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Ismaele Vincent Masiello

ATOMINSTITUT

Direct extraction of path weak values from interferograms without auxiliary qubits

Content

Content

- **Part 1:** Introduction to weak values and weak measurements

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Content

- **Part 1:** Introduction to weak values and weak measurements
- **Part 2:** Weak values based description of a Mach-Zehnder interferometer

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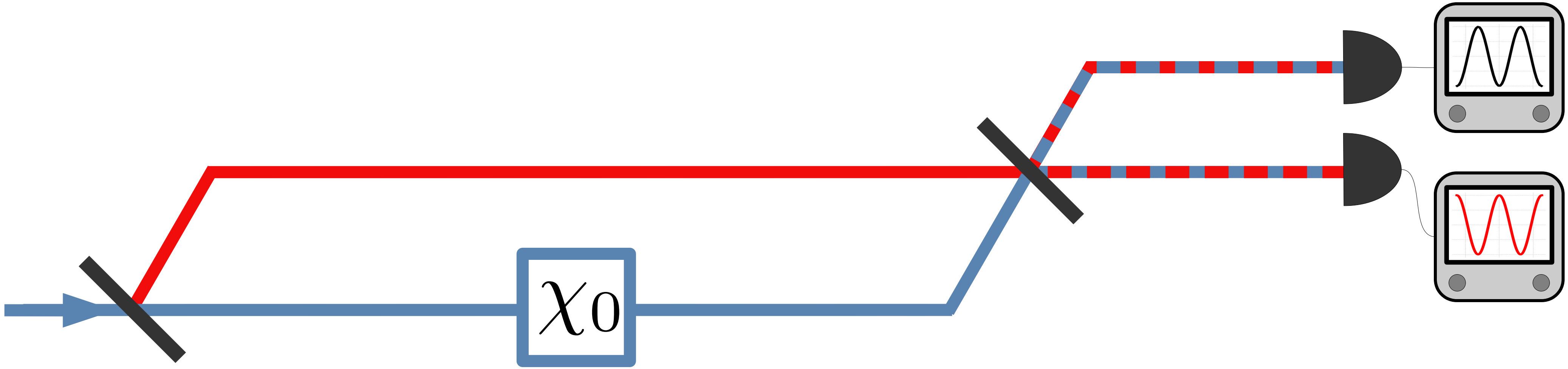
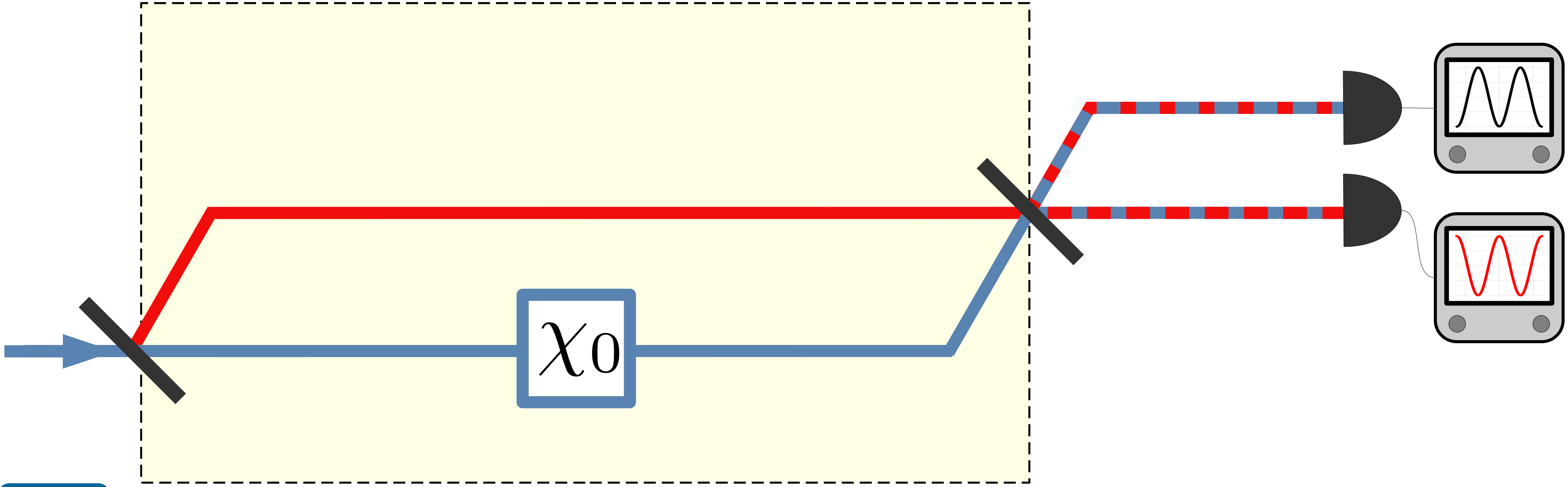
- **Part 1:** Introduction to weak values and weak measurements
- **Part 2:** Weak values based description of a Mach-Zehnder interferometer
- **Part 3:** Experimental measurement of path weak values directly from interferograms

Motivation



Motivation



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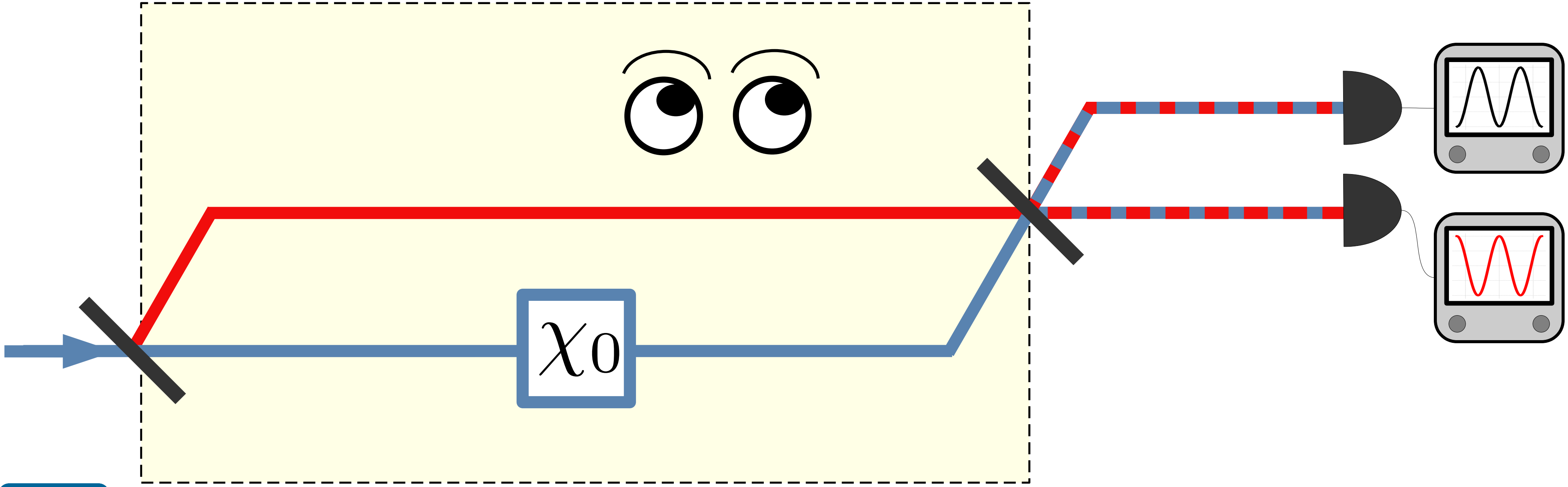
Motivation



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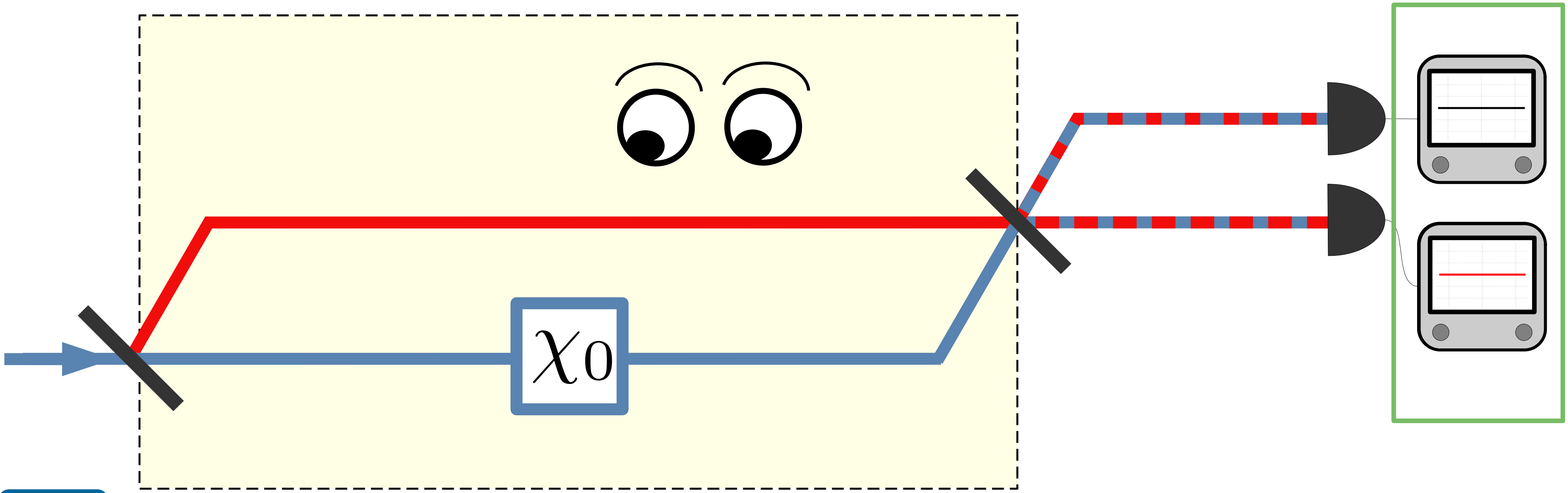
Motivation



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Goal

Use weak values for a new perspective on interferometry, and hopefully some new insight!

Part 1: Introduction to weak values and weak measurement

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

PHYSICAL REVIEW A **76**, 044103 (2007)

Complex weak values in quantum measurement

Richard Jozsa

PHYSICAL REVIEW A **85**, 012107 (2012)

Significance of the imaginary part of the weak value

J. Dressel and A. N. Jordan

Weak value

Definition:

$$A_w = \frac{\langle \psi_{\mathrm{f}} | \hat{A} | \psi_{\mathrm{in}} \rangle}{\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle}$$

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Weak value

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Weak value

Definition:

$$A_w = \frac{\langle \psi_\mathrm{f} | \hat{A} | \psi_\mathrm{in} \rangle}{\langle \psi_\mathrm{f} | \psi_\mathrm{in} \rangle}$$

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_\mathrm{in}\rangle$
- Post-selected state: $|\psi_\mathrm{f}\rangle$

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Weak value

Definition:

$$A_w = \frac{\langle\psi_{\mathrm{f}}|\hat{A}|\psi_{\mathrm{in}}\rangle}{\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle}$$

Expectation value:

$$\langle\hat{A}\rangle = \frac{\langle\psi|\hat{A}|\psi\rangle}{\langle\psi|\psi\rangle}$$

- Observable: $\hat{A}$
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Weak value

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$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_\mathrm{in}\rangle$
- Post-selected state: $|\psi_\mathrm{f}\rangle$
- Complex number

Weak value

$$A_w = A_w^{\Re} + \mathrm{i}A_w^{\Im}$$

Real Imaginary

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_{\mathrm{in}}\rangle$
- Post-selected state: $|\psi_{\mathrm{f}}\rangle$
- Complex number

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Weak value

$$A_w = A_w^{\Re} + \mathrm{i}A_w^{\Im}$$

Real

Imaginary

Observable in the limit of minimal disturbance

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_{\mathrm{in}}\rangle$
- Post-selected state: $|\psi_{\mathrm{f}}\rangle$
- Complex number

