Coherent multiphoton exchange between a neutron and an oscillating magnetic field

Johann Summhammer
Atominstitut der Österreichischen Universitäten, Schüttelstraße 115, A-1020 Vienna, Austria
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We calculate a neutron's transition amplitudes to various energy and spin states when it interacts with a magnetic field oscillating in time. Two field configurations are analyzed: the oscillating component normal to the static component when the rotating-wave approximation is inapplicable and the oscillating and static components parallel to each other. The two cases require different mechanisms of angular momentum conservation. Experimental tests are proposed including neutron interferometry and double crystal diffraction, the latter permitting observation of quantized energy transfer from fields oscillating at only a few hundred kilohertz.

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I. INTRODUCTION

In many experiments with polarized neutron beams the neutron can be considered as a classical particle with a magnetic dipole moment. This is especially true of the various magnetic-field configurations used to turn the polarization of the beam into a desired direction. The evolution of the dipole in the field is then given by the familiar Bloch equation

$$\frac{\partial \boldsymbol{\mu}}{\partial t}=\gamma \boldsymbol{\mu}\times\mathbf{B}\ , \tag{1}$$

where $\boldsymbol{\mu}$ is the magnetic dipole moment of the neutron, γ is the gyromagnetic factor, and $\mathbf{B}$ is the magnetic induction. The physical picture behind this equation is a description of the development of the dipole in the rest frame of the particle. In the present paper we want to focus on an effect which is not obvious from this equation. This is the change of the neutron's energy by interaction with a time-dependent magnetic field. In particular, we will focus on the experimental situation where a neutron traverses a region with a sinusoidally-time-dependent magnetic field. This represents a region containing electromagnetic radiation of a single mode. Consequently, one expects that the neutron's change of energy should be describable in terms of how many photons it has either absorbed or emitted. In our analysis of this situation we will start from the Schrödinger equation, thereby assuming the neutron a quantum-mechanical two-level system, but treating the magnetic field classically. This will be adequate as long as the electromagnetic field is a superposition of a wide range of different number states with a high mean photon number. Then an exchange of a few photons with the neutron has virtually no influence on the state of the field itself. The electromagnetic fields produced by ac-driven electric coils as normally used with polarized neutrons are well covered by this assumption. However, in order to obtain a better physical understanding, we will try to connect our results to the classical picture of Eq. (1).

In recent years there have been a few experiments where the exchange of energy with the electromagnetic field played an essential role. The one dealing most closely with multiphoton exchange was the reconstruction of the energy-level diagram of neutrons in an oscillating magnetic field [1]. By proper choice of oscillating frequency and field amplitude, Larmor precession could be stopped. In analogy to the same effect arising with atoms in light, the term "dressed neutrons" was coined. The experiment relied on a time-averaged measurement of polarization and therefore could not resolve the energy spectrum of the neutrons after the interaction. A very similar experimental layout was used to observe the Berry phase accumulated by Larmor precession in an oscillating field [2]. The time-resolved neutron detection of that experiment would have permitted some reconstruction of the energy spectrum resulting from the interaction. Up to now, such measurements were done in three experiments, which aimed at finding devices for accelerating or decelerating neutrons. Two of them showed the neutron's change of energy occurring in resonant spin flip [3,4]. The third employed a strong magnetic guide field whose strength varied sinusoidally as a function of time. The energy picked up by the neutron was the difference in Zeeman energy at the time of entrance and at the time of exit of the field region [5]. Then, there were three typical experiments involving neutron interferometry. The first used polarized neutrons and resonant spin flip in one arm to demonstrate coherent superposition of spin-up and spin-down states [6]. Another one employed resonant spin flip at two slightly different frequencies in both arms to obtain interference beats as a result of superposition of different neutron energies [7]. And in a very recent experiment the scalar Aharanov-Bohm effect was demonstrated by having the neutron in one arm enter a coil, then switching on a current and switching it off again before the neutron left the coil [8]. The phase shift induced in the wave function was due to the neutron's different energy level while the field was on.

What is characteristic of all these experiments is that they used essentially only two different configurations of the magnetic field. In those experiments where resonant spin flip was used, the magnetic field had a static component and an oscillating component orthogonal to it.

The other configuration consisted of a static component with the oscillating one parallel to it. The first of these configurations has until now always been described in the rotating-wave approximation (RWA), by replacing the oscillating field component with a rotating one. While being a good approximation when the static component is much larger than the time-dependent one, that description only covers one-photon exchange. For the conditions prevailing in the neutron interferometry experiments [6,7], this was not really sufficient. The second configuration has been briefly described in Ref. [1] without going into the details of the transformation of the wave function. It is therefore the purpose of this paper to fill this gap and present an accurate description of the neutron wave function when the particle traverses a sharply bounded region with a static magnetic field and an additional oscillating one, both components being spatially homogeneous. We will calculate the amplitudes for the individual photon-exchange processes and consider their relative phases as well. The latter is useful for interferometry experiments. We begin in Sec. II with the configuration where the oscillating field component is orthogonal to the static one. In Sec. III we look at the case where both components are parallel. Especially in this latter case, it will be seen that under realistic conditions multiphoton processes can become dominant. In fact, processes involving less than four photons can be completely suppressed. We also emphasize that the two configurations require different explanations for angular momentum conservation during the photon-exchange processes. In the discussion in Sec. IV we look at different experimental methods permitting the observation of multiphoton exchange. Aside from interferometry, a method using double crystal diffraction deserves mention already here, because it permits direct observation of quantized energy exchange in fields oscillating at only a few hundred kilohertz.

