundamental frequency of the bottle increase, decrease, or stay the same as the air heats up on a warm day? Explain. **(b)** Find the fundamental frequency if the speed of sound in air increases to 348 m/s. Assume the bottle is 26.0 cm tall, and that it contains water to a depth of 6.5 cm.

15 Fluids

These hot-air balloonists are experiencing the thrill of free-floating flight as they pass close to one of the famous "mitten" rock formations in Monument Valley, Utah. This earliest form of human flight, pioneered by the Montgolfier brothers in France in 1783, relies on the basic principles of fluid physics presented in this chapter. For example, the heated air inside the balloon has a lower density than the surrounding air, and this difference in density results in a buoyant force whose magnitude is given by Archimedes' Principle. Although the balloonists in this photo may not be thinking about density and buoyant forces, their flight is a vivid illustration of physics in action.

When we speak of *fluids* in physics, we refer to substances that can readily flow from place to place, and that take on the shape of a container rather than retain a shape of their own. Thus, when we use the term *fluids,* we are referring to both liquids and gases.

It is hard to think of a subject more relevant to our everyday lives than fluids. After all, we begin life as a fluid-filled cell suspended in a fluid. We live our independent lives immersed in a fluid that we breathe. In fact, fluids coursing through our circulatory system are literally the lifeblood of our existence. If it were not for the gases in our atmosphere and the liquid water on the Earth's surface, we could not exist.

In this chapter, we examine some of the fundamental physical principles that apply to fluids. All of these principles derive from the basic physics we have learned to this point. For example, straightforward considerations of force and weight lead to an understanding of buoyancy. Similarly, the work–energy theorem results in an understanding of how fluids behave when they flow. As such, fluids provide a wonderful opportunity for us to apply our knowledge of physics to a whole new array of interesting physical systems.

TABLE 15–1 Densities of Common Substances

Substance	Density (kg/m^3)
Gold	19,300
Mercury	13,600
Lead	11,300
Silver	10,500
Iron	7860
Aluminum	2700
Ebony (wood)	1220
Ethylene glycol (antifreeze)	1114
Whole blood (37 °C)	1060
Seawater	1025
Freshwater	1000
Olive oil	920
Ice	917
Ethyl alcohol	806
Cherry (wood)	800
Balsa (wood)	120
Styrofoam	100
Oxygen	1.43
Air	1.29
Helium	0.179

15–1 Density

The properties of a fluid can be hard to pin down, given that it can flow, change shape, and either split into smaller portions or combine into a larger system. Thus, one of the best ways to quantify a fluid is in terms of its **density.** Specifically, the density, ρ, of a material (fluid or not) is defined as the mass, M, per volume, V:

Definition of Density, ρ

$$\rho = M/V \qquad 15\text{–}1$$

SI unit: kg/m^3

The denser a material, the more mass it has in any given volume. Note, however, that the density of a substance is the same regardless of the total amount we have in a system.

To get a feel for densities in common substances, we start with water. For example, to fill a cubic container one meter on a side would take over 2000 pounds of water. More precisely, water has the following density:

$$\rho_w = \text{density of water} = 1000\ \text{kg/m}^3$$

A gallon (1 gallon $= 3.79\ \text{L} = 3.79 \times 10^{-3}\ \text{m}^3$) of water, then, has a mass of

$$M = \rho V = (1000\ \text{kg/m}^3)(3.79 \times 10^{-3}\ \text{m}^3) = 3.79\ \text{kg}$$

As a rule of thumb, a gallon of water weighs just over 8 pounds.

In comparison, the helium in a helium-filled balloon has a density of only about $0.179\ \text{kg/m}^3$, and the density of the air in your room is roughly $1.29\ \text{kg/m}^3$. On the higher end of the density scale, solid gold "weighs" in with a hefty $19{,}300\ \text{kg/m}^3$. Further examples of densities for common materials are given in Table 15–1.

CONCEPTUAL CHECKPOINT 15–1 THE WEIGHT OF AIR

One day you look in your refrigerator and find nothing but a dozen eggs (44 g each). A quick measurement shows that the inside of the refrigerator is 1.0 m by 0.60 m by 0.75 m. Is the weight of the *air* in your refrigerator **(a)** much less than, **(b)** about the same as, or **(c)** much more than the weight of the *eggs*?

REASONING AND DISCUSSION

At first it might seem that the "thin air" in the refrigerator weighs practically nothing compared with a carton full of eggs. A brief calculation shows this is not the case. For the eggs, we have

$$m_{\text{eggs}} = 12(44\ \text{g}) = 0.53\ \text{kg}$$

For the air,

$$m_{\text{air}} = \rho V = (1.29\ \text{kg/m}^3)(1.0\ \text{m} \times 0.60\ \text{m} \times 0.75\ \text{m}) = 0.58\ \text{kg}$$

Thus, the air, with a mass of 0.58 kg (1.28 lb), actually weighs slightly more than the eggs, which have a mass of 0.53 kg (1.17 lb)!

ANSWER

(b) The air and the eggs weigh about the same.

15–2 Pressure

If you have ever pushed a button, or pressed a key on a keyboard, you have applied pressure. Now, you might object that you simply exerted a force on the button, or the key, which is correct. That force is spread out over an area, however. For example, when you press a button, the tip of your finger contacts the button over a small but finite area. **Pressure,** P, is a measure of the amount of force, F, per area A:

Definition of Pressure, P

$$P = F/A \qquad 15\text{–}2$$

SI unit: N/m^2

Pressure is increased if the force applied to a given area is increased, or if a given force is applied to a smaller area. For example, if you press your finger against a balloon, not much happens—your finger causes a small indentation. On the other hand, if you push a needle against the balloon with the same force, you get an explosive pop. The difference is that the same force applied to the small area of a needle tip causes a large enough pressure to rupture the balloon.

PROBLEM-SOLVING NOTE

Pressure Is Force per Area

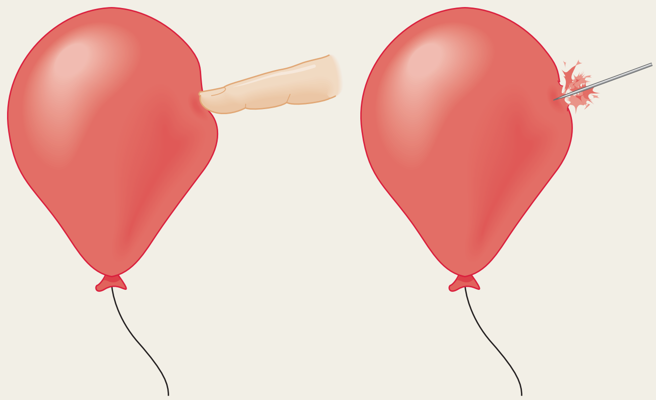
Remember that pressure is proportional to the applied force and *inversely* proportional to the area over which it acts.

EXAMPLE 15–1 POPPING A BALLOON

Find the pressure exerted on the skin of a balloon if you press with a force of 2.1 N using **(a)** your finger or **(b)** a needle. Assume the area of your fingertip is $1.0 \times 10^{-4}\ \mathrm{m}^2$, and the area of the needle tip is $2.5 \times 10^{-7}\ \mathrm{m}^2$. **(c)** Find the minimum force necessary to pop the balloon with the needle, given that the balloon pops with a pressure of $3.0 \times 10^5\ \mathrm{N/m^2}$.

PICTURE THE PROBLEM

Our sketch shows a balloon deformed by the press of a finger and of a needle. The difference is not the amount of force that is applied but the area over which it is applied. In the case of the needle, the force may be sufficient to pop the balloon.



STRATEGY

a, b. The force F and the area A are given in the problem statement. We can find the pressure, then, by applying its definition, $P = F/A$.

c. Rearrange $P = F/A$ to find the force corresponding to a given pressure and area.

SOLUTION

Part (a)

1. Calculate the pressure exerted by the finger:
$$P = \frac{F}{A} = \frac{2.1\ \mathrm{N}}{1.0 \times 10^{-4}\ \mathrm{m}^2} = 2.1 \times 10^4\ \mathrm{N/m^2}$$

Part (b)

2. Calculate the pressure exerted by the needle:
$$P = \frac{F}{A} = \frac{2.1\ \mathrm{N}}{2.5 \times 10^{-7}\ \mathrm{m}^2} = 8.4 \times 10^6\ \mathrm{N/m^2}$$

Part (c)

3. Solve Equation 15–2 for the force:
$$F = PA$$

4. Substitute numerical values:
$$F = (3.0 \times 10^5\ \mathrm{N/m^2})(2.5 \times 10^{-7}\ \mathrm{m}^2) = 0.075\ \mathrm{N}$$

INSIGHT

Note that the pressure exerted by the needle in part (b) is 400 times greater than the pressure due to the finger in part (a). This increase in pressure with decreasing area accounts for the sharp tips to be found on such disparate objects as nails, pens and pencils, and syringes.

PRACTICE PROBLEM

Find the area that a force of 2.1 N would have to act on to produce a pressure of $3.0 \times 10^5\ \mathrm{N/m^2}$. [**Answer:** $A = 7.0 \times 10^{-6}\ \mathrm{m}^2$]

Some related homework problems: Problem 8, Problem 9

An interesting example of force, area, and pressure in nature is provided by the family of small aquatic birds referred to as rails, and in particular by the gallinule. This bird has exceptionally long toes that are spread out over a large area. The result is that the weight of the bird causes only a relatively small pressure as it walks on the soft, muddy shorelines encountered in its habitat. In some species, the pressure exerted while walking is so small that the birds can actually walk across lily pads without sinking into the water.

REAL-WORLD PHYSICS: BIO

Walking on lily pads

▲ This bird exerts only a small pressure on the lily pad on which it walks because its weight is spread out over a large area by its long toes. Since the pressure is not enough to sink a lily pad, the bird can seemingly "walk on water."

Atmospheric Pressure and Gauge Pressure

We are all used to working under pressure—about 14.7 pounds per square inch to be precise. This is **atmospheric pressure,** P_{at}, a direct result of the weight of the air above us. In SI units, atmospheric pressure has the following value:

Atmospheric Pressure, P_{at}

$$P_{at} = 1.01 \times 10^5\ \mathrm{N/m^2} \qquad 15\text{–}3$$

SI unit: $\mathrm{N/m^2}$

A shorthand unit for $\mathrm{N/m^2}$ is the **pascal** (Pa):

$$1\ \mathrm{Pa} = 1\ \mathrm{N/m^2} \qquad 15\text{–}4$$

The pascal honors the pioneering studies of fluids by the French scientist Blaise Pascal (1623–1662). Thus, atmospheric pressure can be written as

$$P_{at} = 101\ \mathrm{kPa}$$

In British units, pressure is measured in pounds per square inch, and

$$P_{at} = 14.7\ \mathrm{lb/in^2}$$

Finally, a common unit for atmospheric pressure in weather forecasting is the **bar,** defined as follows:

$$1\ \mathrm{bar} = 10^5\ \mathrm{Pa} \approx 1P_{at}$$

EXERCISE 15–1

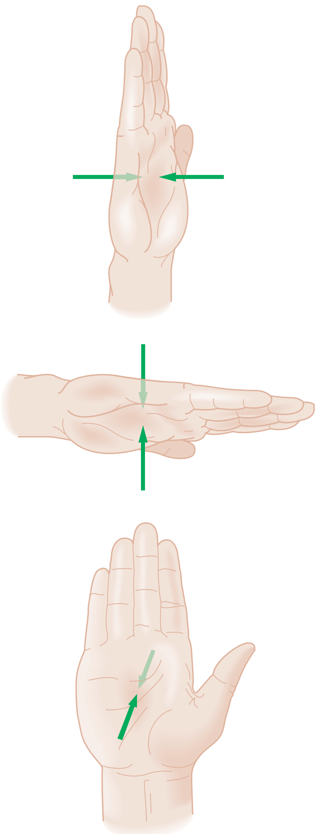
Find the force exerted on the palm of your hand by atmospheric pressure. Assume your palm measures 0.080 m by 0.10 m.

SOLUTION

Applying Equations 15–3 and 15–2, we find

$$F = P_{at}A = (1.01 \times 10^5\ \mathrm{Pa})(0.080\ \mathrm{m})(0.10\ \mathrm{m}) = 810\ \mathrm{N}$$

Thus, the atmosphere pushes on the palm of your hand with a force of approximately 180 pounds! Of course, it also pushes on the back of your hand with essentially the same force, but in the opposite direction.



▲ **FIGURE 15–1 Pressure is the same in all directions**

The forces exerted on the two sides of a hand cancel, regardless of the hand's orientation. Hence, pressure acts equally in all directions.

Figure 15–1 illustrates the forces exerted on your hand by atmospheric pressure. If your hand is vertical, atmospheric pressure pushes to the right and to the

▲ The air around you exerts a force of about 14.7 pounds on every square inch of your body. Because this force is the same in all directions and is opposed by an equal pressure inside your body, you are generally unaware of it. However, when the air is pumped out of a sealed can (left), atmospheric pressure produces an inward force that is unopposed. The resulting collapse of the can vividly illustrates the pressure that is all around us. An air splint (right) utilizes the same principle of unequal pressure. A plastic sleeve is placed around an injured limb and inflated to a pressure greater than that of the atmosphere—and thus of the body's internal pressure. The increased external pressure retards bleeding from the injured area, and also tends to immobilize the limb in case it might be fractured. (Air splints are carried by hikers and others who might be in need of emergency treatment far from professional medical facilities. Before it is inflated, the air splint is about the size and weight of a credit card.)

left equally, so your hand feels zero net force. If your hand is horizontal, atmospheric pressure exerts upward and downward forces on your hand that are essentially the same in magnitude, again giving zero net force. This cancellation of forces occurs no matter what the orientation of your hand; thus, we can conclude the following:

> The pressure in a fluid acts equally in all directions, and acts at right angles to any surface.

In many cases we are interested in the difference between a given pressure and atmospheric pressure. For example, a flat tire does not have zero pressure in it; the pressure in the tire is atmospheric pressure. To inflate the tire to 241 kPa (35 lb/in^2), the pressure inside the tire must be greater than atmospheric pressure by this amount; that is, $P = 241 \text{ kPa} + P_{at} = 342 \text{ kPa}$.

To deal with such situations, we introduce the **gauge pressure,** P_g, defined as follows:

$$P_g = P - P_{at} \qquad 15\text{–}5$$

It is the gauge pressure, then, that is determined by a tire gauge. Many problems in this chapter refer to the gauge pressure. Hence it must be remembered that the actual pressure in these cases is greater by the amount P_{at}.

PROBLEM-SOLVING NOTE

Gauge Pressure

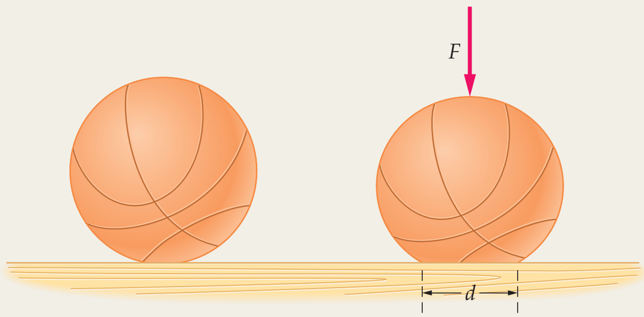
If a problem gives you the gauge pressure, recall that the actual pressure is the gauge pressure *plus* atmospheric pressure.

EXAMPLE 15–2 PRESSURING THE BALL: ESTIMATE THE GAUGE PRESSURE

Estimate the gauge pressure in a basketball by pushing down on it and noting the area of contact it makes with the surface on which it rests.

PICTURE THE PROBLEM

Our sketch shows the basketball both in its original state, and when a force F pushes downward on it. In the latter case, the ball flattens out on the bottom, forming a circular area of contact with the floor of diameter d.



STRATEGY

To solve this problem, we have to make reasonable estimates of the force applied to the ball and the area of contact.

Suppose, for example, that we push down with a moderate force of 22 N (about 5 lb). The circular area of contact will probably have a diameter of about 2.0 centimeters. This can be verified by carrying out the experiment. Thus, given $F = 22 \text{ N}$ and $A = \pi(d/2)^2$ we can find the gauge pressure.

SOLUTION

1. Using these estimates, calculate the gauge pressure, P_g:

$$P_g = \frac{F}{A} = \frac{22 \text{ N}}{\pi\left(\dfrac{0.020 \text{ m}}{2}\right)^2} = 7.0 \times 10^4 \text{ Pa}$$

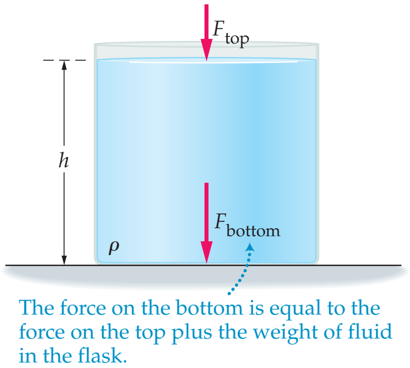
INSIGHT

Given that a pressure of one atmosphere, $P_{at} = 101 \text{ kPa} = 1.01 \times 10^5 \text{ Pa}$, corresponds to 14.7 lb/in^2, it follows that the gauge pressure of the ball, $P_g = 7.0 \times 10^4 \text{ Pa}$, corresponds to a pressure of roughly 10 lb/in^2. Thus, a basketball will typically have a gauge pressure in the neighborhood of 10 lb/in^2, and hence a total pressure inside the ball of about 25 lb/in^2.

PRACTICE PROBLEM

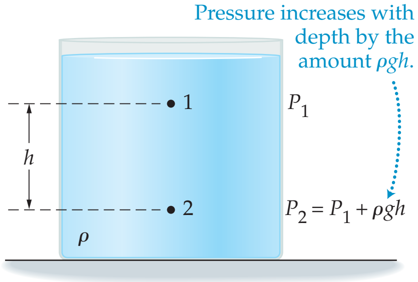
What is the diameter of the circular area of contact if a basketball with a 12 lb/in^2 gauge pressure is pushed down with a force of 44 N (about 10 lb)? [**Answer:** $d = 2.6$ cm]

Some related homework problems: Problem 11, Problem 12



▲ **FIGURE 15–2 Pressure and the weight of a fluid**

The force pushing down on the bottom of the flask is greater than the force pushing down on the surface of the fluid. The difference in force is the weight of fluid in the flask.



▲ **FIGURE 15–3 Pressure variation with depth**

If point 2 is deeper than point 1 by the amount h, its pressure is greater by the amount ρgh.

15–3 Static Equilibrium in Fluids: Pressure and Depth

Countless war movies have educated us on the perils of taking a submarine too deep. The hull creaks and groans, rivets start to pop, water begins to spray into the ship, and the captain keeps a close eye on the depth gauge. But what causes the pressure to increase as a submarine dives, and how much does it go up for a given increase in depth?

The answer to the first question is that the increased pressure is due to the added weight of water pressing on the submarine as it goes deeper. To see how this works, consider a cylindrical container filled to a height h with a fluid of density ρ, as in **Figure 15–2**. The top surface of the fluid is open to the atmosphere, with a pressure P_{at}. If the cross-sectional area of the container is A, the downward force exerted on the top surface by the atmosphere is

$$F_{top} = P_{at}A$$

Now, at the bottom of the container, the downward force is F_{top} *plus* the weight of the fluid. Recalling that $M = \rho V$, and that $V = hA$ for a cylinder of height h and area A, this weight is

$$W = Mg = \rho V g = \rho(hA)g$$

Hence, we have

$$F_{bottom} = F_{top} + W = P_{at}A + \rho(hA)g$$

Finally, the pressure at the bottom is obtained by dividing F_{bottom} by the area A:

$$P_{bottom} = \frac{F_{bottom}}{A} = \frac{P_{at}A + \rho(hA)g}{A} = P_{at} + \rho gh$$

Of course, this relation holds not only for the bottom of the container, but for *any depth h below the surface*. Thus, the answer to the second question is that if the depth increases by the amount h, the pressure increases by the amount ρgh. At a depth h below the surface of a fluid, then, the pressure P is given by

$$P = P_{at} + \rho gh \qquad 15\text{–}6$$

This expression holds for any fluid with constant density ρ and a pressure P_{at} at its upper surface.

REAL-WORLD PHYSICS
Pressure at the wreck of the *Titanic*

EXERCISE 15–2

The *Titanic* was found in 1985 lying on the bottom of the North Atlantic at a depth of 2.5 miles. What is the pressure at this depth?

SOLUTION

Applying Equation 15–6 with $\rho = 1025\ \text{kg/m}^3$, we have

$$P = P_{at} + \rho gh = 1.01 \times 10^5\ \text{Pa} + (1025\ \text{kg/m}^3)(9.81\ \text{m/s}^2)\left(\frac{1609\ \text{m}}{1\ \text{mi}}\right)$$
$$= 4.1 \times 10^7\ \text{Pa}$$

This is about 400 atmospheres.

The relation $P = P_{at} + \rho gh$ can be applied to any two points in a fluid. For example, if the pressure at one point is P_1, the pressure P_2 at a depth h below that point is the following:

Dependence of Pressure on Depth

$$P_2 = P_1 + \rho gh \qquad 15\text{–}7$$

This relation is illustrated in **Figure 15–3**, and utilized in the next Example.

PROBLEM-SOLVING NOTE
Pressure Depends Only on Depth

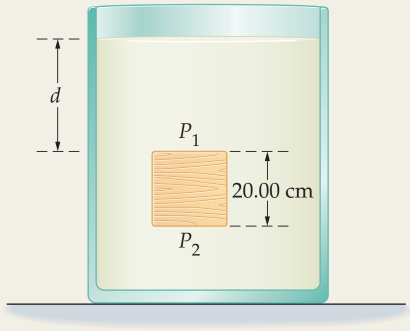
The pressure in a static fluid depends only on the depth of the fluid. It is independent of the shape of the container.

EXAMPLE 15–3 PRESSURE AND DEPTH

A cubical box 20.00 cm on a side is completely immersed in a fluid. At the top of the box the pressure is 105.0 kPa; at the bottom the pressure is 106.8 kPa. What is the density of the fluid?

PICTURE THE PROBLEM

Our sketch shows the box at an unknown depth d below the surface of the fluid. The important dimension for this problem is the height of the box, which is 20.00 cm. We are also given that the pressures at the top and bottom of the box are $P_1 = 105.0$ kPa and $P_2 = 106.8$ kPa, respectively.



STRATEGY

The pressures at the top and bottom of the box are related by $P_2 = P_1 + \rho g h$. Since the pressures and the height of the box are given, this relation can be solved for the unknown density, ρ.

SOLUTION

1. Solve $P_2 = P_1 + \rho g h$ for the density: $$\rho = \frac{P_2 - P_1}{gh}$$

2. Substitute numerical values: $$\rho = \frac{(1.068 \times 10^5\ \text{Pa}) - (1.050 \times 10^5\ \text{Pa})}{(9.81\ \text{m/s}^2)(0.2000\ \text{m})} = 920\ \text{kg/m}^3$$

INSIGHT

Comparing with Table 15–1, it appears that the fluid in question may be olive oil. If the box had been immersed in water instead, with its greater density, the difference in pressure between the top and bottom of the box would have been greater as well.

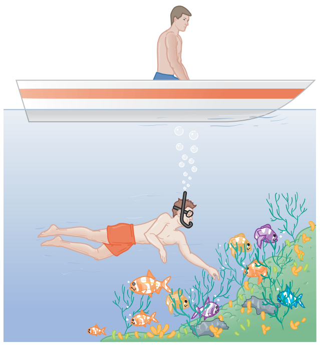
PRACTICE PROBLEM

Given the density obtained above, what is the depth of fluid, d, at the top of the box? [**Answer:** $d = 0.44$ m]

Some related homework problems: Problem 15, Problem 20

CONCEPTUAL CHECKPOINT 15–2 THE SIZE OF BUBBLES

One day, while swimming below the surface of the ocean, you let out a small bubble of air from your mouth. As the bubble rises toward the surface, does its diameter **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

As the bubble rises, the pressure in the surrounding water decreases. This allows the air in the bubble to expand and occupy a larger volume.

ANSWER

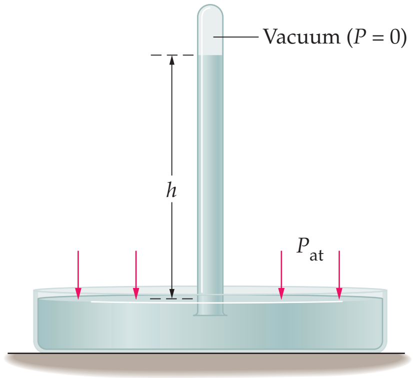
(a) The diameter of the bubble increases.

REAL-WORLD PHYSICS

The barometer

An interesting application of the variation of pressure with depth is the **barometer,** which can be used to measure atmospheric pressure. We consider here the simplest type of barometer, which was first proposed by Evangelista Torricelli (1608–1647) in 1643. First, fill a long glass tube—open at one end and closed at the other—with a fluid of density ρ. Next, invert the tube and place its open end below the surface of the same fluid in a bowl, as shown in **Figure 15–4**. Some of the fluid in the tube will flow into the bowl, leaving an empty space (vacuum) at the top. Enough will remain, however, to create a difference in level, h, between the fluid in the bowl and that in the tube.

The basic idea of the barometer is that this height difference is directly related to the atmospheric pressure that pushes down on the fluid in the bowl. To see how



▲ **FIGURE 15–4 A simple barometer**
Atmospheric pressure, P_{at}, is related to the height of fluid in the tube by the relation $P_{at} = \rho g h$.

this works, first note that the pressure in the vacuum at the top of the tube is zero. Hence, the pressure in the tube at a depth h below the vacuum is $0 + \rho g h = \rho g h$. Now, at the level of the fluid in the bowl we know that the pressure is one atmosphere, P_{at}. Therefore, it follows that

$$P_{at} = \rho g h$$

If these pressures were not the same, there would be a pressure difference between the fluid in the tube and that in the bowl, resulting in a net force and a flow of fluid. Thus, a measurement of h immediately gives atmospheric pressure.

A fluid that is often used in such a barometer is mercury (Hg), with a density of $\rho = 1.3595 \times 10^4\ \mathrm{kg/m^3}$. The corresponding height for a column of mercury is

$$h = \frac{P_{at}}{\rho g} = \frac{1.013 \times 10^5\ \mathrm{Pa}}{(1.3595 \times 10^4\ \mathrm{kg/m^3})(9.81\ \mathrm{m/s^2})} = 760\ \mathrm{mm}$$

In fact, atmospheric pressure is *defined* in terms of millimeters of mercury (mmHg):

$$1\ \text{atmosphere} = P_{at} = 760\ \mathrm{mmHg}$$

Table 15–2 summarizes the various expressions we have developed for atmospheric pressure.

TABLE 15–2 Atmospheric Pressure

1 atmosphere	$= P_{at}$
	$= 760$ mmHg (definition)
	$= 14.7\ \mathrm{lb/in^2}$
	$= 101$ kPa
	$= 101\ \mathrm{kN/m^2}$
	~ 1 bar $= 100$ kPa

Water Seeks Its Own Level

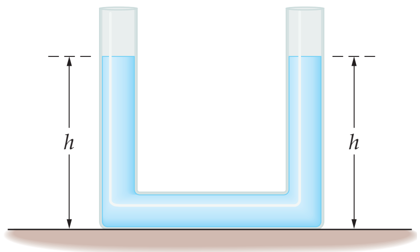
We are all familiar with the aphorism that water seeks its own level. In order for this to hold true, however, it is necessary that the pressure at the surface of the water (or other fluid) be the same everywhere on the surface. This was not the case for the barometer just discussed, where the pressure was P_{at} on one portion of the surface and zero on another. Let's take a moment, then, to consider the level assumed by a fluid with constant pressure on its surface. In doing so, we shall apply considerations involving force, pressure, and energy.

First, the force–pressure point of view. In **Figure 15–5 (a)** we show a U-shaped tube containing a quantity of fluid of density ρ. The fluid rises to the same level in each arm of the U, where it is open to the atmosphere. Therefore, the pressure at the base of each arm is the same: $P_{at} + \rho g h$. Thus, the fluid in the horizontal section of the U is pushed with equal force from each side, giving zero net force. As a result, the fluid remains at rest.

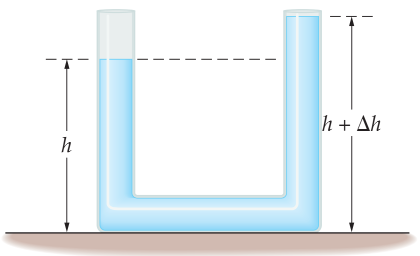
On the other hand, in **Figure 15–5 (b)** the two arms of the U are filled to different levels. Therefore, the pressure at the base of the two arms is different, with the greater pressure at the base of the right arm. The fluid in the horizontal section, then, will experience a net force to the left, causing it to move in that direction. This will tend to equalize the fluid levels of the two arms.

We can arrive at the same conclusion on the basis of energy minimization. Consider a U tube that is initially filled to the same level in both arms, as in Figure 15–5 (a). Now, consider moving a small element of fluid from one arm to the other, to create different levels, as in **Figure 15–6**. In moving this fluid element to the other arm, it is necessary to lift it upward. This, in turn, causes its potential energy to increase. Since nothing else in the system has changed its position, the only change in potential energy is the increase experienced by the element. Thus, we conclude that the system has a minimum energy when the fluid levels are the same and a higher energy when the levels are different. Just as a ball rolls to the bottom of a hill, where its energy is minimized, the fluid seeks its own level and a minimum energy.

If two different liquids, with different densities, are combined in the same U tube, the levels in the arms are not the same. Still, the pressures at the base of each arm must be equal, as before. This is discussed in the next Example.



(a) A fluid is in equilibrium when the levels are equal



(b) Unequal levels result in unequal pressures and fluid flow

◀ **FIGURE 15–5 Fluids seek their own level**
(a) When the levels are equal, the pressure is the same at the base of each arm of the U tube. As a result, the fluid in the horizontal section of the U is in equilibrium. **(b)** With unequal heights, the pressures are different. In this case, the pressure is greater at the base of the right arm, hence fluid will flow toward the left and the levels will equalize.

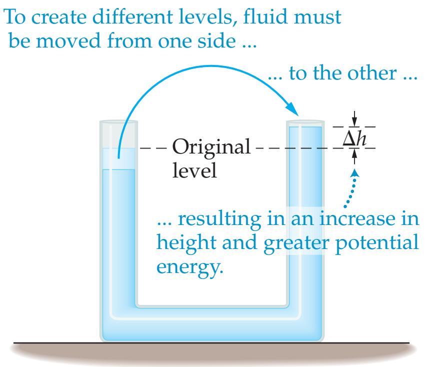


FIGURE 15–6 Gravitational potential energy of a fluid

In order to create unequal levels in the two arms of the U tube, an element of fluid must be raised by the height Δh. This increases the gravitational potential energy of the system. The lowest potential energy corresponds to equal levels.

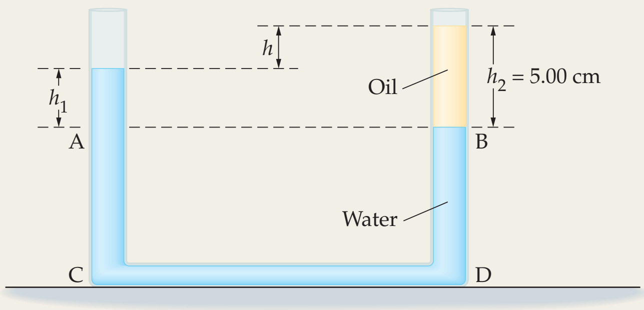
The containers shown here are connected at the bottom by a hollow tube, which allows fluid to flow freely between them. As a result the fluid level is the same in each container regardless of its shape and size.

EXAMPLE 15–4 OIL AND WATER DON'T MIX

A U-shaped tube is filled mostly with water, but a small amount of vegetable oil has been added to one side, as shown in the sketch. The density of the water is $1.00 \times 10^3\ \mathrm{kg/m^3}$, and the density of the vegetable oil is $9.20 \times 10^2\ \mathrm{kg/m^3}$. If the depth of the oil is 5.00 cm, what is the difference in level h between the top of the oil on one side of the U and the top of the water on the other side?

PICTURE THE PROBLEM

The U-shaped tube and the relevant dimensions are shown in our sketch. In particular, note that the depth of the oil is 5.00 cm, and that the oil rises to a greater height on its side of the U than the water does on its side. This is due to the oil having the lower density. Both sides of the U are open to the atmosphere, so the pressure at the top surfaces is P_{at}.



INTERACTIVE FIGURE

STRATEGY

For the system to be in equilibrium, it is necessary that the pressure be the same at the bottom of each side of the U; that is, at points C and D. If the pressure is the same at C and D, it will remain equal as one moves up through the water to the points A and B. Above this point the pressures will differ because of the presence of the oil.

Therefore, setting the pressure at point A equal to the pressure at point B determines the depth h_1 in terms of the known depth h_2. It follows that the difference in level between the two sides of the U is simply $h = h_2 - h_1$.

SOLUTION

1. Find the pressure at point A, where the depth of the water is h_1: $P_{\mathrm{A}} = P_{\mathrm{at}} + \rho_{\mathrm{water}} g h_1$
2. Find the pressure at point B, where the depth of the oil is $h_2 = 5.00$ cm: $P_{\mathrm{B}} = P_{\mathrm{at}} + \rho_{\mathrm{oil}} g h_2$
3. Set P_{A} equal to P_{B}: $P_{\mathrm{at}} + \rho_{\mathrm{water}} g h_1 = P_{\mathrm{at}} + \rho_{\mathrm{oil}} g h_2$
4. Solve for the depth of the water, h_1, and substitute numerical values:
$$h_1 = h_2\left(\frac{\rho_{\mathrm{oil}}}{\rho_{\mathrm{water}}}\right)$$
$$= (5.00\ \mathrm{cm})\left(\frac{9.20 \times 10^2\ \mathrm{kg/m^3}}{1.00 \times 10^3\ \mathrm{kg/m^3}}\right) = 4.60\ \mathrm{cm}$$
5. Calculate the difference in levels between the water and oil sides of the U: $h = h_2 - h_1 = 5.00\ \mathrm{cm} - 4.60\ \mathrm{cm} = 0.40\ \mathrm{cm}$

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INSIGHT

Note that the weight of the height h_1 of water is equal to the weight of the height h_2 of oil. This is a special case, however, and is due to the fact that our U has equal diameters on its two sides. If the oil side of the U had been wider, for example, the weight of the oil would have been greater, though the difference in height still would have been $h = 0.40$ cm. It is the pressure that matters in a system like this, not the weight. (For a similar situation, see the photo on page 507.)

PRACTICE PROBLEM

Find the pressure at points A and B. [**Answer:** $P_A = P_B = P_{at} + 451$ Pa]

Some related homework problems: Problem 18, Problem 24, Problem 25

Pascal's Principle

Recall from Equation 15–6 that if the surface of a fluid of density ρ is exposed to the atmosphere with a pressure P_{at}, the pressure at a depth h below the surface is

$$P = P_{at} + \rho g h$$

Suppose, now, that atmospheric pressure is increased from P_{at} to $P_{at} + \Delta P$. As a result, the pressure at the depth h is

$$P = P_{at} + \Delta P + \rho g h = (P_{at} + \rho g h) + \Delta P$$

Thus, by increasing the pressure at the top of the fluid by the amount ΔP, we have increased it by the same amount everywhere in the fluid. This is **Pascal's principle:**

> An external pressure applied to an enclosed fluid is transmitted unchanged to every point within the fluid.

REAL-WORLD PHYSICS
The hydraulic lift

A classic example of Pascal's principle at work is the **hydraulic lift,** which is shown schematically in **Figure 15–7**. Here we see two cylinders, one of cross-sectional area A_1, the other of cross-sectional area $A_2 > A_1$. The cylinders, each of which is fitted with a piston, are connected by a tube and filled with a fluid. Initially the pistons are at the same level and exposed to the atmosphere.

Now, suppose we push down on piston 1 with the force F_1. This increases the pressure in that cylinder by the amount

$$\Delta P = \frac{F_1}{A_1}$$

By Pascal's principle, the pressure in cylinder 2 increases by the *same* amount. Therefore, the increased upward force on piston 2 due to the fluid is

$$F_2 = (\Delta P)A_2$$

Substituting the increase in pressure, $\Delta P = F_1/A_1$, we find

$$F_2 = \left(\frac{F_1}{A_1}\right)A_2 = F_1\left(\frac{A_2}{A_1}\right) > F_1 \qquad 15\text{–}8$$

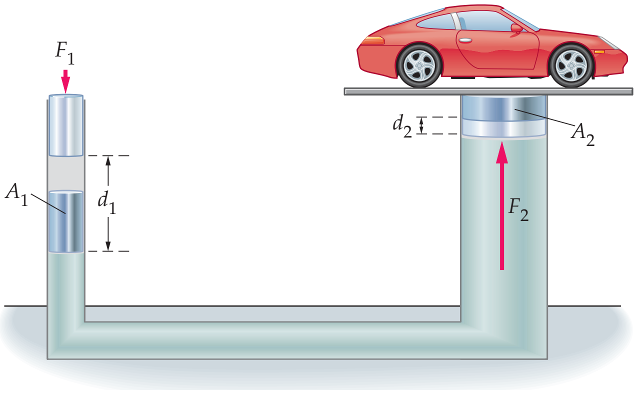


FIGURE 15–7 A hydraulic lift
A force F_1 exerted on the small piston causes a much larger force, F_2, to act on the large piston.

To be specific, let's assume that A_2 is 100 times greater than A_1. Then, by pushing down on piston 1 with a force F_1, we push upward on piston 2 with a force of $100F_1$. Our force has been magnified 100 times!

If this sounds too good to be true, rest assured that we are not getting something for nothing. Just as with a lever, there is a tradeoff between the distance through which a force must be applied and the force magnification. This is illustrated in Figure 15–7, where we show piston 1 being pushed down through a distance d_1. This displaces a volume of fluid equal to A_1d_1. The same volume flows into cylinder 2, where it causes piston 2 to rise through a distance d_2. Equating the two volumes, we have

$$A_1d_1 = A_2d_2$$

or

$$d_2 = d_1\left(\frac{A_1}{A_2}\right)$$

Thus, in the example just given, if we move piston 1 down a distance d_1, piston 2 rises a distance $d_2 = d_1/100$. Our force at piston 2 has been magnified 100 times, but the distance it moves has been reduced 100 times.

EXERCISE 15–3

To inspect a 14,500-N car, it is raised with a hydraulic lift. If the radius of the small piston in Figure 15–7 is 4.0 cm, and the radius of the large piston is 17 cm, find the force that must be exerted on the small piston to lift the car.

SOLUTION

Solving Equation 15–8 for F_1, and noting that the area is πr^2, we find

$$F_1 = F_2\left(\frac{A_1}{A_2}\right) = (14{,}500\ \mathrm{N})\left[\frac{\pi(0.040\ \mathrm{m})^2}{\pi(0.17\ \mathrm{m})^2}\right] = 800\ \mathrm{N}$$

15–4 Archimedes' Principle and Buoyancy

The fact that a fluid's pressure increases with depth leads to many interesting consequences. Among them is the fact that a fluid exerts a net upward force on any object it surrounds. This is referred to as a **buoyant force.**

To see the origin of buoyancy, consider a cubical block immersed in a fluid of density ρ, as in **Figure 15–8**. The surrounding fluid exerts normal forces on all of its faces. Clearly, the horizontal forces pushing to the right and to the left are equal, hence they cancel and have no effect on the block.

The situation is quite different for the vertical forces, however. Note, for example, that the downward force exerted on the top face is less than the upward force exerted on the lower face, since the pressure at the lower face is greater. This difference in forces gives rise to a net upward force—the buoyant force.

Let's calculate the buoyant force acting on the block. First, we assume that the cubical block is of length L on a side, and that the pressure on the top surface is P_1. The downward force on the block, then, is

$$F_1 = P_1A = P_1L^2$$

Note that we have used the fact that the area of a square face of side L is L^2. Next, we consider the bottom face. The pressure there is given by Equation 15–7, with a difference in depth of $h = L$:

$$P_2 = P_1 + \rho gL$$

Therefore, the upward force exerted on the bottom face of the cube is

$$F_2 = P_2A = (P_1 + \rho gL)L^2 = P_1L^2 + \rho gL^3 = F_1 + \rho gL^3$$

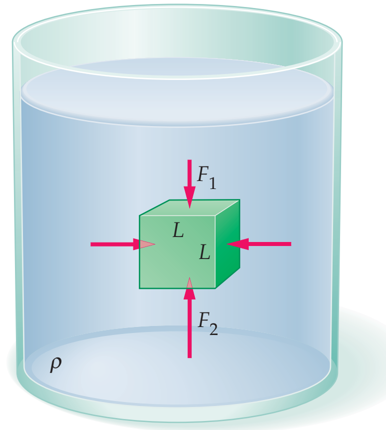


FIGURE 15–8 Buoyant force due to a fluid

A fluid surrounding an object exerts a buoyant force in the upward direction. This is due to the fact that pressure increases with depth, and hence the upward force on the object, F_2, is greater than the downward force, F_1. Forces acting to the left and to the right cancel.

PROBLEM-SOLVING NOTE

The Buoyant Force

Note that the buoyant force is equal to the weight of displaced fluid. It does not depend on the weight of the object that displaces the fluid.

If we take upward as the positive direction, the net vertical force exerted by the fluid on the block—that is, the buoyant force, F_b—is

$$F_b = F_2 - F_1 = \rho g L^3$$

As expected, the block experiences a net upward force from the surrounding fluid.

The precise value of the buoyant force is of some significance, as we now show. First, note that the volume of the cube is L^3. It follows that $\rho g L^3$ is the weight of *fluid* that would occupy the same volume as the cube. Therefore, the buoyant force is equal to the weight of fluid that is displaced by the cube. This is a special case of **Archimedes' principle:**

> An object completely immersed in a fluid experiences an upward buoyant force equal in magnitude to the weight of fluid displaced by the object.

More generally, if a volume V of an object is immersed in a fluid of density ρ_{fluid}, the buoyant force can be expressed as follows:

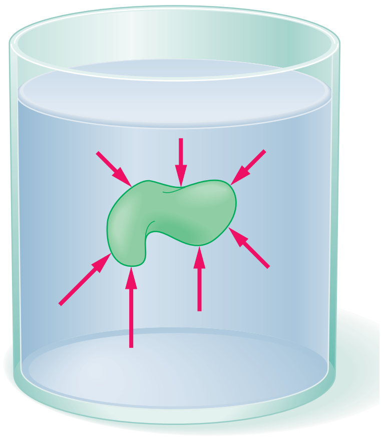
Buoyant Force When a Volume V Is Submerged in a Fluid of Density ρ_{fluid}

$$F_b = \rho_{\text{fluid}} g V \qquad 15\text{–}9$$

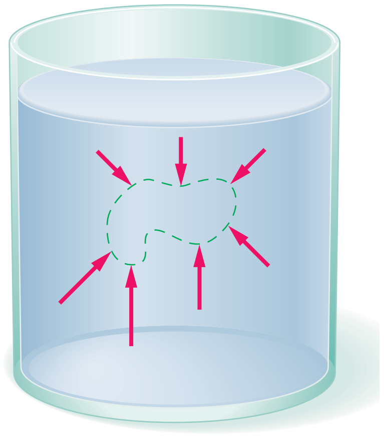
SI unit: N

The volume V may be the total volume of the object, or any fraction of the total volume.

To see that Archimedes' principle is completely general, consider the submerged object shown in **Figure 15–9 (a)**. If we were to replace this object with an equivalent volume of fluid, as in **Figure 15–9 (b)**, the container would hold nothing but fluid and would be in static equilibrium. As a result, we conclude that the net buoyant force acting on this "fluid object" must be upward and equal in magnitude to its weight. Now here is the key idea: Since the original object and the fluid object occupy the same position, the forces acting on their surfaces are identical, and hence the net buoyant force is the same for both objects. Therefore, the original object experiences a buoyant force equal to the weight of fluid that it displaces—that is, equal to the weight of the fluid object.



(a) The forces acting on an object surrounded by fluid

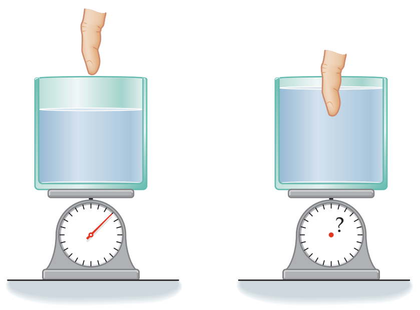


(b) The same forces act when the object is replaced by fluid

FIGURE 15–9 Buoyant force equals the weight of displaced fluid
The buoyant force acting on the object in **(a)** is equal to the weight of the "fluid object" (with the same size and shape) in **(b)**. This is because the fluid in (b) is at rest; hence the buoyant force acting on the fluid object must cancel its weight. The same forces act on the original object in (a), however, and therefore the buoyant force it experiences is also equal to the weight of the fluid object.

CONCEPTUAL CHECKPOINT 15–3 HOW IS THE SCALE READING AFFECTED?

A flask of water rests on a scale. If you dip your finger into the water, without touching the flask, does the reading on the scale **(a)** increase, **(b)** decrease, or **(c)** stay the same?



REASONING AND DISCUSSION

Your finger experiences an upward buoyant force when it is dipped into the water. By Newton's third law, the water experiences an equal and opposite reaction force acting downward. This downward force is transmitted to the scale, which in turn gives a higher reading.

Another way to look at this result is to note that when you dip your finger into the water, its depth increases. This results in a greater pressure at the bottom of the flask, and hence a greater downward force on the flask. The scale reads this increased downward force.

ANSWER

(a) The reading on the scale increases.

15–5 Applications of Archimedes' Principle

In this section we consider a variety of applications of Archimedes' principle. We begin with situations in which an object is fully immersed. Later we consider systems in which an object floats.

Complete Submersion

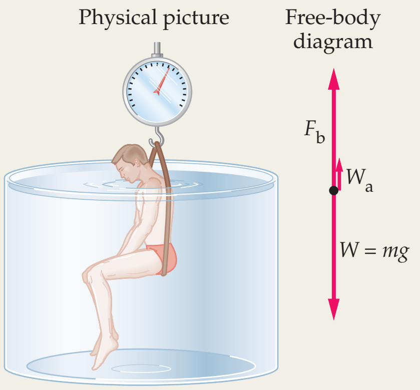
An interesting application of complete submersion can be found in an apparatus commonly used in determining a person's body-fat percentage. We consider the basic physics of the apparatus and the measurement procedure in the next Example. Following the Example, we derive the relation between overall body density and the body-fat percentage.

EXAMPLE 15–5 MEASURING THE BODY'S DENSITY

REAL-WORLD PHYSICS BIO A person who weighs 720.0 N in air is lowered into a tank of water to about chin level. He sits in a harness of negligible mass suspended from a scale that reads his apparent weight. He now exhales as much air as possible and dunks his head underwater, submerging his entire body. If his apparent weight while submerged is 34.3 N, find **(a)** his volume and **(b)** his density.

PICTURE THE PROBLEM

The scale, the tank, and the person are shown in our sketch. We also show the free-body diagram for the person. Note that the weight of the person in air is designated by $W = mg = 720.0$ N, and the apparent weight in water, which is the upward force exerted by the scale on the person, is designated by $W_a = 34.3$ N. The buoyant force exerted by the water on the person is F_b.



STRATEGY

To find the volume, V_p, and density, ρ_p, of the person, we must use two separate conditions. They are as follows:

a. When the person is submerged, the surrounding water exerts an upward buoyant force given by Archimedes' principle: $F_b = \rho_{water}V_p g$. This relation, and Newton's second law, can be used to determine V_p.

b. The weight of the person in air is $W = mg = \rho_p V_p g$. Combining this relation with the volume, V_p, found in part (a) allows us to determine the density, ρ_p.

SOLUTION

Part (a)

1. Apply Newton's second law to the person. Note that the person remains at rest, and therefore the net force acting on him is zero:

$$W_a + F_b - W = 0$$

2. Substitute $F_b = \rho_{water}V_p g$ and solve for V_p:

$$W_a + \rho_{water}V_p g - W = 0$$

$$V_p = \frac{W - W_a}{\rho_{water}g} = \frac{720.0\text{ N} - 34.3\text{ N}}{(1.00 \times 10^3\text{ kg/m}^3)(9.81\text{ m/s}^2)}$$

$$= 6.99 \times 10^{-2}\text{ m}^3$$

Part (b)

3. Use $W = \rho_p V_p g$ to solve for the density of the person, ρ_p:

$$W = \rho_p V_p g$$

$$\rho_p = \frac{W}{V_p g} = \frac{720.0\text{ N}}{(6.99 \times 10^{-2}\text{ m}^3)(9.81\text{ m/s}^2)}$$

$$= 1050\text{ kg/m}^3$$

INSIGHT

As in Conceptual Checkpoint 15–3, the water exerts an upward buoyant force on the person, and an equal and opposite reaction force acts downward on the tank and water, making it press against the floor with a greater force.

In addition, notice that the density of the person (1050 kg/m^3) is only slightly greater than the density of seawater (1025 kg/m^3), as given in Table 15–1.

PRACTICE PROBLEM

The person can float in water if his lungs are partially filled with air, increasing the volume of his body. What volume must his body have to just float? [**Answer:** $V_p = 7.34 \times 10^{-2}\text{ m}^3$]

Some related homework problems: Problem 43, Problem 44

▲ This device, known as the Bod Pod, measures the body-fat percentage of a person inside it by varying the air pressure in the chamber and measuring the corresponding changes in the person's apparent weight. Archimedes' principle is at work here, just as it is in Example 15–5.

Once the overall density of the body is determined, the percentage of body fat can be obtained by noting that body fat has a density of $\rho_f = 9.00 \times 10^2 \text{ kg/m}^3$, whereas the lean body mass (muscles and bone) has a density of $\rho_1 = 1.10 \times 10^3 \text{ kg/m}^3$. Suppose, for example, that a fraction x_f of the total body mass M is fat mass, and a fraction $(1 - x_f)$ is lean mass; that is, the fat mass is $m_f = x_f M$ and the lean mass is $m_1 = (1 - x_f)M$. The total volume of the body is $V = V_f + V_1$. Using the fact that $V = m/\rho$, we can write the total volume as $V = m_f/\rho_f + m_1/\rho_1 = x_f M/\rho_f + (1 - x_f)M/\rho_1$. Combining these results, the overall density of a person's body, ρ_p, is

$$\rho_p = \frac{M}{V} = \frac{1}{\dfrac{x_f}{\rho_f} + \dfrac{(1 - x_f)}{\rho_1}}$$

Solving for the body-fat fraction, x_f, yields

$$x_f = \frac{1}{\rho_p}\left(\frac{\rho_1 \rho_f}{\rho_1 - \rho_f}\right) - \frac{\rho_f}{\rho_1 - \rho_f}$$

Finally, substituting the values for ρ_f and ρ_1, we find

$$x_f = \frac{(4950 \text{ kg/m}^3)}{\rho_p} - 4.50$$

This result is known as *Siri's formula*. For example, if $\rho_p = 900 \text{ kg/m}^3$ (all fat), we find $x_f = 1$; if $\rho_p = 1100 \text{ kg/m}^3$ (no fat), we find $x_f = 0$. In the case of Example 15–5 where $\rho_p = 1050 \text{ kg/m}^3$, we find that this person's body-fat fraction is $x_f = 0.214$, for a percentage of 21.4%. This is a reasonable value for a healthy adult male.

REAL-WORLD PHYSICS: BIO

Measuring body fat

A recent refinement to the measurement of body-fat percentage is the Bod Pod, an egg-shaped, air-tight chamber in which a person sits comfortably—high and dry, surrounded only by air. This device works on the same physical principle as submerging a person in water, only it uses air instead of water. Since air is about a thousand times less dense than water, the measurements of apparent weight must be roughly a thousand times more sensitive. Fortunately, this is possible with today's technology, allowing for a much more convenient means of measurement.

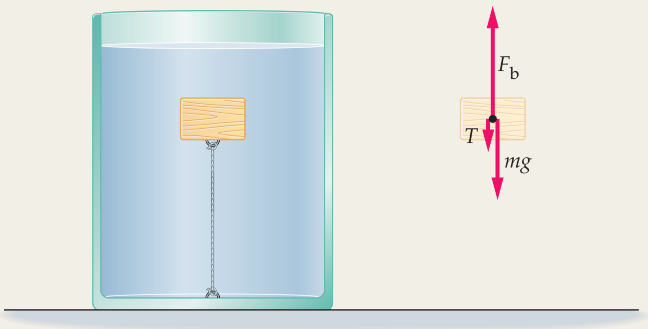
Next we consider a low-density object, such as a piece of wood, held down below the surface of a denser fluid.

ACTIVE EXAMPLE 15–1 FIND THE TENSION IN THE STRING

A piece of wood with a density of 706 kg/m^3 is tied with a string to the bottom of a water-filled flask. The wood is completely immersed, and has a volume of $8.00 \times 10^{-6} \text{ m}^3$. What is the tension in the string?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply Newton's second law to the wood: $F_b - T - mg = 0$
2. Solve for the tension, T: $T = F_b - mg$
3. Calculate the weight of the wood: $mg = 0.0554 \text{ N}$
4. Calculate the buoyant force: $F_b = 0.0785 \text{ N}$
5. Subtract to obtain the tension: $T = 0.0231 \text{ N}$



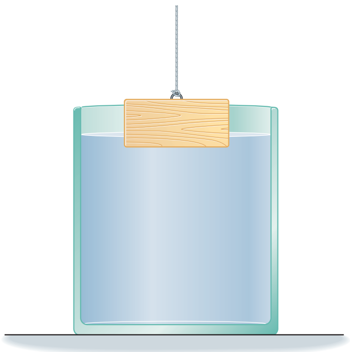
INSIGHT

Since the wood floats in water, its buoyant force when completely immersed is greater than its weight.

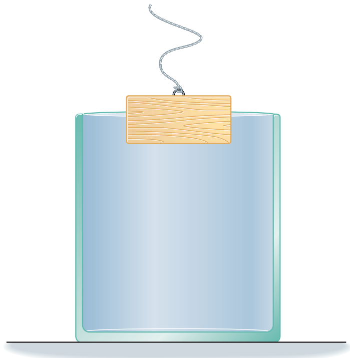
YOUR TURN

What is the tension in the string if the piece of wood has a density of 822 kg/m^3?

(Answers to **Your Turn** *problems can be found in the back of the book.)*



(a) Some water is displaced, but not enough to float the wood



(b) More water is displaced, and now the wood floats

▲ **FIGURE 15–10 Floatation**

(a) The block of wood displaces some water, but not enough to equal its weight. Thus, the block would not float at this position. **(b)** The weight of displaced water equals the weight of the block in this case. The block floats now.

▲ The water of the Dead Sea is unusually dense because of its great salt content. As a result, swimmers can float higher in the water than they are accustomed and engage in recreational activities that we don't ordinarily associate with a dip in the ocean.

Floatation

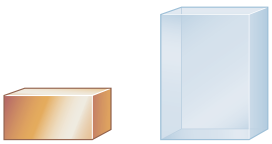
When an object floats, the buoyant force acting on it equals its weight. For example, suppose we slowly lower a block of wood into a flask of water. At first, as in **Figure 15–10 (a)**, only a small amount of water is displaced and the buoyant force is a fraction of the block's weight. If we were to release the block now, it would drop farther into the water. As we continue to lower the block, more water is displaced, increasing the buoyant force.

Eventually, we reach the situation pictured in **Figure 15–10 (b)**, where the block begins to float. In this case, the buoyant force equals the weight of the wood. This, in turn, means that the weight of the displaced water is equal to the weight of the wood. In general,

> An object floats when it displaces an amount of fluid whose weight is equal to the weight of the object.

This is illustrated in **Figure 15–11 (a)**, where we show the volume of water equal to the weight of a block of wood. Similarly, in **Figure 15–11 (b)** we show the amount of water necessary to have the same weight as a block of metal. Clearly, if the metal is completely submerged, the buoyant force is only a fraction of its weight, and so it sinks. On the other hand, if the metal is formed into the shape of a bowl, as in **Figure 15–11 (c)**, it can displace a volume of water equal to its weight and float.

(a) Block of wood and water of equal weight



(b) Block of metal and water of equal weight

(c) Same block of metal as in (b), but shaped like a bowl to displace more water

▲ **FIGURE 15–11 Floating an object that is more dense than water**

(a) A wood block and the volume of water that has the same weight. Because the wood has a larger volume than this, it floats. **(b)** A metal block and the volume of water that has the same weight. Since the metal displaces less water than this, it sinks. **(c)** If the metal in (b) is shaped like a bowl, it can displace more water than the volume of the metal itself. In fact, it can displace enough water to float.

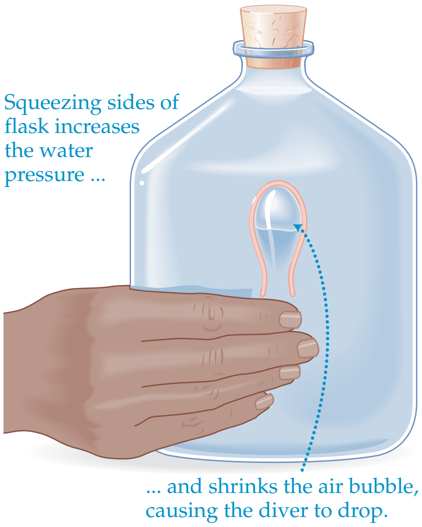
▲ Although steel is denser than water, a ship's bowl-like hull (top) displaces enough water to allow it to float, so long as it is not too heavily loaded. (The boundary between the red and black areas of the hull is the Plimsoll line, which indicates where the ship should ride in the water when carrying its maximum safe load—see Conceptual Checkpoint 15–4.) For balloons (bottom), the key to buoyancy is the lower density and greater volume of hot air. As the air in the balloon is heated, it expands the bag, increasing the volume of air that the balloon displaces. At the same time, heated air spills out of the balloon, decreasing its weight. Eventually, the average density of the balloon plus the hot air it contains becomes lower than that of the surrounding air, and it starts to float.

Another way to change the buoyancy of an object is to alter its overall density. Consider, for example, the *Cartesian diver* shown in **Figure 15–12.** As illustrated, the diver is simply a small glass tube with an air bubble trapped inside. Initially, the overall density of the tube and the air bubble is less than the density of water, and the diver floats. When the bottle containing the diver is squeezed, however, the pressure in the water rises, and the air bubble is compressed to a smaller volume. Now, the overall density of the tube and air bubble is greater than that of water, and the diver descends. By adjusting the pressure on the bottle, the diver can be made to float at any depth in the bottle.

REAL-WORLD PHYSICS: BIO
The swim bladder in fish

The same principle applies to the swim bladder of ray-finned bony fishes. The swim bladder is basically an air sac whose volume can be controlled by the fish. By adjusting the size of the swim bladder, the fish can give itself "neutral buoyancy"—that is, the fish can float without effort at a given depth, just like the Cartesian diver. Similar considerations apply to certain diving sea mammals, such as the bottlenose dolphin, Weddell seal, elephant seal, and blue whale. All of these animals are capable of diving to great depths—in fact, some of the seals have been observed at depths of nearly 400 m. In order to conserve energy on their long dives, they take advantage of the fact that the pressure of the surrounding water compresses their bodies and flattens the air sacs in their lungs. Just as with the Cartesian diver, this decreases their buoyancy to the point where they begin to sink. As a result, they can glide effortlessly to the desired depth, saving energy for the swim back to the surface.

REAL-WORLD PHYSICS: BIO
Diving sea mammals



▲ **FIGURE 15–12 A Cartesian diver**
A Cartesian diver floats because of the bubble of air trapped within it. When the bottle is squeezed, increasing the pressure in the water, the bubble is reduced in size and the diver sinks.

ACTIVE EXAMPLE 15–2 FLOATING A BLOCK OF WOOD

How much water (density 1.00×10^3 kg/m^3) must be displaced to float a cubical block of wood (density 655 kg/m^3) that is 15.0 cm on a side?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the volume of the wood: $V_{\text{wood}} = 3.38 \times 10^{-3}$ m^3
2. Find the weight of the wood: $\rho_{\text{wood}} V_{\text{wood}} g = 21.7$ N
3. Write an expression for the weight of a volume of water: $\rho_{\text{water}} V_{\text{water}} g$
4. Set the weight of water equal to the weight of the wood: $\rho_{\text{water}} V_{\text{water}} g = 21.7$ N
5. Solve for the volume of water: $V_{\text{water}} = 2.21 \times 10^{-3}$ m^3

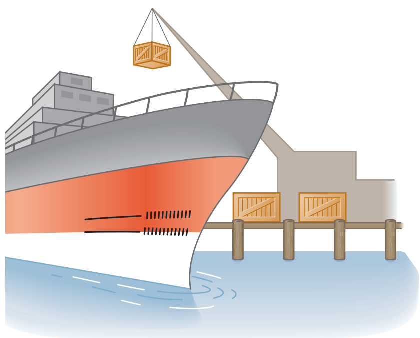
INSIGHT
As expected, only a fraction of the wood must be submerged in order for it to float.

YOUR TURN
What volume of water must be displaced if the density of the wood is 955 kg/m^3? Compare this volume to the volume of the wood itself.

(Answers to **Your Turn** *problems can be found in the back of the book.)*

CONCEPTUAL CHECKPOINT 15–4 THE PLIMSOLL MARK

On the side of a cargo ship you may see a horizontal line indicating "maximum load." (It is sometimes known as the "Plimsoll mark," after the nineteenth-century British legislator who caused it to be adopted.) When a ship is loaded to capacity, the maximum load line is at water level. The ship shown here has two maximum load lines, one for freshwater and one for salt water. Which line should be marked "maximum load for salt water": **(a)** the top line or **(b)** the bottom line?



REASONING AND DISCUSSION
If a ship sails from freshwater into salt water it floats higher, just as it is easier for you to float in an ocean than in a lake. The reason is that salt water is denser than freshwater; hence less of it needs to be displaced to provide a given buoyant force. Since the ship floats higher in salt water, the bottom line should be used to indicate maximum load.

ANSWER
(b) The bottom line should be used in salt water.

Tip of the Iceberg

As we have seen, an object floats when its weight is equal to the weight of the fluid it displaces. Let's use this condition to determine just how much of a floating object is submerged. We will then apply our result to the classic case of an iceberg.

Consider, then, a solid of density ρ_s floating in a fluid of density ρ_f, as in **Figure 15–13**. If the solid has a volume V_s, its total weight is

$$W_s = \rho_s V_s g$$

Similarly, the weight of a volume V_f of displaced fluid is

$$W_f = \rho_f V_f g$$

Equating these weights, we find the following:

$$W_s = W_f$$
$$\rho_s V_s g = \rho_f V_f g$$

Canceling g, and solving for the volume of displaced fluid, we have

$$V_f = V_s(\rho_s/\rho_f)$$

Since, by definition, the volume of displaced fluid, V_f, is the same as the volume of the solid that is submerged, V_{sub}, we find

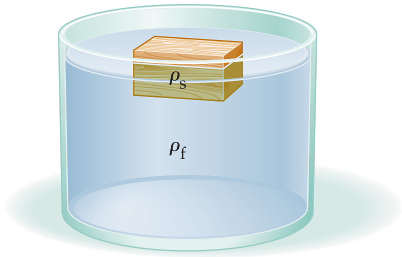
Submerged Volume V_{sub} for a Solid of Volume V_s and Density ρ_s Floating in a Fluid of Density ρ_f

$$V_{sub} = V_s(\rho_s/\rho_f) \qquad 15\text{–}10$$

SI unit: m^3

Note that this relation agrees with the results of Active Example 15–2.

We now apply this result to ice floating in water.



▲ **FIGURE 15–13 Submerged volume of a floating object**
A solid, of volume V_s and density ρ_s, floats in a fluid of density ρ_f. The volume of the solid that is submerged is $V_{sub} = V_s(\rho_s/\rho_f)$.

▲ Most people know that the bulk of an iceberg lies below the surface of the water. But as with ships and swimmers, the actual proportion that is submerged depends on whether the water is fresh or salt (see Example 15–6).

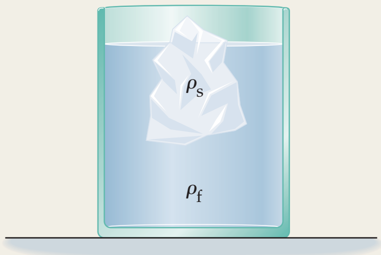
EXAMPLE 15–6 THE TIP OF THE ICEBERG

REAL-WORLD PHYSICS

What percentage of a floating chunk of ice projects above the level of the water? Assume a density of 917 kg/m^3 for the ice and $1.00 \times 10^3\ kg/m^3$ for the water.

PICTURE THE PROBLEM

Our sketch shows a chunk of ice floating in a glass of water. In this case the solid object is ice, with a density $\rho_s = \rho_{ice} = 917\ kg/m^3$, and the fluid is water, with a density $\rho_f = \rho_{water} = 1.00 \times 10^3\ kg/m^3$.



STRATEGY

We can apply Equation 15–10 to this system. First, the fraction of the total volume of the ice, V_s, that is submerged is $V_{sub}/V_s = \rho_s/\rho_f$. Hence the fraction that is above the water is $1 - V_{sub}/V_s = 1 - \rho_s/\rho_f$. Multiplying this fraction by 100 yields the percentage above water.

SOLUTION

1. Calculate the fraction of the total volume of the ice that is submerged: $\dfrac{V_{sub}}{V_s} = \dfrac{\rho_s}{\rho_f} = \dfrac{917\ kg/m^3}{1.00 \times 10^3\ kg/m^3} = 0.917$

2. Calculate the fraction of the ice that is above water: $1 - \dfrac{\rho_s}{\rho_f} = 1 - 0.917 = 0.083$

3. Multiply by 100 to obtain a percentage: $100\left(1 - \dfrac{\rho_s}{\rho_f}\right) = 100(0.083) = 8.3\%$

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INSIGHT

Because we seek a percentage, it is not necessary to know the total volume of the ice. Thus, our result that 8.3% of the ice is above the water applies whether we are talking about an ice cube in a drinking glass, or an iceberg floating in a freshwater lake. If an iceberg floats in the ocean, however, it will float higher due to the higher density of seawater. We consider this case in the following Practice Problem.

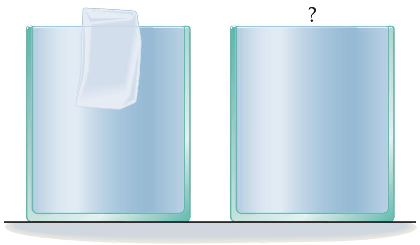
PRACTICE PROBLEM

What percentage of an ice chunk is above water level if it floats in seawater? (The density of seawater can be found in Table 15–1.) [**Answer:** 10.5%]

Some related homework problems: Problem 46, Problem 47

CONCEPTUAL CHECKPOINT 15–5 THE NEW WATER LEVEL I

A cup is filled to the brim with water and a floating ice cube. When the ice melts, which of the following occurs? **(a)** Water overflows the cup, **(b)** the water level decreases, or **(c)** the water level remains the same.



REASONING AND DISCUSSION

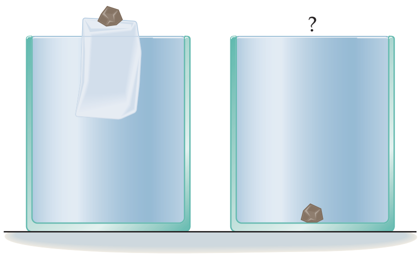
Since the ice cube floats, it displaces a volume of water equal to its weight. But when it melts, it becomes water, and its weight is the same. Hence, the melted water fills exactly the same volume that the ice cube displaced when floating. As a result, the water level is unchanged.

ANSWER

(c) The water level remains the same.

CONCEPTUAL CHECKPOINT 15–6 THE NEW WATER LEVEL II

A cup is filled to the brim with water and a floating ice cube. Resting on top of the ice cube is a small pebble. When the ice melts, which of the following occurs? **(a)** Water overflows the cup, **(b)** the water level decreases, or **(c)** the water level remains the same.



REASONING AND DISCUSSION

We know from the previous Conceptual Checkpoint that the ice itself makes no difference to the water level. As for the pebble, when it floats on the ice it displaces an amount of water equal to its *weight*. When the ice melts, the pebble drops to the bottom of the cup, where it displaces a volume of water equal to its own *volume*. Since the volume of the pebble is less than the volume of water with the same weight, we conclude that less water is displaced after the ice melts. Hence, the water level decreases.

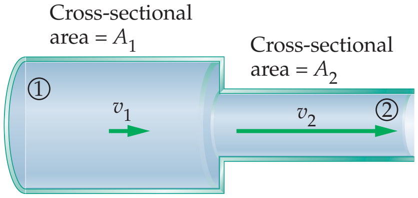
ANSWER

(b) The water level decreases.

15–6 Fluid Flow and Continuity

Suppose you want to water the yard, but you don't have a spray nozzle for the end of the hose. Without a nozzle the water flows rather slowly from the hose and hits the ground within half a meter. But if you place your thumb over the end of the hose, narrowing the opening to a fraction of its original size, the water sprays out with a high speed and a large range. Why does decreasing the size of the opening have this effect?

To answer this question, we begin by considering a simple system that shows the same behavior. Imagine, then, that a fluid flows with a speed v_1 through a cylindrical pipe of cross-sectional area A_1, as in the left-hand portion of Figure 15–14. If the pipe narrows to a cross-sectional area A_2, as in the right-hand portion of Figure 15–14, the fluid will flow with a new speed, v_2.



▲ **FIGURE 15–14 Fluid flow through a pipe of varying diameter**

As a fluid flows from a large pipe to a small pipe, the same mass of fluid passes a given point in a given amount of time. Thus, the speed in the small pipe is greater than it is in the large pipe.

We can find the speed in the narrow section of the pipe by assuming that any amount of fluid that passes point 1 in a given time, Δt, must also flow past point 2 in the same time. If this were not the case, the system would be gaining or losing fluid. To find the mass of fluid passing point 1 in the time Δt, note that the fluid moves through a distance $v_1 \Delta t$ in this time. As a result, the volume of fluid going past point 1 is

$$\Delta V_1 = A_1 v_1 \Delta t$$

Hence, the mass of fluid passing point 1 is

$$\Delta m_1 = \rho_1 \Delta V_1 = \rho_1 A_1 v_1 \Delta t$$

Similarly, the mass passing point 2 in the time Δt is

$$\Delta m_2 = \rho_2 \Delta V_2 = \rho_2 A_2 v_2 \Delta t$$

Note that we have allowed for the possibility of the fluid having different densities at points 1 and 2.

Finally, equating these two masses yields the relation between v_1 and v_2:

$$\Delta m_1 = \Delta m_2$$
$$\rho_1 A_1 v_1 \Delta t = \rho_2 A_2 v_2 \Delta t$$

Canceling Δt we find

Equation of Continuity

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad 15\text{–}11$$

This relation is referred to as the **equation of continuity.**

Most gases are readily compressed, which means that their densities can change. In contrast, most liquids are practically incompressible, so their densities are essentially constant. Unless stated otherwise, we will assume all liquids discussed in this text to be perfectly incompressible. Thus, for liquids, ρ_1 and ρ_2 are the same in Equation 15–11, and the equation of continuity reduces to the following:

Equation of Continuity for an Incompressible Fluid

$$A_1 v_1 = A_2 v_2 \qquad 15\text{–}12$$

We next apply this relation to the case of water flowing through the nozzle of a fire hose.

▲ Narrowing the opening in a hose with a nozzle (or thumb) increases the velocity of flow, as one would expect from the equation of continuity.

PROBLEM-SOLVING NOTE

Continuity of Flow

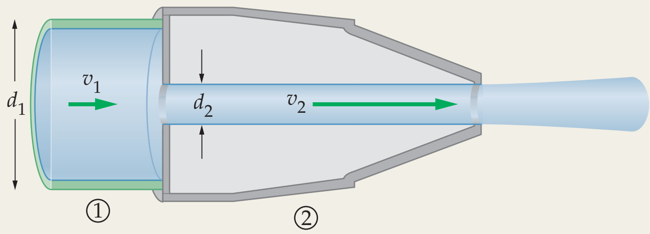
The speed of an incompressible fluid is inversely proportional to the area through which it flows.

EXAMPLE 15–7 SPRAY I

Water travels through a 9.6-cm diameter fire hose with a speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5 cm. What is the speed of the water coming out of the nozzle?

PICTURE THE PROBLEM

In our sketch, we label the speed of the water in the hose with v_1 and the speed of the water coming out the nozzle with v_2. We are given that $v_1 = 1.3$ m/s. We also know that the diameter of the hose is $d_1 = 9.6$ cm and the diameter of the nozzle is $d_2 = 2.5$ cm.



STRATEGY

We can find the water speed in the nozzle by applying $A_1 v_1 = A_2 v_2$. In addition, we assume that the hose and nozzle are circular in cross section; hence, their areas are given by $A = \pi d^2/4$, where d is the diameter.

SOLUTION

1. Solve Equation 15–12 for v_2, the speed of the water in the nozzle: $v_2 = v_1(A_1/A_2)$

2. Replace the areas with $A = \pi d^2/4$: $v_2 = v_1\left(\dfrac{\pi d_1{}^2/4}{\pi d_2{}^2/4}\right) = v_1\left(\dfrac{d_1{}^2}{d_2{}^2}\right)$

2. Substitute numerical values: $v_2 = v_1\left(\dfrac{d_1{}^2}{d_2{}^2}\right) = (1.3\ \text{m/s})\left(\dfrac{9.6\ \text{cm}}{2.5\ \text{cm}}\right)^2 = 19\ \text{m/s}$

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INSIGHT

Note that a small-diameter nozzle can give very high speeds. In fact, the speed depends inversely on the diameter squared.

PRACTICE PROBLEM

What nozzle diameter would be required to give the water a speed of 21 m/s? [**Answer:** $d_2 = 2.4$ cm]

Some related homework problems: Problem 51, Problem 56, Problem 57

15–7 Bernoulli's Equation

In this section, we apply the work–energy theorem to fluids. The result is a relation between the pressure of a fluid, its speed, and its height. This relation is known as **Bernoulli's equation.**

Change in Speed

We begin by considering a system in which the speed of the fluid changes. To be specific, the system of interest is the same as that shown in Figure 15–14. We have already shown that the speed of the fluid increases as it flows from region 1 to region 2; we now investigate the corresponding change in pressure.

Our plan of attack is to first calculate the total work done on the fluid as it moves from one region to the next. This result will depend on the pressure in the fluid. Once the total work is obtained, the work–energy theorem allows us to equate it to the change in kinetic energy of the fluid. This will give the pressure–speed relationship we desire.

Consider an element of fluid of length Δx_1, as shown in **Figure 15–15**. This element is pushed in the direction of motion by the pressure P_1. Thus, the pressure does positive work, ΔW_1, on the fluid element. Noting that the force exerted on the element is $F_1 = P_1 A_1$, and that work is force times distance, the work done on the element is

$$\Delta W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1$$

The volume of the fluid element is $\Delta V_1 = A_1 \Delta x_1$, so the work done by P_1 is

$$\Delta W_1 = P_1 \Delta V_1$$

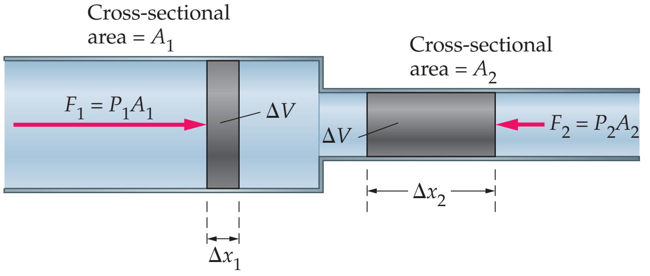


FIGURE 15–15 Work done on a fluid element

As an incompressible fluid element of volume ΔV moves from pipe 1 to pipe 2, the pressure P_1 does a positive work $P_1 \Delta V$ and the pressure P_2 does a negative work $P_2 \Delta V$. Since P_1 is greater than P_2, the net result is that positive work is done, and the fluid element speeds up.

Next, when the fluid element emerges into region 2, it experiences a force opposite to its direction of motion due to the pressure P_2. Thus, P_2 does negative work on the element. Following the same steps given previously, we can write the work done by P_2 as

$$\Delta W_2 = -P_2 \Delta V_2$$

Now, for an incompressible fluid, the volume of the element does not change as it goes from region 1 to region 2. Therefore,

$$\Delta V_1 = \Delta V_2 = \Delta V$$

Using this result, we can write the total work done on the fluid element as follows:

$$\Delta W_{\text{total}} = \Delta W_1 + \Delta W_2 = P_1 \Delta V - P_2 \Delta V = (P_1 - P_2)\Delta V$$

The final step is to equate the total work to the change in kinetic energy:

$$\Delta W_{\text{total}} = (P_1 - P_2)\Delta V = K_{\text{final}} - K_{\text{initial}} = K_2 - K_1 \qquad 15\text{–}13$$

What is the kinetic energy of the fluid element? Well, the mass of the element is

$$\Delta m = \rho \Delta V$$

Thus, its kinetic energy is simply

$$K = \tfrac{1}{2}(\Delta m)v^2 = \tfrac{1}{2}(\rho \Delta V)v^2$$

Using this expression in Equation 15–13, we have

$$\Delta W_{\text{total}} = (P_1 - P_2)\Delta V = \left(\tfrac{1}{2}\rho v_2{}^2 - \tfrac{1}{2}\rho v_1{}^2\right)\Delta V$$

Canceling the common factor ΔV, and rearranging, we find

$$P_1 + \tfrac{1}{2}\rho v_1{}^2 = P_2 + \tfrac{1}{2}\rho v_2{}^2 \qquad 15\text{–}14$$

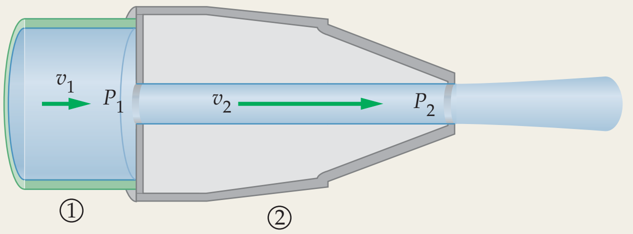
Equation 15–14 is equivalent to saying that $P + \frac{1}{2}\rho v^2$ is constant. Thus, there is a tradeoff between the pressure in a fluid and its speed—as the fluid speeds up, its pressure decreases. If this seems odd, recall that P_1 acts to increase the speed of the fluid element and P_2 acts to decrease its speed. The element will speed up, then, only if P_2 is less than P_1.

EXAMPLE 15–8 SPRAY II

Referring to Example 15–7, suppose the pressure in the fire hose is 350 kPa. **(a)** Is the pressure in the nozzle greater than, less than, or equal to 350 kPa? **(b)** Find the pressure in the nozzle.

PICTURE THE PROBLEM

In our sketch, we use the same numbering system as in Example 15–7; that is, 1 refers to the hose, 2 refers to the nozzle. Therefore, $P_1 = 350$ kPa, and P_2 is to be determined.



STRATEGY

a. The pressure and speed of the fluid are related by $P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2$. This result can be used to compare P_1 to P_2.

b. We used the equation of continuity ($A_1 v_1 = A_2 v_2$) in Example 15–7 to determine v_2. We now use this result, along with $P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2$, to determine P_2.

SOLUTION

Part (a)

1. Apply $P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2$, noting that v_2 is greater than v_1, as determined in Example 15–7:

The pressure and speed are related by $P + \frac{1}{2}\rho v^2 = \text{constant}$. Thus, an increase in speed (v) can occur only when there is a corresponding decrease in pressure (P). **Answer:** P_2 (nozzle) is less than P_1 (hose); that is, $P_2 < 350$ kPa.

Part (b)

2. Solve $P_1 + \frac{1}{2}\rho v_1{}^2 = P_2 + \frac{1}{2}\rho v_2{}^2$ for the pressure in the nozzle, P_2:

$$P_2 = P_1 + \tfrac{1}{2}\rho(v_1{}^2 - v_2{}^2)$$

3. Substitute numerical values, including $v_1 = 1.3$ m/s and $v_2 = 19$ m/s from Example 15–7:

$$P_2 = 350\text{ kPa} + \tfrac{1}{2}(1.00 \times 10^3\text{ kg/m}^3)\left[(1.3\text{ m/s})^2 - (19\text{ m/s})^2\right] = 170\text{ kPa}$$

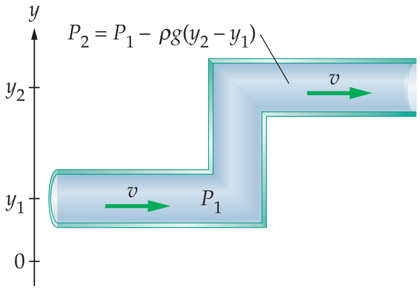
INSIGHT

Note that the pressure in the nozzle is less than the pressure in the hose by roughly a factor of 2. This is because part of the energy associated with the high pressure in the hose has been converted to kinetic energy as the water passes through the nozzle. The connection between pressure and energy will be explored in more detail later in this section.

PRACTICE PROBLEM

What nozzle speed would be required to give a nozzle pressure of 110 kPa? [**Answer:** $v_2 = 22$ m/s]

Some related homework problems: Problem 60, Problem 61



▲ **FIGURE 15–16 Fluid pressure in a pipe of varying elevation**

Fluid of density ρ flows in a pipe of uniform cross-sectional area from height y_1 to height y_2. As it does so, its pressure decreases by the amount $\rho g(y_2 - y_1)$.

Change in Height

If a fluid flows through the pipe shown in **Figure 15–16**, its height increases from y_1 to y_2 as it goes from one region to the next. Since the cross-sectional area of the pipe is constant, however, the speed of the fluid is unchanged, according to Equation 15–12. Thus, the change in kinetic energy of the fluid element shown in Figure 15–16 is zero.

The total work done on the fluid element is the sum of the works done by the pressure in each of the two regions, plus the work done by gravity. As before, the work done by pressure is

$$\Delta W_{\text{pressure}} = \Delta W_1 + \Delta W_2 = (P_1 - P_2)\Delta V$$

As the fluid element rises, gravity does negative work on it. Recalling that the mass of the element is

$$\Delta m = \rho \Delta V$$

the work done by gravity is

$$\Delta W_{\text{gravity}} = -\Delta m g(y_2 - y_1) = -\rho \Delta V g(y_2 - y_1)$$

Setting the total work equal to zero (since $\Delta K = 0$) yields

$$\begin{aligned}\Delta W_{\text{total}} &= \Delta W_{\text{pressure}} + \Delta W_{\text{gravity}} \\ &= (P_1 - P_2)\Delta V - \rho g(y_2 - y_1)\Delta V = 0\end{aligned}$$

Canceling ΔV and rearranging gives

$$P_1 + \rho g y_1 = P_2 + \rho g y_2 \qquad 15\text{–}15$$

In this case, it is $P + \rho g y$ that is constant—hence, pressure decreases as the height within a fluid increases. Note, in fact, that Equation 15–15 is precisely the same as Equation 15–7, which was obtained using force considerations. Here we obtained the result using the work–energy theorem.

EXERCISE 15–4

Water flows with constant speed through a garden hose that goes up a step 20.0 cm high. If the water pressure is 143 kPa at the bottom of the step, what is its pressure at the top of the step?

SOLUTION

Apply Equation 15–15, letting subscript 1 refer to the bottom of the step and subscript 2 refer to the top of the step. Solve for P_2:

$$\begin{aligned}P_2 &= P_1 + \rho g(y_1 - y_2) \\ &= 143\ \text{kPa} + (1.00 \times 10^3\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0 - 0.200\ \text{m}) = 141\ \text{kPa}\end{aligned}$$

This is precisely the pressure difference that would be observed if the water had been at rest.

General Case

In a more general case, both the height of a fluid and its speed may change. Combining the results obtained in Equations 15–14 and 15–15 yields the full form of Bernoulli's equation:

Bernoulli's Equation

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad 15\text{–}16$$

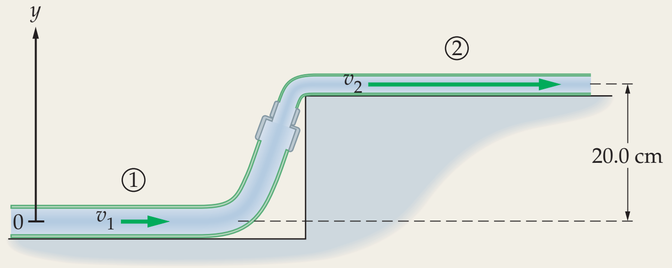
Thus, in general, the quantity $P + \frac{1}{2}\rho v^2 + \rho g y$ is a constant within a fluid. This is basically a statement of energy conservation. For example, recalling the definition of density in Equation 15–1, we find that $\frac{1}{2}\rho v^2$ is $\frac{1}{2}(M/V)v^2 = \left(\frac{1}{2}Mv^2\right)/V$. Clearly, this term represents the kinetic energy per volume of the fluid. Similarly, the term $\rho g y$ can be written as $(M/V)gy = (Mgy)/V$, which is the gravitational potential energy per volume.

Finally, the first term in Bernoulli's equation—the pressure—can also be thought of as an energy per volume. Recall that $P = F/A$. If we multiply numerator and denominator by a distance, d, we have $P = Fd/Ad$. But Fd is the work done by the force F as it acts through the distance d, and Ad is the volume swept out by an area A moved through a distance d. Therefore, the pressure can be thought of as work (energy) per volume: $P = W/V$.

As a result, Bernoulli's equation is simply a restatement of the work–energy theorem in terms of quantities per volume. Of course, this relation holds only as long as we can ignore frictional losses, which would lead to heating. We will consider the energy aspects of heat in Chapter 16.

ACTIVE EXAMPLE 15–3 FIND THE PRESSURE

Repeat Exercise 15–4 with the following additional information: **(a)** the cross-sectional area of the hose on top of the step is half that at the bottom of the step, and **(b)** the speed of the water at the bottom of the step is 1.20 m/s.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Use the continuity equation to find the water's speed on top of the step: $v_2 = 2v_1 = 2.40$ m/s
2. Solve Bernoulli's equation for P_2: $P_2 = P_1 - \rho g(y_2 - y_1) - \frac{1}{2}\rho(v_2^2 - v_1^2)$
3. Substitute numerical values: $P_2 = 139$ kPa

INSIGHT

The pressure on top of the step is less than in Exercise 15–4. This is to be expected because, in this case, the water speeds up as it rises over the step.

YOUR TURN

For what step height will the pressure at the top of the step be equal to atmospheric pressure?

(Answers to **Your Turn** *problems can be found in the back of the book.)*

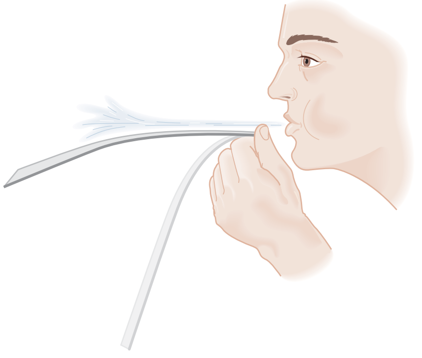
15–8 Applications of Bernoulli's Equation

We now consider a variety of real-world examples that illustrate the application of Bernoulli's equation.

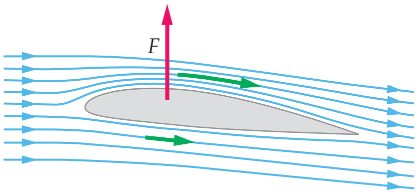
Pressure and Speed

As mentioned, it often seems counterintuitive that a fast-moving fluid should have less pressure than a slow-moving one. Remember, however, that pressure can be thought of as a form of energy. From this point of view, there is an energy tradeoff between pressure and kinetic energy.

Perhaps the easiest way to demonstrate the dependence of pressure on speed is to blow across the top of a piece of paper. If you hold the paper as shown in **Figure 15–17**, then blow over the top surface, the paper will lift upward. The reason

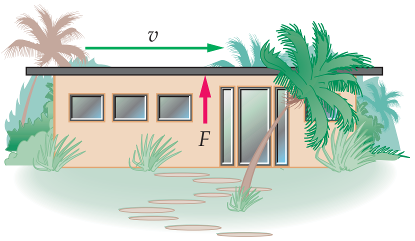


▲ **FIGURE 15–17 The Bernoulli effect on a sheet of paper**
If you hold a piece of paper by its end, it will bend downward. Blowing across the top of the paper reduces the pressure there, resulting in a net upward force which lifts the paper to a nearly horizontal position.



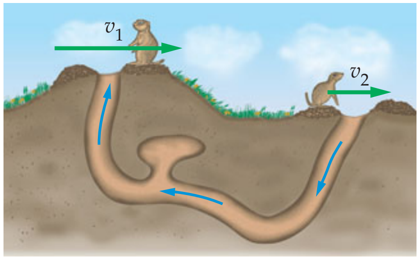
▲ **FIGURE 15–18 Air flow and lift in an airplane wing**

This figure shows the cross section of an airplane wing with air flowing past it. The wing is shaped so that air flows more rapidly over the top of the wing than along the bottom. As a result, the pressure on top of the wing is reduced, and a net upward force (lift) is generated.



▲ **FIGURE 15–19 Force on a roof due to wind speed**

Wind blows across the roof of a house, but the air inside is at rest. The pressure over the roof is therefore less than the pressure inside, resulting in a net upward force on the roof.



▲ **FIGURE 15–20 Air circulation in a prairie dog burrow**

A prairie dog burrow typically has a high mound on one end and a low mound on the other. Since the wind speed increases with height above the ground, the pressure is smaller at the high-mound end of the burrow. The result is a very convenient circulation of fresh air through the burrow.

is that there is a difference in air speed between the top and the bottom of the paper, with the higher speed on top. As a result, the pressure above the paper is lower. This pressure difference, in turn, results in a net upward force, referred to as **lift,** and the paper rises.

A similar example of pressure and speed is provided by the airplane wing. A cross section of a typical wing is shown in **Figure 15–18**. The shape of the wing is designed so that air flows more rapidly over the top surface than the lower surface. As a result, the pressure is less on top. As with the piece of paper, the pressure difference results in a net upward force (lift) on the wing.

Note that lift is a dynamic effect; it requires a flow of air. The greater the speed difference, the greater the upward force.

EXERCISE 15–5

During a windstorm, a 35.5 m/s wind blows across the flat roof of a small home, as in **Figure 15–19**. Find the difference in pressure between the air inside the home and the air just above the roof, assuming the doors and windows of the house are closed. (The density of air is 1.29 kg/m^3.)

SOLUTION

Use Bernoulli's equation with point 1 just under the roof and point 2 just above the roof. Since there is little difference in elevation between these points, $y_1 = y_2 = y$. Thus,

$$P_1 + 0 + \rho g y = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y$$

Solving for the pressure difference, $P_1 - P_2$, we find

$$P_1 - P_2 = \tfrac{1}{2}\rho v_2^2 = \tfrac{1}{2}(1.29\ \text{kg/m}^3)(35.5\ \text{m/s})^2 = 813\ \text{Pa}$$

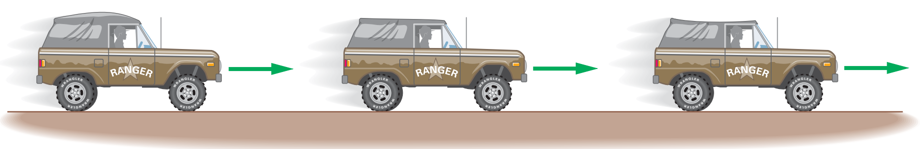
A difference in pressure of 813 Pa might seem rather small, considering that atmospheric pressure is 101 kPa. However, it can still cause a significant force on a relatively large area, such as a roof. If a typical roof has an area of about 150 m^2, for example, a pressure difference of 813 Pa results in an upward force of over 27,000 pounds! This is why roofs are often torn from houses during severe windstorms.

On a lighter note, prairie dogs seem to *benefit* from the effects of Bernoulli's equation. A schematic prairie dog burrow is pictured in **Figure 15–20**. Note that one of the entrance/exit mounds is higher than the other. This is significant because the speed of air flow due to the incessant prairie winds varies with height; the speed goes to zero right at ground level, and increases to its maximum value within a few feet above the surface. As a result, the speed of air over the higher mound is greater than that over the lower mound. This causes the pressure to be less over the higher mound. With a pressure difference between the two mounds, air is drawn through the burrow, giving a form of natural air conditioning.

Similar effects are seen in an atomizer, which sprays perfume in a fine mist. As the bulb shoots a gust of air, as in **Figure 15–21**, it passes through a narrow orifice, which causes the air speed to increase. The pressure decreases as a result, and perfume is drawn up by the pressure difference into the stream of air.

CONCEPTUAL CHECKPOINT 15–7 A RAGTOP ROOF

A small ranger vehicle has a soft, ragtop roof. When the car is at rest, the roof is flat. When the car is cruising at highway speeds with its windows rolled up, does the roof **(a)** bow upward, **(b)** remain flat, or **(c)** bow downward?



REASONING AND DISCUSSION

When the car is in motion, air flows over the top of the roof, while the air inside the car is at rest—since the windows are closed. Thus, there is less pressure over the roof than under it. As a result, the roof bows upward.

ANSWER

(a) The roof bows upward.

▲ We often say that a hurricane or tornado "blew the roof off a house." However, the house at left lost its roof not because of the great pressure exerted by the wind, but rather the opposite. In accordance with the Bernoulli effect, the high speed of the wind passing over the roof created a region of reduced pressure. Normal atmospheric pressure inside the house then blew the roof off. The same phenomenon is exploited by prairie dogs to ventilate their burrows. One end of the burrow is always situated at a greater height than the other. Because the prairie wind blows much faster a few feet above ground level, the pressure at the elevated end of the burrow is reduced. The resulting pressure difference produces a flow of air through the burrow.

Torricelli's Law

Our final example of Bernoulli's equation deals with the speed of a fluid as it flows through a hole in a container. Consider, for example, the tank of water shown in **Figure 15–22**. If a hole is poked through the side of the tank at a depth h below the surface, what is the speed of the water as it emerges?

To answer this question, we apply Bernoulli's equation to the two points shown in the figure. First, at point 1 we note that the water is open to the atmosphere; thus $P_1 = P_{at}$. Next, with the origin at the level of the hole, the height of the water surface is $y_1 = h$. Finally, if the hole is relatively small and the tank is large, the top surface of the water will have essentially zero speed; thus we can set $v_1 = 0$. Collecting these results, we have the following for point 1:

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_{at} + 0 + \rho g h$$

Now for point 2. At this point the height is $y_2 = 0$, by definition of the origin, and the speed of the escaping water is the unknown, v_2. Here is the key step: The pressure P_2 is *atmospheric pressure*, because the hole opens the water to the atmosphere. Thus, for point 2 we have the following:

$$P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 = P_{at} + \tfrac{1}{2}\rho v_2^2 + 0$$

Equating these results yields

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2$$

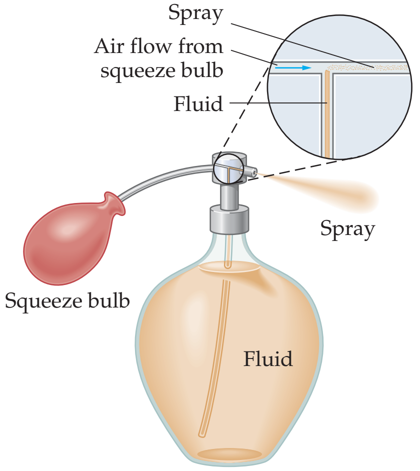
$$P_{at} + \rho g h = P_{at} + \tfrac{1}{2}\rho v_2^2$$

Eliminating P_{at} and ρ we find

$$v_2 = \sqrt{2gh} \qquad \text{15–17}$$

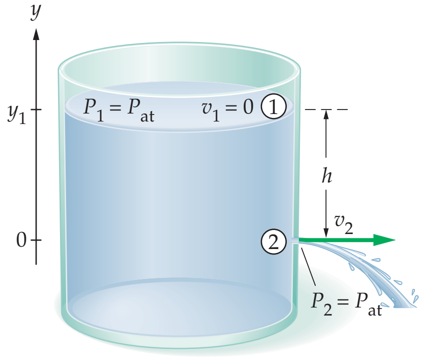
This result is known as **Torricelli's law.**

This expression for v_2 should look familiar; it is the speed of an object that falls freely through a distance h. That is, the water emerges from the tank with the same speed as if it had fallen from the surface of the water to the hole. Similarly, if the emerging stream of water were to be directed upward, as in **Figure 15–23**, it would have just enough speed to rise through a height h—right back to the water's surface. This is precisely what one would expect on the basis of energy conservation.



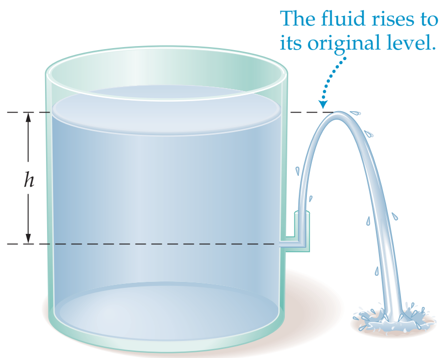
▲ **FIGURE 15–21 An atomizer**

The operation of an "atomizer" can be understood in terms of Bernoulli's equation. The high-speed jet of air created by squeezing the bulb creates a low pressure at the top of the vertical tube. This causes fluid to be drawn up the tube and expelled with the air jet as a fine spray.



▲ **FIGURE 15–22 Fluid emerging from a hole in a container**

Since the fluid exiting the hole is in contact with the atmosphere, the pressure there is just as it is on the top surface of the fluid.



▲ **FIGURE 15–23 Maximum height of a stream of water**

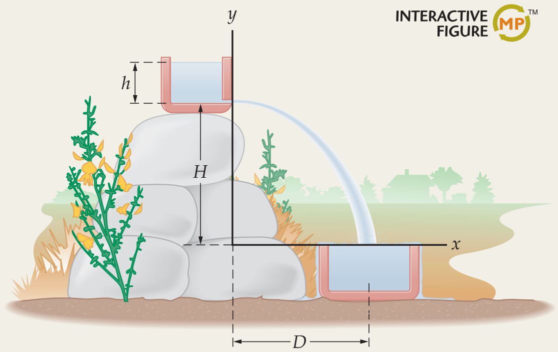
If the fluid emerging from a hole in a container is directed upward, it has just enough speed to reach the surface level of the fluid. This is an example of energy conservation.

EXAMPLE 15–9 A WATER FOUNTAIN

In designing a backyard water fountain, a gardener wants a stream of water to exit from the bottom of one tub and land in a second one, as shown in the sketch. The top of the second tub is 0.500 m below the hole in the first tub, which has water in it to a depth of 0.150 m. How far to the right of the first tub must the second one be placed to catch the stream of water?

PICTURE THE PROBLEM

Our sketch labels the various pertinent quantities for this problem. We know that $h = 0.150$ m and $H = 0.500$ m. The distance D is to be found. We have also chosen an appropriate coordinate system, with the y direction vertical and the x direction horizontal.



STRATEGY

This problem combines Torricelli's law and kinematics. First, we find the speed v of the stream of water as it leaves the first can, using Equation 15–17. Next, we find the time t required for the stream to fall freely through a distance H. Finally, since the stream moves with constant speed in the x direction, the distance D is given by $D = vt$.

SOLUTION

1. Find the speed v of the stream when it leaves the first can: $v = \sqrt{2gh} = \sqrt{2(9.81\ \mathrm{m/s^2})(0.150\ \mathrm{m})} = 1.72\ \mathrm{m/s}$
2. Find the time t for free fall through a height H: $y = H - \frac{1}{2}gt^2 = 0$

$$t = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2(0.500\ \mathrm{m})}{9.81\ \mathrm{m/s^2}}} = 0.319\ \mathrm{s}$$

3. Multiply v times t to find the distance D: $x = vt = (1.72\ \mathrm{m/s})(0.319\ \mathrm{s}) = 0.549\ \mathrm{m} = D$

INSIGHT

Note that our solution for x can also be written as $x = vt = \sqrt{2gh}(\sqrt{2H/g}) = 2\sqrt{hH}$. Thus, if the values of h and H are interchanged, the distance D remains the same. Note also that x is independent of the acceleration of gravity; therefore, the fountain would work just as well on the Moon.

PRACTICE PROBLEM

Find the distance D for $h = 0.500$ m and $H = 0.150$ m. [**Answer:** $D = 2\sqrt{hH} = 0.548$ m, as expected.]

Some related homework problems: Problem 64, Problem 112

*15–9 Viscosity and Surface Tension

To this point we have considered only "ideal" fluids. In particular, we have assumed that fluids flow without frictional losses and that the molecules in a fluid have no interaction with one another. In this section, we consider the consequences that follow from relaxing these assumptions.

Viscosity

When a block slides across a rough floor, it experiences a frictional force opposing the motion. Similarly, a fluid flowing past a stationary surface experiences a force opposing the flow. This tendency to resist flow is referred to as the **viscosity** of a fluid. Fluids like air have low viscosities, thicker fluids like water are more viscous, and fluids like honey and motor oil are characterized by high viscosity.

To be specific, consider a situation of great practical importance—the flow of a fluid through a tube. Examples of this type of system include water flowing through a metal pipe in a house and blood flowing through an artery or a vein. If the fluid were ideal, with zero viscosity, it would flow through the tube with a speed that is the same throughout the fluid, as indicated in **Figure 15–24 (a)**. Real fluids with finite viscosity are found to have flow patterns like the one shown in **Figure 15–24 (b)**. In this case, the fluid is at rest next to the walls of the tube and flows with its greatest speed in the center of the tube. Because adjacent portions of the fluid flow past one another with different speeds, a force must be exerted on the fluid to maintain the flow, just as a force is required to keep a block sliding across a rough surface.

The force causing a viscous fluid to flow is provided by the pressure difference, $P_1 - P_2$, across a given length, L, of tube. Experiments show that the required pressure difference is proportional to the length of the tube and to the average speed, v, of the fluid. In addition, it is inversely proportional to the cross-sectional area, A, of the tube. Combining these observations, the pressure difference can be written in the following form:

$$P_1 - P_2 \propto \frac{vL}{A}$$

The constant of proportionality between the pressure difference and vL/A is related to the **coefficient of viscosity, η,** of a fluid. In fact, the viscosity is *defined* in such a way that the pressure difference is given by the following expression:

$$P_1 - P_2 = 8\pi\eta\frac{vL}{A} \qquad 15\text{–}18$$

From this equation we can see that the dimensions of the coefficient of viscosity are $N \cdot s/m^2$. A common unit in the study of viscous fluids is the **poise,** named for the French physiologist Jean Louis Marie Poiseuille (1799–1869) and defined as

$$1 \text{ poise} = 1 \text{ dyne} \cdot \text{s/cm}^2 = 0.1 \text{ N} \cdot \text{s/m}^2$$

For example, the viscosity of water at room temperature is 1.0055×10^{-3} N·s/m^2 and the viscosity of blood at 37 °C is 2.72×10^{-3} N·s/m^2. Some additional viscosities are given in Table 15–3.

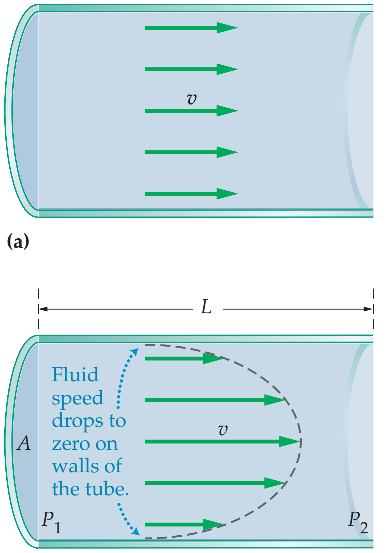
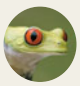


FIGURE 15–24 Fluid flow through a tube
(a) An ideal fluid flows through a tube with a speed that is the same everywhere in the fluid. **(b)** In a fluid with finite viscosity, the speed of the fluid goes to zero on the walls of the tube and reaches its maximum value in the center of the tube. The average speed of the fluid depends on the pressure difference between the ends of the tube, $P_1 - P_2$, the length of the tube, L, the cross-sectional area of the tube, A, and the coefficient of viscosity of the liquid, η.

TABLE 15–3 Viscosities (η) of Various Fluids (N·s/m^2)

Honey	10
Glycerine (20 °C)	1.50
10-wt motor oil (30 °C)	0.250
Whole blood (37 °C)	2.72×10^{-3}
Water (0 °C)	1.79×10^{-3}
Water (20 °C)	1.0055×10^{-3}
Water (100 °C)	2.82×10^{-4}
Air (20 °C)	1.82×10^{-5}

EXAMPLE 15–10 BLOOD SPEED IN THE PULMONARY ARTERY



REAL-WORLD PHYSICS: BIO

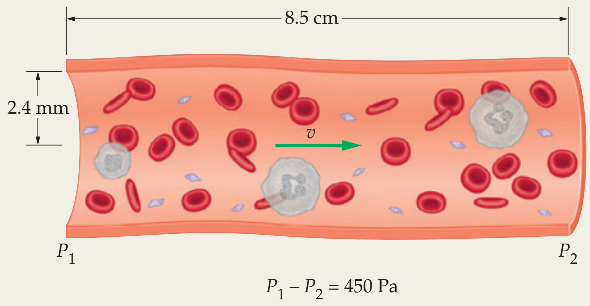
The pulmonary artery, which connects the heart to the lungs, is 8.5 cm long and has a pressure difference over this length of 450 Pa. If the inside radius of the artery is 2.4 mm, what is the average speed of blood in the pulmonary artery?

PICTURE THE PROBLEM

Our sketch shows a schematic representation of the pulmonary artery, not drawn to scale. The length (8.5 cm) and radius (2.4 mm) of the artery are indicated. In addition, we note that the pressure difference between the two ends of the artery is 450 Pa.

STRATEGY

The average speed of the blood can be found by using $P_1 - P_2 = 8\pi\eta(vL/A)$. Note that the pressure difference, $P_1 - P_2$, is given as 450 Pa $= 450$ N/m^2, and that the cross-sectional area of the blood vessel is $A = \pi r^2$.



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SOLUTION

1. Solve Equation 15–18 for the average speed, v: $$v = \frac{(P_1 - P_2)A}{8\pi\eta L}$$

2. Replace the cross-sectional area A with πr^2 and cancel π from the numerator and denominator: $$v = \frac{(P_1 - P_2)r^2}{8\eta L}$$

3. Substitute numerical values: $$v = \frac{(450\ \text{Pa})(0.0024\ \text{m})^2}{8(0.00272\ \text{N}\cdot\text{s/m}^2)(0.085\ \text{m})} = 1.4\ \text{m/s}$$

INSIGHT

The viscosity of blood increases rapidly with its hematocrit value; that is, with the concentration of red blood cells in the whole blood (see Chapter 10 for more information on the hematocrit value of blood). Thus, thick blood, with a high hematocrit value, requires a significantly larger pressure difference for a given rate of blood flow. This higher pressure must be provided by the heart, which consequently works harder with each beat.

PRACTICE PROBLEM

What pressure difference is required to give the blood in this pulmonary artery an average speed of 1.5 m/s? [**Answer:** 480 Pa]

Some related homework problems: Problem 102, Problem 103

A convenient way to characterize the flow of a fluid is in terms of its volume flow rate—the volume of fluid that passes a given point in a given amount of time. Referring to Section 15–6 we see that the volume flow rate of a fluid is simply vA, where v is the average speed of the fluid and A is the cross-sectional area of the tube through which it flows. Solving Equation 15–18 for the average speed gives $v = (P_1 - P_2)A/8\pi\eta L$. Multiplying this result by the cross-sectional area of the tube yields the volume flow rate:

$$\text{volume flow rate} = \frac{\Delta V}{\Delta t} = vA = \frac{(P_1 - P_2)A^2}{8\pi\eta L}$$

Using the fact that the cross-sectional area of the tube is $A = \pi r^2$, where r is its radius, we obtain the result known as **Poiseuille's equation:**

$$\frac{\Delta V}{\Delta t} = \frac{(P_1 - P_2)\pi r^4}{8\eta L} \qquad 15\text{–}19$$

Note that the volume flow rate varies with the fourth power of the tube's radius, thus a small change in radius corresponds to a large change in volume flow rate.

To see the significance of the r^4 dependence, consider an artery that branches into an arteriole that has half the artery's radius. Letting r go to $r/2$ in Poiseuille's equation, and solving for the pressure difference, we find

$$P_1 - P_2 = \frac{8\eta L}{\pi(r/2)^4}\left(\frac{\Delta V}{\Delta t}\right) = 16\left[\frac{8\eta L}{\pi r^4}\left(\frac{\Delta V}{\Delta t}\right)\right]$$

Thus, the pressure difference across a given length of arteriole is 16 times what it is across the same length of artery. In fact, in the human body the pressure drop along an artery is small compared to the rather large pressure drop observed in the arterioles. This is a direct consequence of the increased viscous drag of the blood as it flows through narrower blood vessels.

Similarly, a narrowing, or *stenosis*, of an artery can produce significant increases in blood pressure. For example, a reduction in radius of only 20%, from r to $0.8r$, causes an increase in pressure by a factor of $(1/0.8)^4 \sim 2.4$. Thus, even a small narrowing of an artery can lead to an increased risk for heart disease and stroke.

▲ Surface tension causes the surface of a liquid to behave like an elastic skin or membrane. When a small force is applied to the liquid surface, it tends to stretch, resisting penetration. This phenomenon enables a fishing spider to launch itself vertically upward as much as 4 cm from the water surface, as if from a trampoline. Such an acrobatic maneuver can save the spider from the attack of a predatory fish.

Surface Tension

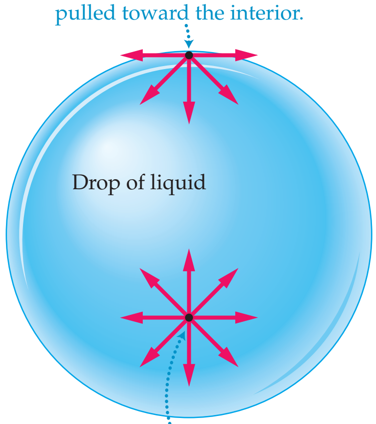
A small insect resting on the surface of a pond or a lake is a common sight in the summertime. If you look carefully, you can see that the insect creates tiny dimples in the water's surface, almost as if it were supported by a thin sheet of rubber. In fact, the surface of water and other fluids behaves in many respects as if it were an elastic membrane. This effect is known as **surface tension.**

To understand the origin of surface tension, we start by noting that the molecules in a fluid exert attractive forces on one another. Thus, a molecule deep within a fluid experiences forces in all directions, as indicated in **Figure 15–25**, due to the molecules that surround it on all sides. The net force on such a molecule is zero. As a molecule nears the surface, however, it experiences a net force away from the surface, since there are no fluid molecules on the other side of the surface to attract it in that direction. It follows that work must be done on a molecule to move it from within a fluid to the surface, and that the *energy* of a fluid is increased for every molecule on its surface.

In general, physical systems tend toward configurations of minimum energy. For example, a ball on a slope rolls downhill, lowering its gravitational potential energy. If the energy of a droplet of liquid is to be minimized, it must have the smallest surface area possible for a given volume; that is, it must have the shape of a sphere. This is the reason small drops of dew are always spherical. Larger drops of water may be distorted by the downward pull of Earth's gravity, but in orbit drops of all sizes are spherical.

Since energy is required to increase the area of a liquid surface, the situation is similar to the energy required to stretch a spring or to stretch a sheet of rubber. Thus, the surface of a liquid behaves as if it were elastic, resisting tendencies to increase its area. For example, if a drop of dew is distorted into an ellipsoid, it quickly returns to its original spherical shape. Similarly, when an insect alights on the surface of a pond, it creates dimples that increase the surface area. The water resists this distortion with a force sufficient to support the weight of the insect—if the insect is not too large. In fact, even a needle or a razor blade can be supported on the surface of water if they are put into place gently, even though they have densities significantly greater than the density of water.

Surface tension, which is important in many biological systems, plays a particularly important role in human breathing. For example, the crucial exchange of oxygen and carbon dioxide between inspired air and the blood occurs across the membranes of small balloonlike structures called *alveoli*. During inhalation, the alveoli expand from a radius of about 0.050 mm to 0.10 mm as they draw in fresh air. The walls of the alveoli are coated with a thin film of water, however, and in order to expand they must push outward against the water's surface tension—like trying to inflate a balloon against the surface tension of the rubber. To reduce the rather large surface tension of water, and make breathing easier, the lungs produce a substance called a *surfactant* that mixes with the water. This surfactant is produced rather late in the development of a fetus, however, and therefore premature infants may experience respiratory distress and even death as a result of too much surface tension in their alveoli.



▲ **FIGURE 15–25 The origin of surface tension**

A molecule in the interior of a fluid experiences attractive forces of equal magnitude in all directions, giving a net force of zero. A molecule near the surface of the fluid experiences a net attractive force toward the interior of the fluid. This causes the surface to be pulled inward, resulting in a surface of minimum area.

REAL-WORLD PHYSICS: BIO

Breathing, alveoli, and premature birth

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
Pressure can be thought of as an application of the concept of force to fluids. We make extensive use of force (Chapters 5 and 6) in Sections 15–2, 15–3, 15–4, and 15–5.	The pressure of an ideal gas, and its connection with temperature and volume, will be explored in detail in Chapter 17.
In Sections 15–7 and 15–8 the connection between a force acting over a distance and the kinetic energy (Chapter 7) is applied to a fluid.	Pressure appears again in Chapter 18 when we consider thermal processes, and the back-and-forth conversion of thermal and mechanical energy.
We also use the concept of gravitational potential energy (Chapter 8) in Sections 15–7 and 15–8. In fact, we show that Bernoulli's equation is basically a statement of energy conservation (Chapter 8) applied to a fluid.	When considering fluid flow in Section 15–6 we introduced the idea of the amount of material passing through a given area in a given time. This is referred to as flux. We generalize this concept to electric fields in Section 19–7 and to magnetic fields in Section 23–2.

CHAPTER SUMMARY

15–1 DENSITY

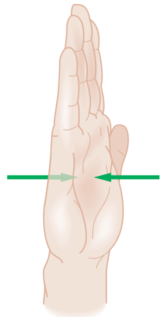
The density, ρ, of a material is its mass M per volume V:

$$\rho = M/V \qquad \text{15–1}$$

15–2 PRESSURE

Pressure, P, is force F per area A:

$$P = F/A \qquad \text{15–2}$$



Atmospheric Pressure
The pressure exerted by the atmosphere is $P_{at} = 1.01 \times 10^5\ \mathrm{N/m^2} \approx 14.7\ \mathrm{lb/in^2}$.

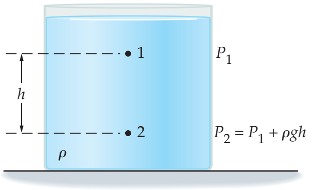
Pascals
Pressure is often given in terms of the pascal (Pa), where $1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$.

Gauge Pressure
Gauge pressure is the difference between the actual pressure and atmospheric pressure:

$$P_g = P - P_{at} \qquad \text{15–5}$$

15–3 STATIC EQUILIBRIUM IN FLUIDS: PRESSURE AND DEPTH

The pressure of a fluid in static equilibrium increases with depth.



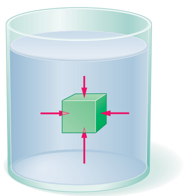
Pressure with Depth
If the pressure at one point in a fluid is P_1, the pressure at a depth h below that point is

$$P_2 = P_1 + \rho gh \qquad \text{15–7}$$

Pascal's Principle
An external pressure applied to an enclosed fluid is transmitted unchanged to every point within the fluid.

15–4 ARCHIMEDES' PRINCIPLE AND BUOYANCY

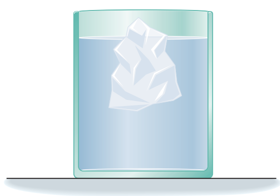
The fact that the pressure in a fluid increases with depth leads to a net upward force on any object that is immersed in the fluid. This upward force is referred to as a buoyant force. The magnitude of the buoyant force is given by Archimedes' principle.



Archimedes' Principle
An object completely immersed in a fluid experiences an upward buoyant force equal in magnitude to the weight of fluid displaced by the object.

15–5 APPLICATIONS OF ARCHIMEDES' PRINCIPLE

Archimedes' principle applies equally well to objects that are completely immersed, partially immersed, or floating.



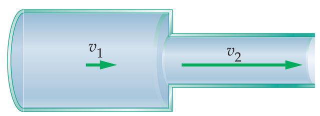
Floatation
An object floats when it displaces an amount of fluid equal to its weight.

Submerged Volume
When a solid of volume V_s and density ρ_s floats in a fluid of density ρ_f, the volume of the solid that is submerged in the fluid, V_{sub}, is

$$V_{sub} = V_s(\rho_s/\rho_f) \qquad \text{15–10}$$

15–6 FLUID FLOW AND CONTINUITY

The speed of a fluid changes as the cross-sectional area of the pipe through which it flows changes.



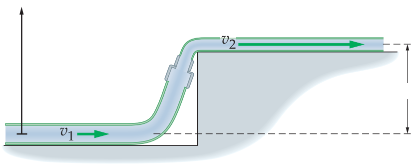
Equation of Continuity, Compressible Flow

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2 \qquad \text{15–11}$$

Equation of Continuity, Incompressible Flow

$$A_1 v_1 = A_2 v_2 \qquad \text{15–12}$$

15–7 BERNOULLI'S EQUATION



Bernoulli's equation can be thought of as energy conservation per volume for a fluid.

Change in Speed
If the speed of a fluid changes from v_1 to v_2, the corresponding pressures are related by

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2 \qquad 15\text{–}14$$

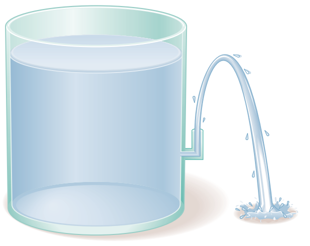
Change in Height
If the height of a fluid changes from y_1 to y_2, the corresponding pressures are related by

$$P_1 + \rho g y_1 = P_2 + \rho g y_2 \qquad 15\text{–}15$$

General Case
If both the height and the speed of a fluid change, the pressures are related by

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad 15\text{–}16$$

15–8 APPLICATIONS OF BERNOULLI'S EQUATION



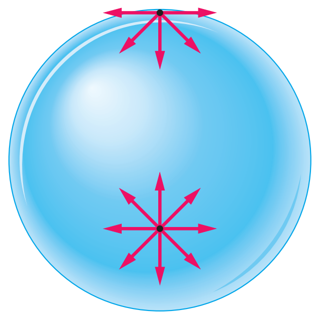
Bernoulli's equation applies to a wide range of everyday situations, including airplane wings and prairie dog burrows.

Torricelli's Law
According to Torricelli's law, if a hole is poked in a container at a depth h below the surface, the fluid exits with the speed

$$v = \sqrt{2gh} \qquad 15\text{–}17$$

Note that this is the same speed as an object in free fall for a distance h.

*15–9 VISCOSITY AND SURFACE TENSION



Viscosity and surface tension are two features of real fluids that are not found in an "ideal" fluid.

Viscosity
Viscosity in a fluid is similar to friction between two solid surfaces. A pressure difference, $P_1 - P_2$, is required to keep a viscous fluid flowing with a constant average speed, v. The relation between the volume flow rate of a fluid, $\Delta V/\Delta t$, and its coefficient of viscosity, η, is given by Poiseuille's equation:

$$\frac{\Delta V}{\Delta t} = \frac{(P_1 - P_2)\pi r^4}{8\eta L} \qquad 15\text{–}19$$

This expression applies to a tube of radius r and length L.

Surface Tension
A fluid tends to pull inward on its surface, resulting in a surface of minimum area. The surface of the fluid behaves much like an elastic membrane enclosing the fluid.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Relate pressure to depth in a static fluid.	The pressure in a static fluid increases uniformly with depth by the amount $\rho g h$. Thus, if point 2 is a depth h below point 1, the pressure there is $P_2 = P_1 + \rho g h$.	Examples 15–3, 15–4
Calculate the buoyant force.	According to Archimedes' principle, the buoyant force is equal to the weight of displaced fluid.	Examples 15–5 Active Examples 15–1, 15–2
Calculate the speed of a fluid as the area through which it flows changes.	If a fluid is incompressible, its speed must vary inversely with the area through which it flows. Thus, if the area changes from A_1 to A_2 the speed of the fluid changes as follows: $A_1 v_1 = A_2 v_2$. This is the equation of continuity.	Example 15–7
Relate the pressure in a fluid to its height and speed.	Bernoulli's principle states that pressure, speed, and height in a fluid are related as follows: $P + \frac{1}{2}\rho v^2 + \rho g y =$ constant. This is simply a statement of energy conservation per volume of the fluid.	Example 15–8 Active Example 15–3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. Suppose you drink a liquid through a straw. Explain why the liquid moves upward, against gravity, into your mouth.
2. Considering your answer to the previous question, is it possible to sip liquid through a straw on the surface of the Moon? Explain.
3. Water towers on the roofs of buildings have metal bands wrapped around them for support. The spacing between bands is smaller near the base of a tower than near its top. Explain.
4. What holds a suction cup in place?
5. Suppose a force of 400 N is required to push the top off a wine barrel. In a famous experiment, Blaise Pascal attached a tall, thin tube to the top of a filled wine barrel, as shown in **Figure 15–26.** Water was slowly added to the tube until the barrel burst. The puzzling result found by Pascal was that the barrel broke when the weight of water in the tube was much less than 400 N. Explain Pascal's observation.

▲ **FIGURE 15–26** Conceptual Question 5 and Problem 19

6. Why is it more practical to use mercury in the barometer shown in Figure 15–4 than water?
7. An object's density can be determined by first weighing it in air, then in water (provided the density of the object is greater than the density of water, so that it is totally submerged when placed in water). Explain how these two measurements can give the desired result.
8. How does a balloonist control the vertical motion of a hot-air balloon?
9. Why is it possible for people to float without effort in Utah's Great Salt Lake?
10. **Physics in the Movies** In the movie *Voyage to the Bottom of the Sea*, the Earth is experiencing a rapid warming. In one scene, large icebergs break up into small, car-size chunks that drop downward through the water and bounce off the hull of the submarine *Seaview*. Is this an example of good, bad, or ugly physics? Explain.
11. One day, while snorkeling near the surface of a crystal-clear ocean, it occurs to you that you could go considerably deeper by simply lengthening the snorkel tube. Unfortunately, this does not work well at all. Why?
12. Since metal is more dense than water, how is it possible for a metal boat to float?
13. A sheet of water passing over a waterfall is thicker near the top than near the bottom. Similarly, a stream of water emerging from a water faucet becomes narrower as it falls. Explain.
14. It is a common observation that smoke rises more rapidly through a chimney when there is a wind blowing outside. Explain.
15. Is it best for an airplane to take off against the wind or with the wind? Explain.
16. If you have a hair dryer and a Ping Pong ball at home, try this demonstration. Direct the air from the dryer in a direction just above horizontal. Next, place the Ping Pong ball in the stream of air. If done just right, the ball will remain suspended in midair. Use the Bernoulli effect to explain this behavior.
17. Suppose a pitcher wants to throw a baseball so that it rises as it approaches the batter. How should the ball be spinning to accomplish this feat? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 15–1 DENSITY

1. • Estimate the weight of the air in your physics classroom.
2. • What weight of water is required to fill a 25-gallon aquarium?
3. • You buy a "gold" ring at a pawn shop. The ring has a mass of 0.014 g and a volume of 0.0022 cm^3. Is the ring solid gold?
4. • Estimate the weight of a treasure chest filled with gold doubloons.
5. • A cube of metal has a mass of 0.347 kg and measures 3.21 cm on a side. Calculate the density and identify the metal.

SECTION 15–2 PRESSURE

6. • What is the downward force exerted by the atmosphere on a football field, whose dimensions are 360 ft by 160 ft?
7. • **BIO Bioluminescence** Some species of dinoflagellate (a type of unicellular plankton) can produce light as the result of biochemical reactions within the cell. This light is an example of bioluminescence. It is found that bioluminescence in dinoflagellates can be triggered by deformation of the cell surface with a pressure as low as one dyne (10^{-5} N) per square centimeter. What is this pressure in **(a)** pascals and **(b)** atmospheres?
8. • A 79-kg person sits on a 3.7-kg chair. Each leg of the chair makes contact with the floor in a circle that is 1.3 cm in diameter. Find the pressure exerted on the floor by each leg of the chair, assuming the weight is evenly distributed.
9. • To prevent damage to floors (and to increase friction), a crutch will often have a rubber tip attached to its end. If the end of the crutch is a circle of radius 1.2 cm without the tip, and the tip is a circle of radius 2.5 cm, by what factor does the tip reduce the pressure exerted by the crutch?
10. • An inflated basketball has a gauge pressure of 9.9 lb/in^2. What is the actual pressure inside the ball?

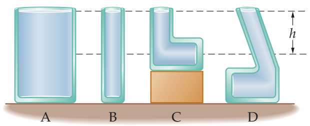
11. •• Suppose that when you ride on your 7.70-kg bike the weight of you and the bike is supported equally by the two tires. If the gauge pressure in the tires is 70.5 lb/in^2 and the area of contact between each tire and the road is 7.13 cm^2, what is your weight?

12. •• **IP** The weight of your 1420-kg car is supported equally by its four tires, each inflated to a gauge pressure of 35.0 lb/in^2. **(a)** What is the area of contact each tire makes with the road? **(b)** If the gauge pressure is increased, does the area of contact increase, decrease, or stay the same? **(c)** What gauge pressure is required to give an area of contact of 116 cm^2 for each tire?

SECTION 15–3 STATIC EQUILIBRIUM IN FLUIDS: PRESSURE AND DEPTH

13. • **CE** Two drinking glasses, 1 and 2, are filled with water to the same depth. Glass 1 has twice the diameter of glass 2. **(a)** Is the weight of the water in glass 1 greater than, less than, or equal to the weight of the water in glass 2? **(b)** Is the pressure at the bottom of glass 1 greater than, less than, or equal to the pressure at the bottom of glass 2?

14. • **CE** **Figure 15–27** shows four containers, each filled with water to the same level. Rank the containers in order of increasing pressure at the depth h. Indicate ties where appropriate.



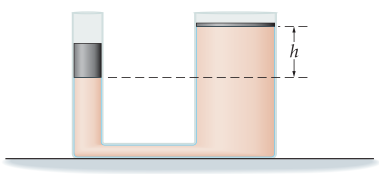
▲ **FIGURE 15–27** Problem 14

15. • Water in the lake behind Hoover Dam is 221 m deep. What is the water pressure at the base of the dam?

16. • In a classroom demonstration, the pressure inside a soft drink can is suddenly reduced to essentially zero. Assuming the can to be a cylinder with a height of 12 cm and a diameter of 6.5 cm, find the net inward force exerted on the vertical sides of the can due to atmospheric pressure.

17. •• As a storm front moves in, you notice that the column of mercury in a barometer rises to only 736 mm. **(a)** What is the air pressure? **(b)** If the mercury in this barometer is replaced with water, to what height does the column of water rise? Assume the same air pressure found in part (a).

18. •• In the hydraulic system shown in **Figure 15–28**, the piston on the left has a diameter of 4.4 cm and a mass of 1.8 kg. The piston on the right has a diameter of 12 cm and a mass of 3.2 kg. If the density of the fluid is 750 kg/m^3, what is the height difference h between the two pistons?



▲ **FIGURE 15–28** Problem 18

19. •• A circular wine barrel 75 cm in diameter will burst if the net upward force exerted on the top of the barrel is 643 N. A tube 1.0 cm in diameter extends into the barrel through a hole in the top, as indicated in Figure 15–26. Initially, the barrel is filled to the top and the tube is empty above that level. What weight of water must be poured into the tube in order to burst the barrel?

20. •• A cylindrical container with a cross-sectional area of 65.2 cm^2 holds a fluid of density 806 kg/m^3. At the bottom of the container the pressure is 116 kPa. **(a)** What is the depth of the fluid? **(b)** Find the pressure at the bottom of the container after an additional 2.05×10^{-3} m^3 of this fluid is added to the container. Assume that no fluid spills out of the container.

21. •• **IP** **Tourist Submarine** A submarine called the *Deep View 66* is currently being developed to take 66 tourists at a time on sightseeing trips to tropical coral reefs. According to guidelines of the American Society of Mechanical Engineers (ASME), to be safe for human occupancy the *Deep View 66* must be able to withstand a pressure of 10.0 N per square millimeter. **(a)** To what depth can the *Deep View 66* safely descend in seawater? **(b)** If the submarine is used in freshwater instead, is its maximum safe depth greater than, less than, or the same as in seawater? Explain.

22. •• **IP** A water storage tower, like the one shown in the accompanying photo, is filled with freshwater to a depth of 6.4 m. What is the pressure at **(a)** 4.5 m and **(b)** 5.5 m below the surface of the water? **(c)** Why are the metal bands on such towers more closely spaced near the base of the tower?

Roof-top water tower.
(Problem 22)

23. •• **IP** You step into an elevator holding a glass of water filled to a depth of 6.9 cm. After a moment, the elevator moves upward with constant acceleration, increasing its speed from 0 to 2.4 m/s in 3.2 s. **(a)** During the period of acceleration, is the pressure exerted on the bottom of the glass greater than, less than, or the same as before the elevator began to move? Explain. **(b)** Find the change in the pressure exerted on the bottom of the glass as the elevator accelerates.

24. •• Suppose you pour water into a container until it reaches a depth of 12 cm. Next, you carefully pour in a 7.2-cm thickness of olive oil so that it floats on top of the water. What is the pressure at the bottom of the container?

25. •• Referring to Example 15–4, suppose that some vegetable oil has been added to both sides of the U tube. On the right side of the tube, the depth of oil is 5.00 cm, as before. On the left side of the tube, the depth of the oil is 3.00 cm. Find the difference in fluid level between the two sides of the tube.

26. •• **IP** As a stunt, you want to sip some water through a very long, vertical straw. **(a)** First, explain why the liquid moves upward, against gravity, into your mouth when you sip. **(b)** What is the tallest straw that you could, in principle, drink from in this way?

27. •• **IP** **BIO** The patient in **Figure 15–29** is to receive an intravenous injection of medication. In order to work properly, the pressure of fluid containing the medication must be 109 kPa at the injection point. **(a)** If the fluid has a density of 1020 kg/m^3, find the height at which the bag of fluid must be suspended above the patient. Assume that the pressure inside the bag is one atmosphere. **(b)** If a less dense fluid is used instead, must the height of suspension be increased or decreased? Explain.

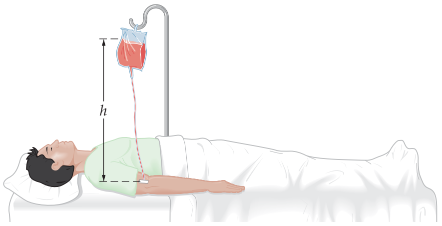


FIGURE 15–29 Problem 27

28. ••• A cylindrical container 1.0 m tall contains mercury to a certain depth, d. The rest of the cylinder is filled with water. If the pressure at the bottom of the cylinder is two atmospheres, what is the depth d?

SECTION 15–4 ARCHIMEDES' PRINCIPLE AND BUOYANCY

29. • **CE Predict/Explain Beebe and Barton** On Wednesday, August 15, 1934, William Beebe and Otis Barton made history by descending in the Bathysphere—basically a steel sphere 4.75 ft in diameter—3028 ft below the surface of the ocean, deeper than anyone had been before. **(a)** As the Bathysphere was lowered, was the buoyant force exerted on it at a depth of 10 ft greater than, less than, or equal to the buoyant force exerted on it at a depth of 50 ft? **(b)** Choose the *best explanation* from among the following:
   I. The buoyant force depends on the density of the water, which is essentially the same at 10 ft and 50 ft.
   II. The pressure increases with depth, and this increases the buoyant force.
   III. The buoyant force decreases as an object sinks below the surface of the water.

30. • **CE** Lead is more dense than aluminum. **(a)** Is the buoyant force on a solid lead sphere greater than, less than, or equal to the buoyant force on a solid aluminum sphere of the same diameter? The spheres are submerged in the same fluid. **(b)** Does your answer to part (a) depend on the fluid that is causing the buoyant force?

31. • **CE** A fish carrying a pebble in its mouth swims with a small, constant velocity in a small bowl. When the fish drops the pebble to the bottom of the bowl, does the water level rise, fall, or stay the same?

32. • A raft is 4.2 m wide and 6.5 m long. When a horse is loaded onto the raft, it sinks 2.7 cm deeper into the water. What is the weight of the horse?

33. • To walk on water, all you need is a pair of water-walking boots shaped like boats. If each boot is 27 cm high and 34 cm wide, how long must they be to support a 75-kg person?

34. •• A 3.2-kg balloon is filled with helium (density $= 0.179\ \mathrm{kg/m^3}$). If the balloon is a sphere with a radius of 4.9 m, what is the maximum weight it can lift?

35. •• A hot-air balloon plus cargo has a mass of 1890 kg and a volume of 11,430 $\mathrm{m^3}$. The balloon is floating at a constant height of 6.25 m above the ground. What is the density of the hot air in the balloon?

36. ••• In the lab you place a beaker that is half full of water (density ρ_w) on a scale. You now use a light string to suspend a piece of metal of volume V in the water. The metal is completely submerged, and none of the water spills out of the beaker. Give a symbolic expression for the change in reading of the scale.

SECTION 15–5 APPLICATIONS OF ARCHIMEDES' PRINCIPLE

37. • **CE Predict/Explain** A block of wood has a steel ball glued to one surface. The block can be floated with the ball "high and dry" on its top surface. **(a)** When the block is inverted, and the ball is immersed in water, does the volume of wood that is submerged increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. When the block is inverted the ball pulls it downward, causing more of the block to be submerged.
   II. The same amount of mass is supported in either case, therefore the amount of the block that is submerged is the same.
   III. When the block is inverted the ball experiences a buoyant force, which reduces the buoyant force that must be provided by the wood.

38. • **CE Predict/Explain** In the preceding problem, suppose the block of wood with the ball "high and dry" is floating in a tank of water. **(a)** When the block is inverted, does the water level in the tank increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Inverting the block makes the block float higher in the water, which lowers the water level in the tank.
   II. The same mass is supported by the water in either case, and therefore the amount of displaced water is the same.
   III. The inverted block floats lower in the water, which displaces more water and raises the level in the tank.

39. • **CE Measuring Density with a Hydrometer** A hydrometer, a device for measuring fluid density, is constructed as shown in Figure 15–30. If the hydrometer samples fluid 1, the small float inside the tube is submerged to level 1. When fluid 2 is sampled, the float is submerged to level 2. Is the density of fluid 1 greater than, less than, or equal to the density of fluid 2? (This is how mechanics test your antifreeze level. Since antifreeze [ethylene glycol] is more dense than water, the higher the density of coolant in your radiator the more antifreeze protection you have.)

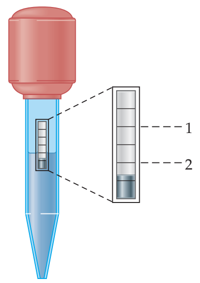


FIGURE 15–30 Problem 39

40. • **CE Predict/Explain** Referring to Active Example 15–1, suppose the flask with the wood tied to the bottom is placed on a scale. At some point the string breaks and the wood rises to the surface where it floats. **(a)** When the wood is floating, is the reading on the scale greater than, less than, or equal to its previous reading? **(b)** Choose the *best explanation* from among the following:
   I. The same mass is supported by the scale before and after the string breaks, and therefore the reading on the scale remains the same.
   II. When the block is floating the water level drops, and this reduces the reading on the scale.
   III. When the block is floating it no longer pulls upward on the flask; therefore, the reading on the scale increases.

41. • **CE** On a planet in a different solar system the acceleration of gravity is greater than it is on Earth. If you float in a pool of

water on this planet, do you float higher than, lower than, or at the same level as when you float in water on Earth?

42. • An air mattress is 2.3 m long, 0.66 m wide, and 14 cm deep. If the air mattress itself has a mass of 0.22 kg, what is the maximum mass it can support in freshwater?

43. •• A solid block is attached to a spring scale. When the block is suspended in air, the scale reads 20.0 N; when it is completely immersed in water, the scale reads 17.7 N. What are **(a)** the volume and **(b)** the density of the block?

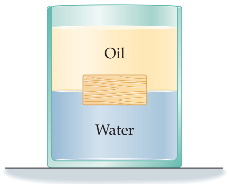
44. •• As in the previous problem, a solid block is suspended from a spring scale. If the reading on the scale when the block is completely immersed in water is 25.0 N, and the reading when it is completely immersed in alcohol of density 806 kg/m^3 is 25.7 N, what are **(a)** the block's volume and **(b)** its density?

45. •• **BIO** A person weighs 756 N in air and has a body-fat percentage of 28.1%. **(a)** What is the overall density of this person's body? **(b)** What is the volume of this person's body? **(c)** Find the apparent weight of this person when completely submerged in water.

46. •• **IP** A log floats in a river with one-fourth of its volume above the water. **(a)** What is the density of the log? **(b)** If the river carries the log into the ocean, does the portion of the log above the water increase, decrease, or stay the same? Explain.

47. •• A person with a mass of 81 kg and a volume of 0.089 m^3 floats quietly in water. **(a)** What is the volume of the person that is above water? **(b)** If an upward force F is applied to the person by a friend, the volume of the person above water increases by 0.0018 m^3. Find the force F.

48. •• **IP** A block of wood floats on water. A layer of oil is now poured on top of the water to a depth that more than covers the block, as shown in **Figure 15–31**. **(a)** Is the volume of wood submerged in water greater than, less than, or the same as before? **(b)** If 90% of the wood is submerged in water before the oil is added, find the fraction submerged when oil with a density of 875 kg/m^3 covers the block.



▲ **FIGURE 15–31** Problem 48

49. •• A piece of lead has the shape of a hockey puck, with a diameter of 7.5 cm and a height of 2.5 cm. If the puck is placed in a mercury bath, it floats. How deep below the surface of the mercury is the bottom of the lead puck?

50. ••• **IP** A lead weight with a volume of 0.82×10^{-5} m^3 is lowered on a fishing line into a lake to a depth of 1.0 m. **(a)** What tension is required in the fishing line to give the weight an upward acceleration of 2.1 m/s^2? **(b)** If the initial depth of the weight is increased to 2.0 m, does the tension found in part (a) increase, decrease, or stay the same? Explain. **(c)** What acceleration will the weight have if the tension in the fishing line is 1.2 N? Give both direction and magnitude.

SECTION 15–6 FLUID FLOW AND CONTINUITY

51. • To water the yard, you use a hose with a diameter of 3.4 cm. Water flows from the hose with a speed of 1.1 m/s. If you partially block the end of the hose so the effective diameter is now 0.57 cm, with what speed does water spray from the hose?

52. • Water flows through a pipe with a speed of 2.1 m/s. Find the flow rate in kg/s if the diameter of the pipe is 3.8 cm.