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Weak value

Definition:

$$A_w = \frac{\langle \psi_{\mathrm{f}} | \hat{A} | \psi_{\mathrm{in}} \rangle}{\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle}$$

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$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

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Weak value

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$$\langle\hat{A}\rangle = \frac{\langle\psi|\hat{A}|\psi\rangle}{\langle\psi|\psi\rangle}$$

- Observable: $\hat{A}$
- Pre-selected state: $|\psi_{\mathrm{in}}\rangle$
- Post-selected state: $|\psi_{\mathrm{f}}\rangle$
- Complex number
- Not bounded by eigenvalues

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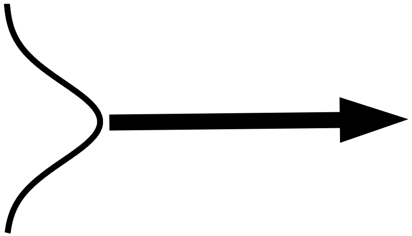
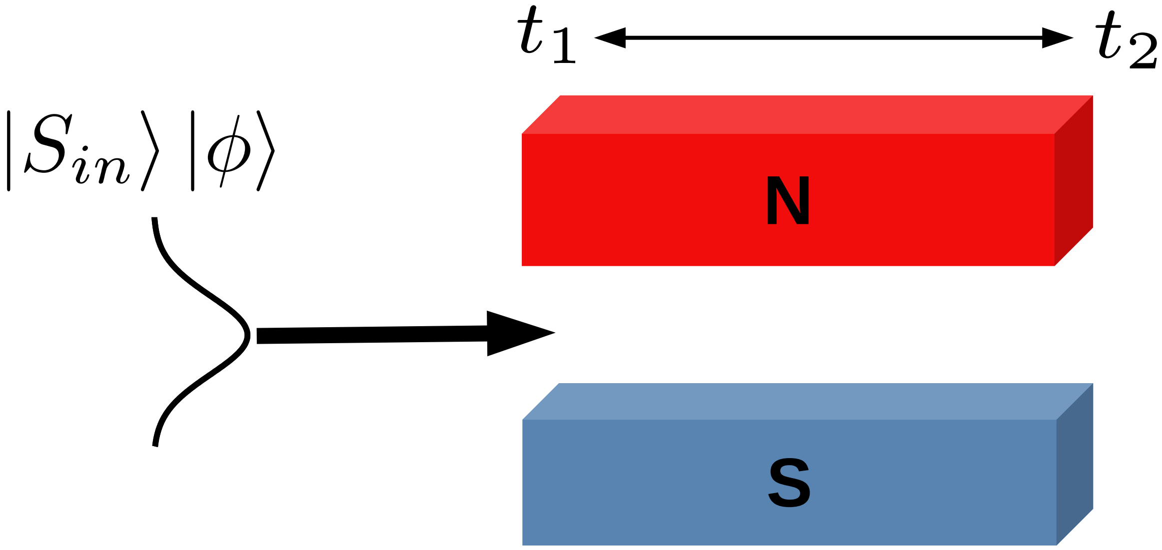
Stern-Gerlach

Spin Position

$$|S_{in}\rangle\,|\phi\rangle$$



Stern-Gerlach



$$\hat{H}_I(t) \propto \hat{\sigma}_z\,\hat{p}_z$$

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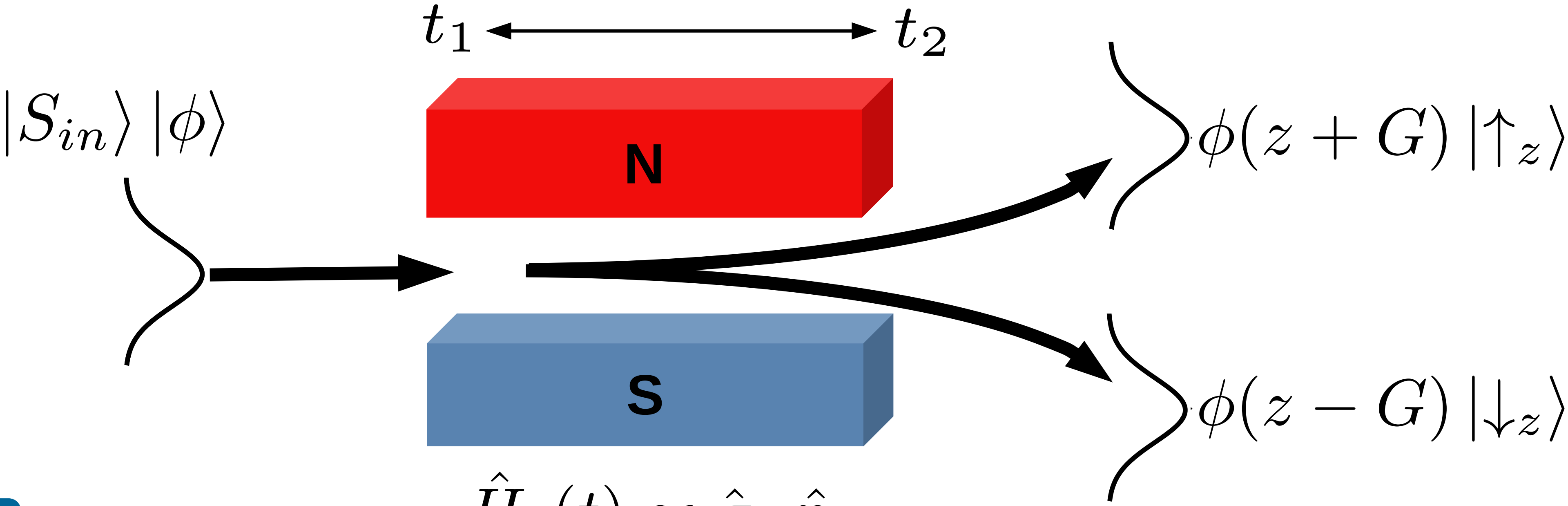
Stern-Gerlach



$$\hat{H}_I(t) \propto \hat{\sigma}_z\,\hat{p}_z$$

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Example of standard measurement (Von Neumann scheme)

Example of standard measurement (Von Neumann scheme)

Two quantum states

$$|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\;|\phi\rangle_{\mathrm{m}}$$

System Meter / Auxiliary

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Example of standard measurement (Von Neumann scheme)

Two quantum states

$$|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\,|\phi\rangle_{\mathrm{m}}$$

System observable $\hat{A}_s$

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Example of standard measurement (Von Neumann scheme)

Two quantum states

$$|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\,|\phi\rangle_{\mathrm{m}}$$

System observable $\hat{A}_s$

$$\hat{A}_s\,|a\rangle_{\mathrm{s}} = a\,|a\rangle_{\mathrm{s}}\ ,\ \{|a\rangle_{\mathrm{s}}\}$$

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Example of standard measurement (Von Neumann scheme)

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System observable $\hat{A}_s$

$$\hat{A}_s\,|a\rangle_{\mathrm{s}} = a\,|a\rangle_{\mathrm{s}}\;,\;\;\{|a\rangle_{\mathrm{s}}\}$$

Interaction Hamiltonian

$$\hat{H}_I(t) \propto \hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}$$

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$\mathrm{e}^{-\mathrm{i}\int_{t_1}^{t_2}\hat{H}_I(t)\mathrm{d}t} = \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}$$

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$e^{-i\int_{t_1}^{t_2}\hat{H}_I(t)\mathrm{d}t} = e^{-iG\hat{A}_\mathrm{s}\hat{p}_\mathrm{m}}$$

Interaction strength

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$e^{-i\int_{t_1}^{t_2}\hat{H}_I(t)\mathrm{d}t} = e^{-iG\hat{A}_s\,\hat{p}_m}$$

$$\hat{\mathbb{I}} = \sum_a |a\rangle_s \langle a|_s$$

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$\mathrm{e}^{-\mathrm{i}\int_{t_1}^{t_2}\hat{H}_I(t)\mathrm{d}t} = \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}$$

$$= \sum_a \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}\,|a\rangle_\mathrm{s}\,\langle a|_\mathrm{s}$$

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$
\begin{aligned}
\mathrm{e}^{-\mathrm{i}\int_{t_1}^{t_2} \hat{H}_I(t)\mathrm{d}t} &= \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}} \\
&= \sum_a \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}} \left|a\right\rangle_\mathrm{s} \left\langle a\right|_\mathrm{s} \\
&= \sum_a \mathrm{e}^{-\mathrm{i}Ga\hat{p}_\mathrm{m}} \left|a\right\rangle_\mathrm{s} \left\langle a\right|_\mathrm{s}
\end{aligned}
$$

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$
\begin{aligned}
\mathrm{e}^{-\mathrm{i}\int_{t_1}^{t_2}\hat{H}_I(t)\mathrm{d}t} &= \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}} \\
&= \sum_a \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}\,|a\rangle_\mathrm{s}\,\langle a|_\mathrm{s} \\
&= \sum_a \mathrm{e}^{-\mathrm{i}G\boxed{a}\hat{p}_\mathrm{m}}\,|a\rangle_\mathrm{s}\,\langle a|_\mathrm{s}
\end{aligned}
$$

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Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$\mathrm{e}^{-\mathrm{i}\int_{t_1}^{t_2}\hat{H}_I(t)\mathrm{d}t} = \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}$$

$$= \sum_a \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}\,|a\rangle_\mathrm{s}\,\langle a|_\mathrm{s}$$

$$= \sum_a \underbrace{\boxed{\mathrm{e}^{-\mathrm{i}Ga\hat{p}_\mathrm{m}}}}_{\text{Translation operator}}\,|a\rangle_\mathrm{s}\,\langle a|_\mathrm{s}$$

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Example of standard measurement (Von Neumann scheme)

Overall effect

Example of standard measurement (Von Neumann scheme)

Overall effect

Eigenvalue

$$\phi_{\mathrm{m}}(x) \rightarrow \phi_{\mathrm{m}}(x + Ga)$$

Interaction strength

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Weak measurement

Effect of the interaction Hamiltonian

$$\mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}\left|\psi_\mathrm{in}\right\rangle_\mathrm{s}\left|\phi\right\rangle_\mathrm{m}$$

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Weak measurement

Effect of the interaction Hamiltonian

$$e^{-iG\hat{A}_s\,\hat{p}_m}\,|\psi_{in}\rangle_s\,|\phi\rangle_m \approx \left(\hat{\mathbb{I}} - iG\hat{A}_s\,\hat{p}_m\right)|\psi_{in}\rangle_s\,|\phi\rangle_m$$

$$G \ll 1$$

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Weak measurement

Effect of the interaction Hamiltonian

$$\langle\psi_{\mathrm{f}}|_{s}\,\mathrm{e}^{-\mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}}\,|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\,|\phi\rangle_{\mathrm{m}} \approx \langle\psi_{\mathrm{f}}|_{s}\left(\hat{\mathbb{I}} - \mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}\right)|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\,|\phi\rangle_{\mathrm{m}}$$

Post-selection

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Weak measurement

Effect of the interaction Hamiltonian

$$\langle\psi_{\mathrm{f}}|_s\, \mathrm{e}^{-\mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}}\, |\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\, |\phi\rangle_{\mathrm{m}} \approx \langle\psi_{\mathrm{f}}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}\right) |\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\, |\phi\rangle_{\mathrm{m}}$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}G\frac{\langle\psi_{\mathrm{f}}|\hat{A}_{\mathrm{s}}|\psi_{\mathrm{in}}\rangle}{\langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle}\hat{p}_{\mathrm{m}}\right) |\phi\rangle_{\mathrm{m}}$$

