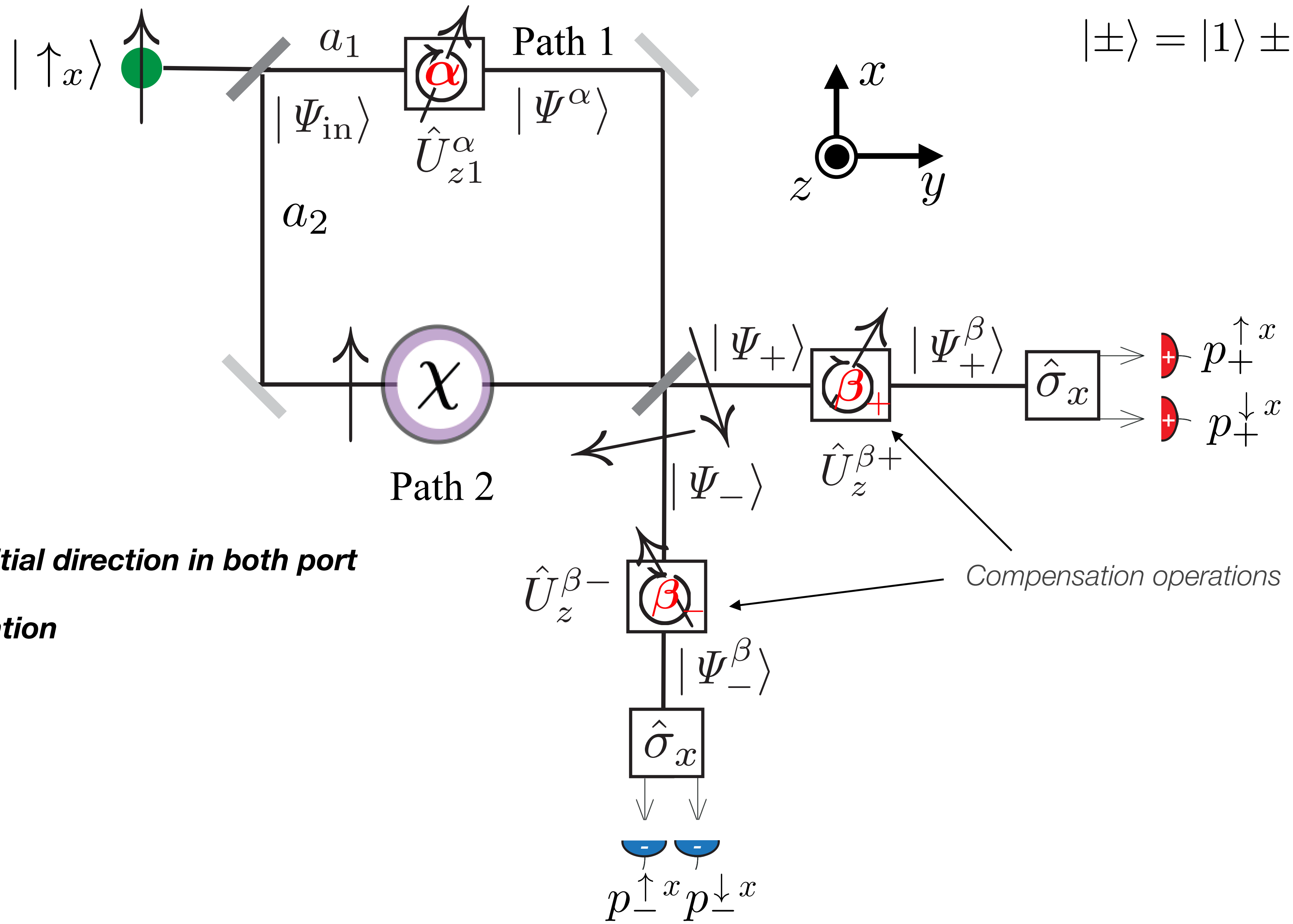
Interference VS Which-Way Measurements: ***path presence***

$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\mathrm{i}\chi}a_2|2\rangle\right) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\text{out}}\rangle = \langle\pm|\psi\rangle\,|\pm\rangle|S_{\pm}\rangle$$

$$|S_{\pm}\rangle = s_{\pm}^{\uparrow x}|\uparrow_x\rangle + s_{\pm}^{\downarrow x}|\downarrow_x\rangle$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$



spin is rotated back to initial direction in both port

by a ***Compensation operation***

H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Interference VS Which-Way Measurements: **path presence**

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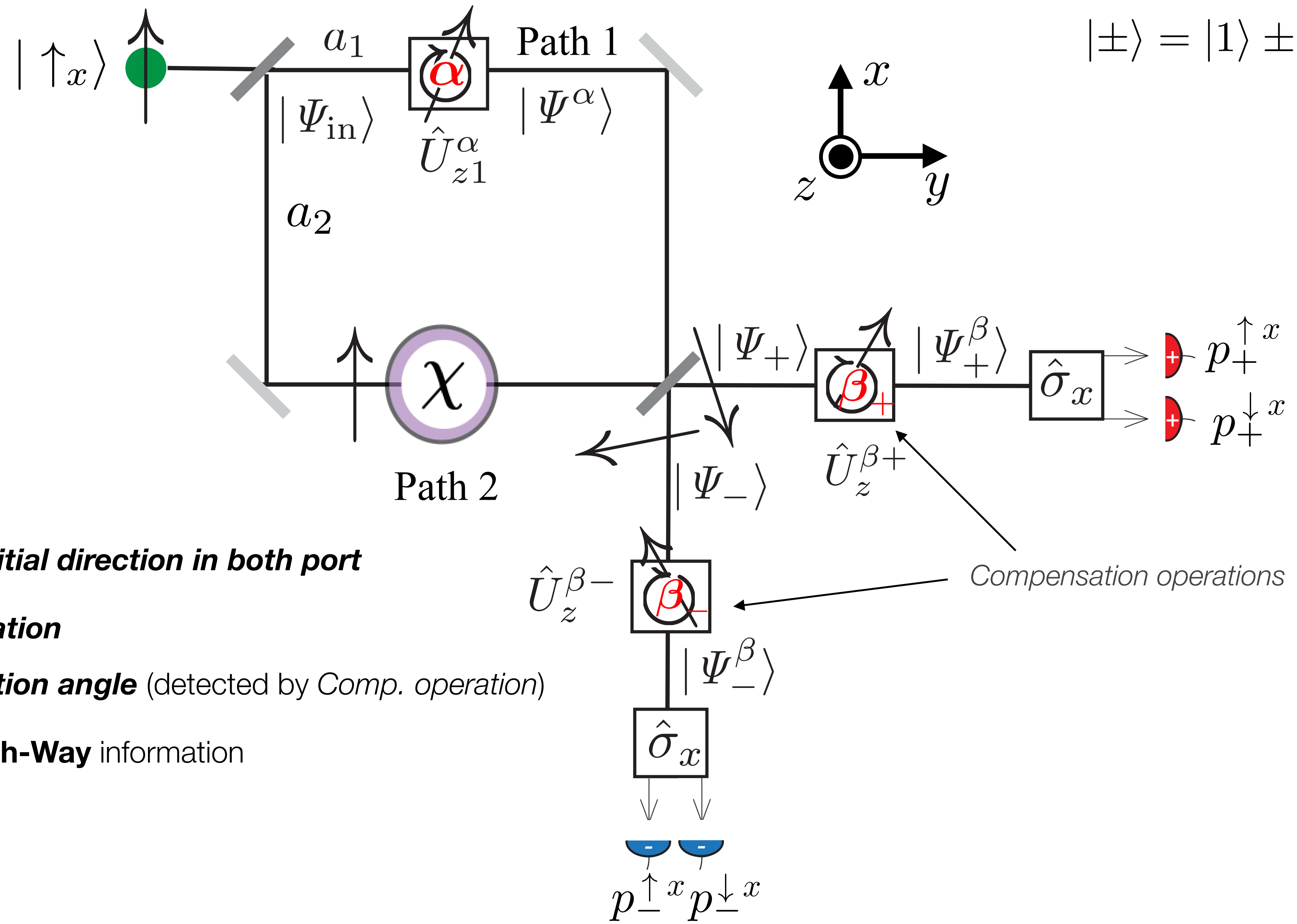

- ***spin is rotated back to initial direction in both port***
  by a ***Compensation operation***
- The accumulated ***spin rotation angle*** (detected by *Comp. operation*)
  must be related to the **Which-Way** information

H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Interference VS Which-Way Measurements: ***path presence***

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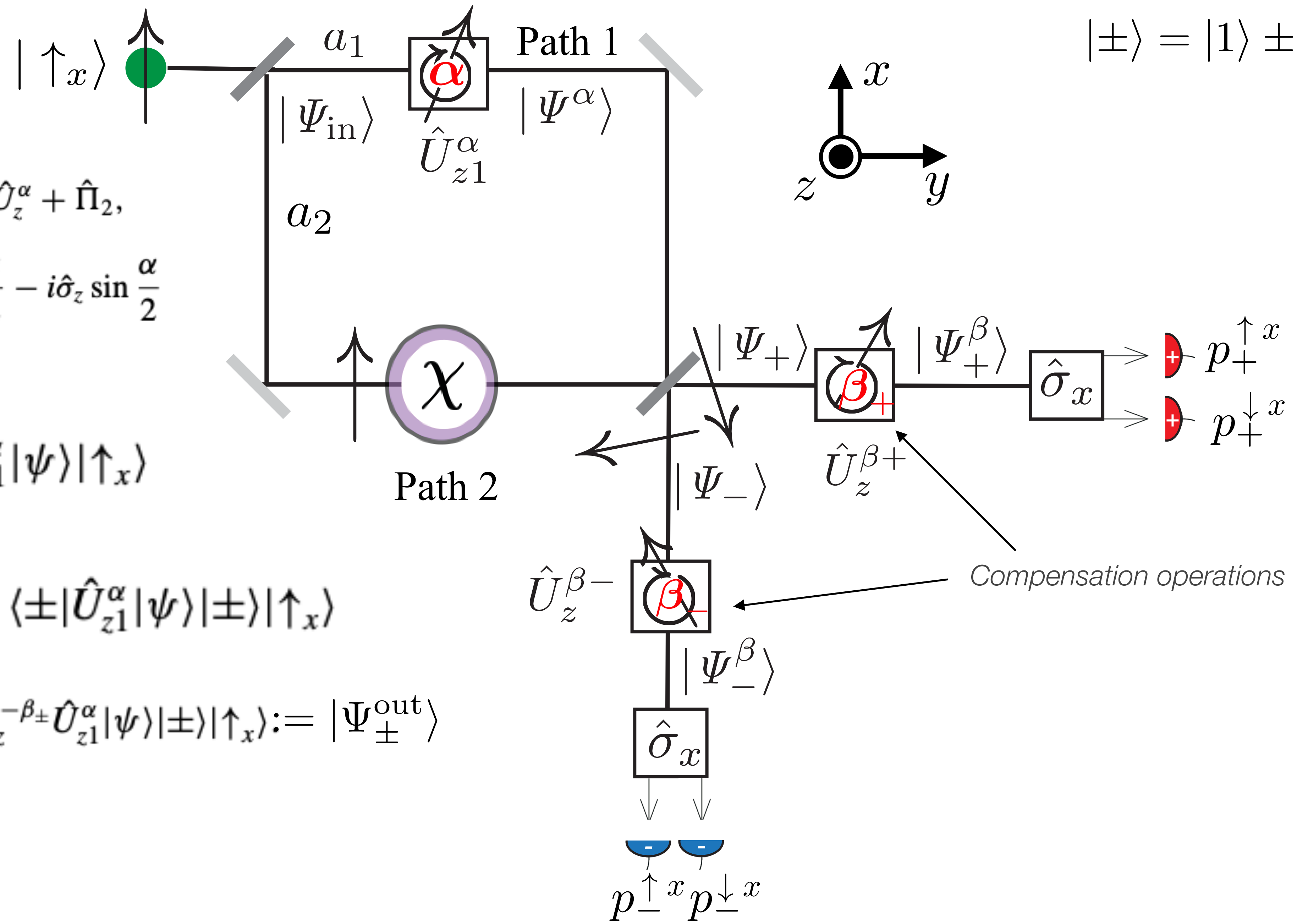
$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = (a_1|1\rangle + e^{i\chi}a_2|2\rangle) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\text{out}}\rangle = \langle \pm|\psi\rangle\, |\pm\rangle |S_{\pm}\rangle$$

$$|S_{\pm}\rangle = s_{\pm}^{\uparrow x}|\uparrow_x\rangle + s_{\pm}^{\downarrow x}|\downarrow_x\rangle$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$

$$\hat{U}_{z1}^{\alpha} = \exp\left(-\frac{i}{2}\alpha\,\hat{\sigma}_z\,\hat{\Pi}_1\right) = \hat{\Pi}_1\hat{U}_z^{\alpha} + \hat{\Pi}_2,$$

$$\hat{U}_z^{\alpha} = \exp\left(-\frac{i}{2}\alpha\,\hat{\sigma}_z\right) = \hat{\mathbb{1}}\cos\frac{\alpha}{2} - i\hat{\sigma}_z\sin\frac{\alpha}{2}$$



$$|\Psi^{\alpha}\rangle = \hat{U}_{z1}^{\alpha}|\Psi_{\text{in}}\rangle = \hat{U}_{z1}^{\alpha}|\psi\rangle|\uparrow_x\rangle$$

$$|\Psi_{\pm}\rangle = |\pm\rangle\langle\pm|\Psi^{\alpha}\rangle = \langle\pm|\hat{U}_{z1}^{\alpha}|\psi\rangle|\pm\rangle|\uparrow_x\rangle$$

$$|\Psi_{\pm}^{\beta}\rangle = \hat{U}_z^{-\beta_{\pm}}|\Psi_{\pm}\rangle = \langle\pm|\hat{U}_z^{-\beta_{\pm}}\hat{U}_{z1}^{\alpha}|\psi\rangle|\pm\rangle|\uparrow_x\rangle := |\Psi_{\pm}^{\text{out}}\rangle$$

H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Interference VS Which-Way Measurements

$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\mathrm{i}\chi} a_2|2\rangle\right) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\text{out}}\rangle = \langle \pm|\psi\rangle\, |\pm\rangle |S_\pm\rangle$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$

$$|S_\pm\rangle = s_\pm^{\uparrow x}|\uparrow_x\rangle + s_\pm^{\downarrow x}|\downarrow_x\rangle$$

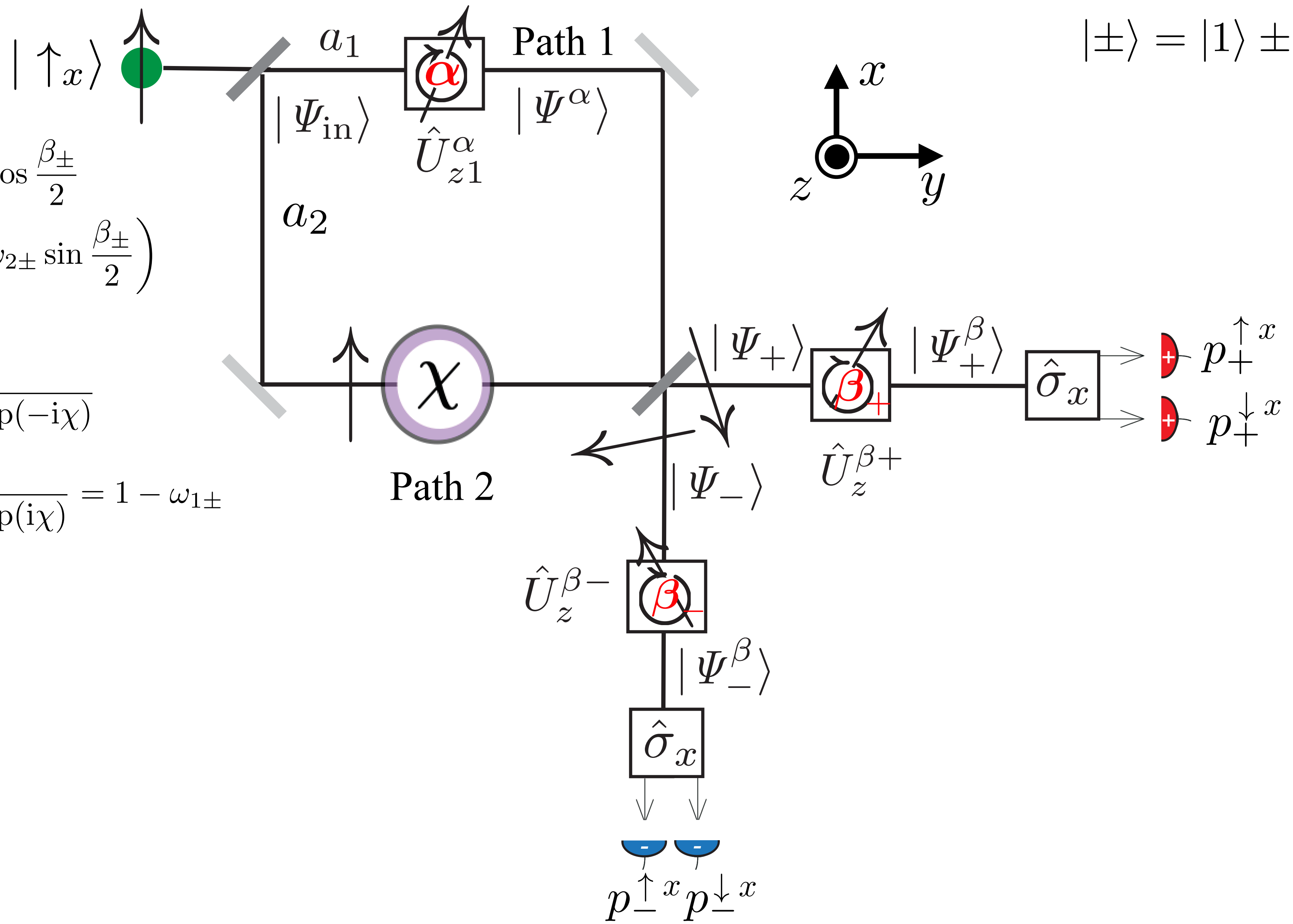
$$s_\pm^{\uparrow x} = \omega_{1\pm} \cos\frac{\alpha - \beta_\pm}{2} + \omega_{2\pm} \cos\frac{\beta_\pm}{2}$$

$$s_\pm^{\downarrow x} = \mathrm{i}\left(-\omega_{1\pm} \sin\frac{\alpha - \beta_\pm}{2} + \omega_{2\pm} \sin\frac{\beta_\pm}{2}\right)$$

Path weak values:

$$\omega_{1\pm} = \frac{\langle \pm|\hat{\Pi}_1|\psi\rangle}{\langle \pm|\psi\rangle} = \frac{1}{1 \pm \frac{a_2}{a_1}\exp(-\mathrm{i}\chi)}$$

$$\omega_{2\pm} = \frac{\langle \pm|\hat{\Pi}_2|\psi\rangle}{\langle \pm|\psi\rangle} = \frac{1}{1 \pm \frac{a_1}{a_2}\exp(\mathrm{i}\chi)} = 1 - \omega_{1\pm}$$



H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Interference VS Which-Way Measurements

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$$|\Psi_{\mathrm{in}}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\mathrm{i}\chi} a_2|2\rangle\right) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\mathrm{out}}\rangle = \langle \pm|\psi\rangle\, |\pm\rangle |S_{\pm}\rangle$$

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$$|S_{\pm}\rangle = s_{\pm}^{\uparrow x}|\uparrow_x\rangle + s_{\pm}^{\downarrow x}|\downarrow_x\rangle$$

$$s_{\pm}^{\uparrow x} = \omega_{1\pm}\cos\frac{\alpha-\beta_{\pm}}{2} + \omega_{2\pm}\cos\frac{\beta_{\pm}}{2}$$

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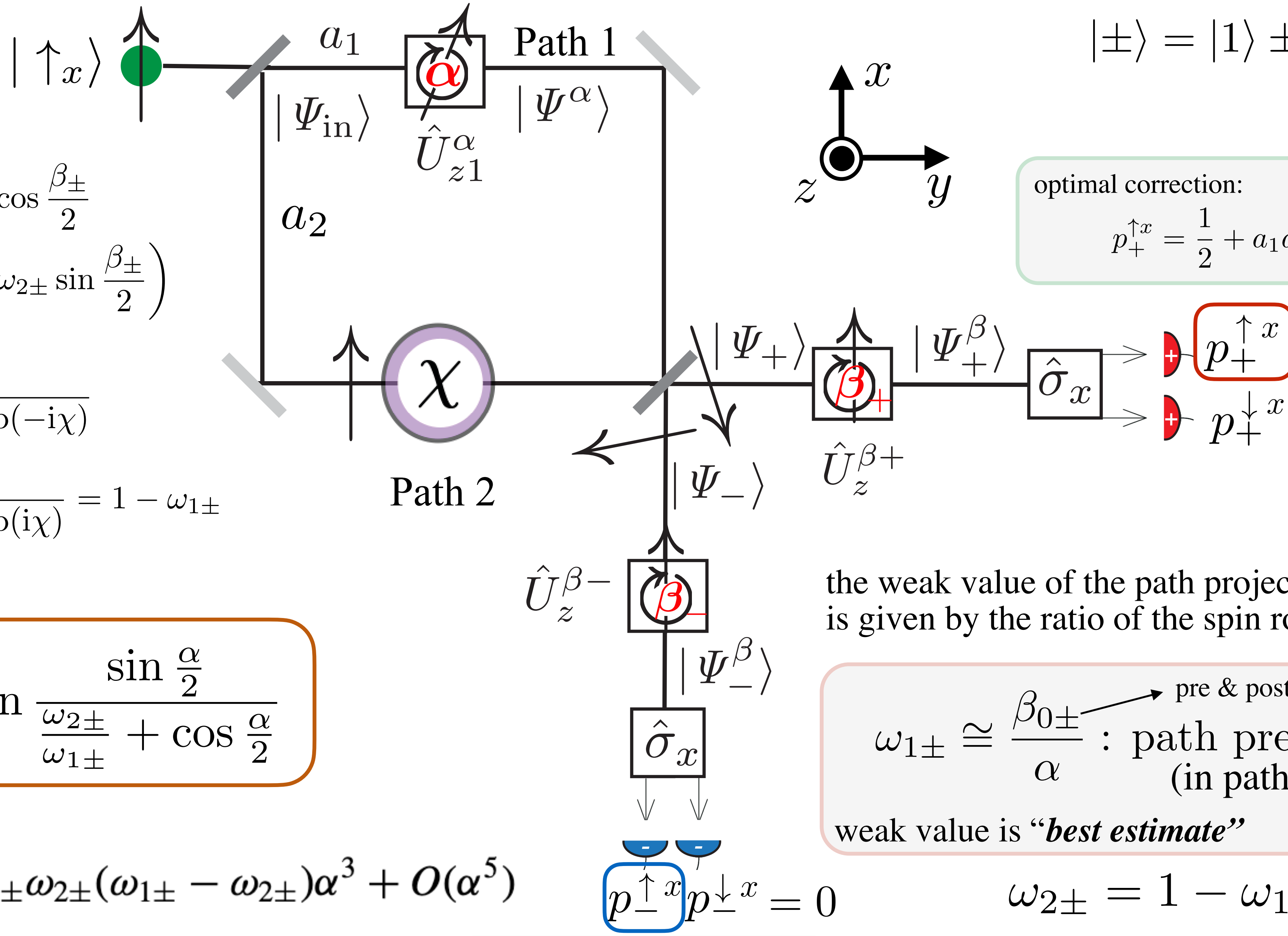


optimal correction:

$$p_+^{\uparrow x} = \frac{1}{2} + a_1 a_2 \cos\chi$$

optimal correction $\quad s_{\pm}^{\downarrow x} = 0$

for all α

$$\beta \to \beta_{0\pm} = 2\arctan\frac{\sin\frac{\alpha}{2}}{\frac{\omega_{2\pm}}{\omega_{1\pm}} + \cos\frac{\alpha}{2}}$$

"post-selection dependent"

for small α

$$\beta_{0\pm} = \omega_{1\pm}\alpha + \frac{1}{24}\omega_{1\pm}\omega_{2\pm}(\omega_{1\pm} - \omega_{2\pm})\alpha^3 + O(\alpha^5)$$

$$p_-^{\uparrow x} = \frac{1}{2} - a_1 a_2 \cos\chi$$

the weak value of the path projection operator is given by the ratio of the spin rotations:

$$\omega_{1\pm} \cong \frac{\beta_{0\pm}}{\alpha} : \text{ path presence (in path 1)}$$

pre & post selection

weak value is "***best estimate***"

$$\omega_{2\pm} = 1 - \omega_{1\pm}$$

$$\beta_{0+} \neq \beta_{0-}$$

H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Interference VS Which-Way Measurements

$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = (a_1|1\rangle + e^{i\chi}a_2|2\rangle) \otimes |\uparrow_x\rangle \qquad |\Psi^{\text{out}}\rangle = \langle\pm|\psi\rangle\,|\pm\rangle|S_\pm\rangle$$

$|S_\pm\rangle = s_\pm^{\uparrow x}|\uparrow_x\rangle$

$s_\pm^{\uparrow x} = \omega_{1\pm}\cos\frac{\alpha}{}$

$s_\pm^{\downarrow x} = \mathrm{i}\left(-\omega_{1\pm}\,\mathrm{s}\right.$

weak values:

$$\omega_{1\pm} = \frac{\langle\pm|\hat{\Pi}_1|\psi\rangle}{\langle\pm|\psi\rangle}$$

$$\omega_{2\pm} = \frac{\langle\pm|\hat{\Pi}_2|\psi\rangle}{\langle\pm|\psi\rangle}$$

optimal correction

for all α

$\beta \rightarrow \beta_{0\pm}$

"post-selection

for small α

$$\beta_{0\pm} = \omega_{1\pm}\alpha + \frac{}{24}\omega_{1\pm}\omega_{2\pm}(\omega_{1\pm} - \omega_{2\pm})\alpha + O(\alpha)$$

