z spin in oscillating magnetic z field
(changing the neutron energy without spin change)

Hartmut Lemmel

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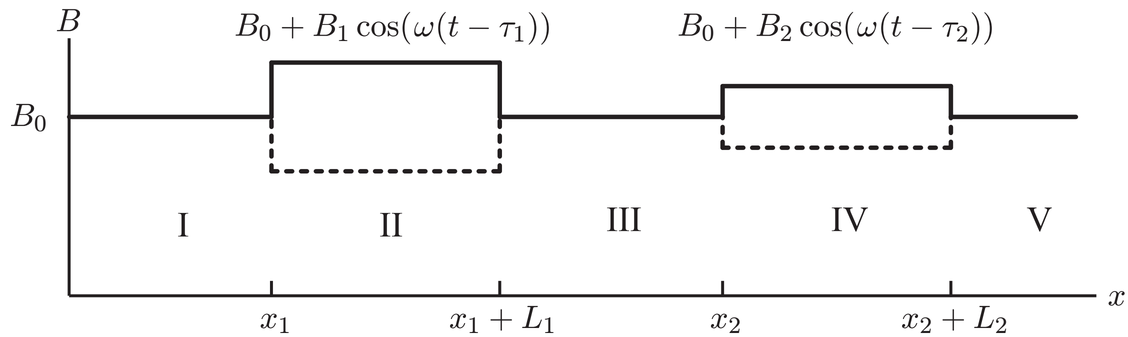


Let a plane wave enter from the left in region I.

$$\psi_I = \mathrm{e}^{\mathrm{i}k_0 x - \mathrm{i}\omega_0 t}$$

The total energy $\hbar\omega_0$ is the sum of the kinetic energy $\hbar^2 k_0^2/(2m)$ and the potential energy μB_0, thus $\omega_0 = \hbar k_0^2/(2m) + \mu B_0/\hbar$. In region II the field oscillates with ω and the wave can gain or lose energy packages of $\hbar\omega$. The single plane wave component ψ_I is therefore split in region II into a fan of equi-distant energy levels and the we obtain in region III a superposition of plane waves with shifted energies, whose amplitudes are given by Bessel functions J_n. The details are worked out in the thesis of Georg Sulyok. His formula (2.28) generalized for arbitrary positions x_1 reads

$$\psi_{III} = \sum_{n=-\infty}^{\infty} (-\mathrm{i})^n J_n\Big(2\alpha_1 \sin\frac{\omega T_1}{2}\Big)\, \mathrm{e}^{-\mathrm{i}n\omega(t_1-\tau_1)}\, \mathrm{e}^{\mathrm{i}k_n x - \mathrm{i}\omega_n t} \tag{1}$$

$$\alpha_1 = \mu B_1/(\hbar\omega) \qquad \text{scaled oscillation amplitude} \tag{2}$$

$$v = \hbar k_0/m \qquad \text{neutron velocity} \tag{3}$$

$$T_1 = L_1/v \qquad \text{time spent in oscillating region} \tag{4}$$

$$t_1 = (x_1 + L_1/2)/v \qquad \text{time when center of oscillating region is passed} \tag{5}$$

$$\tau_1 \qquad \text{starting time of oscillator (phase)} \tag{6}$$

$$\hbar\omega_n = \hbar(\omega_0 + n\omega) \qquad \text{total energy in level } n \tag{7}$$

$$k_n = \sqrt{k_0^2 + n\, 2\omega m/\hbar} \quad \approx k_0 + n\,\delta k, \qquad \delta k = \omega m/(\hbar k_0) = \omega/v = \omega t/x \tag{8}$$

The first order expansion of the root expression is justified by the fact that $\delta k/k_0$ is in the order of 10^{-8} for thermal neutrons and a typical oscillation frequency of 100 kHz.

