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Direct extraction of path weak values from interferograms without auxiliary qubits

Introduction to weak values

References:

Complex weak values in quantum measurement

Richard Jozsa

The significance of the imaginary part of the weak value

J. Dressel and A. N. Jordan

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Introduction to weak values

Weak value:

$$A_w = \frac{\langle \psi_{\mathrm{f}} | \hat{A} | \psi_{\mathrm{in}} \rangle}{\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle}$$

Introduction to weak values

Weak value:

$$A_w = \frac{\langle \psi_{\mathrm{f}} | \hat{A} | \psi_{\mathrm{in}} \rangle}{\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle}$$

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_{\mathrm{in}}\rangle$
- Post-selected State: $|\psi_{\mathrm{f}}\rangle$

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Introduction to weak values

Weak value:

$$A_w = \frac{\langle\psi_\mathrm{f}|\hat{A}|\psi_\mathrm{in}\rangle}{\langle\psi_\mathrm{f}|\psi_\mathrm{in}\rangle}$$

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_\mathrm{in}\rangle$
- Post-selected State: $|\psi_\mathrm{f}\rangle$
- Complex number
- Not bounded by eigenvalues

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Introduction to weak values

Von Neumann measurement

$$|\phi\rangle \otimes |\psi_{\mathrm{in}}\rangle$$

Introduction to weak values

Von Neumann measurement

$$\underset{\text{Detector}}{|\phi\rangle} \otimes \underset{\text{System}}{|\psi_{\text{in}}\rangle}$$

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Von Neumann measurement

$$\underbrace{|\phi\rangle}_{\text{Detector}} \otimes \underbrace{|\psi_{\text{in}}\rangle}_{\text{System}} = |\phi\rangle_d \, |\psi_{\text{in}}\rangle_s$$

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Introduction to weak values

Von Neumann measurement

$$|\phi\rangle \otimes |\psi_{\mathrm{in}}\rangle = |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s$$

$$\hat{H}_I(t) = g(t)\hat{p} \otimes \hat{A}$$

Interaction
Hamiltonian

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$$\mathrm{e}^{-\mathrm{i}\int_{t_1}^{t_2} \hat{H}_I(t)\mathrm{d}t} \, |\phi\rangle_d \, |\psi_{\text{in}}\rangle_s$$

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$$\int_{t_1}^{t_2} g(t)\mathrm{d}t = G$$

Introduction to weak values

Von Neumann measurement

$$\sum_a \langle a|\psi_{\mathrm{in}}\rangle\,|a\rangle = \sum_a c_a\,|a\rangle$$

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Introduction to weak values

Von Neumann measurement

$$\sum_a \langle a|\psi_{\text{in}}\rangle |a\rangle = \sum_a c_a |a\rangle \quad : \quad \hat{A}_s |a\rangle = a |a\rangle$$

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Von Neumann measurement

$$\sum_a \langle a|\psi_{\mathrm{in}}\rangle |a\rangle = \sum_a c_a |a\rangle \quad : \quad \hat{A}_s |a\rangle = a|a\rangle$$

$$\mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} |\phi\rangle_d |\psi_{\mathrm{in}}\rangle_s$$

Introduction to weak values

Von Neumann measurement

$$\sum_a \langle a|\psi_{\mathrm{in}}\rangle |a\rangle = \sum_a c_a |a\rangle \quad : \quad \hat{A}_s |a\rangle = a |a\rangle$$

$$\mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} |\phi\rangle_d |\psi_{\mathrm{in}}\rangle_s = \sum_a \mathrm{e}^{-\mathrm{i}Ga\hat{p}_d} |\phi\rangle_d c_a |a\rangle$$

Introduction to weak values

Von Neumann measurement

$$\sum_a \langle a|\psi_{\text{in}}\rangle |a\rangle = \sum_a c_a |a\rangle \quad : \quad \hat{A}_s |a\rangle = a |a\rangle$$

$$\mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} |\phi\rangle_d |\psi_{\text{in}}\rangle_s = \sum_a \underbrace{\boxed{\mathrm{e}^{-\mathrm{i}Ga\hat{p}_d}}}_{\text{Translation operator}} |\phi\rangle_d c_a |a\rangle$$

Introduction to weak values

Von Neumann measurement

$$\sum_a \underbrace{\langle x|_d}_{\text{Position projection}} \mathrm{e}^{-\mathrm{i}Ga\hat{p}_d} |\phi\rangle_d \, c_a \, |a\rangle$$

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Von Neumann measurement

$$\sum_a \langle x|_d \, \mathrm{e}^{-\mathrm{i}Ga\hat{p}_d} \, |\phi\rangle_d \, c_a \, |a\rangle = \sum_a \phi(x+Ga)c_a \, |a\rangle$$