"Quantum -

Processing "

Workflow

PHYSICAL REVIEW RESEARCH **3**, L012011 (2021)

Letter

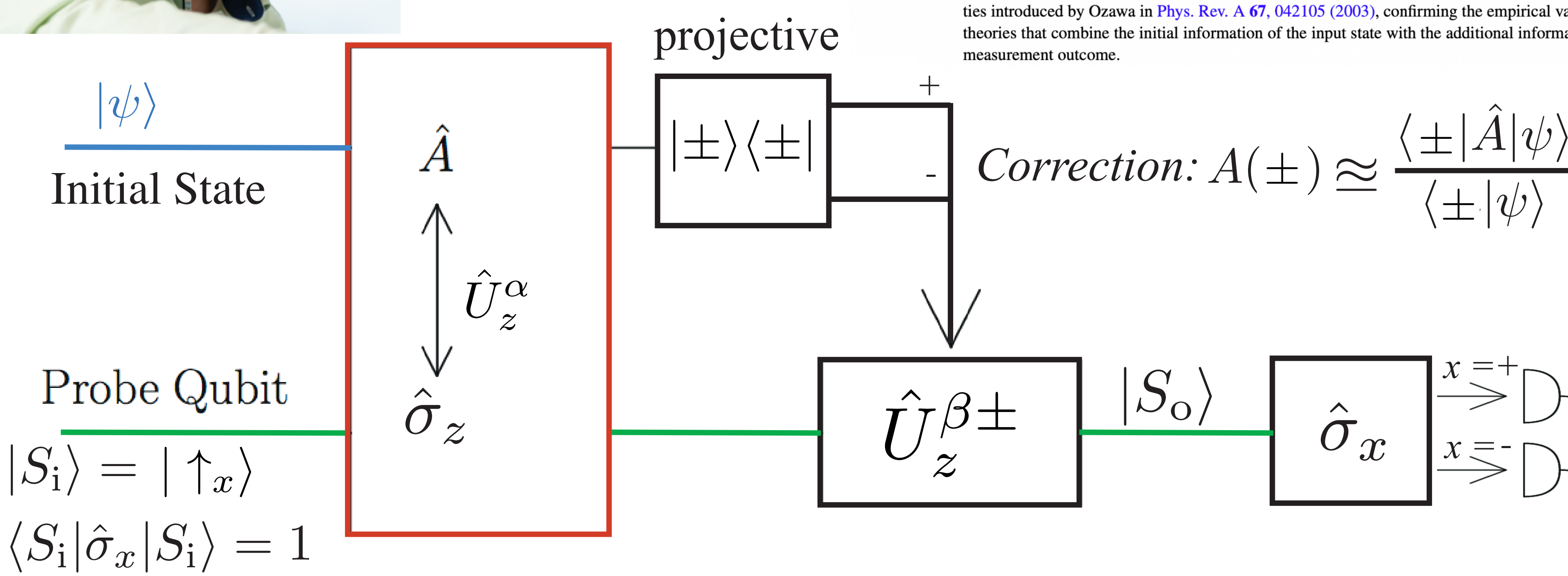
Direct evaluation of measurement uncertainties by feedback compensation of decoherence

Holger F. Hofmann

Graduate School of Advanced Science and Engineering, Hiroshima University, Kagamiyama 1-3-1, Higashi Hiroshima 739-8530, Japan

(Received 23 June 2020; revised 15 November 2020; accepted 25 January 2021; published 3 February 2021)

It is shown that measurement uncertainties can be observed directly by evaluating the feedback compensation of the decoherence induced by the measured system on a probe qubit in a weak interaction occurring between state preparation and measurement. The uncompensated decoherence is described by the measurement uncertainties introduced by Ozawa in Phys. Rev. A **67**, 042105 (2003), confirming the empirical validity of measurement theories that combine the initial information of the input state with the additional information provided by each measurement outcome.



H. F. Hofmann, *Phys. Rev. Research* **3**, L012011 (2021)

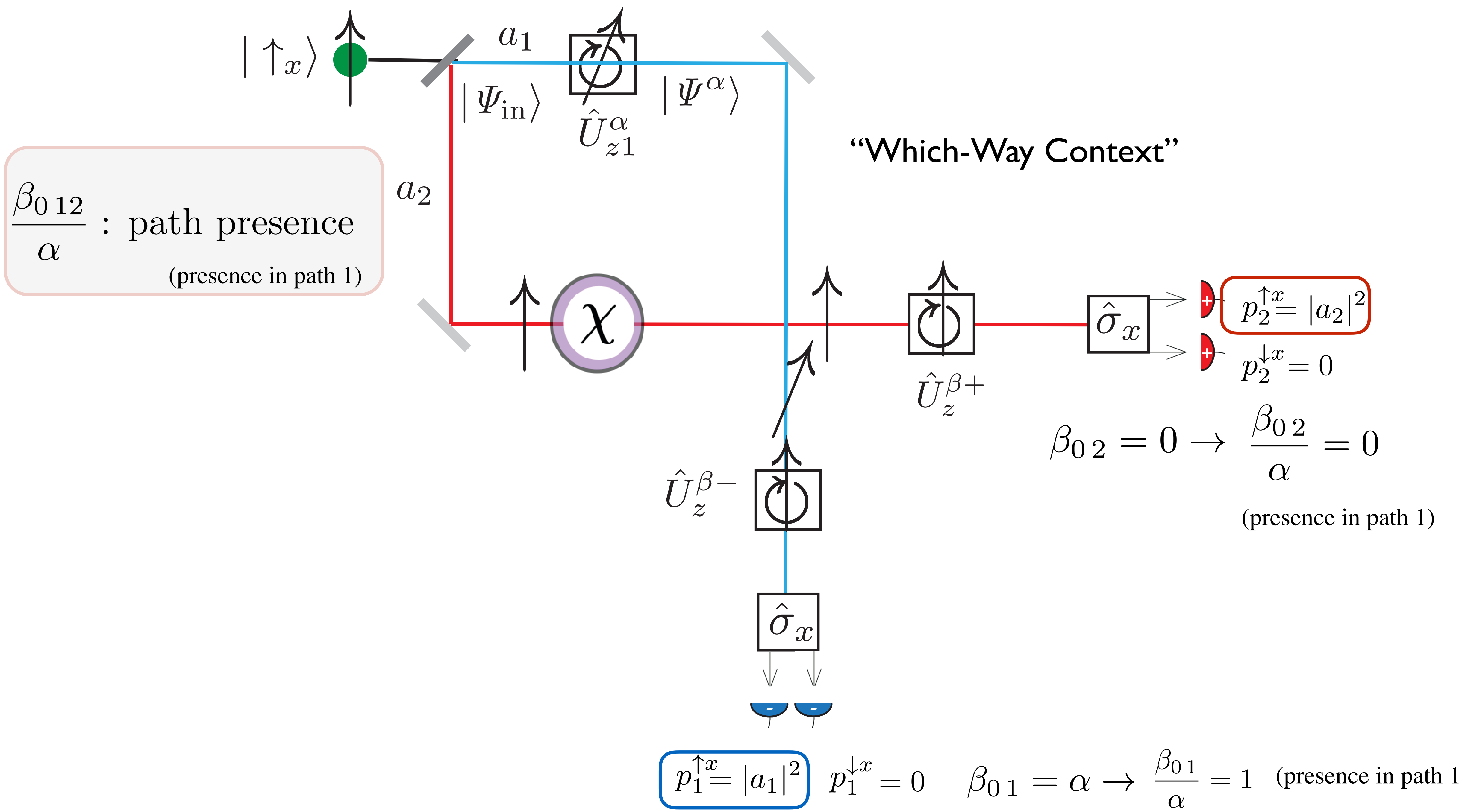
$p_-^{\uparrow x}\, p_-^{\downarrow x} = 0$ $\qquad \omega_{2\pm} = 1 - \omega_{1\pm}$

$p_-^{\uparrow x} = \frac{1}{2} - a_1 a_2 \cos\chi$ $\qquad \beta_{0+} \neq \beta_{0-}$

H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Interference VS Which-Way Measurements

$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\text{i}\chi}a_2|2\rangle\right) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\text{out}}\rangle = \langle\pm|\psi\rangle\,|\pm\rangle|S_{\pm}\rangle$$

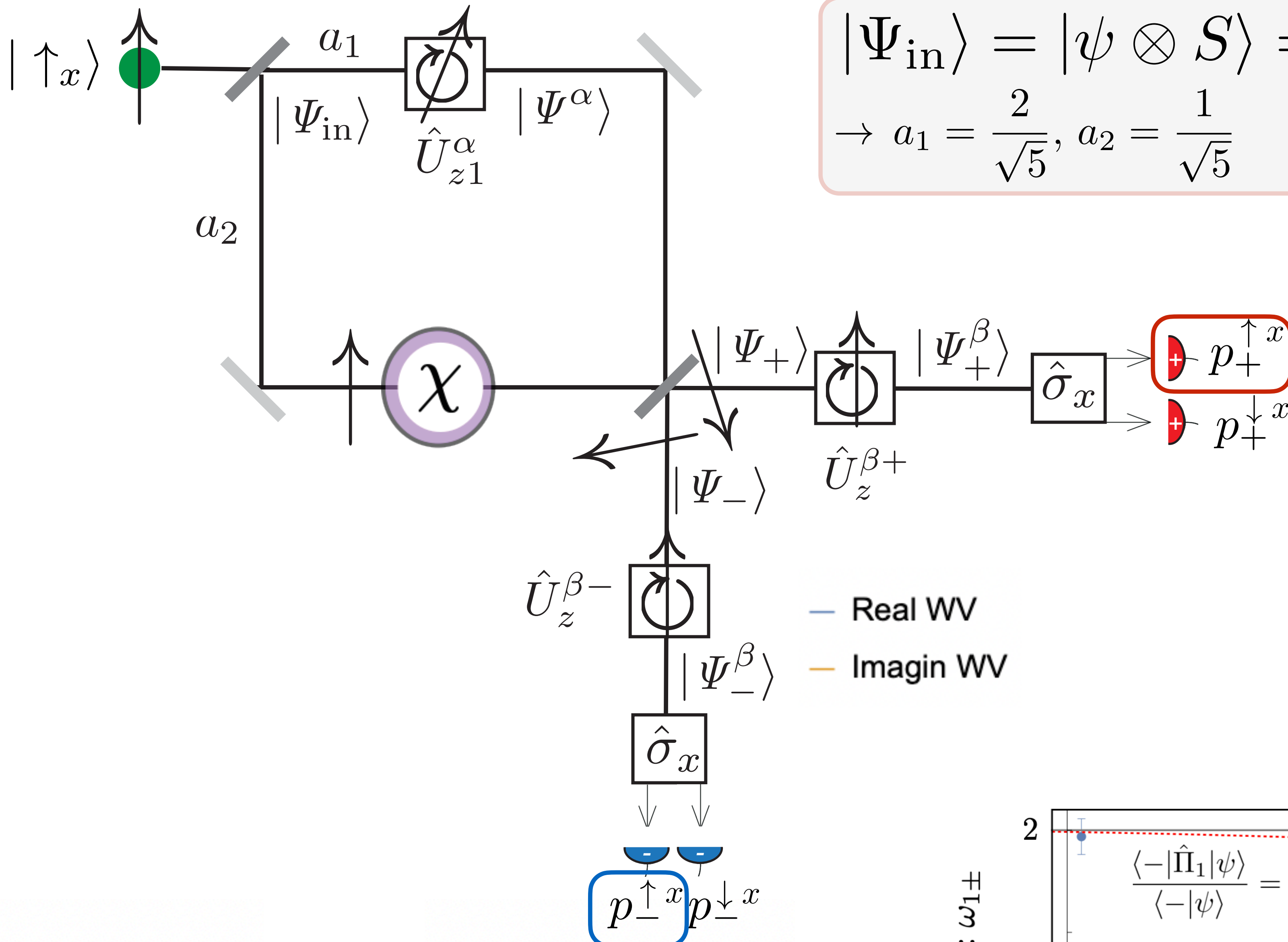


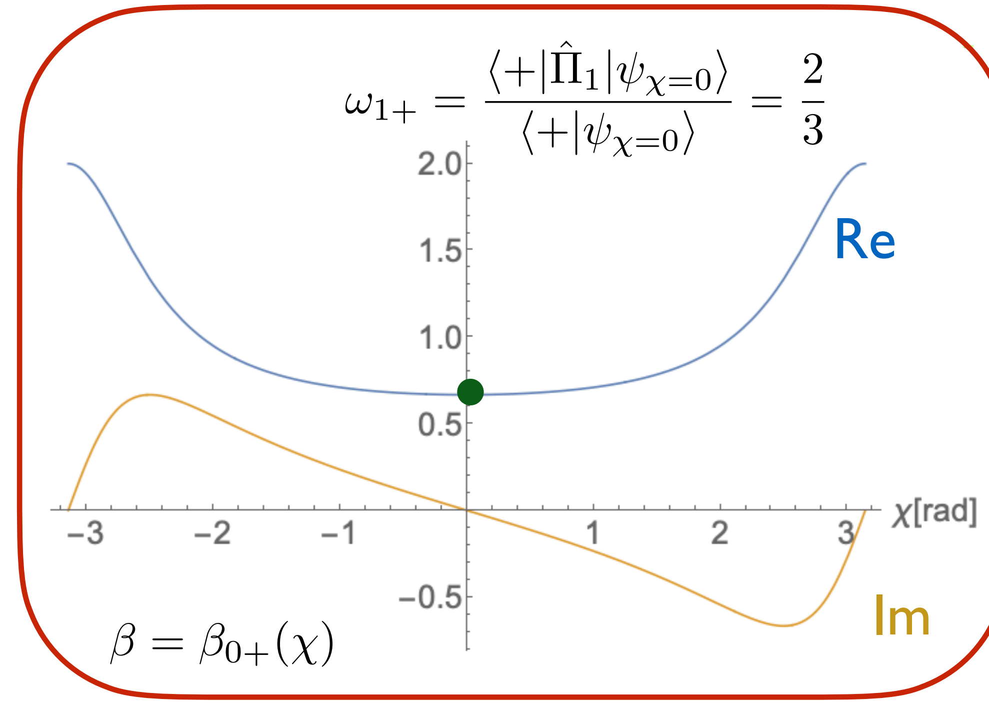
H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

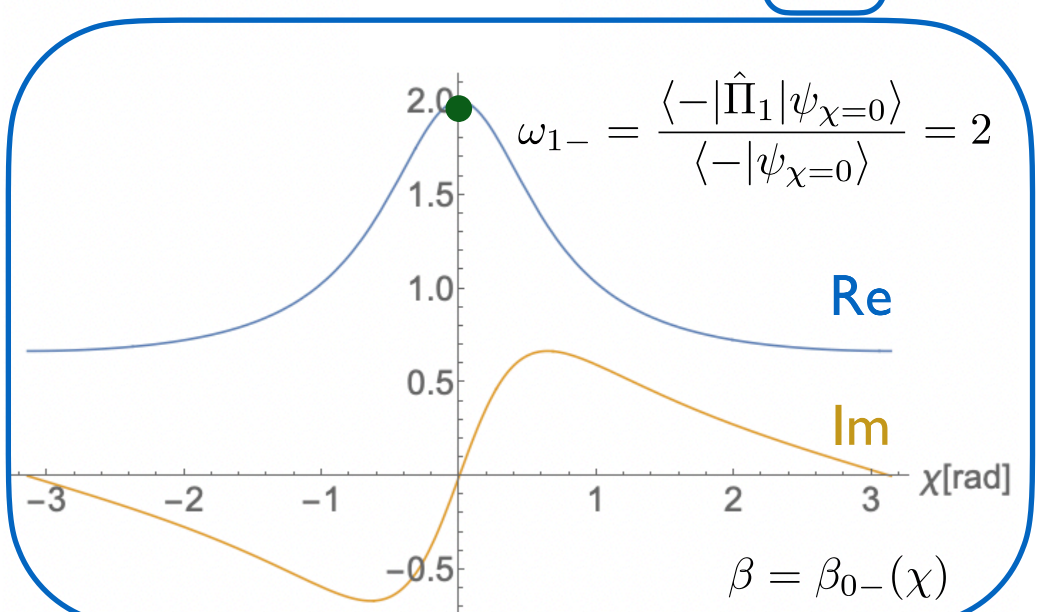
We have already measured …

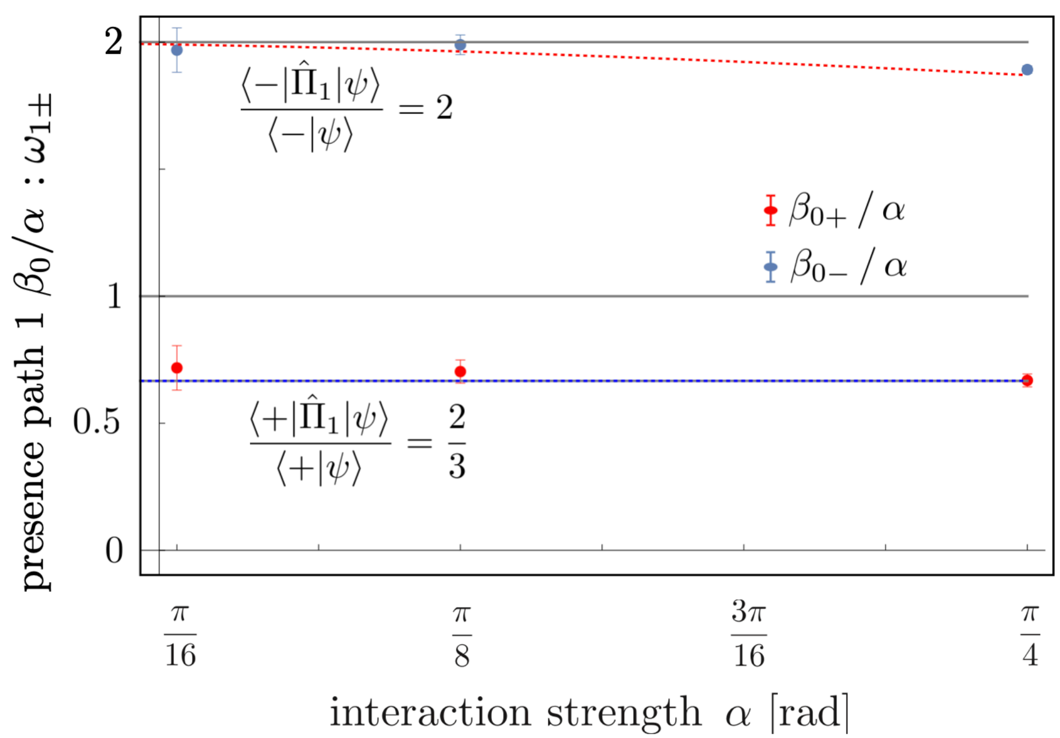
$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\mathrm{i}\chi}a_2|2\rangle\right) \otimes |\uparrow_x\rangle$$

$$\rightarrow a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}}$$









H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

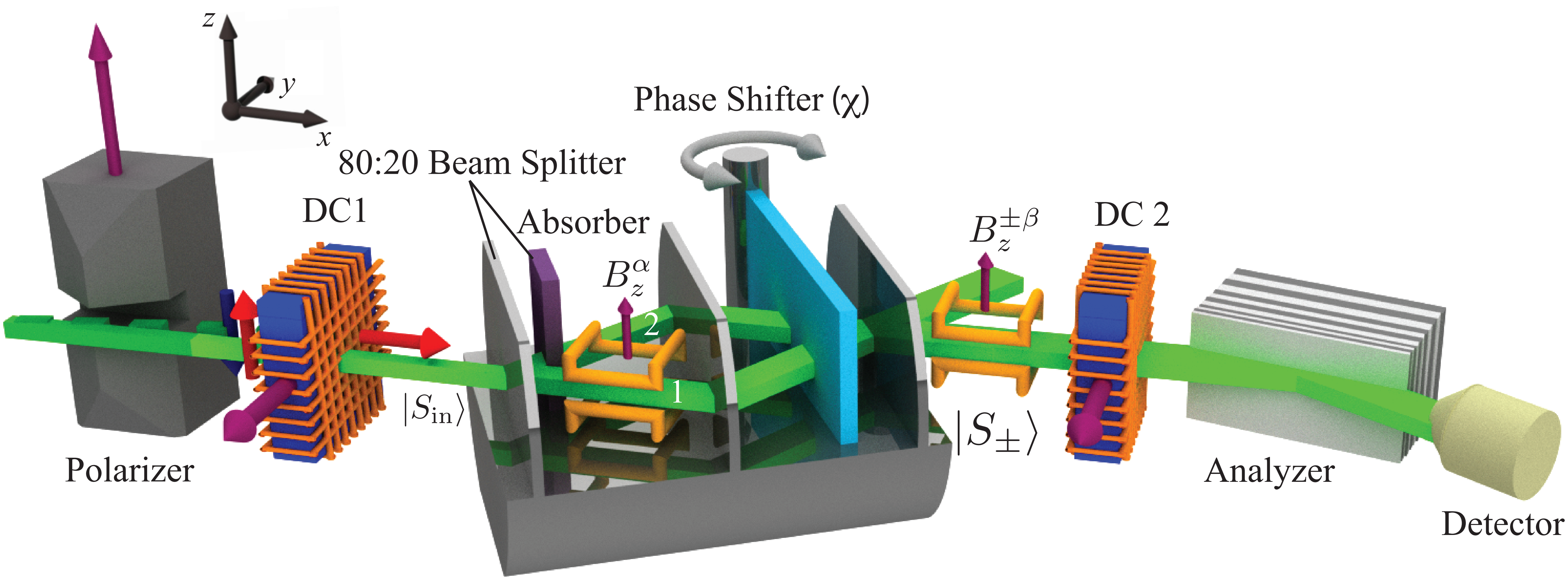
IFM Setup

$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle := \big(a_1|1\rangle + a_2|2\rangle\big)|\uparrow_x\rangle \qquad |\Psi_{\pm}^{\text{out}}\rangle = \langle\pm|\psi\rangle\,|\pm\rangle|S_{\pm}\rangle$$

$\rightarrow a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}}$

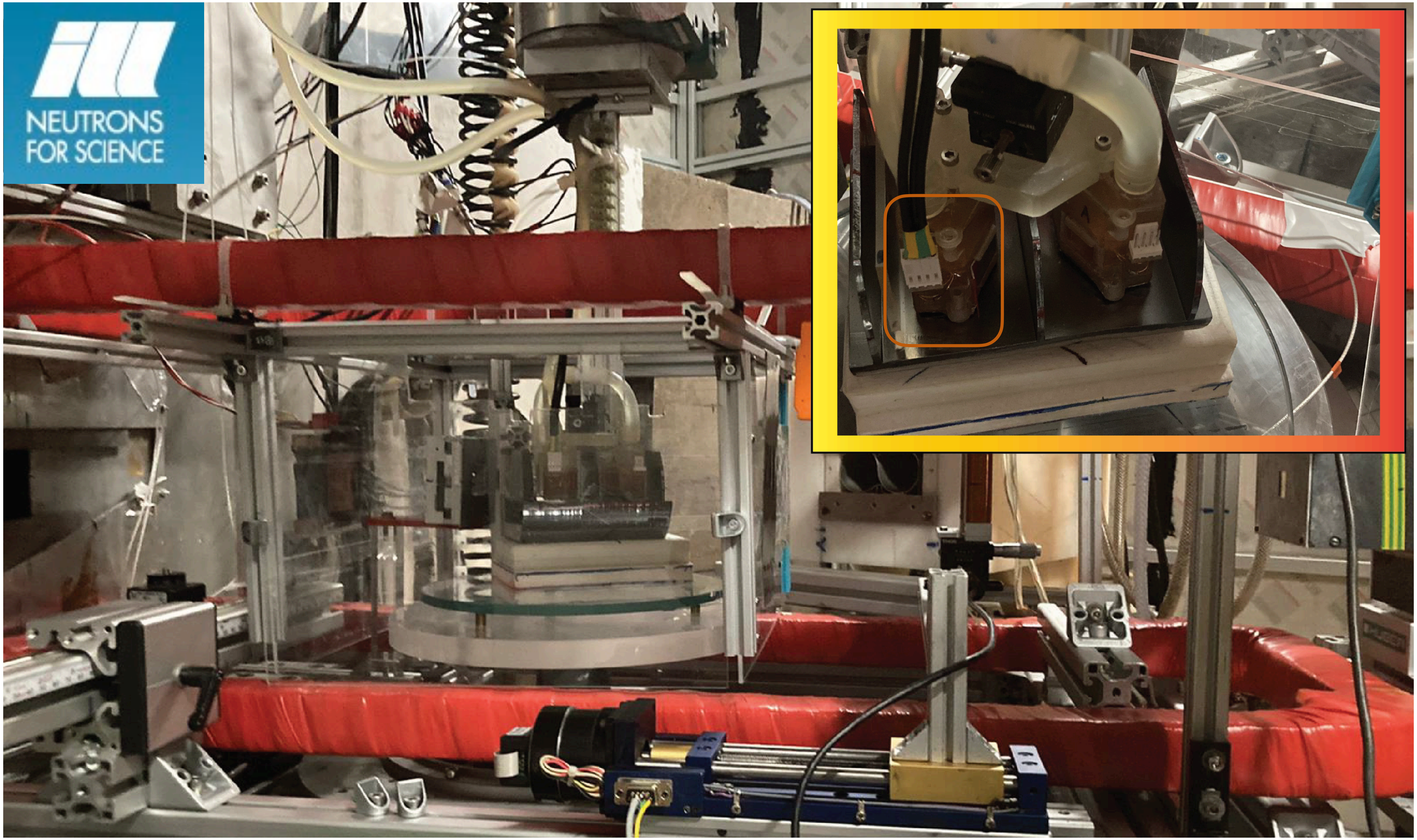
$|\pm\rangle = |1\rangle + e^{i\chi_{\pm}}|2\rangle$