II. ORTHOGONAL FIELDS

We assume a nonrelativistic incident neutron of energy $\hbar\omega_0=(\hbar k_0)^2/2m$ and momentum parallel to the y axis. Here, $\hbar$ is Planck's constant, m is the mass of the neutron, and k_0 is its wave number. The trajectory is divided into the three segments of before, in, and after the oscillating field region. The segments are labeled I, II, and III, respectively. In keeping with experiments we assume a homogeneous magnetic field B_0 parallel to the z axis over all three regions. In region II, which extends from $y=0$ to $y=L$, there exists in addition to the guide field an oscillating component parallel to the x axis:

$$\mathbf{B}(t)=\begin{bmatrix} B_1\cos(\omega t+\varphi) \\ 0 \\ B_0 \end{bmatrix} . \tag{2}$$

Ideally, one wants to solve the Schrödinger equation in the three regions and then match the individual solutions at the boundaries. One would then have the exact dependence of the neutron's wave function on position and time. However, such a solution is analytically very cumbersome, especially in region II. We will therefore try to find a separation of position and time coordinates, thinking of the neutron as of a particle that spends a time $t_F=(mL/\hbar k_0)$ in the field region. The time t_F is the time of flight of a classical particle through region II, when the magnetic potential is negligible with respect to the neutron's energy. This is well justified for neutrons in the thermal and even the cold regimes for laboratory magnetic fields. (For instance, the kinetic energy of very cold neutrons of a wavelength of 100 Å is still two orders of magnitude larger than their potential energy in a field of 1 T.) In addition, when this condition is satisfied, it is possible to neglect reflection at the boundaries. The Schrödinger equation in region II originally reads

$$\left[-\frac{\hbar^2}{2m}\frac{\partial}{\partial y^2}+\mu\begin{bmatrix} B_0 & B_1\cos[\omega(t+t_0)] \\ B_1\cos[\omega(t+t_0)] & -B_0 \end{bmatrix}\right] \times\psi_{\mathrm{II}}(y,t)=i\hbar\frac{\partial}{\partial t}\psi_{\mathrm{II}}(y,t) . \tag{3}$$

From this we obtain the transformation into a purely time-dependent form by the ansatz

$$\psi_{\mathrm{II}}(y,t)=\begin{bmatrix} F^+(t) \\ F^-(t) \end{bmatrix} e^{ik_0y-i\omega_0t} . \tag{4}$$

This is a plane wave with the momentum of the free particle multiplied by a time-dependent spinor. Then Eq. (3) is reduced to

$$i\hbar\frac{\partial}{\partial t}\begin{bmatrix} F^+(t) \\ F^-(t) \end{bmatrix} =\mu\begin{bmatrix} B_0 & B_1\cos[\omega(t+t_0)] \\ B_1\cos[\omega(t+t_0)] & -B_0 \end{bmatrix}\begin{bmatrix} F^+(t) \\ F^-(t) \end{bmatrix} . \tag{5}$$

Both this and the Bloch equation (1) are widely used in nuclear magnetic resonance [9]. Equation (5) is the correct equation for two-level systems which do not propagate from one potential region to the other , but which are exposed to different potentials in temporal sequence. Before applying it to our own case, we should therefore establish what possible errors we commit thereby.

By crossing a potential step into the oscillating field of region II, the state of the incident neutron would be split into a discrete range of different momentum states. (This will be seen explicitly in Sec. III for the more tractable case of parallel fields.) These would accumulate different spatial phases through region II which would ultimately result in different relative phases of the various energy states in region III. Another effect associated with changed momenta would be different times of flight through region II, which in turn would affect the probabilities for the various photon-exchange processes [10]. A measure of the deviation in phase of a given energy state in region III between the purely time-dependent picture and the correct spatiotemporal one is given by the difference between the Larmor angle accumulated by a neutron traversing the field region with its vacuum momentum and one with momentum either increased or decreased by an amount corresponding to the potential energy in the typical magnetic fields of region II. From our numerical example above, it is clear that this is small indeed. We are therefore entitled to start from Eq. (5).

In that equation t represents the time the particle has already spent in region II and hence it only varies between 0 and t_F. The time of entrance into the field is given by t_0. Unfortunately, Eq. (5) has no simple analytic solution except when $B_1 \ll B_0$ or $B_1 \gg B_0$. In the first case the RWA holds such that the oscillating field component can be replaced by one rotating in the x-y plane. We shall not deal with this case here, as it permits only one-photon exchange. The second case is better represented by the configuration of parallel fields discussed in Sec. III. There are a number of treatments of Eq. (5) in connection with two-level systems interacting with electromagnetic radiation [11–14]. A general solution was given by Shirley [15], who also presents the link with the solution obtained when the electromagnetic field is no longer treated classically. Our analysis is based on Shirley's, but we focus on questions relevant with thermal neutrons. According to the Floquet theory of differential equations, a solution to Eq. (5) can be obtained by the ansatz [16]

$$\begin{pmatrix} F^+(t) \\ F^-(t) \end{pmatrix} = \sum_{n=-\infty}^{\infty} \begin{pmatrix} f_n^+ \\ f_n^- \end{pmatrix} e^{-iqt} e^{in\omega(t+t_0)} . \tag{6}$$

The parameter q is called a Floquet coefficient. The coefficients f_n^+ and f_n^- represent spin-up and spin-down states, respectively. When inserting Eq. (6) into Eq. (5) and writing the cosine by complex exponentials, one obtains an infinite set of linear equations. We now introduce dimensionless quantities but will refer to them by the same names as the quantities they represent:

$$\alpha = \frac{\mu B_1}{\hbar\omega} , \tag{7a}$$

$$\beta = \frac{\mu B_0}{\hbar\omega} , \tag{7b}$$

$$\gamma = \frac{q}{\omega} , \tag{7c}$$

$$\tau = \frac{\omega t_F}{2\pi} . \tag{7d}$$

Then the infinite set of equations reads

$$\begin{pmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & (n+1+\beta) & 0 & 0 & \alpha/2 & 0 & 0 & \cdots \\ \cdots & 0 & (n+1-\beta) & \alpha/2 & 0 & 0 & 0 & \cdots \\ \cdots & 0 & \alpha/2 & (n+\beta) & 0 & 0 & \alpha/2 & \cdots \\ \cdots & \alpha/2 & 0 & 0 & (n-\beta) & \alpha/2 & 0 & \cdots \\ \cdots & 0 & 0 & 0 & \alpha/2 & (n-1+\beta) & 0 & \cdots \\ \cdots & 0 & 0 & \alpha/2 & 0 & 0 & (n-1-\beta) & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} \cdots \\ f_{n+1}^+ \\ f_{n+1}^- \\ f_n^+ \\ f_n^- \\ f_{n-1}^+ \\ f_{n-1}^- \\ \cdots \end{pmatrix} = \gamma \begin{pmatrix} \cdots \\ f_{n+1}^+ \\ f_{n+1}^- \\ f_n^+ \\ f_n^- \\ f_{n-1}^+ \\ f_{n-1}^- \\ \cdots \end{pmatrix} . \tag{8}$$