53. • To fill a child's inflatable wading pool, you use a garden hose with a diameter of 2.9 cm. Water flows from this hose with a speed of 1.3 m/s. How long will it take to fill the pool to a depth of 26 cm if the pool is circular and has a diameter of 2.0 m?

54. • **BIO Heart Pump Rate** When at rest, your heart pumps blood at the rate of 5.00 liters per minute (L/min). What are the volume and mass of blood pumped by your heart in one day?

55. •• **BIO Blood Speed in an Arteriole** A typical arteriole has a diameter of 0.030 mm and carries blood at the rate of 5.5×10^{-6} cm^3/s. **(a)** What is the speed of the blood in an arteriole? **(b)** Suppose an arteriole branches into 340 capillaries, each with a diameter of 4.0×10^{-6} m. What is the blood speed in the capillaries? (The low speed in capillaries is beneficial; it promotes the diffusion of materials to and from the blood.)

56. •• **IP** Water flows at the rate of 3.11 kg/s through a hose with a diameter of 3.22 cm. **(a)** What is the speed of water in this hose? **(b)** If the hose is attached to a nozzle with a diameter of 0.732 cm, what is the speed of water in the nozzle? **(c)** Is the number of kilograms per second flowing through the nozzle greater than, less than, or equal to 3.11 kg/s? Explain.

57. •• A river narrows at a rapids from a width of 12 m to a width of only 5.8 m. The depth of the river before the rapids is 2.7 m; the depth in the rapids is 0.85 m. Find the speed of water flowing in the rapids, given that its speed before the rapids is 1.2 m/s. Assume the river has a rectangular cross section.

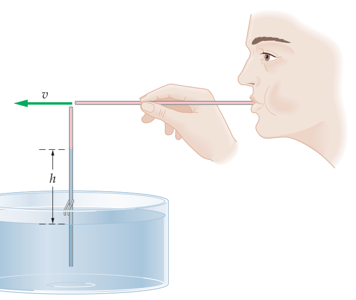
58. •• **BIO How Many Capillaries?** The aorta has an inside diameter of approximately 2.1 cm, compared to that of a capillary, which is about 1.0×10^{-5} m (10 μm). In addition, the average speed of flow is approximately 1.0 m/s in the aorta and 1.0 cm/s in a capillary. Assuming that all the blood that flows through the aorta also flows through the capillaries, how many capillaries does the circulatory system have?

SECTION 15–7 BERNOULLI'S EQUATION

59. • **BIO Plaque in an Artery** The buildup of plaque on the walls of an artery may decrease its diameter from 1.1 cm to 0.75 cm. If the speed of blood flow was 15 cm/s before reaching the region of plaque buildup, find **(a)** the speed of blood flow and **(b)** the pressure drop within the plaque region.

60. • A horizontal pipe contains water at a pressure of 110 kPa flowing with a speed of 1.6 m/s. When the pipe narrows to one-half its original diameter, what are **(a)** the speed and **(b)** the pressure of the water?

61. •• **BIO A Blowhard** Tests of lung capacity show that adults are able to exhale 1.5 liters of air through their mouths in as little as 1.0 second. **(a)** If a person blows air at this rate through a drinking straw with a diameter of 0.60 cm, what is the speed of air in the straw? **(b)** If the air from the straw in part (a) is directed horizontally across the upper end of a second straw that is vertical, as shown in **Figure 15–32**, to what height does water rise in the vertical straw?



▲ **FIGURE 15–32** Problem 61

62. •• **IP** Water flows through a horizontal tube of diameter 2.8 cm that is joined to a second horizontal tube of diameter 1.6 cm. The pressure difference between the tubes is 7.5 kPa. **(a)** Which tube has the higher pressure? **(b)** Which tube has the higher speed of flow? **(c)** Find the speed of flow in the first tube.

63. •• A garden hose is attached to a water faucet on one end and a spray nozzle on the other end. The water faucet is turned on, but the nozzle is turned off so that no water flows through the hose. The hose lies horizontally on the ground, and a stream of water sprays vertically out of a small leak to a height of 0.68 m. What is the pressure inside the hose?

SECTION 15–8 APPLICATIONS OF BERNOULLI'S EQUATION

64. • A water tank springs a leak. Find the speed of water emerging from the hole if the leak is 2.7 m below the surface of the water, which is open to the atmosphere.

65. •• **(a)** Find the pressure difference on an airplane wing if air flows over the upper surface with a speed of 115 m/s, and along the bottom surface with a speed of 105 m/s. **(b)** If the area of the wing is 32 m^2, what is the net upward force exerted on the wing?

66. •• On a vacation flight, you look out the window of the jet and wonder about the forces exerted on the window. Suppose the air outside the window moves with a speed of approximately 170 m/s shortly after takeoff, and that the air inside the plane is at atmospheric pressure. **(a)** Find the pressure difference between the inside and outside of the window. **(b)** If the window is 25 cm by 42 cm, find the force exerted on the window by air pressure.

67. •• **IP** During a thunderstorm, winds with a speed of 47.7 m/s blow across a flat roof with an area of 668 m^2. **(a)** Find the magnitude of the force exerted on the roof as a result of this wind. **(b)** Is the force exerted on the roof in the upward or downward direction? Explain.

68. •• A garden hose with a diameter of 0.63 in. has water flowing in it with a speed of 0.78 m/s and a pressure of 1.2 atmospheres. At the end of the hose is a nozzle with a diameter of 0.25 in. Find **(a)** the speed of water in the nozzle and **(b)** the pressure in the nozzle.

69. ••• **IP** Water flows in a cylindrical, horizontal pipe. As the pipe narrows to half its initial diameter, the pressure in the pipe changes. **(a)** Is the pressure in the narrow region greater than, less than, or the same as the initial pressure? Explain. **(b)** Calculate the change in pressure between the wide and narrow regions of the pipe. Give your answer symbolically in terms of the density of the water, ρ, and its initial speed v.

*SECTION 15–9 VISCOSITY AND SURFACE TENSION

*70. • **BIO Vasodilation** When the body requires an increased blood flow rate in a particular organ or muscle, it can accomplish this by increasing the diameter of arterioles in that area. This is referred to as vasodilation. What percentage increase in the diameter of an arteriole is required to double the volume flow rate of blood, all other factors remaining the same?

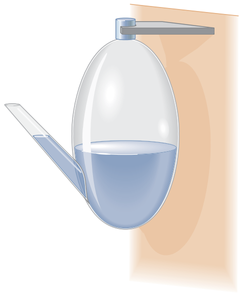
*71. • **BIO** **(a)** Find the volume of blood that flows per second through the pulmonary artery described in Example 15–10. **(b)** If the radius of the artery is reduced by 18%, by what factor is the blood flow rate reduced? Assume that all other properties of the artery remain unchanged.

*72. • **BIO An Occlusion in an Artery** Suppose an occlusion in an artery reduces its diameter by 15%, but the volume flow rate of blood in the artery remains the same. By what factor has the pressure drop across the length of this artery increased?

*73. •• **IP** Water at 20 °C flows through a horizontal garden hose at the rate of $5.0 \times 10^{-4}\ m^3/s$. The diameter of the garden hose is 2.5 cm. **(a)** What is the water speed in the hose? **(b)** What is the pressure drop across a 15-m length of hose? Suppose the cross-sectional area of the hose is halved, but the length and pressure drop remain the same. **(c)** By what factor does the water speed change? **(d)** By what factor does the volume flow rate change? Explain.

GENERAL PROBLEMS

74. • **CE Reading a Weather Glass** A weather glass, as shown in Figure 15–33, is used to give an indication of a change in the weather. Does the water level in the neck of the weather glass move up or move down when a low-pressure system approaches?



▲ **FIGURE 15–33** Problem 74

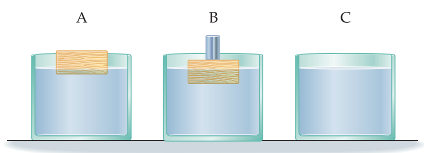
75. • **CE** A helium-filled balloon for a birthday party is being brought home in a car. The balloon is connected to a string, and the passenger holds the lower end of the string in her lap. When the car is at rest at a stop sign the string is vertical. As the car accelerates away from the light, does the string going to the balloon lean forward, lean backward, or remain vertical?

76. • **CE Predict/Explain** A person floats in a boat in a small backyard swimming pool. Inside the boat with the person are some bricks. **(a)** If the person drops the bricks overboard to the bottom of the pool, does the water level in the pool increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. When the bricks sink they displace less water than when they were floating in the boat; hence, the water level decreases.
II. The same mass (boat + bricks + person) is in the pool in either case, and therefore the water level remains the same.
III. The bricks displace more water when they sink to the bottom than they did when they were above the water in the boat; therefore the water level increases.

77. • **CE** A person floats in a boat in a small backyard swimming pool. Inside the boat with the person are several blocks of wood. Suppose the person now throws the blocks of wood into the pool, where they float. **(a)** Does the boat float higher, lower, or at the same level relative to the water? **(b)** Does the water level in the pool increase, decrease, or stay the same?

78. • **CE** The three identical containers in Figure 15–34 are open to the air and filled with water to the same level. A block of wood



▲ **FIGURE 15–34** Problem 78

floats in container A; an identical block of wood floats in container B, supporting a small lead weight; container C holds only water. **(a)** Rank the three containers in order of increasing weight of water they contain. Indicate ties where appropriate. **(b)** Rank the three containers in order of increasing weight of the container plus its contents. Indicate ties where appropriate.

79. • **CE** A pan half-filled with water is placed near the rim of a rotating turntable. Is the normal to the surface of the water in the pan tilted outward away from the axis of rotation, tilted inward toward the axis of rotation, or is the water surface level and the normal vertical? (Refer to Problem 68 in Chapter 6 for a similar situation.)

80. • **CE** In the system described in the previous problem, suppose the temperature is lowered below the freezing point of water as the turntable continues to rotate. The water is now a solid block of ice. If a marble is placed on the surface of the ice and released from rest, will it move toward the axis of rotation, move away from the axis of rotation, or stay where it is released?

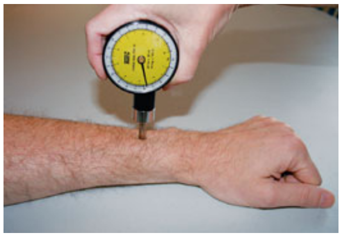
81. • **CE BIO Sphygmomanometer** When a person's blood pressure is taken with a device known as a sphygmomanometer, it is measured on the arm, at approximately the same level as the heart. If the measurement were to be taken on the patient's leg instead, would the reading on the sphygmomanometer be greater than, less than, or the same as when the measurement is made on the arm?

82. • At what depth below the ocean surface is the pressure equal to two atmospheres?

83. • **Supersonic Erosion** In waterjet cutting, a stream of supersonic water is used to slice through materials ranging from sheets of paper to solid steel plates. The water is held in a reservoir at 59,500 psi and allowed to exit through a small orifice at high speed. Find the exit speed of the water, and compare with the speed of sound.

84. • A water main broke on Lake Shore Drive in Chicago on November 8, 2002, shooting water straight upward to a height of 8.0 ft. What was the pressure in the pipe?

85. • **BIO Measuring Pain Threshold** A useful instrument for evaluating fibromyalgia and trigger-point tenderness is the doloriometer or algorimeter. This device consists of a force meter attached to a circular probe that is pressed against the skin until pain is experienced. If the reading of the force meter is 3.25 lb, and the diameter of the circular probe is 1.39 cm, what is the pressure applied to the skin? Give your answer in pascals.



A doloriometer/algorimeter used to evaluate pain threshold. (Problem 85)

86. •• **BIO Power Output of the Heart** The power output of the heart is given by the product of the average blood pressure, $1.33\ \mathrm{N/cm^2}$, and the flow rate, $105\ \mathrm{cm^3/s}$. **(a)** Find the power of the heart. Give your answer in watts. **(b)** How much energy does the heart expend in a day? **(c)** Suppose the energy found in part (b) is used to lift a 72-kg person vertically to a height h. Find h, in meters.

87. •• An above-ground backyard swimming pool is shaped like a large hockey puck, with a circular bottom and a vertical wall forming its perimeter. The diameter of the pool is 4.8 m and its depth is 1.8 m. Find the total outward force exerted on the vertical wall of the pool by the water, assuming the pool is completely filled.

88. •• A solid block is suspended from a spring scale. When the block is in air, the scale reads 35.0 N, when immersed in water the scale reads 31.1 N, and when immersed in oil the scale reads 31.8 N. **(a)** What is the density of the block? **(b)** What is the density of the oil?

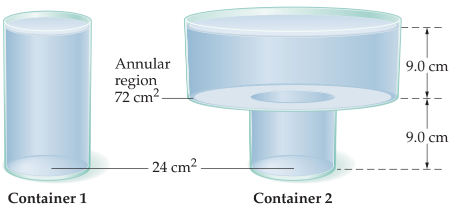
89. •• A wooden block with a density of $710\ \mathrm{kg/m^3}$ and a volume of $0.012\ \mathrm{m^3}$ is attached to the top of a vertical spring whose force constant is $k = 540\ \mathrm{N/m}$. Find the amount by which the spring is stretched or compressed if it and the wooden block are **(a)** in air or **(b)** completely immersed in water. [The density of air may be neglected in part (a).]

90. •• **IP Floating a Ball and Block** A 1.25-kg wooden block has an iron ball of radius 1.22 cm glued to one side. **(a)** If the block floats in water with the iron ball "high and dry," what is the volume of wood that is submerged? **(b)** If the block is now inverted, so that the iron ball is completely immersed, does the volume of wood that is submerged in water increase, decrease, or remain the same? Explain. **(c)** Calculate the volume of wood that is submerged when the block is in the inverted position.

91. •• On a bet, you try to remove water from a glass by blowing across the top of a vertical straw immersed in the water. What is the minimum speed you must give the air at the top of the straw to draw water upward through a height of 1.6 cm?

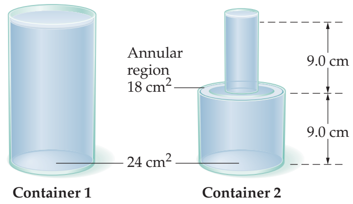
92. •• **The Depth of the Atmosphere** Evangelista Torricelli (1608–1647) was the first to put forward the idea that we live at the bottom of an ocean of air. **(a)** Given the value of atmospheric pressure at the surface of the Earth, and the fact that there is zero pressure in the vacuum of space, determine the depth of the atmosphere, assuming that the density of air and the acceleration of gravity are constant. **(b)** According to this model, what is the atmospheric pressure at the summit of Mt. Everest, 29,035 ft above sea level. (In fact, the density of air and the acceleration of gravity decrease with altitude, so the result obtained here is less than the actual depth of the atmosphere. Still this is a reasonable first estimate.)

93. •• **The Hydrostatic Paradox I** Consider the lightweight containers shown in **Figure 15–35**. Both containers have bases of area $A_{\mathrm{base}} = 24\ \mathrm{cm^2}$ and depths of water equal to 18 cm. As a result, the downward force on the base of container 1 is equal to the downward force on container 2, even though the containers clearly hold different weights of water. This is referred to as the hydrostatic paradox. **(a)** Given that container 2 has an annular (ring-shaped) region of area $A_{\mathrm{ring}} = 72\ \mathrm{cm^2}$, determine the *net* downward force exerted on the container by the water. **(b)** Show that your result from part (a) is equal to the weight of the water in container 2.



▲ **FIGURE 15–35** Problem 93

94. •• **The Hydrostatic Paradox II** Consider the two lightweight containers shown in **Figure 15–36**. As in the previous problem, these containers have equal forces on their bases but contain different weights of water. This is another version of the hydrostatic



▲ **FIGURE 15–36** Problem 94

paradox. **(a)** Determine the *net* downward force exerted by the water on container 2. Note that the bases of the containers have an area $A_{base} = 24\ cm^2$, the annular region has an area $A_{ring} = 18\ cm^2$, and the depth of the water is 18 cm. **(b)** Show that your result from part (a) is equal to the weight of the water in container 2. **(c)** If a hole is poked in the annular region of container 2, how fast will water exit the hole? **(d)** How high above the hole will the stream of water rise?

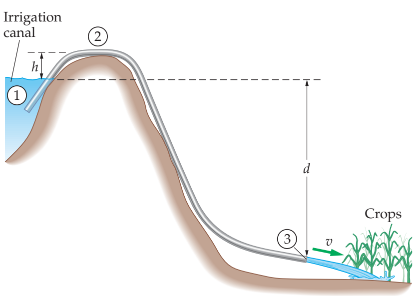
95. •• **IP** A backyard swimming pool is circular in shape and contains water to a uniform depth of 38 cm. It is 2.3 m in diameter and is not completely filled. **(a)** What is the pressure at the bottom of the pool? **(b)** If a person gets into the pool and floats peacefully, does the pressure at the bottom of the pool increase, decrease, or stay the same? **(c)** Calculate the pressure at the bottom of the pool if the floating person has a mass of 72 kg.

96. •• A prospector finds a solid rock composed of granite ($\rho = 2650\ kg/m^3$) and gold. If the volume of the rock is $3.55 \times 10^{-4}\ m^3$, and its mass is 3.81 kg, **(a)** what mass of gold is contained in the rock? What percentage of the rock is gold by **(b)** volume and **(c)** mass?

97. •• **The Maximum Depth of the Earth's Crust** Consider the crustal rocks of the Earth to be a fluid of density $3.0 \times 10^3\ kg/m^3$. Under this assumption, the pressure at a depth h within the crust is $P = P_{at} + \rho g h$. If the greatest pressure crustal rock can sustain before crumbling is $1.2 \times 10^9\ Pa$, find the maximum depth of the Earth's crust. (Below this depth the crust changes from a solid to a plasticlike material.)

98. •• **IP** **(a)** If the tension in the string in Active Example 15–1 is 0.89 N, what is the volume of the wood? Assume that everything else remains the same. **(b)** If the string breaks and the wood floats on the surface, does the water level in the flask rise, drop, or stay the same? Explain. **(c)** Assuming the flask is cylindrical with a cross-sectional area of $62\ cm^2$, find the change in water level after the string breaks.

99. •• **IP** **A Siphon for Irrigation** A siphon is a device that allows water to flow from one level to another. The siphon shown in **Figure 15–37** delivers water from an irrigation canal to a field of crops. To operate the siphon, water is first drawn through the length of the tube. After the flow is started in this way it continues on its own. **(a)** Using points 1 and 3 in Figure 15–37, find the speed v of the water leaving the siphon at its lower end. Give a symbolic answer. **(b)** Is the speed of the water at point 2 greater than, less than, or the same as its speed at point 3? Explain.



▲ **FIGURE 15–37** Problem 99

100. •• A tin can is filled with water to a depth of 39 cm. A hole 11 cm above the bottom of the can produces a stream of water that is directed at an angle of 36° above the horizontal. Find **(a)** the range and **(b)** the maximum height of this stream of water.

101. •• **BIO** A person weighs 685 N in air but only 497 N when standing in water up to the hips. Find **(a)** the volume of each of the person's legs and **(b)** the mass of each leg, assuming they have a density that is 1.05 times the density of water.

*102. •• A horizontal pipe carries oil whose coefficient of viscosity is $0.00012\ N \cdot s/m^2$. The diameter of the pipe is 5.2 cm, and its length is 55 m. **(a)** What pressure difference is required between the ends of this pipe if the oil is to flow with an average speed of 1.2 m/s? **(b)** What is the volume flow rate in this case?

*103. •• **BIO** A patient is given an injection with a hypodermic needle 3.3 cm long and 0.26 mm in diameter. Assuming the solution being injected has the same density and viscosity as water at 20 °C, find the pressure difference needed to inject the solution at the rate of 1.5 g/s.

104. •• **An Airburst over Pennsylvania** On the evening of July 23, 2001, a meteor streaked across the skies of Pennsylvania, creating a spectacular fireball before exploding in the atmosphere with an energy release of 3 kilotons of TNT. The pressure wave from the airburst caused an increase in pressure of 0.50 kPa, enough to shatter some windows. Find the force that this "overpressure" would exert on a 34-in. × 46-in. window. Give your answer in newtons and pounds.

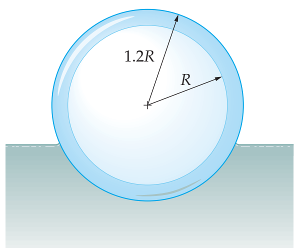
105. •• **Going Over Like a *Mythbuster* Lead Balloon** On one episode of *Mythbusters*, Jamie and Adam try to make a lead balloon that will float when filled with helium. The balloon they constructed was approximately cubical in shape, and 10 feet on a side. They used a thin lead foil, which gave the finished balloon a mass of 11 kg. **(a)** What was the thickness of the foil? **(b)** Would the lead balloon float if filled with helium? **(c)** If the balloon does float, what would be the most mass it could lift in addition to its own mass?

106. ••• **IP** A pan half-filled with water is placed in the back of an SUV. **(a)** When the SUV is driving on the freeway with a constant velocity, is the surface of the water in the pan level, tilted forward, or tilted backward? Explain. **(b)** Suppose the SUV accelerates in the forward direction with a constant acceleration a. Is the surface of the water tilted forward, or tilted backward? Explain. **(c)** Show that the angle of tilt, θ, in part (b) has a magnitude given by $\tan\theta = a/g$, where g is the acceleration of gravity.

107. ••• A wooden block of cross-sectional area A, height H, and density ρ_1 floats in a fluid of density ρ_2. If the block is displaced downward and then released, it will oscillate with simple harmonic motion. Find the period of its motion.

108. ••• A round wooden log with a diameter of 73 cm floats with one-half of its radius out of the water. What is the log's density?

109. ••• The hollow, spherical glass shell shown in **Figure 15–38** has an inner radius R and an outer radius $1.2R$. The density of the glass is ρ_g. What fraction of the shell is submerged when it



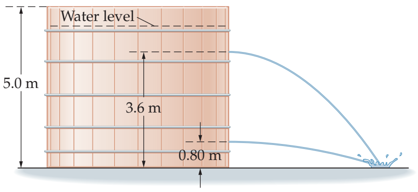
▲ **FIGURE 15–38** Problem 109

floats in a liquid of density $\rho = 1.5\rho_g$? (Assume the interior of the shell is a vacuum.)

110. ••• A geode is a hollow rock with a solid shell and an air-filled interior. Suppose a particular geode weighs twice as much in air as it does when completely submerged in water. If the density of the solid part of the geode is 2500 kg/m^3, what fraction of the geode's volume is hollow?

111. ••• A tank of water filled to a depth d has a hole in its side a height h above the table on which it rests. Show that water emerging from the hole hits the table at a horizontal distance of $2\sqrt{(d - h)h}$ from the base of the tank.

112. ••• The water tank in **Figure 15–39** is open to the atmosphere and has two holes in it, one 0.80 m and one 3.6 m above the floor on which the tank rests. If the two streams of water strike the floor in the same place, what is the depth of water in the tank?



▲ **FIGURE 15–39** Problem 112

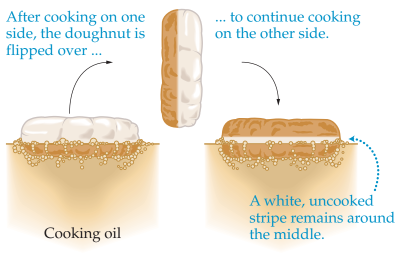
113. ••• A hollow cubical box, 0.29 m on a side, with walls of negligible thickness floats with 35% of its volume submerged. What mass of water can be added to the box before it sinks?

PASSAGE PROBLEMS

Cooking Doughnuts

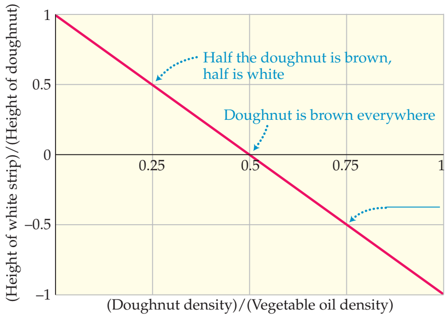
Doughnuts are cooked by dropping the dough into hot vegetable oil until it changes from white to a rich, golden brown. One popular brand of doughnut automates this process; the doughnuts are made on an assembly line that customers can view in operation as they wait to order. Watching the doughnuts cook gives the customers time to develop an appetite as they ponder the physics of the process.

First, the uncooked doughnut is dropped into hot vegetable oil, whose density is $\rho = 919$ kg/m^3. There it browns on one side as it floats on the oil. After cooking for the proper amount of time, a mechanical lever flips the doughnut over so it can cook on the other side. The doughnut floats fairly high in the oil, with less than half of its volume submerged. As a result, the final product has a characteristic white stripe around the middle where the dough is always out of the oil, as indicated in **Figure 15–40**.



▲ **FIGURE 15–40 Steps in cooking a doughnut**

The connection between the density of the doughnut and the height of the white stripe is illustrated in **Figure 15–41**. On the x axis we plot the density of the doughnut as a fraction of the density of the vegetable cooking oil; the y axis shows the height of the white stripe as a fraction of the total height of the doughnut. Notice that the height of the white stripe is plotted for both positive and negative values.



▲ **FIGURE 15–41 Cooking a floating doughnut**

A finished doughnut has a stripe around the middle whose height is related to the density of the doughnut by the straight line shown here. (Problems 114, 115, 116, and 117)

114. • Assuming the doughnut has a cylindrical shape of height H and diameter D, and that the height of the white stripe is $0.22H$, what is the density of the doughnut?

A. 260 kg/m³ **B.** 360 kg/m³

C. 720 kg/m³ **D.** 820 kg/m³

115. • A new doughnut is being planned whose density will be 330 kg/m³. If the height of the doughnut is H, what will be the height of the white stripe?

A. $0.14H$ **B.** $0.24H$

C. $0.28H$ **D.** $0.64H$

116. • Figure 15–41 has comments where the "height" of the white stripe is $0.5H$ and 0. The comment for $-0.5H$ has been left blank, however. Which of the following comments is most appropriate for this case?

A. The doughnut sinks.

B. The white stripe is still present, but has a negative height.

C. Half the doughnut is light brown, half is dark brown.

D. The top and bottom of the doughnut are white, the middle one-half is brown.

117. • Suppose the density of a doughnut is 550 kg/m³. In terms of the height of the doughnut, H, what is the height of the portion of the doughnut that is out of the oil?

A. $0.20H$ **B.** $0.40H$

C. $0.67H$ **D.** $0.80H$

INTERACTIVE PROBLEMS

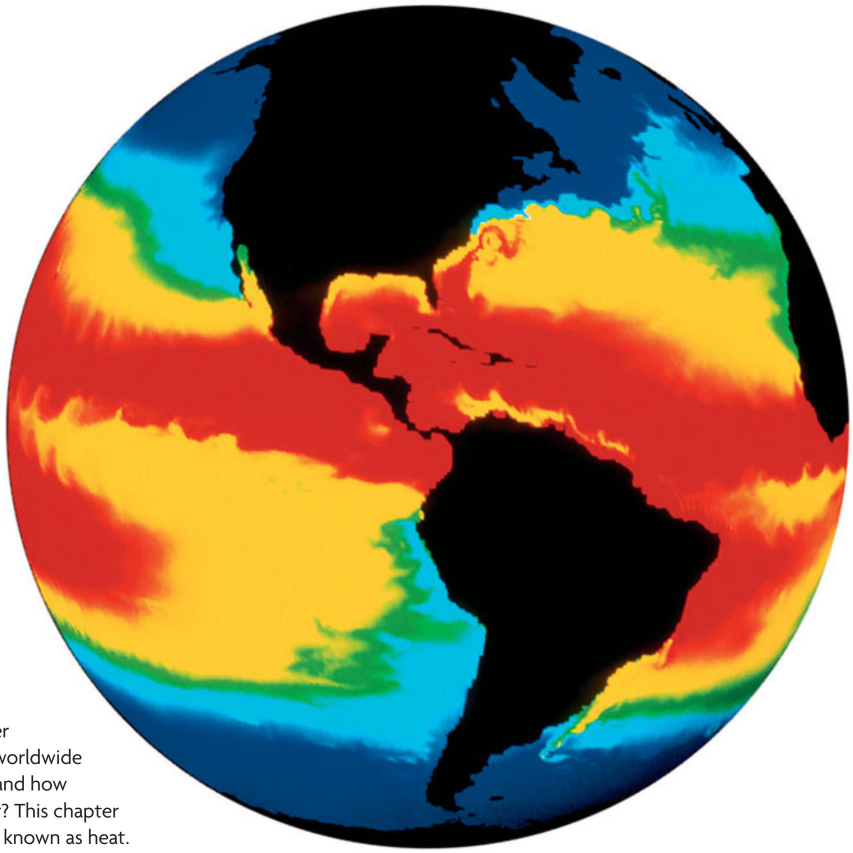
118. •• **IP Referring to Example 15–4** Suppose we use a different vegetable oil that has a higher density than the one in Example 15–4. **(a)** If everything else remains the same, will the height difference, h, increase, decrease, or remain the same? Explain. **(b)** Find the height difference for an oil that has a density of 9.60×10^2 kg/m^3.

119. •• **Referring to Example 15–4** Find the height difference, h, if the depth of the oil is increased to 7.50 cm. Assume everything else in the problem remains the same.

120. •• **Referring to Example 15–9 (a)** Find the height H required to make $D = 0.655$ m. Assume everything else in the problem remains the same. **(b)** Find the depth h required to make $D = 0.455$ m. Assume everything else in the problem remains the same.

121. •• **Referring to Example 15–9** Suppose both h and H are increased by a factor of two. By what factor is the distance D increased?

16 Temperature and Heat



In this computer-generated map displaying the variation in average global ocean temperatures, red represents the hottest and blue the coolest areas. The relatively high-temperature water in equatorial regions warms the air above it, greatly influencing worldwide climate and weather patterns. But what exactly is temperature, and how does thermal energy pass from the warm water to the cooler air? This chapter explores such questions and others related to the phenomenon known as heat.

To this point our study of physics has involved just three physical quantities: mass, length, and time. Every measurement, every calculation, has been in terms of *M*, *L*, and *T*, or some combination of the three. We now add a fourth physical quantity—*temperature*. With the introduction of temperature, we broaden the scope of physics, allowing the study of a wide variety of new physical situations that mechanics alone cannot address.

In this chapter we introduce the concept of temperature and discuss its effects on macroscopic systems. We begin by showing how differences in temperature relate to a particular type of energy transfer we call *heat*. We also discuss the connection between changes in temperature and changes in other physical quantities, such as length, pressure, and volume. Finally, we consider various mechanisms by which thermal energy is exchanged. Later in the text, when we consider the microscopic aspects of temperature, we shall see that temperature is ultimately related to the rapidity of molecular motion.

16–1 Temperature and the Zeroth Law of Thermodynamics

Even as small children we learn to avoid objects that are "hot." We also discover early on that if we forget to wear our coats outside we can become "cold." Later, we associate high values of something called "temperature" with hot objects, and low values with cold objects. These basic notions about temperature carry over into physics, though with a bit more precision.

Similarly, when we put a cool pan of water on a hot stove burner we say that "heat" flows from the hot burner to the cool water. To be more precise, as required in physics, we will define **heat** as follows:

> Heat is the energy that is *transferred* between objects because of a temperature difference.

Therefore, when we say that there is a "transfer of heat" or a "heat flow" from object A to object B, we simply mean that the total energy of object A decreases and the total energy of object B increases. To summarize, an object does not "contain" heat—it has a certain energy content, and the energy it *exchanges* with other objects due to temperature differences is called heat.

Now, objects are said to be in **thermal contact** if heat can flow between them. In general, when a hot object is brought into thermal contact with a cold object, heat will be exchanged. The result is that the hot object cools off—and its molecules move more slowly—while the cold object warms up—and its molecules move more rapidly. After some time in thermal contact, the transfer of heat ceases. At this point, we say that the objects are in **thermal equilibrium.** The study of physical processes involving the transfer of heat is the area of physics referred to as **thermodynamics.** Because thermodynamics deals with the flow of energy within and between objects, it has great practical importance in all areas of the life sciences, physical sciences, and engineering.

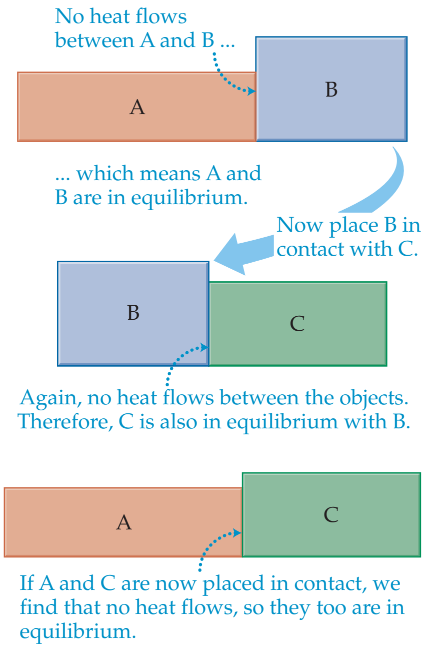
Note that thermal contact and physical contact are not necessarily the same. For example, thermal contact can occur even when there is no physical contact at all—as when you warm your hands near a fire. Various types of thermal contact will be discussed in detail in Section 16–6.

Zeroth Law of Thermodynamics

We now introduce perhaps the most fundamental law obeyed by thermodynamic systems—referred to, appropriately enough, as the zeroth law of thermodynamics. The zeroth law spells out the basic properties of temperature. Its name reflects not only its basic importance, but also its subtlety, in that it was not recognized as a separate law until after the other three laws of thermodynamics had been accepted. Later, in Chapter 18, we introduce the remaining three laws of thermodynamics. These laws enable us to analyze the behavior of engines and refrigerators, and to show, among other things, that perpetual motion machines are not possible.

The basic idea of the zeroth law of thermodynamics is that thermal equilibrium is determined by a single physical quantity—the **temperature.** For example, two objects in thermal contact are in equilibrium when they have the *same* temperature. If one or the other has a higher temperature, heat flows from that object to the other until their temperatures are equal.

This may seem almost too obvious to mention, at least until you give it more thought. Suppose, for example, that you have a piece of metal and a pool of water, and you want to know if heat will flow between them when you put the metal in the pool. You measure the temperature of each, and if they are the same, you can conclude that no heat will flow. If the temperatures are different, however, it follows that there will be a flow of heat. Nothing else matters—not the type of metal, its mass, its shape, the amount of water, whether the water is fresh or salt, and so on—all that matters is one number, the temperature.



▲ **FIGURE 16–1 An illustration of the zeroth law of thermodynamics**

If A and C are each in thermal equilibrium with B, then A will be in thermal equilibrium with C when they are brought into thermal contact.

To summarize, the **zeroth law of thermodynamics** can be stated as follows:

> If object A is in thermal equilibrium with object B, and object C is also in thermal equilibrium with object B, then objects A and C will be in thermal equilibrium if brought into thermal contact.

This is illustrated in **Figure 16–1**. We begin with objects A and B in thermal contact and in equilibrium. Next, object B is separated from A, and placed in contact with C. Objects C and B are also found to be in equilibrium. Hence, by the zeroth law, we are assured that when A and C are placed in contact they also will be in equilibrium.

To apply this principle to our example, let object A be the piece of metal and object C be the pool of water. Object B, then, can be a thermometer, used to measure the temperature of the metal and the water. If A and C are each separately in equilibrium with B—which means that they have the same temperature—they will be in equilibrium with one another.

16–2 Temperature Scales

A variety of temperature scales are commonly used both in everyday situations and in physics. Some are related to familiar reference points, such as the temperature of boiling or freezing water. Others have more complex, historical rationales for their values. Here we consider three of the more frequently used temperature scales. We also examine the connections between them.

Later in this chapter we will discuss some of the physical phenomena—such as thermal expansion—that can be used to construct a thermometer. With a properly calibrated thermometer, we can determine the temperature on any of these scales. In the next chapter, we will explore more fully the question of just what temperature means on a conceptual and microscopic level.

The Celsius Scale

Perhaps the easiest temperature scale to remember is the Celsius scale, named in honor of the Swedish astronomer Anders Celsius (1701–1744). Originally, Celsius assigned zero degrees to boiling water and 100 degrees to freezing water. These values were later reversed by the biologist Carolus Linnaeus (1707–1778). Thus, today we say that water freezes at zero degrees Celsius, which we abbreviate as 0 °C, and boils at 100 °C.

Note that the choice of zero level for a temperature scale is quite arbitrary, as is the number of degrees between any two reference points. In the Celsius scale, as in others, there is no upper limit to the value a temperature may have. There is a lower limit, however. For the Celsius scale, the lowest possible temperature is −273.15 °C, as we shall see later in this section.

One bit of notation should be pointed out before we continue. When we write a Celsius temperature, we give a numerical value followed by the degree symbol, °, and the capital letter C. For example, 5 °C is the temperature five degrees Celsius. On the other hand, if the temperature of an object is *changed* by a given amount, we use the notation C°. Thus, if we increase the temperature by five degrees on the Celsius scale, we say that the change in temperature is 5 C°; that is, five Celsius degrees. This is summarized below:

> A temperature T of five degrees is 5 °C
> (five degrees Celsius)
>
> A temperature change ΔT of five degrees is 5 C°
> (five Celsius degrees)

The Fahrenheit Scale

The Fahrenheit scale was developed by Gabriel Fahrenheit (1686–1736), who chose zero to be the lowest temperature he was able to achieve in his laboratory. He also chose 96 degrees to be body temperature, though why he made this choice is not known. In the modern version of the Fahrenheit scale body temperature is 98.6 °F; in addition, water freezes at 32 °F and boils at 212 °F. Lastly, using the same convention as for °C and C°, we say that an increase of 180 F° is required to bring water from freezing to boiling.

Note that the Fahrenheit scale not only has a different zero than the Celsius scale, it also has a different "size" for its degree. As just noted, 180 Fahrenheit degrees are required for the same change in temperature as 100 Celsius degrees. Hence, the Fahrenheit degrees are smaller by a factor of $100/180 = 5/9$.

To convert between a Fahrenheit temperature, T_F, and a Celsius temperature, T_C, we start by writing a linear relation between them. Thus, let

$$T_F = aT_C + b$$

We would like to determine the constants a and b. This requires two independent pieces of information, which we have in the freezing and boiling points of water. Using the freezing point, we find

$$32\ °F = a(0\ °C) + b = b$$

Thus, b is 32 °F. Next, the boiling point gives

$$212\ °F = a(100\ °C) + 32\ °F$$

Solving for the constant a we find

$$a = (212\ °F - 32\ °F)/(100\ °C) = \frac{180\ F°}{100\ C°} = \tfrac{9}{5}\ F°/C°$$

Combining our results gives the following conversion relationship:

Conversion Between Degrees Celsius and Degrees Fahrenheit

$$T_F = (\tfrac{9}{5}\ F°/C°)T_C + 32\ °F \qquad 16\text{–}1$$

Similarly, this relation can be rearranged to convert from Fahrenheit to Celsius:

Conversion Between Degrees Fahrenheit and Degrees Celsius

$$T_C = (\tfrac{5}{9}\ C°/F°)(T_F - 32\ °F) \qquad 16\text{–}2$$

Since conversion factors like $\frac{9}{5}$ F°/C° are a bit clumsy and tend to clutter up an equation, we will generally drop the degree symbols until the final result. For example, to convert 10 °C to degrees Fahrenheit, we write

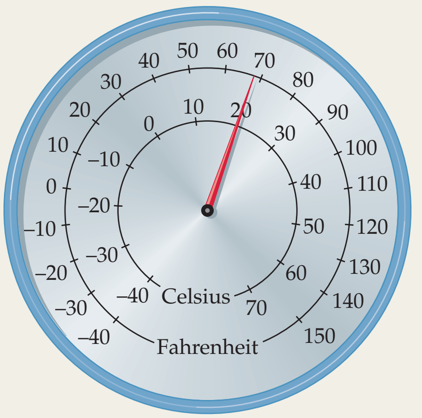
$$T_F = \tfrac{9}{5}T_C + 32 = \tfrac{9}{5}(10) + 32 = 50\ °F$$

EXAMPLE 16–1 TEMPERATURE CONVERSIONS

(a) On a fine spring day you notice that the temperature is 75 °F. What is the corresponding temperature on the Celsius scale? **(b)** If the temperature on a brisk winter morning is −2.0 °C, what is the corresponding Fahrenheit temperature?

PICTURE THE PROBLEM

Our sketch shows a circular thermometer with both a Fahrenheit and a Celsius scale. The range of the scales extends well beyond the temperatures 75 °F and −2.0 °C needed for the problem.



STRATEGY

The conversions asked for in this problem are straightforward applications of the relations between T_F and T_C. In particular, for **(a)** we use $T_C = (5/9)(T_F - 32)$, and for **(b)** we use $T_F = \frac{9}{5}T_C + 32$.

SOLUTION

Part (a)

1. Substitute $T_F = 75$ °F into Equation 16–2: $T_C = \frac{5}{9}(75 - 32) = 24\ °C$

Part (b)

2. Substitute $T_C = -2.0$ °C in Equation 16–1: $T_F = \frac{9}{5}(-2.0) + 32 = 28\ °F$

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INSIGHT

Note that the results given here agree with the scales shown in the drawing.

PRACTICE PROBLEM

Find the Celsius temperature that corresponds to 110 °F. [**Answer:** $T_C = 43$ °C]

Some related homework problems: Problem 1, Problem 2

ACTIVE EXAMPLE 16–1 SAME TEMPERATURE

What temperature is the same on both the Celsius and Fahrenheit scales?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set $T_F = T_C = t$ in Equation 16–1: $t = 9t/5 + 32$
2. Move all terms involving t to the left side of the equation: $-4t/5 = 32$
3. Solve for t: $t = -40$
4. As a check, substitute $T_F = -40$ °F in Equation 16–2: $T_C = (5/9)(-40 - 32) = -40$ °C

INSIGHT

Thus, -40 °F is the same as -40 °C. This is consistent with the scale shown in Example 16–1.

YOUR TURN

Find the Fahrenheit temperature whose numerical value is three times greater than the corresponding Celsius temperature.

(Answers to **Your Turn** *problems can be found in the back of the book.)*

Absolute Zero

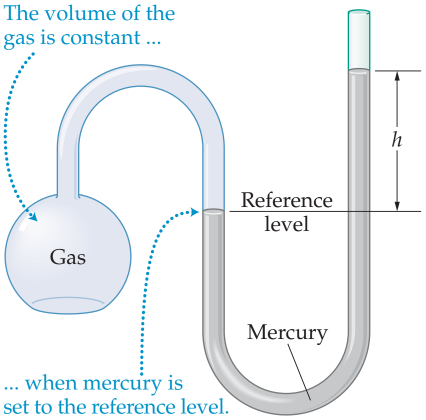
Experiments show conclusively that there is a lowest temperature below which it is impossible to cool an object. This is referred to as **absolute zero.** Though absolute zero can be approached from above arbitrarily closely, it can never be attained.

To give an idea of just where absolute zero is on the Celsius scale, we start with the following observation: If a given volume V of air—say the air in a balloon—is cooled from 100 °C to 0 °C, its volume decreases by roughly $V/4$. Imagine this trend continuing uninterrupted. In this case, cooling from 0 °C to -100 °C would reduce the volume by another $V/4$, from -100 °C to -200 °C by another $V/4$, and finally, from -200 °C to -300 °C by another $V/4$, which brings the volume down to zero. Clearly, it doesn't make sense for the volume to be less than zero, and hence absolute zero must be roughly -300 °C.

This result, though crude, is in the right ballpark. A precise determination of absolute zero can be made with a device known as a **constant-volume gas thermometer.** This instrument is shown in **Figure 16–2**. The basic idea is that by adjusting the level of mercury in the right-hand tube, the level of mercury in the left-hand tube can be set to a fixed reference level. With the mercury so adjusted, the gas occupies a constant volume and its pressure is simply $P_{gas} = P_{at} + \rho g h$ (Equation 15–7), where ρ is the density of mercury.

As the temperature of the gas is changed, the mercury level in the right-hand tube can be readjusted as described. The gas pressure can be determined again, and the process repeated. The results of a series of such measurements are shown in **Figure 16–3**.

Note that as a gas is cooled its pressure decreases, as one would expect. In fact, the decrease in pressure is approximately linear. At low enough temperatures the gas eventually liquefies, and its behavior changes, but if we extrapolate the



▲ **FIGURE 16–2 A constant-volume gas thermometer**

By adjusting the height of mercury in the right-hand tube, the level in the left-hand tube can be set at the reference level. This assures that the gas occupies a constant volume.

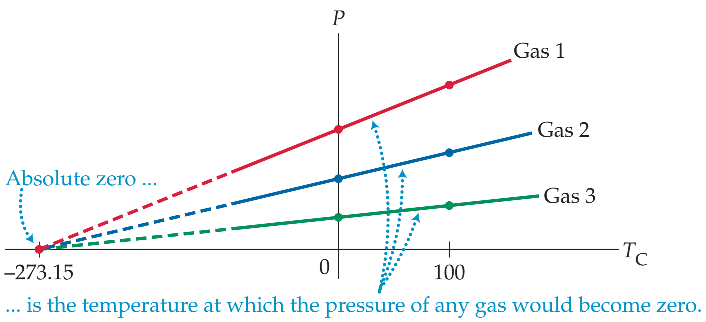


FIGURE 16–3 Determining absolute zero
Different gases have different pressures at any given temperature. However, they all extend down to zero pressure at precisely the same temperature, −273.15 °C. This is the location of absolute zero.

straight line obtained before liquefaction, we see that it reaches zero pressure (the lowest pressure possible) at −273.15 °C.

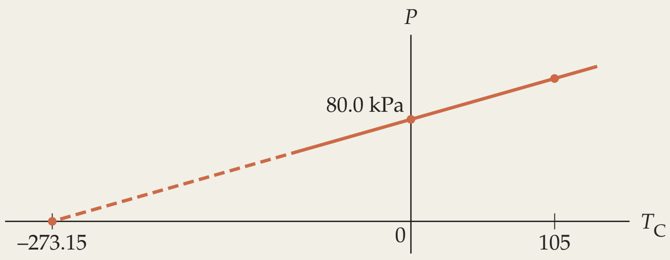
What is remarkable about this result is that it is independent of the gas we use in the thermometer. For example, gas 2 and gas 3 in Figure 16–3 have pressures that are different from one another, and from gas 1. Yet all three gases extrapolate to zero pressure at precisely the same temperature. Thus, we conclude that there is indeed a *unique* value of absolute zero, below which further cooling is not possible.

EXAMPLE 16–2 IT'S A GAS

The gas in a constant-volume gas thermometer has a pressure of 80.0 kPa at 0.00 °C. Assuming ideal behavior, as in Figure 16–3, what is the pressure of this gas at 105 °C?

PICTURE THE PROBLEM

Our sketch plots the pressure of the gas as a function of temperature. Note that at $T_C = 0.00$ °C the pressure is 80.0 kPa, and that at $T_C = -273.15$ °C the pressure extrapolates linearly to zero. We also indicate the point corresponding to 105 °C on the graph.



STRATEGY

We assume that the pressure lies on a straight line, as in Figure 16–3. To find the pressure at $T_C = 105$ °C we simply extend the straight line.

Thus, we start with the information that the pressure increases linearly from 0 to 80.0 kPa when the temperature increases from −273.15 °C to 0.00 °C. This rate of increase in pressure must also apply to an increase in temperature from −273.15 °C to 105 °C. Using this rate we can find the desired pressure.

SOLUTION

1. Calculate the rate at which pressure increases for this gas: $\text{rate} = \dfrac{80.0\ \text{kPa}}{273.15\ \text{C}^\circ} = 0.293\ \text{kPa/C}^\circ$

2. Multiply this rate by the temperature change from −273.15 °C to 105 °C: $(0.293\ \text{kPa/C}^\circ)(378\ \text{C}^\circ) = 111\ \text{kPa}$

INSIGHT

The pressure of this gas increases from slightly less than one atmosphere at 0.00 °C to slightly more than one atmosphere at 105 °C.

An alternative solution to this problem is to calculate the pressure difference from 0.00 °C to 105 °C. Specifically, we can say that $P = 80.0\ \text{kPa} + (0.293\ \text{kPa/C}^\circ)(105\ \text{C}^\circ) = 111\ \text{kPa}$.

PRACTICE PROBLEM

Find the temperature at which the pressure of the gas is 70.0 kPa. [**Answer:** $T_C = -34.1$ °C]

Some related homework problems: Problem 7, Problem 8

The Kelvin Scale

The Kelvin temperature scale, named for the Scottish physicist William Thomson, Lord Kelvin (1824–1907), is based on the existence of absolute zero. In fact, the zero of the Kelvin scale, abbreviated 0 K, is set exactly at absolute zero. Thus, in this scale there are no negative equilibrium temperatures. The Kelvin scale is also chosen to have the same degree size as the Celsius scale.

As mentioned, absolute zero occurs at $-273.15\ °C$, hence the conversion between a Kelvin-scale temperature, T, and a Celsius temperature, T_C, is as follows:

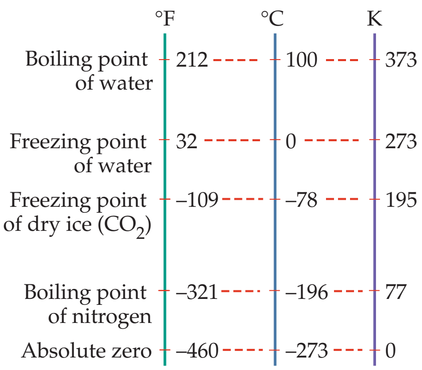
Conversion Between a Celsius Temperature and a Kelvin Temperature

$$T = T_C + 273.15 \qquad 16\text{–}3$$

Note that the difference between the Celsius and Kelvin scales is simply a difference in the zero level.

The notation for the Kelvin scale differs somewhat from that for the Celsius and Fahrenheit scales. In particular, by international agreement, the degree terminology and the degree symbol, °, are not used in the Kelvin scale. Instead, a temperature of 5 K is read simply as 5 kelvin. In addition, a change in temperature of 5 kelvin is written 5 K, the same as for a temperature of 5 kelvin.

Though the Celsius and Fahrenheit scales are the ones most commonly used in everyday situations, the Kelvin scale is used more than any other in physics. This stems from the fact that the Kelvin scale incorporates the significant concept of absolute zero. As a result, the thermal energy of a system depends in a very simple way on the Kelvin temperature. This will be discussed in detail in the next chapter.



▲ **FIGURE 16–4 Temperature scales**

A comparison of the Fahrenheit, Celsius, and Kelvin temperature scales. Physically significant temperatures, such as the freezing and boiling points of water, are indicated for each scale.

EXERCISE 16–1

Convert 55 °F to the Kelvin temperature scale.

SOLUTION

First, convert from °F to °C:

$$T_C = \tfrac{5}{9}(55 - 32) = 13\ °C$$

Next, convert °C to K:

$$T = 13 + 273.15 = 286\ K$$

The three temperature scales presented in this section are shown side by side in **Figure 16–4**. Temperatures of particular interest are indicated as well. This permits a useful visual comparison between the scales.

16–3 Thermal Expansion

Most substances expand when heated. For example, power lines on a hot summer day hang low compared to their appearance on a cold day in winter. In fact, thermal expansion is the basis for many thermometers, including the familiar thermometers used for measuring a fever. The expansion of a liquid, such as mercury or alcohol, results in a column of liquid of variable height within the glass neck of the thermometer. The height is read against markings on the glass, which gives the temperature.

You may wonder what bizarre substance could possibly be an exception to this common response to heating. The most important exception occurs in a substance you drink every day—water. This is just one of the many special properties that sets water apart from most other substances.

In this section we consider the physics of thermal expansion, including linear, area, and volume expansion. We also discuss briefly the unusual thermal behavior of water, and some of its more significant consequences.

Linear Expansion

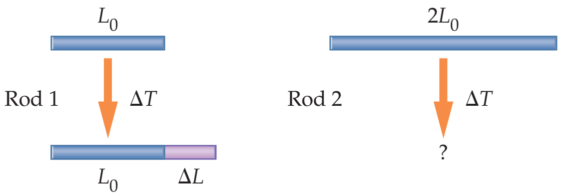
Consider a rod whose length is L_0 at the temperature T_0. Experiments show that when the rod is heated or cooled, its length changes in direct proportion to the temperature change. Thus, if the change in temperature is ΔT, the change in length of the rod, ΔL, is

$$\Delta L = (\text{constant})\Delta T$$

The constant of proportionality depends, among other things, on the substance from which the rod is made.

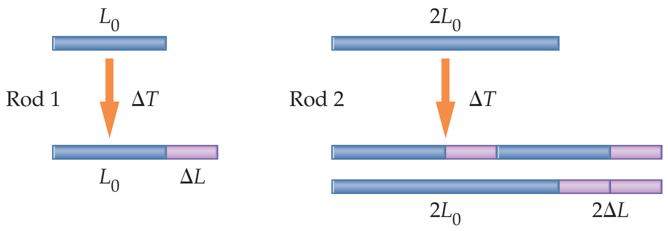
CONCEPTUAL CHECKPOINT 16–1 COMPARE EXPANSIONS

When rod 1 is heated by an amount ΔT, its length increases by ΔL. If rod 2, which is twice as long as rod 1 and made of the same material, is heated by the same amount, does its length increase by **(a)** ΔL, **(b)** $2\Delta L$, or **(c)** $\Delta L/2$?



REASONING AND DISCUSSION

We can imagine rod 2 to be composed of two copies of rod 1 placed end to end, as shown.



When the temperature is increased by ΔT, each copy of rod 1 expands by ΔL. Hence, the total expansion of the two copies is $2\Delta L$, as is the total expansion of rod 2.

ANSWER

(b) The rod that is twice as long expands twice as much; $2\Delta L$.

We conclude, then, on the basis of the preceding Conceptual Checkpoint, that the change in length is proportional to *both* the initial length, L_0, and the temperature change, ΔT. The constant of proportionality is referred to as α, the **coefficient of linear expansion,** and is defined as follows:

Definition of Coefficient of Linear Expansion, α

$$\Delta L = \alpha L_0 \Delta T \qquad 16\text{–}4$$

SI unit for α: $K^{-1} = (C°)^{-1}$

Table 16–1 gives values of α for a variety of substances.

TABLE 16–1 Coefficients of Thermal Expansion near 20 °C

Substance	Coefficient of linear expansion, α (K^{-1})
Lead	29×10^{-6}
Aluminum	24×10^{-6}
Brass	19×10^{-6}
Copper	17×10^{-6}
Iron (steel)	12×10^{-6}
Concrete	12×10^{-6}
Window glass	11×10^{-6}
Pyrex glass	3.3×10^{-6}
Quartz	0.50×10^{-6}
Substance	**Coefficient of volume expansion, β (K^{-1})**
Ether	1.51×10^{-3}
Carbon tetrachloride	1.18×10^{-3}
Alcohol	1.01×10^{-3}
Gasoline	0.95×10^{-3}
Olive oil	0.68×10^{-3}
Water	0.21×10^{-3}
Mercury	0.18×10^{-3}

EXERCISE 16–2

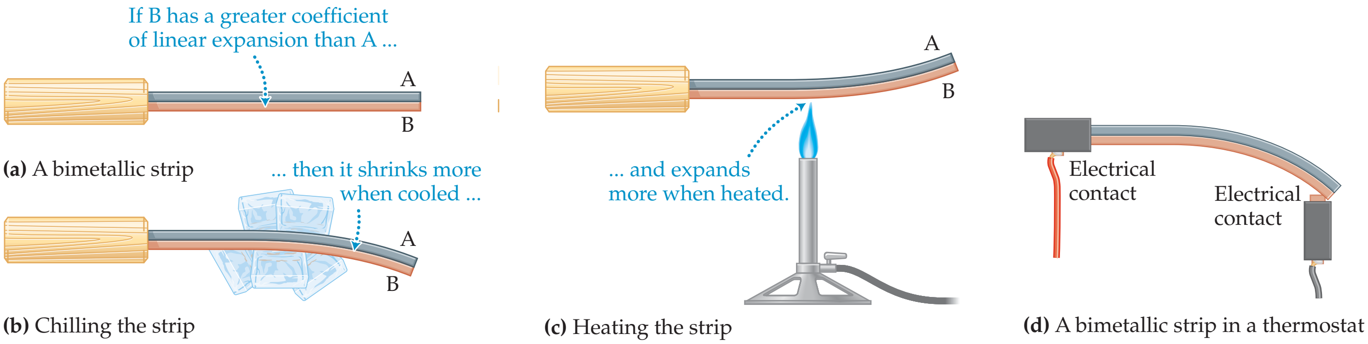
The Eiffel Tower, constructed in 1889 by Alexandre Eiffel, is an impressive latticework structure made of iron. If the tower is 301 m high on a 22 °C day, how much does its height decrease when the temperature cools to 0.0 °C?

SOLUTION

We can calculate the change in height with Equation 16–4. Note that the coefficient of linear expansion for iron, given in Table 16–1, is $12 \times 10^{-6}\ K^{-1}$ and the change in temperature is $\Delta T = -22\ C° = -22\ K$:

$$\Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6}\ K^{-1})(301\ m)(-22\ K) = -7.9\ cm$$

An interesting application of thermal expansion is in the behavior of a bimetallic strip. As the name suggests, a bimetallic strip consists of two metals bonded together to form a linear strip of metal. This is illustrated in **Figure 16–5**. Since two different metals will, in general, have different coefficients of linear



▲ **FIGURE 16–5 A bimetallic strip**

(a) A bimetallic strip composed of metals A and B. **(b)** If metal B has a larger coefficient of linear expansion than metal A, it will shrink more when cooled and **(c)** expand more when heated. **(d)** A bimetallic strip can be used to construct a thermostat. If the temperature falls, the strip bends downward and closes the electric circuit, which then operates a heater. When the temperature rises, the strip deflects in the opposite direction, breaking the circuit and turning off the heater.

▲ The Eiffel Tower in Paris gains about a thirteenth of an inch in height for each Fahrenheit degree that the temperature rises.

REAL-WORLD PHYSICS
Bimetallic strips

REAL-WORLD PHYSICS
Antiscalding devices

REAL-WORLD PHYSICS
Thermal expansion joints

expansion, α, the two sides of the strip will change lengths by different amounts when heated or cooled.

For example, suppose metal B in Figure 16–5 (a) has the larger coefficient of linear expansion. This means that its length will change by greater amounts than metal A for the same temperature change. Hence, if this strip is cooled, the B side will shrink more than the A side, resulting in the strip bending toward the B side, as in Figure 16–5 (b). On the other hand, if the strip is heated, the B side expands by a greater amount than the A side, and the strip curves toward the A side, as in Figure 16–5 (c). Thus, the shape of the bimetallic strip depends sensitively on temperature.

Because of this property, bimetallic strips are used in a variety of thermal applications. For example, a bimetallic strip can be used as a thermometer; as the strip changes its shape it can move a needle to indicate the temperature. Similarly, many thermostats have a bimetallic strip to turn on or shut off a heater. This is shown in Figure 16–5 (d). As the temperature of the room changes, the bimetallic strip deflects in one direction or the other, which either closes or breaks the electric circuit connected to the heater.

Another common use of thermal expansion is the *antiscalding device* for water faucets. An antiscalding device is simply a valve inside a water faucet that is attached to a spring. When the water temperature is at a safe level, the valve permits water to flow freely through the faucet. If the temperature of the water reaches a dangerous level, however, the thermal expansion of the spring is enough to close the valve and stop the flow of water—thus preventing inadvertent scalding. When the water cools down again, the valve reopens and the flow of water resumes.

Finally, thermal expansion can have undesirable effects in some cases. For example, you may have noticed that bridges often have gaps between different sections of the structure. When the air temperature rises in the summer, the sections of the bridge can expand freely into these gaps. If the gaps were not present, the expansion of the different sections could cause the bridge to buckle and warp. Thus, these gaps, referred to as *expansion joints*, are a way of avoiding this type of heat-related damage. Expansion joints can also be found in railroad tracks and oil pipelines, to name just two other examples.

Area Expansion

Since the length of an object changes with temperature, it follows that its area changes as well. To see precisely how the area changes, consider a square piece of metal of length L on a side. The initial area of the square is $A = L^2$. If the temperature of the square is increased by ΔT the length of each side increases from

L to $L + \Delta L = L + \alpha L \Delta T$. As a result, the square has an increased area, A', given by

$$A' = (L + \Delta L)^2 = (L + \alpha L \Delta T)^2$$
$$= L^2 + 2\alpha L^2 \Delta T + \alpha^2 L^2 \Delta T^2$$

Now, if $\alpha \Delta T$ is much less than one—which is certainly the case for typical changes in temperature, ΔT—then $\alpha^2 \Delta T^2$ is even smaller. Hence, if we ignore this small contribution, we find

$$A' \approx L^2 + 2\alpha L^2 \Delta T = A + 2\alpha A \Delta T$$

As a result, the change in area, ΔA, is

$$\Delta A = A' - A \approx 2\alpha A \Delta T \quad \text{16–5}$$

Note the similarity between this relation and Equation 16–4; the length L has been replaced with the area A, and the expansion coefficient has been doubled to 2α.

Though this calculation was done for the simple case of a square, the result applies to an area of any shape. For example, a circular disk of radius r and area $A = \pi r^2$ will increase its area by the amount $2\alpha A \Delta T$ with an increase in temperature of ΔT. What about a washer, however, which is a disk of metal with a circular hole cut out of its center? What happens to the area of the hole as the washer is heated? Does it expand along with everything else, or does the expanding washer "expand into the hole" to make it smaller? We consider this question in the next Conceptual Checkpoint.

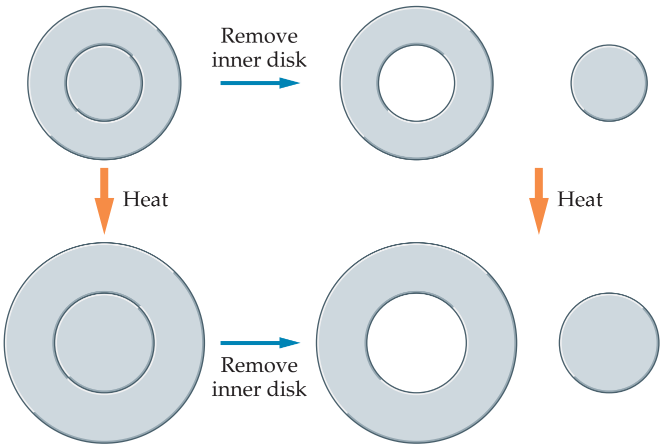
▲ Thermal expansion, though small, is far from negligible in many everyday situations. This is especially true when long objects such as railroad tracks, bridges, or pipelines are involved. Bridges and elevated highways (top) must include expansion joints to prevent the roadway from buckling when it expands in hot weather. Similarly, pipelines (bottom) typically include loops that allow for expansion and contraction when the temperature changes.

CONCEPTUAL CHECKPOINT 16–2 HEATING A HOLE

A washer has a hole in the middle. As the washer is heated, does the hole **(a)** expand, **(b)** shrink, or **(c)** stay the same?

REASONING AND DISCUSSION

To make a washer from a disk of metal, we can cut along a circular curve, as shown, and remove the inner disk. If we now heat the system, both the washer and the inner disk expand. On the other hand, if we had left the inner disk in place and heated the original disk, it would also expand. Removing the *heated* inner disk would create an expanded washer with an expanded hole in the middle. We obtain the same result whether we remove the inner disk and then heat, or heat first and then remove the inner disk.



Thus, heating the washer causes both it and its hole to expand, and they both expand with the same coefficient of linear expansion. Basically, the system behaves the same as if we had produced a photographic enlargement—everything expands.

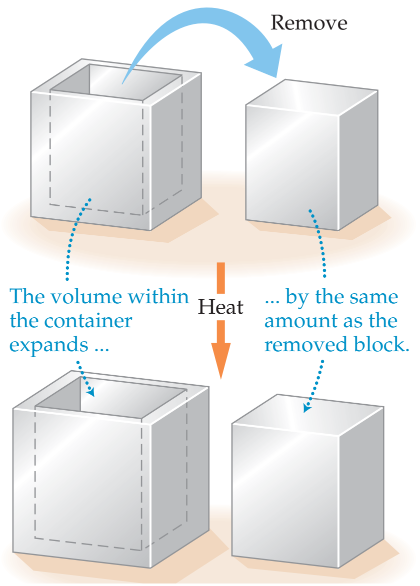
ANSWER

(a) The hole expands along with everything else.

PROBLEM-SOLVING NOTE

Expansion of a Hole

A hole in a material expands the same as if it were made of the material itself. Thus, to find the expansion of a hole in a steel plate, we use the coefficient of linear expansion for steel.



▲ **FIGURE 16–6 Volume expansion**
A portion of a cube is removed to create a container. When heated, the removed portion expands, just as the volume within the container expands.

PROBLEM-SOLVING NOTE
Expansion of a Volume

The empty volume inside a container expands the same as if it were made of the same material as the container. For example, to find the increase in volume of a steel tank, we use the coefficient of linear expansion for steel, just as if the tank were actually filled with steel.

PROBLEM-SOLVING NOTE
Temperature Change

Remember that a change in temperature of 1 C° is the same as a change in temperature of 1 K. Thus, when finding the thermal expansion of an object, the change in temperature ΔT can be expressed in terms of either the Celsius or the Kelvin temperature scale.

Volume Expansion

Just as the hole in a washer increases in area with heating, so does the empty volume within a cup or other container. This is illustrated in **Figure 16–6**, where we show a block of material with a volume removed to convert it into a container. As the system is heated, there will be an expansion of the container, of the volume within it, and of the volume that was removed. As with the area of a hole, the volume within a container expands with the same coefficient of expansion as the container itself.

To calculate the change in volume, consider a cube of length L on a side. The initial volume of the cube is $V = L^3$. Increasing the temperature results in an increased volume given by

$$\begin{aligned} V' &= (L + \Delta L)^3 = (L + \alpha L \Delta T)^3 \\ &= L^3 + 3\alpha L^3 \Delta T + 3\alpha^2 L^3 \Delta T^2 + \alpha^3 L^3 \Delta T^3 \end{aligned}$$

Neglecting the smaller contributions, as we did with the area, we find

$$V' \approx L^3 + 3\alpha L^3 \Delta T = V + 3\alpha V \Delta T$$

Therefore, the change in volume, ΔV, is

$$\Delta V = V' - V \approx 3\alpha V \Delta T$$

This expression, though calculated for a cube, applies to any volume.

In general, volume expansion is described in the same way as linear expansion, but with a **coefficient of volume expansion,** β, defined as follows:

Definition of Coefficient of Volume Expansion, β

$$\Delta V = \beta V \Delta T \qquad 16\text{–}6$$

SI unit for β: $\mathrm{K}^{-1} = (\mathrm{C}°)^{-1}$

Typical values of β are given in Table 16–1 for a number of different liquids. If Table 16–1 lists a value of α for a given substance, but not for β, its change in volume is calculated as follows:

$$\Delta V = \beta V \Delta T \approx 3\alpha V \Delta T \qquad 16\text{–}7$$

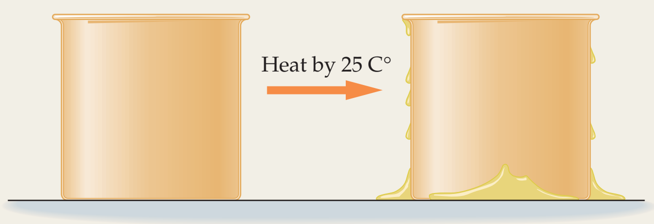
That is, we simply make the identification $\beta = 3\alpha$. For substances that have a specific value for β listed in Table 16–1, we simply use Equation 16–6. This is illustrated in the next Example.

EXAMPLE 16–3 OIL SPILL

A copper flask with a volume of 150 cm^3 is filled to the brim with olive oil. If the temperature of the system is increased from 6.0 °C to 31 °C, how much oil spills from the flask?

PICTURE THE PROBLEM

Our sketch shows the flask filled to the top initially, then spilling over when heated by 25 C°. (*Note:* Since degrees have the same size on the Celsius and Kelvin scales, it follows that $\Delta T = 25\ \mathrm{C}° = 25\ \mathrm{K}$.)



STRATEGY

As the system is heated, both the flask and the olive oil expand. Thus, we start by calculating the expansion of the oil and the flask separately. A quick glance at Table 16–1 shows that the olive oil will expand more, and the difference in expansion volumes is what spills out.

For the copper flask we find α in Table 16–1, then let $\beta = 3\alpha$.

SOLUTION

1. Calculate the change in volume of the olive oil:

$$\Delta V_{oil} = \beta V \Delta T = (0.68 \times 10^{-3}\ \mathrm{K}^{-1})(150\ \mathrm{cm}^3)(25\ \mathrm{K}) = 2.6\ \mathrm{cm}^3$$

2. Calculate the change in volume of the flask:

$$\Delta V_{flask} = 3\alpha V \Delta T = 3(17 \times 10^{-6}\ \mathrm{K}^{-1})(150\ \mathrm{cm}^3)(25\ \mathrm{K}) = 0.19\ \mathrm{cm}^3$$

3. Find the difference in volume expansions. This is the volume of oil that spills out:

$$\Delta V_{oil} - \Delta V_{flask} = 2.6\ \mathrm{cm}^3 - 0.19\ \mathrm{cm}^3 = 2.4\ \mathrm{cm}^3$$

INSIGHT

If the system were cooled, the oil would lose volume more rapidly than the flask. This would result in a drop in oil level.

This Example illustrates why fuel tanks on cars are designed to stop filling before the gas reaches the top—otherwise, the tank could overflow on a hot day.

PRACTICE PROBLEM

Suppose the copper flask is initially filled to the brim with gasoline rather than olive oil. Referring to Table 16–1, do you expect the amount of liquid that spills with heating to be greater than, less than, or the same as in the case of olive oil? Calculate the volume of spilled gasoline for a temperature increase of 25 C°. [**Answer:** More liquid will spill. $\Delta V = 3.4\ \mathrm{cm}^3$]

Some related homework problems: Problem 21, Problem 25

Special Properties of Water

As we have mentioned, water is a substance rich with unusual behavior. For example, in the last chapter we discussed the fact that the solid form of water (ice) is less dense than the liquid form. Hence, icebergs float. What is remarkable about icebergs floating is not that 90% is submerged, but that 10%, or any amount at all, is above the water. The solids of most substances are denser than their liquids; hence when they freeze, their solids immediately sink.

REAL-WORLD PHYSICS

Floating icebergs

Here we consider the unusual *thermal* behavior of water. **Figure 16–7** shows the density of water over a wide range of temperatures. Note that the density is a maximum at about 4 °C. Thus, when you *heat* water from 0 °C to 4 °C it actually *shrinks*, rather than expands, and becomes *more dense*. The reason is that water molecules that were once part of the rather open crystal structure of ice are now able to pack more closely together in the liquid.

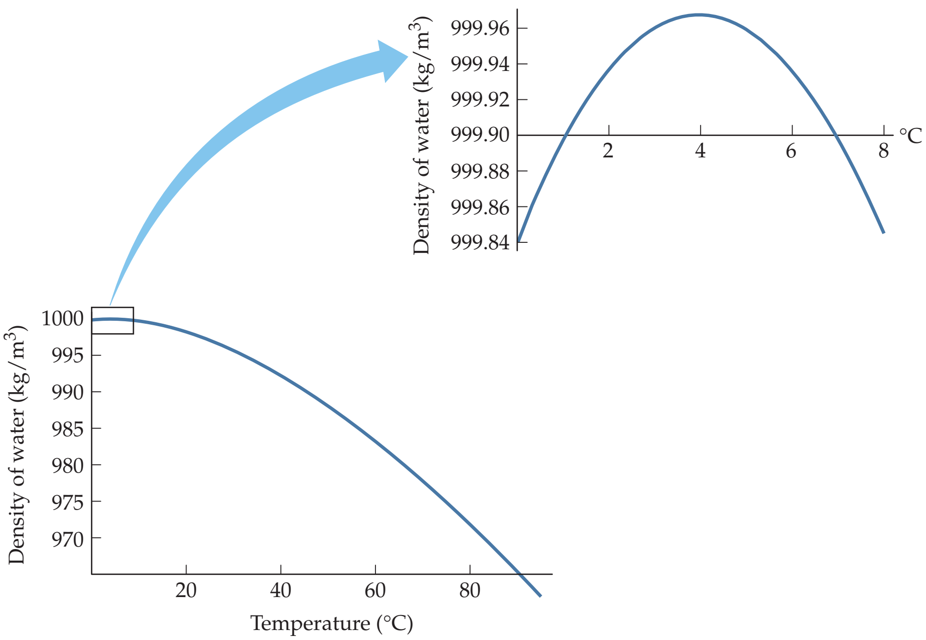
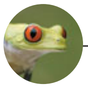


FIGURE 16–7 The unusual behavior of water near 4 °C

The density of water actually *increases* as the water is heated between 0 °C and 4 °C. Maximum density occurs near 4 °C.



REAL-WORLD PHYSICS: BIO

The ecology of lakes

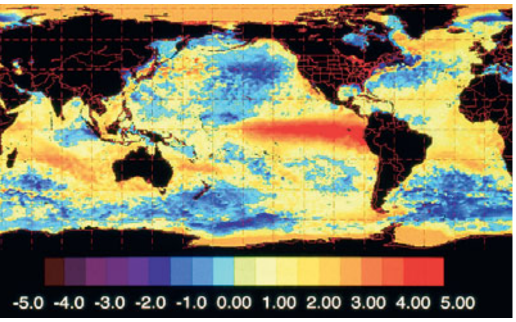
This behavior has significant consequences for the ecology of lakes in northern latitudes. When temperatures drop in the winter, the surface waters of a lake cool first and sink, allowing warmer water to rise to the surface to be cooled in turn. Eventually, a lake can fill with water at 4 °C. Further drops in temperature result in cooler, less dense water near the surface, where it floats until it freezes. Thus, lakes freeze on the top surface first, with the bottom water staying relatively warm at about 4 °C. In addition, the ice and snow on top of a lake act as thermal insulation, slowing the continued growth of ice.

REAL-WORLD PHYSICS

Bursting water pipes

On the other hand, if water had more ordinary behavior—like shrinking when cooled, and a solid form that is more dense than the liquid—a lake would freeze from the bottom up. There would be no insulating layer of ice on top, and if the winter were long enough, and cold enough, the lake could freeze solid. This, of course, would be disastrous for fish and other creatures that live in the water.

Finally, the same physics that is responsible for floating icebergs and ice-capped lakes is to blame for water pipes that burst in the winter. Even a water pipe made of steel is not strong enough to keep from rupturing when the ice forming within it expands outward. Later, when the temperature rises above freezing again, the burst pipe will make itself known by springing a leak.



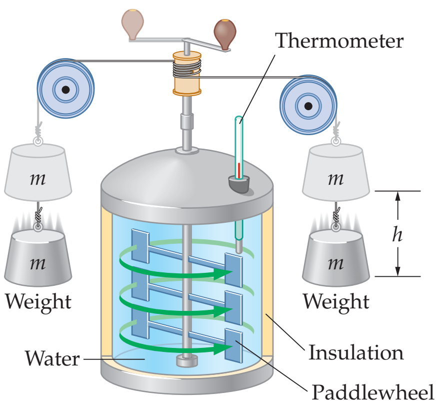
▲ Water exhibits unusual behavior around its freezing point, but above 4 °C it expands with increasing temperature like any normal liquid. This photo shows the expansion of water on a grand scale. It depicts an El Niño event: the appearance of a mass of unusually warm water in the equatorial region of the eastern Pacific Ocean every few years. An El Niño causes worldwide changes in climate and weather patterns. In this satellite image, the warmest water temperatures are represented by red, the coolest by violet. The El Niño is the red spike west of the South American coast. In this region, sea levels are as much as 20 cm higher than their average values, due to thermal expansion of the water.

16–4 Heat and Mechanical Work

In this section we consider the connection between heat and mechanical work. We also discuss the conservation of energy as it regards heat.

As mentioned previously, heat is the energy *transferred* from one object to another. At one time it was thought—erroneously—that an object contained a certain amount of "heat fluid," or caloric, that could flow from one place to another. This idea was overturned by the observations of Benjamin Thompson (1753–1814), also known as Count Rumford, the American-born physicist, spy, and social reformer who at one point in his eclectic career supervised the boring of cannon barrels by large drills. He observed that as long as mechanical work was done to turn the drill bits, they continued to produce heat in unlimited quantities. Clearly, the unlimited heat observed in boring the cannons was not present initially in the metal, but instead was produced by continually turning the drill bit.

With this observation, it became clear that heat was simply another form of energy that must be taken into account when applying conservation of energy. For example, if you rub sandpaper back and forth over a piece of wood, you do work. The energy associated with that work is not lost; instead, it produces an increase in temperature. Taking into account the energy associated with this temperature change, we find that energy is indeed conserved. In fact, no observation has ever indicated a situation in which energy is not conserved.



▲ **FIGURE 16–8 The mechanical equivalent of heat**

A device of this type was used by James Joule to measure the mechanical equivalent of heat.

The equivalence between work and heat was first explored quantitatively by James Prescott Joule (1818–1889), the British physicist. In one of his experiments, Joule observed the increase in temperature in a device similar to that shown in **Figure 16–8**. Here, a total mass $2m$ falls through a certain distance h, during which gravity does the mechanical work, $2mgh$. As the mass falls, it turns the paddles in the water, which results in a slight warming of the water. By measuring the mechanical work, $2mgh$, and the increase in the water's temperature, ΔT, Joule was able to show that energy was indeed conserved—it had been converted from gravitational potential energy to an increased energy of the water, as indicated by a higher temperature. Joule's experiments established the precise amount of mechanical work that has the same effect as a given transfer of heat.

Before Joule's work, heat was measured in a unit called the calorie (cal). In particular, one kilocalorie (kcal) was defined as the amount of heat needed to raise the temperature of 1 kg of water from 14.5 °C to 15.5 °C. With his experiments, Joule was able to show that 1 kcal = 4186 J, or equivalently, that one calorie of heat transfer is the equivalent of 4.186 J of mechanical work. This is referred to as the **mechanical equivalent of heat:**

The Mechanical Equivalent of Heat

$$1 \text{ cal} = 4.186 \text{ J} \qquad 16\text{–}8$$

SI unit: J

In studies of nutrition a different calorie is used. It is the Calorie, with a capital C, and it is simply a kilocalorie; that is, 1 C = 1 kcal. Perhaps this helps people to feel a little better about their calorie intake. After all, a 250-C candy bar sounds a lot better than a 250,000-cal candy bar. They are equivalent, however.

Another common unit for measuring heat is the British thermal unit (Btu). By definition, a Btu is the energy required to heat 1 lb of water from 63 °F to 64 °F. In terms of calories and joules, a Btu is as follows:

$$1 \text{ Btu} = 0.252 \text{ kcal} = 1055 \text{ J} \qquad 16\text{–}9$$

Finally, we shall use the symbol Q to denote heat:

Heat, *Q*

$$Q = \text{heat} = \text{energy transferred due to temperature differences} \qquad 16\text{–}10$$

SI unit: J

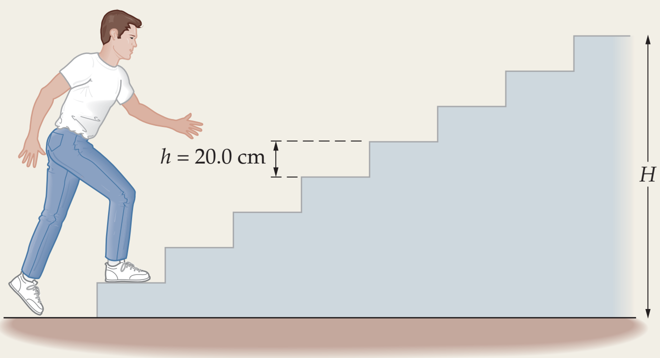
Using the mechanical equivalent of heat as the conversion factor, we will typically give heat in either calories or joules.

EXAMPLE 16–4 STAIR MASTER

A 74.0-kg person drinks a thick, rich, 305-C milkshake. How many stairs must this person climb to work off the shake? Let the height of a stair be 20.0 cm.

PICTURE THE PROBLEM

Our sketch shows the person climbing the stairs after drinking the milkshake. Note that each step has a height $h = 20.0$ cm. In addition, the total height to which the person climbs to work off the milkshake is designated by H. (As we noted in Conceptual Checkpoint 7–1, the horizontal distance is irrelevant here, as it does not affect the work done against gravity.)



STRATEGY

We know that the energy intake by drinking the milkshake is the equivalent of the heat $Q = 305{,}000$ cal. This energy can be converted to joules by using the relation 1 cal = 4.186 J. Finally, we set the energy of the shake equal to the work done against gravity, mgH, in climbing to a height H.

SOLUTION

1. Convert the energy of the milkshake to joules: $Q = 305{,}000 \text{ cal} = 305{,}000 \text{ cal}\left(\dfrac{4.186 \text{ J}}{1 \text{ cal}}\right) = 1.28 \times 10^6 \text{ J}$

2. Equate the energy of the milkshake with the work done against gravity: $Q = mgH$

   Solve for the height H: $H = Q/mg$

3. Substitute numerical values: $H = \dfrac{Q}{mg} = \dfrac{1.28 \times 10^6 \text{ J}}{(74.0 \text{ kg})(9.81 \text{ m/s}^2)} = 1760 \text{ m}$

4. Divide by the height of a stair to get the number of stairs: $\dfrac{1760 \text{ m}}{0.200 \text{ m/stair}} = 8800 \text{ stairs}$

CONTINUED ON NEXT PAGE

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INSIGHT

This is clearly a lot of stairs, and a significant height. In fact, 1760 m is more than a mile. Even assuming a metabolic efficiency of 25%, a height of about a quarter of a mile must be climbed to work off the shake. Put another way, the shake would be enough "fuel" for you to walk to the top of the Empire State Building with a little left over.

PRACTICE PROBLEM

If the person in the problem climbs 100 stairs, how many Calories have been burned? Assume 100% efficiency. [**Answer:** 3.47 C]

Some related homework problems: Problem 27, Problem 28

16–5 Specific Heats

In the previous section, we mentioned that it takes 4186 J of heat to raise the temperature of 1 kg of water by 1 C°. We have to be clear about the fact that we are heating water, however, because the heat required for a 1 C° increase varies considerably from one substance to another. For example, it takes only 128 J of heat to increase the temperature of 1 kg of lead by 1 C°. In general, the heat required for a given increase in temperature is given by the **heat capacity** of a substance.

Heat Capacity

Suppose we add the heat Q to a given object, and its temperature increases by the amount ΔT. The heat capacity, C, of this object is defined as follows:

Definition of Heat Capacity, C

$$C = \frac{Q}{\Delta T} \qquad 16\text{–}11$$

SI unit: J/K = J/C°

Note that the units of heat capacity are joules per kelvin (J/K). Equivalently, since the degree size is the same for the Kelvin and Celsius scales, C can be expressed in units of joules per Celsius degree (J/C°).

The name "heat capacity" is perhaps a bit unfortunate. It derives from the mistaken idea of a "heat fluid," mentioned in the previous section. Objects were imagined to "contain" a certain amount of this nonexistent fluid. Today, we know that an object can readily gain or release heat when it is in thermal contact with other objects—objects cannot be thought of as holding a certain amount of heat.

Instead, the heat capacity should be viewed as the amount of heat necessary for a given temperature change. An object with a large heat capacity, like water, requires a large amount of heat for each increment in temperature. Just the opposite is true for an object with a small heat capacity, like a piece of lead.

To find the heat required for a given ΔT, we simply rearrange Equation 16–11 to solve for Q. This yields

$$Q = C\Delta T \qquad 16\text{–}12$$

It should be noted that the *heat capacity is always positive*—just like a speed. Thus, Equation 16–12 shows that the heat Q and the temperature change ΔT have the same sign. This observation leads to the following sign conventions for Q:

Q is positive if ΔT is positive; that is, if heat is *added* to a system.

Q is negative if ΔT is negative; that is, if heat is *removed* from a system.

EXERCISE 16–3

The heat capacity of 1.00 kg of water is 4186 J/K. What is the temperature change of the water if **(a)** 505 J of heat is added to the system, or **(b)** 1010 J of heat is removed?

SOLUTION

a. Calculate ΔT for $Q = 505$ J:

$$\Delta T = \frac{Q}{C} = \frac{505\text{ J}}{4186\text{ J/K}} = 0.121\text{ K}$$

b. Since heat is removed in this case, $Q = -1010$ J:

$$\Delta T = \frac{Q}{C} = \frac{-1010\text{ J}}{4186\text{ J/K}} = -0.241\text{ K}$$

Specific Heat

Since it takes 4186 J to increase the temperature of one kilogram of water by one degree Celsius, it takes twice that much to make the same temperature change in two kilograms of water, and so on. Thus, the heat capacity varies not only with the type of substance, but also with the mass of the substance.

We can therefore define a new quantity—the **specific heat,** c—that depends only on the substance, and not on its mass, as follows:

Definition of Specific Heat, *c*

$$c = \frac{Q}{m\Delta T} \qquad 16\text{–}13$$

SI unit: $\text{J/(kg}\cdot\text{K)} = \text{J/(Kg}\cdot\text{C}^\circ)$

Thus, for example, the specific heat of water is

$$c_{\text{water}} = 4186\text{ J/(kg}\cdot\text{K)}$$

Specific heats for common substances are listed in Table 16–2. Note that the specific heat of water is by far the largest of any common material. This is just another of the many unusual properties of water. Having such a large specific heat means that water can give off or take in large quantities of heat with little change in temperature. It is for this reason that if you take a bite of a pie that is just out of the oven, you are much more likely to burn your tongue on the fruit filling (which has a high water content) than on the much drier crust.

Water's unusually large specific heat also accounts for the moderate climates experienced in regions near great bodies of water. In particular, the enormous volume and large heat capacity of an ocean serve to maintain a nearly constant temperature in the water, which in turn acts to even out the temperature of adjacent coastal areas. For example, the West Coast of the United States benefits from the moderating effect of the Pacific Ocean, aided by the prevailing breezes that come from the ocean onto the coastal regions. In the Midwest, on the other hand, temperature variations can be considerably greater as the land (with a relatively small specific heat) quickly heats up in the summer and cools off in the winter.

Calorimetry

Let's use the specific heat to solve a practical problem. Suppose a block of mass m_b, specific heat c_b, and initial temperature T_b is dropped into a **calorimeter** (basically, a lightweight, insulated flask) containing water. If the water has a mass m_w, a specific heat c_w, and an initial temperature T_w, find the final temperature of the block and the water. Assume that the calorimeter is light enough that it can be ignored, and that no heat is transferred from the calorimeter to its surroundings.

There are two basic ideas involved in solving this problem: (a) the final temperatures of the block and water are equal, since the system will be in thermal equilibrium; and (b) the total energy of the system is conserved. In particular, the second condition means that the amount of heat lost by the block is equal to the heat gained by the water—or vice versa if the water's initial temperature is higher.

Mathematically, we can write these conditions as follows:

$$Q_b + Q_w = 0 \qquad 16\text{–}14$$

TABLE 16–2 Specific Heats at Atmospheric Pressure

Substance	Specific heat, c [J/(kg · K)]
Water	4186
Ice	2090
Steam	2010
Beryllium	1820
Air	1004
Aluminum	900
Glass	837
Silicon	703
Iron (steel)	448
Copper	387
Silver	234
Gold	129
Lead	128

REAL-WORLD PHYSICS

Water and the climate

▲ These two blocks of metal (aluminum on the left and lead on the right) have equal volumes. In addition, they were heated to equal temperatures before being placed on the block of paraffin wax. Notice, however, that the aluminum melted more wax—and hence gave off more heat—even though the lead block is about 4 times heavier than the aluminum block. The reason is that lead has a very small specific heat; in fact, lead's specific heat is about 7 times smaller than the specific heat of aluminum, as we see in Table 16–2. As a result, even this relatively large mass of lead melts considerably less wax per degree of temperature change than the lightweight aluminum—a direct visual illustration of lead's low specific heat.

PROBLEM-SOLVING NOTE

Heat Flow and Thermal Equilibrium

To find the temperature of thermal equilibrium when two objects with different temperatures are brought into thermal contact, we simply use the idea that the heat that flows *out* of one of the objects flows *into* the other object.

This means that the heat flow from the block is equal and opposite to the heat flow from the water; in other words, energy is conserved. If we write the heats Q in terms of the specific heats and temperatures, letting the final temperature be T, we have

$$m_b c_b (T - T_b) + m_w c_w (T - T_w) = 0$$

Note that for each heat the change in temperature is T_{final} minus $T_{initial}$, as it should be. Solving for the final temperature, T, we find

$$T = \frac{m_b c_b T_b + m_w c_w T_w}{m_b c_b + m_w c_w} \qquad 16\text{–}15$$

This result need not be memorized, of course. Whenever solving a problem of this sort, one simply writes down energy conservation and solves for the desired unknown.

In some cases, we may wish to consider the influence of the container itself. This is illustrated in the following Active Example.

ACTIVE EXAMPLE 16–2 FIND THE FINAL TEMPERATURE

Suppose 550 g of water at 32 °C are poured into a 210-g aluminum can with an initial temperature of 15 °C. Find the final temperature of the system, assuming no heat is exchanged with the surroundings.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write an expression for the heat flow out of the water:	$Q_w = m_w c_w (T - T_w)$
2. Write an expression for the heat flow into the aluminum:	$Q_a = m_a c_a (T - T_a)$
3. Apply energy conservation:	$Q_w + Q_a = 0$
4. Solve for the final temperature:	$T = 31\ °C$

INSIGHT

As one might expect from water's large specific heat, the final common temperature (T) is much closer to the initial temperature of the water (T_w) than that of the aluminum (T_a).

YOUR TURN

Suppose the can is made from iron rather than aluminum. In this case, do you expect the final temperature to be greater than or less than the value for aluminum? Find the final temperature for the case of an iron can.

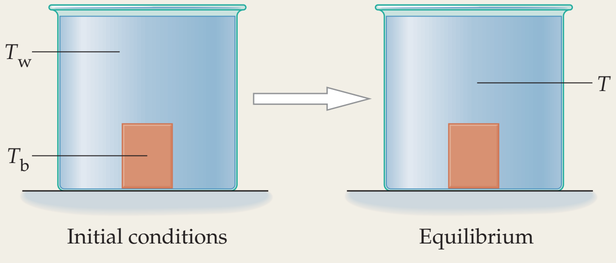
(Answers to **Your Turn** *problems can be found in the back of the book.)*

EXAMPLE 16–5 COOLING OFF

A 0.50-kg block of metal with an initial temperature of 54.5 °C is dropped into a container holding 1.1 kg of water at 20.0 °C. If the final temperature of the block–water system is 21.4 °C, what is the specific heat of the metal? Assume the container can be ignored, and that no heat is exchanged with the surroundings.

PICTURE THE PROBLEM

Initially, when the block is first dropped into the water, the temperatures of the block and water are $T_b = 54.5\ °C$ and $T_w = 20.0\ °C$, respectively. When thermal equilibrium is established, both the block and the water have the same temperature, $T = 21.4\ °C$.



INTERACTIVE FIGURE

STRATEGY

Heat flows from the block to the water. Setting the heat flow *out of the block* plus the heat flow *into the water* equal to zero (conservation of energy) yields the block's specific heat.

SOLUTION

1. Write an expression for the heat flow out of the block. Note that Q_{block} is negative, since T is less than T_b:

$$Q_{\text{block}} = m_b c_b (T - T_b)$$

2. Write an expression for the heat flow into the water. Note that Q_{water} is positive, since T is greater than T_w:

$$Q_{\text{water}} = m_w c_w (T - T_w)$$

3. Set the sum of the heats equal to zero:

$$Q_{\text{block}} + Q_{\text{water}} = m_b c_b (T - T_b) + m_w c_w (T - T_w) = 0$$

4. Solve for the specific heat of the block, c_b:

$$c_b = \frac{m_w c_w (T - T_w)}{m_b (T_b - T)}$$

5. Substitute numerical values:

$$c_b = \frac{(1.1\ \text{kg})[4186\ \text{J/(kg}\cdot\text{K)}](21.4\ ^\circ\text{C} - 20.0\ ^\circ\text{C})}{(0.50\ \text{kg})(54.5\ ^\circ\text{C} - 21.4\ ^\circ\text{C})}$$

$$= 390\ \text{J/(kg}\cdot\text{K)}$$

INSIGHT

We note from Table 16–2 that the block is probably made of copper.

In addition, note that the final temperature is much closer to the initial temperature of the water than to the initial temperature of the block. This is due in part to the fact that the mass of the water is about twice that of the block; more important, however, is the fact that the water's specific heat is more than 10 times greater than that of the block. In the following Practice Problem we set the mass of the water equal to the mass of the block, so that we can see clearly the effect of the different specific heats.

PRACTICE PROBLEM

If the mass of the water is also 0.50 kg, what is the equilibrium temperature? [**Answer:** $T = 23\ ^\circ\text{C}$. Still much closer to the water's initial temperature than to the block's.]

Some related homework problems: Problem 37, Problem 38

16–6 Conduction, Convection, and Radiation

Heat can be exchanged in a variety of ways. The Sun, for example, warms the Earth from across 93 million miles of empty space by a process known as radiation. As the sunlight strikes the ground and raises its temperature, the ground-level air gets warmer and begins to rise, producing a further exchange of heat by means of convection. Finally, if you walk across the ground in bare feet, you will feel the warming effect of heat entering your body by conduction. In this section we consider each of these three mechanisms of heat exchange in detail.

Conduction

Perhaps the most familiar form of heat exchange is **conduction,** which is the flow of heat directly through a physical material. For example, if you hold one end of a metal rod and put the other end in a fire, it doesn't take long before you begin to feel warmth on your end. The heat you feel is transported along the rod by conduction.

Let's consider this observation from a microscopic point of view. To begin, when you placed one end of the rod into the fire, the high temperature at that location caused the molecules to vibrate with an increased amplitude. These molecules in turn jostle their neighbors and cause them to vibrate with greater amplitude as well. Eventually, the effect travels from molecule to molecule across the length of the rod, resulting in the macroscopic phenomenon of conduction.

If you were to repeat the experiment, this time with a wooden rod, the hot end of the rod would heat up so much that it might even catch on fire, but your end would still be comfortably cool. Thus, conduction depends on the type of material involved. Some materials, called **conductors,** conduct heat very well, whereas other materials, called **insulators,** conduct heat poorly.

Just how much heat flows as a result of conduction? To answer this question we consider the simple system shown in **Figure 16–9**. Here we show a rod of

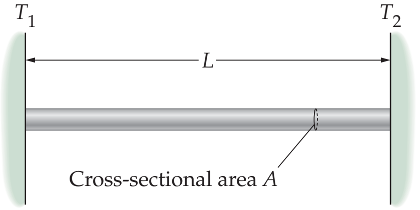


FIGURE 16–9 Heat conduction through a rod
The amount of heat that flows through a rod of length L and cross-sectional area A per time is proportional to $A(T_2 - T_1)L$.

TABLE 16–3 Thermal Conductivities

Substance	Thermal conductivity, k[W/(m · K)]
Silver	417
Copper	395
Gold	291
Aluminum	217
Steel, low carbon	66.9
Lead	34.3
Stainless steel—alloy 302	16.3
Ice	1.6
Concrete	1.3
Glass	0.84
Water	0.60
Asbestos	0.25
Wood	0.10
Wool	0.040
Air	0.0234

▲ Maintaining proper body temperature in an environment that is often too hot or too cold is a problem for many animals. When the sand is blazing hot, this lizard (left) keeps its contact with the ground to a minimum. By standing on two legs instead of four, it reduces conduction of heat from the ground to its body. Polar bears (right) have the opposite problem. The loss of precious body heat to their surroundings is retarded by their thick fur, which is actually made up of hollow fibers. Air trapped within these fibers provides enhanced insulation, just as it does in our thermal blankets and double-paned windows.

length L and cross-sectional area A, with one end at the temperature T_1 and the other at the temperature $T_2 > T_1$. Experiments show that the amount of heat Q that flows through this rod:

- increases in proportion to the rod's cross-sectional area, A;
- increases in proportion to the temperature difference, $\Delta T = T_2 - T_1$;
- increases steadily with time, t;
- decreases with the length of the rod, L.

Combining these observations in a mathematical expression gives:

Heat Flow by Conduction

$$Q = kA\left(\frac{\Delta T}{L}\right)t \qquad 16\text{–}16$$

The constant k is referred to as the **thermal conductivity** of the rod. It varies from material to material, as indicated in Table 16–3.

PROBLEM-SOLVING NOTE

Area and Length in Heat Conduction

When applying Equation 16–16, note that A is the area through which the heat flows. Thus, the plane of the area A is perpendicular to the direction of heat flow. The length L is the distance from the high-temperature side of an object to its low-temperature side.

CONCEPTUAL CHECKPOINT 16–3 THE FEEL OF TILE

You get up in the morning and walk barefoot from the bedroom to the bathroom. In the bedroom you walk on carpet, but in the bathroom the floor is tile. Does the tile feel **(a)** warmer, **(b)** cooler, or **(c)** the same temperature as the carpet?

REASONING AND DISCUSSION

Everything in the house is at the same temperature, so it might seem that the carpet and the tile would feel the same. As you probably know from experience, however, the tile feels cooler. The reason is that tile has a much larger thermal conductivity than the carpet, which is actually a fairly good insulator. As a result, more heat flows from your skin to the tile than from your skin to the carpet. To your feet, then, it is as if the tile were much cooler than the carpet.

To get an idea of the thermal conductivities that would apply in this case, let's examine Table 16–3. For the tile, we might expect a thermal conductivity of roughly 0.84, the value appropriate for glass. For the carpet, the thermal conductivity might be as low as 0.04, the thermal conductivity of wool. Thus, the tile could have a thermal conductivity that is 20 times larger than that of the carpet.

ANSWER

(b) The tile feels cooler.

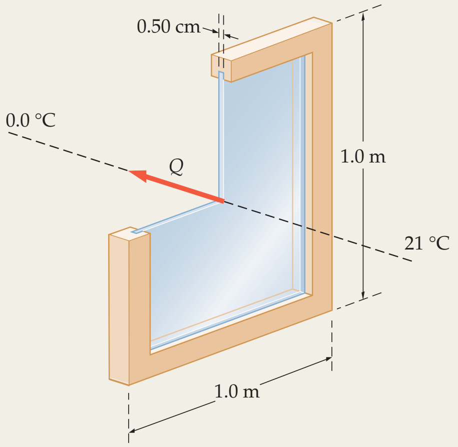
Thermal conductivity is an important consideration when insulating a home. We consider some of these issues in the next Example.

EXAMPLE 16–6 WHAT A PANE!

One of the windows in a house has the shape of a square 1.0 m on a side. The glass in the window is 0.50 cm thick. **(a)** How much heat is lost through this window in one day if the temperature in the house is 21 °C and the temperature outside is 0.0 °C? **(b)** Suppose all the dimensions of the window—height, width, thickness—are doubled. If everything else remains the same, by what factor does the heat flow change?

PICTURE THE PROBLEM

The glass from the window is shown in our sketch, along with its relevant dimensions. Heat flows from the 21 °C side of the window to the 0.0 °C side.



STRATEGY

a. The heat flow is given by $Q = kA(\Delta T/L)t$ (Equation 16–16). Note that the area is $A = (1.0\text{ m})^2$ and that the length over which heat is conducted is, in this case, the thickness of the glass. Thus, $L = 0.0050$ m. The temperature difference is $\Delta T = 21\text{ C}° = 21$ K, and the thermal conductivity of glass (from Table 16–3) is $0.84\text{ W/(m}\cdot\text{K)}$. Also, recall from Section 7–4 that 1 W = 1 J/s.

b. Doubling all dimensions increases the thickness by a factor of 2 and increases the area by a factor of 4; that is, $L \rightarrow 2L$ and $A \rightarrow (2 \times \text{height}) \times (2 \times \text{width}) = 4A$. Use these results in $Q = kA(\Delta T/L)t$.

SOLUTION

Part (a)

1. Calculate the heat flow for a given time, t:

$$Q = kA\left(\frac{\Delta T}{L}\right)t$$

$$= [0.84\text{ W/(m}\cdot\text{K)}](1.0\text{ m})^2\left(\frac{21\text{ K}}{0.0050\text{ m}}\right)t = (3500\text{ W})t$$

2. Substitute the number of seconds in a day, 86,400 s, for the time t in the expression for Q:

$$Q = (3500\text{ W})t = (3500\text{ W})(86{,}400\text{ s}) = 3.0 \times 10^8\text{ J}$$

Part (b)

3. Replace L with $2L$ and A with $4A$ in Step 1. The result is a doubling of the heat flow, Q:

$$Q = kA\left(\frac{\Delta T}{L}\right)t \rightarrow k(4A)\left[\frac{\Delta T}{(2L)}\right]t \rightarrow 2\left[kA\left(\frac{\Delta T}{L}\right)t\right] = 2Q$$

INSIGHT

Q is a sizable amount of heat, roughly equivalent to the energy released in burning a gallon of gasoline. A considerable reduction in heat loss can be obtained by using a double-paned window, which has an insulating layer of air (actually argon or krypton) sandwiched between the two panes of glass. This is discussed in more detail later in this section, and is explored in Homework Problems 53 and 91.

PRACTICE PROBLEM

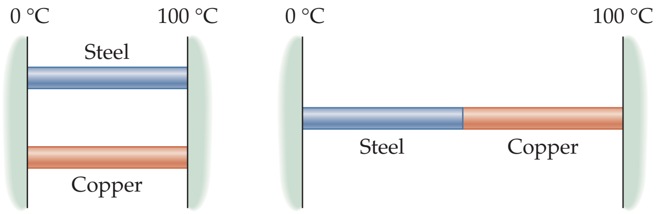
Suppose the window is replaced with a plate of solid silver. How thick must this plate be to have the same heat flow in a day as the glass? [**Answer:** The silver must have a thickness of $L = 2.5$ m.]

Some related homework problems: Problem 49, Problem 50

Now we consider the heat flow through a combination of two different materials with different thermal conductivities.

CONCEPTUAL CHECKPOINT 16–4 COMPARE THE HEAT FLOW

Two metal rods are to be used to conduct heat from a region at 100 °C to a region at 0 °C. The rods can be placed in parallel, as shown on the left, or in series, as on the right. Is the heat conducted in the parallel arrangement **(a)** greater than, **(b)** less than, or **(c)** the same as the heat conducted with the rods in series?



REASONING AND DISCUSSION

The parallel arrangement conducts more heat for two reasons. First, the cross-sectional area available for heat flow is twice as large for the parallel rods. A greater cross-sectional area gives a greater heat flow—everything else being equal. Second, more heat flows through each rod in the parallel configuration because they both have the full temperature difference of 100 C° between their ends. In the series configuration, each rod has a smaller temperature difference between its ends, so less heat flows.

ANSWER

(a) More heat is conducted when the rods are in parallel.

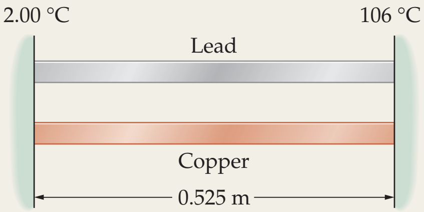
In the next Example, we consider the case of heat conduction when two rods are placed in parallel. In the homework we shall consider the corresponding case of the same two rods conducting heat in series.

EXAMPLE 16–7 PARALLEL RODS

Two 0.525-m rods, one lead the other copper, are connected between metal plates held at 2.00 °C and 106 °C. The rods have a square cross section, 1.50 cm on a side. How much heat flows through the two rods in 1.00 s? Assume that no heat is exchanged between the rods and the surroundings, except at the ends.

PICTURE THE PROBLEM

The lead and the copper rods, each 0.525 m long, are shown in the sketch. Note that both rods have a temperature difference of 104 C° = 104 K between their ends.



STRATEGY

The heat flowing through each rod can be calculated using $Q = kA(\Delta T/L)t$ and the value of k given in Table 16–3. The total heat flow is simply the sum of that calculated for each rod.

SOLUTION

1. Calculate the heat flow in one second through the lead rod:

$$Q_l = k_l A\left(\frac{\Delta T}{L}\right)t$$

$$= [34.3\ \mathrm{W/(m\cdot K)}](0.0150\ \mathrm{m})^2\left(\frac{104\ \mathrm{K}}{0.525\ \mathrm{m}}\right)(1.00\ \mathrm{s})$$

$$= 1.53\ \mathrm{J}$$

2. Calculate the heat flow in one second through the copper rod:

$$Q_c = k_c A\left(\frac{\Delta T}{L}\right)t$$

$$= [395\ \mathrm{W/(m\cdot K)}](0.0150\ \mathrm{m})^2\left(\frac{104\ \mathrm{K}}{0.525\ \mathrm{m}}\right)(1.00\ \mathrm{s})$$

$$= 17.6\ \mathrm{J}$$

3. Sum the heats found in Steps 1 and 2 to get the total heat:

$$Q_{\text{total}} = Q_l + Q_c = 1.53\ \mathrm{J} + 17.6\ \mathrm{J} = 19.1\ \mathrm{J}$$

INSIGHT

As our results show, the copper rod is by far the better conductor of heat. It is also a very good conductor of electricity, as we shall see in Chapter 21.

PRACTICE PROBLEM

What temperature difference would be required for the total heat flow in one second to be 15.0 J?
[**Answer:** ΔT = 81.5 C° = 81.5 K]

Some related homework problems: Problem 54, Problem 55

As we shall see in Homework Problem 56, the heat conducted by the same two rods connected in series is only 1.41 J, which is less than either of the two rods conducts when placed in parallel. This verifies the conclusion stated in Conceptual Checkpoint 16–4.

REAL-WORLD PHYSICS
Insulated windows

An application of thermal conductivity in series can be found in the *insulated window*. Most homes today have insulated windows as a means of increasing their energy efficiency. If you look closely at one of these windows, you will see that it is actually constructed from two panes of glass separated by an air-filled gap. Thus, heat flows through three different materials in series as it passes into or out of a home. The fact that the thermal conductivity of air is about 40 times smaller

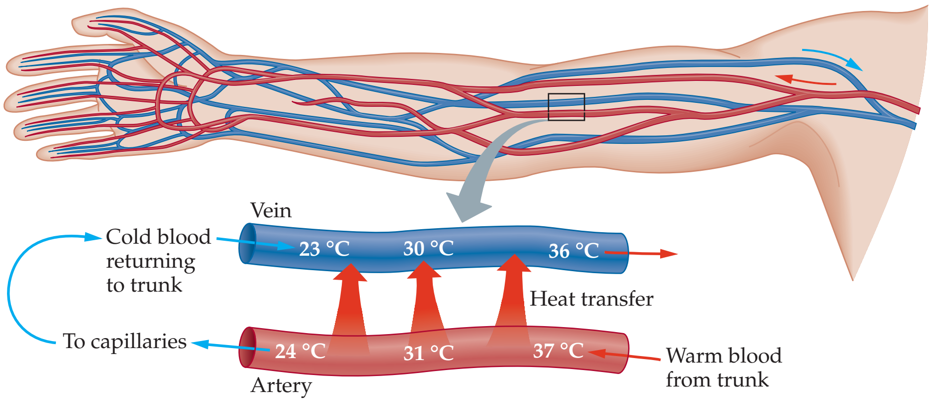


FIGURE 16–10 Countercurrent heat exchange in the human arm
Arteries bringing warm blood to the limbs lie close to veins returning cooler blood to the body. This arrangement assures that a temperature difference (gradient) is maintained over the entire length that the vessels run parallel to one another, maximizing heat exchange between the warm arterial blood and the cooler venous blood.

than that of glass means that the insulated window results in significantly less heat flow than would be experienced with a single pane of glass.

REAL-WORLD PHYSICS: BIO
Countercurrent exchange

As a final example of conduction, we note that many biological systems transfer heat by a mechanism known as *countercurrent exchange*. Consider, for example, an egret or other wading bird standing in cool water all day. As warm blood flows through the arteries on its way to the legs and feet of the bird, it passes through constricted regions where the legs join to the body. Here, where the arteries and veins are packed closely together, the body-temperature arterial blood flowing into the legs transfers heat to the much cooler venous blood returning to the body. Thus, the counter-flowing streams of blood serve to maintain the core body temperature of the bird, while at the same time keeping the legs and feet at much cooler temperatures. The feet still receive the oxygen and nutrients carried by the blood, but they stay at a relatively low temperature to reduce the amount of heat lost to the water.

REAL-WORLD PHYSICS: BIO
Cold hands and feet

Similar effects occur in humans. It is common, for example, to hear complaints that a person's hands or feet are cold. There is good reason for this, since they are in fact much cooler than the core body temperature. Just as with the wading birds, the warm arterial blood flowing to the hands and feet exchanges heat with the cool venous blood flowing in the opposite direction (**Figure 16–10**). This helps to reduce the heat loss to our surroundings, and to maintain the desired temperature in the core of the body.

Many wading birds use countercurrent exchange in their circulatory systems. This mechanism allows them to keep the temperature of their legs well below that of their bodies. In this way they reduce the conductive loss of body heat to the water.

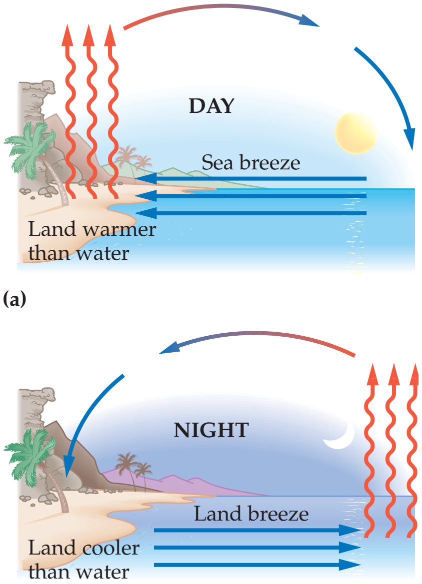
Convection

Suppose you want to heat a small room. To do so, you bring a portable electric heater into the room and turn it on. As the heating coils get red hot, they heat the air in their vicinity; as this air warms, it expands, becoming less dense. Because of its lower density, the warm air rises, to be replaced by cold dense air descending from overhead. This sets up a circulating flow of air that transports heat from the heating coils to the air throughout the room. Heat exchange of this type is referred to as **convection.**

In general, convection occurs when a fluid is unevenly heated. As with the room heater, the warm portions of the fluid rise because of their lower density and the cool portions sink because of their higher density. Thus, in convection, temperature differences result in a flow of fluid. It is this physical flow of matter that carries heat throughout the system.

Convection occurs on an enormous range of length scales. For example, the same type of uneven heating produced by an electric heater in a room can occur in the atmosphere of the Earth as well. The common seashore occurrence of sea breezes during the day and land breezes in the evening is one such example. (See **Figure 16–11** for an illustration of this effect.) On a larger scale, the Sun causes

When you light a candle, it heats the air near the flame as it burns. Because hot air is less dense—and hence more buoyant—than cool air, a circulation pattern is established with hot air rising and being replaced from below by cool, oxygenated air. Thus, convection is necessary for a candle to continue burning. When the burning candle in this jar is dropped, it suddenly finds itself in free fall—an essentially "weightless" environment where buoyancy has no effect. As a result, convection ceases and the flame is quickly extinguished as it consumes all the oxygen in its immediate vicinity.



(a)

(b)

FIGURE 16–11 Alternating land and sea breezes

(a) During the day, the sun warms the land more rapidly than the water. This is because the land, which is mostly rocks, has a lower specific heat than the water. The warm land heats the air above it, which becomes less dense and rises. Cooler air from over the water flows in to take its place, producing a "sea breeze." **(b)** At night, the land cools off more rapidly than the water—again because of its lower specific heat. Now it is the air above the relatively warm water that rises and is replaced by cooler air from over the land, producing a "land breeze."

greater warming near the equator than near the poles; as a result, warm equatorial air rises, cool polar air descends, and global convection patterns are established. Similar convection patterns occur in ocean waters, and plate tectonics is believed to be caused, at least in part, by convection currents in the Earth's mantle. The Sun also has convection currents, due to the intense heating that occurs in its interior, and disturbances in these currents are often visible as sunspots.

Radiation

Though convection and conduction occur primarily in specific situations, *all* objects give off energy as a result of **radiation.** The energy radiated by an object is in the form of electromagnetic waves (Chapter 25), which include visible light as well as infrared and ultraviolet radiation. Thus, unlike convection and conduction, radiation has no need for a physical material to mediate the energy transfer, since electromagnetic waves can propagate through empty space—that is, through a vacuum. Therefore, the heat you feel radiated from a hot furnace would reach you even if the air were suddenly removed—just as radiant energy from the Sun reaches the Earth across 150 million kilometers of vacuum.

REAL-WORLD PHYSICS
Using color to measure temperature

REAL-WORLD PHYSICS
Temperatures of the stars

Since radiation can include visible light, it is often possible to "see" the temperature of an object. This is the physical basis of the **optical pyrometer,** invented by Josiah Wedgwood (1730–1795), the renowned English potter. When objects are about 800 °C they appear to be a dull "red hot." Examples include the heating coils in a range or oven. The filament in an incandescent lightbulb glows "yellow hot" at about 3000 °C, about the same temperature as the surface of the red supergiant star Betelgeuse in the constellation Orion. The surface of the Sun, in comparison, is about 6000 °C. Very hot stars, with surface temperatures in the vicinity of 10,000 to 30,000 °C, are "blue hot" and actually appear bluish in the night sky. Rigel, also in the constellation Orion, is an example of such a star. Look above and to the left of Orion's "belt" to see the red Betelgeuse, and below and to the right to see the blue Rigel.

The energy radiated per time by an object—that is, the radiated power, P—is proportional to the surface area, A, over which the radiation occurs. It also depends on the temperature of the object. In fact, the dependence is on the fourth power of

Stars have colors that are indicative of their surface temperatures. Thus, the blue supergiant Rigel (lower right) in the constellation Orion is much hotter than the red supergiant Betelgeuse (upper left).

Red-hot volcanic lava is just hot enough (about 1000 °C) to radiate in the visible range. Even when it cools enough to stop glowing, it still emits energy, but most of it is in the form of invisible infrared radiation. Infrared radiation is also given off by the finned "radiators" attached to the supports of the Alaska pipeline. These fins are designed to prevent melting of the environmentally sensitive permafrost over which the pipeline runs. They function much like the radiator that cools your car engine, absorbing heat from the warm oil in the pipeline and dissipating it by radiation and convection into the atmosphere. (What design features can you see that facilitate this function?)

the temperature, T^4, where T is the Kelvin-scale temperature. Thus, for instance, if T is doubled, the radiated power increases by a factor of 16. All this behavior is contained in the **Stefan–Boltzmann law:**

Stefan–Boltzmann Law for Radiated Power, *P*

$$P = e\sigma AT^4 \qquad 16\text{–}17$$

SI unit: W

The constant σ in this expression is a fundamental physical constant, the **Stefan–Boltzmann constant:**

$$\sigma = 5.67 \times 10^{-8}\ \mathrm{W/(m^2 \cdot K^4)} \qquad 16\text{–}18$$

The other constant in the Stefan–Boltzmann law is the **emissivity,** e. The emissivity is a dimensionless number between 0 and 1 that indicates how effective the object is in radiating energy. A value of 1 means that the object is a perfect radiator. In general, a dark-colored object will have an emissivity near 1, and a light-colored object will have an emissivity closer to 0.

▲ Wear black in the desert? It actually helps, by radiating heat away from the wearer more efficiently.

EXERCISE 16–4

Calculate the radiated power from a sphere with a radius of 5.00 cm at the temperature 355 K. Assume the emissivity is unity.

SOLUTION

Using $A = 4\pi r^2$ for a sphere, we have

$$P = e\sigma AT^4 = (1)[5.67 \times 10^{-8}\ \mathrm{W/(m^2 \cdot K^4)}]4\pi(0.0500\ \mathrm{m})^2(355\ \mathrm{K})^4 = 28.3\ \mathrm{W}$$

Experiments show that objects absorb radiation from their surroundings according to the same law, the Stefan–Boltzmann law, by which they emit radiation. Thus, if the temperature of an object is T, and its surroundings are at the temperature T_s, the *net* power radiated by the object is

Net Radiated Power, P_{net}

$$P_{\text{net}} = e\sigma A(T^4 - T_s^{\,4}) \qquad 16\text{–}19$$

SI unit: W

If the object's temperature is greater than its surroundings, it radiates more energy than it absorbs and P_{net} is positive. On the other hand, if its temperature is less than the surroundings, it absorbs more energy than it radiates and P_{net} is negative. When the object has the same temperature as its surroundings, it is in equilibrium and the net power is zero.

PROBLEM-SOLVING NOTE

Radiated Power

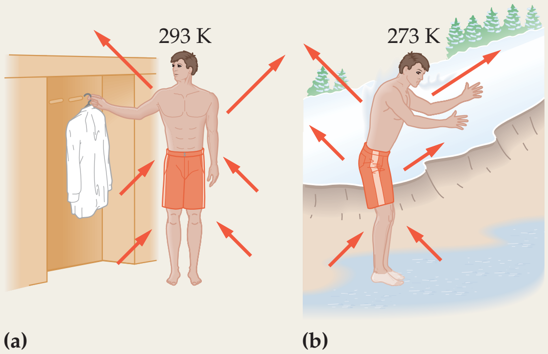
To correctly calculate the radiated power, Equations 16–17 and 16–19, the temperatures must be expressed in the Kelvin scale.

EXAMPLE 16–8 HUMAN POLAR BEARS

On New Year's Day, several human "polar bears" prepare for their annual dip into the icy waters of Narragansett Bay. One of these hardy souls has a surface area of 1.15 m^2 and a surface temperature of 303 K (~30 °C). Find the net radiated power from this person **(a)** in a dressing room, where the temperature is 293 K (~20 °C), and **(b)** outside, where the temperature is 273 K (~0 °C). Assume an emissivity of 0.900 for the person's skin.

PICTURE THE PROBLEM

Our sketch shows the person radiating power in a room where the surroundings are at 293 K, and outside where the temperature is 273 K. The person also absorbs radiation from the surroundings; hence the net radiated power is greater when the surroundings are cooler.



STRATEGY

A straightforward application of $P_{\text{net}} = e\sigma A(T^4 - T_s^{\,4})$ can be used to find the net power for both parts (a) and (b). The only difference is the temperature of the surroundings, T_s.

CONTINUED ON NEXT PAGE

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SOLUTION

Part (a)

1. Calculate the net power using Equation 16–19 and $T_s = 293$ K:

$$P_{net} = e\sigma A(T^4 - T_s^4) = (0.900)[5.67 \times 10^{-8}\ \mathrm{W/(m^2 \cdot K^4)}](1.15\ \mathrm{m^2}) \times [(303\ \mathrm{K})^4 - (293\ \mathrm{K})^4] = 62.1\ \mathrm{W}$$

Part (b)

2. Calculate the net power using Equation 16–19 and $T_s = 273$ K:

$$P_{net} = e\sigma A(T^4 - T_s^4) = (0.900)[5.67 \times 10^{-8}\ \mathrm{W/(m^2 \cdot K^4)}](1.15\ \mathrm{m^2}) \times [(303\ \mathrm{K})^4 - (273\ \mathrm{K})^4] = 169\ \mathrm{W}$$

INSIGHT

In the warm room the net radiated power is roughly that of a small lightbulb (about 60 W); outdoors, the net radiated power has more than doubled, and is comparable to that of a 150-W lightbulb.

PRACTICE PROBLEM

What temperature must the surroundings have if the net radiated power is to be 155 W? [**Answer:** $T = 276\ \mathrm{K} \approx 3\ °\mathrm{C}$]

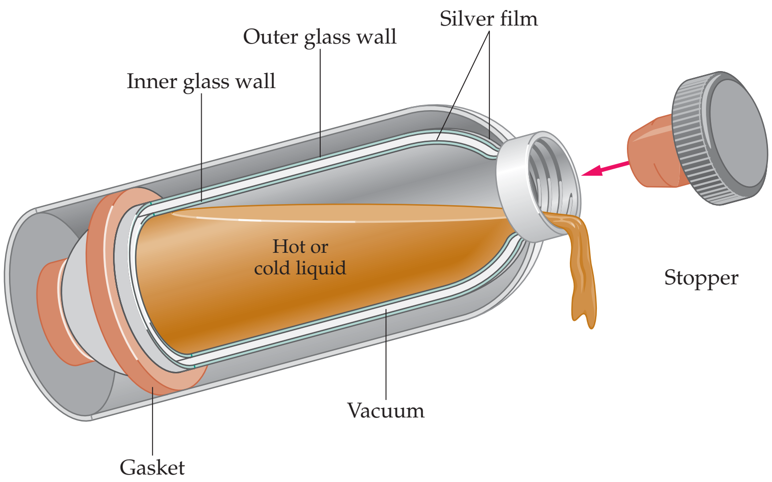
Some related homework problems: Problem 51, Problem 59

REAL-WORLD PHYSICS

Thermos bottles and the Dewar

Note that the same emissivity e applies to both the emission and absorption of energy. Thus, a perfect emitter ($e = 1$) is also a perfect absorber. Such an object is referred to as a **blackbody.** As we shall see later, in Chapter 30, the study of blackbody radiation near the turn of the twentieth century ultimately led to one of the most fundamental revolutions in science—the introduction of quantum physics.

The opposite of a blackbody is an ideal reflector, which absorbs *no* radiation ($e = 0$). It follows that an ideal reflector also radiates no energy. This is why the inside of a Thermos bottle is highly reflective. As an almost ideal reflector, the inside of the bottle radiates very little of the energy contained in the hot liquid that it holds. In addition to its shiny interior, a Thermos bottle also has a vacuum between its inner and outer walls, as shown in **Figure 16–12**. This limits the flow of heat to radiation only, since convection and conduction cannot occur in a vacuum. This type of double-walled insulating container was invented by Sir James Dewar (1842–1923), a Scottish physicist and chemist.



▲ **FIGURE 16–12 The Thermos bottle**

The hot or cold liquid stored in a Thermos bottle is separated from the outside world by a vacuum between the inner and outer glass walls. In addition, the inner glass wall has a reflective coating so that it is a good reflector and a poor radiator.

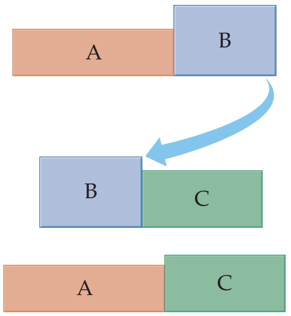
THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The concept of work (Chapter 7) plays a key role in this chapter, and particularly in Section 16–4 where we relate mechanical work to heat. Energy and energy conservation (Chapter 8) are important in our discussion of specific heats in Section 16–5. We extend the concept of the flux of a fluid (Section 15–6) to heat flux in Section 16–6, where we consider the amount of heat that flows across an area in a given time. We also use the concept of power (Chapter 7) in talking about the amount of energy radiated by an object in a given amount of time.	We make extensive use of the concept of temperature throughout Chapters 17 and 18. Heat plays an important role in Chapter 18, where we consider the heat and work exchanged during different types of thermal processes. The flux of a fluid flowing through an area, or the flux of heat transferred between two objects in thermal contact, is a useful analogy in understanding the flux of electric fields in Section 19–7 and magnetic fields in Section 23–2.

CHAPTER SUMMARY

16–1 TEMPERATURE AND THE ZEROTH LAW OF THERMODYNAMICS

This section defines several new terms dealing with heat and temperature.



Heat
Heat is the energy transferred between objects because of a temperature difference.

Thermal Contact
Objects are in thermal contact if heat can flow between them.

Thermal Equilibrium
Objects that are in thermal contact, but have no heat exchange between them, are said to be in thermal equilibrium.

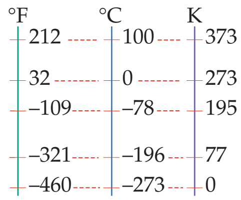
Thermodynamics
The study of physical processes involving the transfer of heat.

Zeroth Law of Thermodynamics
If objects A and C are in thermal equilibrium with object B, they are in thermal equilibrium with each other.

Temperature
Temperature is the quantity that determines whether or not two objects will be in thermal equilibrium.

16–2 TEMPERATURE SCALES

Temperature is commonly measured in terms of several different scales.



Celsius Scale
In the Celsius scale, water freezes at 0 °C and boils at 100 °C.

Fahrenheit Scale
In the Fahrenheit scale, water freezes at 32 °F and boils at 212 °F.

Absolute Zero
The lowest temperature attainable is referred to as absolute zero. It is impossible to cool an object to a temperature lower than absolute zero, which is −273.15 °C.

Kelvin Scale
In the Kelvin scale, absolute zero is 0 K. In addition, water freezes at 273.15 K and boils at 373.15 K. The degree size is the same for the Kelvin and Celsius scales.

Conversion Relations
The following relations convert between Celsius temperatures, T_C, Fahrenheit temperatures, T_F, and Kelvin temperatures, T:

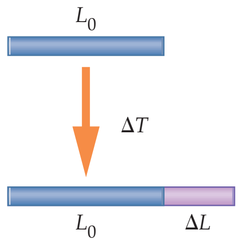
$$T_F = \tfrac{9}{5}T_C + 32 \qquad 16\text{–}1$$

$$T_C = \tfrac{5}{9}(T_F - 32) \qquad 16\text{–}2$$

$$T = T_C + 273.15 \qquad 16\text{–}3$$

16–3 THERMAL EXPANSION

Most, though not all, substances expand when heated.



Linear Expansion

When an object of length L_0 is heated by the amount ΔT, its length increases by ΔL:

$$\Delta L = \alpha L_0 \Delta T \quad 16\text{–}4$$

The constant α is the coefficient of linear expansion (Table 16–1).

Volume Expansion

When an object of volume V is heated by the amount ΔT, its volume increases by ΔV:

$$\Delta V = \beta V \Delta T \quad 16\text{–}6$$

The constant β is the coefficient of volume expansion (Table 16–1).

Relation between α and β

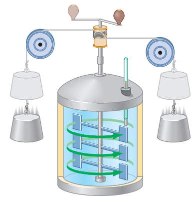
If β is not listed for a particular substance, but α is listed, the volume expansion can be calculated using

$$\beta = 3\alpha$$

Special Properties of Water

Water is unusual in that it contracts when heated from 0 °C to 4 °C.

16–4 HEAT AND MECHANICAL WORK



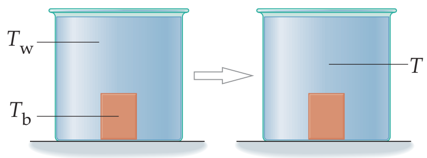
An important step forward in the understanding of heat was the recognition that it is a form of energy.

Mechanical Equivalent of Heat

$$1 \text{ cal} = 4.186 \text{ J} \quad 16\text{–}8$$

16–5 SPECIFIC HEATS

Heat Capacity



The heat capacity of an object is the heat Q divided by the associated temperature change, ΔT:

$$C = \frac{Q}{\Delta T} \quad 16\text{–}11$$

Specific Heat

The specific heat is the heat capacity per unit mass. Thus, the specific heat is independent of the quantity of a material in a given object:

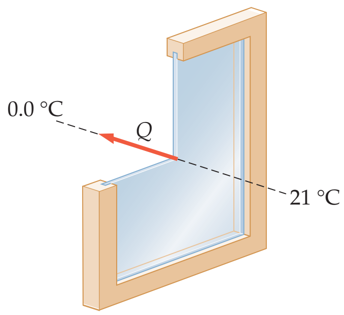
$$c = \frac{Q}{m\Delta T} \quad 16\text{–}13$$

Conservation of Energy

In a system in which no heat is exchanged with the surroundings, the heat that flows out of one object equals the heat that flows into a second object. This is energy conservation.

16–6 CONDUCTION, CONVECTION, AND RADIATION

This section considers three common mechanisms of heat exchange.



Conduction

In conduction, heat flows through a material with no bulk motion. The heat flows as a result of the interactions of individual atoms with their neighbors.

If the thermal conductivity of a material is k, its cross-sectional area is A, and its length L, the heat exchanged in the time t is

$$Q = kA\left(\frac{\Delta T}{L}\right)t \quad 16\text{–}16$$

Convection

Convection is heat exchange due to the bulk motion of an unevenly heated fluid.

Radiation

Radiation is the heat exchange due to electromagnetic radiation, such as infrared rays and light.

The energy per time, or power P, radiated by an object with a surface area A at the Kelvin temperature T is

$$P = e\sigma AT^4 \quad 16\text{–}17$$

where e is the emissivity (a constant between 0 and 1) and σ is the Stefan–Boltzmann constant, $\sigma = 5.67 \times 10^{-8}\ \text{W}/(\text{m}^2 \cdot \text{K}^4)$.

Since an object will also absorb radiation from its surroundings at the temperature T_s, the net radiated power is

$$P_{\text{net}} = e\sigma A(T^4 - T_s^{\,4}) \quad 16\text{–}19$$

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Calculate the change in length of an object due to a change in temperature.	The change in an object's length is proportional to its initial length, L_0, and to the change in temperature, ΔT. In particular, $\Delta L = \alpha L_0 \Delta T$, where α is the coefficient of thermal expansion.	Exercise 16–2
Determine the change in volume of an object.	If Table 16–1 gives a value of β for the substance in question, apply Equation 16–6. If a value of α is given instead, let $\beta = 3\alpha$ and use Equation 16–6.	Example 16–3
Relate the temperature change of an object to the amount of heat it absorbs or releases.	The heat capacity of an object determines the amount of heat, Q, it can absorb or give off for a given change in temperature, ΔT. The relationship is $Q = C\Delta T$.	Example 16–5, Active Example 16–2
Find the heat flow as a result of conduction.	Heat flow through a material is proportional to the temperature difference, the area through which the heat flows, and the time of flow; it is inversely proportional to the length over which the heat flows. Specifically, $Q = kA\Delta Tt/L$, where k is the thermal conductivity.	Examples 16–6, 16–7
Calculate the power given off by radiation.	The power, P, radiated by an object is proportional to its area, A, and to the fourth power of its temperature, T. The complete expression is $P = e\sigma AT^4$, where e is the emissivity and σ is the Stefan–Boltzmann constant.	Example 16–8

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. A cup of hot coffee is placed on the table. Is it in thermal equilibrium? What condition determines when the coffee is in equilibrium?

2. We know that −40 °F is the same as −40 °C. Is there a temperature for which the Kelvin and Celsius scales agree? Explain.

3. To find the temperature at the core of the Sun, you consult some Web sites on the Internet. One site says the temperature is about 15 million °C, another says it is 15 million kelvin. Is this a serious discrepancy? Explain.

4. Is it valid to say that a hot object contains more heat than a cold object?

5. If the glass in a glass thermometer had the same coefficient of volume expansion as mercury, the thermometer would not be very useful. Explain.

6. Suppose the glass in a glass thermometer expands more with temperature than the mercury it holds. What would happen to the mercury level as the temperature increased?

7. When a mercury-in-glass thermometer is inserted into a hot liquid the mercury column first drops and then rises. Explain this behavior.

8. Sometimes the metal lid on a glass jar has been screwed on so tightly that it is very difficult to open. Explain why holding the lid under hot running water often loosens it enough for easy opening.

9. Why do you hear creaking and groaning sounds in a house, particularly at night as the air temperature drops?

10. Two different objects receive different amounts of heat but experience the same increase in temperature. Give at least two possible reasons for this behavior.

11. Two different objects receive the same amount of heat. Give at least two reasons why their temperature changes may not be the same.

12. The specific heat of concrete is greater than that of soil. Given this fact, would you expect a major-league baseball field or the parking lot that surrounds it to cool off more in the evening following a sunny day?

13. Extending the result of the previous question to a larger scale, would you expect daytime winds to generally blow from a city to the surrounding suburbs or from the suburbs to the city? Explain.

14. When you touch a piece of metal and a piece of wood that are both at room temperature the metal feels cooler. Why?

15. After lighting a wooden match, you can hold onto the end of it for some time, until the flame almost reaches your fingers. Why aren't you burned as soon as the match is lit?

16. The rate of heat flow through a slab does *not* depend on which of the following? **(a)** The temperature difference between opposite faces of the slab. **(b)** The thermal conductivity of the slab. **(c)** The thickness of the slab. **(d)** The cross-sectional area of the slab. **(e)** The specific heat of the slab.

17. If a lighted match is held beneath a balloon inflated with air, the balloon quickly bursts. If, instead, the lighted match is held beneath a balloon filled with water, the balloon remains intact, even if the flame comes in contact with the balloon. Explain.

18. Updrafts of air allow hawks and eagles to glide effortlessly, all the while gaining altitude. What causes the updrafts?

19. When penguins huddle together during an Antarctic storm, they are warmer than if they are well separated. Explain.

20. **BIO** The fur of polar bears consists of hollow fibers. (Sometimes algae will grow in the hollow regions, giving the fur a green cast.) Explain why hollow hairs can be beneficial to the polar bears.

21. Object 2 has twice the emissivity of object 1, though they have the same size and shape. If the two objects radiate the same power, what is the ratio of their Kelvin temperatures?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 16–2 TEMPERATURE SCALES

1. • **Lowest Temperature on Earth** The official record for the lowest temperature ever recorded on Earth was set at Vostok, Antarctica, on July 21, 1983. The temperature on that day fell to −89.2 °C, well below the temperature of dry ice. What is this temperature in degrees Fahrenheit?

2. • More than likely, there is a glowing incandescent lightbulb in your room at this moment. The filament of that bulb, with a temperature of about 4500 °F, is almost half as hot as the surface of the Sun. What is this temperature in degrees Celsius?

3. • Normal body temperature for humans is 98.6 °F. What is the corresponding temperature in **(a)** degrees Celsius and **(b)** kelvins?

4. • What is the temperature 1.0 K on the Fahrenheit scale?

5. • The temperature at the surface of the Sun is about 6000 K. Convert this temperature to the **(a)** Celsius and **(b)** Fahrenheit scales.

6. • One day you notice that the outside temperature increased by 27 F° between your early morning jog and your lunch at noon. What is the corresponding change in temperature in the **(a)** Celsius and **(b)** Kelvin scales?

7. • The gas in a constant-volume gas thermometer has a pressure of 93.5 kPa at 105 °C. **(a)** What is the pressure of the gas at 50.0 °C? **(b)** At what temperature does the gas have a pressure of 115 kPa?

8. • **IP** A constant-volume gas thermometer has a pressure of 80.3 kPa at −10.0 °C and a pressure of 86.4 kPa at 10.0 °C. **(a)** At what temperature does the pressure of this system extrapolate to zero? **(b)** What are the pressures of the gas at the freezing and boiling points of water? **(c)** In general terms, how would your answers to parts (a) and (b) change if a different constant-volume gas thermometer is used? Explain.

9. •• **Greatest Change in Temperature** A world record for the greatest change in temperature was set in Spearfish, SD, on January 22, 1943. At 7:30 A.M. the temperature was −4.0 °F; two minutes later the temperature was 45 °F. Find the average rate of temperature change during those two minutes in kelvins per second.

10. •• We know that −40 °C corresponds to −40 °F. What temperature has the same value in both the Fahrenheit and Kelvin scales?

11. •• When the bulb of a constant-volume gas thermometer is placed in a beaker of boiling water at 100 °C, the pressure of the gas is 227 mmHg. When the bulb is moved to an ice-salt mixture, the pressure of the gas drops to 162 mmHg. Assuming ideal behavior, as in Figure 16–3, what is the Celsius temperature of the ice-salt mixture?

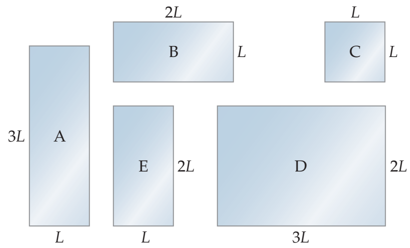
SECTION 16–3 THERMAL EXPANSION

12. • **CE** Bimetallic strip A is made of copper and steel; bimetallic strip B is made of aluminum and steel. **(a)** Referring to Table 16–1, which strip bends more for a given change in temperature? **(b)** Which of the metals listed in Table 16–1 would give the greatest amount of bend when combined with steel in a bimetallic strip?

13. • **CE** Referring to Table 16–1, which would be more accurate for all-season outdoor use: a tape measure made of steel or one made of aluminum?

14. • **CE Predict/Explain** A brass plate has a circular hole whose diameter is slightly smaller than the diameter of an aluminum ball. If the ball and the plate are always kept at the same temperature, **(a)** should the temperature of the system be increased or decreased in order for the ball to fit through the hole? **(b)** Choose the *best explanation* from among the following:
    - **I.** The aluminum ball changes its diameter more with temperature than the brass plate, and therefore the temperature should be decreased.
    - **II.** Changing the temperature won't change the fact that the ball is larger than the hole.
    - **III.** Heating the brass plate makes its hole larger, and that will allow the ball to pass through.

15. • **CE** Figure 16–13 shows five metal plates, all at the same temperature and all made from the same material. They are all placed in an oven and heated by the same amount. Rank the plates in order of increasing expansion in **(a)** the vertical and **(b)** the horizontal direction. Indicate ties where appropriate.



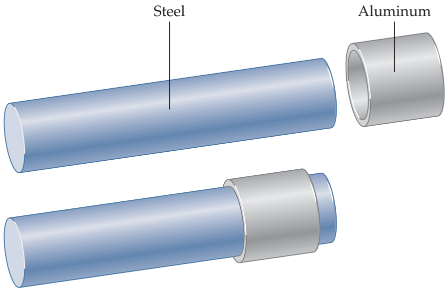
▲ **FIGURE 16–13** Problems 15 and 16

16. • **CE** Referring to Problem 15, rank the metal plates in order of increasing expansion in area. Indicate ties where appropriate.

17. • **Longest Suspension Bridge** The world's longest suspension bridge is the Akashi Kaikyo Bridge in Japan. The bridge is 3910 m long and is constructed of steel. How much longer is the bridge on a warm summer day (30.0 °C) than on a cold winter day (−5.00 °C)?

18. •• A hole in an aluminum plate has a diameter of 1.178 cm at 23.00 °C. **(a)** What is the diameter of the hole at 199.0 °C? **(b)** At what temperature is the diameter of the hole equal to 1.176 cm?

19. •• **IP** It is desired to slip an aluminum ring over a steel bar (Figure 16–14). At 10.00 °C the inside diameter of the ring is 4.000 cm and the diameter of the rod is 4.040 cm. **(a)** In order for the ring to slip over the bar, should the ring be heated or cooled? Explain. **(b)** Find the temperature of the ring at which it fits over the bar. The bar remains at 10.0 °C.



▲ **FIGURE 16–14** Problems 19 and 78

20. •• At 12.25 °C a brass sleeve has an inside diameter of 2.19625 cm and a steel shaft has a diameter of 2.19893 cm. It is desired to shrink-fit the sleeve over the steel shaft. **(a)** To what temperature must the sleeve be heated in order for it to slip over the shaft? **(b)** Alternatively, to what temperature must the shaft be cooled before it is able to slip through the sleeve?

21. •• Early in the morning, when the temperature is 5.0 °C, gasoline is pumped into a car's 51-L steel gas tank until it is filled to the top. Later in the day the temperature rises to 25 °C. Since the volume of gasoline increases more for a given temperature increase than the volume of the steel tank, gasoline will spill out of the tank. How much gasoline spills out in this case?