$\chi_+ = 0,\ \chi_- = \pi$



H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

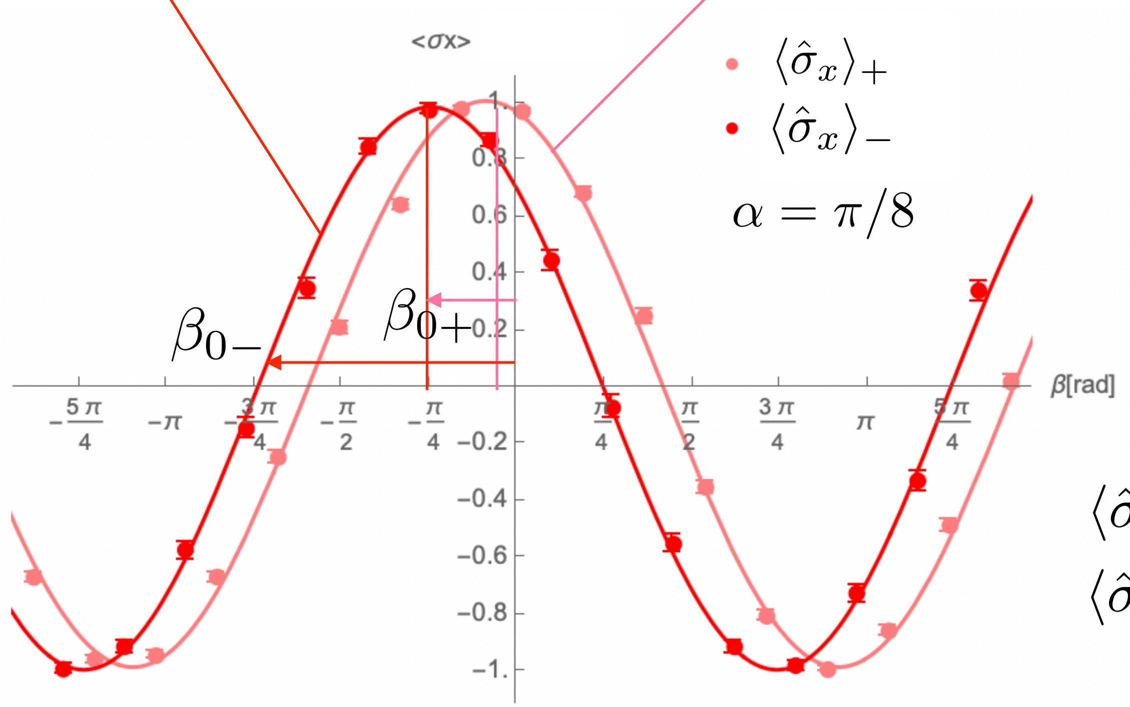
Experimental Setup @ S 18, ILL Grenoble



H. Lemmel, N. Geerits, A. Danner, H. F. Hofmann, and S. Sponar, *Physical Review Research* **4**, 023075 (2022)

Additional Notes on Ozawa-Hall Error: i) Values of Errors

$$\langle\hat{\sigma}_x\rangle_- = \frac{I^{\uparrow x}_{\chi=\pi} - I^{\downarrow x}_{\chi=\pi}}{I^{\uparrow x}_{\chi=\pi} + I^{\downarrow x}_{\chi=\pi}} \cdot C_{\mathrm{Corr}} \qquad \langle\hat{\sigma}_x\rangle_+ = \frac{I^{\uparrow x}_{\chi=0} - I^{\downarrow x}_{\chi=0}}{I^{\uparrow x}_{\chi=0} + I^{\downarrow x}_{\chi=0}} \cdot C_{\mathrm{Corr}}$$



best estimates(@ $\alpha = \pi/8$) :

$$\frac{\beta_{0+}}{\alpha} = 0.703(45)$$ [Theory: 2/3]

$$\frac{\beta_{0-}}{\alpha} = 1.989(38)$$ [Theory: 2]

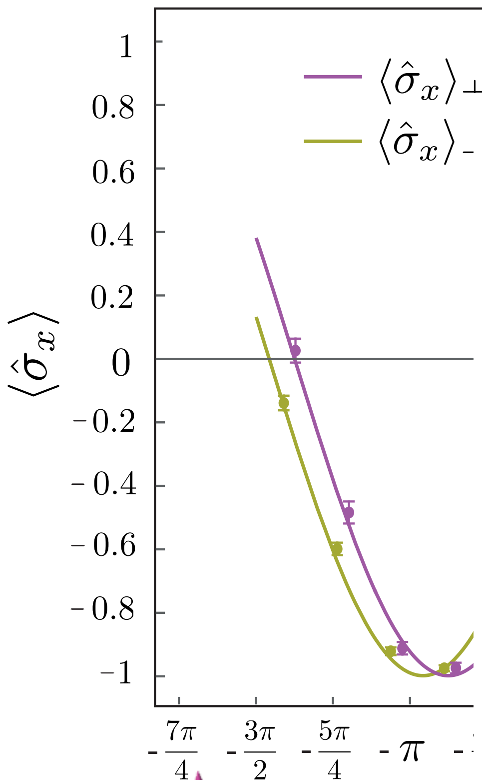
$$\langle\hat{\sigma}_x\rangle_+ = \cos(\beta_+ - \omega_{1+}\,\alpha)$$
$$\langle\hat{\sigma}_x\rangle_- = \cos(\beta_- - \omega_{1-}\,\alpha)$$

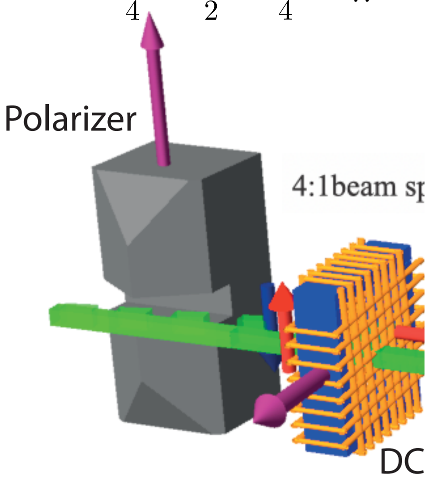
Interference Measurements

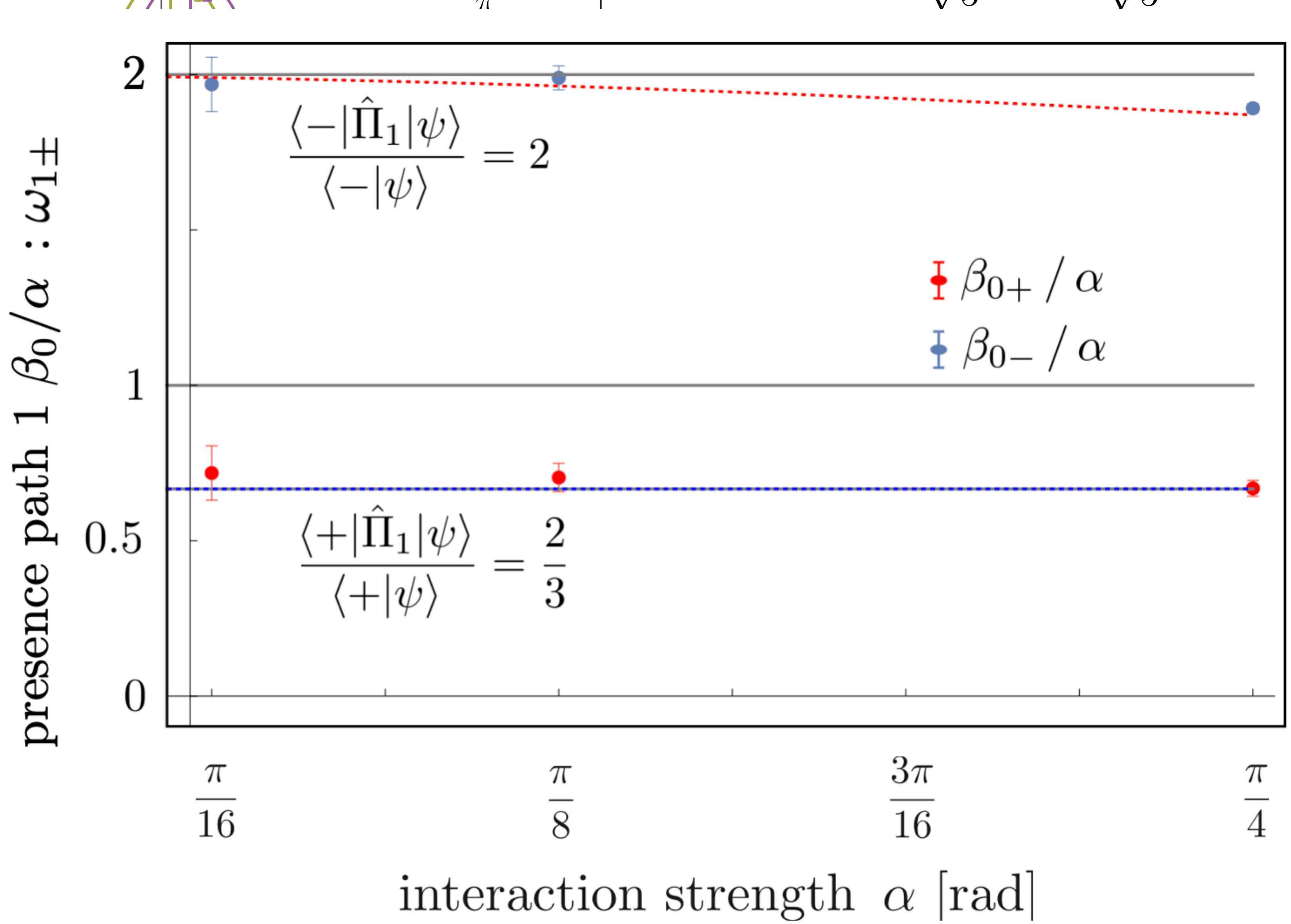
$$|\Psi_{\text{in}}\rangle = |\psi \otimes S\rangle = \big(a_1|1\rangle + a_2|2\rangle\big) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\text{out}}\rangle = \langle \pm|\psi\rangle\, |\pm\rangle |S_{\pm}\rangle$$

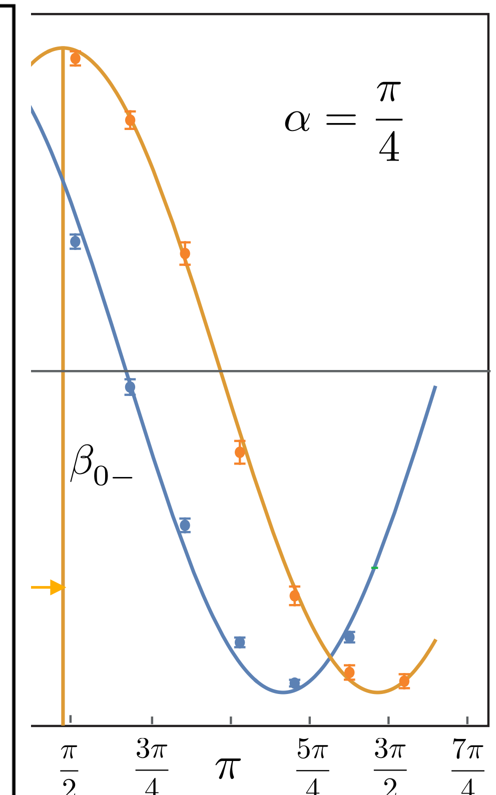
$$\rightarrow a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}} \qquad |\pm\rangle = |1\rangle + e^{\mathrm{i}\chi_\pm}|2\rangle$$

$$\chi_+ = 0,\ \chi_- = \pi$$









$$\ldots\mathrm{s}(\beta_+ - \omega_{1+}\,\alpha)$$
$$\langle\sigma_x\rangle_- = \cos(\beta_- - \omega_{1-}\,\alpha)$$

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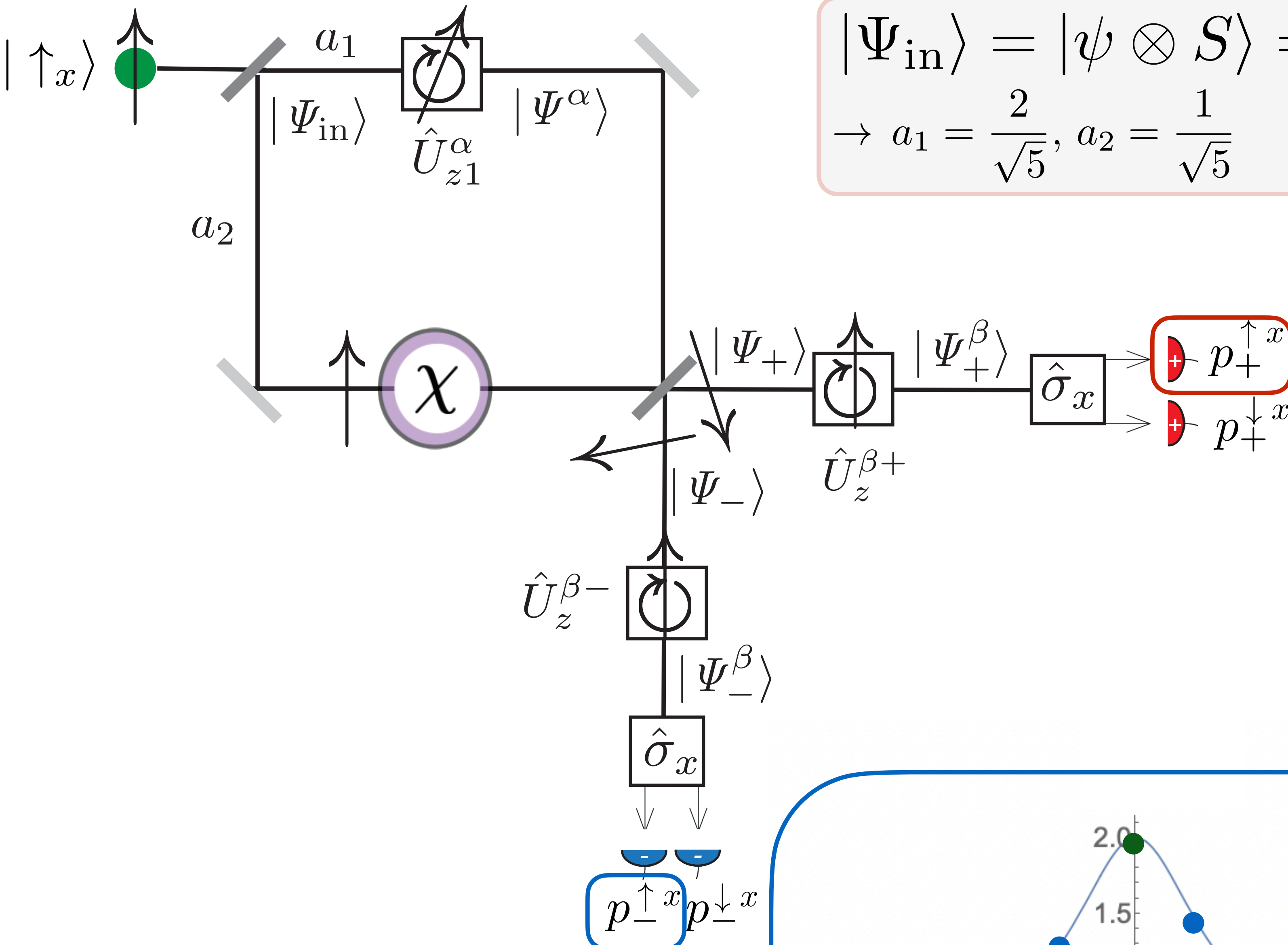
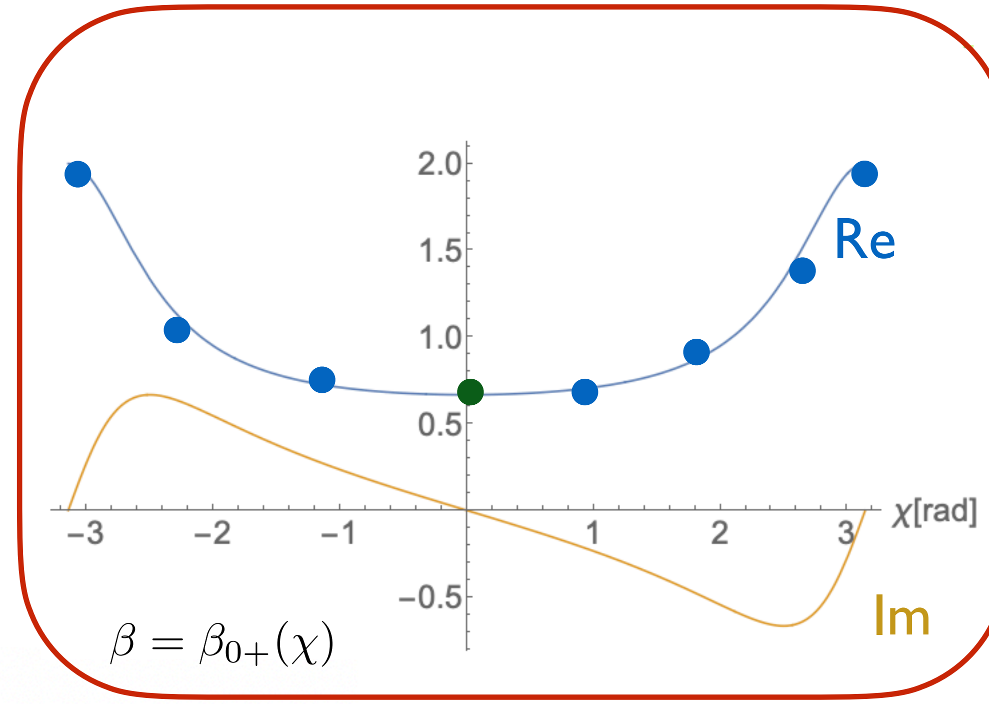
ATOMINSTITUT

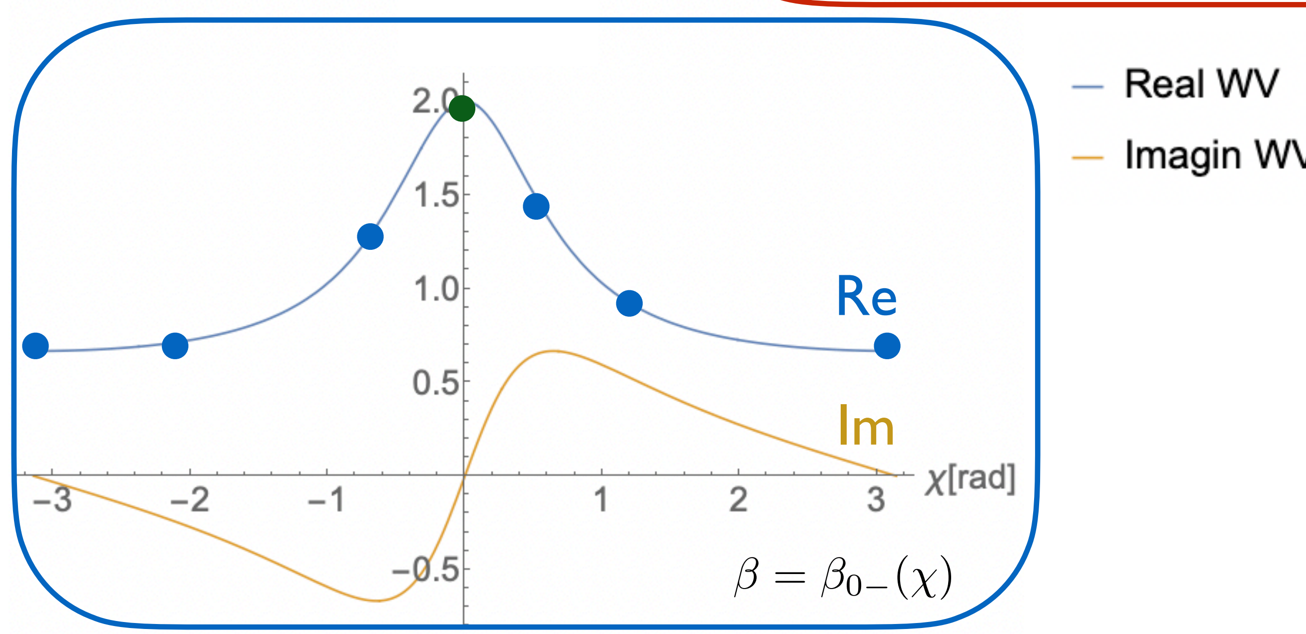
We shall measure …

$$|\Psi_{\rm in}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\mathrm{i}\chi} a_2|2\rangle\right) \otimes |\uparrow_x\rangle$$

$$\rightarrow\ a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}}$$

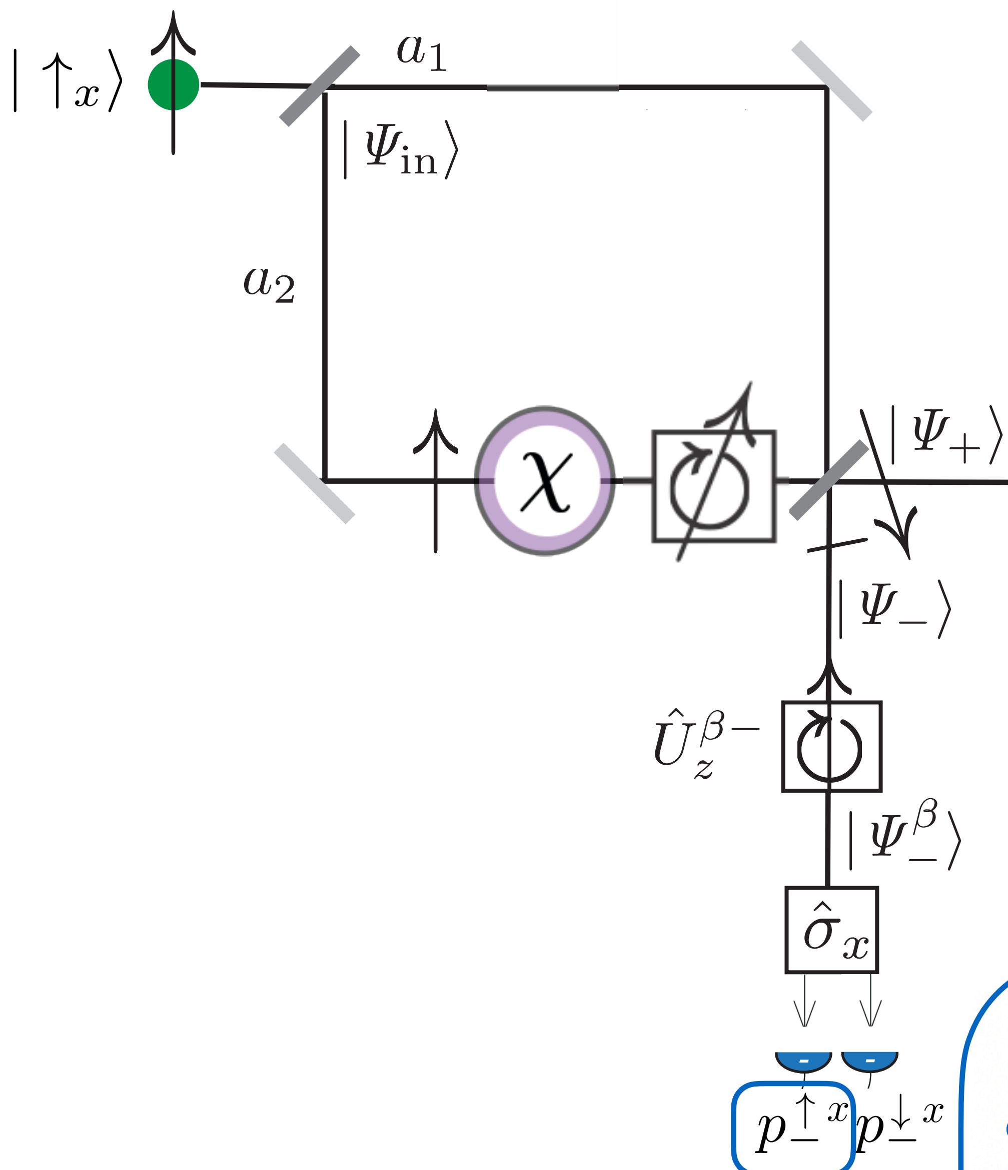


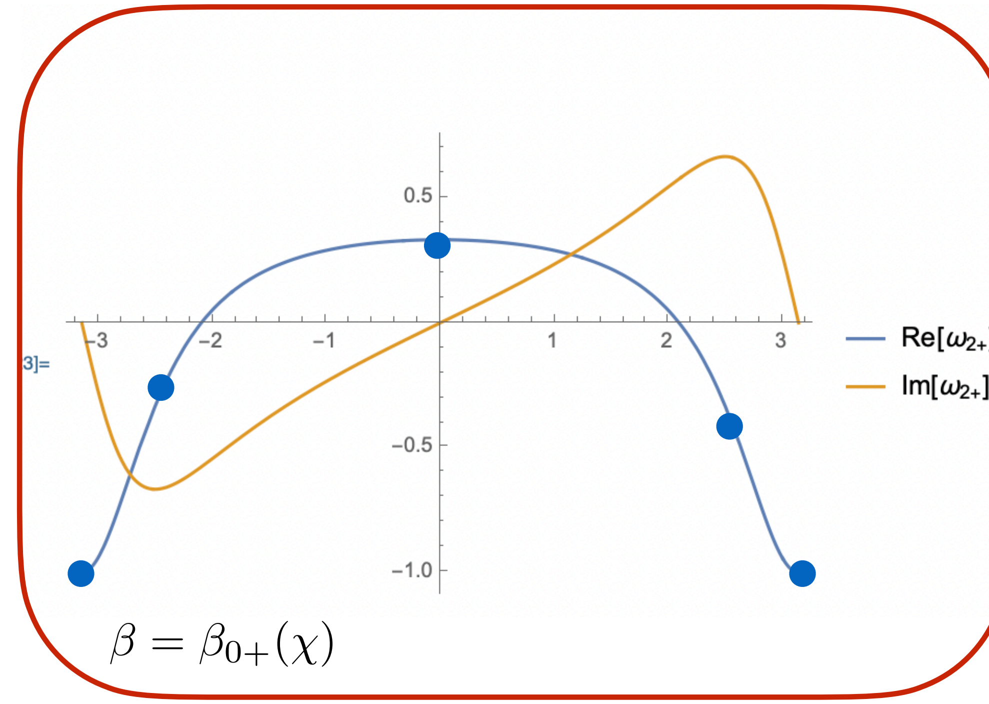


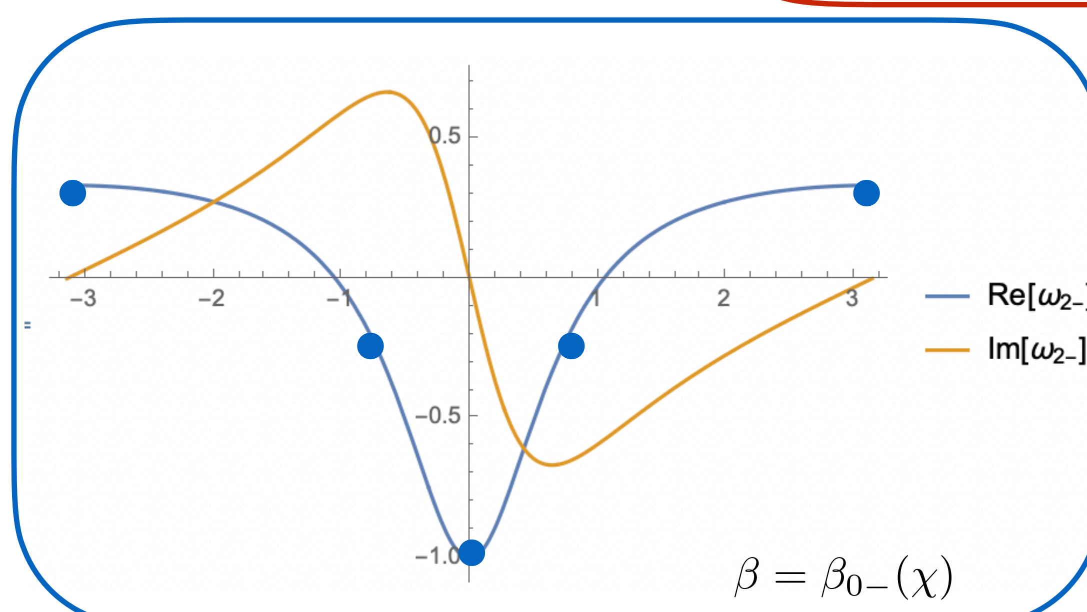


Additional Notes on Ozawa-Hall Error iii)Variation of initial state

$$|\Psi_{\rm in}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{i\chi}a_2|2\rangle\right) \otimes |\uparrow_x\rangle$$
$$\rightarrow a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}}$$