Position projection

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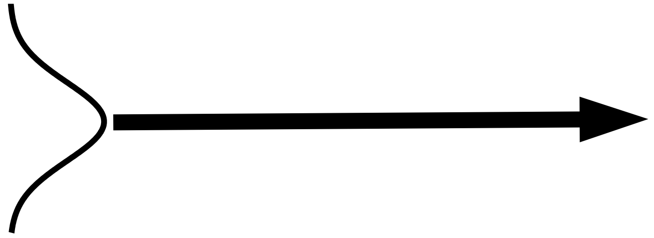
Stern-Gerlach:

Position Spin

$$|\phi\rangle \otimes |S\rangle$$



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Stern-Gerlach:

Position Spin

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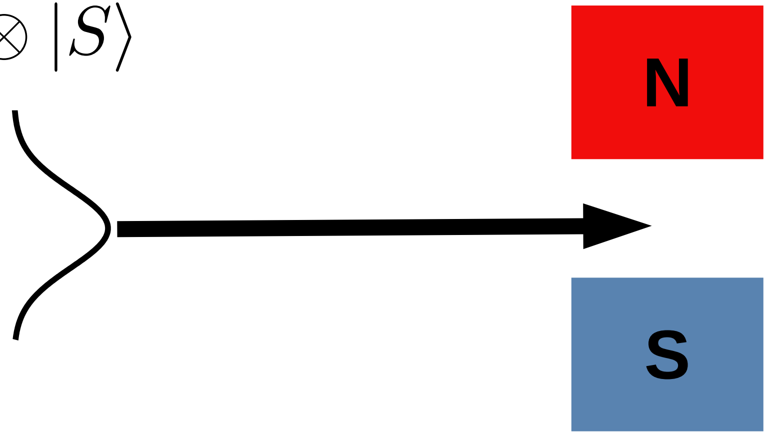

$$\hat{H}_I(t) = g(t)\hat{p}_z \otimes \hat{\sigma}_z$$

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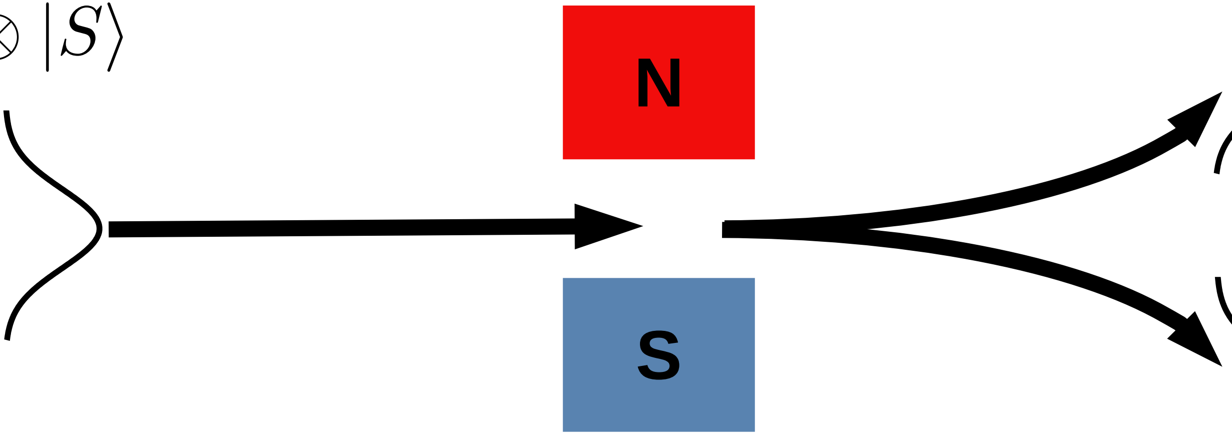
Stern-Gerlach:

Position Spin

$$|\phi\rangle \otimes |S\rangle$$



$$\phi(x+G)c_\uparrow |\uparrow_z\rangle$$

$$\phi(x-G)c_\downarrow |\downarrow_z\rangle$$

$$\hat{H}_I(t) = g(t)\hat{p}_z \otimes \hat{\sigma}_z$$

Introduction to weak values

Weak measurement:

$$\mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s}\left|\phi\right\rangle_d\left|\psi_{\mathrm{in}}\right\rangle_s$$

Introduction to weak values

Weak measurement:

$$\langle\psi_{\mathrm{f}}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} \, |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s$$

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Weak measurement:

$$\langle \psi_{\rm f}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} \, |\phi\rangle_d \, |\psi_{\rm in}\rangle_s \underset{G \ll 1}{\approx}$$

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Weak measurement:

$$\langle\psi_\mathrm{f}|_s\, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s}\, |\phi\rangle_d\, |\psi_\mathrm{in}\rangle_s \underset{G \ll 1}{\approx} \langle\psi_\mathrm{f}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{p}_d\hat{A}_s\right) |\phi\rangle_d\, |\psi_\mathrm{in}\rangle_s$$