22. •• Some cookware has a stainless steel interior ($\alpha = 17.3 \times 10^{-6}\ \mathrm{K}^{-1}$) and a copper bottom ($\alpha = 17.0 \times 10^{-6}\ \mathrm{K}^{-1}$) for better heat distribution. Suppose an 8.0-in. pot of this construction is heated to 610 °C on the stove. If the initial temperature of the pot is 22 °C, what is the difference in diameter change for the copper and the steel?

23. •• **IP** You construct two wire-frame cubes, one using copper wire, the other using aluminum wire. At 23 °C the cubes enclose equal volumes of 0.016 m^3. **(a)** If the temperature of the cubes is increased, which cube encloses the greater volume? **(b)** Find the difference in volume between the cubes when their temperature is 97 °C.

24. •• A copper ball with a radius of 1.5 cm is heated until its diameter has increased by 0.19 mm. Assuming an initial temperature of 22 °C, find the final temperature of the ball.

25. •• **IP** An aluminum saucepan with a diameter of 23 cm and a height of 6.0 cm is filled to the brim with water. The initial temperature of the pan and water is 19 °C. The pan is now placed on a stove burner and heated to 88 °C. **(a)** Will water overflow from the pan, or will the water level in the pan decrease? Explain. **(b)** Calculate the volume of water that overflows or the drop in water level in the pan, whichever is appropriate.

SECTION 16–4 HEAT AND MECHANICAL WORK

26. • **BIO** **Sleeping Metabolic Rate** When people sleep, their metabolic rate is about $2.6 \times 10^{-4}\ \mathrm{C/(s \cdot kg)}$. How many Calories does a 75-kg person metabolize while getting a good night's sleep of 8.0 hr?

27. •• **BIO** An exercise machine indicates that you have worked off 2.5 Calories in a minute-and-a-half of running in place. What was your power output during this time? Give your answer in both watts and horsepower.

28. •• **BIO** During a workout, a person repeatedly lifts a 12-lb barbell through a distance of 1.3 ft. How many "reps" of this lift are required to burn off 150 C?

29. •• **IP** Consider the apparatus that Joule used in his experiments on the mechanical equivalent of heat, shown in Figure 16–8. Suppose both blocks have a mass of 0.95 kg and that they fall through a distance of 0.48 m. **(a)** Find the expected rise in temperature of the water, given that 6200 J are needed for every 1.0 C° increase. Give your answer in Celsius degrees. **(b)** Would the temperature rise in Fahrenheit degrees be greater than or less than the result in part (a)? Explain. **(c)** Find the rise in temperature in Fahrenheit degrees.

30. •• **BIO** It was shown in Example 16–8 that a typical person radiates about 62 W of power at room temperature. Given this result, how long does it take for a person to radiate away the energy acquired by consuming a 230-Calorie doughnut?

SECTION 16–5 SPECIFIC HEATS

31. • **CE** **Predict/Explain** Two objects are made of the same material but have different temperatures. Object 1 has a mass m, and object 2 has a mass $2m$. If the objects are brought into thermal contact, **(a)** is the temperature change of object 1 greater than, less than, or equal to the temperature change of object 2? **(b)** Choose the *best explanation* from among the following:

I. The larger object gives up more heat, and therefore its temperature change is greatest.

II. The heat given up by one object is taken up by the other object. Since the objects have the same heat capacity, the temperature changes are the same.

III. One object loses heat of magnitude Q, the other gains heat of magnitude Q. With the same magnitude of heat involved, the smaller object has the greater temperature change.

32. • **CE** **Predict/Explain** A certain amount of heat is transferred to 2 kg of aluminum, and the same amount of heat is transferred to 1 kg of ice. Referring to Table 16–2, **(a)** is the increase in temperature of the aluminum greater than, less than, or equal to the increase in temperature of the ice? **(b)** Choose the *best explanation* from among the following:

I. Twice the specific heat of aluminum is less than the specific heat of ice, and hence the aluminum has the greater temperature change.

II. The aluminum has the smaller temperature change since its mass is less than that of the ice.

III. The same heat will cause the same change in temperature.

33. • Suppose 79.3 J of heat are added to a 111-g piece of aluminum at 22.5 °C. What is the final temperature of the aluminum?

34. • How much heat is required to raise the temperature of a 55-g glass ball by 15 C°?

35. • Estimate the heat required to heat a 0.15-kg apple from 12 °C to 36 °C. (Assume the apple is mostly water.)

36. • A 5.0-g lead bullet is fired into a fence post. The initial speed of the bullet is 250 m/s, and when it comes to rest, half its kinetic energy goes into heating the bullet. How much does the bullet's temperature increase?

37. •• **IP** Silver pellets with a mass of 1.0 g and a temperature of 85 °C are added to 220 g of water at 14 °C. **(a)** How many pellets must be added to increase the equilibrium temperature of the system to 25 °C? Assume no heat is exchanged with the surroundings. **(b)** If copper pellets are used instead, does the required number of pellets increase, decrease, or stay the same? Explain. **(c)** Find the number of copper pellets that are required.

38. •• A 235-g lead ball at a temperature of 84.2 °C is placed in a light calorimeter containing 177 g of water at 21.5 °C. Find the equilibrium temperature of the system.

39. •• If 2200 J of heat are added to a 190-g object, its temperature increases by 12 C°. **(a)** What is the heat capacity of this object? **(b)** What is the object's specific heat?

40. •• An 97.6-g lead ball is dropped from rest from a height of 4.57 m. The collision between the ball and the ground is totally inelastic. Assuming all the ball's kinetic energy goes into heating the ball, find its change in temperature.

41. •• To determine the specific heat of an object, a student heats it to 100 °C in boiling water. She then places the 38.0-g object in a 155-g aluminum calorimeter containing 103 g of water. The aluminum and water are initially at a temperature of 20.0 °C, and are thermally insulated from their surroundings. If the final temperature is 22.0 °C, what is the specific heat of the object? Referring to Table 16–2, identify the material in the object.

42. •• **IP** At the local county fair, you watch as a blacksmith drops a 0.50-kg iron horseshoe into a bucket containing 25 kg of water. **(a)** If the initial temperature of the horseshoe is 450 °C, and the initial temperature of the water is 23 °C, what is the equilibrium temperature of the system? Assume no heat is exchanged with the surroundings. **(b)** Suppose the 0.50-kg iron horseshoe had been a 1.0-kg lead horseshoe instead. Would the equilibrium temperature in this case be greater than, less than, or the same as in part (a)? Explain.

43. •• The ceramic coffee cup in **Figure 16–15**, with $m = 116$ g and $c = 1090\ \mathrm{J/(kg \cdot K)}$, is initially at room temperature (24.0 °C). If 225 g of 80.3 °C coffee and 12.2 g of 5.00 °C cream are added to the cup, what is the equilibrium temperature of the system? Assume that no heat is exchanged with the surroundings, and that the specific heat of coffee and cream are the same as the specific heat of water.

▲ **FIGURE 16–15** Problem 43

SECTION 16–6 CONDUCTION, CONVECTION, AND RADIATION

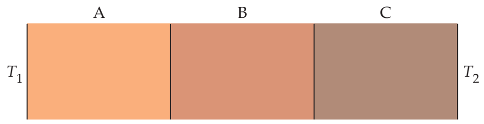
44. • **CE Predict/Explain** In a popular lecture demonstration, a sheet of paper is wrapped around a rod that is made from wood on one half and metal on the other half. If held over a flame, the paper on one half of the rod is burned while the paper on the other half is unaffected. **(a)** Is the burned paper on the wooden half of the rod, or on the metal half of the rod? **(b)** Choose the *best explanation* from among the following:

I. The metal will be hotter to the touch than the wood; therefore the metal side will be burnt.

II. The metal conducts heat better than the wood, and hence the paper on the metal half is unaffected.

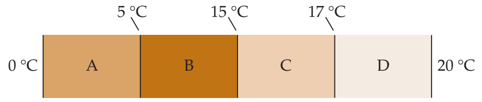
III. The metal has the smaller specific heat; hence it heats up more and burns the paper on its half of the rod.

45. • **CE** **Figure 16–16** shows a composite slab of three different materials with equal thickness but different thermal conductivities. The opposite sides of the composite slab are held at the fixed temperatures T_1 and T_2. Given that $k_B > k_A > k_C$, rank the materials in order of the temperature difference across them, starting with the smallest. Indicate ties where appropriate.



▲ **FIGURE 16–16** Problem 45

46. • **CE** Heat is transferred from an area where the temperature is 20 °C to an area where the temperature is 0 °C through a composite slab consisting of four different materials, each with the same thickness. The temperatures at the interface between each of the materials are given in **Figure 16–17**. Rank the four materials in order of increasing thermal conductivity. Indicate ties where appropriate.



▲ **FIGURE 16–17** Problem 46

47. • **CE** On a sunny day identical twins wear different shirts. Twin 1 wears a dark shirt; twin 2 wears a light-colored shirt. Which twin has the warmer shirt?

48. • **CE** Two bowls of soup with identical temperatures are placed on a table. Bowl 1 has a metal spoon in it; bowl 2 does not. After a few minutes, is the temperature of the soup in bowl 1 greater than, less than, or equal to the temperature of the soup in bowl 2?

49. • A glass window 0.35 cm thick measures 84 cm by 36 cm. How much heat flows through this window per minute if the inside and outside temperatures differ by 15 C°?

50. • To compare the relative efficiency of air and glass as insulators, repeat the previous problem with a 0.35-cm-thick layer of air instead of glass. By what factor is the rate of heat transfer reduced?

51. • **BIO** Assuming your skin temperature is 37.2 °C and the temperature of your surroundings is 21.8 °C, determine the length of time required for you to radiate away the energy gained by eating a 306-Calorie ice cream cone. Let the emissivity of your skin be 0.915 and its area be 1.22 $\mathrm{m^2}$.

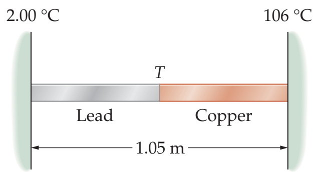
52. • Find the heat that flows in 1.0 s through a lead brick 15 cm long if the temperature difference between the ends of the brick is 9.5 C°. The cross-sectional area of the brick is 14 $\mathrm{cm^2}$.

53. •• Consider a double-paned window consisting of two panes of glass, each with a thickness of 0.500 cm and an area of 0.725 $\mathrm{m^2}$, separated by a layer of air with a thickness of 1.75 cm. The temperature on one side of the window is 0.00 °C; the temperature on the other side is 20.0 °C. In addition, note that the thermal conductivity of glass is roughly 36 times greater than that of air. **(a)** Approximate the heat transfer through this window by ignoring the glass. That is, calculate the heat flow per second through 1.75 cm of air with a temperature difference of 20.0 C°. (The exact result for the complete window is 19.1 J/s.) **(b)** Use the approximate heat flow found in part (a) to find an approximate temperature difference across each pane of glass. (The exact result is 0.157 C°.)

54. •• **IP** Two metal rods of equal length—one aluminum, the other stainless steel—are connected in parallel with a temperature of 20.0 °C at one end and 118 °C at the other end. Both rods have a circular cross section with a diameter of 3.50 cm. **(a)** Determine the length the rods must have if the combined rate of heat flow through them is to be 27.5 J per second. **(b)** If the length of the rods is doubled, by what factor does the rate of heat flow change?

55. •• Two cylindrical metal rods—one copper, the other lead—are connected in parallel with a temperature of 21.0 °C at one end and 112 °C at the other end. Both rods are 0.650 m in length, and the lead rod is 2.76 cm in diameter. If the combined rate of heat flow through the two rods is 33.2 J/s, what is the diameter of the copper rod?

56. •• **IP** Two metal rods—one lead, the other copper—are connected in series, as shown in **Figure 16–18**. These are the same two rods that were connected in parallel in Example 16–7. Note that each rod is 0.525 m in length and has a square cross section 1.50 cm on a side. The temperature at the lead end of the rods is 2.00 °C; the temperature at the copper end is 106 °C. **(a)** The average temperature of the two ends is 54.0 °C. Is the temperature in the middle, at the lead-copper interface, greater than, less than, or equal to 54.0 °C? Explain. **(b)** Given that the heat flow through each of these rods in 1.00 s is 1.41 J, find the temperature at the lead-copper interface.



▲ **FIGURE 16–18** Problem 56

57. •• **IP** Consider two cylindrical metal rods with equal cross section—one lead, the other aluminum—connected in series. The temperature at the lead end of the rods is 20.0 °C; the temperature at the aluminum end is 80.0 °C. **(a)** Given that the temperature at the lead-aluminum interface is 50.0 °C, and that the lead rod is 14 cm long, what condition can you use to find the length of the aluminum rod? **(b)** Find the length of the aluminum rod.

58. •• A copper rod 81 cm long is used to poke a fire. The hot end of the rod is maintained at 105 °C and the cool end has a constant temperature of 21 °C. What is the temperature of the rod 25 cm from the cool end?

59. •• Two identical objects are placed in a room at 21 °C. Object 1 has a temperature of 98 °C, and object 2 has a temperature of 23 °C. What is the ratio of the net power emitted by object 1 to that radiated by object 2?

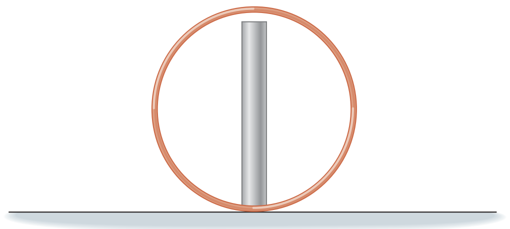
60. ••• A block has the dimensions L, $2L$, and $3L$. When one of the $L \times 2L$ faces is maintained at the temperature T_1 and the other $L \times 2L$ face is held at the temperature T_2, the rate of heat conduction through the block is P. Answer the following questions in terms of P. **(a)** What is the rate of heat conduction in this block if one of the $L \times 3L$ faces is held at the temperature T_1 and the other $L \times 3L$ face is held at the temperature T_2? **(b)** What is the rate of heat conduction in this block if one of the $2L \times 3L$ faces is held at the temperature T_1 and the other $2L \times 3L$ face is held at the temperature T_2?

GENERAL PROBLEMS

61. • **CE** A steel tape measure is marked in such a way that it gives accurate length measurements at a normal room temperature of 20 °C. If this tape measure is used outdoors on a cold day when the temperature is 0°C, are its measurements too long, too short, or accurate?

62. • **CE Predict/Explain** A pendulum is made from an aluminum rod with a mass attached to its free end. If the pendulum is cooled, **(a)** does the pendulum's period increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. The period of a pendulum depends only on its length and the acceleration of gravity. It is independent of mass and temperature.
   II. Cooling makes everything move more slowly, and hence the period of the pendulum increases.
   III. Cooling shortens the aluminum rod, which decreases the period of the pendulum.

63. • **CE** A copper ring stands on edge with a metal rod placed inside it, as shown in **Figure 16–19**. As this system is heated, will the rod ever touch the top of the ring? Answer yes or no for the case of a rod that is made of **(a)** copper, **(b)** aluminum, and **(c)** steel.



▲ **FIGURE 16–19** Problems 63 and 64

64. • **CE** Referring to the copper ring in the previous problem, imagine that initially the ring is hotter than room temperature, and that an aluminum rod that is colder than room temperature fits snugly inside the ring. When this system reaches thermal equilibrium at room temperature, is the rod (**A**, firmly wedged in the ring; or **B**, can it be removed easily)?

65. • **CE Predict/Explain** The specific heat of alcohol is about half that of water. Suppose you have 0.5 kg of alcohol at the temperature 20 °C in one container, and 0.5 kg of water at the temperature 30 °C in a second container. When these fluids are poured into the same container and allowed to come to thermal equilibrium, **(a)** is the final temperature greater than, less than, or equal to 25 °C? **(b)** Choose the *best explanation* from among the following:
   I. The low specific heat of alcohol pulls in more heat, giving a final temperature that is less than 25°.
   II. More heat is required to change the temperature of water than to change the temperature of alcohol. Therefore, the final temperature will be greater than 25°.
   III. Equal masses are mixed together; therefore, the final temperature will be 25°, the average of the two initial temperatures.

66. • **CE** Hot tea is poured from the same pot into two identical mugs. Mug 1 is filled to the brim; mug 2 is filled only halfway. Is the rate of cooling of mug 1 (**A**, greater than; **B**, less than; or **C**, equal to) the rate of cooling of mug 2?

67. • **Making Steel Sheets** In the continuous-caster process, steel sheets 25.4 cm thick, 2.03 m wide, and 10.0 m long are produced at a temperature of 872 °C. What are the dimensions of a steel sheet once it has cooled to 20.0 °C?

68. • **The Coldest Place in the Universe** The Boomerang nebula holds the distinction of having the lowest recorded temperature in the universe, a frigid −272 °C. What is this temperature in kelvins?

69. • When technicians work on a computer, they often ground themselves to prevent generating a spark. If an electrostatic discharge does occur, it can cause temperatures as high as 1500 °C in a localized area of a circuit. Temperatures this high can melt aluminum, copper, and silicon. What is this temperature in **(a)** degrees Fahrenheit and **(b)** kelvins?

70. • **CE** Two objects at the same initial temperature absorb equal amounts of heat. If the final temperature of the objects is different, it may be because they differ in which of the following

properties: mass; coefficient of expansion; thermal conductivity; specific heat?

71. • **BIO** **The Hottest Living Things** From the surreal realm of deep-sea hydrothermal vents 200 miles offshore from Puget Sound, comes a newly discovered hyperthermophilic—or extreme heat-loving—microbe that holds the record for the hottest existence known to science. This microbe is tentatively known as Strain 121 for the temperature at which it thrives: 121 °C. (At sea level, water at this temperature would boil vigorously, but the extreme pressures at the ocean floor prevent boiling from occurring.) What is this temperature in degrees Fahrenheit?

72. •• **CE** The heat Q will warm 1 g of material A by 1 C°, the heat $2Q$ will warm 3 g of material B by 3 C°, the heat $3Q$ will warm 3 g of material C by 1 C°, and the heat $4Q$ will warm 4 g of material D by 2 C°. Rank these materials in order of increasing specific heat. Indicate ties where appropriate.

73. •• In many biological systems it is of more interest to know how much heat is required to raise the temperature of a given *volume* of material rather than a given *mass* of material. Calculate the heat needed to raise the temperature of one cubic meter of **(a)** air and **(b)** water by one degree Celsius. Compare with the corresponding specific heats (for a given mass) listed in Table 16–2.

74. •• **BIO** **Brain Power** As you read this problem, your brain is consuming about 22 W of power. **(a)** How many steps with a height of 21 cm must you climb to expend a mechanical energy equivalent to one hour of brain operation? **(b)** A typical human brain, which is 77% water, has a mass of 1.4 kg. Assuming that the 22 W of brain power is converted to heat, what temperature rise would you estimate for the brain in one hour of operation? Ignore the significant heat transfer that occurs between a human head and its surroundings, as well as the 23% of the brain that is not water.

75. •• **BIO** **The Cricket Thermometer** The rate of chirping of the snowy tree cricket (*Oecanthus fultoni* Walker) varies with temperature in a predictable way. A linear relationship provides a good match to the chirp rate, but an even more accurate relationship is the following:

$$N = (5.63 \times 10^{10})e^{-(6290\text{ K})/T}$$

In this expression, N is the number of chirps in 13.0 s and T is the temperature in kelvins. If a cricket is observed to chirp 185 times in 60.0 s, what is the temperature in degrees Fahrenheit?

76. •• If heat is transferred to 150 g of water at a constant rate for 2.5 min, its temperature increases by 13 C°. When heat is transferred at the same rate for the same amount of time to a 150-g object of unknown material, its temperature increases by 61 C°. **(a)** From what material is the object made? **(b)** What is the heating rate?

77. •• **IP** A pendulum consists of a large weight suspended by a steel wire that is 0.9500 m long. **(a)** If the temperature increases, does the period of the pendulum increase, decrease, or stay the same? Explain. **(b)** Calculate the change in length of the pendulum if the temperature increase is 150.0 C°. **(c)** Calculate the period of the pendulum before and after the temperature increase. (Assume that the coefficient of linear expansion for the wire is $12.00 \times 10^{-6}\text{ K}^{-1}$, and that $g = 9.810\text{ m/s}^2$ at the location of the pendulum.)

78. •• **IP** Once the aluminum ring in Problem 19 is slipped over the bar, the ring and bar are allowed to equilibrate at a temperature of 22 °C. The ring is now stuck on the bar. **(a)** If the temperatures of both the ring and the bar are changed together, should the system be heated or cooled to remove the ring? **(b)** Find the temperature at which the ring can be removed.

79. •• A steel plate has a circular hole with a diameter of 1.000 cm. In order to drop a Pyrex glass marble 1.003 cm in diameter through the hole in the plate, how much must the temperature of the system be raised? (Assume the plate and the marble are always at the same temperature.)

80. •• A 226-kg rock sits in full sunlight on the edge of a cliff 5.25 m high. The temperature of the rock is 30.2 °C. If the rock falls from the cliff into a pool containing 6.00 m^3 of water at 15.5 °C, what is the final temperature of the rock–water system? Assume that the specific heat of the rock is $1010\text{ J/(kg}\cdot\text{K)}$.

81. •• Water going over Iguaçu Falls on the border of Argentina and Brazil drops through a height of about 72 m. Suppose that all the gravitational potential energy of the water goes into raising its temperature. Find the increase in water temperature at the bottom of the falls as compared with the top.

82. •• **IP** A 0.22-kg steel pot on a stove contains 2.1 L of water at 22 °C. When the burner is turned on, the water begins to boil after 8.5 minutes. **(a)** At what rate is heat being transferred from the burner to the pot of water? **(b)** At this rate of heating, would it take more time or less time for the water to start to boil if the pot were made of gold rather than steel?

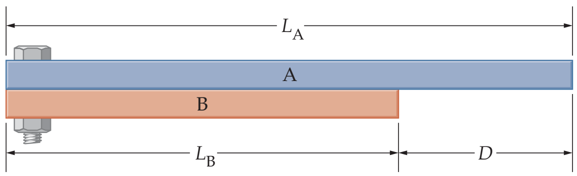
83. •• **BIO** Suppose you could convert the 525 Calories in the cheeseburger you ate for lunch into mechanical energy with 100% efficiency. **(a)** How high could you throw a 0.145-kg baseball with the energy contained in the cheeseburger? **(b)** How fast would the ball be moving at the moment of release?

84. •• You turn a crank on a device similar to that shown in Figure 16–8 and produce a power of 0.18 hp. If the paddles are immersed in 0.65 kg of water, for what length of time must you turn the crank to increase the temperature of the water by 5.0 C°?

85. •• **IP** **BIO** **Heat Transport in the Human Body** The core temperature of the human body is 37.0 °C, and the skin, with a surface area of 1.40 m^2, has a temperature of 34.0 °C. **(a)** Find the rate of heat transfer out of the body under the following assumptions: (i) The average thickness of tissue between the core and the skin is 1.20 cm; (ii) the thermal conductivity of the tissue is that of water. **(b)** Without repeating the calculation of part (a), what rate of heat transfer would you expect if the skin temperature were to fall to 31.0 °C? Explain.

86. •• **The Solar Constant** The surface of the Sun has a temperature of 5500 °C. **(a)** Treating the Sun as a perfect blackbody, with an emissivity of 1.0, find the power that it radiates into space. The radius of the Sun is 7.0×10^8 m, and the temperature of space can be taken to be 3.0 K. **(b)** The *solar constant* is the number of watts of sunlight power falling on a square meter of the Earth's upper atmosphere. Use your result from part (a) to calculate the solar constant, given that the distance from the Sun to the Earth is 1.5×10^{11} m.

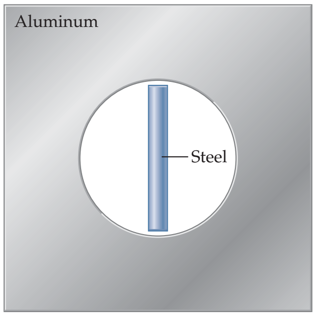
87. ••• Bars of two different metals are bolted together, as shown in **Figure 16–20**. Show that the distance D does not change with temperature if the lengths of the two bars have the following ratio: $L_A/L_B = \alpha_B/\alpha_A$.



▲ **FIGURE 16–20** Problem 87

88. • • • A grandfather clock has a simple brass pendulum of length L. One night, the temperature in the house is 25.0 °C and the period of the pendulum is 1.00 s. The clock keeps correct time at this temperature. If the temperature in the house quickly drops to 17.1 °C just after 10 P.M., and stays at that value, what is the actual time when the clock indicates that it is 10 A.M. the next morning?

89. • • • **IP** A sheet of aluminum has a circular hole with a diameter of 10.0 cm. A 9.99-cm-long steel rod is placed inside the hole, along a diameter of the circle, as shown in **Figure 16–21**. It is desired to change the temperature of this system until the steel rod just touches both sides of the circle. **(a)** Should the temperature of the system be increased or decreased? Explain. **(b)** By how much should the temperature be changed?



▲ **FIGURE 16–21** Problem 89

90. • • • A layer of ice has formed on a small pond. The air just above the ice is at −5.4 °C, the water-ice interface is at 0 °C, and the water at the bottom of the pond is at 4.0 °C. If the total depth from the top of the ice to the bottom of the pond is 1.4 m, how thick is the layer of ice? *Note*: The thermal conductivity of ice is 1.6 W/(m · C°) and that of water is 0.60 W/(m · C°).

91. • • • **A Double-Paned Window** An energy-efficient double-paned window consists of two panes of glass, each with thickness L_1 and thermal conductivity k_1, separated by a layer of air of thickness L_2 and thermal conductivity k_2. Show that the equilibrium rate of heat flow through this window per unit area, A, is

$$\frac{Q}{At} = \frac{(T_2 - T_1)}{2L_1/k_1 + L_2/k_2}$$

In this expression, T_1 and T_2 are the temperatures on either side of the window.

PASSAGE PROBLEMS

Faster than a Speeding Bullet

The SR-71 Blackbird, which is 107 feet 5.0 inches long, is a remarkable aircraft in many ways. For example, on July 28, 1976, it set an altitude record for sustained horizontal flight of 85,068.997 feet. On the same day, it set a closed-course speed record of 2193.167 miles per hour, which—literally—is faster than a speeding bullet.

Called the "Blackbird" because of its distinctive black paint job, the SR-71 becomes very hot due to air resistance when it flies at supersonic speeds. Its typical cruising speed is 3.2 times the speed of sound, referred to as Mach 3.2. The extreme heating at these speeds results in a number of interesting consequences, including the fact that the Blackbird is too hot to touch for about 30 minutes after it lands. In addition, pilots have been known to heat their lunch by holding it against the windshield, which reaches temperatures comparable to an oven.

Of course, temperatures this high also result in significant thermal expansion. For example, portions of the upper and lower inboard wing skin of the SR-71 are corrugated, to allow expansion during flight. Similarly, the fuel tanks in the fuselage and wings are designed to seal in flight, when they are expanded by the heat. As a result, the SR-71 leaks fuel constantly when on the ground, and takes off with a light fuel load; it is refueled seven minutes later by a KC-135Q tanker. One final consequence of thermal expansion: When the Blackbird lands, it is a full 8.0 inches longer than when it took off. No wonder pilots say this is one "hot" airplane.

92. • How hot is the Blackbird when it lands, assuming it is 8.0 inches longer than at takeoff, its coefficient of linear expansion is $22 \times 10^{-6}\,\mathrm{K}^{-1}$, and its temperature at takeoff is 23 °C?

A. 280 °C **B.** 310 °C
C. 560 °C **D.** 3400 °C

93. • **Predict** If the SR-71 were painted white instead of black, would its in-flight temperature be greater than, less than, or equal to its temperature with black paint?

94. • **Explain** Choose the *best explanation* for the previous problem from among the following:

A. Heating by air resistance is the same for any color of paint; therefore, the plane will have the same temperature regardless of color.

B. Black is a more efficient radiator of heat than white. Therefore, the black paint radiates more heat, and allows the airplane to stay cooler.

C. Black objects are generally hotter than white ones, all other things being equal. Therefore, the plane would be cooler with white paint.

95. • How long is the Blackbird when it is 120 °C?

A. 107 ft 7.8 in. **B.** 107 ft 8.2 in.
C. 108 ft 0.8 in. **D.** 108 ft 1.4 in.

INTERACTIVE PROBLEMS

96. • • **Referring to Example 16–5** Suppose the mass of the block is to be increased enough to make the final temperature of the system equal to 22.5 °C. What is the required mass? Everything else in Example 16–5 remains the same.

97. • • **Referring to Example 16–5** Suppose the initial temperature of the block is to be increased enough to make the final temperature of the system equal to 22.5 °C. What is the required initial temperature? Everything else remains the same as in Example 16–5.

98. • • **IP** Suppose the lead rod is replaced with a second copper rod. **(a)** Will the heat that flows in 1.00 s increase, decrease, or stay the same? Explain. **(b)** Find the heat that flows in 1.00 s with two copper rods. Everything else remains the same as in Example 16–7.

99. • • **IP** Suppose the temperature of the hot plate is to be changed to give a total heat flow of 25.2 J in 1.00 s. **(a)** Should the new temperature of the hot plate be greater than or less than 106 °C? Explain. **(b)** Find the required temperature of the hot plate. Everything else is the same as in Example 16–7.

17 Phases and Phase Changes

Though many people are surprised to learn that a liquid can boil and freeze at the same time, it is in fact true. At a unique combination of temperature and pressure called the triple point, all three phases of matter—solid, liquid, and gas—coexist in equilibrium. In this chapter we'll learn more about the three phases of matter, their differences and similarities, and what happens when a substance changes from one phase to another.

The matter we come into contact with every day is in one of three forms: solid, liquid, or gas. The air we breathe is a gas, the water we drink is a liquid, and the salt on our popcorn is a crystalline solid. In this chapter we consider these three *phases* of matter in detail.

We begin by considering the behavior of a gas, which is the easiest phase to describe physically. In particular, we show that an "ideal gas"—one that has no interactions between its molecules—is a good approximation to real gases. In addition, we show how the kinetic theory of gases allows us to relate the temperature of a substance to the kinetic energy of its molecules.

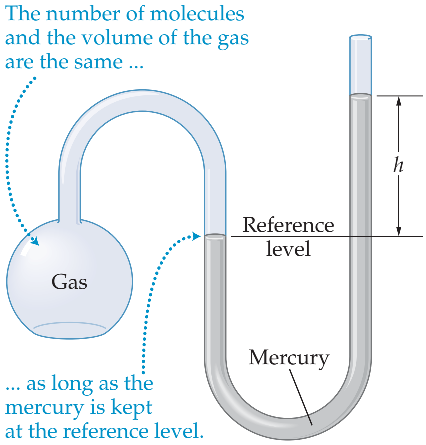
Next, we discuss some of the mechanical properties associated with the solid phase of matter. In particular, we explore the relationship between a force applied to a solid and the resulting deformation. For example, we shall determine the amount of stretch in your arm bones when you carry a heavy suitcase through an airport.

Finally, we turn our attention to the behavior of a substance when it changes from one phase to another—as when solid ice melts to form liquid water or water boils to produce gaseous clouds of steam. Remarkably, when a material is changing phase, the addition of heat does not result in a higher temperature. A significant amount of heat must first be absorbed by the material so that it can complete its phase change. Only after the change is accomplished can its temperature begin to rise again.

17–1 Ideal Gases

In Chapter 16 we discussed the thermal behavior of gases. In particular, we saw that the pressure of a constant volume of gas decreases linearly with decreasing temperature over a wide range of temperatures. Eventually, however, when the temperature is low enough, real gases change to liquid and then solid form, and their behavior changes. These changes are due to interactions between the molecules in a gas. Hence, the weaker these interactions, the wider the range of temperatures over which the simple, linear gas behavior persists. We now consider a simplified model of a gas, the **ideal gas,** in which intermolecular interactions are vanishingly small.

Though ideal gases do not actually exist in nature, the behavior of real gases is generally quite well approximated by that of an ideal gas, especially when the gas is dilute. By studying the simple "ideal" case, we can gain considerable insight into the workings of a real gas. This is similar to the kind of idealizations we made in mechanics. For example, we often considered a surface to be perfectly frictionless, when real surfaces have at least some friction, and we imagined springs to obey Hooke's law exactly, though real springs show some deviations. The ideal gas plays an analogous role in our study of thermodynamics.



▲ **FIGURE 17–1 A constant-volume gas thermometer**

A device like this can be used as a thermometer. Note that both the volume of the gas and the number of molecules in the gas remain constant.

Equation of State

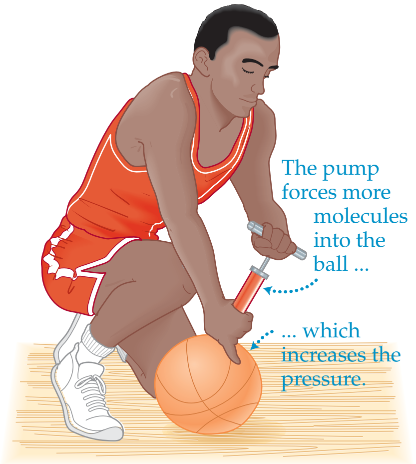
We can describe the way the pressure, P, of an ideal gas depends on temperature, T, number of molecules, N, and volume, V, from just a few simple observations. First, imagine holding the number of molecules and the volume of a gas constant, as in the constant-volume gas thermometer shown in **Figure 17–1**. As we have already noted, the pressure of a gas under these conditions varies linearly with temperature. Therefore, we conclude the following:

$$P = (\text{constant})T$$
$$(\textit{fixed volume}, V;\ \textit{fixed number of molecules}, N)$$

In this expression, the constant multiplying the temperature depends on the number of molecules in the gas and its volume; the temperature T is measured on the Kelvin scale.

Second, imagine you have a basketball that is just slightly underinflated, as in **Figure 17–2**. The ball has the size and shape of a basketball, but is a bit too squishy. To increase the pressure you "pump" more molecules from the atmosphere into the ball. Thus, while the temperature and volume of the gas in the ball remain constant, its pressure increases as the number of molecules increases:

$$P = (\text{constant})N$$
$$(\textit{fixed volume}, V;\ \textit{fixed temperature}, T)$$



▲ **FIGURE 17–2 Inflating a basketball**

A hand pump can be used to increase the number of molecules inside a basketball. This raises the pressure of the gas in the ball, causing it to inflate.

Our third and final observation concerns the volume dependence of pressure. Returning to the basketball, suppose that instead of pumping it up you sit on it, as pictured in **Figure 17–3**. This deforms it a bit, *reducing* its volume. At the same time, the pressure of the gas in the ball *increases*. Thus, when the number of molecules and temperature remain constant, the pressure varies *inversely* with volume:

$$P = \frac{(\text{constant})}{V}$$
$$(\textit{fixed number of molecules}, N;\ \textit{fixed temperature}, T)$$

Combining these observations, we arrive at the following mathematical expression for the pressure of a gas:

$$P = k\frac{NT}{V}$$

▶ **FIGURE 17–3 Increasing pressure by decreasing volume**

Sitting on a basketball reduces its volume and increases the pressure of the gas it contains.

The constant k in this expression is a fundamental constant of nature, the **Boltzmann constant,** named for the Austrian physicist Ludwig Boltzmann (1844–1906):

Boltzmann Constant, *k*

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K} \qquad 17\text{–}1$$

SI unit: J/K

PROBLEM-SOLVING NOTE

Ideal Gas Law

When using the equation of state for an ideal gas, remember that the temperature must always be given in terms of the Kelvin scale.

In general, a relationship between the thermal properties of a substance is referred to as an **equation of state.** Rearranging slightly, the equation of state for an ideal gas can be written as

Equation of State for an Ideal Gas

$$PV = NkT \qquad 17\text{–}2$$

We apply this result to the gas contained in a person's lungs in the next Example.

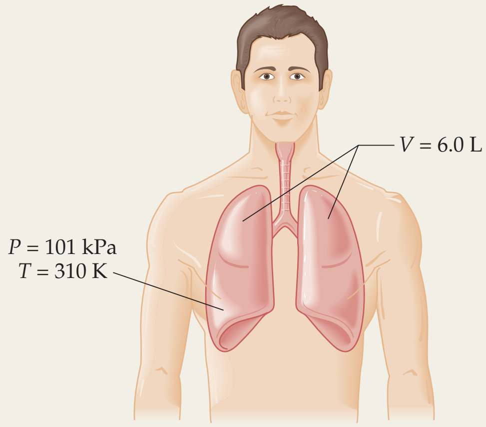
EXAMPLE 17–1 TAKE A DEEP BREATH

REAL-WORLD PHYSICS: BIO

A person's lungs might hold 6.0 L ($1\ \mathrm{L} = 10^{-3}\ \mathrm{m}^3$) of air at body temperature (310 K) and atmospheric pressure (101 kPa). **(a)** Given that the air is 21% oxygen, find the number of oxygen molecules in the lungs. **(b)** If the person now climbs to the top of a mountain, where the air pressure is considerably less than 101 kPa, does the number of molecules in the lungs increase, decrease, or stay the same?

PICTURE THE PROBLEM

Our sketch shows a person's lungs, with their combined volume of $V = 6.0$ L. In addition, we indicate that the pressure in the lungs is $P = 101$ kPa and the temperature is $T = 310$ K.



STRATEGY

a. We will treat the air in the lungs as an ideal gas. Given the volume, temperature, and pressure of the gas, we can use the equation of state, $PV = NkT$, to solve for the number, N.

Finally, only 21% of the molecules in the air are oxygen. Therefore, we multiply N by 0.21 to find the number of oxygen molecules.

b. We apply $PV = NkT$ again, but this time with a reduced pressure P. Note that V and T remain the same, since they are determined by the person's body.

SOLUTION

Part (a)

1. Solve the equation of state for the number of molecules: $PV = NkT$ or $N = PV/kT$

2. Substitute numerical values to find the number of molecules in the lungs:

$$N = \frac{PV}{kT} = \frac{(1.01 \times 10^5\ \mathrm{Pa})(6.0 \times 10^{-3}\ \mathrm{m}^3)}{(1.38 \times 10^{-23}\ \mathrm{J/K})(310\ \mathrm{K})} = 1.4 \times 10^{23}$$

3. Multiply N by 0.21 to find the number of molecules that are oxygen, O_2: $0.21N = 0.21(1.4 \times 10^{23}) = 2.9 \times 10^{22}$

Part (b)

From Step 1 we see that $N = PV/kT$. As a result, if the volume V of a person's lungs remains the same, along with the body temperature T, it follows that the number of molecules N will decrease linearly with pressure. For example, at an altitude of 10,000 ft the air pressure is about 70% of its value at sea level, meaning that the number of oxygen molecules in the lungs is only 70% of its usual value. No wonder people have a hard time "catching their breath" at high altitude.

INSIGHT

As one might expect, the number of molecules in even an average-size pair of lungs is enormous. In fact, a number this large is difficult to compare to anything we are familiar with. For example, the number of stars in the Milky Way galaxy is estimated to be "only" about 10^{11}. Thus, if each star in the Milky Way were itself a galaxy of 10^{11} stars, then all of these stars combined would just begin to approach the number of oxygen molecules in your lungs at this very moment.

PRACTICE PROBLEM

If the person takes a particularly deep breath, so that the lungs hold a total of 1.5×10^{23} molecules, what is the new volume of the lungs? [**Answer:** $V = 6.4 \times 10^{-3}\ m^3 = 6.4$ L]

Some related homework problems: Problem 5, Problem 6, Problem 83

Another common way to write the ideal-gas equation of state is in terms of the number of **moles** in a gas—as opposed to using the number of molecules, N, as in Equation 17–2. In the SI system of units, the mole (mol) is defined in terms of the most abundant isotope of carbon, which is referred to as carbon-12. The definition is as follows:

> A mole is the amount of a substance that contains as many elementary entities as there are atoms in 12 g of carbon-12.

The phrase "elementary entities" refers to molecules, which may contain more than one atom, as in water (H_2O), or just a single atom, as in carbon (C) and helium (He). Experiments show that the number of atoms in a mole of carbon-12 is 6.022×10^{23}. This number is known as **Avogadro's number,** N_A, named for the Italian physicist and chemist Amedeo Avogadro (1776–1856).

Avogadro's Number, N_A

$$N_A = 6.022 \times 10^{23}\ \text{molecules/mol} \qquad 17\text{–}3$$

SI unit: mol^{-1}; the number of molecules is dimensionless

Examples of one mole of various substances are shown in **Figure 17–4**.

Now, if we let n denote the number of moles in a gas, the number of molecules it contains is

$$N = nN_A$$

Substituting this into the ideal-gas equation of state yields

$$PV = nN_AkT$$

The constants N_A and k are combined to form the **universal gas constant,** R, defined as follows:

Universal Gas Constant, R

$$R = N_Ak = (6.022 \times 10^{23}\ \text{molecules/mol})(1.38 \times 10^{-23}\ \text{J/K})$$
$$= 8.31\ \text{J/(mol}\cdot\text{K)} \qquad 17\text{–}4$$

SI unit: $\text{J/(mol}\cdot\text{K)}$

Thus, an alternative form of the equation of state is

Equation of State for an Ideal Gas

$$PV = nRT \qquad 17\text{–}5$$

This relation is used in the following Active Example.

▲ **FIGURE 17–4 Moles of various substances**

Counting atoms or molecules is obviously hard to do, but the mole concept provides a useful way of dealing with the difficulty. A mole of any substance contains the same number of elementary entities (atoms or molecules).

This number, referred to as Avogadro's number, has the value $N_A = 6.022 \times 10^{23}$. To count out N_A molecules of a gas at standard temperature and pressure, for example, all you need do is measure out a volume of gas equal to 22.4 liters. Similarly, you can effectively count one mole of any substance by measuring out a mass in grams that is equal to the atomic mass of that substance. Thus, the mole provides a convenient bridge between the realm of atoms and molecules and the macroscopic world of observable masses and volumes. This photo shows molar amounts of four different substances: hydrogen, copper, mercury, and sulfur.

ACTIVE EXAMPLE 17–1 THE AMOUNT OF AIR IN A BASKETBALL

How many moles of air are in an inflated basketball? Assume that the pressure in the ball is 171 kPa, the temperature is 293 K, and the diameter of the ball is 30.0 cm.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve $PV = nRT$ for the number of moles, n: $n = PV/RT$
2. Calculate the volume of the ball: $V = 4\pi r^3/3 = 0.0141\ m^3$
3. Substitute numerical values: $n = 0.990$ mol

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INSIGHT

Thus, an inflated basketball contains approximately one mole of air.

In this calculation we used $PV = nRT$ because we were interested in the number of *moles* (n) contained in the gas. In contrast, we used $PV = NkT$ in Example 17–1 because we wanted to know the number of *molecules* (N) in the gas.

YOUR TURN

Suppose we were to halve both the temperature and the diameter of the ball. By what factor would the number of moles contained within it change?

(Answers to **Your Turn** *problems are given in the back of the book.)*

As mentioned before, a mole of anything has precisely the same number (N_A) of particles. What differs from substance to substance is the mass of one mole. For example, one mole of helium atoms has a mass of 4.00260 g, and one mole of copper atoms has a mass of 63.546 g. In general, we define the **atomic or molecular mass,** M, of a substance to be the mass in grams of one mole of that substance. Thus, the atomic mass for helium is $M = 4.00260\ \mathrm{g/mol}$ and for copper it is $M = 63.546\ \mathrm{g/mol}$. The periodic table in Appendix E gives the atomic masses for all elements.

Note that the atomic mass provides a convenient bridge between the macroscopic world, where we measure the mass of a substance in grams, and the microscopic world, where the number of molecules is typically 10^{23} or greater. As we have seen, if you measure out a mass of copper equal to 63.546 g you have, in effect, counted out $N_A = 6.022 \times 10^{23}$ atoms of copper. It follows that the mass of an individual copper atom, m, is the atomic mass of copper divided by Avogadro's number; that is,

$$m = \frac{M}{N_A} \qquad 17\text{–}6$$

We use this relation to find the masses of a copper atom and an oxygen molecule in the following Exercise.

EXERCISE 17–1

Find the masses of **(a)** a copper atom and **(b)** a molecule of oxygen, O_2. Atomic masses are listed in Appendix E.

SOLUTION

a. The atomic mass of copper is 63.546 g/mol. Thus, a copper atom has the mass

$$m_{Cu} = \frac{M}{N_A} = \frac{63.546\ \mathrm{g/mol}}{6.022 \times 10^{23}\ \mathrm{atoms/mol}} = 1.055 \times 10^{-22}\ \mathrm{g/atom}$$

b. Since the atomic mass of oxygen is 16.00 g/mol (Appendix E), the molecular mass of O_2 is 32.00 g/mol. Therefore, the mass of O_2 is

$$m_{O_2} = \frac{M}{N_A} = \frac{32.00\ \mathrm{g/mol}}{6.022 \times 10^{23}\ \mathrm{molecules/mol}} = 5.314 \times 10^{-23}\ \mathrm{g/molecule}$$

Finally, we consider a common situation to further illustrate the general features of the ideal-gas equation of state.

CONCEPTUAL CHECKPOINT 17–1 AIR PRESSURE

Feeling a bit cool, you turn up the thermostat in your living room. A short time later the air is warmer. Assuming the room is well sealed, is the pressure of the air **(a)** greater than, **(b)** less than, or **(c)** the same as before you turned up the heat?

REASONING AND DISCUSSION

We assume that the number of molecules and the volume occupied by the molecules are approximately constant. Thus, increasing T, while holding N and V fixed, leads to an increased pressure, P.

ANSWER

(a) The air pressure increases.

Isotherms

Historically, the ideal-gas equation of state was arrived at piece by piece, as a result of the combined efforts of a number of researchers. For example, the English scientist Robert Boyle (1627–1691) established the fact that the pressure of a gas varies inversely with volume—as long as temperature and the number of molecules are held constant. This is known as **Boyle's law**:

$$P_iV_i = P_fV_f$$

$$(\textit{fixed number of molecules, } N\textit{; fixed temperature, } T) \qquad \text{17–7}$$

To see that Boyle's law is consistent with $PV = NkT$, note that if N and T are constant, then so too is PV. Another way of saying that PV is a constant is to say that its initial value must be equal to its final value. This is Boyle's law.

When N and T are constant, the equation of state $PV = NkT$ implies that

$$P = \frac{NkT}{V} = \frac{\text{constant}}{V}$$

This result is plotted in **Figure 17–5**, where we show P as a function of V. Note that the larger the temperature, T, the larger the constant in the numerator. Therefore, the curves farther from the origin correspond to higher temperatures, as indicated.

Each of the curves in Figure 17–5 corresponds to a different, fixed temperature. As a result, these curves are known as **isotherms,** which means, literally, "constant temperature." We shall return to the topic of isotherms in the next chapter, when we consider various thermal processes.

PROBLEM-SOLVING NOTE

Pressure and Volume in an Isotherm

If the temperature of an ideal gas is constant, the pressure and volume change in such a way that their product has a constant value.

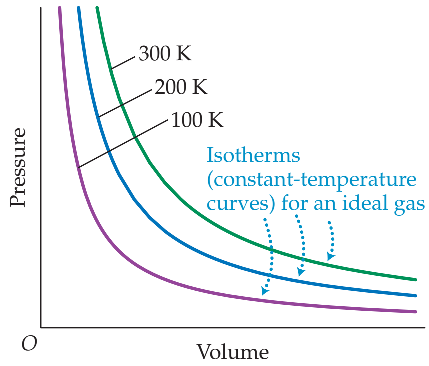


FIGURE 17–5 Ideal-gas isotherms

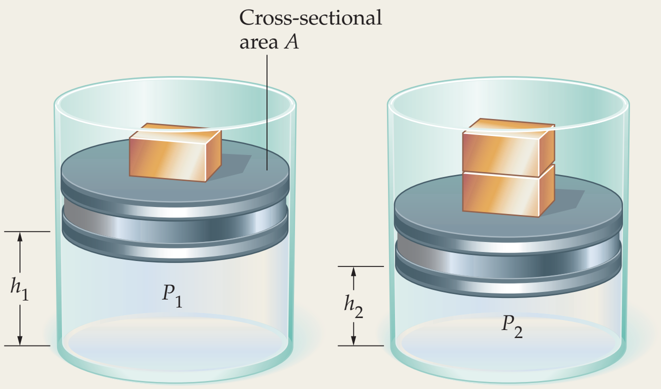
Pressure-versus-volume isotherms for an ideal gas. Each isotherm is of the form $P = NkT/V = \text{constant}/V$. The three isotherms shown are for the temperatures 100 K, 200 K, and 300 K.

EXAMPLE 17–2 UNDER PRESSURE

A cylindrical flask of cross-sectional area A is fitted with an airtight piston that is free to slide up and down. Contained within the flask is an ideal gas. Initially the pressure applied by the piston is 130 kPa and the height of the piston above the base of the flask is 25 cm. When additional mass is added to the piston, the pressure increases to 170 kPa. Assuming the system is always at the temperature 290 K, find the new height of the piston.

PICTURE THE PROBLEM

The physical system is shown in our sketch. The initial pressure in the flask is $P_1 = 130$ kPa, and the initial height of the piston is $h_1 = 25$ cm. When additional mass is placed on the piston, the pressure increases to $P_2 = 170$ kPa and the height decreases to h_2. The temperature remains constant.



STRATEGY

Since the temperature is held constant in this system, it follows that $PV = NkT$ is also constant. Thus, as in Boyle's law, $P_1V_1 = P_2V_2$. This relation can be used to find V_2 (since we are given P_1, P_2, and V_1).

Next, the cylindrical volume is related to the height by $V = Ah$. Note that the area A is the same in both cases, hence it will cancel, allowing us to solve for the height, h_2.

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SOLUTION

1. Set the initial and final values of PV equal to one another, then solve for V_2:
$$P_1V_1 = P_2V_2$$
$$V_2 = V_1(P_1/P_2)$$

2. Substitute $V_1 = Ah_1$ and $V_2 = Ah_2$, and solve for h_2:
$$Ah_2 = Ah_1(P_1/P_2)$$
$$h_2 = h_1(P_1/P_2)$$

3. Substitute numerical values:
$$h_2 = h_1\frac{P_1}{P_2} = (25\text{ cm})\frac{130\text{ kPa}}{170\text{ kPa}} = 19\text{ cm}$$

INSIGHT

In order to keep the temperature of this system constant, it is necessary to allow some heat to flow out of the cylindrical flask as the gas is compressed by the additional weight. Thus, the flask is not thermally insulated during this experiment.

PRACTICE PROBLEM

What pressure would be required to change the height of the piston to 29 cm? [**Answer:** $P = 110$ kPa]

Some related homework problems: Problem 18, Problem 19

▲ Charles's law states that the volume of a gas at constant pressure is proportional to its Kelvin temperature. For example, this balloon was fully inflated at room temperature (293 K) and atmospheric pressure. When cooled by vapors from liquid nitrogen (77 K), however, the volume of the air inside the balloon decreased markedly, causing it to shrivel.

Finally, recall that in Conceptual Checkpoint 15–2 we considered the behavior of an air bubble rising from a swimmer in shallow water. At the time, we said that the diameter of the bubble increases as it rises because the pressure of the surrounding water is decreasing. This is certainly the case, but now we can be more precise. If we assume that the water temperature is constant, and that no gas molecules are added to or removed from the bubble, then the volume of the bubble has the following dependence:

$$V = \frac{NkT}{P} = \frac{\text{constant}}{P}$$

This is just our isotherm, again, and we see that the volume indeed increases as pressure decreases.

Constant Pressure

Another aspect of ideal-gas behavior was discovered by the French scientist Jacques Charles (1746–1823) and later studied in greater detail by fellow Frenchman Joseph Gay-Lussac (1778–1850). Known today as **Charles's law,** their result is that the volume of a gas divided by its temperature is constant, as long as the pressure and number of molecules are constant:

$$\frac{V_i}{T_i} = \frac{V_f}{T_f}$$
$$(\textit{fixed number of molecules, } N\textit{; fixed pressure, } P) \qquad 17\text{–}8$$

As with Boyle's law, this result follows immediately from the ideal-gas equation of state. Solving Equation 17–2 for V/T, we have

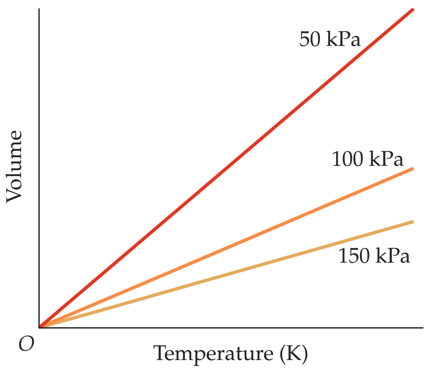
$$\frac{V}{T} = \frac{Nk}{P}$$

If N and P are constant, then so is the quantity V/T.

Charles's law can be rewritten as a linear relation between volume and temperature:

$$V = (\text{constant})T$$

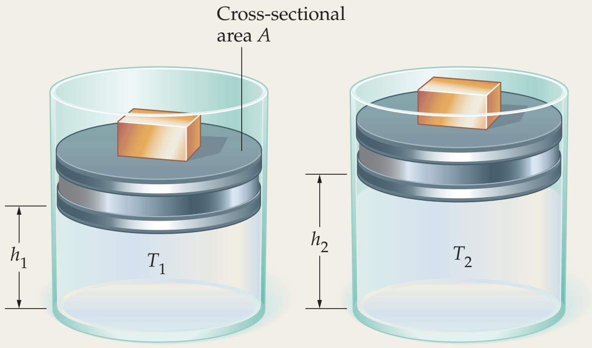
The constant in this expression is Nk/P. This result is illustrated in **Figure 17–6**, where we see the volume of an ideal gas vanishing as the temperature approaches absolute zero.



◀ **FIGURE 17–6 Volume versus temperature for an ideal gas at constant pressure** The temperature in this plot is the Kelvin temperature, T; hence $T = 0$ corresponds to absolute zero. At absolute zero the volume attains its lowest possible value—zero.

ACTIVE EXAMPLE 17–2 FIND THE PISTON HEIGHT

Consider again the system described in Example 17–2. In this case the temperature is changed from an initial value of 290 K to a final value of 330 K. The pressure exerted on the gas remains constant at 130 kPa, and the initial height of the piston is 25 cm. Find the final height of the piston.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Set the initial value of V/T equal to the final value of V/T: $V_1/T_1 = V_2/T_2$
2. Solve for V_2: $V_2 = V_1(T_2/T_1)$
3. Use $V = Ah$ to solve for the height, h_2: $h_2 = h_1(T_2/T_1)$
4. Substitute numerical values: $h_2 = 28 \text{ cm}$

INSIGHT

The volume of the gas, and hence the height of the piston, increased in direct proportion to the increase in temperature. This is to be expected when the pressure is constant.

YOUR TURN

At what temperature is the piston height equal to 19 cm?

(Answers to **Your Turn** *problems are given in the back of the book.)*

PROBLEM-SOLVING NOTE

Volume and Temperature in an Ideal Gas

If the pressure of an ideal gas is constant, the ratio of the volume and temperature remains constant. This is true no matter what the value of the pressure.

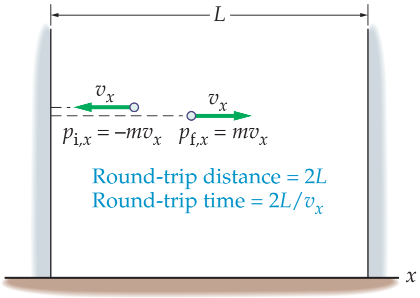
17–2 Kinetic Theory

We can readily measure the pressure and temperature of a gas using a pressure gauge and a thermometer. These are *macroscopic* quantities that apply to the gas as a whole. It is not so easy, however, to measure *microscopic* quantities, such as the position or velocity of an individual molecule. Still, there must be some connection between what happens on the microscopic level and what we observe on the macroscopic level. This connection is described by the **kinetic theory of gases.**

In the kinetic theory, we imagine a gas to be made up of a collection of molecules moving about inside a container of volume V. In particular, we assume the following:

- The container holds a very large number N of identical molecules, each of mass m. The molecules themselves can be thought of as "point particles"—that is, the volume of the molecules is negligible compared to the volume of the container, and the size of the molecules is negligible compared to the distance between them. (This is why dilute real gases are the best approximation to the ideal case.)
- The molecules move about the container in a random manner. They obey Newton's laws of motion at all times.
- When molecules hit the walls of the container or collide with one another, they bounce elastically. Other than these collisions, the molecules have no interactions.

With these basic assumptions we can relate the pressure of a gas to the behavior of the molecules themselves.



▲ **FIGURE 17–7 Force exerted by a molecule on the wall of a container**

A molecule bounces off a wall of a container, changing its momentum from $-mv_x$ to $+mv_x$; the change in momentum is $2mv_x$. A round trip will be completed in the time $\Delta t = 2L/v_x$, so the average force exerted on the molecule by the wall is $F = \Delta p/\Delta t = 2mv_x/(2L/v_x) = mv_x{}^2/L$.

The Origin of Pressure

As we shall see, the pressure exerted by a gas is due to the innumerable collisions between gas molecules and the walls of their container. Each collision results in a change of momentum for a given molecule, just like throwing a ball at a wall and having it bounce back. The total change in momentum of the molecules in a given time, divided by the time, is simply the force a wall must exert on the gas to contain it (see Section 9–2). The average of this force over time, and over the area of a wall, is the pressure of the gas.

To be specific, imagine a container that is a cube of length L on a side. Its volume, then, is $V = L^3$. In addition, consider a given molecule that happens to be moving in the negative x direction toward a wall, as in **Figure 17–7**. If its speed is v_x, its initial momentum is $p_{i,x} = -mv_x$. After bouncing from the wall, it moves in the positive x direction with the same speed (since the collision is elastic), and hence its final momentum is $p_{f,x} = +mv_x$. As a result, the molecule's change in momentum is

$$\Delta p_x = p_{f,x} - p_{i,x} = mv_x - (-mv_x) = 2mv_x$$

The wall exerts a force on the molecule to cause this momentum change.

After the bounce, the molecule travels to the other side of the container and back before bouncing off the same wall again. The time required for this round trip of length $2L$ is

$$\Delta t = 2L/v_x$$

Thus, by Newton's second law, the average force exerted by the wall on the molecule is

$$F = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x{}^2}{L}$$

The average pressure exerted by this wall, then, is simply the force divided by the area. Since the area of the wall is $A = L^2$, we have

$$P = \frac{F}{A} = \frac{(mv_x{}^2/L)}{L^2} = \frac{mv_x{}^2}{L^3} = \frac{mv_x{}^2}{V} \qquad \text{17–9}$$

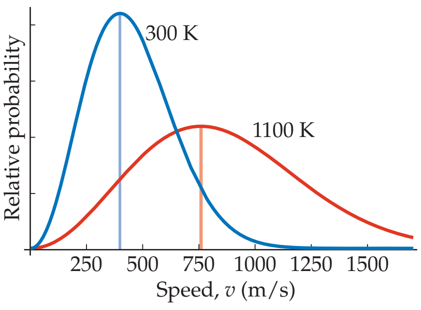
Note that we have used the fact that the volume of the container is L^3.

In this calculation we assumed that the molecule moves in the x direction. This was merely to simplify the derivation. If, instead, the molecule moves at some angle to the x axis, the calculation applies to its x component of motion. The final conclusions are unchanged.

Speed Distribution of Molecules

In deriving Equation 17–9, we considered a single molecule with a particular speed. Other molecules, of course, will have different speeds. In addition, the speed of any given molecule changes with time as it collides with other molecules in the gas. What remains constant, however, is the overall **distribution of speeds.**

This is illustrated in **Figure 17–8**, where we present the results obtained by the Scottish physicist James Clerk Maxwell (1831–1879). This plot shows the relative probability that an O_2 molecule will have a given speed. For example, on the curve labeled 300 K, the most probable speed is about 390 m/s. When the temperature is increased to 1100 K, the most probable speed increases to roughly 750 m/s. Other speeds occur as well, from speeds near zero to those that are very large, but these have much lower probabilities.



▲ **FIGURE 17–8 The Maxwell speed distribution**

The Maxwell distribution of molecular speeds for O_2 at the temperatures $T = 300$ K and $T = 1100$ K. Note that the most probable speed increases with increasing temperature.

Thus, in Equation 17–9, the term $v_x{}^2$ should be replaced with the average of $v_x{}^2$ over all the molecules in the gas. Writing this average as $(v_x{}^2)_{av}$, we have

$$P = \frac{m(v_x{}^2)_{av}}{V}$$

Since all N molecules in the gas follow the same distribution, the pressure exerted by the gas as a whole is N times this result:

$$P = N\frac{m(v_x{}^2)_{av}}{V} = \left(\frac{N}{V}\right)m(v_x{}^2)_{av} \qquad \text{17–10}$$

Of course, there is nothing special about the x direction—Equation 17–10 applies equally well with $(v_y{}^2)_{av}$ or $(v_z{}^2)_{av}$ in place of $(v_x{}^2)_{av}$. Thus, it would be preferable to express the pressure of the gas in terms of the overall speed of the molecules, rather than in terms of a single component. The speed squared of a molecule is

$$v^2 = v_x{}^2 + v_y{}^2 + v_z{}^2$$

Hence, the average of v^2 is

$$(v^2)_{av} = (v_x{}^2)_{av} + (v_y{}^2)_{av} + (v_z{}^2)_{av}$$

Since the x, y, and z directions are equivalent, it follows that

$$(v_x{}^2)_{av} = (v_y{}^2)_{av} = (v_z{}^2)_{av}$$

As a result,

$$(v^2)_{av} = (v_x{}^2)_{av} + (v_y{}^2)_{av} + (v_z{}^2)_{av} = 3(v_x{}^2)_{av}$$

or, equivalently,

$$(v_x{}^2)_{av} = \tfrac{1}{3}(v^2)_{av}$$

Using this replacement in Equation 17–10 yields

$$P = \tfrac{1}{3}\left(\frac{N}{V}\right)m(v^2)_{av}$$

The last part of this expression, $m(v^2)_{av}$, is simply twice the average kinetic energy of a molecule. Thus, using K for the kinetic energy, as in Chapters 7 and 8, we can write the pressure as follows:

Pressure in the Kinetic Theory of Gases

$$P = \tfrac{1}{3}\left(\frac{N}{V}\right)2K_{av} = \tfrac{2}{3}\left(\frac{N}{V}\right)\left(\tfrac{1}{2}mv^2\right)_{av} \qquad 17\text{–}11$$

To summarize, using the kinetic theory, we have shown that the pressure of a gas is proportional to the number of molecules and inversely proportional to the volume. We discussed both of these dependences before when considering the ideal gas. In addition, we see that

> The pressure of a gas is directly proportional to the average kinetic energy of its molecules.

This is the key connection between microscopic behavior and macroscopic observables.

Kinetic Energy and Temperature

If we compare the ideal-gas equation of state, $PV = NkT$, with the result from kinetic theory, Equation 17–11, we find

$$PV = NkT = \tfrac{2}{3}N\left(\tfrac{1}{2}mv^2\right)_{av}$$

As a result,

$$\tfrac{2}{3}\left(\tfrac{1}{2}mv^2\right)_{av} = kT$$

Equivalently,

Kinetic Energy and Temperature

$$\left(\tfrac{1}{2}mv^2\right)_{av} = K_{av} = \tfrac{3}{2}kT \qquad 17\text{–}12$$

This is one of the most important results of kinetic theory. It says that the average kinetic energy of a gas molecule is directly proportional to the Kelvin temperature, T. Thus, when we heat a gas, what happens on the microscopic level is that the molecules move with speeds that are, on average, greater. Similarly, cooling a gas causes the molecules to slow.

EXERCISE 17–2

Find the average kinetic energy of oxygen molecules in the air. Assume the air is at a temperature of 21 °C.

SOLUTION

Using Equation 17–12, and the fact that 21 °C = 294 K, we find

$$K_{av} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(294\ \text{K}) = 6.09 \times 10^{-21}\ \text{J}$$

Note that this is also the average kinetic energy of nitrogen molecules in the air. In fact, the type of molecule is not important; all that matters is the temperature of the gas.

Returning to Equation 17–12, we find with a slight rearrangement that

$$(v^2)_{av} = 3kT/m$$

Now, the square root of $(v^2)_{av}$ is given a special name—it is called the **root mean square** (rms) speed. For gas molecules, then, the rms speed, v_{rms}, is the following:

RMS Speed of a Gas Molecule

$$v_{rms} = \sqrt{(v^2)_{av}} = \sqrt{\frac{3kT}{m}} \qquad 17\text{–}13$$

SI unit: m/s

Rewriting this in terms of the molecular mass, M, we have

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{(M/N_A)}} = \sqrt{\frac{3N_AkT}{M}}$$

Finally, using $N_Ak = R$ yields

$$v_{rms} = \sqrt{\frac{3RT}{M}} \qquad 17\text{–}14$$

The rms speed is one of the characteristic speeds of the Maxwell speed distribution. As shown in **Figure 17–9**, v_{rms} is slightly greater than the most probable speed, v_{mp} and the average speed, v_{av}. (Homework Problem 88 illustrates the difference between v_{av} and v_{rms} for a simple system of molecules.)

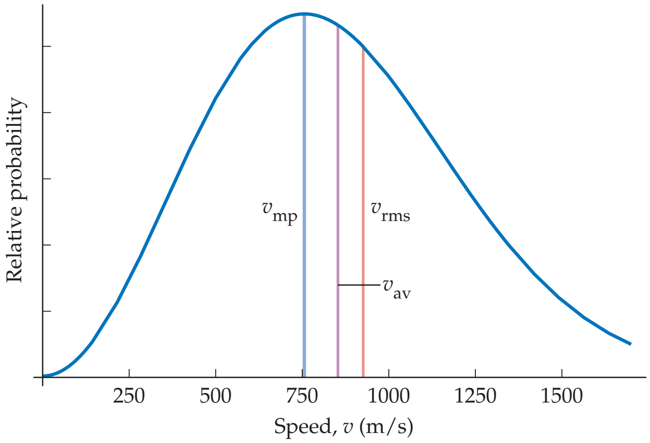


FIGURE 17–9 Most probable, average, and rms speeds

Characteristic speeds for O_2 at the temperature $T = 1100$ K. From left to right, the indicated speeds are the most probable speed, v_{mp}, the average speed, v_{av}, and the rms speed, v_{rms}.

EXAMPLE 17–3 FRESH AIR

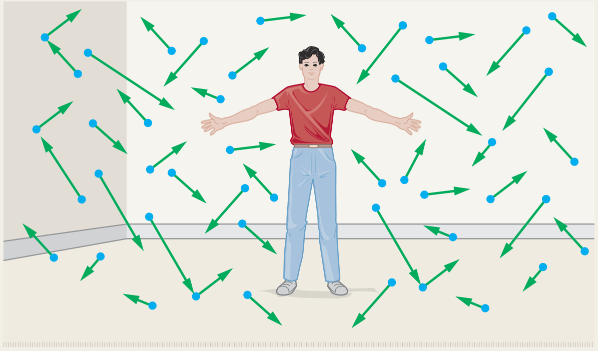
The atmosphere is composed primarily of nitrogen N_2 (78%) and oxygen O_2 (21%). **(a)** Is the rms speed of N_2 (28.0 g/mol) greater than, less than, or the same as the rms speed of O_2 (32.0 g/mol)? **(b)** Find the rms speed of N_2 and O_2 at 293 K.

PICTURE THE PROBLEM

Our sketch shows molecules in the air bouncing off the walls of a room—they also bounce off a person in the room. Note that the molecules move in random directions with a variety of speeds. Most of the molecules are N_2, but about 21% are O_2.

STRATEGY

The rms speeds are calculated by straightforward substitution into the relation $v_{rms} = \sqrt{3RT/M}$. The only point to be careful about is the molecular mass of each molecule. In particular, note that for nitrogen the molecular mass is 28.0 g/mol = 0.0280 kg/mol, and similarly for oxygen, the molecular mass is 32.0 g/mol = 0.0320 kg/mol.



SOLUTION

Part (a)

Since both molecules are at the same temperature, they have the same kinetic energy. The nitrogen molecule has less mass, however; thus, if it is to have the same kinetic energy, it must have a higher speed. This is also what one would expect on the basis of Equation 17–14.

Part (b)

1. To find the rms speed of N_2, substitute $M = 0.0280$ kg/mol in Equation 17–14:

$$v_{rms} = \sqrt{\frac{3[8.31\ \mathrm{J/(mol\cdot K)}](293\ \mathrm{K})}{0.0280\ \mathrm{kg/mol}}} = 511\ \mathrm{m/s}$$

2. To find the rms speed of O_2, substitute $M = 0.0320$ kg/mol in Equation 17–14:

$$v_{rms} = \sqrt{\frac{3[8.31\ \mathrm{J/(mol\cdot K)}](293\ \mathrm{K})}{0.0320\ \mathrm{kg/mol}}} = 478\ \mathrm{m/s}$$

INSIGHT

As expected, N_2 has the higher rms speed. For comparison, the speed of sound at 293 K is 343 m/s, which is just over 750 mi/h. Thus, the molecules in the air are bouncing off your skin with the speed of a supersonic jet.

Such high speeds can help to promote chemical reactions, which usually depend on collisions between molecules to take place. In general, higher temperatures mean higher molecular speeds and increased rates of chemical reactions.

PRACTICE PROBLEM

One of the substances found in air is monatomic and has an rms speed of 428 m/s, at 293 K. What is this substance? [**Answer:** $M = 0.0399$ kg/mol; thus the substance is argon, Ar, which comprises about 0.94% of the atmosphere].

Some related homework problems: Problem 26, Problem 27

Chemical reactions usually depend on collisions between molecules to take place. Though the molecules in the air are not moving fast enough to trigger chemical reactions, high molecular speeds can help to promote chemical reactions in many circumstances. In general, higher temperatures mean higher molecular speeds and increased rates of chemical reactions. This is why the rate at which a cricket chirps increases with increasing temperature, making it a novel type of thermometer. (See the Passage Problem in Chapter 13 and Problem 75 in Chapter 16.)

REAL-WORLD PHYSICS: BIO

Chemical reactions and temperature

CONCEPTUAL CHECKPOINT 17–2 COMPARE SPEEDS

Each of two containers of equal volume holds an ideal gas. Container A has twice as many molecules as container B. If the gas pressure is the same in the two containers, is the rms speed of the molecules in container A **(a)** greater than, **(b)** less than, or **(c)** the same as the rms speed of the molecules in container B?

REASONING AND DISCUSSION

Since P and V are the same, you might think the rms speeds are the same as well. Recall, however, that *the pressure of a gas is caused by the collisions of gas molecules with the walls of the container*. If more molecules occupy a given volume, and they bounce off the walls with the same speed as in a container with fewer molecules, the pressure will be greater. Therefore, in order for the pressure in the two containers to be the same, it is necessary that the rms speed of the molecules in container A be less than the rms speed in container B.

Mathematically, we note that $P_A = P_B$ and $V_A = V_B$; hence $P_A V_A = P_B V_B$. Using the ideal-gas relation, $PV = NkT$, we can write this condition as $N_A k T_A = N_B k T_B$. Thus, if $N_A = 2N_B$, it follows that $T_A = T_B/2$. Since the temperature is lower in container A, its molecules have the smaller rms speed.

ANSWER

(b) The rms speed is less in container A.

The Internal Energy of an Ideal Gas

The internal energy of a substance is the sum of all its potential and kinetic energies. In an ideal gas there are no interactions between molecules, other than perfectly elastic collisions; hence, there is no potential energy. As a result, the total energy of the system is the sum of the kinetic energy of each of its molecules. Thus, for an ideal gas of N pointlike molecules—that is, a monatomic gas—the internal energy is simply

Internal Energy of a Monatomic Ideal Gas

$$U = \tfrac{3}{2}NkT \qquad 17\text{–}15$$

SI unit: J

In terms of moles, this result is

$$U = \tfrac{3}{2}nRT \qquad 17\text{–}16$$

We shall return to this result in the next chapter.

EXERCISE 17–3

A basketball at 290 K holds 0.95 mol of air molecules. What is the internal energy of the air in the ball?

SOLUTION

Applying Equation 17–16 we find

$$U = \tfrac{3}{2}nRT = \tfrac{3}{2}(0.95\ \text{mol})[8.31\ \text{J}/(\text{mol}\cdot\text{K})](290\ \text{K}) = 3400\ \text{J}$$

This is roughly the kinetic energy a basketball would have if you dropped it from a height of 700 m.

If an ideal gas is diatomic, there are additional contributions to the internal energy. For example, a diatomic molecule, which is shaped somewhat like a dumbbell, can have rotational kinetic energy. Diatomic molecules can also vibrate along the line joining the two atoms, which is yet another contribution to the total energy. Thus, the results in Equations 17–15 and 17–16 apply only to the simplest case, the ideal monatomic gas.

17–3 Solids and Elastic Deformation

The defining characteristic of a solid object is that it has a particular shape. For example, when Michelangelo carved the statue of David from a solid block of marble, he was confident it would retain its shape long after his work was done. Liquids and gases do not behave in this way. In contrast, they assume the shape of the container into which they are placed. On a molecular level, these differences arise from the fact that the intermolecular forces in a solid are strong enough to practically immobilize its molecules, whereas the intermolecular forces in liquids and gases are so weak the molecules can move about relatively freely.

Even so, the shape of a solid *can* be changed—though usually only slightly—if it is acted on by a force. In this section we consider various types of deformations that can occur in solids and the way these deformations are related to the forces that cause them.

Changing the Length of a Solid

A useful physical model for a solid is a lattice of small balls representing molecules connected to one another by springs representing the intermolecular forces. Pulling on a solid rod with a force F, for example, causes each "intermolecular spring" in the direction of the force to expand by an amount proportional to F. The net result is that the entire solid increases its length by an amount proportional to the force, $\Delta L \propto F$, as indicated in **Figure 17–10**.

The stretch ΔL is also proportional to the initial length of the rod, L_0. To see why, we first note that each intermolecular spring expands by the same amount, giving a total stretch that is proportional to the number of such springs. But the

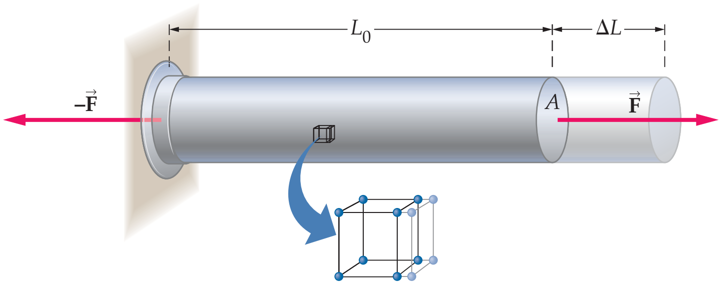


FIGURE 17–10 Stretching a rod

Equal and opposite forces applied to the ends of a rod cause it to stretch. On the atomic level, the forces stretch the "intermolecular springs" in the solid, resulting in an overall increase in length. The stretch is proportional to the force F and the initial length L_0, and inversely proportional to the cross-sectional area A.

number of intermolecular springs is proportional to the total initial length of the rod. Thus, it follows that $\Delta L \propto FL_0$.

Finally, the amount of stretch for a given force F is inversely proportional to the cross-sectional area A of the rod. For example, a rod with a cross-sectional area $2A$ is like two rods of cross-sectional area A placed side by side. Thus, applying a force F to a rod of area $2A$ is equivalent to applying a force $F/2$ to two rods of area A. The result is half the stretch when the area is doubled; that is, $\Delta L \propto FL_0/A$. Solving this relation for the amount of force F required to produce a given stretch ΔL we find $F \propto (\Delta L/L_0)A$, or as an equality

$$F = Y\left(\frac{\Delta L}{L_0}\right)A \qquad \text{17–17}$$

The proportionality constant Y in this expression is **Young's modulus,** named for the English physicist Thomas Young (1773–1829). Comparing the two sides of Equation 17–17, we see that Young's modulus has the units of force per area $(\mathrm{N/m^2})$.

Typical values for Young's modulus are given in Table 17–1. Notice that the values vary from material to material, but are all rather large. This means that a large force is required to cause even a small stretch in a solid. Of course, Equation 17–17 applies equally well to a compression or a stretch. However, some materials have a slightly different Young's modulus for compression and stretching. For example, human bones under tension (stretching) have a Young's modulus of $1.6 \times 10^{10}\ \mathrm{N/m^2}$, while bones under compression have a slightly smaller Young's modulus of $9.4 \times 10^9\ \mathrm{N/m^2}$.

TABLE 17–1 Young's Modulus for Various Materials

Material	Young's modulus, Y $(\mathrm{N/m^2})$
Tungsten	36×10^{10}
Steel	20×10^{10}
Copper	11×10^{10}
Brass	9.0×10^{10}
Aluminum	6.9×10^{10}
Pyrex glass	6.2×10^{10}
Lead	1.6×10^{10}
Bone	
Tension	1.6×10^{10}
Compression	0.94×10^{10}
Nylon	0.37×10^{10}

PROBLEM-SOLVING NOTE

Area and Young's Modulus

When using Young's modulus, remember that the area A is the area that is at right angles to the applied force.

EXAMPLE 17–4 STRETCHING A BONE

REAL-WORLD PHYSICS: BIO

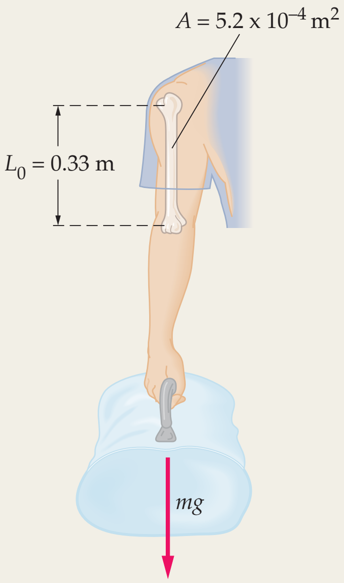
At the local airport, a person carries a 21-kg duffel bag in one hand. Assuming the humerus (the upper arm bone) supports the entire weight of the bag, determine the amount by which it stretches. (The humerus may be assumed to be 33 cm in length and to have an effective cross-sectional area of $5.2 \times 10^{-4}\ \mathrm{m^2}$.)

PICTURE THE PROBLEM

The humerus is oriented vertically, hence the weight of the duffel bag applies tension to the bone. The Young's modulus in this case is $1.6 \times 10^{10}\ \mathrm{N/m^2}$. In addition, we note that the initial length of the bone is $L_0 = 0.33\ \mathrm{m}$ and its cross-sectional area is $A = 5.2 \times 10^{-4}\ \mathrm{m^2}$.

STRATEGY

We can find the amount of stretch by solving the relation $F = Y(\Delta L/L_0)A$ for the quantity ΔL. Note that the force applied to the bone is simply the weight of the suitcase, $F = mg$, with $m = 21$ kg. (We ignore the relatively small weight of the forearm and hand.)



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SOLUTION

1. Solve Equation 17–17 for the amount of stretch, ΔL: $\Delta L = \dfrac{FL_0}{YA}$
2. Calculate the force applied to the humerus: $F = mg = (21\text{ kg})(9.81\text{ m/s}^2) = 210\text{ N}$
3. Substitute numerical values into the expression for ΔL:

$$\Delta L = \frac{FL_0}{YA} = \frac{(210\text{ N})(0.33\text{ m})}{(1.6 \times 10^{10}\text{ N/m}^2)(5.2 \times 10^{-4}\text{ m}^2)} = 8.3 \times 10^{-6}\text{ m}$$

INSIGHT

We find that the amount of stretch is imperceptibly small. The reason for this, of course, is that Young's modulus is such a large number. If the bone had been compressed rather than stretched, its change in length, though still minuscule, would have been greater by a factor of 16/9.4.

PRACTICE PROBLEM

Suppose the humerus is reduced uniformly in size by a factor of 2. This means that both the length and diameter are halved. What is the amount of stretch in this case? [**Answer:** Since $L_0 \rightarrow L_0/2$ and $A \rightarrow A/4$, the stretch is doubled. Thus, $\Delta L \rightarrow 2\,\Delta L = 1.7 \times 10^{-5}$ m.]

Some related homework problems: Problem 37, Problem 38

There is a straightforward connection between Equation 17–17 and Hooke's law for a spring (Equation 6–4). To see the connection, we rewrite Equation 17–17 as follows:

$$F = \left(\frac{YA}{L_0}\right)\Delta L$$

Notice that the force required to cause a certain stretch is proportional to the stretch—just as in Hooke's law. In fact, if we identify ΔL with the displacement x of a spring from equilibrium and YA/L_0 with the force constant k, we have Hooke's law:

$$F = \left(\frac{YA}{L_0}\right)\Delta L = kx$$

Thus, we see that the force constant of a spring depends on the Young's modulus, Y, of the material from which it is made, the cross-sectional area A of the wire, and the length of the wire, L_0.

▶ The cables in this suspension bridge have significant forces pulling on them from either end. As a result, they are under tension, just like the rod in Figure 17–10. It follows that the length of each cable increases by an amount that is proportional to its initial length. The dam, on the other hand, experiences forces that tend to compress it. Still, the magnitude of its change in length can be described in terms of Equation 17–17, with the appropriate value of Young's modulus.

CONCEPTUAL CHECKPOINT 17–3 COMPARE FORCE CONSTANTS

Two identical springs are connected end to end. Is the force constant of the resulting compound spring **(a)** greater than, **(b)** less than, or **(c)** equal to that of a single spring?

REASONING AND DISCUSSION

It might seem that the force constant would be the same, since we simply have twice the length of the same spring. On the other hand, it might seem that the force constant is greater, since two springs are exerting a force rather than just one. In fact, the force constant decreases.

The reason for the reduced force constant is that if we apply a force F to the compound spring, each individual spring stretches by a certain amount. The compound spring, then, stretches by twice this amount. By Hooke's law, $F = kx$, a spring that stretches twice as far for the same applied force has half the force constant.

We can also obtain this result by recalling that the force constant is $k = YA/L_0$. Thus, if the length of a spring is doubled—as in the compound spring—the force constant is halved.

ANSWER

(b) The force constant of the compound spring is less, by a factor of 2.

Changing the Shape of a Solid

Another type of deformation, referred to as a **shear deformation,** changes the *shape* of a solid. Consider a book of thickness L_0 resting on a table, as shown in **Figure 17–11**. A force F is applied to the right on the top cover of the book, and static friction applies a force F to the left on the bottom cover of the book. The result is that the book remains at rest but becomes slanted by the amount Δx. The force required to cause a given amount of slant is proportional to Δx, inversely proportional to the thickness of the book L_0, and proportional to the surface area A of the book's cover; that is, $F \propto A\Delta x/L_0$. Writing this as an equality, we have

$$F = S\left(\frac{\Delta x}{L_0}\right)A \qquad 17\text{–}18$$

The constant of proportionality in this case is the **shear modulus,** S. Like Young's modulus, the shear modulus has the units N/m^2. Typical values of the shear modulus are collected in Table 17–2. As with Young's modulus, the shear modulus is large in magnitude, meaning that most solids require a large force to cause even a small amount of shear.

TABLE 17–2 Shear Modulus for Various Materials

Material	Shear modulus, S (N/m^2)
Tungsten	15×10^{10}
Steel	8.1×10^{10}
Bone	8.0×10^{10}
Copper	4.2×10^{10}
Brass	3.5×10^{10}
Aluminum	2.4×10^{10}
Lead	0.54×10^{10}

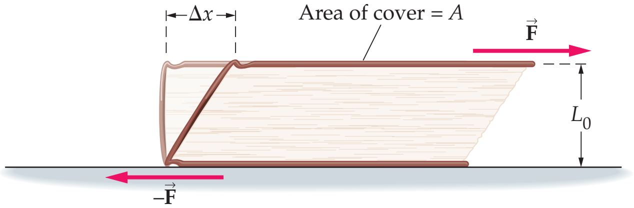

FIGURE 17–11 Shear deformation
Equal and opposite forces applied to the top and bottom of a book result in a shear deformation. The amount of deformation is proportional to the force F and the thickness of the book L_0, and inversely proportional to the area A.

Equations 17–17 and 17–18 are similar in structure, but it is important to be aware of their differences as well. For example, the term L_0 in the Young's modulus equation refers to the length of a solid measured in the direction of the applied force. In contrast, L_0 in the shear modulus equation refers to the thickness of the solid as measured in a direction perpendicular to the applied force. Similarly, the area A in Equation 17–17 is the cross-sectional area of the solid, and this area is perpendicular to the applied force. On the other hand, the area A in Equation 17–18 is the area of the solid in the plane of the applied force.

PROBLEM-SOLVING NOTE

Area and the Shear Modulus

When using the shear modulus, remember that the area A is the area that is parallel to the applied force.

ACTIVE EXAMPLE 17–3 DEFORMING A STACK OF PANCAKES: FIND THE SHEAR MODULUS

A horizontal force of 1.2 N is applied to the top of a stack of pancakes 13 cm in diameter and 9.0 cm high. The result is a shear deformation of 2.5 cm. What is the shear modulus for these pancakes?

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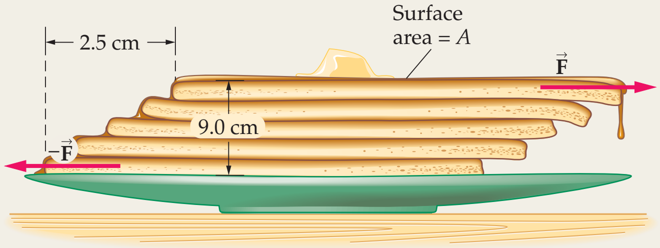


SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 17–18 for the shear modulus: $S = FL_0/A\Delta x$
2. Calculate the area of the pancakes: $A = \pi d^2/4 = 0.013\ \text{m}^2$
3. Substitute numerical values: $S = 330\ \text{N/m}^2$

INSIGHT

Notice the small value of the pancakes' shear modulus, especially when compared to the shear modulus of a typical metal. This is a reflection of the fact that the pancake stack is easily deformed.

YOUR TURN

Suppose the stack of pancakes is doubled in height. By what factor does the shear deformation change?

(Answers to **Your Turn** *problems are given in the back of the book.)*

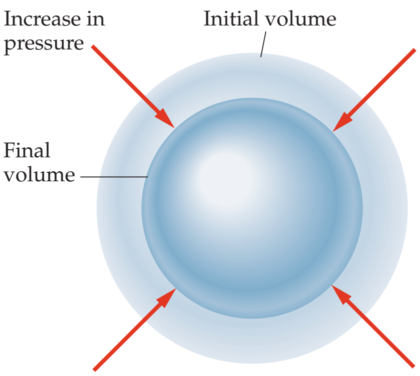


FIGURE 17–12 Changing the volume of a solid

As the pressure surrounding an object increases, its volume decreases. The amount of volume change is proportional to the initial volume and to the change in pressure.

Changing the Volume of a Solid

If a piece of Styrofoam is taken deep into the ocean, the tremendous pressure of the water causes it to shrink to a fraction of its original volume. This is an extreme example of the volume change that occurs in all solids when the pressure of their surroundings is changed. The general situation is illustrated in Figure 17–12, where we show a spherical solid whose volume decreases by the amount ΔV when the pressure acting on it increases by the amount ΔP. Experiments show that the pressure difference required to cause a given change in volume, ΔV, is proportional to ΔV and inversely proportional to the initial volume of the object, V_0. Therefore, we can write ΔP as follows:

$$\Delta P = -B\left(\frac{\Delta V}{V_0}\right) \tag{17–19}$$

The constant of proportionality in this case, B, is called the **bulk modulus.** As with Young's modulus and the shear modulus, the bulk modulus is defined to be a positive quantity; hence the minus sign in Equation 17–19. For example, if the pressure increases ($\Delta P > 0$), the volume will decrease ($\Delta V < 0$) and the quantity $-\Delta V$ will be positive. Since ΔP is equal to $B(-\Delta V/V_0)$, and V_0 is always positive, it follows that B must be positive as well.

Table 17–3 gives a list of representative values of the bulk modulus. Note the large magnitudes of B, indicating that even small volume changes require large changes in pressure. Finally, the units of B are N/m^2, as is clear from Equation 17–19.

TABLE 17–3 Bulk Modulus for Various Materials

Material	Bulk modulus, B (N/m^2)
Gold	22×10^{10}
Tungsten	20×10^{10}
Steel	16×10^{10}
Copper	14×10^{10}
Aluminum	7.0×10^{10}
Brass	6.1×10^{10}
Ice	0.80×10^{10}
Water	0.22×10^{10}
Oil	0.17×10^{10}

ACTIVE EXAMPLE 17–4 A GOLD DOUBLOON: FIND THE CHANGE IN VOLUME

A gold doubloon 6.1 cm in diameter and 2.0 mm thick is dropped over the side of a pirate ship. When it comes to rest on the ocean floor at a depth of 770 m, how much has its volume changed?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Solve Equation 17–19 for the change in volume: $\Delta V = -V_0\,\Delta P/B$
2. Calculate the initial volume of the doubloon (a cylinder): $V_0 = 5.8 \times 10^{-6}\ \text{m}^3$
3. Find the change in pressure due to the depth of seawater. Use Equation 15–7: $\Delta P = \rho g h = 7.7 \times 10^6\ \text{N/m}^2$
4. Substitute numerical values: $\Delta V = -2.0 \times 10^{-10}\ \text{m}^3$

INSIGHT

The change in volume is imperceptibly small. Clearly, enormous pressures must be applied to metals to cause a significant change in volume.

YOUR TURN

Suppose the "gold" doubloon is actually made of brass. Do you expect the change in volume in this case to be greater than or less than for gold? Verify your answer with a calculation.

(Answers to **Your Turn** *problems are given in the back of the book.)*

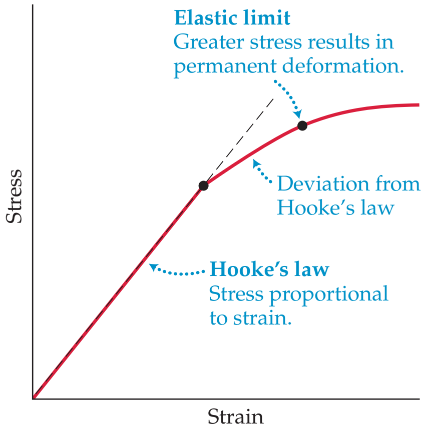
▲ Styrofoam has a very small bulk modulus, which means that even a relatively small increase in pressure can cause a large decrease in volume. The Styrofoam cup at left was immersed to a depth of 1955 m, where the water pressure is nearly 200 atm.

Stress and Strain

Equation 17–19 appears to differ from Equations 17–17 and 17–18 because of the absence of the area A on the right-hand side of the equation. When one recalls that pressure is a force per area, however, we can see that the area in Equation 17–19 is contained in the denominator of the left-hand side. Similarly, we can rewrite Equation 17–17 as $F/A = Y(\Delta L/L_0)$ and Equation 17–18 as $F/A = S(\Delta x/L_0)$. Written in this way, each of these equations states that a deformation of a particular type is proportional to a corresponding applied force per area.

In general, we refer to an applied force per area as a **stress** and the resulting deformation as a **strain.** If the stress applied to an object is not too large, the proportional relationship between the strain and stress is found to hold, and a plot of strain versus stress gives a straight line, as indicated in **Figure 17–13**. This straight-line relationship is simply a generalization of Hooke's law—extending the result for a spring to any solid object. As the stress becomes larger, the strain eventually begins to increase at a rate that is greater than the straight line.

A change in behavior occurs when the stress reaches the elastic limit. For stresses less than the elastic limit, the deformation of an object is reversible; that is, the deformation vanishes as the stress that caused it is reduced to zero. This is just like a spring returning to its equilibrium position when the applied force vanishes. When a deformation is reversible, we say that it is an **elastic deformation.** For stresses greater than the elastic limit, on the other hand, the object becomes permanently deformed, and it will not return to its original size and shape when the stress is removed. This is like a spring that has been stretched too far, or a car fender that has been dented. If the stress on an object is increased even further, the object eventually tears apart or fractures.



▲ **FIGURE 17–13 Stress versus strain**

When stress and strain are small they are proportional, as given by Hooke's law. Larger stresses can result in deviations from Hooke's law. Stresses greater than the elastic limit cause permanent deformation of the material.

17–4 Phase Equilibrium and Evaporation

To this point, we have studied the behavior of substances when they are in a single phase of matter. For example, we considered the properties of liquids in Chapter 15, and in this chapter we have turned our attention to the characteristics of gases and solids. It is common, however, for a substance to exist in more than one phase at a time. In particular, if a substance has two or more phases that coexist in a steady, stable fashion, we say that the substance exhibits **phase equilibrium.**

To see just what this means in a specific case, consider a closed container that is partially filled with a liquid, as in **Figure 17–14**. The container is kept at the constant temperature T_0, and initially the volume above the liquid is empty of

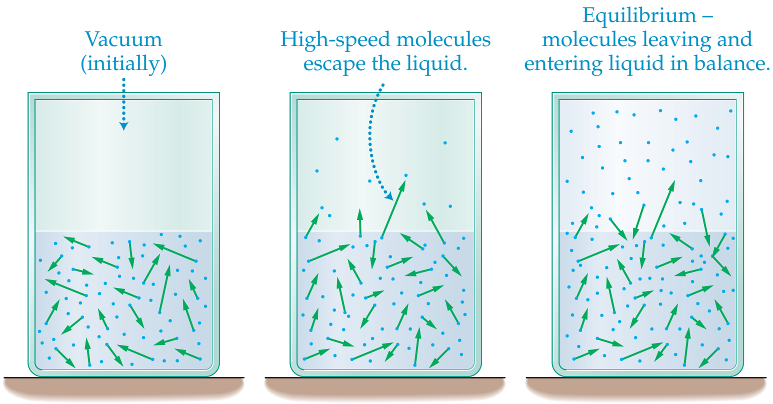


FIGURE 17–14 A liquid in equilibrium with its vapor

Initially, a liquid is placed in a container, and the volume above it is a vacuum. High-speed molecules in the liquid are able to escape into the upper region of the container, forming a low-density gas. As the gas becomes more dense, the number of molecules leaving the liquid is balanced by the number returning to the liquid. At this point the system is in equilibrium.

molecules—it is a vacuum. Soon, however, some of the faster molecules in the liquid begin to escape the relatively strong intermolecular forces of their neighbors in the liquid and start to form a low-density gas. Occasionally a gas molecule collides with the liquid and reenters it, but initially, more molecules are entering the gas than returning to the liquid.

This process continues until the gas is dense enough that the number of molecules returning to the liquid equals the number entering the gas. There is a constant "flow" of molecules in both directions, but when *phase equilibrium* is reached, these flows cancel, and the number of molecules in each phase remains constant. The pressure of the gas when equilibrium is established is referred to as the **equilibrium vapor pressure.** In **Figure 17–15** we plot the equilibrium vapor pressure of water.

What happens when we change the temperature? Well, if we increase the temperature, there will be more high-speed molecules in the liquid that can escape into the gas. Thus, to have an equal number of gas molecules returning to the liquid, it will be necessary for the pressure of the vapor to be greater. Thus, the equilibrium vapor pressure increases with temperature. This is also illustrated in Figure 17–15.

For each temperature there is just one equilibrium vapor pressure—that is, just one pressure where the precise balance between the phases is established. Thus, when we plot the equilibrium vapor pressure versus temperature, as in Figure 17–15, the result is a curve—the **vapor-pressure curve.** The significance of this curve is that it determines the boiling point of a liquid. In particular:

> A liquid boils at the temperature at which its vapor pressure equals the external pressure.

Note in Figure 17–15 that a vapor pressure equal to atmospheric pressure, $P_{at} = 101$ kPa, occurs when the temperature of the water is 100 °C, as expected.

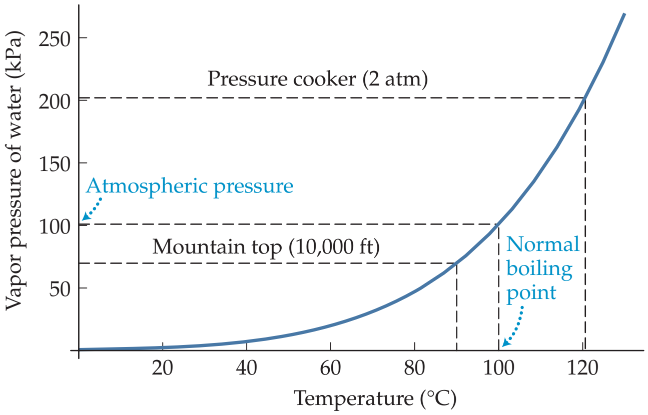


FIGURE 17–15 The vapor-pressure curve for water

The vapor pressure of water increases with increasing temperature. In particular, at $T = 100$ °C the vapor pressure is one atmosphere, 101 kPa.

CONCEPTUAL CHECKPOINT 17–4 BOILING TEMPERATURE

When water boils at the top of a mountain, is its temperature **(a)** higher than, **(b)** lower than, or **(c)** equal to 100 °C?

REASONING AND DISCUSSION

At the top of a mountain, air pressure is less than it is at sea level. Therefore, according to Figure 17–15, the boiling temperature will be lower as well.

For example, the atmospheric pressure at the top of a 10,000-ft mountain is roughly three-quarters what it is at sea level. The corresponding boiling point, as shown in Figure 17–15, is about 90 °C. Thus, cooking food in boiling water may be very difficult at such altitudes. Similarly, by measuring the boiling temperature of a pot of water on the summit of an uncharted mountain, it is possible to gain a rough estimate of its altitude.

ANSWER:

(b) The temperature of the water is lower than 100 °C.

An *autoclave* (which is basically an elaborate pressure cooker) sterilizes surgical tools by using the same principle discussed in Conceptual Checkpoint 17–4, only in the opposite direction. If surgical tools were heated in boiling water open to the atmosphere, they would experience a temperature of 100 °C. In the autoclave, which is a sealed vessel, the pressure rises to values significantly greater than atmospheric pressure. As a result, the water has a much higher boiling temperature, and the sterilization is more effective. In the pressure cooker, this elevated temperature results in a reduced cooking time.

REAL-WORLD PHYSICS: BIO
The autoclave

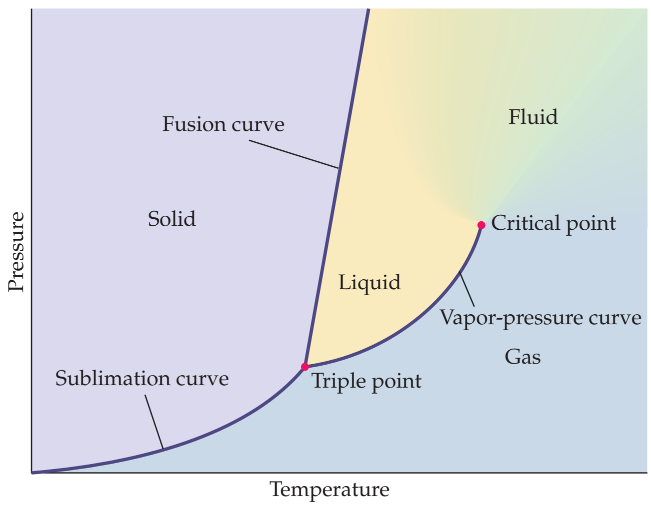
REAL-WORLD PHYSICS
The pressure cooker

Another way to increase the boiling temperature of water is to add a pinch of salt. The salt dissolves into sodium and chloride ions in the water. These ions have a strong interaction with the water molecules, making it harder for them to break free of the liquid and enter the vapor phase. As a result, the boiling temperature rises. This is why salt is often added to water when boiling eggs; the result is a higher temperature of boiling, which helps to solidify any material that might leak out of small cracks in the eggs.

REAL-WORLD PHYSICS
Adding salt to boiling water

The vapor-pressure curve separates areas of liquid and gas on a graph of pressure versus temperature, as shown in **Figure 17–16**. Notice that this curve comes to an end at a finite temperature and pressure. This end point is called the **critical point.** Beyond the critical point there is no longer a distinction between liquid and gas. They are one and the same, and are referred to, simply, as a fluid. Thus, as mentioned before, the liquid and gas phases are very similar—the liquid is just more dense than the gas and its molecules are somewhat less mobile.

A curve similar to the vapor-pressure curve indicates where the solid and liquid phases are in equilibrium. It is referred to as the **fusion curve.** Similarly, equilibrium between the solid and gas phases occurs along the **sublimation curve.** All



◀ **FIGURE 17–16 A typical phase diagram**
This phase diagram shows the regions corresponding to each of the three common phases of matter. Note that all three phases are in equilibrium at the triple point. In addition, the liquid and gas phases become indistinguishable beyond the critical point, where they are referred to as a fluid.

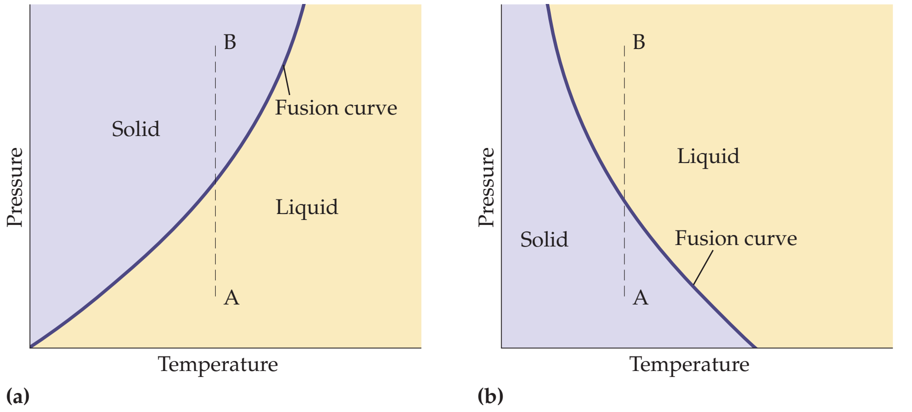


FIGURE 17–17 The solid–liquid phase boundary

Fusion curves for a typical substance **(a)** and for water **(b).** In (a), we note that an increase in pressure at constant temperature results in a liquid being converted to a solid. For the case of water, increasing the pressure exerted on ice at constant temperature can result in the ice melting.

three equilibrium curves are shown in Figure 17–16, on a plot that is referred to as a **phase diagram.**

Note that the phase diagram also indicates that there is one particular temperature and pressure where all three phases are in equilibrium. This point is called the **triple point.** In water, the triple point occurs at the temperature $T = 273.16$ K and the pressure $P = 611.2$ Pa. At this temperature and pressure, ice, water, and steam are all in equilibrium with one another.

Finally, there is one feature of the fusion line that is of particular interest. In most substances, this line has a positive slope, as in Figure 17–16. This means that as the pressure is increased, the melting temperature of the substance also increases. This is sensible, because a solid is generally more dense than the corresponding liquid. Hence, if you apply pressure to a liquid—with the temperature held constant—the system will tend to become more dense and eventually solidify. This is indicated in **Figure 17–17 (a)**, where we see that an increase in pressure at constant temperature (as from A to B) results in crossing the fusion curve into the solid region.

REAL-WORLD PHYSICS
Ice melting under pressure

There are exceptions to this rule, however, and it will probably come as no surprise that water is one such exception. This is related to the fact that ice is less dense than water. As a result, the fusion curve for water has a negative slope coming out of the triple point. This is illustrated in **Figure 17–17 (b)**. What this means is that if the temperature is held constant, ice will melt when the pressure applied to it is increased. Thus, the pressure due to an ice skate's blade, for example, can cause the ice beneath it to melt.

REAL-WORLD PHYSICS
Frost wedging and frost heaving

The expansion of water as it freezes has important implications in geology as well. Suppose, for example, that water finds its way into cracks and fissures on a rocky cliff. If the temperature dips below freezing, the water in these cracks will begin to freeze and expand. This tends to split the rock farther apart in a process known as *frost wedging*. Over time, the repeated freezing and melting of water can break a "solid" rock cliff into a pile of debris, forming a talus slope at the base of the cliff. Similar effects occur in *frost heaving*, where expanding ice lifts rocks that were buried in the soil to the surface—a phenomenon well known in areas with long, cold winters.

The repeated expansion and contraction of water as it freezes and thaws over long periods of time can cause the ground to crack and buckle. Eventually, the resulting cracks may combine to form polygonal networks referred to as patterned ground (left), a common feature in arctic regions of the Earth. The same type of patterned ground has recently been observed on the surface of Mars (right), giving a strong indication that water may exist just below the surface in certain parts of the planet.

Finally, the expansion of ice can have devastating effects in living systems. If the blood cells of an organism were to freeze, for example, the resulting expansion could rupture the cells. The icefish, a transparent fish found in southern polar seas, has a high concentration of glycoproteins in its bloodstream that serve as a biological antifreeze, inhibiting the formation and growth of ice crystals.

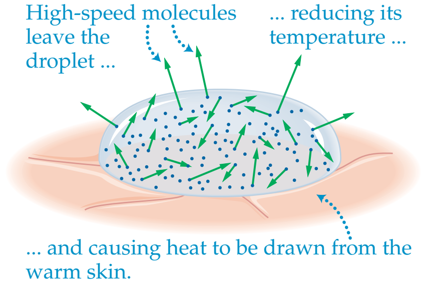
REAL-WORLD PHYSICS: BIO

Biological antifreeze

Evaporation

If you think about it a moment, you realize that evaporation is a bit odd. After all, when you are hot and sweaty from physical exertion, steam rises from your skin. But how can this be, since water boils at 100 °C? How can a skin temperature of only 30 or 35 °C result in steam?

Recall that if a liquid is placed in a closed container with a vacuum above it, some of its fastest molecules will break loose and form a gas. When the gas becomes dense enough, its pressure rises to the equilibrium vapor pressure of the liquid, and the system attains equilibrium. But what if you open the container and let a breeze blow across it, removing much of the gas? In that case, the release of molecules from the liquid into the gas—that is, **evaporation**—continues without reaching equilibrium. As the molecules are continually removed, the liquid progressively evaporates until none is left. This is the basic mechanism of evaporation.



▲ **FIGURE 17–18 A droplet of sweat resting on the skin**

High-speed molecules in a droplet of sweat are able to escape the droplet and become part of the atmosphere. The average speed of the molecules that remain in the droplet is reduced, so the temperature of the droplet is reduced as well.

Let's investigate how evaporation helps to cool us when we exercise or work up a sweat. First, consider a droplet of sweat on the skin, as illustrated in **Figure 17–18**. As mentioned before, the high-speed molecules in the drop are the ones that will escape from the liquid into the surrounding air. The breeze takes these molecules away as soon as they escape; hence, the chance of their reentering the drop of sweat is very small.

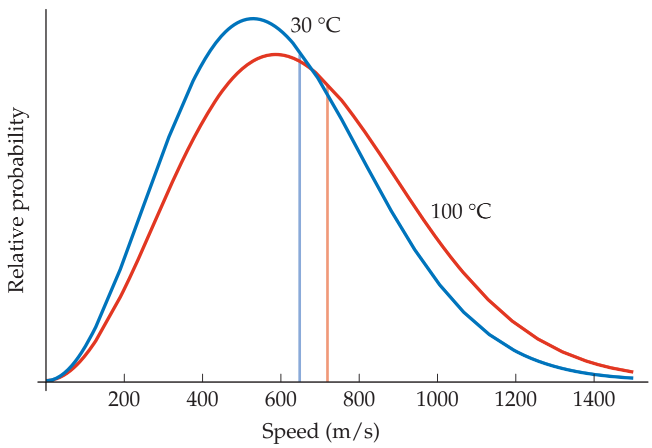
REAL-WORLD PHYSICS: BIO

Cooling the body with evaporation

Now, what does this mean for the molecules that are left behind in the sweat droplet? Well, since the droplet is preferentially losing high-speed molecules, the average kinetic energy of the remaining molecules must decrease. As we know from kinetic theory, this means that the temperature of the droplet must also decrease. Since the droplet is now cooler than its surroundings, including the skin on which it rests, it draws in heat from the body. This warms the droplet, increasing the speed of its molecules, and continuing the evaporation process at more or less the same rate. Thus, sweat droplets are an effective means of drawing heat from the body and releasing it into the surrounding air in the form of high-speed water molecules.

▲ Although the body temperature of this athlete is well below the boiling point of water, the water contained in his sweat is evaporating rapidly from his skin. (Since water vapor is invisible, the "steam" we see in this photo, though it indicates the presence of evaporation, is not the water vapor itself. Rather, it is a cloud of tiny droplets that form when the water vapor loses heat to the cold air around it and condenses back to the liquid state.)

To look at this a bit more quantitatively, consider the Maxwell speed distribution for water molecules in a sweat droplet at 30 °C. This is shown in **Figure 17–19**. The rms speed at this temperature is 648 m/s. Also shown in Figure 17–19 is the speed distribution for water molecules at 100 °C. In this case the rms speed is 719 m/s, only slightly greater than that at 30 °C. Thus, it is clear that if water molecules at 100 °C have enough speed to escape into the gas phase, many molecules at 30 °C will be able to escape as well.



◀ **FIGURE 17–19 Speed distribution for water**

The blue curve shows the speed distribution for water at 30 °C, and the red curve corresponds to 100 °C. Note that the rms speed for 100 °C is only slightly greater than that for 30 °C.

▲ Human beings keep cool by sweating. Because the most energetic molecules are the ones most likely to escape by evaporation, significant quantities of heat are removed from the body when perspiration evaporates from the skin. Dogs, lacking sweat glands, nevertheless take advantage of the same mechanism to help regulate body temperature. In hot weather they pant to promote evaporation from the tongue. Of course, sitting in a cool pond can also help. (What mechanism is involved here?)

REAL-WORLD PHYSICS
Stability of planetary atmospheres

The Evaporating Atmosphere

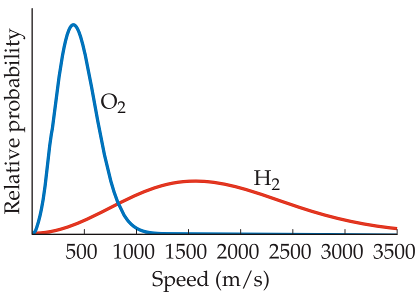
The atmosphere of a planet or a moon can evaporate in much the same way a drop of sweat evaporates from your forehead. In the case of an atmosphere, however, it is the force of gravity that must be overcome. If an astronomical body has a weak gravitational field, some molecules in its atmosphere may be moving rapidly enough to escape; that is, some molecules may have speeds in excess of the escape speed for that body.

Consider the Earth, for example. If a rocket is to escape the Earth, it must have a speed of at least 11,200 m/s. Recall from Chapter 12, however, that the escape speed is independent of the mass of the rocket. Thus, it applies equally well to molecules and rockets. As a result, molecules moving faster than 11,200 m/s may escape the Earth.

Now, let's compare the escape speed to the speeds of some of the molecules in the Earth's atmosphere. We have already seen in this chapter that the speed of nitrogen and oxygen are on the order of the speed of sound; that is, several hundred meters per second. This is much below the Earth's escape speed. Thus, the odds against a nitrogen or oxygen molecule having enough speed to escape the Earth are truly astronomical. Good thing, too, since this is why these molecules have persisted in our atmosphere for billions of years.

On the other hand, consider a lightweight molecule like hydrogen, H_2. The fact that its average kinetic energy is the same as that of all the other molecules in the air (see Equation 17–12) means that its speed will be much greater than the speed of, say, oxygen or nitrogen. This is illustrated in **Figure 17–20**, where we show the speed distribution for both O_2 and H_2. Clearly, it is very likely to find an H_2 molecule with a speed on the order of a couple thousand meters per second. Because of the higher speeds for H_2, the probability that an H_2 molecule will have enough speed to escape the Earth is about 300 orders of magnitude greater than the corresponding probability for O_2. It is no surprise, then, that Earth's atmosphere contains essentially no hydrogen.

On Jupiter, however, gravity is more intense than on Earth and the temperature is less. As a result, not even hydrogen moves quickly enough to escape. In fact, Jupiter's atmosphere is composed mostly of H_2 and He.



▲ **FIGURE 17–20 Speed distribution for O_2 and H_2 at 20 °C**
The typical speeds of an H_2 molecule (red curve) are much greater than those for an O_2 molecule (blue curve). In fact, some H_2 molecules move fast enough to escape from the Earth's atmosphere.

At the other extreme, the Moon has a rather weak gravitational field. In fact, it is unable to maintain any atmosphere at all. Whatever atmosphere it may have had early in its history has long since evaporated. You might say that the Moon's atmosphere is "lost in space."

17–5 Latent Heats

When two phases coexist something surprising happens—the temperature remains the same even when you add a small amount of heat. How can that be?

To understand this behavior, let's start by considering an ice cube initially at the temperature $-10\ °C$. Adding an amount of heat Q results in a temperature increase ΔT given by the relation $Q = mc_{ice}\,\Delta T$, as discussed in Chapter 16. When the ice cube's temperature reaches 0 °C, however, adding more heat does not cause an additional increase in temperature. Instead, the heat goes into converting some of the ice into water. On a microscopic level, the added heat causes some of the molecules in the solid ice to vibrate with enough energy to break loose from neighboring molecules and become part of the liquid.

Thus, as long as any ice remains in a cup of water, *and* the water and ice are in equilibrium, you can be sure that both the ice and the water are at 0 °C. If heat is added to the system, the amount of ice decreases; if heat is removed, the amount of ice increases. The amount of heat required to completely convert 1 kg of ice to water is referred to as the **latent heat,** L. In general, the latent heat is defined as follows:

> The latent heat, L, is the heat that must be added to or removed from one kilogram of a substance to convert it from one phase to another.
>
> During the conversion process, the temperature of the system remains constant.

Just as with the specific heat, the latent heat is always a positive quantity.

In mathematical form, we can say that the heat Q required to convert a mass m from one phase to another is $Q = mL$. This gives the following relation:

Definition of Latent Heat, *L*

$$L = Q/m \qquad 17\text{–}20$$

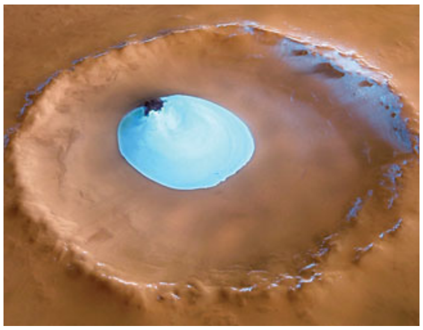
SI unit: J/kg

The value of the latent heat depends on which phases are involved. For example, the latent heat to melt (or fuse) a substance is referred to as the **latent heat of fusion,** L_f. Similarly, the latent heat required to convert a liquid to a gas is the **latent heat of vaporization,** L_v, and the latent heat needed to convert a solid directly to a gas is the **latent heat of sublimation,** L_s. Typical latent heats are given in Table 17–4.

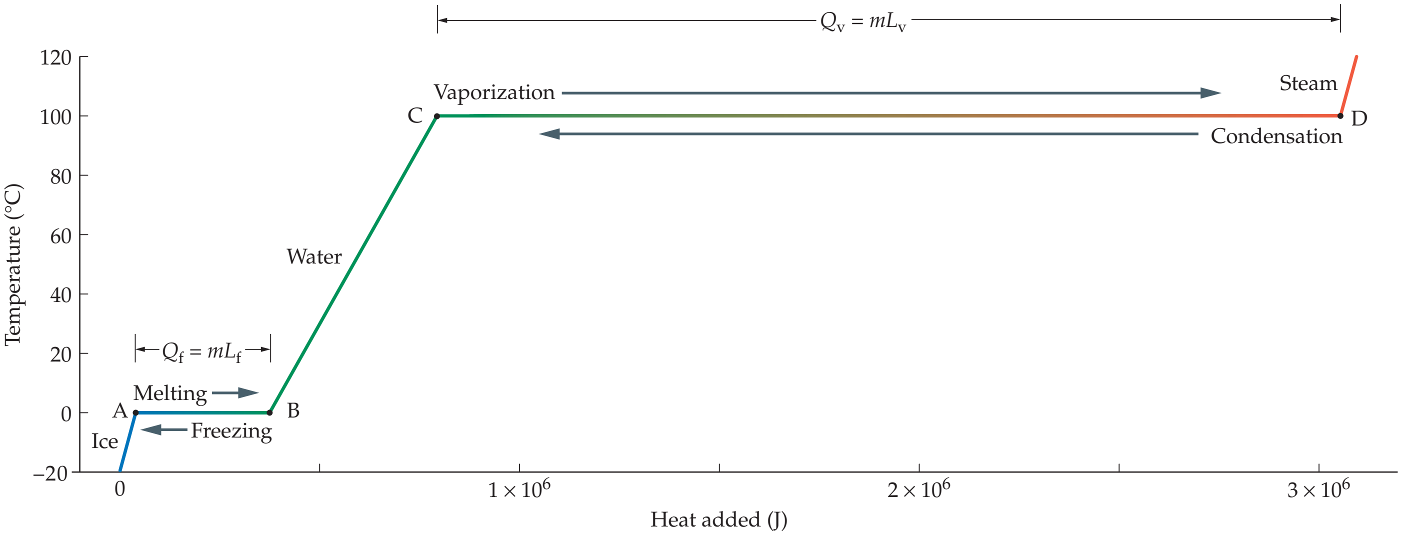
The relationship between the temperature of a substance and the heat added to it is illustrated in Figure 17–21. Initially 1 kg of H_2O is in the form of ice at $-20\ °C$.

TABLE 17–4 Latent Heats for Various Materials

Material	Latent heat of fusion, L_f (J/kg)	Latent heat of vaporization, L_v (J/kg)
Water	33.5×10^4	22.6×10^5
Ammonia	33.2×10^4	13.7×10^5
Copper	20.7×10^4	47.3×10^5
Benzene	12.6×10^4	3.94×10^5
Ethyl alcohol	10.8×10^4	8.55×10^5
Gold	6.28×10^4	17.2×10^5
Nitrogen	2.57×10^4	2.00×10^5
Lead	2.32×10^4	8.59×10^5
Oxygen	1.39×10^4	2.13×10^5



▲ This recently discovered ice lake on the surface of Mars lies on the floor of an impact crater. It grows or shrinks with the Martian seasons. Because atmospheric pressure is so low on Mars, however, the ice does not melt during the Martian summer—instead, it sublimates directly to the vapor phase. On Mars, water ice would have behavior similar to that of dry ice here on Earth.



▲ **FIGURE 17–21 Temperature versus heat added or removed**

The temperature of $m = 1.000$ kg of water as heat is added to or removed from the system. Note that the temperature stays the same—even as heat is added—when the system is changing from one phase to another. The points A, B, C, and D are referred to in Homework Problems 62 and 63.

As heat is added to the ice, its temperature rises until it begins to melt at 0 °C. The temperature then remains constant until the latent heat of fusion is supplied to the system. When all the ice has melted to water at 0 °C, continued heating results in a renewed increase in temperature. When the temperature of the water rises to 100 °C, boiling begins and the temperature again remains constant—this time until an amount of heat equal to the latent heat of vaporization is added to the system. Finally, with the entire system converted to steam, continued heating again produces an increasing temperature.

CONCEPTUAL CHECKPOINT 17–5 SEVERITY OF A BURN

Both water at 100 °C and steam at 100 °C can cause serious burns. Is a burn produced by steam likely to be **(a)** more severe than, **(b)** less severe than, or **(c)** the same as a burn produced by water?

REASONING AND DISCUSSION

As the water or steam comes into contact with the skin, it cools from 100 °C to a skin temperature of something like 35 °C. For the case of water, this means that a certain amount of heat is transferred to the skin, which can cause a burn. The steam, on the other hand, must first give off the heat required for it to condense to water at 100 °C. After that, the condensed water cools to body temperature, as before. Thus, the heat transferred to the skin will be larger in the case of steam, resulting in a more serious burn.

ANSWER

(a) The steam burn is worse.

As a numerical example of using latent heat, let's calculate the heat energy required to raise the temperature of 0.550 kg of ice from −20.0 °C to water at 20.0 °C; that is, from point A to point D in **Figure 17–22**. The way to approach a problem like this is to take it one step at a time—that is, each phase, and each conversion from one phase to another, should be treated separately.

Thus, the first step (A to B in Figure 17–22) is to find the heat necessary to warm the ice from −20.0 °C to 0 °C. Using the specific heat of ice, $c_{ice} = 2090\ \mathrm{J/(kg \cdot C^\circ)}$, we find

$$Q_1 = mc_{ice}\,\Delta T = (0.550\ \mathrm{kg})[2090\ \mathrm{J/(kg \cdot C^\circ)}](20.0\ \mathrm{C^\circ}) = 23{,}000\ \mathrm{J}$$

The second step in the process (B to C) is to melt the ice at 0 °C. The heat required for this is found using the latent heat of fusion for water ($L_f = 33.5 \times 10^4\ \mathrm{J/kg}$):

$$Q_2 = mL_f = (0.550\ \mathrm{kg})(33.5 \times 10^4\ \mathrm{J/kg}) = 184{,}000\ \mathrm{J}$$

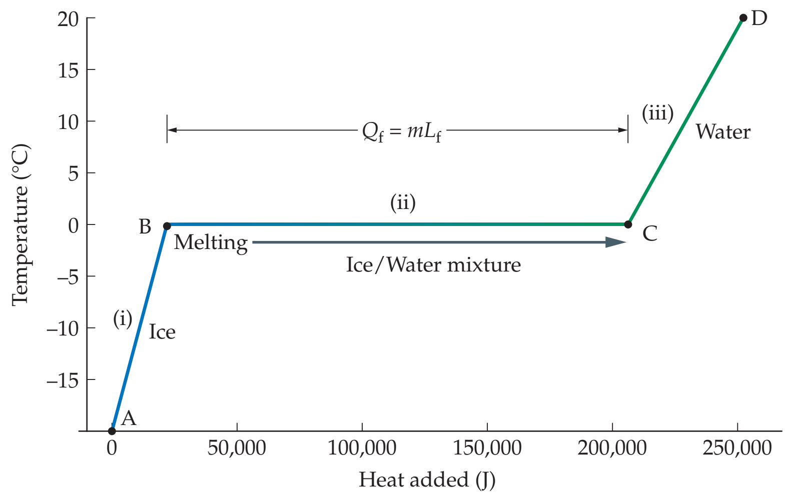


FIGURE 17–22 Heat required for a given change in temperature

The amount of heat required to raise 0.550 kg of H_2O from ice at −20.0 °C to water at 20.0 °C is the heat difference between points A and D. To calculate this heat we sum the following three heats: (i) the heat to warm the ice from −20.0 °C to 0 °C; (ii) the heat to melt all the ice; (iii) the heat to warm the water from 0 °C to 20.0 °C.

Finally, the third step (C to D) is to heat the water at 0 °C to 20.0 °C. This time we use the specific heat for water, $c_{\text{water}} = 4186 \text{ J}/(\text{kg} \cdot \text{C}°)$:

$$Q_3 = mc_{\text{water}}\,\Delta T = (0.550 \text{ kg})[4186 \text{ J}/(\text{kg}\cdot\text{C}°)](20.0 \text{ C}°) = 46{,}000 \text{ J}$$

The total heat required for this process, then, is

$$Q_{\text{total}} = Q_1 + Q_2 + Q_3$$
$$= 23{,}000 \text{ J} + 184{,}000 \text{ J} + 46{,}000 \text{ J} = 253{,}000 \text{ J}$$

We consider a similar problem in the following Example.

PROBLEM-SOLVING NOTE

Specific Heats Versus Latent Heats

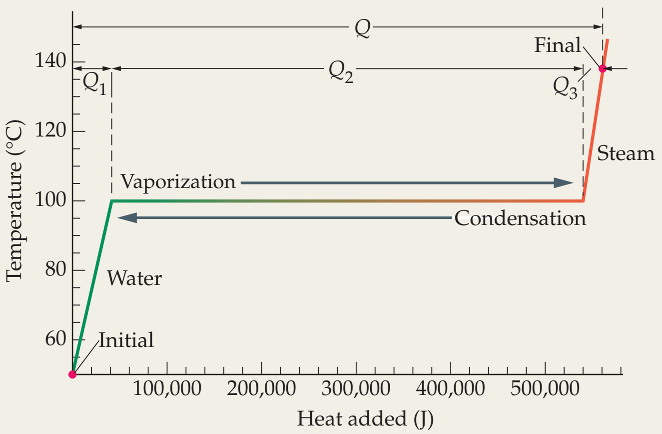
In solving problems involving specific heats and latent heats, recall that specific heats give the heat related to a *change in temperature* in a given phase, and latent heats give the heat related to a *change in phase* at a given temperature.

EXAMPLE 17–5 STEAM HEAT

To make steam, you add 5.60×10^5 J of heat to 0.220 kg of water at an initial temperature of 50.0 °C. Find the final temperature of the steam.

PICTURE THE PROBLEM

Our sketch shows the temperature–versus–heat-added curve for water. The initial point for this system, placed at the origin, is at 50.0 °C. As we shall see, adding the given amount of heat, $Q = 5.60 \times 10^5$ J, raises the temperature to the point labeled "final" in the plot.



STRATEGY

To find the final temperature, we first calculate the amount of heat that must be added to heat the water to 100 °C. If this is less than the total heat added, we continue by calculating the amount of heat needed to vaporize all the water. If the sum of these two heats is still less than the total heat added to the water, we calculate the increase in temperature when the remaining heat is added to the steam.

SOLUTION

1. Calculate the heat that must be added to the water to heat it to 100 °C. Call the result Q_1:

$$Q_1 = mc_{\text{water}}\,\Delta T$$
$$= (0.220 \text{ kg})[4186 \text{ J}/(\text{kg}\cdot\text{C}°)](50.0 \text{ C}°)$$
$$= 4.60 \times 10^4 \text{ J}$$

2. Next, calculate the heat that must be added to the water to convert it to steam. Let this result be Q_2:

$$Q_2 = mL_f = (0.220 \text{ kg})(22.6 \times 10^5 \text{ J/kg})$$
$$= 4.97 \times 10^5 \text{ J}$$

3. Determine the heat that is still to be added to the system. Let this remaining heat be Q_3:

$$Q_3 = 5.60 \times 10^5 \text{ J} - Q_1 - Q_2$$
$$= 5.60 \times 10^5 \text{ J} - 4.60 \times 10^4 \text{ J} - 4.97 \times 10^5 \text{ J}$$
$$= 17{,}000 \text{ J}$$

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4. Use Q_3 to find the increase in temperature of the steam:

$$Q_3 = mc_{\text{steam}}\,\Delta T$$

$$\Delta T = \frac{Q_3}{mc_{\text{steam}}} = \frac{17{,}000\ \text{J}}{(0.220\ \text{kg})[(2010\ \text{J})/(\text{kg}\cdot\text{C}^\circ)]} = 38\ \text{C}^\circ$$

INSIGHT

Thus, the system ends up completely converted to steam at a temperature of 138 °C. If the amount of heat added to the system had been greater than Q_1, but less than $Q_1 + Q_2$, the final temperature of the system would have been 100 °C. In this case, the final state of the system is a mixture of liquid water and steam.

PRACTICE PROBLEM

Find the final temperature if the amount of heat added to the system is 3.40×10^5 J. [**Answer:** In this case, only 0.130 kg of water vaporizes into steam. Hence the final temperature is 100 °C, with water and steam in equilibrium].

Some related homework problems: Problem 57, Problem 60

REAL-WORLD PHYSICS

Homemade ice cream

A pleasant application of latent heat is found in the making of homemade ice cream. As you may know, it is necessary to add salt to the ice–water mixture surrounding the container holding the ingredients for the ice cream. The dissolved salt molecules interact with water molecules in the liquid, impairing their ability to interact with one another and freeze. This means that ice and water are no longer in equilibrium at 0 °C; a lower temperature is required. The result is that ice begins to melt in the ice–water mixture, and in the process of melting it draws the required latent heat from its surroundings—which include the ice cream. Thus, the salt together with the ice produces a temperature lower than the melting temperature of ice alone.

17–6 Phase Changes and Energy Conservation

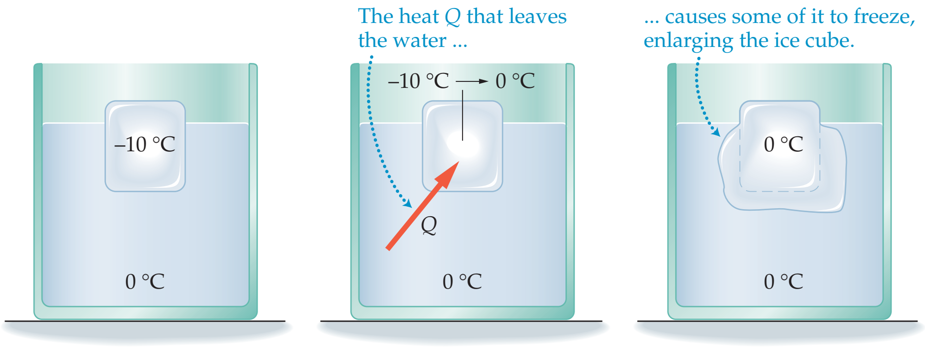
In the last section, we considered problems in which a given amount of heat is simply added to or removed from a system. We now turn to a more interesting type of problem involving energy conservation. In these problems, heat is exchanged within a system—that is, between its parts—but not with the external world. As a result, the total energy of the system is constant. Still, the heat flow within the system can cause changes in temperatures and phases.

The basic idea in solving energy conservation problems is the following:

> Set the magnitude of the heat *lost* by one part of the system equal to the magnitude of the heat *gained* by another.

For example, consider the following problem: A 0.0420-kg ice cube at −10.0 °C is placed in a Styrofoam cup containing 0.350 kg of water at 0 °C. Assuming the cup can be ignored, and that no heat is exchanged with the surroundings, find the mass of ice in the system when the equilibrium temperature of 0 °C is reached.

The initial setup for this problem is illustrated in **Figure 17–23 (a)**. Since the water is warmer than the ice, it follows that heat flows into the ice from the water,



(a) Initial **(b)** Heat goes from the warmer water into the colder ice **(c)** Final

▶ **FIGURE 17–23 Water freezing to form ice**

An ice cube at −10.0 °C is placed in water at 0 °C. Since the ice is colder than the water, heat flows *from* the water *into* the ice. Heat that leaves the water results in the formation of additional ice in the system.

as indicated in **Figure 17–23 (b)**. The amount of heat will be just enough to raise the temperature of the ice from −10.0 °C to 0 °C. Thus,

$$Q = \text{heat gained by the ice}$$
$$= mc_{ice}\,\Delta T = (0.0420\text{ kg})[2090\text{ J}/(\text{kg}\cdot\text{C}°)](10.0\text{ C}°) = 878\text{ J}$$

By energy conservation, we can say that the heat lost by the water has the same magnitude as the heat gained by the ice. (In general, in these problems it is simplest to calculate the magnitude of each heat—so that all the heats are positive—and then set the "lost" and "gained" magnitudes equal.) In this case, then, we have

$$\text{heat lost by the water} = \text{heat gained by the ice} = 878\text{ J}$$

Thus, 878 J of heat are removed from the water.

Now, since the water is already at 0 °C, removing heat from it does not lower its temperature—instead, it merely converts some of the water to ice. How much is converted? The amount is determined by the latent heat of fusion. In particular, we have

$$Q = \text{heat lost by the water}$$
$$= mL_f = 878\text{ J}$$

In this expression, m is the mass of water that has been converted to ice. Solving for the mass yields

$$m = \frac{Q}{L_f} = \frac{878\text{ J}}{33.5 \times 10^4\text{ J/kg}} = 0.00262\text{ kg}$$

Thus, the final amount of ice in the system is 0.0420 kg + 0.00262 kg = 0.0446 kg. This is illustrated in **Figure 17–23 (c)**.

Finally, we consider a system in which a quantity of ice completely melts in warm water. The result is a container of liquid water at an intermediate temperature. Still, the basic idea is to apply energy conservation.

PROBLEM-SOLVING NOTE

Determining Equilibrium

In problems involving two different phases—like solid and liquid—it may not be clear in advance whether both phases or only one is present in the equilibrium state. It may be necessary to assume one case or the other and proceed with the calculation based on that assumption. If the result obtained in this way is not physical, the assumption must be changed.

EXAMPLE 17–6 WARM PUNCH

A large punch bowl holds 3.95 kg of lemonade (which is essentially just water) at 20.0 °C. A 0.0450-kg ice cube at 0 °C is placed in the lemonade. What is the final temperature of the system, and how much ice (if any) remains when the system reaches equilibrium? Ignore any heat exchange with the bowl or the surroundings.

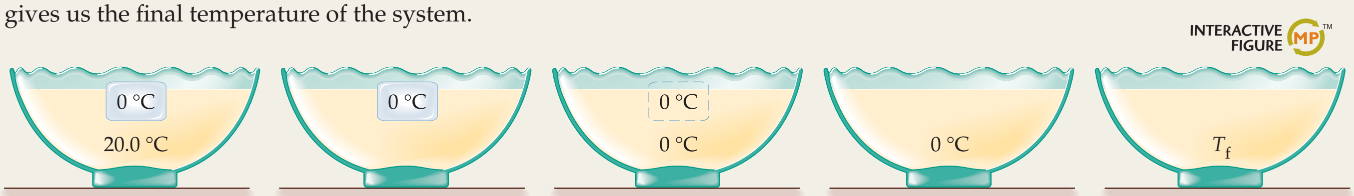
PICTURE THE PROBLEM

Our sketch shows the various stages imagined for this problem, starting with the initial condition in which the ice is at 0 °C and the lemonade (water) is at 20.0 °C. As one might guess from the large amount of water and the small amount of ice, all of the ice will melt. Therefore, the final condition is a container of liquid water at a final temperature, T_f.

STRATEGY

To apply energy conservation to this problem, we first calculate the heat that would be lost by the water, Q_w, if we cooled it to 0 °C. We then imagine using part of this heat, Q_{ice}, to melt the ice.

As we shall see, a great deal of heat, $Q = Q_w - Q_{ice}$, is left after the ice is melted. We imagine adding this heat back into the system, which now contains 3.95 kg + 0.0450 kg of water at 0 °C. Calculating the increase in temperature caused by the heat Q gives us the final temperature of the system.



Initial condition

Calculate heat Q_w lost by water, 20 °C ⟶ 0 °C

Calculate heat Q_{ice} needed to melt ice cube

Calculate remaining heat $Q = Q_w - Q_{ice}$

Calculate final temperature when heat Q is added to water and melted ice

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SOLUTION

1. Find the heat lost by the water, Q_w, if it is cooled to 0 °C:

$$Q_w = m_{water}c_{water}\,\Delta T = (3.95\text{ kg})[4186\text{ J/(kg}\cdot\text{C°)}](20.0\text{ C°}) = 3.31 \times 10^5\text{ J}$$

2. Calculate the amount of heat, Q_{ice}, needed to melt all the ice:

$$Q_{ice} = m_{ice}L_f = (0.0450\text{ kg})(33.5 \times 10^4\text{ J/kg}) = 1.51 \times 10^4\text{ J}$$

3. Determine the amount of heat, Q, that is left:

$$Q = Q_w - Q_{ice} = 3.31 \times 10^5\text{ J} - 1.51 \times 10^4\text{ J} = 3.16 \times 10^5\text{ J}$$

4. Use the heat Q to warm the 3.95 kg + 0.0450 kg of water at 0 °C to its final temperature:

$$Q = (m_{water} + m_{ice})c_{water}\,\Delta T$$

$$\Delta T = \frac{Q}{(m_{water} + m_{ice})c_{water}} = \frac{3.16 \times 10^5\text{ J}}{(3.95\text{ kg} + 0.0450\text{ kg})[4186\text{ J/(kg}\cdot\text{C°)}]} = 18.9\text{ C°}$$

INSIGHT

Therefore, the final temperature of the system is 18.9 °C. As expected, the relatively small ice cube did not lower the temperature of the system very much.

PRACTICE PROBLEM

What would the final temperature of the system be if the ice cube's mass were 0.0750 kg? [**Answer:** $T_f = 18.2$ °C]

Some related homework problems: Problem 67, Problem 70

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

We use the concept of temperature (Chapter 16) throughout this chapter. We also make use of pressure (Chapter 15) in our discussion of ideal gases in Sections 17–1 and 17–2.

Notice that kinetic energy (Chapter 7) plays a central role in our understanding of an ideal gas. In particular, we make a clear and specific connection in Section 17–2 between the mechanical concept of kinetic energy and the thermodynamic concept of temperature.

We gain a deeper understanding of Hooke's law (Chapter 6) in Section 17–3 when we discuss the elastic deformation of solids.

LOOKING AHEAD

The equation of state for an ideal gas (Section 17–1) plays an important role in Chapter 18, especially when considering thermal processes (Section 18–3) and specific heats at constant pressure and volume (Section 18–4).

The concept of a phase change is used in Chapter 21 when we discuss the fact that certain materials become superconducting (have zero electrical resistance) below a critical temperature.

Phase changes appear again in Chapter 32 when we discuss the fundamental forces of nature and how they evolved as the universe cooled over time.

CHAPTER SUMMARY

17–1 IDEAL GASES

An ideal gas is a simplified model of a real gas in which interactions between molecules are ignored.

Equation of State

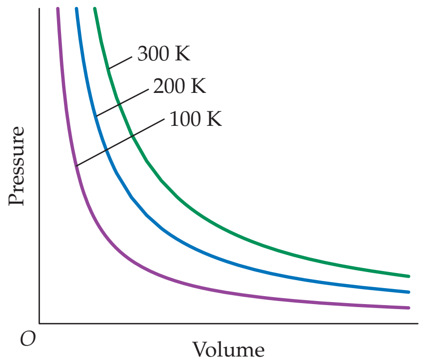
The equation of state for an ideal gas is

$$PV = NkT \qquad 17\text{–}2$$

In this expression, N is the number of molecules, T is the Kelvin temperature, and $k = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant.

In terms of the universal gas constant, $R = 8.31\ \mathrm{J/(mol \cdot K)}$, and the number of moles in the gas, n, the ideal-gas equation of state is

$$PV = nRT \qquad 17\text{–}5$$



Avogadro's Number and Moles

The number of molecules in a mole (mol) is Avogadro's number, $N_A = 6.022 \times 10^{23}$.

Molecular Mass

If the mass of an individual molecule is m, its molecular mass, M, is

$$M = N_A m \qquad 17\text{–}6$$

Isotherms and Boyle's Law

If the temperature and number of molecules are held constant, the pressure and volume of an ideal gas satisfy Boyle's law:

$$PV = \text{constant} \qquad 17\text{–}7$$

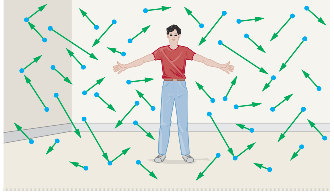
Constant Pressure and Charles's Law

If the pressure and number of molecules are held constant, the temperature and volume of an ideal gas satisfy Charles's law:

$$\frac{V}{T} = \text{constant} \qquad 17\text{–}8$$

17–2 KINETIC THEORY

In kinetic theory, a gas is imagined to be comprised of a large number of point-like molecules bouncing off the walls of a container.



The Origin of Pressure

The pressure exerted by a gas is a result of the momentum transfers that occur every time a molecule bounces off a wall of a container.

Speed Distribution of Molecules

The molecules in a gas have a range of speeds. The Maxwell distribution indicates which speeds are most likely to occur in a given gas.