Real WV

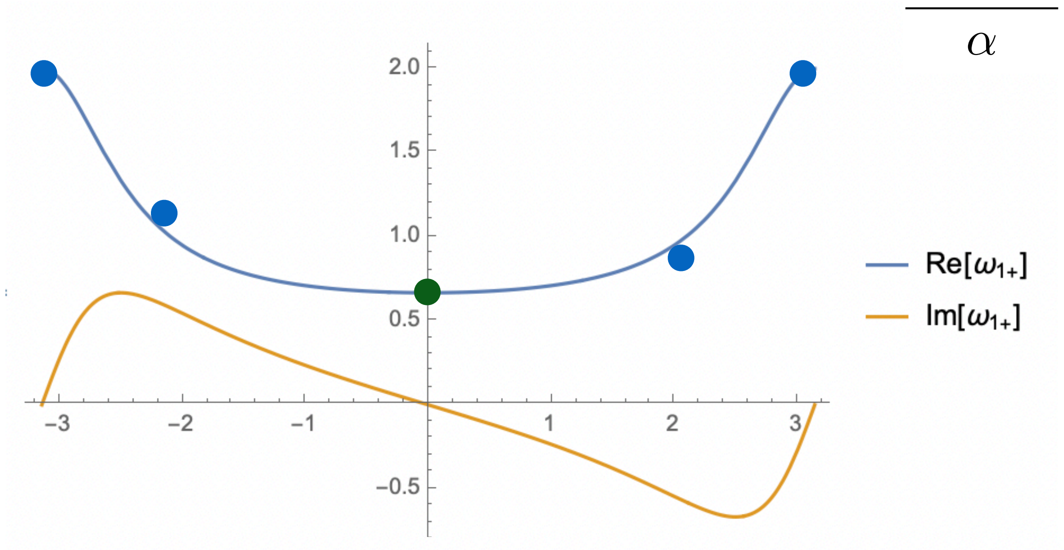
Imagin WV

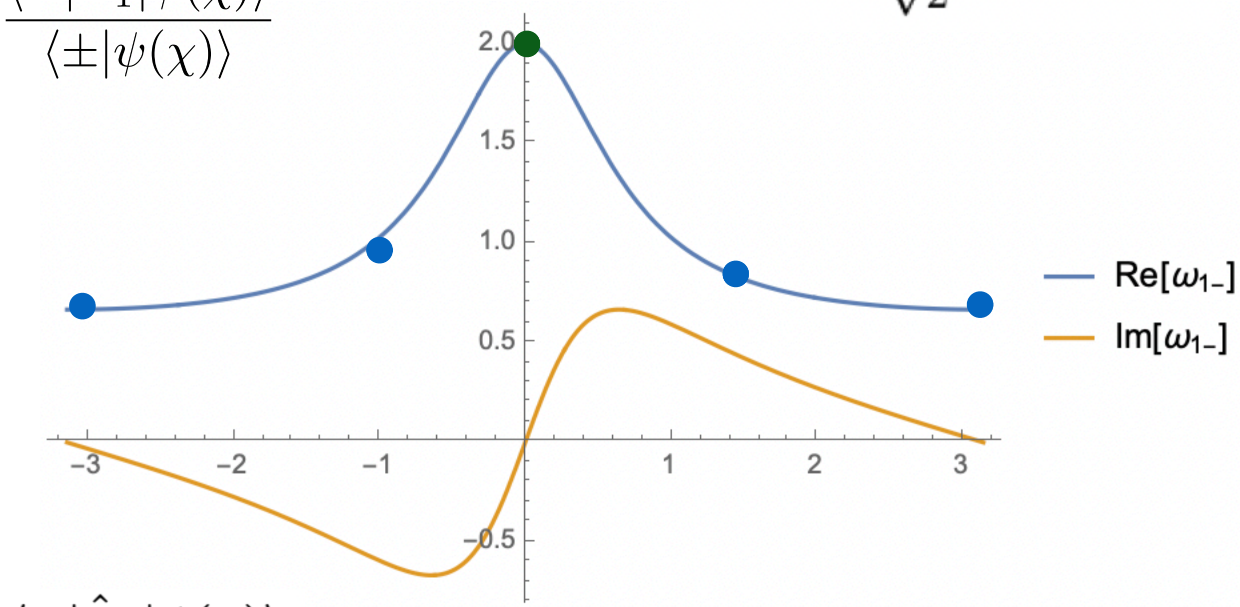
Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

- *path presence for* $|\psi\rangle := |\psi(\chi)\rangle = \frac{2}{\sqrt{5}}|1\rangle + e^{i\chi}\frac{1}{\sqrt{5}}|2\rangle$

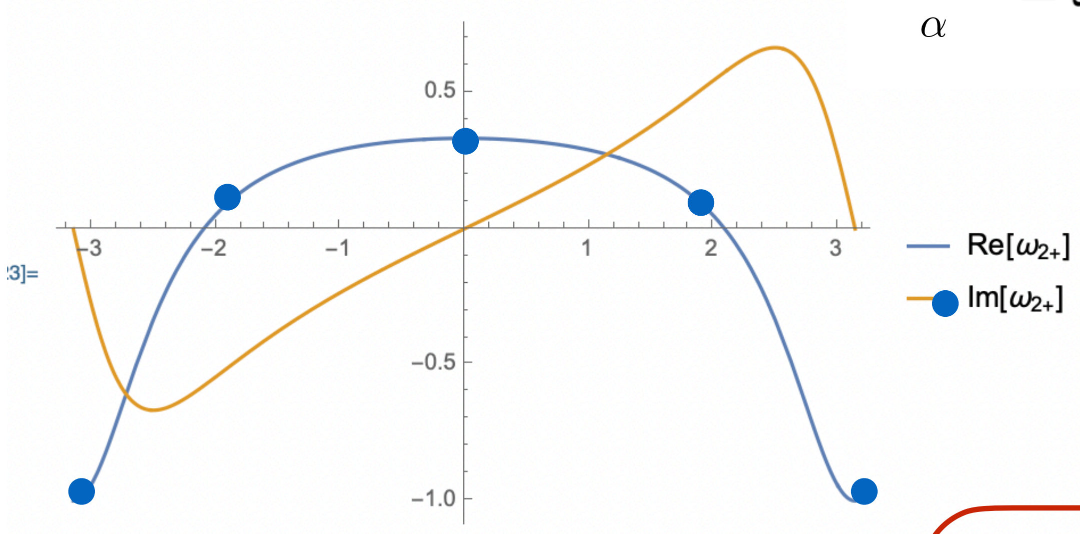
$$|+\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle),$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|1\rangle - |2\rangle).$$

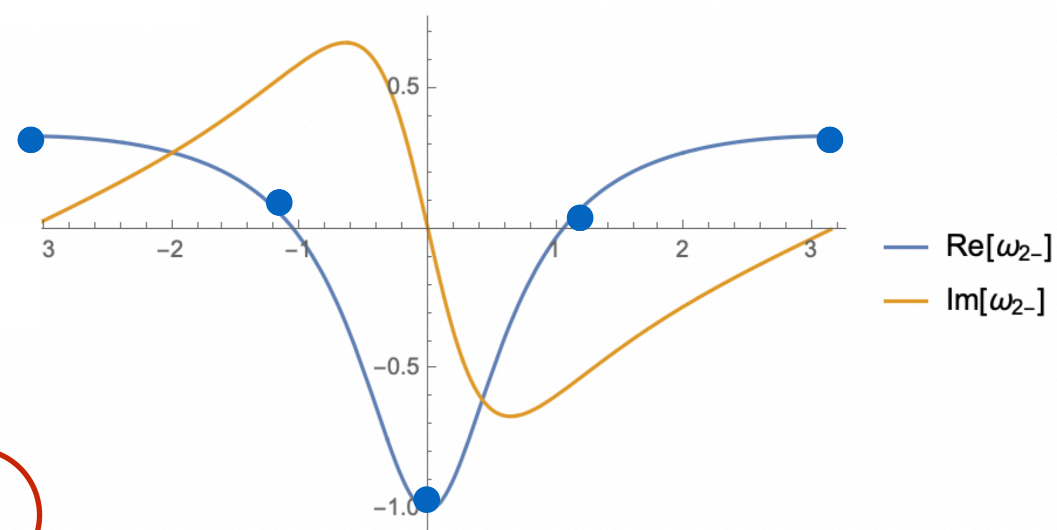
$$\frac{\beta^{(1)}_{0\pm}(\chi)}{\alpha} = \omega_{1\pm}(\chi) \cong \Re\frac{\langle\pm|\hat{\Pi}_1|\psi(\chi)\rangle}{\langle\pm|\psi(\chi)\rangle}$$



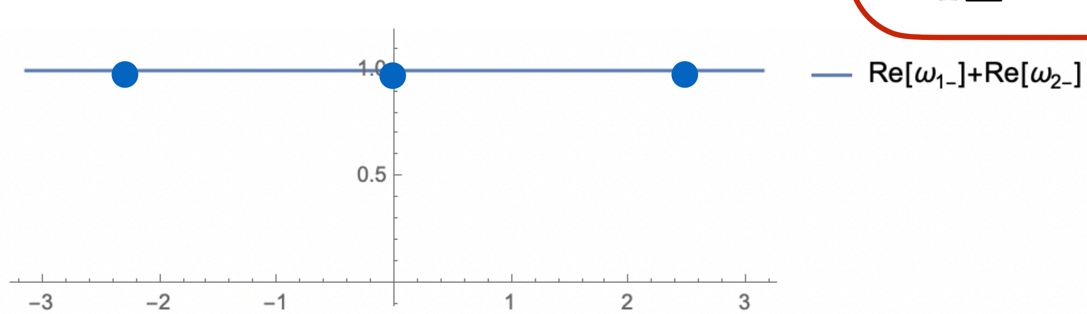


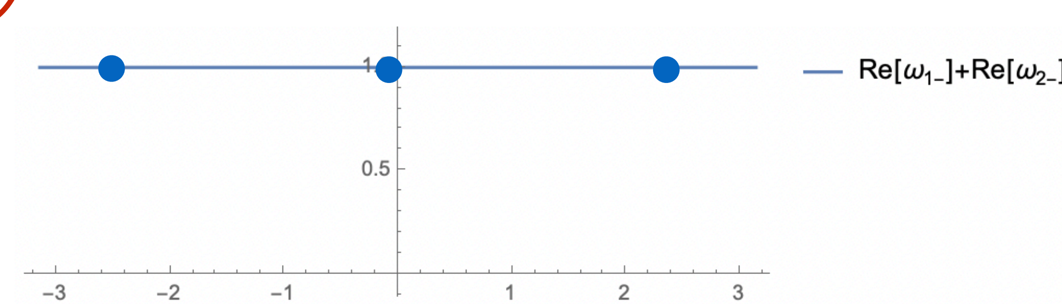
$$\frac{\beta^{(2)}_{0\pm}(\chi)}{\alpha} = \omega_{2\pm}(\chi) \cong \Re\frac{\langle\pm|\hat{\Pi}_2|\psi(\chi)\rangle}{\langle\pm|\psi(\chi)\rangle}$$





$$\boxed{\omega_{1\pm} + \omega_{2\pm} = 1}$$





Ozawa Error

PHYSICAL REVIEW A **67**, 042105 (2003)

Universally valid reformulation of the Heisenberg uncertainty principle on noise and disturbance in measurement

Masanao Ozawa
Graduate School of Information Sciences, Tôhoku University, Aoba-ku, Sendai, 980-8579, Japan
(Received 9 October 2002; published 11 April 2003)

The Heisenberg uncertainty principle states that the product of the noise in a position measurement and the momentum disturbance caused by that measurement should be no less than the limit set by Planck's constant $\hbar/2$ as demonstrated by Heisenberg's thought experiment using a γ-ray microscope. Here it is shown that this common assumption is not universally true: a universally valid trade-off relation between the noise and the disturbance has an additional correlation term, which is redundant when the intervention brought by the measurement is independent of the measured object, but which allows the noise-disturbance product much

Measurement Uncertainty:

$$\epsilon(A)\eta(B) + \epsilon(A)\Delta B + \eta(B)\Delta A \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|$$

- Definitions of ERROR $\epsilon(A)$ & DISTURBANCE $\eta(B)$

describing accuracy of quantum measuring process

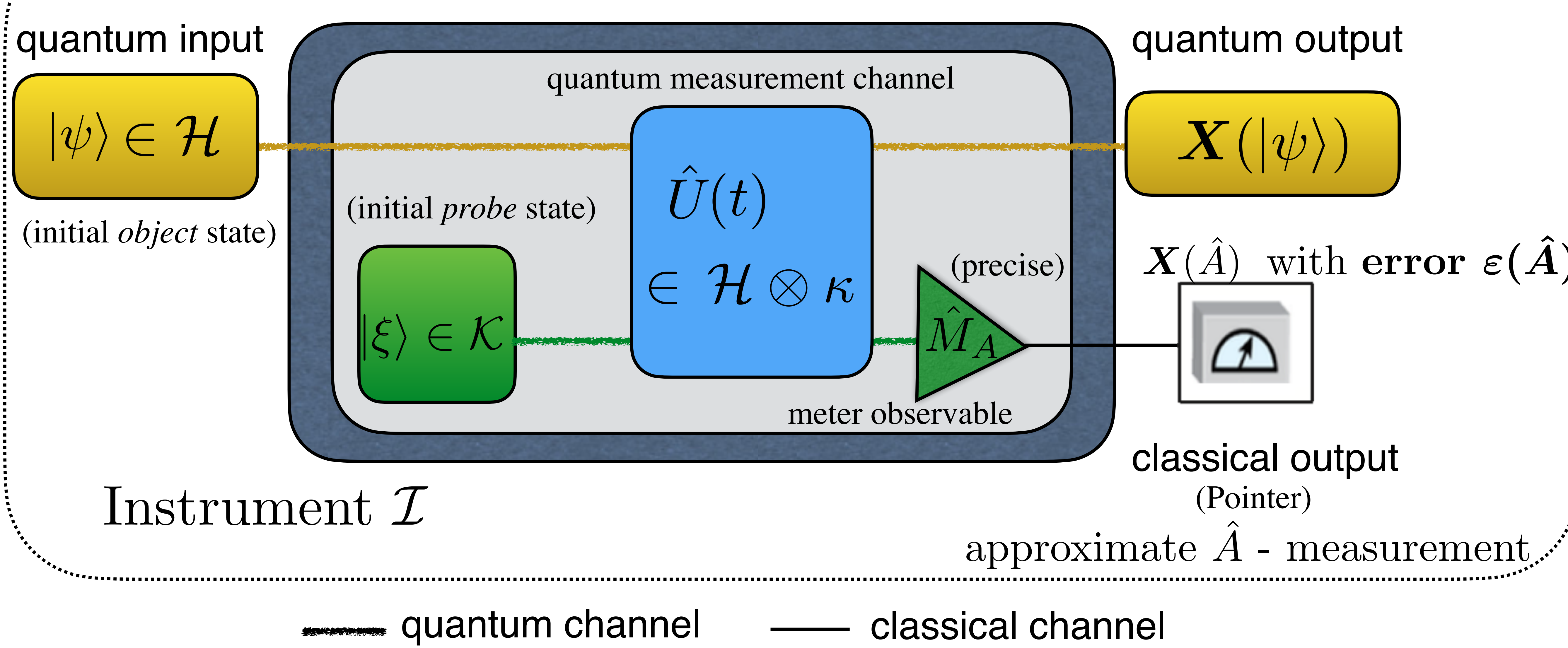
effect on another measurement incompatible (non-commuting) observable

Ozawa‘s **operator-based** Definitions of Error & Disturbance

Indirect Measurement Model:

$\hat{A},\ \hat{B},\ |\psi\rangle \in \mathcal{H}$, Hilbert space of *object* system
$\hat{M}_A,\ |\xi\rangle \in \mathcal{K}$, Hilbert space of *probe* system

$$\varepsilon(\hat{A}) = \langle \psi \otimes \xi | (\hat{U}(t)^\dagger (I \otimes \hat{M}_A)\hat{U}(t) - \hat{A} \otimes I)^2 | \xi \otimes \psi \rangle^{1/2}$$

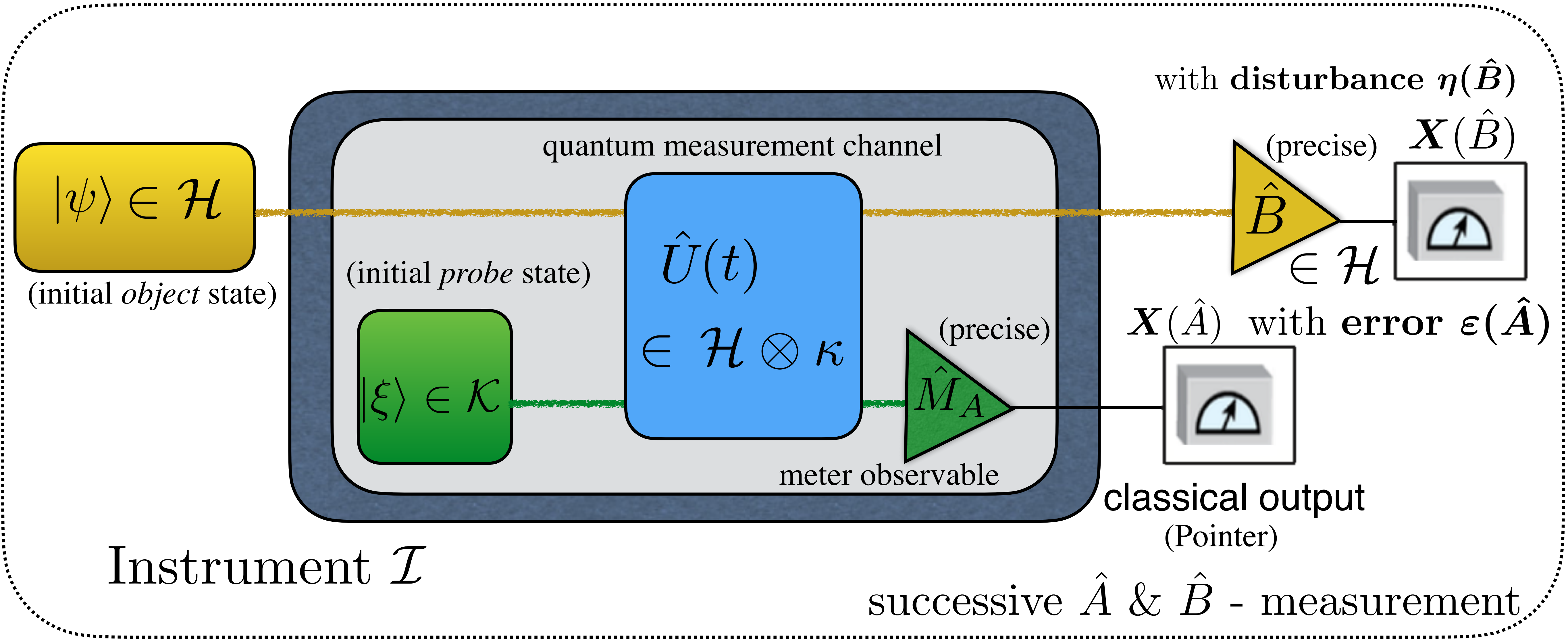


Ozawa, J. Math. Phys. **25**, 79–87 (1984).

Ozawa's ***operator-based*** *Definitions of Error & Disturbance*

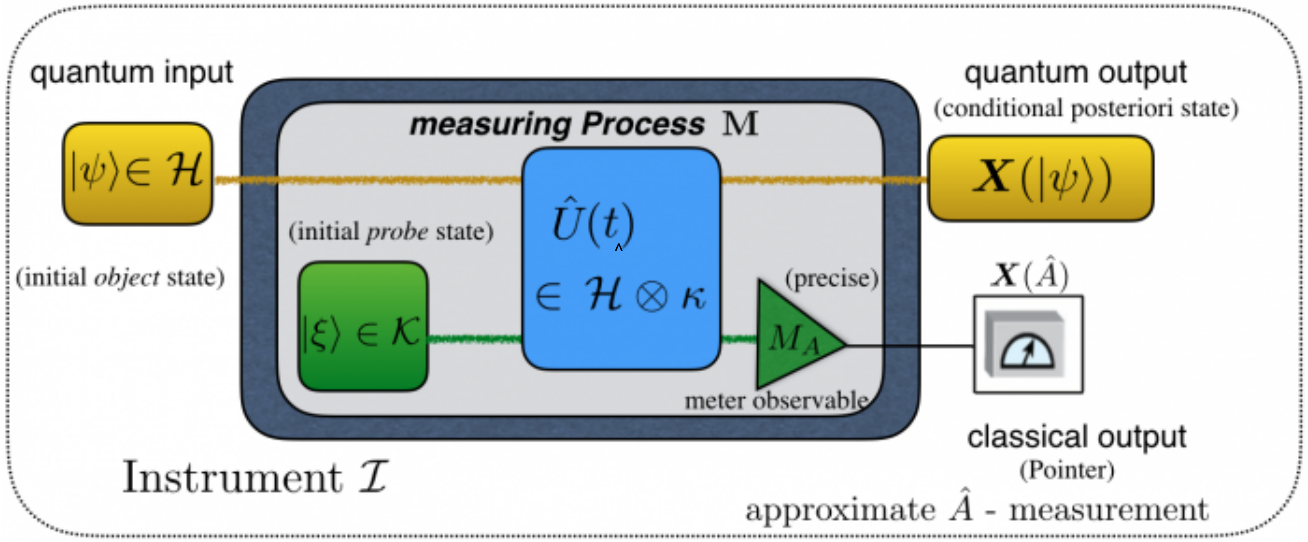
Indirect Measurement Model:

$\hat{A},\ \hat{B},\ |\psi\rangle \in \mathcal{H}$, Hilbert space of *object* system
$\hat{M}_A,\ |\xi\rangle \in \mathcal{K}$, Hilbert space of *probe* system



Ozawa, J. Math. Phys. **25**, 79–87 (1984), M. Ozawa, *Phys. Rev. A* **67,** 042105 (2003).