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Weak measurement:

$$\langle\psi_{\mathrm{f}}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} \, |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s \approx \langle\psi_{\mathrm{f}}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{p}_d\hat{A}_s\right) |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}G\frac{\langle\psi_{\mathrm{f}}|\hat{A}|\psi_{\mathrm{in}}\rangle}{\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle}\hat{p}_d\right) |\phi\rangle_d$$

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Weak measurement:

$$\langle\psi_{\mathrm{f}}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} \, |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s \approx \langle\psi_{\mathrm{f}}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{p}_d\hat{A}_s\right) |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}G\boxed{\frac{\langle\psi_{\mathrm{f}}|\hat{A}|\psi_{\mathrm{in}}\rangle}{\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle}}\hat{p}_d\right) |\phi\rangle_d$$

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Weak measurement:

$$\langle\psi_{\mathrm{f}}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} \, |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s \approx \langle\psi_{\mathrm{f}}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{p}_d\hat{A}_s\right) |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}GA_w\hat{p}_d\right) |\phi\rangle_d$$

Introduction to weak values

Weak measurement:

$$\langle\psi_{\mathrm{f}}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{p}_d\hat{A}_s} \, |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s \approx \langle\psi_{\mathrm{f}}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{p}_d\hat{A}_s\right) |\phi\rangle_d \, |\psi_{\mathrm{in}}\rangle_s$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}GA_w\hat{p}_d\right) |\phi\rangle_d$$

$$\approx \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\mathrm{e}^{-\mathrm{i}GA_w\hat{p}_d}\right) |\phi\rangle_d$$

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Weak measurement:

$$\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left\langle x\right|_d \left(\mathrm{e}^{-\mathrm{i}GA_w\hat{p}_d}\right) \left|\phi\right\rangle_d \approx \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle\, \phi\left(x - GA_w\right)$$

Introduction to weak values

Weak measurement:

$$\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \langle x|_d \left(\mathrm{e}^{-\mathrm{i}GA_w\hat{p}_d}\right)|\phi\rangle_d \approx \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle\,\phi\left(x - GA_w\right)$$

The sense in which a "weak measurement" of a spin-$\frac{1}{2}$ particle's spin component yields a value 100

I. M. Duck and P. M. Stevenson

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Introduction to weak values

Weak measurement:

$$\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \, \langle x|_d \left(\mathrm{e}^{-\mathrm{i}GA_w\hat{p}_d}\right)|\phi\rangle_d \approx \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \, \phi\left(x - GA_w\right)$$

$$A_w = A_w^{\Re} + \mathrm{i}A_w^{\Im}$$

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Weak measurement:

$$\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle \, \langle x|_d \left(\mathrm{e}^{-\mathrm{i} G A_w \hat{p}_d} \right) |\phi\rangle_d \approx \langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle \, \phi \left(x - G A_w \right)$$

$$A_w = A_w^{\Re} + \mathrm{i} A_w^{\Im}$$

For a real valued detector function $\phi(x)$

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Weak measurement:

$$\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \, \langle x|_d \left(\mathrm{e}^{-\mathrm{i}GA_w\hat{p}_d}\right) |\phi\rangle_d \approx \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \, \phi\left(x - GA_w\right)$$

$$A_w = A_w^{\Re} + \mathrm{i}A_w^{\Im}$$

For a real valued detector function $\phi(x)$

$$\langle\hat{x}\rangle_f = \langle\hat{x}\rangle_i + G\,A_w^{\Re}$$

$$\langle\hat{p}\rangle_f = \langle\hat{p}\rangle_i + G\,A_w^{\Im}\,\mathrm{Var}(\hat{p})$$

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IS THE WEAK VALUE ALWAYS WEAK?

Is the weak value always weak?

Expectation value:

$$\langle \psi_{\mathrm{in}} | \hat{A} | \psi_{\mathrm{in}} \rangle$$

Is the weak value always weak?

Expectation value:

$$\langle \psi_{\text{in}} | \hat{A} | \psi_{\text{in}} \rangle$$

$$\hat{\mathbb{I}} = \sum_k |\psi_{\text{f}}^k\rangle \langle \psi_{\text{f}}^k|$$

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Is the weak value always weak?

Expectation value:

$$\langle \psi_{\text{in}} | \hat{A} | \psi_{\text{in}} \rangle = \sum_k \langle \psi_{\text{in}} | \psi_{\text{f}}^k \rangle \langle \psi_{\text{f}}^k | \hat{A} | \psi_{\text{in}} \rangle$$

$$\hat{\mathbb{I}} = \sum_k | \psi_{\text{f}}^k \rangle \langle \psi_{\text{f}}^k |$$

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Is the weak value always weak?