For large positive or negative values of n, the coefficients f_n^+ and f_n^- will tend to zero such that this infinite set of eigenvalue equations can be suitably truncated and solved numerically. The numerical solution is much facilitated, furthermore, by recognizing that the system (8) is comprised of two independent but interlaced sets of equations. These can be written as

$$\alpha f_{2n+1}^- + 2(\beta+2n) f_{2n}^+ + \alpha f_{2n-1}^- = 2\gamma f_{2n}^+ , \tag{9a}$$

$$\alpha f_{2n+1}^+ + 2(-\beta+2n) f_{2n}^- + \alpha f_{2n-1}^+ = 2\gamma f_{2n}^- . \tag{9b}$$

In Eq. (9a) the even f^+ components are related to the two neighboring odd f^- components and in Eq. (9b) it is the other way around. This is physically relevant. Since the f^+ components are associated with the spin-up state of the neutron and the f^- components with the spin-down state, these equations imply that a change of the neutron's energy by an odd number of photonic quanta $\hbar\omega$ will be accompanied by a spin flip. And, conversely, a change in energy by an even multiple of $\hbar\omega$ will leave the neutron's spin state unaffected. The eigenvectors of Eqs. (9a) and (9b) are

$$(\ldots, f_{2n+2}^+, f_{2n+1}^-, f_{2n}^+, f_{2n-1}^-, f_{2n-2}^+, \ldots) \tag{10a}$$

and

$$(\ldots, f_{2n+2}^-, f_{2n+1}^+, f_{2n}^-, f_{2n-1}^+, f_{2n-2}^-, \ldots) . \tag{10b}$$

A practical advantage of decomposing system (8) into Eqs. (9a) and (9b) is that the eigenvectors (10a) and (10b) are numerically more stable as a function of cutoff at $-n_{\max}$ and $+n_{\max}$ than calculating directly eigenvectors for Eq. (8) [17]. The two sets of eigenvalue equations (9a) and (9b) do not have the same eigenvalues. An eigenvector to the complete system (8) is thus obtained by taking any solution (10a) or (10b) and writing it in the form of the eigenvector to the system (8) by inserting zeros for those components which are not contained in the vector chosen. The eigenvalue is then also an eigenvalue to the eigenvector of the system (8). It follows from the Floquet theory that an eigenvalue γ of the system (9a) has a counterpart eigenvalue $-\gamma$ in system (9b). From the periodic structure of system (8) it can be seen that when γ is an eigenvalue, so must be γ plus an arbitrary integer [15]. Thus the complete set of eigenvalues is defined by γ_0+j and $-\gamma_0-j$, j running through all integers, and γ_0 being the smallest positive eigenvalue. Since there are infinitely many eigenvalues and hence eigenvectors, it might seem that there are infinitely many solutions to the ansatz (6). But when inserting into Eq. (6) an eigenvalue γ_0+j and the corresponding eigenvector, the result is the same as when inserting the eigenvalue γ_0 and its respective eigenvector. This is because the coefficients of any two eigenvectors whose eigenvalues differ by an integer j are shifted by j positions relative to each other, but are otherwise identical. Hence there are only two physically significant eigenvectors. They can be taken as the ones corresponding to the eigenvalues γ_0 and $-\gamma_0$, respectively. We will denote these eigenvectors by g and h.

The general solution in the field region is then a linear combination of these two independent solutions:

$$\psi_{\text{field}}(t,t_0)=\sum_{n=-\infty}^{\infty}\left[c\begin{bmatrix}g_n^+\\ g_n^-\end{bmatrix}e^{-i\gamma_0\omega t}+d\begin{bmatrix}h_n^+\\ h_n^-\end{bmatrix}e^{+i\gamma_0\omega t}\right]\times e^{in\omega(t+t_0)} . \tag{11}$$

The complex constants c and d are obtained by matching Eq. (11) for $t=0$ to the initial state of the neutron incident from region I, given by the spin amplitudes z^+ and z^-:

$$\begin{bmatrix}z^+\\ z^-\end{bmatrix}=\sum_{n=-\infty}^{\infty}\left[c\begin{bmatrix}g_n^+\\ g_n^-\end{bmatrix}+d\begin{bmatrix}h_n^+\\ h_n^-\end{bmatrix}\right]e^{in\omega t_0} . \tag{12}$$

Using expressions for c and d derived from this equation, one can then match Eq. (11) to the general state expected in region III, which must be periodic with the frequency of the oscillating field, ω,

$$\psi_{\text{final}}(t_0)=\sum_{n=-\infty}^{\infty}\begin{bmatrix}T_n^+\\ T_n^-\end{bmatrix}e^{in\omega t_0} , \tag{13}$$

with the T_n being transition amplitudes. In particular, T_n^+ is the probability amplitude of finding the neutron in the spin-up state with an energy differing from its original energy by $n\hbar\omega$. The T_n^- have an analogous meaning for the spin-down state. They are obtained by equating (11) and (13) for $t=t_F$, multiplying the resulting equation by $\exp(-in\omega t_0)$ and integrating over t_0. Then, with the use of the dimensionless time of flight τ [Eq. (7d)], the transition amplitudes can be expressed as a function of the original state of the neutron:

$$T_n^+=\sum_{j=-\infty}^{\infty}e^{ij2\pi\tau}[g_j^+e^{-i2\pi\gamma_0\tau}(z^+h_{n-j}^- - z^-h_{n-j}^+) + h_j^+e^{+i2\pi\gamma_0\tau}(z^-g_{n-j}^+ - z^+g_{n-j}^-)] , \tag{14a}$$

$$T_n^-=\sum_{j=-\infty}^{\infty}e^{ij2\pi\tau}[g_j^-e^{-i2\pi\gamma_0\tau}(z^+h_{n-j}^- - z^-h_{n-j}^+) + h_j^-e^{+i2\pi\gamma_0\tau}(z^-g_{n-j}^+ - z^+g_{n-j}^-)] . \tag{14b}$$