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Weak measurement

Effect of the interaction Hamiltonian

$$\langle\psi_\mathrm{f}|_s\, \mathrm{e}^{-\mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}}\, |\psi_\mathrm{in}\rangle_\mathrm{s}\, |\phi\rangle_\mathrm{m} \approx \langle\psi_\mathrm{f}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{A}_\mathrm{s}\,\hat{p}_\mathrm{m}\right) |\psi_\mathrm{in}\rangle_\mathrm{s}\, |\phi\rangle_\mathrm{m}$$

$$= \langle\psi_\mathrm{f}|\psi_\mathrm{in}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}G\boxed{\frac{\langle\psi_\mathrm{f}|\hat{A}_\mathrm{s}|\psi_\mathrm{in}\rangle}{\langle\psi_\mathrm{f}|\psi_\mathrm{in}\rangle}}\hat{p}_\mathrm{m}\right) |\phi\rangle_\mathrm{m}$$

Weak value of $\hat{A}_s$

Weak measurement

Effect of the interaction Hamiltonian

$$\langle\psi_{\mathrm{f}}|_{s}\, \mathrm{e}^{-\mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}}\, |\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\, |\phi\rangle_{\mathrm{m}} \approx \langle\psi_{\mathrm{f}}|_{s} \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}\right) |\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\, |\phi\rangle_{\mathrm{m}}$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}GA_{w}\hat{p}_{\mathrm{m}}\right) |\phi\rangle_{\mathrm{m}}$$

Weak measurement

Effect of the interaction Hamiltonian

$$\langle\psi_{\mathrm{f}}|_s \, \mathrm{e}^{-\mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}} \,|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\,|\phi\rangle_{\mathrm{m}} \approx \langle\psi_{\mathrm{f}}|_s \left(\hat{\mathbb{I}} - \mathrm{i}G\hat{A}_{\mathrm{s}}\,\hat{p}_{\mathrm{m}}\right)|\psi_{\mathrm{in}}\rangle_{\mathrm{s}}\,|\phi\rangle_{\mathrm{m}}$$

$$= \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\hat{\mathbb{I}} - \mathrm{i}GA_w\hat{p}_{\mathrm{m}}\right)|\phi\rangle_{\mathrm{m}}$$

$$\approx \langle\psi_{\mathrm{f}}|\psi_{\mathrm{in}}\rangle \left(\boxed{\mathrm{e}^{-\mathrm{i}GA_w\hat{p}_{\mathrm{m}}}}\right)|\phi\rangle_{\mathrm{m}}$$

Translation operator

Weak measurement

Overall effect

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Weak measurement

Overall effect

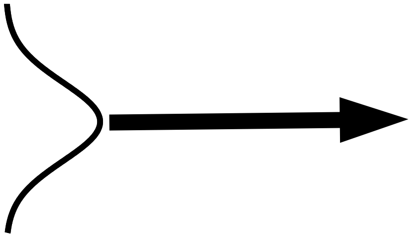
Weak value

$$\phi(x) \rightarrow \phi\left(x + G A_w\right)$$

Interaction strength

Stern-Gerlach weak measurement

$$|S_{in}\rangle\,|\phi\rangle$$



Stern-Gerlach weak measurement

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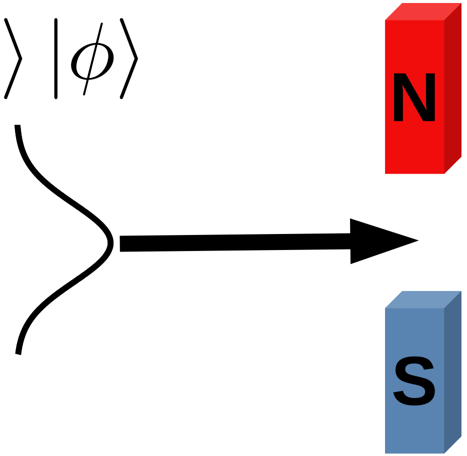
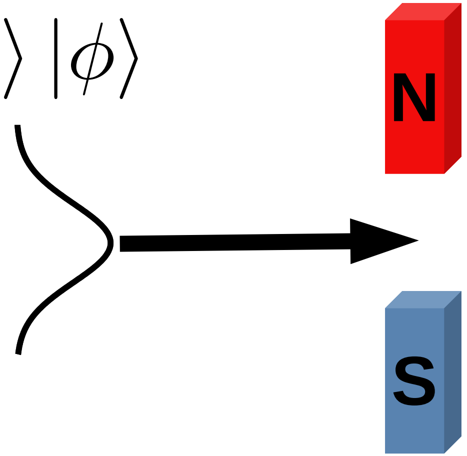


$$\hat{H}_I(t) \propto \hat{\sigma}_z\,\hat{p}_z$$

Stern-Gerlach weak measurement

$$G \ll 1$$

$$|S_{in}\rangle \, |\phi\rangle$$



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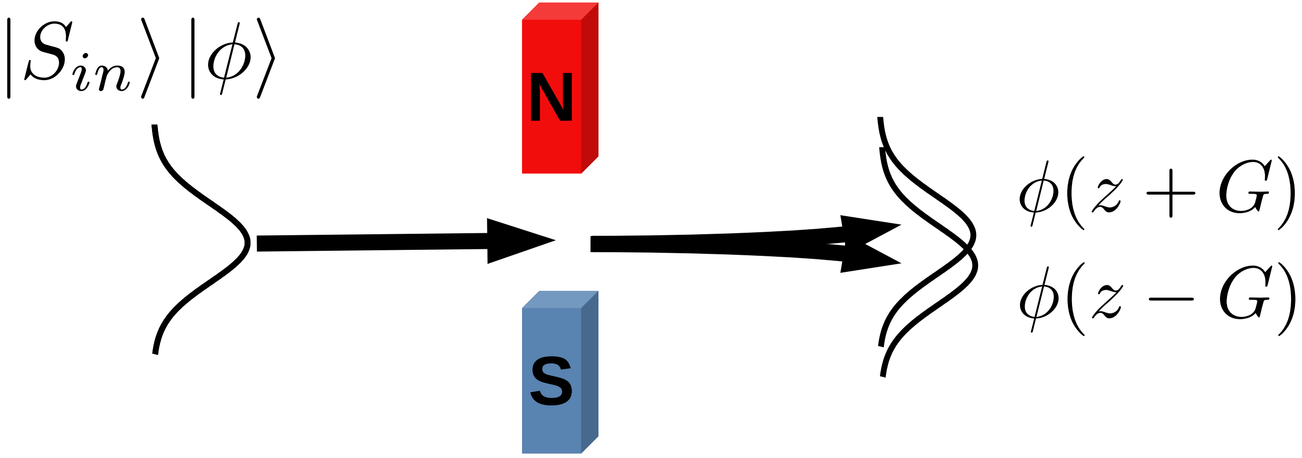

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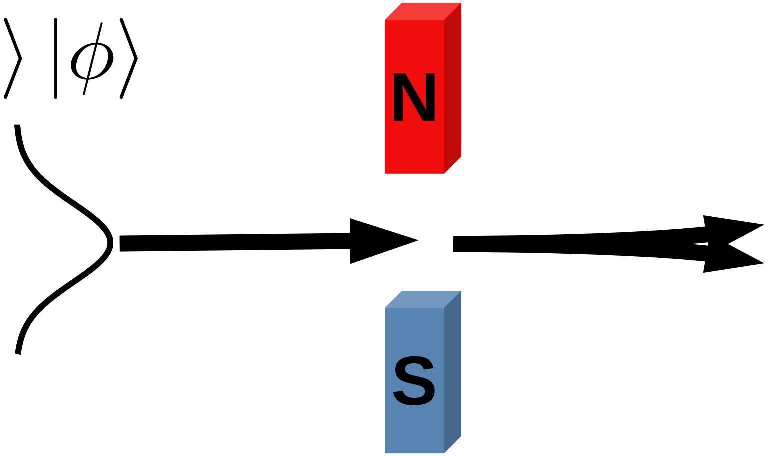

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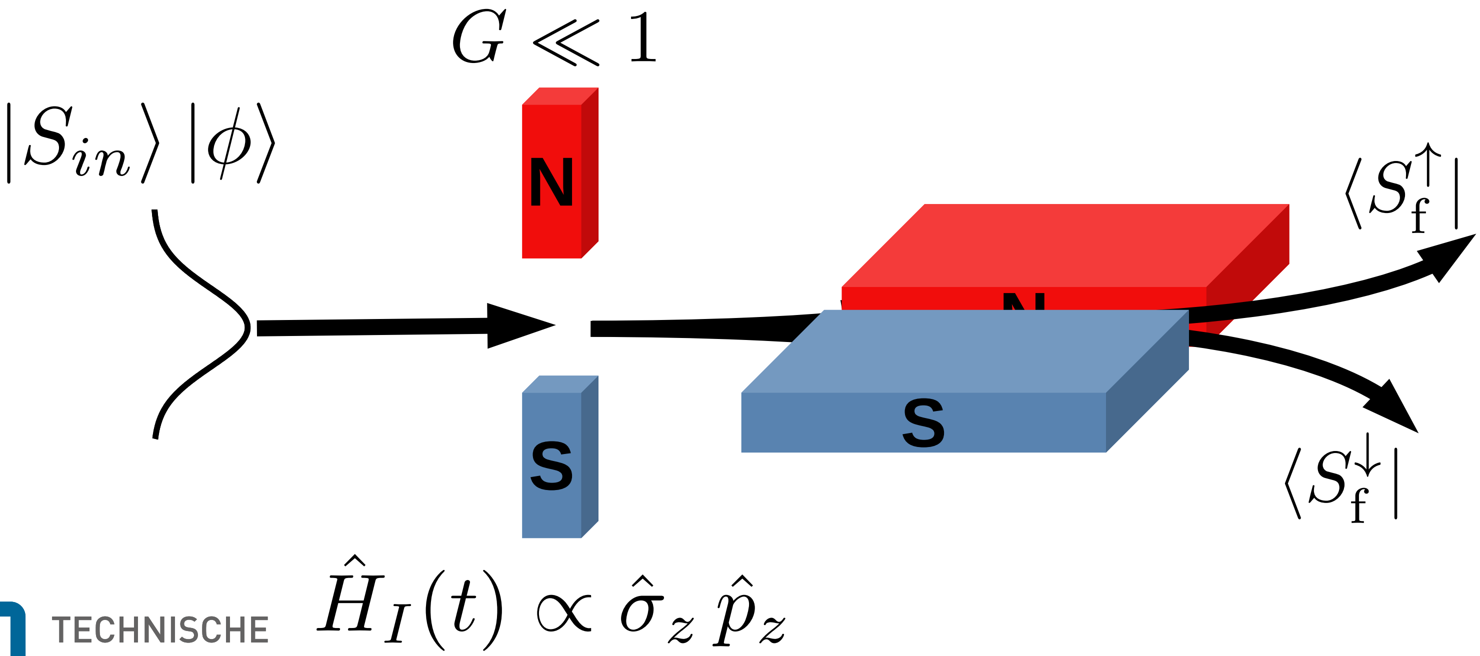
Stern-Gerlach weak measurement



