Assignment 3

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Contents

Theoretical exercises

1

a:

Show that $Cov(z_t, \varepsilon_{yt}) \neq 0$.

- Recall the formula for covariance: $Cov(z_t, \varepsilon_{yt}) = E(z_t\varepsilon_{yt}) - E(z_t)E(\varepsilon_{yt})$. Because $\varepsilon_{yt} \sim WN(0, \sigma_y^2)$, we obtain: $Cov(z_t, \varepsilon_{yt}) = E(z_t\varepsilon_{yt})$.
- Next, expand the the expression for y_t in the expression for z_t: $E(z_t\varepsilon_{yt}) = E[(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}] + \gamma_{21}y_{t-1} + \gamma_{22}z_{t-1} + \varepsilon_{zt})\varepsilon_{yt}] = *$.
- Now distribute ε_{yt} over the system: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt} + \gamma_{21}y_{t-1}\varepsilon_{yt} + \gamma_{22}z_{t-1}\varepsilon_{yt} + \varepsilon_{zt}\varepsilon_{yt})$
- Expand the expectation operator to a sum: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt}) + E(\gamma_{21}y_{t-1}\varepsilon_{yt}) + E(\gamma_{22}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{zt}\varepsilon_{yt})$.
- Exploit intertemporal independence and that ε_{yt} and ε_{zt} are independent: $* = E(-b_{21}[(b_{12}z_t + \gamma_{11}y_{t-1} + \gamma_{12}z_{t-1} + \varepsilon_{yt}]\varepsilon_{yt})$
- Distibute ε_{yt}: $* = -b_{21}E([(b_{12}z_t\varepsilon_{yt} + \gamma_{11}y_{t-1}\varepsilon_{yt} + \gamma_{12}z_{t-1}\varepsilon_{yt} + \varepsilon_{yt}\varepsilon_{yt}])$
- Expand the expectation: $-b_{21}[E(b_{12}z_t\varepsilon_{yt}) + E(\gamma_{11}y_{t-1}\varepsilon_{yt}) + E(\gamma_{12}z_{t-1}\varepsilon_{yt}) + E(\varepsilon_{yt}^2)]$
- What remains after exploiting independence is $-b_{21}E(\varepsilon_{yt}^2) = -b_{21}\sigma_y^2 \neq 0$ QED.

Empirical exercises

Do exercises 10a-10g in the textbook (p.340)

- Remark 1: It is possible that the values you obtain for the F-statistics, p-values and correlations are different than those reported since the sample is extended. However, the main conclusions should be the same.
- Remark 2: Exercise d. is optional and so is the part on the forecast error variance in e. (but you could use the command fevd in STATA to answer these questions).
- Remark 3: You find the appropriate specifications for the variables st , Δlip, and Δur described in the text to exercise 9 (p.339).

```
## -- Attaching packages ------------------------------------ tidyverse 1.2.1 --

## v ggplot2 3.1.0     v purrr   0.2.5
## v tibble  2.0.1     v dplyr   0.7.8
## v tidyr   0.8.2     v stringr 1.3.1
## v readr   1.3.1     v forcats 0.3.0

## -- Conflicts --------------------------------------- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()    masks stats::lag()

## Loading required package: MASS

##
## Attaching package: 'MASS'

## The following object is masked from 'package:dplyr':
##
##     select

## Loading required package: strucchange

## Loading required package: zoo

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric

## Loading required package: sandwich

##
## Attaching package: 'strucchange'

## The following object is masked from 'package:stringr':
##
##     boundary

## Loading required package: urca

## Loading required package: lmtest
```

10:

Estimate the three-VAR beginning in 1961Q1 and use the ordering such that Δlip_t is causally prior to Δur_t and that Δur_t is causally prior to s_t.

We begin by defining the variables we are going to include in our analysis.

Do we need to check for stationarity here?

The lag length is already determined to be 3.

10 a:

If you perform a test to determine whether s_t Granger causes Δlip_t, you should find that the F-statistic is 2.44 with a prob-value of 0.065. How do you interpret this result?

10 b:

Verify that s_t Granger causes $\Delta unemp_t$. You should find that the F statistic is 5.93 with a prob value of less than 0.001.

10 c:

It turns out that the correlation coefficient between e_{1t} and e_{2t} is -0.72. The correlation between e_{1t} and e_{3t} is -0.11 and between e_{2t} and e_{3t} is 0.10. Explain why the ordering of a Choleski composition is likely to be important for obtaining the impulse responses.

10 e:

Now estimate the model using the levels of lip_t and ur_t. Do you now find a lag length of 5 appropriate?

10 f:

Obtain the impulse response function from the model using $\Delta lip_t, \Delta ur_t$ and s_t. Show that a positive shock to the industrial production induces a decline in the unemployment rate that lasts six quarters. Then, Δur_t overshoots its long run level before returning to zero.

10 g:

Reverse the ordering and explain why the results depend on whether or not Δlip_t proceeds Δur_t