Expectation value:

$$\langle \psi_{\mathrm{in}} | \hat{A} | \psi_{\mathrm{in}} \rangle = \sum_k \langle \psi_{\mathrm{in}} | \psi_{\mathrm{f}}^k \rangle \langle \psi_{\mathrm{f}}^k | \hat{A} | \psi_{\mathrm{in}} \rangle$$

Is the weak value always weak?

Expectation value:

$$\langle\psi_{\rm in}|\hat{A}|\psi_{\rm in}\rangle = \sum_k \langle\psi_{\rm in}|\psi_{\rm f}^k\rangle \langle\psi_{\rm f}^k|\hat{A}|\psi_{\rm in}\rangle$$

$$= \sum_k |\langle\psi_{\rm f}|\psi_{\rm in}^k\rangle|^2 \frac{\langle\psi_{\rm f}^k|\hat{A}|\psi_{\rm in}\rangle}{\langle\psi_{\rm f}^k|\psi_{\rm in}\rangle}$$

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Is the weak value always weak?

Expectation value:

$$\langle\psi_{\mathrm{in}}|\hat{A}|\psi_{\mathrm{in}}\rangle = \sum_k \langle\psi_{\mathrm{in}}|\psi_{\mathrm{f}}^k\rangle\,\langle\psi_{\mathrm{f}}^k|\hat{A}|\psi_{\mathrm{in}}\rangle$$

$$= \sum_k |\,\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}^k\rangle\,|^2 \frac{\langle\psi_{\mathrm{f}}^k|\hat{A}|\psi_{\mathrm{in}}\rangle}{\langle\psi_{\mathrm{f}}^k|\psi_{\mathrm{in}}\rangle}$$

$$= \sum_k |\,\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}^k\rangle\,|^2 A_w^k$$

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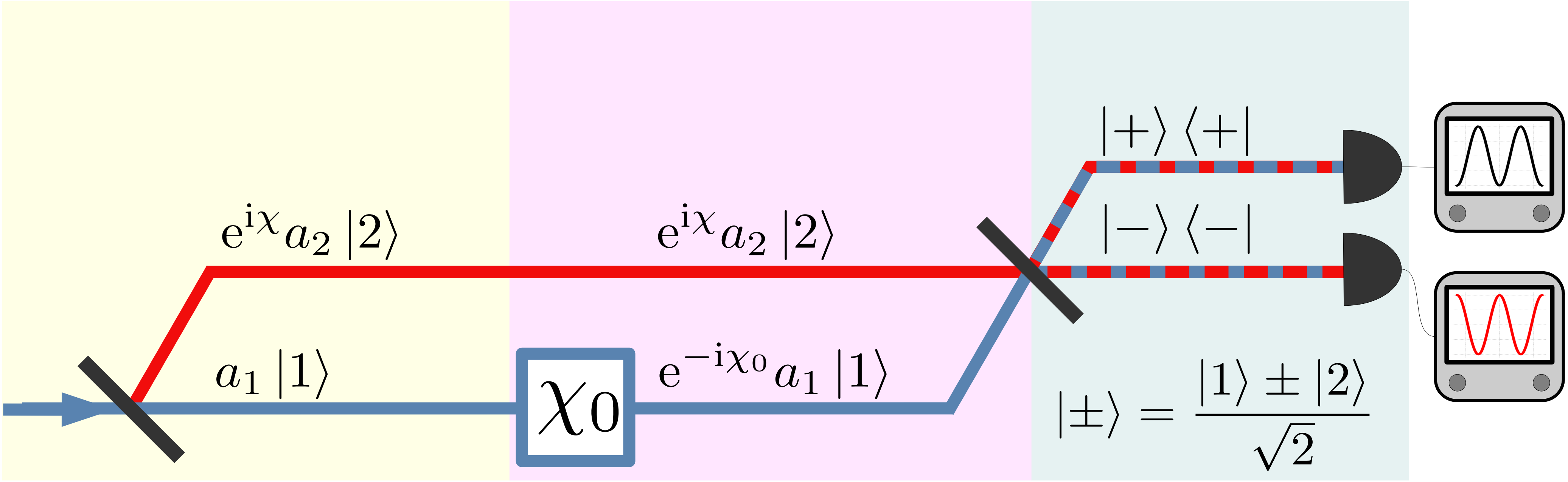
CAN WEAK VALUES DESCRIBE INTERFEROMETRY?

Weak values and interferometry

Standard interferometry formalism



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Weak values and interferometry

Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi,\chi_0) = \left|\langle\pm|\left(\mathrm{e}^{-\mathrm{i}\chi_0}a_1|1\rangle + \mathrm{e}^{\mathrm{i}\chi}a_2|2\rangle\right)\right|^2 = \frac{1}{2} \pm a_1 a_2 \cos(\chi+\chi_0)$$

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