Kinetic Energy and Temperature

Kinetic theory relates the average kinetic energy of the molecules in a gas to the Kelvin temperature of the gas, T:

$$\left(\tfrac{1}{2}mv^2\right)_{\text{av}} = K_{\text{av}} = \tfrac{3}{2}kT \qquad 17\text{–}12$$

RMS Speed

The rms (root mean square) speed of the molecules in a gas at the Kelvin temperature T is

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{M}} \qquad 17\text{–}13$$

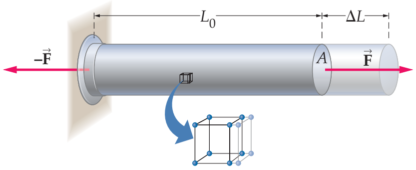
Internal Energy of an Ideal Gas

The internal energy of a monatomic ideal gas is

$$U = \tfrac{3}{2}NkT = \tfrac{3}{2}nRT \qquad 17\text{–}15$$

17–3 SOLIDS AND ELASTIC DEFORMATION

When a force is applied to a solid, its size and shape may change.



Changing the Length of a Solid

The force required to change the length of a solid by the amount ΔL is

$$F = Y\left(\frac{\Delta L}{L_0}\right)A \qquad 17\text{–}17$$

In this expression, Y is Young's modulus, L_0 is the initial length parallel to the applied force, and A is the cross-sectional area perpendicular to the applied force.

Shear Deformation

The force required to shear, or deform, a solid by the amount Δx is

$$F = S\left(\frac{\Delta x}{L_0}\right)A \qquad 17\text{–}18$$

In this expression, S is the shear modulus, L_0 is the initial length perpendicular to the applied force, and A is the cross-sectional area parallel to the applied force.

Changing the Volume of a Solid
The change in pressure required to change the volume of a solid by the amount ΔV is

$$\Delta P = -B\left(\frac{\Delta V}{V_0}\right) \qquad \text{17–19}$$

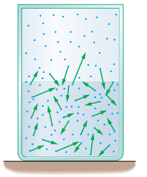
In this expression, B is the bulk modulus and V_0 is the initial volume.

Stress and Strain
The applied force per area is the stress; the resulting deformation is the strain.

Elastic Deformation
An elastic deformation is one in which a solid returns to its original size and shape when the stress is removed.

17–4 PHASE EQUILIBRIUM AND EVAPORATION

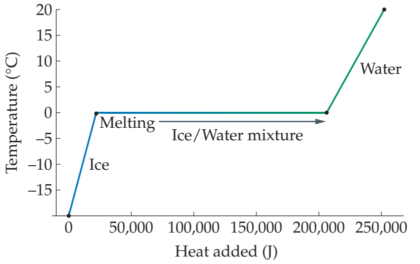
The three most common phases of matter are the solid, liquid, and gas. Solids maintain a definite shape, whereas gases and liquids flow to take on the shape of their container.



Equilibrium Between Phases
When phases are in equilibrium, the number of molecules in each phase remains constant.

Evaporation
Evaporation occurs when some molecules in a liquid have speeds great enough to allow them to escape into the gas phase.

17–5 LATENT HEATS

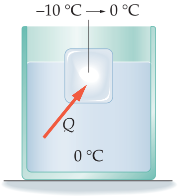


The latent heat, L, is the amount of heat per unit mass that must be added to or removed from a substance to convert it from one phase to another.

Latent Heat of Fusion
The heat required for melting or freezing is called the latent heat of fusion, L_f.

Latent Heat of Vaporization
The heat required for vaporizing or condensing is the latent heat of vaporization, L_v.

Latent Heat of Sublimation
The heat required to sublime a solid directly to a gas, or to condense a gas to a solid, is the latent heat of sublimation, L_s.



17–6 PHASE CHANGES AND ENERGY CONSERVATION

When heat is exchanged within a system, with no exchanges with the surroundings, the energy of the system is conserved.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find pressure, volume, or temperature in an ideal gas.	These basic quantities are related by the ideal-gas equation of state, $PV = NkT = nRT$.	Examples 17–1, 17–2 Active Examples 17–1, 17–2
Relate the rms speed of a gas to the absolute temperature T.	Absolute temperature is directly related to the average kinetic energy, $3kT/2 = K_{av}$. From this connection we obtain the result $v_{rms} = \sqrt{3kT/m}$.	Example 17–3
Find the strain produced by a given stress.	Strain is proportional to stress, at least for small stress. The basic relations are $F/A = Y(\Delta L/L_0)$, $F/A = S(\Delta x/L_0)$, and $\Delta P = -B(\Delta V/V_0)$, where Y, S, and B are the Young's modulus, the shear modulus, and the bulk modulus, respectively.	Example 17–4, Active Examples 17–3, 17–4
Calculate the heat associated with a change in phase.	A certain amount of heat, called the latent heat, L, must be added to or taken away from a system to change it from one phase to another. The process occurs at constant temperature. The amount of heat involved in a change of phase is given by $Q = mL$, where m is the mass that changes phase.	Examples 17–5, 17–6

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. At the beginning of a typical airline flight you are instructed about the proper use of oxygen masks that will fall from the ceiling if the cabin pressure suddenly drops. You are advised that the oxygen masks are working properly, even if the bags do not fully inflate. In fact, the bags expand to their fullest if cabin pressure is lost at high altitude, but expand only partially if the plane is at low altitude. Explain.
2. How is the air pressure in a tightly sealed house affected by operating the furnace? Explain.
3. The average speed of air molecules in your room is on the order of the speed of sound. What is their average velocity?
4. Is it possible to change both the pressure and the volume of an ideal gas without changing the average kinetic energy of its molecules? If your answer is no, explain why not. If your answer is yes, give a specific example.
5. **An Airport at Great Elevation** One of the highest airports in the world is located in La Paz, Bolivia. Pilots prefer to take off from this airport in the morning or the evening, when the air is quite cold. Explain.
6. A camping stove just barely boils water on a mountaintop. When the stove is used at sea level, will it be able to boil water? Explain your answer.
7. An autoclave is a device used to sterilize medical instruments. It is essentially a pressure cooker that heats the instruments in water under high pressure. This ensures that the sterilization process occurs at temperatures greater than the normal boiling point of water. Explain why the autoclave produces such high temperatures.
8. As the temperature of ice is increased, it changes first into a liquid and then into a vapor. On the other hand, dry ice, which is solid carbon dioxide, changes directly from a solid to a vapor as its temperature is increased. How might one produce liquid carbon dioxide?
9. **BIO** Isopropyl alcohol is sometimes rubbed onto a patient's arms and legs to lower their body temperature. Why is this effective?
10. If you toss an ice cube into a swimming pool, is the water in the pool now at 0 °C? Explain.
11. A drop of water on a kitchen counter evaporates in a matter of minutes. However, only a relatively small fraction of the molecules in the drop move rapidly enough to escape through the drop's surface. Why, then, does the entire drop evaporate rather than just a small fraction of it?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 17–1 IDEAL GASES

1. • **CE (a)** Is the number of molecules in one mole of N_2 greater than, less than, or equal to the number of molecules in one mole of O_2? **(b)** Is the mass of one mole of N_2 greater than, less than, or equal to the mass of one mole of O_2?
2. • **CE** Is the number of atoms in one mole of helium greater than, less than, or equal to the number of atoms in one mole of oxygen? Helium consists of individual atoms, He, and oxygen is a diatomic gas, O_2.
3. • **CE Predict/Explain** If you put a helium-filled balloon in the refrigerator, **(a)** will its volume increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Lowering the temperature of an ideal gas at constant pressure results in a reduced volume.
   II. The same amount of gas is in the balloon; therefore, its volume remains the same.
   III. The balloon can expand more in the cool air of the refrigerator, giving an increased volume.
4. • **CE** Two containers hold ideal gases at the same temperature. Container A has twice the volume and half the number of molecules as container B. What is the ratio P_A/P_B, where P_A is the pressure in container A and P_B is the pressure in container B?
5. • Standard temperature and pressure (STP) is defined as a temperature of 0 °C and a pressure of 101.3 kPa. What is the volume occupied by one mole of an ideal gas at STP?
6. • **BIO** After emptying her lungs, a person inhales 4.1 L of air at 0.0 °C and holds her breath. How much does the volume of the air increase as it warms to her body temperature of 37 °C?
7. • In the morning, when the temperature is 286 K, a bicyclist finds that the absolute pressure in his tires is 501 kPa. That afternoon he finds that the pressure in the tires has increased to 554 kPa. Ignoring expansion of the tires, find the afternoon temperature.
8. • An automobile tire has a volume of $0.0185\ m^3$. At a temperature of 294 K the absolute pressure in the tire is 212 kPa. How many moles of air must be pumped into the tire to increase its pressure to 252 kPa, given that the temperature and volume of the tire remain constant?
9. • **Amount of Helium in a Blimp** The Goodyear blimp *Spirit of Akron* is 62.6 m long and contains $7023\ m^3$ of helium. When the temperature of the helium is 285 K, its absolute pressure is 112 kPa. Find the mass of the helium in the blimp.
10. • A compressed-air tank holds $0.500\ m^3$ of air at a temperature of 285 K and a pressure of 880 kPa. What volume would the air occupy if it were released into the atmosphere, where the pressure is 101 kPa and the temperature is 303 K?
11. • A typical region of interstellar space may contain 10^6 atoms per cubic meter (primarily hydrogen) at a temperature of 100 K. What is the pressure of this gas?
12. •• **CE** Four ideal gases have the following pressures, P, volumes, V, and mole numbers, n: gas A, $P = 100$ kPa, $V = 1\ m^3$,

$n = 10$ mol; gas B, $P = 200$ kPa, $V = 2\ m^3$, $n = 20$ mol; gas C, $P = 50$ kPa, $V = 1\ m^3$, $n = 50$ mol; gas D, $P = 50$ kPa, $V = 4\ m^3$ $n = 5$ mol. Rank these gases in order of increasing temperature. Indicate ties where appropriate.

13. •• A balloon contains 3.7 liters of nitrogen gas at a temperature of 87 K and a pressure of 101 kPa. If the temperature of the gas is allowed to increase to 24 °C and the pressure remains constant, what volume will the gas occupy?

14. •• **IP** A balloon is filled with helium at a pressure of 2.4×10^5 Pa. The balloon is at a temperature of 18 °C and has a radius of 0.25 m. **(a)** How many helium atoms are contained in the balloon? **(b)** Suppose we double the number of helium atoms in the balloon, keeping the pressure and the temperature fixed. By what factor does the radius of the balloon increase? Explain.

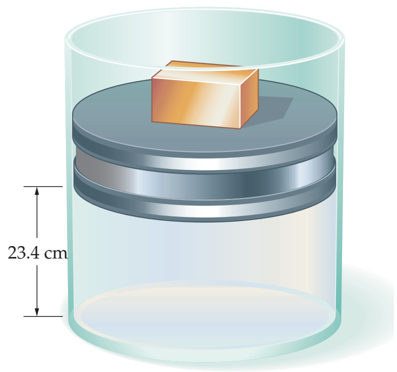
15. •• **IP** A gas has a temperature of 310 K and a pressure of 101 kPa. **(a)** Find the volume occupied by 1.25 mol of this gas, assuming it is ideal. **(b)** Assuming the gas molecules can be approximated as small spheres of diameter 2.5×10^{-10} m, determine the fraction of the volume found in part (a) that is occupied by the molecules. **(c)** In determining the properties of an ideal gas, we assume that molecules are points of zero volume. Discuss the validity of this assumption for the case considered here.

16. •• A 515-cm^3 flask contains 0.460 g of a gas at a pressure of 153 kPa and a temperature of 322 K. What is the molecular mass of this gas?

17. •• **IP The Atmosphere of Mars** On Mars, the average temperature is −64°F and the average atmospheric pressure is 0.92 kPa. **(a)** What is the number of molecules per volume in the Martian atmosphere? **(b)** Is the number of molecules per volume on the Earth greater than, less than, or equal to the number per volume on Mars? Explain your reasoning. **(c)** Estimate the number of molecules per volume in Earth's atmosphere.

18. •• The air inside a hot-air balloon has an average temperature of 79.2 °C. The outside air has a temperature of 20.3 °C. What is the ratio of the density of air in the balloon to the density of air in the surrounding atmosphere?

19. •• A cylindrical flask is fitted with an airtight piston that is free to slide up and down, as shown in **Figure 17–24**. A mass rests on top of the piston. The initial temperature of the system is 313 K and the pressure of the gas is held constant at 137 kPa. The temperature is now increased until the height of the piston rises from 23.4 cm to 26.0 cm. What is the final temperature of the gas?



▲ **FIGURE 17–24** Problems 19 and 20

20. •• Consider the system described in Problem 19. Contained within the flask is an ideal gas at a constant temperature of 313 K. Initially the pressure applied by the piston and the mass is 137 kPa and the height of the piston above the base of the flask is 23.4 cm. When additional mass is added to the piston, the height of the piston decreases to 20.0 cm. Find the new pressure applied by the piston.

21. ••• One mole of a monatomic ideal gas has an initial pressure of 210 kPa, an initial volume of $1.2 \times 10^{-3}\ m^3$, and an initial temperature of 350 K. The gas now undergoes three separate processes: (i) a constant-temperature expansion that triples its volume; (ii) a constant-pressure compression to its initial volume; and (iii) a constant-volume increase in pressure to its initial pressure. At the end of these three processes, the gas is back at its initial pressure, volume, and temperature. Plot these processes on a pressure-versus-volume graph, showing the values of P and V at the end points of each process.

SECTION 17–2 KINETIC THEORY

22. • **CE Predict/Explain** The air in your room is composed mostly of oxygen (O_2) and nitrogen (N_2) molecules. The oxygen molecules are more massive than the nitrogen molecules. **(a)** Is the rms speed of the O_2 molecules greater than, less than, or equal to the rms speed of the N_2 molecules? **(b)** Choose the *best explanation* from among the following:

I. The more massive oxygen molecules have greater momentum and therefore greater speed.

II. Equal temperatures for the oxygen and nitrogen molecules imply they have equal rms speeds.

III. The temperature is the same for both molecules, and hence their average kinetic energies are equal. As a result, the more massive oxygen molecules have lower speeds.

23. • **CE** If the translational speed of molecules in an ideal gas is doubled, by what factor does the Kelvin temperature change? Explain.

24. • **CE** A piston held at the temperature T contains a gas mixture with molecules of three different types; A, B, and C. The corresponding molecular masses are $m_C > m_B > m_A$. Rank these molecular types in order of increasing **(a)** average kinetic energy and **(b)** rms speed. Indicate ties where appropriate.

25. • The molecules in a tank of hydrogen have the same rms speed as the molecules in a tank of oxygen. State whether each of the following statements is true, false, or unknowable with the given information: **(a)** the pressures are the same; **(b)** the hydrogen is at the higher temperature; **(c)** the hydrogen has the higher pressure; **(d)** the temperatures are the same; **(e)** the oxygen is at the higher temperature.

26. • At what temperature is the rms speed of H_2 equal to the rms speed that O_2 has at 313 K?

27. • Suppose a planet has an atmosphere of pure ammonia at 0.0 °C. What is the rms speed of the ammonia molecules?

28. •• **IP** Three moles of oxygen gas (that is, 3.0 mol of O_2) are placed in a portable container with a volume of 0.0035 m^3. If the temperature of the gas is 295 °C, find **(a)** the pressure of the gas and **(b)** the average kinetic energy of an oxygen molecule. **(c)** Suppose the volume of the gas is doubled, while the temperature and number of moles are held constant. By what factor do your answers to parts (a) and (b) change? Explain.

29. •• **IP** The rms speed of O_2 is 1550 m/s at a given temperature. **(a)** Is the rms speed of H_2O at this temperature greater than, less than, or equal to 1550 m/s? Explain. **(b)** Find the rms speed of H_2O at this temperature.

30. •• **IP** An ideal gas is kept in a container of constant volume. The pressure of the gas is also kept constant. **(a)** If the number of molecules in the gas is doubled, does the rms speed increase, decrease, or stay the same? Explain. **(b)** If the initial rms speed is 1300 m/s, what is the final rms speed?

31. •• What is the temperature of a gas of CO_2 molecules whose rms speed is 329 m/s?

32. •• The rms speed of a sample of gas is increased by 1%. **(a)** What is the percent change in the temperature of the gas? **(b)** What is the percent change in the pressure of the gas, assuming its volume is held constant?

33. •• **Enriching Uranium** In naturally occurring uranium atoms, 99.3% are ^{238}U (atomic mass = 238 u, where u = 1.6605×10^{-27} kg) and only 0.7% are ^{235}U (atomic mass = 235 u). Uranium-fueled reactors require an enhanced proportion of ^{235}U. Since both isotopes of uranium have identical chemical properties, they can be separated only by methods that depend on their differing masses. One such method is gaseous diffusion, in which uranium hexafluoride (UF_6) gas diffuses through a series of porous barriers. The lighter $^{235}UF_6$ molecules have a slightly higher rms speed at a given temperature than the heavier $^{238}UF_6$ molecules, and this allows the two isotopes to be separated. Find the ratio of the rms speeds of the two isotopes at 23.0 °C.

34. ••• A 350-mL spherical flask contains 0.075 mol of an ideal gas at a temperature of 293 K. What is the average force exerted on the walls of the flask by a single molecule?

SECTION 17–3 SOLIDS AND ELASTIC DEFORMATION

35. • **CE** A brick has faces with the following dimensions: face 1 is 1 cm by 2 cm; face 2 is 2 cm by 3 cm; face 3 is 1 cm by 3 cm. On which face should the brick be placed if it is to have the smallest change in dimensions due to its own weight? Explain.

36. • **CE Predict/Explain** A hollow cylindrical rod (rod 1) and a solid cylindrical rod (rod 2) are made of the same material. The two rods have the same length and the same outer radius. If the same compressional force is applied to each rod, **(a)** is the change in length of rod 1 greater than, less than, or equal to the change in length of rod 2? **(b)** Choose the *best explanation* from among the following:

I. The solid rod has the larger effective cross-sectional area, since the empty part of the hollow rod doesn't resist compression. Therefore, the solid rod has the smaller change in length.

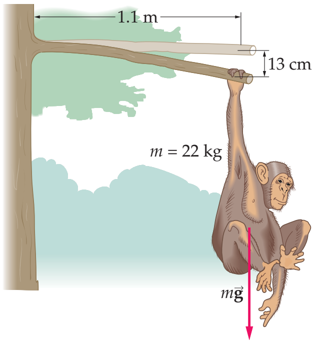
II. The rods have the same outer radius and hence the same cross-sectional area. As a result, their change in length is the same.

III. The walls of the hollow rod are hard and resist compression more than the uniform material in the solid rod. Therefore the hollow rod has the smaller change in length.

37. • A rock climber hangs freely from a nylon rope that is 14 m long and has a diameter of 8.3 mm. If the rope stretches 4.6 cm, what is the mass of the climber?

38. • **BIO** To stretch a relaxed biceps muscle 2.5 cm requires a force of 25 N. Find the Young's modulus for the muscle tissue, assuming it to be a uniform cylinder of length 0.24 m and cross-sectional area 47 cm^2.

39. • A 22-kg chimpanzee hangs from the end of a horizontal, broken branch 1.1 m long, as shown in **Figure 17–25**. The branch is a uniform cylinder 4.6 cm in diameter, and the end of the branch supporting the chimp sags downward through a vertical distance of 13 cm. What is the shear modulus for this branch?



▲ **FIGURE 17–25** Problem 39

40. • **The Marianas Trench** The deepest place in all the oceans is the Marianas Trench, where the depth is 10.9 km and the pressure is 1.10×10^8 Pa. If a copper ball 15.0 cm in diameter is taken to the bottom of the trench, by how much does its volume decrease?

41. •• **CE** Four cylindrical rods with various cross-sectional areas and initial lengths are stretched by an applied force, as in Figure 17–10. The resulting change in length of each rod is given in the following table. Rank these rods in order of increasing Young's modulus. Indicate ties where appropriate.

Rod	Applied Force	Cross-Sectional Area	Initial Length	Change in Length
1	F	A	L	ΔL
2	F	$2A$	$2L$	ΔL
3	$2F$	$2A$	L	$2\Delta L$
4	$3F$	A	$L/2$	ΔL

42. •• **IP** A steel wire 4.7 m long stretches 0.11 cm when it is given a tension of 360 N. **(a)** What is the diameter of the wire? **(b)** If it is desired that the stretch be less than 0.11 cm, should its diameter be increased or decreased? Explain.

43. •• **BIO Spiderweb** An orb weaver spider with a mass of 0.26 g hangs vertically by one of its threads. The thread has a Young's modulus of $4.7 \times 10^9\ N/m^2$ and a radius of 9.8×10^{-6} m. **(a)** What is the fractional increase in the thread's length caused by the spider? **(b)** Suppose a 76-kg person hangs vertically from a nylon rope. What radius must the rope have if its fractional increase in length is to be the same as that of the spider's thread?

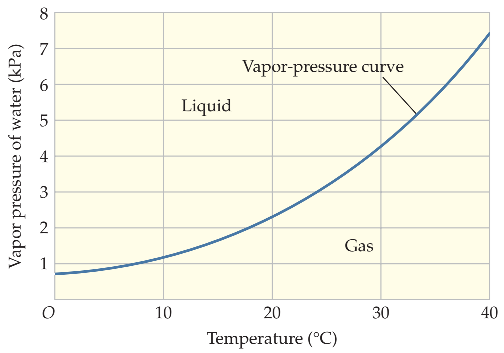
44. •• **IP** Two rods of equal length (0.55 m) and diameter (1.7 cm) are placed end to end. One rod is aluminum, the other is brass. If a compressive force of 8400 N is applied to the rods, **(a)** how much does their combined length decrease? **(b)** Which of the rods changes its length by the greatest amount? Explain.

45. •• A piano wire 0.82 m long and 0.93 mm in diameter is fixed on one end. The other end is wrapped around a tuning peg 3.5 mm in diameter. Initially the wire, whose Young's modulus is $2.4 \times 10^{10}\ N/m^2$, has a tension of 14 N. Find the tension in the wire after the tuning peg has been turned through one complete revolution.

SECTION 17–4 PHASE EQUILIBRIUM AND EVAPORATION

46. • The formation of ice from water is accompanied by which of the following: **(a)** an absorption of heat by the water; **(b)** an increase in temperature; **(c)** a decrease in volume; **(d)** a removal of heat from the water; **(e)** a decrease in temperature?

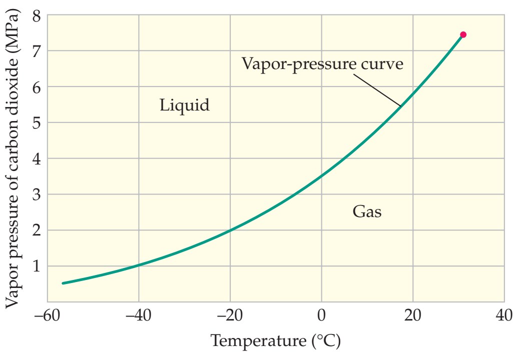
47. • **Vapor Pressure for Water** **Figure 17–26** shows a portion of the vapor-pressure curve for water. Referring to the figure, estimate the pressure that would be required for water to boil at 30 °C.



▲ **FIGURE 17–26** Problems 47 and 48

48. • Using the vapor-pressure curve given in Figure 17–26, find the temperature at which water boils when the pressure is 1.5 kPa.

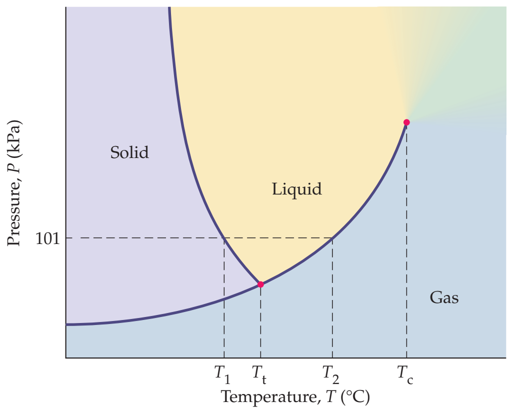
49. • **IP** **The Vapor Pressure of CO_2** A portion of the vapor-pressure curve for carbon dioxide is given in **Figure 17–27**. **(a)** Estimate the pressure at which CO_2 boils at 0 °C. **(b)** If the temperature is increased, does the boiling pressure increase, decrease, or stay the same? Explain.



▲ **FIGURE 17–27** Problems 49 and 50

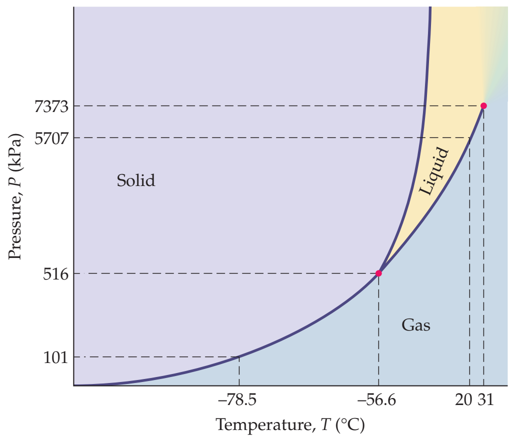
50. • **IP** Referring to the vapor-pressure curve for carbon dioxide given in Figure 17–27, **(a)** estimate the temperature at which CO_2 boils when the pressure is 1.5×10^6 Pa. **(b)** If the pressure is increased, does the boiling temperature increase, decrease, or stay the same? Explain.

51. •• **Phase Diagram for Water** The phase diagram for water is shown in **Figure 17–28**. **(a)** What is the temperature T_1 on the phase diagram? **(b)** What is the temperature T_2 on the phase diagram? **(c)** What happens to the melting/freezing temperature of water if atmospheric pressure is *decreased*? Justify your answer by referring to the phase diagram. **(d)** What happens to the boiling/condensation temperature of water if atmospheric pressure is *increased*? Justify your answer by referring to the phase diagram.



▲ **FIGURE 17–28** Problems 51, 53, and 54

52. •• **Phase Diagram for CO_2** The phase diagram for CO_2 is shown in **Figure 17–29**. **(a)** What is the phase of CO_2 at $T = 20$ °C and $P = 500$ kPa? **(b)** What is the phase of CO_2 at $T = -80$ °C and $P = 120$ kPa? **(c)** For reasons of economy and convenience, bulk CO_2 is often transported in liquid form in pressurized tanks. Using the phase diagram, determine the minimum pressure required to keep CO_2 in the liquid phase at 20 °C.



▲ **FIGURE 17–29** Problems 52 and 55

53. •• A sample of water ice at atmospheric pressure has a temperature just below the freezing point. Refer to Figure 17–28 to answer the following questions. **(a)** What phase changes occur if the temperature of the system is increased while the pressure is held constant? **(b)** Suppose, instead, that the temperature of the system is held constant just below the freezing point while the pressure is decreased. What phase changes occur now?

54. •• A sample of liquid water at atmospheric pressure has a temperature just above the freezing point. Refer to Figure 17–28 to answer the following questions. **(a)** What phase changes occur if the temperature of the system is increased while the pressure is held constant? **(b)** Suppose, instead, that the temperature of the system is held constant just above the freezing point while the pressure is decreased. What phase changes occur now?

55. •• A sample of solid carbon dioxide at a pressure of 5707 kPa has a temperature just below the freezing point. Refer to Figure 17–29 to answer the following questions. **(a)** What phase changes occur if the temperature of the system is increased while the pressure is held constant? **(b)** Suppose, instead, that the temperature of the system is held constant just below the freezing point while the pressure is decreased. What phase changes occur now?

SECTION 17–5 LATENT HEATS

56. •• **CE** Four liquids are at their freezing temperature. Heat is now removed from each of the liquids until it becomes completely solidified. The amount of heat that must be removed, Q, and the mass, m, of each of the liquids are as follows: liquid A, $Q = 33{,}500$ J, $m = 0.100$ kg; liquid B, $Q = 166{,}000$ J, $m = 0.500$ kg; liquid C, $Q = 31{,}500$ J, $m = 0.250$ kg; liquid D,

$Q = 5400$ J, $m = 0.0500$ kg. Rank these liquids in order of increasing latent heat of fusion. Indicate ties where appropriate.

57. • How much heat must be removed from 0.96 kg of water at 0 °C to make ice cubes at 0 °C?

58. • A heat transfer of 9.5×10^5 J is required to convert a block of ice at −15 °C to water at 15 °C. What was the mass of the block of ice?

59. • How much heat must be added to 1.75 kg of copper to change it from a solid at 1358 K to a liquid at 1358 K?

60. •• **IP** A 1.1-kg block of ice is initially at a temperature of −5.0 °C. **(a)** If 5.2×10^5 J of heat are added to the ice, what is the final temperature of the system? Find the amount of ice, if any, that remains. **(b)** Suppose the amount of heat added to the ice block is doubled. By what factor must the mass of the ice be increased if the system is to have the same final temperature? Explain.

61. •• **IP** Referring to the previous problem, suppose the amount of heat added to the block of ice is reduced by a factor of 2 to 2.6×10^5 J. Note that this amount of heat is still sufficient to melt at least some of the ice. **(a)** Do you expect the temperature increase in this case to be one-half that found in the previous problem? Explain. **(b)** What is the final temperature of the system in this case? Find the amount of ice, if any, that remains.

62. •• Figure 17–21 shows a temperature-versus-heat plot for 1.000 kg of water. **(a)** Calculate the heat corresponding to the points A, B, C, and D. **(b)** Calculate the slope of the line from point B to point C. Show that this slope is equal to $1/c$, where c is the specific heat of liquid water.

63. •• **IP** Suppose the 1.000 kg of water in Figure 17–21 starts at point A at time zero. Heat is added to this system at the rate of 12,250 J/s. How long does it take for the system to reach **(a)** point B, **(b)** point C, and **(c)** point D? **(d)** Describe the physical state of the system at time $t = 63.00$ s.

64. •• Figure 17–22 shows a temperature-versus-heat plot for 0.550 kg of water. **(a)** Calculate the slope of the line from point A to point B. Show that the slope is equal to $1/mc$, where c is the specific heat of ice. **(b)** Calculate the slope of the line from point C to point D. Show that the slope is equal to $1/mc$, where c is the specific heat of liquid water.

65. •• **BIO** In Conceptual Checkpoint 17–5 we pointed out that steam can cause more serious burns than water at the same temperature. Here we examine this effect quantitatively, noting that flesh becomes badly damaged when its temperature reaches 50.0 °C. **(a)** Calculate the heat released as 12.5 g of liquid water at 100 °C is cooled to 50.0 °C. **(b)** Calculate the heat released when 12.5 g of steam at 100 °C is condensed and cooled to 50.0 °C. **(c)** Find the mass of flesh that can be heated from 37.0 °C (normal body temperature) to 50.0 °C for the cases considered in parts (a) and (b). (The average specific heat of flesh is 3500 J/kg · K.)

66. •• When you go out to your car one cold winter morning you discover a 0.58-cm-thick layer of ice on the windshield, which has an area of 1.6 m^2. If the temperature of the ice is −2.0 °C, and its density is 917 kg/m^3, find the heat required to melt all the ice.

SECTION 17–6 PHASE CHANGES AND ENERGY CONSERVATION

67. •• A large punch bowl holds 3.99 kg of lemonade (which is essentially water) at 20.5 °C. A 0.0550-kg ice cube at −10.2 °C is placed in the lemonade. What is the final temperature of the system, and the amount of ice (if any) remaining? Ignore any heat exchange with the bowl or the surroundings.

68. •• A 155-g aluminum cylinder is removed from a liquid nitrogen bath, where it has been cooled to −196 °C. The cylinder is immediately placed in an insulated cup containing 80.0 g of water at 15.0 °C. What is the equilibrium temperature of this system? If your answer is 0 °C, determine the amount of water that has frozen. The average specific heat of aluminum over this temperature range is 653 J/(kg · K).

69. •• An 825-g iron block is heated to 352 °C and placed in an insulated container (of negligible heat capacity) containing 40.0 g of water at 20.0 °C. What is the equilibrium temperature of this system? If your answer is 100 °C, determine the amount of water that has vaporized. The average specific heat of iron over this temperature range is 560 J/(kg · K).

70. •• **IP** A 35-g ice cube at 0.0 °C is added to 110 g of water in a 62-g aluminum cup. The cup and the water have an initial temperature of 23 °C. **(a)** Find the equilibrium temperature of the cup and its contents. **(b)** Suppose the aluminum cup is replaced with one of equal mass made from silver. Is the equilibrium temperature with the silver cup greater than, less than, or the same as with the aluminum cup? Explain.

71. •• A 48-g block of copper at −12 °C is added to 110 g of water in a 75-g aluminum cup. The cup and the water have an initial temperature of 4.1 °C. **(a)** Find the equilibrium temperature of the cup and its contents. **(b)** What mass of ice, if any, is present when the system reaches equilibrium?

72. •• A 0.075-kg ice cube at 0.0 °C is dropped into a Styrofoam cup holding 0.33 kg of water at 14 °C. **(a)** Find the final temperature of the system and the amount of ice (if any) remaining. Assume the cup and the surroundings can be ignored. **(b)** Find the initial temperature of the water that would be enough to *just barely* melt all of the ice.

73. •• To help keep her barn warm on cold days, a farmer stores 865 kg of warm water in the barn. How many hours would a 2.00-kilowatt electric heater have to operate to provide the same amount of heat as is given off by the water as it cools from 20.0 °C to 0 °C and then freezes at 0 °C?

GENERAL PROBLEMS

74. • **CE** Plastic bubble wrap is used as a protective packing material. Is the bubble wrap more effective on a cold day or on a warm day? Explain.

75. • **CE** Two adjacent rooms in a hotel are equal in size and connected by an open door. Room 1 is warmer than room 2. Which room contains more air? Explain.

76. • **CE** As you go up in altitude, do you expect the ratio of oxygen to nitrogen in the atmosphere to increase, decrease; or stay the same? Explain.

77. • **CE Predict/Explain** Suppose the Celsius temperature of an ideal gas is doubled from 100 °C to 200 °C. **(a)** Does the average kinetic energy of the molecules in this gas increase by a factor that is greater than, less than, or equal to 2? **(b)** Choose the *best explanation* from among the following:
   I. Changing the temperature from 100 °C to 200 °C goes beyond the boiling point, which will increase the kinetic energy by more than a factor of 2.
   II. The average kinetic energy is directly proportional to the temperature, so doubling the temperature doubles the kinetic energy.

III. Doubling the Celsius temperature from 100 °C to 200 °C changes the Kelvin temperature from 373.15 K to 473.15 K, which is an increase of less than a factor of 2.

78. • **CE Predict/Explain** Suppose the absolute temperature of an ideal gas is doubled from 100 K to 200 K. **(a)** Does the average speed of the molecules in this gas increase by a factor that is greater than, less than, or equal to 2? **(b)** Choose the *best explanation* from among the following:

I. Doubling the Kelvin temperature doubles the average kinetic energy, but this implies an increase in the average speed by a factor of $\sqrt{2} = 1.414\ldots$, which is less than 2.

II. The Kelvin temperature is the one we use in the ideal-gas law, and therefore doubling it also doubles the average speed of the molecules.

III. The change in average speed depends on the mass of the molecules in the gas, and hence doubling the Kelvin temperature generally results in an increase in speed that is greater than a factor of 2.

79. •**Largest Raindrops** Atmospheric scientists studying clouds in the Marshall Islands have observed what they believe to be the world's largest raindrops, with a radius of 0.52 cm. How many molecules are in these monster drops?

80. •**Cooling Computers** Researchers are developing "heat exchangers" for laptop computers that take heat from the laptop—to keep it from being damaged by overheating—and use it to vaporize methanol. Given that 5100 J of heat is removed from the laptop when 4.6 g of methanol is vaporized, what is the latent heat of vaporization for methanol?

81. •• **Scuba Tanks** In scuba diving circles, "an 80" refers to a scuba tank that holds 80 cubic feet of air, a standard amount for recreational diving. Given that a scuba tank is a cylinder 2 feet long and half a foot in diameter, determine **(a)** the volume of a tank and **(b)** the pressure in a tank when 80 cubic feet of air is compressed into its relatively small volume. **(c)** What is the mass of air in a tank that holds 80 cubic feet of air. Assume the temperature is 21 °C and that the walls of the tank are of negligible thickness.

82. •• A reaction vessel contains 8.06 g of H_2 and 64.0 g of O_2 at a temperature of 125 °C and a pressure of 101 kPa. **(a)** What is the volume of the vessel? **(b)** The hydrogen and oxygen are now ignited by a spark, initiating the reaction $2\,H_2 + O_2 \rightarrow 2\,H_2O$. This reaction consumes all the hydrogen and oxygen in the vessel. What is the pressure of the resulting water vapor when it returns to its initial temperature of 125 °C?

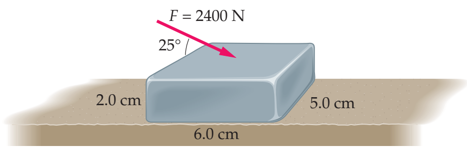
83. •• A bicycle tire with a radius of 0.68 m has a gauge pressure of 42 lb/in^2. Treating the tire as a hollow hoop with a cross-sectional area of 0.0028 m^2, find the number of air molecules in the tire when its temperature is 24 °C.

84. •• Peter catches a 4.8-kg striped bass on a fishing line 0.54 mm in diameter and begins to reel it in. He fishes from a pier well above the water, and his fish hangs vertically from the line out of the water. The fishing line has a Young's modulus of $5.1 \times 10^9\ \text{N/m}^2$. **(a)** What is the fractional increase in length of the fishing line if the fish is at rest? **(b)** What is the fractional increase in the fishing line's length when the fish is pulled upward with a constant speed of 1.5 m/s? **(c)** What is the fractional increase in the fishing line's length when the fish is pulled upward with a constant acceleration of 1.5 m/s^2?

85. •• **IP** You use a steel socket wrench 28 cm long to loosen a rusty bolt, applying a force F at the end of the handle. The handle undergoes a shear deformation of 0.11 mm. **(a)** If the cross-sectional area of the handle is 2.3 cm^2, what is the magnitude of the applied force F? **(b)** If the cross-sectional area of the handle is doubled, by what factor does the shear deformation change? Explain.

86. •• A steel ball (density = 7860 kg/m^3) with a diameter of 6.4 cm is tied to an aluminum wire 82 cm long and 2.5 mm in diameter. The ball is whirled about in a vertical circle with a tangential speed of 7.8 m/s at the top of the circle and 9.3 m/s at the bottom of the circle. Find the amount of stretch in the wire **(a)** at the top and **(b)** at the bottom of the circle.

87. •• A lead brick with the dimensions shown in **Figure 17–30** rests on a rough solid surface. A force of 2400 N is applied as indicated. Find **(a)** the change in height of the brick and **(b)** the amount of shear deformation.



▲ **FIGURE 17–30** Problem 87

88. •• **IP** Five molecules have the following speeds: 221 m/s, 301 m/s, 412 m/s, 44.0 m/s, and 182 m/s. **(a)** Find v_{av} for these molecules. **(b)** Do you expect $(v^2)_{av}$ to be greater than, less than, or equal to $(v_{av})^2$? Explain. **(c)** Calculate $(v^2)_{av}$ and comment on your results. **(d)** Calculate v_{rms} and compare with v_{av}.

89. •• **(a)** Find the amount of heat that must be extracted from 1.5 kg of steam at 110 °C to convert it to ice at 0.0 °C. **(b)** What speed would this 1.5-kg block of ice have if its translational kinetic energy were equal to the thermal energy calculated in part (a)?

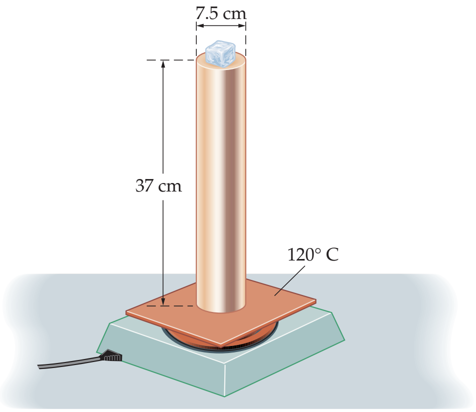
90. •• When water freezes into ice it expands in volume by 9.05%. Suppose a volume of water is in a household water pipe or a cavity in a rock. If the water freezes, what pressure must be exerted on it to keep its volume from expanding? (If the pipe or rock cannot supply this pressure, the pipe will burst and the rock will split.)

91. •• Suppose the 0.550 kg of ice in Figure 17–22 starts at point A. How much ice is left in the system after **(a)** 5.00×10^4 J, **(b)** 1.00×10^5 J, and **(c)** 1.50×10^5 J of heat are added to the system?

92. ••• Students on a spring break picnic bring a cooler that contains 5.1 kg of ice at 0.0 °C. The cooler has walls that are 3.8 cm thick and are made of Styrofoam, which has a thermal conductivity of 0.030 W/(m · C°). The surface area of the cooler is 1.5 m^2, and it rests in the shade where the air temperature is 21 °C. **(a)** Find the rate at which heat flows into the cooler. **(b)** How long does it take for the ice in the cooler to melt?

93. ••• A 5.5-kg block of ice at −1.5 °C slides on a horizontal surface with a coefficient of kinetic friction equal to 0.062. The initial speed of the block is 6.9 m/s and its final speed is 5.5 m/s. Assuming that all the energy dissipated by kinetic friction goes into melting a small mass m of the ice, and that the rest of the ice block remains at −1.5 °C , determine the value of m.

94. ••• A cylindrical copper rod 37 cm long and 7.5 cm in diameter is placed upright on a hot plate held at a constant temperature of 120 °C, as indicated in **Figure 17–31**. A small depression on top of the rod holds a 25-g ice cube at an initial temperature of 0.0 °C. How long does it take for the ice cube to melt? Assume there is no heat loss through the vertical surface of the rod, and that the thermal conductivity of copper is 390 W/(m · C°).

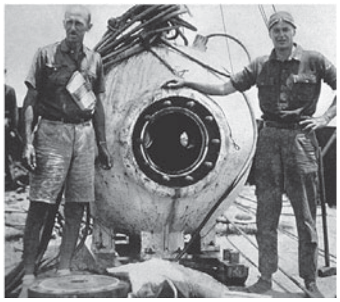


▲ **FIGURE 17–31** Problem 94

PASSAGE PROBLEMS

Diving in the Bathysphere

The American naturalist Charles William Beebe (1877–1962) set a world record in 1934 when he and Otis Barton (1899–1992) made a dive to a depth of 923 m below the surface of the ocean. The dive was made just 10 miles from Nonsuch Island, off the coast of Bermuda, in a device known as the bathysphere, designed and built by Barton. The bathysphere was basically a steel sphere 4.75 ft in diameter with three small ports made of fused quartz. Lowered into the ocean on a steel cable whose radius was 1.85 cm, the bathysphere also carried bottles of oxygen, chemicals to absorb carbon dioxide, and a telephone line to the surface.



Charles William Beebe (left) and Otis Barton with the bathysphere in 1934. (Problems 95, 96, 97, and 98)

Beebe was fascinated by the new forms of life he and Barton encountered on their numerous dives. At one point he saw a "creature, several feet long, dart toward the window, turn sideways and—explode. At the flash, which was so strong that it illumined my face . . . I saw the great red shrimp and the outpouring fluid of flame." No wonder he considered the ocean depths to be "a world as strange as that of Mars."

The dives were not without their risks, however. It was not uncommon, for example, to have the bathysphere return to the surface partially filled with water after a window seal failed. On one deep dive, water began to stream rapidly into the sphere. Beebe quickly called to the surface and asked—not to be raised quickly—but to be lowered more rapidly, in the hope that increasing water pressure would force the leaking window into its seals to stop the leak. It worked, showing that Beebe was not only an exceptional naturalist, but also a cool-headed scientist with a good knowledge of basic physics!

95. • What pressure did the bathysphere experience at its record depth?

A. 9.37 atm **B.** 89.6 atm

C. 91.9 atm **D.** 92.9 atm

96. • How many moles of air did the bathysphere contain when it was sealed at the surface, assuming a temperature of 297 K and ignoring the thickness of the metal shell? (*Note:* A resting person breathes roughly 0.5 mol of air per minute.)

A. 65.2 mol **B.** 270 mol

C. 392 mol **D.** 523 mol

97. • How much did the volume of the bathysphere decrease as it was lowered to its record depth? (For simplicity, treat the bathysphere as a solid metal sphere.)

A. $9.0 \times 10^{-5}\ m^3$ **B.** $9.2 \times 10^{-5}\ m^3$

C. $1.1 \times 10^{-4}\ m^3$ **D.** $3.8 \times 10^{-4}\ m^3$

98. • Suppose the bathysphere and its occupants had a combined mass of 12,700 kg. How much did the cable stretch when the bathysphere was at a depth of 923 m? (Neglect the weight of the cable itself, but include the effects of the bathysphere's buoyancy.)

A. 47 cm **B.** 48 cm

C. 52 cm **D.** 53 cm

INTERACTIVE PROBLEMS

99. •• **Referring to Example 17–6 (a)** Find the final temperature of the system if *two* 0.0450-kg ice cubes are added to the warm lemonade. The temperature of the ice is 0 °C; the temperature and mass of the warm lemonade are 20.0 °C and 3.95 kg, respectively. **(b)** How many 0.0450-kg ice cubes at 0 °C must be added to the original warm lemonade if the final temperature of the system is to be at least as cold as 15.0 °C?

100. •• **Referring to Example 17–6 (a)** Find the final temperature of the system if a single 0.045-kg ice cube at 0 °C is added to 2.00 kg of lemonade at 1.00 °C. **(b)** What initial temperature of the lemonade will be just high enough to melt all of the ice in a single ice cube and result in an equilibrium temperature of 0 °C? The mass of the lemonade is 2.00 kg and the temperature of the ice cube is 0 °C.

18 The Laws of Thermodynamics

Every day, plants and animals take small, simple molecules and use them to create large, complex molecules such as proteins and DNA. These, in turn, are assembled into even more highly-ordered structures—a butterfly grows from a formless egg to a complex organism, blooms on a plant progress from minute buds to fully formed flowers. Yet, one of the most profound and far-reaching of physical laws holds that all processes must decrease the amount of order in the universe. Are living things exempt from this principle? Do they really make the universe more ordered? This chapter explores the laws of thermodynamics, and considers the question of whether they apply to biological organisms.

In this chapter we discuss the fundamental laws of nature that govern thermodynamic processes. One of these laws—the one dealing with temperature—was first introduced in Chapter 16. The others are presented here for the first time.

Of particular interest are the first and second laws of thermodynamics. The first law extends the basic principle of energy conservation to include the type of energy transfer we call heat. The real heart of thermodynamics, however, is embodied in the second law. This law introduces a fundamentally new concept to physics—the idea that there is a directionality to the behavior of nature. Melting ice, cooling lava, and the crumbling ruins of the Parthenon all illustrate the second law in action.

Finally, the third law of thermodynamics states that absolute zero is, in fact, the lowest temperature possible. Great efforts have been made over the years to reach lower and lower temperatures, and with remarkable success, but the third law sets the ultimate limit that we can only approach, but never attain.

18–1 The Zeroth Law of Thermodynamics

Though the zeroth law of thermodynamics has already been presented in Chapter 16, we repeat it here so that all the laws of thermodynamics can be collected together in one chapter. As you recall, the zeroth law states the conditions under which objects will be in thermal equilibrium with one another. To be precise:

Zeroth Law of Thermodynamics
If object A is in thermal equilibrium with object C, and object B is separately in thermal equilibrium with object C, then objects A and B will be in thermal equilibrium if they are placed in thermal contact.

The physical quantity that is equal when two objects are in thermal equilibrium is the temperature. In particular, if two objects have the same temperature, we can be assured that *no heat will flow* when they are placed in thermal contact. On the other hand, if heat does flow between two objects, it follows that they are not in thermal equilibrium, and they do not have the same temperature.

Any temperature scale can be used to determine whether objects will be in thermal equilibrium. As we saw in the previous chapter, however, the Kelvin scale is particularly significant in physics. For example, the average kinetic energy of a gas molecule is directly proportional to the Kelvin temperature, as is the volume of an ideal gas. In this chapter we present additional illustrations of the special significance of the Kelvin scale.

18–2 The First Law of Thermodynamics

The first law of thermodynamics is a statement of energy conservation that specifically includes heat. For example, consider the system shown in **Figure 18–1**. The internal energy of this system—that is, the sum of all its potential and kinetic energies—has the initial value U_i. If an amount of heat Q flows into the system, its internal energy increases to the final value $U_f = U_i + Q$. Thus,

$$\Delta U = U_f - U_i = Q \qquad 18\text{–}1$$

Of course, if heat is removed from the system, its internal energy decreases. We can take this into account by giving Q a *positive* value when the system *gains* heat, and a *negative* value when it *loses* heat.

Similarly, suppose the system under consideration does a work W on the external world, as in **Figure 18–2**. If the system is insulated so that no heat can flow in or out, the energy to do the work must come from the internal energy of the system. Thus, if the initial internal energy is U_i, the final internal energy is $U_f = U_i - W$. Therefore,

$$\Delta U = U_f - U_i = -W \qquad 18\text{–}2$$

On the other hand, if work is done *on* the system, its internal energy increases. Thus, we use the following sign convention for the work: W has a *positive* value when the system *does work on the external world*, and it has a *negative* value when *work is done on the system*. These sign conventions are summarized in Table 18–1.

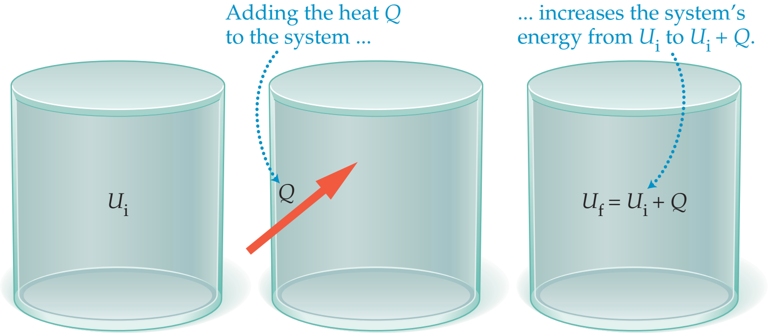


FIGURE 18–1 The internal energy of a system
A system initially has the internal energy U_i (left). After the heat Q is added, the system's new internal energy is $U_f = U_i + Q$ (right). The system has rigid walls; hence, it can do no work on the external world.

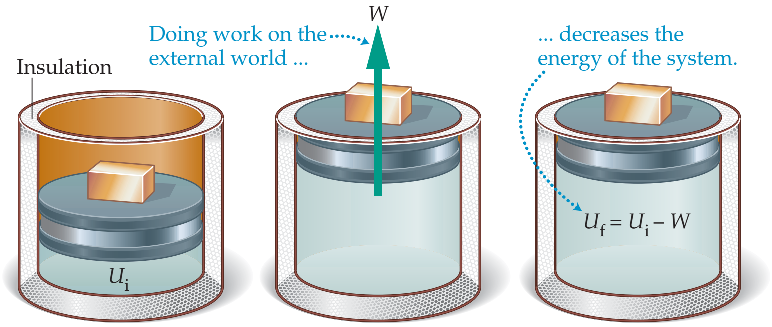


FIGURE 18–2 Work and internal energy
A system initially has the internal energy U_i (left). After the system does the work W on the external world, its remaining internal energy is $U_f = U_i - W$ (right). Note that the insulation guarantees that no heat is gained or lost by the system.

TABLE 18–1 Signs of Q and W

Q positive	System *gains* heat
Q negative	System *loses* heat
W positive	Work done *by* system
W negative	Work done *on* system

PROBLEM-SOLVING NOTE

Proper Signs for Q and W

When applying the first law of thermodynamics, it is important to determine the proper signs for Q, W, and ΔU.

Combining the results in Equations 18–1 and 18–2 yields the first law of thermodynamics:

First Law of Thermodynamics

The change in a system's internal energy, ΔU, is related to the heat Q and the work W as follows:

$$\Delta U = Q - W \qquad 18\text{–}3$$

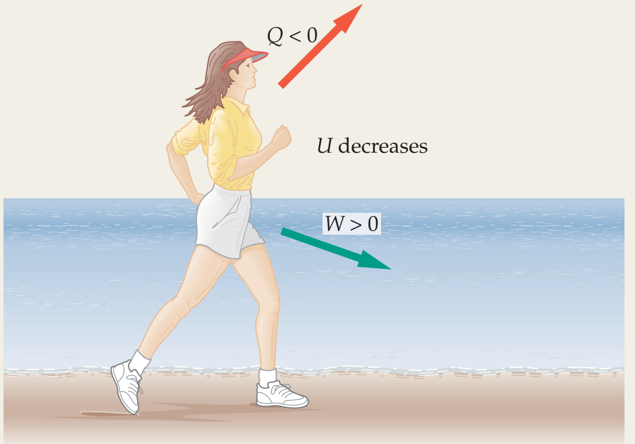
If you apply the sign conventions given here, it is straightforward to verify that adding heat to a system, and/or doing work on it, increases the internal energy. On the other hand, if the system does work, and/or heat is removed, its internal energy decreases. Example 18–1 gives a specific numerical application of the first law.

EXAMPLE 18–1 HEAT, WORK, AND INTERNAL ENERGY

(a) Jogging along the beach one day, you do 4.3×10^5 J of work and give off 3.8×10^5 J of heat. What is the change in your internal energy? **(b)** Switching over to walking, you give off 1.2×10^5 J of heat and your internal energy decreases by 2.6×10^5 J. How much work have you done while walking?

PICTURE THE PROBLEM

Our sketch shows a person jogging along the beach. The fact that the person does work on the external world means that W is positive. As for the heat, the fact that heat is given off by the person means that Q is negative.



STRATEGY

The signs of W and Q have been determined in our sketch, and the magnitudes are given in the problem statement. To find ΔU for part (a), we simply use the first law of thermodynamics, $\Delta U = Q - W$. To find the work W for part (b), we solve the first law for W, which yields $W = Q - \Delta U$.

SOLUTION

Part (a)

1. Calculate ΔU, using $Q = -3.8 \times 10^5$ J and $W = 4.3 \times 10^5$ J:

$$\Delta U = Q - W$$
$$= (-3.8 \times 10^5\ \text{J}) - 4.3 \times 10^5\ \text{J} = -8.1 \times 10^5\ \text{J}$$

Part (b)

2. Solve $\Delta U = Q - W$ for W:

$$W = Q - \Delta U$$

3. Substitute $Q = -1.2 \times 10^5$ J and $\Delta U = -2.6 \times 10^5$ J:

$$W = -1.2 \times 10^5\ \text{J} - (-2.6 \times 10^5\ \text{J}) = 1.4 \times 10^5\ \text{J}$$

INSIGHT

Note the importance of using the correct signs for Q, W, and ΔU. When the proper signs are used, the first law is simply a way of keeping track of all the energy exchanges—including mechanical work and heat—that can occur in a given system.

PRACTICE PROBLEM

After walking for a few minutes you begin to run, doing 5.1×10^5 J of work and decreasing your internal energy by 8.8×10^5 J. How much heat have you given off? [**Answer:** $Q = -3.7 \times 10^5$ J. The negative sign means you have given off heat, as expected.]

Some related homework problems: Problem 3, Problem 5

Just looking at the first law, $\Delta U = Q - W$, it is easy to get the false impression that U, Q, and W are basically the same type of physical quantity. Certainly, they are all measured in the same units (J). In other respects, however, they are quite different. For example, the heat Q represents energy that flows through thermal contact. In contrast, the work W indicates a transfer of energy by the action of a force through a distance.

The most important distinction between these quantities, however, is the way they depend on the **state of a system,** which is determined by its temperature, pressure, and volume. The internal energy, for example, depends only on the state of a system, and not on how the system is brought to that state. A simple example is the ideal gas, where U depends only on the temperature T, and not on any previous values T may have had. Since U depends only on the state of a system—whether the system is an ideal gas or something more complicated—it is referred to as a **state function.**

On the other hand, Q and W are *not* state functions; they depend on the precise way—that is, on the process—by which a system is changed from one state, A, to another state, B. For example, one process connecting the states A and B may result in a heat $Q_1 = 19$ J and a work $W_1 = 32$ J. With a different process connecting the *same two states*, we may find a heat $Q_2 = -24\ \mathrm{J} \neq Q_1$, and a work $W_2 = -11\ \mathrm{J} \neq W_1$. Still, the difference in internal energy—which depends only on the initial and final states—must be the same for all processes connecting A and B. It follows that

$$\Delta U_{AB} = Q_1 - W_1 = Q_2 - W_2 = -13\ \mathrm{J}$$

Therefore, the energy of this system always decreases by 13 J when it changes from state A to state B.

We now turn our attention to specific types of thermal processes that can change the energy of a system.

18–3 Thermal Processes

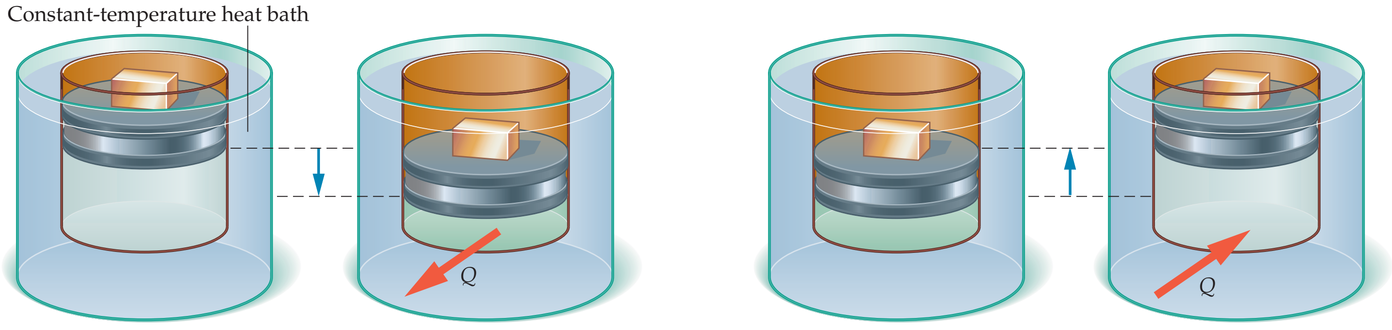
In this section we consider a variety of thermodynamic processes that can be used to change the state of a system. Among these are ones that take place at constant pressure, constant volume, or constant temperature. We also consider processes in which no heat is allowed to flow into or out of the system.

All of the processes discussed in this section are assumed to be **quasi-static,** which is a fancy way of saying they occur so slowly that at any given time the system and its surroundings are essentially in equilibrium. Thus, in a quasi-static process, the pressure and temperature are always uniform throughout the system. Furthermore, we assume that the system in question is free from friction or other dissipative forces.

These assumptions can be summarized by saying that the processes we consider are **reversible.** To be precise, a reversible process is defined by the following condition:

> For a process to be reversible, it must be possible to return both the system and its surroundings to exactly the same states they were in before the process began.

The fact that both the system and its surroundings must be returned to their initial states is the key element of this definition. For example, if there is friction between a piston and the cylinder in which it slides, a reversible process is not possible. Even if the piston is returned to its original location, the heat generated by friction



(a) Compressing at constant temperature expels heat.

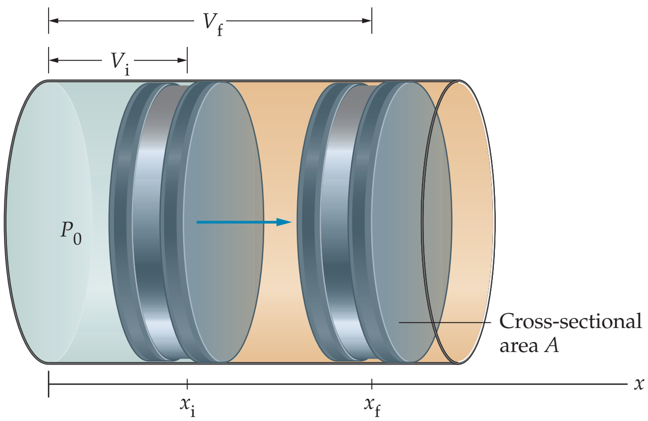
(b) Expanding at constant temperature draws in heat.

▲ FIGURE 18–3 An idealized reversible process

(a) A piston is slowly moved downward, compressing a gas. In order for its temperature to remain constant, a heat Q goes from the gas into the constant-temperature heat bath, which may be nothing more than a large volume of water. **(b)** As the piston is allowed to slowly rise back to its initial position, it draws the same amount of heat Q from the heat bath that it gave to the bath in (a). Hence, both the system (gas) and the surroundings (heat bath) return to their initial states.

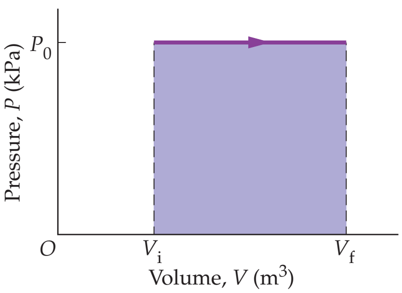
will warm the cylinder and eventually flow into the surrounding air. Thus, it is not possible to "undo" the effects of friction, and such a process is said to be **irreversible.** Even without friction, a process can be irreversible if it occurs rapidly enough to cause effects such as turbulence, or if the system is far from equilibrium.

In practice, then, all real processes are irreversible to some extent. It is still possible, however, to have a process that closely approximates a perfectly reversible process, just as we can have systems that are practically free of friction. For example, in **Figure 18–3 (a)**, we consider a "frictionless" piston that is slowly forced downward, while the gas in the cylinder is kept at constant temperature. An amount of heat, Q, goes from the gas to its surroundings in order to keep the temperature from rising. In **Figure 18–3 (b)** the piston is slowly moved back upward, drawing in the same heat Q from its surroundings to keep the temperature from dropping. In an "ideal" case like this, the process is reversible, since the system and its surroundings are left unchanged. We shall assume that all the processes described in this section are reversible.



▶ FIGURE 18–4 Work done by an expanding gas

A gas in a cylinder of cross-sectional area A expands with a constant pressure of P_0 from an initial volume $V_i = Ax_i$ to a final volume $V_f = Ax_f$. As it expands, it does the work $W = P_0(V_f - V_i)$.



▲ FIGURE 18–5 A constant-pressure process

A PV plot representing the constant-pressure process shown in Figure 18–4. The area of the shaded region, $P_0(V_f - V_i)$, is equal to the work done by the expanding gas in Figure 18–4.

Constant Pressure/Constant Volume

We begin by considering a process that occurs at **constant pressure.** To be specific, suppose that a gas with the pressure P_0 is held in a cylinder of cross-sectional area A, as in **Figure 18–4**. If the piston moves outward, so that the volume of the gas increases from an initial value V_i to a final value V_f, the process can be represented graphically as shown in **Figure 18–5**. Here we plot pressure P versus volume V in a PV plot; the process just described is the horizontal line segment.

As the gas expands, it does work on the piston. First, the gas exerts a force on the piston equal to the pressure times the area:

$$F = P_0A$$

Second, the gas moves the piston from the position x_i to the position x_f, where $V_i = Ax_i$ and $V_f = Ax_f$, as indicated in Figure 18–4. Thus, the work done by the gas is the force times the distance through which the piston moves:

$$W = F(x_f - x_i) = P_0A(x_f - x_i) = P_0(Ax_f - Ax_i) = P_0(V_f - V_i)$$

In general, if the pressure, P, is constant, and the volume changes by the amount ΔV, the work done by the gas is

$$W = P\Delta V \qquad (\textit{constant pressure}) \qquad 18\text{–}4$$

EXERCISE 18–1

A gas with a constant pressure of 150 kPa expands from a volume of 0.76 m^3 to a volume of 0.92 m^3. How much work does the gas do?

SOLUTION

Applying Equation 18–4 we find

$$W = P\Delta V = (150\text{ kPa})(0.92\text{ m}^3 - 0.76\text{ m}^3) = 24{,}000\text{ J}$$

This is the energy required to raise the temperature of 1.0 kg of water by 5.7 C°.

Looking closely at the PV plot in Figure 18–5, we see that the work done by the gas is equal to the area under the horizontal line representing the constant-pressure process. In particular, the shaded region is a rectangle of height P_0 and width $V_f - V_i$, and therefore its area is

$$\text{area} = P_0(V_f - V_i) = W$$

Though this result was obtained for the special case of constant pressure, it applies to any process; that is, *the work done by an expanding gas is equal to the area under the curve representing the process in a PV plot*. This result is applied in the next Example.

PROBLEM-SOLVING NOTE

Work and the PV Diagram

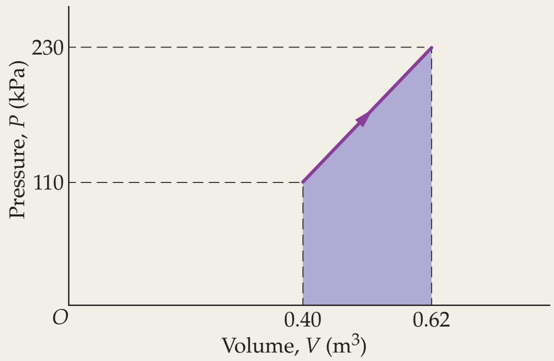
When finding the work done by calculating the area on a PV diagram, recall that a pressure of 1 Pa times a volume of 1 m^3 gives an energy equal to 1 J.

EXAMPLE 18–2 WORK AREA

A gas expands from an initial volume of 0.40 m^3 to a final volume of 0.62 m^3 as the pressure increases linearly from 110 kPa to 230 kPa. Find the work done by the gas.

PICTURE THE PROBLEM

The sketch shows the process for this problem. As the volume increases from $V_i = 0.40\text{ m}^3$ to $V_f = 0.62\text{ m}^3$, the pressure increases linearly from $P_i = 110$ kPa to $P_f = 230$ kPa.



STRATEGY

The work done by this gas is equal to the shaded area in the sketch. We can calculate this area as the sum of the area of a rectangle plus the area of a triangle.

In particular, the rectangle has a height P_i and a width $(V_f - V_i)$. Similarly, the triangle has a height $(P_f - P_i)$ and a base of $(V_f - V_i)$.

SOLUTION

1. Calculate the area of the rectangular portion of the total area:

$$A_{\text{rectangle}} = P_i(V_f - V_i) = (110\text{ kPa})(0.62\text{ m}^3 - 0.40\text{ m}^3) = 2.4 \times 10^4\text{ J}$$

2. Next, calculate the area of the triangular portion of the total area:

$$A_{\text{triangle}} = \tfrac{1}{2}(P_f - P_i)(V_f - V_i) = \tfrac{1}{2}(230\text{ kPa} - 110\text{ kPa})(0.62\text{ m}^3 - 0.40\text{ m}^3) = 1.3 \times 10^4\text{ J}$$

3. Sum these areas to find the work done by the gas:

$$W = A_{\text{rectangle}} + A_{\text{triangle}} = 2.4 \times 10^4\text{ J} + 1.3 \times 10^4\text{ J} = 3.7 \times 10^4\text{ J}$$

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INSIGHT

We could also have solved this problem by noting that since the pressure varies linearly, its average value is simply $P_{av} = \frac{1}{2}(P_f + P_i) = 170\text{ kPa}$. The work done by the gas, then, is $W = P_{av}\,\Delta V = (170\text{ kPa})(0.22\text{ m}^3) = 3.7 \times 10^4\text{ J}$, as before.

PRACTICE PROBLEM

Suppose the pressure varies linearly from 110 kPa to 260 kPa. How much work does the gas do in this case?
[**Answer:** $W = 4.1 \times 10^4\text{ J}$]

Some related homework problems: Problem 18, Problem 19

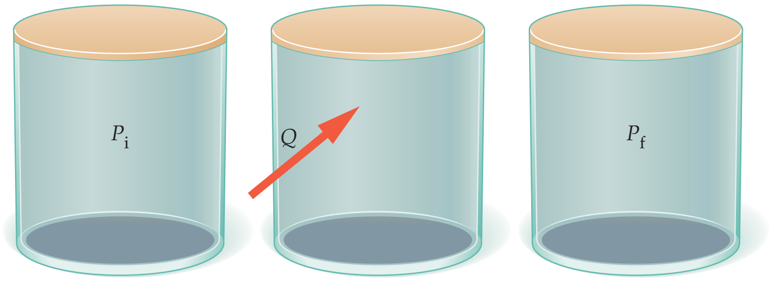


FIGURE 18–6 Adding heat to a system of constant volume

Heat is added to a system of constant volume, increasing its pressure from P_i to P_f. Since there is no displacement of the walls, there is no work done in this process. Therefore, $W = 0$ and the change in internal energy is simply equal to the heat added to or removed from the system, $\Delta U = Q$.

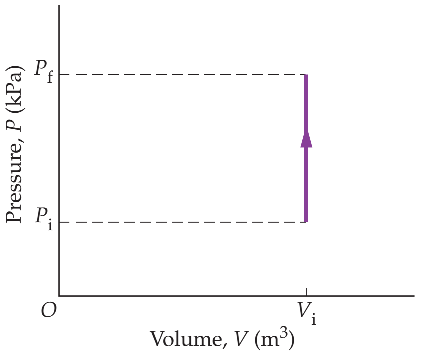


FIGURE 18–7 A constant-volume process

The pressure increases from P_i to P_f, just as in Figure 18–6, while the volume remains constant at its initial value, V_i. The area under this process is zero, as is the work.

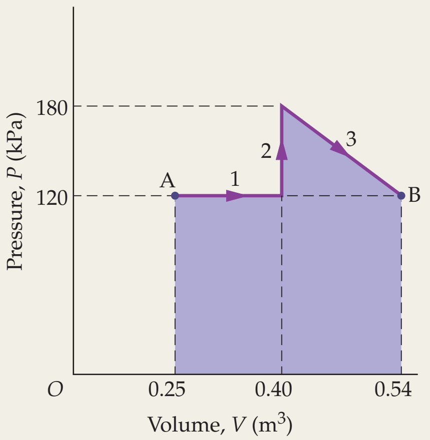
Next, we consider a **constant-volume** process. Suppose, for example, that heat is added to a gas in a container of fixed volume, as in **Figure 18–6**, causing the pressure to increase. Since there is no displacement of any of the walls, it follows that the force exerted by the gas does no work. Thus, for *any* constant-volume process, we have

$$W = 0 \qquad (\textit{constant volume})$$

From the first law of thermodynamics, $\Delta U = Q - W$, it follows that $\Delta U = Q$; that is, the change in internal energy is equal to the amount of heat that is added to or removed from the system. Note that zero work in a constant-volume process is consistent with our earlier statement that the work is equal to the area under the curve representing the process. For example, in **Figure 18–7** we show a constant-volume process in which the pressure is increased from P_i to P_f. Since this line is vertical, the area under it is zero; that is, $W = 0$ as expected.

ACTIVE EXAMPLE 18–1 FIND THE TOTAL WORK

A gas undergoes the three-part process shown here, connecting the states A and B. Find the total work done by the gas during this process.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the work done during part 1 of the total process: 18,000 J
2. Calculate the work done during part 2 of the total process: 0 J
3. Calculate the work done during part 3 of the total process: 21,000 J
4. Sum the results to find the total work: 39,000 J

INSIGHT

Note that the work done by the gas would be different if the process connecting A and B were a constant-pressure expansion with a pressure of 120 kPa. In that case, the total work done would be 35,000 J. Thus, the work W—which is not a state function—depends on the process, as expected.

YOUR TURN

Describe a process connecting the states A and B for which the work done by the gas is 58,000 J.

(Answers to **Your Turn** *problems are given in the back of the book.)*

Isothermal Processes

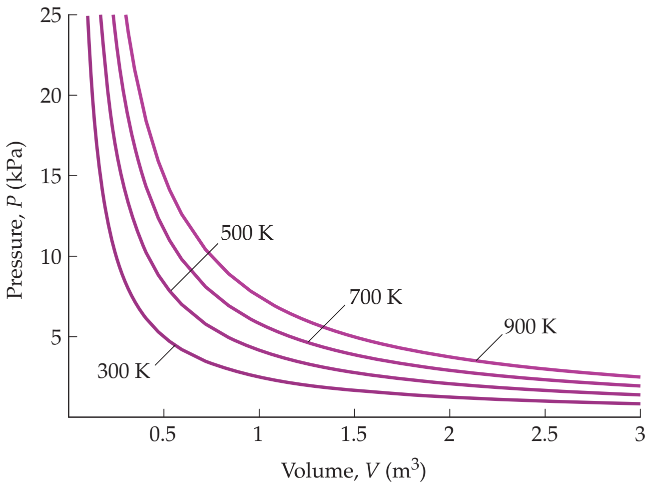
Another common process is one that takes place at constant temperature; that is, an **isothermal process.** For an ideal gas the isotherm has a relatively simple form. In particular, if T is constant, it follows that PV is a constant as well:

$$PV = NkT = \text{constant}$$

Thus, ideal-gas isotherms have the following pressure-volume relationship:

$$P = \frac{NkT}{V} = \frac{\text{constant}}{V}$$

This is illustrated in **Figure 18–8**, where we show several isotherms corresponding to different temperatures. (Recall that we've seen isotherms before, in Figure 17–5.)



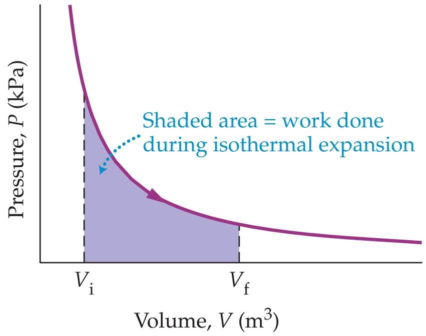
▲ **FIGURE 18–8 Isotherms on a *PV* plot**

These isotherms are for one mole of an ideal gas at the temperatures 300 K, 500 K, 700 K, and 900 K. Notice that each isotherm has the shape of a hyperbola. As the temperature is increased, however, the isotherms move farther from the origin. Thus, the pressure corresponding to a given volume increases with temperature, as one would expect.

For any given isotherm, such as the one shown in **Figure 18–9**, the work done by an expanding gas is equal to the area under the curve, as usual. In particular, the work in expanding from V_i to V_f in Figure 18–9 is equal to the shaded area. This area may be derived by using the methods of calculus. The result is found to be

$$W = NkT \ln\left(\frac{V_f}{V_i}\right) = nRT \ln\left(\frac{V_f}{V_i}\right) \qquad (\textit{constant temperature}) \qquad 18\text{–}5$$

Note that "ln" stands for the natural logarithm; that is, log to the base e. This result is utilized in the next Example.



▲ **FIGURE 18–9 An isothermal expansion**

In an isothermal expansion from the volume V_i to the volume V_f, the work done is equal to the shaded area. For n moles of an ideal gas at the temperature T, the work done by the gas is $W = nRT\ln(V_f/V_i)$.

EXAMPLE 18–3 HEAT FLOW

A cylinder holds 0.50 mol of a monatomic ideal gas at a temperature of 310 K. As the gas expands isothermally from an initial volume of 0.31 m^3 to a final volume of 0.45 m^3, determine the amount of heat that must be added to the gas to maintain a constant temperature.

PICTURE THE PROBLEM

The physical process is illustrated in the drawing on the left (next page). Note that heat flows into the gas as it expands in order to keep its temperature constant at $T = 310$ K. The graph on the right (next page) is a PV plot showing the same process. The work done by the expanding gas is equal to the shaded area from $V_i = 0.31\ m^3$ to $V_f = 0.45\ m^3$.

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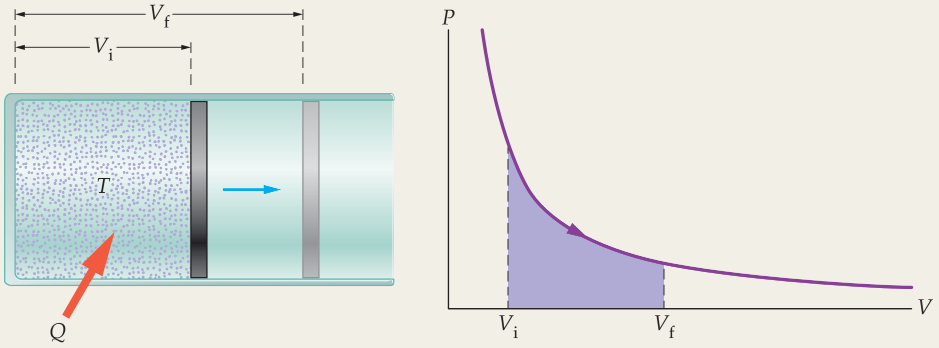


STRATEGY

We can use the first law, $\Delta U = Q - W$, to find the heat Q in terms of W and ΔU. We can find W and ΔU as follows:

First, the work W is found using $W = nRT\ln(V_f/V_i)$.

Next, recall that the internal energy of an ideal gas depends only on the temperature. For example, the internal energy of a monatomic ideal gas is $U = \frac{3}{2}nRT$. Since the temperature is constant in this process, there is no change in internal energy; that is, $\Delta U = 0$.

SOLUTION

1. Solve the first law of thermodynamics for the heat, Q: $\Delta U = Q - W$
   $Q = \Delta U + W$
2. Calculate the work done by the expanding gas: $W = nRT\ln\left(\frac{V_f}{V_i}\right)$
   $= (0.50\text{ mol})[8.31\text{ J/(mol}\cdot\text{K)}](310\text{ K})\ln\left(\frac{0.45\text{ m}^3}{0.31\text{ m}^3}\right)$
   $= 480\text{ J}$
3. Calculate the change in the gas's internal energy: $\Delta U = \frac{3}{2}nR(T_f - T_i) = 0$
4. Substitute numerical values to find Q: $Q = \Delta U + W = 0 + 480\text{ J} = 480\text{ J}$

INSIGHT

We find, then, that when an ideal gas undergoes an isothermal expansion, the heat gained by the gas is equal to the work it does; that is, $Q = W$. This is a direct consequence of the first law of thermodynamics, $\Delta U = Q - W$, and the fact that $\Delta U = 0$ for an ideal gas at constant temperature.

PRACTICE PROBLEM

Find the final volume of this gas when it has expanded enough to do 590 J of work. [**Answer:** $V_f = 0.49\text{ m}^3$]

Some related homework problems: Problem 20, Problem 21

Note that if a gas is compressed at constant temperature, work is done *on* the gas, rather than *by* the gas. As a result, we expect the work to be negative. This is consistent with Equation 18–5, since in a compression the final volume is less than the initial volume. Thus, $V_f/V_i < 1$, and hence $W = nRT\ln(V_f/V_i)$ is negative.

Finally, we consider one last point regarding isothermal processes.

CONCEPTUAL CHECKPOINT 18–1 IDEAL OR NOT?

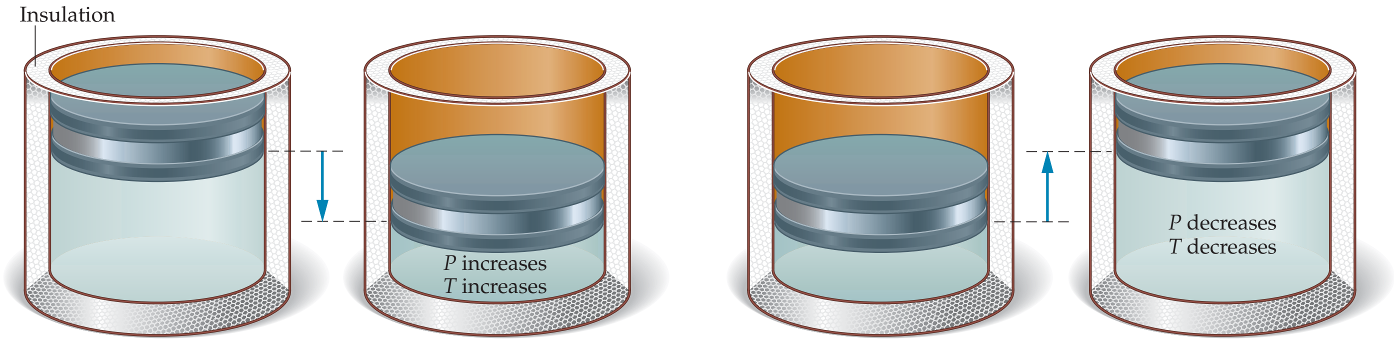
The internal energy of a certain gas increases when it is compressed isothermally. Is the gas ideal?

REASONING AND DISCUSSION

In an isothermal process, the internal energy of an ideal gas is unchanged, since the temperature is constant. Hence, if the internal energy of this gas changes during an isothermal compression, it must not be ideal.

ANSWER

No, the gas is not ideal.



(a) Compressing with no heat flow increases P and T.

(b) Expanding with no heat flow decreases P and T.

▲ **FIGURE 18–10 An adiabatic process**

In adiabatic processes, no heat flows into or out of the system. In the cases shown in this figure, heat flow is prevented by insulation. **(a)** An adiabatic compression increases both the pressure and the temperature. **(b)** An adiabatic expansion results in a decrease in pressure and temperature.

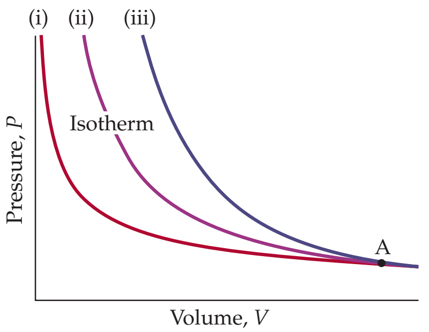
Adiabatic Processes

The final process we consider is one in which no heat flows into or out of the system. Such a process, in which $Q = 0$, is said to be **adiabatic.** One way to produce an adiabatic process is illustrated in **Figure 18–10 (a)**. Here we see a cylinder that is insulated well enough that no heat can pass through the insulation (adiabatic means, literally, "not passable"). When the piston is pushed downward in the cylinder—decreasing the volume—the gas heats up and its pressure increases. Similarly, in **Figure 18–10 (b)**, an adiabatic expansion causes the temperature of the gas to decrease, as does the pressure.

What does an adiabatic process look like on a PV plot? Certainly, its general shape must be similar to that of an isotherm; in particular, as the volume is decreased, the pressure increases. However, it can't be identical to an isotherm because, as we have pointed out, the temperature changes during an adiabatic process. The comparison between an adiabatic curve and an isothermal curve is the subject of the next Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 18–2 PRESSURE VERSUS VOLUME

A certain gas has an initial volume and pressure given by point A in the PV plot. If the gas is compressed isothermally, its pressure rises as indicated by the curve labeled "isotherm." If, instead, the gas is compressed adiabatically from point A, does its pressure follow **(a)** curve i, **(b)** curve ii, or **(c)** curve iii?



REASONING AND DISCUSSION

In an isothermal compression, some heat flows out of the system in order for its temperature to remain constant. No heat flows out of the system in the adiabatic process, however, and therefore its temperature rises. As a result, the pressure is greater for any given volume when the compression is adiabatic, as compared to isothermal. It follows, then, that the adiabatic curve, or adiabat, is represented by curve iii.

ANSWER

(c) As volume is decreased, the pressure on an adiabat rises more rapidly than on an isotherm.

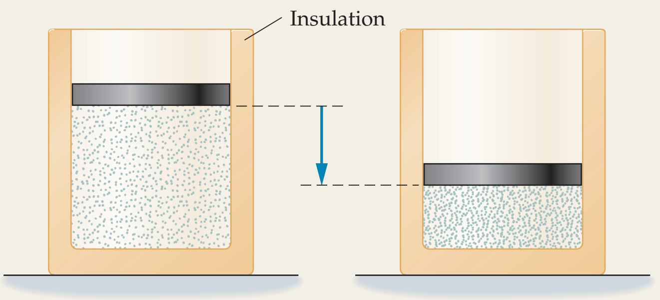
As we have seen, then, an adiabatic curve is similar to an isotherm, only steeper. The precise mathematical relationship describing an adiabat will be presented in the next section.

EXAMPLE 18–4 WORK INTO ENERGY

When a certain gas is compressed adiabatically, the amount of work done on it is 640 J. Find the change in internal energy of the gas.

PICTURE THE PROBLEM

Our sketch shows a piston being pushed downward, compressing a gas in an insulated cylinder. The insulation ensures that no heat can flow—as required in an adiabatic process.



STRATEGY

We know that 640 J of work is done on the gas, and we know that no heat is exchanged ($Q = 0$) in an adiabatic process. Thus, we can find ΔU by substituting Q and W into the first law of thermodynamics. One note of caution: Be careful to use the correct sign for the work. In particular, recall that work done *on* a system is negative.

SOLUTION

1. Identify the work and heat for this process: $W = -640\text{ J}$, $Q = 0$
2. Substitute Q and W into the first law of thermodynamics to find the change in internal energy, ΔU: $\Delta U = Q - W = 0 - (-640\text{ J}) = 640\text{ J}$

INSIGHT

Because no energy can enter or leave the system in the form of heat, all the work done on the system goes into increasing its internal energy. This follows from the first law of thermodynamics, $\Delta U = Q - W$, which for $Q = 0$ reduces to $\Delta U = -W$. As a result, the temperature of the gas increases.

A familiar example of this type of effect is the heating that occurs when you pump air into a tire or a ball—the work done on the pump appears as an increased temperature. The effect occurs in the reverse direction as well. When air is let out of a tire, for example, it does work on the atmosphere as it expands, producing a cooling effect that can be quite noticeable. In extreme cases, the cooling can be great enough to create frost on the valve stem of the tire.

PRACTICE PROBLEM

If a system's internal energy decreases by 470 J in an adiabatic process, how much work was done by the system? [**Answer:** $W = +470\text{ J}$]

Some related homework problems: Problem 17, Problem 24

An adiabatic process can occur when the system is thermally insulated, as in Figure 18–10, or in a system where the change in volume occurs rapidly. For example, if an expansion or compression happens quickly enough, there is no time for heat flow to occur. As a result, the process is adiabatic, even if there is no insulation.

REAL-WORLD PHYSICS
Adiabatic heating and diesel engines

An example of a rapid process is shown in **Figure 18–11**. Here, a piston is fitted into a cylinder that contains a certain volume of gas and a small piece of tissue paper. If the piston is driven downward rapidly, by a sharp impulsive blow, for example, the gas is compressed before heat has a chance to flow. As a result, the temperature of the gas rises rapidly. In fact, the rise in temperature can be enough for the paper to burst into flames.

The same principle applies to the operation of a diesel engine. As you may know, a diesel differs from a standard internal combustion engine in that it has no spark plugs. It doesn't need them. Instead of using a spark to ignite the fuel in a cylinder, it uses adiabatic heating. Fuel and air are admitted into the cylinder, then the piston rapidly compresses the air-fuel mixture. Just as with the piece of paper in Figure 18–11, the rising temperature is sufficient to ignite the fuel and run the engine.

Adiabatic heating is one of the mechanisms being considered to explain the fascinating and enigmatic phenomenon known as *sonoluminescence*. Sonoluminescence occurs when an intense, high-frequency sound wave causes a small gas bubble in water to pulsate. When the sound wave collapses the bubble to its minimum size, which is about a thousandth of a millimeter, the bubble gives off an extremely short burst of light. The light is mostly in the ultraviolet (see Chapter 25), but enough is in the visible range of light to make the bubble appear blue to the eye. For an object to give off light in the ultraviolet it must be extremely hot (see

TABLE 18–2 Thermodynamic Processes and Their Characteristics

Constant pressure	$W = P\Delta V$	$Q = \Delta U + P\Delta V$
Constant volume	$W = 0$	$Q = \Delta U$
Isothermal (constant temperature)	$W = Q$	$\Delta U = 0$
Adiabatic (no heat flow)	$W = -\Delta U$	$Q = 0$

Chapter 30). In fact, it is estimated that the temperature inside a collapsing bubble is at least 10,000 °F, about the same temperature as the surface of the Sun.

The characteristics of constant-pressure, constant-volume, isothermal, and adiabatic processes are summarized in Table 18–2.

18–4 Specific Heats for an Ideal Gas: Constant Pressure, Constant Volume

Recall that the specific heat of a substance is the amount of heat needed to raise the temperature of 1 kg of the substance by 1 Celsius degree. As we know, however, the amount of heat depends on the type of process used to raise the temperature. Thus, we should specify, for example, whether a specific heat applies to a process at constant pressure or constant volume.

If a substance is heated or cooled while open to the atmosphere, the process occurs at constant (atmospheric) pressure. This has been the case in all the specific heat discussions to this point; thus it was not necessary to make a distinction between different types of specific heats. We now wish to consider constant-volume processes as well, and the relationship between constant-volume and constant-pressure specific heats.

A constant-volume process is illustrated in **Figure 18–12**. Here we see an ideal gas of mass m in a container of fixed volume V. A heat Q flows into the container. As a result of this added heat, the temperature of the gas rises by the amount ΔT, and its pressure increases as well. Now, the specific heat at constant volume, c_v, is defined by the following relation:

$$Q_v = mc_v\Delta T$$

In what follows, it will be more convenient to use the **molar specific heat,** denoted by a capital letter C, which is defined in terms of the number of moles rather than the mass of the substance. Thus, if a gas contains n moles, its molar specific heat at constant volume is given by

$$Q_v = nC_v\Delta T$$

Similarly, a constant-pressure process is illustrated in **Figure 18–13**. In this case, the gas is held in a container with a moveable piston that applies a constant pressure P. As a heat Q is added to the gas, its temperature increases, which causes the piston to rise—after all, if the piston didn't rise, the pressure of the gas would increase. If the temperature of the gas increases by the amount ΔT, the molar specific heat at constant pressure is given by

$$Q_p = nC_p\Delta T$$

We would now like to obtain a relation between C_p and C_v.

FIGURE 18–11 Adiabatic heating

When a piston that fits snugly inside a cylinder is pushed downward rapidly, the temperature of the gas within the cylinder increases before there is time for heat to flow out of the system. Thus, the process is essentially adiabatic. As a result, the temperature of the gas can increase enough to ignite bits of paper in the cylinder. In a diesel engine, the same principle is used to ignite an air-gasoline mixture without a spark plug.

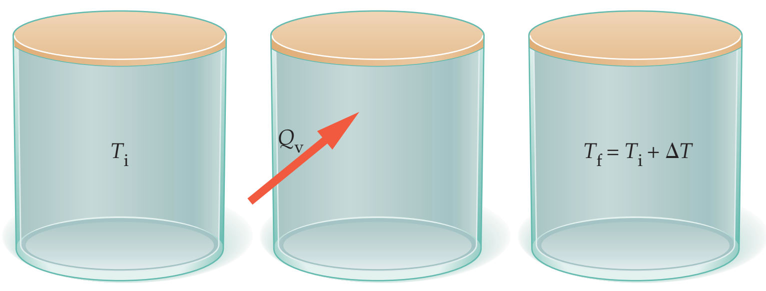


FIGURE 18–12 Heating at constant volume

If the heat Q_v is added to n moles of gas, and the temperature rises by ΔT, the molar specific heat at constant volume is $C_v = Q_v/n\Delta T$. No mechanical work is done at constant volume.

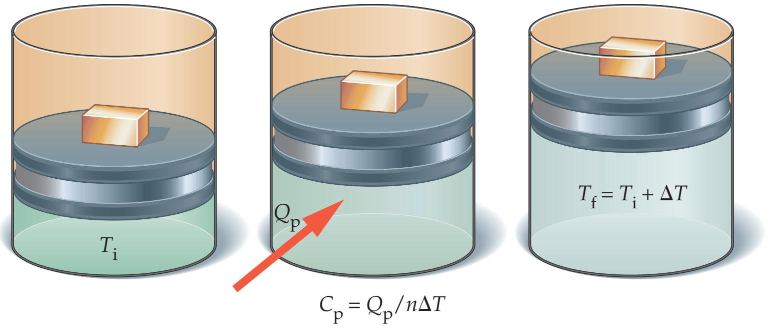


FIGURE 18–13 Heating at constant pressure
If the heat Q_p is added to n moles of gas, and the temperature rises by ΔT, the molar specific heat at constant pressure is $C_p = Q_p/n\Delta T$. Note that the heat Q_p must increase the temperature *and* do mechanical work by lifting the piston.

Before we carry out the mathematics, let's consider the qualitative relationship between these specific heats. This is addressed in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 18–3 COMPARING SPECIFIC HEATS

How does the molar specific heat at constant pressure, C_p, compare with the molar specific heat at constant volume, C_v? **(a)** $C_p > C_v$; **(b)** $C_p = C_v$; **(c)** $C_p < C_v$.

REASONING AND DISCUSSION

In a constant-volume process, as in Figure 18–12, the heat that is added to a system goes entirely into increasing the temperature, since no work is done. On the other hand, at constant pressure the heat added to a system increases the temperature *and* does mechanical work. This is illustrated in Figure 18–13 where we see that the heat must not only raise the temperature, but also supply enough energy to lift the piston. Thus, more heat is required in the constant-pressure process, and hence that specific heat is greater.

ANSWER

(a) The specific heat at constant pressure is greater than the specific heat at constant volume.

We turn now to a detailed calculation of C_v for a monatomic ideal gas. To begin, rearrange the first law of thermodynamics, $\Delta U = Q - W$, to solve for the heat, Q:

$$Q = \Delta U + W$$

Recall from the previous section, however, that the work is zero, $W = 0$, for any constant-volume process. Hence, for constant volume we have

$$Q_v = \Delta U$$

Finally, noting that $U = \frac{3}{2}NkT = \frac{3}{2}nRT$ yields

$$Q_v = \Delta U = \frac{3}{2}nR\Delta T$$

Comparing with the definition of the molar specific heat, we find

Molar Specific Heat for a Monatomic Ideal Gas at Constant Volume

$$C_v = \frac{3}{2}R \qquad 18\text{–}6$$

Now we perform a similar calculation for constant pressure. In this case, referring again to the previous section, we find that $W = P\Delta V$. Since we are considering an ideal gas, in which $PV = nRT$, it follows that

$$W = P\Delta V = nR\Delta T$$

Combining this with the first law of thermodynamics yields

$$Q_p = \Delta U + W$$
$$= \frac{3}{2}nR\Delta T + nR\Delta T = \frac{5}{2}nR\Delta T$$

Applying the definition of molar specific heat yields

PROBLEM-SOLVING NOTE

Constant Volume Versus Constant Pressure

The heat required to increase the temperature of an ideal gas depends on whether the process is at constant pressure or constant volume. More heat is required when the process occurs at constant pressure.

Molar Specific Heat for a Monatomic Ideal Gas at Constant Pressure

$$C_p = \tfrac{5}{2}R \qquad \text{18–7}$$

As expected, the specific heat at constant pressure is larger than the specific heat at constant volume, and the difference is precisely the extra contribution due to the work done in lifting the piston in the constant-pressure case. In particular, we see that

$$C_p - C_v = R \qquad \text{18–8}$$

Though this relation was derived for a monatomic ideal gas, it holds for all ideal gases, regardless of the structure of their molecules. It is also a good approximation for most real gases, as can be seen in Table 18–3.

TABLE 18–3 $C_p - C_v$ for Various Gases

Helium	0.995 R
Nitrogen	1.00 R
Oxygen	1.00 R
Argon	1.01 R
Carbon dioxide	1.01 R
Methane	1.01 R

EXERCISE 18–2

Find the heat required to raise the temperature of 0.200 mol of a monatomic ideal gas by 5.00 C° at **(a)** constant volume and **(b)** constant pressure.

SOLUTION

Applying Equations 18–6 and 18–7, we find

a. $Q_v = \tfrac{3}{2}nR\Delta T = \tfrac{3}{2}(0.200\text{ mol})[8.31\text{ J}/(\text{mol}\cdot\text{K})](5.00\text{ K}) = 12.5\text{ J}$

b. $Q_p = \tfrac{5}{2}nR\Delta T = \tfrac{5}{2}(0.200\text{ mol})[8.31\text{ J}/(\text{mol}\cdot\text{K})](5.00\text{ K}) = 20.8\text{ J}$

Adiabatic Processes

We return now briefly to a consideration of adiabatic processes. As we shall see, the relationship between C_p and C_v is important in determining the behavior of a system undergoing an adiabatic process.

Figure 18–14 shows an adiabatic curve and two isotherms. As mentioned before, the adiabatic curve is steeper and it cuts across the isotherms. For the isotherms, we recall that the curves are described by the equation

$$PV = \text{constant} \qquad (\textit{isothermal})$$

A similar equation applies to adiabats. In this case, using calculus, it can be shown that the appropriate equation is

$$PV^{\gamma} = \text{constant} \qquad (\textit{adiabatic}) \qquad \text{18–9}$$

In this expression, the constant γ is the ratio C_p/C_v:

$$\gamma = \frac{C_p}{C_v} = \frac{\tfrac{5}{2}R}{\tfrac{3}{2}R} = \frac{5}{3}$$

This value of γ applies to monatomic ideal gases—and is a good approximation for monatomic real gases as well. The value of γ is different, however, for gases that are diatomic, triatomic, and so on.

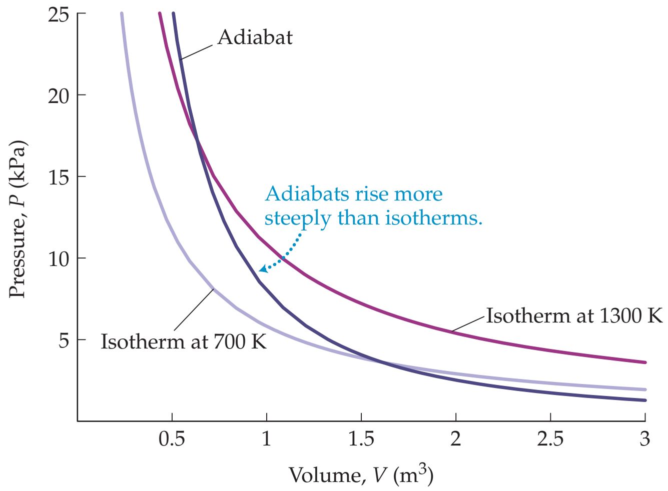


FIGURE 18–14 A comparison between isotherms and adiabats

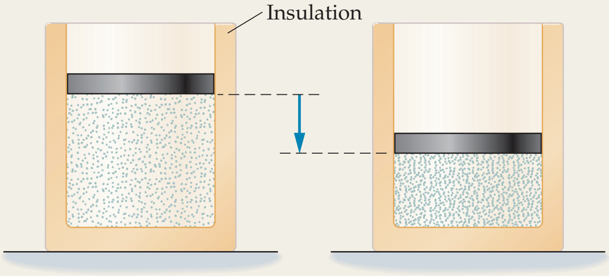
Two isotherms are shown, one for 700 K and one for 1300 K. An adiabat is also shown. Note that the adiabat is a steeper curve than the isotherms.

EXAMPLE 18–5 HOT AIR

A container with an initial volume of 0.0625 m^3 holds 2.50 moles of a monatomic ideal gas at a temperature of 315 K. The gas is now compressed adiabatically to a volume of 0.0350 m^3. Find **(a)** the final pressure and **(b)** the final temperature of the gas.

PICTURE THE PROBLEM

Our sketch shows a gas being compressed from an initial volume of 0.0625 m^3 to a final volume of 0.0350 m^3. The gas starts with a temperature of 315 K, but because no heat can flow outward through the insulation (adiabatic process), the work done on the gas results in an increased temperature.



STRATEGY

a. We can find the final pressure as follows: First, find the initial pressure using the ideal-gas equation of state, $P_i V_i = nRT_i$.
Next, let $P_i V_i^{\gamma} = P_f V_f^{\gamma}$, since this is an adiabatic process. Solve this relation for the final pressure.

b. Use the final pressure and volume to find the final temperature, using the ideal-gas relation, $P_f V_f = nRT_f$.

SOLUTION

Part (a)

1. Find the initial pressure, using $PV = nRT$:

$$P_i = \frac{nRT_i}{V_i} = \frac{(2.50 \text{ mol})[8.31 \text{ J}/(\text{mol}\cdot\text{K})](315 \text{ K})}{0.0625 \text{ m}^3} = 105 \text{ kPa}$$

2. Use $PV^{\gamma} =$ constant to find a relation for P_f:

$$P_i V_i^{\gamma} = P_f V_f^{\gamma}$$
$$P_f = P_i (V_i/V_f)^{\gamma}$$

3. Substitute numerical values:

$$P_f = (105 \text{ kPa})(0.0625 \text{ m}^3/0.0350 \text{ m}^3)^{5/3} = 276 \text{ kPa}$$

Part (b)

4. Use $PV = nRT$ to solve for the final temperature:

$$T_f = \frac{P_f V_f}{nR} = \frac{(276 \text{ kPa})(0.0350 \text{ m}^3)}{(2.50 \text{ mol})[8.31 \text{ J}/(\text{mol}\cdot\text{K})]} = 465 \text{ K}$$

INSIGHT

Thus, decreasing the volume of the gas by a factor of roughly two has increased its pressure from 105 kPa to 276 kPa and increased its temperature from 315 K to 465 K. This is a specific example of adiabatic heating. Adiabatic cooling is the reverse effect, where the temperature of a gas decreases as its volume increases. For example, if the gas in this system is expanded back to its initial volume of 0.0625 m^3, its temperature will drop from 465 K to 315 K.

PRACTICE PROBLEM

To what volume must the gas be compressed to yield a final pressure of 425 kPa? [**Answer:** $V_f = 0.0270 \text{ m}^3$]

Some related homework problems: Problem 39, Problem 40

REAL-WORLD PHYSICS
Rain shadows

Adiabatic heating and cooling can have important effects on the climate of a given region. For example, moisture-laden winds blowing from the Pacific Ocean into western Oregon are deflected upward when they encounter the Cascade Mountains. As the air rises, the atmospheric pressure decreases (see Chapter 15), allowing the air to expand and undergo adiabatic cooling. The result is that the moisture in the air condenses to form clouds and precipitation on the west side of the mountains. (In some cases, where the air holds relatively little moisture, this mechanism may result in isolated, lens-shaped clouds just above the peak of a mountain, as shown in the photograph on the next page.) When the winds continue on to the east side of the mountains they have little moisture remaining; thus, eastern Oregon is in the *rain shadow* of the Cascade Mountains. In addition, as the air descends on the east side of the mountains, it undergoes adiabatic heating. These are the primary reasons why the summers in western Oregon are moist and mild, while the summers in eastern Oregon are hot and dry.

18–5 The Second Law of Thermodynamics

Have you ever warmed your hands by pressing them against a block of ice? Probably not. But if you think about it, you might wonder why it doesn't work. After all, the first law of thermodynamics would be satisfied if energy simply flowed from the ice to your hands. The ice would get colder while your hands got warmer, and the energy of the universe would remain the same.

As we know, however, this sort of thing just doesn't happen—the spontaneous flow of heat is *always* from warmer objects to cooler objects, and never in the reverse direction. This simple observation, in fact, is one of many ways of expressing the **second law of thermodynamics:**

Second Law of Thermodynamics: Heat Flow

When objects of different temperatures are brought into thermal contact, the spontaneous flow of heat that results is always from the high-temperature object to the low-temperature object. Spontaneous heat flow never proceeds in the reverse direction.

Thus, the second law of thermodynamics is more restrictive than the first law; it says that of all the processes that conserve energy, only those that proceed in a certain direction actually occur. In a sense, the second law implies a definite "directionality" to the behavior of nature. For this reason, the second law is sometimes referred to as the "arrow of time."

For example, suppose you saw a movie that showed a snowflake landing on a person's hand and melting to a small drop of water. Nothing would seem particularly noteworthy about the scene from a physics point of view. But if the movie showed a drop of water on a person's hand suddenly freeze into the shape of a snowflake, then lift off the person's hand into the air, it wouldn't take long to realize the film was running backward. It is clear in which direction time should "flow."

We shall study further consequences of the second law in the next few sections. As we do so, we shall find other more precise, but equivalent, ways of stating the second law.

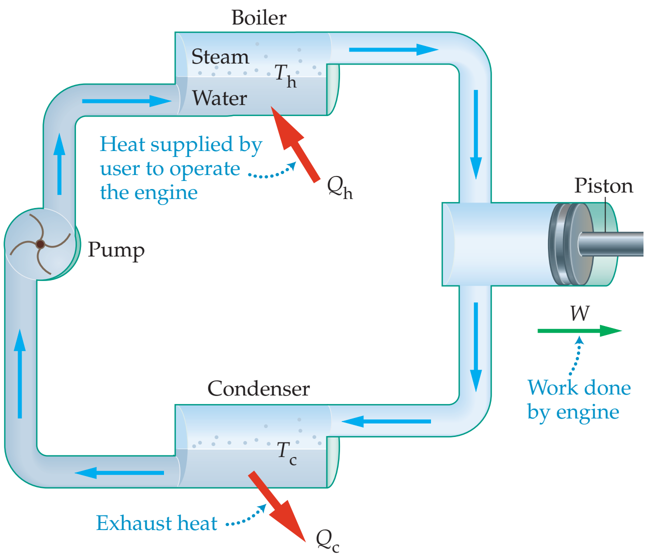
▲ A spectacular lenticular (lens-shaped) cloud floats above a mountain in Tierra del Fuego, at the southern tip of Chile. Lenticular clouds are often seen "parked" above and just downwind of high mountain peaks, even when there are no other clouds in the sky. The reason is that as moisture-laden winds are deflected upward by the mountain, the moisture they contain cools due to adiabatic expansion and condenses to form a cloud.

18–6 Heat Engines and the Carnot Cycle

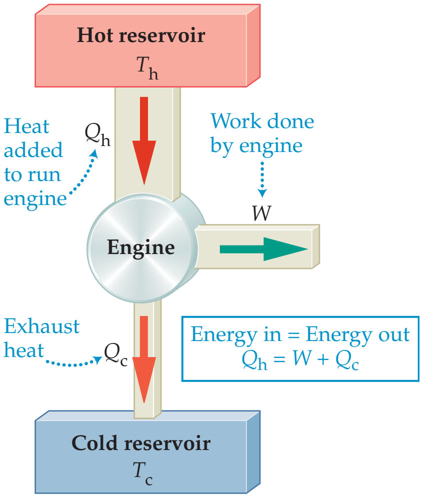
A **heat engine,** simply put, is a device that converts heat into work. The classic example of this type of device is the steam engine, whose basic elements are illustrated in **Figure 18–15**. First, some form of fuel (oil, wood, coal, etc.) is used to vaporize water in the boiler. The resulting steam is then allowed to enter the engine itself, where it expands against a piston, doing mechanical work. As the piston moves, it causes gears or wheels to rotate, which delivers the mechanical work to

REAL-WORLD PHYSICS

The steam engine



◀ **FIGURE 18–15 A schematic steam engine**
The basic elements of a steam engine are a boiler—where heat converts water to steam—and a piston that can be displaced by the expanding steam. In some engines, the steam is simply exhausted into the atmosphere after it has expanded against the piston. More sophisticated engines send the exhaust steam to a condenser, where it is cooled and condensed back to liquid water, then recycled to the boiler.



▲ **FIGURE 18–16 A schematic heat engine**

The engine absorbs a heat Q_h from the hot reservoir, performs the work W, and gives off the heat Q_c to the cold reservoir. Energy conservation gives $Q_h = W + Q_c$, where Q_h and Q_c are the magnitudes of the hot- and cold-temperature heats.

the external world. After leaving the engine, the steam proceeds to the condenser, where it gives off heat to the cool air in the atmosphere and condenses to liquid form.

What all heat engines have in common are: (i) A high-temperature region, or reservoir, that supplies heat to the engine (the boiler in the steam engine); (ii) a low-temperature reservoir where "waste" heat is released (the condenser in the steam engine); and (iii) an engine that operates in a cyclic fashion. These features are illustrated schematically in **Figure 18–16**. In addition, though not shown in the figure, heat engines have a working substance (steam in the steam engine) that causes the engine to operate.

To begin our analysis, we note that a certain amount of heat, Q_h, is supplied to the engine from the high temperature or "hot" reservoir during each cycle. Of this heat, a fraction appears as work, W, and the rest is given off as waste heat, Q_c, at a relatively low temperature to the "cold" reservoir. There is no change in energy for the engine, because it returns to its initial state at the completion of each cycle. Letting Q_h and Q_c denote magnitudes, so that both quantities are positive, energy conservation can be written as $Q_h = W + Q_c$, or

$$W = Q_h - Q_c \qquad \text{18–10}$$

As we shall see, the second law of thermodynamics requires that heat engines must *always* exhaust a finite amount of heat to a cold reservoir.

Of particular interest for any engine is its **efficiency,** e, which is simply the fraction of the heat supplied to the engine that appears as work. Thus, we define the efficiency to be

$$e = \frac{W}{Q_h} \qquad \text{18–11}$$

Using the energy-conservation result just derived in Equation 18–10, we find

Efficiency of a Heat Engine, *e*

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad \text{18–12}$$

SI unit: dimensionless

For example, if $e = 0.20$, we say that the engine is 20% efficient. In this case, 20% of the input heat is converted to work, $W = 0.20Q_h$, and 80% goes to waste heat, $Q_c = 0.80Q_h$. Efficiency, then, can be thought of as the ratio of how much you receive (work) to how much you have to pay to run the engine (input heat).

▲ At left, a modern version of Hero's engine, invented by the Greek mathematician and engineer Hero of Alexandria. In this simple heat engine, the steam that escapes from a heated vessel of water is directed tangentially, causing the vessel to rotate. This converts the thermal energy supplied to the water into mechanical energy, in the form of rotational motion. At right, a steam engine of slightly more recent design hauls passengers up and down Mt. Washington in New Hampshire. Note in the photo that the locomotive is belching two clouds, one black and one white. Can you explain their origin?

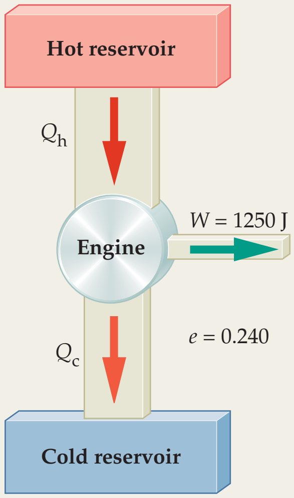
EXAMPLE 18–6 HEAT INTO WORK

A heat engine with an efficiency of 24.0% performs 1250 J of work. Find **(a)** the heat absorbed from the hot reservoir, and **(b)** the heat given off to the cold reservoir.

PICTURE THE PROBLEM

Our sketch shows a schematic of the heat engine. We know the amount of work that is done and the efficiency of the engine. We seek the heats Q_h and Q_c.

Note that an efficiency of 24.0% means that $e = 0.240$.



STRATEGY

a. We can find the heat absorbed from the hot reservoir directly from the definition of efficiency, $e = W/Q_h$.

b. We can find Q_c by using energy conservation, $W = Q_h - Q_c$, or by using the expression for efficiency in terms of the heats, $e = 1 - Q_c/Q_h$.

SOLUTION

Part (a)

1. Use $e = W/Q_h$ to solve for the heat Q_h:

$$e = W/Q_h$$
$$Q_h = \frac{W}{e} = \frac{1250 \text{ J}}{0.240} = 5210 \text{ J}$$

Part (b)

2. Use energy conservation to solve for Q_c:

$$W = Q_h - Q_c$$
$$Q_c = Q_h - W = 5210 \text{ J} - 1250 \text{ J} = 3960 \text{ J}$$

3. Use the efficiency, expressed in terms of Q_h and Q_c, to find Q_c:

$$e = 1 - Q_c/Q_h$$
$$Q_c = (1 - e)Q_h = (1 - 0.240)(5210 \text{ J}) = 3960 \text{ J}$$

INSIGHT

Note that when the efficiency of a heat engine is less than one-half (50%), as in this case, the amount of heat given off as waste to the cold reservoir is more than the amount of heat converted to work.

PRACTICE PROBLEM

What is the efficiency of a heat engine that does 1250 J of work and gives off 5250 J of heat to the cold reservoir?
[**Answer:** $e = 0.192$]

Some related homework problems: Problem 45, Problem 46

A temperature difference is essential to the operation of a heat engine. As heat flows from the hot to the cold reservoir in Figure 18–16, for example, the heat engine is able to tap into that flow and convert part of it to work—the greater the efficiency of the engine, the more heat converted to work. The second law of thermodynamics imposes limits, however, on the maximum efficiency a heat engine can have. We explore these limits next.

The Carnot Cycle and Maximum Efficiency

In 1824, the French engineer Sadi Carnot (1796–1832) published a book entitled *Reflections on the Motive Power of Fire* in which he considered the following question: Under what conditions will a heat engine have maximum efficiency? To address this question, let's consider a heat engine that operates between a single hot reservoir at the fixed temperature T_h and a single cold reservoir at the fixed temperature T_c. Carnot's result, known today as **Carnot's theorem,** can be expressed as follows:

Carnot's Theorem

If an engine operating between two constant-temperature reservoirs is to have maximum efficiency, it must be an engine in which all processes are reversible.

In addition, all reversible engines operating between the same two temperatures, T_c and T_h, have the same efficiency.

We should point out that no real engine can ever be perfectly reversible, just as no surface can be perfectly frictionless. Nonetheless, the concept of a reversible engine is a useful idealization.

Carnot's theorem is remarkable for a number of reasons. First, consider what the theorem says: No engine, no matter how sophisticated or technologically advanced, can exceed the efficiency of a reversible engine. We can strive to improve the technology of heat engines, but there is an upper limit to the efficiency that can never be exceeded. Second, the theorem is just as remarkable for what it does not say. It says nothing, for example, about the working substance that is used in the engine—it is as valid for a liquid or solid working substance as for one that is gaseous. Furthermore, it says nothing about the type of reversible engine that is used, what the engine is made of, or how it is constructed. Diesel engine, jet engine, rocket engine—none of these things matter. In fact, all that *does* matter are the two temperatures, T_c and T_h.

Recall that the efficiency of a heat engine can be written as follows:

$$e = 1 - \frac{Q_c}{Q_h}$$

Since the efficiency e depends only on the temperatures T_c and T_h, according to Carnot's theorem, it follows that Q_c/Q_h must also depend only on T_c and T_h. In fact, Lord Kelvin used this observation to propose that, instead of using a thermometer to measure temperature, we measure the efficiency of a heat engine and from this determine the temperature. Thus, he suggested that we *define* the ratio of the temperatures of two reservoirs, T_c/T_h, to be equal to the ratio of the heats Q_c/Q_h:

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

If we choose the size of a degree in this temperature scale to be equal to 1 C°, then we have, in fact, defined the Kelvin temperature scale discussed in Chapter 16. Thus, if T_h and T_c are given in kelvins, the maximum efficiency of a heat engine is:

Maximum Efficiency of a Heat Engine

$$e_{max} = 1 - \frac{T_c}{T_h} \qquad 18\text{–}13$$

PROBLEM-SOLVING NOTE

Maximum Efficiency

The maximum efficiency a heat engine can have is determined solely by the temperature of the hot (T_h) and cold (T_c) reservoirs. The numerical value of this efficiency is $e = 1 - T_c/T_h$. Remember, however, that the temperatures must be expressed in the Kelvin scale for this expression to be valid.

Suppose for a moment that we *could* construct an ideal engine, perfectly reversible and free from all forms of friction. Would this ideal engine have 100% efficiency? No, it would not. From Equation 18–13 we can see that the only way the efficiency of a heat engine could be 100% (that is, $e_{max} = 1$) would be for T_c to be 0 K. As we shall see in the last section of this chapter, this is ruled out by the third law of thermodynamics. Hence, the maximum efficiency will always be less than 100%. No matter how perfect the engine, some of the input heat will always be wasted—given off as Q_c—rather than converted to work.

Since the efficiency is defined to be $e = W/Q_h$, it follows that the maximum work a heat engine can do with the input heat Q_h is

$$W_{max} = e_{max}Q_h = \left(1 - \frac{T_c}{T_h}\right)Q_h \qquad 18\text{–}14$$

If the hot and cold reservoirs have the same temperature, so that $T_c = T_h$, it follows that the maximum efficiency is zero. As a result, the amount of work that such an engine can do is also zero. As mentioned before, a heat engine requires different temperatures in order to operate. For example, for a fixed T_c, the higher the temperature of T_h the greater the efficiency.

Finally, even though Carnot's theorem may seem quite different from the second law of thermodynamics, they are, in fact, equivalent. It can be shown, for example, that if Carnot's theorem were violated, it would be possible for heat to flow spontaneously from a cold object to a hot object.

CONCEPTUAL CHECKPOINT 18–4 COMPARING EFFICIENCIES

Suppose you have a heat engine that can operate in one of two different modes. In mode 1, the temperatures of the two reservoirs are $T_c = 200\text{ K}$ and $T_h = 400\text{ K}$; in mode 2, the temperatures are $T_c = 400\text{ K}$ and $T_h = 600\text{ K}$. Is the efficiency of mode 1 **(a)** greater than, **(b)** less than, or **(c)** equal to the efficiency of mode 2?

REASONING AND DISCUSSION

At first, you might think that since the temperature difference is the same in the two modes, the efficiency is the same as well. This is not the case, however, since efficiency depends on the *ratio* of the two temperatures ($e = 1 - T_c/T_h$) rather than on their difference. In this case, the efficiency of mode 1 is $e_1 = 1 - 1/2 = 1/2$ and the efficiency of mode 2 is $e_2 = 1 - 2/3 = 1/3$. Thus, mode 1, even though it operates at the lower temperatures, is more efficient.

ANSWER

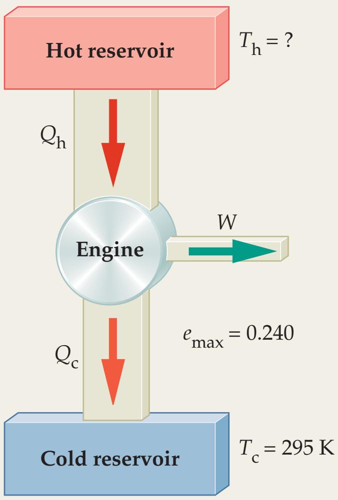
(a) The efficiency of mode 1 is greater than the efficiency of mode 2.

ACTIVE EXAMPLE 18–2 FIND THE TEMPERATURE

If the heat engine in Example 18–6 is operating at its maximum efficiency of 24.0%, and its cold reservoir is at a temperature of 295 K, what is the temperature of the hot reservoir?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the efficiency, e, in terms of the hot and cold temperatures: $e = 1 - T_c/T_h$
2. Solve for T_h: $T_h = T_c/(1 - e)$
3. Substitute numerical values for T_c and e to find T_h: $T_h = 388\text{ K}$



INSIGHT

Though an efficiency of 24.0% may seem low, it is characteristic of many real engines.

YOUR TURN

What is the efficiency of this heat engine if T_h is increased by 20 K? What is the efficiency if, instead, T_c is reduced by 20 K?

(Answers to **Your Turn** *problems are given in the back of the book.)*

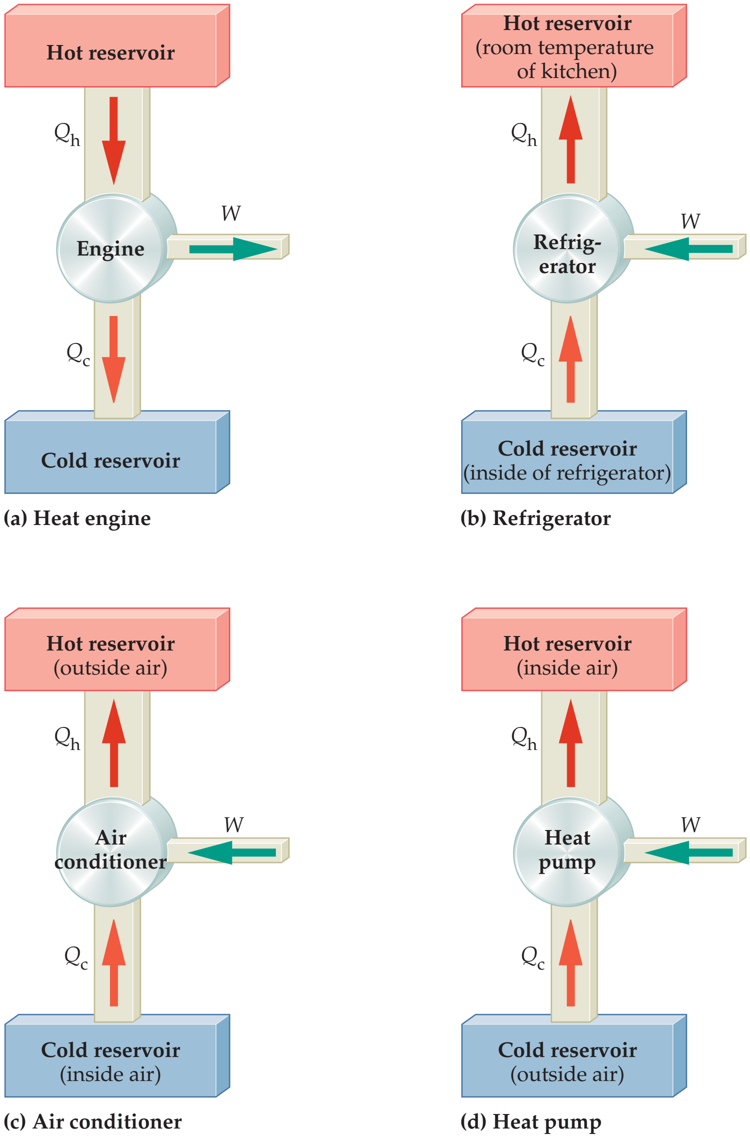
We can summarize the conclusions of this section as follows: The first law of thermodynamics states that you cannot get something for nothing. To be specific, you cannot get more work out of a heat engine than the amount of heat you put in. The best you can do is break even. The second law of thermodynamics is more restrictive than the first law; it says that you can't even break even—some of the input heat must be wasted. It's a law of nature.

18–7 Refrigerators, Air Conditioners, and Heat Pumps

When we stated the second law of thermodynamics in Section 18–5, we said that the spontaneous flow of heat is always from high temperature to low temperature. The key word here is "spontaneous." It *is* possible for heat to flow "uphill,"

FIGURE 18–17 Cooling and heating devices

Schematic comparison of a generalized heat engine **(a)** compared with a refrigerator, an air conditioner, and a heat pump. In the refrigerator **(b)**, a work W is done to remove a heat Q_c from the cold reservoir (the inside of the refrigerator). By energy conservation, the heat given off to the hot reservoir (the kitchen) is $Q_h = W + Q_c$. An air conditioner **(c)** removes heat from the cool air inside a house and exhausts it into the hot air of the atmosphere. A heat pump **(d)** moves heat from a cold reservoir to a hot reservoir, just like an air conditioner. The difference is that the reservoirs are switched, so that heat is pumped into the house rather than out to the atmosphere.



from a cold object to a hot one, but it doesn't happen spontaneously—work must be done on the system to make it happen, just as work must be done to pump water from a well. Refrigerators, air conditioners, and heat pumps are devices that use work to transfer heat from a cold object to a hot object.

REAL-WORLD PHYSICS
Refrigerators

Let us first compare the operation of a heat engine, Figure 18–17 (a), and a **refrigerator,** Figure 18–17 (b). Note that all the arrows are reversed in the refrigerator; in effect, a refrigerator is a heat engine running backward. In particular, the refrigerator uses a work W to remove a certain amount of heat, Q_c, from the cold reservoir (the interior of the refrigerator). It then exhausts an even larger amount of heat, Q_h, to the hot reservoir (the air in the kitchen). By energy conservation, it follows that

$$Q_h = Q_c + W$$

Thus, as a refrigerator operates, it warms the kitchen at the same time that it cools the food stored within it.

To design an effective refrigerator, you would like it to remove as much heat from its interior as possible for the smallest amount of work. After all, the work is supplied by electrical energy that must be paid for each month. Thus, we define the **coefficient of performance, COP, for a refrigerator** as an indicator of its effectiveness:

Coefficient of Performance for a Refrigerator, COP

$$\text{COP} = \frac{Q_c}{W} \qquad 18\text{–}15$$

SI unit: dimensionless

Typical values for the coefficient of performance are in the range 2 to 6.

EXERCISE 18–3

A refrigerator has a coefficient of performance of 2.50. How much work must be supplied to this refrigerator in order to remove 225 J of heat from its interior?

SOLUTION

Solving Equation 18–15 for the work yields

$$W = \frac{Q_c}{\text{COP}} = \frac{225\text{ J}}{2.50} = 90.0\text{ J}$$

Thus, 90.0 J of work removes 225 J of heat from the refrigerator, and exhausts 90.0 J + 225 J = 315 J of heat into the kitchen.

An **air conditioner,** Figure 18–17 (c), is basically a refrigerator in which the cold reservoir is the room that is being cooled. To be specific, the air conditioner uses electrical energy to "pump" heat from the cool room to the warmer air outside. As with the refrigerator, more heat is exhausted to the hot reservoir than is removed from the cold reservoir; that is, $Q_h = Q_c + W$, as before.

REAL-WORLD PHYSICS
Air conditioners

CONCEPTUAL CHECKPOINT 18–5 ROOM TEMPERATURE

You haven't had time to install your new air conditioner in the window yet, so as a short-term measure you decide to place it on the dining-room table and turn it on to cool things off a bit. As a result, does the air in the dining room **(a)** get warmer, **(b)** get cooler, or **(c)** stay at the same temperature?

REASONING AND DISCUSSION

You might think the room stays at the same temperature, since the air conditioner draws heat from the room as usual, but then exhausts heat back into the room that would normally be sent outside. However, the motor of the air conditioner is doing work in order to draw heat from the room, and the heat that would normally be exhausted outdoors is equal to the heat drawn from the room *plus* the work done by the motor: $Q_h = Q_c + W$. Thus, the net effect is that the motor of the air conditioner is continually adding heat to the room, causing it to get warmer.

ANSWER

(a) The air in the dining room gets warmer.

Finally, a **heat pump** can be thought of as an air conditioner with the reservoirs switched. As we see in Figure 18–17 (d), a heat pump does a work W to remove an amount of heat Q_c from the cold reservoir of outdoor air, then exhausts a heat Q_h into the hot reservoir of air in the room. Just as with the refrigerator and the air conditioner, the heat going to the hot reservoir is $Q_h = Q_c + W$.

REAL-WORLD PHYSICS
Heat pumps

In an **ideal,** reversible heat pump with only two operating temperatures, T_c and T_h, the Carnot relationship $Q_c/Q_h = T_c/T_h$ holds, just as it does for a heat engine. Thus, if you want to add a heat Q_h to a room, the work that must be done to accomplish this is

$$W = Q_h - Q_c = Q_h\left(1 - \frac{Q_c}{Q_h}\right) = Q_h\left(1 - \frac{T_c}{T_h}\right) \qquad 18\text{–}16$$

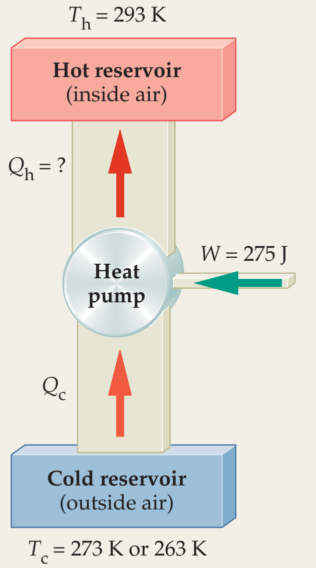
We use this result in the next Example.

EXAMPLE 18–7 PUMPING HEAT

An ideal heat pump, one that satisfies the Carnot relation given in Equation 18–16, is used to heat a room that is at 293 K. If the pump does 275 J of work, how much heat does it supply to the room if the outdoor temperature is **(a)** 273 K or **(b)** 263 K?

PICTURE THE PROBLEM

Our sketch shows the heat pump doing 275 J of mechanical work to transfer the heat Q_h to the hot reservoir at the temperature 293 K. The temperature of the cold reservoir is 273 K for part (a) and 263 K for part (b). We wish to determine Q_h for each of these cases.



STRATEGY

For an ideal heat pump, we know that $W = Q_h(1 - T_c/T_h)$. Therefore, given the hot and cold temperatures, as well as the mechanical work, it is straightforward to determine the heat delivered to the hot reservoir, Q_h.

SOLUTION

1. Solve Equation 18–16 for the heat Q_h:

$$Q_h = W/(1 - T_c/T_h)$$

Part (a)

2. Substitute $W = 275$ J, $T_c = 273$ K, and $T_h = 293$ K into the expression for Q_h:

$$Q_h = \frac{W}{1 - \dfrac{T_c}{T_h}} = \frac{275\text{ J}}{1 - \dfrac{273\text{ K}}{293\text{ K}}} = 4030\text{ J}$$

Part (b)

3. Substitute $W = 275$ J, $T_c = 263$ K, and $T_h = 293$ K into the expression for Q_h:

$$Q_h = \frac{W}{1 - \dfrac{T_c}{T_h}} = \frac{275\text{ J}}{1 - \dfrac{263\text{ K}}{293\text{ K}}} = 2690\text{ J}$$

INSIGHT

As one might expect, the same amount of work provides less heat when the outside temperature is lower. That is, more work must be done on a colder day to provide the same heat to the inside air.

In addition, note that if 275 J of heat is supplied to an electric heater, then 275 J of heat is given to the air in the room. When that same energy is used to run a heat pump, a good deal more than 275 J of heat is added to the room.

PRACTICE PROBLEM

How much work must be done by this heat pump to supply 2550 J of heat on a day when the outside temperature is 253 K? [**Answer:** $W = 348$ J]

Some related homework problems: Problem 58, Problem 59

Since the purpose of a heat pump is to add heat to a room, and we want to add as much heat as possible for the least work, the **coefficient of performance, COP, for a heat pump** is defined as follows:

Coefficient of Performance for a Heat Pump, COP

$$\text{COP} = \frac{Q_h}{W} \qquad 18\text{–}17$$

SI unit: dimensionless

The COP for a heat pump, which is usually in the range of 3 to 4, depends on the inside and outside temperatures. We use the COP in the next Exercise.

EXERCISE 18–4

A heat pump with a coefficient of performance equal to 3.5 supplies 2500 J of heat to a room. How much work is required?

SOLUTION

Solving Equation 18–17 for the work, W, we find

$$W = \frac{Q_h}{\text{COP}} = \frac{2500\ \text{J}}{3.5} = 710\ \text{J}$$

18–8 Entropy

In this section we introduce a new quantity that is as fundamental to physics as energy or temperature. This quantity is referred to as the entropy, and it is related to the amount of disorder in a system. For example, a messy room has more entropy than a neat one, a pile of bricks has more entropy than a building constructed from the bricks, a freshly laid egg has more entropy than one that is just about to hatch, and a puddle of water has more entropy than the block of ice from which it melted. We begin by considering the connection between entropy and heat, and later develop more fully the connection between disorder and entropy.

When discussing heat engines, we saw that if an engine is reversible it satisfies the following relation:

$$\frac{Q_c}{Q_h} = \frac{T_c}{T_h}$$

Rearranging slightly, we can rewrite this as

$$\frac{Q_c}{T_c} = \frac{Q_h}{T_h}$$

Notice that the quantity Q/T is the same for both the hot and the cold reservoirs. This relationship prompted the German physicist Rudolf Clausius (1822–1888) to propose the following definition: The **entropy,** S, is a quantity whose change is given by the heat Q divided by the absolute temperature T:

Definition of Entropy Change, ΔS

$$\Delta S = \frac{Q}{T} \qquad \text{18–18}$$

SI unit: J/K

PROBLEM-SOLVING NOTE

Calculating Entropy Change

When calculating the entropy change, $\Delta S = Q/T$, be sure to convert the temperature T to kelvins. It is *only* when this is done that the correct value for ΔS can be obtained.

For this definition to be valid, the heat Q must be transferred reversibly at the fixed Kelvin temperature T. Note that if heat is added to a system $(Q > 0)$, the entropy of the system increases; if heat is removed from a system $(Q < 0)$, its entropy decreases.

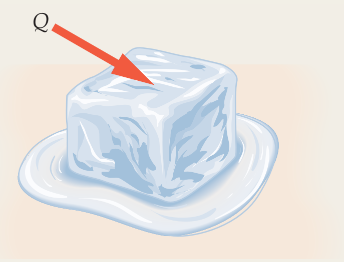
Entropy is a state function, just like the internal energy, U. This means that the value of S depends only on the state of a system, and not on how the system gets to that state. It follows, then, that the *change* in entropy, ΔS, depends only on the initial and final states of a system. Thus, if a process is irreversible—so that Equation 18–18 *does not* hold—we can still calculate ΔS by using a reversible process to connect the same initial and final states.

EXAMPLE 18–8 MELTS IN YOUR HAND

(a) Calculate the change in entropy when a 0.125-kg chunk of ice melts at 0 °C. Assume the melting occurs reversibly. **(b)** Suppose heat is now drawn reversibly from the 0 °C meltwater, causing a decrease in entropy of 112 J/K. How much ice freezes in the process?

PICTURE THE PROBLEM

In our sketch we show a 0.125-kg chunk of ice at the temperature 0 °C. As the ice absorbs the heat Q from its surroundings, it melts to water at 0 °C. Because the system absorbs heat, its entropy increases. When heat is drawn out of the meltwater, the entropy will decrease.



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STRATEGY

a. The entropy change is $\Delta S = Q/T$, where $T = 0\text{ °C} = 273\text{ K}$. To find the heat Q, we note that to melt the ice we must add to it the latent heat of fusion, L_f. Thus, the heat is $Q = mL_f$, where $L_f = 33.5 \times 10^4\text{ J/kg}$.

b. The magnitude of heat withdrawn from the meltwater is $Q = T|\Delta S|$, where $\Delta S = -112\text{ J/K}$. Using this heat, we can calculate the mass of re-frozen ice using $m = Q/L_f$.

SOLUTION

Part (a)

1. Find the heat that must be absorbed by the ice for it to melt:

$$Q = mL_f = (0.125\text{ kg})(33.5 \times 10^4\text{ J/kg}) = 4.19 \times 10^4\text{ J}$$

2. Calculate the change in entropy:

$$\Delta S = \frac{Q}{T} = \frac{4.19 \times 10^4\text{ J}}{273\text{ K}} = 153\text{ J/K}$$

Part (b)

3. Find the magnitude of the heat that is removed from the meltwater:

$$Q = T|\Delta S| = (273\text{ K})(112\text{ J/K}) = 3.06 \times 10^4\text{ J}$$

4. Use this heat to determine the amount of water that is re-frozen:

$$m = \frac{Q}{L_f} = (3.06 \times 10^4\text{ J})/(33.5 \times 10^4\text{ J/kg}) = 0.0913\text{ kg}$$

INSIGHT

Note that we were careful to convert the temperature of the system from 0 °C to 273 K before we applied $\Delta S = Q/T$. This must always be done when calculating the entropy change. If we had neglected to do the conversion in this case, we would have found an infinite increase in entropy—which is clearly unphysical.

In addition, we see that the entropy change is positive in part (a) and negative in part (b). This illustrates the general rule that entropy increases when heat is added to a system, and decreases when it is removed. Finally, we could have retained the negative sign of the heat in part (b). If we had, the mass in Step 4 would have been negative. This sounds odd at first, but the *minus sign* would simply indicate that *mass is removed* from the water to become ice. As it is, we avoided the sign issue by simply noting that removing heat results in the formation of ice.

PRACTICE PROBLEM

Find the mass of ice that would be required to give an entropy change of 275 J/K. [**Answer:** $m = 0.224$ kg]

Some related homework problems: Problem 66, Problem 67

Let's apply the definition of entropy change to the case of a reversible heat engine. First, a heat Q_h leaves the hot reservoir at the temperature T_h. Thus, the entropy of this reservoir decreases by the amount Q_h/T_h:

$$\Delta S_h = -\frac{Q_h}{T_h}$$

Recall that Q_h is the magnitude of the heat leaving the hot reservoir; hence, the minus sign is used to indicate a decrease in entropy. Similarly, heat is added to the cold reservoir; hence, its entropy increases by the amount Q_c/T_c:

$$\Delta S_c = \frac{Q_c}{T_c}$$

The total entropy change for this system is

$$\Delta S_{\text{total}} = \Delta S_h + \Delta S_c = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c}$$

Since we know that $Q_h/T_h = Q_c/T_c$ it follows that the total entropy change vanishes:

$$\Delta S_{\text{total}} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} = 0$$

What is special about a reversible engine, then, is the fact that its entropy does not change.

On the other hand, a real engine will always have a lower efficiency than a reversible engine operating between the same temperatures. This means that in a real engine less of the heat from the hot reservoir is converted to work; hence, more heat is given off as waste heat to the cold reservoir. Thus, for a given value of Q_h, the heat Q_c is greater in an irreversible engine than in a reversible one. As a result, instead of $Q_c/T_c = Q_h/T_h$, we have

$$\frac{Q_c}{T_c} > \frac{Q_h}{T_h}$$

Therefore, if an engine is irreversible, the total entropy change is positive:

$$\Delta S_{total} = -\frac{Q_h}{T_h} + \frac{Q_c}{T_c} > 0$$

In general, *any irreversible process results in an increase of entropy*.

These results can be summarized in the following general statement:

Entropy in the Universe

The total entropy of the universe *increases* whenever an *irreversible* process occurs.

The total entropy of the universe is *unchanged* whenever a *reversible* process occurs.

Since all *real* processes are irreversible (with reversible processes being a useful idealization), the total entropy of the universe continually increases. Thus, in terms of the entropy, the universe moves in only one direction—toward an ever-increasing entropy. This is quite different from the behavior with regard to energy, which remains constant no matter what type of process occurs.

In fact, this statement about entropy in the universe is yet another way of expressing the second law of thermodynamics. Recall, for example, that our original statement of the second law said that heat flows spontaneously from a hot object to a cold object. During this flow of heat the entropy of the universe increases, as we show in the next Example. Hence, the direction in which heat flows is seen to be the result of the general principle of entropy increase in the universe. Again, we see a directionality in nature—the ever-present "arrow of time."

PROBLEM-SOLVING NOTE

Entropy Change

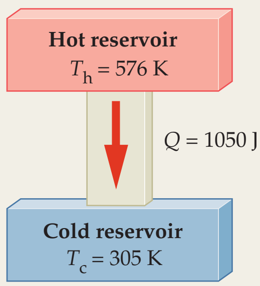
Though it is tempting to treat entropy like energy, setting the final value equal to the initial value, this is not the case in general. Only in a reversible process is the entropy unchanged—otherwise it increases. Still, the entropy of part of a system can decrease, as long as the entropy of other parts increases by the same amount or more.

EXAMPLE 18–9 ENTROPY IS NOT CONSERVED!

A hot reservoir at the temperature 576 K transfers 1050 J of heat irreversibly to a cold reservoir at the temperature 305 K. Find the change in entropy of the universe.

PICTURE THE PROBLEM

The relevant physical situation is shown in our sketch. Note that the heat $Q = 1050$ J is transferred from the hot reservoir at the temperature $T_h = 576$ K directly to the cold reservoir at the temperature $T_c = 305$ K.



STRATEGY

As the heat Q leaves the hot reservoir, its entropy *decreases* by Q/T_h. When the same heat Q enters the cold reservoir, its entropy *increases* by the amount Q/T_c. Summing these two contributions gives the entropy change of the universe.