If ψ_{III} passes region IV the plane wave component of each level n is again spread over the neighboring levels.

$$\mathrm{e}^{\mathrm{i}k_n x - \mathrm{i}\omega_n t} \;\rightarrow\; \sum_m (-\mathrm{i})^m J_m\left(2\alpha_2 \sin\frac{\omega T_2}{2}\right)\, \mathrm{e}^{-\mathrm{i}m\omega(t_2-\tau_2)}\, \mathrm{e}^{\mathrm{i}k_{n+m} x - \mathrm{i}\omega_{n+m} t} \tag{9}$$

Inserting Eq. (9) in (1) we obtain behind the second oscillating region

$$\psi_V = \sum_n (-\mathrm{i})^n J_n \left(2\alpha_1 \sin \frac{\omega T_1}{2}\right) \mathrm{e}^{-\mathrm{i}n\omega(t_1-\tau_1)} \times \sum_m (-\mathrm{i})^m J_m \left(2\alpha_2 \sin \frac{\omega T_2}{2}\right) \mathrm{e}^{-\mathrm{i}m\omega(t_2-\tau_2)}\, \mathrm{e}^{\mathrm{i}k_{n+m}x-\mathrm{i}\omega_{n+m}t} \tag{10}$$

Replacing the index m by $m-n$ and swapping the summations we obtain

$$\psi_V = \sum_m \mathrm{e}^{\mathrm{i}k_m x-\mathrm{i}\omega_m t}(-\mathrm{i})^m\, \mathrm{e}^{-\mathrm{i}m\omega(t_2-\tau_2)} \times \sum_n \mathrm{e}^{\mathrm{i}n\omega(t_2-\tau_2-t_1+\tau_1)} J_{m-n}\left(2\alpha_2 \sin\frac{\omega T_2}{2}\right) J_n\left(2\alpha_1 \sin\frac{\omega T_1}{2}\right) \tag{11}$$

The summation rule for Bessel functions $J_m(u \pm v) = \sum_n J_{m\mp n}(u)J_n(v)$ applies to the second sum if the exponent vanishes for all n, which is the case if the phases of the oscillators are equal (neutron arrival time $t_{1,2}$ minus oscillator starting time $\tau_{1,2}$).

$$t_2 - \tau_2 = t_1 - \tau_1 \tag{12}$$

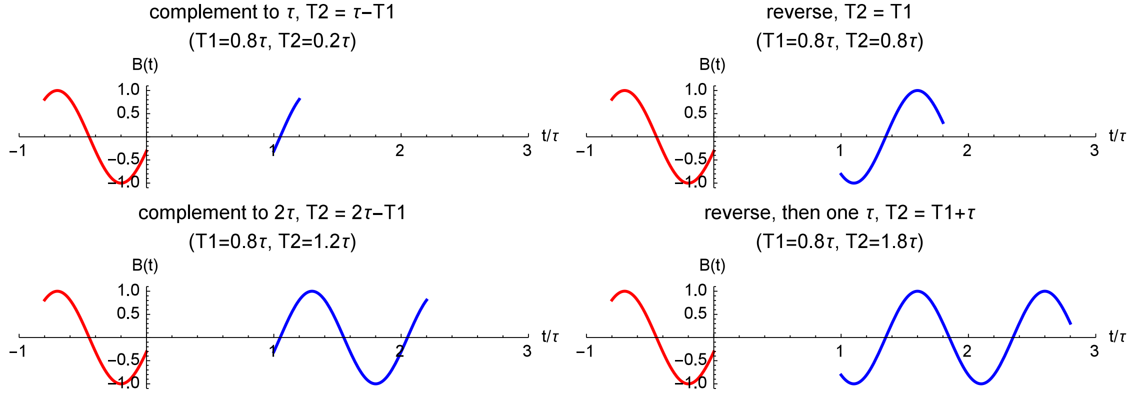
$$\psi_V = \sum_m \mathrm{e}^{\mathrm{i}k_m x-\mathrm{i}\omega_m t}(-\mathrm{i})^m\, \mathrm{e}^{-\mathrm{i}m\omega(t_2-\tau_2)} J_m\left(2\alpha_1 \sin\frac{\omega T_1}{2} + 2\alpha_2 \sin\frac{\omega T_2}{2}\right) \tag{13}$$