If originally the neutrons are in the spin-up state ($z^-=0$, $z^+=1$, neglecting the phase), these amplitudes reduce to

$$S_n^+=\sum_{j=-\infty}^{\infty}e^{ij2\pi\tau}(g_j^+h_{n-j}^-e^{-i2\pi\gamma_0\tau}-h_j^+g_{n-j}^-e^{+i2\pi\gamma_0\tau}) , \tag{15a}$$

$$S_n^-=\sum_{j=-\infty}^{\infty}e^{ij2\pi\tau}(g_j^-h_{n-j}^-e^{-i2\pi\gamma_0\tau}-h_j^-g_{n-j}^-e^{+i2\pi\gamma_0\tau}) . \tag{15b}$$

It is sufficient to discuss these amplitudes because, if the initial state were spin down, the magnitudes of the corresponding transition amplitudes would be the same. But before doing so, we will try to relate this formally derived result to the one which would have been afforded by the Bloch equation [Eq. (1)]. There, a dipole passing the oscillating field region would undergo rotation. It would be quite a complex one, because the magnetic field changes magnitude as well as direction as a function of time. But at the exit of the oscillating field region, the polarization would be the same as that obtainable from Eq. (11). However, the Bloch equation would not yield any information about the energy of the neutron. By contrast, that information is provided naturally by the quantum description because it demands that the general state of a neutron in the time-independent potential of region III must be given by a superposition of different energy eigenstates. We expressed this by the ansatz (13). There we made further use of the fact that the field is periodic in time and that the solution in region III would have to have the same periodicity. Therefore, the difference between initial and final states of the neutron could only be in multiples of the lowest frequency of the magnetic field multiplied by Planck's constant. This is how the concept of photon exchange enters our analysis despite treating the field classically. The essential element is the wave description of the system that is subjected to a periodic potential. This can even be a classical wave as in a recent experiment showing multiphoton resonance in a light wave by switching its polarization periodically [18]. As was also pointed out in Ref. [1], the description of quantized energy transfer from an oscillating magnetic field need therefore not involve quantization of the field itself. However, when the electromagnetic field has only a low photon density, deviations between classical and quantized field description become apparent. But presently it seems difficult to observe such effects with neutrons, due to their small magnetic moment. For a clear account of how the quantized field of high mean photon number becomes equivalent to the classical one, but also on the differences that would occur in the Hamiltonian of Eq. (8), we refer to Shirley [15]. We now continue discussing the results of Eqs. (14) and (15).

As was observed in connection with Eqs. (10a) and (10b) and as is well known from nuclear magnetic resonance [9], the non-spin-flip amplitudes S^+ will vanish for odd n. This fits with a simple picture of angular momentum conservation: Pairs of photons of opposite polarization have no angular momentum and when the neutron exchanges such pairs with the field, its spin state is unaffected. It is also noteworthy that with even-number photon exchange the magnitudes for emission and for absorption are equal:

$$|S_{2n}^{+}|=|S_{-2n}^{+}| \ . \tag{16}$$

The spin-flip amplitudes S^- vanish for even-number photon exchange. Spin flip occurs only by exchanging an odd number of photons with the field, again permitting a simple visualization in terms of angular momentum conservation, when considering that neutrons have spin $\frac{1}{2}$ while photons have spin 1.

A graphical presentation of the general behavior of the transition amplitudes is difficult because they are governed by three independent parameters: the amplitude of the oscillating field (α), the static component of the field (β), and the time of flight through the field region (τ). We just present an example with experimental conditions easily achieved in one arm of a single-crystal neutron interferometer (Fig. 1). The value of the static magnetic field is chosen to fit the one-photon resonance condition ($\beta=0.5$). The dashed vertical line indicates roughly the parameters of an experiment already done [7]. One notices that significant amplitudes even for five- and six-photon exchange can be obtained. Measurability is, of course, simplified by the temporal modulation of the interference pattern being directly proportional to the transition amplitudes and not just to their squares. This is in contrast to experiments with two-level atoms, where observation of an exchange of two photons [19] and three photons [20] is done by measuring the absorbed power

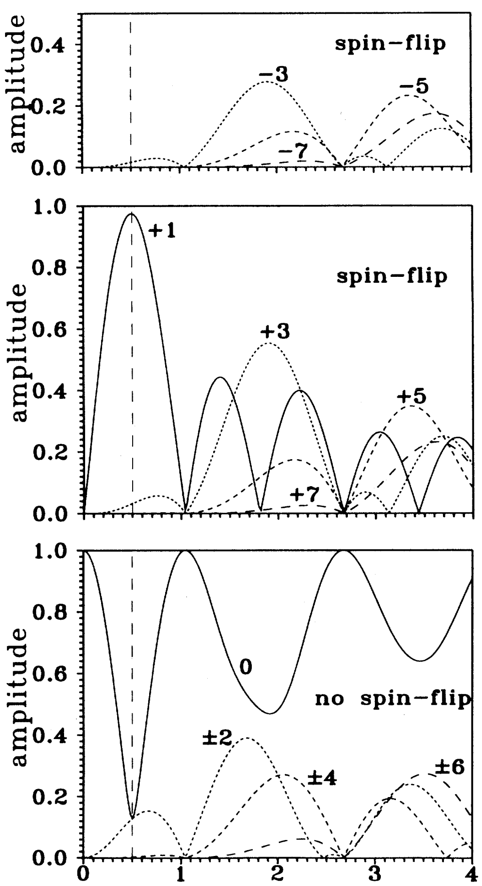

FIG. 1. Energy and spin transition amplitudes for a neutron passing a magnetic field with a static component and a sinusoidally oscillating component normal to it, as a function of the strength of the oscillating component. The neutron originally is polarized parallel to the static field component. Numbers indicate the number of absorbed (+) or emitted (−) photons. Only an exchange of an odd number of photons is accompanied by a spin transition. Strength of the static field component: $\beta=0.5$. Time of flight through the field region: $\tau=1$. The dashed vertical line indicates the conditions of experiment of Ref. [7].