SOLUTION

1. Calculate the entropy change of the hot reservoir: $\Delta S_h = -\frac{Q}{T_h} = -\frac{1050\ \text{J}}{576\ \text{K}} = -1.82\ \text{J/K}$

2. Calculate the entropy change of the cold reservoir: $\Delta S_c = \frac{Q}{T_c} = \frac{1050\ \text{J}}{305\ \text{K}} = 3.44\ \text{J/K}$

3. Sum these contributions to obtain the entropy change of the universe:

$$\Delta S_{universe} = \Delta S_h + \Delta S_c = -\frac{Q}{T_h} + \frac{Q}{T_c}$$
$$= -1.82\ \text{J/K} + 3.44\ \text{J/K} = 1.62\ \text{J/K}$$

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INSIGHT

Note that the decrease in entropy of the hot reservoir is more than made up for by the increase in entropy of the cold reservoir. This is a general result.

PRACTICE PROBLEM

What amount of heat must be transferred between these reservoirs for the entropy of the universe to increase by 1.50 J/K? [**Answer:** $Q = 972$ J]

Some related homework problems: Problem 68, Problem 72

When certain processes occur, it sometimes appears as if the entropy of the universe has decreased. On closer examination, however, it can always be shown that there is a larger increase in entropy elsewhere that results in an overall increase. This issue is addressed in the next Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 18–6 ENTROPY CHANGE

You put a tray of water in the kitchen freezer, and some time later it has turned to ice. Has the entropy of the universe **(a)** increased, **(b)** decreased, or **(c)** stayed the same?

REASONING AND DISCUSSION

It might seem that the entropy of the universe has decreased. After all, heat is removed from the water to freeze it, and, as we know, removing heat from an object lowers its entropy. On the other hand, we also know that the freezer does work to draw heat from the water; hence, it exhausts more heat into the kitchen than it absorbs from the water. Detailed calculations always show that the entropy of the heated air in the kitchen increases by more than the entropy of the water decreases, thus the entropy of the universe increases—as it must for *any* real process.

ANSWER

(a) The entropy of the universe has increased.

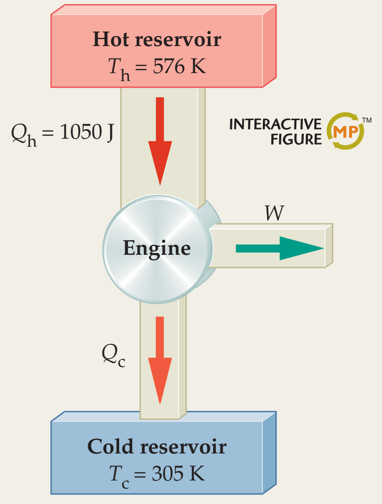
As the entropy in the universe increases, the amount of work that can be done is diminished. For example, the heat flow in Example 18–9 resulted in an increase in the entropy of the universe by the amount 1.62 J/K. If this same heat had been used in a reversible engine, however, it could have done work, and since the engine was reversible, the entropy of the universe would have stayed the same. In the next Active Example, we calculate the work that could be done with a reversible engine.

ACTIVE EXAMPLE 18–3 FIND THE WORK

Suppose a reversible heat engine operates between the two heat reservoirs described in Example 18–9. Find the amount of work done by such an engine when 1050 J of heat is drawn from the hot reservoir.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the efficiency of this engine: $e = 1 - T_c/T_h = 0.470$
2. Multiply the efficiency by Q_h to find the work done: $W = eQ_h = 494$ J



INSIGHT

Since this engine is reversible, its total entropy change must be zero. The decrease in entropy of the hot reservoir is $-(1050\text{ J})/576\text{ K} = -1.82$ J/K. It follows that the increase in entropy of the cold reservoir must have the same magnitude. The amount of heat that flows into the cold reservoir, Q_c, is $Q_h - W = 1050\text{ J} - 494\text{ J} = 556$ J. This heat causes an entropy increase equal to $556\text{ J}/305\text{ K} = +1.82$ J/K, as expected. Thus, the reason the engine exhausts the heat Q_c is to produce zero net change in entropy. If the engine were irreversible, it would exhaust a heat greater than 556 J, and this would create a net increase in entropy and a reduction in the amount of work done. Clearly, then, a reversible engine produces the maximum amount of work.

YOUR TURN

Suppose this engine is not reversible, and that only 455 J of work is done when 1050 J of heat is drawn from the hot reservoir. What is the entropy increase of the universe in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Note that when 1050 J of heat is simply transferred from the hot reservoir to the cold reservoir, as in Example 18–9, the entropy of the universe increases by 1.62 J/K. When this same heat is transferred reversibly, with an ideal engine, the entropy of the universe stays the same, but 494 J of work is done. The connection between the entropy increase of the irreversible process and the work done by a reversible engine is very simple:

$$W = T_c \, \Delta S_{\text{universe}} = (305\text{ K})(1.62\text{ J/K}) = 494\text{ J}$$

To see that this expression is valid in general, recall that in Example 18–9 the total change in entropy is $\Delta S_{\text{universe}} = Q/T_c - Q/T_h$. That is, the heat $Q_h = Q$ is withdrawn from the hot reservoir (lowering the entropy by the amount Q/T_h), and the same heat $Q_c = Q$ is added to the cold reservoir (increasing the entropy by the larger amount, Q/T_c). If we multiply this increase in entropy by the temperature of the cold reservoir, T_c, we have $T_c \, \Delta S_{\text{universe}} = Q - QT_c/T_h = Q(1 - T_c/T_h)$. Recalling that the efficiency of an ideal engine is $e = 1 - T_c/T_h$, we see that $T_c \, \Delta S_{\text{universe}} = Qe$. Finally, the work done by an ideal engine is $W = eQ_h$, or in this case, $W = eQ$, since $Q_h = Q$. Therefore, we see that $W = T_c \, \Delta S_{\text{universe}}$, as expected.

In general, a process in which the entropy of the universe increases is one in which less work is done than if the process had been reversible. Thus, we lose forever the ability for that work to be done, because to restore the universe to its former state would mean lowering its entropy, which cannot be done. Thus, with every increase in entropy, there is that much less work that can be done by the universe.

For this reason, entropy is sometimes referred to as a measure of the "quality" of energy. When an irreversible process occurs, and the entropy of the universe increases, we say that the energy of the universe has been "degraded" because less of it is available to do work. This process of increasing degradation of energy and increasing entropy in the universe is a continuing aspect of nature.

18–9 Order, Disorder, and Entropy

In the previous section we considered entropy from the point of view of thermodynamics. We saw that as heat flows from a hot object to a cold object the entropy of the universe increases. In this section we show that entropy can also be thought of as a measure of the amount of **disorder** in the universe.

We begin with the situation of heat flow from a hot to a cold object. In **Figure 18–18 (a)** we show two bricks, one hot and the other cold. As we know from kinetic

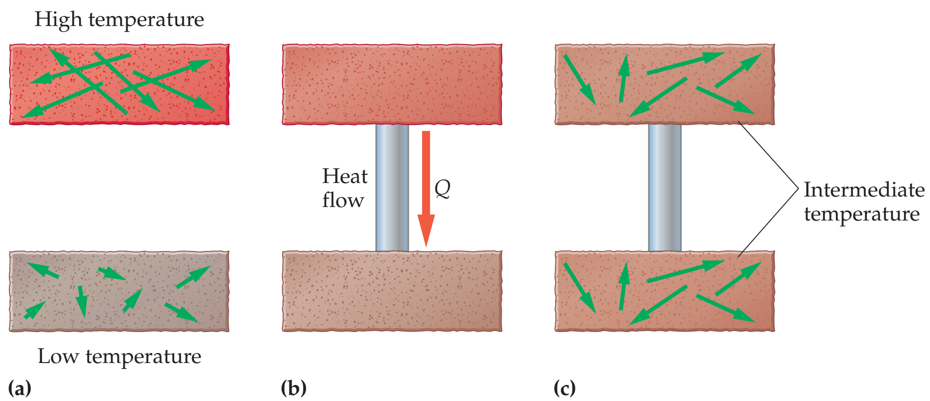


FIGURE 18–18 Heat flow and disorder
(a) Initially, two bricks have different temperatures, and hence different average kinetic energies. **(b)** Heat flows from the hot brick to the cold brick. **(c)** The final result is that both bricks have the same intermediate temperature, and all the molecules have the same average kinetic energy. Thus, the initial orderly segregation of molecules by kinetic energy has been lost.

theory, the molecules in the hot brick have more kinetic energy than the molecules in the cold brick. This means that the system is rather orderly, in that all the high-kinetic-energy molecules are grouped together in the hot brick, and all the low-kinetic-energy molecules are grouped together in the cold brick. There is a definite regularity, or order, to the distribution of the molecular speeds.

Now bring the bricks into thermal contact, as in **Figure 18–18 (b)**. The result is a flow of heat from the hot brick to the cold brick until the temperatures become equal. The final result is indicated in **Figure 18–18 (c)**. During the heat transfer the entropy of the universe increases, as we know, and the system loses the nice orderly distribution it had in the beginning. Now, all the molecules have the same average kinetic energy; hence, the system is randomized, or disordered. We are led to the following conclusion:

> As the entropy of a system increases, its disorder increases as well; that is, an *increase* in entropy is the same as a *decrease* in order.

Note that if heat had flowed in the opposite direction—from the cold brick to the hot brick—the ordered distribution of molecules would have been reinforced, rather than lost.

▲ All processes that occur spontaneously increase the entropy of the universe. In this case, the movements of the water molecules become more random and chaotic when they reach the tumultuous, swirling pool at the bottom of the falls. In addition, some of their kinetic energy is converted into thermal energy—the most disordered and degraded form of energy.

To take another example, consider the 0.125-kg chunk of ice discussed in Example 18–8. As we saw there, the entropy of the universe increases as the ice melts. Now let's consider what happens on the molecular level. To begin, the molecules are well ordered in their crystalline positions. As heat is absorbed, however, the molecules begin to free themselves from the ice and move about randomly in the growing puddle of water. Thus, the regular order of the solid is lost. Again, we see that as entropy increases, so too does the disorder of the molecules.

Thus, the second law of thermodynamics can be stated as the principle that the disorder of the universe is continually increasing. Everything that happens in the universe is simply making it a more disorderly place. And there is nothing you can do to prevent it—nothing you can do will result in the universe being more ordered. Just as freezing a tray of water to make ice actually results in more entropy—and more disorder—in the universe, so does any action you take.

Heat Death

If one carries the previous discussion to its logical conclusion, it seems that the universe is "running down." That is, the disorder of the universe constantly increases, and as it does, the amount of energy available to do work decreases. If this process continues, might there come a day when no more work can be done? And if that day does come, what then?

This is one possible scenario for the fate of the universe, and it is referred to as the "heat death" of the universe. In this scenario, heat continues to flow from hotter regions in space (like stars) to cooler regions (like planets) until, after many billions of years, all objects in the universe have the same temperature. With no temperature differences left, there can be no work done, and the universe would cease to do anything of particular interest. Not a pretty picture, but certainly a possibility. The universe may simply continue with its present expansion until the stars burn out and the galaxies fade away like the dying embers of a scattered campfire.

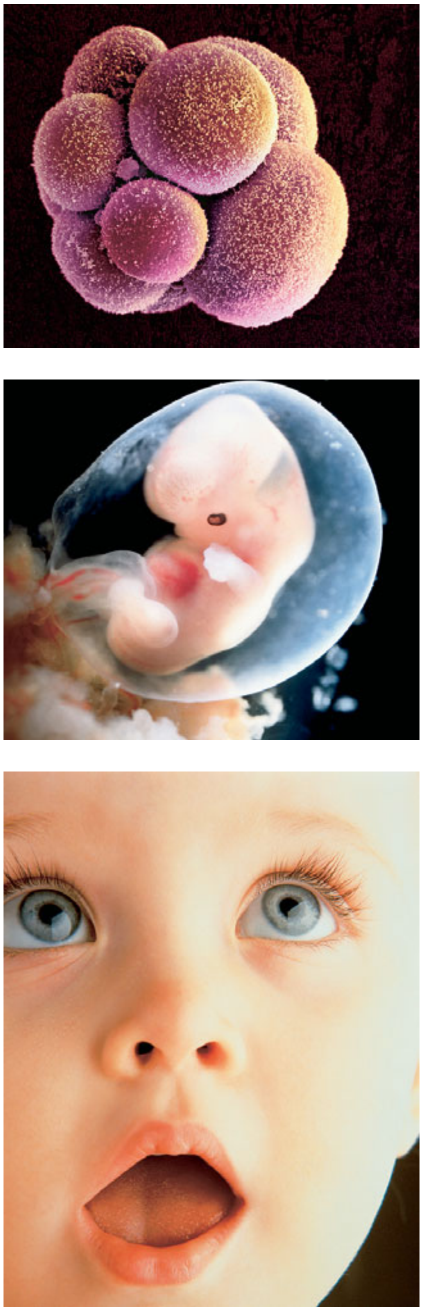
Living Systems

REAL-WORLD PHYSICS: BIO
Entropy and life

So far we have focused on the rather gloomy prospect of the universe constantly evolving toward greater disorder. Is it possible, however, that life is an exception to this rule? After all, we know that an embryo utilizes simple raw materials to produce a complex, highly ordered living organism. Similarly, the well-known biological aphorism "ontogeny recapitulates phylogeny," while not strictly correct, expresses the fact that the development of an individual organism from embryo to adult often reflects certain aspects of the evolutionary development of the species as a whole. Thus, over time, species often evolve toward more complex forms. Finally, living systems are able to use disordered raw materials in the environment to produce orderly structures in which to live. It seems, then, that there are many ways in which living systems produce increasing order.

This conclusion is flawed, however, since it fails to take into account the entropy of the environment in which the organism lives. It is similar to the conclusion that a freezer violates the second law of thermodynamics because it reduces the entropy of water as it freezes it into ice. This analysis neglects the fact that the freezer exhausts heat into the room, increasing the entropy of the air by an amount that is greater than the entropy decrease of the water. In the same way, living organisms constantly give off heat to the atmosphere as a by-product of their metabolism, increasing its entropy. Thus, if we build a house from a pile of bricks—decreasing the entropy of the bricks—the heat we give off during our exertions increases the entropy of the atmosphere more than enough to give a net increase in entropy.

Finally, all living organisms can be thought of as heat engines, tapping into the flow of energy from one place to another to produce mechanical work. Plants, for example, tap into the flow of energy from the high temperature of the Sun to the cold of deep space and use a small fraction of this energy to sustain themselves and reproduce. Animals consume plants and generate heat within their bodies as they metabolize their food. A fraction of the energy released by the metabolism is in turn converted to mechanical work. Living systems, then, obey the same laws of physics as steam engines and refrigerators—they simply produce different results as they move the universe toward greater disorder.



▲ Many species, including humans, develop from a single fertilized egg into a complex multicellular organism. In the process they create large, intricately ordered molecules such as proteins and DNA from smaller, simpler precursors. In the metabolic processes of living things, however, heat is produced, increasing the entropy of the universe as a whole. Thus, the second law is not violated.

18–10 The Third Law of Thermodynamics

Finally, we consider the **third law of thermodynamics,** which states that there is no temperature lower than absolute zero, and that absolute zero is unattainable. It is possible to cool an object to temperatures arbitrarily close to absolute zero—experiments have reached temperatures as low as 4.5×10^{-10} K—but no object can ever be cooled to precisely 0 K.

As an analogy to cooling toward absolute zero, imagine walking toward a wall, with each step half the distance between you and the wall. Even if you take an infinite number of steps, you will still not reach the wall. You can get arbitrarily close, of course, but you never get all the way there.

The same sort of thing happens when cooling. To cool an object, you can place it in thermal contact with an object that is colder. Heat transfer will occur, with your object ending up cooler and the other object ending up warmer. In particular, suppose you had a collection of objects at 0 K to use for cooling. You put your object in contact with one of the 0-K objects and your object cools, while the 0-K object warms slightly. You continue this process, each time throwing away the "warmed up" 0-K object and using a new one. Each time you cool your object it gets closer to 0 K, without ever actually getting there.

In light of this discussion, we can express the third law of thermodynamics as follows:

The Third Law of Thermodynamics

It is impossible to lower the temperature of an object to absolute zero in a finite number of steps.

As with the second law of thermodynamics, this law can be expressed in a number of different but equivalent ways. The essential idea, however, is always the same: Absolute zero is the limiting temperature and, though it can be approached arbitrarily closely, it can never be attained.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concepts of heat, work, and temperature (Chapters 16 and 17) are used throughout this chapter.

Our understanding of ideal gases (Chapter 17) is put to good use in Sections 18–3 and 18–4, where we study common thermal processes.

Specific heat (Chapter 16) is revisited in Section 18–4, where we extend the concept to processes that occur at constant pressure or constant volume.

LOOKING AHEAD

Chapter 21 shows how an electric current can generate heat, much like the heating caused by friction as one object slides against another.

In Chapter 23 we shall see how the mechanical work produced by a heat engine can be converted to electrical energy by a generator, in the form of an electric current.

The expansion of the universe is discussed in Chapter 32. During the expansion, the temperature of the universe has decreased, very much as it does for an ideal gas undergoing an adiabatic expansion.

CHAPTER SUMMARY

18–1 THE ZEROTH LAW OF THERMODYNAMICS

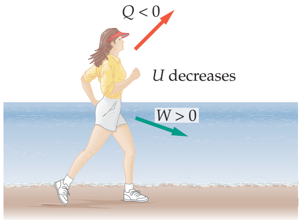
When two objects have the same temperature, they are in thermal equilibrium.

18–2 THE FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics is a statement of energy conservation that includes heat.

If U is the internal energy of an object, Q is the heat added to it, and W is the work done by the object, the first law of thermodynamics can be written as follows:

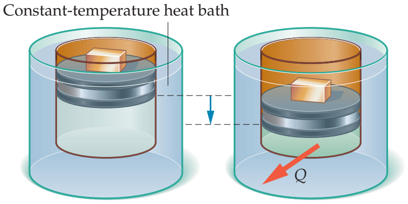
$$\Delta U = Q - W \qquad 18\text{–}3$$



State Function
The internal energy U depends only on the state of a system; that is, on its temperature, pressure, and volume. The value of U is independent of how a system is brought to a certain state.

18–3 THERMAL PROCESSES

Quasi-Static
A quasi-static process is one in which a system is moved slowly from one state to another. The change of state is so slow that the system may be considered to be in equilibrium at any given time during the process.



Reversible
In a reversible process it is possible to return the system and its surroundings to their initial states.

Irreversible
Irreversible processes cannot be "undone." When the system is returned to its initial state, the surroundings have been altered.

Work
In general, the work done during a process is equal to the area under the process curve in a PV plot.

Constant Pressure
In a PV plot, a constant-pressure process is represented by a horizontal line. The work done at constant pressure is $W = P\Delta V$.

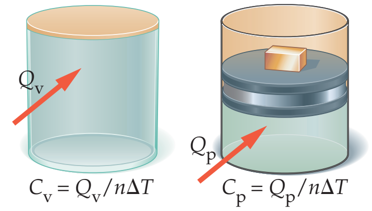
Constant Volume
In a PV plot, a constant-volume process is represented by a vertical line. The work done at constant volume is zero; $W = 0$.

Isothermal Process
In a PV plot, an isothermal process is represented by PV = constant. The work done in an isothermal expansion from V_i to V_f is $W = nRT\ln(V_f/V_i)$.

Adiabatic Process
An adiabatic process occurs with no heat transfer; that is, $Q = 0$.

18–4 SPECIFIC HEATS FOR AN IDEAL GAS: CONSTANT PRESSURE, CONSTANT VOLUME

Specific heats have different values depending on whether they apply to a process at constant pressure or a process at constant volume.



Molar Specific Heat
The molar specific heat, C, is defined by $Q = nC\Delta T$, where n is the number of moles.

Constant Volume
The molar specific heat for an ideal monatomic gas at constant volume is

$$C_v = \tfrac{3}{2}R \qquad 18\text{–}6$$

Constant Pressure
The molar specific heat for an ideal monatomic gas at constant pressure is

$$C_p = \tfrac{5}{2}R \qquad 18\text{–}7$$

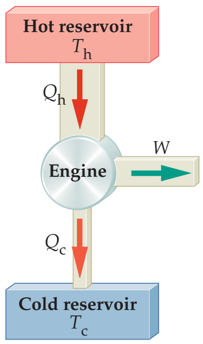
Adiabatic Process
In a PV plot, an adiabatic process is represented by $PV^{\gamma} =$ constant, where γ is the ratio C_p/C_v. For a monatomic, ideal gas, $\gamma = 5/3$.

18–5 THE SECOND LAW OF THERMODYNAMICS

When objects of different temperatures are brought into thermal contact, the spontaneous flow of heat that results is always from the high-temperature object to the low-temperature object. Spontaneous heat flow never proceeds in the reverse direction.

18–6 HEAT ENGINES AND THE CARNOT CYCLE

A heat engine is a device that converts heat into work; for example, a steam engine.



Efficiency
The efficiency e of a heat engine that takes in the heat Q_h from a hot reservoir, exhausts a heat Q_c to a cold reservoir, and does the work W is

$$e = \frac{W}{Q_h} = \frac{Q_h - Q_c}{Q_h} = 1 - \frac{Q_c}{Q_h} \qquad 18\text{–}12$$

Carnot's Theorem
If an engine operating between two constant-temperature reservoirs is to have maximum efficiency, it must be an engine in which all processes are reversible. In addition, all reversible engines operating between the same two temperatures have the same efficiency.

Maximum Efficiency
The maximum efficiency of a heat engine operating between the Kelvin temperatures T_h and T_c is

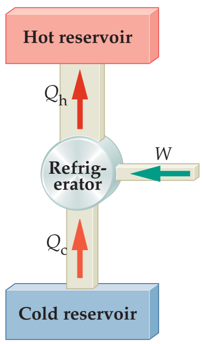
$$e_{max} = 1 - \frac{T_c}{T_h} \qquad 18\text{–}13$$

Maximum Work
If a heat engine takes in the heat Q_h from a hot reservoir, the maximum work it can do is

$$W_{max} = e_{max}Q_h = \left(1 - \frac{T_c}{T_h}\right)Q_h \qquad 18\text{–}14$$

18–7 REFRIGERATORS, AIR CONDITIONERS, AND HEAT PUMPS

Refrigerators, air conditioners, and heat pumps are devices that use work to make heat flow from a cold region to a hot region.



Coefficient of Performance
The coefficient of performance of a refrigerator or air conditioner doing the work W to remove a heat Q_c from a cold reservoir is

$$\text{COP} = \frac{Q_c}{W} \qquad 18\text{–}15$$

Heat Pump

In an ideal heat pump, the work W that must be done to deliver a heat Q_h to a hot reservoir at the temperature T_h by extracting a heat Q_c from a cold reservoir at the temperature T_c is

$$W = Q_h - Q_c = Q_h\left(1 - \frac{Q_c}{Q_h}\right) = Q_h\left(1 - \frac{T_c}{T_h}\right) \qquad 18\text{–}16$$

The coefficient of performance for a heat pump is

$$\mathrm{COP} = \frac{Q_h}{W} \qquad 18\text{–}17$$

18–8 ENTROPY

Like the internal energy U, the entropy S is a state function.

Change in Entropy

The change in entropy during a reversible exchange of the heat Q at the Kelvin temperature T is

$$\Delta S = \frac{Q}{T} \qquad 18\text{–}18$$

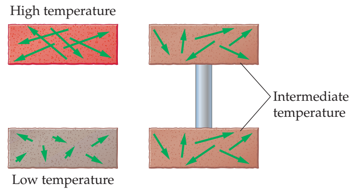
Entropy in the Universe

The total entropy of the universe increases whenever an irreversible process occurs. *Note:* Entropy is not conserved.

In an idealized reversible process the entropy of the universe is unchanged.

18–9 ORDER, DISORDER, AND ENTROPY

Entropy is a measure of the disorder of a system. As entropy increases, a system becomes more disordered.



Heat Death

A possible fate of the universe is heat death, in which everything is at the same temperature and no more work can be done.

18–10 THE THIRD LAW OF THERMODYNAMICS

It is impossible to lower the temperature of an object to absolute zero in a finite number of steps.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Relate the heat and work exchanged by a system to its change in internal energy.	The heat, work, and internal energy of a system are related by the first law of thermodynamics; $\Delta U = Q - W$.	Example 18–1
Calculate the work done by a system during a given process in terms of the corresponding PV plot.	The work done by an expanding system is equal to the area under the curve that represents the process on a PV plot. If the system is compressed, the work is equal to the negative of the area, meaning that work is done on the system rather than by it.	Example 18–2 Active Example 18–1
Find the efficiency of a heat engine.	If a heat engine takes in a heat Q_h and does the work W, its efficiency, e, is given by $e = W/Q_h$. Since W is the difference between the heat going into the engine, Q_h, and the heat leaving the engine, Q_c, the efficiency can also be written as $e = (Q_h - Q_c)/Q_h = 1 - Q_c/Q_h$. Finally, the efficiency can also be related to the absolute temperature of the hot (T_h) and cold (T_c) reservoirs as follows: $e = 1 - T_c/T_h$.	Example 18–6 Active Example 18–2
Determine the change in entropy of a system.	If a system exchanges a heat Q at the absolute temperature T, its entropy S changes by the following amount: $\Delta S = Q/T$. If heat is added to the system, its entropy increases; if heat is removed from the system, its entropy decreases.	Examples 18–8, 18–9

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

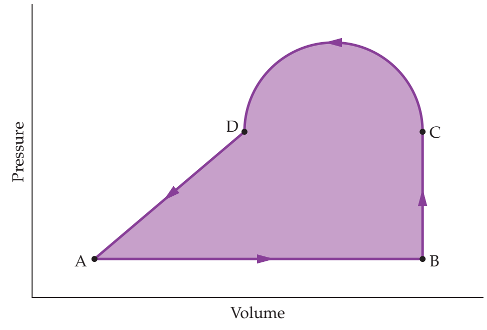
1. If an engine has a reverse gear, does this make it reversible?
2. The temperature of a substance is held fixed. Is it possible for heat to flow **(a)** into or **(b)** out of this system? For each case, give an explanation if your answer is no. If your answer is yes, give a specific example.
3. A substance is thermally insulated, so that no heat can flow between it and its surroundings. Is it possible for the temperature of this substance to **(a)** increase or **(b)** decrease? For each case, give an explanation if your answer is no. If your answer is yes, give a specific example.
4. Heat is added to a substance. Is it safe to conclude that the temperature of the substance will rise? Give an explanation if your answer is no. If your answer is yes, give a specific example.
5. The temperature of a substance is increased. Is it safe to conclude that heat was added to the substance? Give an explanation if your answer is no. If your answer is yes, give a specific example.
6. Are there thermodynamic processes in which all the heat absorbed by an ideal gas goes completely into mechanical work? If so, give an example.
7. Is it possible to convert a given amount of mechanical work completely into heat? Explain.
8. An ideal gas is held in an insulated container at the temperature T. All the gas is initially in one-half of the container, with a partition separating the gas from the other half of the container, which is a vacuum. If the partition ruptures, and the gas expands to fill the entire container, what is its final temperature?
9. Which of the following processes are approximately reversible? **(a)** Lighting a match. **(b)** Pushing a block up a frictionless inclined plane. **(c)** Frying an egg. **(d)** Swimming from one end of a pool to the other. **(e)** Stretching a spring by a small amount. **(f)** Writing a report for class.
10. Which law of thermodynamics would be violated if heat were to spontaneously flow between two objects of equal temperature?
11. Heat engines always give off a certain amount of heat to a low-temperature reservoir. Would it be possible to use this "waste" heat as the heat input to a second heat engine, and then use the "waste" heat of the second engine to run a third engine, and so on?
12. A heat pump uses 100 J of energy as it operates for a given time. Is it possible for the heat pump to deliver more than 100 J of heat to the inside of the house in this same time? Explain.
13. If you clean up a messy room, putting things back where they belong, you decrease the room's entropy. Does this violate the second law of thermodynamics? Explain.
14. Which law of thermodynamics is most pertinent to the statement that "all the king's horses and all the king's men couldn't put Humpty Dumpty back together again?"
15. Which has more entropy: **(a)** popcorn kernels, or the resulting popcorn; **(b)** two eggs in a carton, or an omelet made from the eggs; **(c)** a pile of bricks, or the resulting house; **(d)** a piece of paper, or the piece of paper after it has been burned?

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 18–2 THE FIRST LAW OF THERMODYNAMICS

1. • **CE** Give the change in internal energy of a system if **(a)** $W = 50$ J, $Q = 50$ J; **(b)** $W = -50$ J, $Q = -50$ J; or **(c)** $W = 50$ J, $Q = -50$ J.
2. • **CE** A gas expands, doing 100 J of work. How much heat must be added to this system for its internal energy to increase by 200 J?
3. • A swimmer does 6.7×10^5 J of work and gives off 4.1×10^5 J of heat during a workout. Determine ΔU, W, and Q for the swimmer.
4. • When 1210 J of heat are added to one mole of an ideal monatomic gas, its temperature increases from 272 K to 276 K. Find the work done by the gas during this process.
5. • Three different processes act on a system. **(a)** In process A, 42 J of work are done on the system and 77 J of heat are added to the system. Find the change in the system's internal energy. **(b)** In process B, the system does 42 J of work and 77 J of heat are added to the system. What is the change in the system's internal energy? **(c)** In process C, the system's internal energy decreases by 120 J while the system performs 120 J of work on its surroundings. How much heat was added to the system?
6. • An ideal gas is taken through the four processes shown in Figure 18–19. The changes in internal energy for three of these processes are as follows: $\Delta U_{AB} = +82$ J; $\Delta U_{BC} = +15$ J; $\Delta U_{DA} = -56$ J. Find the change in internal energy for the process from C to D.



▲ **FIGURE 18–19** Problems 6 and 80

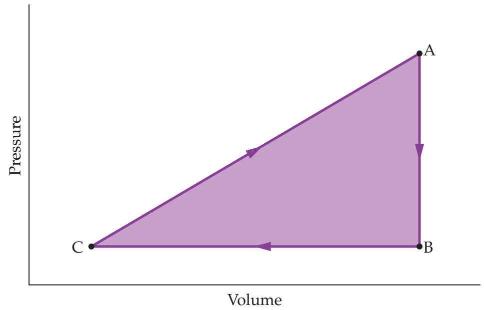
7. •• A basketball player does 2.43×10^5 J of work during her time in the game, and evaporates 0.110 kg of water. Assuming a latent heat of 2.26×10^6 J/kg for the perspiration (the same as for water), determine **(a)** the change in the player's internal energy and **(b)** the number of nutritional calories the player has converted to work and heat.
8. •• **IP** One mole of an ideal monatomic gas is initially at a temperature of 263 K. **(a)** Find the final temperature of the gas if 3280 J of heat are added to it and it does 722 J of work. **(b)** Suppose the amount of gas is doubled to two moles. Does the final temperature found in part (a) increase, decrease, or stay the same? Explain.

9. •• **IP Energy from Gasoline** Burning a gallon of gasoline releases 1.19×10^8 J of internal energy. If a certain car requires 5.20×10^5 J of work to drive one mile, **(a)** how much heat is given off to the atmosphere each mile, assuming the car gets 25.0 miles to the gallon? **(b)** If the miles per gallon of the car is increased, does the amount of heat released to the atmosphere increase, decrease, or stay the same? Explain.

10. •• A cylinder contains 4.0 moles of a monatomic gas at an initial temperature of 27 °C. The gas is compressed by doing 560 J of work on it, and its temperature increases by 130 °C. How much heat flows into or out of the gas?

11. •• An ideal gas is taken through the three processes shown in Figure 18–20. Fill in the missing entries in the following table:

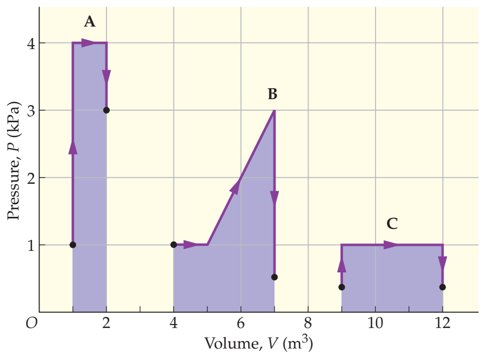
	Q	W	ΔU
A → B	−53.0 J	(a)	(b)
B → C	−325 J	−130 J	(c)
C → A	(e)	82.7 J	(d)



▲ **FIGURE 18–20** Problems 11 and 91

SECTION 18–3 THERMAL PROCESSES

12. • **CE** Figure 18–21 shows three different multistep processes, labeled A, B, and C. Rank these processes in order of increasing work done by a gas that undergoes the process. Indicate ties where appropriate.



▲ **FIGURE 18–21** Problems 12 and 76

13. • A system consisting of an ideal gas at the constant pressure of 110 kPa gains 920 J of heat. Find the change in volume of the system if the internal energy of the gas increases by **(a)** 550 J or **(b)** 660 J.

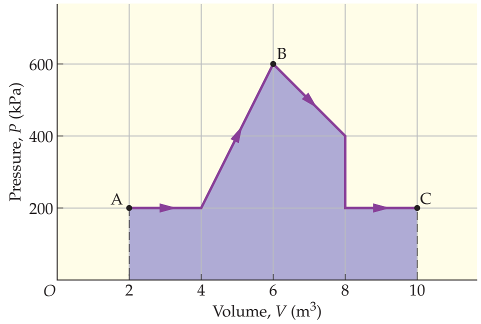
14. • An ideal gas is compressed at constant pressure to one-half its initial volume. If the pressure of the gas is 120 kPa, and 790 J of work is done on it, find the initial volume of the gas.

15. • As an ideal gas expands at constant pressure from a volume of 0.74 m^3 to a volume of 2.3 m^3 it does 93 J of work. What is the gas pressure during this process?

16. • The volume of a monatomic ideal gas doubles in an isothermal expansion. By what factor does its pressure change?

17. •• **IP (a)** If the internal energy of a system increases as the result of an adiabatic process, is work done on the system or by the system? **(b)** Calculate the work done on or by the system in part (a) if its internal energy increases by 670 J.

18. •• **(a)** Find the work done by a monatomic ideal gas as it expands from point A to point C along the path shown in Figure 18–22. **(b)** If the temperature of the gas is 220 K at point A, what is its temperature at point C? **(c)** How much heat has been added to or removed from the gas during this process?

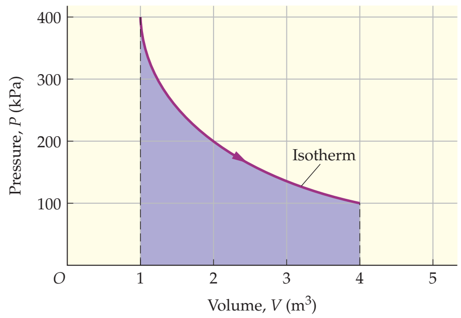


▲ **FIGURE 18–22** Problems 18 and 19

19. •• **IP** A fluid expands from point A to point B along the path shown in Figure 18–22. **(a)** How much work is done by the fluid during this expansion? **(b)** Does your answer to part (a) depend on whether the fluid is an ideal gas? Explain.

20. •• **IP** If 8.00 moles of a monatomic ideal gas at a temperature of 245 K are expanded isothermally from a volume of 1.12 L to a volume of 4.33 L, calculate **(a)** the work done and **(b)** the heat flow into or out of the gas. **(c)** If the number of moles is doubled, by what factors do your answers to parts (a) and (b) change? Explain.

21. •• Suppose 145 moles of a monatomic ideal gas undergo an isothermal expansion from 1.00 m^3 to 4.00 m^3, as shown in Figure 18–23. **(a)** What is the temperature at the beginning and at the end of this process? **(b)** How much work is done by the gas during this expansion?



▲ **FIGURE 18–23** Problems 21, 22, and 79

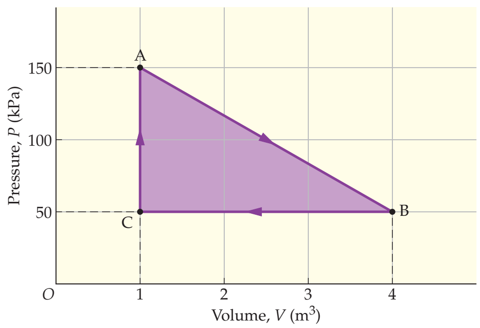
22. •• **IP** A system consisting of 121 moles of a monatomic ideal gas undergoes the isothermal expansion shown in Figure 18–23. **(a)** During this process, does heat enter or leave the system? Explain. **(b)** Is the magnitude of the heat exchanged with the gas from 1.00 m^3 to 2.00 m^3 greater than, less than, or the same as it is from 3.00 m^3 to 4.00 m^3? Explain. Calculate the heat

exchanged with the gas **(c)** from 1.00 m^3 to 2.00 m^3 and **(d)** from 3.00 m^3 to 4.00 m^3.

23. •• **IP** **(a)** A monatomic ideal gas expands at constant pressure. Is heat added to the system or taken from the system during this process? **(b)** Find the heat added to or taken from the gas in part (a) if it expands at a pressure of 130 kPa from a volume of 0.76 m^3 to a volume of 0.93 m^3.

24. •• During an adiabatic process, the temperature of 3.92 moles of a monatomic ideal gas drops from 485 °C to 205 °C. For this gas, find **(a)** the work it does, **(b)** the heat it exchanges with its surroundings, and **(c)** the change in its internal energy.

25. •• An ideal gas follows the three-part process shown in **Figure 18–24**. At the completion of one full cycle, find **(a)** the net work done by the system, **(b)** the net change in internal energy of the system, and **(c)** the net heat absorbed by the system.



▲ **FIGURE 18–24** Problems 25 and 27

26. •• With the pressure held constant at 210 kPa, 49 mol of a monatomic ideal gas expands from an initial volume of 0.75 m^3 to a final volume of 1.9 m^3. **(a)** How much work was done by the gas during the expansion? **(b)** What were the initial and final temperatures of the gas? **(c)** What was the change in the internal energy of the gas? **(d)** How much heat was added to the gas?

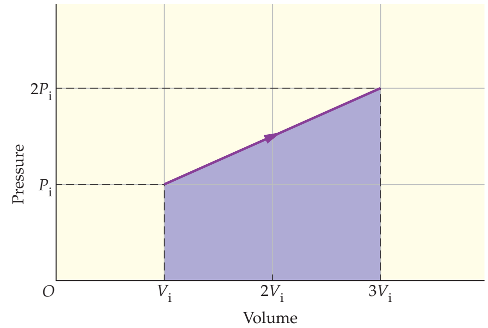
27. •• **IP** Suppose 67.5 moles of an ideal monatomic gas undergo the series of processes shown in Figure 18–24. **(a)** Calculate the temperature at the points A, B, and C. **(b)** For each process, $A \rightarrow B$, $B \rightarrow C$, and $C \rightarrow A$, state whether heat enters or leaves the system. Explain in each case. **(c)** Calculate the heat exchanged with the gas during each of the three processes.

28. •• A gas is contained in a cylinder with a pressure of 140 kPa and an initial volume of 0.66 m^3. How much work is done by the gas as it **(a)** expands at constant pressure to twice its initial volume, or **(b)** is compressed to one-third its initial volume?

29. •• A system expands by 0.75 m^3 at a constant pressure of 125 kPa. Find the heat that flows into or out of the system if its internal energy **(a)** increases by 65 J or **(b)** decreases by 1850 J. In each case, give the direction of heat flow.

30. •• **IP** An ideal monatomic gas is held in a perfectly insulated cylinder fitted with a movable piston. The initial pressure of the gas is 110 kPa, and its initial temperature is 280 K. By pushing down on the piston, you are able to increase the pressure to 140 kPa. **(a)** During this process, did the temperature of the gas increase, decrease, or stay the same? Explain. **(b)** Find the final temperature of the gas.

31. ••• A certain amount of a monatomic ideal gas undergoes the process shown in **Figure 18–25**, in which its pressure doubles and its volume triples. In terms of the number of moles, n, the initial pressure, P_i, and the initial volume, V_i, determine **(a)** the work done by the gas W, **(b)** the change in internal energy of the gas U, and **(c)** the heat added to the gas Q.



▲ **FIGURE 18–25** Problem 31

32. ••• An ideal gas doubles its volume in one of three different ways: (i) at constant pressure; (ii) at constant temperature; (iii) adiabatically. Explain your answers to each of the following questions: **(a)** In which expansion does the gas do the most work? **(b)** In which expansion does the gas do the least work? **(c)** Which expansion results in the highest final temperature? **(d)** Which expansion results in the lowest final temperature?

SECTION 18–4 SPECIFIC HEATS FOR AN IDEAL GAS: CONSTANT PRESSURE, CONSTANT VOLUME

33. • **CE Predict/Explain** You plan to add a certain amount of heat to a gas in order to raise its temperature. **(a)** If you add the heat at constant volume, is the increase in temperature greater than, less than, or equal to the increase in temperature if you add the heat at constant pressure? **(b)** Choose the *best explanation* from among the following:

I. The same amount of heat increases the temperature by the same amount, regardless of whether the volume or the pressure is held constant.
II. All the heat goes into raising the temperature when added at constant volume; none goes into mechanical work.
III. Holding the pressure constant will cause a greater increase in temperature than simply having a fixed volume.

34. • Find the amount of heat needed to increase the temperature of 3.5 mol of an ideal monatomic gas by 23 K if **(a)** the pressure or **(b)** the volume is held constant.

35. • **(a)** If 535 J of heat are added to 45 moles of a monatomic gas at constant volume, how much does the temperature of the gas increase? **(b)** Repeat part (a), this time for a constant-pressure process.

36. • A system consists of 2.5 mol of an ideal monatomic gas at 325 K. How much heat must be added to the system to double its internal energy at **(a)** constant pressure or **(b)** constant volume?

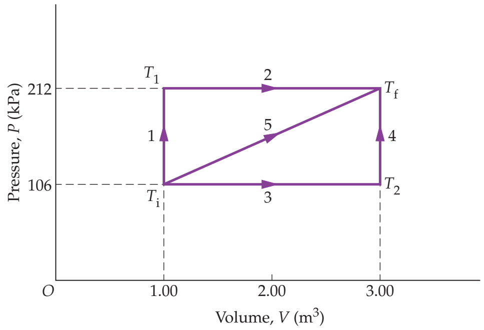
37. • Find the change in temperature if 170 J of heat are added to 2.8 mol of an ideal monatomic gas at **(a)** constant pressure or **(b)** constant volume.

38. •• **IP** A cylinder contains 18 moles of a monatomic ideal gas at a constant pressure of 160 kPa. **(a)** How much work does the gas do as it expands 3200 cm^3, from 5400 cm^3 to 8600 cm^3? **(b)** If the gas expands by 3200 cm^3 again, this time from 2200 cm^3 to 5400 cm^3, is the work it does greater than, less than, or equal to the work found in part (a)? Explain. **(c)** Calculate the work done as the gas expands from 2200 cm^3 to 5400 cm^3.

39. •• **IP** The volume of a monatomic ideal gas doubles in an adiabatic expansion. By what factor do **(a)** the pressure and **(b)** the temperature of the gas change? **(c)** Verify your answers to parts (a) and (b) by considering 135 moles of gas with an initial pressure of 330 kPa and an initial volume of 1.2 m^3. Find the pressure and temperature of the gas after it expands adiabatically to a volume of 2.4 m^3.

40. •• A monatomic ideal gas is held in a thermally insulated container with a volume of 0.0750 m^3. The pressure of the gas is 105 kPa, and its temperature is 317 K. **(a)** To what volume must the gas be compressed to increase its pressure to 145 kPa? **(b)** At what volume will the gas have a temperature of 295 K?

41. ••• Consider the expansion of 60.0 moles of a monatomic ideal gas along processes 1 and 2 in **Figure 18–26**. On process 1 the gas is heated at constant volume from an initial pressure of 106 kPa to a final pressure of 212 kPa. On process 2 the gas expands at constant pressure from an initial volume of 1.00 m^3 to a final volume of 3.00 m^3. **(a)** How much heat is added to the gas during these two processes? **(b)** How much work does the gas do during this expansion? **(c)** What is the change in the internal energy of the gas?



▲ **FIGURE 18–26** Problems 41, 42, and 81

42. ••• Referring to Problem 41, suppose the gas is expanded along processes 3 and 4 in Figure 18–26. On process 3 the gas expands at constant pressure from an initial volume of 1.00 m^3 to a final volume of 3.00 m^3. On process 4 the gas is heated at constant volume from an initial pressure of 106 kPa to a final pressure of 212 kPa. **(a)** How much heat is added to the gas during these two processes? **(b)** How much work does the gas do during this expansion? **(c)** What is the change in the internal energy of the gas?

SECTION 18–6 HEAT ENGINES AND THE CARNOT CYCLE

43. • **CE** A Carnot engine operates between a hot reservoir at the Kelvin temperature T_h and a cold reservoir at the Kelvin temperature T_c. **(a)** If both temperatures are doubled, does the efficiency of the engine increase, decrease, or stay the same? Explain. **(b)** If both temperatures are increased by 50 K, does the efficiency of the engine increase, decrease, or stay the same? Explain.

44. • **CE** A Carnot engine can be operated with one of the following four sets of reservoir temperatures: **A**, 400 K and 800 K; **B**, 400 K and 600 K; **C**, 800 K and 1200 K; and **D**, 800 K and 1000 K. Rank these reservoir temperatures in order of increasing efficiency of the Carnot engine. Indicate ties where appropriate.

45. • What is the efficiency of an engine that exhausts 870 J of heat in the process of doing 340 J of work?

46. • An engine receives 690 J of heat from a hot reservoir and gives off 430 J of heat to a cold reservoir. What are **(a)** the work done and **(b)** the efficiency of this engine?

47. • A Carnot engine operates between the temperatures 410 K and 290 K. **(a)** How much heat must be given to the engine to produce 2500 J of work? **(b)** How much heat is discarded to the cold reservoir as this work is done?

48. • A nuclear power plant has a reactor that produces heat at the rate of 838 MW. This heat is used to produce 253 MW of mechanical power to drive an electrical generator. **(a)** At what rate is heat discarded to the environment by this power plant? **(b)** What is the thermal efficiency of the plant?

49. • At a coal-burning power plant a steam turbine is operated with a power output of 548 MW. The thermal efficiency of the power plant is 32.0%. **(a)** At what rate is heat discarded to the environment by this power plant? **(b)** At what rate must heat be supplied to the power plant by burning coal?

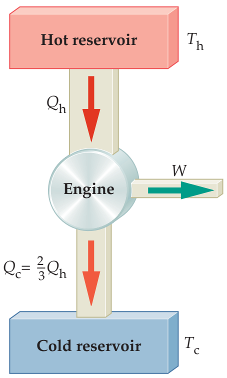
50. •• **IP** If a heat engine does 2700 J of work with an efficiency of 0.18, find **(a)** the heat taken in from the hot reservoir and **(b)** the heat given off to the cold reservoir. **(c)** If the efficiency of the engine is increased, do your answers to parts (a) and (b) increase, decrease, or stay the same? Explain.

51. •• **IP** The efficiency of a particular Carnot engine is 0.300. **(a)** If the high-temperature reservoir is at a temperature of 545 K, what is the temperature of the low-temperature reservoir? **(b)** To increase the efficiency of this engine to 40.0%, must the temperature of the low-temperature reservoir be increased or decreased? Explain. **(c)** Find the temperature of the low-temperature reservoir that gives an efficiency of 0.400.

52. •• During each cycle a reversible engine absorbs 2500 J of heat from a high-temperature reservoir and performs 2200 J of work. **(a)** What is the efficiency of this engine? **(b)** How much heat is exhausted to the low-temperature reservoir during each cycle? **(c)** What is the ratio, T_h/T_c, of the two reservoir temperatures?

53. ••• The operating temperatures for a Carnot engine are T_c and $T_h = T_c + 55$ K. The efficiency of the engine is 11%. Find T_c and T_h.

54. ••• A certain Carnot engine takes in the heat Q_h and exhausts the heat $Q_c = 2Q_h/3$, as indicated in **Figure 18–27**. **(a)** What is the efficiency of this engine? **(b)** Using the Kelvin temperature scale, find the ratio T_c/T_h.



▲ **FIGURE 18–27** Problem 54

SECTION 18–7 REFRIGERATORS, AIR CONDITIONERS, AND HEAT PUMPS

55. • **CE Predict/Explain** **(a)** If the temperature in the kitchen is decreased, is the cost (work needed) to freeze a dozen ice cubes greater than, less than, or equal to what it was before the kitchen was cooled? **(b)** Choose the *best explanation* from among the following:

I. The difference in temperature between the inside and the outside of the refrigerator is decreased, and hence less work is required to freeze the ice.

II. The same amount of ice is frozen in either case, which requires the same amount of heat to be removed and hence the same amount of work.

III. Cooling the kitchen means that the refrigerator must do more work, both to freeze the ice cubes and to warm the kitchen.

56. • The refrigerator in your kitchen does 480 J of work to remove 110 J of heat from its interior. **(a)** How much heat does the refrigerator exhaust into the kitchen? **(b)** What is the refrigerator's coefficient of performance?

57. • A refrigerator with a coefficient of performance of 1.75 absorbs 3.45×10^4 J of heat from the low-temperature reservoir during each cycle. **(a)** How much mechanical work is required to operate the refrigerator for a cycle? **(b)** How much heat does the refrigerator discard to the high-temperature reservoir during each cycle?

58. •• To keep a room at a comfortable 21.0 °C, a Carnot heat pump does 345 J of work and supplies it with 3240 J of heat. **(a)** How much heat is removed from the outside air by the heat pump? **(b)** What is the temperature of the outside air?

59. •• An air conditioner is used to keep the interior of a house at a temperature of 21 °C while the outside temperature is 32 °C. If heat leaks into the house at the rate of 11 kW, and the air conditioner has the efficiency of a Carnot engine, what is the mechanical power required to keep the house cool?

60. •• A reversible refrigerator has a coefficient of performance equal to 10.0. What is its efficiency?

61. •• A freezer has a coefficient of performance equal to 4.0. How much electrical energy must this freezer use to produce 1.5 kg of ice at −5.0 °C from water at 15 °C?

62. •• If a Carnot engine has an efficiency of 0.23, what is its coefficient of performance if it is run backward as a heat pump?

SECTION 18–8 ENTROPY

63. • **CE Predict/Explain (a)** If you rub your hands together, does the entropy of the universe increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. Rubbing hands together draws heat from the surroundings, and therefore lowers the entropy.
II. No mechanical work is done by the rubbing, and hence the entropy does not change.
III. The heat produced by rubbing raises the temperature of your hands and the air, which increases the entropy.

64. • **CE Predict/Explain (a)** An ideal gas is expanded slowly and isothermally. Does its entropy increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. Heat must be added to the gas to maintain a constant temperature, and this increases the entropy of the gas.
II. The temperature of the gas remains constant, which means its entropy also remains constant.
III. As the gas is expanded its temperature and entropy will decrease.

65. • **CE Predict/Explain (a)** A gas is expanded reversibly and adiabatically. Does its entropy increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. The process is reversible, and no heat is added to the gas. Therefore, the entropy of the gas remains the same.
II. Expanding the gas gives it more volume to occupy, and this increases its entropy.
III. The gas is expanded with no heat added to it, and hence its temperature will decrease. This, in turn, will lower its entropy.

66. • Find the change in entropy when 1.85 kg of water at 100 °C is boiled away to steam at 100 °C.

67. • Determine the change in entropy that occurs when 3.1 kg of water freezes at 0 °C.

68. •• **CE** You heat a pan of water on the stove. Rank the following temperature increases in order of increasing entropy change. Indicate ties where appropriate: **A**, 25 °C to 35 °C; **B**, 35 °C to 45 °C; **C**, 45 °C to 50 °C; and **D**, 50 °C to 55 °C.

69. •• On a cold winter's day heat leaks slowly out of a house at the rate of 20.0 kW. If the inside temperature is 22 °C, and the outside temperature is −14.5 °C, find the rate of entropy increase.

70. •• An 88-kg parachutist descends through a vertical height of 380 m with constant speed. Find the increase in entropy produced by the parachutist, assuming the air temperature is 21 °C.

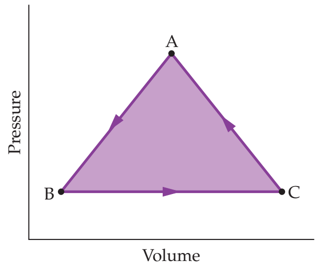
71. •• **IP** Consider the air-conditioning system described in Problem 59. **(a)** Does the entropy of the universe increase, decrease, or stay the same as the air conditioner keeps the imperfectly insulated house cool? Explain. **(b)** What is the rate at which the entropy of the universe changes during this process?

72. •• A heat engine operates between a high-temperature reservoir at 610 K and a low-temperature reservoir at 320 K. In one cycle, the engine absorbs 6400 J of heat from the high-temperature reservoir and does 2200 J of work. What is the net change in entropy as a result of this cycle?

GENERAL PROBLEMS

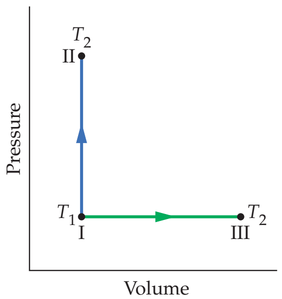
73. • **CE** An ideal gas is held in an insulated container at the temperature T. All the gas is initially in one-half of the container, with a partition separating the gas from the other half of the container, which is a vacuum. If the partition ruptures, and the gas expands to fill the entire container, is the final temperature greater than, less than, or equal to T? Explain.

74. • **CE** Consider the three-process cycle shown in **Figure 18–28**. For each process in the cycle, **(a)** A → B, **(b)** B → C, and **(c)** C → A, state whether the work done by the system is positive, negative, or zero.



▲ **FIGURE 18–28** Problem 74

75. • **CE** An ideal gas has the pressure and volume indicated by point I in **Figure 18–29**. At this point its temperature is T_1. The temperature of the gas can be increased to T_2 by using the constant-volume process, I → II, or the constant-pressure process, I → III. Is the entropy change for the process I → II greater than, less than, or equal to the entropy change on the process I → III? Explain.



▲ **FIGURE 18–29** Problem 75

76. • Find the work done by a monatomic ideal gas on each of the three multipart processes, A, B, and C, shown in Figure 18–21.

77. • Heat is added to a 0.14-kg block of ice at 0 °C, increasing its entropy by 87 J/K. How much ice melts?

78. • The heat that goes into a particular Carnot engine is 4.00 times greater than the work it performs. What is the engine's efficiency?

79. •• **IP** Consider 132 moles of a monatomic gas undergoing the isothermal expansion shown in Figure 18–23. **(a)** What is the temperature T of this expansion? **(b)** Does the entropy of the gas increase, decrease, or stay the same during the process? Explain. **(c)** Calculate the change in entropy for the gas, ΔS, if it is nonzero. **(d)** Calculate the work done by the gas during this process, and compare to $T\Delta S$.

80. •• **IP** Consider a monatomic ideal gas that undergoes the four processes shown in Figure 18–19. Is the work done by the gas positive, negative, or zero on process **(a)** AB, **(b)** BC, **(c)** CD, and **(d)** DA? Explain in each case. **(e)** If the heat added to the gas on process AB is 27 J, how much work does the gas do during that process?

81. •• **IP** Referring to Figure 18–26, suppose 60.0 moles of a monatomic ideal gas are expanded along process 5. **(a)** How much work does the gas do during this expansion? **(b)** What is the change in the internal energy of the gas? **(c)** How much heat is added to the gas during this process?

82. •• **IP** Engine A has an efficiency of 66%. Engine B absorbs the same amount of heat from the hot reservoir and exhausts twice as much heat to the cold reservoir. **(a)** Which engine has the greater efficiency? Explain. **(b)** What is the efficiency of engine B?

83. •• A freezer with a coefficient of performance of 3.88 is used to convert 1.75 kg of water to ice in one hour. The water starts at a temperature of 20.0 °C, and the ice that is produced is cooled to a temperature of −5.00 °C. **(a)** How much heat must be removed from the water for this process to occur? **(b)** How much electrical energy does the freezer use during this hour of operation? **(c)** How much heat is discarded into the room that houses the freezer?

84. •• Suppose 1800 J of heat are added to 3.6 mol of argon gas at a constant pressure of 120 kPa. Find the change in **(a)** internal energy and **(b)** temperature for this gas. **(c)** Calculate the change in volume of the gas. (Assume that the argon can be treated as an ideal monatomic gas.)

85. •• **Entropy and the Sun** The surface of the Sun has a temperature of 5500 °C and the temperature of deep space is 3.0 K. **(a)** Find the entropy increase produced by the Sun in one day, given that it radiates heat at the rate of 3.80×10^{26} W. **(b)** How much work could have been done if this heat had been used to run an ideal heat engine?

86. • The following table lists results for various processes involving n moles of a monatomic ideal gas. Fill in the missing entries.

	Q	**W**	**ΔU**
Constant pressure	$\frac{5}{2}nR\Delta T$	(a)	$\frac{3}{2}nR\Delta T$
Adiabatic	(b)	$-\frac{3}{2}nR\Delta T$	(c)
Constant volume	$\frac{3}{2}nR\Delta T$	(d)	(e)
Isothermal	(f)	$nRT\ln(V_f/V_i)$	(g)

87. •• A cylinder with a movable piston holds 2.75 mol of argon at a constant temperature of 295 K. As the gas is compressed isothermally, its pressure increases from 101 kPa to 121 kPa. Find **(a)** the final volume of the gas, **(b)** the work done by the gas, and **(c)** the heat added to the gas.

88. •• An inventor claims a new cyclic engine that uses organic grape juice as its working material. According to the claims, the engine absorbs 1250 J of heat from a 1010-K reservoir and performs 1120 J of work each cycle. The waste heat is exhausted to the atmosphere at a temperature of 302 K. **(a)** What is the efficiency that is implied by these claims? **(b)** What is the efficiency of a reversible engine operating between the same high and low temperatures used by this engine? (Should you invest in this invention?)

89. •• A nonreversible heat engine operates between a high-temperature reservoir at $T_h = 810$ K and a low-temperature reservoir at $T_c = 320$ K. During each cycle the engine absorbs 660 J of heat from the high-temperature reservoir and performs 250 J of work. **(a)** Calculate the total entropy change ΔS_{tot} for one cycle. **(b)** How much work would a reversible heat engine perform in one cycle if it operated between the same two temperatures and absorbed the same amount of heat? **(c)** Show that the difference in work between the nonreversible engine and the reversible engine is equal to $T_c\Delta S_{tot}$.

90. •• **IP** A small dish containing 530 g of water is placed outside for the birds. During the night the outside temperature drops to −5.0 °C and stays at that value for several hours. **(a)** When the water in the dish freezes at 0 °C, does its entropy increase, decrease, or stay the same? Explain. **(b)** Calculate the change in entropy that occurs as the water freezes. **(c)** When the water freezes, is there an entropy change anywhere else in the universe? If so, specify where the change occurs.

91. •• **IP** An ideal gas is taken through the three processes shown in Figure 18–20. Fill in the missing entries in the following table:

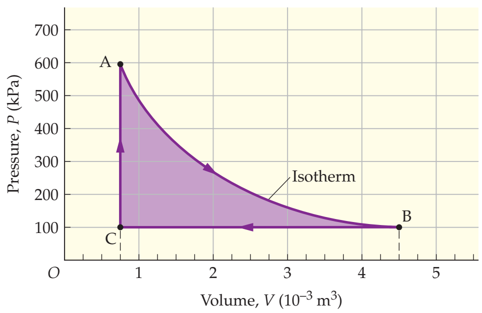
	Q	**W**	**ΔU**
A → B	(a)	(b)	−38.0 J
B → C	(c)	−89.0 J	−134 J
C → A	229 J	(d)	(e)

92. ••• Which would make the greater change in the efficiency of a Carnot heat engine: **(a)** raising the temperature of the high-temperature reservoir by ΔT, or **(b)** lowering the temperature of the low-temperature reservoir by ΔT? Justify your answer by calculating the change in efficiency for each of these cases.

93. ••• One mole of an ideal monatomic gas follows the three-part cycle shown in **Figure 18–30**. **(a)** Fill in the following table:

	Q	**W**	**ΔU**
A → B			
B → C			
C → A			

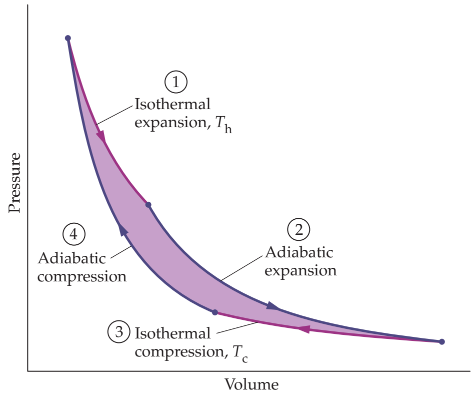
(b) What is the efficiency of this cycle?



▲ **FIGURE 18–30** Problem 93

94. ••• When a heat Q is added to a monatomic ideal gas at constant pressure, the gas does a work W. Find the ratio, W/Q.

95. ••• **The Carnot Cycle** **Figure 18–31** shows an example of a Carnot cycle. The cycle consists of the following four processes: (1) an isothermal expansion from V_1 to V_2 at the temperature T_h ; (2) an adiabatic expansion from V_2 to V_3 during which the temperature drops from T_h to T_c; (3) an isothermal compression from V_3 to V_4 at the temperature T_c; and (4) an adiabatic compression from V_4 to V_1 during which the temperature increases from T_c to T_h. Show that the efficiency of this cycle is $e = 1 - T_c/T_h$, as expected.



▲ **FIGURE 18–31** Problem 95

96. ••• A Carnot engine and a Carnot refrigerator operate between the same two temperatures. Show that the coefficient of performance, COP, for the refrigerator is related to the efficiency, e, of the engine by the following expression; $\text{COP} = (1 - e)/e$.

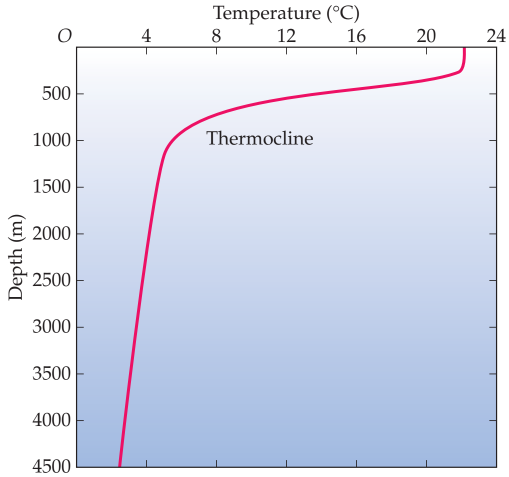
PASSAGE PROBLEMS

Energy from the Ocean

Whenever two objects are at different temperatures, thermal energy can be extracted with a heat engine. A case in point is the ocean, where one "object" is the warm water near the surface, and the other is the cold water at considerable depth. Tropical seas, in particular, can have significant temperature differences between the sun-warmed surface waters, and the cold, dark water 1000 m or more below the surface. A typical oceanic "temperature profile" is shown in **Figure 18–32**, where we see a rapid change in temperature—a thermocline—between depths of approximately 400 m and 900 m.

The idea of tapping this potential source of energy has been around for a long time. In 1870, for example, Captain Nemo in Jules Verne's *Twenty Thousand Leagues Under the Sea*, said, "I owe all to the ocean; it produces electricity, and electricity gives heat, light, motion, and, in a word, life to the Nautilus." Just 11 years later, the French physicist Jacques Arsene d'Arsonval proposed a practical system referred to as Ocean Thermal Energy Conversion (OTEC), and in 1930 Georges Claude, one of d'Arsonval's students, built and operated the first experimental OTEC system off the coast of Cuba.

OTEC systems, which are potentially low-cost and carbon neutral, can provide not only electricity, but also desalinated water as part of the process. In fact, an OTEC plant generating 2 MW of electricity is expected to produce over 14,000 cubic feet of desalinated water a day. Today, the governments of Hawaii, Japan, and Australia are actively pursuing plans for OTEC systems. The National Energy Laboratory of Hawaii Authority (NELHA), for example, operated a test facility near Kona, Hawaii, from 1992 to 1998, and plans further tests in the future.



▲ **FIGURE 18–32** Temperature versus depth for ocean waters in the tropics (Problems 97, 98, and 99)

97. • Suppose an OTEC system operates with surface water at 22 °C and deep water at 4.0 °C. What is the maximum efficiency this system could have?

A. 6.10% **B.** 8.20%
C. 9.40% **D.** 18.0%

98. • If 1500 kg of water at 22 °C is cooled to 4.0 °C, how much energy is released? (For comparison, the energy released in burning a gallon of gasoline is 1.3×10^8 J.)

A. 2.5×10^7 J **B.** 1.1×10^8 J
C. 1.4×10^8 J **D.** 1.6×10^8 J

99. • If we go deeper for colder water, where the temperature is only 2.0 °C, what is the maximum efficiency now?

A. 6.78% **B.** 9.09%
C. 9.32% **D.** 19.0%

INTERACTIVE PROBLEMS

100. •• **IP Referring to Active Example 18–3** Suppose we lower the temperature of the cold reservoir to 295 K; the temperature of the hot reservoir is still 576 K. **(a)** Is the new efficiency of the engine greater than, less than, or equal to 0.470? Explain. **(b)** What is the new efficiency? **(c)** Find the work done by this engine when 1050 J of heat is drawn from the hot reservoir.

101. •• **IP Referring to Active Example 18–3** Suppose the temperature of the hot reservoir is increased by 16 K, from 576 K to 592 K, *and* that the temperature of the cold reservoir is also increased by 16 K, from 305 K to 321 K. **(a)** Is the new efficiency greater than, less than, or equal to 0.470? Explain. **(b)** What is the new efficiency? **(c)** What is the change in entropy of the hot reservoir when 1050 J of heat is drawn from it? **(d)** What is the change in entropy of the cold reservoir?

PHYSICS IN

PERSPECTIVE

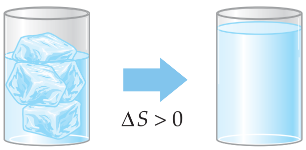
Entropy and Thermo-dynamics

The behavior of heat engines may seem unrelated to the fate of the universe. However, it led physicists to discover a new physical quantity: entropy. The future of the universe is shaped by the fact that the total entropy can only increase. Our fate is sealed.

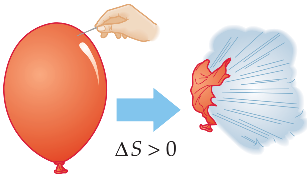
1 Spontaneous processes cannot cause a decrease in entropy

Fundamentally, entropy (S) is randomness or disorder. A process that occurs spontaneously—without a driving input of energy—cannot result in a net increase in order (decrease in entropy).

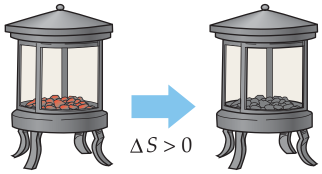
Irreversible processes: $\Delta S > 0$
An *irreversible* process runs spontaneously in just one direction—for instance, ice melts in warm water; warm water doesn't spontaneously form ice cubes. Irreversible processes always cause a net increase in entropy.



Ice melts in warm water



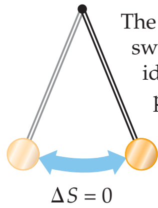
Air leaves a popped balloon



Cooling embers heat their surroundings

Reversible processes: $\Delta S = 0$
If a process can run spontaneously in either direction—so that a movie of it would look equally realistic run forward or backward—it is *reversible* and causes zero entropy change.

In practice, reversibility is an idealization—real processes are never completely reversible.

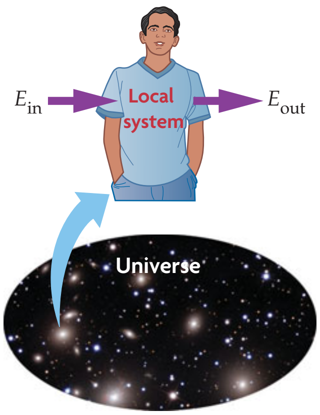


The to-and-fro swinging of an ideal frictionless pendulum is a reversible process

2 Entropy can decrease locally but must increase overall

An input of energy can be used to drive *nonspontaneous* processes that reduce disorder (entropy). That is what your body does with the energy it gains from food.

However, the universe as a whole cannot gain or lose energy, so its total entropy cannot decrease. This means that every process that decreases entropy locally must cause a larger entropy increase elsewhere.



Local system:
Input of energy can drive a decrease in entropy: $\Delta S < 0$.

Universe:
$\Delta E = 0$ (energy is conserved), so
$\Delta S > 0$ (total entropy can only change by increasing)

3 The second law puts entropy in thermodynamic terms

The second law of thermodynamics—that heat moves from hotter to colder objects—actually implies all that we've said about entropy. In fact, the change in entropy ΔS can be defined in terms of the thermodynamic quantities heat Q and temperature T:

$$\Delta S = \frac{Q}{T}$$

Change in system's entropy

Heat entering or leaving system (positive if heat enters system)

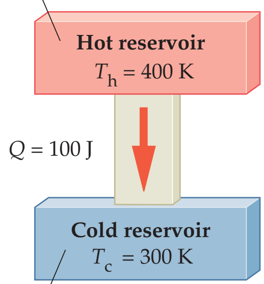
System's temperature

As the example at right shows, the fact that temperature T is in the denominator means that the transfer of a given amount of heat Q causes a greater magnitude of entropy change for a colder object than for a hotter one.

Therefore, a flow of heat from a hotter to a colder object causes a net increase in entropy—as we would predict from the fact that this process is spontaneous and irreversible.

Loss of heat → entropy decrease.

$$\Delta S_h = \frac{Q}{T_h} = \frac{-100\ \text{J}}{400\ \text{K}} = -0.25\ \text{J/K}$$



$$\Delta S_c = \frac{Q}{T_c} = \frac{100\ \text{J}}{300\ \text{K}} = 0.33\ \text{J/K}$$

Gain of heat → entropy increase.

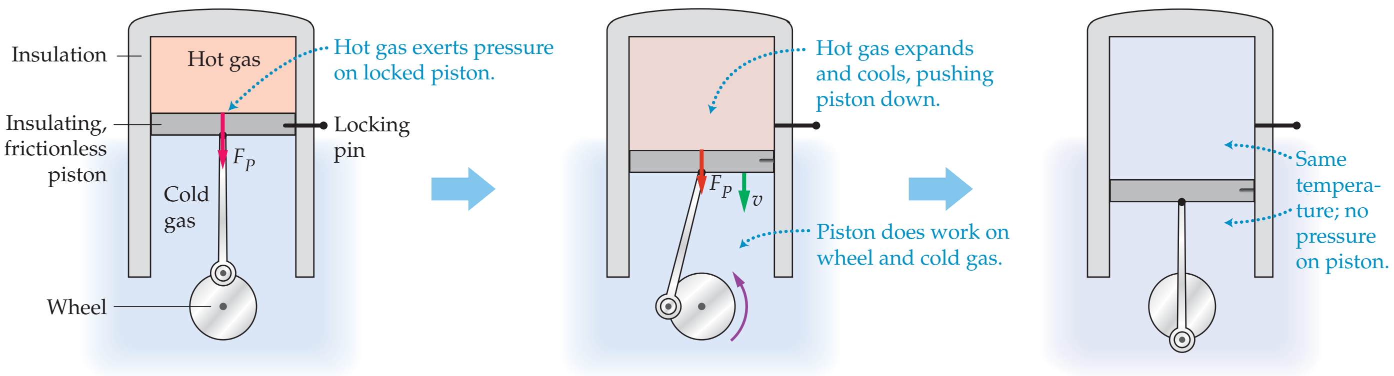
4 A temperature difference can be exploited to do work ...

The tendency of hotter and colder objects to come to the same temperature can be tapped to do work, as in this example:

Initial state: Gases at different temperatures are separated by a locked piston.

Piston unlocked: Pressure difference causes piston to move, doing work on wheel.

Final state: Gases at same temperature; no more work can be done.



The expansion shown above is a single process, not a cycle, so this piston-cylinder does not constitute a heat engine.

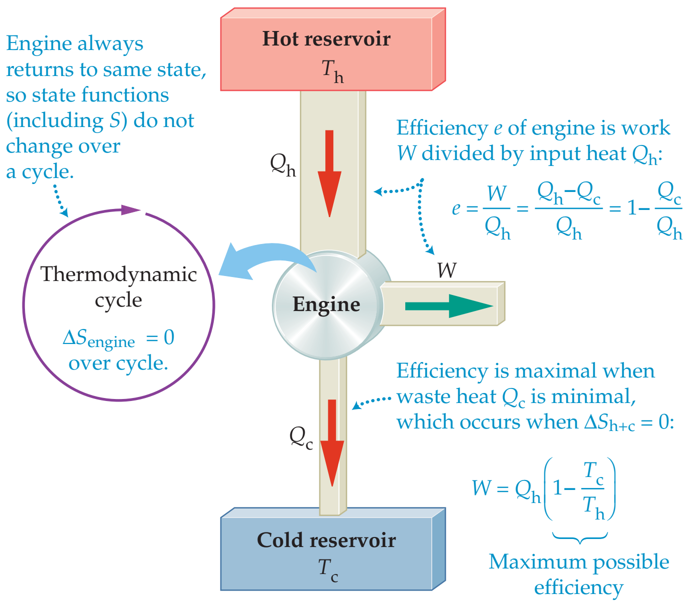
5 ... but entropy sets the limit of efficiency for a heat engine

A heat engine is a device that converts part of a heat flow into work. Entropy sets an absolute limit on the efficiency of this process.

To see why, we start with the fact that a heat engine operates on a thermodynamic cycle—it starts in a particular state, goes through a series of processes involving heat and work, and returns to its original state. (Think of the cyclic operation of a cylinder in a car engine.)

Because entropy S is a state function, the engine's entropy returns to its original value at the end of each cycle—so over the course of a cycle, the entropy change ΔS_{engine} of a heat engine is zero. Therefore, the entropy of the engine's *environment*—specifically, of the hot and cold reservoir ($S_{\text{h+c}}$)—must increase or stay the same ($\Delta S_{\text{h+c}} \geq 0$).

The engine will have the highest efficiency $e = W/Q_{\text{h}}$ when $\Delta S_{\text{h+c}} = 0$, because higher values of $\Delta S_{\text{h+c}}$ entail more waste heat (Q_{c}) and thus yield less work W. To be more efficient than this, an engine would have to cause a net *decrease* in entropy, which is impossible. Actual engines all have $\Delta S_{\text{h+c}} \geq 0$.



6 Entropy spells the death of the universe

The night sky shows us a universe of stars and galaxies separated by cold, nearly empty space. Over time, the inexorable growth of entropy will erase these differences, leaving a universe that is uniform in temperature and density—unable ever again to create stars or give rise to life.

Nevertheless, the energy content of the universe will remain the same as at its birth.

19 Electric Charges, Forces, and Fields

Amber, a form of fossilized tree resin long used to make beautiful beads and other ornaments, has also made contributions to two different sciences. Pieces of amber have preserved prehistoric insects and pollen grains for modern students of evolution. And over 2500 years ago, amber provided Greek scientists with their first opportunity to study electric forces—the subject of this chapter.

We are all made up of electric charges. Every atom in every human body contains both positive and negative charges held together by an attractive force that is similar to gravity—only vastly stronger. Our atoms are bound together by electric forces to form molecules; these molecules, in turn, interact with one another to produce solid bones and liquid blood. In a very real sense, we are walking, talking manifestations of electricity.

In this chapter, we discuss the basic properties of electric charge. Among these are that electric charge comes in discrete units (quantization) and that the total amount of charge in the universe remains constant (conservation). In addition, we present the force law that describes the interactions between electric charges. Finally, we introduce the idea of an electric *field*, and show how it is related to the distribution of charge.

19–1 Electric Charge

The effects of electric charge have been known since at least 600 B.C. About that time, the Greeks noticed that amber—a solid, translucent material formed from the fossilized resin of extinct coniferous trees—has a peculiar property. If a piece of amber is rubbed with animal fur, it attracts small, lightweight objects. This phenomenon is illustrated in **Figure 19–1**.

For some time, it was thought that amber was unique in its ability to become "charged." Much later, it was discovered that other materials can behave in this way as well. For example, if glass is rubbed with a piece of silk, it too can attract small objects. In this respect, glass and amber seem to be the same. It turns out, however, that these two materials have different types of charge.

To see this, imagine suspending a small, charged rod of amber from a thread, as in **Figure 19–2**. If a second piece of charged amber is brought near the rod, as shown in Figure 19–2 (a), the rod rotates away, indicating a repulsive force between the two pieces of amber. Thus, "like" charges repel. On the other hand, if a piece of charged glass is brought near the amber rod, the amber rotates toward the glass, indicating an attractive force. This is illustrated in Figure 19–2 (b). Clearly, then, the *different* charges on the glass and amber attract one another. We refer to different charges as being the "opposite" of one another, as in the familiar expression "opposites attract."

We know today that the two types of charge found on amber and glass are, in fact, the only types that exist, and we still use the purely arbitrary names—**positive** (+) charge and **negative** (−) charge—proposed by Benjamin Franklin (1706–1790) in 1747. In accordance with Franklin's original suggestion, the charge of amber is negative, and the charge of glass is positive (the opposite of negative). Calling the different charges + and − is actually quite useful mathematically; for example, an object that contains an equal amount of positive and negative charge has zero net charge. Objects with zero net charge are said to be electrically **neutral.**

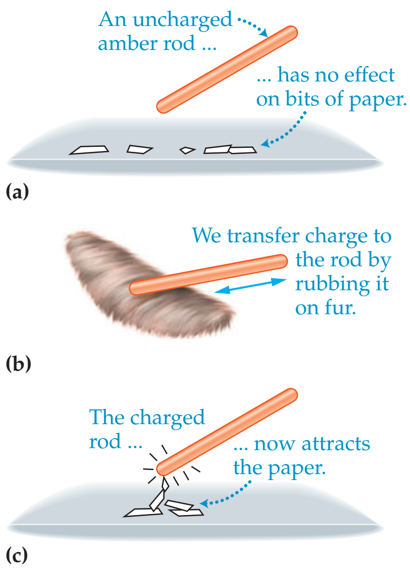


FIGURE 19–1 Charging an amber rod
An uncharged amber rod **(a)** exerts no force on scraps of paper. When the rod is rubbed against a piece of fur **(b)**, it becomes charged and then attracts the paper **(c)**.

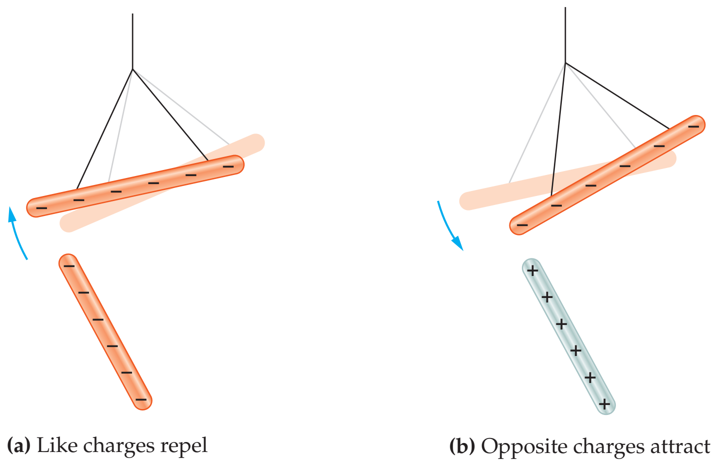


FIGURE 19–2 Likes repel, opposites attract
A charged amber rod is suspended by a string. According to the convention introduced by Benjamin Franklin, the charge on the amber is designated as negative. **(a)** When another charged amber rod is brought near the suspended rod, it rotates away, indicating a repulsive force between like charges. **(b)** When a charged glass rod is brought close to the suspended amber rod, the amber rotates toward the glass, indicating an attractive force and the existence of a second type of charge, which we designate as positive.

A familiar example of an electrically neutral object is the atom. Atoms have a small, dense nucleus with a positive charge surrounded by a cloud of negatively charged electrons (from the Greek word for amber, *elektron*). A pictorial representation of an atom is shown in **Figure 19–3**.

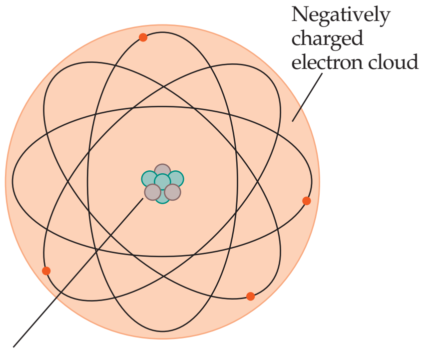
All electrons have exactly the same electric charge. This charge is very small and is defined to have a magnitude, *e*, given by the following:

Magnitude of an Electron's Charge, *e*

$$e = 1.60 \times 10^{-19}\ \mathrm{C} \qquad 19\text{–}1$$

SI unit: coulomb, C

In this expression, C is a unit of charge referred to as the **coulomb,** named for the French physicist Charles-Augustin de Coulomb (1736–1806). (The precise definition



▲ **FIGURE 19–3 The structure of an atom**
A crude representation of an atom, showing the positively charged nucleus at its center and the negatively charged electrons orbiting about it. More accurately, the electrons should be thought of as forming a "cloud" of negative charge surrounding the nucleus.

of the coulomb is in terms of electric current, which we shall discuss in Chapter 21.) Clearly, the charge on an electron, which is negative, is $-e$. This is one of the defining, or *intrinsic*, properties of the electron. Another intrinsic property of the electron is its mass, m_e:

$$m_e = 9.11 \times 10^{-31}\ \mathrm{kg} \tag{19–2}$$

In contrast, the charge on a proton—one of the main constituents of nuclei—is *exactly* $+e$. Therefore, the net charge on atoms, which have equal numbers of electrons and protons, is precisely zero. The mass of the proton is

$$m_p = 1.673 \times 10^{-27}\ \mathrm{kg} \tag{19–3}$$

Note that this is about 2000 times larger than the mass of the electron. The other main constituent of the nucleus is the neutron, which, as its name implies, has zero charge. Its mass is slightly larger than that of the proton:

$$m_n = 1.675 \times 10^{-27}\ \mathrm{kg} \tag{19–4}$$

Since the magnitude of the charge per electron is 1.60×10^{-19} C/electron, it follows that the number of electrons in 1 C of charge is enormous:

$$\frac{1\ \mathrm{C}}{1.60 \times 10^{-19}\ \mathrm{C/electron}} = 6.25 \times 10^{18}\ \mathrm{electrons}$$

As we shall see when we consider the force between charges, a coulomb is a significant amount of charge; even a powerful lightning bolt delivers only 20 to 30 C. A more common unit of charge is the microcoulomb, μC, where 1 μC $= 10^{-6}$ C. Still, the amount of charge contained in everyday objects is very large, even in units of the coulomb, as we show in the following Exercise.

EXERCISE 19–1

Find the amount of positive electric charge in one mole of helium atoms. (Note that the nucleus of a helium atom consists of two protons and two neutrons.)

SOLUTION

Since each helium atom contains two positive charges of magnitude e, the total positive charge in a mole is

$$N_A(2e) = (6.02 \times 10^{23})(2)(1.60 \times 10^{-19}\ \mathrm{C}) = 1.93 \times 10^{5}\ \mathrm{C}$$

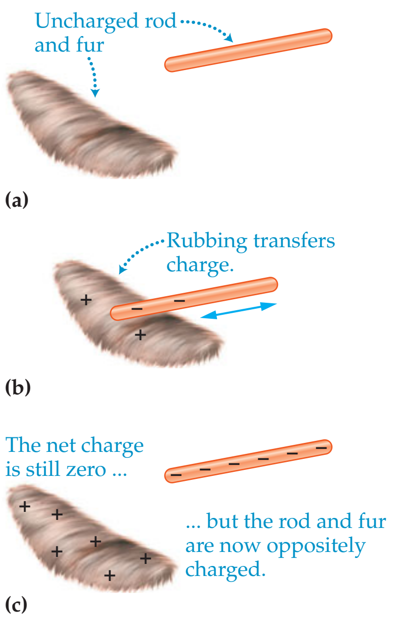
Thus, a mere 4 g of helium contains almost 200,000 C of positive charge, and the same amount of negative charge, as well.

Charge Separation

How is it that rubbing a piece of amber with fur gives the amber a charge? Originally, it was thought that the friction of rubbing *created* the observed charge. We now know, however, that rubbing the fur across the amber simply results in a *transfer* of charge from the fur to the amber—with the total amount of charge remaining unchanged. This is indicated in **Figure 19–4**. Before charging, the fur and the amber are both neutral. During the rubbing process some electrons are transferred from the fur to the amber, giving the amber a net negative charge, and leaving the fur with a net positive charge. At no time during this process is charge ever created or destroyed. This, in fact, is an example of one of the fundamental conservation laws of physics:

Conservation of Electric Charge
The total electric charge of the universe is constant. No physical process can result in an increase or decrease in the total amount of electric charge in the universe.

When charge is transferred from one object to another, it is generally due to the movement of electrons. In a typical solid, the nuclei of the atoms are fixed in



▲ **FIGURE 19–4 Charge transfer**
(a) Initially, an amber rod and a piece of fur are electrically neutral; that is, they each contain equal quantities of positive and negative charge. **(b)** As they are rubbed together, charge is transferred from one to the other. **(c)** In the end, the fur and the rod have charges of equal magnitude but opposite sign.

position. The outer electrons of these atoms, however, are often weakly bound and fairly easily separated. As a piece of fur rubs across amber, for example, some of the electrons that were originally part of the fur are separated from their atoms and deposited onto the amber. The atom that loses an electron is now a **positive ion,** and the atom that receives an extra electron becomes a **negative ion.** This is charging by separation.

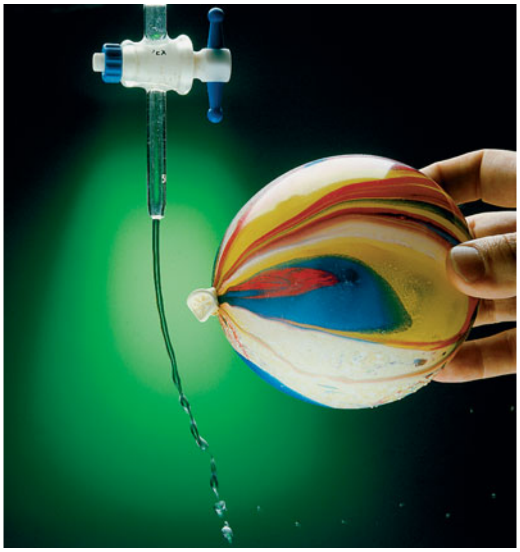
In general, when two materials are rubbed together, the magnitude *and* sign of the charge that each material acquires depend on how strongly it holds onto its electrons. For example, if silk is rubbed against glass, the glass acquires a positive charge, as was mentioned earlier in this section. It follows that electrons have moved from the glass to the silk, giving the silk a *negative* charge. If silk is rubbed against amber, however, the silk becomes *positively* charged, as electrons in this case pass from the silk to the amber.

These results can be understood by referring to Table 19–1, which presents the relative charging due to rubbing—also known as **triboelectric charging**—for a variety of materials. The more plus signs associated with a material, the more readily it gives up electrons and becomes positively charged. Similarly, the more minus signs for a material, the more readily it acquires electrons. For example, we know that amber becomes negatively charged when rubbed against fur, but a greater negative charge is obtained if rubber, PVC, or Teflon is rubbed with fur instead. In general, when two materials in Table 19–1 are rubbed together, the one higher in the list becomes positively charged, and the one lower in the list becomes negatively charged. The greater the separation on the list, the greater the magnitude of the charge.

Charge separation occurs not only when one object is rubbed against another, but also when objects collide. For example, colliding crystals of ice in a rain cloud can cause charge separation that may ultimately result in bolts of lightning to bring the charges together again. Similarly, particles in the rings of Saturn are constantly undergoing collisions and becoming charged as a result. In fact, when the *Voyager* spacecraft examined the rings of Saturn, it observed electrostatic discharges, similar to lightning bolts on Earth. In addition, ghostly radial "spokes" that extend across the rings of Saturn—which cannot be explained by gravitational forces alone—are also the result of electrostatic interactions.

TABLE 19–1 Triboelectric Charging

Material	Relative charging with rubbing
Rabbit fur	++++++
Glass	+++++
Human hair	++++
Nylon	+++
Silk	++
Paper	+
Cotton	−
Wood	−−
Amber	−−−
Rubber	−−−−
PVC	−−−−−
Teflon	−−−−−−



▲ The Van de Graaff generator (left) that these children are touching can produce very large charges of static electricity. Since they are clearly not frightened, why is their hair standing on end? On a smaller scale, if you rub a balloon against a cloth surface, the balloon acquires a negative electric charge. The balloon can then attract a stream of water (right), even though water molecules themselves are electrically neutral. This phenomenon occurs because the water molecules, though they have no net charge, are polar: one end of the molecule has a slight positive charge and the other a slight negative charge. Under the influence of the balloon's negative charge, the water molecules orient themselves so that their positive ends point toward the balloon. This alignment ensures that the electrical attraction between the balloon and the positive part of each molecule exceeds the repulsion between the balloon and the negative part of each molecule.

CONCEPTUAL CHECKPOINT 19–1 COMPARE THE MASS

Is the mass of an amber rod after charging with fur **(a)** greater than, **(b)** less than, or **(c)** the same as its mass before charging?

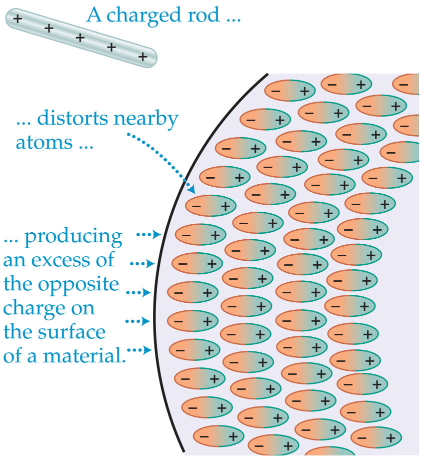
REASONING AND DISCUSSION

Since an amber rod becomes negatively charged, it has acquired electrons from the fur. Each electron has a small, but nonzero, mass. Therefore, the mass of the rod increases ever so slightly as it is charged.

ANSWER

(a) The mass of the amber rod is greater after charging.

Since electrons always have the charge $-e$, and protons always have the charge $+e$, it follows that all objects must have a net charge that is an integral multiple of e. This conclusion was confirmed early in the twentieth century by the American physicist Robert A. Millikan (1868–1953) in a classic series of experiments. He found that the charge on an object can be $\pm e$, $\pm 2e$, $\pm 3e$, and so on, but never $1.5e$ or $-9.3847e$, for example. We describe this restriction by saying that electric charge is **quantized.**



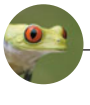
▲ **FIGURE 19–5 Electrical polarization**
When a charged rod is far from a neutral object, the atoms in the object are undistorted, as in Figure 19–3. As the rod is brought closer, however, the atoms distort, producing an excess of one type of charge on the surface of the object (in this case a negative charge). This induced charge is referred to as a polarization charge. Because the sign of the polarization charge is the opposite of the sign of the charge on the rod, there is an attractive force between the rod and the object.

Polarization

We know that charges of opposite sign attract, but it is also possible for a charged rod to attract small objects that have zero net charge. The mechanism responsible for this attraction is called **polarization.**

To see how polarization works, consider **Figure 19–5**. Here we show a positively charged rod held close to an enlarged view of a neutral object. An atom near the surface of the neutral object will become elongated because the negative electrons in it are attracted to the rod while the positive protons are repelled. As a result, a net negative charge develops on the surface near the rod—the so-called polarization charge. The attractive force between the rod and this *induced* polarization charge leads to a net attraction between the rod and the entire neutral object.

Of course, the same conclusion is reached if we consider a negative rod held near a neutral object—except in this case the polarization charge is positive. Thus, the effect of polarization is to give rise to an attractive force regardless of the sign of the charged object. It is for this reason that both charged amber and charged glass attract neutral objects—even though their charges are opposite.



REAL-WORLD PHYSICS: BIO

Bacterial infection from endoscopic surgery

A potentially dangerous, and initially unsuspected, medical application of polarization occurs in endoscopic surgery. In these procedures, a tube carrying a small video camera is inserted into the body. The resulting video image is produced by electrons striking the inside surface of a computer monitor's screen, which is kept positively charged to attract the electrons. Minute airborne particles in the operating room—including dust, lint, and skin cells—are polarized by the positive charge on the screen, and are attracted to its exterior surface.

The problem comes when a surgeon touches the screen to point out an important feature to others in the medical staff. Even the slightest touch can transfer particles—many of which carry bacteria—from the screen to the surgeon's finger and from there to the patient. In fact, the surgeon's finger doesn't even have to touch the screen—as the finger approaches the screen, it too becomes polarized, and hence, it can attract particles from the screen, or directly from the air. Situations like these have resulted in infections, and surgeons are now cautioned not to bring their fingers near the video monitor.

19–2 Insulators and Conductors

Suppose you rub one end of an amber rod with fur, being careful not to touch the other end. The result is that the rubbed portion becomes charged, whereas the other end remains neutral. In particular, the negative charge transferred to the rubbed end stays put; it does not move about from one end of the rod to the other. Materials like amber, in which charges are not free to move, are referred to as **insulators.** Most insulators are nonmetallic substances, and most are also good thermal insulators.

In contrast, most metals are good **conductors** of electricity, in the sense that they allow charges to move about more or less freely. For example, suppose an uncharged metal sphere is placed on an insulating base. If a charged rod is brought into contact with the sphere, as in **Figure 19–6 (a)**, some charge will be transferred to the sphere at the point of contact. The charge does not stay put, however. Since the metal is a good conductor of electricity, the charges are free to move about the sphere, which they do because of their mutual repulsion. The result is a uniform distribution of charge over the surface of the sphere, as shown in **Figure 19–6 (b)**. Note that the insulating base prevents charge from flowing away from the sphere into the ground.

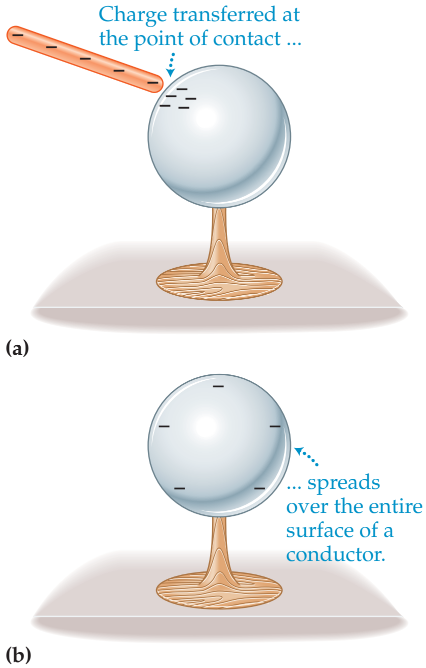
On a microscopic level, the difference between conductors and insulators is that the atoms in conductors allow one or more of their outermost electrons to become detached. These detached electrons, often referred to as "conduction electrons," can move freely throughout the conductor. In a sense, the conduction electrons behave almost like gas molecules moving about within a container. Insulators, in contrast, have very few, if any, free electrons; the electrons are bound to their atoms and cannot move from place to place within the material.

Some materials have properties that are intermediate between those of a good conductor and a good insulator. These materials, referred to as **semiconductors,** can be fine-tuned to display almost any desired degree of conductivity by controlling the concentration of the various components from which they are made. The great versatility of semiconductors is one reason they have found such wide areas of application in electronics and computers.

Exposure to light can sometimes determine whether a given material is an insulator or a conductor. An example of such a **photoconductive** material is selenium, which conducts electricity when light shines on it but is an insulator when in the dark. Because of this special property, selenium plays a key role in the production of photocopies. To see how, we first note that at the heart of every photocopier is a selenium-coated aluminum drum. Initially, the selenium is given a positive charge and kept in the dark—which causes it to retain its charge. When flash lamps illuminate a document to be copied, an image of the document falls on the drum. Where the document is light, the selenium is illuminated and becomes a conductor, and the positive charge flows away into the aluminum drum, leaving the selenium uncharged. Where the document is dark, the selenium is not illuminated, meaning that it is an insulator, and its charge remains in place. At this point, a negatively charged "toner" powder is wiped across the drum, where it sticks to those positively charged portions of the drum that were not illuminated. Next, the drum is brought into contact with paper, transferring the toner to it. Finally, the toner is fused into the paper fibers with heat, the drum is cleaned of excess toner, and the cycle repeats. Thus, a slight variation in electrical properties due to illumination is the basis of an entire technology.

The operation of a laser printer is basically the same as that of a photocopier, with the difference that in the laser printer the selenium-coated drum is illuminated with a computer-controlled laser beam. As the laser sweeps across the selenium, the computer turns the beam on and off to produce areas that will print light or dark, respectively.

▲ People who work with electricity must be careful to use gloves made of nonconducting materials. Rubber, an excellent insulator, is often used for this purpose.



▲ **FIGURE 19–6 Charging a conductor**
(a) When an uncharged metal sphere is touched by a charged rod, some charge is transferred at the point of contact. **(b)** Because like charges repel, and charges move freely on a conductor, the transferred charge quickly spreads out and covers the entire surface of the sphere.

REAL-WORLD PHYSICS
Photocopiers and laser printers

19–3 Coulomb's Law

We have already discussed the fact that electric charges exert forces on one another. The precise law describing these forces was first determined by Coulomb in the late 1780s. His result is remarkably simple. Suppose, for example, that an idealized point charge q_1 is separated by a distance r from another point charge q_2. Both charges are at rest; that is, the system is **electrostatic.** According to Coulomb's law, the magnitude of the electrostatic force between these charges is proportional to the product of the magnitude of the charges, $|q_1||q_2|$, and inversely proportional to the square of the distance, r^2, between them:

Coulomb's Law for the Magnitude of the Electrostatic Force Between Point Charges

$$F = k\frac{|q_1||q_2|}{r^2} \qquad 19\text{–}5$$

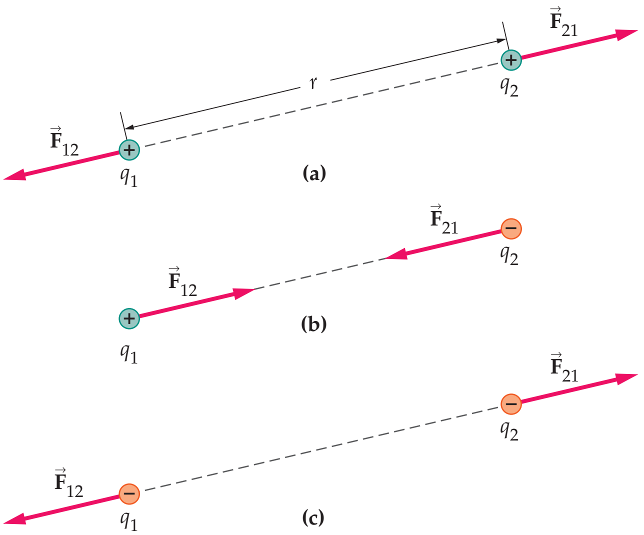
SI unit: newton, N

In this expression, the proportionality constant k has the value

$$k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2} \qquad 19\text{–}6$$

Note that the units of k are simply those required for the force F to have the units of newtons.

The direction of the force in Coulomb's law is along the line connecting the two charges. In addition, we know from the observations described in Section 19–1 that like charges repel and opposite charges attract. These properties are illustrated in **Figure 19–7**, where force vectors are shown for charges of various signs. Thus, when applying Coulomb's law, we first calculate the magnitude of the force using Equation 19–5, and then determine its direction with the "likes repel, opposites attract" rule.



▶ **FIGURE 19–7 Forces between point charges**
The forces exerted by two point charges on one another are always along the line connecting the charges. If the charges have the same sign, as in **(a)** and **(c)**, the forces are repulsive; that is, each charge experiences a force that points away from the other charge. Charges of opposite sign, as in **(b)**, experience attractive forces. Notice that in all cases the forces exerted on the two charges form an action–reaction pair. That is, $\vec{F}_{21} = -\vec{F}_{12}$.

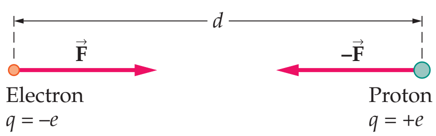
Finally, note how Newton's third law applies to each of the cases shown in Figure 19–7. For example, the force exerted on charge 1 by charge 2, $\vec{F}_{12}$, is always equal in magnitude and opposite in direction to the force exerted on charge 2 by charge 1, $\vec{F}_{21}$; that is, $\vec{F}_{21} = -\vec{F}_{12}$.

CONCEPTUAL CHECKPOINT 19–2 WHERE DO THEY COLLIDE?

An electron and a proton, initially separated by a distance d, are released from rest simultaneously. The two particles are free to move. When they collide, are they **(a)** at the midpoint of their initial separation, **(b)** closer to the initial position of the proton, or **(c)** closer to the initial position of the electron?

REASONING AND DISCUSSION

Because of Newton's third law, the forces exerted on the electron and proton are equal in magnitude and opposite in direction. For this reason, it might seem that the particles meet at the midpoint. The masses of the particles, however, are quite different. In fact, as mentioned in Section 19–1, the mass of the proton is about 2000 times greater than the mass of the electron; therefore, the proton's acceleration ($a = F/m$) is about 2000 times less than the electron's acceleration. As a result, the particles collide near the initial position of the proton. More specifically, they collide at the location of the center of mass of the system, which remains at rest throughout the process.



ANSWER

(b) The particles collide near the initial position of the proton.

It is interesting to note the similarities and differences between Coulomb's law, $F = k|q_1||q_2|/r^2$, and Newton's law of gravity, $F = Gm_1m_2/r^2$. In each case, the force decreases as the square of the distance between the two objects. In addition, both forces depend on a product of intrinsic quantities: in the case of the electric force the intrinsic quantity is the charge; in the case of gravity it is the mass.

Equally significant, however, are the differences. In particular, the force of gravity is always attractive, whereas the electric force can be attractive or repulsive. As a result, the net electric force between neutral objects, such as the Earth and the Moon, is essentially zero because attractive and repulsive forces cancel one another. Since gravity is always attractive, however, the net gravitational force between the Earth and the Moon is nonzero. Thus, in astronomy, gravity rules, and electric forces play hardly any role.

Just the opposite is true in atomic systems. To see this, let's compare the electric and gravitational forces between a proton and an electron in a hydrogen atom. Taking the distance between the two particles to be the radius of hydrogen, $r = 5.29 \times 10^{-11}$ m, we find that the gravitational force has a magnitude

$$\begin{aligned} F_g &= G\frac{m_e m_p}{r^2} \\ &= (6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(9.11 \times 10^{-31}\ \mathrm{kg})(1.673 \times 10^{-27}\ \mathrm{kg})}{(5.29 \times 10^{-11}\ \mathrm{m})^2} \\ &= 3.63 \times 10^{-47}\ \mathrm{N} \end{aligned}$$

Similarly, the magnitude of the electric force between the electron and the proton is

$$\begin{aligned} F_e &= k\frac{|q_1||q_2|}{r^2} \\ &= (8.99 \times 10^{9}\ \mathrm{N \cdot m^2/C^2})\frac{|-1.60 \times 10^{-19}\ \mathrm{C}||1.60 \times 10^{-19}\ \mathrm{C}|}{(5.29 \times 10^{-11}\ \mathrm{m})^2} \\ &= 8.22 \times 10^{-8}\ \mathrm{N} \end{aligned}$$

Taking the ratio, we find that the electric force is greater than the gravitational force by a factor of

$$\begin{aligned} \frac{F_e}{F_g} &= \frac{8.22 \times 10^{-8}\ \mathrm{N}}{3.63 \times 10^{-47}\ \mathrm{N}} = 2.26 \times 10^{39} \\ &= 2{,}260{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000{,}000 \end{aligned}$$

This huge factor explains why a small piece of charged amber can lift bits of paper off the ground, even though the entire mass of the Earth is pulling downward on the paper.

Clearly, then, the force of gravity plays essentially no role in atomic systems. The reason gravity dominates in astronomy is that, even though the force is incredibly weak, it always attracts, giving a larger net force the larger the astronomical body. The electric force, on the other hand, is very strong but cancels for neutral objects.

Next, we use the electric force to get an idea of the speed of an electron in a hydrogen atom and the frequency of its orbital motion.

PROBLEM-SOLVING NOTE

Distance Dependence of the Coulomb Force

The Coulomb force has an inverse-square dependence on distance. Be sure to divide the product of the charges, $k|q_1||q_2|$, by r^2 when calculating the force.

EXAMPLE 19–1 THE BOHR ORBIT

In an effort to better understand the behavior of atomic systems, the Danish physicist Niels Bohr (1885–1962) introduced a simple model for the hydrogen atom. In the Bohr model, as it is known today, the electron is imagined to move in a circular orbit about a stationary proton. The force responsible for the electron's circular motion is the electric force of attraction between the electron and the proton. **(a)** Given that the radius of the electron's orbit is 5.29×10^{-11} m, and its mass is $m_e = 9.11 \times 10^{-31}$ kg, find the electron's speed. **(b)** What is the frequency of the electron's orbital motion?

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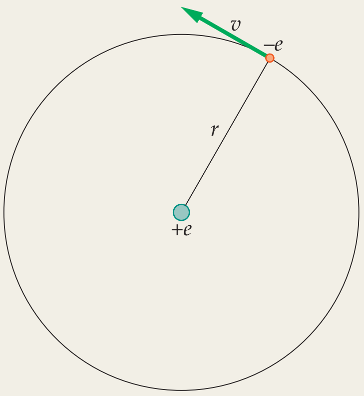
PICTURE THE PROBLEM

Our sketch shows the electron moving with a speed v in its orbit of radius r. Because the proton is so much more massive than the electron, it is essentially stationary at the center of the orbit. Note that the electron has a charge $-e$ and the proton has a charge $+e$.



STRATEGY

a. The idea behind this model is that a force is required to make the electron move in a circular path, and this force is provided by the electric force of attraction between the electron and the proton. Thus, as with any circular motion, we set the force acting on the electron equal to its mass times its centripetal acceleration. This allows us to solve for the centripetal acceleration, $a_{cp} = v^2/r$ (Equation 6–14), which in turn gives us the speed v.

b. The frequency of the electron's orbital motion is $f = 1/T$, where T is the period of the motion; that is, the time for one complete orbit. The time for an orbit, in turn, is the circumference divided by the speed, or $T = C/v = 2\pi r/v$. Taking the inverse immediately yields the frequency.

SOLUTION

Part (a)

1. Set the Coulomb force between the electron and proton equal to the centripetal force required for the electron's circular orbit:

$$k\frac{|q_1||q_2|}{r^2} = m_e a_{cp}$$

$$k\frac{e^2}{r^2} = m_e\frac{v^2}{r}$$

2. Solve for the speed of the electron, v:

$$v = e\sqrt{\frac{k}{m_e r}}$$

3. Substitute numerical values:

$$v = (1.60 \times 10^{-19}\,\mathrm{C})\sqrt{\frac{8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}}{(9.11 \times 10^{-31}\,\mathrm{kg})(5.29 \times 10^{-11}\,\mathrm{m})}}$$

$$= 2.19 \times 10^6\,\mathrm{m/s}$$

Part (b)

4. Calculate the time for one orbit, T, which is the distance ($C = 2\pi r$) divided by the speed (v):

$$T = \frac{C}{v} = \frac{2\pi r}{v} = \frac{2\pi(5.29 \times 10^{-11}\,\mathrm{m})}{2.19 \times 10^6\,\mathrm{m/s}} = 1.52 \times 10^{-16}\,\mathrm{s}$$

5. Take the inverse of T to find the frequency:

$$f = \frac{1}{T} = \frac{1}{1.52 \times 10^{-16}\,\mathrm{s}} = 6.58 \times 10^{15}\,\mathrm{Hz}$$

INSIGHT

If you could travel around the world at this speed, your trip would take only about 18 s, but your centripetal acceleration would be a more-than-lethal 75,000 times the acceleration of gravity. As it is, the centripetal acceleration of the electron in this "Bohr" orbit around the proton is about 10^{22} times greater than the acceleration of gravity on the surface of the Earth.

The frequency of the orbit is also incredibly large. We won't encounter frequencies this high again until we study light waves in Chapter 25.

PRACTICE PROBLEM

The second Bohr orbit has a radius that is four times the radius of the first orbit. What is the speed of an electron in this orbit? [**Answer:** $v = 1.09 \times 10^6$ m/s]

Some related homework problems: Problem 19, Problem 28, Problem 37

Another indication of the strength of the electric force is given in the following Exercise.

EXERCISE 19–2

Find the electric force between two 1.00-C charges separated by 1.00 m.

SOLUTION

Substituting $q_1 = q_2 = 1.00$ C and $r = 1.00$ m in Coulomb's law, we find

$$F = k\frac{|q_1||q_2|}{r^2} = (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{(1.00\,\mathrm{C})(1.00\,\mathrm{C})}{(1.00\,\mathrm{m})^2} = 8.99 \times 10^9\,\mathrm{N}$$

Exercise 19–2 shows that charges of one coulomb exert a force of about a million tons on one another when separated by a distance of a meter. If the charge in your body could be separated into a pile of positive charge on one side of the room and a pile of negative charge on the other side, the force needed to hold them apart would be roughly 10^{10} tons! Thus, everyday objects are never far from electrical neutrality, since disturbing neutrality requires such tremendous forces.

Superposition of Forces

The electric force, like all forces, is a vector quantity. Hence, when a charge experiences forces due to two or more other charges, the net force on it is simply the *vector* sum of the forces taken individually. For example, in **Figure 19–8**, the total force on charge 1, $\vec{\mathbf{F}}_1$, is the vector sum of the forces due to charges 2, 3, and 4:

$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13} + \vec{\mathbf{F}}_{14}$$

This is referred to as the **superposition** of forces.

Notice that the total force acting on a given charge is the sum of interactions involving just *two* charges at a time, with the force between each pair of charges given by Coulomb's law. For example, the total force acting on charge 1 in Figure 19–8 is the sum of the forces between q_1 and q_2, q_1 and q_3, and q_1 and q_4. Therefore, superposition of forces can be thought of as the generalization of Coulomb's law to systems containing more than two charges. In our first numerical Example of superposition, we consider three charges in a line.

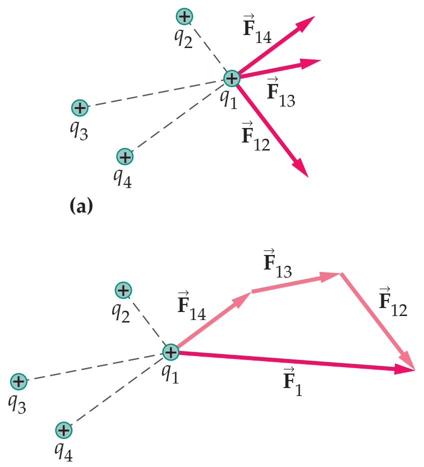


FIGURE 19–8 Superposition of forces

(a) Forces are exerted on q_1 by the charges q_2, q_3, and q_4. These forces are $\vec{\mathbf{F}}_{12}$, $\vec{\mathbf{F}}_{13}$, and $\vec{\mathbf{F}}_{14}$, respectively. **(b)** The net force acting on q_1, which we label $\vec{\mathbf{F}}_1$, is the vector sum of $\vec{\mathbf{F}}_{12}$, $\vec{\mathbf{F}}_{13}$, and $\vec{\mathbf{F}}_{14}$.

EXAMPLE 19–2 NET FORCE

A charge $q_1 = -5.4\ \mu\text{C}$ is at the origin, and a charge $q_2 = -2.2\ \mu\text{C}$ is on the x axis at $x = 1.00$ m. Find the net force acting on a charge $q_3 = +1.6\ \mu\text{C}$ located at $x = 0.75$ m.

PICTURE THE PROBLEM

The physical situation is shown in our sketch, with each charge at its appropriate location. Notice that the forces exerted on charge q_3 by the charges q_1 and q_2 are in opposite directions. We give the force on q_3 due to q_1 the label $\vec{\mathbf{F}}_{31}$, and the force on q_3 due to q_2 the label $\vec{\mathbf{F}}_{32}$.

STRATEGY

The net force on q_3 is the vector sum of the forces due to q_1 and q_2. In particular, note that $\vec{\mathbf{F}}_{31}$ points in the negative x direction $(-\hat{\mathbf{x}})$, whereas $\vec{\mathbf{F}}_{32}$ points in the positive x direction $(\hat{\mathbf{x}})$. The magnitude of $\vec{\mathbf{F}}_{31}$ is $k|q_1||q_3|/r^2$, with $r = 0.75$ m. Similarly, the magnitude of $\vec{\mathbf{F}}_{32}$ is $k|q_2||q_3|/r^2$, with $r = 0.25$ m.

SOLUTION

1. Find the force acting on q_3 due to q_1. Since this force is in the negative x direction, as indicated in the sketch, we give it a negative sign:

$$\begin{aligned}\vec{\mathbf{F}}_{31} &= -k\frac{|q_1||q_3|}{r^2}\hat{\mathbf{x}}\\ &= -(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\\ &\quad\times \frac{(5.4 \times 10^{-6}\ \text{C})(1.6 \times 10^{-6}\ \text{C})}{(0.75\ \text{m})^2}\hat{\mathbf{x}}\\ &= -0.14\ \text{N}\,\hat{\mathbf{x}}\end{aligned}$$

2. Find the force acting on q_3 due to q_2. Since this force is in the positive x direction, as indicated in the sketch, we give it a positive sign:

$$\begin{aligned}\vec{\mathbf{F}}_{32} &= k\frac{|q_2||q_3|}{r^2}\hat{\mathbf{x}}\\ &= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\\ &\quad\times \frac{(2.2 \times 10^{-6}\ \text{C})(1.6 \times 10^{-6}\ \text{C})}{(0.25\ \text{m})^2}\hat{\mathbf{x}}\\ &= 0.51\ \text{N}\,\hat{\mathbf{x}}\end{aligned}$$

3. Superpose these forces to find the total force, $\vec{\mathbf{F}}_3$, acting on q_3:

$$\begin{aligned}\vec{\mathbf{F}}_3 &= \vec{\mathbf{F}}_{31} + \vec{\mathbf{F}}_{32} = -0.14\ \text{N}\,\hat{\mathbf{x}} + 0.51\ \text{N}\,\hat{\mathbf{x}}\\ &= 0.37\ \text{N}\,\hat{\mathbf{x}}\end{aligned}$$

INSIGHT

The net force acting on q_3 has a magnitude of 0.37 N, and it points in the positive x direction. As usual, notice that we use only magnitudes for the charges in the numerator of Coulomb's law.

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PRACTICE PROBLEM

Find the net force on q_3 if it is at the location $x = 0.25$ m. [**Answer:** $\vec{\mathbf{F}}_3 = -1.2\ \mathrm{N}\,\hat{\mathbf{x}}$]

Some related homework problems: Problem 23, Problem 26, Problem 27

ACTIVE EXAMPLE 19–1 FIND THE LOCATION OF ZERO NET FORCE

In Example 19–2, the net force acting on the charge q_3 is to the right. To what value of x should q_3 be moved for the net force on it to be zero?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the magnitude of the force due to q_1: $F_{31} = k|q_1||q_3|/x^2$
2. Write the magnitude of the force due to q_2: $F_{32} = k|q_2||q_3|/(1.00\ \mathrm{m} - x)^2$
3. Set these forces equal to one another, and cancel common terms: $|q_1|/x^2 = |q_2|/(1.00\ \mathrm{m} - x)^2$
4. Take the square root of both sides and solve for x: $x = 0.61$ m

INSIGHT

Therefore, if q_3 is placed between $x = 0.61$ m and $x = 1.00$ m, the net force acting on it is to the right, in agreement with Example 19–2. On the other hand, if q_3 is placed between $x = 0$ and $x = 0.61$ m, the net force acting on it is to the left. This agrees with the result in the Practice Problem of Example 19–2.

YOUR TURN

If the magnitude of each charge in this system is doubled, does the point of zero net force move to the right, move to the left, or remain in the same place? Explain.

(Answers to **Your Turn** *problems are given in the back of the book.)*

PROBLEM-SOLVING NOTE

Determining the Direction of the Electric Force

When determining the total force acting on a charge, begin by calculating the magnitude of each of the individual forces acting on it. Next, assign appropriate directions to the forces based on the principle that "opposites attract, likes repel" and perform a vector sum.

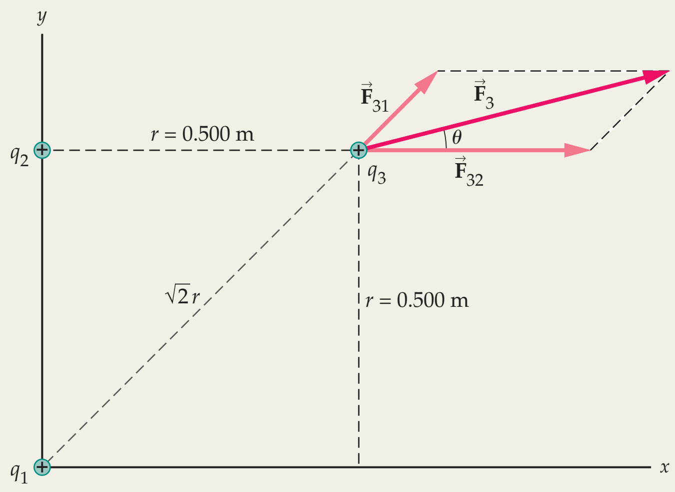
Next we consider systems in which the individual forces are not along the same line. In such cases, it is often useful to resolve the individual force vectors into components and then perform the required vector sum component by component. This technique is illustrated in the following Example and Conceptual Checkpoint.

EXAMPLE 19–3 SUPERPOSITION

Three charges, each equal to $+2.90\ \mu\mathrm{C}$, are placed at three corners of a square 0.500 m on a side, as shown in the diagram. Find the magnitude and direction of the net force on charge 3.

PICTURE THE PROBLEM

The positions of the three charges are shown in the sketch. We also show the force produced by charge 1, $\vec{\mathbf{F}}_{31}$, and the force produced by charge 2, $\vec{\mathbf{F}}_{32}$. Note that $\vec{\mathbf{F}}_{31}$ is 45.0° above the x axis and that $\vec{\mathbf{F}}_{32}$ is in the positive x direction. Also, the distance from charge 2 to charge 3 is $r = 0.500$ m, and the distance from charge 1 to charge 3 is $\sqrt{2}r$.



STRATEGY

To find the net force, we first calculate the magnitudes of $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$ and then their components. Summing these components yields the components of the net force, $\vec{\mathbf{F}}_3$. Once we know the components of $\vec{\mathbf{F}}_3$, we can calculate its magnitude and direction in the same way as for any other vector.

SOLUTION

1. Find the magnitude of $\vec{\mathbf{F}}_{31}$:

$$F_{31} = k\frac{|q_1||q_3|}{\left(\sqrt{2}r\right)^2}$$
$$= (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{(2.90 \times 10^{-6}\,\mathrm{C})^2}{2(0.500\,\mathrm{m})^2}$$
$$= 0.151\,\mathrm{N}$$

2. Find the magnitude of $\vec{\mathbf{F}}_{32}$:

$$F_{32} = k\frac{|q_2||q_3|}{r^2}$$
$$= (8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2})\frac{(2.90 \times 10^{-6}\,\mathrm{C})^2}{(0.500\,\mathrm{m})^2}$$
$$= 0.302\,\mathrm{N}$$

3. Calculate the components of $\vec{\mathbf{F}}_{31}$ and $\vec{\mathbf{F}}_{32}$:

$$F_{31,x} = F_{31}\cos 45.0° = (0.151\,\mathrm{N})(0.707) = 0.107\,\mathrm{N}$$
$$F_{31,y} = F_{31}\sin 45.0° = (0.151\,\mathrm{N})(0.707) = 0.107\,\mathrm{N}$$
$$F_{32,x} = F_{32}\cos 0° = (0.302\,\mathrm{N})(1) = 0.302\,\mathrm{N}$$
$$F_{32,y} = F_{32}\sin 0° = (0.302\,\mathrm{N})(0) = 0$$

4. Find the components of $\vec{\mathbf{F}}_3$:

$$F_{3,x} = F_{31,x} + F_{32,x} = 0.107\,\mathrm{N} + 0.302\,\mathrm{N} = 0.409\,\mathrm{N}$$
$$F_{3,y} = F_{31,y} + F_{32,y} = 0.107\,\mathrm{N} + 0 = 0.107\,\mathrm{N}$$

5. Find the magnitude of $\vec{\mathbf{F}}_3$:

$$F_3 = \sqrt{F_{3,x}{}^2 + F_{3,y}{}^2}$$
$$= \sqrt{(0.409\,\mathrm{N})^2 + (0.107\,\mathrm{N})^2} = 0.423\,\mathrm{N}$$

6. Find the direction of $\vec{\mathbf{F}}_3$:

$$\theta = \tan^{-1}\left(\frac{F_{3,y}}{F_{3,x}}\right) = \tan^{-1}\left(\frac{0.107\,\mathrm{N}}{0.409\,\mathrm{N}}\right) = 14.7°$$

INSIGHT

Thus, the net force on charge 3 has a magnitude of 0.423 N and points in a direction 14.7° above the x axis. Note that charge 1, which is $\sqrt{2}$ times farther away from charge 3 than is charge 2, produces only half as much force as charge 2.

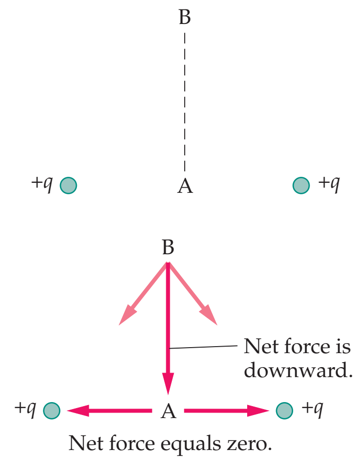
PRACTICE PROBLEM

Find the magnitude and direction of the net force on charge 3 if its magnitude is doubled to 5.80 μC. Assume that charge 1 and charge 2 are unchanged. [**Answer:** $F_3 = 2(0.423\,\mathrm{N}) = 0.846\,\mathrm{N}$, $\theta = 14.7°$. Note that the angle is unchanged.]

Some related homework problems: Problem 31, Problem 32

CONCEPTUAL CHECKPOINT 19–3 COMPARE THE FORCE

A charge $-q$ is to be placed at either point A or point B in the accompanying figure. Assume points A and B lie on a line that is midway between the two positive charges. Is the net force experienced at point A **(a)** greater than, **(b)** equal to, or **(c)** less than the net force experienced at point B?



REASONING AND DISCUSSION

Point A is closer to the two positive charges than is point B. As a result, the force exerted by each positive charge will be greater when the charge $-q$ is placed at A. The *net* force, however, is zero at point A, since the equal attractive forces due to the two positive charges cancel, as shown in the diagram.

At point B, on the other hand, the attractive forces combine to give a net downward force. Hence, the charge $-q$ will experience a greater net force at point B.

ANSWER

(c) The net force at point A is less than the net force at point B.

Spherical Charge Distributions

Although Coulomb's law is stated in terms of point charges, it can be applied to any type of charge distribution by using the appropriate mathematics. For example, suppose a sphere has a charge Q distributed uniformly over its surface. If a point charge q is outside the sphere, a distance r from its center, the methods of calculus show that the magnitude of the force between the point charge and the sphere is simply

$$F = k\frac{|q||Q|}{r^2}$$

In situations like this, the spherical charge distribution behaves the same as if all its charge were concentrated in a point at its center. For point charges inside a charged spherical shell, the net force exerted by the shell is zero. In general, the electrical behavior of spherical *charge* distributions is analogous to the gravitational behavior of spherical *mass* distributions.

PROBLEM-SOLVING NOTE

Spherical Charge Distributions

Remember that a uniform spherical charge distribution can be replaced with a point charge only when considering points outside the charge distribution.

In the next Active Example, we consider a system in which a charge Q is distributed uniformly over the surface of a sphere. In such a case it is often convenient to specify the amount of *charge per area* on the sphere. This is referred to as the **surface charge density,** σ. If a sphere has an area A and a surface charge density σ, its total charge is

$$Q = \sigma A \qquad 19\text{–}7$$

Note that the SI unit of σ is C/m^2. If the radius of the sphere is R, then $A = 4\pi R^2$, and $Q = \sigma(4\pi R^2)$.

ACTIVE EXAMPLE 19–2 FIND THE FORCE EXERTED BY A SPHERE

An insulating sphere of radius $R = 0.10$ m has a uniform surface charge density equal to $5.9\ \mu C/m^2$. A point charge of magnitude $0.71\ \mu C$ is 0.45 m from the center of the sphere. Find the magnitude of the force exerted by the sphere on the point charge.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Find the area of the sphere: $A = 0.13\ m^2$
2. Calculate the total charge on the sphere: $Q = 0.77\ \mu C$
3. Use Coulomb's law to calculate the magnitude of the force between the sphere and the point charge: $F = 0.024$ N

INSIGHT

As long as the point charge is outside the sphere, and the charge distribution remains spherically uniform, the sphere may be treated as a point charge.

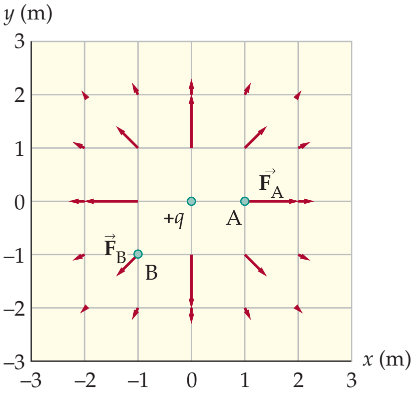
YOUR TURN

Suppose the sphere in this problem is replaced by one with half the radius, but with the same surface charge density. Is the force exerted by this sphere greater than, less than, or the same as the force exerted by the original sphere? Explain.

(Answers to **Your Turn** *problems are given in the back of the book.)*

19–4 The Electric Field

You have probably encountered the notion of a "force field" in various science fiction novels and movies. A concrete example of a force field is provided by the force between electric charges. Consider, for example, a positive point charge q at the origin of a coordinate system, as in Figure 19–9. If a positive "test charge," q_0, is placed at point A, the force exerted on it by q is indicated by the vector $\vec{\mathbf{F}}_A$. On the other hand, if the test charge is placed at point B, the force it experiences there is $\vec{\mathbf{F}}_B$. At every point in space there is a corresponding force. In this sense, Figure 19–9 allows us to visualize the "force field" associated with the charge q.



▲ **FIGURE 19–9 An electrostatic force field**

The positive charge $+q$ at the origin of this coordinate system exerts a different force on a given charge at every point in space. Here we show the force vectors associated with q for a grid of points.

Since the magnitude of the force at every point in Figure 19–9 is proportional to q_0 (due to Coulomb's law), it is convenient to divide by q_0 and define a *force per* *charge* at every point in space that is independent of q_0. We refer to the force per charge as the electric field, $\vec{\mathbf{E}}$. Its precise definition is as follows:

Definition of the Electric Field, $\vec{\mathbf{E}}$

If a test charge q_0 experiences a force $\vec{\mathbf{F}}$ at a given location, the electric field $\vec{\mathbf{E}}$ at that location is

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0} \qquad \text{19–8}$$

SI unit: N/C

It should be noted that this definition applies whether the force $\vec{\mathbf{F}}$ is due to a single charge, as in Figure 19–9, or to a group of charges. In addition, it is assumed that the test charge is small enough that it does not disturb the position of any other charges in the system.

To summarize, *the electric field is the force per charge at a given location*. Therefore, if we know the electric field vector $\vec{\mathbf{E}}$ at a given point, the force that a charge q experiences at that point is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} \qquad \text{19–9}$$

Notice that the direction of the force depends on the sign of the charge. In particular,

- A positive charge experiences a force *in the direction* of $\vec{\mathbf{E}}$.
- A negative charge experiences a force *in the opposite direction* of $\vec{\mathbf{E}}$.

Finally, the magnitude of the force is the product of the magnitudes of q and $\vec{\mathbf{E}}$:

- The magnitude of the force acting on a charge q is $F = |q|E$.

As we continue in this chapter, we will determine the electric field for a variety of different charge distributions. In some cases, $\vec{\mathbf{E}}$ will decrease with distance as $1/r^2$ (a point charge), in other cases as $1/r$ (a line of charge), and in others $\vec{\mathbf{E}}$ will be a constant (a charged plane). Before we calculate the electric field itself, however, we first consider the force exerted on charges by a constant electric field.

PROBLEM-SOLVING NOTE

The Force Exerted by an Electric Field

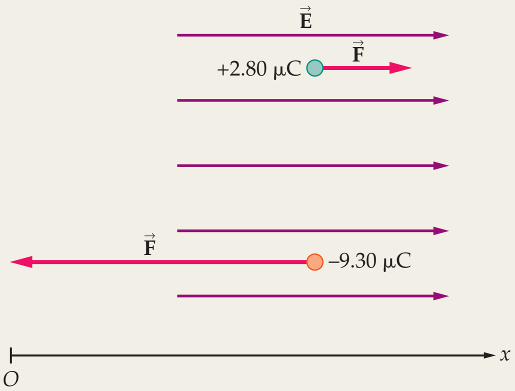
The force exerted on a charge by an electric field can point in only one of two directions—parallel or antiparallel to the direction of the field.

EXAMPLE 19–4 FORCE FIELD

In a certain region of space, a uniform electric field has a magnitude of 4.60×10^4 N/C and points in the positive x direction. Find the magnitude and direction of the force this field exerts on a charge of **(a)** $+2.80\ \mu\text{C}$ and **(b)** $-9.30\ \mu\text{C}$.

PICTURE THE PROBLEM

In our sketch we indicate the uniform electric field and the two charges mentioned in the problem. Note that the positive charge experiences a force in the positive x direction (the direction of $\vec{\mathbf{E}}$), and the negative charge experiences a force in the negative x direction (opposite to $\vec{\mathbf{E}}$).



STRATEGY

To find the magnitude of each force, we use $F = |q|E$. The direction has already been indicated in our sketch.

SOLUTION

Part (a)

1. Find the magnitude of the force on the $+2.80$-μC charge: $F = |q|E = (2.80 \times 10^{-6}\ \text{C})(4.60 \times 10^4\ \text{N/C}) = 0.129\ \text{N}$

Part (b)

2. Find the magnitude of the force on the -9.30-μC charge: $F = |q|E = (9.30 \times 10^{-6}\ \text{C})(4.60 \times 10^4\ \text{N/C}) = 0.428\ \text{N}$

INSIGHT

To summarize, the force on the $+2.80$-μC charge is of magnitude 0.129 N in the positive x direction; the force on the -9.30-μC charge is of magnitude 0.428 N in the negative x direction.

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PRACTICE PROBLEM

If the $+2.80\text{-}\mu C$ charge experiences a force of 0.25 N, what is the magnitude of the electric field? [**Answer:** $E = 8.9 \times 10^4$ N/C]

Some related homework problems: Problem 44, Problem 47

REAL-WORLD PHYSICS

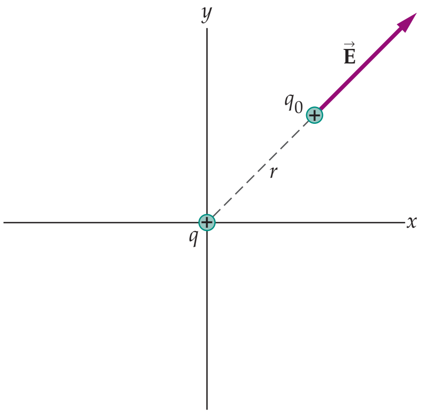
Electrodialysis for water purification

The fact that charges of opposite sign experience forces in opposite directions in an electric field is used to purify water in the process known as **electrodialysis.** This process depends on the fact that most minerals that dissolve in water dissociate into positive and negative ions. Probably the most common example is table salt (NaCl), which dissociates into positive sodium ions (Na^+) and negative chlorine ions (Cl^-). When brackish water is passed through a strong electric field in an electrodialysis machine, the mineral ions move in opposite directions and pass through two different types of semipermeable membrane—one that allows only positive ions to pass through, the other only negative ions. This process leaves water that is purified of dissolved minerals and suitable for drinking.

The Electric Field of a Point Charge

Perhaps the simplest example of an electric field is the field produced by an idealized point charge. To be specific, suppose a positive point charge q is at the origin in **Figure 19–10**. If a positive test charge q_0 is placed a distance r from the origin, the force it experiences is directed away from the origin and is of magnitude

$$F = k\frac{|q||q_0|}{r^2}$$



▲ **FIGURE 19–10 The electric field of a point charge**
The electric field $\vec{E}$ due to a positive charge q at the origin is radially outward. Its magnitude is $E = k|q|/r^2$.

Applying our definition of the electric field in Equation 19–8, we find that the magnitude of the field is

$$E = \frac{F}{q_0} = \frac{\left(k\frac{|q||q_0|}{r^2}\right)}{q_0} = k\frac{|q|}{r^2}$$

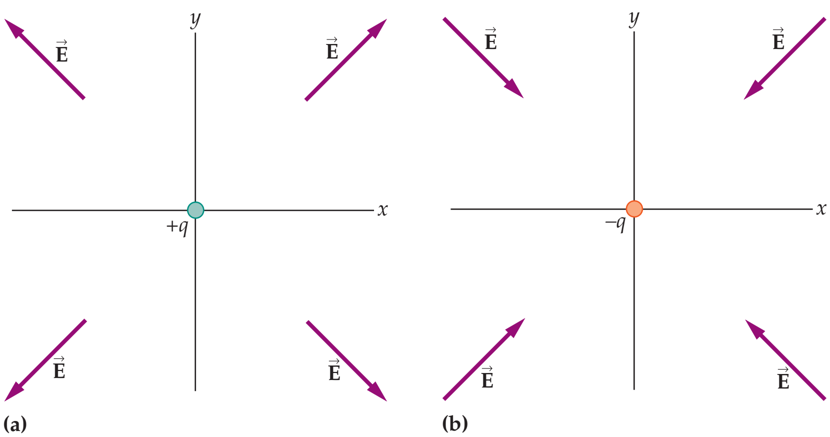
Since a positive charge experiences a force that is radially outward, that too is the direction of $\vec{E}$.

In general, then, we can say that the electric field a distance r from a point charge q has the following magnitude:

Magnitude of the Electric Field Due to a Point Charge

$$E = k\frac{|q|}{r^2} \qquad 19\text{–}10$$

If the charge q is positive, the field points radially outward from the charge; if it is negative, the field is radially inward. This is illustrated in **Figure 19–11**. Thus, to



▶ **FIGURE 19–11 The direction of the electric field**
(a) The electric field due to a positive charge at the origin points radially outward. **(b)** If the charge at the origin is negative, the electric field is radially inward.

determine the electric field due to a point charge, we first use Equation 19–10 to find its magnitude, and then use the rule illustrated in Figure 19–11 to find its direction.

EXERCISE 19–3

Find the electric field produced by a 1.0-μC point charge at a distance of **(a)** 0.75 m and **(b)** 1.5 m.

SOLUTION

a. Applying Equation 19–10 with $q = 1.0\ \mu\text{C}$ and $r = 0.75$ m yields

$$E = k\frac{|q|}{r^2} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.0 \times 10^{-6}\ \text{C})}{(0.75\ \text{m})^2} = 1.6 \times 10^4\ \text{N/C}$$

b. Noting that E depends on $1/r^2$, we see that doubling the distance from 0.75 m to 1.5 m results in a reduction in the electric field by a factor of 4:

$$E = \tfrac{1}{4}(1.6 \times 10^4\ \text{N/C}) = 0.40 \times 10^4\ \text{N/C}$$

Superposition of Fields

Many electrical systems consist of more than two charges. In such cases, the total electric field can be found by using superposition—just as when we find the total force due to a system of charges. In particular, the total electric field is found by calculating the vector sum—often using components—of the electric fields due to each charge separately.

For example, let's calculate the total electric field at point P in **Figure 19–12**. First we sketch the directions of the fields $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ due to the charges $q_1 = +q$ and $q_2 = +q$, respectively. In particular, if a positive test charge is at point P, the force due to q_1 is down and to the right, whereas the force due to q_2 is up and to the right. From the geometry of the figure we see that $\vec{\mathbf{E}}_1$ is at an angle θ below the x axis, and—by symmetry—$\vec{\mathbf{E}}_2$ is at the same angle θ above the axis. Since the two charges have the same magnitude, and the distances from P to the charges are the same, it follows that $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ have the same magnitude:

$$E_1 = E_2 = E = k\frac{|q|}{d^2}$$

To find the net electric field $\vec{\mathbf{E}}_{\text{net}}$, we use components. First, consider the y direction. In this case, we have $E_{1,y} = -E \sin\theta$ and $E_{2,y} = +E \sin\theta$. Hence, the y component of the net electric field is zero:

$$E_{\text{net},y} = E_{1,y} + E_{2,y} = -E\sin\theta + E\sin\theta = 0$$

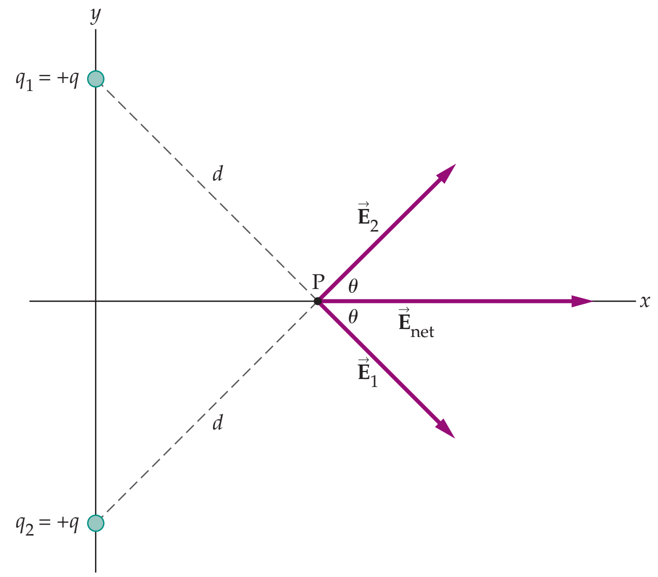


FIGURE 19–12 Superposition of the electric field
The net electric field at the point P is the vector sum of the fields due to the charges q_1 and q_2. Note that $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ point away from the charges q_1 and q_2, respectively. This is as expected, since both of these charges are positive.

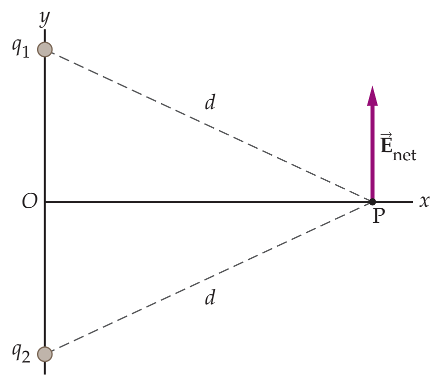
Referring again to Figure 19–12, it is apparent that this result could have been anticipated by symmetry considerations. Finally, we determine the x component of E_{net}:

$$E_{\text{net},x} = E_{1,x} + E_{2,x} = E\cos\theta + E\cos\theta = 2E\cos\theta$$

Thus, the net electric field at P is in the positive x direction, as shown in Figure 19–12, and has a magnitude equal to $2E\cos\theta$.