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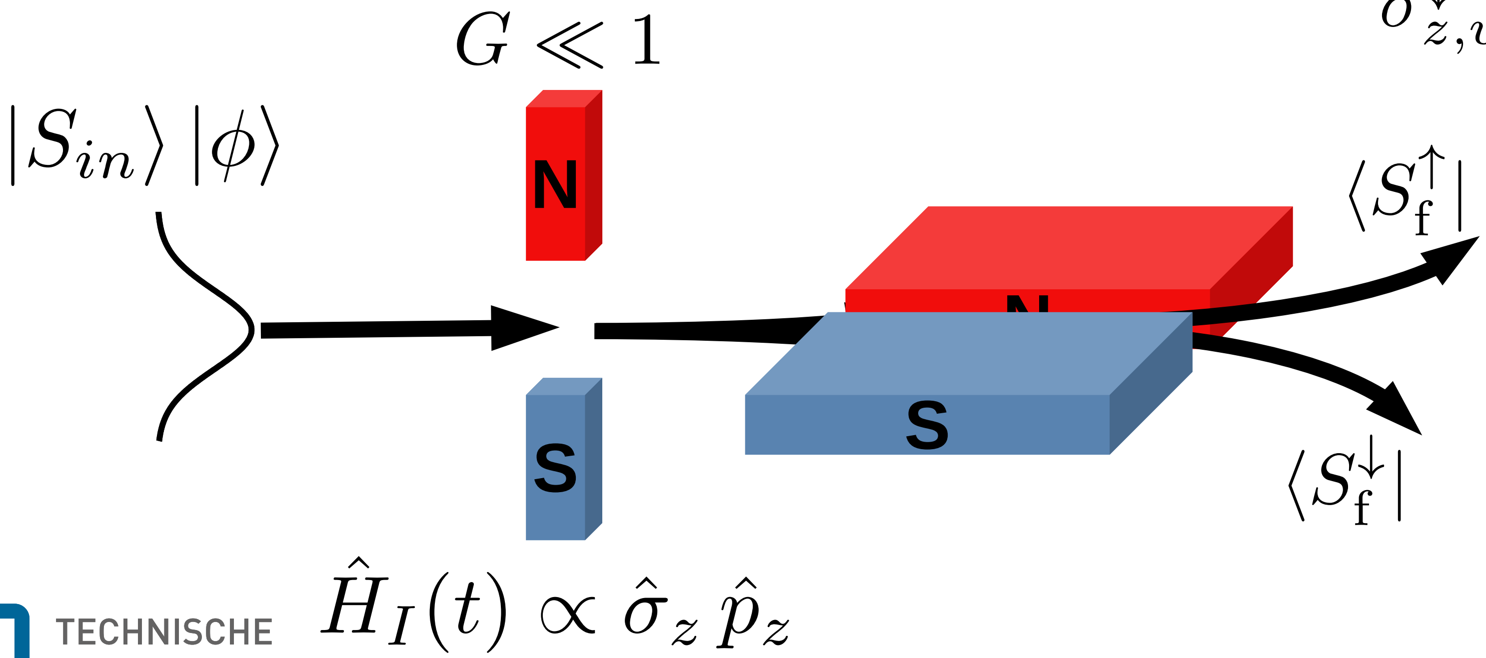
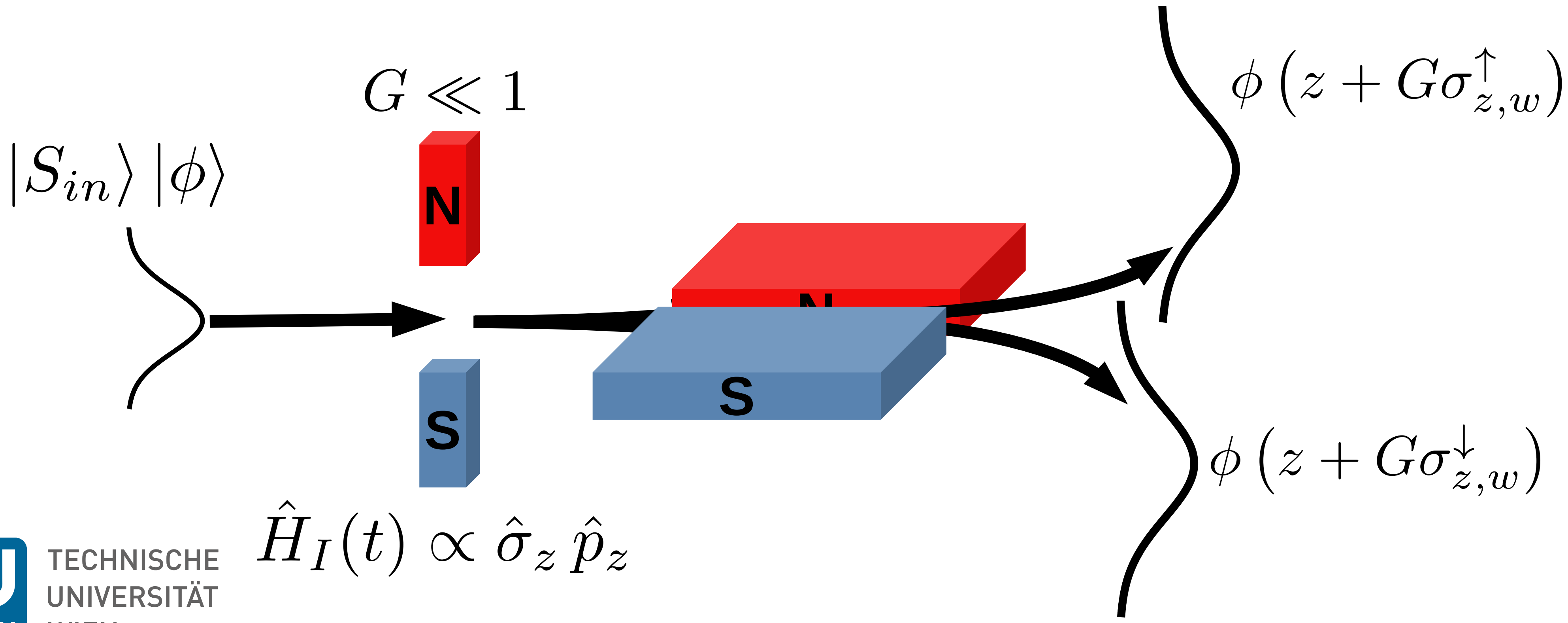
Stern-Gerlach weak measurement

Spin weak value:

$$\sigma^{\updownarrow}_{z,w} = \frac{\langle S^{\updownarrow}_{\mathrm{f}} | \hat{\sigma}_z | S_{\mathrm{in}} \rangle}{\langle S^{\updownarrow}_{\mathrm{f}} | S_{\mathrm{in}} \rangle}$$



Stern-Gerlach weak measurement



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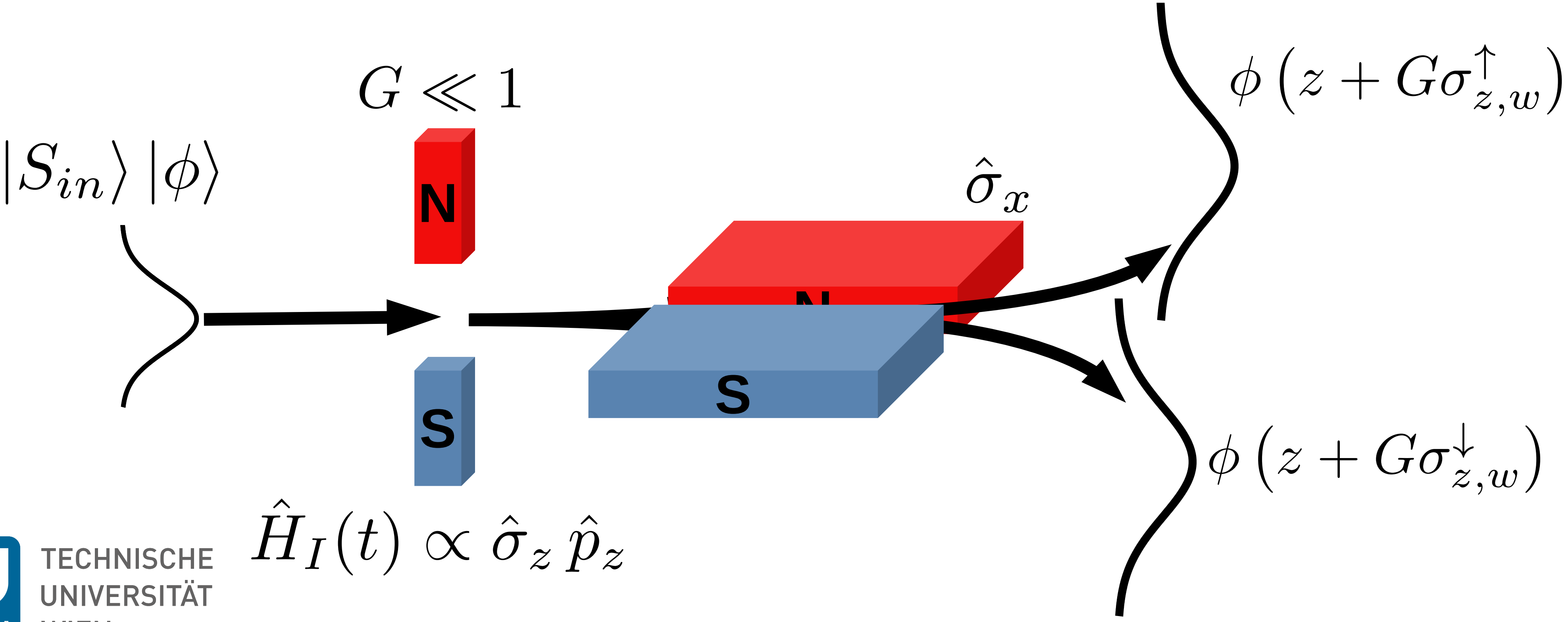
Stern-Gerlach weak measurement



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First paper on weak values

VOLUME 60, NUMBER 14 PHYSICAL REVIEW LETTERS 4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin-$\frac{1}{2}$ Particle Can Turn Out to be 100

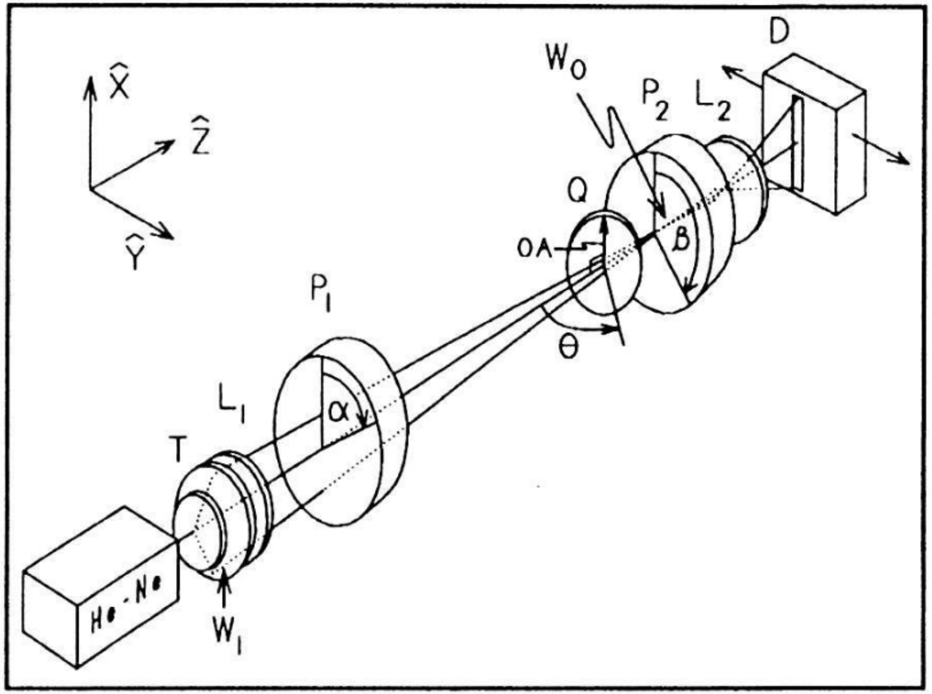
Yakir Aharonov, David Z. Albert, and Lev Vaidman

First measurement of weak values

4 MARCH 1991

Realization of a Measurement of a "Weak Value"

N. W. M. Ritchie, J. G. Story, and Randall G. Hulet



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What are they good for?

Theory and experiment

- Quantum amplification
- Quantum paradoxes
- Negative quasi-probability distributions
- Uncertainty relations

Theory and experiment

- Quantum amplification
- Quantum paradoxes
- Negative quasi-probability distributions
- Uncertainty relations
- And more...