Alternatively, we can invert the sign of the second Bessel argument in (11) using the relation $J_n(z) = (-1)^n J_n(-z)$. Then the sign of α_2 is inverted in comparison to (13), and the oscillator phases are shifted by π.

$$t_2 - \tau_2 = t_1 - \tau_1 + \pi/\omega \tag{14}$$

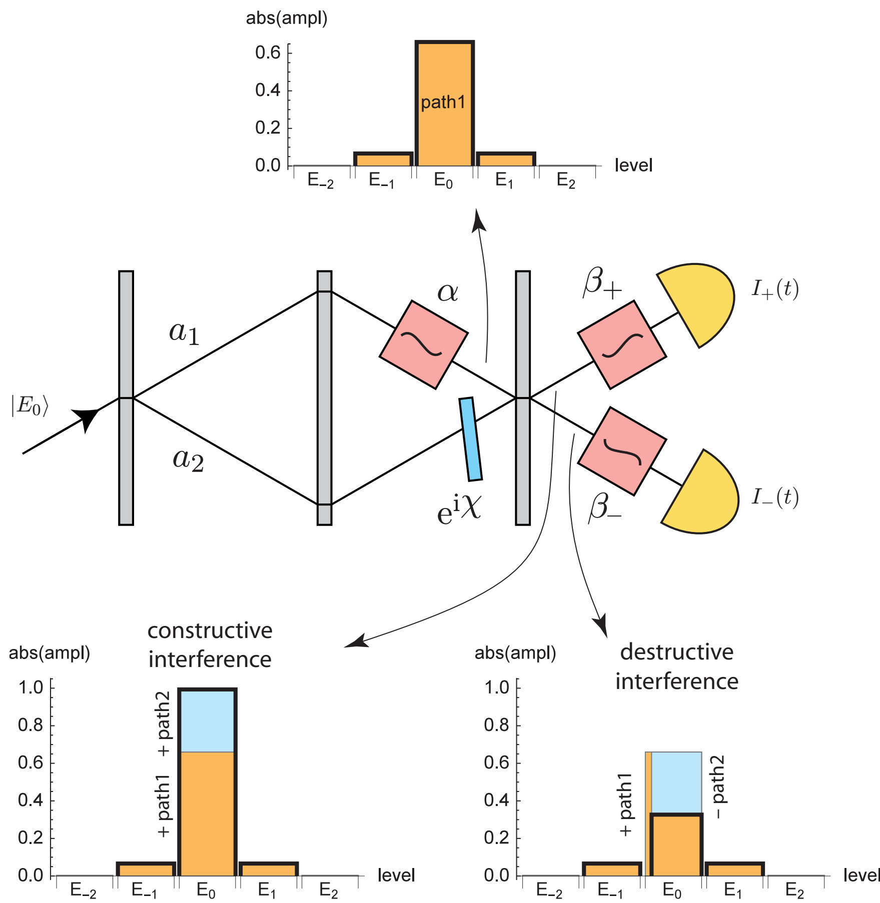
$$\psi_V = \sum_m \mathrm{e}^{\mathrm{i}k_m x-\mathrm{i}\omega_m t}(-\mathrm{i})^m\, \mathrm{e}^{-\mathrm{i}m\omega(t_2-\tau_2)} J_m\left(-2\alpha_1 \sin\frac{\omega T_1}{2} + 2\alpha_2 \sin\frac{\omega T_2}{2}\right) \tag{15}$$

Since $J_m(0) = \delta_{m,0}$ the wave function is reduced to the original single plane wave ψ_I if the arguments of all Bessel functions vanish. For a single oscillating region this is the case (cf. Eq. 1) if $\sin(\omega T_1/2) = 0$, meaning that the time T_1 spent in the oscillator equals a multiple of the oscillation period $2\pi/\omega$. In the case of two oscillating regions the Bessel arguments in Eq. (13) or (15) have to vanish. If the amplitudes are equal $\alpha_2 = \alpha_1$ these arguments can be written as a beating function $4\alpha_1 \sin((T_2 \pm T_1)\omega/4)\cos((T_2 \mp T_1)\omega/4)$ respectively. The sine term vanishes if $T_2 = 2n\tau \mp T_1$ and the cosine term vanishes if $T_2 = (2n+1)\tau \pm T_1$ where $\tau = 2\pi/\omega$ denotes the oscillation period. These four different cases are illustrated below. The curves represent the magnetic fields which act on the neutron in the first (red) and second (blue) oscillator respectively.