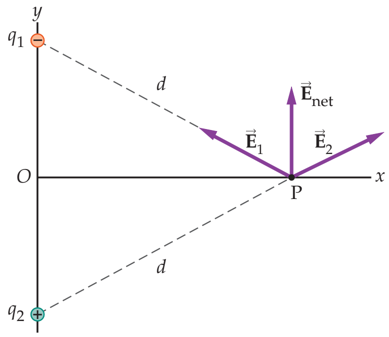
CONCEPTUAL CHECKPOINT 19–4 THE SIGN OF THE CHARGES

Two charges, q_1 and q_2, have equal magnitudes q and are placed as shown in the figure to the right. The net electric field at point P is vertically upward. Do we conclude that **(a)** q_1 is positive, q_2 is negative; **(b)** q_1 is negative, q_2 is positive; or **(c)** q_1 and q_2 have the same sign?



REASONING AND DISCUSSION

If the net electric field at P is vertically upward, the x components of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ must cancel, and the y components must both be in the positive y direction. The only way for this to happen is to have q_1 negative and q_2 positive, as shown in the following diagram.



With this choice, a positive test charge at P is attracted to q_1 (so that $\vec{\mathbf{E}}_1$ is up and to the left) and repelled from q_2 (so that $\vec{\mathbf{E}}_2$ is up and to the right).

ANSWER

(b) q_1 is negative, q_2 is positive.

We conclude this section by considering the same physical system presented in Example 19–3, this time from the point of view of the electric field.

EXAMPLE 19–5 SUPERPOSITION IN THE FIELD

Two charges, each equal to $+2.90\ \mu\text{C}$, are placed at two corners of a square 0.500 m on a side, as shown in the sketch. Find the magnitude and direction of the net electric field at a third corner of the square, the point labeled 3 in the sketch.

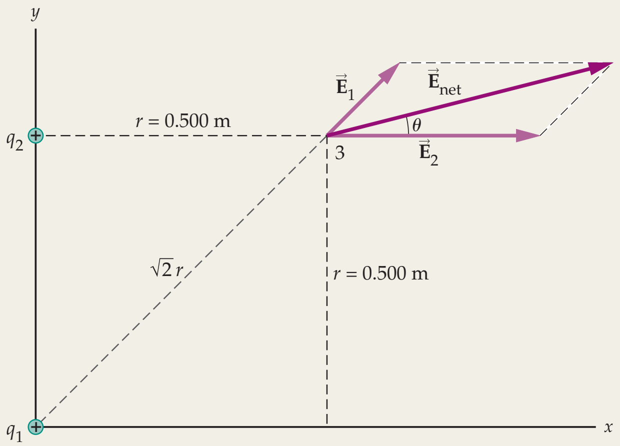
PICTURE THE PROBLEM

The positions of the two charges are shown in the sketch. We also show the electric field produced by each charge. The key difference between this sketch and the one in Example 19–3 is that in this case there is no charge at point 3; the electric field still exists there, even though it has no charge on which to exert a force.

STRATEGY

In analogy with Example 19–3, we first calculate the magnitudes of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$ and then their components. Summing these components yields the components of the net electric field, $\vec{\mathbf{E}}_{\text{net}}$. Once we know the components of $\vec{\mathbf{E}}_{\text{net}}$, we find its magnitude and direction in the same way as for $\vec{\mathbf{F}}_{\text{net}}$ in Example 19–3.

INTERACTIVE FIGURE



SOLUTION

1. Find the magnitude of $\vec{\mathbf{E}}_1$:

$$E_1 = k\frac{|q_1|}{(\sqrt{2}r)^2}$$
$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(2.90 \times 10^{-6}\ \mathrm{C})}{2(0.500\ \mathrm{m})^2}$$
$$= 5.21 \times 10^4\ \mathrm{N/C}$$

2. Find the magnitude of $\vec{\mathbf{E}}_2$:

$$E_2 = k\frac{|q_2|}{r^2}$$
$$= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(2.90 \times 10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2}$$
$$= 1.04 \times 10^5\ \mathrm{N/C}$$

3. Calculate the components of $\vec{\mathbf{E}}_1$ and $\vec{\mathbf{E}}_2$:

$$E_{1,x} = E_1 \cos 45.0^\circ$$
$$= (5.21 \times 10^4\ \mathrm{N/C})(0.707) = 3.68 \times 10^4\ \mathrm{N/C}$$
$$E_{1,y} = E_1 \sin 45.0^\circ$$
$$= (5.21 \times 10^4\ \mathrm{N/C})(0.707) = 3.68 \times 10^4\ \mathrm{N/C}$$
$$E_{2,x} = E_2 \cos 0^\circ$$
$$= (1.04 \times 10^5\ \mathrm{N/C})(1) = 1.04 \times 10^5\ \mathrm{N/C}$$
$$E_{2,y} = E_2 \sin 0^\circ = (1.04 \times 10^5\ \mathrm{N/C})(0) = 0$$

4. Find the components of $\vec{\mathbf{E}}_{\mathrm{net}}$:

$$E_{\mathrm{net},x} = E_{1,x} + E_{2,x}$$
$$= 3.68 \times 10^4\ \mathrm{N/C} + 1.04 \times 10^5\ \mathrm{N/C}$$
$$= 1.41 \times 10^5\ \mathrm{N/C}$$
$$E_{\mathrm{net},y} = E_{1,y} + E_{2,y}$$
$$= 3.68 \times 10^4\ \mathrm{N/C} + 0 = 3.68 \times 10^4\ \mathrm{N/C}$$

5. Find the magnitude of $\vec{\mathbf{E}}_{\mathrm{net}}$:

$$E_{\mathrm{net}} = \sqrt{E_{\mathrm{net},x}{}^2 + E_{\mathrm{net},y}{}^2}$$
$$= \sqrt{(1.41 \times 10^5\ \mathrm{N/C})^2 + (3.68 \times 10^4\ \mathrm{N/C})^2}$$
$$= 1.46 \times 10^5\ \mathrm{N/C}$$

6. Find the direction of $\vec{\mathbf{E}}_{\mathrm{net}}$:

`

$$\theta = \tan^{-1}(E_{\mathrm{net},y}/E_{\mathrm{net},x})$$
$$= \tan^{-1}\left(\frac{3.68 \times 10^4\ \mathrm{N/C}}{1.41 \times 10^5\ \mathrm{N/C}}\right) = 14.6^\circ$$

INSIGHT

Note that, as one would expect, the direction of the net electric field is the same as the direction of the net force in Example 19–3 (except for a small discrepancy in the last decimal place due to rounding off in the calculations). In addition, the magnitude of the force exerted by the electric field on a charge of 2.90 μC is $F = qE_{\mathrm{net}} = (2.90\ \mu\mathrm{C})(1.46 \times 10^5\ \mathrm{N/C}) = 0.423\ \mathrm{N}$, the same as was found in Example 19–3.

PRACTICE PROBLEM

Find the magnitude and direction of the net electric field at the bottom right corner of the square. [**Answer:** $E_{\mathrm{net}} = 1.46 \times 10^5\ \mathrm{N/C}$, $\theta = -14.6^\circ$]

Some related homework problems: Problem 50, Problem 51

REAL-WORLD PHYSICS: BIO

Electric fish

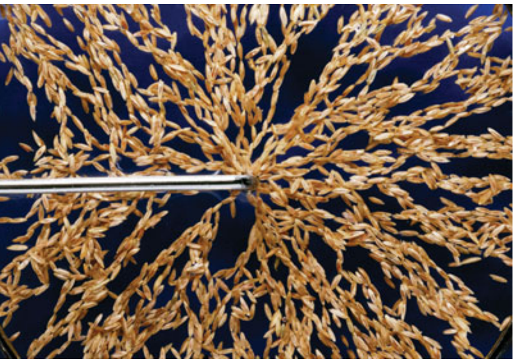
Many aquatic creatures are capable of producing electric fields. For example, African freshwater fishes in the family Mormyridae can generate weak electric fields from modified tail muscles and are able to detect variations in this field as they move through their environment. With this capability, these nocturnal feeders have an electrical guidance system that assists them in locating obstacles, enemies, and food. Much stronger electric fields are produced by electric eels and electric skates. In particular, the electric eel *Electrophorus electricus* generates electric fields great enough to kill small animals and to stun larger animals, including humans.

REAL-WORLD PHYSICS: BIO
Electrical shark repellent

Sharks are well known for their sensitivity to weak electric fields in their surroundings. In fact, they possess specialized organs for this purpose, known as the ampullae of Lorenzini, which assist in the detection of prey. Recently, this sensitivity has been put to use as a method of repelling sharks in order to protect swimmers and divers. A device called the SharkPOD (Protective Oceanic Device) consists of two metal electrodes, one attached to a diver's air tank, the other to one of the diver's fins. These electrodes produce a strong electric field that completely surrounds the diver and causes sharks to turn away out to a distance of up to 7 m.

The SharkPOD was used in the 2000 Summer Olympic Games in Sydney to protect swimmers competing in the triathlon. The swimming part of the event was held in Sydney harbor, where great white sharks are a common sight. To protect the swimmers, divers wearing the SharkPOD swam along the course, a couple meters below the athletes. The race was completed without incident.

19–5 Electric Field Lines



▲ **FIGURE 19–13 Grass seeds in an electric field**
Grass seeds aligning with electric field lines.

When looking at plots like those in Figures 19–09 and 19–11, it is tempting to imagine a pictorial representation of the electric field. This thought is reinforced when one considers a photograph like **Figure 19–13**, which shows grass seeds suspended in oil. Because of polarization effects, the grass seeds tend to align in the direction of the electric field, much like the elongated atoms shown in Figure 19–5. In this case, the seeds are aligned radially, due to the electric field of the charged rod seen "end on" in the middle of the photograph. Clearly, a set of radial lines would seem to represent the electric field in this case.

In fact, an entirely consistent method of drawing electric field lines is obtained by using the following set of rules:

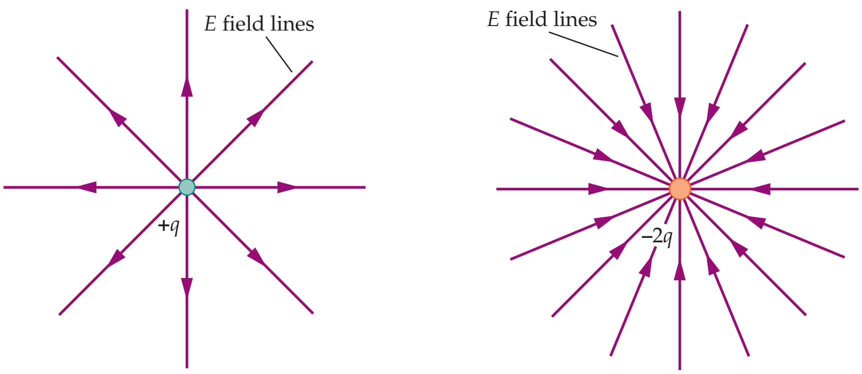
Rules for Drawing Electric Field Lines
Electric field lines:

1. Point in the *direction* of the electric field vector $\vec{\mathbf{E}}$ at every point;
2. *Start* at positive (+) charges or at infinity;
3. *End* at negative (−) charges or at infinity;
4. Are more *dense* where $\vec{\mathbf{E}}$ has a greater magnitude. In particular, the number of lines entering or leaving a charge is proportional to the magnitude of the charge.

We now show how these rules are applied.

For example, the electric field lines for two different point charges are presented in **Figure 19–14**. First, we know that the electric field points directly away from the charge ($+q$) in Figure 19–14 (a); hence from rule 1 the field lines are radial. In agreement with rule 2 the field lines start on a + charge, and in agreement

▶ **FIGURE 19–14 Electric field lines for a point charge**
(a) Near a positive charge the field lines point radially away from the charge. The lines start on the positive charge and end at infinity. **(b)** Near a negative charge the field lines point radially inward. They start at infinity and end on a negative charge and are more dense where the field is more intense. Notice that the number of lines drawn for part (b) is twice the number drawn for part (a), a reflection of the relative magnitudes of the charges.



(a) *E* field lines point away from positive charges **(b)** *E* field lines point toward negative charges with rule 3 they end at infinity. Finally, as anticipated from rule 4, the field lines are closer together near the charge, where the field is more intense. Similar considerations apply to Figure 19–14 (b), where the charge is $-2q$. In this case, however, the direction of the field lines is reversed and the number of lines is doubled.

CONCEPTUAL CHECKPOINT 19–5 INTERSECT OR NOT?

Which of the following statements is correct: Electric field lines **(a)** can or **(b)** cannot intersect?

REASONING AND DISCUSSION

By definition, electric field lines are always tangent to the electric field. Since the electric force, and hence the electric field, can point in only one direction at any given location, it follows that field lines cannot intersect. If they did, the field at the intersection point would have two conflicting directions.

ANSWER

(b) Electric field lines cannot intersect.

Figure 19–15 shows examples of electric field lines for various combinations of charges. In systems like these, we draw a set of curved field lines that are tangent to the electric field vector, $\vec{\mathbf{E}}$, at every point. This is illustrated for a variety of points in Figure 19–15 (a), and similar considerations apply to all such field diagrams. In addition, note that the magnitude of $\vec{\mathbf{E}}$ is greater in those regions of Figure 19–15 where the field lines are more closely packed together. Clearly, then, we expect an intense electric field between the charges in Figure 19–15 (b) and a vanishing field between the charges in Figure 19–15 (c).

Of particular interest is the $+q$ and $-q$ charge combination in Figure 19–15 (a). In general, a system of equal and opposite charges separated by a nonzero distance is known as an **electric dipole.** The total charge of the dipole is zero, but because the charges are separated, the electric field does not vanish. Instead, the field lines form "loops" that are characteristic of dipoles.

Many molecules are polar—water is a common example—which means they have an excess of positive charge near one end and a corresponding excess of

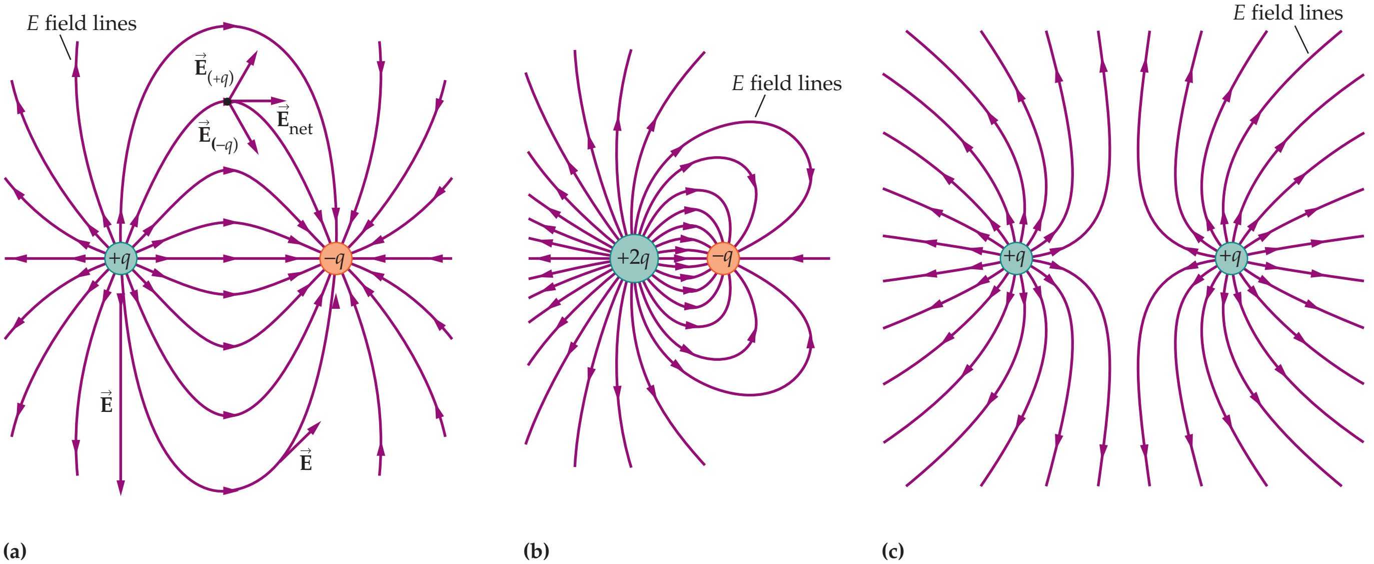


FIGURE 19–15 Electric field lines for systems of charges

(a) The electric field lines for a dipole form closed loops that become more widely spaced with distance from the charges. Note that at each point in space, the electric field vector $\vec{\mathbf{E}}$ is tangent to the field lines. **(b)** In a system with a net charge, some field lines extend to infinity. If the charges have opposite signs, some field lines start on one charge and terminate on the other charge. **(c)** All of the field lines in a system with charges of the same sign extend to infinity.

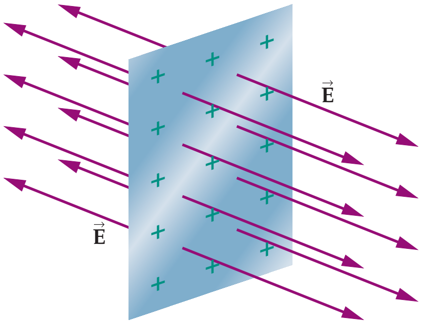


FIGURE 19–16 The electric field of a charged plate
The electric field near a large charged plate is uniform in direction and magnitude.

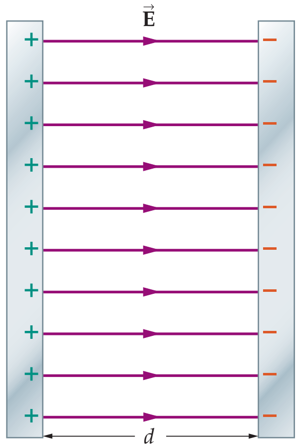


FIGURE 19–17 A parallel-plate capacitor
In the ideal case, the electric field is uniform between the plates and zero outside.

negative charge near the other end. As a result, they produce an electric dipole field. Similarly, a typical bar magnet produces a *magnetic* dipole field, as we shall see in Chapter 22.

Finally, the electric field representations in Figures 19–14 and 19–15 are two-dimensional “slices” through the full field, which is three-dimensional. Therefore, one should imagine a similar set of field lines in these figures coming out of the page and going into the page.

Parallel-Plate Capacitor

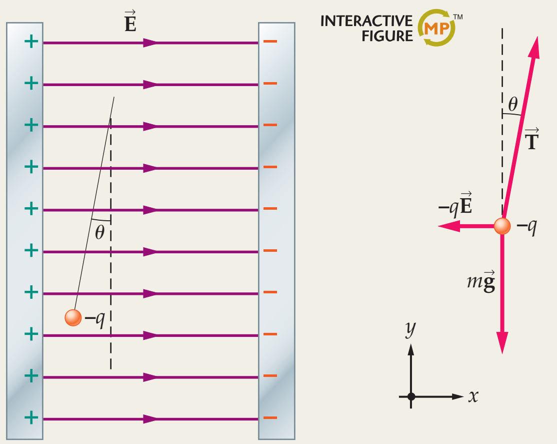
A particularly simple and important field picture results when charge is spread uniformly over a large plate, as illustrated in **Figure 19–16**. At points that are not near the edge of the plate, the electric field is uniform in both direction and magnitude. That is, the field points in a single direction—perpendicular to the plate—and its magnitude is independent of the distance from the plate. This result can be proved using Gauss’s law, as we show in Section 19–7.

If two such conducting plates with opposite charge are placed parallel to one another and separated by a distance d, as in **Figure 19–17**, the result is referred to as a **parallel-plate capacitor.** The field for such a system is uniform between the plates, and zero outside the plates. This is the ideal case, which is exactly true for an infinite plate and a good approximation for real plates. The field lines are illustrated in Figure 19–17. Parallel-plate capacitors are discussed further in the next chapter and will be of particular interest in Chapters 21 and 24, when we consider electric circuits.

EXAMPLE 19–6 DANGLING BY A THREAD

The electric field between the plates of a parallel-plate capacitor is horizontal, uniform, and has a magnitude E. A small object of mass 0.0250 kg and charge $-3.10\ \mu\text{C}$ is suspended by a thread between the plates, as shown in the sketch. The thread makes an angle of 10.5° with the vertical. Find **(a)** the tension in the thread and **(b)** the magnitude of the electric field.

PICTURE THE PROBLEM
Our sketch shows the thread making an angle $\theta = 10.5°$ with the vertical. The inset to the right shows the free-body diagram for the suspended object, as well as our choice of positive x and y directions. Note that we label the charge of the object $-q$, where $q = 3.10\ \mu\text{C}$, in order to clearly indicate its sign.



STRATEGY
The relevant physical principle in this problem is that because the object is at rest, the net force acting on it must vanish. Thus, setting the x and y components of the net force equal to zero yields two conditions, which can be used to solve for the two unknowns, T and E.

SOLUTION

1. Set the net force in the x direction equal to zero: $-qE + T\sin\theta = 0$
2. Set the net force in the y direction equal to zero: $T\cos\theta - mg = 0$

Part (a)

3. Because we know all the quantities in the y force equation except for the tension, we use it to solve for T:

$$T = \frac{mg}{\cos\theta} = \frac{(0.0250\ \text{kg})(9.81\ \text{m/s}^2)}{\cos(10.5°)} = 0.249\ \text{N}$$

Part (b)

4. Now use the x force equation to find the magnitude of the electric field, E:

$$E = \frac{T \sin \theta}{q} = \frac{(0.249\ \text{N}) \sin(10.5°)}{3.10 \times 10^{-6}\ \text{C}} = 1.46 \times 10^4\ \text{N/C}$$

INSIGHT

As expected, the negatively charged object is attracted to the positively charged plate. This means that the electric force exerted on it is opposite in direction to the electric field.

PRACTICE PROBLEM

Suppose the electric field between the plates is 2.50×10^4 N/C, but the charge of the object and the angle of the thread with the vertical are the same as before. Find the tension in the thread and the mass of the object. [**Answer:** $T = 0.425$ N, $m = 0.0426$ kg]

Some related homework problems: Problem 60, Problem 94

The fact that a charge experiences a force when it passes between two charged plates finds application in a wide variety of devices. For example, the image you see on many television screens is produced when a beam of electrons strikes the screen from behind and illuminates individual red, blue, or green *pixels*. Which pixels are illuminated and which remain dark is controlled by parallel charged plates that deflect the electron beam up or down and left or right. Thus, sending the appropriate electrical signals to the deflection plates makes the beam of electrons "paint" any desired picture on the screen.

REAL-WORLD PHYSICS

Television screens and ink-jet printers

Similar deflection plates are used in an ink-jet printer. In this case, the beam in question is not a beam of electrons but rather a beam of electrically charged ink droplets. The beam of droplets can be deflected as desired, so that individual letters can be constructed from a series of closely spaced dots on the page. A typical printer might produce as many as 600 dots per inch—that is, 600 dpi.

19–6 Shielding and Charging by Induction

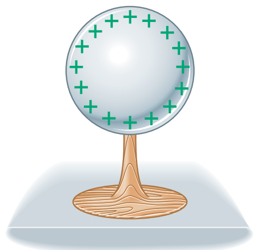
In a perfect conductor there are enormous numbers of electrons completely free to move about within the conductor. This simple fact has some rather interesting consequences. Consider, for example, a solid metal sphere attached to an insulating base as in **Figure 19–18**. Suppose a positive charge Q is placed on the sphere. The question is: How does this charge distribute itself on the sphere when it is in equilibrium—that is, when all the charges are at rest? In particular, does the charge spread itself uniformly throughout the volume of the sphere, or does it concentrate on the surface?

The answer is that the charge concentrates on the surface, as shown in Figure 19–18 (a), but let's investigate why this should be the case. First, assume the opposite—that the charge is spread uniformly throughout the sphere's volume, as indicated in Figure 19–18 (b). If this were the case, a charge at location A would experience an outward force due to the spherical distribution of charge between it and the center of the sphere. Since charges are free to move, the charge at A would respond to this force by moving toward the surface. Clearly, then, a uniform distribution of charge within the sphere's volume is not in equilibrium. In fact, the argument that a charge at point A will move toward the surface can be applied to any charge within the sphere. Thus, the net result is that *all* the excess charge Q moves onto the surface of the sphere which, in turn, allows the individual charges to be spread as far from one another as possible.

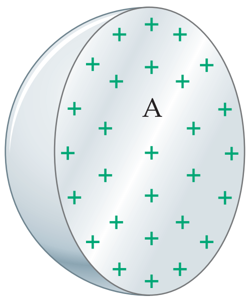
The preceding result holds no matter what the shape of the conductor. In general,

Excess Charge on a Conductor

Excess charge placed on a conductor, whether positive or negative, moves to the exterior surface of the conductor.



(a)



(b)

FIGURE 19–18 Charge distribution on a conducting sphere

(a) A charge placed on a conducting sphere distributes itself uniformly on the surface of the sphere; none of the charge is within the volume of the sphere. **(b)** If the charge were distributed uniformly throughout the volume of a sphere, individual charges, like that at point A, would experience a force due to other charges in the volume. Since charges are free to move in a conductor, they will respond to these forces by moving as far from one another as possible—that is, to the surface of the conductor.

We specify the exterior surface in this statement because a conductor may contain one or more cavities. When an excess charge is applied to such a conductor, all the charge ends up on the exterior surface, and none on the interior surfaces.

Electrostatic Shielding

The ability of electrons to move freely within a conductor has another important consequence; namely, the electric field within a conductor vanishes.

Zero Field within a Conductor

When electric charges are at rest, the electric field within a conductor is zero; $E = 0$.

By *within* a conductor, we mean a location in the actual material of the conductor, as opposed to a location in a cavity within the material.

The best way to see the validity of this statement is to again consider the opposite. If there were a nonzero field within a conductor, electrons would move in response to the field. They would continue to move until finally the field was reduced to zero, at which point the system would be in equilibrium and no more charges would move. Thus, equilibrium and $E = 0$ within a conductor go hand in hand.

REAL-WORLD PHYSICS

Electrical shielding

A straightforward extension of this idea explains the phenomenon of **shielding,** in which a conductor "shields" its interior from external electric fields. For example, in **Figure 19–19 (a)** we show an uncharged, conducting metal sphere placed in an electric field. Because the positive ions in the metal do not move, the field tends to move negative charges to the left and leave excess positive charges on the right; hence, it causes the sphere to have an **induced** negative charge on its left half and an induced positive charge on its right half. The total charge on the sphere, of course, is still zero. Since field lines end on (−) charges and begin on (+) charges, the external electric field ends on the left half of the sphere and starts up again on the right half. In between, within the conductor, the field is zero, as expected. Thus, the conductor has shielded its interior from the applied electric field.

Shielding occurs whether the conductor is solid, as in Figure 19–19, or hollow. In fact, even a thin sheet of metal foil formed into the shape of a box will shield its interior from external electric fields. This effect is put to use in numerous electrical devices, which often have a metal foil or wire mesh enclosure surrounding the sensitive electrical circuits. In this way, a given device can be isolated from the effects of other nearby devices that might otherwise interfere with its operation.

Notice also in Figure 19–19 that the field lines bend slightly near the surface of the sphere. In fact, on closer examination, as in **Figure 19–19 (b)**, we see that the field lines always contact the surface at right angles. This is true for any conductor:

Electric Fields at Conductor Surfaces

Electric field lines contact conductor surfaces at right angles.

If an electric field contacted a conducting surface at an angle other than 90°, the result would be a component of force parallel to the surface. This would result in a movement of electrons and, hence, would not correspond to equilibrium. Instead, electrons would move until the parallel component of the electric field was canceled.

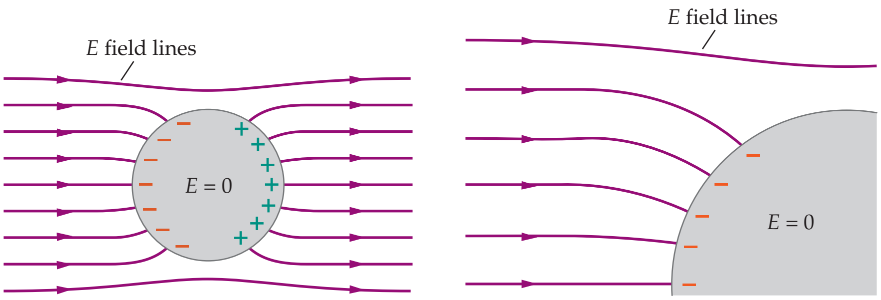


FIGURE 19–19 Electric field near a conducting surface

(a) When an uncharged conductor is placed in an electric field, the field induces opposite charges on opposite sides of the conductor. The net charge on the conductor is still zero, however. The induced charges produce a field within the conductor that *exactly* cancels the external field, leading to $E = 0$ inside the conductor. This is an example of electrical shielding. **(b)** Electric field lines meet the surface of a conductor at right angles.

(a) The electric field E vanishes inside a conductor

(b) E field lines meet a conducting surface at right angles

Further examples of field lines near conducting surfaces are shown in **Figure 19–20**. Notice that the field lines are more densely packed near a sharp point, indicating that the field is more intense in such regions. This effect illustrates the basic principle behind the operation of lightning rods. If you look closely, you will notice that all lightning rods have a sharply pointed tip. During an electrical storm, the electric field at the tip becomes so intense that electric charge is given off into the atmosphere. In this way, a lightning rod acts to discharge the area near a house—by giving off a steady stream of charge—thus preventing a strike by a bolt of lightning, which transfers charge in one sudden blast. Sharp points on the rigging of a ship at sea can also give off streams of charge during a storm, often producing glowing lights referred to as Saint Elmo's fire.

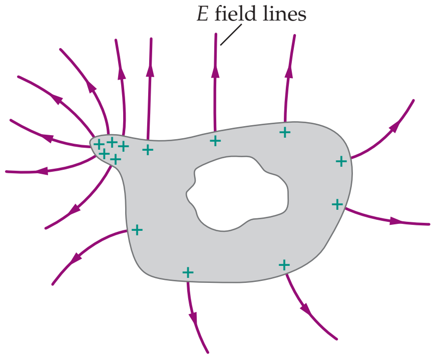
REAL-WORLD PHYSICS

Lightning rods and Saint Elmo's fire

The same principle is used to clean the air we breathe, in devices known as *electrostatic precipitators*. In an electrostatic precipitator, smoke and other airborne particles in a smokestack are given a charge as they pass by sharply pointed electrodes—like lightning rods—within the stack. Once the particles are charged, they are removed from the air by charged plates that exert electrostatic forces on them. The resultant emission from the smokestack contains drastically reduced amounts of potentially harmful particulates.

REAL-WORLD PHYSICS

Electrostatic precipitation

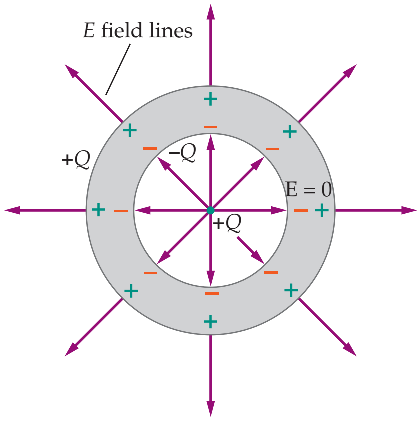


▲ **FIGURE 19–20 Intense electric field near a sharp point**

Electric charges and field lines are more densely packed near a sharp point. This means that the electric field is more intense in such regions as well. (Note that there are no electric charges on the interior surface surrounding the cavity.)

▲ In a dramatic science-museum demonstration of electrical shielding (left), the metal bars of a cage provide excellent protection from an artificially generated lightning bolt. A more practical safeguard is the lightning rod (right). Lightning rods always have sharp points, because that is where the electric field of a conductor is most intense. At the tip, the field can become so strong that charge leaks away into the atmosphere rather than building up to levels that will attract a lightning strike. If a strike does occur, it is conducted to the ground through the lightning rod, rather than through some part of the building itself.

One final note regarding shielding is that it works in one direction only: A conductor shields its interior from external fields, but it does not shield the external world from fields within it. This phenomenon is illustrated in **Figure 19–21** for the case of an uncharged conductor. First, the charge $+Q$ in the cavity induces a charge $-Q$ on the interior surface, in order for the field in the conductor to be zero. Since the conductor is uncharged, a charge $+Q$ will be induced on its exterior surface. As a result, the external world will experience a field due, ultimately, to the charge $+Q$ within the cavity of the conductor.

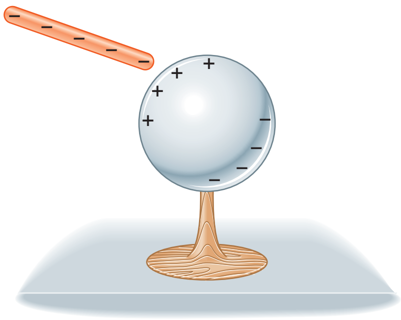


▲ **FIGURE 19–21 Shielding works in only one direction**

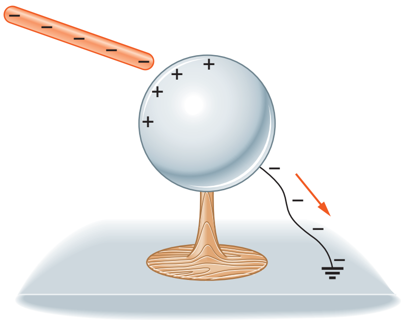
A conductor does not shield the external world from charges it encloses. Still, the electric field is zero within the conductor itself.

Charging by Induction

One way to charge an object is to touch it with a charged rod; but since electric forces can act at a distance, it is also possible to charge an object without making direct physical contact. This type of charging is referred to as **charging by induction.**



(a)



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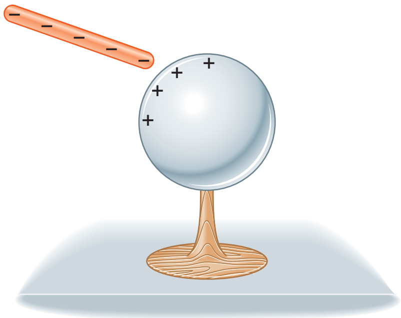
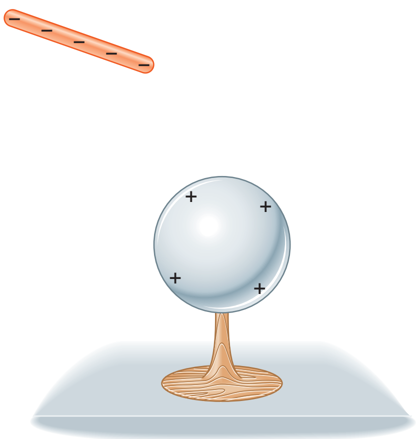


(c)



(d)

FIGURE 19–22 Charging by induction

(a) A charged rod induces + and − charges on opposite sides of the conductor. **(b)** When the conductor is grounded, charges that are repelled by the rod enter the ground. There is now a net charge on the conductor. **(c)** Removing the grounding wire, with the charged rod still in place, traps the net charge on the conductor. **(d)** The charged rod can now be removed, and the conductor retains a charge that is opposite in sign to that on the charged rod.

To see how this type of charging works, consider an uncharged metal sphere on an insulating base. If a negatively charged rod is brought close to the sphere without touching it, as in **Figure 19–22 (a)**, electrons in the sphere are repelled. An induced positive charge is produced on the near side of the sphere, and an induced negative charge on the far side. At this point the sphere is still electrically neutral, however.

The key step in this charging process, shown in **Figure 19–22 (b)**, is to connect the sphere to the ground using a conducting wire. As one might expect, this is referred to as **grounding** the sphere, and is indicated by the symbol ⏚. (A table of electrical symbols can be found in Appendix D.) Since the ground is a fairly good conductor of electricity, and since the Earth can receive or give up practically unlimited numbers of electrons, the effect of grounding the sphere is that the electrons repelled by the charged rod enter the ground. Now the sphere has a net positive charge. With the rod kept in place, the grounding wire is removed, as in **Figure 19–22 (c)**, trapping the net positive charge on the sphere. The rod can now be pulled away, as in **Figure 19–22 (d)**.

Notice that the *induced* charge on the sphere is opposite in sign to the charge on the rod. In contrast, when the sphere is charged by *touch* as in Figure 19–6, it acquires a charge with the same sign as the charge on the rod.

19–7 Electric Flux and Gauss's Law

In this section, we introduce the idea of an electric flux and show that it can be used to calculate the electric field. The precise connection between electric flux and the charges that produce the electric field is provided by Gauss's law.

Electric Flux

Consider a uniform electric field $\vec{\mathbf{E}}$, as in **Figure 19–23 (a)**, passing through an area A that is perpendicular to the field. Looking at the electric field lines with their arrows, we can easily imagine a "flow" of electric field through the area. Though there is no actual flow, of course, the analogy is a useful one. It is with this in mind that we define an **electric flux,** Φ, for this case as follows:

$$\Phi = EA$$

On the other hand, if the area A is parallel to the field lines, as in **Figure 19–23 (b)**, none of the $\vec{\mathbf{E}}$ lines pierce the area, and hence there is no flux of electric field:

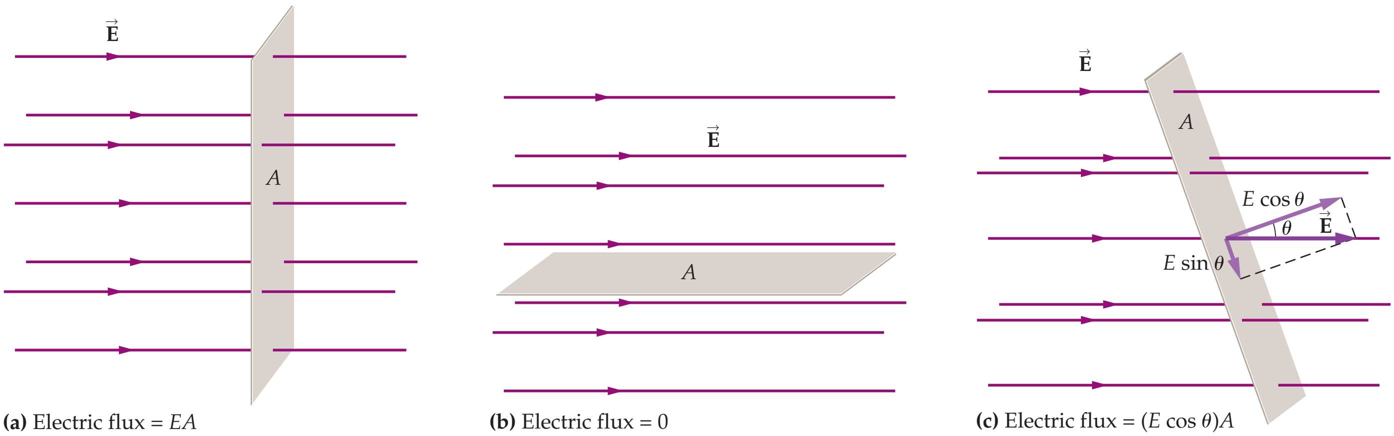
$$\Phi = 0$$

In an intermediate case, as shown in **Figure 19–23 (c)**, the $\vec{\mathbf{E}}$ lines pierce the area A at an angle θ away from the perpendicular. As a result, the component of $\vec{\mathbf{E}}$ perpendicular to the surface is $E\cos\theta$, and the component parallel to the surface is $E\sin\theta$. Since only the perpendicular component of $\vec{\mathbf{E}}$ causes a flux (the parallel component does not pierce the area), the flux in the general case is the following:

Definition of Electric Flux, Φ

$$\Phi = EA\cos\theta \qquad 19\text{–}11$$

SI unit: $N \cdot m^2/C$



(a) Electric flux = EA **(b)** Electric flux = 0 **(c)** Electric flux = $(E \cos\theta)A$

▲ **FIGURE 19–23 Electric flux**

(a) When an electric field $\vec{E}$ passes perpendicularly through the plane of an area A, the electric flux is $\Phi = EA$. **(b)** When the plane of an area is parallel to $\vec{E}$, so that no field lines "pierce" the area, the electric flux is zero, $\Phi = 0$. **(c)** When the normal to the plane of an area is tilted at an angle θ away from the electric field $\vec{E}$, only the perpendicular component of $\vec{E}$, $E \cos\theta$, contributes to the electric flux. Thus, the flux is $\Phi = (E \cos\theta)A$.

Finally, if the surface through which the flux is calculated is *closed*, the sign of the flux is as follows:

- The flux is *positive* for field lines that *leave* the enclosed volume of the surface.
- The flux is *negative* for field lines that *enter* the enclosed volume of the surface.

Gauss's Law

As a simple example of electric flux, consider a positive point charge q and a spherical surface of radius r centered on the charge, as in **Figure 19–24**. The electric field on the surface of the sphere has the constant magnitude

$$E = k\frac{q}{r^2}$$

Since the electric field is everywhere perpendicular to the spherical surface, it follows that the electric flux is simply E times the area $A = 4\pi r^2$ of the sphere:

$$\Phi = EA = \left(k\frac{q}{r^2}\right)(4\pi r^2) = 4\pi kq$$

We will often find it convenient to express k in terms of another constant, ε_0, as follows: $k = 1/(4\pi\varepsilon_0)$. This new constant, which we call the **permittivity of free space**, is

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \qquad \text{19–12}$$

In terms of ε_0, the flux through the spherical surface reduces to

$$\Phi = 4\pi kq = \frac{q}{\varepsilon_0}$$

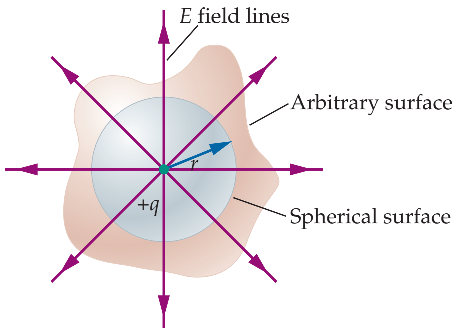
Thus, we find the very simple result that the electric flux through a sphere that encloses a charge q is the charge divided by the permittivity of free space, ε_0. This is a nice result, but what makes it truly remarkable is that it is equally true for *any* surface that encloses the charge q. For example, if one were to calculate the electric flux through the closed irregular surface also shown in Figure 19–24—which would be a difficult task—the result, nonetheless, would still be simply q/ε_0. This, in fact, is a special case of Gauss's law:

Gauss's Law

If a charge q is enclosed by an arbitrary surface, the total electric flux through the surface, Φ, is

$$\Phi = \frac{q}{\varepsilon_0} \qquad \text{19–13}$$

SI unit: $\mathrm{N \cdot m^2/C}$



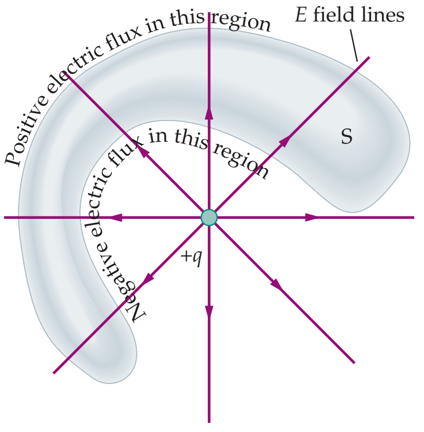
▲ **FIGURE 19–24 Electric flux for a point charge**

The electric flux through the spherical surface surrounding a positive point charge q is $\Phi = EA = (kq/r^2)(4\pi r^2) = q/\varepsilon_0$. The electric flux through an arbitrary surface is the same as for the sphere. The calculation of the flux, however, would be much more difficult for this surface.

Note that we use q rather than $|q|$ in Equation 19–13. This is because the electric flux can be positive or negative, depending on the sign of the charge. In particular, if the charge q is positive, the field lines leave the enclosed volume and the flux is positive; if the charge is negative, the field lines enter the enclosed volume and the flux is negative.

CONCEPTUAL CHECKPOINT 19–6 SIGN OF THE ELECTRIC FLUX

Consider the surface S shown in the sketch. Is the electric flux through this surface **(a)** negative, **(b)** positive, or **(c)** zero?



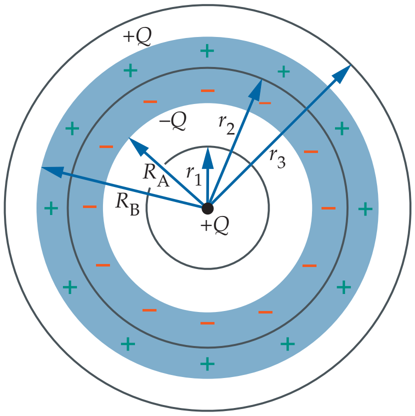
REASONING AND DISCUSSION

Because the surface S encloses no charge, the net electric flux through it must be zero, by Gauss's law. The fact that a charge $+q$ is nearby is irrelevant, because it is outside the volume enclosed by the surface.

We can explain why the flux vanishes in another way. Notice that the flux on portions of S near the charge is negative, since field lines enter the enclosed volume there. On the other hand, the flux is positive on the outer portions of S where field lines exit the volume. The combination of these positive and negative contributions is a net flux of zero.

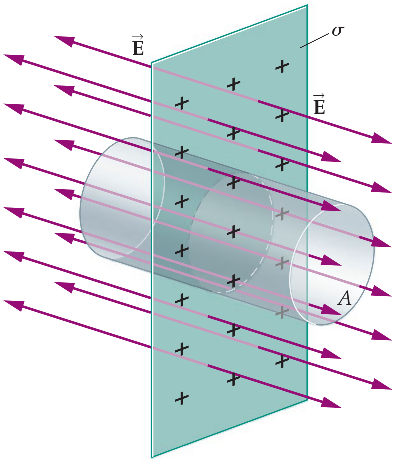
ANSWER

(c) The electric flux through the surface S is zero.



▲ **FIGURE 19–25 Gauss's law applied to a spherical shell**

A simple system with three different Gaussian surfaces.



▲ **FIGURE 19–26 Gauss's law applied to a sheet of charge**

A charged sheet of infinite extent and the Gaussian surface used to calculate the electric field.

Although Gauss's law holds for an arbitrarily complex surface, its real utility is seen when the system in question has a simple symmetry. For example, consider a point charge $+Q$ in the center of a hollow, conducting spherical shell, as illustrated in **Figure 19–25**. The shell has an inside radius R_A and an outside radius R_B, and is uncharged. To calculate the field inside the cavity, where $r < R_A$, we consider an imaginary spherical surface—a so-called **Gaussian surface**—with radius r_1, and centered on the charge $+Q$, as indicated in Figure 19–25. The electric flux through this Gaussian surface is $\Phi = E(4\pi r_1^{\,2}) = Q/\varepsilon_0$. Therefore, the magnitude of the electric field, as expected, is

$$E = \frac{Q}{4\pi\varepsilon_0 r_1^{\,2}} = k\frac{Q}{r_1^{\,2}}$$

Note, in particular, that the charges on the spherical shell do not affect the electric flux through this Gaussian surface, since they are not contained within the surface.

Next, consider a Gaussian surface within the shell, with $R_A < r_2 < R_B$, as in Figure 19–25. Since the field within a conductor is zero, $E = 0$, it follows that the electric flux for this surface is zero: $\Phi = EA = 0$. This means that the net charge within the Gaussian surface is also zero; that is, the induced charge on the inner surface of the shell is $-Q$.

Finally, consider a spherical Gaussian surface that encloses the entire spherical shell, with a radius $r_3 > R_B$. In this case, also shown in Figure 19–25, the flux is $\Phi = E(4\pi r_3^{\,2}) = (\text{enclosed charge})/\varepsilon_0$. What is the enclosed charge? Well, we know that the spherical shell is uncharged—the induced charges of $+Q$ and $-Q$ on its outer and inner surfaces sum to zero—hence the total enclosed charge is simply $+Q$ from the charge at the center of the shell. Therefore, $\Phi = E(4\pi r_3^{\,2}) = Q/\varepsilon_0$, and

$$E = \frac{Q}{4\pi\varepsilon_0 r_3^{\,2}} = k\frac{Q}{r_3^{\,2}}$$

Note that the field outside the shell is the same as if the shell were not present, showing that the conducting shell does not shield the external world from charges within it, in agreement with our conclusions in the previous section.

Gaussian surfaces do not need to be spherical, however. Consider, for example, a thin sheet of charge that extends to infinity, as in **Figure 19–26**. We expect the field to be at right angles to the sheet because, by symmetry, there is no reason for it to tilt in one direction more than in any other direction. Hence, we choose our Gaussian surface to be a cylinder, as in Figure 19–26. With this choice, no field lines pierce the curved surface of the cylinder. The electric flux through this Gaussian surface, then, is due solely to the contributions of the two end caps, each of area A. Hence, the flux is $\Phi = E(2A)$. If the charge per area on the sheet is σ, the enclosed charge is σA, and hence Gauss's law states that

$$\Phi = E(2A) = \frac{(\sigma A)}{\varepsilon_0}$$

Canceling the area, we find

$$E = \frac{\sigma}{2\varepsilon_0}$$

Note that E does not depend in any way on the distance from the sheet, as was mentioned in Section 19–5.

We conclude this chapter with an additional example of Gauss's law in action.

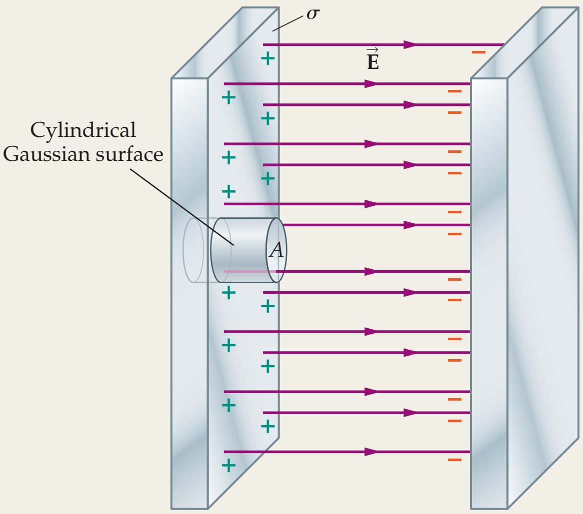
PROBLEM-SOLVING NOTE

Applying Gauss's Law

Gauss's law is useful only when the electric field is constant on a given surface. It is only in such cases that the electric flux can be calculated with ease.

ACTIVE EXAMPLE 19–3 FIND THE ELECTRIC FIELD

Use the cylindrical Gaussian surface shown in the diagram to calculate the electric field between the metal plates of a parallel-plate capacitor. Each plate has a charge per area of magnitude σ.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Calculate the electric flux through the curved surface of the cylinder: 0
2. Calculate the electric flux through the end caps of the cylinder: $0 + EA$
3. Determine the charge enclosed by the cylinder: σA
4. Apply Gauss's law to find the field, E: $E = \sigma/\varepsilon_0$

INSIGHT

Note that the electric field is zero within the metal of the plates (because they are conductors). It is for this reason that the electric flux through the left end cap of the Gaussian surface is zero.

YOUR TURN

Suppose we extend the Gaussian surface so that its right end cap is within the metal of the right plate. The left end of the Gaussian surface remains in its original location. What is the electric flux through this new Gaussian surface? Explain.

(Answers to **Your Turn** *problems are given in the back of the book.)*

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

The concept of a conserved quantity, like energy (Chapter 8) or momentum (Chapter 9), appears again in Section 19–1, where we show that electric charge is also a conserved quantity.

Coulomb's law for the electrostatic force between two charges, Equation 19–5, is virtually identical to Newton's law of universal gravitation between two masses (Chapter 12), but with electric charge replacing mass.

The electric force, like all forces, is a vector quantity. Therefore, the material on vector addition (Chapter 3) again finds use in Sections 19–3, 19–4, and 19–5.

LOOKING AHEAD

The electric force is conservative, and hence it has an associated electric potential energy. We will determine this potential energy in Chapter 20. We will also point out the close analogy between the electric potential energy and the gravitational potential energy of Chapter 12.

When electric charge flows from one location to another, it produces an electric current. We consider electric circuits with direct current (dc) in Chapter 21, and circuits with alternating current (ac) in Chapter 24.

Coulomb's law comes up again in atomic physics, where it plays a key role in the Bohr model of the hydrogen atom in Section 31–3.

CHAPTER SUMMARY

19–1 ELECTRIC CHARGE

Electric charge is one of the fundamental properties of matter. Electrons have a negative charge, $-e$, and protons have a positive charge, $+e$. An object with zero net charge, like a neutron, is said to be electrically neutral.

Magnitude of an Electron's Charge
The charge on an electron has the following magnitude:

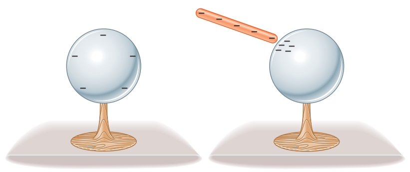
$$e = 1.60 \times 10^{-19}\ \mathrm{C} \qquad 19\text{–}1$$

The SI unit of charge is the coulomb, C.

Charge Conservation
The total charge in the universe is constant.

Charge Quantization
Charge comes in quantized amounts that are always integer multiples of e.

19–2 INSULATORS AND CONDUCTORS



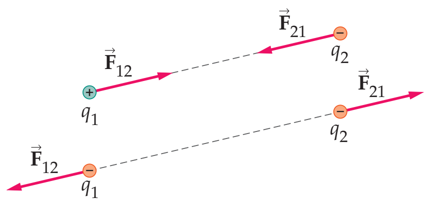
An insulator does not allow electrons within it to move from atom to atom. In conductors, each atom gives up one or more electrons that are then free to move throughout the material. Semiconductors have properties that are intermediate between those of insulators and conductors.

19–3 COULOMB'S LAW

Electric charges exert forces on one another along the line connecting them: Like charges repel, opposite charges attract.

Coulomb's Law
The magnitude of the force between two point charges, q_1 and q_2, separated by a distance r is

$$F = k\frac{|q_1||q_2|}{r^2} \qquad 19\text{–}5$$



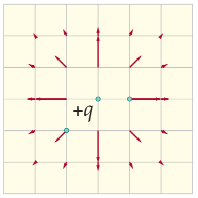
The constant k in this expression is

$$k = 8.99 \times 10^{9}\ \mathrm{N \cdot m^2/C^2} \qquad 19\text{–}6$$

Superposition
The electric force on one charge due to two or more other charges is the vector sum of each individual force.

Spherical Charge Distributions
A spherical distribution of charge, when viewed from outside, behaves the same as an equivalent point charge at the center of the sphere.

19–4 THE ELECTRIC FIELD



The electric field is the force per charge at a given location in space.

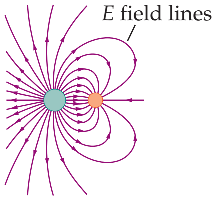
Direction of $\vec{\mathbf{E}}$
$\vec{\mathbf{E}}$ points in the direction of the force experienced by a *positive* test charge.

Point Charge
The electric field a distance r from a point charge q has a magnitude given by

$$E = k\frac{|q|}{r^2} \tag{19–10}$$

Superposition
The total electric field due to two or more charges is given by the vector sum of the fields due to each charge individually.

19–5 ELECTRIC FIELD LINES

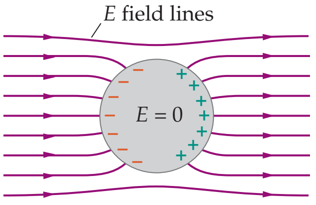


The electric field can be visualized by drawing lines according to a given set of rules.

Rules for Drawing Electric Field Lines
Electric field lines (1) point in the direction of the electric field vector $\vec{\mathbf{E}}$ at all points; (2) start at + charges or infinity; (3) end at − charges or infinity; and (4) are more dense the greater the magnitude of $\vec{\mathbf{E}}$.

Parallel-Plate Capacitor
A parallel-plate capacitor consists of two oppositely charged, conducting parallel plates separated by a finite distance. The field between the plates is uniform in direction (perpendicular to the plates) and magnitude.

19–6 SHIELDING AND CHARGING BY INDUCTION



Ideal conductors have a range of interesting behaviors that arise because they have an enormous number of electrons that are free to move.

Excess Charge
Any excess charge placed on a conductor moves to its exterior surface.

Zero Field in a Conductor
The electric field within a conductor in equilibrium is zero.

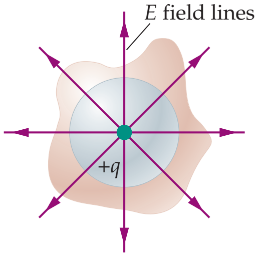
Shielding
A conductor shields a cavity within it from external electric fields.

Electric Fields at Conductor Surfaces
Electric field lines contact conductor surfaces at right angles.

Charging by Induction
A conductor can be charged without direct physical contact with another charged object. This is charging by induction.

Grounding
Connecting a conductor to the ground is referred to as grounding. The ground itself is a good conductor, and it can give up or receive an unlimited number of electrons.

19–7 ELECTRIC FLUX AND GAUSS'S LAW



Gauss's law relates the charge enclosed by a given surface to the electric flux through the surface.

Electric Flux
If an area A is tilted at an angle θ to an electric field $\vec{\mathbf{E}}$, the electric flux through the area is

$$\Phi = EA\cos\theta \tag{19–11}$$

Gauss's Law

Gauss's law states that if a charge q is enclosed by a surface, the electric flux through the surface is

$$\Phi = \frac{q}{\varepsilon_0} \qquad 19\text{–}13$$

The constant appearing in this equation is the permittivity of free space, ε_0:

$$\varepsilon_0 = \frac{1}{4\pi k} = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2} \qquad 19\text{–}12$$

Gauss's law is used to calculate the electric field in highly symmetric systems.

PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the electric force exerted by one or more point charges.	The magnitude of the electric force between point charges is $F = k\lvert q_1\rvert\lvert q_2\rvert/r^2$; the direction of the force is given by the expression "opposites attract, likes repel." When more than one charge exerts a force on a given charge, the net force is the vector sum of the individual forces.	Examples 19–1, 19–2, 19–3 Active Examples 19–1, 19–2
Find the electric force due to a spherical distribution of charge.	For points outside a spherical distribution of charge, the spherical distribution behaves the same as a point charge of the same amount at the center of the sphere.	Active Example 19–2
Calculate the force exerted by an electric field.	An electric field, $\vec{\mathbf{E}}$, produces a force, $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$, on a point charge q. The direction of the force is in the direction of the field if the charge is positive and opposite to the field if the charge is negative.	Example 19–4
Find the electric field due to one or more point charges.	The electric field due to a point charge q has a magnitude given by $E = k\lvert q\rvert/r^2$. The direction of the field is radially outward if q is positive, and radially inward if q is negative. When a group of point charges is being considered, the total electric field is the vector sum of the individual fields.	Example 19–5
Calculate the electric field using Gauss's law.	The electric field can be found by setting the electric flux through a given surface equal to the charge enclosed by the surface divided by ε_0.	Active Example 19–3

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. When an object that was neutral becomes charged, does the total charge of the universe change? Explain.
2. The fact that the electron has a negative charge and the proton has a positive charge is due to a convention established by Benjamin Franklin. Would there have been any significant consequences if Franklin had chosen the opposite convention? Is there any advantage to naming charges plus and minus as opposed to, say, A and B?
3. Explain why a comb that has been rubbed through your hair attracts small bits of paper, even though the paper is uncharged.
4. Small bits of paper are attracted to an electrically charged comb, but as soon as they touch the comb they are strongly repelled. Explain this behavior.
5. A charged rod is brought near a suspended object, which is repelled by the rod. Can we conclude that the suspended object is charged? Explain.
6. A charged rod is brought near a suspended object, which is attracted to the rod. Can we conclude that the suspended object is charged? Explain.
7. Describe some of the similarities and differences between Coulomb's law and Newton's law of gravity.
8. A point charge $+Q$ is fixed at a height H above the ground. Directly below this charge is a small ball with a charge $-q$ and a mass m. When the ball is at a height h above the ground, the net force (gravitational plus electrical) acting on it is zero. Is this a stable equilibrium for the object? Explain.
9. Four identical point charges are placed at the corners of a square. A fifth point charge placed at the center of the square experiences zero net force. Is this a stable equilibrium for the fifth charge? Explain.
10. A proton moves in a region of constant electric field. Does it follow that the proton's velocity is parallel to the electric field? Does it follow that the proton's acceleration is parallel to the electric field? Explain.
11. Describe some of the differences between charging by induction and charging by contact.
12. A system consists of two charges of equal magnitude and opposite sign separated by a distance d. Since the total electric

charge of this system is zero, can we conclude that the electric field produced by the system is also zero? Does your answer depend on the separation d? Explain.

13. The force experienced by charge 1 at point A is different in direction and magnitude from the force experienced by charge 2 at point B. Can we conclude that the electric fields at points A and B are different? Explain.

14. Can an electric field exist in a vacuum? Explain.

15. Explain why electric field lines never cross.

16. Charge q_1 is inside a closed Gaussian surface; charge q_2 is just outside the surface. Does the electric flux through the surface depend on q_1? Does it depend on q_2? Explain.

17. In the previous question, does the electric field at a point on the Gaussian surface depend on q_1? Does it depend on q_2? Explain.

18. Gauss's law can tell us how much charge is contained within a Gaussian surface. Can it tell us where inside the surface it is located? Explain.

19. Explain why Gauss's law is not very useful in calculating the electric field of a charged disk.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 19–1 ELECTRIC CHARGE

1. • **CE Predict/Explain** An electrically neutral object is given a positive charge. **(a)** In principle, does the object's mass increase, decrease, or stay the same as a result of being charged? **(b)** Choose the *best explanation* from among the following:
   I. To give the object a positive charge we must remove some of its electrons; this will reduce its mass.
   II. Since electric charges have mass, giving the object a positive charge will increase its mass.
   III. Charge is conserved, and therefore the mass of the object will remain the same.

2. • **CE Predict/Explain** An electrically neutral object is given a negative charge. **(a)** In principle, does the object's mass increase, decrease, or stay the same as a result of being charged? **(b)** Choose the *best explanation* from among the following:
   I. To give the object a negative charge we must give it more electrons, and this will increase its mass.
   II. A positive charge increases an object's mass; a negative charge decreases its mass.
   III. Charge is conserved, and therefore the mass of the object will remain the same.

3. • **CE (a)** Based on the materials listed in Table 19–1, is the charge of the rubber balloon shown on page 655 more likely to be positive or negative? Explain. **(b)** If the charge on the balloon is reversed, will the stream of water deflect toward or away from the balloon? Explain.

4. • **CE** This problem refers to the information given in Table 19–1. **(a)** If rabbit fur is rubbed against glass, what is the sign of the charge each acquires? Explain. **(b)** Repeat part (a) for the case of glass and rubber. **(c)** Comparing the situations described in parts (a) and (b), in which case is the magnitude of the triboelectric charge greater? Explain.

5. • Find the net charge of a system consisting of 4.9×10^7 electrons.

6. • Find the net charge of a system consisting of **(a)** 6.15×10^6 electrons and 7.44×10^6 protons or **(b)** 212 electrons and 165 protons.

7. • How much negative electric charge is contained in 2 moles of carbon?

8. • Find the total electric charge of 1.5 kg of **(a)** electrons and **(b)** protons.

9. • A container holds a gas consisting of 1.85 moles of oxygen molecules. One in a million of these molecules has lost a single electron. What is the net charge of the gas?

10. • **The Charge on Adhesive Tape** When adhesive tape is pulled from a dispenser, the detached tape acquires a positive charge and the remaining tape in the dispenser acquires a negative charge. If the tape pulled from the dispenser has 0.14 μC of charge per centimeter, what length of tape must be pulled to transfer 1.8×10^{13} electrons to the remaining tape?

11. •• **CE** Four pairs of conducting spheres, all with the same radius, are shown in **Figure 19–27**, along with the net charge placed on them initially. The spheres in each pair are now brought into contact, allowing charge to transfer between them. Rank the pairs of spheres in order of increasing magnitude of the charge transferred. Indicate ties where appropriate.

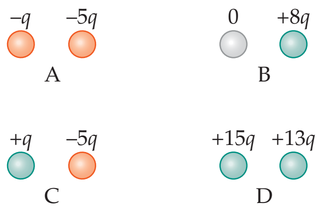


FIGURE 19–27 Problem 11

12. •• A system of 1525 particles, each of which is either an electron or a proton, has a net charge of -5.456×10^{-17} C. **(a)** How many electrons are in this system? **(b)** What is the mass of this system?

SECTION 19–3 COULOMB'S LAW

13. • **CE** A charge $+q$ and a charge $-q$ are placed at opposite corners of a square. Will a third point charge experience a greater force if it is placed at one of the empty corners of the square, or at the center of the square? Explain.

14. • **CE** Repeat the previous question, this time with charges $+q$ and $+q$ at opposite corners of a square.

15. • **CE** Consider the three electric charges, A, B, and C, shown in **Figure 19–28**. Rank the charges in order of increasing magnitude of the net force they experience. Indicate ties where appropriate.

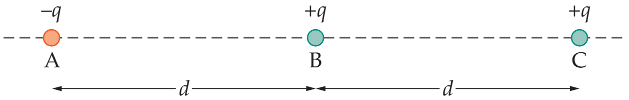


FIGURE 19–28 Problems 15 and 46

16. • **CE Predict/Explain** Suppose the charged sphere in Active Example 19–2 is made from a conductor, rather than an insulator. **(a)** Do you expect the magnitude of the force between the point charge and the conducting sphere to be greater than, less than, or equal to the force between the point charge and an insulating sphere? **(b)** Choose the *best explanation* from among the following:
   I. The conducting sphere will allow the charges to move, resulting in a greater force.
   II. The charge of the sphere is the same whether it is conducting or insulating, and therefore the force is the same.
   III. The charge on a conducting sphere will move as far away as possible from the point charge. This results in a reduced force.

17. • At what separation is the electrostatic force between a $+11.2\text{-}\mu\text{C}$ point charge and a $+29.1\text{-}\mu\text{C}$ point charge equal in magnitude to 1.57 N?

18. • The attractive electrostatic force between the point charges $+8.44 \times 10^{-6}$ C and Q has a magnitude of 0.975 N when the separation between the charges is 1.31 m. Find the sign and magnitude of the charge Q.

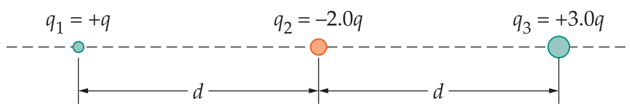
19. • If the speed of the electron in Example 19–1 were 7.3×10^5 m/s, what would be the corresponding orbital radius?

20. • **IP** Two point charges, the first with a charge of $+3.13 \times 10^{-6}$ C and the second with a charge of -4.47×10^{-6} C, are separated by 25.5 cm. **(a)** Find the magnitude of the electrostatic force experienced by the positive charge. **(b)** Is the magnitude of the force experienced by the negative charge greater than, less than, or the same as that experienced by the positive charge? Explain.

21. • When two identical ions are separated by a distance of 6.2×10^{-10} m, the electrostatic force each exerts on the other is 5.4×10^{-9} N. How many electrons are missing from each ion?

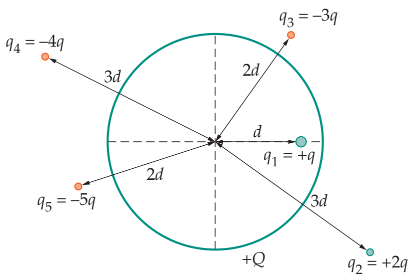
22. • A sphere of radius 4.22 cm and uniform surface charge density $+12.1\ \mu\text{C/m}^2$ exerts an electrostatic force of magnitude 46.9×10^{-3} N on a point charge of $+1.95\ \mu$C. Find the separation between the point charge and the center of the sphere.

23. • Given that $q = +12\ \mu$C and $d = 16$ cm, find the direction and magnitude of the net electrostatic force exerted on the point charge q_1 in **Figure 19–29**.



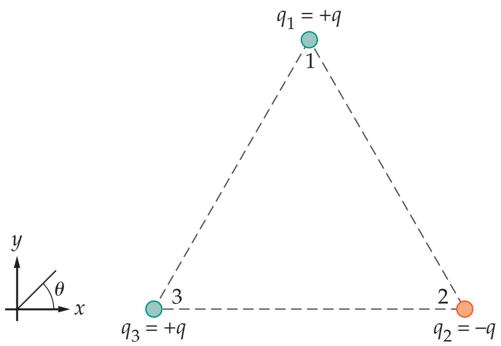
▲ **FIGURE 19–29** Problems 23, 26, 27, and 58

24. •• **CE** Five point charges, $q_1 = +q$, $q_2 = +2q$, $q_3 = -3q$, $q_4 = -4q$, and $q_5 = -5q$, are placed in the vicinity of an insulating spherical shell with a charge $+Q$, distributed uniformly over its surface, as indicated in **Figure 19–30**. Rank the point charges in order of increasing magnitude of the force exerted on them by the sphere. Indicate ties where appropriate.



▲ **FIGURE 19–30** Problem 24

25. •• **CE** Three charges, $q_1 = +q$, $q_2 = -q$, and $q_3 = +q$, are at the vertices of an equilateral triangle, as shown in **Figure 19–31**. **(a)** Rank the three charges in order of increasing magnitude of the electric force they experience. Indicate ties where appropriate. **(b)** Give the direction angle, θ, of the net electric force experienced by charge 1. Note that θ is measured counterclockwise from the positive x axis. **(c)** Repeat part (b) for charge 2. **(d)** Repeat part (b) for charge 3.



▲ **FIGURE 19–31** Problems 25 and 82

26. •• **IP** Given that $q = +12\ \mu$C and $d = 19$ cm, **(a)** find the direction and magnitude of the net electrostatic force exerted on the point charge q_2 in Figure 19–29. **(b)** How would your answers to part (a) change if the distance d were tripled?

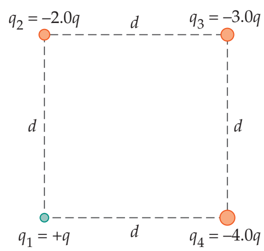
27. •• Suppose the charge q_2 in Figure 19–29 can be moved left or right along the line connecting the charges q_1 and q_3. Given that $q = +12\ \mu$C, find the distance from q_1 where q_2 experiences a net electrostatic force of zero. (The charges q_1 and q_3 are separated by a fixed distance of 32 cm.)

28. •• Find the orbital radius for which the kinetic energy of the electron in Example 19–1 is 1.51 eV. (*Note:* 1 eV = 1 electron volt = 1.6×10^{-19} J.)

29. •• A point charge $q = -0.35$ nC is fixed at the origin. Where must a proton be placed in order for the electric force acting on it to be exactly opposite to its weight? (Let the y axis be vertical and the x axis be horizontal.)

30. •• A point charge $q = -0.35$ nC is fixed at the origin. Where must an electron be placed in order for the electric force acting on it to be exactly opposite to its weight? (Let the y axis be vertical and the x axis be horizontal.)

31. •• Find the direction and magnitude of the net electrostatic force exerted on the point charge q_2 in **Figure 19–32**. Let $q = +2.4\ \mu$C and $d = 33$ cm.



▲ **FIGURE 19–32** Problems 31 and 32

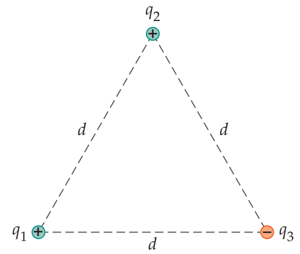
32. •• **IP** **(a)** Find the direction and magnitude of the net electrostatic force exerted on the point charge q_3 in Figure 19–32. Let

$q = +2.4\ \mu C$ and $d = 27$ cm. **(b)** How would your answers to part (a) change if the distance d were doubled?

33. •• **IP** Two point charges lie on the x axis. A charge of $+9.9\ \mu C$ is at the origin, and a charge of $-5.1\ \mu C$ is at $x = 10.0$ cm. **(a)** At what position x would a third charge q_3 be in equilibrium? **(b)** Does your answer to part (a) depend on whether q_3 is positive or negative? Explain.

34. •• A system consists of two positive point charges, q_1 and $q_2 > q_1$. The total charge of the system is $+62.0\ \mu C$, and each charge experiences an electrostatic force of magnitude 85.0 N when the separation between them is 0.270 m. Find q_1 and q_2.

35. •• **IP** The point charges in **Figure 19–33** have the following values: $q_1 = +2.1\ \mu C$, $q_2 = +6.3\ \mu C$, $q_3 = -0.89\ \mu C$. **(a)** Given that the distance d in Figure 19–33 is 4.35 cm, find the direction and magnitude of the net electrostatic force exerted on the point charge q_1. **(b)** How would your answers to part (a) change if the distance d were doubled? Explain.



▲ **FIGURE 19–33** Problems 35, 36, 48, and 59

36. •• Referring to Problem 35, suppose that the magnitude of the net electrostatic force exerted on the point charge q_2 in Figure 19–33 is 0.65 N. **(a)** Find the distance d. **(b)** What is the direction of the net force exerted on q_2?

37. •• **IP** **(a)** If the nucleus in Example 19–1 had a charge of $+2e$ (as would be the case for a nucleus of helium), would the speed of the electron be greater than, less than, or the same as that found in the Example? Explain. (Assume the radius of the electron's orbit is the same.) **(b)** Find the speed of the electron for a nucleus of charge $+2e$.

38. •• Four point charges are located at the corners of a square with sides of length a. Two of the charges are $+q$, and two are $-q$. Find the magnitude and direction of the net electric force exerted on a charge $+Q$, located at the center of the square, for each of the following two arrangements of charge: **(a)** The charges alternate in sign ($+q, -q, +q, -q$) as you go around the square; **(b)** the two positive charges are on the top corners, and the two negative charges are on the bottom corners.

39. •• **IP** Two identical point charges in free space are connected by a string 7.6 cm long. The tension in the string is 0.21 N. **(a)** Find the magnitude of the charge on each of the point charges. **(b)** Using the information given in the problem statement, is it possible to determine the sign of the charges? Explain. **(c)** Find the tension in the string if $+1.0\ \mu C$ of charge is transferred from one point charge to the other. Compare with your result from part (a).

40. ••• Two spheres with uniform surface charge density, one with a radius of 7.2 cm and the other with a radius of 4.7 cm, are separated by a center-to-center distance of 33 cm. The spheres have a combined charge of $+55\ \mu C$ and repel one another with a force of 0.75 N. What is the surface charge density on each sphere?

41. ••• Point charges, q_1 and q_2, are placed on the x axis, with q_1 at $x = 0$ and q_2 at $x = d$. A third point charge, $+Q$, is placed at $x = 3d/4$. If the net electrostatic force experienced by the charge $+Q$ is zero, how are q_1 and q_2 related?

SECTION 19–4 THE ELECTRIC FIELD

42. • **CE** Two electric charges are separated by a finite distance. Somewhere between the charges, on the line connecting them, the net electric field they produce is zero. **(a)** Do the charges have the same or opposite signs? Explain. **(b)** If the point of zero field is closer to charge 1, is the magnitude of charge 1 greater than or less than the magnitude of charge 2? Explain.

43. • What is the magnitude of the electric field produced by a charge of magnitude $7.50\ \mu C$ at a distance of **(a)** 1.00 m and **(b)** 2.00 m?

44. • A $+5.0$-μC charge experiences a 0.44-N force in the positive y direction. If this charge is replaced with a -2.7-μC charge, what force will it experience?

45. • Two point charges lie on the x axis. A charge of $+6.2\ \mu C$ is at the origin, and a charge of $-9.5\ \mu C$ is at $x = 10.0$ cm. What is the net electric field at **(a)** $x = -4.0$ cm and at **(b)** $x = +4.0$ cm?

46. •• **CE** The electric field on the dashed line in Figure 19–28 vanishes at infinity, but also at two different points a finite distance from the charges. Identify the regions in which you can find $E = 0$ at a finite distance from the charges: region 1, to the left of point A; region 2, between points A and B; region 3, between points B and C; region 4, to the right of point C.

47. •• An object with a charge of $-3.6\ \mu C$ and a mass of 0.012 kg experiences an upward electric force, due to a uniform electric field, equal in magnitude to its weight. **(a)** Find the direction and magnitude of the electric field. **(b)** If the electric charge on the object is doubled while its mass remains the same, find the direction and magnitude of its acceleration.

48. •• **IP** Figure 19–33 shows a system consisting of three charges, $q_1 = +5.00\ \mu C$, $q_2 = +5.00\ \mu C$, and $q_3 = -5.00\ \mu C$, at the vertices of an equilateral triangle of side $d = 2.95$ cm. **(a)** Find the magnitude of the electric field at a point halfway between the charges q_1 and q_2. **(b)** Is the magnitude of the electric field halfway between the charges q_2 and q_3 greater than, less than, or the same as the electric field found in part (a)? Explain. **(c)** Find the magnitude of the electric field at the point specified in part (b).

49. •• Two point charges of equal magnitude are 7.5 cm apart. At the midpoint of the line connecting them, their combined electric field has a magnitude of 45 N/C. Find the magnitude of the charges.

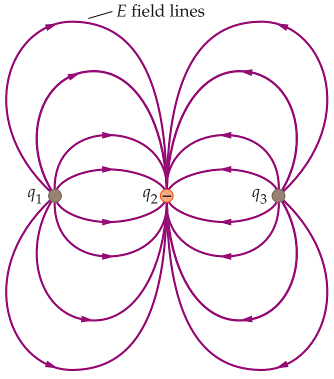
50. •• **IP** A point charge $q = +4.7\ \mu C$ is placed at each corner of an equilateral triangle with sides 0.21 m in length. **(a)** What is the magnitude of the electric field at the midpoint of any of the three sides of the triangle? **(b)** Is the magnitude of the electric field at the center of the triangle greater than, less than, or the same as the magnitude at the midpoint of a side? Explain.

51. ••• **IP** Four point charges, each of magnitude q, are located at the corners of a square with sides of length a. Two of the charges are $+q$, and two are $-q$. The charges are arranged in one of the following two ways: (1) The charges alternate in sign ($+q, -q, +q, -q$) as you go around the square; (2) the top two corners of the square have positive charges ($+q, +q$), and the bottom two corners have negative charges ($-q, -q$). **(a)** In which case will the electric field at the center of the square have the greatest magnitude? Explain. **(b)** Calculate the electric field at the center of the square for each of these two cases.

52. ••• The electric field at the point $x = 5.00$ cm and $y = 0$ points in the positive x direction with a magnitude of 10.0 N/C. At the point $x = 10.0$ cm and $y = 0$ the electric field points in the positive x direction with a magnitude of 15.0 N/C. Assuming this electric field is produced by a single point charge, find **(a)** its location and **(b)** the sign and magnitude of its charge.

SECTION 19–5 ELECTRIC FIELD LINES

53. • **IP** The electric field lines surrounding three charges are shown in **Figure 19–34**. The center charge is $q_2 = -10.0\ \mu C$. **(a)** What are the signs of q_1 and q_3? **(b)** Find q_1. **(c)** Find q_3.



▲ **FIGURE 19–34** Problems 53 and 56

54. • Make a qualitative sketch of the electric field lines produced by two equal positive charges, $+q$, separated by a distance d.

55. • Make a qualitative sketch of the electric field lines produced by two charges, $+q$ and $-q$, separated by a distance d.

56. •• Referring to Figure 19–34, suppose q_2 is not known. Instead, it is given that $q_1 + q_2 = -2.5\ \mu C$. Find q_1, q_2, and q_3.

57. •• Make a qualitative sketch of the electric field lines produced by the four charges, $+q$, $-q$, $+q$, and $-q$, arranged clockwise on the four corners of a square with sides of length d.

58. •• Sketch the electric field lines for the system of charges shown in Figure 19–29.

59. •• Sketch the electric field lines for the system of charges described in Problem 35.

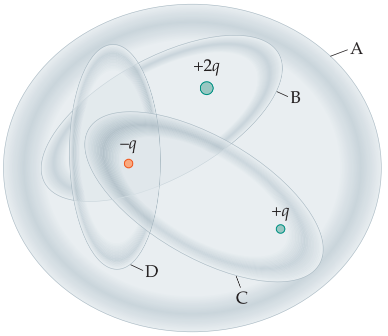
60. •• Suppose the magnitude of the electric field between the plates in Example 19–6 is changed, and a new object with a charge of $-2.05\ \mu C$ is attached to the string. If the tension in the string is 0.450 N, and the angle it makes with the vertical is 16°, what are **(a)** the mass of the object and **(b)** the magnitude of the electric field?

SECTION 19–7 ELECTRIC FLUX AND GAUSS'S LAW

61. • **CE Predict/Explain** Gaussian surface 1 has twice the area of Gaussian surface 2. Both surfaces enclose the same charge Q. **(a)** Is the electric flux through surface 1 greater than, less than, or the same as the electric flux through surface 2? **(b)** Choose the *best explanation* from among the following:
   I. Gaussian surface 2 is closer to the charge, since it has the smaller area. It follows that it has the greater electric flux.
   II. The two surfaces enclose the same charge, and hence they have the same electric flux.
   III. Electric flux is proportional to area. As a result, Gaussian surface 1 has the greater electric flux.

62. • **CE** Suppose the conducting shell in Figure 19–25—which has a point charge $+Q$ at its center—has a nonzero net charge. How much charge is on the inner and outer surface of the shell when the net charge of the shell is **(a)** $-2Q$, **(b)** $-Q$, and **(c)** $+Q$?

63. • **CE** Rank the Gaussian surfaces shown in **Figure 19–35** in order of increasing electric flux, starting with the most negative. Indicate ties where appropriate.



▲ **FIGURE 19–35** Problems 63 and 77

64. • A uniform electric field of magnitude 25,000 N/C makes an angle of 37° with a plane surface of area $0.0153\ m^2$. What is the electric flux through this surface?

65. • A surface encloses the charges $q_1 = 3.2\ \mu C$, $q_2 = 6.9\ \mu C$, and $q_3 = -4.1\ \mu C$. Find the electric flux through this surface.

66. • **IP** A uniform electric field of magnitude 6.00×10^3 N/C points upward. An empty, closed shoe box has a top and bottom that are 35.0 cm by 25.0 cm, vertical ends that are 25.0 cm by 20.0 cm, and vertical sides that are 20.0 cm by 35.0 cm. **(a)** Which side of the box has the greatest positive electric flux? Which side has the greatest negative electric flux? Which sides have zero electric flux? **(b)** Calculate the electric flux through each of the six sides of the box.

67. • **BIO Nerve Cells** Nerve cells are long, thin cylinders along which electrical disturbances (nerve impulses) travel. The cell membrane of a typical nerve cell consists of an inner and an outer wall separated by a distance of 0.10 μm. The electric field within the cell membrane is 7.0×10^5 N/C. Approximating the cell membrane as a parallel-plate capacitor, determine the magnitude of the charge density on the inner and outer cell walls.

68. •• The electric flux through each of the six sides of a rectangular box are as follows:

$$\Phi_1 = +150.0\ N \cdot m^2/C; \qquad \Phi_2 = +250.0\ N \cdot m^2/C;$$
$$\Phi_3 = -350.0\ N \cdot m^2/C; \qquad \Phi_4 = +175.0\ N \cdot m^2/C;$$
$$\Phi_5 = -100.0\ N \cdot m^2/C; \qquad \Phi_6 = +450.0\ N \cdot m^2/C.$$

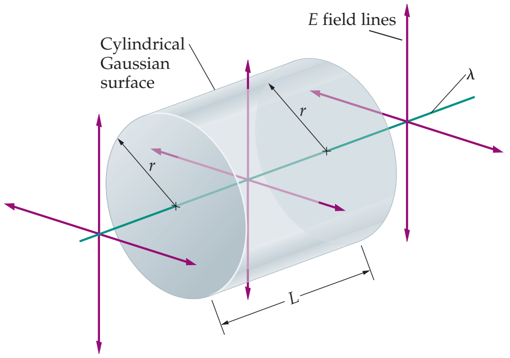
How much charge is in this box?

69. •• Consider a spherical Gaussian surface and three charges: $q_1 = 1.61\ \mu C$, $q_2 = -2.62\ \mu C$, and $q_3 = 3.91\ \mu C$. Find the electric flux through the Gaussian surface if it completely encloses **(a)** only charges q_1 and q_2, **(b)** only charges q_2 and q_3, and **(c)** all three charges. **(d)** Suppose a fourth charge, Q, is added to the situation described in part (c). Find the sign and magnitude of Q required to give zero electric flux through the surface.

70. ••• A thin wire of infinite extent has a charge per unit length of λ. Using the cylindrical Gaussian surface shown in **Figure 19–36**, show that the electric field produced by this wire at a radial distance r has a magnitude given by

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Note that the direction of the electric field is always radially away from the wire.



▲ **FIGURE 19–36** Problems 70 and 87

GENERAL PROBLEMS

71. • **CE Predict/Explain** An electron and a proton are released from rest in space, far from any other objects. The particles move toward each other, due to their mutual electrical attraction. **(a)** When they meet, is the kinetic energy of the electron greater than, less than, or equal to the kinetic energy of the proton? **(b)** Choose the *best explanation* from among the following:
   I. The proton has the greater mass. Since kinetic energy is proportional to mass, it follows that the proton will have the greater kinetic energy.
   II. The two particles experience the same force, but the light electron moves farther than the massive proton. Therefore, the work done on the electron, and hence its kinetic energy, is greater.
   III. The same force acts on the two particles. Therefore, they will have the same kinetic energy and energy will be conserved.

72. • **CE Predict/Explain** In Conceptual Checkpoint 19–3, suppose the charge to be placed at either point A or point B is $+q$ rather than $-q$. **(a)** Is the magnitude of the net force experienced by the movable charge at point A greater than, less than, or equal to the magnitude of the net force at point B? **(b)** Choose the *best explanation* from among the following:
   I. Point B is farther from the two fixed charges. As a result, the net force at point B is less than at point A.
   II. The net force at point A cancels, just as it does in Conceptual Checkpoint 19–3. Therefore, the nonzero net force at point B is greater in magnitude than the zero net force at point A.
   III. The net force is greater in magnitude at point A because at that location the movable charge experiences a net repulsion from each of the fixed charges.

73. • **CE** An electron (charge $= -e$) orbits a helium nucleus (charge $= +2e$). Is the magnitude of the force exerted on the helium nucleus by the electron greater than, less than, or the same as the magnitude of the force exerted on the electron by the helium nucleus? Explain.

74. • **CE** In the operating room, technicians and doctors must take care not to create an electric spark, since the presence of the oxygen gas used during an operation increases the risk of a deadly fire. Should the operating-room personnel wear shoes that are conducting or nonconducting? Explain.

75. • **CE** Under normal conditions, the electric field at the surface of the Earth points downward, into the ground. What is the sign of the electric charge on the ground?

76. • **CE** Two identical spheres are made of conducting material. Initially, sphere 1 has a net charge of $+35Q$ and sphere 2 has a net charge of $-26Q$. If the spheres are now brought into contact, what is the final charge on sphere 1? Explain.

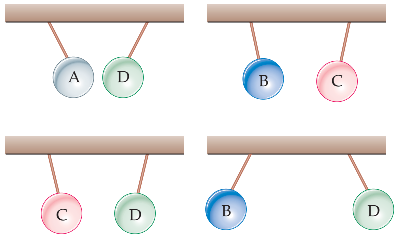
77. • **CE** A Gaussian surface for the charges shown in Figure 19–35 has an electric flux equal to $+3q/\varepsilon_0$. Which charges are contained within this Gaussian surface?

78. • A proton is released from rest in a uniform electric field of magnitude 1.08×10^5 N/C. Find the speed of the proton after it has traveled **(a)** 1.00 cm and **(b)** 10.0 cm.

79. • **BIO Ventricular Fibrillation** If a charge of 0.30 C passes through a person's chest in 1.0 s, the heart can go into ventricular fibrillation—a nonrhythmic "fluttering" of the ventricles that results in little or no blood being pumped to the body. If this rate of charge transfer persists for 4.5 s, how many electrons pass through the chest?

80. • A point charge at the origin of a coordinate system produces the electric field $\vec{\mathbf{E}} = (36{,}000\ \mathrm{N/C})\hat{\mathbf{x}}$ on the x axis at the location $x = -0.75$ m. Determine the sign and magnitude of the charge.

81. •• **CE** Four lightweight, plastic spheres, labeled A, B, C, and D, are suspended from threads in various combinations, as illustrated in **Figure 19–37**. It is given that the net charge on sphere D is $+Q$, and that the other spheres have net charges of $+Q$, $-Q$, or 0. From the results of the four experiments shown in Figure 19–37, and the fact that the spheres have equal masses, determine the net charge of **(a)** sphere A, **(b)** sphere B, and **(c)** sphere C.



▲ **FIGURE 19–37** Problem 81

82. •• Find **(a)** the direction and **(b)** the magnitude of the net electric field at the center of the equilateral triangle in Figure 19–31. Give your answers in terms of the angle θ, as defined in Figure 19–31, and E, the magnitude of the electric field produced by any *one* of the charges at the center of the triangle.

83. •• At the moment, the number of electrons in your body is essentially the same as the number of protons, giving you a net charge of zero. Suppose, however, that this balance of charges is off by 1% in both you and your friend, who is 1 meter away. Estimate the magnitude of the electrostatic force each of you experiences, and compare it with your weight.

84. •• A small object of mass 0.0150 kg and charge 3.1 μC hangs from the ceiling by a thread. A second small object, with a charge of 4.2 μC, is placed 1.2 m vertically below the first charge. Find **(a)** the electric field at the position of the upper charge due to the lower charge and **(b)** the tension in the thread.

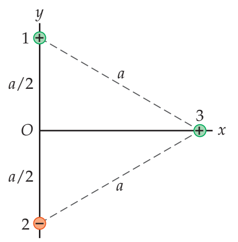
85. •• **IP** Consider a system of three point charges on the x axis. Charge 1 is at $x = 0$, charge 2 is at $x = 0.20$ m, and charge 3 is at $x = 0.40$ m. In addition, the charges have the following values: $q_1 = -19\ \mu$C, $q_2 = q_3 = +19\ \mu$C. **(a)** The electric field vanishes at some point on the x axis between $x = 0.20$ m and $x = 0.40$ m. Is the point of zero field (i) at $x = 0.30$ m, (ii) to the left of $x = 0.30$ m, or (iii) to the right of $x = 0.30$ m? Explain. **(b)** Find the point where $E = 0$ between $x = 0.20$ m and $x = 0.40$ m.

86. •• **IP** Consider the system of three point charges described in the previous problem. **(a)** The electric field vanishes at two different points on the x axis. One point is between $x = 0.20$ m and $x = 0.40$ m. Is the second point located to the left of charge 1 or to the right of charge 3? Explain. **(b)** Find the value of x at the second point where $E = 0$.

87. •• The electric field at a radial distance of 47.7 cm from the thin charged wire shown in Figure 19–36 has a magnitude of 35,400 N/C. **(a)** Using the result given in Problem 70, what is the magnitude of the charge per length on this wire? **(b)** At what distance from the wire is the magnitude of the electric field equal to $\frac{1}{2}$(35,400 N/C)?

88. •• A system consisting entirely of electrons and protons has a net charge of 1.84×10^{-15} C and a net mass of 4.56×10^{-23} kg. How many **(a)** electrons and **(b)** protons are in this system?

89. •• **IP** Three charges are placed at the vertices of an equilateral triangle of side $a = 0.93$ m, as shown in **Figure 19–38**. Charges 1 and 3 are $+7.3\ \mu$C; charge 2 is $-7.3\ \mu$C. **(a)** Find the magnitude and direction of the net force acting on charge 3. **(b)** If charge 3 is moved to the origin, will the net force acting on it there be greater than, less than, or equal to the net force found in part (a)? Explain. **(c)** Find the net force on charge 3 when it is at the origin.



▲ **FIGURE 19–38** Problems 89 and 90

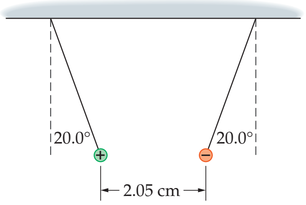
90. •• **IP** Consider the system of three charges described in the previous problem and shown in Figure 19–38. **(a)** Do you expect the net force acting on charge 1 to have a magnitude greater than, less than, or the same as the magnitude of the net force acting on charge 2? Explain. **(b)** Find the magnitude of the net force acting on charge 1. **(c)** Find the magnitude of the net force acting on charge 2.

91. •• **IP BIO Cell Membranes** The cell membrane in a nerve cell has a thickness of $0.12\ \mu$m. **(a)** Approximating the cell membrane as a parallel-plate capacitor with a surface charge density of 5.9×10^{-6} C/m^2, find the electric field within the membrane. **(b)** If the thickness of the membrane were doubled, would your answer to part (a) increase, decrease, or stay the same? Explain.

92. •• A square with sides of length L has a point charge at each of its four corners. Two corners that are diagonally opposite have charges equal to $+2.25\ \mu$C; the other two diagonal corners have charges Q. Find the magnitude and sign of the charges Q such that each of the $+2.25$-μC charges experiences zero net force.

93. •• **IP** Suppose a charge $+Q$ is placed on the Earth, and another charge $+Q$ is placed on the Moon. **(a)** Find the value of Q needed to "balance" the gravitational attraction between the Earth and the Moon. **(b)** How would your answer to part (a) change if the distance between the Earth and the Moon were doubled? Explain.

94. •• Two small plastic balls hang from threads of negligible mass. Each ball has a mass of 0.14 g and a charge of magnitude q. The balls are attracted to each other, and the threads attached to the balls make an angle of 20.0° with the vertical, as shown in **Figure 19–39**. Find **(a)** the magnitude of the electric force acting on each ball, **(b)** the tension in each of the threads, and **(c)** the magnitude of the charge on the balls.



▲ **FIGURE 19–39** Problem 94

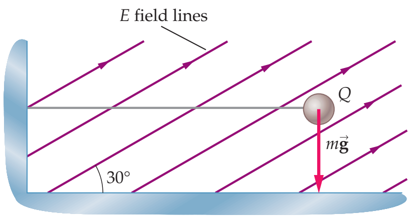
95. •• A small sphere with a charge of $+2.44\ \mu$C is attached to a relaxed horizontal spring whose force constant is 89.2 N/m. The spring extends along the x axis, and the sphere rests on a frictionless surface with its center at the origin. A point charge $Q = -8.55\ \mu$C is now moved slowly from infinity to a point $x = d > 0$ on the x axis. This causes the small sphere to move to the position $x = 0.124$ m. Find d.

96. •• Twelve identical point charges q are equally spaced around the circumference of a circle of radius R. The circle is centered at the origin. One of the twelve charges, which happens to be on the positive x axis, is now moved to the center of the circle. Find **(a)** the direction and **(b)** the magnitude of the net electric force exerted on this charge.

97. •• **BIO Nerve Impulses** When a nerve impulse propagates along a nerve cell, the electric field within the cell membrane changes from 7.0×10^5 N/C in one direction to 3.0×10^5 N/C in the other direction. Approximating the cell membrane as a parallel-plate capacitor, find the magnitude of the change in charge density on the walls of the cell membrane.

98. •• **IP The Electric Field of the Earth** The Earth produces an approximately uniform electric field at ground level. This electric field has a magnitude of 110 N/C and points radially inward, toward the center of the Earth. **(a)** Find the surface charge density (sign and magnitude) on the surface of the Earth. **(b)** Given that the radius of the Earth is 6.38×10^6 m, find the total electric charge on the Earth. **(c)** If the Moon had the same amount of electric charge distributed uniformly over its surface, would its electric field at the surface be greater than, less than, or equal to 110 N/C? Explain.

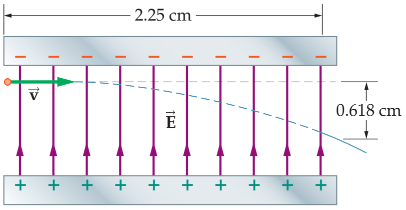
99. •• An object of mass $m = 3.1$ g and charge $Q = +48\ \mu$C is attached to a string and placed in a uniform electric field that is inclined at an angle of 30.0° with the horizontal **(Figure 19–40)**. The object is in static equilibrium when the string is horizontal. Find **(a)** the magnitude of the electric field and **(b)** the tension in the string.



▲ **FIGURE 19–40** Problem 99

100. •• Four identical charges, $+Q$, occupy the corners of a square with sides of length a. A fifth charge, q, can be placed at any desired location. Find the location of the fifth charge, and the value of q, such that the net electric force acting on each of the original four charges, $+Q$, is zero.

101. •• **Figure 19–41** shows an electron entering a parallel-plate capacitor with a speed of 5.45×10^6 m/s. The electric field of the



▲ **FIGURE 19–41** Problem 101

capacitor has deflected the electron downward by a distance of 0.618 cm at the point where the electron exits the capacitor. Find **(a)** the magnitude of the electric field in the capacitor and **(b)** the speed of the electron when it exits the capacitor.

102. ••• Two identical conducting spheres are separated by a fixed center-to-center distance of 45 cm and have different charges. Initially, the spheres attract each other with a force of 0.095 N. The spheres are now connected by a thin conducting wire. After the wire is removed, the spheres are positively charged and repel one another with a force of 0.032 N. Find **(a)** the final and **(b)** the initial charges on the spheres.

PASSAGE PROBLEMS

Bumblebees and Static Cling

Have you ever pulled clothes from a dryer only to have them "cling" together? Have you ever walked across a carpet and had a "shocking" experience when you touched a doorknob? If so, you already know a lot about static electricity.

Ben Franklin showed that the same kind of spark we experience on a carpet, when scaled up in size, is responsible for bolts of lightning. His insight led to the invention of lightning rods to conduct electricity safely away from a building into the ground. Today, we employ static electricity in many technological applications, ranging from photocopiers to electrostatic precipitators that clean emissions from smokestacks. We even use electrostatic salting machines to give potato chips the salty taste we enjoy!

Living organisms also use static electricity—in fact, static electricity plays an important role in the pollination process. Imagine a bee busily flitting from flower to flower. As air rushes over its body and wings it acquires an electric charge—just as you do when your feet rub against a carpet. A bee might have only 93.0 pC of charge, but that's more than enough to attract grains of pollen from a distance, like a charged comb attracting bits of paper. The result is a bee covered with grains of pollen, as illustrated in the accompanying photo, unwittingly transporting pollen from one flower to another. So, the next time you experience annoying static cling in your clothes, just remember that the same force helps pollinate the plants that we all need for life on Earth.

▲ A white-tailed bumblebee with static cling. (Problems 103, 104, 105, and 106)

103. • How many electrons must be transferred away from a bee to produce a charge of +93.0 pC?

A. 1.72×10^{-9} **B.** 5.81×10^{8}
C. 1.02×10^{20} **D.** 1.49×10^{29}

104. • Suppose two bees, each with a charge of 93.0 pC, are separated by a distance of 1.20 cm. Treating the bees as point charges, what is the magnitude of the electrostatic force experienced by the bees? (In comparison, the weight of a 0.140-g bee is 1.37×10^{-3} N.)

A. 6.01×10^{-17} N **B.** 6.48×10^{-9} N
C. 5.40×10^{-7} N **D.** 5.81×10^{-3} N

105. • The force required to detach a grain of pollen from an avocado stigma is approximately 4.0×10^{-8} N. What is the maximum distance at which the electrostatic force between a bee and a grain of pollen is sufficient to detach the pollen? Treat the bee and pollen as point charges, and assume the pollen has a charge opposite in sign and equal in magnitude to the bee.

A. 4.7×10^{-7} m **B.** 1.9 mm
C. 4.4 cm **D.** 220 m

106. • The Earth produces an electric field of magnitude 110 N/C. What force does this electric field exert on a bee carrying a charge of 93.0 pC? (Again, for comparison, the weight of a bee is approximately 1.37×10^{-3} N.)

A. 1.76×10^{-17} N **B.** 8.45×10^{-13} N
C. 1.02×10^{-8} N **D.** 1.13×10^{-6} N

INTERACTIVE PROBLEMS

107. •• **IP Referring to Example 19–5** Suppose $q_1 = +2.90\ \mu C$ is no longer at the origin, but is now on the y axis between $y = 0$ and $y = 0.500$ m. The charge $q_2 = +2.90\ \mu C$ is at $x = 0$ and $y = 0.500$ m, and point 3 is at $x = y = 0.500$ m. **(a)** Is the magnitude of the net electric field at point 3, which we call E_{net}, greater than, less than, or equal to its previous value? Explain. **(b)** Is the angle θ that E_{net} makes with the x axis greater than, less than, or equal to its previous value? Explain. Find the new values of **(c)** E_{net} and **(d)** θ if q_1 is at $y = 0.250$ m.

108. •• **IP Referring to Example 19–5** In this system, the charge q_1 is at the origin, the charge q_2 is at $x = 0$ and $y = 0.500$ m, and point 3 is at $x = y = 0.500$ m. Suppose that $q_1 = +2.90\ \mu C$, but that q_2 is increased to a value greater than $+2.90\ \mu C$. As a result, do **(a)** E_{net} and **(b)** θ increase, decrease, or stay the same? Explain. If $E_{net} = 1.66 \times 10^{5}$ N/C, find **(c)** q_2 and **(d)** θ.

109. •• **IP Referring to Example 19–6** The magnitude of the charge is changed until the angle the thread makes with the vertical is $\theta = 15.0°$. The electric field is 1.46×10^{4} N/C and the mass of the object is 0.0250 kg. **(a)** Is the new magnitude of q greater than or less than its previous value? Explain. **(b)** Find the new value of q.

110. •• **Referring to Example 19–6** Suppose the magnitude of the electric field is adjusted to give a tension of 0.253 N in the thread. This will also change the angle the thread makes with the vertical. **(a)** Find the new value of E. **(b)** Find the new angle between the thread and the vertical.

20 Electric Potential and Electric Potential Energy

Neon signs like the one shown here have been a familiar advertising tool since 1912, when the first commercial sign was sold to a Parisian barber. These signs do in fact contain neon, which gives the red color, as well as other noble gases such as argon, helium, krypton, and xenon to produce a wide range of brilliant colors. A typical neon sign operates with a difference in electric potential of about 10,000 volts. In contrast, muscle contractions in the human heart produce electric potentials that are only about a thousandth of a volt. The physics underlying electric potential—and its application to everything from neon signs to EKGs—is the subject of this chapter.

When paramedics try to revive a heart-attack victim, they apply a jolt of electrical energy to the person's heart with a device known as a defibrillator. Before they can use the defibrillator, however, they must wait a few seconds for it to be "charged up." What exactly is happening as the defibrillator charges? The answer is that electric charge is building up on a capacitor, and in the process storing electrical energy. When the defibrillator is activated, the energy stored in the capacitor is released in a sudden surge of power that is often capable of saving a person's life. In this chapter we develop the concept of electrical energy and discuss how both energy and charge can be stored in a capacitor.

20–1 Electric Potential Energy and the Electric Potential

Electric and gravitational forces have many similarities. One of the most important of these is that both forces are conservative. As a result, there must be an **electric potential energy,** U, associated with the electric force, just as there is a gravitational potential energy, $U = mgy$, due to the force of gravity. (Recall that conservative forces and potential energies are discussed in Chapter 8.)

To illustrate the concept of electric potential energy, consider a uniform electric field, $\vec{\mathbf{E}}$, as shown in **Figure 20–1 (a)**. A positive test charge q_0 is placed in this field, where it experiences a downward electric force of magnitude $F = q_0E$. If the charge is moved upward through a distance d, the electric force and the displacement are in opposite directions; hence, the work done by the electric force is negative:

$$W = -q_0Ed$$

Using our definition of potential energy change given in Equation 8–1, $\Delta U = -W$, we find that the potential energy of the charge is changed by the amount

$$\Delta U = -W = q_0Ed \qquad \text{20–1}$$

Note that the electric potential energy increases, just as the gravitational potential energy of a ball increases when it is raised against the force of gravity to a higher altitude, as indicated in **Figure 20–1 (b)**.

On the other hand, if the charge q_0 is negative, the electric force acting on it will be upward. In this case, the electric force does *positive* work as the charge is raised through the distance d, and the change in potential energy is *negative*. Thus, the change in potential energy depends on the sign of a charge as well as on its magnitude.

The electric force depends on charge in the same way as does the change in electric potential energy. In the last chapter we found it convenient to define an electric field, $\vec{\mathbf{E}}$, as the force per charge:

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0}$$

Similarly, it is useful to define a quantity that is equal to the change in electric potential energy per charge, $\Delta U/q_0$. This quantity, which is independent of the test charge q_0, is referred to as the **electric potential,** V:

Definition of Electric Potential, *V*

$$\Delta V = \frac{\Delta U}{q_0} = \frac{-W}{q_0} \qquad \text{20–2}$$

SI unit: joule/coulomb = volt, V

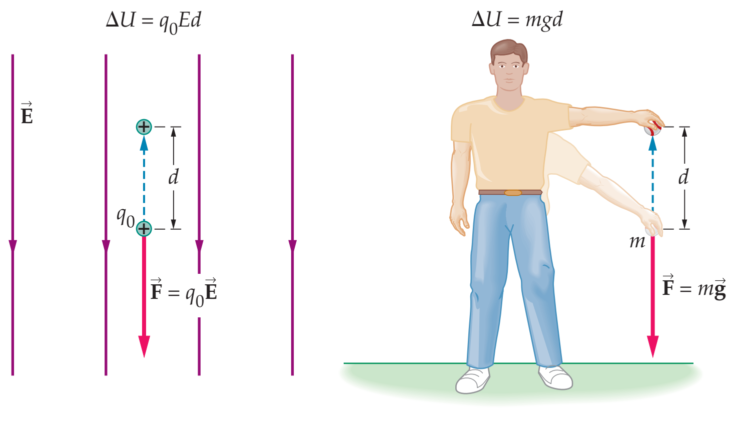
In common usage, the electric potential is often referred to simply as the "potential."

PROBLEM-SOLVING NOTE

A "Potential" Cause of Confusion

When reading a problem statement, be sure to note whether it refers to the electric potential energy, U, the electric potential, V, or the potential—which is just a shorthand for the electric potential. The properties of these quantities are summarized below:

Quantity	Symbol	Units	Meaning
Electric potential energy	U	joules	energy associated with electric force
Electric potential	V	volts	electric potential energy per charge
Potential (shorthand for electric potential)	V	volts	same as electric potential



(a) Moving a charge in an electric field

(b) Moving a mass in a gravitational field

FIGURE 20–1 Change in electric potential energy
(a) A positive test charge q_0 experiences a downward force due to the electric field $\vec{\mathbf{E}}$. If the charge is moved upward a distance d, the work done by the electric field is $-q_0Ed$. At the same time, the electric potential energy of the system increases by q_0Ed. The situation is analogous to that of an object in a gravitational field. **(b)** If a ball is lifted against the force exerted by gravity, the gravitational potential energy of the system increases.

PROBLEM-SOLVING NOTE

Electric Potential and Its Unit of Measurement

Be careful to distinguish between the electric potential V and the corresponding unit of measurement, the volt V, since both are designated by the same symbol. For example, in the expression $\Delta V = 12$ V, the first V refers to the electric potential, the second V refers to the units of the volt. Similarly, when someone says he has a "12-volt battery," what he really means is that the battery produces a difference in electric potential, ΔV, of 12 volts.

Notice that our definition gives only the *change* in electric potential. As with gravitational potential energy, the electric potential can be set to zero at any desired location—only changes in electric potential are measurable. In addition, just as the potential energy is a scalar (that is, simply a number) so too is the potential V.

Potential energy is measured in joules, and charge is measured in coulombs; hence, the SI units of electric potential are joules per coulomb. This combination of units is referred to as the volt, in honor of Alessandro Volta (1745–1827), who invented a predecessor to the modern battery. To be specific, the volt (V) is defined as follows:

$$1 \text{ V} = 1 \text{ J/C} \qquad \text{20–3}$$

Rearranging slightly, we see that energy can be expressed as charge times voltage: $1 \text{ J} = (1 \text{ C})(1 \text{ V})$.

A convenient and commonly used unit of energy in atomic systems is the **electron volt** (eV), defined as the product of the electron charge and a potential difference of 1 volt:

$$1 \text{ eV} = (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.60 \times 10^{-19} \text{ J}$$

It follows that the electron volt is the energy change an electron experiences when it moves through a potential difference of 1 V. As we shall see in later chapters, typical atomic energies are in the eV range, whereas typical energies in nuclear systems are in the MeV (10^6 eV) range. Even the MeV is a minuscule energy, however, when compared to the joule.

EXERCISE 20–1

Find the change in electric potential energy, ΔU, as a charge of **(a)** 2.20×10^{-6} C or **(b)** -1.10×10^{-6} C moves from a point A to a point B, given that the change in electric potential between these points is $\Delta V = V_B - V_A = 24.0$ V.

SOLUTION

a. Solving $\Delta V = \Delta U/q_0$ for ΔU, we find

$$\Delta U = q_0\,\Delta V = (2.20 \times 10^{-6} \text{ C})(24.0 \text{ V}) = 5.28 \times 10^{-5} \text{ J}$$

b. Similarly, using $q_0 = -1.10 \times 10^{-6}$ C we obtain

$$\Delta U = q_0\,\Delta V = (-1.10 \times 10^{-6} \text{ C})(24.0 \text{ V}) = -2.64 \times 10^{-5} \text{ J}$$

Electric Field and the Rate of Change of Electric Potential

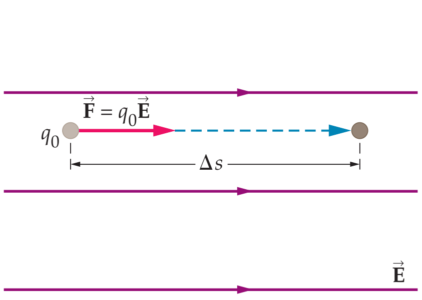
There is a connection between the electric field and the electric potential that is both straightforward and useful. To obtain this relation, let's apply the definition $\Delta V = -W/q_0$ to the case of a test charge that moves through a distance Δs in the direction of the electric field, as in **Figure 20–2**. The work done by the electric field in this case is simply the magnitude of the electric force, $F = q_0E$, times the distance, Δs:

$$W = q_0E\Delta s$$

Therefore, the change in electric potential is

$$\Delta V = \frac{-W}{q_0} = \frac{-(q_0E\Delta s)}{q_0} = -E\Delta s$$

Solving for the electric field, we find



▲ **FIGURE 20–2 Electric field and electric potential**

As a charge q_0 moves in the direction of the electric field, $\vec{E}$, the electric potential, V, decreases. In particular, if the charge moves a distance Δs, the electric potential decreases by the amount $\Delta V = -E\Delta s$.

Connection Between the Electric Field and the Electric Potential

$$E = -\frac{\Delta V}{\Delta s} \qquad \text{20–4}$$

SI unit: volts/meter, V/m

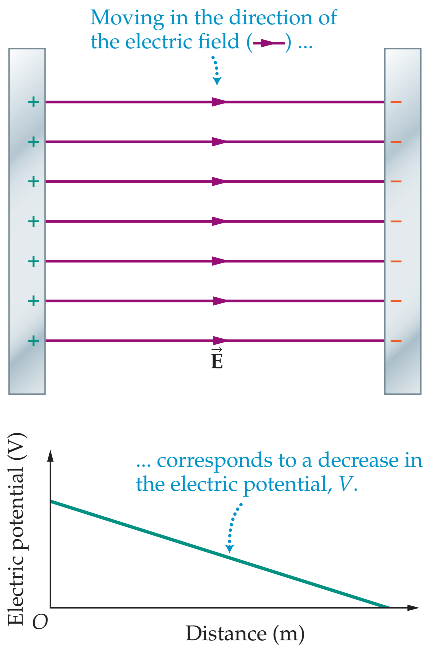


FIGURE 20–3 The electric potential for a constant electric field
As a general rule, the electric potential, V, decreases as one moves in the direction of the electric field. In the case shown here, the electric field is constant; as a result, the electric potential decreases uniformly with distance. We have arbitrarily set the potential equal to zero at the right-hand plate.

This relation shows that the electric field, which can be expressed in units of N/C, also has the units of volts per meter. That is,

$$1\ \mathrm{N/C} = 1\ \mathrm{V/m} \qquad \text{20–5}$$

To summarize, the electric field depends on *the rate of change of the electric potential with position*. In terms of our gravitational analogy, you can think of the potential V as the height of a hill and the electric field E as the slope of the hill.

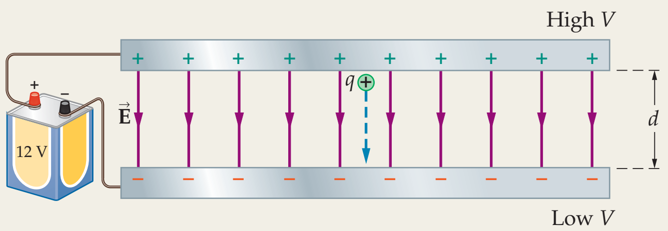
In addition, it follows from Equation 20–4 that *the electric potential decreases as one moves in the direction of the electric field*. Specifically, notice that the change in potential, $\Delta V = -E\Delta s$, is negative when E and Δs are in the same direction—that is, when both E and Δs are positive or both are negative. For example, in cases where the electric field is *constant*, as between the plates of a parallel-plate capacitor (Section 19–5), the electric potential decreases *linearly* in the direction of the field. These observations are illustrated in **Figure 20–3**.

EXAMPLE 20–1 PLATES AT DIFFERENT POTENTIALS

A uniform electric field is established by connecting the plates of a parallel-plate capacitor to a 12-V battery. **(a)** If the plates are separated by 0.75 cm, what is the magnitude of the electric field in the capacitor? **(b)** A charge of $+6.24 \times 10^{-6}$ C moves from the positive plate to the negative plate. Find the change in electric potential energy for this charge. (In electrical systems we shall assume that gravity can be ignored, unless specifically instructed otherwise.)

PICTURE THE PROBLEM

Our sketch shows the parallel-plate capacitor connected to a 12-V battery. The battery guarantees that the potential difference between the plates is 12 V, with the positive plate at the higher potential. The separation of the plates is $d = 0.75\ \mathrm{cm} = 0.0075\ \mathrm{m}$, and the charge that moves from the positive to the negative plate is $q = +6.24 \times 10^{-6}$ C.



STRATEGY

a. The electric field can be calculated using $E = -\Delta V/\Delta s$. Note that if one moves in the direction of the field from the positive plate to the negative plate ($\Delta s = 0.75$ cm), the electric potential decreases by 12 V; that is, $\Delta V = -12$ V.

b. The change in electric potential energy is $\Delta U = q\,\Delta V$.

SOLUTION

Part (a)

1. Substitute $\Delta s = 0.0075$ m and $\Delta V = -12$ V in $E = -\Delta V/\Delta s$:

$$E = -\frac{\Delta V}{\Delta s} = -\frac{(-12\ \mathrm{V})}{0.0075\ \mathrm{m}} = 1600\ \mathrm{V/m}$$

Part (b)

2. Evaluate $\Delta U = q\Delta V$:

$$\Delta U = q\Delta V = (6.24 \times 10^{-6}\ \mathrm{C})(-12\ \mathrm{V}) = -7.5 \times 10^{-5}\ \mathrm{J}$$

INSIGHT

Note that the electric potential energy of the system decreases as the positive charge moves in the direction of the electric field, just as the gravitational potential energy of a ball decreases when it falls. In the next section, we show how this decrease in electric potential energy shows up as an increase in the charge's kinetic energy, just as the kinetic energy of a falling ball increases.

PRACTICE PROBLEM

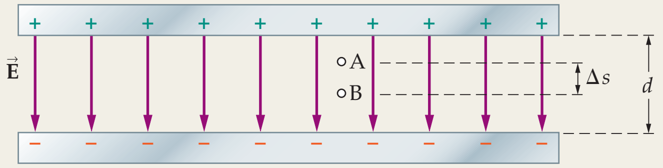
Find the separation of the plates that results in an electric field of 2.0×10^3 V/m. [**Answer:** The separation should be 0.60 cm. Note that decreasing the separation *increases* the field.]

Some related homework problems: Problem 7, Problem 10

A similar problem is considered in the following Active Example.

ACTIVE EXAMPLE 20–1 FIND THE ELECTRIC FIELD AND POTENTIAL DIFFERENCE

The electric potential at point B in the parallel-plate capacitor shown here is less than the electric potential at point A by 4.50 V. The separation between points A and B is 0.120 cm, and the separation between the plates is 2.55 cm. Find **(a)** the electric field within the capacitor and **(b)** the potential difference between the plates.



SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Calculate the electric field using $E = -\Delta V/\Delta s$, with $\Delta V = V_B - V_A = -4.50$ V and $\Delta s = 0.120$ cm: $E = 3750$ V/m

Part (b)

2. Calculate $\Delta V = V_{(-)} - V_{(+)}$ using $\Delta V = -E\,\Delta s$, with $E = 3750$ V/m and $\Delta s = 2.55$ cm: $\Delta V = -95.6$ V

INSIGHT

Notice that in part (b) we use the same electric field found in part (a). This is valid because the field within an ideal parallel-plate capacitor is uniform; thus, the same value of E applies everywhere within the capacitor.

Finally, the negative value of ΔV in part (b) simply indicates that the electric potential of the negative plate is less than that of the positive plate by 95.6 V. Put another way, the electric potential increases as we move "against" the electric field, just as the gravitational potential energy increases when we move "against" gravity—that is, when we move uphill.

YOUR TURN

Is the electric potential energy of an electron at point A greater than or less than its electric potential energy at point B? Calculate the difference in electric potential energy as an electron moves from point A to point B.

(Answers to **Your Turn** *problems are given in the back of the book.)*

CONCEPTUAL CHECKPOINT 20–1 CONSTANT ELECTRIC POTENTIAL

In a certain region of space the electric potential V is known to be constant. Is the electric field in this region **(a)** positive, **(b)** zero, or **(c)** negative?

REASONING AND DISCUSSION

The electric field is related to the *rate of change* of the electric potential with position, not to the value of the potential. Since the rate of change of a constant potential is zero, so too is the electric field.

In particular, if one moves a distance Δs from one point to another in this region, the change in potential is zero; $\Delta V = 0$. Hence, the electric field vanishes; $E = -\Delta V/\Delta s = 0$.

ANSWER

(b) The electric field is zero.

Finally, in our calculations of the potential difference $\Delta V = -E\Delta s$ in the examples to this point, we have always taken Δs to be a displacement in the direction of the electric field. More generally, Δs can be a displacement in any direction, as long as we replace E with the component of $\vec{\mathbf{E}}$ in the direction of Δs. Thus, for example, the change in potential as we move in the x direction is $\Delta V = -E_x\,\Delta x$, and the corresponding change as we move in the y direction, is $\Delta V = -E_y\,\Delta y$. We shall point out the utility of these expressions in Section 20–4.

20–2 Energy Conservation

When a ball is dropped in a gravitational field, its gravitational potential energy decreases as it falls. At the same time, its kinetic energy increases. If nonconservative forces such as air resistance can be ignored, we know that the decrease in gravitational potential energy is equal to the increase in kinetic energy—in other words, the total energy of the ball is conserved.

Because the electric force is also conservative, the same considerations apply to a charged object in an electric field. Therefore, ignoring other forces, we can say that the total energy of an electric charge must be conserved. As a result, the sum of its kinetic and electric potential energies must be the same at any two points, say A and B:

$$K_A + U_A = K_B + U_B$$

Expressing the kinetic energy as $\frac{1}{2}mv^2$, we can write energy conservation as

$$\tfrac{1}{2}mv_A^{\,2} + U_A = \tfrac{1}{2}mv_B^{\,2} + U_B$$

This expression applies to any conservative force. In the case of a uniform gravitational field the potential energy is $U = mgy$; for an ideal spring it is $U = \frac{1}{2}kx^2$. When dealing with an electrical system, we can use Equation 20–2 to express the electric potential energy in terms of the electric potential as follows:

$$U = qV$$

To be specific, suppose a particle of mass $m = 1.75 \times 10^{-5}$ kg and charge $q = 5.20 \times 10^{-5}$ C is released from rest at a point A. As the particle moves to another point, B, the electric potential decreases by 60.0 V; that is, $V_A - V_B = 60.0$ V. The particle's speed at point B can be found using energy conservation. To do so, we first solve for the kinetic energy at point B:

$$\tfrac{1}{2}mv_B^{\,2} = \tfrac{1}{2}mv_A^{\,2} + U_A - U_B$$

$$= \tfrac{1}{2}mv_A^{\,2} + q(V_A - V_B)$$

Next we set $v_A = 0$, since the particle starts at rest, and solve for v_B:

$$\tfrac{1}{2}mv_B^{\,2} = q(V_A - V_B)$$

$$v_B = \sqrt{\frac{2q(V_A - V_B)}{m}} = \sqrt{\frac{2(5.20 \times 10^{-5}\ \text{C})(60.0\ \text{V})}{1.75 \times 10^{-5}\ \text{kg}}} = 18.9\ \text{m/s} \qquad \text{20–6}$$

Thus, the decrease in electric potential energy appears as an increase in kinetic energy—and a corresponding increase in speed.

PROBLEM-SOLVING NOTE

Energy Conservation and Electric Potential Energy

Energy conservation applies to charges in an electric field. Therefore, to relate the speed or kinetic energy of a charge to its location simply requires setting the initial energy equal to the final energy. Note that the potential energy for a charge q is $U = qV$.

ACTIVE EXAMPLE 20–2 FIND THE SPEED WITH A RUNNING START

Suppose the particle described in the preceding discussion has an initial speed of 5.00 m/s at point A. What is its speed at point B?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the equation for energy conservation: $\frac{1}{2}mv_A^{\,2} + qV_A = \frac{1}{2}mv_B^{\,2} + qV_B$
2. Solve for the kinetic energy at point B: $\frac{1}{2}mv_B^{\,2} = \frac{1}{2}mv_A^{\,2} + q(V_A - V_B)$
3. Solve for v_B: $v_B = \sqrt{v_A^{\,2} + 2q(V_A - V_B)/m}$
4. Substitute numerical values: $v_B = 19.5$ m/s

INSIGHT

The final speed is not 5.00 m/s greater than 18.9 m/s; in fact, it is only 0.6 m/s greater. As usual, this is due to the fact that the kinetic energy depends on v^2 rather than on v.

YOUR TURN

What initial speed is required to obtain a final speed of 18.9 m/s + 5.00 m/s = 23.9 m/s?

(Answers to **Your Turn** *problems are given in the back of the book.)*

Notice that in the preceding discussions a positive charge moves to a region where the electric potential is less, and its speed increases. As one might expect, the situation is just the opposite for a negative charge. In particular, a negative charge will move to a region of higher electric potential with an increase in speed.

For example, if the charge in Equation 20–6 is changed in sign to -5.20×10^{-5} C, and the potential difference is changed in sign to $V_A - V_B = -60.0$ V, the final speed remains the same. In general:

> *Positive* charges accelerate in the direction of *decreasing* electric potential.
>
> *Negative* charges accelerate in the direction of *increasing* electric potential.

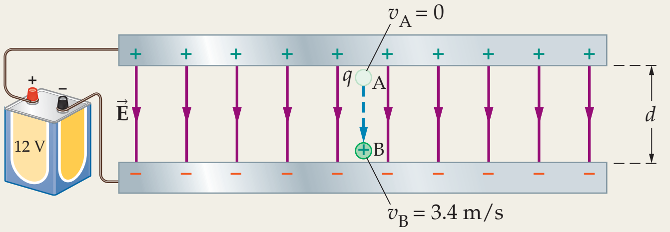
In both cases, however, the charge moves to a region of lower electric potential energy.

EXAMPLE 20–2 FROM PLATE TO PLATE

Suppose the charge in Example 20–1 is released from rest at the positive plate and that it reaches the negative plate with a speed of 3.4 m/s. What are **(a)** the mass of the charge and **(b)** its final kinetic energy?

PICTURE THE PROBLEM

The physical situation is the same as in Example 20–1. In this case, however, we know that the charge starts at rest at the positive plate, $v_A = 0$, and hits the negative plate with a speed of $v_B = 3.4$ m/s. Note that the electric potential of the positive plate is 12 V greater than that of the negative plate. Therefore $V_A - V_B = 12$ V.



STRATEGY

a. The energy of the charge is conserved as it moves from one plate to the other. Setting the initial energy equal to the final energy gives us an equation in which there is only one unknown, the mass of the charge.

b. The final kinetic energy is simply $\frac{1}{2}mv_B^2$.

SOLUTION

Part (a)

1. Apply energy conservation to this system: $\frac{1}{2}mv_A^2 + qV_A = \frac{1}{2}mv_B^2 + qV_B$

2. Solve for the mass, m: $$m = \frac{2q(V_A - V_B)}{v_B^2 - v_A^2}$$

3. Substitute numerical values: $$m = \frac{2(6.24 \times 10^{-6}\ \text{C})(12\ \text{V})}{(3.4\ \text{m/s})^2 - 0} = 1.3 \times 10^{-5}\ \text{kg}$$

Part (b)

4. Calculate the final kinetic energy: $$K_B = \tfrac{1}{2}mv_B^2 = \tfrac{1}{2}(1.3 \times 10^{-5}\ \text{kg})(3.4\ \text{m/s})^2 = 7.5 \times 10^{-5}\ \text{J}$$

INSIGHT

We see that the final kinetic energy is precisely equal to the decrease in electric potential energy calculated in Example 20–1, as expected by energy conservation.

PRACTICE PROBLEM

If the mass of the charge had been 5.2×10^{-5} kg, what would be its **(a)** final speed and **(b)** final kinetic energy? [**Answer: (a)** $v_B = 1.7$ m/s, **(b)** $K_B = 7.5 \times 10^{-5}$ J. Note that the final kinetic energy is the same, regardless of the mass, because energy, not speed, is conserved.]

Some related homework problems: Problem 18, Problem 19

CONCEPTUAL CHECKPOINT 20–2 FINAL SPEED

An electron, with a charge of -1.60×10^{-19} C, accelerates from rest through a potential difference V. A proton, with a charge of $+1.60 \times 10^{-19}$ C, accelerates from rest through a potential difference $-V$. Is the final speed of the electron **(a)** greater than, **(b)** less than, or **(c)** the same as the final speed of the proton?

REASONING AND DISCUSSION

The electron and proton have charges of equal magnitude, and therefore they have equal changes in electric potential energy. As a result, their final kinetic energies are equal. Since the electron has less mass than the proton, however, its speed must be greater.

ANSWER

(a) The electron is moving faster than the proton.

20–3 The Electric Potential of Point Charges

Consider a point charge, $+q$, that is fixed at the origin of a coordinate system, as in **Figure 20–4**. Suppose, in addition, that a positive test charge, $+q_0$, is held at rest at point A, a distance r_A from the origin. At this location the test charge experiences a repulsive force with a magnitude given by Coulomb's law, $F = k|q_0||q|/r_A{}^2$.

If the test charge is now released, the repulsive force between it and the charge $+q$ will cause it to accelerate away from the origin. When it reaches an arbitrary point B, its kinetic energy will have increased by the same amount that its electric potential energy has decreased. Thus, we conclude that the electric potential energy is greater at point A than at point B. In fact, the methods of integral calculus can be used to show that the difference in electric potential energy between points A and B is

$$U_A - U_B = \frac{kq_0q}{r_A} - \frac{kq_0q}{r_B}$$

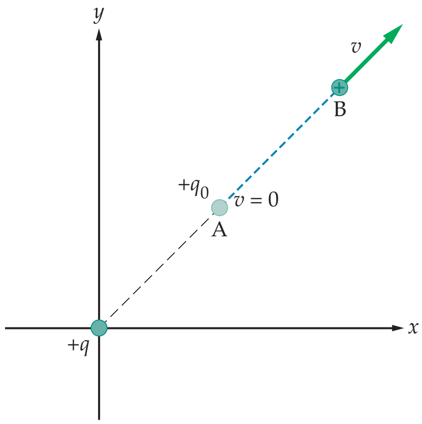


FIGURE 20–4 Energy conservation in an electrical system

A test charge, $+q_0$, is released from rest at point A. When it reaches point B, its kinetic energy will have increased by the same amount that its electric potential energy has decreased.

Note that the electric potential energy for point charges depends inversely on their separation, the same as the distance dependence of the gravitational potential energy for point masses (Equation 12–9).

The corresponding change in electric potential is found by dividing the electric potential energy by the test charge, q_0:

$$V_A - V_B = \frac{1}{q_0}(U_A - U_B) = \frac{kq}{r_A} - \frac{kq}{r_B}$$

If the test charge is moved infinitely far away from the origin, so that $r_B \to \infty$, the term kq/r_B vanishes, and the difference in electric potential becomes

$$V_A - V_B = \frac{kq}{r_A}$$

Since the potential can be set to zero at any convenient location, we choose to set V_B equal to zero; in other words, *we choose the electric potential to be zero infinitely far from a given charge*. With this choice, the potential of a point charge is $V_A = kq/r_A$. Dropping the subscript, we see that the electric potential at an arbitrary distance r is given by the following:

Electric Potential for a Point Charge

$$V = \frac{kq}{r} \qquad 20\text{–}7$$

SI unit: volt, V

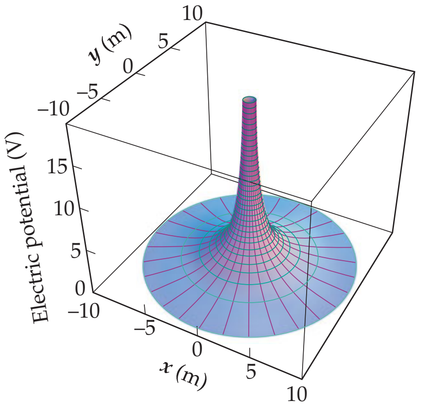
Recall that this expression for V actually represents a *change* in potential; in particular, V is the change in potential from a distance of infinity to a distance r. The corresponding difference in electric potential energy for the test charge q_0 is simply $U = q_0V$; that is,

Electric Potential Energy for Point Charges q and q_0 Separated by a Distance r

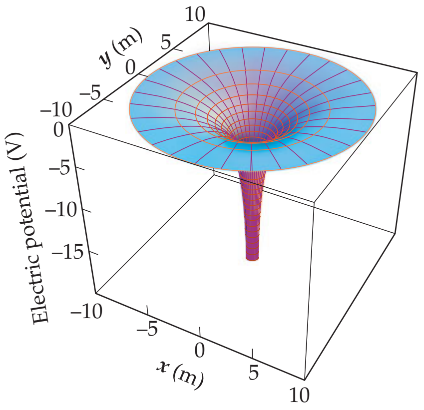
$$U = q_0V = \frac{kq_0q}{r} \qquad 20\text{–}8$$

SI unit: joule, J

Note that *the electric potential energy of two charges separated by an infinite distance is zero.*



(a) Electric potential near a positive charge



(b) Electric potential near a negative charge

▲ **FIGURE 20–5 The electric potential of a point charge**

Electric potential near **(a)** a positive and **(b)** a negative charge at the origin. In the case of the positive charge, the electric potential forms a "potential hill" near the charge. Near the negative charge we observe a "potential well."

One final point in regard to $V = kq/r$ and $U = kq_0q/r$ is that r is a *distance* and hence is always a positive quantity. For example, suppose a charge q is at the origin of a coordinate system. It follows that the potential at the point $x = 1$ m is the same as the potential at $x = -1$ m, since in both cases the distance is $r = 1$ m.

EXERCISE 20–2

Find the electric potential produced by a point charge of 6.80×10^{-7} C at a distance of 2.60 m.

SOLUTION

Substituting $q = 6.80 \times 10^{-7}$ C and $r = 2.60$ m in $V = kq/r$ yields

$$V = \frac{kq}{r} = \frac{(8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})(6.80 \times 10^{-7}\ \mathrm{C})}{2.60\ \mathrm{m}} = 2350\ \mathrm{V}$$

Thus, the potential at $r = 2.60$ m due to this point charge is 2350 V greater than the potential at infinity.

Note that V depends on the sign of the charge in question. This is shown in **Figure 20–5**, which represents the electric potential for a positive and a negative charge at the origin. The potential for the positive charge increases to positive infinity near the origin and decreases to zero far away, forming a "potential hill." On the other hand, the potential for the negative charge approaches negative infinity near the origin, forming a "potential well."

Thus, if the charge at the origin is positive, a positive test charge will move away from the origin, as if sliding "downhill" on the electric potential surface. Similarly, if the charge at the origin is negative, a positive test charge will move toward the origin, which again means that it slides downhill, this time into a potential well. Negative test charges, in contrast, always tend to slide "uphill" on electric potential curves, like bubbles rising in water.

Superposition of the Electric Potential

Like many physical quantities, the electric potential obeys a simple superposition principle. In particular:

> The total electric potential due to two or more charges is equal to the algebraic sum of the potentials due to each charge separately.

By *algebraic sum* we mean that the potential of a given charge may be positive or negative, and hence the *algebraic sign* of each potential must be taken into account when calculating the total potential. In particular, positive and negative contributions may cancel to give zero potential at a given location.

Finally, because the electric potential is a scalar, its superposition is as simple as adding numbers of various signs. This, in general, is easier than adding vectors, as is required for the superposition of electric fields.

EXAMPLE 20–3 TWO POINT CHARGES

A charge $q = 4.11 \times 10^{-9}$ C is placed at the origin, and a second charge equal to $-2q$ is placed on the x axis at the location $x = 1.00$ m. **(a)** Find the electric potential midway between the two charges. **(b)** The electric potential vanishes at some point between the charges; that is, for a value of x between 0 and 1.00 m. Find this value of x.

PICTURE THE PROBLEM

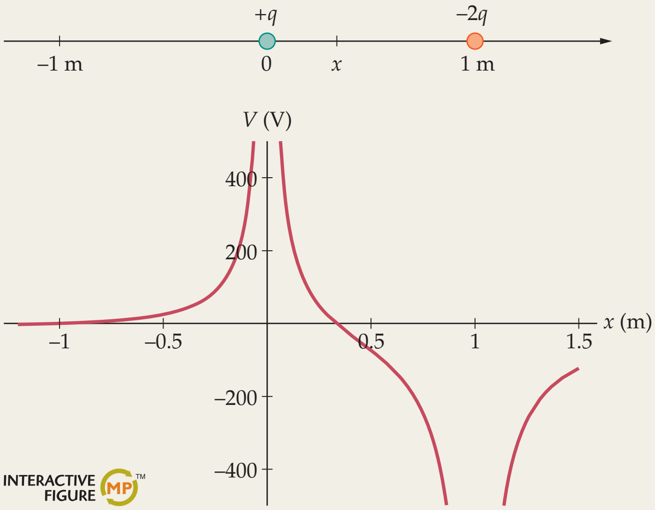
Our sketch shows the two charges placed on the x axis as described in the problem statement. Clearly, the electric potential is large and positive near the origin, and large and negative near $x = 1.00$ m. Thus, it follows that V must vanish at some point between $x = 0$ and $x = 1.00$ m.

A plot of V as a function of x is shown as an aid in visualizing the calculations given in this Example.

STRATEGY

a. As indicated in the sketch, an arbitrary point between $x = 0$ and $x = 1.00$ m is a distance x from the charge $+q$ and a distance $1.00\text{ m} - x$ from the charge $-2q$. Thus, by superposition, the total electric potential at a point x is $V = kq/x + k(-2q)/(1.00\text{ m} - x)$.

b. Setting $V = 0$ allows us to solve for the unknown, x.



INTERACTIVE FIGURE

SOLUTION

Part (a)

1. Use superposition to write an expression for V at an arbitrary point x between $x = 0$ and $x = 1.00$ m:

$$V = \frac{kq}{x} + \frac{k(-2q)}{1.00\text{ m} - x}$$

2. Substitute numerical values into the expression for V. Note that the midway point between the charges is $x = 0.500$ m:

$$V = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.11 \times 10^{-9}\ \text{C})}{0.500\text{ m}} + \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-2)(4.11 \times 10^{-9}\ \text{C})}{1.00\text{ m} - 0.500\text{ m}}$$
$$= -73.9\ \text{N}\cdot\text{m/C} = -73.9\text{ V}$$

Part (b)

3. Set the expression for V in Step 1 equal to zero, and simplify by canceling the common factor, kq:

$$V = \frac{kq}{x} + \frac{k(-2q)}{1.00\text{ m} - x} = 0$$
$$\frac{1}{x} = \frac{2}{1.00\text{ m} - x}$$

4. Solve for x:

$$1.00\text{ m} - x = 2x$$
$$x = \tfrac{1}{3}(1.00\text{ m}) = 0.333\text{ m}$$

INSIGHT

Suppose a small positive test charge is released from rest at $x = 0.500$ m. In which direction will it move? From the point of view of the Coulomb force, we know it will move to the right—repelled by the positive charge at the origin and attracted to the negative charge at $x = 1.00$ m. We come to the same conclusion when considering the electric potential, since we know that a positive test charge will "slide downhill" on the potential curve.

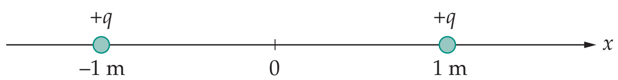
PRACTICE PROBLEM

The electric potential in this system also vanishes at a point on the negative x axis. Find this point. [**Answer:** $V = 0$ at $x = -1.00$ m. Note that the point $x = -1.00$ m is 1.00 m from the charge $+q$, and 2.00 m from the charge $-2q$. Hence, at $x = -1.00$ m we have $V = kq/(1.00\text{ m}) + k(-2q)/(2.00\text{ m}) = 0$.]

Some related homework problems: Problem 31, Problem 33

CONCEPTUAL CHECKPOINT 20–3 A PEAK OR A VALLEY?

Two point charges, each equal to $+q$, are placed on the x axis at $x = -1$ m and $x = +1$ m. As one moves along the x axis, does the potential look like a peak or a valley near the origin?



PROBLEM-SOLVING NOTE

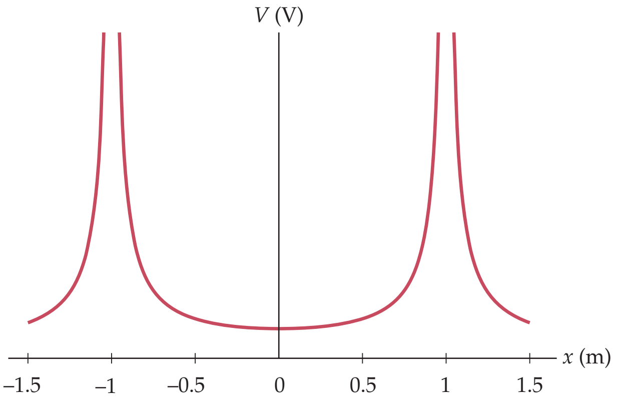
Superposition for the Electric Potential

Recall that to find the total electric potential due to a system of charges, you need only sum the potentials due to each charge separately—being careful to take into account the appropriate sign. Note that there are no components to deal with, as there are when we superpose electric fields.

REASONING AND DISCUSSION

We know that the potential is large and positive near each of the charges. As you move away from the charges, the potential tends to decrease. In particular, at very large positive or negative values of x the potential approaches zero.