J. Dressel. Weak values as interference phenomena. *Phys. Rev. A*, 91:032116.

J. Dressel, M. Malik, F. M. Miatto, A. N. Jordan, and R. W. Boyd. Colloquium: Understanding quantum weak values: Basics and applications. *Rev. Mod. Phys.*, 86:307–316.

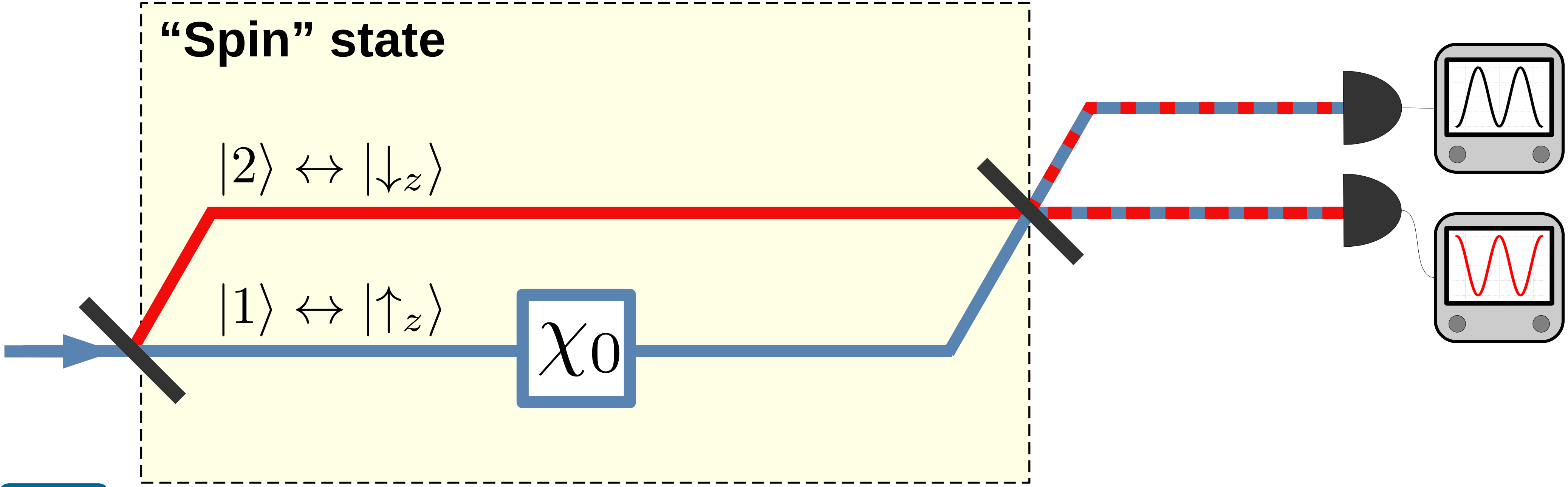
M. J. W. Hall. Prior information: How to circumvent the standard joint-measurement uncertainty relation. *Phys. Rev. A*, 69:052113.

“In between” evolution of a system

- Minimal disturbance
- Non-commuting observables

Interferometry

Two level system = spinor



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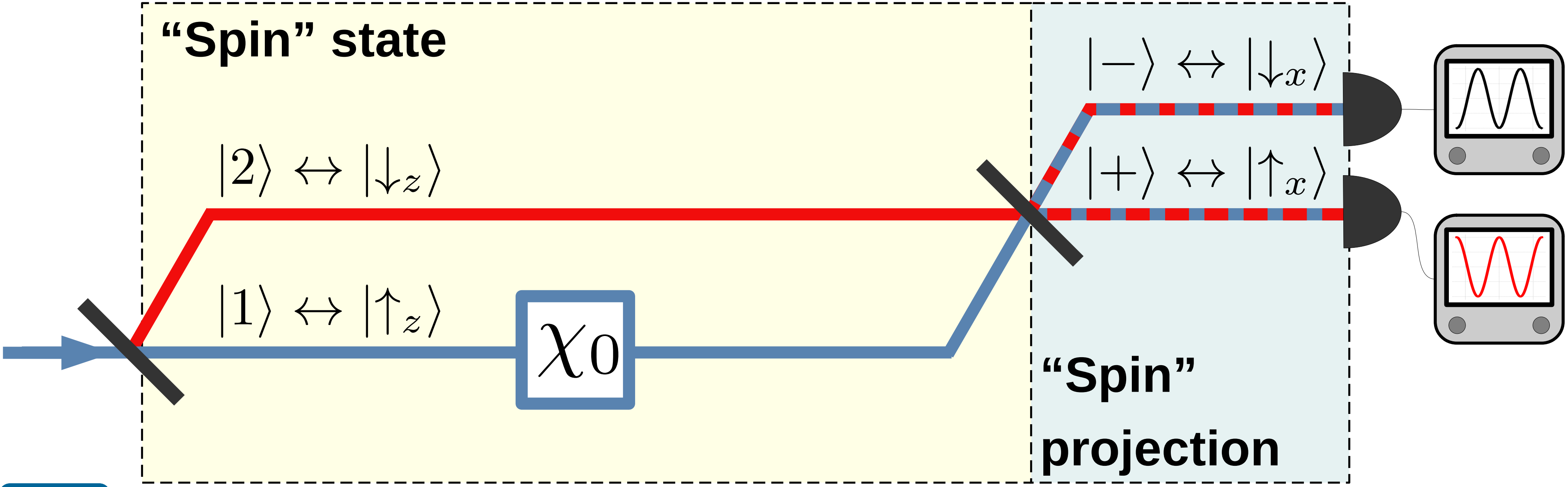
Interferometry

Two level system = spinor



Do we actually need a weak measurement?

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Weak value

Definition:

$$A_w = \frac{\langle \psi_{\mathrm{f}} | \hat{A} | \psi_{\mathrm{in}} \rangle}{\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle}$$

Weak value

Definition:

$$A_w = \frac{\langle \psi_{\mathrm{f}} | \hat{A} | \psi_{\mathrm{in}} \rangle}{\langle \psi_{\mathrm{f}} | \psi_{\mathrm{in}} \rangle}$$

- No mention of interaction strength
- No mention of meter/auxiliary state

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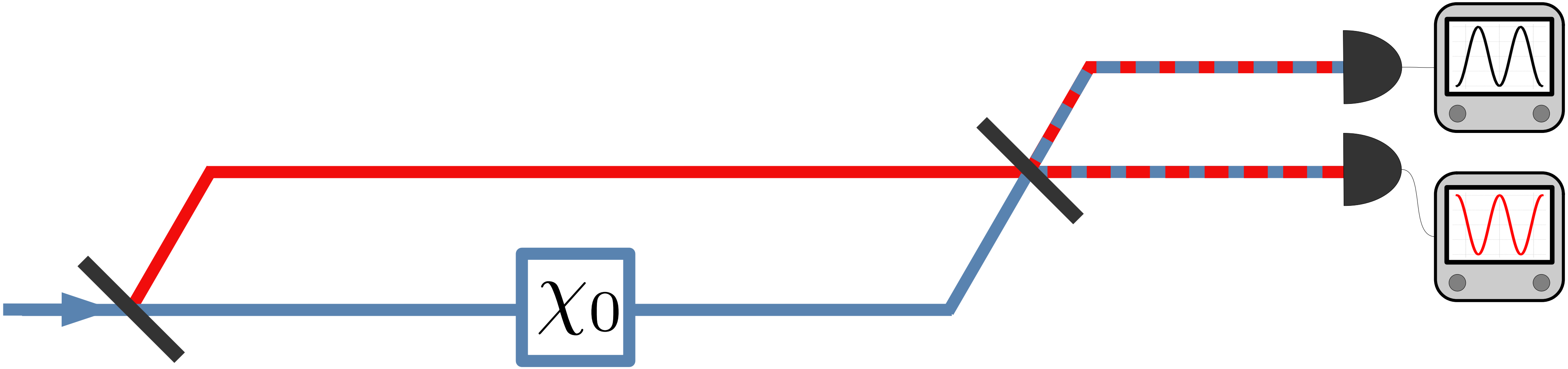
Part 2: Weak values based description of interferometry

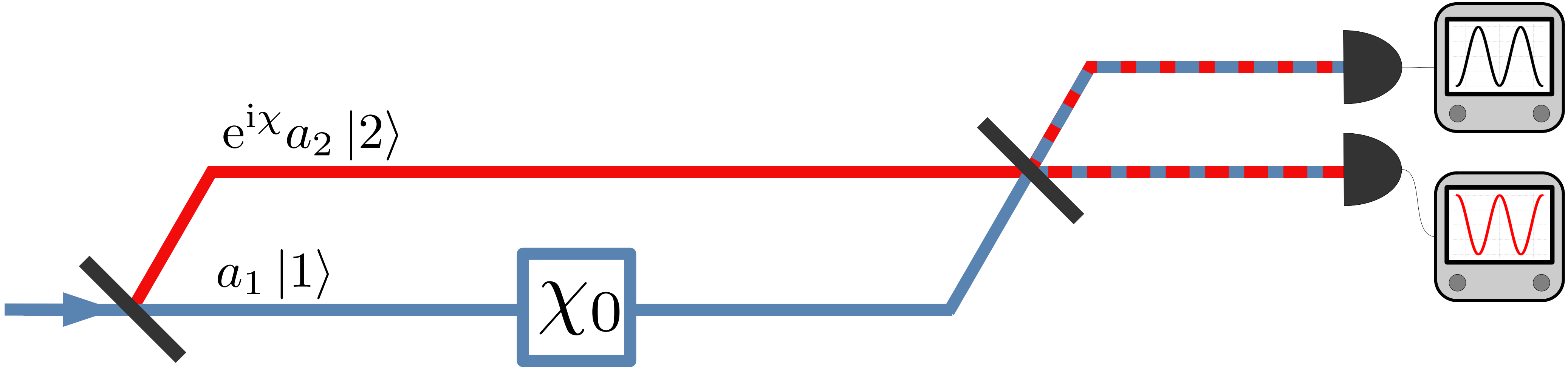
Weak values and interferometry

Standard interferometry formalism



Weak values and interferometry

Standard interferometry formalism



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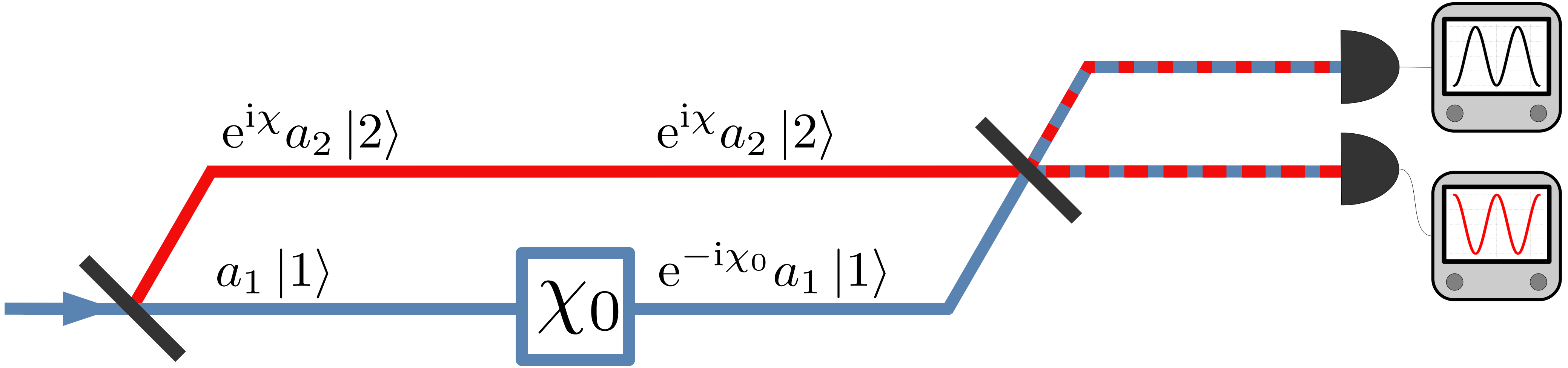
Weak values and interferometry