I. DETERMINING THE PATH PRESENCE

Assume an oscillator in path 1 of a neutron interferometer for path marking and another oscillator in either exit beam for compensating (undoing) the energy spreading. All oscillators have the same frequency ω and the same passing time T. Both parameters are tuned in a way that $\sin(\omega T/2)$ is roughly at its maximum ≈ 1. The oscillation amplitudes are denoted by α, β_+ and β_- respectively, as indicated in the figure. Let α be so weak that it is sufficient to take only levels $-1 \leq n \leq 1$ into account. The oscillator in path 1 creates a certain amplitude ratio between level 0 and ± 1. In case of constructive interference the population of level 0 is raised by the contribution of path 2, in case of destructive interference it is lowered. In either case the amplitude ratio between level 0 and ± 1 is changed, so we expect a $\beta_\pm \neq \alpha$ for an optimal compensation.



The wave function in the detectors can be written as

$$\psi_{det} = \sum_{n=-\infty}^{\infty} e^{ik_n x - i\omega_n t} (-i)^n \, e^{-in\omega(t_2-\tau_2)} \Big(a_1 J_n \left(\bar{\alpha} - \bar{\beta}\pm\right) + a_2 \, e^{i\chi} J_n \left(-\bar{\beta}_\pm\right) \Big) \tag{16}$$

with $\bar{\alpha} = 2\alpha \sin(\omega T/2)$ and $\bar{\beta}_\pm = 2\beta \sin(\omega T/2)$. We assume $\chi = 0$ for the '+'-detector and $\chi = \pi$ for the '−'-detector. With the relation $J_{-n}(z) = J_n(-z)$ and $k_n \approx k_0 + n\,\delta k$ we can combine the $+n$ and $-n$ terms of the sum.

$$\psi_{det} = \frac{1}{\sqrt{2}} \, e^{ik_0 x - i\omega_0 t} \left[c(0) + \sum_{n=1}^{\infty} c(n) \cos(n\,x\,\delta k - n\,\omega\,t - n\,\omega\,(t_2 - \tau_2)) \right] \tag{17}$$

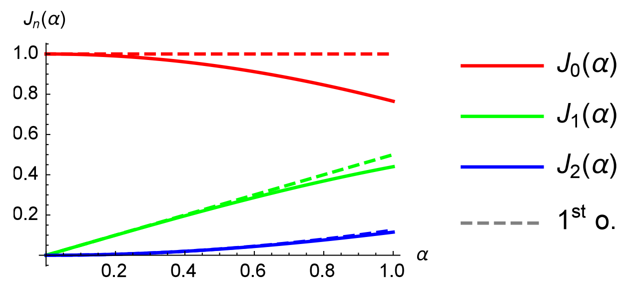
$$c(n) = a_1 J_n(\bar{\alpha} - \bar{\beta}_\pm) + a_2 \, e^{i\chi} J_n(-\bar{\beta}_\pm) \tag{18}$$

For perfect compensation, we demand that

$$c(0) = a_1 J_0(\bar{\alpha} - \bar{\beta}_\pm) + a_2\, \mathrm{e}^{\mathrm{i}\chi} J_0(-\bar{\beta}_\pm) = a_1 + a_2\, \mathrm{e}^{\mathrm{i}\chi} \tag{19}$$

$$c(n \geq 1) = a_1 J_n(\bar{\alpha} - \bar{\beta}_\pm) + a_2\, \mathrm{e}^{\mathrm{i}\chi} J_n(-\bar{\beta}_\pm) = 0 \tag{20}$$

We restrict ourselves to small Bessel arguments $\lessapprox 0.2$.