Ozawa Error



$$\varepsilon(\hat{A}) = \langle \psi \otimes \xi | (\hat{U}(t)^\dagger (I \otimes \hat{M}_A)\hat{U}(t) - \hat{A} \otimes I)^2 | \xi \otimes \psi \rangle^{1/2}$$

local proj. meas:

- $\hat{P}^M_\lambda = (\hat{I} \otimes \underbrace{\sum_\lambda \lambda |\lambda\rangle\langle\lambda|)}_{\hat{M}_A}$

family of projection ***operators***: $\{\hat{O}_\lambda\}$

$$\hat{O}_\lambda = \hat{O}^\dagger_\lambda \hat{O}_\lambda = \langle \xi | \hat{U}^\dagger(t) \hat{P}^M_\lambda \hat{U}(t) | \xi \rangle$$

POVM

acting on object-space $\mathcal{H}^{\mathrm{obj}}$ (U on $\mathcal{H}^{\mathrm{obj}} \otimes \mathcal{H}^{\mathrm{app}}$)

$$\epsilon^2(A) = \sum_\lambda ||O_\lambda(\lambda - A)|\psi\rangle||^2$$

$$||\ldots|| = ||X|\psi\rangle|| = \langle \psi | X^\dagger X | \psi \rangle^{\frac{1}{2}}$$

If the O_λ are mutually orthogonal **projection operators** sum and norm can be exchanged

$$\epsilon^{\cancel{2}}(A) = ||(\sum_\lambda \lambda O_\lambda - A)|\psi\rangle||^{\cancel{2}}$$

output operator: $O_A = \sum_\lambda \lambda O_\lambda$

"Approximator of operator A"

$$\epsilon(A) = ||(O_A - A)|\psi\rangle|| \longleftrightarrow \epsilon(A) = \sqrt{\langle \psi | (O_A - A)^2 | \psi \rangle}$$

M. Ozawa, *Phys. Rev. A* **67,** 042105 (2003).

Error and Disturbance for Spin 1/2 Measurements

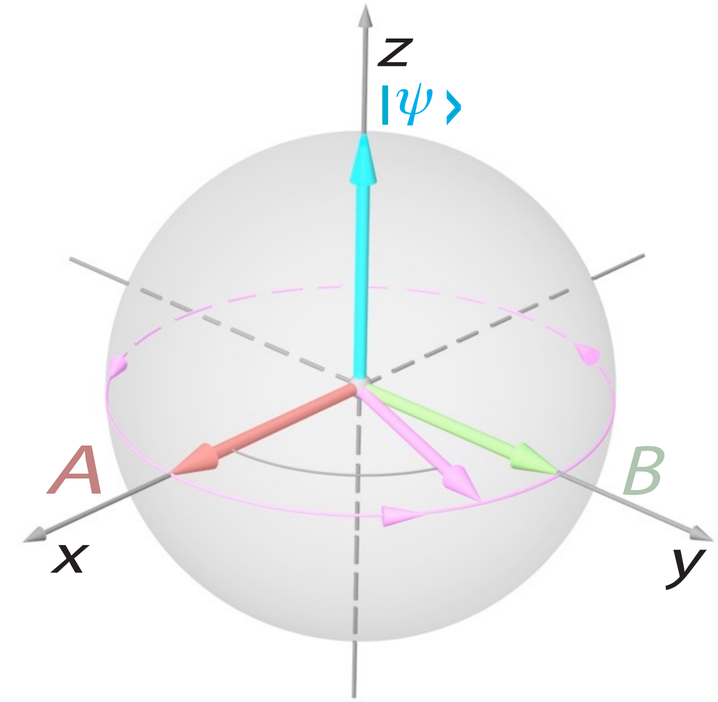
- **set of observables and initial state**

$$A = \sigma_x$$

$$B = \sigma_y$$

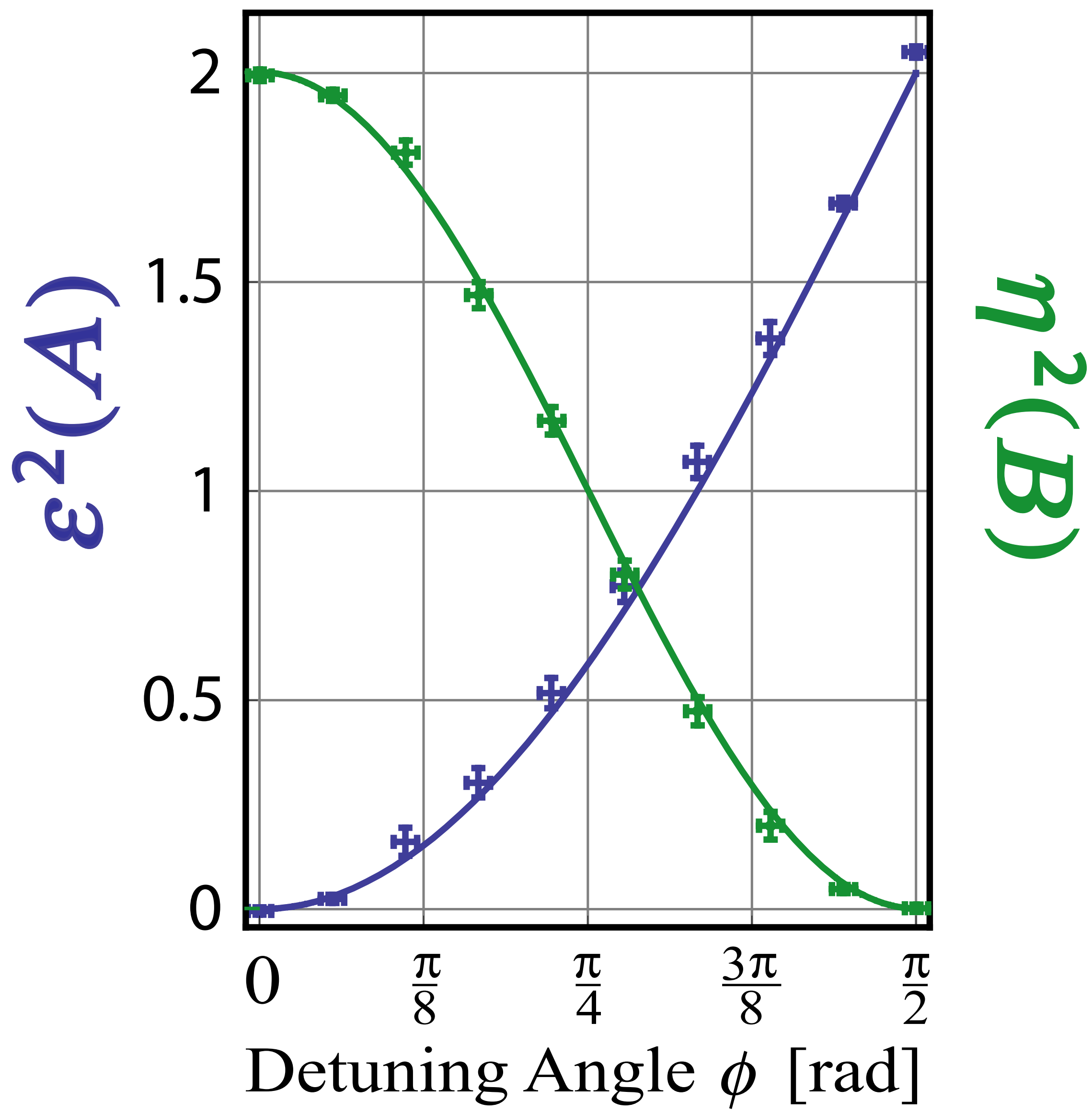
$$O_A = \sigma_\phi = \sigma_x \sin\phi + \sigma_y \cos\phi$$

$$|\psi\rangle = |+z\rangle$$



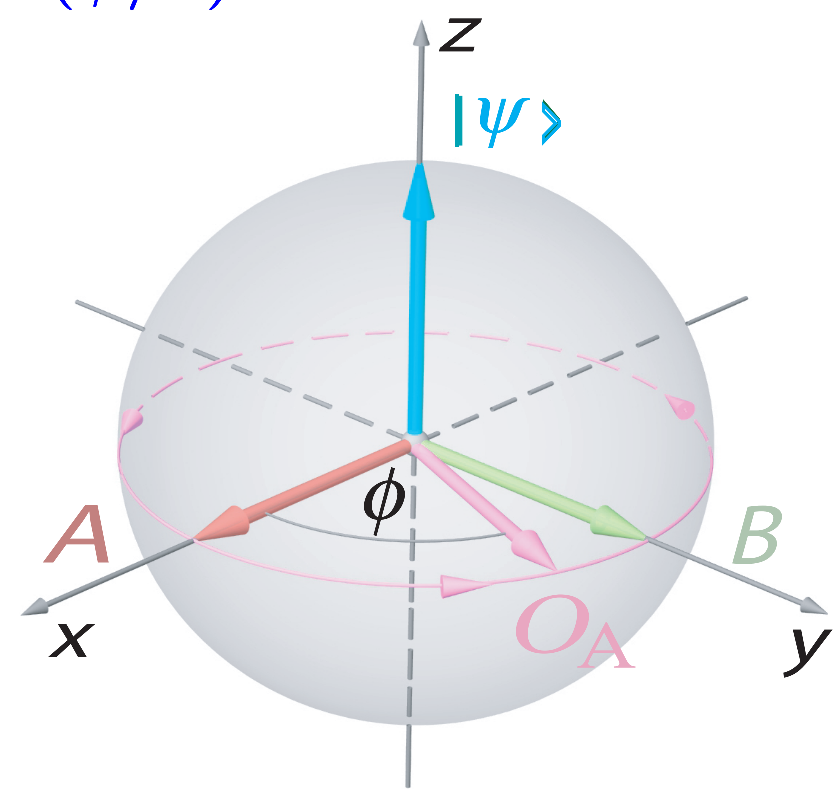
First Experimental Results of Ozawa's Error and Disturbance

- **error, disturbance: trade - off**



$$\epsilon(A) = \sqrt{\langle\psi|(O_A - A)^2|\psi\rangle}$$

$$\epsilon(A)^2 = 4\sin^2(\phi/2)$$



$$\eta(B)^2 = 2\cos^2\phi$$

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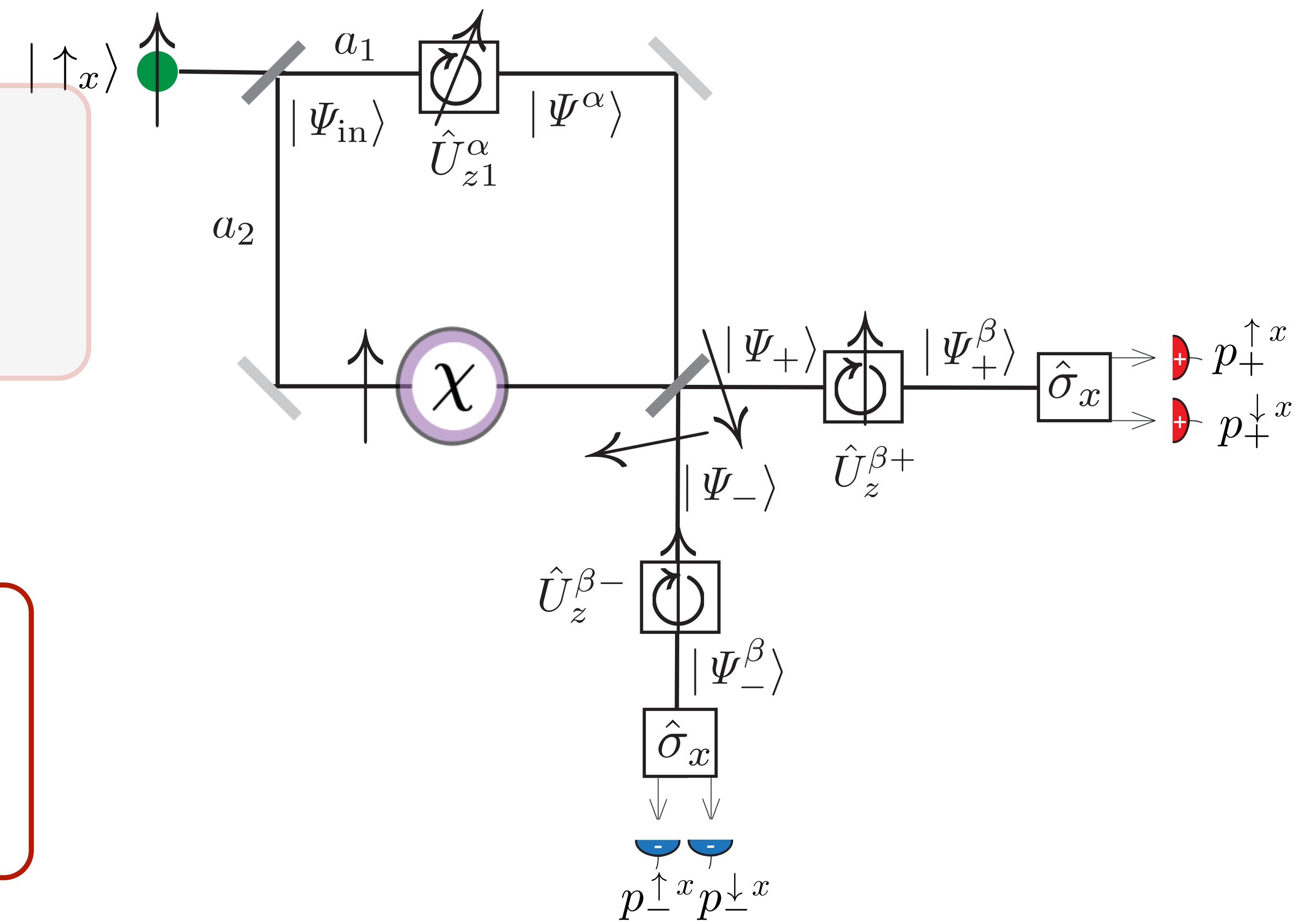
Jacqueline Erhart[1], Stephan Sponar[1], Georg Sulyok[1], Gerald Badurek[1], Masanao Ozawa[2] and Yuji Hasegawa[1★] NATURE PHYSICS. VOL **8**, 185, MARCH 2012.

Ozawa Error

$$|\Psi_{\mathrm{in}}\rangle = |\psi \otimes S\rangle = \big(a_1|1\rangle + a_2|2\rangle\big) \otimes |\uparrow_x\rangle \qquad |\Psi_{\pm}^{\mathrm{out}}\rangle = \langle \pm|\psi\rangle\, |\pm\rangle |S_\pm\rangle$$

$$\epsilon(A) = \sqrt{\langle\psi|(O_A - A)^2|\psi\rangle}$$

$$\overset{A}{\varepsilon^2(\hat{\Pi}_1)} = \langle\psi| \Bigg(\overbrace{\sum_\pm \frac{\beta_\pm}{\alpha} |\pm\rangle\langle\pm|}^{O_A} - \overset{A}{\hat{\Pi}_1} \Bigg)^2 |\psi\rangle$$



$$\varepsilon^2(\hat{\Pi}_1) = \sum_\pm \langle\psi| \left(\hat{\Pi}_1^\dagger - \frac{\beta_\pm}{\alpha} \right) |\pm\rangle\langle\pm| \left(\hat{\Pi}_1 - \frac{\beta_\pm}{\alpha} \right) |\psi\rangle$$

$$= \sum_\pm \boxed{p_\pm} \left| \langle \hat{\Pi}_1 \rangle_{\mathrm{w}}^{\pm,\psi(\chi)} - \boxed{\frac{\beta_\pm}{\alpha}} \right|^2$$

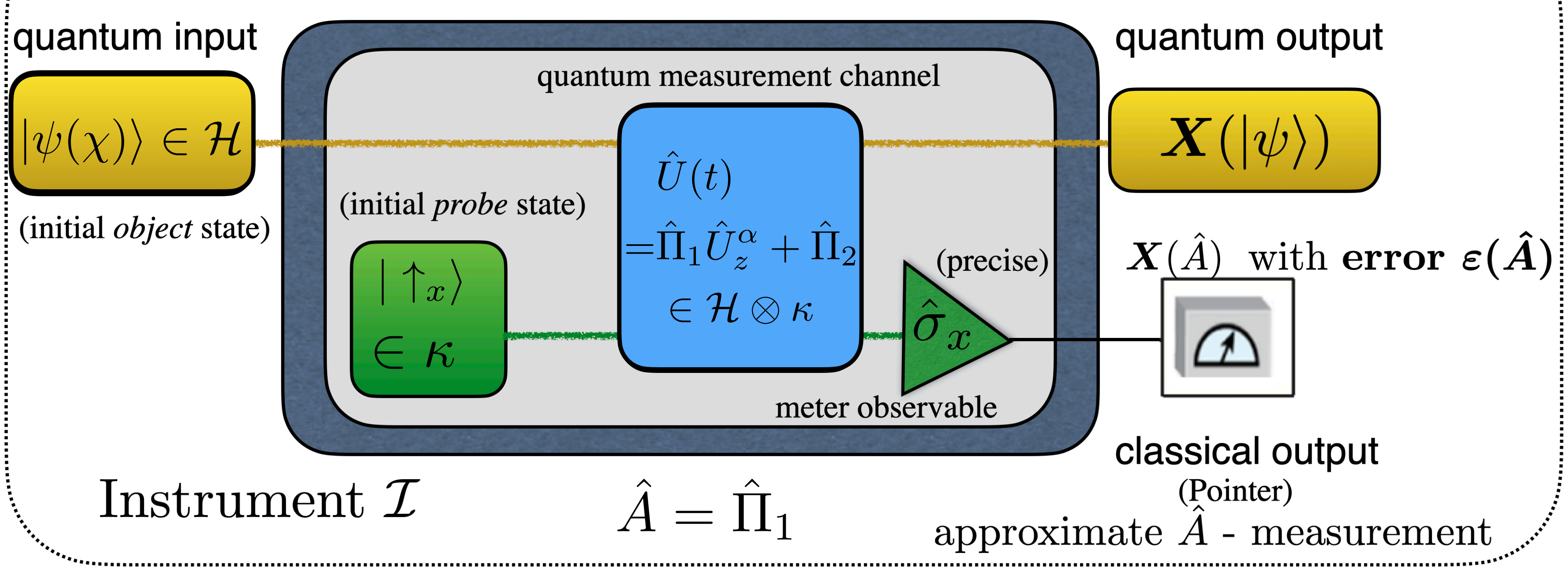
exp. data

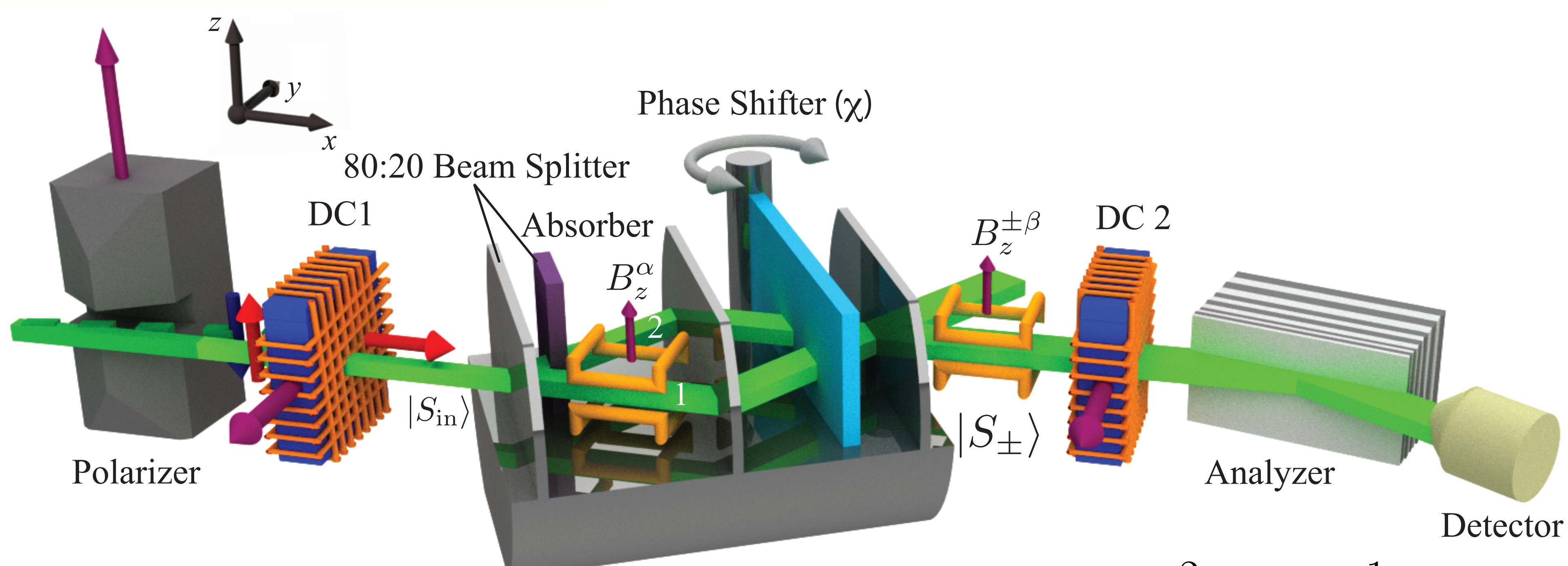
$$p_\pm = |\langle\pm|\psi\rangle|^2 = \frac{1}{2} \pm a_1 a_2 \cos\chi$$

$$\langle \hat{\Pi}_1 \rangle_{\mathrm{w}}^{\pm,\psi(\chi)} = \frac{\langle\pm|\hat{\Pi}_1|\psi(\chi)\rangle}{\langle\pm|\psi(\chi)\rangle}$$

Ozawa's **operator-based** Definitions of Error & Disturbance

$$\varepsilon(\hat{A}) = \langle \psi \otimes \xi | (\hat{U}(t)^\dagger (I \otimes \hat{M}_A) \hat{U}(t) - \hat{A} \otimes I)^2 | \xi \otimes \psi \rangle^{1/2}$$





$$|\Psi_{\text{in}}\rangle = |\psi(\chi) \otimes S\rangle = \big(a_1|1\rangle + a_2|2\rangle\big)|\uparrow_x\rangle \quad \rightarrow a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}}$$

Additional Notes on Ozawa-Hall Error: i) Values of Errors

The same conclusion can be drawn using the Ozawa-Hall theory. Ozawa [12] introduced a general concept of measurement errors and it was shown by Hall that these errors correspond to the uncertainty of an estimate of a physical property based on the outcome of an arbitrary measurement [13]. This uncertainty is given by the statistical deviation between the operator of interest and the estimated value of that operator. In our case, it reads

$$\varepsilon^2(\hat{\Pi}_1) = \langle\psi|\left[\hat{\Pi}_1 - \sum_{\pm}\frac{\beta_\pm}{\alpha}|\pm\rangle\langle\pm|\right]^2|\psi\rangle, \qquad (23)$$

$$\varepsilon^2(\hat{\Pi}_1) = \sum_{\pm}\langle\psi|\left(\hat{\Pi}_1^\dagger - \frac{\beta_\pm}{\alpha}\right)|\pm\rangle\langle\pm|\left(\hat{\Pi}_1 - \frac{\beta_\pm}{\alpha}\right)|\psi\rangle$$

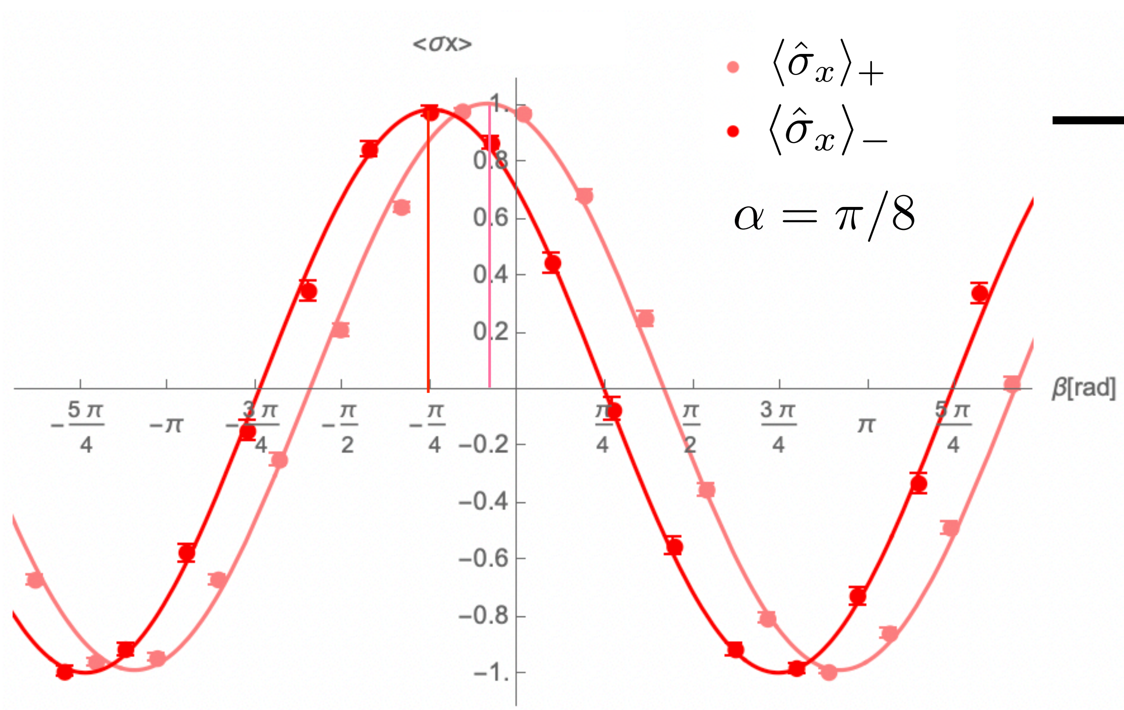
$$\boxed{= \sum_{\pm} p_\pm \left|\langle\hat{\Pi}_1\rangle_{\mathrm{w}}^{\pm,\psi(\chi)} - \boxed{\frac{\beta_\pm}{\alpha}}\right|^2} \qquad (25)$$

exp. data

where $p_\pm$ denotes the statistical probability in the undisturbed system of finding the neutron in the final state $|\pm\rangle$:

$$p_\pm = |\langle\pm|\psi\rangle|^2 = \tfrac{1}{2} \pm a_1 a_2 \cos\chi\,. \qquad (26)$$