Standard interferometry formalism



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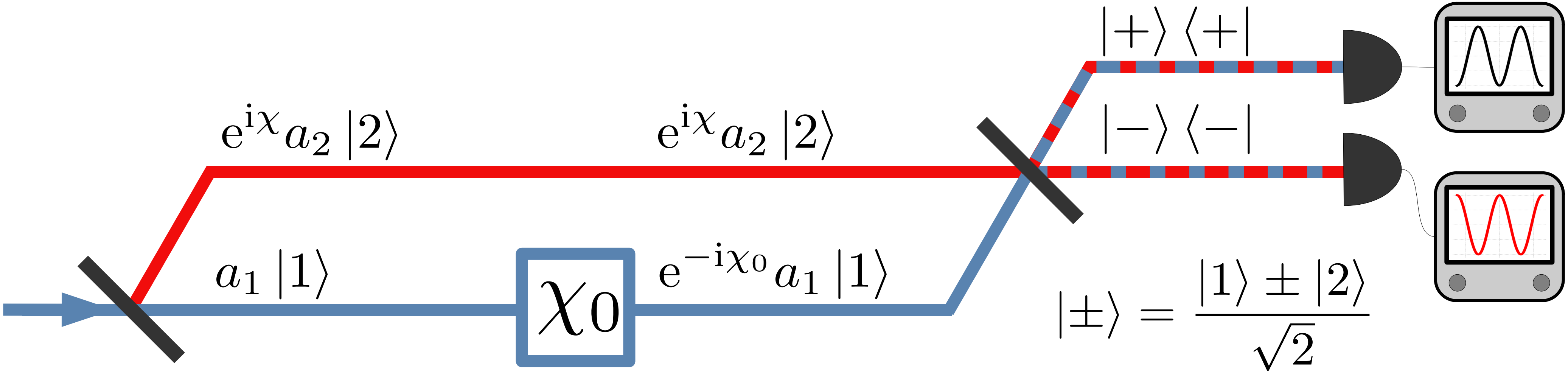
Weak values and interferometry

Standard interferometry formalism



$$|\pm\rangle = \frac{|1\rangle \pm |2\rangle}{\sqrt{2}}$$

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Weak values and interferometry

Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi,\chi_0) = \quad \left(\mathrm{e}^{-\mathrm{i}\chi_0} a_1 \left|1\right\rangle + \mathrm{e}^{\mathrm{i}\chi} a_2 \left|2\right\rangle\right)$$

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Weak values and interferometry

Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi, \chi_0) = \langle \pm | \left(\mathrm{e}^{-\mathrm{i}\chi_0} a_1 |1\rangle + \mathrm{e}^{\mathrm{i}\chi} a_2 |2\rangle \right)$$

Weak values and interferometry

Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi,\chi_0) = \left|\langle\pm|\left(\mathrm{e}^{-\mathrm{i}\chi_0} a_1 |1\rangle + \mathrm{e}^{\mathrm{i}\chi} a_2 |2\rangle\right)\right|^2$$

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Weak values and interferometry

Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi,\chi_0) = \left|\langle\pm| \left(\mathrm{e}^{-\mathrm{i}\chi_0} a_1 |1\rangle + \mathrm{e}^{\mathrm{i}\chi} a_2 |2\rangle\right)\right|^2 = \frac{1}{2} \pm a_1 a_2 \cos(\chi + \chi_0)$$

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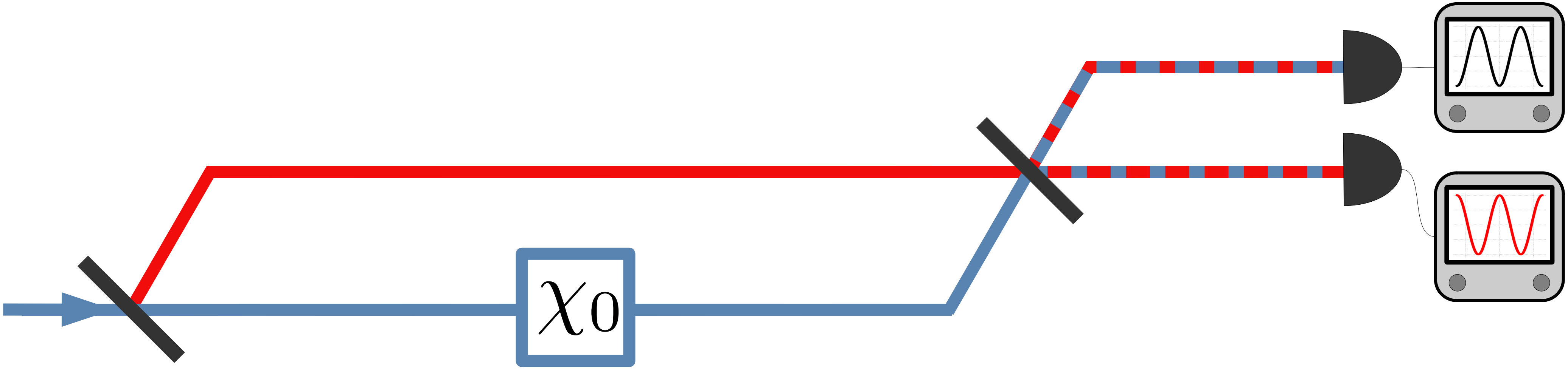
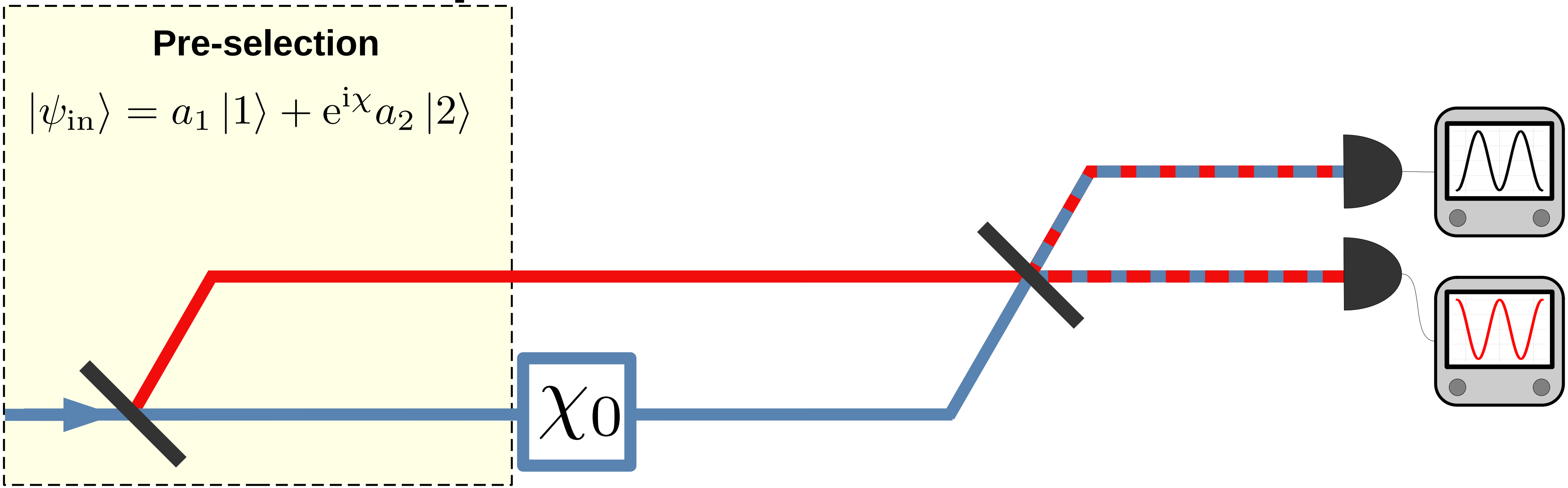
Weak values and interferometry