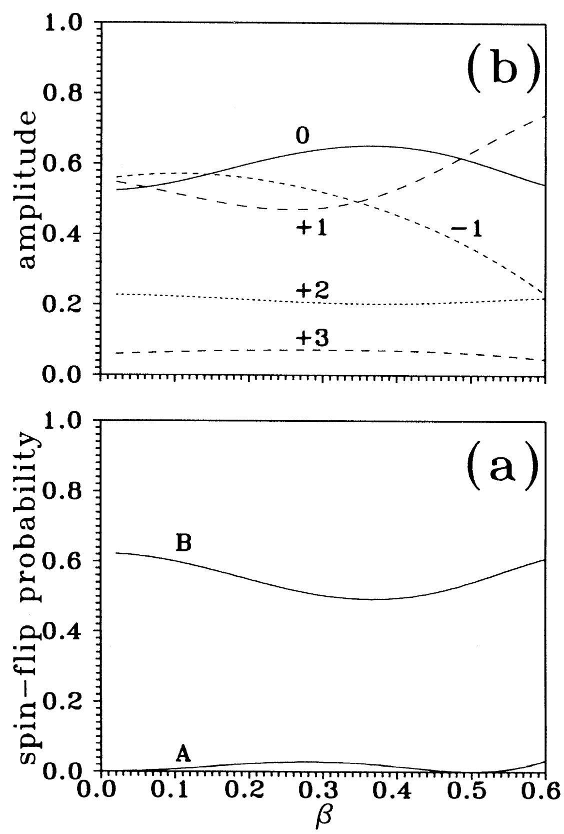

FIG. 2. "Dressed-neutron" effect. Spin-flip probability and the various energy transition amplitudes become quite independent of the static magnetic-field component at a certain strength of the oscillating component (here, $\alpha\approx1.04$). (a) A given time of flight results in a more or less constant spin-flip probability. Curve A, $\tau=1$, curve B, $\tau=1.25$. (b) Only slow variation of various energy transition amplitudes. Parameters as in (a), curve B.

which is proportional to the transition probabilities (see also [21] and references therein). The most interesting situation is in the range $\alpha \approx \beta$ to $\alpha \approx 4\beta$. For smaller values of α, one approaches the case of the RWA, enabling only one-photon exchange and most often used for spin-flip devices [3,22], although spin-flip devices far away from this condition and for applications other than interferometry have also been tested [23]. And for values of α much larger than β, the static guide field becomes increasingly unimportant. The transition amplitudes are then better calculated from a purely oscillating field discussed in Sec. III.

In Fig. 1 the first one-photon absorption resonance is the strongest and one-photon emission is virtually absent. This is due to the choice $\beta=0.5$ and $\tau=1$. But it should be mentioned that with another choice of β, the one-photon emission can also be made large. In typical nuclear magnetic resonance (NMR) conditions one has $\tau \gg 1$. Then the one-photon resonance occurs at small values of α and is very narrow. One has then only a small Bloch-Siegert shift [24], implying that β has to be changed only a little from 0.5 to obtain maximum spin flip. Under conditions as in Fig. 1, the Bloch-Siegert shift is quite large. For the given τ, the maximum spin-flip probability of 0.964 is reached at $\beta=0.469$ and $\alpha=0.499$. The amplitude for one-photon absorption is then 0.981 and that for one-photon emission is 0.031.

It is worth considering the influence of the eigenvalue γ_0. When $\gamma_0=0$, the transition amplitudes are periodic in the time of flight τ, the period being that of the oscillating field. An appropriate choice of τ leads to a complete suppression of spin flip and of any photon-exchange processes, as seen in Fig. 1 at $\alpha=1.04$ and at $\alpha=2.68$. This is the "dressed-neutron" effect analyzed experimentally in Ref. [1], where due to interaction with the photons of the field, the apparent gyromagnetic factor of the neutron tends to zero and Larmor precession comes to a halt. In a wide range of the static field, the eigenvalue stays small and so does the spin-flip probability. This is shown in Fig. 2(a), curve A. But with another choice of τ, one can similarly achieve a certain finite spin-flip probability, which is also quite independent of β, as shown in Fig. 2(a), curve B. Then the transition amplitudes also vary only slowly [Fig. 2(b)].

III. PARALLEL FIELDS

Now we consider the case where the oscillating magnetic field is parallel to the static field. Just as in Sec. II we assume a static field throughout space and a region with an additional oscillating component extending from $y=0$ to $y=L$. Both fields are parallel to the z axis. The incident neutron shall again have momentum in vacuum of $\hbar k_0$ parallel to the y axis. The Schrödinger equation in the region with the oscillating field is

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2}+\mu\begin{bmatrix} B_0+B_1\cos(\omega t) & 0 \\ 0 & -B_0-B_1\cos(\omega t)\end{bmatrix}\right] \times \psi_{\mathrm{II}}(y,t)=i\hbar\frac{\partial}{\partial t}\psi_{\mathrm{II}}(y,t) . \quad (17)$$

The two spin components are decoupled. It therefore suffices to solve the equation for one component, as was done in analytical detail in another context in Ref. [25]. A solution for two-level atoms is for instance given in Ref. [11]. Different from the previous case, Eq. (17) has a simple closed solution without having to neglect the space coordinate. We will present this solution here, because especially in interferometry it is useful to know the phase as a function of space and time coordinates. The incident plane wave present in region I is

$$\psi_{\mathrm{I}}(y,t)=\begin{bmatrix} ae^{iK^+y} \\ be^{iK^-y}\end{bmatrix} e^{-i\omega_0 t} , \quad (18)$$

with

$$K^{+(-)}=\left[k_0^2-(+)\frac{2\mu B_0 m}{\hbar^2}\right]^{1/2} \quad (19)$$

and

$$\omega_0=\frac{\hbar k_0^2}{2m} . \quad (20)$$

We neglect reflection with the same arguments as in Sec. II. The complex constants a and b depend on the initial polarization. Region II contains the oscillating field. A solution of Eq. (17) is

$$\psi'_{\mathrm{II}}(y,t)=\begin{bmatrix} ue^{iK^+y-i\alpha\sin(\omega t)} \\ ve^{iK^-y+i\alpha\sin(\omega t)}\end{bmatrix} e^{-i\omega_0 t} , \quad (21)$$

where α is defined as before in Eq. (7a) and u and v are constants. It must however be matched to the incident plane wave at $y=0$ for all times. This leads to an ansatz consisting of a sum of differently excited states, the excitation energy being in units of the photon energy:

$$\psi_{\mathrm{II}}(y,t)=\sum_{n=-\infty}^{\infty}\begin{bmatrix} c_n e^{iK_n^+y-i\alpha\sin(\omega t)} \\ d_n e^{iK_n^-y+i\alpha\sin(\omega t)}\end{bmatrix} e^{-i\omega_n t} \quad (22)$$