Interference Context: **individual "postselection-based"** feedback comp:



best estimates(@ $\alpha = \pi/8$) :

$$\frac{\beta_{0+}}{\alpha} = 0.703(45) \quad \text{[Theory: 2/3]}$$

$$\frac{\beta_{0-}}{\alpha} = 1.989(38) \quad \text{[Theory: 2]}$$

(not shown in paper; only $\alpha = \pi/16$ & $\pi/4$)

Additional Notes on Ozawa-Hall Error i) Values of Errors

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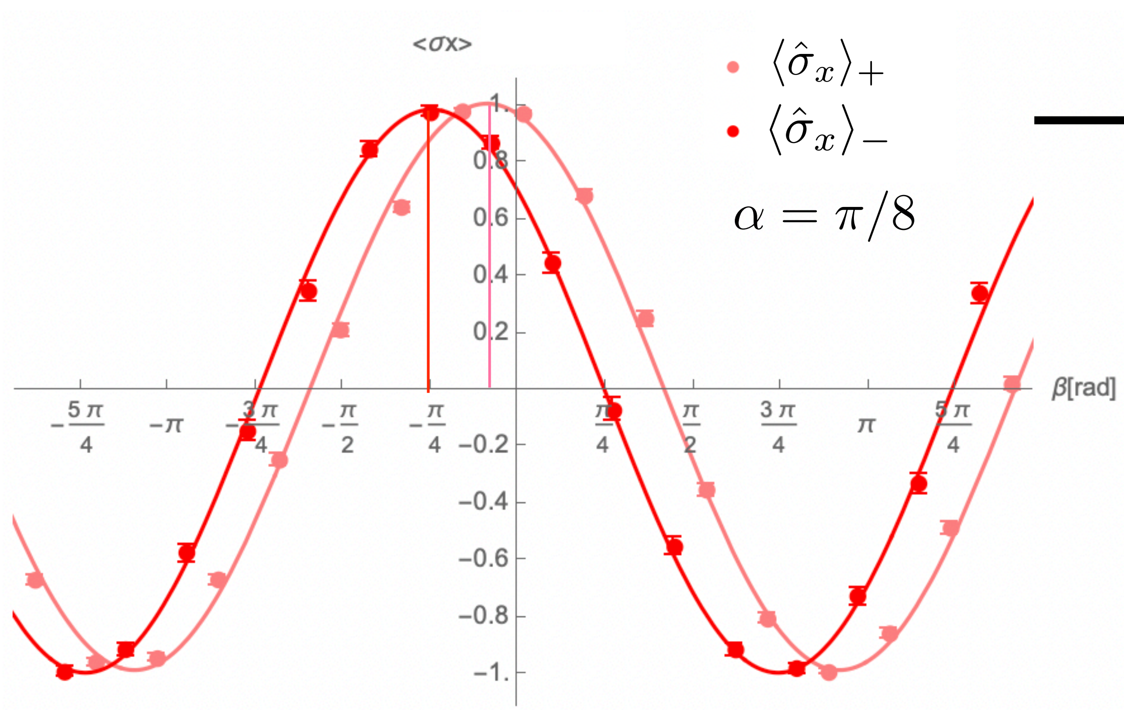


(not shown in paper; only $\alpha = \pi/16$ & $\pi/4$)

best estimates(@ $\alpha = \pi/8$) :

$$\frac{\beta_{0+}}{\alpha} = 0.703(45)$$ [Theory: 2/3]

$$\frac{\beta_{0-}}{\alpha} = 1.989(38)$$ [Theory: 2]

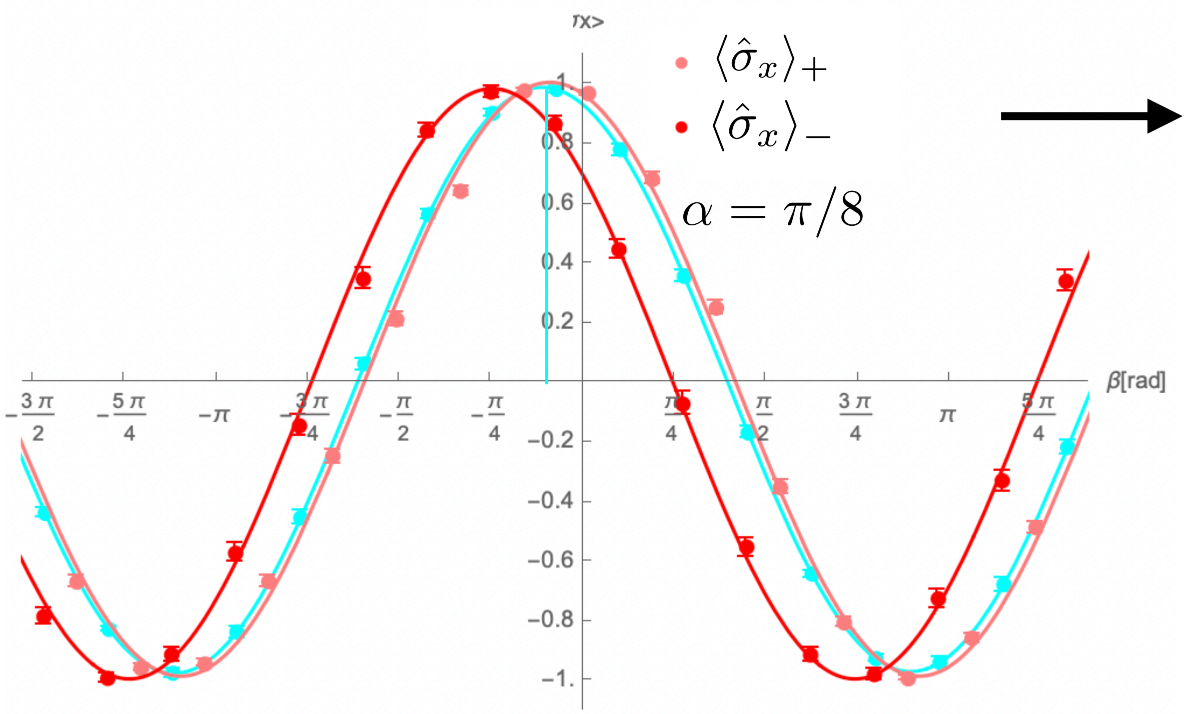
$$\varepsilon^2(\hat{\Pi}_1) = 0.0012(29)$$

$$\varepsilon(\hat{\Pi}_1) = 0.034(42)$$

[Theory: 0]

Averages …

Interference Context: **common "postselection-based"** feedback comp:



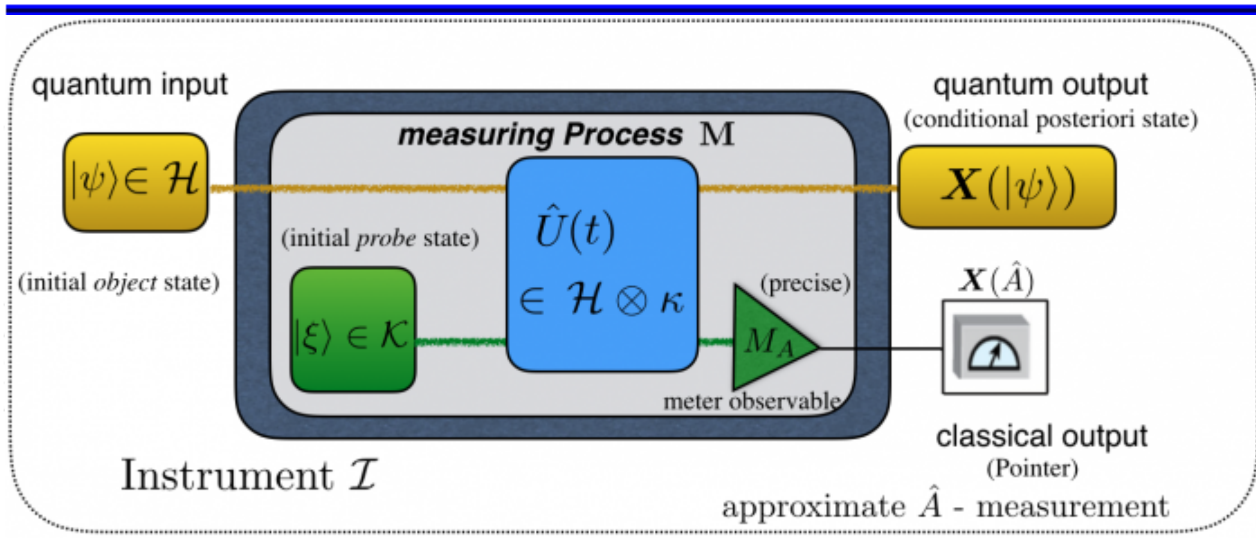
best estimates(@ $\alpha = \pi/8$) :

$$\frac{\bar{\beta}_0}{\alpha} = 0.831(41)$$ [Theory: 4/5]

(not shown in paper; only $\alpha = \pi/16$ & $\pi/4$)

$$\overline{\langle\hat{\sigma}_x\rangle}_\pm = p_+\langle\hat{\sigma}_x\rangle_+ + p_-\langle\hat{\sigma}_x\rangle_-$$

Averages …



$$\varepsilon^2(\hat{\Pi}_1) = \langle\psi|\left[\hat{\Pi}_1 - \sum_{\pm} \frac{\beta_\pm}{\alpha} |\pm\rangle\langle\pm|\right]^2 |\psi\rangle, \qquad (23)$$

"postselection-based" feedback comp:

$$\varepsilon^2(\hat{\Pi}_1) = \sum_{\pm} \langle\psi|\left(\hat{\Pi}_1^\dagger - \frac{\beta_\pm}{\alpha}\right)|\pm\rangle\langle\pm|\left(\hat{\Pi}_1 - \frac{\beta_\pm}{\alpha}\right)|\psi\rangle$$

$$\boxed{= \sum_{\pm} p_\pm \left| w_{1\pm} - \frac{\beta_\pm}{\alpha} \right|^2,} \qquad (25)$$

where $p_\pm$ denotes the statistical probability in the undisturbed system of finding the neutron in the final state $|\pm\rangle$:

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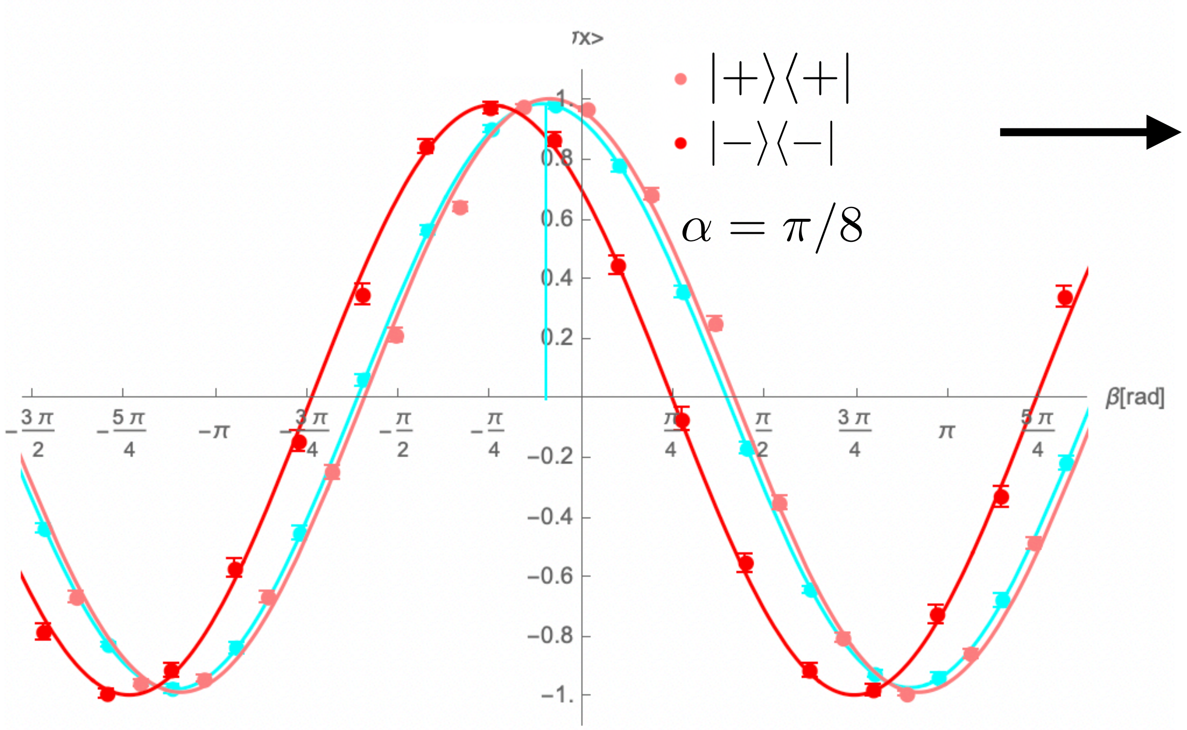
Interference Context: **common "postselection-based"** feedback comp:



(not shown in paper; only $\alpha = \pi/16$ & $\pi/4$)

best estimates($@\,\alpha = \pi/8$) :

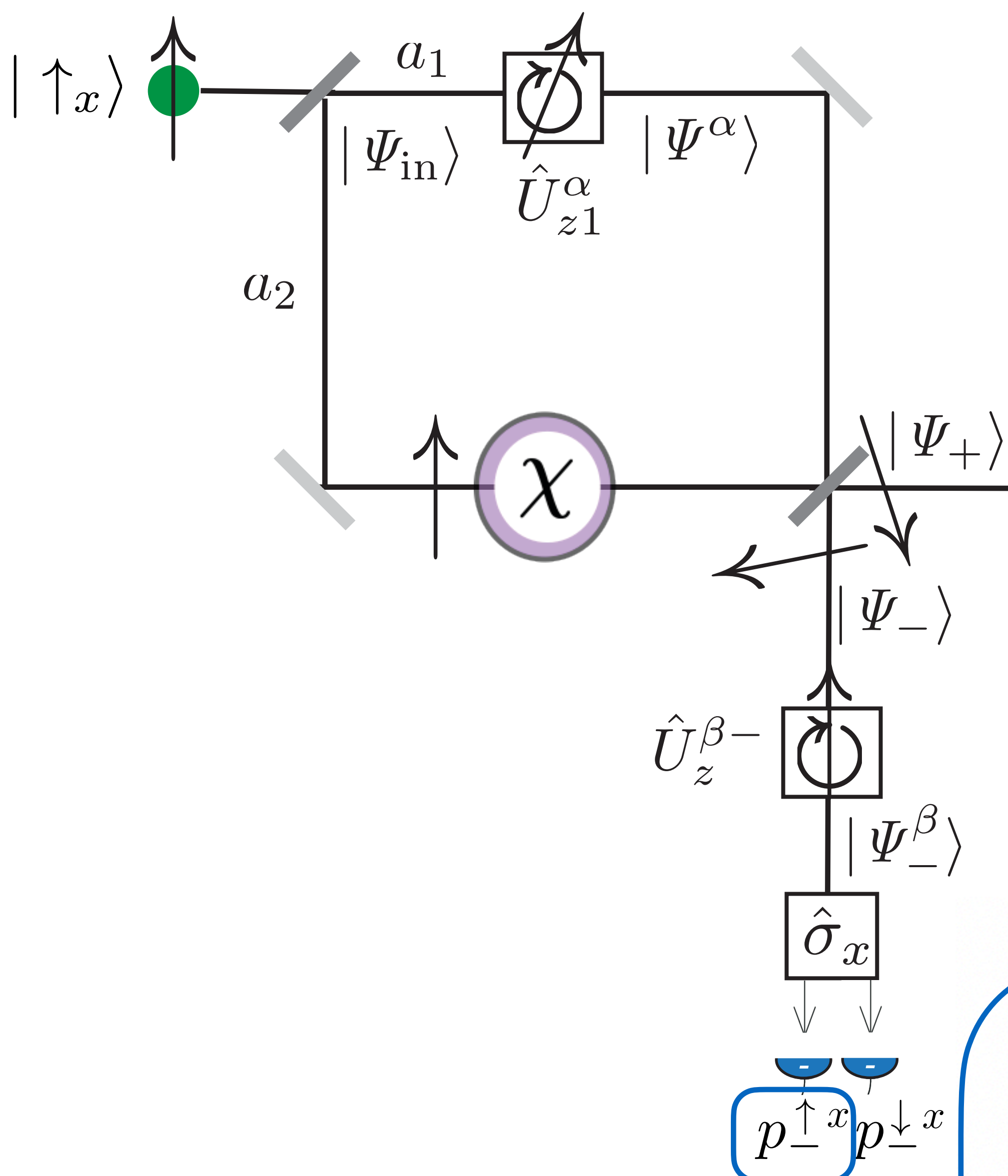
$$\frac{\bar{\beta}_0}{\alpha} = 0.831(41) \quad \text{[Theory: 4/5]}$$

$$\varepsilon^2(\hat{\Pi}_1) = 0.161(13) \quad \text{[Theory: 0.16]}$$

$$\varepsilon(\hat{\Pi}_1) = 0.401(3) \quad \text{[Theory: 0.4]}$$

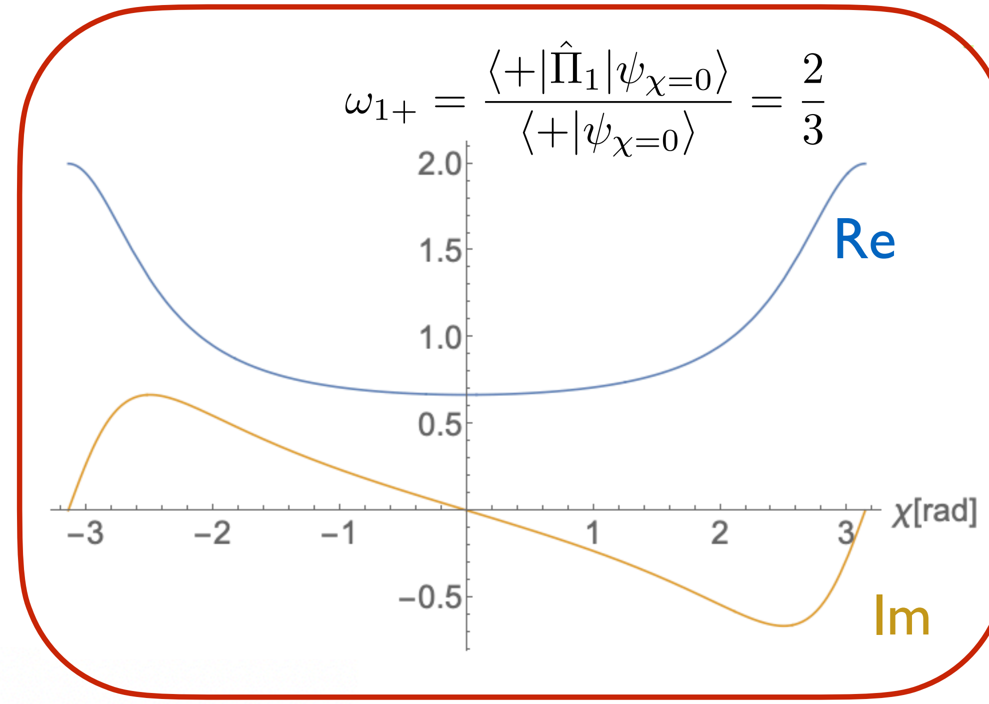
Variance $\Delta(\bar{\omega}_1) = \frac{2}{5}$

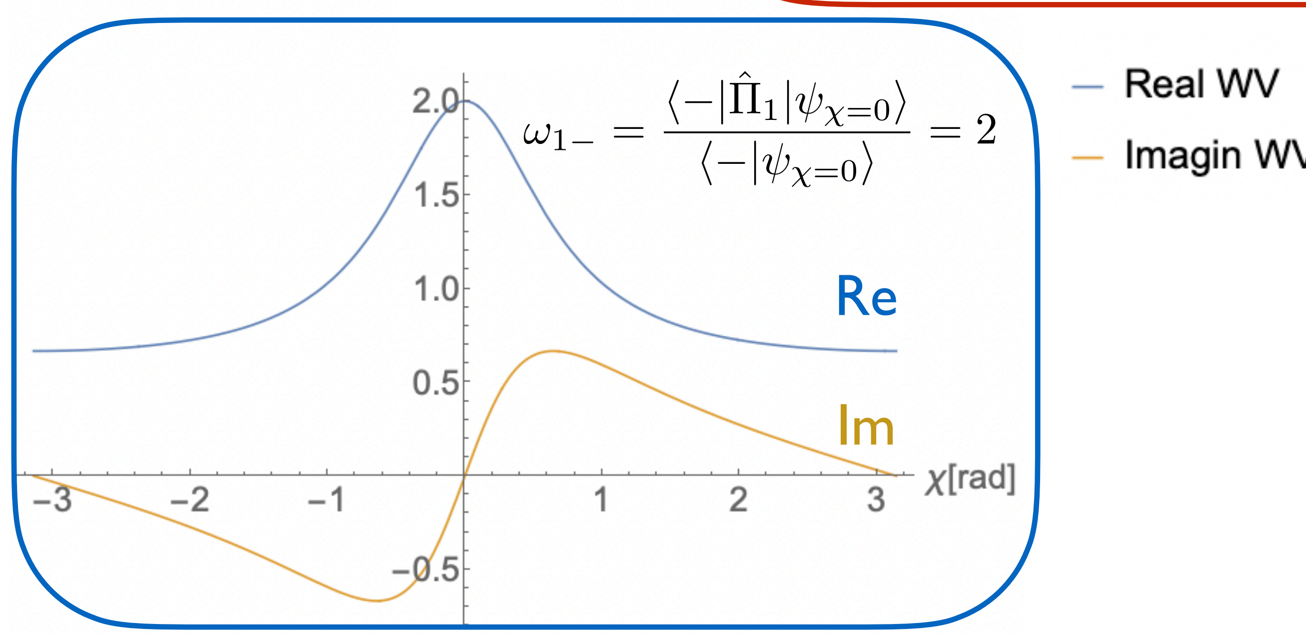
Additional Notes on Ozawa-Hall Error iii) Variation of initial state



$$|\Psi_{\mathrm{in}}\rangle = |\psi \otimes S\rangle = \left(a_1|1\rangle + e^{\mathrm{i}\chi} a_2|2\rangle\right) \otimes |\uparrow_x\rangle$$

$$\rightarrow\ a_1 = \frac{2}{\sqrt{5}},\ a_2 = \frac{1}{\sqrt{5}}$$





Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

Ozawa's universally valid measurement uncertainty relation

$$\boldsymbol{\epsilon(A)\eta(B) + \epsilon(A)\Delta B + \eta(B)\Delta A \geq \frac{1}{2}|\langle\psi|[A,B]|\psi\rangle|}$$

$$|\psi\rangle := |\psi(\chi)\rangle = \frac{2}{\sqrt{5}}|1\rangle + e^{\mathrm{i}\chi}\frac{1}{\sqrt{5}}|2\rangle \qquad \hat{A} = \hat{\Pi}_1 = |1\rangle\langle 1| \qquad \hat{B} = |+\rangle\langle +| - |-\rangle\langle -| := \hat{X} \;\; (\text{or } \hat{\sigma}_x^{\mathrm{p}})$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$

- *simultaneous measurement:*

  $\eta(B) \rightarrow \varepsilon(B)$

Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

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$$|\pm\rangle = |1\rangle \pm |2\rangle$$

- **Error**: $\varepsilon^2(\hat{\Pi}_1) = \langle\psi(\chi)|\left[\hat{\Pi}_1 - \sum_{\pm}\frac{\beta_\pm}{\alpha}(\chi)|\pm\rangle\langle\pm|\right]^2|\psi(\chi)\rangle$ (23)

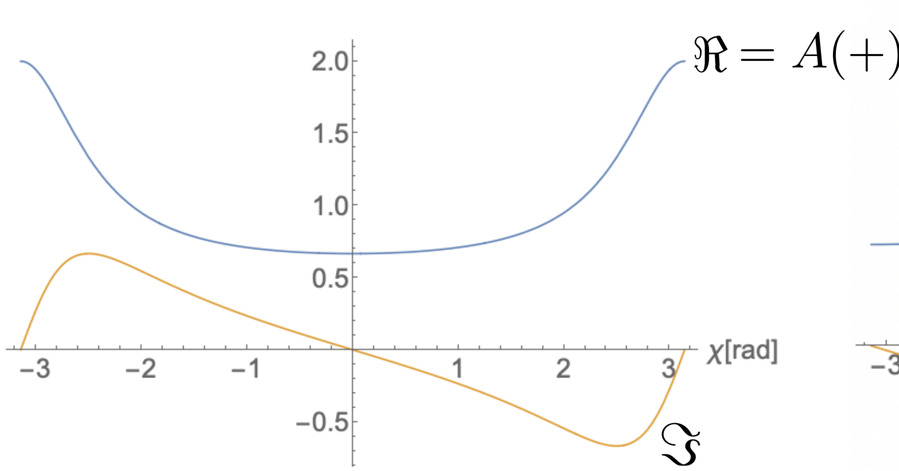
best estimate weak value path projector :