Between $x = -1$ m and $x = +1$ m the potential has its lowest value when you are as far away from the two charges as possible. This occurs at the origin. Moving slightly to the left or the right simply brings you closer to one of the two charges, resulting in an increase in the potential; therefore, the potential has a minimum (bottom of a valley) at the origin. A plot of the potential for this case is shown below.



Finally, the electric field is associated with the slope of this curve, as mentioned in relation to Equation 20–4. Clearly, the field has a large magnitude near the charges and is zero at the origin.

ANSWER

Near the origin, the potential looks like a valley.

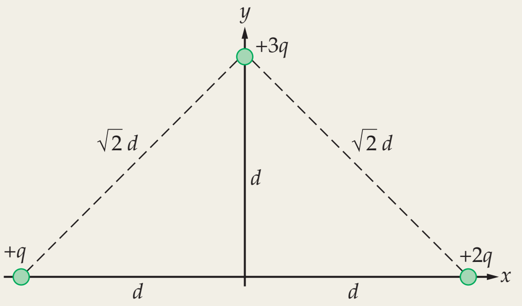
Superposition applies equally well to the electric potential energy. The following Example illustrates superposition for both the electric potential and the electric potential energy.

EXAMPLE 20–4 FLY AWAY

Two charges, $+q$ and $+2q$, are held in place on the x axis at the locations $x = -d$ and $x = +d$, respectively. A third charge, $+3q$, is released from rest on the y axis at $y = d$. **(a)** Find the electric potential due to the first two charges at the initial location of the third charge. **(b)** Find the initial electric potential energy of the third charge. **(c)** What is the kinetic energy of the third charge when it has moved infinitely far away from the other two charges?

PICTURE THE PROBLEM

As indicated in the sketch, the third charge, $+3q$, is separated from the other two charges by the initial distance, $\sqrt{2}d$. When the third charge is released, the repulsive forces due to $+q$ and $+2q$ will cause it to move away to an infinite distance.



STRATEGY

a. The electric potential at the initial position of the third charge is the sum of the potentials due to $+q$ and $+2q$, each at a distance of $\sqrt{2}d$.

b. The initial electric potential energy of the third charge, U_i, is simply its charge, $+3q$, times the potential, V, found in part (a).

c. We can find the final kinetic energy using energy conservation; $U_i + K_i = U_f + K_f$. Because $K_i = 0$ (third charge starts at rest) and $U_f = 0$ (third charge infinitely far away), we find that $K_f = U_i$.

SOLUTION

Part (a)

1. Calculate the net electric potential at the initial position of the third charge:

$$V_i = \frac{k(+q)}{\sqrt{2}d} + \frac{k(+2q)}{\sqrt{2}d} = \frac{3kq}{\sqrt{2}d}$$

Part (b)

2. Multiply V by $(+3q)$ to find the initial electric potential energy, U_i, of the third charge:

$$U_i = (+3q)V_i = (+3q)\frac{3kq}{\sqrt{2}d} = \frac{9kq^2}{\sqrt{2}d}$$

Part (c)

3. Use energy conservation to find the final (infinite separation) kinetic energy:

$$U_i + K_i = U_f + K_f$$
$$\frac{9kq^2}{\sqrt{2}d} + 0 = 0 + K_f$$
$$K_f = \frac{9kq^2}{\sqrt{2}d}$$

INSIGHT

If the third charge had started closer to the other two charges, it would have been higher up on the "potential hill," and hence its kinetic energy at infinity would have been greater, as shown in the following Practice Problem.

PRACTICE PROBLEM

Suppose the third charge is released from rest just above the origin. What is its final kinetic energy in this case? [**Answer:** $K_f = U_i = 9kq^2/d$. As expected, this is greater than $9kq^2/\sqrt{2}d$.]

Some related homework problems: Problem 34, Problem 39

The electric potential energy for a pair of charges, q_1 and q_2, separated by a distance r is $U = kq_1q_2/r$. In systems that contain more than two charges, the total electric potential energy is the sum of terms like $U = kq_1q_2/r$ for each pair of charges in the system. This procedure is illustrated in the following Active Example.

ACTIVE EXAMPLE 20–3 FIND THE ELECTRIC POTENTIAL ENERGY

A system consists of the charges $-q$ at $(-d, 0)$, $+2q$ at $(d, 0)$, and $+3q$ at $(0, d)$. What is the total electric potential energy of the system?

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Write the electric potential energy between $-q$ and $+2q$: $-kq(2q)/2d$
2. Write the electric potential energy between $-q$ and $+3q$: $-kq(3q)/\sqrt{2}d$
3. Write the electric potential energy between $+2q$ and $+3q$: $k(2q)(3q)/\sqrt{2}d$
4. Sum the contributions from each pair of charges to find the total electric potential energy: $U = 3kq^2/\sqrt{2}d - kq^2/d$

YOUR TURN

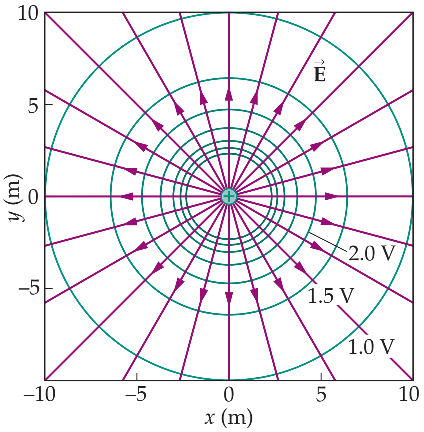
What is the total electric potential energy of this system if the charge $+3q$ is moved from $(0, d)$ to $(0, -d)$?

(Answers to **Your Turn** *problems are given in the back of the book.)*

20–4 Equipotential Surfaces and the Electric Field

A contour map is a useful tool for serious hikers and backpackers. The first thing you notice when looking at a map such as the one shown in Figure 8–13 is a series of closed curves—the contours—each denoting a different altitude. When the contours are closely spaced, the altitude changes rapidly, indicating steep terrain. Conversely, widely spaced contours indicate a fairly flat surface.

A similar device can help visualize the electric potential due to one or more electric charges. Consider, for example, a single positive charge located at the origin. As we saw in the previous section, the electric potential due to this charge approaches zero far from the charge and rises to form an infinitely high "potential hill" near the charge. A three-dimensional representation of the potential is plotted



▲ **FIGURE 20–6 Equipotentials for a point charge**

Equipotential surfaces for a positive point charge located at the origin. Near the origin, where the equipotentials are closely spaced, the potential varies rapidly with distance and the electric field is large. This is a top view of Figure 20–5 (a).

in Figure 20–5. The same potential is shown as a contour map in **Figure 20–6**. In this case, the contours, rather than representing altitude, indicate the value of the potential. Since the value of the potential at any point on a given contour is equal to the value at any other point on the same contour, we refer to the contours as **equipotential surfaces,** or simply, **equipotentials.**

An equipotential plot also contains important information about the magnitude and direction of the electric field. For example, in Figure 20–6 we know that the electric field is more intense near the charge, where the equipotentials are closely spaced, than it is far from the charge, where the equipotentials are widely spaced. This simply illustrates that the electric field, $E = -\Delta V/\Delta s$, depends on the rate of change of the potential with position—the greater the change in potential, ΔV, over a given distance, Δs, the larger the magnitude of E.

The direction of the electric field is given by the minus sign in $E = -\Delta V/\Delta s$. For example, if the electric potential decreases over a distance Δs, it follows that ΔV is negative; $\Delta V < 0$. Thus, $-\Delta V$ is positive, and hence $E > 0$. To summarize:

The electric field points in the direction of decreasing electric potential.

This is also illustrated in Figure 20–6, where we see that the electric field points away from the charge $+q$, in the direction of decreasing electric potential.

Not only does the field point in the direction of decreasing potential, it is, in fact, *perpendicular* to the equipotential surfaces:

The electric field is always perpendicular to the equipotential surfaces.

To see why this is the case, we note that zero work is done when a charge is moved perpendicular to an electric field. That is, the work $W = Fd\cos\theta$ is zero when the angle θ is 90°. If zero work is done, it follows from $\Delta V = -W/q_0$ (Equation 20–2) that there is no change in potential. Therefore, the potential is constant (equipotential) in a direction perpendicular to the electric field.

CONCEPTUAL CHECKPOINT 20–4 EQUIPOTENTIAL SURFACES

Is it possible for equipotential surfaces to intersect?

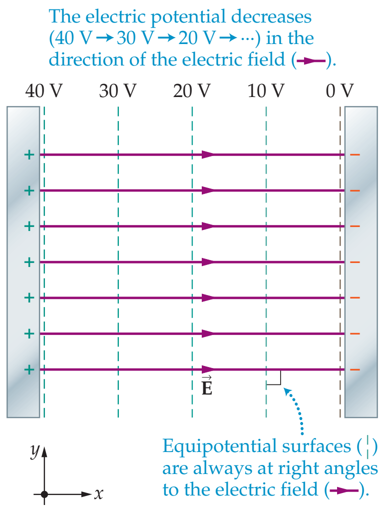
REASONING AND DISCUSSION

To answer this question, it is useful to consider the analogous case of contours on a contour map (Figure 8–13). As we know, each contour corresponds to a different altitude. Because each point on the map has only a single value of altitude, it follows that it is impossible for contours to intersect.

Precisely the same reasoning applies to the electric potential, and hence equipotential surfaces never intersect.

ANSWER

No. Equipotential surfaces cannot intersect.

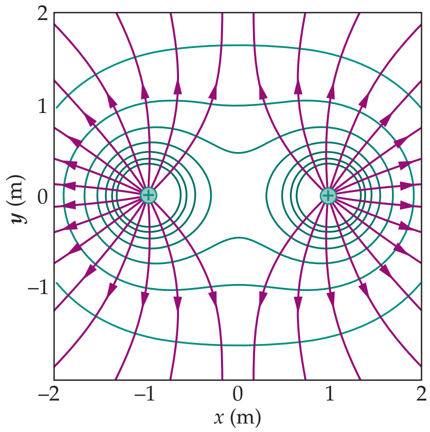


◀ **FIGURE 20–7 Equipotential surfaces for a uniform electric field**

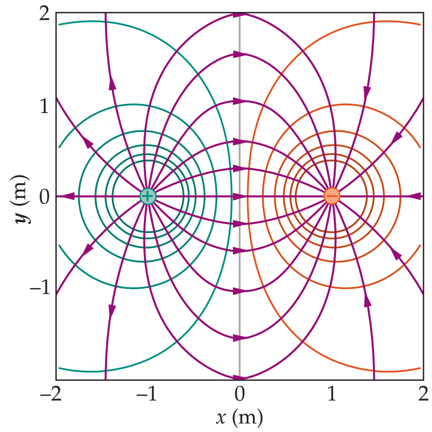
The electric field is always perpendicular to equipotential surfaces, and points in the direction of decreasing electric potential. In this case the electric field is (i) uniform and (ii) horizontal. As a result, (i) the electric potential decreases at a uniform rate, and (ii) the equipotential surfaces are vertical.

The graphical relationship between equipotentials and electric fields can be illustrated with a few examples. We begin with the simple case of a uniform electric field, as between the plates of a parallel-plate capacitor. In **Figure 20–7**, we plot both the electric field lines, which point in the positive x direction, and the corresponding equipotential surfaces. As expected, the field lines are perpendicular to the equipotentials, and $\vec{\mathbf{E}}$ points in the direction of decreasing V.

To see this last result more clearly, note that $\vec{\mathbf{E}} = E\hat{\mathbf{x}}$ in Figure 20–7, from which it follows that $E_x = E > 0$ and $E_y = 0$. Referring to Section 20–1, we see



(a) Equipotentials for two positive charges



(b) Equipotentials for a dipole

FIGURE 20–8 Equipotential surfaces for two point charges

(a) In the case of two equal positive charges, the electric field between them is weak because the field produced by one charge effectively cancels the field produced by the other. As a result, the electric potential is practically constant between the charges. **(b)** For equal charges of opposite sign (a dipole), the electric field is strong between the charges, and the potential changes rapidly.

that if we move in the positive x direction, the change in electric potential is negative, $\Delta V = -E_x\,\Delta x = -E\Delta x < 0$, as expected. On the other hand, if we move in the y direction, the change in potential is zero, $\Delta V = -E_y\,\Delta y = 0$. This is why the equipotentials, which are always perpendicular to $\vec{\mathbf{E}}$, are parallel to the y axis in Figure 20–7.

Figure 20–8 (a) shows a similar plot for the case of two positive charges of equal magnitude. Notice that the electric field lines always cross the equipotentials at right angles. In addition, the electric field is more intense where the equipotential surfaces are closely spaced. In the region midway between the two charges, where the electric field is essentially zero, the potential is practically constant.

Finally, in **Figure 20–8 (b)** we show the equipotentials for two charges of opposite sign, one $+q$ (at $x = -1$ m) and the other $-q$ (at $x = +1$ m), forming an electric dipole. In this case, the amber-color equipotentials denote negative values of V. Teal-color equipotentials correspond to positive V, and the tan-color equipotential has the value $V = 0$. The electric field is nonzero between the charges, even though the potential is zero there—recall that $E = -\Delta V/\Delta s$ is related to the *rate of change of* V, not to its value. The relatively large number of equipotential surfaces between the charges shows that V is indeed *changing rapidly* in that region.

Ideal Conductors

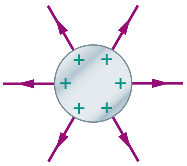
A charge placed on an ideal conductor is free to move. As a result, a charge can be moved from one location on a conductor to another with no work being done. Since the work is zero, the change in potential is also zero; therefore, every point on or within an ideal conductor is at the same potential:

> Ideal conductors are equipotential surfaces; every point on or within such a conductor is at the same potential.

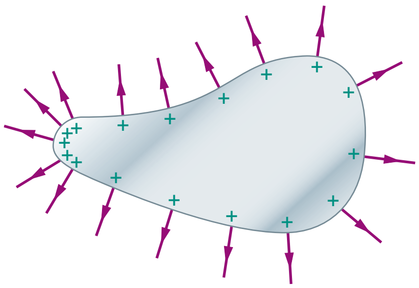
For example, when a charge Q is placed on a conductor, as in **Figure 20–9**, it distributes itself over the surface in such a way that the potential is the same everywhere. If the conductor has the shape of a sphere, the charge spreads uniformly over its surface, as in Figure 20–9 (a). If, however, the conductor has a sharp end and a blunt end, as in Figure 20–9 (b), the charge is more concentrated near the sharp end, which results in a large electric field. We pointed out this effect earlier, in Section 19–6.

To see why this should be the case, consider a conducting sphere of radius R with a charge Q distributed uniformly over its surface, as in **Figure 20–10 (a)**. The charge density on this sphere is $\sigma = Q/4\pi R^2$, and the potential at its surface is the same as for a point charge Q at the center of the sphere; that is, $V = kQ/R$. Writing this in terms of the surface charge density, we have

$$V = \frac{kQ}{R} = \frac{k\sigma(4\pi R^2)}{R} = 4\pi k\sigma R$$



(a) Uniform sphere, uniform charge distribution



(b) Charge is concentrated at pointed end

FIGURE 20–9 Electric charges on the surface of ideal conductors

(a) On a spherical conductor, the charge is distributed uniformly over the surface. **(b)** On a conductor of arbitrary shape, the charge is more concentrated, and the electric field is more intense, where the conductor is more sharply curved. Note that in all cases the electric field is perpendicular to the surface of the conductor.

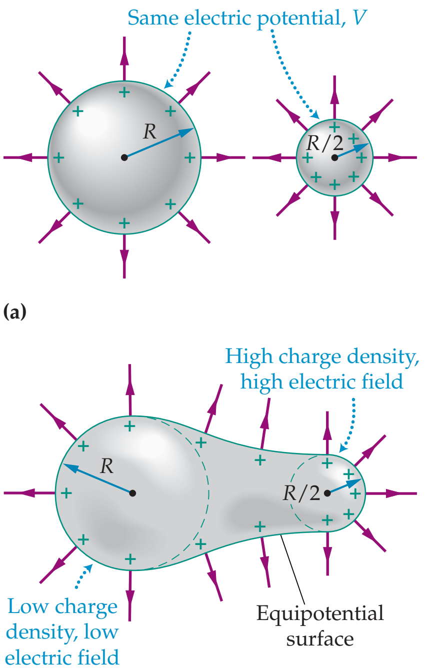


FIGURE 20–10 Charge concentration near points
(a) If two spheres of different radii have the same electric potential at their surfaces, the sphere with the smaller radius of curvature has the greater charge density and the greater electric field. **(b)** An arbitrarily shaped conductor can be approximated by spheres with the same potential at the surface and varying radii of curvature. It follows that the more sharply curved end of a conductor has a greater charge density and a more intense field.

Now, consider a sphere of radius $R/2$. For this sphere to have the *same potential* as the large sphere, it must have twice the charge density, 2σ. In particular, letting σ go to 2σ and R go to $R/2$ in the expression $V = 4\pi k\sigma R$ yields the same result as for the large sphere:

$$V = 4\pi k(2\sigma)(R/2) = 4\pi k\sigma R$$

Clearly, then, the smaller the radius of a sphere with potential V, the greater its charge density.

The electric field is greater for the small sphere as well. At the surface of the large sphere in Figure 20–10 (a) the electric field has a magnitude given by

$$E = \frac{kQ}{R^2} = \frac{k\sigma(4\pi R^2)}{R^2} = 4\pi k\sigma$$

The small sphere in Figure 20–10 (a) has twice the charge density, hence it has twice the electric field at its surface.

The relevance of these results for a conductor of arbitrary shape can be seen in **Figure 20–10 (b)**. Here we show that even an arbitrarily shaped conductor has regions that approximate a spherical surface. In this case, the left end of the conductor is approximately a portion of a sphere of radius R, and the right end is approximately a sphere of radius $R/2$. Noting that every point on a conductor is at the *same potential*, we see that the situation in Figure 20–10 (b) is similar to that in Figure 20–10 (a). It follows that the sharper end of the conductor has the greater charge density and the greater electric field. If a conductor has a sharp point, as in a lightning rod, the corresponding radius of curvature is very small, which can result in an enormous electric field. We consider this possibility in more detail in the next section.

Finally, we note that because the surface of an ideal conductor is an equipotential, the electric field must meet the surface at right angles. This is also shown in Figures 20–9 and 20–10.

Electric Potential and the Human Body

REAL-WORLD PHYSICS: BIO
Electrocardiograph

The human body is a relatively good conductor of electricity, but it is not an ideal conductor. If it were, the entire body would be one large equipotential surface. Instead, muscle activity, the beating of the heart, and nerve impulses in the brain all lead to slight differences in electric potential from one point on the skin to another.

In the case of the heart, the powerful waves of electrical activity as the heart muscles contract result in potential differences that are typically in the range of 1 mV. These potential differences can be detected and displayed with an instrument known as an **electrocardiograph,** abbreviated ECG or EKG. (The K derives from the original Dutch name for the device.) A typical EKG signal is displayed in **Figure 20–11 (a)**.

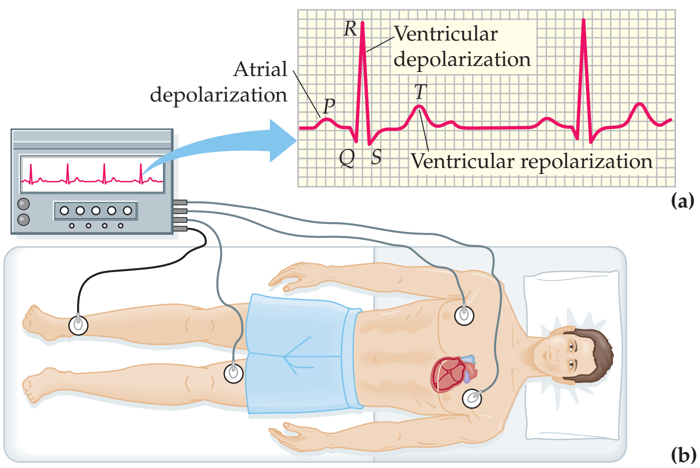
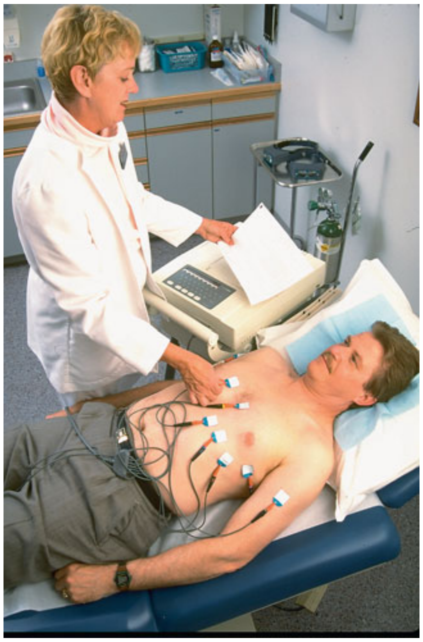


FIGURE 20–11 The electrocardiograph
(a) A typical electrocardiograph tracing, with the major features labeled. The EKG records the electrical activity that accompanies the rhythmic contraction and relaxation of heart muscle tissue. The main pumping action of the heart is associated with the *QRS* complex: contraction of the ventricles begins just after the *R* peak. **(b)** The simplest arrangement of electrodes, or "leads," for an EKG. More precise information can be obtained by the use of additional leads—typically, a dozen in modern practice.

To understand the various features of an EKG signal, we note that a heartbeat begins when the heart's natural "pacemaker," the sinoatrial (SA) node in the right atrium, triggers a wave of muscular contraction across both atria that pumps blood into the ventricles. This activity gives rise to the pulse known as the *P wave* in Figure 20–11 (a). Following this contraction, the atrioventricular (AV) node, located between the ventricles, initiates a more powerful wave of contraction in the ventricles, sending oxygen-poor blood to the lungs and oxygenated blood to the rest of the body. These events are reflected in the series of pulses known as the *QRS complex*. Other features in the EKG signal can be interpreted as well. For example, the *T wave* is associated with repolarization of the ventricles in preparation for the next contraction cycle. Even small irregularities in the magnitude, shape, sequence, or timing of these features can provide an experienced physician with essential clues for the diagnosis of many cardiac abnormalities and pathologies. For example, an inverted *T* wave often indicates inadequate blood supply to the heart muscle (ischemia), possibly caused by a heart attack (myocardial infarction).

To record these signals, electrodes are attached to the body in a variety of locations. In the simplest case, illustrated in **Figure 20–11 (b)**, three electrodes are connected in an arrangement known as *Einthoven's triangle*, after the Dutch pioneer of these techniques. Two of these three electrodes are attached to the shoulders, the third to the left groin. For convenience, the two shoulder electrodes are often connected to the wrists, and the groin electrode is connected to the left ankle—the signal at these locations is practically the same as for the locations in Figure 20–11 (b) because of the high conductivity of the human limbs. By convention, the electrode on the right leg is used as the ground. In current applications of the EKG, it is typical to use 12 electrodes to gain more detailed information about the heart's activity.

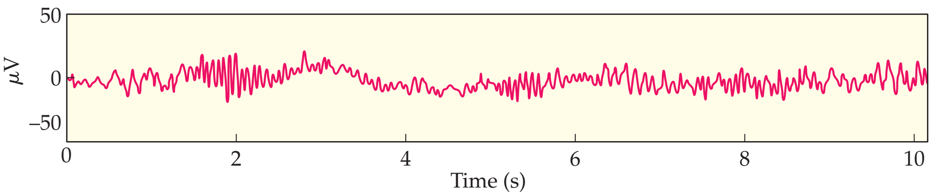


▲ An electrocardiograph can be made with as few as three electrodes and a ground. However, more detailed information about the heart's electrical activity can be obtained by using additional "leads" at precise anatomical locations. Modern EKGs often use a 12-lead array.

REAL-WORLD PHYSICS: BIO

The electroencephalograph

Electrical activity in the brain can be detected and displayed with an **electroencephalograph** or EEG. In a typical application of this technique, a regular array of 8 to 16 electrodes is placed around the head. The potential differences in this case—in the range of 0.1 mV—are much smaller than those produced by the heart, and much more complex to interpret. One of the key characteristics of an EEG signal, such as the one shown in **Figure 20–12**, is the frequency of the waves. For example, waves at 0.5 to 3.5 Hz, referred to as *D waves*, are common during sleep. A relaxed brain produces *a waves*, with frequencies in the range of 8 to 13 Hz, and an alert brain generates *b waves*, with frequencies greater than 13 Hz. Finally, *q waves*—with frequencies of 5 to 8 Hz—are common in newborns but indicate severe stress in adults.



◀ **FIGURE 20–12 The electroencephalograph**
A typical EEG signal.

20–5 Capacitors and Dielectrics

A **capacitor** gets its name from the fact that it has a *capacity* to store both electric charge and energy. Capacitors are a common and important element in modern electronic devices. They can provide large bursts of energy to a circuit, or protect delicate circuitry from excess charge originating elsewhere.

In general, a capacitor is nothing more than two conductors, referred to as *plates*, separated by a finite distance. When the plates of a capacitor are connected to the terminals of a battery, they become charged—one plate acquires a charge $+Q$, the other an equal and opposite charge, $-Q$. The greater the charge Q for a given voltage V, the greater the **capacitance,** C, of the capacitor.

To be specific, suppose a certain battery produces a potential difference of V volts between its terminals. When this battery is connected to a capacitor, a charge of magnitude Q appears on each plate. If a different battery, with a voltage of $2V$, is connected to the same capacitor, the charge on the plates doubles in magnitude to $2Q$. Thus, the charge Q is proportional to the applied voltage V. We define the constant of proportionality to be the capacitance C:

$$Q = CV$$

Solving for the capacitance, we have

Definition of Capacitance, *C*

$$C = \frac{Q}{V} \qquad 20\text{–}9$$

SI unit: coulomb/volt = farad, F

In this expression, Q is the magnitude of the charge on either plate, and V is the magnitude of the voltage difference between the plates. By definition, then, the capacitance is always a positive quantity.

As we can see from the relation $C = Q/V$, the units of capacitance are coulombs per volt. In the SI system this combination of units is referred to as the farad (F), in honor of the English physicist Michael Faraday (1791–1867), a pioneering researcher into the properties of electricity and magnetism. In particular,

$$1\text{ F} = 1\text{ C/V} \qquad 20\text{–}10$$

Just as the coulomb is a rather large unit of charge, so too is the farad a rather large unit of capacitance. More typical values for capacitance are in the picofarad ($1\text{ pF} = 10^{-12}\text{ F}$) to microfarad ($1\ \mu\text{F} = 10^{-6}\text{ F}$) range.

EXERCISE 20–3

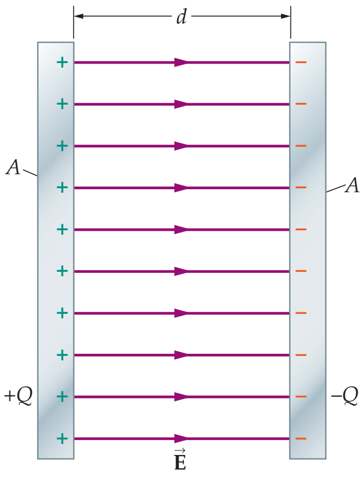
A capacitor of $0.75\ \mu\text{F}$ is charged to a voltage of 16 V. What is the magnitude of the charge on each plate of the capacitor?

SOLUTION

Using $Q = CV$, we find

$$Q = CV = (0.75 \times 10^{-6}\text{ F})(16\text{ V}) = 1.2 \times 10^{-5}\text{ C}$$

A bucket of water provides a useful analogy when thinking about capacitors. In this analogy we make the following identifications: (i) The cross-sectional area of the bucket is the capacitance C; (ii) the amount of water in the bucket is the charge Q; and (iii) the depth of the water is the voltage difference V between the plates. Therefore, just as a wide bucket can hold more water than a narrow bucket—when filled to the same level—a large capacitor can hold more charge than a small capacitor when they both have the same potential difference. Charging a capacitor, then, is like pouring water into a bucket—if the capacitance is large, a large amount of charge can be placed on the plates with little potential difference between them.



▲ **FIGURE 20–13 A parallel-plate capacitor**

A parallel-plate capacitor, with plates of area A, separation d, and charges of magnitude Q. The capacitance of such a capacitor is $C = \varepsilon_0 A/d$.

Parallel-Plate Capacitor

A particularly simple capacitor is the parallel-plate capacitor, first introduced in Section 19–5. In this device, two parallel plates of area A are separated by a distance d, as indicated in **Figure 20–13**. We would like to determine the capacitance of such a capacitor. As we shall see, the capacitance is related in a simple way to the parameters A and d.

To begin, we note that when a charge $+Q$ is placed on one plate, and a charge $-Q$ is placed on the other, a uniform electric field is produced between the plates. The magnitude of this field is given by Gauss's law, and was determined in Active Example 19–3. There we found that $E = \sigma/\varepsilon_0$. Noting that the charge per area is $\sigma = Q/A$, we have

$$E = \frac{Q}{\varepsilon_0 A} \qquad 20\text{–}11$$

The corresponding potential difference between the plates is $\Delta V = -E\Delta s = -(Q/\varepsilon_0 A)d$. The magnitude of this potential difference is simply $V = (Q/\varepsilon_0 A)d$.

Now that we have determined the potential difference for a parallel-plate capacitor with a charge Q on its plates, we can apply $C = Q/V$ to find an expression for the capacitance:

$$C = \frac{Q}{V} = \frac{Q}{(Q/\varepsilon_0 A)d}$$

Canceling the charge Q and rearranging slightly give the final result:

Capacitance of a Parallel-Plate Capacitor

$$C = \frac{\varepsilon_0 A}{d} \qquad 20\text{–}12$$

As mentioned previously, the capacitance depends in a straightforward way on the area of the plates, A, and their separation, d. The capacitance does not depend separately on the amount of charge on the plates, Q, or the potential difference between the plates, V, but instead depends only on the *ratio* of these two quantities. We use this expression for capacitance in the next Example.

PROBLEM-SOLVING NOTE

Using Magnitudes with Capacitance

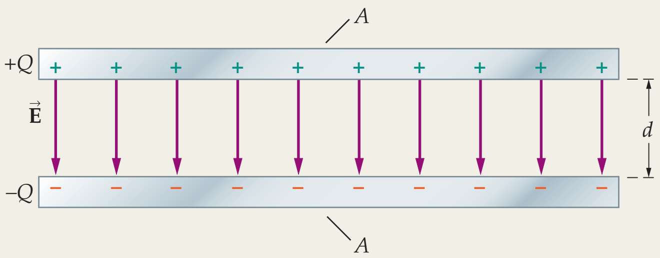
Note that the capacitance C is always positive. Therefore, when applying a relation like $C = Q/V$, we always use magnitudes for the charge and the electric potential.

EXAMPLE 20–5 ALL CHARGED UP

A parallel-plate capacitor is constructed with plates of area $0.0280\ \text{m}^2$ and separation 0.550 mm. **(a)** Find the magnitude of the charge on each plate of this capacitor when the potential difference between the plates is 20.1 V. **(b)** What is the magnitude of the electric field between the plates of the capacitor?

PICTURE THE PROBLEM

The capacitor is shown in our sketch. Note that the plates of the capacitor have an area $A = 0.0280\ \text{m}^2$, a separation $d = 0.550$ mm, and a potential difference $V = 20.1$ V. The charge on each plate has a magnitude Q. The electric field is uniform, and points from the positive plate to the negative plate.



STRATEGY

a. The charge on the plates is given by $Q = CV$. We know the potential difference, V, but we must determine the capacitance, C. We can do this using the relation $C = \varepsilon_0 A/d$, along with the given information for A and d.

b. We can find the magnitude of the electric field with Equation 20–11; that is, $E = Q/\varepsilon_0 A$. As in part (a), the charge on the plates is $Q = CV$. In addition, using the symbolic expression $C = \varepsilon_0 A/d$ will allow us to simplify the final result for E.

SOLUTION

Part (a)

1. Calculate the capacitance of the capacitor:

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0280\ \text{m}^2)}{0.550 \times 10^{-3}\ \text{m}}$$
$$= 4.51 \times 10^{-10}\ \text{F}$$

2. Find the charge on the plates of the capacitor:

$$Q = CV = (4.51 \times 10^{-10}\ \text{F})(20.1\ \text{V}) = 9.06 \times 10^{-9}\ \text{C}$$

Part (b)

3. Calculate the magnitude of the electric field using $E = Q/\varepsilon_0 A$. Use symbols to simplify the final expression:

$$E = \frac{Q}{\varepsilon_0 A} = \frac{CV}{\varepsilon_0 A} = \frac{(\varepsilon_0 A/d)V}{\varepsilon_0 A} = \frac{V}{d}$$

4. Substitute numerical values:

$$E = \frac{V}{d} = \frac{20.1\ \text{V}}{0.550 \times 10^{-3}\ \text{m}} = 36{,}500\ \text{V/m}$$

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INSIGHT

Notice the relatively small values for both the capacitance, C, and the magnitude of the charge on a plate, Q. The total amount of charge in the capacitor is zero, $+Q + (-Q) = 0$, but the fact that there is a charge separation—with one plate positive and the other negative—means that the capacitor stores energy, as we shall see in the next section.

If a capacitor is connected to a battery, the battery maintains a constant voltage between the plates of the capacitor. It follows from $Q = CV$ that any change in the capacitance—by changing the plate area, A, or separation, d, for example—results in a different amount of charge on the plates.

Finally, by simplifying the expression for E, and concerning ourselves only with magnitudes, we have obtained a result that is consistent with Equation 20–4, with $\Delta V = V$ and $\Delta s = d$.

PRACTICE PROBLEM

What separation d is necessary to give an increased charge of magnitude 2.00×10^{-8} C on each plate of this capacitor? Assume all other quantities in the system remain the same. [**Answer:** $d = 0.249$ mm. Note that a *smaller* separation results in a *greater* amount of stored charge.]

Some related homework problems: Problem 50, Problem 51

We see from Equation 20–12 that the capacitance of a parallel-plate capacitor increases with the area A of its plates—basically, the greater the area, the more room there is in the capacitor to hold charge. Again, this is like pouring water into a bucket with a large cross-sectional area. On the other hand, the capacitance decreases with increasing separation d. The reason for this dependence is that with the electric field between the plates constant (as we saw in Section 19–5), the potential difference between the plates is proportional to their separation: $V = Ed$. Thus, the capacitance, which is inversely proportional to the potential difference $(C = Q/V)$, is also inversely proportional to the plate separation—the greater the separation, the greater the potential difference required to store a given amount of charge. All capacitors, regardless of their design, share these general features. Thus, to produce a large capacitance, one would like to have plates of large area close together. This is often accomplished by inserting a thin piece of paper between two large sheets of metal foil. The foil is then rolled up tightly to form a compact, large-capacity capacitor. Examples of common capacitors are shown in **Figure 20–14**.

▶ **FIGURE 20–14 Capacitors**
Capacitors come in a variety of physical forms (left), with many different types of dielectric (insulating material) occupying the space between their plates. Capacitors are also rated for the maximum voltage that can be applied to them before the dielectric breaks down, allowing a spark to jump across the gap between the plates. A variable air capacitor (right), often used in early radios, has interleaved plates that can be rotated to vary their area of overlap. This changes the effective area of the capacitor, and hence its capacitance.

CONCEPTUAL CHECKPOINT 20–5 CHARGE ON THE PLATES

A parallel-plate capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the plates are pulled away from each other, increasing their separation, does the magnitude of the charge on the plates **(a)** increase, **(b)** decrease, or **(c)** remain the same?

REASONING AND DISCUSSION

Since the capacitance of a parallel-plate capacitor is $C = \varepsilon_0 A/d$, increasing the separation, d, decreases the capacitance. With a smaller value of C, and a constant value for V, the charge $Q = CV$ will decrease. The same general behavior can be expected with any capacitor.

ANSWER

(b) The charge on the plates decreases.

Sometimes a capacitor is first connected to a battery to be charged and is then disconnected. In this case, the charge on the plates is "trapped"—it has no place to go—and hence Q must remain constant. If the capacitance is changed now, the result is a different potential difference, $V = Q/C$, between the plates.

Dielectrics

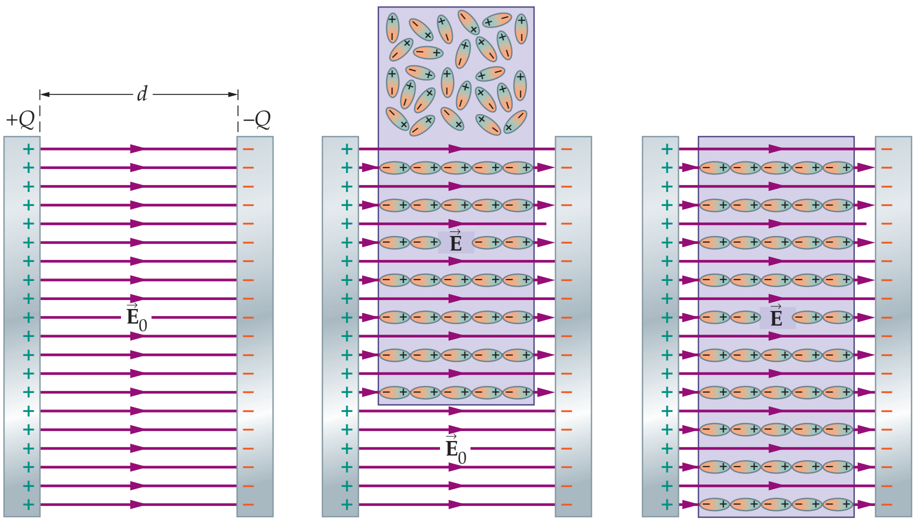
One way to increase the capacitance of a capacitor is to insert an insulating material, referred to as a **dielectric,** between its plates. With a dielectric in place, a capacitor can store more charge or energy in the same volume. Thus, dielectrics play an important role in miniaturizing electronic devices.

To see how this works, consider the parallel-plate capacitor shown in **Figure 20–15 (a)**. Initially the plates are separated by a vacuum and connected to a battery, giving the plates the charges $+Q$ and $-Q$. The battery is now removed, and the charge on the plates remains constant. The electric field between the plates is uniform and has a magnitude E_0. If the distance between the plates is d, the corresponding potential difference is $V_0 = E_0 d$, and the capacitance is

$$C_0 = \frac{Q}{V_0}$$

Now, insert a dielectric slab, as illustrated in **Figures 20–15 (b)** and **(c)**. If the molecules in the dielectric have a permanent dipole moment, they will align with the field, as shown in Figures 20–15 (b). Even without a permanent dipole moment, however, the molecules will become polarized by the field (see Section 19–1). This polarization leads to the same type of alignment, although the effect is weaker. The result of this alignment is a positive charge on the surface of the slab near the negative plate and a negative charge on the surface near the positive plate.

Recalling that electric field lines terminate on negative charges and start on positive charges, we can see from Figure 20–15 (c) that fewer field lines exist within



(a) Capacitor without dielectric

(b) Inserting a dielectric

(c) A dielectric reduces the electric field within a capacitor

FIGURE 20–15 The effect of a dielectric on the electric field of a capacitor
When a dielectric is placed in the electric field between the plates of a capacitor, the molecules of the dielectric tend to become oriented with their positive ends pointing toward the negatively charged plate and their negative ends pointing toward the positively charged plate. The result is a buildup of positive charge on one surface of the dielectric and of negative charge on the other. Since field lines start on positive charges and end on negative charges, we see that the number of field lines within the dielectric is reduced. Thus, within the dielectric the applied electric field $\vec{E}_0$ is partially canceled. Because the strength of the electric field is less, the voltage between the plates is less as well. Since V is smaller while Q remains the same, the capacitance, $C = Q/V$, is increased by the dielectric.

the dielectric. Consequently, there is a reduced field, E, in a dielectric, which we characterize with a dimensionless **dielectric constant,** κ, as follows:

$$E = \frac{E_0}{\kappa} \qquad 20\text{–}13$$

TABLE 20–1 Dielectric Constants

Substance	Dielectric constant, κ
Water	80.4
Neoprene rubber	6.7
Pyrex glass	5.6
Mica	5.4
Paper	3.7
Mylar	3.1
Teflon	2.1
Air	1.00059
Vacuum	1

In the case of a vacuum, $\kappa = 1$, and $E = E_0$, as before. For an insulating material, however, the value of κ is greater than one. For example, paper has a dielectric constant of roughly 4, which means that the electric field within paper is about one-quarter what it would be in a vacuum. Typical values of κ are listed in Table 20–1.

Thus, a dielectric reduces the field between the plates of a capacitor by a factor of κ. This, in turn, decreases the *potential difference* between the plates by the same factor:

$$V = Ed = \left(\frac{E_0}{\kappa}\right)d = \frac{E_0 d}{\kappa} = \frac{V_0}{\kappa}$$

Finally, since the potential difference is smaller, the capacitance must be larger:

$$C = \frac{Q}{V} = \frac{Q}{(V_0/\kappa)} = \kappa\frac{Q}{V_0} = \kappa C_0 \qquad 20\text{–}14$$

In effect, the dielectric partially shields one plate from the other, making it easier to build up a charge on the plates. If the space between the plates of a capacitor is filled with paper, for example, the capacitance will be about four times larger than if the space had been a vacuum.

The relation $C = \kappa C_0$ applies to any capacitor. For the special case of a parallel-plate capacitor filled with a dielectric, we have

PROBLEM-SOLVING NOTE

The Effects of a Dielectric

Dielectrics reduce the electric field in a capacitor, which results in a reduced potential difference between the plates. As a result, a dielectric always increases the capacitance.

Capacitance of a Parallel-Plate Capacitor Filled with a Dielectric

$$C = \frac{\kappa\varepsilon_0 A}{d} \qquad 20\text{–}15$$

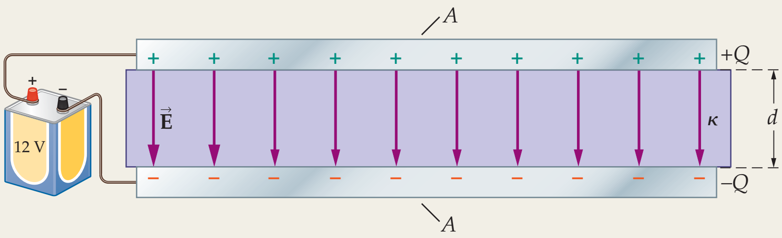
We apply this relation in the next Example.

EXAMPLE 20–6 EVEN MORE CHARGED UP

A parallel-plate capacitor is constructed with plates of area 0.0280 m^2 and separation 0.550 mm. The space between the plates is filled with a dielectric with dielectric constant κ. When the capacitor is connected to a 12.0-V battery, each of the plates has a charge of magnitude 3.62×10^{-8} C. What is the value of the dielectric constant, κ?

PICTURE THE PROBLEM

The sketch shows the capacitor with a dielectric material inserted between the plates. In other respects, the capacitor is the same as the one considered in Example 20–5.



STRATEGY

Since we are given the potential difference V and the charge Q, we can find the capacitance using $C = Q/V$. Next, we relate the capacitance to the physical characteristics of the capacitor with $C = \kappa\varepsilon_0 A/d$. Using the given values for A and d, we solve for κ.

SOLUTION

1. Determine the value of the capacitance:
$$C = \frac{Q}{V} = \frac{(3.62 \times 10^{-8}\ \text{C})}{12.0\ \text{V}} = 3.02 \times 10^{-9}\ \text{F}$$

2. Solve $C = \kappa\varepsilon_0 A/d$ for the dielectric constant, κ:
$$C = \kappa\varepsilon_0 A/d$$
$$\kappa = Cd/\varepsilon_0 A$$

3. Substitute numerical values to find κ:
$$\kappa = \frac{Cd}{\varepsilon_0 A}$$
$$= \frac{(3.02 \times 10^{-9}\ \text{F})(0.550 \times 10^{-3}\ \text{m})}{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.0280\ \text{m}^2)} = 6.70$$

INSIGHT

Comparing our result with the dielectric constants given in Table 20–1, we see that the dielectric may be neoprene rubber.

PRACTICE PROBLEM

If a different dielectric with a smaller dielectric constant is inserted into the capacitor, does the charge on the plates increase, decrease, or remain the same? Find the charge on the plates for $\kappa = 3.5$. [**Answer:** The charge decreases to $Q = 1.89 \times 10^{-8}$ C.]

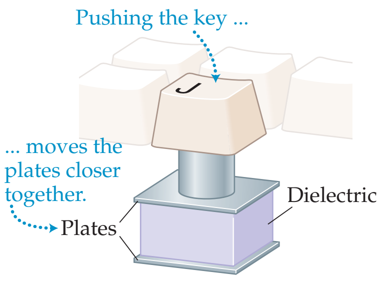
Some related homework problems: Problem 53, Problem 54

The fact that the capacitance of a capacitor depends on the separation of its plates finds a number of interesting applications. For example, if you have ever typed on a computer keyboard, you have probably been utilizing the phenomenon of capacitance without realizing it. Many computer keyboards are designed in such a way that each key is connected to the upper plate of a parallel-plate capacitor, as illustrated in **Figure 20–16**. When you depress a given key, the separation between the plates of that capacitor decreases, and the corresponding capacitance increases. The circuitry of the computer can detect this change in capacitance, thereby determining which key you have pressed.

Another, less well-known application of capacitance is the theremin, a musical instrument that you play without touching! Two antennas on the theremin are used to control the sound it makes; one antenna adjusts the volume, the other adjusts the pitch. When a person places a hand near one of the antennas, the effect is similar to that of a parallel-plate capacitor, with the hand playing the role of one plate and the antenna playing the role of the other plate. Changing the separation between hand and antenna changes the capacitance, which the theremin's circuitry then converts into a corresponding change of volume or pitch. Theremins have been used to provide "ethereal" music for a number of science fiction films, and some popular bands use theremins in their musical arrangements.

REAL-WORLD PHYSICS

Computer keyboards



▲ **FIGURE 20–16 Capacitance and the computer keyboard**

The keys on many computer keyboards form part of a parallel-plate capacitor. Depressing the key changes the plate separation. The corresponding change in capacitance can be detected by the computer's circuitry.

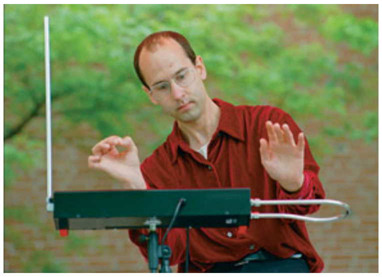
Dielectric Breakdown

If the electric field applied to a dielectric is large enough, it can literally tear the atoms apart, allowing the dielectric to conduct electricity. This condition is referred to as **dielectric breakdown.** The maximum field a dielectric can withstand before breakdown is called the **dielectric strength.** Typical values are given in Table 20–2.

For example, if the electric field in air exceeds about 3,000,000 V/m, dielectric breakdown will occur, leading to a spark on a small scale or a bolt of lightning on a larger scale. Next time you walk across a carpet and get a shock when reaching for the doorknob, think about the fact that you have just produced an electric field of roughly *3 million volts per meter!* The sharp tip of a lightning rod, which has a high electric field in its vicinity, helps initiate and guide lightning to the ground, or to dissipate charge harmlessly so that no lightning occurs at all. Saint Elmo's fire—the glow of light around the rigging of a ship in a storm—is another example of dielectric breakdown in air.

REAL-WORLD PHYSICS

The theremin—a musical instrument you play without touching



▲ A musician plays a theremin at an outdoor concert.

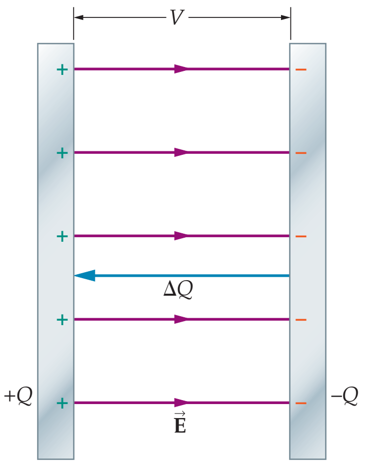
20–6 Electrical Energy Storage

As mentioned in the previous section, capacitors store more than just charge—they also store energy. To see how, consider a capacitor that has charges of magnitude Q on its plates, and a potential difference of V. Now, imagine transferring a small amount of charge, ΔQ, from one plate to the other, as in **Figure 20–17**. Since this charge must be moved across a potential difference of V, the change in electric potential energy is $\Delta U = (\Delta Q)V$. Thus, the potential energy of the capacitor increases by $(\Delta Q)V$ when the magnitude of the charge on its plates is increased from Q to $Q + \Delta Q$. As more charge is transferred from one plate to the other, more electric potential energy is stored in the capacitor.

To find the total electric energy stored in a capacitor, we must take into account the fact that the potential difference between the plates increases as the charge on

TABLE 20–2 Dielectric Strengths

Substance	Dielectric Strength (V/m)
Mica	100×10^6
Teflon	60×10^6
Paper	16×10^6
Pyrex glass	14×10^6
Neoprene rubber	12×10^6
Air	3.0×10^6



▲ **FIGURE 20–17 The energy required to charge a capacitor**

A capacitor has a charge of magnitude Q on its plates and a potential difference V between the plates. Transferring a small charge increment, $+\Delta Q$, from the negative plate to the positive plate increases the electric potential energy of the capacitor by the amount $\Delta U = (\Delta Q)V$.

the plates increases. In fact, recalling that the potential difference is given by $V = Q/C$, it is clear that V increases linearly with the charge, as illustrated in **Figure 20–18**. In particular, if the final potential difference is V, the average potential during charging is $\frac{1}{2}V$. Therefore, the total energy U stored in a capacitor with charge Q and potential difference V can be written as follows:

$$U = QV_{av} = \tfrac{1}{2}QV \qquad 20\text{–}16$$

Equivalently, since $Q = CV$, the energy stored in a capacitor of capacitance C and voltage V is

$$U = \tfrac{1}{2}CV^2 \qquad 20\text{–}17$$

Finally, using $V = Q/C$, we find that the energy stored in a capacitor of charge Q and capacitance C is

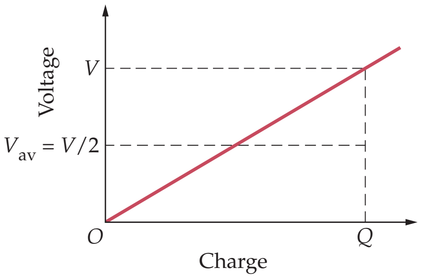
$$U = \frac{Q^2}{2C} \qquad 20\text{–}18$$

All these expressions are equivalent; they simply give the energy in terms of different variables.

REAL-WORLD PHYSICS

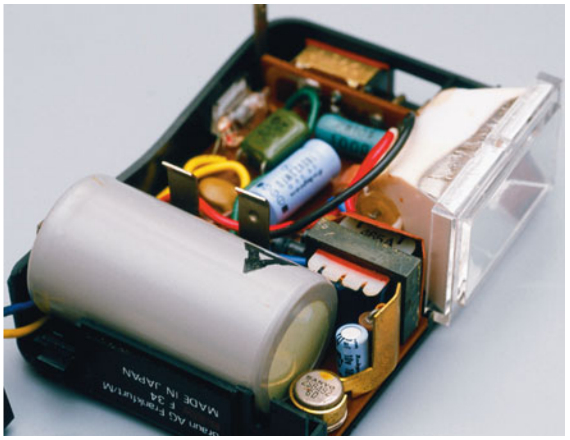
The electronic flash

The energy stored in a capacitor can be put to a number of practical uses. Any time you take a flash photograph, for example, you are triggering the rapid release of energy from a capacitor. The flash unit typically contains a capacitor with a capacitance of 100 to 400 μF. When fully charged to a voltage of about 300 V, the capacitor contains roughly 15 J of energy. Activating the flash causes the stored energy, which took several seconds to accumulate, to be released in less than a millisecond. Because of the rapid release of energy, the power output of a flash unit is impressively large—about 10 to 20 kW. This is far in excess of the power provided by the battery that operates the unit. Similar considerations apply to the defibrillator used in the treatment of heart attack victims, as we show in the next Example.



▲ **FIGURE 20–18 The voltage of a capacitor being charged**

The voltage V between the plates of a capacitor increases linearly with the charge Q on the plates, $V = Q/C$. Therefore, if a capacitor is charged to a final voltage of V, the average voltage during charging is $V_{av} = \frac{1}{2}V$.



▲ An electronic flash unit like the one at left includes a capacitor (gray) that can store a large amount of charge. When the charge is released, the resulting flash can be as brief as a millisecond or less, allowing photographers to "freeze" motion, as in the photo at right. Even faster strobe units can be used to photograph explosions, shock waves, or speeding bullets.

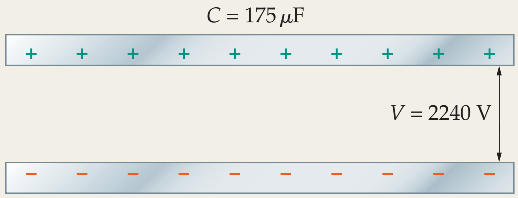
EXAMPLE 20–7 THE DEFIBRILLATOR: DELIVERING A SHOCK TO THE SYSTEM

REAL WORLD PHYSICS: BIO

When a person's heart undergoes ventricular fibrillation—a rapid, uncoordinated twitching of the heart muscles—it often takes a strong jolt of electrical energy to restore the heart's regular beating and save the person's life. The device that delivers this jolt of energy is known as a defibrillator, and it uses a capacitor to store the necessary energy. In a typical defibrillator, a 175-μF capacitor is charged until the potential difference between the plates is 2240 V. **(a)** What is the magnitude of the charge on each plate of the fully charged capacitor? **(b)** Find the energy stored in the charged-up defibrillator.

PICTURE THE PROBLEM

Our sketch shows a simplified representation of a capacitor. The values of the capacitance and the potential difference are indicated.



STRATEGY

a. We can find the charge stored on the capacitor plates using $Q = CV$.

b. The energy stored in the capacitor can be determined immediately using $U = \frac{1}{2}CV^2$. In addition, now that we know the charge on each plate of the capacitor, the energy can also be found with the relations $U = \frac{1}{2}QV$ and $U = Q^2/2C$.

SOLUTION

Part (a)

1. Use $Q = CV$ to find the charge on the plates: $Q = CV = (175 \times 10^{-6}\ \text{F})(2240\ \text{V}) = 0.392\ \text{C}$

Part (b)

2. Find the stored energy using $U = \frac{1}{2}CV^2$: $U = \frac{1}{2}CV^2 = \frac{1}{2}(175 \times 10^{-6}\ \text{F})(2240\ \text{V})^2 = 439\ \text{J}$

3. As a check, use $U = \frac{1}{2}QV$: $U = \frac{1}{2}QV = \frac{1}{2}(0.392\ \text{C})(2240\ \text{V}) = 439\ \text{J}$

4. Finally, use the relation $U = Q^2/2C$: $U = \dfrac{Q^2}{2C} = \dfrac{(0.392\ \text{C})^2}{2(175 \times 10^{-6}\ \text{F})} = 439\ \text{J}$

INSIGHT

Of the 439 J stored in the defibrillator's capacitor, typically about 200 J will actually pass through the person's body in a pulse lasting about 2 ms. The power delivered by the pulse is approximately $P = U/t = (200\ \text{J})/(0.002\ \text{s}) = 100\ \text{kW}$. This is significantly larger than the power delivered by the battery, which can take up to 30 s to fully charge the capacitor.

PRACTICE PROBLEM

Suppose the defibrillator is "fired" when the voltage is only half its maximum value of 2240 V. How much energy is stored in this case? [**Answer:** $E = (439\ \text{J})/4 = 110\ \text{J}$]

Some related homework problems: Problem 64, Problem 69

REAL-WORLD PHYSICS: BIO

Capacitor hazards

A defibrillator uses a capacitor to deliver a shock to a person's heart, restoring it to normal function. Capacitors can have the opposite effect as well, and it is for this reason that they can be quite dangerous, even in electrical devices that are turned off and unplugged from the wall. For example, a television set contains a number of capacitors, some of which store significant amounts of charge and energy. When a TV is unplugged, the capacitors retain their charge for long periods of time. Therefore, if you reach into the back of an unplugged television set there is a danger that you may come in contact with the terminals of a capacitor, which would then discharge its stored energy through your body. The resulting shock could be harmful or even fatal.

Finally, we have discussed many examples of energy stored in a capacitor, but where exactly is the energy located? The answer is that the energy can be thought of as stored in the electric field, E, between the plates. To be specific, consider the relation

$$\text{energy} = \tfrac{1}{2}QV$$

In the case of a parallel-plate capacitor of area A and separation d, we know that $Q = \varepsilon_0 EA$ (Equation 20–11) and $V = Ed$. Thus, the energy stored in the capacitor can be written as

$$U = \text{energy} = \tfrac{1}{2}(\varepsilon_0 EA)(Ed) = \tfrac{1}{2}\varepsilon_0 E^2(Ad)$$

We have grouped A and d together because the product Ad is simply the total volume between the plates. Therefore, the **energy density** (energy per volume) is given by the following:

$$u_E = \text{electric energy density} = \frac{\text{electric energy}}{\text{volume}} = \tfrac{1}{2}\varepsilon_0 E^2 \qquad \text{20–19}$$

This result, though derived for a capacitor, is valid for any electric field, whether it occurs within a capacitor or anyplace else.

▲ A jolt of electric current from a defibrillator can restore normal heartbeat when the heart muscle has begun to twitch irregularly or has stopped beating altogether. A capacitor is used to store electricity, discharging it in a burst lasting only a couple of milliseconds.

THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK

In Section 20–1 we introduce the concepts of the electric potential and the electric potential energy. The development of these concepts parallels that of the gravitational potential energy for a uniform gravitational field (Chapter 8).

Energy conservation (Chapter 8) is used in Section 20–2, this time with the electric potential energy.

We also return to the idea of equipotential curves—curves along which the potential energy is constant—in direct analogy to the contour maps discussed at the end of Chapter 8.

The electric potential of a point charge is developed in Section 20–3. The results are almost identical to those obtained for the gravitational potential energy of a point mass in Chapter 12.

LOOKING AHEAD

Electrical energy is generalized to direct-current (dc) electric circuits in Chapter 21.

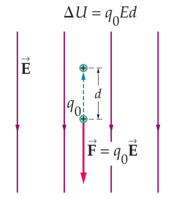
We will also see the important role that capacitors play in dc circuits in Chapter 21, and in alternating-current (ac) circuits in Chapter 24.

The concept of electrical energy is generalized yet again in Chapter 23, where we show how an electric motor can convert electrical energy to mechanical energy. We also show that the reverse process is possible, with a generator converting mechanical energy to electrical energy.

The electric potential energy for a point charge is applied to Bohr's model of the hydrogen atom in Chapter 31. With this energy we can determine the colors of light that hydrogen atoms emit.

CHAPTER SUMMARY

20–1 ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL



The electric force is conservative, just like the force of gravity. As a result, there is a potential energy U associated with the electric force.

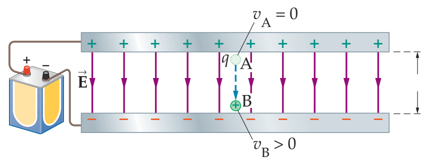
Electric Potential Energy, U
The change in electric potential energy is defined by $\Delta U = -W$, where W is the work done by the electric field.

Electric Potential, V
The change in electric potential is defined to be $\Delta V = \Delta U/q_0$.

Relation Between the Electric Field and the Electric Potential
The electric field is related to the rate of change of the electric potential. In particular, if the electric potential changes by the amount ΔV with a displacement Δs, the electric field in the direction of the displacement is

$$E = -\frac{\Delta V}{\Delta s} \qquad 20\text{–}4$$

20–2 ENERGY CONSERVATION



Another consequence of the fact that the electric force is conservative is that the total energy of an object is conserved—as long as nonconservative forces like friction can be ignored.

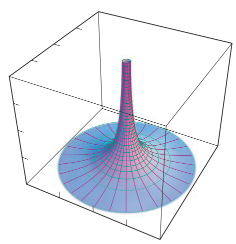
Energy Conservation
As usual, energy conservation can be expressed as follows:

$$\tfrac{1}{2}mv_A^{\,2} + U_A = \tfrac{1}{2}mv_B^{\,2} + U_B$$

In the case of the electric force, the potential energy is $U = q_0V$.

Direction of Acceleration
Positive charges accelerate in the direction of decreasing electric potential; negative charges accelerate in the direction of increasing electric potential.

20–3 THE ELECTRIC POTENTIAL OF POINT CHARGES


Electric potential of a positive point charge

If we define the electric potential of a point charge q to be zero at an infinite distance from the charge, the electric potential at a distance r is

$$V = \frac{kq}{r} \qquad 20\text{–}7$$

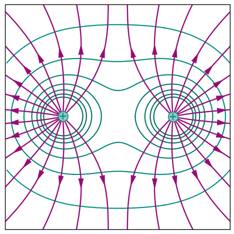
Electric Potential Energy
We define the electric potential energy of two charges, q_0 and q, to be zero when the separation between them is infinite. When the charges are separated by a distance r, the potential energy of the system is

$$U = \frac{kq_0q}{r} \qquad 20\text{–}8$$

Superposition
The electric potential of two or more point charges is simply the algebraic sum of the potentials due to each charge separately.

The total electric potential energy of two or more point charges is the sum of the potential energies due to each pair of charges.

20–4 EQUIPOTENTIAL SURFACES AND THE ELECTRIC FIELD



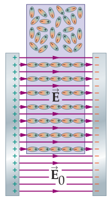
Equipotentials for two positive charges

Equipotential surfaces are defined as surfaces on which the electric potential is constant. Different equipotential surfaces correspond to different values of the potential.

Electric Field
The electric field is always perpendicular to the equipotential surfaces, and it points in the direction of decreasing electric potential.

Ideal Conductors
Ideal conductors are equipotential surfaces; every point on or within an ideal conductor is at the same potential. The electric field, therefore, is perpendicular to the surface of a conductor.

20–5 CAPACITORS AND DIELECTRICS



A capacitor is a device that stores electric charge.

Capacitance
Capacitance is defined as the amount of charge Q stored in a capacitor per volt of potential difference V between the plates of the capacitor. Thus,

$$C = \frac{Q}{V} \qquad 20\text{–}9$$

Parallel-Plate Capacitor
The capacitance of a parallel-plate capacitor, with plates of area A and separation d, is

$$C = \frac{\varepsilon_0 A}{d} \qquad 20\text{–}12$$

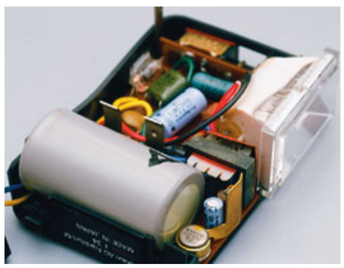
Dielectrics
A dielectric is an insulating material that increases the capacitance of a capacitor.

Dielectric Constant
A dielectric is characterized by the dimensionless dielectric constant, κ. In particular, the electric field in a dielectric is reduced by the factor κ, $E = E_0/\kappa$; the potential difference between capacitor plates is decreased by the factor κ, $V = V_0/\kappa$; and the capacitance is increased by the factor κ:

$$C = \kappa C_0 \qquad 20\text{–}14$$

Dielectric Breakdown/Dielectric Strength
A large electric field can cause a dielectric material to conduct electricity. This condition is referred to as dielectric breakdown. The strength of electric field required for dielectric breakdown is called the dielectric strength of the material.

20–6 ELECTRICAL ENERGY STORAGE



A capacitor, in addition to storing charge, also stores electrical energy.

Energy Stored in a Capacitor
The electrical energy stored in a capacitor can be expressed as follows:

$$U = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = Q^2/2C \qquad 20\text{–}16, 17, 18$$

Electric Energy Density of an Electric Field

Electric energy can be thought of as stored in the electric field. The electrical energy per volume, referred to as the electric energy density, is given by the following relation:

$$u_E = \text{electric energy density} = \tfrac{1}{2}\varepsilon_0 E^2 \qquad 20\text{–}19$$

PROBLEM-SOLVING SUMMARY

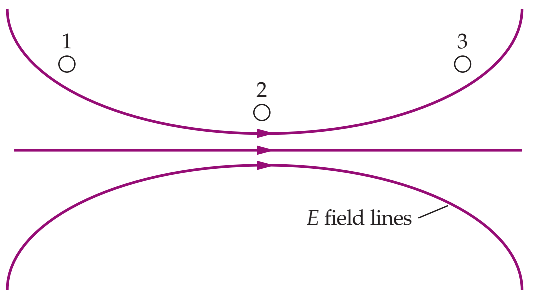
Type of Problem	Relevant Physical Concepts	Related Examples
Find the electric field corresponding to a change in electric potential.	The electric field is related to the change of electric potential with distance. The precise relation is $E = -\Delta V/\Delta s$.	Example 20–1 Active Example 20–1
Find the kinetic energy or speed of a particle moving in an electric field.	Apply energy conservation, including the electric potential energy, $U = qV$.	Examples 20–2, 20–4 Active Example 20–2
Calculate the electric potential due to a system of point charges.	The electric potential of a single point charge q at a distance r is $V = kq/r$. For a system of point charges, the total electric potential is the algebraic sum of the potentials calculated for each charge separately.	Examples 20–3, 20–4 Active Example 20–3
Determine the charge on the plates of a capacitor, or the potential difference between the plates.	The charge Q and potential difference V are related to the capacitance C by the expression $C = Q/V$.	Examples 20–5, 20–6
Determine the amount of energy stored in a capacitor.	The energy stored in a capacitor is given by three equivalent expressions: $U = \frac{1}{2}QV$; $U = \frac{1}{2}CV^2$; $U = Q^2/2C$.	Example 20–7

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com MP™

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. In one region of space the electric potential has a positive constant value. In another region of space the potential has a negative constant value. What can be said about the electric field within each of these two regions of space?
2. Two like charges a distance r apart have a positive electric potential energy. Conversely, two unlike charges a distance r apart have a negative electric potential energy. Explain the physical significance of these observations.
3. If the electric field is zero in some region of space is the electric potential zero there as well? Explain.
4. Sketch the equipotential surface that goes through point 1 in **Figure 20–19**. Repeat for point 2 and for point 3.



▲ **FIGURE 20–19** Conceptual Question 4 and Problems 43 and 44

5. How much work is required to move a charge from one location on an equipotential to another point on the same equipotential? Explain.
6. It is known that the electric potential is constant on a given two-dimensional surface. What can be said about the electric field on this surface?
7. Explain why equipotentials are always perpendicular to the electric field.
8. Two charges are at locations that have the same value of the electric potential. Is the electric potential energy the same for these charges? Explain.
9. A capacitor is connected to a battery and fully charged. What becomes of the charge on the capacitor when it is disconnected from the battery? What becomes of the charge when the two terminals of the capacitor are connected to one another?
10. It would be unwise to unplug a television set, take off the back, and reach inside. The reason for the danger is that if you happen to touch the terminals of a high-voltage capacitor you could receive a large electrical shock—even though the set is unplugged. Why?
11. On which of the following quantities does the capacitance of a capacitor depend: **(a)** the charge on the plates; **(b)** the separation of the plates; **(c)** the voltage difference between the plates; **(d)** the electric field between the plates; or **(e)** the area of the plates?
12. We say that a capacitor stores charge, yet the total charge in a capacitor is zero; that is, $Q + (-Q) = 0$. In what sense does a capacitor store charge if the net charge within it is zero?
13. The plates of a particular parallel-plate capacitor are uncharged. Is the capacitance of this capacitor zero? Explain.

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 20–1 ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL

1. • **CE** An electron is released from rest in a region of space with a nonzero electric field. As the electron moves, does it experience an increasing or decreasing electric potential? Explain.

2. • A uniform electric field of magnitude 4.1×10^5 N/C points in the positive x direction. Find the change in electric potential energy of a 4.5-μC charge as it moves from the origin to the points **(a)** (0, 6.0 m); **(b)** (6.0 m, 0); and **(c)** (6.0 m, 6.0 m).

3. • A uniform electric field of magnitude 6.8×10^5 N/C points in the positive x direction. Find the change in electric potential between the origin and the points **(a)** (0, 6.0 m); **(b)** (6.0 m, 0); and **(c)** (6.0 m, 6.0 m).

4. • **BIO Electric Potential Across a Cell Membrane** In a typical living cell, the electric potential inside the cell is 0.070 V lower than the electric potential outside the cell. The thickness of the cell membrane is 0.10 μm. What are the magnitude and direction of the electric field within the cell membrane?

5. • A computer monitor accelerates electrons and directs them to the screen in order to create an image. If the accelerating plates are 1.05 cm apart, and have a potential difference of 25,500 V, what is the magnitude of the uniform electric field between them?

6. • Find the change in electric potential energy for an electron that moves from one accelerating plate to the other in the computer monitor described in the previous problem.

7. • A parallel-plate capacitor has plates separated by 0.75 mm. If the electric field between the plates has a magnitude of **(a)** 1.2×10^5 V/m or **(b)** 2.4×10^4 N/C, what is the potential difference between the plates?

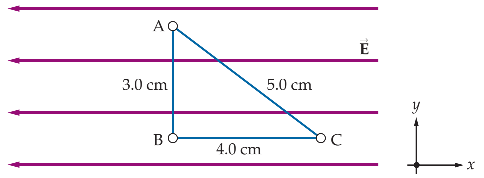
8. • When an ion accelerates through a potential difference of 2140 V, its electric potential energy decreases by 1.37×10^{-15} J. What is the charge on the ion?

9. • **The Electric Potential of the Earth** The Earth has a vertical electric field with a magnitude of approximately 100 V/m near its surface. What is the magnitude of the potential difference between a point on the ground and a point on the same level as the top of the Washington Monument (555 ft high)?

10. •• A uniform electric field with a magnitude of 6350 N/C points in the positive x direction. Find the change in electric potential energy when a +12.5-μC charge is moved 5.50 cm in **(a)** the positive x direction, **(b)** the negative x direction, and **(c)** the positive y direction.

11. •• **IP** A spark plug in a car has electrodes separated by a gap of 0.025 in. To create a spark and ignite the air-fuel mixture in the engine, an electric field of 3.0×10^6 V/m is required in the gap. **(a)** What potential difference must be applied to the spark plug to initiate a spark? **(b)** If the separation between electrodes is increased, does the required potential difference increase, decrease, or stay the same? Explain. **(c)** Find the potential difference for a separation of 0.050 in.

12. •• A uniform electric field with a magnitude of 1200 N/C points in the negative x direction, as shown in **Figure 20–20**.



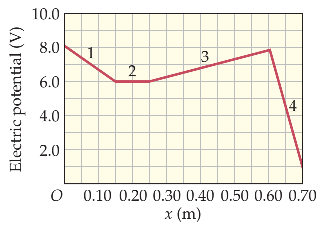
▲ **FIGURE 20–20** Problems 12 and 21

(a) What is the difference in electric potential, $\Delta V = V_B - V_A$, between points A and B? **(b)** What is the difference in electric potential, $\Delta V = V_B - V_C$, between points B and C? **(c)** What is the difference in electric potential, $\Delta V = V_C - V_A$, between points C and A? **(d)** From the information given in this problem, is it possible to determine the value of the electric potential at point A? If so, determine V_A; if not, explain why.

13. •• **A Charged Battery** A typical 12-V car battery can deliver 7.5×10^5 C of charge. If the energy supplied by the battery could be converted entirely to kinetic energy, what speed would it give to a 1400-kg car that is initially at rest?

14. •• **IP BIO The Sodium Pump** Living cells actively "pump" positive sodium ions (Na^+) from inside the cell to outside the cell. This process is referred to as pumping because work must be done on the ions to move them from the negatively charged inner surface of the membrane to the positively charged outer surface. Given that the electric potential is 0.070 V higher outside the cell than inside the cell, and that the cell membrane is 0.10 μm thick, **(a)** calculate the work that must be done (in joules) to move one sodium ion from inside the cell to outside. **(b)** If the thickness of the cell membrane is increased, does your answer to part (a) increase, decrease, or stay the same? Explain. (It is estimated that as much as 20% of the energy we consume in a resting state is used in operating this "sodium pump.")

15. •• **IP** The electric potential of a system as a function of position along the x axis is given in **Figure 20–21**. **(a)** In which of the regions, 1, 2, 3, or 4, do you expect E_x to be greatest? In which region does E_x have its greatest magnitude? Explain. **(b)** Calculate the value of E_x in each of the regions, 1, 2, 3, and 4.



▲ **FIGURE 20–21** Problems 15 and 94

16. ••• Points A and B have electric potentials of 332 V and 149 V, respectively. When an electron released from rest at point A arrives at point C, its kinetic energy is K_A. When the electron is released from rest at point B, however, its kinetic energy when it reaches point C is $K_B = 2K_A$. What are **(a)** the electric potential at point C and **(b)** the kinetic energy K_A?

SECTION 20–2 ENERGY CONSERVATION

17. • **CE Predict/Explain** An electron is released from rest in a region of space with a nonzero electric field. **(a)** As the electron

moves, does the electric potential energy of the system increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:

I. Because the electron has a negative charge its electric potential energy doesn't decrease, as one might expect, but increases instead.

II. As the electron begins to move, its kinetic energy increases. The increase in kinetic energy is equal to the decrease in the electric potential energy of the system.

III. The electron will move perpendicular to the electric field, and hence its electric potential energy will remain the same.

18. • Calculate the speed of **(a)** a proton and **(b)** an electron after each particle accelerates from rest through a potential difference of 275 V.

19. • The electrons in a TV picture tube are accelerated from rest through a potential difference of 25 kV. What is the speed of the electrons after they have been accelerated by this potential difference?

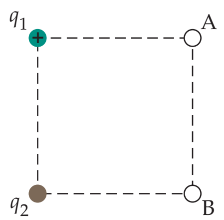
20. • Find the potential difference required to accelerate protons from rest to 10% of the speed of light. (At this point, relativistic effects start to become significant.)

21. •• **IP** A particle with a mass of 3.8 g and a charge of $+0.045\ \mu C$ is released from rest at point A in Figure 20–20. **(a)** In which direction will this charge move? **(b)** What speed will it have after moving through a distance of 5.0 cm? The electric field has a magnitude of 1200 N/C. **(c)** Suppose the particle continues moving for another 5.0 cm. Will its increase in speed for the second 5.0 cm be greater than, less than, or equal to its increase in speed in the first 5.0 cm? Explain.

22. •• A proton has an initial speed of 4.0×10^5 m/s. **(a)** What potential difference is required to bring the proton to rest? **(b)** What potential difference is required to reduce the initial speed of the proton by a factor of 2? **(c)** What potential difference is required to reduce the initial kinetic energy of the proton by a factor of 2?

SECTION 20–3 THE ELECTRIC POTENTIAL OF POINT CHARGES

23. • In **Figure 20–22**, it is given that $q_1 = +Q$. **(a)** What value must q_2 have if the electric potential at point A is to be zero? **(b)** With the value for q_2 found in part (a), is the electric potential at point B positive, negative, or zero? Explain.



▲ **FIGURE 20–22** Problems 23, 24, and 25

24. • **CE** The charge q_1 in Figure 20–22 has the value $+Q$. **(a)** What value must q_2 have if the electric potential at point B is to be zero? **(b)** With the value for q_2 found in part (a), is the electric potential at point A positive, negative, or zero? Explain.

25. • **CE** It is given that the electric potential is zero at the center of the square in Figure 20–22. **(a)** If $q_1 = +Q$, what is the value of the charge q_2? **(b)** Is the electric potential at point A positive, negative, or zero? Explain. **(c)** Is the electric potential at point B positive, negative, or zero? Explain.

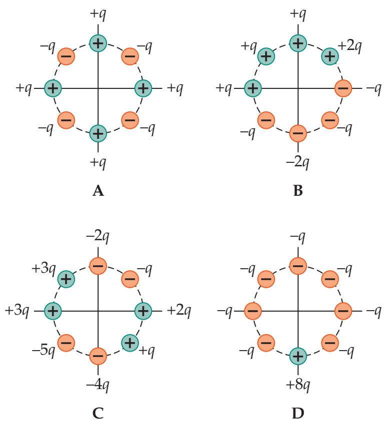
26. • The electric potential 1.1 m from a point charge q is 2.8×10^4 V. What is the value of q?

27. • A point charge of $-7.2\ \mu C$ is at the origin. What is the electric potential at **(a)** (3.0 m, 0); **(b)** $(-3.0\text{ m}, 0)$; and **(c)** (3.0 m, −3.0 m)?

28. • **The Bohr Atom** The hydrogen atom consists of one electron and one proton. In the Bohr model of the hydrogen atom, the electron orbits the proton in a circular orbit of radius 0.529×10^{-10} m. What is the electric potential due to the proton at the electron's orbit?

29. • How far must the point charges $q_1 = +7.22\ \mu C$ and $q_2 = -26.1\ \mu C$ be separated for the electric potential energy of the system to be −126 J?

30. •• **CE** Four different arrangements of point charges are shown in **Figure 20–23**. In each case the charges are the same distance from the origin. Rank the four arrangements in order of increasing electric potential at the origin, taking the potential at infinity to be zero. Indicate ties where appropriate.



▲ **FIGURE 20–23** Problem 30

31. •• **IP** Point charges $+4.1\ \mu C$ and $-2.2\ \mu C$ are placed on the x axis at (11 m, 0) and (−11 m, 0), respectively. **(a)** Sketch the electric potential on the x axis for this system. **(b)** Your sketch should show one point on the x axis between the two charges where the potential vanishes. Is this point closer to the $+4.1$-μC charge or closer to the -2.2-μC charge? Explain. **(c)** Find the point referred to in part (b).

32. •• **IP** **(a)** In the previous problem, find the point to the left of the negative charge where the electric potential vanishes. **(b)** Is the electric field at the point found in part (a) positive, negative, or zero? Explain.

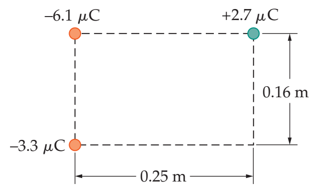
33. •• A dipole is formed by point charges $+3.6\ \mu C$ and $-3.6\ \mu C$ placed on the x axis at (0.25 m, 0) and (−0.25 m, 0), respectively. **(a)** Sketch the electric potential on the x axis for this system. **(b)** At what positions on the x axis does the potential have the value 7.5×10^5 V?

34. •• A charge of 3.05 μC is held fixed at the origin. A second charge of 3.05 μC is released from rest at the position (1.25 m, 0.570 m). **(a)** If the mass of the second charge is 2.16 g, what is its speed when it moves infinitely far from the origin? **(b)** At what distance from the origin does the second charge attain half the speed it will have at infinity?

35. •• **IP** A charge of 20.2 μC is held fixed at the origin. **(a)** If a -5.25-μC charge with a mass of 3.20 g is released from rest at the position (0.925 m, 1.17 m), what is its speed when it is halfway to the origin? **(b)** Suppose the -5.25-μC charge is released from rest at the point $x = \frac{1}{2}(0.925\text{ m})$ and $y = \frac{1}{2}(1.17\text{ m})$. When it is halfway to the origin, is its speed greater than, less than, or equal to the speed found in part (a)? Explain. **(c)** Find the speed of the charge for the situation described in part (b).

36. •• A charge of $-2.205\ \mu C$ is located at (3.055 m, 4.501 m), and a charge of $1.800\ \mu C$ is located at (-2.533 m, 0). **(a)** Find the electric potential at the origin. **(b)** There is one point on the line connecting these two charges where the potential is zero. Find this point.

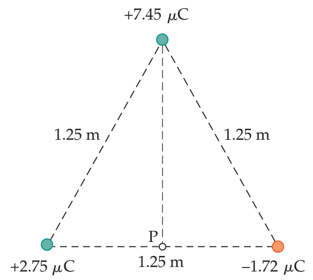
37. •• **IP** **Figure 20–24** shows three charges at the corners of a rectangle. **(a)** How much work must be done to move the $+2.7\text{-}\mu C$ charge to infinity? **(b)** Suppose, instead, that we move the $-6.1\text{-}\mu C$ charge to infinity. Is the work required in this case greater than, less than, or the same as when we moved the $+2.7\text{-}\mu C$ charge to infinity? Explain. **(c)** Calculate the work needed to move the $-6.1\text{-}\mu C$ charge to infinity.



▲ **FIGURE 20–24** Problems 37 and 38

38. •• How much work must be done to move the three charges in Figure 20–24 infinitely far from one another?

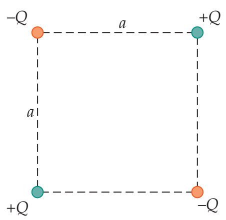
39. •• **(a)** Find the electric potential at point P in **Figure 20–25**. **(b)** Suppose the three charges shown in Figure 20–25 are held in place. A fourth charge, with a charge of $+6.11\ \mu C$ and a mass of 4.71 g, is released from rest at point P. What is the speed of the fourth charge when it has moved infinitely far away from the other three charges?



▲ **FIGURE 20–25** Problems 39 and 91

40. ••• A square of side a has a charge $+Q$ at each corner. What is the electric potential energy of this system of charges?

41. ••• A square of side a has charges $+Q$ and $-Q$ alternating from one corner to the next, as shown in **Figure 20–26**. Find the electric potential energy for this system of charges.



▲ **FIGURE 20–26** Problems 41 and 100

SECTION 20–4 EQUIPOTENTIAL SURFACES AND THE ELECTRIC FIELD

42. • **CE Predict/Explain** A positive charge is moved from one location on an equipotential to another point on the same equipotential. **(a)** Is the work done on the charge positive, negative, or zero? **(b)** Choose the *best explanation* from among the following:

I. The electric field is perpendicular to an equipotential, therefore the work done in moving along an equipotential is zero.

II. Because the charge is positive the work done on it is also positive.

III. It takes negative work to keep the positive charge from accelerating as it moves along the equipotential.

43. • **CE Predict/Explain** **(a)** Is the electric potential at point 1 in **Figure 20–19** greater than, less than, or equal to the electric potential at point 3? **(b)** Choose the *best explanation* from among the following:

I. The electric field lines point to the right, indicating that the electric potential is greater at point 3 than at point 1.

II. The value of the electric potential is large where the electric field lines are close together, and small where they are widely spaced. Therefore, the electric potential is the same at points 1 and 3.

III. The electric potential decreases as we move in the direction of the electric field, as shown in Figure 20–3. Therefore, the electric potential is greater at point 1 than at point 3.

44. • **CE Predict/Explain** Imagine sketching a large number of equipotential surfaces in Figure 20–19, with a constant difference in electric potential between adjacent surfaces. **(a)** Would the equipotentials at point 2 be more closely spaced, be less closely spaced, or have the same spacing as equipotentials at point 1? **(b)** Choose the *best explanation* from among the following:

I. When electric field lines are close together, the corresponding equipotentials are far apart.

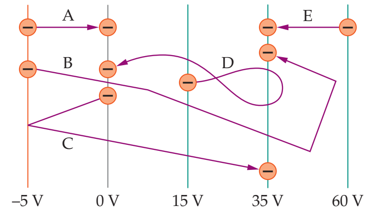
II. Equipotential surfaces, by definition, always have equal spacing between them.

III. The electric field is more intense at point 2 than at point 1, which means the equipotential surfaces are more closely spaced in that region.

45. • Two point charges are on the x axis. Charge 1 is $+q$ and is located at $x = -1.0$ m; charge 2 is $-2q$ and is located at $x = 1.0$ m. Make sketches of the equipotential surfaces for this system **(a)** out to a distance of about 2.0 m from the origin and **(b)** far from the origin. In each case, indicate the direction in which the potential increases.

46. • Two point charges are on the x axis. Charge 1 is $+q$ and is located at $x = -1.0$ m; charge 2 is $+2q$ and is located at $x = 1.0$ m. Make sketches of the equipotential surfaces for this system **(a)** out to a distance of about 2.0 m from the origin and **(b)** far from the origin. In each case, indicate the direction in which the potential increases.

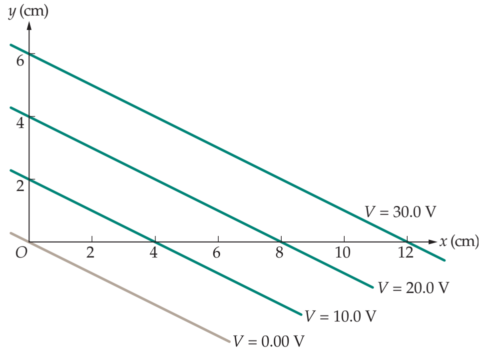
47. •• **CE** **Figure 20–27** shows a series of equipotentials in a particular region of space, and five different paths along which an electron is moved. **(a)** Does the electric field in this region point to the right, to the left, up, or down? Explain. **(b)** For each path, indicate whether the work done on the electron by the electric field is positive, negative, or zero. **(c)** Rank the paths in order of increasing amount of work done on the electron by the electric field. Indicate ties where appropriate. **(d)** Is the electric field near path A greater than, less than, or equal to the electric field near path E? Explain.



▲ **FIGURE 20–27** Problem 47

48. •• **IP** Consider a region in space where a uniform electric field $E = 6500$ N/C points in the negative x direction. **(a)** What is the orientation of the equipotential surfaces? Explain. **(b)** If you move in the positive x direction, does the electric potential increase or decrease? Explain. **(c)** What is the distance between the +14-V and the +16-V equipotentials?

49. •• A given system has the equipotential surfaces shown in **Figure 20–28**. **(a)** What are the magnitude and direction of the electric field? **(b)** What is the shortest distance one can move to undergo a change in potential of 5.00 V?



▲ **FIGURE 20–28** Problems 49 and 92

SECTION 20–5 CAPACITORS AND DIELECTRICS

50. • A 0.40-μF capacitor is connected to a 9.0-V battery. How much charge is on each plate of the capacitor?

51. • It is desired that 5.8 μC of charge be stored on each plate of a 3.2-μF capacitor. What potential difference is required between the plates?

52. • To operate a given flash lamp requires a charge of 32 μC. What capacitance is needed to store this much charge in a capacitor with a potential difference between its plates of 9.0 V?

53. •• A parallel-plate capacitor is made from two aluminum-foil sheets, each 6.3 cm wide and 5.4 m long. Between the sheets is a Teflon strip of the same width and length that is 0.035 mm thick. What is the capacitance of this capacitor? (The dielectric constant of Teflon is 2.1.)

54. •• A parallel-plate capacitor is constructed with circular plates of radius 0.056 m. The plates are separated by 0.25 mm, and the space between the plates is filled with a dielectric with dielectric constant κ. When the charge on the capacitor is 1.2 μC the potential difference between the plates is 750 V. Find the value of the dielectric constant, κ.

55. •• **IP** A parallel-plate capacitor has plates with an area of 0.012 m^2 and a separation of 0.88 mm. The space between the plates is filled with a dielectric whose dielectric constant is 2.0. **(a)** What is the potential difference between the plates when the charge on the capacitor plates is 4.7 μC? **(b)** Will your answer to part (a) increase, decrease, or stay the same if the dielectric constant is increased? Explain. **(c)** Calculate the potential difference for the case where the dielectric constant is 4.0.

56. •• **IP** Consider a parallel-plate capacitor constructed from two circular metal plates of radius R. The plates are separated by a distance of 1.5 mm. **(a)** What radius must the plates have if the capacitance of this capacitor is to be 1.0 μF? **(b)** If the separation between the plates is increased, should the radius of the plates be increased or decreased to maintain a capacitance of 1.0 μF? Explain. **(c)** Find the radius of the plates that gives a capacitance of 1.0 μF for a plate separation of 3.0 mm.

57. •• A parallel-plate capacitor has plates of area 3.45×10^{-4} m^2. What plate separation is required if the capacitance is to be 1630 pF? Assume that the space between the plates is filled with **(a)** air or **(b)** paper.

58. •• **IP** A parallel-plate capacitor filled with air has plates of area 0.0066 m^2 and a separation of 0.45 mm. **(a)** Find the magnitude of the charge on each plate when the capacitor is connected to a 12-V battery. **(b)** Will your answer to part (a) increase, decrease, or stay the same if the separation between the plates is increased? Explain. **(c)** Calculate the magnitude of the charge on the plates if the separation is 0.90 mm.

59. •• Suppose that after walking across a carpeted floor you reach for a doorknob and just before you touch it a spark jumps 0.50 cm from your finger to the knob. Find the minimum voltage needed between your finger and the doorknob to generate this spark.

60. •• **(a)** What plate area is required if an air-filled, parallel-plate capacitor with a plate separation of 2.6 mm is to have a capacitance of 22 pF? **(b)** What is the maximum voltage that can be applied to this capacitor without causing dielectric breakdown?

61. •• **Lightning** As a crude model for lightning, consider the ground to be one plate of a parallel-plate capacitor and a cloud at an altitude of 550 m to be the other plate. Assume the surface area of the cloud to be the same as the area of a square that is 0.50 km on a side. **(a)** What is the capacitance of this capacitor? **(b)** How much charge can the cloud hold before the dielectric strength of the air is exceeded and a spark (lightning) results?

62. ••• A parallel-plate capacitor is made from two aluminum-foil sheets, each 3.00 cm wide and 10.0 m long. Between the sheets is a mica strip of the same width and length that is 0.0225 mm thick. What is the maximum charge that can be stored in this capacitor? (The dielectric constant of mica is 5.4, and its dielectric strength is 1.00×10^8 V/m.)

SECTION 20–6 ELECTRICAL ENERGY STORAGE

63. • Calculate the work done by a 3.0-V battery as it charges a 7.8-μF capacitor in the flash unit of a camera.

64. • **BIO Defibrillator** An automatic external defibrillator (AED) delivers 125 J of energy at a voltage of 1050 V. What is the capacitance of this device?

65. •• **IP BIO Cell Membranes** The membrane of a living cell can be approximated by a parallel-plate capacitor with plates of area 4.75×10^{-9} m^2, a plate separation of 8.5×10^{-9} m, and a dielectric with a dielectric constant of 4.5. **(a)** What is the energy stored in such a cell membrane if the potential difference across it is 0.0725 V? **(b)** Would your answer to part (a) increase, decrease, or stay the same if the thickness of the cell membrane is increased? Explain.

66. •• A 0.22-μF capacitor is charged by a 1.5-V battery. After being charged, the capacitor is connected to a small electric motor. Assuming 100% efficiency, **(a)** to what height can the motor lift a 5.0-g mass? **(b)** What initial voltage must the capacitor have if it is to lift a 5.0-g mass through a height of 1.0 cm?

67. •• Find the electric energy density between the plates of a 225-μF parallel-plate capacitor. The potential difference between the plates is 345 V, and the plate separation is 0.223 mm.

68. •• What electric field strength would store 17.5 J of energy in every 1.00 mm^3 of space?

69. •• An electronic flash unit for a camera contains a capacitor with a capacitance of 890 μF. When the unit is fully charged and ready for operation, the potential difference between the capacitor plates is 330 V. **(a)** What is the magnitude of the charge on each plate of the fully charged capacitor? **(b)** Find the energy stored in the "charged-up" flash unit.

70. ••• A parallel-plate capacitor has plates with an area of 405 cm^2 and an air-filled gap between the plates that is 2.25 mm thick. The capacitor is charged by a battery to 575 V and then is disconnected from the battery. (a) How much energy is stored in the capacitor? (b) The separation between the plates is now increased to 4.50 mm. How much energy is stored in the capacitor now? (c) How much work is required to increase the separation of the plates from 2.25 mm to 4.50 mm? Explain your reasoning.

GENERAL PROBLEMS

71. • **CE** A proton is released from rest in a region of space with a nonzero electric field. As the proton moves, does it experience an increasing or decreasing electric potential? Explain.

72. • **CE Predict/Explain** A proton is released from rest in a region of space with a nonzero electric field. **(a)** As the proton moves, does the electric potential energy of the system increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. As the proton begins to move, its kinetic energy increases. The increase in kinetic energy is equal to the decrease in the electric potential energy of the system.
   II. Because the proton has a positive charge, its electric potential energy will always increase.
   III. The proton will move perpendicular to the electric field, and hence its electric potential energy will remain the same.

73. • **CE** In the Bohr model of the hydrogen atom, a proton and an electron are separated by a constant distance r. **(a)** Would the electric potential energy of the system increase, decrease, or stay the same if the electron is replaced with a proton? Explain. **(b)** Suppose, instead, that the proton is replaced with an electron. Would the electric potential energy of the system increase, decrease, or stay the same? Explain.

74. • **CE** The plates of a parallel-plate capacitor have constant charges of $+Q$ and $-Q$. Do the following quantities increase, decrease, or remain the same as the separation of the plates is increased? **(a)** The electric field between the plates; **(b)** the potential difference between the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

75. • **CE** A parallel-plate capacitor is connected to a battery that maintains a constant potential difference V between the plates. If the plates of the capacitor are pulled farther apart, do the following quantities increase, decrease, or remain the same? **(a)** The electric field between the plates; **(b)** the charge on the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

76. • **CE** The plates of a parallel-plate capacitor have constant charges of $+Q$ and $-Q$. Do the following quantities increase, decrease, or remain the same as a dielectric is inserted between the plates? **(a)** The electric field between the plates; **(b)** the potential difference between the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

77. • **CE** A parallel-plate capacitor is connected to a battery that maintains a constant potential difference V between the plates. If a dielectric is inserted between the plates of the capacitor, do the following quantities increase, decrease, or remain the same? **(a)** The electric field between the plates; **(b)** the charge on the plates; **(c)** the capacitance; **(d)** the energy stored in the capacitor.

78. • Find the difference in electric potential, $\Delta V = V_B - V_A$, between the points A and B for the following cases: **(a)** The electric field does 0.052 J of work as you move a $+5.7\text{-}\mu C$ charge from A to B. **(b)** The electric field does -0.052 J of work as you move a $-5.7\text{-}\mu C$ charge from A to B. **(c)** You perform 0.052 J of work as you slowly move a $+5.7\text{-}\mu C$ charge from A to B.

79. • The separation between the plates of a parallel-plate capacitor is doubled and the area of the plates is halved. How is the capacitance affected?

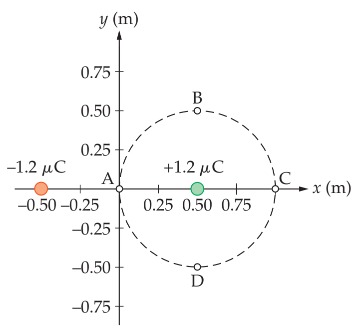
80. • A parallel-plate capacitor is connected to a battery that maintains a constant potential difference between the plates. If the spacing between the plates is doubled, how is the magnitude of charge on the plates affected?

81. •• **CE** Two point charges are placed on the x axis. The charge $+2q$ is at $x = 1.5$ m, and the charge $-q$ is at $x = -1.5$ m. **(a)** There is a point on the x axis between the two charges where the electric potential is zero. Where is this point? **(b)** The electric potential also vanishes at a point in one of the following regions: region 1, x between 1.5 m and 5.0 m; region 2, x between -1.5 m and -3.0 m; region 3, x between -3.5 m and -5.0 m. Identify the appropriate region. **(c)** Find the value of x referred to in part (b).

82. •• A charge of 24.5 μC is located at (4.40 m, 6.22 m), and a charge of -11.2 μC is located at (-4.50 m, 6.75 m). What charge must be located at (2.23 m, -3.31 m) if the electric potential is to be zero at the origin?

83. •• **The Bohr Model** In the Bohr model of the hydrogen atom (see Problem 28) what is the smallest amount of work that must be done on the electron to move it from its circular orbit, with a radius of 0.529×10^{-10} m, to an infinite distance from the proton? This value is referred to as the ionization energy of hydrogen.

84. •• **IP** A $+1.2\text{-}\mu C$ charge and a $-1.2\text{-}\mu C$ charge are placed at (0.50 m, 0) and (-0.50 m, 0), respectively. **(a)** In **Figure 20–29**, at which of the points A, B, C, or D is the electric potential smallest in value? At which of these points does it have its greatest value? Explain. **(b)** Calculate the electric potential at points A, B, C, and D.



▲ **FIGURE 20–29** Problems 84, 85, and 103

85. •• Repeat Problem 84 for the case where both charges are $+1.2$ μC.

86. •• How much work is required to bring three protons, initially infinitely far apart, to a configuration where each proton is 1.5×10^{-15} m from the other two? (This is a typical separation for protons in a nucleus.)

87. •• A point charge $Q = +87.1$ μC is held fixed at the origin. A second point charge, with mass $m = 0.0576$ kg and charge $q = -2.87$ μC, is placed at the location (0.323 m, 0). **(a)** Find the electric potential energy of this system of charges. **(b)** If the second charge is released from rest, what is its speed when it reaches the point (0.121 m, 0)?

88. •• **Electron Escape Speed** An electron is at rest just above the surface of a sphere with a radius of 2.7 mm and a uniformly distributed positive charge of 1.8×10^{-15} C. Like a rocket blasting off from the Earth, the electron is given an initial speed v_e radially

outward from the sphere. If the electron coasts to infinity, where its kinetic energy drops to zero, what is the escape speed, v_e?

89. •• **Quark Model of the Neutron** According to the quark model of fundamental particles, neutrons—the neutral particles in an atom's nucleus—are composed of three quarks. Two of these quarks are "down" quarks, each with a charge of $-e/3$; the third quark is an "up" quark, with a charge of $+2e/3$. This gives the neutron a net charge of zero. What is the electric potential energy of these three quarks, assuming they are equidistant from one another, with a separation distance of 1.3×10^{-15} m? (Quarks are discussed in Chapter 32.)

90. •• A parallel-plate capacitor is charged to an electric potential of 325 V by moving 3.75×10^{16} electrons from one plate to the other. How much work is done in charging the capacitor?

91. •• **IP** The three charges shown in Figure 20–25 are held in place as a fourth charge, q, is brought from infinity to the point P. The charge q starts at rest at infinity and is also at rest when it is placed at the point P. **(a)** If q is a positive charge, is the work required to bring it to the point P positive, negative, or zero? Explain. **(b)** Find the value of q if the work needed to bring it to point P is -1.3×10^{-11} J.

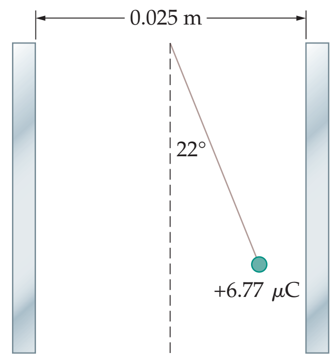
92. •• **(a)** In Figure 20–28 we see that the electric potential increases by 10.0 V as one moves 4.00 cm in the positive x direction. Use this information to calculate the x component of the electric field. (Ignore the y direction for the moment.) **(b)** Apply the same reasoning as in part (a) to calculate the y component of the electric field. **(c)** Combine the results from parts (a) and (b) to find the magnitude and direction of the electric field for this system.

93. •• **IP BIO Electric Catfish** The electric catfish (*Malapterurus electricus*) is an aggressive fish, 1.0 m in length, found today in tropical Africa (and depicted in Egyptian hieroglyphics). The catfish is capable of generating jolts of electricity up to 350 V by producing a positively charged region of muscle near the head and a negatively charged region near the tail. **(a)** For the same amount of charge, can the catfish generate a higher voltage by separating the charge from one end of its body to the other, as it does, or from one side of the body to the other? Explain. **(b)** Estimate the charge generated at each end of a catfish as follows: Treat the catfish as a parallel-plate capacitor with plates of area 1.8×10^{-2} m^2, separation 1.0 m, and filled with a dielectric with a dielectric constant $\kappa = 95$.

94. •• As a $+6.2$-μC charge moves along the x axis from $x = 0$ to $x = 0.70$ m, the electric potential it experiences is shown in Figure 20–21. Find the approximate location(s) of the charge when its electric potential energy is **(a)** 2.6×10^{-5} J and **(b)** 4.3×10^{-5} J.

95. •• **IP Computer Keyboards** Many computer keyboards operate on the principle of capacitance. As shown in Figure 20–16, each key forms a small parallel-plate capacitor whose separation is reduced when the key is depressed. **(a)** Does depressing a key increase or decrease its capacitance? Explain. **(b)** Suppose the plates for each key have an area of 47.5 mm^2 and an initial separation of 0.550 mm. In addition, let the dielectric have a dielectric constant of 3.75. If the circuitry of the computer can detect a change in capacitance of 0.425 pF, what is the minimum distance a key must be depressed to be detected?

96. •• **IP** A point charge of mass 0.081 kg and charge $+6.77$ μC is suspended by a thread between the vertical parallel plates of a parallel-plate capacitor, as shown in **Figure 20–30**. **(a)** If the charge deflects to the right of vertical, as indicated in the figure, which of the two plates is at the higher electric potential? **(b)** If the angle of deflection is 22°, and the separation between the plates is 0.025 m, what is the potential difference between the plates?



▲ **FIGURE 20–30** Problems 96 and 101

97. •• **BIO Cell Membranes and Dielectrics** Many cells in the body have a cell membrane whose inner and outer surfaces carry opposite charges, just like the plates of a parallel-plate capacitor. Suppose a typical cell membrane has a thickness of 8.1×10^{-9} m, and its inner and outer surfaces carry charge densities of -0.58×10^{-3} C/m^2 and $+0.58 \times 10^{-3}$ C/m^2, respectively. In addition, assume that the material in the cell membrane has a dielectric constant of 5.5. **(a)** Find the direction and magnitude of the electric field within the cell membrane. **(b)** Calculate the potential difference between the inner and outer walls of the membrane, and indicate which wall of the membrane has the higher potential.

98. •• Long, long ago, on a planet far, far away, a physics experiment was carried out. First, a 0.250-kg ball with zero net charge was dropped from rest at a height of 1.00 m. The ball landed 0.552 s later. Next, the ball was given a net charge of 7.75 μC and dropped in the same way from the same height. This time the ball fell for 0.680 s before landing. What is the electric potential at a height of 1.00 m above the ground on this planet, given that the electric potential at ground level is zero? (Air resistance can be ignored.)

99. •• **Rutherford's Planetary Model of the Atom** In 1911, Ernest Rutherford developed a planetary model of the atom, in which a small positively charged nucleus is orbited by electrons. The model was motivated by an experiment carried out by Rutherford and his graduate students, Geiger and Marsden. In this experiment, they fired alpha particles with an initial speed of 1.75×10^7 m/s at a thin sheet of gold. (Alpha particles are obtained from certain radioactive decays. They have a charge of $+2e$ and a mass of 6.64×10^{-27} kg.) How close can the alpha particles get to a gold nucleus (charge $= +79e$), assuming the nucleus remains stationary? (This calculation sets an upper limit on the size of the gold nucleus. See Chapter 31 for further details.)

100. ••• **IP** **(a)** One of the $-Q$ charges in Figure 20–26 is given an outward "kick" that sends it off with an initial speed v_0 while the other three charges are held at rest. If the moving charge has a mass m, what is its speed when it is infinitely far from the other charges? **(b)** Suppose the remaining $-Q$ charge, which also has a mass m, is now given the same initial speed, v_0. When it is infinitely far away from the two $+Q$ charges, is its speed greater than, less than, or the same as the speed found in part (a)? Explain.

101. ••• Figure 20–30 shows a charge $q = +6.77$ μC with a mass $m = 0.071$ kg suspended by a thread of length $L = 0.022$ m between the plates of a capacitor. **(a)** Plot the electric potential energy of the system as a function of the angle θ the thread makes with the vertical. (The electric field between the plates has a magnitude $E = 4.16 \times 10^4$ V/m.) **(b)** Repeat part (a) for

the case of the gravitational potential energy of the system. **(c)** Show that the total potential energy of the system (electric plus gravitational) is a minimum when the angle θ satisfies the equilibrium condition for the charge, $\tan\theta = qE/mg$. This relation implies that $\theta = 22°$.

102. ••• The electric potential a distance r from a point charge q is 2.70×10^4 V. One meter farther away from the charge the potential is 6140 V. Find the charge q and the initial distance r.

103. ••• Referring to Problem 84, calculate and plot the electric potential on the circle centered at (0.50 m, 0). Give your results in terms of the angle θ, defined as follows: θ is the angle measured counterclockwise from a vertex at the center of the circle, with $\theta = 0$ at point C.

104. ••• When the potential difference between the plates of a capacitor is increased by 3.25 V, the magnitude of the charge on each plate increases by 13.5 μC. What is the capacitance of this capacitor?

105. ••• The electric potential a distance r from a point charge q is 155 V, and the magnitude of the electric field is 2240 N/C. Find the values of q and r.

PASSAGE PROBLEMS

BIO The Electric Eel

Of the many unique and unusual animals that inhabit the rainforests of South America, including howler monkeys, freshwater dolphins, and deadly piranhas, one stands out because of its mastery of electricity. The electric eel (*Electrophorus electricus*), one of the few creatures on Earth able to generate, store, and discharge electricity, can deliver a powerful series of high-voltage discharges reaching 650 V. These jolts of electricity are so strong, in fact, that electric eels have been known to topple a horse crossing a stream 20 feet away, and to cause respiratory paralysis, cardiac arrhythmia, and even death in humans.

Though similar in appearance to an eel, the electric "eel" is actually more closely related to catfish. They are found primarily in the Amazon and Orinoco river basins, where they navigate the slow-moving, muddy water with low-voltage electric organ discharges (EOD), saving the high-voltage EODs for stunning prey and defending against predators. Obligate air breathers, electric eels obtain about 80% of their oxygen by gulping air at the water's surface. Even so, they are able to attain lengths of 2.5 m and a mass of 20 kg.

The organs that produce the eel's electricity take up most of its body, and consist of thousands of modified muscle cells—called electroplaques—stacked together like the cells in a battery. Each electroplaque is capable of generating a voltage of 0.15 V, and together they produce a positive charge near the head of the eel and a negative charge near its tail.

106. • Electric eels produce an electric field within their body. In which direction does the electric field point?

A. toward the head **B.** toward the tail

C. upward **D.** downward

107. • As a rough approximation, consider an electric eel to be a parallel-plate capacitor with plates of area 1.8×10^{-2} m^2 separated by 2.0 m and filled with a dielectric whose dielectric constant is $\kappa = 95$. What is the capacitance of the eel in this model?

A. 8.0×10^{-14} F **B.** 7.6×10^{-12} F

C. 1.5×10^{-11} F **D.** 9.3×10^{-8} F

108. • In terms of the parallel-plate model of the previous problem, how much charge does an electric eel generate at each end of its body when it produces a voltage of 650 V?

A. 1.2×10^{-14} C **B.** 5.2×10^{-11} C

C. 4.9×10^{-9} C **D.** 6.1×10^{-5} C

109. • How much energy is stored by an electric eel when it is charged up to 650 V. Use the same parallel-plate model discussed in the previous two problems.

A. 1.8×10^{-17} J **B.** 1.7×10^{-8} J

C. 1.6×10^{-6} J **D.** 2.0×10^{-2} J

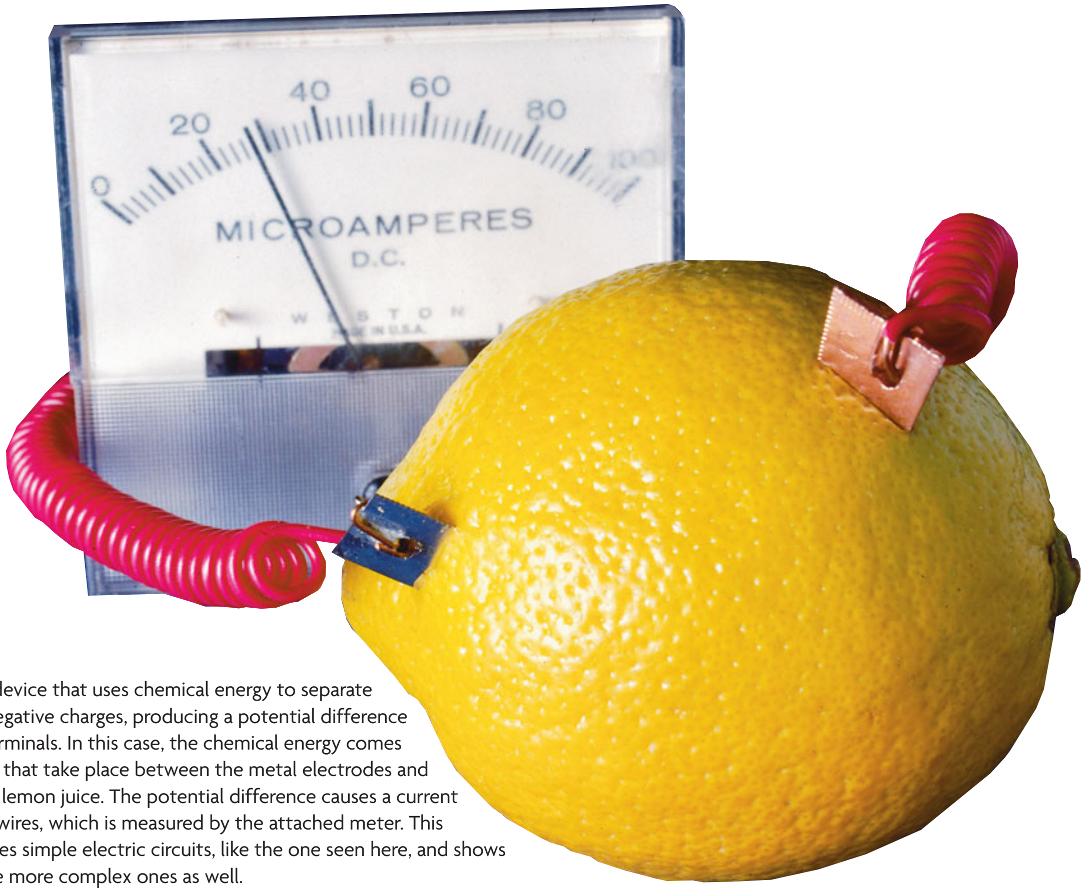
INTERACTIVE PROBLEMS

110. •• IP **Referring to Example 20–3** Suppose the charge $-2q$ at $x = 1.00$ m is replaced with a charge $-3q$, where $q = 4.11 \times 10^{-9}$ C. The charge $+q$ is at the origin. **(a)** Is the electric potential positive, negative, or zero at the point $x = 0.333$ m? Explain. **(b)** Find the point between $x = 0$ and $x = 1.00$ m where the electric potential vanishes. **(c)** Is there a point in the region $x < 0$ where the electric potential passes through zero?

111. •• **Referring to Example 20–3** Suppose we can change the location of the charge $-2q$ on the x axis. The charge $+q$ (where $q = 4.11 \times 10^{-9}$C) is still at the origin. **(a)** Where should the charge $-2q$ be placed to ensure that the electric potential vanishes at $x = 0.500$ m? **(b)** With the location of $-2q$ found in part (a), where does the electric potential pass through zero in the region $x < 0$?

112. •• IP **Referring to Example 20–3** Suppose the charge $+q$ at the origin is replaced with a charge $+5q$, where $q = 4.11 \times 10^{-9}$ C. The charge $-2q$ is still at $x = 1.00$ m. **(a)** Is there a point in the region $x < 0$ where the electric potential passes through zero? **(b)** Find the location between $x = 0$ and $x = 1.00$ m where the electric potential passes through zero. **(c)** Find the location in the region $x > 1.00$ m where the electric potential passes through zero.

21 Electric Current and Direct-Current Circuits



A battery is a device that uses chemical energy to separate positive and negative charges, producing a potential difference between its terminals. In this case, the chemical energy comes from reactions that take place between the metal electrodes and the acid in the lemon juice. The potential difference causes a current to flow in the wires, which is measured by the attached meter. This chapter explores simple electric circuits, like the one seen here, and shows how to analyze more complex ones as well.

As you read this paragraph, your heart is pumping blood through the arteries and veins in your body. In a way, your heart is acting like a battery in an electric circuit: A battery causes electric charge to flow through a closed circuit of wires; your heart causes blood to flow through your body. Just as the flow of blood is important to life, the flow of electric charge is of central importance to modern technology. In this chapter we consider some of the basic properties of moving electric charges, and we apply these results to simple electric circuits.

21–1 Electric Current

A flow of electric charge from one place to another is referred to as an **electric current.** Often, the charge is carried by electrons moving through a metal wire. Though the analogy should not be pushed too far, the electrons flowing through a wire are much like water molecules flowing through a garden hose or blood cells flowing through an artery.

To be specific, suppose a charge ΔQ flows past a given point in a wire in a time Δt. In such a case, we say that the electric current, I, in the wire is:

Definition of Electric Current, *I*

$$I = \frac{\Delta Q}{\Delta t} \qquad \text{21–1}$$

SI unit: coulomb per second, C/s = ampere, A

The unit of current, the ampere (A) or *amp* for short, is named for the French physicist André-Marie Ampère (1775–1836) and is defined simply as 1 coulomb per second:

$$1\ \mathrm{A} = 1\ \mathrm{C/s}$$

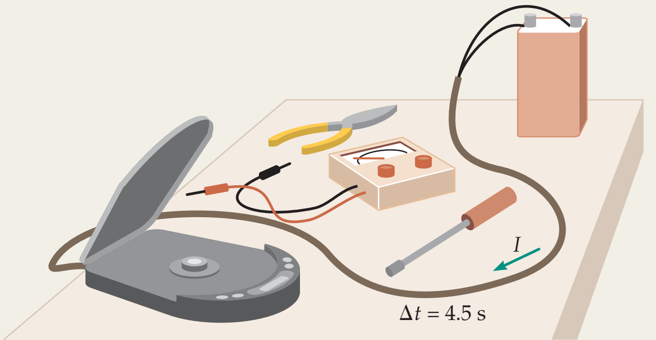
The following Example shows that the number of electrons involved in typical electric circuits, with currents of roughly an amp, is extremely large—not unlike the large number of water molecules flowing through a garden hose.

EXAMPLE 21–1 MEGA BLASTER

The disk drive in a portable CD player is connected to a battery that supplies it with a current of 0.22 A. How many electrons pass through the drive in 4.5 s?

PICTURE THE PROBLEM

Our sketch shows the CD drive with a current $I = 0.22$ A flowing through it. Also indicated is the time $\Delta t = 4.5$ s during which the current flows.



STRATEGY

Since we know both the current, I, and the length of time, Δt, we can use the definition of current, $I = \Delta Q/\Delta t$, to find the charge, ΔQ, that flows through the player. Once we know the charge, the number of electrons, N, is simply ΔQ divided by the magnitude of the electron's charge: $N = \Delta Q/e$.

SOLUTION

1. Calculate the charge, ΔQ, that flows through the drive: $\Delta Q = I\,\Delta t = (0.22\ \mathrm{A})(4.5\ \mathrm{s}) = 0.99\ \mathrm{C}$

2. Divide by the magnitude of the electron's charge, e, to find the number of electrons: $N = \dfrac{\Delta Q}{e} = \dfrac{0.99\ \mathrm{C}}{1.60 \times 10^{-19}\ \mathrm{C/electron}} = 6.2 \times 10^{18}$ electrons

INSIGHT

Thus, even a modest current flowing for a brief time corresponds to the transport of an extremely large number of electrons.

PRACTICE PROBLEM

How long must this current last if 7.5×10^{18} electrons are to flow through the disk drive? [**Answer:** 5.5 s]

Some related homework problems: Problem 1, Problem 2

When charge flows through a closed path and returns to its starting point, we refer to the closed path as an *electric circuit*. In this chapter we consider **direct-current circuits,** also known as dc circuits, in which the current always flows in the same direction. Circuits with currents that periodically reverse their direction

▲ Electric currents are not confined to the wires in our houses and machines, but occur in nature as well. A lightning bolt is simply an enormous, brief current. It flows when the difference in electric potential between cloud and ground (or cloud and cloud) becomes so great that it exceeds the breakdown strength of air. An enormous quantity of charge then leaps across the gap in a fraction of a second. Some organisms, such as this electric torpedo ray, have internal organic "batteries" that can produce significant electric potentials. The resulting current is used to stun their prey.

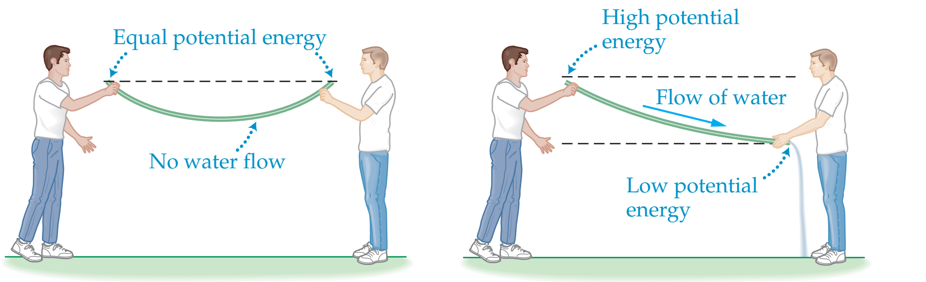
are referred to as **alternating-current circuits.** These AC circuits are considered in detail in Chapter 24.

Batteries and Electromotive Force

Although electrons move rather freely in metal wires, they do not flow unless the wires are connected to a source of electrical energy. A close analogy is provided by water in a garden hose. Imagine that you and a friend each hold one end of a garden hose filled with water. If the two ends are held at the same level, as in **Figure 21–1 (a)**, the water does not flow. If, however, one end is raised above the other, as in **Figure 21–1 (b)**, water flows from the high end—where the gravitational potential energy is high—to the low end.

▶ **FIGURE 21–1 Water flow as an analogy for electric current**

Water can flow quite freely through a garden hose, but if both ends are at the same level **(a)**, there is no flow. If the ends are held at different levels **(b)**, the water flows from the region where the gravitational potential energy is high to the region where it is low.



(a) Equal potential energy → no flow

(b) Water flows from high potential energy to low

A **battery** performs a similar function in an electric circuit. To put it simply, a battery uses chemical reactions to produce a difference in electric potential between its two ends, or **terminals.** The symbol for a battery is . The terminal corresponding to a high electric potential is denoted by a +, and the terminal corresponding to a low electric potential is denoted by a −. When the battery is connected to a circuit, electrons move in a closed path from the negative terminal of the battery, through the circuit, and back to the positive terminal.

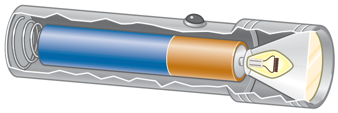
A simple example of an electrical system is shown in **Figure 21–2 (a)**, where we show a battery, a switch, and a lightbulb as they might be connected in a flashlight. In the schematic circuit shown in **Figure 21–2 (b)**, the switch is "open"—creating an **open circuit**—which means there is no closed path through which the electrons can flow. As a result, the light is off. When the switch is closed—which "closes" the circuit—charge flows around the circuit, causing the light to glow.

A mechanical analog to the flashlight circuit is shown in **Figure 21–3**. In this system, the person raising the water from a low to a high level is analogous to the battery, the paddle wheel is analogous to the lightbulb, and the water is analogous to the electric charge. Notice that the person does work in raising the water; later, as the water falls to its original level, it does work on the external world by turning the paddle wheel.

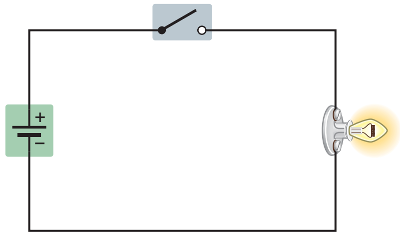
When a battery is disconnected from a circuit and carries no current, the difference in electric potential between its terminals is referred to as its *electromotive force*, or *emf* ($\mathcal{E}$). It follows that the units of emf are the same as those of electric potential, namely, volts. Clearly, then, the electromotive force is not really a force at all. Instead, the emf determines the amount of work a battery does to move a certain amount of charge around a circuit (like the person lifting water in Figure 21–3). To be specific, the magnitude of the work done by a battery of emf $\mathcal{E}$ as a charge ΔQ moves from one of its terminals to the other is given by Equation 20–2:

$$W = \Delta Q\mathcal{E}$$

We apply this relation to a flashlight circuit in the following Active Example.



(a) A simple flashlight



(b) Circuit diagram for flashlight

▲ **FIGURE 21–2 The flashlight: A simple electric circuit**

(a) A simple flashlight, consisting of a battery, a switch, and a lightbulb. **(b)** When the switch is in the open position, the circuit is "broken," and no charge can flow. When the switch is closed, electrons flow through the circuit and the light glows.

ACTIVE EXAMPLE 21–1 OPERATING A FLASHLIGHT: FIND THE CHARGE AND THE WORK

A battery with an emf of 1.5 V delivers a current of 0.44 A to a flashlight bulb for 64 s (see Figure 21–2). Find **(a)** the charge that passes through the circuit and **(b)** the work done by the battery.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

Part (a)

1. Use the definition of current, $I = \Delta Q/\Delta t$, to find the charge that flows through the circuit: $\Delta Q = 28\text{ C}$

Part (b)

2. Once we know ΔQ, we can use $W = \Delta Q\mathcal{E}$ to find the work: $W = 42\text{ J}$

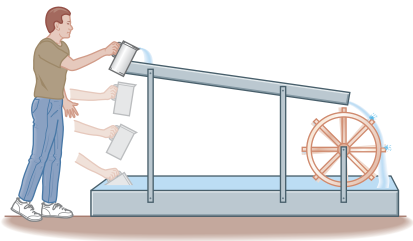
INSIGHT

Note that the more charge a battery moves through a circuit, the more work it does. Similarly, the greater the emf, the greater the work. We can see, then, that a car battery that operates at 12 volts and delivers several amps of current does much more work than a flashlight battery—as expected.

YOUR TURN

How long must the flashlight battery operate to do 150 J of work?

(Answers to **Your Turn** *problems are given in the back of the book.)*



▲ **FIGURE 21–3 A mechanical analog to the flashlight circuit**

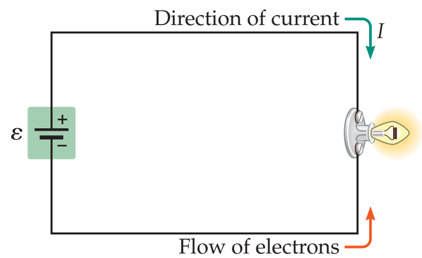
The person lifting the water corresponds to the battery in Figure 21–2, and the paddle wheel corresponds to the lightbulb.

The emf of a battery is the potential difference it can produce between its terminals under ideal conditions. In real batteries, however, there is always some internal loss, leading to a potential difference that is less than the ideal value. In fact, the greater the current flowing through a battery, the greater the reduction in potential difference between its terminals, as we shall see in Section 21–4. Only when the current is zero can a real battery produce its full emf. Because most batteries have relatively small internal losses, we shall treat batteries as ideal—always producing a potential difference precisely equal to $\mathcal{E}$—unless specifically stated otherwise.

When we draw an electric circuit, it will be useful to draw an arrow indicating the flow of current. By convention, the direction of the current arrow is given in terms of a positive test charge, in much the same way that the direction of the electric field is determined:

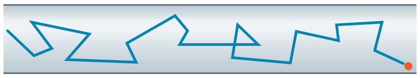
> The direction of the current in an electric circuit is the direction in which a *positive* test charge would move.

Of course, in typical circuits the charges that flow are actually *negatively* charged electrons. As a result, the flow of electrons and the current arrow point in opposite directions, as indicated in **Figure 21–4**. Notice that a positive charge will flow from



▲ **FIGURE 21–4 Direction of current and electron flow**

In the flashlight circuit, electrons flow from the negative terminal of the battery to the positive terminal. The direction of the current, I, is just the opposite: from the positive terminal to the negative terminal.



▲ **FIGURE 21–5 Path of an electron in a wire**

Typical path of an electron as it bounces off atoms in a metal wire. Because of the tortuous path the electron follows, its average velocity is rather small.

a region of high electric potential, near the positive terminal of the battery, to a region of low electric potential, near the negative terminal, as one would expect.

Finally, surprising as it may seem, electrons move rather slowly through a typical wire. They suffer numerous collisions with the atoms in the wire, and hence their path is rather tortuous and roundabout, as indicated in **Figure 21–5**. Like a car contending with a series of speed bumps, the electron's average speed, or **drift speed** as it is often called, is limited by the repeated collisions—in fact, their average speed is commonly about 10^{-4} m/s. Thus, if you switch on the headlights of a car, for example, an electron leaving the battery will take about an hour to reach the lightbulb, yet the lights seem to shine from the instant the switch is turned on. How is this possible?

The answer is that as an electron begins to move away from the battery, it exerts a force on its neighbors, causing them to move in the same general direction and, in turn, to exert a force on their neighbors, and so on. This process generates a propagating influence that travels through the wire at nearly the speed of light. The phenomenon is analogous to a bowling ball hitting one end of a line of balls; the effect of the colliding ball travels through the line at roughly the speed of sound, although the individual balls have very little displacement. Similarly, the electrons in a wire move with a rather small average velocity as they collide with and bounce off the atoms making up the wire, whereas the influence they have on one another races ahead and causes the light to shine.

21–2 Resistance and Ohm's Law

Electrons flow through metal wires with relative ease. In the ideal case, nothing about the wire would prevent their free motion. Real wires, however, under normal conditions, always affect the electrons to some extent, creating a **resistance** to their motion in much the same way that friction slows a box sliding across the floor.

In order to cause electrons to move against the resistance of a wire, it is necessary to apply a potential difference between its ends. For a wire with constant resistance, R, the potential difference, V, necessary to create a current, I, is given by Ohm's law:

Ohm's Law

$$V = IR \qquad \text{21–2}$$

SI unit: volt, V

Ohm's law is named for the German physicist Georg Simon Ohm (1789–1854).

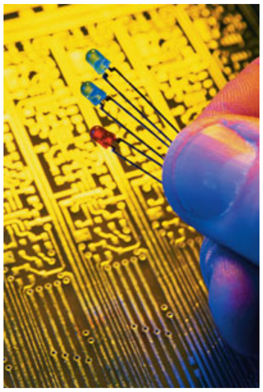
It should be noted at the outset that Ohm's law is not a law of nature but more on the order of a useful rule of thumb—like Hooke's law for springs or the ideal-gas laws that approximate the behavior of real gases. Materials that are well approximated by Ohm's law are said to be "ohmic" in their behavior; they show a simple linear relationship between the voltage applied to them and the current that results. In particular, if one plots current versus voltage for an ohmic material, the result is a straight line, with a constant slope equal to $1/R$. Nonohmic materials, on the other hand, have more complex relationships between voltage and current. A plot of current versus voltage for a nonohmic material is nonlinear; hence, the material does not have a constant resistance. (As an example, see Problem 9.) It is precisely these "nonlinearities," however, that can make such materials so useful in the construction of electronic devices, including the ubiquitous light-emitting diodes (LEDs).

Solving Ohm's law for the resistance, we find

$$R = \frac{V}{I}$$

From this expression it is clear that the units of resistance are volts per amp. In particular, we define 1 volt per amp to be 1 **ohm.** Letting the Greek letter omega, Ω, designate the ohm, we have

$$1\ \Omega = 1\ \text{V/A}$$



▲ A light-emitting diode (LED) is a relatively small, nonohmic device (top), but groups of LEDs can be used to form displays of practically any size (bottom). Because LEDs are extremely durable, and predicted to last 20 years or more, they are becoming the illumination of choice in high-reliability applications such as traffic lights, emergency exit signs, and brake lights. You'll probably see several on your way home today.

A device for measuring resistance is called an ohmmeter. We describe the operation of an ohmmeter in Section 21–8.

EXERCISE 21–1

A potential difference of 24 V is applied to a 150-Ω resistor. How much current flows through the resistor?

SOLUTION

Solving Ohm's law for the current, I, we find

$$I = \frac{V}{R} = \frac{24\text{ V}}{150\ \Omega} = \frac{24\text{ V}}{150\text{ V/A}} = 0.16\text{ A}$$

In an electric circuit a resistor is signified by a zigzag line:  . The straight lines in a circuit indicate ideal wires of zero resistance. To indicate the resistance of a real wire or device, we simply include a resistor of the appropriate value in the circuit.

Resistivity

Suppose you have a piece of wire of length L and cross-sectional area A. The resistance of this wire depends on the particular material from which it is constructed. If the wire is made of copper, for instance, its resistance will be less than if it is made from iron. The quantity that characterizes the resistance of a given material is its **resistivity,** ρ. For a wire of given dimensions, the greater the resistivity, the greater the resistance.

The resistance of a wire also depends on its length and area. To understand the dependence on L and A, consider again the analogy of water flowing through a hose. If the hose is very long, the resistance it presents to the water will be correspondingly large, whereas a wider hose—one with a greater cross-sectional area—will offer less resistance to the water. After all, water flows more easily through a short fire hose than through a long soda straw; hence, the resistance of a hose—and similarly a piece of wire—should be proportional to L and inversely proportional to A; that is, proportional to (L/A).

Combining these observations, we can write the resistance of a wire of length L, area A, and resistivity ρ in the following way:

Definition of Resistivity, ρ

$$R = \rho\left(\frac{L}{A}\right) \qquad 21\text{–}3$$

Since the units of L are m and the units of A are m^2, it follows that the units of resistivity are $(\Omega \cdot \text{m})$. Typical values for ρ are given in Table 21–1. Notice the enormous range in values of ρ, with the resistivity of an insulator like rubber about 10^{21} times greater than the resistivity of a good conductor like silver.

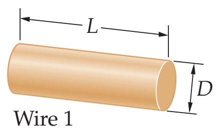
TABLE 21–1 Resistivities

Substance	Resistivity, ρ $(\Omega \cdot \text{m})$
Insulators	
Quartz (fused)	7.5×10^{17}
Rubber	$1 \text{ to } 100 \times 10^{13}$
Glass	$1 \text{ to } 10{,}000 \times 10^{9}$
Semiconductors	
Silicon*	0.10 to 60
Germanium*	0.001 to 0.5
Conductors	
Lead	22×10^{-8}
Iron	9.71×10^{-8}
Tungsten	5.6×10^{-8}
Aluminum	2.65×10^{-8}
Gold	2.20×10^{-8}
Copper	1.68×10^{-8}
Silver	1.59×10^{-8}

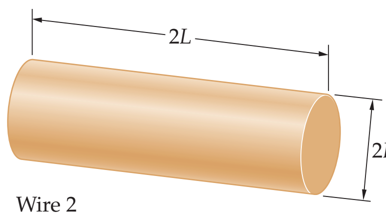
*The resistivity of a semiconductor varies greatly with the type and amount of impurities it contains. This property makes them particularly useful in electronic applications.

CONCEPTUAL CHECKPOINT 21–1 COMPARE THE RESISTANCE

Wire 1 has a length L and a circular cross section of diameter D. Wire 2 is constructed from the same material as wire 1 and has the same shape, but its length is $2L$, and its diameter is $2D$. Is the resistance of wire 2 **(a)** the same as that of wire 1, **(b)** twice that of wire 1, or **(c)** half that of wire 1?



Wire 1



Wire 2

REASONING AND DISCUSSION

First, the resistance of wire 1 is

$$R_1 = \rho\left(\frac{L}{A}\right) = \rho\frac{L}{(\pi D^2/4)}$$

Note that we have used the fact that the area of a circle of diameter D is $\pi D^2/4$. For wire 2 we replace L with $2L$ and D with $2D$:

$$R_2 = \rho\frac{2L}{[\pi(2D)^2/4]} = \left(\tfrac{1}{2}\right)\rho\frac{L}{(\pi D^2/4)} = \tfrac{1}{2}R_1$$

CONTINUED ON NEXT PAGE

Thus, increasing the length by a factor of 2 increases the resistance by a factor of 2; on the other hand, increasing the diameter by a factor of 2 increases the area, and decreases the resistance, by a factor of 4. Overall, then, the resistance of wire 2 is half that of wire 1.

ANSWER

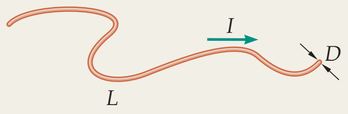
(c) The resistance of wire 2 is half that of wire 1; $R_2 = R_1/2$.

EXAMPLE 21–2 A CURRENT-CARRYING WIRE

A current of 1.82 A flows through a copper wire 1.75 m long and 1.10 mm in diameter. Find the potential difference between the ends of the wire. (The value of ρ for copper may be found in Table 21–1.)

PICTURE THE PROBLEM

The wire carries a current $I = 1.82$ A, and its total length L is 1.75 m. We assume that the wire has a circular cross section, with a diameter $D = 1.10$ mm.



STRATEGY

We know from Ohm's law that the potential difference associated with a current I and a resistance R is $V = IR$. We are given the current in the wire, but not the resistance. The resistance is easily determined, however, using $R = \rho(L/A)$ with $A = \pi D^2/4$. Thus, we first calculate R and then substitute the result into $V = IR$ to obtain the potential difference.

SOLUTION

1. Calculate the resistance of the wire:

$$R = \rho\left(\frac{L}{A}\right) = \rho\left(\frac{L}{\pi D^2/4}\right)$$

$$= (1.72 \times 10^{-8}\ \Omega \cdot \mathrm{m})\left[\frac{1.75\ \mathrm{m}}{\pi(0.00110\ \mathrm{m})^2/4}\right] = 0.0317\ \Omega$$

2. Multiply R by the current, I, to find the potential difference:

$$V = IR = (1.82\ \mathrm{A})(0.0317\ \Omega) = 0.0577\ \mathrm{V}$$

INSIGHT

Copper is an excellent conductor; therefore, both the resistance and the potential difference are quite small.

PRACTICE PROBLEM

What diameter of copper wire is needed for there to be a potential difference of 0.100 V? Assume that all other quantities remain the same. [**Answer:** 0.835 mm]

Some related homework problems: Problem 17, Problem 18

Temperature Dependence and Superconductivity

We know from everyday experience that a wire carrying an electric current can become warm—even quite hot, as in the case of a burner on a stove or the filament in an incandescent lightbulb. This follows from our earlier discussion of the fact that electrons collide with the atoms in a wire as they flow through an electric circuit. These collisions cause the atoms to jiggle with greater kinetic energy about their equilibrium positions. As a result, the temperature of the wire increases (see Section 17–2, and Equation 17–21 in particular). For example, the wire filament in an incandescent lightbulb can reach temperatures of roughly 2800 °C (in comparison, the surface of the Sun has a temperature of about 5500 °C), and the heating coil on a stove has a temperature of about 750 °C.

As a wire is heated, its resistivity tends to increase. This is because atoms that are jiggling more rapidly are more likely to collide with electrons and slow their progress through the wire. In fact, many metals show an approximately linear increase of ρ over a wide range of temperature. Once the dependence of ρ on T is known for a given material, the change in resistivity can be used as a means of measuring temperature.

REAL-WORLD PHYSICS
The bolometer

The first practical application of this principle was in a device known as the **bolometer.** Invented in 1880, the bolometer is an extremely sensitive thermometer that uses the temperature variation in the resistivity of platinum, nickel, or bismuth as a means of detecting temperature changes as small as 0.0001 C°. Soon after its invention, a bolometer was used to detect infrared radiation from the stars.

Some materials, like semiconductors, actually show a drop in resistivity as temperature is increased. This is because the resistivity of a semiconductor is strongly dependent on the number of electrons that are free to move about and conduct a current. As the temperature is increased in a semiconductor, more electrons are able to break free from their atoms, leading to an increased current and a reduced resistivity. Electronic devices incorporating such temperature-dependent semiconductors are known as **thermistors.** The digital fever thermometer so common in today's hospitals uses a thermistor to provide accurate measurements of a patient's temperature.

REAL-WORLD PHYSICS

Thermistors and fever thermometers

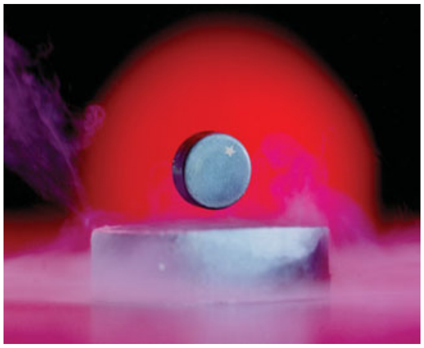
Since resistivity typically increases with temperature, it follows that if a wire is cooled below room temperature, its resistivity will *decrease*. Quite surprising, however, was a discovery made in the laboratory of Heike Kamerlingh-Onnes in 1911. Measuring the resistance of a sample of mercury at temperatures just a few degrees above absolute zero, researchers found that at about 4.2 K the resistance of the mercury suddenly dropped to zero—not just to a very small value, but to *zero*. At this temperature, we say that the mercury becomes **superconducting,** a hitherto unknown phase of matter. Since that time many different superconducting materials have been discovered, with various different **critical temperatures,** T_c, at which superconductivity begins. Today we know that superconductivity is a result of quantum effects (Chapter 30).

REAL-WORLD PHYSICS

Superconductors and high-temperature superconductivity

When a material becomes superconducting, a current can flow through it with absolutely no resistance. In fact, if a current is initiated in a superconducting ring of material, it will flow undiminished for as long as the ring is kept cool enough. In some cases, circulating currents have been maintained for years, with absolutely no sign of diminishing.

In 1986 a new class of superconductors was discovered that has zero resistance at temperatures significantly greater than those of any previously known superconducting materials. At the moment, the highest temperature at which superconductivity has been observed is about 125 K. Since the discovery of these "high-T_c" superconductors, hopes have been raised that it may one day be possible to produce room-temperature superconductors. The practical benefits of such a breakthrough, including power transmission with no losses, improved MRI scanners, and magnetically levitated trains, could be immense.



▲ When cooled below their critical temperature, superconductors not only lose their resistance to current flow but also exhibit new magnetic properties, such as repelling an external magnetic field. Here, a superconductor (bottom) levitates a small permanent magnet.

21–3 Energy and Power in Electric Circuits

When a charge ΔQ moves across a potential difference V, its electrical potential energy, U, changes by the amount

$$\Delta U = (\Delta Q)V$$

Recalling that power is the rate at which energy changes, $P = \Delta U/\Delta t$, we can write the electrical power as follows:

$$P = \frac{\Delta U}{\Delta t} = \frac{(\Delta Q)V}{\Delta t}$$

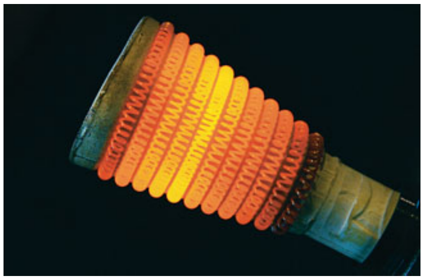
Since the electric current is given by $I = \Delta Q/\Delta t$, we have:

Electrical Power

$$P = IV \qquad 21\text{–}4$$

SI unit: watt, W

Thus, a current of 1 amp flowing across a potential difference of 1 V produces a power of 1 W.



▲ The heating element of an electric space heater is nothing more than a length of resistive wire coiled up for compactness. As electric current flows through the wire, the power it dissipates ($P = I^2R$) is converted to heat and light. The coils near the center are the hottest, and hence they glow with a higher-frequency, yellowish light.

EXERCISE 21–2

A handheld electric fan operates on a 3.00-V battery. If the power generated by the fan is 2.24 W, what is the current supplied by the battery?