with

$$K_n^{+(-)}=\left[k_0^2+\frac{2m}{\hbar}n\omega-(+)\frac{2\mu B_0 m}{\hbar^2}\right]^{1/2} \quad (23)$$

and

$$\omega_n=\omega_0+n\omega . \quad (24)$$

As a result of matching, the excitation amplitudes c_n and d_n are obtained as Bessel functions:

$$c_n=a(-1)^n J_n(\alpha) , \quad (25a)$$

$$d_n=bJ_n(\alpha) . \quad (25b)$$

From this the wave function in region III can be calculated by matching at the boundary $y=L$ for all times. The general solution in region III is similar to the one of Sec. II [Eq. (13)]

$$\psi_{\mathrm{III}}(y,t)=\sum_{n=-\infty}^{\infty}\begin{bmatrix} T_n^+ e^{iK_n^+y} \\ T_n^- e^{iK_n^-y}\end{bmatrix} e^{-i\omega_n t} , \quad (26)$$

where the transition amplitudes T_n^+, T_n^- turn out to be

$$\begin{bmatrix} T_n^+ \\ T_n^- \end{bmatrix} = \begin{bmatrix} a(-1)^n e^{i(K_j^+ - K_n^+)L} \\ b e^{i(K_j^- - K_n^-)L} \end{bmatrix} \sum_{j=-\infty}^{\infty} J_j(\alpha) J_{j-n}(\alpha) . \tag{27}$$

A further reduction of this result can be obtained by the approximation

$$(K_j^+ - K_n^+)L \approx k_0 L (j-n) \frac{\omega}{2\omega_0} , \tag{28}$$

which is also valid for $(K_j^- - K_n^-)$. It should be mentioned that this approximation is implicitly contained in the solution found in Sec. II, by not having taken into account the spatial coordinate. It amounts to neglecting the higher orders of the different momenta of spin-up and spin-down states due to Zeeman splitting of a neutron of given energy. Rewriting Eq. (28) in terms of the dimensionless time of flight τ [Eq. (7d)] of a neutron of momentum $\hbar k_0$ over a distance L and making use of the summation laws for Bessel functions [26], one finally obtains

$$\begin{bmatrix} T_n^+ \\ T_n^- \end{bmatrix} = \begin{bmatrix} a \\ b(-1)^n \end{bmatrix} e^{-in(2\pi\tau+\pi)/2} J_n(2\alpha \sin(\pi\tau)) . \tag{29}$$

As was to be expected from the absence of off-diagonal terms in the Hamiltonian, there is no change in the occupation probabilities of the two levels when a neutron passes this kind of oscillating field. A monoenergetic neutron originally in the spin-up state will not change its spin state, but will turn into a superposition of different energy states. Just as in the case of orthogonal fields, it is clear that this result could not have been obtained from the Bloch equation, which only covers the dynamics of the polarization vector. What is interesting, moreover, in Eq. (29) when compared to the result of Sec. II, Eqs. (14), is that there are no restrictions on the number of photons exchanged. In contrast to the orthogonal field configuration, the neutron may now absorb or emit an odd number of photons without spin flip. Therefore, a visualization of angular momentum conservation by just considering neutron plus exchange photons is not possible. Rather, the field must take up some angular momentum, too. This question also arises in experiments with two-level atoms [21].

The probabilities of emission and of absorption of a given number of photons are the same, independent of whether an even or an odd number of photons are exchanged. The transition amplitudes are periodic as a function of the time of flight through the oscillating field, the periodicity being twice the cycle of the oscillating field. This is reminiscent of the 4π periodicity of spinor rotation [27,28]. But the transition probabilities are periodic with each cycle of the field. When the time of flight is exactly one cycle, all photon-exchange processes stop, independent of the strength α of the oscillating field. The transition amplitudes do not depend on the static component of the magnetic field. But its influence is of course manifest in the phases in region III. The magnitudes of the transition amplitudes do not depend separately on α and τ, as can be seen from the argument in the Bessel functions. It is proportional to the maximum difference in the Zeeman energy a neutron may be exposed to in the oscillating field. A given value of this quantity can be achieved by infinitely many different combinations of field strength and time of flight. Therefore, all possible experimental results can be depicted by one generic set of curves, shown in Fig. 3. From these curves one notes that whenever the amplitudes for n- and for $(n+2)$-photon exchange are equal, the amplitude for $(n+1)$-photon exchange has either a local maximum or it vanishes. This can be exploited to simultaneously suppress $(n+1)$-, $(n+2)$-, and $(n+3)$-photon exchange for any n. We will analyze this effect for $n=0$. The original neutron beam has to be polarized normal to the direction of the oscillating field, i.e., $|a|=|b|$, in Eq. (29). Then the amplitude of the oscillating field and the time of flight through the region have to be such that there is no two-photon exchange as at $\alpha \approx 2.67$ in Fig. 3. Furthermore, a static spin-turn device rotating the polarization vector by 90° around an axis in the x-y plane has to be positioned at the proper distance behind the oscillating field region. In this way one can make the spin-up component contain only even transition amplitudes and the

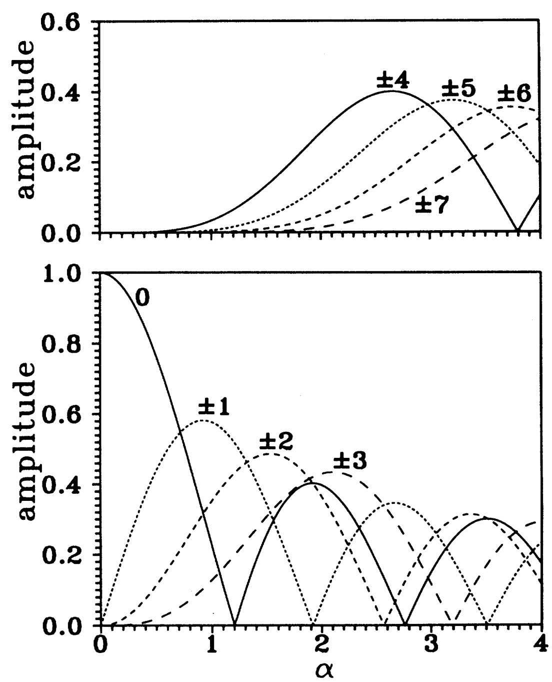

FIG. 3. Energy transition amplitudes for a neutron passing a magnetic field consisting of a static component and a sinusoidally oscillating component parallel to it, as a function of the strength of the oscillating field. The neutron originally is polarized parallel to the field. No spin transitions occur, but even- and odd-number photon exchange is possible. Amplitudes for absorption and emission of a given number of photons are equal. The static field component has no influence on the magnitudes of the transition amplitudes. Time of flight through the field region: $\tau=0.5$.