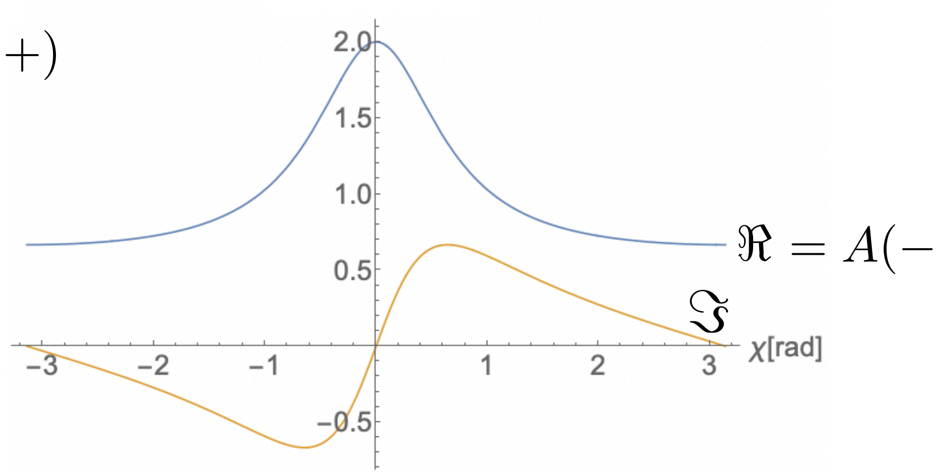
$$\frac{\beta_\pm}{\alpha}(\chi) = \omega_{1\pm}(\chi) \underset{\lim\alpha\to 0}{\cong} \Re\frac{\langle\pm|\hat{\Pi}_1|\psi(\chi)\rangle}{\langle\pm|\psi(\chi)\rangle}$$

The optimal estimates after the interference measurements are given by

$$\frac{\beta_+(\chi)}{\alpha} = A(+) = c_1^2 - \frac{c_1c_2\cos(\chi)}{1+2c_1c_2\cos(\chi)}(c_1^2 - c_2^2)$$

$$\frac{\beta_-(\chi)}{\alpha} = A(-) = c_1^2 + \frac{c_1c_2\cos(\chi)}{1-2c_1c_2\cos(\chi)}(c_1^2 - c_2^2)$$





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Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

Ozawa's universally valid measurement uncertainty relation

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$$|\psi(\chi)\rangle = \underbrace{\frac{2}{\sqrt{5}}}_{:=c_1}|1\rangle + e^{i\chi}\underbrace{\frac{1}{\sqrt{5}}}_{:=c_2}|2\rangle \qquad \hat{A} = \hat{\Pi}_1 = |1\rangle\langle 1| \qquad \hat{B} = |+\rangle\langle +| - |-\rangle\langle -| := \hat{X}$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$

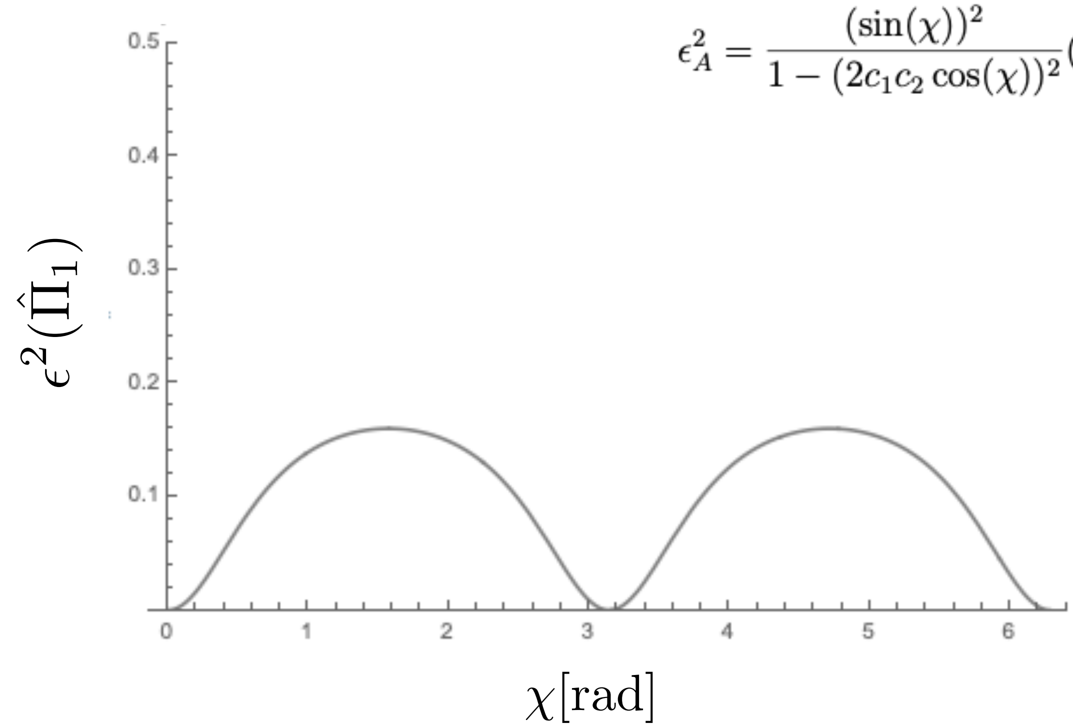
Error: $\varepsilon^2(\hat{\Pi}_1) = \langle\psi(\chi)|\left[\hat{\Pi}_1 - \sum \frac{\beta_\pm}{\alpha}(\chi)|\pm\rangle\langle\pm|\right]^2|\psi(\chi)\rangle \quad (23)$

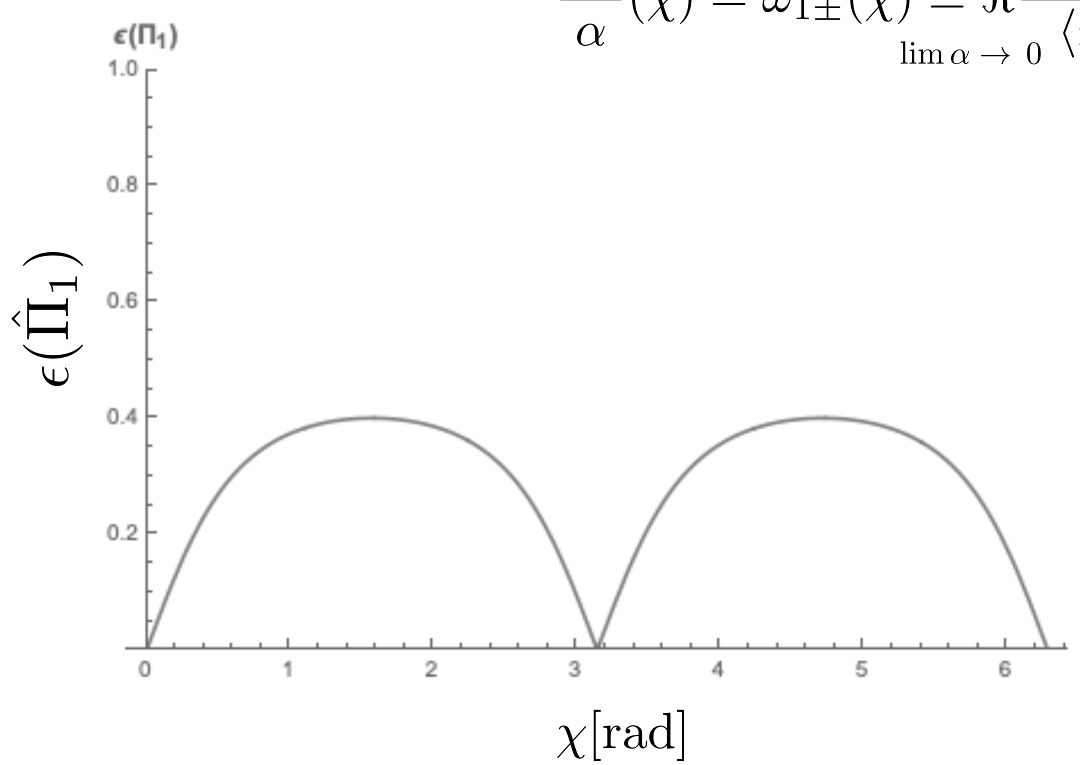
best estimate weak value path projector :

$$\frac{\beta_\pm}{\alpha}(\chi) = \omega_{1\pm}(\chi) \underset{\lim \alpha \to 0}{\cong} \Re\frac{\langle\pm|\hat{\Pi}_1|\psi(\chi)\rangle}{\langle\pm|\psi(\chi)\rangle}$$

Theory: The predicted result for the interference fringes is

$$\epsilon_A^2 = \frac{(\sin(\chi))^2}{1 - (2c_1c_2\cos(\chi))^2}(c_1c_2)^2.$$





Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

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$$|\pm\rangle = |1\rangle \pm |2\rangle$$

Error: $\varepsilon^2(\hat{\Pi}_1) = \langle\psi(\chi)|\left[\hat{\Pi}_1 - \sum \frac{\beta_\pm}{\alpha}(\chi)|\pm\rangle\langle\pm|\right]^2|\psi(\chi)\rangle \quad (23)$

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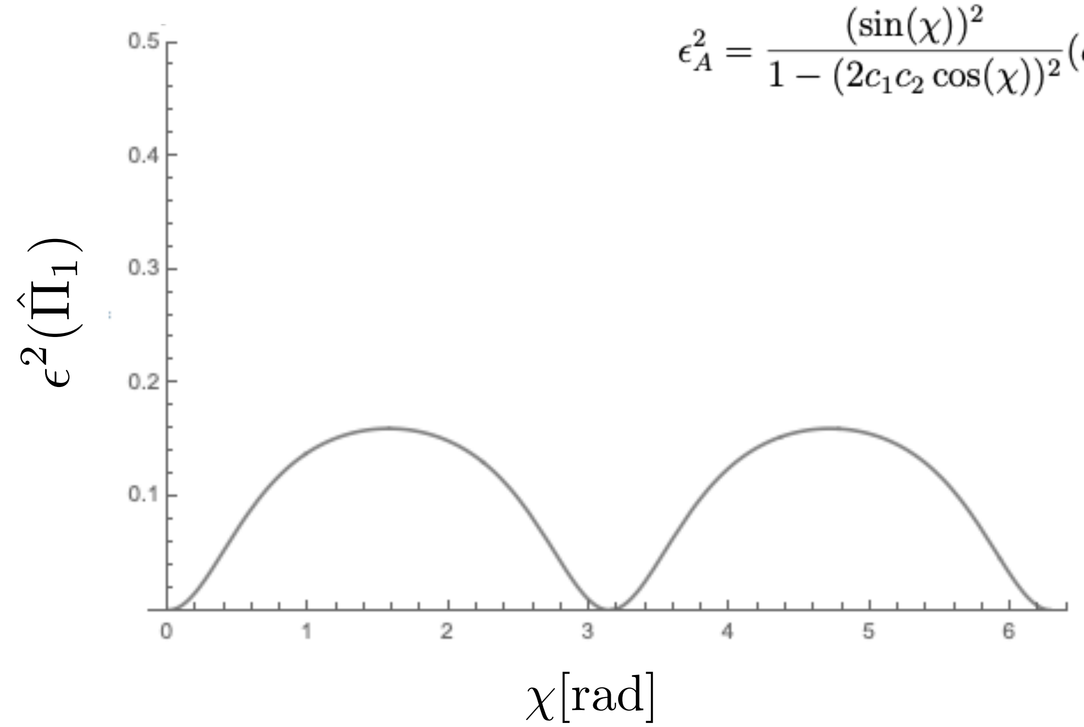

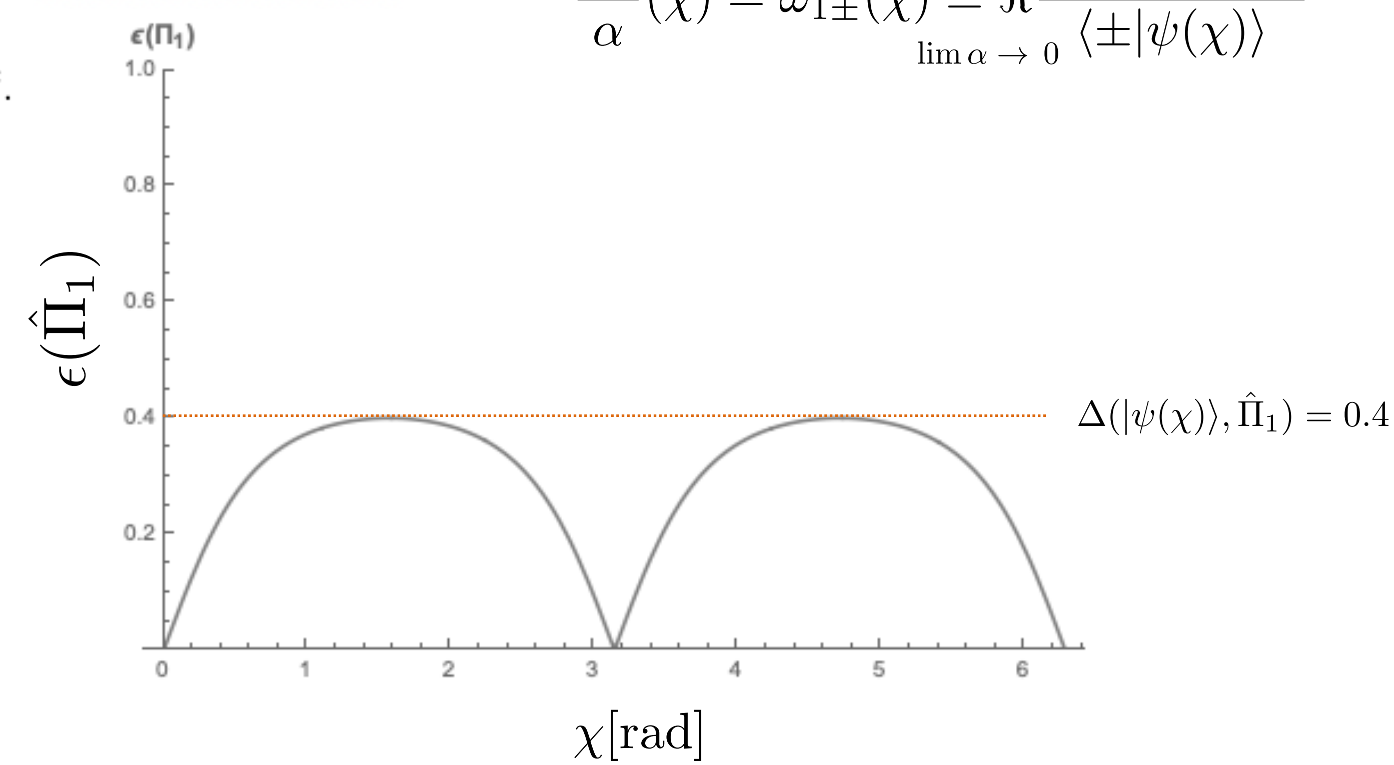

Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

$$|\psi(\chi)\rangle = \underbrace{\frac{2}{\sqrt{5}}}_{:=c_1}|1\rangle + e^{i\chi}\underbrace{\frac{1}{\sqrt{5}}}_{:=c_2}|2\rangle \qquad \hat{A} = \hat{\Pi}_1 = |1\rangle\langle 1|$$

Theory:

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Measurement:

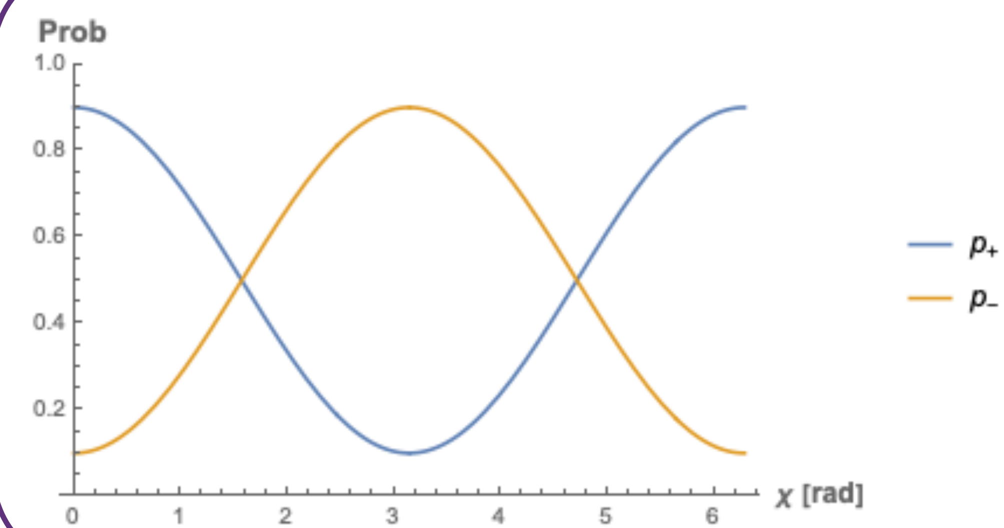
$$= \sum_{\pm}\boxed{p_\pm}\left|\langle\hat{\Pi}_1\rangle_{\mathrm{w}}^{\pm,\psi(\chi)} - \boxed{\frac{\beta_\pm}{\alpha}}\right|^2$$

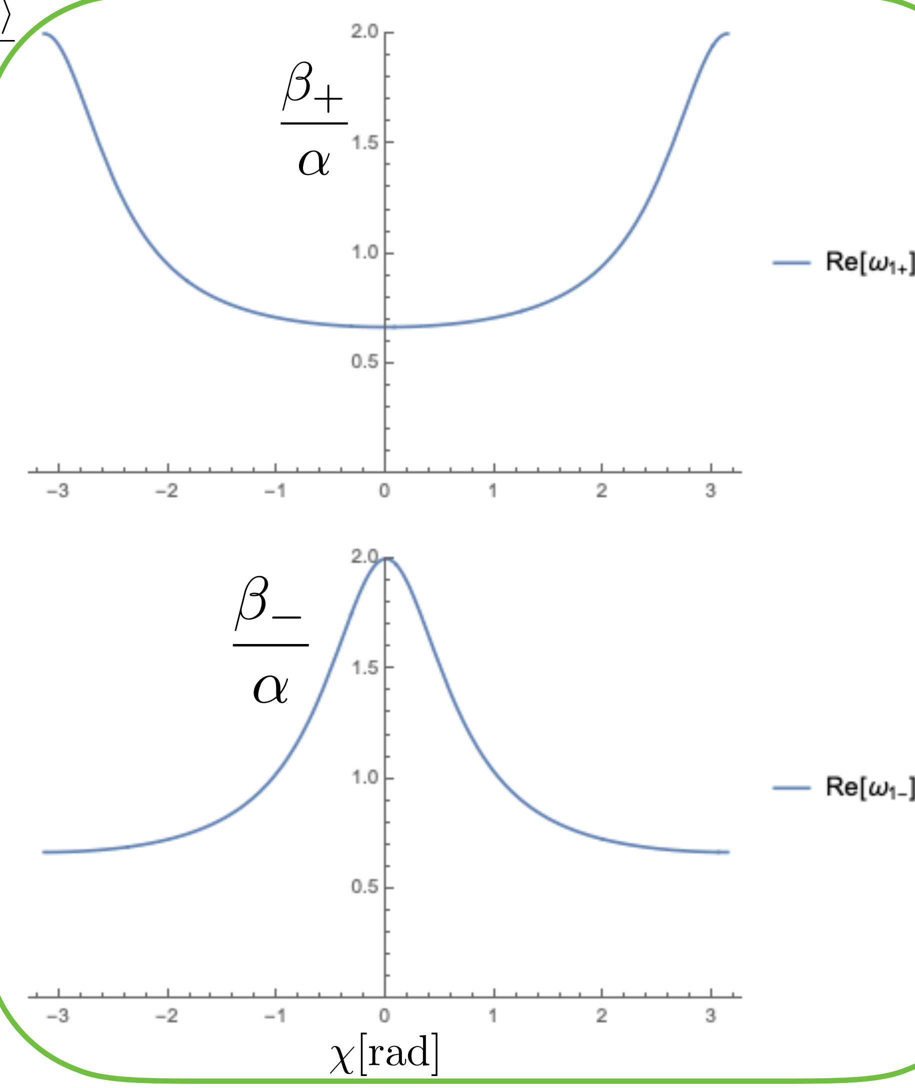
exp. data

$$p_\pm = |\langle\pm|\psi\rangle|^2 = \frac{1}{2} \pm a_1 a_2 \cos\chi$$

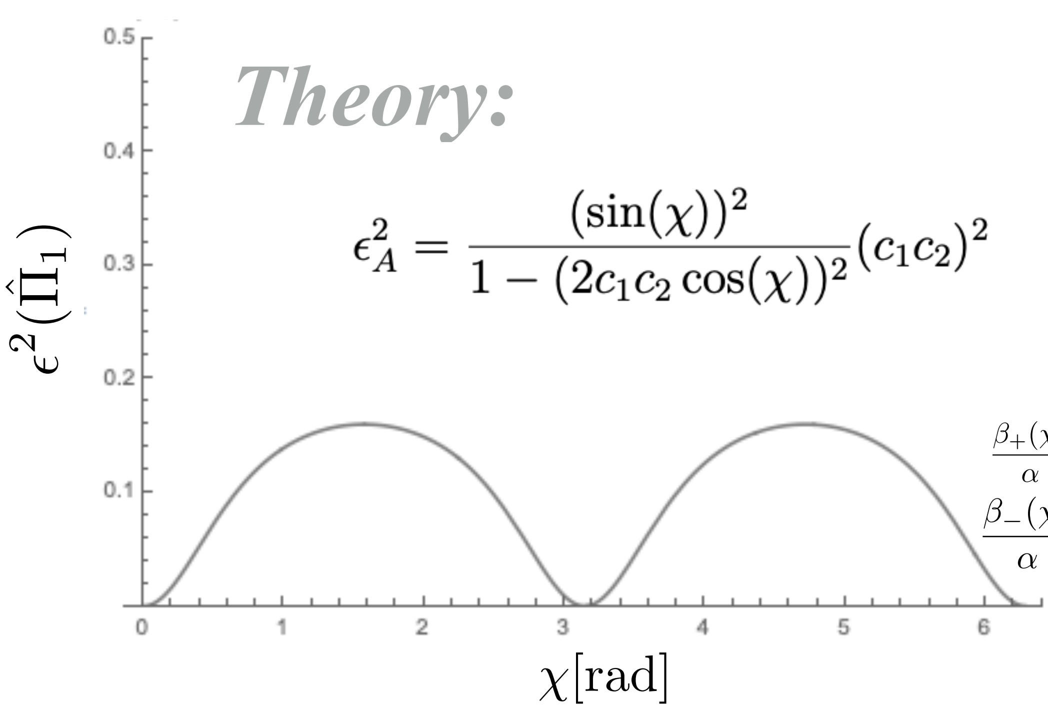
$$\langle\hat{\Pi}_1\rangle_{\mathrm{w}}^{\pm,\psi(\chi)} = \frac{\langle\pm|\hat{\Pi}_1|\psi(\chi)\rangle}{\langle\pm|\psi(\chi)\rangle}$$

Measurement:





$$p_+ = \frac{1}{2} + c_1c_2\cos(\chi)$$
$$p_- = \frac{1}{2} - c_1c_2\cos(\chi).$$



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$$\frac{\beta_+(\chi)}{\alpha} = A(+) = c_1^2 - \frac{c_1c_2\cos(\chi)}{1+2c_1c_2\cos(\chi)}(c_1^2-c_2^2)$$
$$\frac{\beta_-(\chi)}{\alpha} = A(-) = c_1^2 + \frac{c_1c_2\cos(\chi)}{1-2c_1c_2\cos(\chi)}(c_1^2-c_2^2)$$

Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

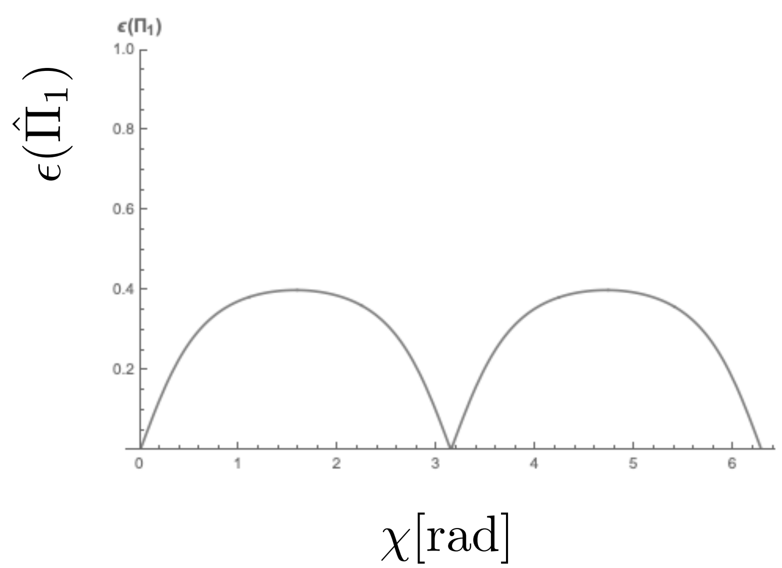
Ozawa's universally valid measurement uncertainty relation