Weak value picture



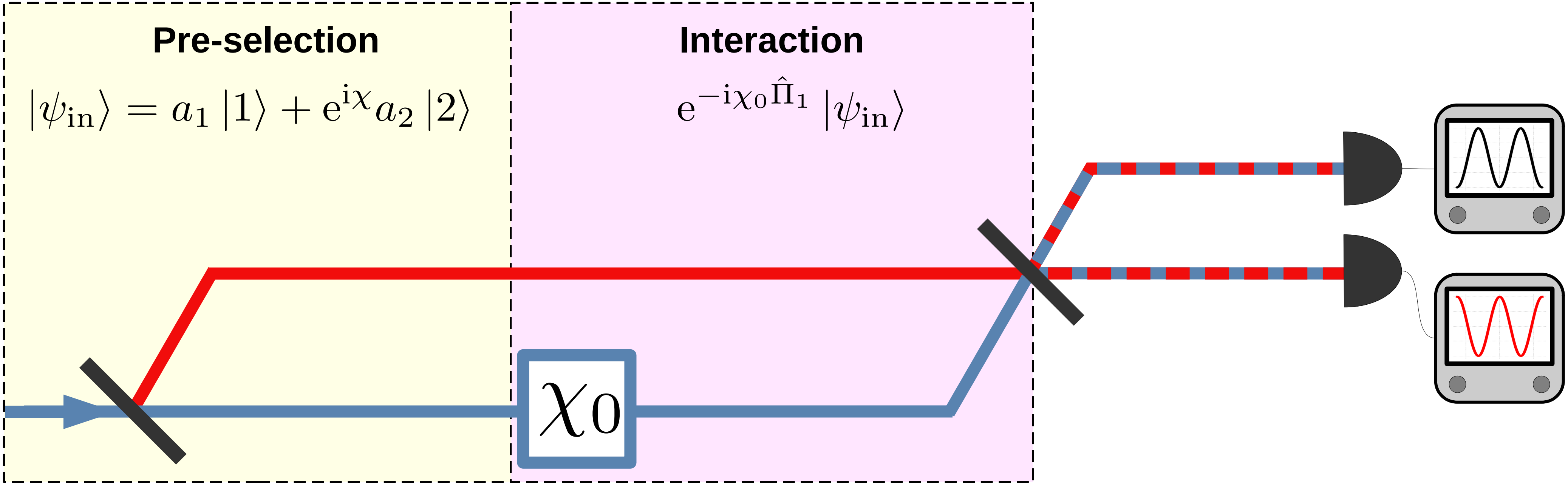
Weak values and interferometry

Weak value picture



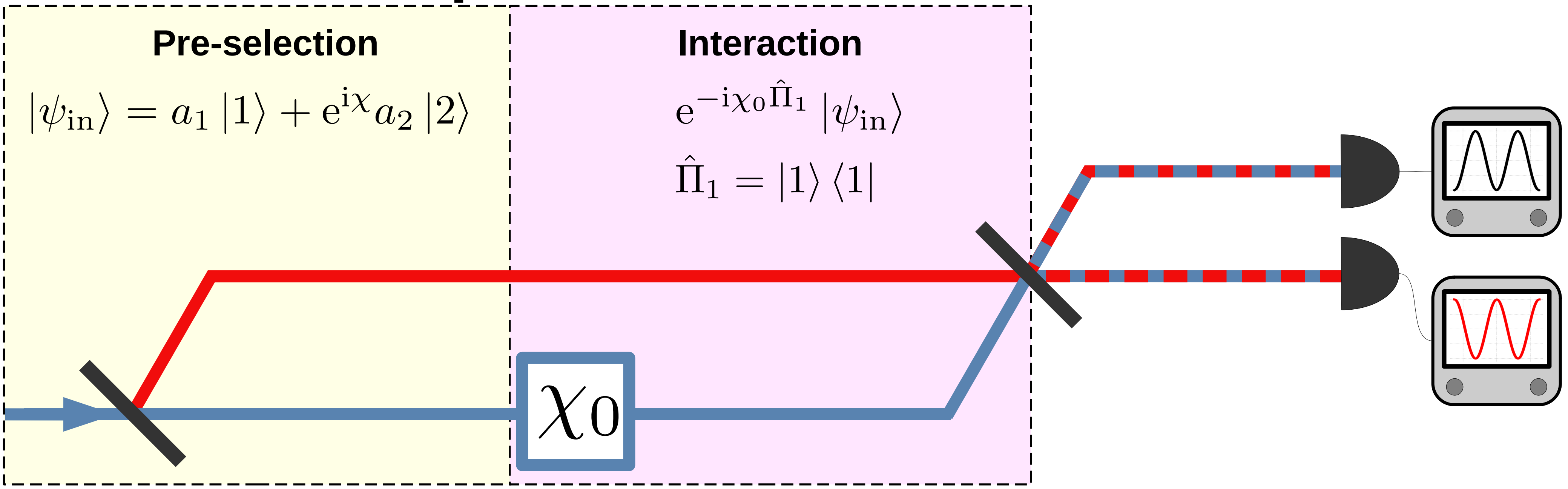
Weak values and interferometry

Weak value picture



Weak values and interferometry

Weak value picture



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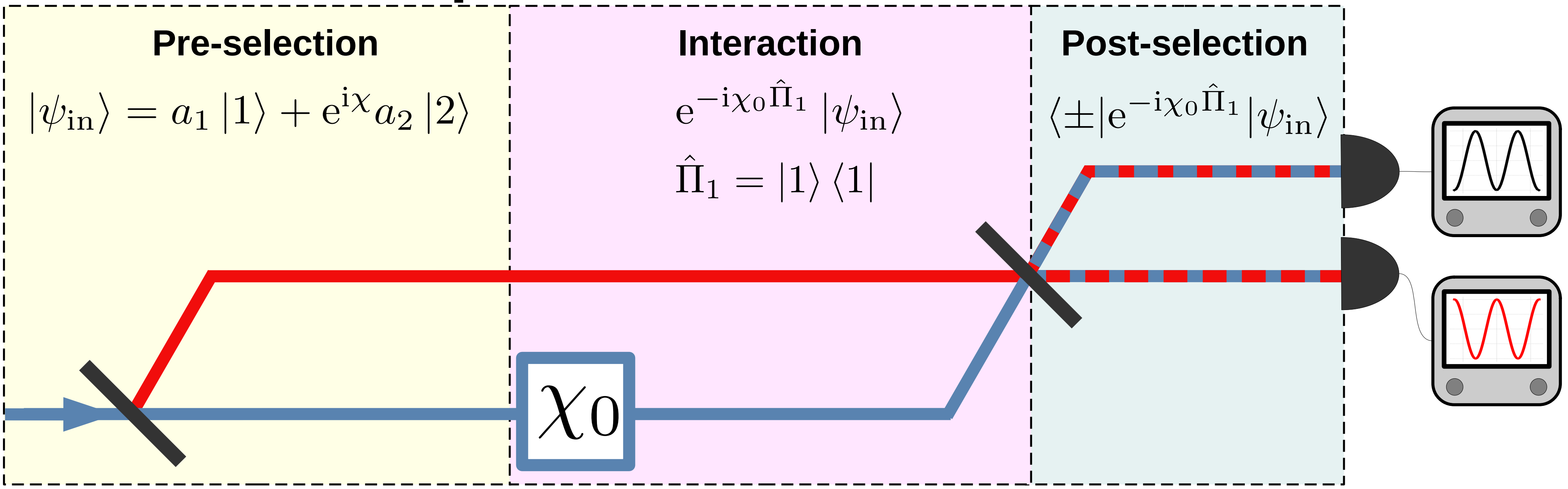
Weak values and interferometry

Weak value picture



Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi, \chi_0) = \left| \langle \pm | \mathrm{e}^{-\mathrm{i}\chi_0 \hat{\Pi}_1} | \psi_{\mathrm{in}} \rangle \right|^2$$

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Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi,\chi_0) = \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0\hat{\Pi}_1}|\psi_{\mathrm{in}}\rangle\right|^2 = \left|\langle\pm|1+\left(\mathrm{e}^{-\mathrm{i}\chi_0}-1\right)\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2$$

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Weak values and interferometry

Weak value picture

Measured intensity

$$
\begin{aligned}
I_\pm(\chi,\chi_0) &= \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0\hat{\Pi}_1}|\psi_{\mathrm{in}}\rangle\right|^2 = \left|\langle\pm|1+\left(\mathrm{e}^{-\mathrm{i}\chi_0}-1\right)\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2 \\
&= \left|\langle\pm|\psi_{\mathrm{in}}\rangle\left[1+\left(\mathrm{e}^{-\mathrm{i}\chi_0}-1\right)\frac{\langle\pm|\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle}{\langle\pm|\psi_{\mathrm{in}}\rangle}\right]\right|^2
\end{aligned}
$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi, \chi_0) = \left|\langle \pm | \mathrm{e}^{-\mathrm{i}\chi_0 \hat{\Pi}_1} | \psi_{\mathrm{in}} \rangle\right|^2 = \left|\langle \pm | 1 + \left(\mathrm{e}^{-\mathrm{i}\chi_0} - 1\right) \hat{\Pi}_1 | \psi_{\mathrm{in}} \rangle\right|^2$$

$$= \left|\langle \pm | \psi_{\mathrm{in}} \rangle \left[1 + \left(\mathrm{e}^{-\mathrm{i}\chi_0} - 1\right) \boxed{\frac{\langle \pm | \hat{\Pi}_1 | \psi_{\mathrm{in}} \rangle}{\langle \pm | \psi_{\mathrm{in}} \rangle}}\right]\right|^2$$

Path weak value $w_{\pm,1}$

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Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi,\chi_0) = \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0\hat{\Pi}_1}|\psi_{\mathrm{in}}\rangle\right|^2 = \left|\langle\pm|1+\left(\mathrm{e}^{-\mathrm{i}\chi_0}-1\right)\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2$$

$$= \left|\langle\pm|\psi_{\mathrm{in}}\rangle\left[1+\left(\mathrm{e}^{-\mathrm{i}\chi_0}-1\right)w_{\pm,1}\right]\right|^2$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi,\chi_0) = \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0\hat{\Pi}_1}|\psi_{\mathrm{in}}\rangle\right|^2 = \left|\langle\pm|1 + \left(\mathrm{e}^{-\mathrm{i}\chi_0} - 1\right)\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2$$

$$= \left|\langle\pm|\psi_{\mathrm{in}}\rangle\left[1 + \left(\mathrm{e}^{-\mathrm{i}\chi_0} - 1\right) w_{\pm,1}\right]\right|^2$$

$$= |\langle\pm|\psi_{\mathrm{in}}\rangle|^2 \left[1 + 2\left(|w_{\pm,1}|^2 - w^{\Re}_{\pm,1}\right)(1 - \cos\chi_0) + 2w^{\Im}_{\pm,1}\sin\chi_0\right]$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi,\chi_0) = |\langle \pm|\psi_{\rm in}\rangle|^2 \left[1 + 2\left(|w_{\pm,1}|^2 - w^{\Re}_{\pm,1}\right)(1-\cos\chi_0) + 2w^{\Im}_{\pm,1}\sin\chi_0\right]$$

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Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi, \chi_0) = |\langle \pm|\psi_{\text{in}}\rangle|^2 \left[1 + 2\left(\boxed{|w_{\pm,1}|^2} - w^{\Re}_{\pm,1}\right)(1 - \cos\chi_0) + 2w^{\Im}_{\pm,1}\sin\chi_0\right]$$

Amplitude square

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Weak values and interferometry

Weak value picture

Measured intensity

Real part

$$I_\pm(\chi,\chi_0) = |\langle \pm|\psi_{\text{in}}\rangle|^2 \left[1 + 2\left(\boxed{|w_{\pm,1}|^2} - \boxed{w^{\Re}_{\pm,1}}\right)(1-\cos\chi_0) + 2w^{\Im}_{\pm,1}\sin\chi_0\right]$$

Amplitude
square

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 \left[1 + 2 \left(\underbrace{\boxed{|w_{\pm,1}|^2}}_{\text{Amplitude square}} - \overbrace{\boxed{w^{\Re}_{\pm,1}}}^{\text{Real part}} \right) (1 - \cos\chi_0) + 2 \underbrace{\boxed{w^{\Im}_{\pm,1}}}_{\text{Imaginary part}} \sin\chi_0 \right]$$

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Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi,\chi_0) = |\langle\pm|\psi_{\text{in}}\rangle|^2\left[1 + 2\left(|w_{\pm,1}|^2 - w^{\Re}_{\pm,1}\right)(1-\cos\boxed{\chi_0}) + 2w^{\Im}_{\pm,1}\sin\boxed{\chi_0}\right]$$

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Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi,\chi_0) = \boxed{|\langle\pm|\psi_{\text{in}}\rangle|^2}\left[1 + 2\left(|w_{\pm,1}|^2 - w^{\Re}_{\pm,1}\right)(1-\cos\chi_0) + 2w^{\Im}_{\pm,1}\sin\chi_0\right]$$

$$\boxed{I_{\pm}(\chi,0) = |\langle\pm|\psi_{\text{in}}\rangle|^2}$$

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Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi,\chi_0) = \boxed{|\langle\pm|\psi_{\text{in}}\rangle|^2}\left[1 + 2\boxed{\left(|w_{\pm,1}|^2 - w^{\Re}_{\pm,1}\right)}(1-\cos\chi_0) + 2w^{\Im}_{\pm,1}\sin\chi_0\right]$$

$$\boxed{I_\pm(\chi,0) = |\langle\pm|\psi_{\text{in}}\rangle|^2}$$

$$\boxed{\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = |w_{\pm,1}|^2 - w^{\Re}_{\pm,1}}$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_\pm(\chi,\chi_0) = \boxed{|\langle\pm|\psi_{\mathrm{in}}\rangle|^2}\left[1 + 2\boxed{\left(|w_{\pm,1}|^2 - w^{\Re}_{\pm,1}\right)}(1-\cos\chi_0) + 2\boxed{w^{\Im}_{\pm,1}}\sin\chi_0\right]$$

$$\boxed{I_\pm(\chi,0) = |\langle\pm|\psi_{\mathrm{in}}\rangle|^2}$$

$$\boxed{\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = |w_{\pm,1}|^2 - w^{\Re}_{\pm,1}}$$

$$\boxed{\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}}$$

Weak values and interferometry

Almost there...