spin-down component only odd ones or the other way around. This is due to the factor $(-1)^n$ in Eq. (29). If a subsequent polarizer selects the spin state consisting of even components, the resulting beam contains only the following components:

$$\ldots, -6, -4, 0, +4, +6, \cdots .$$

IV. DISCUSSION

For the measurement of multiphoton exchange, various methods can be envisaged. Interferometry is the most accurate one, because it is sensitive directly to the transition amplitudes. They are obtained by placing the region with the oscillating magnetic field in only one arm and measuring the spectrum of the time-dependent interference signal. In Fig. 4 a possible layout for the experiment is shown. Similar experiments not aimed at measuring photon-exchange processes have already been done [6–8]. With the available intensities, visibilities of the interference pattern of a few percent can be resolved. This would permit observation of multiphoton-exchange probabilities as low as 10^{-3} to 10^{-4}.

Such an experiment again accentuates the question of whether multiphoton exchange constitutes determination of the neutron's path [29]. It has been shown that this is not the case with coherent states of the electromagnetic field of high mean photon number as produced by a coil driven by an ac current [30]. However, if the electromagnetic field is a superposition of only a few different number states, it does become sensitive to an increase or decrease by a few photons [31]. Multiphoton exchange may therefore facilitate such path detection, which is nondestructive as the neutron is not absorbed in a nuclear reaction. But as mentioned above, with present technology an experimental realization seems difficult.

Another question contrasting quantum and classical concepts is raised by the linearity of the interference signal and the square root of photon-exchange probabilities. If only 1% of the neutrons of the respective path exchange photons with the field, this will result in a 10% modulation of the interference intensity. Hence, minima and maxima of the count rate differ by much more than the number of neutrons that may have exchanged photons with the field. One may therefore observe substantial temporal intensity modulations, but would have hardly found neutrons with changed energy, had one chosen to measure that. This problem also arises in partial neutron absorption in one path of the interferometer [32,33].

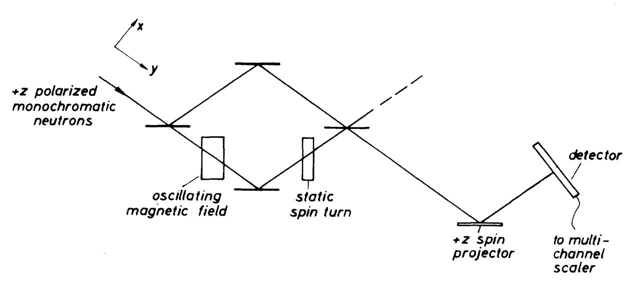

FIG. 4. Measurement of multiphoton exchange by neutron interferometry. A static magnetic field parallel to the z axis across the whole arrangement ensures definition of spin eigenstates. In the interferometer, one path passes through a region with an additional oscillating magnetic field. When measuring energy transitions without spin transition, the static spin-turn device, which turns spin up into spin down and vice versa, remains inactive. It is switched on when energy transitions with simultaneous spin transition are to be measured. The various transition amplitudes are obtained by Fourier analysis of the time-dependent count rate registered by the detector.

It is worth assessing the suitable range of parameters for such an experiment. For instance, with an oscillating field of 20 kHz and an amplitude of 50 G parallel to the static field and extending over 2 cm along the beam, neutrons with a wavelength of 2 Å would be exposed to exchange amplitudes as shown for $\alpha=2.1$ in Fig. 3. Note that under these conditions the neutron's change in energy is much smaller than the spread in energy of even a very monochromatic incident beam, which is usually below 1% [34]. However, the different energy components of the incident beam can be considered incoherent [35] and thus do not complicate the interpretation of the experiment.

Another method of measuring multiphoton exchange, which would show the quantized change in velocity when the neutron absorbs or emits photons, makes use of ultracold neutrons (e.g., [36] and references therein). Monochromatic pulsed neutrons pass a region with an oscillating magnetic field where they are split into packets of different mean velocity, depending on the number of photons exchanged. These different packets could then be detected by time-of-flight methods. For neutrons with an original velocity of 6 m/s, exchange of one photon of 1 MHz with the field results in a change in velocity of around 1.1%. The monochromacy of the original beam would thus have to be better than 1%. An oscillating field of higher frequency could be used, but this makes it more difficult to reach the large field strengths necessary for appreciable multiphoton-exchange probabilities.

Still another experimental possibility would rely on the effects of dynamical diffraction in a double crystal arrangement as sketched in Fig. 5. This is similar to the experiments showing one-photon exchange in resonant spin flip, which used Bragg backscattering and frequencies between 30 and 60 MHz [3,4]. At such high frequencies it is not easily possible to obtain large enough fields for multiphoton exchange. But with the experiment of Fig. 5 frequencies of a few kilohertz would sufficiently change the neutron's momentum normal to the boundaries of the field region to give rise to different propagation paths within the Borrmann fan in the second crystal. The high sensitivity of trajectories within the Borrmann fan towards momentum changes has already been exploited in experiments on the neutron's magnetic charge [37] and on its effective mass in the crystal [38]. In the present arrangement, neutrons would emerge at different points at the back side of the second crystal, depending on the number of exchanged photons. The experimental parameters can be estimated as follows. The width w_1 of the slit behind the first crystal determines the minimum spread of momentum parallel to the y axis as $\Delta p_y > 2\hbar/w_1$. (We are using the full widths of the position and momentum distributions here, rather than the standard deviations of the uncertainty relation.) In order to achieve the minimum width of the momentum distribution, the first crystal must be made thick enough such that at the back surface the Borrmann-fan separation of the plane-wave components of the incident beam is sufficiently large. A simple calculation by means of the dynamical theory of diffraction of neutrons in crystals [39] shows that for $w_1 = 1$ mm and a wavelength of 3 Å the thickness of the first crystal need only be between 15 to 20 mm, when using the (111) plane of a silicon crystal as the Bragg plane and when exploiting the central part of the Borrmann fan. For highest sensitivity the boundaries of the oscillating field region should be chosen parallel to the Bragg plane, because any change of energy