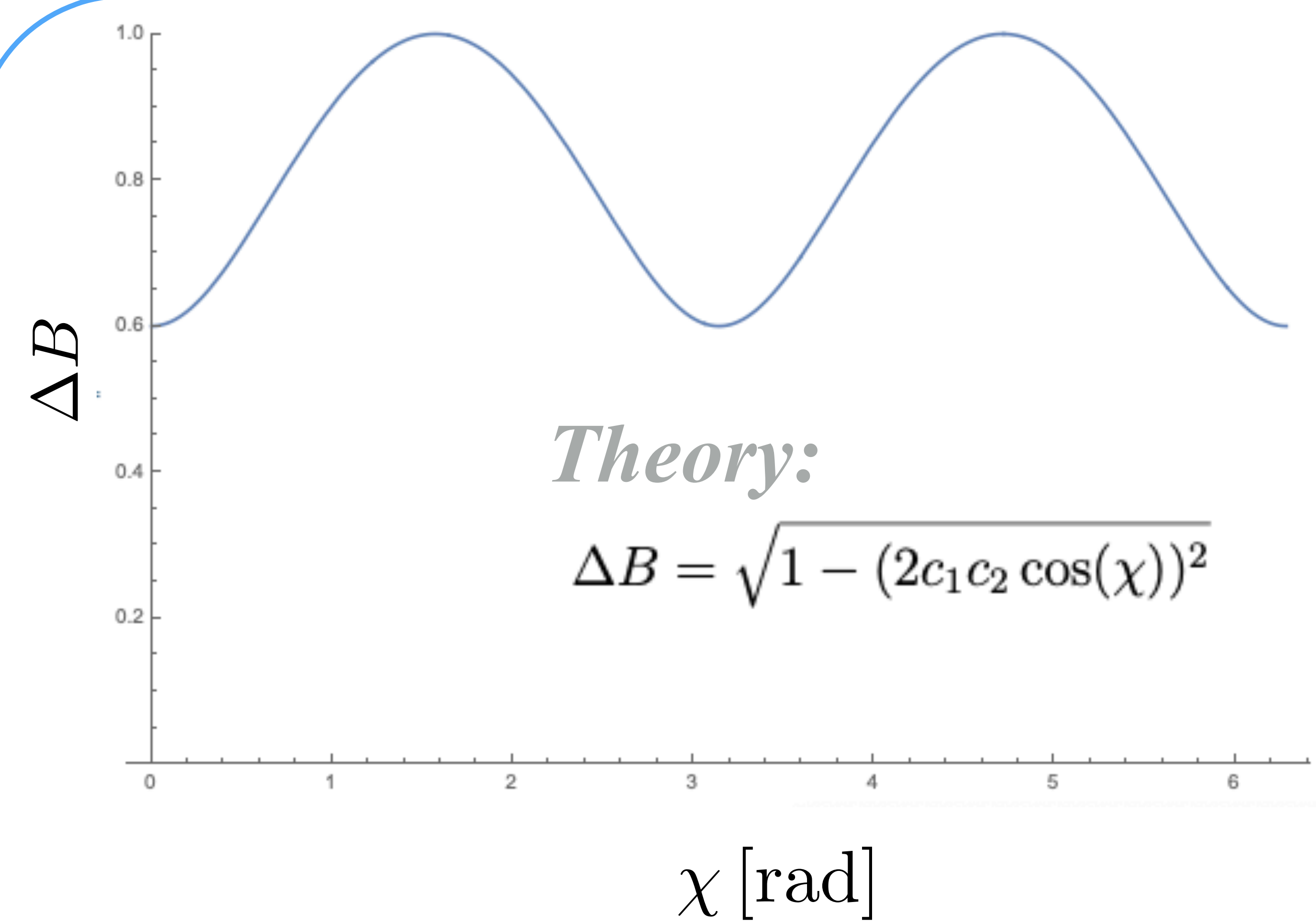
$$\epsilon(A)\eta(B) + \boxed{\epsilon(A)}\boxed{\Delta B} + \eta(B)\Delta A \geq \frac{1}{2}|\langle\psi|[A, B]|\psi\rangle|$$

$$|\psi(\chi)\rangle = \underbrace{\frac{2}{\sqrt{5}}}_{:= c_1}|1\rangle + e^{i\chi}\underbrace{\frac{1}{\sqrt{5}}}_{:= c_2}|2\rangle$$

$$\hat{A} = \hat{\Pi}_1 \qquad \hat{B} = |+\rangle\langle+| - |-\rangle\langle-| := \hat{X} = \hat{\sigma}_x$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$





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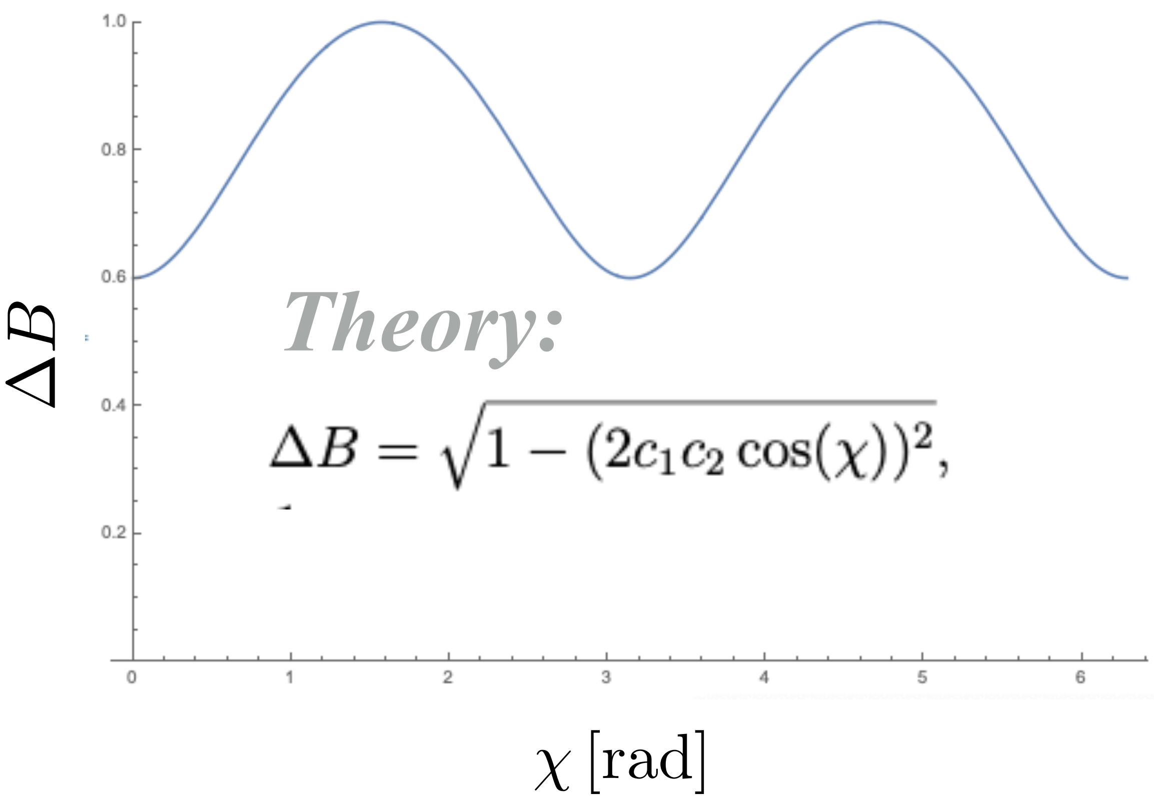


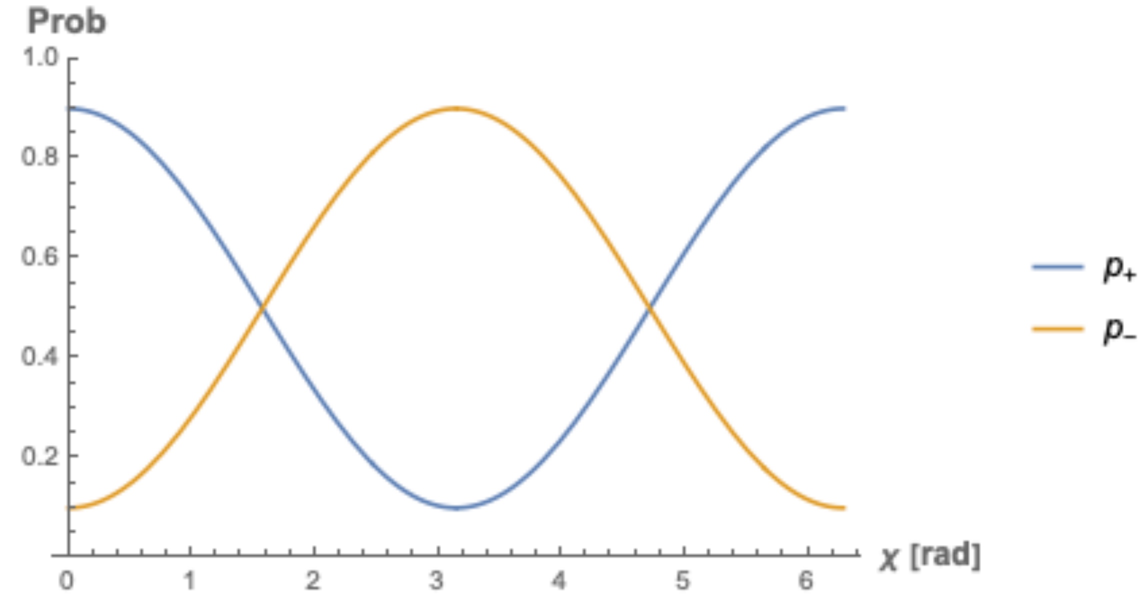
Theory:

$$\Delta B = \sqrt{1 - (2c_1c_2\cos(\chi))^2},$$

$$p_+ = \frac{1}{2} + c_1c_2\cos(\chi)$$

$$p_- = \frac{1}{2} - c_1c_2\cos(\chi).$$

Measurement:



$$\Delta B = \Delta\hat{\sigma}_x = \sqrt{\langle\psi(\chi)|\underbrace{\hat{\sigma}_x^2}_{\mathbb{1}}|\psi(\chi)\rangle - \langle\psi(\chi)|\hat{\sigma}_x|\psi(\chi)\rangle^2}$$

$$= \sqrt{1 - \langle\psi(\chi)|\hat{\sigma}_x|\psi(\chi)\rangle^2}$$

$$= \sqrt{1 - (\langle\psi(\chi)|+\rangle\langle+|\psi(\chi)\rangle - \langle\psi(\chi)|-\rangle\langle-|\psi(\chi)\rangle)^2}$$

$$= \sqrt{1 - (p_+ - p_-)^2}$$

$$= \sqrt{1 - (2c_1c_2\cos(\chi))^2}$$

Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

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$$|\psi(\chi)\rangle = \frac{2}{\sqrt{5}}|1\rangle + e^{i\chi}\frac{1}{\sqrt{5}}|2\rangle$$

$$\hat{A} = \hat{\Pi}_1 = |1\rangle\langle 1|$$
$$\hat{B} = |+\rangle\langle +| - |-\rangle\langle -| := \hat{X} = \hat{\sigma}_x$$

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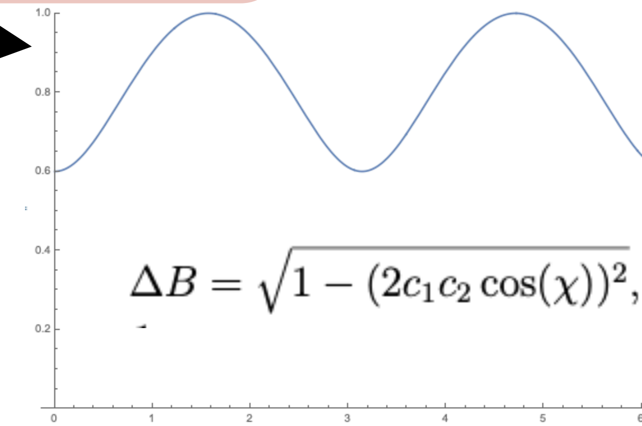
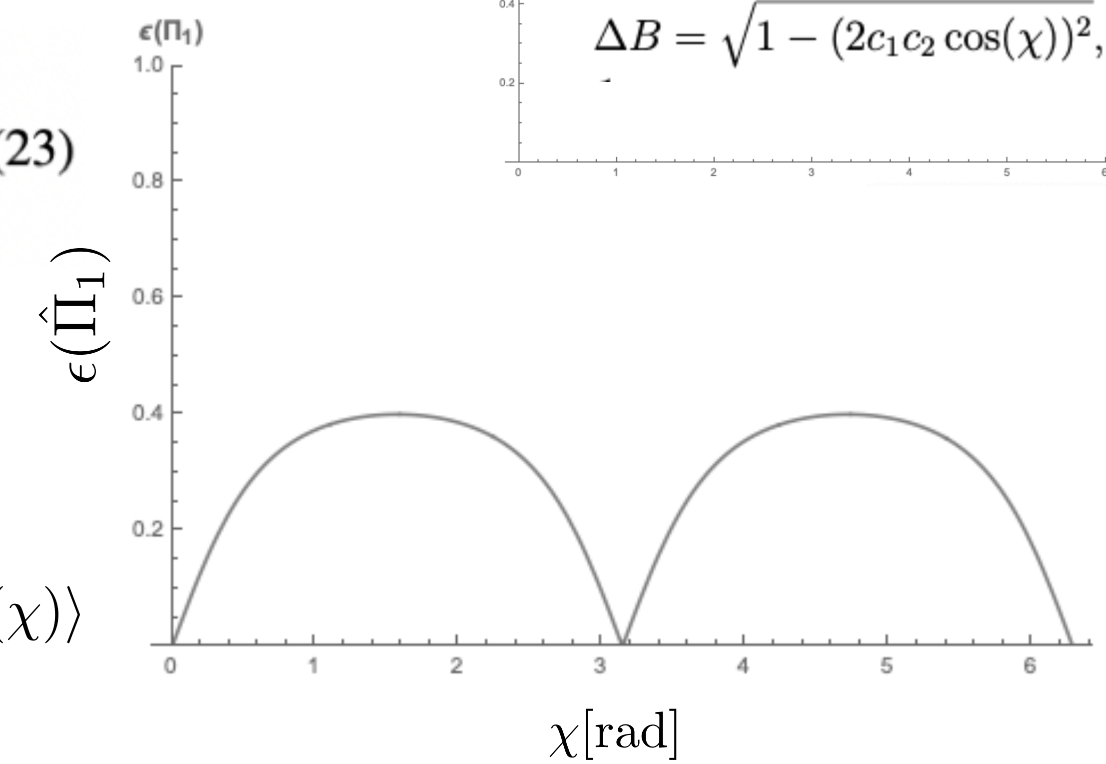


$$\Delta B = \sqrt{1-(2c_1c_2\cos(\chi))^2},$$

$$\varepsilon^2(A) = ||\Big(A - \sum_\lambda \lambda\, O_\lambda\Big)\psi(\chi)\rangle||^2$$

$$O_\lambda = |\pm\rangle\langle\pm|$$

- **Error:** $$\varepsilon^2(\hat{\Pi}_1) = \langle\psi(\chi)|\left[\hat{\Pi}_1 - \sum_{\pm}\frac{\beta_\pm}{\alpha}(\chi)|\pm\rangle\langle\pm|\right]^2|\psi(\chi)\rangle \quad (23)$$



- **Disturbance:** $$\eta^2(B) = \sum_\lambda \langle\psi(\chi)|[O_\lambda, B]^\dagger[O_\lambda, B]|\psi(\chi)\rangle$$
$$= \sum_\lambda ||[O_\lambda, B]|\psi(\chi)\rangle||^2 = 0 \quad \forall|\psi(\chi)\rangle$$

$$\updownarrow$$

$$[|\pm\rangle\langle\pm|, B] = 0$$

Eigenvectors of B are *Approximators* of A

Additional Notes on Ozawa-Hall Error iii) Error - Disturbance Scenario

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$$|\psi(\chi)\rangle = \underbrace{\frac{2}{\sqrt{5}}}_{:=c_1}|1\rangle + e^{\mathrm{i}\chi}\underbrace{\frac{1}{\sqrt{5}}}_{:=c_2}|2\rangle \qquad \hat{A} = \hat{\Pi}_1 = |1\rangle\langle 1| \qquad \hat{B} = |+\rangle\langle+| - |-\rangle\langle-| := \hat{X} = \hat{\sigma}_x$$

$$|\pm\rangle = |1\rangle \pm |2\rangle$$

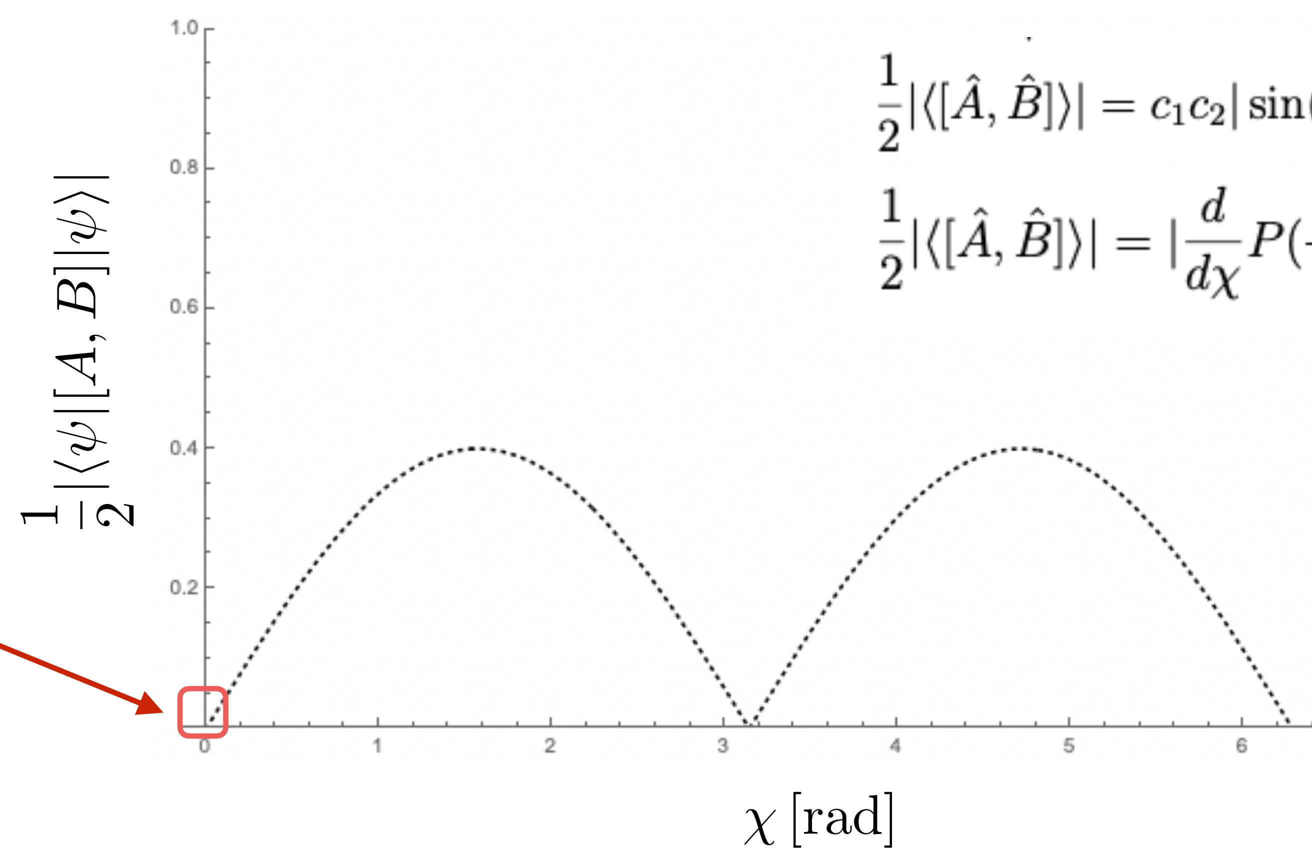
$$[A,B] = c_1c_2e^{i\chi} - c_1c_2e^{-i\chi} = 2c_1c_2\sin(\chi)$$

$$\frac{1}{2}|\langle[\hat{A},\hat{B}]\rangle| = c_1c_2|\sin(\chi)|$$

$$\frac{1}{2}|\langle[\hat{A},\hat{B}]\rangle| = |\frac{d}{d\chi}P(+)|$$

limit:

$$\frac{1}{2}|\langle\psi(\chi=0)|[\hat{\Pi}_1,\hat{X}]|\psi(\chi=0)\rangle| = 0$$



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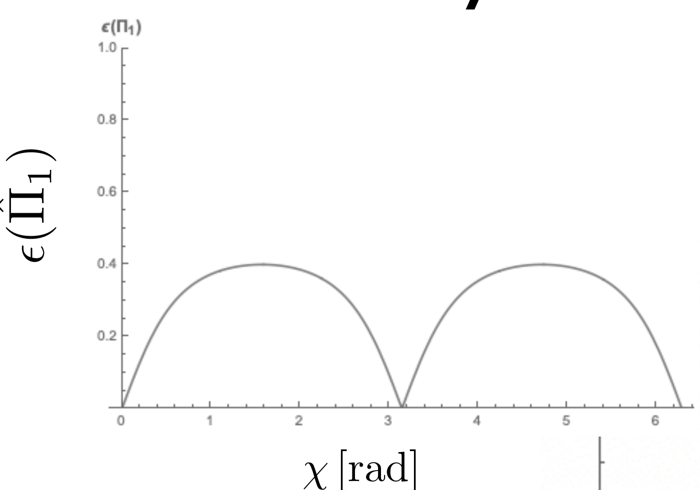

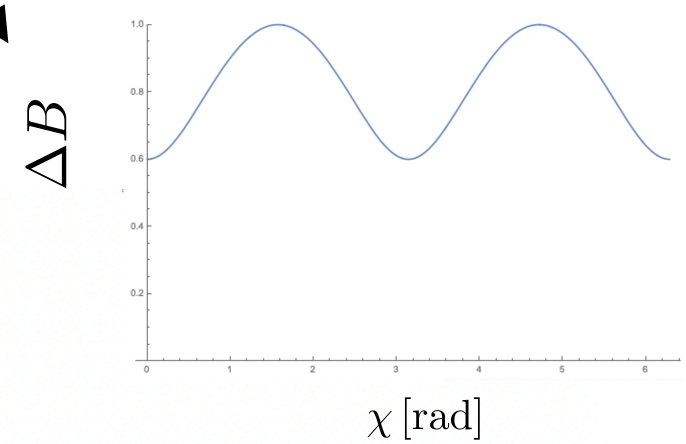

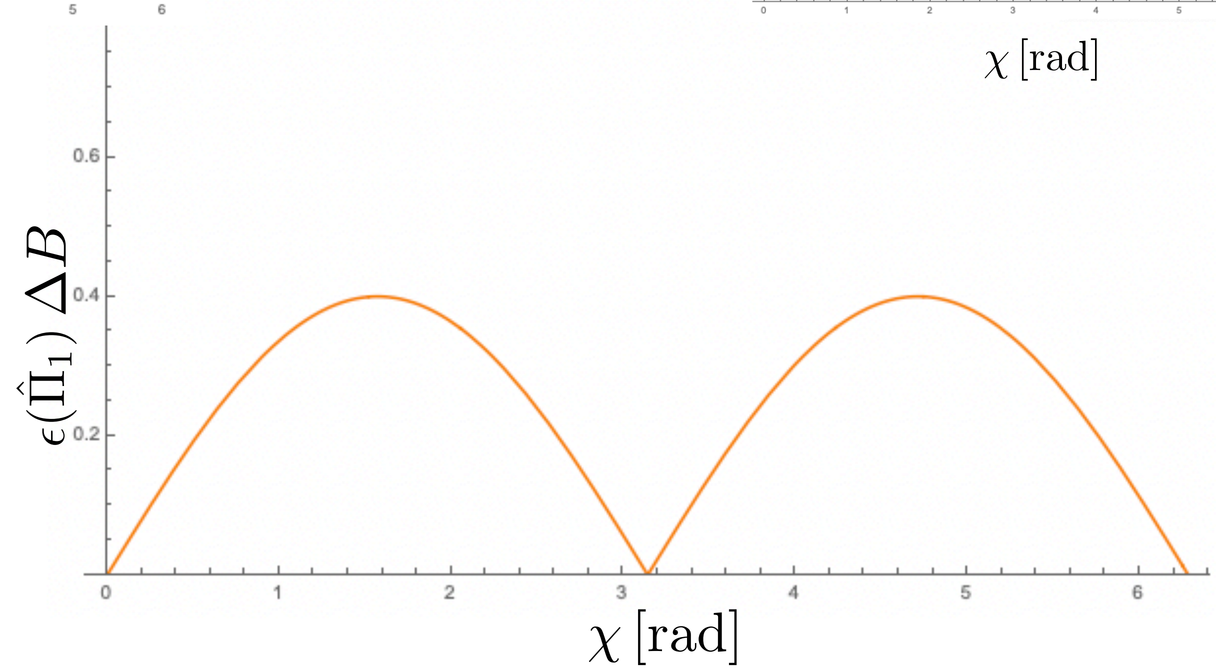

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$$\psi(\chi) = \frac{2}{\sqrt{5}}|1\rangle + e^{i\chi}\frac{1}{\sqrt{5}}|2\rangle \qquad |\pm\rangle = |1\rangle \pm |2\rangle$$

$$\boxed{\varepsilon(A)\Delta(B) = \frac{1}{2}|\langle\psi(\chi)|[A,B]|\psi(\chi)\rangle|, \quad \forall|\psi(\chi)\rangle}$$

$$\frac{1}{2}|\langle\psi(\chi=0)|[\hat{\Pi}_1,\hat{X}]|\psi(\chi=0)\rangle| = 0$$