Then $J_0 \approx 1$, which fulfills condition $c(0)$, and $J_{n\geq 2} \approx 0$ fulfills $c(n \geq 2) = 0$. The remaining condition $c(1) = 0$ reads with $J_1(\alpha) \approx \alpha/2$

$$0 = a_1(\bar{\alpha} - \bar{\beta}_\pm) + a_2\, \mathrm{e}^{\mathrm{i}\chi}(-\bar{\beta}_\pm) \tag{21}$$

and we obtain

$$\frac{\bar{\beta}_\pm}{\bar{\alpha}} = \frac{\beta_\pm}{\alpha} = \frac{1}{1 + \mathrm{e}^{\mathrm{i}\chi}\frac{a_2}{a_1}} \tag{22}$$

which equals the weak value of the path 1 projection operator, cf. path presence paper 2022 eq. (10). (The different sign of χ comes from the different ansatz.)

II. EXPERIMENTAL FEASIBILITY

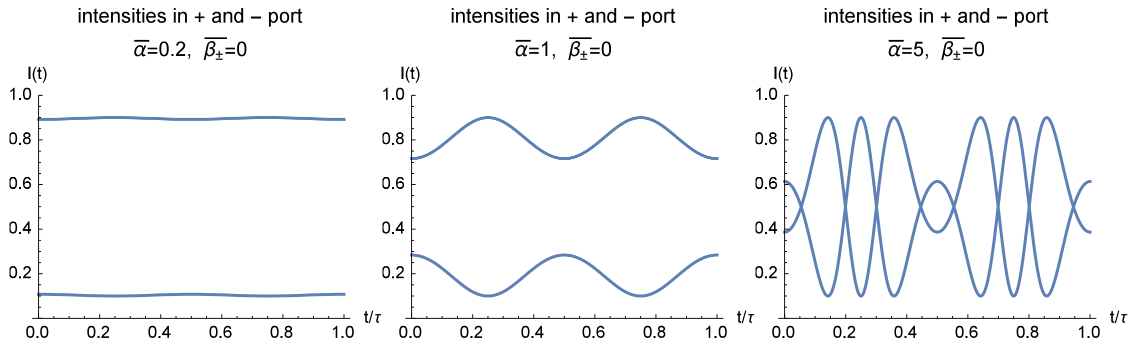
In general, the mixing of energy levels creates a beating pattern in time in the intensity measured at the detector. The intensity is given by the modulus squared of Eq. (16). With perfect compensation, i.e. using $\beta_\pm$ given by Eq. (22), only the initial energy state is populated and we obtain to first order

$$I_{opt} = \frac{1}{2}\left[a_1^2 + a_2^2 + 2a_1a_2\Big(\cos\chi - \bar{\alpha}\sin\chi\cos\big(x\delta k - \omega(t + t_2 - \tau_2)\big)\Big)\right] \tag{23}$$

The time-dependent term vanishes for $\chi = 0$ or $\chi = \pi$ and the result matches the empty interferometer result. On the other hand, if we apply no compensation at all ($\bar{\beta}_\pm = 0$) the intensity can be calculated rigorously, not only for small $\bar{\alpha}$, cf. Sulyok (2.66):

$$I_{\beta=0} = \frac{1}{2}\left[a_1^2 + a_2^2 + 2a_1a_2\cos\Big(\chi + \bar{\alpha}\cos\big(x\delta k - \omega(t + t_2 - \tau_2)\big)\Big)\right] \tag{24}$$

It is plotted below for $\chi = 0$ and $\chi = \pi$ respectively and for different values of $\bar{\alpha}$ (using arbitrary values for the phase in the time domain, $x = 0$, $t_2 = \tau_2 = 0$).



Unfortunately, in the limit of small $\bar{\alpha}$ (left figure), the signature of the first side levels is barely visible. Sulyok measured more in the regime of the right figure.