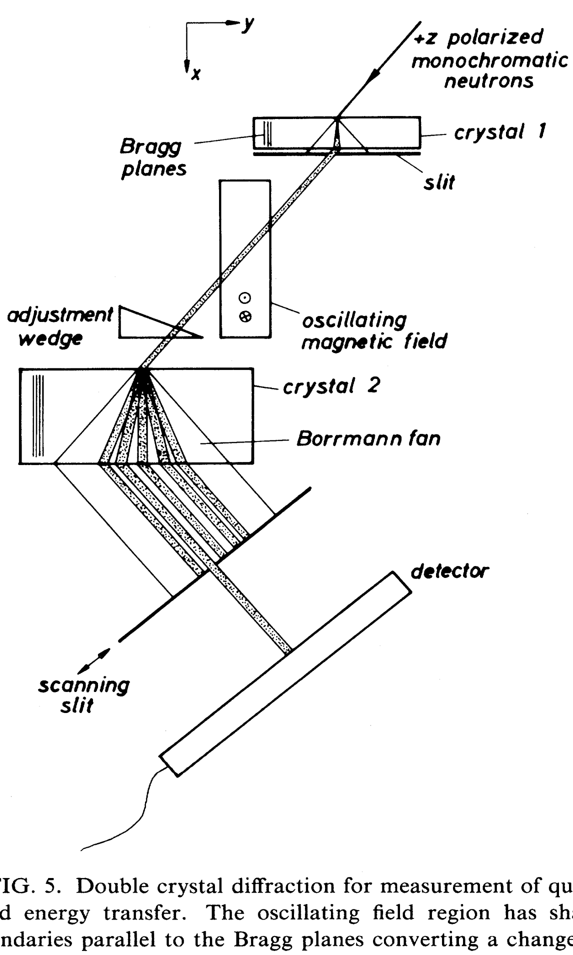

FIG. 5. Double crystal diffraction for measurement of quantized energy transfer. The oscillating field region has sharp boundaries parallel to the Bragg planes converting a change of the neutron's energy into a change of momentum normal to the Bragg planes. Each momentum gives rise to two distinct trajectories within the Borrmann fan in the crystal. The direction of the trajectories can be controlled by the adjustment wedge. Energy change due to photons of a few hundred kilohertz can be resolved.

is then translated into a momentum change normal to the Bragg plane. A separation of propagation direction in the second crystal will then occur when the photon energy is sufficiently high to produce a change in the y component of the neutron's momentum larger than Δp_y. Since the magnitude of the y component of the momentum is typically $\Delta p_y/2$, it must be increased to at least $3\Delta p_y/2$. This leads to a lower bound for the frequency of the oscillating field of $\nu_{\min} = \hbar/(mdw_1)$, where d is the spacing of the Bragg planes. This formula is an approximation to first order. But note that the wavelength of the neutrons does not enter, as only the momentum normal to the Bragg plane is relevant. With the presently considered parameters, one obtains $\nu_{\min} = 116.3$ kHz. This corresponds to a photon energy of about 0.5 neV. At this frequency one would expect a separation of trajectories in the second crystal only around the central part of the Borrmann fan. In order to make energy changes by more than one photon visible, it will be necessary to insert an adjustable wedge as in Fig. 5. The wedge could be made of aluminum [40], bending the neutron trajectory just a little such that the energy state with the desired number of absorbed or emitted photons propagates parallel to the Bragg planes. This permits high resolution with respect to the two neighboring energy states. However, all noncentral trajectories in the second crystal are an overlap of two different energy states of the neutron, because dynamical diffraction is symmetric about the exact Bragg condition. Furthermore, it should be mentioned that the proposed experiment could also be used to test low-energy quantization with other periodic interactions, e.g., when the beam is chopped between the two crystals.

A final experimental possibility of investigating multiphoton exchange would be passing a polarized beam through the oscillating field and analyzing the spectrum of the resulting time-dependent polarization in an arrangement quite similar to the experiment on "dressed neutrons" [1] or to a recent experiment on Berry's phase [2]. However, the obtainable information is limited. Because this method relies on rotation of the polarization vector in the oscillating field, it would for instance not be possible to investigate the occurrence of even- and odd-number photon exchange without spin flip in the case of parallel fields.

V. CONCLUSION

We have analytically calculated the energy and spin transition amplitudes of a neutron passing a spatially homogeneous region in which there is a magnetic field consisting of a static and a sinusoidally-time-dependent component. We concentrated on the case of static and time-dependent fields orthogonal to each other and on the case where both components are parallel. The first case is known from NMR, but we looked at the situation where the oscillating field component is large enough to render the rotating-wave approximation inapplicable. We found significant amplitudes for absorption or emission of several photons by the neutron, when static and oscillating field components are comparable. A neutron originally in a spin eigenstate of the static field component experiences spin flip when it exchanges an odd number of photons with the field; and even number of exchanged photons results in no change of spin state. Angular momentum conservation could thus be understood by just considering neutron plus absorbed or emitted photons. We also identified criteria when the probability of spin flip becomes quite independent of the static field component. In the case of parallel fields we found large amplitudes for the exchange of several photons with easily achievable experimental parameters. The magnitude of the exchange amplitudes turned out to be independent of the static field component. In this configuration a neutron originally in a spin eigenstate will not undergo spin flip, and yet have amplitudes for the exchange of an even as well as an odd number of photons. Especially when an odd number of photons are exchanged, the mechanism of angular momentum conservation requires that the field as a whole also take up some angular momentum. For the observation of photon-exchange processes, we suggested four different experiments: Interferometry would permit measurement of the amplitudes rather than the probabilities and could thus be very sensitive. The amplitudes would show up in the Fourier spectrum of the time-dependent interference signal. A direct observation of quantized energy transfer in magnetic fields oscillating at only a few hundred kilohertz would be possible by exploiting the Borrmann fan in double crystal diffraction. With ultracold neutrons, energy transfer in the megahertz range would already permit observation by time-of-flight methods. Finally, some information on the exchange amplitudes could be obtained by analyzing the time-dependent behavior of the resulting polarization vector when passing appropriately polarized neutrons through a region with an oscillating magnetic field.

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