SOLUTION

Solving $P = IV$ for the current, we obtain

$$I = \frac{P}{V} = \frac{2.24\ \text{W}}{3.00\ \text{V}} = 0.747\ \text{A}$$

The expression $P = IV$ applies to any electrical system. In the special case of a resistor, the electrical power is dissipated in the form of heat. Applying Ohm's law $(V = IR)$ to this case, we can write the power dissipated in a resistor as

$$P = IV = I(IR) = I^2R \qquad 21\text{–}5$$

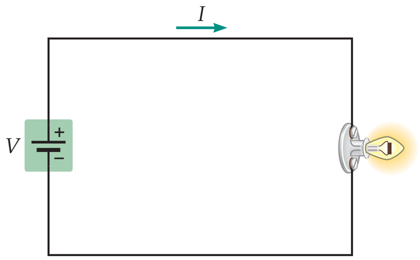
Similarly, using Ohm's law to solve for the current, $I = V/R$, we have

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R} \qquad 21\text{–}6$$

These relations also apply to incandescent lightbulbs, which are basically resistors that become hot enough to glow.

CONCEPTUAL CHECKPOINT 21–2 COMPARE LIGHTBULBS

A battery that produces a potential difference V is connected to a 5-W lightbulb. Later, the 5-W lightbulb is replaced with a 10-W lightbulb. **(a)** In which case does the battery supply more current? **(b)** Which lightbulb has the greater resistance?



REASONING AND DISCUSSION

a. To compare the currents, we need consider only the relation $P = IV$. Solving for the current yields $I = P/V$. When the voltage V is the same, it follows that the greater the power, the greater the current. In this case, then, the current in the 10-W bulb is twice the current in the 5-W bulb.

b. We now consider the relation $P = V^2/R$, which gives resistance in terms of voltage and power. In fact, $R = V^2/P$. Again, with V the same, it follows that the smaller the power, the greater the resistance. Thus, the resistance of the 5-W bulb is twice that of the 10-W bulb.

ANSWER

(a) When the battery is connected to the 10-W bulb, it delivers twice as much current as when it is connected to the 5-W bulb. **(b)** The 5-W bulb has twice as much resistance as the 10-W bulb.

On a microscopic level, the power dissipated by a resistor is the result of incessant collisions between electrons moving through the circuit and atoms making up the resistor. Specifically, the electric potential difference produced by the battery causes electrons to accelerate until they bounce off an atom of the resistor. At this point the electrons transfer energy to the atoms, causing them to jiggle more rapidly. The increased kinetic energy of the atoms is reflected in an increased temperature of the resistor (see Section 17–2). After each collision, the potential difference accelerates the electrons again and the process repeats—like a car bouncing through a series of speed bumps—resulting in a continuous transfer of energy from the electrons to the atoms.

EXAMPLE 21–3 HEATED RESISTANCE

A battery with an emf of 12 V is connected to a 545-Ω resistor. How much energy is dissipated in the resistor in 65 s?

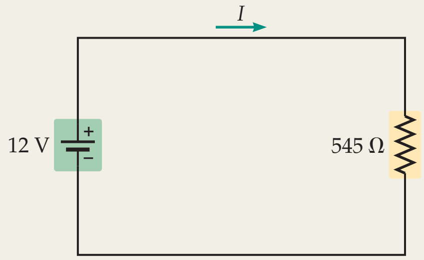
PICTURE THE PROBLEM

The circuit, consisting of a battery and a resistor, is shown in our sketch. We show the current flowing from the positive terminal of the 12-V battery, through the 545-Ω resistor, and into the negative terminal of the battery.

STRATEGY

We know that a current flowing through a resistor dissipates power (energy per time), which means that the energy it dissipates in a given time is simply the power multiplied by the time: $\Delta U = P\,\Delta t$. The time is given $(\Delta t = 65\text{ s})$, and the power can be found using $P = IV$, $P = I^2R$, or $P = V^2/R$. The last expression is most convenient in this case, because the problem statement gives us the voltage and resistance.

To summarize, we first calculate the power, then multiply by the time.



SOLUTION

1. Calculate the power dissipated in the resistor: $P = V^2/R = (12\text{ V})^2/(545\ \Omega) = 0.26\text{ W}$

2. Multiply the power by the time to find the energy dissipated: $\Delta U = P\,\Delta t = (0.26\text{ W})(65\text{ s}) = 17\text{ J}$

INSIGHT

The current in this circuit is $I = V/R = 0.022$ A. Using this result, we find that the power is $P = I^2R = IV = 0.26$ W, as expected.

PRACTICE PROBLEM

How much energy is dissipated in the resistor if the voltage is doubled to 24 V? [**Answer:** $4(17\text{ J}) = 68\text{ J}$]

Some related homework problems: Problem 29, Problem 32

◀ The battery testers now often built into battery packages (left) employ a tapered graphite strip. The narrow end (at bottom in the right-hand photo) has the highest resistance, and thus produces the most heat when a current flows through the strip. The heat is used to produce the display on the front that indicates the strength of the battery—if the current is sufficient to warm even the top of the strip, where the resistance is lowest, the battery is fresh.

REAL-WORLD PHYSICS

"Battery check" meters

A commonly encountered application of resistance heating is found in the "battery check" meters often included with packs of batteries. To operate one of these meters, you simply press the contacts on either end of the meter against the corresponding terminals of the battery to be checked. This allows a current to flow through the main working element of the meter—a tapered strip of graphite.

The reason the strip is tapered is to provide a variation in resistance. According to the relation given in Equation 21–3, $R = \rho(L/A)$, the smaller the cross-sectional area A of the strip, the larger the resistance R. It follows that the narrow end has a higher resistance than the wide end. Because the same current I flows through all parts of the strip, the power dissipated is expressed most conveniently in the form $P = I^2R$. It follows that at the narrow end of the strip, where R is largest, the heating due to the current will be the greatest. Pressing the meter against the terminals of the battery, then, results in an overall warming of the graphite strip, with the narrow end warmer than the wide end.

The final element in the meter is a thin layer of liquid crystal (similar to the material used in LCD displays) that responds to small increases in temperature. In particular, this liquid crystal is black and opaque at room temperature but transparent when heated slightly. The liquid crystal is placed in front of a colored background, which can be seen in those regions where the graphite strip is warm enough to make the liquid crystal transparent. If the battery is weak, only the narrow portion of the strip becomes warm enough, and the meter shows only a small stripe of color. A strong battery, on the other hand, heats the entire strip enough to make the liquid crystal transparent, resulting in a colored stripe the full length of the meter.

Energy Usage

When you get a bill from the local electric company, you will find the number of kilowatt-hours of electricity that you have used. Notice that a kilowatt-hour (kWh) has the units of energy:

$$1 \text{ kilowatt-hour} = (1000 \text{ W})(3600 \text{ s}) = (1000 \text{ J/s})(3600 \text{ s})$$
$$= 3.6 \times 10^6 \text{ J}$$

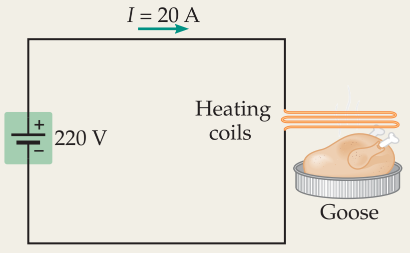
Thus, the electric company is charging for the amount of energy you use—as one would expect—and not for the rate at which you use it. The following Example considers the energy and monetary cost for a typical everyday situation.

EXAMPLE 21–4 YOUR GOOSE IS COOKED

A holiday goose is cooked in the kitchen oven for 4.00 h. Assume that the stove draws a current of 20.0 A, operates at a voltage of 220.0 V, and uses electrical energy that costs \$0.068 per kWh. How much does it cost to cook your goose?

PICTURE THE PROBLEM

We show a schematic representation of the stove cooking the goose in our sketch. The current in the circuit is 20.0 A, and the voltage difference across the heating coils is 220 V.



STRATEGY

The cost is simply the energy usage (in kWh) times the cost per kilowatt-hour (\$0.068). To find the energy used, we note that energy is power multiplied by time. The time is given, and the power associated with a current I and a voltage V is $P = IV$.

Thus, we find the power, multiply by the time, and then multiply by \$0.068 to find the cost.

SOLUTION

1. Calculate the power delivered to the stove: $P = IV = (20.0 \text{ A})(220.0 \text{ V}) = 4.40 \text{ kW}$
2. Multiply power by time to determine the total energy supplied to the stove during the 4.00 h of cooking: $\Delta U = P\,\Delta t = (4.40 \text{ kW})(4.00 \text{ h}) = 17.6 \text{ kWh}$
3. Multiply by the cost per kilowatt-hour to find the total cost of cooking: $\text{cost} = (17.6 \text{ kWh})(\$0.068/\text{kWh}) = \$1.20$

INSIGHT

Thus, your goose can be cooked for just over a dollar.

PRACTICE PROBLEM

If the voltage and current are reduced by a factor of 2 each, how long must the goose be cooked to use the same amount of energy? [**Answer:** $4(4.00 \text{ h}) = 16.0 \text{ h}$. *Note*: You should be able to answer a question like this by referring to your previous solution, without repeating the calculation in detail.]

Some related homework problems: Problem 30, Problem 31

21–4 Resistors in Series and Parallel

Electric circuits often contain a number of resistors connected in various ways. In this section we consider simple circuits containing only resistors and batteries. For each type of circuit considered, we calculate the **equivalent resistance** produced by a group of individual resistors.

Resistors in Series

When resistors are connected one after the other, end to end, we say that they are in *series*. **Figure 21–6 (a)** shows three resistors, R_1, R_2, and R_3, connected in series. The three resistors acting together have the same effect—that is, they draw the same current—as a single resistor, referred to as the equivalent resistor, R_{eq}. This equivalence is illustrated in **Figure 21–6 (b)**. We now calculate the value of the equivalent resistance.

The first thing to notice about the circuit in Figure 21–6 (a) is that the same current I must flow through each of the resistors—there is no other place for the current to go. As a result, the potential differences across the three resistors are

$V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$, respectively. Since the total potential difference from point A to point B must be the emf of the battery, $\mathcal{E}$, it follows that

$$\mathcal{E} = V_1 + V_2 + V_3$$

Writing each of the potentials in terms of the current and resistance, we find

$$\mathcal{E} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$$

Now, let's compare this expression with the result we obtain for the equivalent circuit in Figure 21–6 (b). In this circuit, the potential difference across the battery is $V = IR_{\text{eq}}$. Since this potential must be the same as the emf of the battery, we have

$$\mathcal{E} = IR_{\text{eq}}$$

Comparing this expression with $\mathcal{E} = I(R_1 + R_2 + R_3)$, we see that the equivalent resistance is simply the sum of the individual resistances:

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

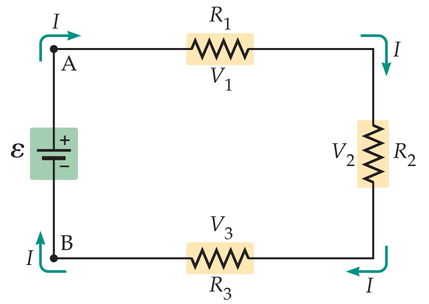
In general, for any number of resistors in series, the equivalent resistance is

Equivalent Resistance for Resistors in Series

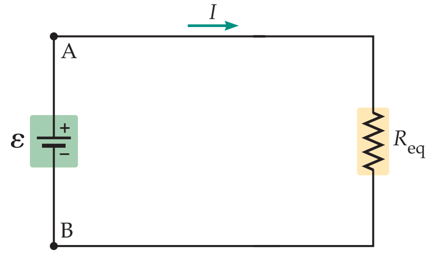
$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots = \sum R \qquad \text{21–7}$$

SI unit: ohm, Ω

Note that the equivalent resistance is greater than the greatest resistance of any of the individual resistors. Connecting the resistors in series is like making a single resistor increasingly longer; as its length increases so does its resistance.



(a) Three resistors in series



(b) Equivalent resistance has the same current

FIGURE 21–6 Resistors in series

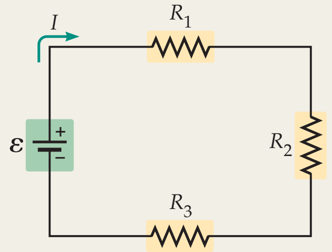
(a) Three resistors, R_1, R_2, and R_3, connected in series. Note that the same current I flows through each resistor. **(b)** The equivalent resistance, $R_{\text{eq}} = R_1 + R_2 + R_3$, has the same current I flowing through it as the current I in the original circuit.

EXAMPLE 21–5 THREE RESISTORS IN SERIES

A circuit consists of three resistors connected in series to a 24.0-V battery. The current in the circuit is 0.0320 A. Given that $R_1 = 250.0\ \Omega$ and $R_2 = 150.0\ \Omega$, find **(a)** the value of R_3 and **(b)** the potential difference across each resistor.

PICTURE THE PROBLEM

The circuit is shown in our sketch. Note that the 24.0-V battery delivers the same current, $I = 0.0320$ A, to each of the three resistors. This is the key characteristic of a series circuit.



STRATEGY

a. First, we can obtain the equivalent resistance of the circuit using Ohm's law (as in Equation 21–2); $R_{\text{eq}} = \mathcal{E}/I$. Since the resistors are in series, we also know that $R_{\text{eq}} = R_1 + R_2 + R_3$. We can solve this relation for the only unknown, R_3.

b. We can then calculate the potential difference across each resistor using Ohm's law, $V = IR$.

SOLUTION

Part (a)

1. Use Ohm's law to find the equivalent resistance of the circuit: $R_{\text{eq}} = \dfrac{\mathcal{E}}{I} = \dfrac{24.0\ \text{V}}{0.0320\ \text{A}} = 7.50 \times 10^2\ \Omega$

2. Set R_{eq} equal to the sum of the individual resistances, and solve for R_3:

$$R_{\text{eq}} = R_1 + R_2 + R_3$$
$$R_3 = R_{\text{eq}} - R_1 - R_2$$
$$= 7.50 \times 10^2\ \Omega - 250.0\ \Omega - 150.0\ \Omega = 3.50 \times 10^2\ \Omega$$

Part (b)

3. Use Ohm's law to determine the potential difference across R_1: $V_1 = IR_1 = (0.0320\ \text{A})(250.0\ \Omega) = 8.00\ \text{V}$

4. Find the potential difference across R_2: $V_2 = IR_2 = (0.0320\ \text{A})(150.0\ \Omega) = 4.80\ \text{V}$

5. Find the potential difference across R_3: $V_3 = IR_3 = (0.0320\ \text{A})(3.50 \times 10^2\ \Omega) = 11.2\ \text{V}$

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INSIGHT

Note that the greater the resistance, the greater the potential difference. In addition, the sum of the individual potential differences is 8.00 V + 4.80 V + 11.2 V = 24.0 V, as expected.

PRACTICE PROBLEM

Find the power dissipated in each resistor. [**Answer:** $P_1 = 0.256\text{ W}$, $P_2 = 0.154\text{ W}$, $P_3 = 0.358\text{ W}$]

Some related homework problems: Problem 43, Problem 44

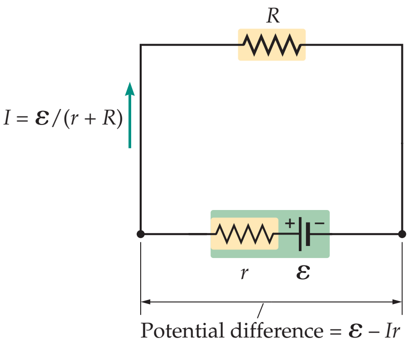


FIGURE 21–7 The internal resistance of a battery

Real batteries always dissipate some energy in the form of heat. These losses can be modeled by a small "internal" resistance, r, within the battery. As a result, the potential difference between the terminals of a real battery is less than its ideal emf, $\mathcal{E}$. For example, if a battery produces a current I, the potential difference between its terminals is $\mathcal{E} - Ir$. In the case shown here, a battery is connected in series with the resistor R. Instead of producing the current $I = \mathcal{E}/R$, as in the ideal case, it produces the current $I = \mathcal{E}/(r + R)$.

An everyday example of resistors in series is the **internal resistance,** r, of a battery. As was mentioned in Section 21–1, real batteries have internal losses that cause the potential difference between their terminals to be less than $\mathcal{E}$ and to depend on the current in the battery. The simplest way to model a real battery is to imagine it to consist of an ideal battery of emf $\mathcal{E}$ in series with an internal resistance r, as shown in **Figure 21–7**. If this battery is then connected to an external resistance, R, the equivalent resistance of the circuit is $r + R$. As a result, the current flowing through the circuit is $I = \mathcal{E}/(r + R)$, and the potential difference between the terminals of the battery is $\mathcal{E} - Ir$. Thus, we see that the potential difference produced by the battery is less than $\mathcal{E}$ by an amount that is proportional to the current I. Only in the limit of zero current, or zero internal resistance, will the battery produce its full emf. (See Problems 51, 54, 116, and 121 for examples of batteries with internal resistance.)

Another application of resistors in series is the three-way lightbulb circuit shown in **Figure 21–8**. In this circuit, the two resistors represent two different filaments within a single bulb that are connected to a constant potential difference V. At the "high" setting, the lower-resistance filament, R_1, is connected to the electrical outlet via terminals A and B, and the brightest light is obtained $(P = V^2/R)$. At the "middle" setting, the higher-resistance filament, R_2, is connected to the outlet via terminals B and C, resulting in a dimmer light. Finally, at the "low" setting, both filaments are connected in series via terminals A and C. This setting gives the greatest equivalent resistance, and thus the lowest light output.

An alternative method of producing a three-way lightbulb is to connect the resistors in parallel. This will be discussed in the next subsection.

REAL-WORLD PHYSICS

Three-way lightbulbs

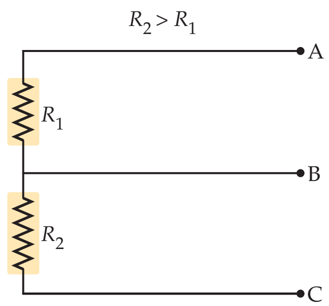


FIGURE 21–8 A three-way bulb

The circuit diagram for a three-way lightbulb. For the brightest light, terminals A and B are connected to the household electrical line, so the current passes through the low-resistance filament R_1. For intermediate brightness, terminals B and C are used, so the current passes through the higher-resistance filament R_2. For the lowest light output, terminals A and C are used, so the current passes through both R_1 and R_2 in series.

Resistors in Parallel

Resistors are in *parallel* when they are connected across the same potential difference, as in **Figure 21–9 (a)**. In a case like this, the current has parallel paths through which it can flow. As a result, the total current in the circuit, I, is equal to the sum of the currents through each of the three resistors:

$$I = I_1 + I_2 + I_3$$

Since the potential difference is the same for each of the resistors, it follows that the currents flowing through them are as follows:

$$I_1 = \frac{\mathcal{E}}{R_1}, \quad I_2 = \frac{\mathcal{E}}{R_2}, \quad I_3 = \frac{\mathcal{E}}{R_3}$$

Summing these three currents, we find

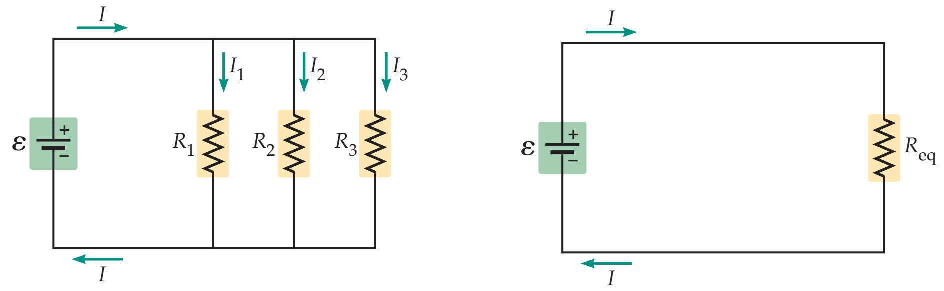
$$I = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} + \frac{\mathcal{E}}{R_3} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \qquad 21\text{–}8$$

Now, in the equivalent circuit shown in **Figure 21–9 (b)**, Ohm's law gives $\mathcal{E} = IR_{\text{eq}}$, or

$$I = \mathcal{E}\left(\frac{1}{R_{\text{eq}}}\right) \qquad 21\text{–}9$$

Comparing Equations 21–8 and 21–9, we find that the equivalent resistance for three resistors in parallel is

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$



(a) Three resistors in parallel

(b) Equivalent resistance has the same current

FIGURE 21–9 Resistors in parallel

(a) Three resistors, R_1, R_2, and R_3, connected in parallel. Note that each resistor is connected across the same potential difference $\mathcal{E}$. **(b)** The equivalent resistance, $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$, has the same current flowing through it as the total current I in the original circuit.

In general, for any number of resistors in parallel, we have:

Equivalent Resistance for Resistors in Parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R} \qquad 21\text{–}10$$

SI unit: ohm, Ω

PROBLEM-SOLVING NOTE

The Equivalent Resistance of Resistors in Parallel

After summing the inverse of resistors in parallel, remember to take one more inverse at the end of your calculation to find the equivalent resistance.

As a simple example, consider a circuit with two identical resistors R connected in parallel. The equivalent resistance in this case is given by

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

Solving for R_{eq}, we find $R_{eq} = \frac{1}{2}R$. If we connect three such resistors in parallel, the corresponding result is

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

In this case, $R_{eq} = \frac{1}{3}R$. Thus, the more resistors we connect in parallel, the smaller the equivalent resistance. Each time we add a new resistor in parallel with the others, we give the battery a new path through which current can flow—analogous to opening an additional lane of traffic on a busy highway. Stated another way, giving the current multiple paths through which it can flow is equivalent to using a wire with a greater cross-sectional area. From either point of view, the fact that more current flows with the same potential difference means that the equivalent resistance has been reduced.

Finally, if any one of the resistors in a parallel connection is equal to zero, the equivalent resistance is also zero. This situation is referred to as a **short circuit,** and is illustrated in **Figure 21–10**. In this case, all of the current flows through the path of zero resistance.

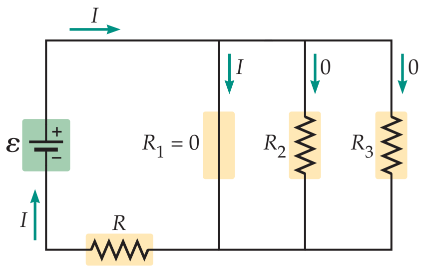


FIGURE 21–10 A short circuit

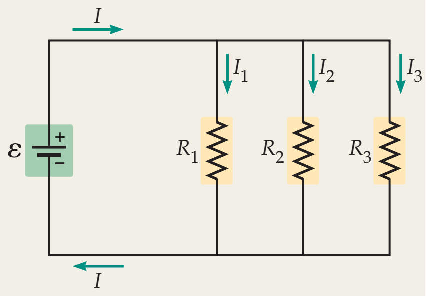
If one of the resistors in parallel with others is equal to zero, all the current flows through that portion of the circuit, giving rise to a short circuit. In this case, resistors R_2 and R_3 are "shorted out," and the current in the circuit is $I = \mathcal{E}/R$.

EXAMPLE 21–6 THREE RESISTORS IN PARALLEL

Consider a circuit with three resistors, $R_1 = 250.0\ \Omega$, $R_2 = 150.0\ \Omega$, and $R_3 = 350.0\ \Omega$, connected in parallel with a 24.0-V battery. Find **(a)** the total current supplied by the battery and **(b)** the current through each resistor.

PICTURE THE PROBLEM

The accompanying sketch indicates the parallel connection of the resistors with the battery. Notice that each of the resistors experiences precisely the same potential difference; namely, the 24.0 V produced by the battery. This is the feature that characterizes parallel connections.



STRATEGY

a. We can find the total current from $I = \mathcal{E}/R_{eq}$, where $1/R_{eq} = 1/R_1 + 1/R_2 + 1/R_3$.

b. For each resistor, the current is given by Ohm's law, $I = \mathcal{E}/R$.

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SOLUTION

Part (a)

1. Find the equivalent resistance of the circuit:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$= \frac{1}{250.0\ \Omega} + \frac{1}{150.0\ \Omega} + \frac{1}{350.0\ \Omega} = 0.01352\ \Omega^{-1}$$

$$R_{eq} = (0.01352\ \Omega^{-1})^{-1} = 73.96\ \Omega$$

2. Use Ohm's law to find the total current:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{24.0\ \text{V}}{73.96\ \Omega} = 0.325\ \text{A}$$

Part (b)

3. Calculate I_1 using $I_1 = \mathcal{E}/R_1$ with $\mathcal{E} = 24.0$ V:

$$I_1 = \frac{\mathcal{E}}{R_1} = \frac{24.0\ \text{V}}{250.0\ \Omega} = 0.0960\ \text{A}$$

4. Repeat the preceding calculation for resistors 2 and 3:

$$I_2 = \frac{\mathcal{E}}{R_2} = \frac{24.0\ \text{V}}{150.0\ \Omega} = 0.160\ \text{A}$$

$$I_3 = \frac{\mathcal{E}}{R_3} = \frac{24.0\ \text{V}}{350.0\ \Omega} = 0.0686\ \text{A}$$

INSIGHT

As expected, the smallest resistor, R_2, carries the greatest current. The three currents combined yield the total current, as they must; that is, $I_1 + I_2 + I_3 = 0.0960\ \text{A} + 0.160\ \text{A} + 0.0686\ \text{A} = 0.325\ \text{A} = I$.

PRACTICE PROBLEM

Find the power dissipated in each resistor. [**Answer:** $P_1 = 2.30$ W, $P_2 = 3.84$ W, $P_3 = 1.65$ W]

Some related homework problems: Problem 45, Problem 46

In comparing Examples 21–5 and 21–6 note the differences in the power dissipated in each circuit. First, the total power dissipated in the parallel circuit is much greater than that dissipated in the series circuit. This is due to the fact that the equivalent resistance of the parallel circuit is smaller than the equivalent resistance of the series circuit, and the power delivered by a voltage V to a resistance R is inversely proportional to the resistance ($P = V^2/R$). In addition, note that the smallest resistor, R_2, has the smallest power in the series circuit but the largest power in the parallel circuit. These issues are explored further in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 21–3 SERIES VERSUS PARALLEL

Two identical lightbulbs are connected to a battery, either in series or in parallel. Are the bulbs in series **(a)** brighter than, **(b)** dimmer than, or **(c)** the same brightness as the bulbs in parallel?

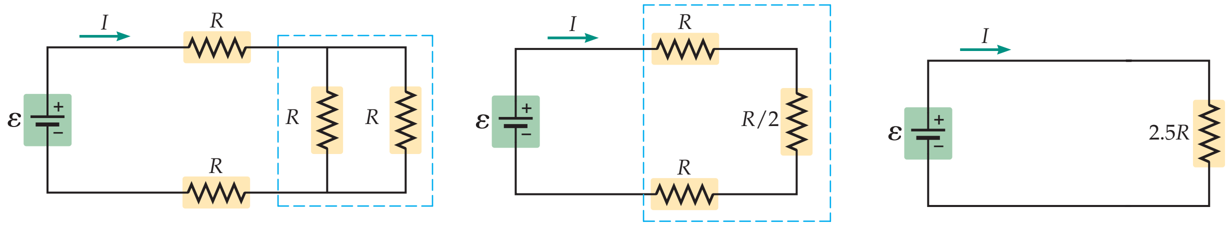
REASONING AND DISCUSSION

Both sets of lightbulbs are connected to the same potential difference, V; hence, the power delivered to the bulbs is V^2/R_{eq}, where R_{eq} is twice the resistance of a bulb in the series circuit and half the resistance of a bulb in the parallel circuit. As a result, more power is converted to light in the parallel circuit.

ANSWER

(b) The bulbs connected in series are dimmer than the bulbs connected in parallel.

Finally, note that a three-way lightbulb can also be produced by simply wiring two filaments in parallel. For example, one filament might have a power of 50 W and the second filament a power of 100 W. One setting of the switch sends current through the 50-W filament, the next setting sends current through the 100-W filament, and the third setting connects the two filaments in parallel. With the third connection, each filament produces the same power as before—since each is connected to the same potential difference—giving a total power of 50 W + 100 W = 150 W.



(a) Replace parallel resistors **(b)** Replace series resistors **(c)** Final equivalent resistance

▲ **FIGURE 21–11 Analyzing a complex circuit of resistors**

(a) The two vertical resistors are in parallel with one another; hence, they can be replaced with their equivalent resistance, $R/2$. **(b)** Now the circuit consists of three resistors in series. The equivalent resistance of these three resistors is 2.5 R. **(c)** The original circuit reduced to a single equivalent resistance.

Combination Circuits

The rules we have developed for series and parallel resistors can be applied to more complex circuits as well. For example, consider the circuit shown in **Figure 21–11 (a)**, where four resistors, each equal to R, are connected in a way that combines series and parallel features. To analyze this circuit, we first note that the two vertically oriented resistors are connected in parallel with one another. Therefore, the equivalent resistance of this unit is given by $1/R_{eq} = 1/R + 1/R$, or $R_{eq} = R/2$. Replacing these two resistors with $R/2$ yields the circuit shown in **Figure 21–11 (b)**, which consists of three resistors in series. As a result, the equivalent resistance of the entire circuit is $R + R/2 + R = 2.5R$, as indicated in **Figure 21–11 (c)**. Similar methods can be applied to a wide variety of circuits.

PROBLEM-SOLVING NOTE

Analyzing a Complex Circuit

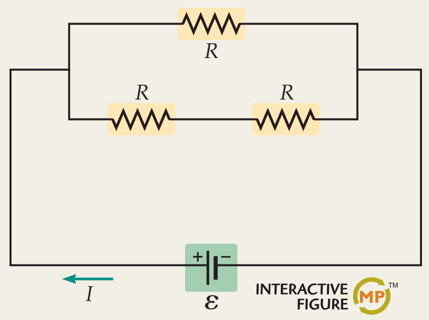
When considering an electric circuit with resistors in series and parallel, work from the smallest units of the circuit outward to ever larger units.

EXAMPLE 21–7 COMBINATION SPECIAL

In the circuit shown in the diagram, the emf of the battery is 12.0 V, and each resistor has a resistance of 200.0 Ω. Find **(a)** the current supplied by the battery to this circuit and **(b)** the current through the lower two resistors.

PICTURE THE PROBLEM

The circuit for this problem has three resistors connected to a battery. Note that the lower two resistors are in series with one another, and in parallel with the upper resistor. The battery has an emf of 12.0 V.



INTERACTIVE FIGURE

STRATEGY

a. The current supplied by the battery, I, is given by Ohm's law, $I = \mathcal{E}/R_{eq}$, where R_{eq} is the equivalent resistance of the three resistors. To find R_{eq}, we first note that the lower two resistors are in series, giving a net resistance of $2R$. Next, the upper resistor, R, is in parallel with $2R$. Calculating this equivalent resistance yields the desired R_{eq}.

b. Because the voltage across the lower two resistors is $\mathcal{E}$, the current through them is $I_{lower} = \mathcal{E}/R_{eq,lower} = \mathcal{E}/2R$.

SOLUTION

Part (a)

1. Calculate the equivalent resistance of the lower two resistors: $R_{eq,lower} = R + R = 2R$

2. Calculate the equivalent resistance of R in parallel with $2R$:
$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} = \frac{3}{2R}$$
$$R_{eq} = \tfrac{2}{3}R = \tfrac{2}{3}(200.0\ \Omega) = 133.3\ \Omega$$

3. Find the current supplied by the battery, I:
$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{12.0\ \text{V}}{133.3\ \Omega} = 0.0900\ \text{A}$$

Part (b)

4. Use $\mathcal{E}$ and $R_{eq,lower}$ to find the current in the lower two resistors:
$$I_{lower} = \frac{\mathcal{E}}{R_{eq,lower}} = \frac{12.0\ \text{V}}{2(200.0\ \Omega)} = 0.0300\ \text{A}$$

INSIGHT

Note that the total resistance of the three 200.0-Ω resistors is less than 200.0 Ω—in fact, it is only 133.3 Ω. We also see that 0.0300 A flows through the lower two resistors, and therefore twice that much—0.0600 A—flows through the upper resistor.

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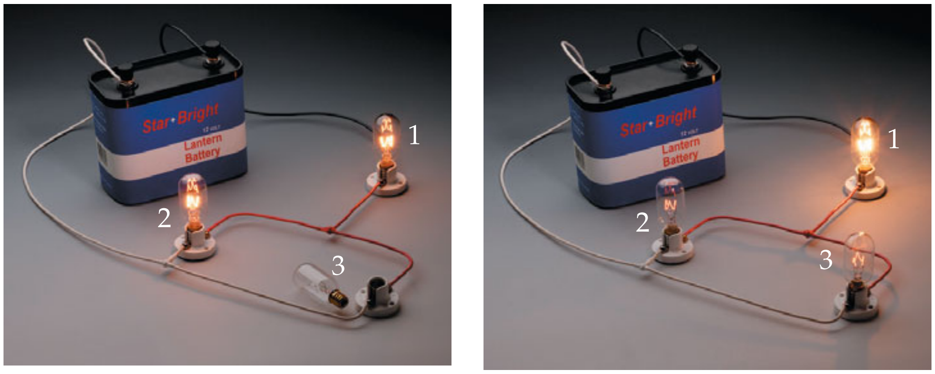
PRACTICE PROBLEM

Suppose the upper resistor is changed from R to $2R$, and the lower two resistors remain the same. **(a)** Will the current supplied by the battery increase, decrease, or stay the same? **(b)** Find the new current. [**Answer:** (a) The current will decrease because there is greater resistance to its flow; (b) 0.0600 A.]

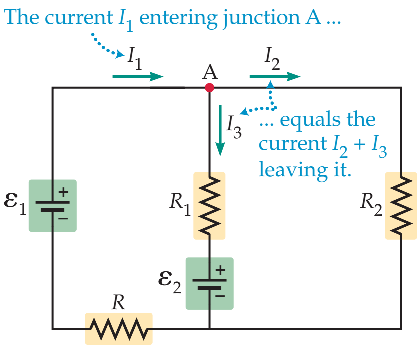
Some related homework problems: Problem 48, Problem 49, Problem 51



▶ The electric circuit in these photos starts with two identical lightbulbs (1 and 2) in series with a battery, as we see on the left. The bulbs are equally bright. Now, *before you examine the photo to the right*, consider the effect of adding a third identical bulb (3) to the circuit by placing it in the empty socket. What happens to the brightness of bulbs 1 and 2? As you can see, adding bulb 3 creates a new path for the current and increases the total current in the circuit by a factor of 4/3 (check this yourself). The current passing through bulb 1 is equally split between bulbs 2 and 3, however, and the new current in bulb 2 is now only $\frac{1}{2}(4/3) = 2/3$ of its original value. Thus, bulb 1 brightens and bulb 2 becomes dimmer.

21–5 Kirchhoff's Rules

To find the currents and voltages in a general electric circuit, we use two rules first introduced by the German physicist Gustav Kirchhoff (1824–1887). The *Kirchhoff rules* are simply ways of expressing charge conservation (the junction rule) and energy conservation (the loop rule) in a closed circuit. Since these conservation laws are always obeyed in nature, the Kirchhoff rules are completely general.



▲ **FIGURE 21–12 Kirchhoff's junction rule**

Kirchhoff's junction rule states that the sum of the currents entering a junction must equal the sum of the currents leaving the junction. In this case, for the junction labeled A, $I_1 = I_2 + I_3$, or $I_1 - I_2 - I_3 = 0$.

The Junction Rule

The junction rule follows from the observation that the current entering any point in a circuit must equal the current leaving that point. If this were not the case, charge would either build up or disappear from a circuit.

As an example, consider the circuit shown in **Figure 21–12**. At point A, three wires join to form a **junction.** (In general, a *junction* is any point in a circuit where three or more wires meet.) The current carried by each of the three wires is indicated in the figure. Notice that the current entering the junction is I_1; the current leaving the junction is $I_2 + I_3$. Setting the incoming and outgoing currents equal, we have $I_1 = I_2 + I_3$, or equivalently

$$I_1 - I_2 - I_3 = 0$$

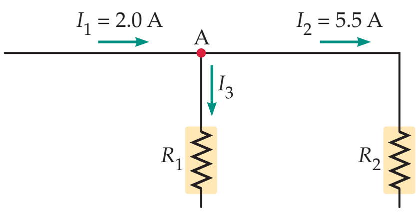
This is Kirchhoff's junction rule applied to the junction at point A.

In general, if we associate a $+$ sign with currents entering a junction and a $-$ sign with currents leaving a junction, Kirchhoff's junction rule can be stated as follows:

> The algebraic sum of all currents meeting at any junction in a circuit must equal zero.

In the example just discussed, I_1 enters the junction $(+)$, I_2 and I_3 leave the junction $(-)$; hence, the algebraic sum of currents at the junction is $I_1 - I_2 - I_3$. Setting this sum equal to zero recovers our previous result.

In some cases we may not know the direction of all the currents meeting at a junction in advance. When this happens, we simply choose a direction for the unknown currents, apply the junction rule, and continue as usual. If the value we obtain for a given current is negative, it simply means that the direction we chose was wrong; the current actually flows in the opposite direction.



▲ **FIGURE 21–13 A specific application of Kirchhoff's junction rule**

Applying Kirchhoff's junction rule to the junction A, $I_1 - I_2 - I_3 = 0$, yields the result $I_3 = -3.5$ A. The minus sign indicates that I_3 flows opposite to the direction shown; that is, I_3 is actually upward.

For example, suppose we know both the direction and magnitude of the currents I_1 and I_2 in **Figure 21–13**. To find the third current, we apply the junction rule—but first we must choose a direction for I_3. If we choose I_3 to point downward, as shown in the figure, the junction rule gives

$$I_1 - I_2 - I_3 = 0$$

Solving for I_3, we have

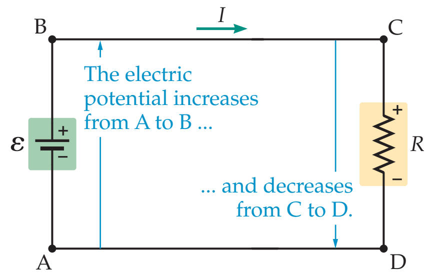
$$I_3 = I_1 - I_2 = 2.0\text{ A} - 5.5\text{ A} = -3.5\text{ A}$$

Since I_3 is negative, we conclude that the actual direction of this current is upward; that is, the 2.0-A current and the 3.5-A current enter the junction and combine to yield the 5.5-A current that leaves the junction.

The Loop Rule

Imagine taking a day hike on a mountain path. First, you gain altitude to reach a scenic viewpoint; later you descend below your starting point into a valley; finally, you gain altitude again and return to the trailhead. During the hike you sometimes increase your gravitational potential energy, and sometimes you decrease it, but the net change at the end of the hike is zero—after all, you return to the same altitude from which you started. Kirchhoff's loop rule is an application of the same idea to an electric circuit.

For example, consider the simple circuit shown in **Figure 21–14**. The electric potential increases by the amount $\mathcal{E}$ in going from point A to point B, since we move from the low-potential $(-)$ terminal of the battery to the high-potential $(+)$ terminal. This is like gaining altitude in the hiking analogy. Next, there is no potential change as we go from point B to point C, since these points are connected by an ideal wire. As we move from point C to point D, however, the potential does change—recall that a potential difference is required to force a current through a resistor. We label the potential difference across the resistor ΔV_{CD}. Finally, there is no change in potential between points D and A, since they too are connected by an ideal wire.



▲ **FIGURE 21–14 Kirchhoff's loop rule**
Kirchhoff's loop rule states that as one moves around a closed loop in a circuit, the algebraic sum of all potential differences must be zero. The electric potential increases as one moves from the $-$ to the $+$ plate of a battery; it decreases as one moves through a resistor in the direction of the current.

We can now apply the idea that the net change in electric potential (the analog to gravitational potential energy in the hike) must be zero around any closed loop. In this case, we have

$$\mathcal{E} + \Delta V_{CD} = 0$$

Thus, we find that $\Delta V_{CD} = -\mathcal{E}$; that is, the electric potential *decreases* as one moves across the resistor *in the direction of the current*. To indicate this drop in potential, we label the side where the current enters the resistor with a $+$ (indicating high potential) and the side where the current leaves the resistor with a $-$ (indicating low potential). Finally, we can use Ohm's law to set the magnitude of the potential drop equal to IR and find the current in the circuit:

$$|\Delta V_{CD}| = \mathcal{E} = IR$$
$$I = \frac{\mathcal{E}}{R}$$

PROBLEM-SOLVING NOTE

Applying Kirchhoff's Rules

When applying Kirchhoff's rules, be sure to use the appropriate sign for currents and potential differences.

This, of course, is the expected result.

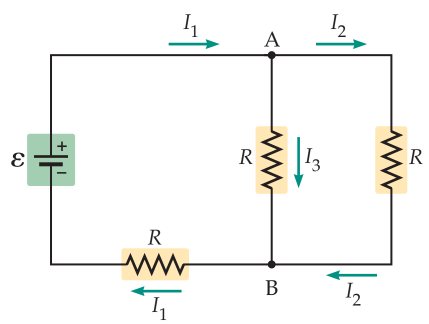
In general, Kirchhoff's loop rule can be stated as follows:

> The algebraic sum of all potential differences around any closed loop in a circuit is zero.

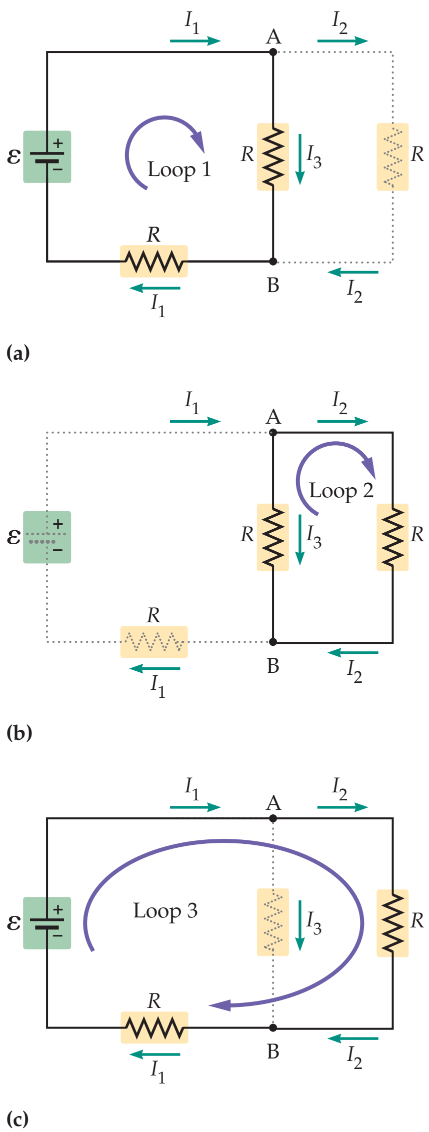
We now consider a variety of applications in which both the junction rule and the loop rule are used to find the various currents and potentials in a circuit.

Applications

We begin by considering the relatively simple circuit shown in **Figure 21–15**. The currents and voltages in this circuit can be found by considering various parallel and series combinations of the resistors, as we did in the previous section. Thus, Kirchhoff's rules are not strictly needed in this case. Still, applying the rules to this circuit illustrates many of the techniques that can be used when studying more complex circuits.



▲ **FIGURE 21–15 Analyzing a simple circuit**
A simple circuit that can be studied using either equivalent resistance or Kirchhoff's rules.



▲ **FIGURE 21–16 Using loops to analyze a circuit**

Three loops associated with the circuit in Figure 21–15.

Let's suppose that all the resistors have the value $R = 100.0\ \Omega$, and that the emf of the battery is $\mathcal{E} = 15.0$ V. The equivalent resistance of the resistors can be obtained by noting that the vertical resistors are connected in parallel with one another and in series with the horizontal resistor. The vertical resistors combine to give a resistance of $R/2$, which, when added to the horizontal resistor, gives an equivalent resistance of $R_{eq} = 3R/2 = 150.0\ \Omega$. The current in the circuit, then, is $I = \mathcal{E}/R_{eq} = 15.0\text{ V}/150.0\ \Omega = 0.100$ A.

Now we approach the same problem from the point of view of Kirchhoff's rules. First, we apply the junction rule to point A:

$$I_1 - I_2 - I_3 = 0 \qquad \text{(junction A)} \qquad 21\text{–}11$$

Note that current I_1 splits at point A into currents I_2 and I_3, which combine again at point B to give I_1 flowing through the horizontal resistor. We can apply the junction rule to point B, which gives $-I_1 + I_2 + I_3 = 0$, but since this differs from Equation 21–11 by only a minus sign, no new information is gained.

Next, we apply the loop rule. Since there are three unknowns, I_1, I_2, and I_3, we need three independent equations for a full solution. One has already been given by the junction rule; thus, we expect that two loop equations will be required to complete the solution. To begin, we consider loop 1, which is shown in **Figure 21–16 (a)**. We choose to move around this loop in the clockwise direction. (If we were to choose the counterclockwise direction instead, the same information would be obtained.) For loop 1, then, we have an increase in potential as we move across the battery, a drop in potential across the vertical resistor of I_3R, and another drop in potential across the horizontal resistor, this time of magnitude I_1R. Applying the loop rule, we find the following:

$$\mathcal{E} - I_3R - I_1R = 0 \qquad \text{(loop 1)} \qquad 21\text{–}12$$

Similarly, we can apply the loop rule to loop 2, shown in **Figure 21–16 (b)**. In this case we cross the right-hand vertical resistor in the direction of the current, implying a drop in potential, and we cross the left-hand vertical resistor against the current, implying an increase in potential. Therefore, the loop rule gives

$$I_3R - I_2R = 0 \qquad \text{(loop 2)} \qquad 21\text{–}13$$

There is a third possible loop, shown in **Figure 21–16 (c)**, but the information it gives is not different from that already obtained. In fact, *any two of the three loops* complete our solution.

Note that R cancels in Equation 21–13; hence, we see that $I_3 - I_2 = 0$, or $I_3 = I_2$. Substituting this result into the junction rule (Equation 21–11), we obtain

$$\begin{aligned} I_1 - I_2 - I_3 &= I_1 - I_2 - I_2 \\ &= I_1 - 2I_2 = 0 \end{aligned}$$

Solving this equation for I_2 gives us $I_2 = I_1/2 = I_3$. Finally, using the first loop equation (Equation 21–12), we find

$$\mathcal{E} - (I_1/2)R - I_1R = \mathcal{E} - \tfrac{3}{2}I_1R = 0$$

Note that the only unknown in this equation is current I_1. Solving for this current, we find

$$I_1 = \frac{\mathcal{E}}{\frac{3}{2}R} = \frac{15.0\text{ V}}{\frac{3}{2}(100.0\ \Omega)} = 0.100\text{ A}$$

As expected, our result using Kirchhoff's rules agrees with the result obtained previously. Finally, the other two currents in the circuit are $I_2 = I_3 = I_1/2 = 0.0500$ A.

EXERCISE 21–3

Write the loop equation for loop 3 in Figure 21–16 (c).

SOLUTION

Proceeding in a clockwise direction, as indicated in the figure, we find

$$\mathcal{E} - I_2R - I_1R = 0$$

Since I_2 and I_3 are equal (loop 2), it follows that loop 1 ($\mathcal{E} - I_3R - I_1R = 0$) and loop 3 ($\mathcal{E} - I_2R - I_1R = 0$) give the same information. If we proceed in a counterclockwise direction around loop 3, we find

$$-\mathcal{E} + I_2R + I_1R = 0$$

Notice that this result is the same as the clockwise result except for an overall minus sign, and, therefore, it contains no new information. In general, it does not matter in which direction we choose to go around a loop.

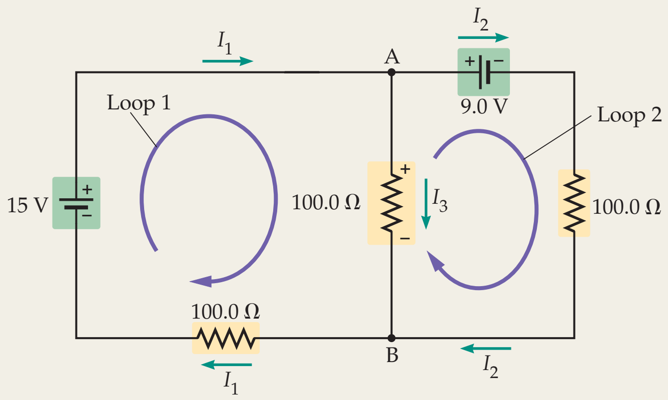
Clearly, the Kirchhoff approach is more involved than the equivalent-resistance method. However, it is not possible to analyze all circuits in terms of equivalent resistances. In such cases, Kirchhoff's rules are the only option, as illustrated in the next Active Example.

ACTIVE EXAMPLE 21–2 TWO LOOPS, TWO BATTERIES: FIND THE CURRENTS

Find the currents in the circuit shown.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*

1. Apply the junction rule to point A: $I_1 - I_2 - I_3 = 0$
2. Apply the loop rule to loop 1 (let $R = 100.0\ \Omega$): $15\text{ V} - I_3R - I_1R = 0$
3. Apply the loop rule to loop 2 (let $R = 100.0\ \Omega$): $-9.0\text{ V} - I_2R + I_3R = 0$
4. Solve for I_1, I_2, and I_3: $I_1 = 0.070\text{ A}$, $I_2 = -0.010\text{ A}$, $I_3 = 0.080\text{ A}$



INSIGHT

Note that I_2 is negative. This means that its direction is opposite to that shown in the circuit diagram.

YOUR TURN

Suppose the polarity of the 9.0-V battery is reversed. What are the currents in this case?

(Answers to **Your Turn** *problems are given in the back of the book.)*

21–6 Circuits Containing Capacitors

To this point we have considered only resistors and batteries in electric circuits. Capacitors, which can also play an important role, are represented by a set of parallel lines (reminiscent of a parallel-plate capacitor): ⊣⊢. We now investigate simple circuits involving batteries and capacitors, leaving for the next section circuits that combine all three circuit elements.

Capacitors in Parallel

The simplest way to combine capacitors, as we shall see, is by connecting them in parallel. For example, **Figure 21–17 (a)** shows three capacitors connected in parallel with a battery of emf ε. As a result, each capacitor has the same potential difference, ε, between its plates. The magnitudes of the charges on each capacitor are as follows:

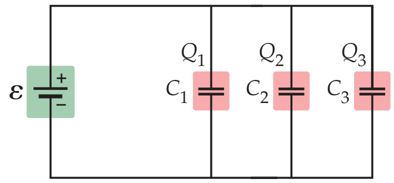
$$Q_1 = C_1\mathcal{E}, \qquad Q_2 = C_2\mathcal{E}, \qquad Q_3 = C_3\mathcal{E}$$

As a result, the total charge on the three capacitors is

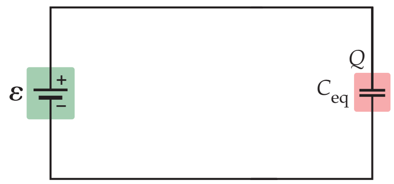
$$Q = Q_1 + Q_2 + Q_3 = \mathcal{E}C_1 + \mathcal{E}C_2 + \mathcal{E}C_3 = (C_1 + C_2 + C_3)\mathcal{E}$$

If an equivalent capacitor is used to replace the three in parallel, as in **Figure 21–17 (b)**, the charge on its plates must be the same as the total charge on the individual capacitors:

$$Q = C_{eq}\mathcal{E}$$



(a) Three capacitors in parallel



(b) Equivalent capacitance with same total charge

▲ **FIGURE 21–17 Capacitors in parallel**

(a) Three capacitors, C_1, C_2, and C_3, connected in parallel. Note that each capacitor is connected across the same potential difference, $\mathcal{E}$. **(b)** The equivalent capacitance, $C_{eq} = C_1 + C_2 + C_3$, has the same charge on its plates as the total charge on the three original capacitors.

Comparing $Q = C_{eq}\mathcal{E}$ with $Q = (C_1 + C_2 + C_3)\mathcal{E}$, we see that the equivalent capacitance is simply

$$C_{eq} = C_1 + C_2 + C_3$$

In general, the equivalent capacitance of capacitors connected in parallel is the sum of the individual capacitances:

Equivalent Capacitance for Capacitors in Parallel

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C \qquad \text{21–14}$$

SI unit: farad, F

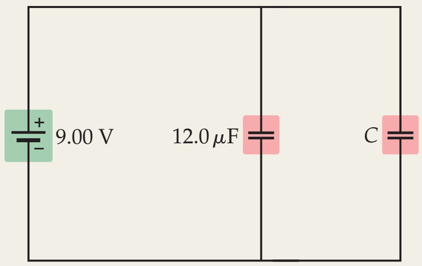
Thus, connecting capacitors in parallel produces an equivalent capacitance greater than the greatest individual capacitance. It is as if the plates of the individual capacitors are connected together to give one large set of plates, with a correspondingly large capacitance.

EXAMPLE 21–8 ENERGY IN PARALLEL

Two capacitors, one 12.0 μF and the other of unknown capacitance C, are connected in parallel across a battery with an emf of 9.00 V. The total energy stored in the two capacitors is 0.0115 J. What is the value of the capacitance C?

PICTURE THE PROBLEM

The circuit, consisting of one 9.00-V battery and two capacitors, is illustrated in the diagram. The total energy of 0.0115 J stored in the two capacitors is the same as the energy stored in the equivalent capacitance for this circuit.



STRATEGY

Recall from Chapter 20 that the energy stored in a capacitor can be written as $U = \frac{1}{2}CV^2$. It follows, then, that for an equivalent capacitance, C_{eq}, the energy is $U = \frac{1}{2}C_{eq}V^2$. Since we know the energy and voltage, we can solve this relation for the equivalent capacitance. Finally, the equivalent capacitance is the sum of the individual capacitances, $C_{eq} = 12.0\ \mu\text{F} + C$. We use this relation to solve for C.

SOLUTION

1. Solve $U = \frac{1}{2}C_{eq}V^2$ for the equivalent capacitance:

$$U = \frac{1}{2}C_{eq}V^2$$

$$C_{eq} = \frac{2U}{V^2}$$

2. Substitute numerical values to find C_{eq}:

$$C_{eq} = \frac{2U}{V^2} = \frac{2(0.0115\ \text{J})}{(9.00\ \text{V})^2} = 284\ \mu\text{F}$$

3. Solve for C in terms of the equivalent capacitance:

$$C_{eq} = 12.0\ \mu\text{F} + C$$

$$C = C_{eq} - 12.0\ \mu\text{F} = 284\ \mu\text{F} - 12.0\ \mu\text{F} = 272\ \mu\text{F}$$

INSIGHT

The energy stored in the 12.0-μF capacitor is $U = \frac{1}{2}CV^2 = 0.000486$ J. In comparison, the 272-μF capacitor stores an energy equal to 0.0110 J. Thus, the larger capacitor stores the greater amount of energy. Though this may seem only natural, one needs to be careful. When we examine capacitors in *series* later in this section, we shall find exactly the opposite result.

PRACTICE PROBLEM

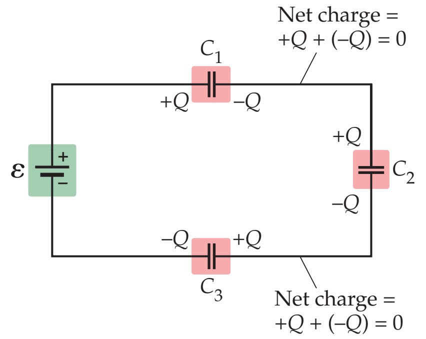
What is the total charge stored on the two capacitors? [**Answer:** $Q = C_{eq}\mathcal{E} = 2.56 \times 10^{-3}$ C]

Some related homework problems: Problem 72, Problem 73

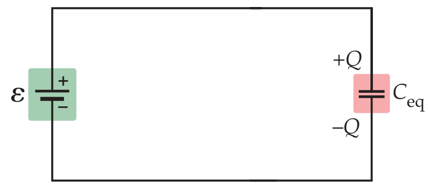
REAL-WORLD PHYSICS

"Touch-sensitive" lamps

Although you probably haven't realized it, when you turn on a "touch sensitive" lamp, you are part of a circuit with capacitors in parallel. In fact, you are one of the capacitors! When you touch such a lamp, a small amount of charge moves onto your body—your body is like the plate of a capacitor. Because you have effectively increased the plate area—as always happens when capacitors are connected in parallel—the capacitance of the circuit increases. The electronic circuitry in the lamp senses this increase in capacitance and triggers the switch to turn the light on or off.



(a) Three capacitors in series



(b) Equivalent capacitance with same total charge

▲ **FIGURE 21–18 Capacitors in series**
(a) Three capacitors, C_1, C_2, and C_3, connected in series. Note that each capacitor has the same magnitude charge on its plates. **(b)** The equivalent capacitance, $1/C_{eq} = 1/C_1 + 1/C_2 + 1/C_3$, has the same charge as the original capacitors.

Capacitors in Series

You have probably noticed from Equation 21–14 that capacitors connected in *parallel* combine in the same way as resistors connected in *series*. Similarly, capacitors connected in *series* obey the same rules as resistors connected in *parallel*, as we now show.

Consider three capacitors—initially uncharged—connected in series with a battery, as in **Figure 21–18 (a)**. The battery causes the left plate of C_1 to acquire a positive charge, $+Q$. This charge, in turn, attracts a negative charge $-Q$ onto the right plate of the capacitor. Because the capacitors start out uncharged, there is zero net charge between C_1 and C_2. As a result, the negative charge $-Q$ on the right plate of C_1 leaves a corresponding positive charge $+Q$ on the upper plate of C_2. The charge $+Q$ on the upper plate of C_2 attracts a negative charge $-Q$ onto its lower plate, leaving a corresponding positive charge $+Q$ on the right plate of C_3. Finally, the positive charge on the right plate of C_3 attracts a negative charge $-Q$ onto its left plate. The result is that all three capacitors have charge of the same magnitude on their plates.

With the same charge Q on all the capacitors, the potential difference for each is as follows:

$$V_1 = \frac{Q}{C_1}, \qquad V_2 = \frac{Q}{C_2}, \qquad V_3 = \frac{Q}{C_3}$$

Since the total potential difference across the three capacitors must equal the emf of the battery, we have

$$\mathcal{E} = V_1 + V_2 + V_3 = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right) \qquad \text{21–15}$$

An equivalent capacitor connected to the same battery, as in **Figure 21–18 (b)**, will satisfy the relation $Q = C_{eq}\mathcal{E}$, or

$$\mathcal{E} = Q\left(\frac{1}{C_{eq}}\right) \qquad \text{21–16}$$

A comparison of Equations 21–15 and 21–16 yields the result

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Thus, in general, we have the following rule for combining capacitors in series:

Equivalent Capacitance for Capacitors in Series

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C} \qquad \text{21–17}$$

SI unit: farad, F

It follows, then, that the equivalent capacitance of a group of capacitors connected in series is less than the smallest individual capacitance. In this case, it is as if the plate separations of the individual capacitors add to give a larger effective separation, and a correspondingly smaller capacitance.

More complex circuits, with some capacitors in series and others in parallel, can be handled in the same way as was done earlier with resistors. This is illustrated in the following Active Example.

PROBLEM-SOLVING NOTE

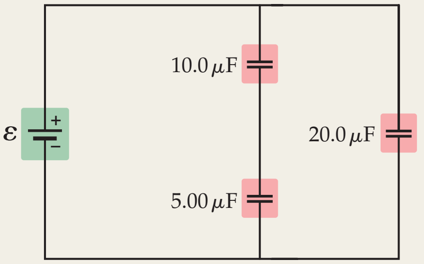
Finding the Equivalent Capacitance of a Circuit

When calculating the equivalent capacitance of capacitors in series, be sure to take one final inverse at the end of your calculation to find C_{eq}. Also, when considering circuits with capacitors in both series and parallel, start with the smallest units of the circuit and work your way out to the larger units.

ACTIVE EXAMPLE 21–3 FIND THE EQUIVALENT CAPACITANCE AND THE STORED ENERGY

Consider the electric circuit shown here, consisting of a 12.0-V battery and three capacitors connected partly in series and partly in parallel. Find **(a)** the equivalent capacitance of this circuit and **(b)** the total energy stored in the capacitors.

SOLUTION *(Test your understanding by performing the calculations indicated in each step.)*



Part (a)

1. Find the equivalent capacitance of a 10.0-μF capacitor in series with a 5.00-μF capacitor: 3.33 μF
2. Find the equivalent capacitance of a 3.33-μF capacitor in parallel with a 20.0-μF capacitor: $C_{\mathrm{eq}} = 23.3\ \mu\mathrm{F}$

Part (b)

3. Calculate the stored energy using $U = \frac{1}{2}C_{\mathrm{eq}}V^2$: $U = 1.68 \times 10^{-3}\ \mathrm{J}$

INSIGHT

Notice that the 10.0-μF capacitor and the 5.00-μF capacitor are connected in series. As you might expect, one of these capacitors stores twice as much energy as the other. Which is it? Check the Your Turn question for the answer.

YOUR TURN

Is the energy stored in the 10.0-μF capacitor greater than or less than the energy stored in the 5.0-μF capacitor? Explain. Check your answer by calculating the energy stored in each of the capacitors.

(Answers to **Your Turn** *problems are given in the back of the book.)*

21–7 *RC* Circuits

When the switch is closed on a circuit containing only batteries and capacitors, the charge on the capacitor plates appears almost instantaneously—essentially at the speed of light. This is not the case, however, in circuits that also contain resistors. In these situations, the resistors limit the rate at which charge can flow, and an appreciable amount of time may be required before the capacitors acquire a significant charge. A useful analogy is the amount of time needed to fill a bucket with water. If you use a fire hose, which has little resistance to the flow of water, the bucket fills almost instantly. If you use a garden hose, which presents a much greater resistance to the water, filling the bucket may take a minute or more.

The simplest example of such a circuit, a so-called ***RC* circuit,** is shown in Figure 21–19. Initially (before $t = 0$) the switch is open, and there is no current in the resistor or charge on the capacitor. At $t = 0$ the switch is closed and current begins to flow. If the resistor was not present, the capacitor would immediately take on the charge $Q = C\mathcal{E}$. The effect of the resistor, however, is to slow the charging process—in fact, the larger the resistance, the longer it takes for the capacitor to charge. One way to think of this is to note that as long as a current flows in the circuit, as in Figure 21–19 (b), there is a potential drop across the resistor; hence, the potential difference between the plates of the capacitor is less than the emf of the battery. With less voltage across the capacitor there will be less charge on its plates compared with the charge that would result if the plates were connected directly to the battery.

The methods of calculus can be used to show that the charge on the capacitor in Figure 21–19 varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$

In this expression, e is Euler's number ($e = 2.718\ldots$) or, more precisely, the base of natural logarithms (see Appendix A). The quantity τ is referred to as the **time constant** of the circuit. The time constant is related to the resistance and capacitance of a circuit by the following simple relation: $\tau = RC$. As we shall see, τ can be thought of as a characteristic time for the behavior of an RC circuit.

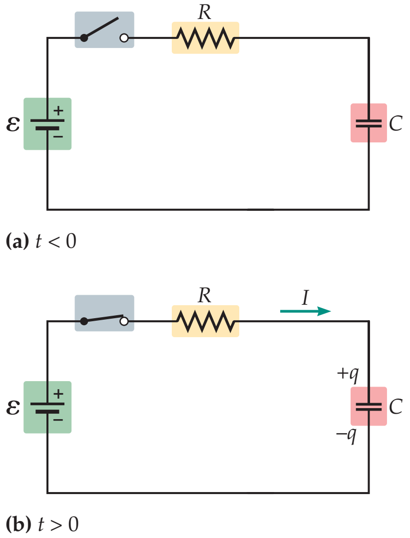


FIGURE 21–19 A typical *RC* circuit

(a) Before the switch is closed ($t < 0$) there is no current in the circuit and no charge on the capacitor. **(b)** After the switch is closed ($t > 0$), current flows and the charge on the capacitor builds up over a finite time. As $t \to \infty$ the charge on the capacitor approaches $Q = C\mathcal{E}$.

For example, at time $t = 0$ the exponential term is $e^{-0/\tau} = e^0 = 1$; therefore, the charge on the capacitor is zero at $t = 0$, as expected:

$$q(0) = C\mathcal{E}(1 - 1) = 0$$

In the opposite limit, $t \to \infty$, the exponential vanishes: $e^{-\infty/\tau} = 0$. Thus the charge in this limit is $C\mathcal{E}$:

$$q(t \to \infty) = C\mathcal{E}(1 - 0) = C\mathcal{E}$$

This is just the charge Q the capacitor would have had from $t = 0$ on if there had been no resistor in the circuit. Finally, at time $t = \tau$ the charge on the capacitor is $q = C\mathcal{E}(1 - e^{-1}) = C\mathcal{E}(1 - 0.368) = 0.632C\mathcal{E}$, which is 63.2% of its final charge. The charge on the capacitor as a function of time is plotted in **Figure 21–20**.

Before we continue, let's check to see that the quantity $\tau = RC$ is in fact a time. Suppose, for example, that the resistor and capacitor in an RC circuit have the values $R = 120\ \Omega$ and $C = 3.5\ \mu\text{F}$, respectively. Multiplying R and C we find

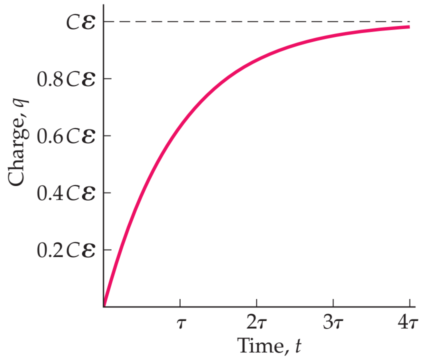
$$\tau = RC = (120\ \text{ohm})(3.5 \times 10^{-6}\ \text{farad})$$
$$= \left(\frac{120\ \text{volt}}{\text{coulomb/second}}\right)\left(\frac{3.5 \times 10^{-6}\ \text{coulomb}}{\text{volt}}\right) = 4.2 \times 10^{-4}\ \text{second}$$

The tick marks on the horizontal axis in Figure 21–20 indicate the times τ, 2τ, 3τ, and 4τ. Notice that the capacitor is almost completely charged by the time $t = 4\tau$.

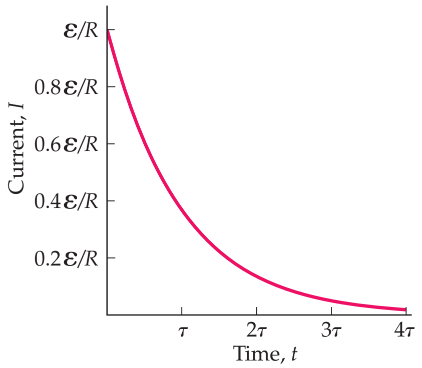
Figure 21–20 also shows that the charge on the capacitor increases rapidly initially, indicating a large current in the circuit. Eventually, the charging slows down, because the greater the charge on the capacitor, the harder it is to transfer additional charge against the electrical repulsive force. Later, the charge barely changes with time, which means that the current is essentially zero. In fact, the mathematical expression for the current—again derived from calculus—is the following:

$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad \text{21–19}$$

This expression is plotted in **Figure 21–21**, where we see that significant variation in the current occurs over times ranging from $t = 0$ to $t \sim 4\tau$. At time $t = 0$ the current is $I(0) = \mathcal{E}/R$, which is the value it would have if the capacitor were replaced by an ideal wire. As $t \to \infty$, the current approaches zero, as expected: $I(t \to \infty) \to 0$. In this limit, the capacitor is essentially fully charged, so that no more charge can flow onto its plates. Thus, in this limit, the capacitor behaves like an open switch.



▲ **FIGURE 21–20 Charge versus time for the *RC* circuit in Figure 21–19**
The horizontal axis shows time in units of the characteristic time, $\tau = RC$. The vertical axis shows the magnitude of the charge on the capacitor in units of $C\mathcal{E}$.



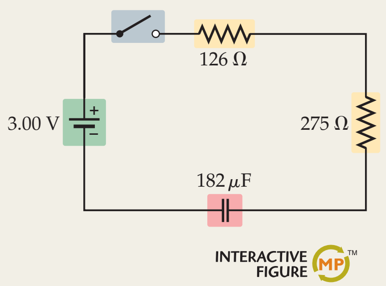
▲ **FIGURE 21–21 Current versus time for the *RC* circuit in Figure 21–19**
Initially the current is $\mathcal{E}/R$, the same as if the capacitor were not present. The current approaches zero after a period equal to several time constants, $\tau = RC$.

EXAMPLE 21–9 CHARGING A CAPACITOR

A circuit consists of a 126-Ω resistor, a 275-Ω resistor, a 182-μF capacitor, a switch, and a 3.00-V battery all connected in series. Initially the capacitor is uncharged and the switch is open. At time $t = 0$ the switch is closed. **(a)** What charge will the capacitor have a long time after the switch is closed? **(b)** At what time will the charge on the capacitor be 80.0% of the value found in part (a)?

PICTURE THE PROBLEM

The circuit described in the problem statement is shown with the switch in the open position. Once the switch is closed at $t = 0$, current will flow in the circuit and charge will begin to accumulate on the capacitor plates.



STRATEGY

a. A long time after the switch is closed, the current stops and the capacitor is fully charged. At this point, the voltage across the capacitor is equal to the emf of the battery. Therefore, the charge on the capacitor is $Q = C\mathcal{E}$.

b. To find the time when the charge will be 80.0% of the full charge, $Q = C\mathcal{E}$, we can set $q(t) = C\mathcal{E}(1 - e^{-t/\tau}) = 0.800C\mathcal{E}$ and solve for the desired time, t.

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SOLUTION

Part (a)

1. Evaluate $Q = C\mathcal{E}$ for this circuit:	$Q = C\mathcal{E} = (182\ \mu\text{F})(3.00\ \text{V}) = 546\ \mu\text{C}$

Part (b)

2. Set $q(t) = 0.800C\mathcal{E}$ in $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$ and cancel $C\mathcal{E}$:	$q(t) = 0.800C\mathcal{E} = C\mathcal{E}(1 - e^{-t/\tau})$ $0.800 = 1 - e^{-t/\tau}$
3. Solve for t in terms of the time constant τ:	$e^{-t/\tau} = 1 - 0.800 = 0.200$ $t = -\tau \ln(0.200)$
4. Calculate τ and use the result to find the time t:	$\tau = RC = (126\ \Omega + 275\ \Omega)(182\ \mu F) = 73.0\ \text{ms}$ $t = -(73.0\ \text{ms}) \ln(0.200)$ $= -(73.0\ \text{ms})(-1.61) = 118\ \text{ms}$

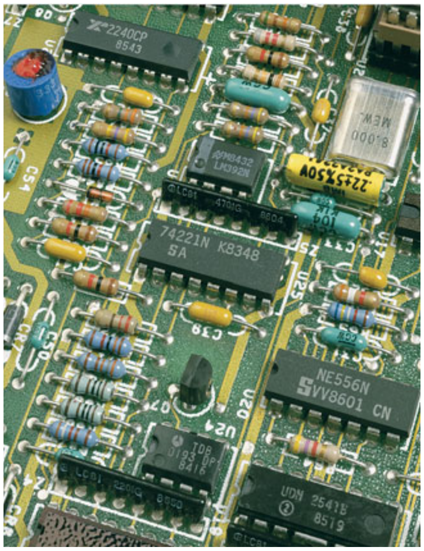
INSIGHT

Note that the time required for the charge on a capacitor to reach 80.0% of its final value is 1.61 time constants. This result is independent of the values of R and C in an RC circuit.

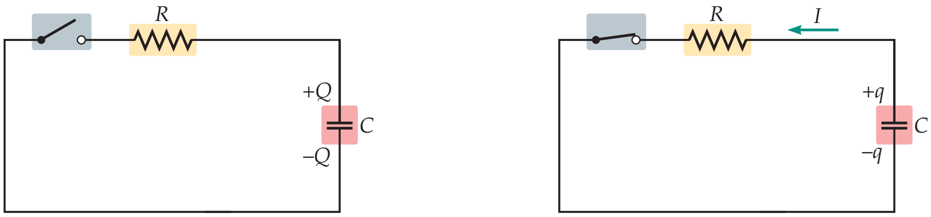
PRACTICE PROBLEM

What is the current in this circuit at the time found in part (b)? [**Answer:** $I(t) = (\mathcal{E}/R)e^{-t/\tau} = [(3.00\ \text{V})/(126\ \Omega + 275\ \Omega)](0.200) = (7.48\ \text{mA})(0.200) = 1.50\ \text{mA}$]

Some related homework problems: Problem 79, Problem 82



▲ A modern-day circuit board incorporates numerous resistors (cylinders with colored bands) and capacitors (yellow cylinders and metal container).



(a) $t < 0$ **(b)** $t > 0$

▲ **FIGURE 21–22 Discharging a capacitor**

(a) A charged capacitor is connected to a resistor. Initially the circuit is open, and no current can flow. **(b)** When the switch is closed, current flows from the + plate of the capacitor to the − plate. The charge remaining on the capacitor approaches zero after several time units, RC.

PROBLEM-SOLVING NOTE

The Limiting Behavior of Capacitors

Capacitors in dc circuits act like short circuits at $t = 0$ and open circuits as $t \to \infty$.

Similar behavior occurs when a charged capacitor is allowed to discharge, as in **Figure 21–22**. In this case, the initial charge on the capacitor is Q. If the switch is closed at $t = 0$, the charge for later times is

$$q(t) = Qe^{-t/\tau} \qquad 21\text{–}20$$

Like charging, the discharging of a capacitor occurs with a characteristic time $\tau = RC$.

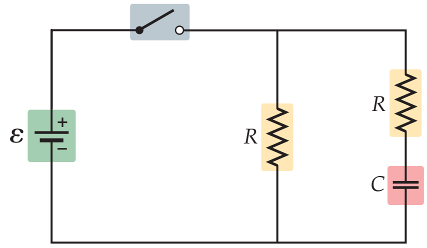
To summarize, circuits with resistors and capacitors have the following general characteristics:

- Charging and discharging occur over a finite, characteristic time given by the time constant, $\tau = RC$.
- At $t = 0$ current flows freely through a capacitor being charged; it behaves like a short circuit.
- As $t \to \infty$ the current flowing into a capacitor approaches zero. In this limit, a capacitor behaves like an open switch.

We explore these features further in the following Conceptual Checkpoint.

CONCEPTUAL CHECKPOINT 21–4 CURRENT IN AN *RC* CIRCUIT

What current flows through the battery in this circuit **(a)** immediately after the switch is closed and **(b)** a long time after the switch is closed?



REASONING AND DISCUSSION

a. Immediately after the switch is closed, the capacitor acts like a short circuit; that is, as if the battery were connected to two resistors R in parallel. The equivalent resistance in this case is $R/2$; therefore, the current is $I = \mathcal{E}/(R/2) = 2\mathcal{E}/R$.

b. After current has been flowing in the circuit for a long time, the capacitor acts like an open switch. Now current can flow only through the one resistor, R; hence, the current is $I = \mathcal{E}/R$, half of its initial value.

ANSWER

(a) The current is $2\mathcal{E}/R$; **(b)** the current is $\mathcal{E}/R$.

REAL-WORLD PHYSICS

Delay circuits in windshield wipers and turn signals

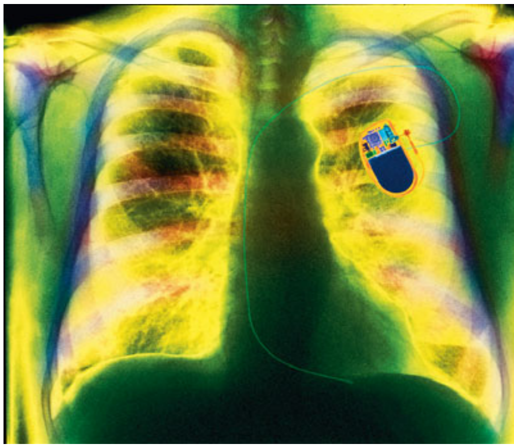
The fact that *RC* circuits have a characteristic time makes them useful in a variety of different applications. On a rather mundane level, *RC* circuits are used to determine the time delay on windshield wipers. When you adjust the delay knob in your car, you change a resistance or a capacitance, which in turn changes the time constant of the circuit. This results in a greater or a smaller delay. The blinking rate of turn signals is also determined by the time constant of an *RC* circuit.

REAL-WORLD PHYSICS: BIO

Pacemakers

A more critical application of *RC* circuits is the heart pacemaker. In the simplest case, these devices use an *RC* circuit to deliver precisely timed pulses directly to the heart. The more sophisticated pacemakers available today can even "sense" when a patient's heart rate falls below a predetermined value. The pacemaker then begins sending appropriate pulses to the heart to increase its rate. Many pacemakers can even be reprogrammed after they are surgically implanted to respond to changes in a patient's condition.

Normally, the heart's rate of beating is determined by its own natural pacemaker, the sinoatrial or SA node, located in the upper right chamber of the heart. If the SA node is not functioning properly, it may cause the heart to beat slowly or irregularly. To correct the problem, a pacemaker is implanted just under the collarbone, and an electrode is introduced intravenously via the cephalic vein. The distal end of the electrode is positioned, with the aid of fluoroscopic guidance, in the right ventricular apex. From that point on, the operation of the pacemaker follows the basic principles of electric circuits, as described in this chapter.



▲ An X-ray showing a pacemaker installed in a person's chest. The timing of the electrical pulses that keep the heart beating regularly is determined by an *RC* circuit powered by a small, long-lived battery.

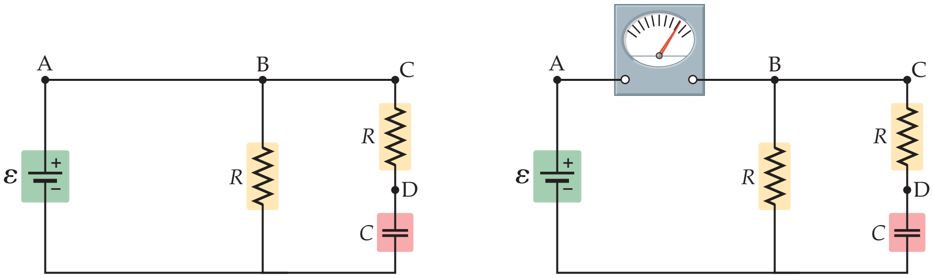
*21–8 Ammeters and Voltmeters

Devices for measuring currents and voltages in a circuit are referred to as **ammeters** and **voltmeters,** respectively. In each case, the ideal situation is for the meter to measure the desired quantity without altering the characteristics of the circuit being studied. This is accomplished in different ways for these two types of meters, as we shall see.

First, the ammeter is designed to measure the flow of current through a particular portion of a circuit. For example, we may want to know the current flowing between points A and B in the circuit shown in **Figure 21–23 (a)**. To measure this current, we insert the ammeter into the circuit in such a way that all the current flowing from A to B must also flow through the meter. This is done by connecting the meter "in series" with the other circuit elements between A and B, as indicated in **Figure 21–23 (b)**.

If the ammeter has a finite resistance—which must be the case for real meters—the presence of the meter in the circuit will alter the current it is intended to measure. Thus, an *ideal* ammeter would be one with zero resistance. In practice, if the resistance of the ammeter is much less than the other resistances in the circuit, its reading will be reasonably accurate.

▲ A typical digital multimeter, which can measure resistance (teal settings), current (yellow settings), or voltage (red settings). This meter is measuring the voltage of a "9 volt" battery.



(a) Typical electric circuit

(b) Measuring the current between A and B

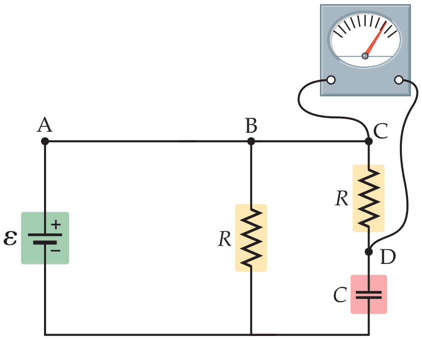
▲ **FIGURE 21–23 Measuring the current in a circuit**

To measure the current flowing between points A and B in **(a)**, an ammeter is inserted into the circuit, as shown in **(b)**. An ideal ammeter would have zero resistance.

Second, a voltmeter measures the potential drop between any two points in a circuit. Referring again to the circuit in Figure 21–23 (a), we may wish to know the difference in potential between points C and D. To measure this voltage, we connect the voltmeter "in parallel" to the circuit at the appropriate points, as in **Figure 21–24**.

A real voltmeter always allows some current to flow through it, which means that the current flowing through the circuit is less than before the meter was connected. As a result, the measured voltage is altered from its ideal value. An *ideal* voltmeter, then, would be one in which the resistance is infinite, so that the current it draws from the circuit is negligible. In practical situations it is sufficient that the resistance of the meter be much greater than the resistances in the circuit.

Sometimes the functions of an ammeter, voltmeter, and ohmmeter are combined in a single device called a **multimeter.** Adjusting the settings on a multimeter allows a variety of circuit properties to be measured.



Measuring the voltage between C and D

◀ **FIGURE 21–24 Measuring the voltage in a circuit**

The voltage difference between points C and D can be measured by connecting a voltmeter in parallel to the original circuit. An ideal voltmeter would have infinite resistance.

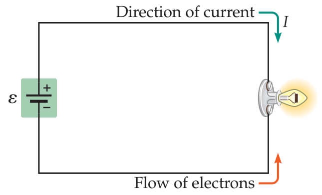
THE BIG PICTURE PUTTING PHYSICS IN CONTEXT

LOOKING BACK	LOOKING AHEAD
The concept of electric potential energy (Chapter 20) is used in Section 21–3, where we talk about the energy associated with an electric circuit. We also discuss the power of an electric circuit in Section 21–3. For this we refer back to mechanics, where power was originally introduced in Chapter 7. Capacitors, first introduced in Chapter 20, are used in dc circuits in Section 21–6.	A dc circuit with a current flowing through it will play an important role in our discussion of magnetism in Chapter 22. We will also consider the magnetic force exerted on a current-carrying wire in Chapter 22. In Chapter 24 we extend our discussion of electric circuits from those in which the current flows in only one direction (dc) to circuits in which the current alternates in direction (ac, or alternating current). We will again use resistors and capacitors in the ac circuits. A simple dc circuit appears in Chapter 30, where we discuss the photoelectric effect and its importance in the development of quantum mechanics.

CHAPTER SUMMARY

21–1 ELECTRIC CURRENT

Electric current is the flow of electric charge.



Definition
If a charge ΔQ passes a given point in the time Δt, the corresponding electric current is

$$I = \frac{\Delta Q}{\Delta t} \qquad 21\text{–}1$$

Ampere
The unit of current is the ampere, or amp for short. By definition, 1 amp is one coulomb per second; $1\ \mathrm{A} = 1\ \mathrm{C/s}$.

Battery
A battery is a device that uses chemical reactions to produce a potential difference between its two terminals.

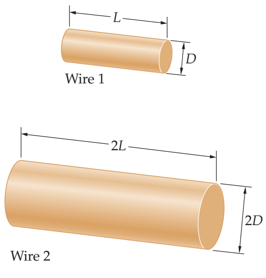
Electromotive Force
The electromotive force, or emf, $\mathcal{E}$, is the potential difference between the terminals of a battery under ideal conditions.

Work Done by a Battery
As a battery moves a charge ΔQ around a circuit, it does the work $W = (\Delta Q)\mathcal{E}$.

Direction of Current
By definition, the direction of the current I in a circuit is the direction in which *positive* charges would move. The actual charge carriers, however, are generally electrons; hence, they move in the opposite direction to I.

21–2 RESISTANCE AND OHM'S LAW

When electrons move through a wire, they encounter resistance to their motion. In order to move electrons against this resistance, it is necessary to apply a potential difference between the ends of the wire.



Ohm's Law
To produce a current I through a wire with resistance R the following potential difference, V, is required:

$$V = IR \qquad 21\text{–}2$$

Resistivity
The resistivity ρ of a material determines how much resistance it gives to the flow of electric current.

Resistance of a Wire
The resistance of a wire of length L, cross-sectional area A, and resistivity ρ is

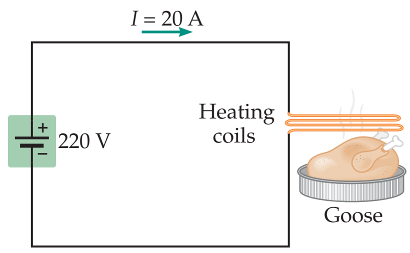
$$R = \rho\left(\frac{L}{A}\right) \qquad 21\text{–}3$$

Temperature Dependence
The resistivity of most metals increases approximately linearly with temperature.

Superconductivity
Below a certain critical temperature, T_c, certain materials lose all electrical resistance. A current flowing in a superconductor can continue undiminished as long as its temperature is maintained below T_c.

21–3 ENERGY AND POWER IN ELECTRIC CIRCUITS

In general, energy is required to cause an electric current to flow through a circuit. The rate at which the energy must be supplied is the power.



Electrical Power
If a current I flows across a potential difference V, the corresponding electrical power is

$$P = IV \qquad 21\text{–}4$$

Power Dissipation in a Resistor

If a potential difference V produces a current I in a resistor R, the electrical power converted to heat is

$$P = I^2R = V^2/R \qquad \text{21–5, 21–6}$$

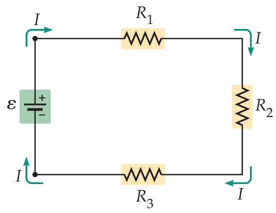
Energy Usage and the Kilowatt-Hour

The energy equivalent of one kilowatt-hour (kWh) is

$$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

21–4 RESISTORS IN SERIES AND PARALLEL

Resistors connected end to end—so that the same current flows through each one—are said to be in series. Resistors connected across the same potential difference—allowing parallel paths for the current to flow—are said to be connected in parallel.



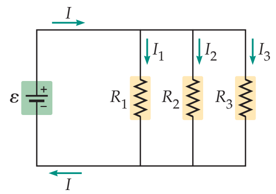
Series

The equivalent resistance, R_{eq}, of resistors connected in series is equal to the sum of the individual resistances:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum R \qquad \text{21–7}$$

Parallel

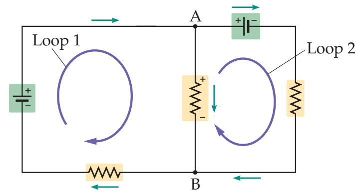
The equivalent resistance, R_{eq}, of resistors connected in parallel is given by the following:



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum \frac{1}{R} \qquad \text{21–10}$$

21–5 KIRCHHOFF'S RULES

Kirchhoff's rules are statements of charge conservation and energy conservation as applied to closed electric circuits.



Junction Rule (Charge Conservation)

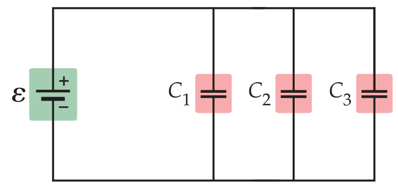
The algebraic sum of all currents meeting at a junction must equal zero. Currents entering the junction are taken to be positive; currents leaving are taken to be negative.

Loop Rule (Energy Conservation)

The algebraic sum of all potential differences around a closed loop is zero. The potential increases in going from the − to the + terminal of a battery and decreases when crossing a resistor in the direction of the current.

21–6 CIRCUITS CONTAINING CAPACITORS

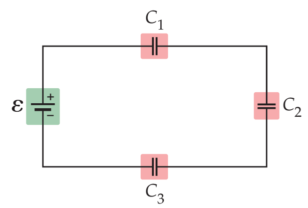
Capacitors connected end to end—so that the same charge is on each one—are said to be in series. Capacitors connected across the same potential difference are said to be connected in parallel.



Parallel

The equivalent capacitance, C_{eq}, of capacitors connected in parallel is equal to the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum C \qquad \text{21–14}$$



Series

The equivalent capacitance, C_{eq}, of capacitors connected in series is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots = \sum \frac{1}{C} \qquad \text{21–17}$$

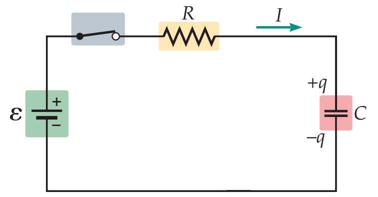
21–7 *RC* CIRCUITS

In circuits containing both resistors and capacitors, there is a characteristic time, $\tau = RC$, during which significant changes occur. This time is referred to as the time constant. The simplest such circuit, known as an *RC* circuit, consists of one resistor and one capacitor connected in series.

Charging a Capacitor

The charge on a capacitor in an RC circuit varies with time as follows:

$$q(t) = C\mathcal{E}(1 - e^{-t/\tau}) \qquad 21\text{–}18$$



The corresponding current is given by

$$I(t) = \left(\frac{\mathcal{E}}{R}\right)e^{-t/\tau} \qquad 21\text{–}19$$

Discharging a Capacitor

If a capacitor in an RC circuit starts with a charge Q at time $t = 0$, its charge at all later times is

$$q(t) = Qe^{-t/\tau} \qquad 21\text{–}20$$

Behavior near $t = 0$

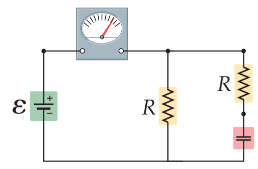
Just after the switch is closed in an RC circuit, capacitors behave like ideal wires—that is, they offer no resistance to the flow of current.

Behavior as $t \to \infty$

Long after the switch is closed in an RC circuit, capacitors behave like open circuits.

*21–8 AMMETERS AND VOLTMETERS

Ammeters and voltmeters are devices for measuring currents and voltages, respectively, in electric circuits.

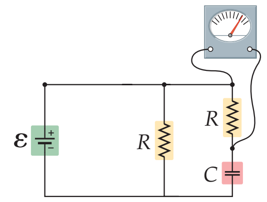


Ammeter

An ammeter is connected in series with the section of the circuit in which the current is to be measured. In the ideal case, an ammeter's resistance is zero.

Voltmeter

A voltmeter is connected in parallel with the portion of the circuit to be measured. In the ideal case, a voltmeter's resistance is infinite.



PROBLEM-SOLVING SUMMARY

Type of Problem	Relevant Physical Concepts	Related Examples
Find the work done by a battery.	The work done by a battery is the charge that passes through the battery times the emf of the battery: $W = \Delta Q\mathcal{E}$.	Active Example 21–1
Relate resistance to resistivity.	The resistance of a wire is its resistivity, ρ, times its length, divided by its cross-sectional area: $R = \rho(L/A)$.	Example 21–2
Relate the power in an electric circuit to the current, voltage, and resistance.	The basic definition of electrical power is current times voltage: $P = IV$. Using Ohm's law when appropriate, the power can also be expressed as $P = I^2R$ and $P = V^2/R$.	Examples 21–3, 21–4
Determine the equivalent resistance of resistors in series and parallel.	Resistors in series simply add: $R_{eq} = R_1 + R_2 + \cdots$; resistors in parallel add in terms of inverses: $1/R_{eq} = 1/R_1 + 1/R_2 + \cdots$.	Examples 21–5, 21–6, 21–7
Find the current in a circuit containing resistors that are not simply in series or parallel.	Apply Kirchhoff's junction rule (the algebraic sum of currents at a junction must be zero) and loop rule (the algebraic sum of potential difference around a loop is zero).	Active Example 21–2
Determine the equivalent capacitance of capacitors in series and parallel.	Capacitors in parallel simply add: $C_{eq} = C_1 + C_2 + \cdots$; capacitors in series add in terms of inverses: $1/C_{eq} = 1/C_1 + 1/C_2 + \cdots$.	Example 21–8 Active Example 21–3
Find the charge and the current in an RC circuit as a function of time.	The charge and current in an RC circuit during charging vary exponentially with time as follows: $q(t) = C\mathcal{E}(1 - e^{-t/\tau})$; $I(t) = (\mathcal{E}/R)e^{-t/\tau}$. The characteristic time is $\tau = RC$.	Example 21–9

CONCEPTUAL QUESTIONS

For instructor-assigned homework, go to www.masteringphysics.com

(Answers to odd-numbered Conceptual Questions can be found in the back of the book.)

1. What is the direction of the electric current produced by an electron that falls toward the ground?

2. Your body is composed of electric charges. Does it follow, then, that you produce an electric current when you walk?

3. Suppose you charge a comb by rubbing it through your hair. Do you produce a current when you walk across the room carrying the comb?

4. Suppose you charge a comb by rubbing it through the fur on your dog's back. Do you produce a current when you walk across the room carrying the comb?

5. An electron moving through a wire has an average drift speed that is very small. Does this mean that its instantaneous velocity is also very small?

6. Are car headlights connected in series or parallel? Give an everyday observation that supports your answer.

7. Give an example of how four resistors of resistance R can be combined to produce an equivalent resistance of R.

8. Is it possible to connect a group of resistors of value R in such a way that the equivalent resistance is less than R? If so, give a specific example.

9. What physical quantity do resistors connected in series have in common?

10. What physical quantity do resistors connected in parallel have in common?

11. Explain how electrical devices can begin operating almost immediately after you throw a switch, even though individual electrons in the wire may take hours to reach the device.

12. Explain the difference between resistivity and resistance.

13. Explain why birds can roost on high-voltage wire without being electrocuted.

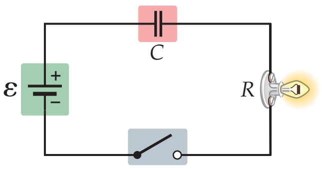
14. List two electrical applications that would benefit from room-temperature superconductors. List two applications for which room-temperature superconductivity would not be beneficial.

15. On what basic conservation laws are Kirchhoff's rules based?

16. What physical quantity do capacitors connected in series have in common?

17. What physical quantity do capacitors connected in parallel have in common?

18. Consider the circuit shown in **Figure 21–25**, in which a light of resistance R and a capacitor of capacitance C are connected in series. The capacitor has a large capacitance, and is initially uncharged. The battery provides enough power to light the bulb when connected to the battery directly. Describe the behavior of the light after the switch is closed.



▲ **FIGURE 21–25** Conceptual Question 18

PROBLEMS AND CONCEPTUAL EXERCISES

Note: Answers to odd-numbered Problems and Conceptual Exercises can be found in the back of the book. **IP** *denotes an integrated problem, with both conceptual and numerical parts;* **BIO** *identifies problems of biological or medical interest;* **CE** *indicates a conceptual exercise.* **Predict/Explain** *problems ask for two responses:* **(a)** *your prediction of a physical outcome, and* **(b)** *the best explanation among three provided. On all problems, red bullets (•, ••, •••) are used to indicate the level of difficulty.*

SECTION 21–1 ELECTRIC CURRENT

1. • How many coulombs of charge are in one ampere-hour?

2. • A flashlight bulb carries a current of 0.18 A for 78 s. How much charge flows through the bulb in this time? How many electrons?

3. • The picture tube in a particular television draws a current of 15 A. How many electrons strike the viewing screen every second?

4. • **IP** A car battery does 260 J of work on the charge passing through it as it starts an engine. **(a)** If the emf of the battery is 12 V, how much charge passes through the battery during the start? **(b)** If the emf is doubled to 24 V, does the amount of charge passing through the battery increase or decrease? By what factor?

5. • Highly sensitive ammeters can measure currents as small as 10.0 fA. How many electrons per second flow through a wire with a 10.0-fA current?

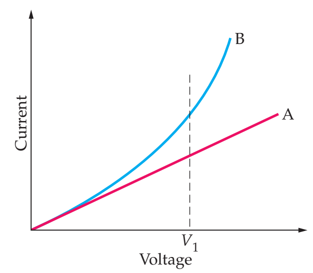
6. •• A television set connected to a 120-V outlet consumes 78 W of power. **(a)** How much current flows through the television? **(b)** How long does it take for 10 million electrons to pass through the TV?

7. •• **BIO** **Pacemaker Batteries** Pacemakers designed for long-term use commonly employ a lithium–iodine battery capable of supplying 0.42 A·h of charge. **(a)** How many coulombs of charge can such a battery supply? **(b)** If the average current produced by the pacemaker is 5.6 μA, what is the expected lifetime of the device?

SECTION 21–2 RESISTANCE AND OHM'S LAW

8. • **CE** A conducting wire is quadrupled in length and tripled in diameter. **(a)** Does its resistance increase, decrease, or stay the same? Explain. **(b)** By what factor does its resistance change?

9. • **CE** **Figure 21–26** shows a plot of current versus voltage for two different materials, A and B. Which of these materials satisfies Ohm's law? Explain.



▲ **FIGURE 21–26** Problems 9 and 10

10. • **CE Predict/Explain** Current-versus-voltage plots for two materials, A and B, are shown in Figure 21–26. **(a)** Is the resistance of material A greater than, less than, or equal to the resistance of material B at the voltage V_1? **(b)** Choose the *best explanation* from among the following:

I. Curve B is higher in value than curve A.

II. A larger slope means a larger value of I/V, and hence a smaller value of R.

III. Curve B has the larger slope at the voltage V_1 and hence the larger resistance.

11. • **CE** Two cylindrical wires are made of the same material and have the same length. If wire B is to have nine times the resistance of wire A, what must be the ratio of their radii, r_B/r_A?

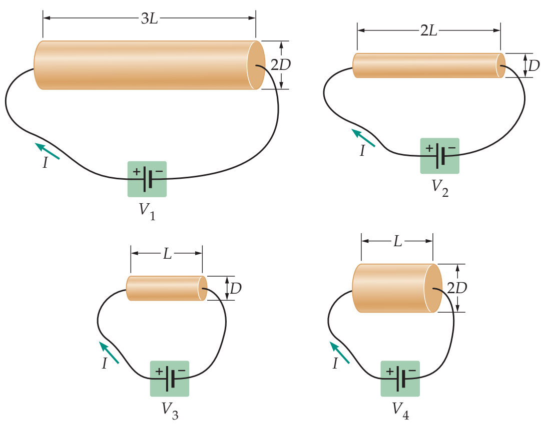
12. • A silver wire is 5.9 m long and 0.49 mm in diameter. What is its resistance?

13. • When a potential difference of 18 V is applied to a given wire, it conducts 0.35 A of current. What is the resistance of the wire?

14. • The tungsten filament of a lightbulb has a resistance of 0.07 Ω. If the filament is 27 cm long, what is its diameter?

15. • What is the resistance of 6.0 mi of copper wire with a diameter of 0.55 mm?

16. •• **CE** The four conducting cylinders shown in **Figure 21–27** are all made of the same material, though they differ in length and/or diameter. They are connected to four different batteries, which supply the necessary voltages to give the circuits the same current, I. Rank the four voltages, V_1, V_2, V_3, and V_4, in order of increasing value. Indicate ties where appropriate.



▲ **FIGURE 21–27** Problem 16

17. •• **IP** A bird lands on a bare copper wire carrying a current of 32 A. The wire is 8 gauge, which means that its cross-sectional area is 0.13 cm^2. **(a)** Find the difference in potential between the bird's feet, assuming they are separated by a distance of 6.0 cm. **(b)** Will your answer to part (a) increase or decrease if the separation between the bird's feet increases? Explain.

18. •• A current of 0.96 A flows through a copper wire 0.44 mm in diameter when it is connected to a potential difference of 15 V. How long is the wire?

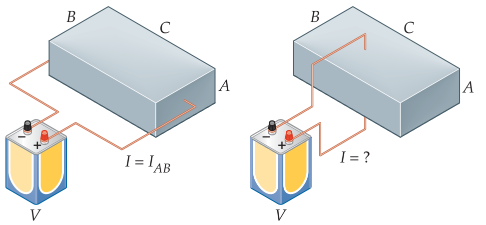
19. •• **IP BIO Current Through a Cell Membrane** A typical cell membrane is 8.0 nm thick and has an electrical resistivity of $1.3 \times 10^7\ \Omega \cdot \text{m}$. **(a)** If the potential difference between the inner and outer surfaces of a cell membrane is 75 mV, how much current flows through a square area of membrane 1.0 μm on a side? **(b)** Suppose the thickness of the membrane is doubled, but the resistivity and potential difference remain the same. Does the current increase or decrease? By what factor?

20. •• When a potential difference of 12 V is applied to a wire 6.9 m long and 0.33 mm in diameter, the result is an electric current of 2.1 A. What is the resistivity of the wire?

21. •• **IP (a)** What is the resistance per meter of an aluminum wire with a cross-sectional area of $2.4 \times 10^{-7}\ \text{m}^2$. **(b)** Would your answer to part (a) increase, decrease, or stay the same if the diameter of the wire were increased? Explain. **(c)** Repeat part (a) for a wire with a cross-sectional area of $3.6 \times 10^{-7}\ \text{m}^2$.

22. •• **BIO Resistance and Current in the Human Finger** The interior of the human body has an electrical resistivity of $0.15\ \Omega \cdot \text{m}$. **(a)** Estimate the resistance for current flowing the length of your index finger. (For this calculation, ignore the much higher resistivity of your skin.) **(b)** Your muscles will contract when they carry a current greater than 15 mA. What voltage is required to produce this current through your finger?

23. ••• Consider a rectangular block of metal of height A, width B, and length C, as shown in **Figure 21–28**. If a potential difference V is maintained between the two $A \times B$ faces of the block, a current I_{AB} is observed to flow. Find the current that flows if the same potential difference V is applied between the two $B \times C$ faces of the block. Give your answer in terms of I_{AB}.



▲ **FIGURE 21–28** Problem 23

SECTION 21–3 ENERGY AND POWER IN ELECTRIC CIRCUITS

24. • **CE** Light A has four times the power rating of light B when operated at the same voltage. **(a)** Is the resistance of light A greater than, less than, or equal to the resistance of light B? Explain. **(b)** What is the ratio of the resistance of light A to the resistance of light B?

25. • **CE** Two lightbulbs operate on the same potential difference. Bulb A has four times the power output of bulb B. **(a)** Which bulb has the greater current passing through it? Explain. **(b)** What is the ratio of the current in bulb A to the current in bulb B?

26. • **CE** Two lightbulbs operate on the same current. Bulb A has four times the power output of bulb B. **(a)** Is the potential difference across bulb A greater than or less than the potential difference across bulb B? Explain. **(b)** What is the ratio of the potential difference across bulb A to that across bulb B?

27. • A 75-V generator supplies 3.8 kW of power. How much current does the generator produce?

28. • A portable CD player operates with a current of 22 mA at a potential difference of 4.1 V. What is the power usage of the player?

29. • Find the power dissipated in a 25-Ω electric heater connected to a 120-V outlet.

30. • The current in a 120-V reading lamp is 2.6 A. If the cost of electrical energy is $0.075 per kilowatt-hour, how much does it cost to operate the light for an hour?

31. • It costs 2.6 cents to charge a car battery at a voltage of 12 V and a current of 15 A for 120 minutes. What is the cost of electrical energy per kilowatt-hour at this location?

32. •• **IP** A 75-W lightbulb operates on a potential difference of 95 V. Find **(a)** the current in the bulb and **(b)** the resistance of the bulb. **(c)** If this bulb is replaced with one whose resistance is half the value found in part (b), is its power rating greater than or less than 75 W? By what factor?

33. •• **Rating Car Batteries** Car batteries are rated by the following two numbers: **(1)** cranking amps = current the battery can produce for 30.0 seconds while maintaining a terminal voltage of at least 7.2 V and **(2)** reserve capacity = number of minutes the battery can produce a 25-A current while maintaining a terminal voltage of at least 10.5 V. One particular battery is advertised as having 905 cranking amps and a 155-minute reserve capacity. Which of these two ratings represents the greater amount of energy delivered by the battery?

SECTION 21–4 RESISTORS IN SERIES AND PARALLEL

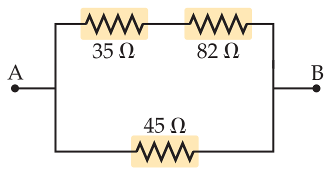
34. • **CE Predict/Explain** A dozen identical lightbulbs are connected to a given emf. **(a)** Will the lights be brighter if they are connected in series or in parallel? **(b)** Choose the *best explanation* from among the following:
   I. When connected in parallel each bulb experiences the maximum emf and dissipates the maximum power.
   II. Resistors in series have a larger equivalent resistance and dissipate more power.
   III. Resistors in parallel have a smaller equivalent resistance and dissipate less power.

35. • **CE Predict/Explain** A *fuse* is a device to protect a circuit from the effects of a large current. The fuse is a small strip of metal that burns through when the current in it exceeds a certain value, thus producing an open circuit. **(a)** Should a fuse be connected in series or in parallel with the circuit it is intended to protect? **(b)** Choose the *best explanation* from among the following:
   I. Either connection is acceptable; the main thing is to have a fuse in the circuit.
   II. The fuse should be connected in parallel, otherwise it will interrupt the current in the circuit.
   III. With the fuse connected in series, the current in the circuit drops to zero as soon as the fuse burns through.

36. • **CE** A circuit consists of three resistors, $R_1 < R_2 < R_3$, connected in series to a battery. Rank these resistors in order of increasing **(a)** current through them and **(b)** potential difference across them. Indicate ties where appropriate.

37. • **CE Predict/Explain** Two resistors are connected in parallel. **(a)** If a third resistor is now connected in parallel with the original two, does the equivalent resistance of the circuit increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:
   I. Adding a resistor generally tends to increase the resistance, but putting it in parallel tends to decrease the resistance; therefore the effects offset and the resistance stays the same.
   II. Adding more resistance to the circuit will increase the equivalent resistance.
   III. The third resistor gives yet another path for current to flow in the circuit, which means that the equivalent resistance is less.

38. • Find the equivalent resistance between points A and B for the group of resistors shown in **Figure 21–29**.



▲ **FIGURE 21–29** Problems 38 and 115

39. • What is the minimum number of 65-Ω resistors that must be connected in parallel to produce an equivalent resistance of 11 Ω or less?

40. •• Four lightbulbs (A, B, C, D) are connected together in a circuit of unknown arrangement. When each bulb is removed one at a time and replaced, the following behavior is observed:

	A	B	C	D
A removed	*	on	on	on
B removed	on	*	on	off
C removed	off	off	*	off
D removed	on	off	on	*

Draw a circuit diagram for these bulbs.

41. •• Your toaster has a power cord with a resistance of 0.020 Ω connected in series with a 9.6-Ω nichrome heating element. If the potential difference between the terminals of the toaster is 120 V, how much power is dissipated in **(a)** the power cord and **(b)** the heating element?

42. •• A hobbyist building a radio needs a 150-Ω resistor in her circuit, but has only a 220-Ω, a 79-Ω, and a 92-Ω resistor available. How can she connect these resistors to produce the desired resistance?

43. •• A circuit consists of a 12.0-V battery connected to three resistors (42 Ω, 17 Ω, and 110 Ω) in series. Find **(a)** the current that flows through the battery and **(b)** the potential difference across each resistor.

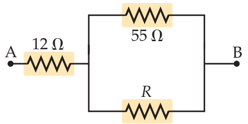
44. •• **IP** Three resistors, 11 Ω, 53 Ω, and R, are connected in series with a 24.0-V battery. The total current flowing through the battery is 0.16 A. **(a)** Find the value of resistance R. **(b)** Find the potential difference across each resistor. **(c)** If the voltage of the battery had been greater than 24.0 V, would your answer to part (a) have been larger or smaller? Explain.

45. •• A circuit consists of a battery connected to three resistors (65 Ω, 25 Ω, and 170 Ω) in parallel. The total current through the resistors is 1.8 A. Find **(a)** the emf of the battery and **(b)** the current through each resistor.

46. •• **IP** Three resistors, 22 Ω, 67 Ω, and R, are connected in parallel with a 12.0-V battery. The total current flowing through the battery is 0.88 A. **(a)** Find the value of resistance R. **(b)** Find the current through each resistor. **(c)** If the total current in the battery had been greater than 0.88 A, would your answer to part (a) have been larger or smaller? Explain.

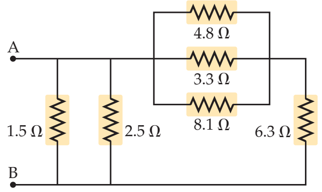
47. •• An 89-Ω resistor has a current of 0.72 A and is connected in series with a 130-Ω resistor. What is the emf of the battery to which the resistors are connected?

48. •• The equivalent resistance between points A and B of the resistors shown in **Figure 21–30** is 26 Ω. Find the value of resistance R.



▲ **FIGURE 21–30** Problems 48, 52, and 98

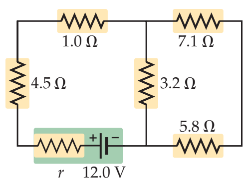
49. •• Find the equivalent resistance between points A and B shown in **Figure 21–31**.



▲ **FIGURE 21–31** Problems 49 and 53

50. •• How many 65-W lightbulbs can be connected in parallel across a potential difference of 85 V before the total current in the circuit exceeds 2.1 A?

51. •• The circuit in **Figure 21–32** includes a battery with a finite internal resistance, $r = 0.50\ \Omega$. **(a)** Find the current flowing through the 7.1-Ω and the 3.2-Ω resistors. **(b)** How much current flows through the battery? **(c)** What is the potential difference between the terminals of the battery?



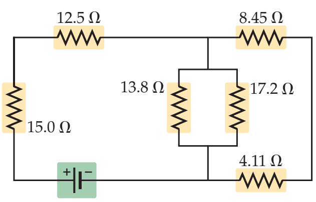
▲ **FIGURE 21–32** Problems 51 and 54

52. •• **IP** A 12-V battery is connected to terminals A and B in Figure 21–30. **(a)** Given that $R = 85\ \Omega$, find the current in each resistor. **(b)** Suppose the value of R is increased. For each resistor in turn, state whether the current flowing through it increases or decreases. Explain.

53. •• **IP** The terminals A and B in Figure 21–31 are connected to a 9.0-V battery. **(a)** Find the current flowing through each resistor. **(b)** Is the potential difference across the 6.3-Ω resistor greater than, less than, or the same as the potential difference across the 1.5-Ω resistor? Explain.

54. •• **IP** Suppose the battery in Figure 21–32 has an internal resistance $r = 0.25\ \Omega$. **(a)** How much current flows through the battery? **(b)** What is the potential difference between the terminals of the battery? **(c)** If the 3.2-Ω resistor is increased in value, will the current in the battery increase or decrease? Explain.

55. ••• **IP** The current flowing through the 8.45-Ω resistor in **Figure 21–33** is 1.52 A. **(a)** What is the voltage of the battery? **(b)** If the

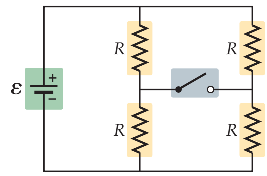


▲ **FIGURE 21–33** Problems 55 and 56

17.2-Ω resistor is increased in value, will the current provided by the battery increase, decrease, or stay the same? Explain.

56. ••• The current in the 13.8-Ω resistor in Figure 21–33 is 0.795 A. Find the current in the other resistors in the circuit.

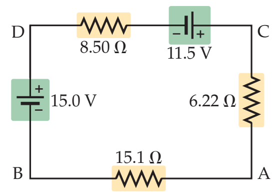
57. ••• **IP** Four identical resistors are connected to a battery as shown in **Figure 21–34**. When the switch is open, the current through the battery is I_0. **(a)** When the switch is closed, will the current through the battery increase, decrease, or stay the same? Explain. **(b)** Calculate the current that flows through the battery when the switch is closed. Give your answer in terms of I_0.



▲ **FIGURE 21–34** Problem 57

SECTION 21–5 KIRCHHOFF'S RULES

58. • Find the magnitude and direction (clockwise or counterclockwise) of the current in **Figure 21–35**.

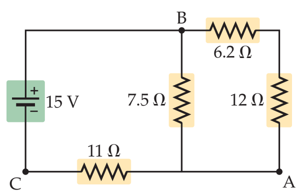


▲ **FIGURE 21–35** Problems 58, 59, and 60

59. • **IP** Suppose the polarity of the 11.5-V battery in Figure 21–35 is reversed. **(a)** Do you expect this to increase or decrease the amount of current flowing in the circuit? Explain. **(b)** Calculate the magnitude and direction (clockwise or counterclockwise) of the current in this case.

60. •• **IP** It is given that point A in Figure 21–35 is grounded ($V = 0$). **(a)** Is the potential at point B greater than or less than zero? Explain. **(b)** Is the potential at point C greater than or less than zero? Explain. **(c)** Calculate the potential at point D.

61. •• Consider the circuit shown in **Figure 21–36**. Find the current through each resistor using **(a)** the rules for series and parallel resistors and **(b)** Kirchhoff's rules.

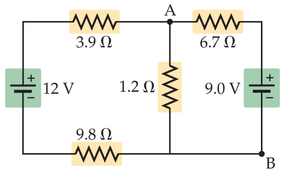


▲ **FIGURE 21–36** Problems 61 and 62

62. •• Suppose point A is grounded ($V = 0$) in Figure 21–36. Find the potential at points B and C.

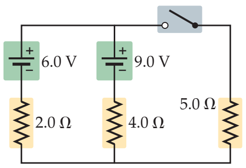
63. •• **IP** **(a)** Find the current in each resistor in **Figure 21–37**. **(b)** Is the potential at point A greater than, less than, or equal to the

potential at point B? Explain. **(c)** Determine the potential difference between the points A and B.



▲ **FIGURE 21–37** Problem 63

64\. ••• Two batteries and three resistors are connected as shown in **Figure 21–38**. How much current flows through each battery when the switch is **(a)** closed and **(b)** open?



▲ **FIGURE 21–38** Problem 64

SECTION 21–6 CIRCUITS CONTAINING CAPACITORS

65\. • **CE** Two capacitors, $C_1 = C$ and $C_2 = 2C$, are connected to a battery. **(a)** Which capacitor stores more energy when they are connected to the battery in series? Explain. **(b)** Which capacitor stores more energy when they are connected in parallel? Explain.

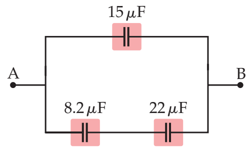
66\. • **CE Predict/Explain** Two capacitors are connected in series. **(a)** If a third capacitor is now connected in series with the original two, does the equivalent capacitance increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:

- **I.** Adding a capacitor generally tends to increase the capacitance, but putting it in series tends to decrease the capacitance; therefore, the net result is no change.
- **II.** Adding a capacitor in series will increase the total amount of charge stored, and hence increase the equivalent capacitance.
- **III.** Adding a capacitor in series decreases the equivalent capacitance since each capacitor now has less voltage across it, and hence stores less charge.

67\. • **CE Predict/Explain** Two capacitors are connected in parallel. **(a)** If a third capacitor is now connected in parallel with the original two, does the equivalent capacitance increase, decrease, or remain the same? **(b)** Choose the *best explanation* from among the following:

- **I.** Adding a capacitor tends to increase the capacitance, but putting it in parallel tends to decrease the capacitance; therefore, the net result is no change.
- **II.** Adding a capacitor in parallel will increase the total amount of charge stored, and hence increase the equivalent capacitance.
- **III.** Adding a capacitor in parallel decreases the equivalent capacitance since each capacitor now has less voltage across it, and hence stores less charge.

68\. • Find the equivalent capacitance between points A and B for the group of capacitors shown in **Figure 21–39**.

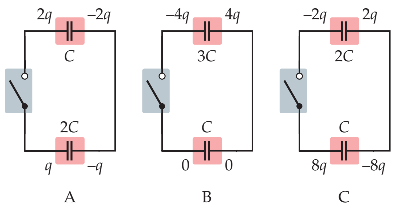


▲ **FIGURE 21–39** Problems 68 and 72

69\. • A 12-V battery is connected to three capacitors in series. The capacitors have the following capacitances: 4.5 μF, 12 μF, and 32 μF. Find the voltage across the 32-μF capacitor.

70\. •• **CE** You conduct a series of experiments in which you connect the capacitors C_1 and $C_2 > C_1$ to a battery in various ways. The experiments are as follows: **A**, C_1 alone connected to the battery; **B**, C_2 alone connected to the battery; **C**, C_1 and C_2 connected to the battery in series; **D**, C_1 and C_2 connected to the battery in parallel. Rank these four experiments in order of increasing equivalent capacitance. Indicate ties where appropriate.

71\. •• **CE** Three different circuits, each containing a switch and two capacitors, are shown in **Figure 21–40**. Initially, the plates of the capacitors are charged as shown. The switches are then closed, allowing charge to move freely between the capacitors. Rank the circuits in order of increasing final charge on the left plate of **(a)** the upper capacitor and **(b)** the lower capacitor. Indicate ties where appropriate.



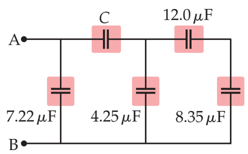
▲ **FIGURE 21–40** Problem 71

72\. •• Terminals A and B in Figure 21–39 are connected to a 9.0-V battery. Find the energy stored in each capacitor.

73\. •• **IP** Two capacitors, one 7.5 μF and the other 15 μF, are connected in parallel across a 15-V battery. **(a)** Find the equivalent capacitance of the two capacitors. **(b)** Which capacitor stores more charge? Explain. **(c)** Find the charge stored on each capacitor.

74\. •• **IP** Two capacitors, one 7.5 μF and the other 15 μF, are connected in series across a 15-V battery. **(a)** Find the equivalent capacitance of the two capacitors. **(b)** Which capacitor stores more charge? Explain. **(c)** Find the charge stored on each capacitor.

75\. •• The equivalent capacitance of the capacitors shown in **Figure 21–41** is 9.22 μF. Find the value of capacitance C.

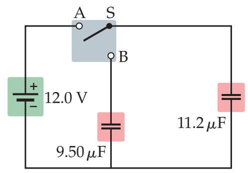


▲ **FIGURE 21–41** Problems 75 and 118

76\. ••• Two capacitors, C_1 and C_2, are connected in series and charged by a battery. Show that the energy stored in C_1 plus the energy stored in C_2 is equal to the energy stored in the equivalent capacitor, C_{eq}, when it is connected to the same battery.

77\. ••• With the switch in position A, the 11.2-μF capacitor in **Figure 21–42** is fully charged by the 12.0-V battery, and the

9.50-μF capacitor is uncharged. The switch is now moved to position B. As a result, charge flows between the capacitors until they have the same voltage across their plates. Find this voltage.



▲ **FIGURE 21–42** Problem 77

SECTION 21–7 *RC* CIRCUITS

78. • The switch on an RC circuit is closed at $t = 0$. Given that $\mathcal{E} = 9.0$ V, $R = 150\ \Omega$, and $C = 23\ \mu$F, how much charge is on the capacitor at time $t = 4.2$ ms?

79. • The capacitor in an RC circuit ($R = 120\ \Omega, C = 45\ \mu$F) is initially uncharged. Find **(a)** the charge on the capacitor and **(b)** the current in the circuit one time constant ($\tau = RC$) after the circuit is connected to a 9.0-V battery.

80. •• **CE** Three RC circuits have the emf, resistance, and capacitance given in the accompanying table. Initially, the switch on the circuit is open and the capacitor is uncharged. Rank these circuits in order of increasing **(a)** initial current (immediately after the switch is closed) and **(b)** time for the capacitor to acquire half its final charge. Indicate ties where appropriate.

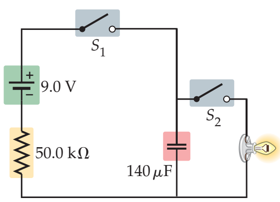
	$\mathcal{E}$ (V)	R (Ω)	C (μF)
Circuit A	12	4	3
Circuit B	9	3	1
Circuit C	9	9	2

81. •• Consider an RC circuit with $\mathcal{E} = 12.0$ V, $R = 175\ \Omega$, and $C = 55.7\ \mu$F. Find **(a)** the time constant for the circuit, **(b)** the maximum charge on the capacitor, and **(c)** the initial current in the circuit.

82. •• The resistor in an RC circuit has a resistance of 145 Ω. **(a)** What capacitance must be used in this circuit if the time constant is to be 3.5 ms? **(b)** Using the capacitance determined in part (a), calculate the current in the circuit 7.0 ms after the switch is closed. Assume that the capacitor is uncharged initially and that the emf of the battery is 9.0 V.

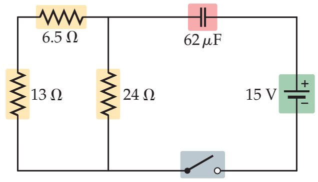
83. •• A flash unit for a camera has a capacitance of 1500 μF. What resistance is needed in this RC circuit if the flash is to charge to 90% of its full charge in 21 s?

84. •• **Figure 21–43** shows a simplified circuit for a photographic flash unit. This circuit consists of a 9.0-V battery, a 50.0-kΩ resistor, a 140-μF capacitor, a flashbulb, and two switches. Initially, the capacitor is uncharged and the two switches are open. To charge the unit, switch S_1 is closed; to fire the flash, switch S_2 (which is connected to the camera's shutter) is closed. How long does it take to charge the capacitor to 5.0 V?



▲ **FIGURE 21–43** Problem 84

85. •• **IP** Consider the RC circuit shown in **Figure 21–44**. Find **(a)** the time constant and **(b)** the initial current for this circuit. **(c)** It is desired to increase the time constant of this circuit by adjusting the value of the 6.5-Ω resistor. Should the resistance of this resistor be increased or decreased to have the desired effect? Explain.



▲ **FIGURE 21–44** Problems 85 and 119

86. ••• The capacitor in an RC circuit is initially uncharged. In terms of R and C, determine **(a)** the time required for the charge on the capacitor to rise to 50% of its final value and **(b)** the time required for the initial current to drop to 10% of its initial value.

GENERAL PROBLEMS

87. • **CE** A given car battery is rated as 250 amp-hours. Is this rating a measure of energy, power, charge, voltage, or current? Explain.

88. • **CE Predict/Explain** The resistivity of tungsten increases with temperature. **(a)** When a light containing a tungsten filament heats up, does its power consumption increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. The voltage is unchanged, and therefore an increase in resistance implies a reduced power, as we can see from $P = V^2/R$.
   II. Increasing the resistance increases the power, as is clear from $P = I^2R$.
   III. The power consumption is independent of resistance, as we can see from $P = IV$.

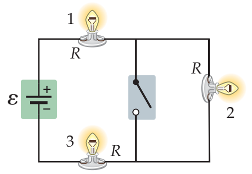
89. • **CE** A cylindrical wire is to be doubled in length, but it is desired that its resistance remain the same. **(a)** Must its radius be increased or decreased? Explain. **(b)** By what factor must the radius be changed?

90. • **CE Predict/Explain** An electric space heater has a power rating of 500 W when connected to a given voltage V. **(a)** If two of these heaters are connected in series to the same voltage, is the power consumed by the two heaters greater than, less than, or equal to 1000 W? **(b)** Choose the *best explanation* from among the following:
   I. Each heater consumes 500 W; therefore two of them will consume 500 W + 500 W = 1000 W.
   II. The voltage is the same, but the resistance is doubled by connecting the heaters in series. Therefore, the power consumed ($P = V^2/R$) is less than 1000 W.
   III. Connecting two heaters in series doubles the resistance. Since power depends on the resistance squared, it follows that the power consumed is greater than 1000 W.

91. • **CE** Two resistors, $R_1 = R$ and $R_2 = 2R$, are connected to a battery. **(a)** Which resistor dissipates more power when they are connected to the battery in series? Explain. **(b)** Which resistor dissipates more power when they are connected in parallel? Explain.

92. • **CE** Consider the circuit shown in **Figure 21–45**, in which three lights, each with a resistance R, are connected in series. The circuit also contains an open switch. **(a)** When the switch is closed, does the intensity of light 2 increase, decrease, or stay

the same? Explain. **(b)** Do the intensities of lights 1 and 3 increase, decrease, or stay the same when the switch is closed? Explain.

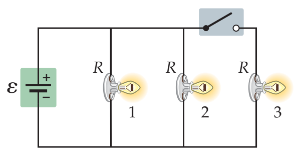


▲ **FIGURE 21–45** Problems 92, 93, and 94

93. • **CE Predict/Explain** **(a)** Referring to Problem 92 and the circuit in Figure 21–45, does the current supplied by the battery increase, decrease, or remain the same when the switch is closed? **(b)** Choose the *best explanation* from among the following:
   I. The current decreases because only two resistors can draw current from the battery when the switch is closed.
   II. Closing the switch makes no difference to the current since the second resistor is still connected to the battery as before.
   III. Closing the switch shorts out the second resistor, decreases the total resistance of the circuit, and increases the current.

94. • **CE Predict/Explain** **(a)** Referring to Problem 92 and the circuit in Figure 21–45, does the total power dissipated in the circuit increase, decrease, or remain the same when the switch is closed? **(b)** Choose the *best explanation* from among the following:
   I. Closing the switch shorts out one of the resistors, which means that the power dissipated decreases.
   II. The equivalent resistance of the circuit is reduced by closing the switch, but the voltage remains the same. Therefore, from $P = V^2/R$ we see that the power dissipated increases.
   III. The power dissipated remains the same because power, $P = IV$, is independent of resistance.

95. • **CE** Consider the circuit shown in **Figure 21–46**, in which three lights, each with a resistance R, are connected in parallel. The circuit also contains an open switch. **(a)** When the switch is closed, does the intensity of light 3 increase, decrease, or stay the same? Explain. **(b)** Do the intensities of lights 1 and 2 increase, decrease, or stay the same when the switch is closed? Explain.



▲ **FIGURE 21–46** Problems 95, 96, and 97

96. • **CE Predict/Explain** **(a)** When the switch is closed in the circuit shown in Figure 21–46, does the current supplied by the battery increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. The current increases because three resistors are drawing current from the battery when the switch is closed, rather than just two.
   II. Closing the switch makes no difference to the current because the voltage is the same as before.
   III. Closing the switch decreases the current because an additional resistor is added to the circuit.

97. • **CE Predict/Explain** **(a)** When the switch is closed in the circuit shown in Figure 21–46, does the total power dissipated in the circuit increase, decrease, or stay the same? **(b)** Choose the *best explanation* from among the following:
   I. Closing the switch adds one more resistor to the circuit. This makes it harder for the battery to supply current, which decreases the power dissipated.
   II. The equivalent resistance of the circuit is reduced by closing the switch, but the voltage remains the same. Therefore, from $P = V^2/R$ we see that the power dissipated increases.
   III. The power dissipated remains the same because power, $P = IV$, is independent of resistance.

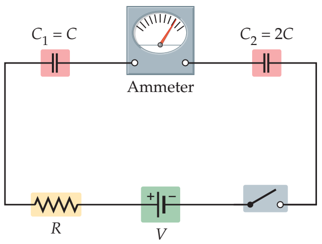
98. • Suppose that points A and B in Figure 21–30 are connected to a 12-V battery. Find the power dissipated in each of the resistors assuming that $R = 65\ \Omega$.

99. • You are given resistors of 413 Ω, 521 Ω, and 146 Ω. Describe how these resistors must be connected to produce an equivalent resistance of 255 Ω.

100. • You are given capacitors of 18 μF, 7.2 μF, and 9.0 μF. Describe how these capacitors must be connected to produce an equivalent capacitance of 22 μF.

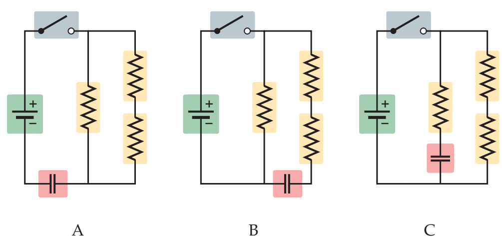
101. • Suppose your car carries a charge of 85 μC. What current does it produce as it travels from Dallas to Fort Worth (35 mi) in 0.75 h?

102. •• **CE** The circuit shown in **Figure 21–47** shows a resistor and two capacitors connected in series with a battery of voltage V. The circuit also has an ammeter and a switch. Initially, the switch is open and both capacitors are uncharged. The following questions refer to a time long after the switch is closed and current has ceased to flow. **(a)** In terms of V, what is the voltage across the capacitor C_1? **(b)** In terms of CV, what is the charge on the right plate of C_2? **(c)** What is the net charge that flowed through the ammeter during charging? Give your answer in terms of CV.



▲ **FIGURE 21–47** Problem 102

103. •• **CE** The three circuits shown in **Figure 21–48** have identical batteries, resistors, and capacitors. Initially, the switches are open and the capacitors are uncharged. Rank the circuits in order of increasing **(a)** final charge on the capacitor and **(b)** time for the current to drop to 90% of its initial value. Indicate ties where appropriate.

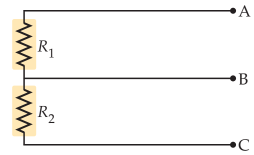


▲ **FIGURE 21–48** Problem 103

104. •• It is desired to construct a 5.0-Ω resistor from a 1.2-m length of tungsten wire. What diameter is needed for this wire?

105. •• **Electrical Safety Codes** For safety reasons, electrical codes have been established that limit the amount of current a wire of a given size can carry. For example, an 18-gauge (cross-sectional area = 1.17 mm^2), rubber-insulated extension cord with copper wires can carry a maximum current of 5.0 A. Find the voltage drop in a 12-ft, 18-gauge extension cord carrying a current of 5.0 A. (*Note:* In an extension cord, the current must flow through two lengths—down and back.)

106. •• **A Three-Way Lightbulb** A three-way lightbulb has two filaments with resistances R_1 and R_2 connected in series. The resistors are connected to three terminals, as indicated in **Figure 21–49**, and the light switch determines which two of the three terminals are connected to a potential difference of 120 V at any given time. When terminals A and B are connected to 120 V the bulb uses 75.0 W of power. When terminals A and C are connected to 120 V the bulb uses 50.0 W of power. **(a)** What is the resistance R_1? **(b)** What is the resistance R_2? **(c)** How much power does the bulb use when 120 V is connected to terminals B and C?

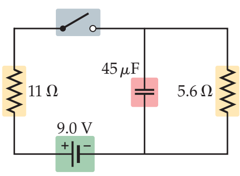


▲ **FIGURE 21–49** Problem 106

107. •• A portable CD player uses a current of 7.5 mA at a potential difference of 3.5 V. **(a)** How much energy does the player use in 35 s? **(b)** Suppose the player has a mass of 0.65 kg. For what length of time could the player operate on the energy required to lift it through a height of 1.0 m?

108. •• An electrical heating coil is immersed in 4.6 kg of water at 22 °C. The coil, which has a resistance of 250 Ω, warms the water to 32 °C in 15 min. What is the potential difference at which the coil operates?

109. •• **IP** Consider the circuit shown in **Figure 21–50**. **(a)** Is the current flowing through the battery immediately after the switch is closed greater than, less than, or the same as the current flowing through the battery long after the switch is closed? Explain. **(b)** Find the current flowing through the battery immediately after the switch is closed. **(c)** Find the current in the battery long after the switch is closed.



▲ **FIGURE 21–50** Problems 109 and 116

110. •• A silver wire and a copper wire have the same volume and the same resistance. Find the ratio of their radii, r_{silver}/r_{copper}.

111. •• Two resistors are connected in series to a battery with an emf of 12 V. The voltage across the first resistor is 2.7 V and the current through the second resistor is 0.15 A. Find the resistance of the two resistors.

112. •• **BIO Pacemaker Pulses** A pacemaker sends a pulse to a patient's heart every time the capacitor in the pacemaker charges to a voltage of 0.25 V. It is desired that the patient receive 75 pulses per minute. Given that the capacitance of the pacemaker is 110 μF and that the battery has a voltage of 9.0 V, what value should the resistance have?

113. •• A long, thin wire has a resistance R. The wire is now cut into three segments of equal length, which are connected in parallel. In terms of R, what is the equivalent resistance of the three wire segments?

114. •• Three resistors (R, $\frac{1}{2}R$, $2R$) are connected to a battery. **(a)** If the resistors are connected in series, which one has the greatest rate of energy dissipation? **(b)** Repeat part (a), this time assuming that the resistors are connected in parallel.

115. •• **IP** Suppose we connect a 12.0-V battery to terminals A and B in Figure 21–29. **(a)** Is the current in the 45-Ω resistor greater than, less than, or the same as the current in the 35-Ω resistor? Explain. **(b)** Calculate the current flowing through each of the three resistors in this circuit.

116. •• **IP** Suppose the battery in Figure 21–50 has an internal resistance of 0.73 Ω. **(a)** What is the potential difference across the terminals of the battery when the switch is open? **(b)** When the switch is closed, does the potential difference of the battery increase or decrease? Explain. **(c)** Find the potential difference across the battery after the switch has been closed a long time.

117. •• **National Electric Code** In the United States, the National Electric Code sets standards for maximum safe currents in insulated copper wires of various diameters. The accompanying table gives a portion of the code. Notice that wire diameters are identified by the *gauge* of the wire, and that 1 mil = 10^{-3} in. Find the maximum power dissipated per length in **(a)** an 8-gauge wire and **(b)** a 10-gauge wire.

Gauge	Diameter (mils)	Safe current (A)
8	129	35
10	102	25

118. ••• **IP** A 15.0-V battery is connected to terminals A and B in Figure 21–41. **(a)** Given that C = 15.0 μF, find the charge on each of the capacitors. **(b)** Find the total energy stored in this system. **(c)** If the 7.22-μF capacitor is increased in value, will the total energy stored in the circuit increase or decrease? Explain.

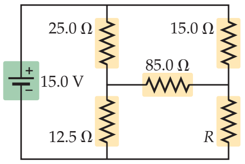
119. ••• **IP** The switch in the RC circuit shown in Figure 21–44 is closed at $t = 0$. **(a)** How much power is dissipated in each resistor just after $t = 0$ and in the limit $t \rightarrow \infty$? **(b)** What is the charge on the capacitor at the time $t = 0.35$ ms? **(c)** How much energy is stored in the capacitor in the limit $t \rightarrow \infty$? **(d)** If the voltage of the battery is doubled, by what factor does your answer to part (c) change? Explain.

120. ••• Two resistors, R_1 and R_2, are connected in parallel and connected to a battery. Show that the power dissipated in R_1 plus the power dissipated in R_2 is equal to the power dissipated in the equivalent resistor, R_{eq}, when it is connected to the same battery.

121. ••• A battery has an emf $\mathcal{E}$ and an internal resistance r. When the battery is connected to a 25-Ω resistor, the current through the battery is 0.65 A. When the battery is connected to a 55-Ω resistor, the current is 0.45 A. Find the battery's emf and internal resistance.

122. ••• When two resistors, R_1 and R_2, are connected in series across a 6.0-V battery, the potential difference across R_1 is 4.0 V. When R_1 and R_2 are connected in parallel to the same battery, the current through R_2 is 0.45 A. Find the values of R_1 and R_2.

123. ••• The circuit shown in **Figure 21–51** is known as a Wheatstone bridge. Find the value of the resistor R such that the current through the 85.0-Ω resistor is zero.



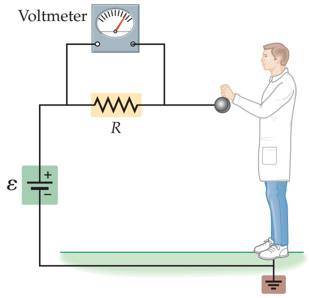
▲ **FIGURE 21–51** Problem 123

PASSAGE PROBLEMS

BIO Footwear Safety

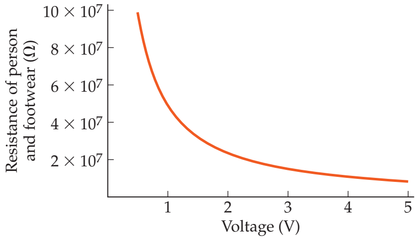
The American National Standards Institute (ANSI) specifies safety standards for a number of potential workplace hazards. For example, ANSI requires that footwear provide protection against the effects of compression from a static weight, impact from a dropped object, puncture from a sharp tool, and cuts from saws. In addition, to protect against the potentially lethal effects of an electrical shock, ANSI provides standards for the electrical resistance that a person and footwear must offer to the flow of electric current.

Specifically, regulation ANSI Z41-1999 states that the resistance of a person and his or her footwear must be tested with the circuit shown in **Figure 21–52**. In this circuit, the voltage supplied by the battery is $\mathcal{E} = 50.0$ V and the resistance in the circuit is $R = 1.00$ MΩ. Initially the circuit is open and no current flows. When a person touches the metal sphere attached to the battery, however, the circuit is closed and a small current flows through the person, the shoes, and back to the battery. The amount of current flowing through the person can be determined by using a voltmeter to measure the voltage drop V across the resistor R. To be safe, the current should not exceed 150 μA.



▲ **FIGURE 21–52** Problems 124, 125, 126, and 127

Notice that the experimental setup in Figure 21–52 is a dc circuit with two resistors in series—the resistance R and the resistance of the person and footwear, R_{pf}. It follows that the current in the circuit is $I = \mathcal{E}/(R + R_{pf})$. We also know that the current is $I = V/R$, where V is the reading of the voltmeter. These relations can be combined to relate the voltage V to the resistance R_{pf}, with the result shown in **Figure 21–53**. According to ANSI regulations, Type II footwear must give a resistance R_{pf} in the range of 0.1×10^7 Ω to 100×10^7 Ω.



▲ **FIGURE 21–53** Problems 124, 125, 126, and 127

124. • Suppose the voltmeter measures a potential difference of 3.70 V across the resistor. What is the current that flows through the person's body?

A. 3.70×10^{-6} A **B.** 5.00×10^{-5} A

C. 0.0740 A **D.** 3.70 A

125. • What is the resistance of the person and footwear when the voltmeter reads 3.70 V?

A. 1.25×10^7 Ω **B.** 1.35×10^7 Ω

C. 4.63×10^7 Ω **D.** 1.71×10^8 Ω

126. • The resistance of a given person and footwear is 4.00×10^7 Ω. What is the reading on the voltmeter when this person is tested?

A. 0.976 V **B.** 1.22 V

C. 1.25 V **D.** 50.0 V

127. • Suppose that during one test a person's shoes become wet when water spills onto the floor. When this happens, do you expect the reading on the voltmeter to increase, decrease, or stay the same?

INTERACTIVE PROBLEMS

128. •• **Referring to Example 21–7** Suppose the three resistors in this circuit have the values $R_1 = 100.0$ Ω, $R_2 = 200.0$ Ω, and $R_3 = 300.0$ Ω, and that the emf of the battery is 12.0 V. (The resistor numbers are given in the Interactive Figure.) **(a)** Find the potential difference across each resistor. **(b)** Find the current that flows through each resistor.

129. •• **Referring to Example 21–7** Suppose $R_1 = R_2 = 225$ Ω and $R_3 = R$. The emf of the battery is 12.0 V. (The resistor numbers are given in the Interactive Figure.) **(a)** Find the value of R such that the current supplied by the battery is 0.0750 A. **(b)** Find the value of R that gives a potential difference of 2.65 V across resistor 2.

130. •• **IP Referring to Example 21–9** Suppose the resistance of the 126-Ω resistor is reduced by a factor of 2. The other resistor is 275 Ω, the capacitor is 182 μF, and the battery has an emf of 3.00 V. **(a)** Does the final value of the charge on the capacitor increase, decrease, or stay the same? Explain. **(b)** Does the time for the capacitor to charge to 80.0% of its final value increase, decrease, or stay the same? Explain. **(c)** Find the time referred to in part (b).

131. •• **IP Referring to Example 21–9** Suppose the capacitance of the 182-μF capacitor is reduced by a factor of 2. The two resistors are 126 Ω and 275 Ω, and the battery has an emf of 3.00 V. **(a)** Find the final value of the charge on the capacitor. **(b)** Does the time for the capacitor to charge to 80.0% of its final value increase, decrease, or stay the same? Explain. **(c)** Find the time referred to in part (b).