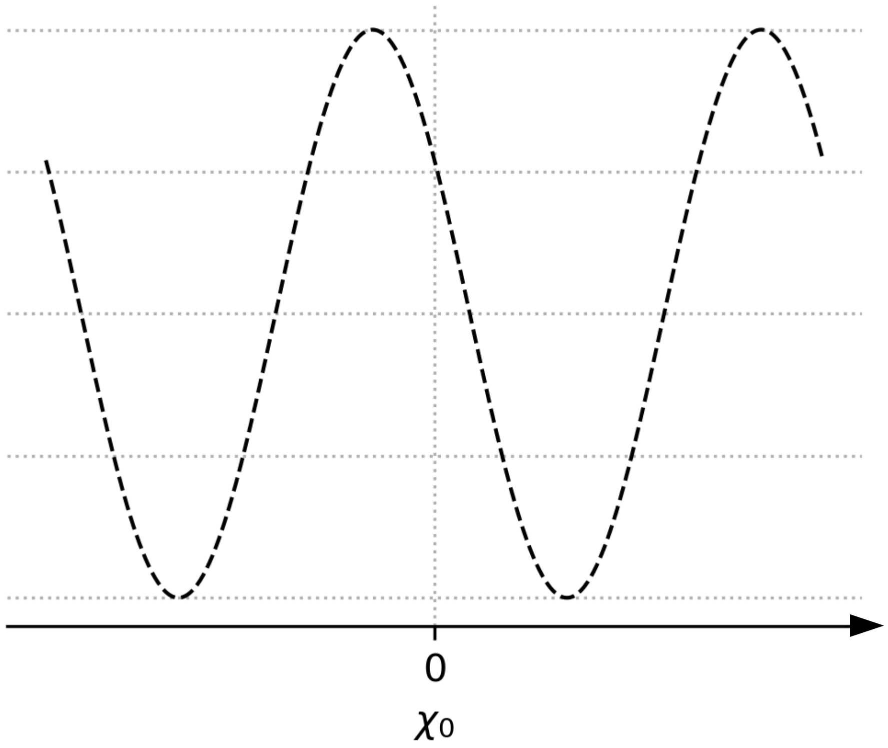
$$\frac{I_{\pm}(\chi,\frac{\pi}{2}) - I_{\pm}(\chi,\frac{3\pi}{2})}{4I_{\pm}(\chi,0)} = w^{\Im}_{\pm,1}$$

Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi, \frac{\pi}{2}) - I_\pm(\chi, \frac{3\pi}{2})}{4I_\pm(\chi, 0)} = w^{\Im}_{\pm,1}$$

Interferogram



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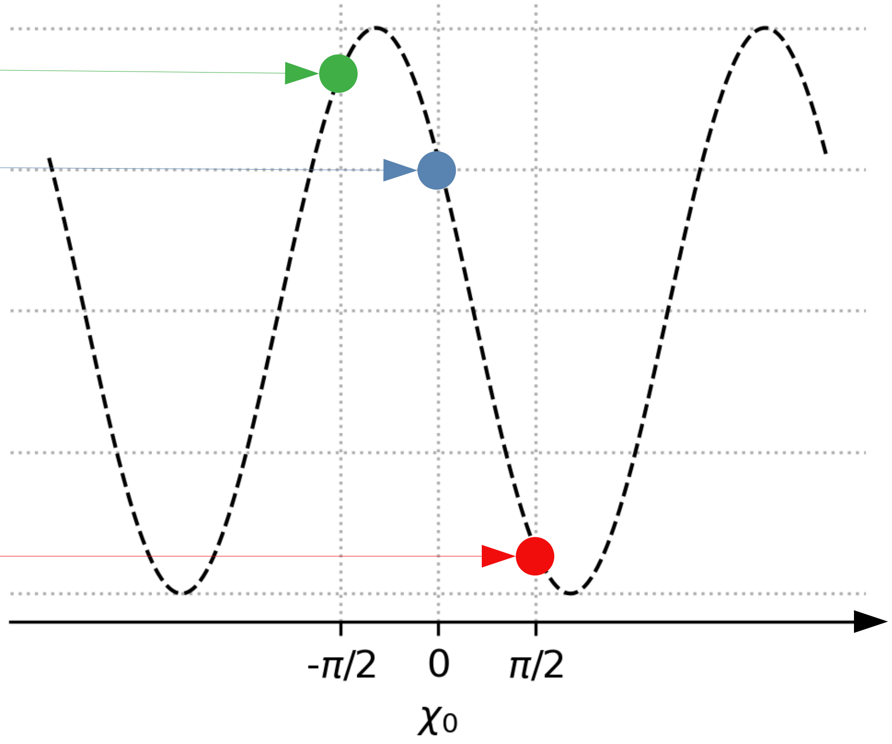
Weak values and interferometry

Almost there...

$$\frac{\boxed{I_\pm(\chi,\frac{\pi}{2})} - \boxed{I_\pm(\chi,\frac{3\pi}{2})}}{4\boxed{I_\pm(\chi,0)}} = w^{\Im}_{\pm,1}$$

Interferogram



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Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi, \frac{\pi}{2}) - I_\pm(\chi, \frac{3\pi}{2})}{4 I_\pm(\chi, 0)} = w^{\Im}_{\pm,1}$$

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Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}$$

$$\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = |w_{\pm,1}|^2 - w^{\Re}_{\pm,1}$$

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Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}$$

$$\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = \boxed{\left|w_{\pm,1}\right|^2} - w^{\Re}_{\pm,1}$$

$$\boxed{{w^{\Re}_{\pm,1}}^2 + {w^{\Im}_{\pm,1}}^2}$$

Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}$$

$$\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = \underbrace{\boxed{\left|w_{\pm,1}\right|^2}}_{\boxed{{w^{\Re}_{\pm,1}}^2 + {w^{\Im}_{\pm,1}}^2}} - w^{\Re}_{\pm,1} \longleftarrow$$

2nd order equation

Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}$$

$$\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = \underbrace{\boxed{\left|w_{\pm,1}\right|^2}}_{\boxed{{w^{\Re}_{\pm,1}}^2 + {w^{\Im}_{\pm,1}}^2}} - w^{\Re}_{\pm,1} \longleftarrow \text{2}^{\text{nd}}\text{ order equation}$$

2nd order equation

2 solutions

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Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w_{\pm,1}^{\Im}$$

$$\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} = \boxed{\left|w_{\pm,1}\right|^2} - w_{\pm,1}^{\Re}$$

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Weak values and interferometry

Almost there...

$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}$$

$$-\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} + |w_{\pm,1}|^2 = w^{\Re}_{\pm,1}$$

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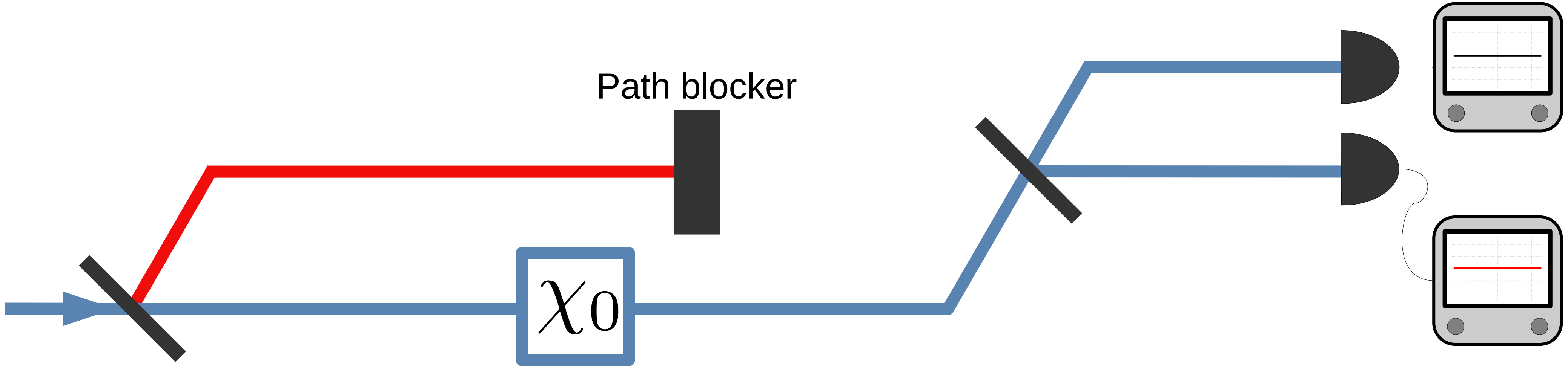
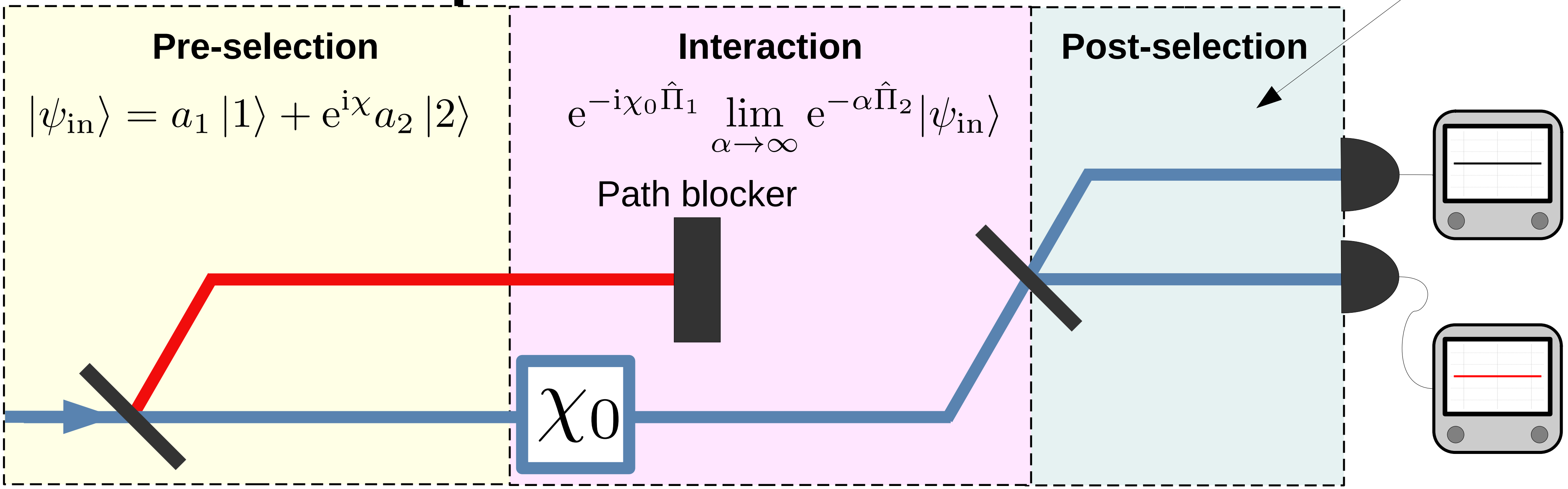
Weak values and interferometry



Weak values and interferometry

Weak value picture

$$\langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha\to\infty} e^{-\alpha \hat{\Pi}_2} | \psi_{\mathrm{in}} \rangle$$



Weak values and interferometry

Measured intensity

$$I_{\pm}^{Bl.\,2} = \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0\hat{\Pi}_1} \lim_{\alpha\to\infty} \mathrm{e}^{-\alpha\hat{\Pi}_2}|\psi_{\mathrm{in}}\rangle\right|^2$$

$$= \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0}\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2 = \left|\langle\pm|\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2$$

$$= \left|\langle\pm|\psi_{\mathrm{in}}\rangle\right|^2 \left|w_{\pm,1}\right|^2$$

Weak values and interferometry

Measured intensity

$$
\begin{aligned}
I_{\pm}^{Bl.\,2} &= \left| \langle \pm | \mathrm{e}^{-\mathrm{i}\chi_0 \hat{\Pi}_1} \lim_{\alpha\to\infty} \mathrm{e}^{-\alpha \hat{\Pi}_2} | \psi_{\mathrm{in}} \rangle \right|^2 \\
&= \left| \langle \pm | \mathrm{e}^{-\mathrm{i}\chi_0} \hat{\Pi}_1 | \psi_{\mathrm{in}} \rangle \right|^2 = \left| \langle \pm | \hat{\Pi}_1 | \psi_{\mathrm{in}} \rangle \right|^2 \\
&= | \langle \pm | \psi_{\mathrm{in}} \rangle |^2 \boxed{| w_{\pm,1} |^2}
\end{aligned}
$$

Weak values and interferometry

Measured intensity

$$
\begin{aligned}
I_{\pm}^{Bl.\,2} &= \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0\hat{\Pi}_1}\lim_{\alpha\to\infty}\mathrm{e}^{-\alpha\hat{\Pi}_2}|\psi_{\mathrm{in}}\rangle\right|^2 \\
&= \left|\langle\pm|\mathrm{e}^{-\mathrm{i}\chi_0}\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2 = \left|\langle\pm|\hat{\Pi}_1|\psi_{\mathrm{in}}\rangle\right|^2 \\
&= \boxed{\left|\langle\pm|\psi_{\mathrm{in}}\rangle\right|^2}\boxed{\left|w_{\pm,1}\right|^2}
\end{aligned}
$$

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Weak values and interferometry

We got there!

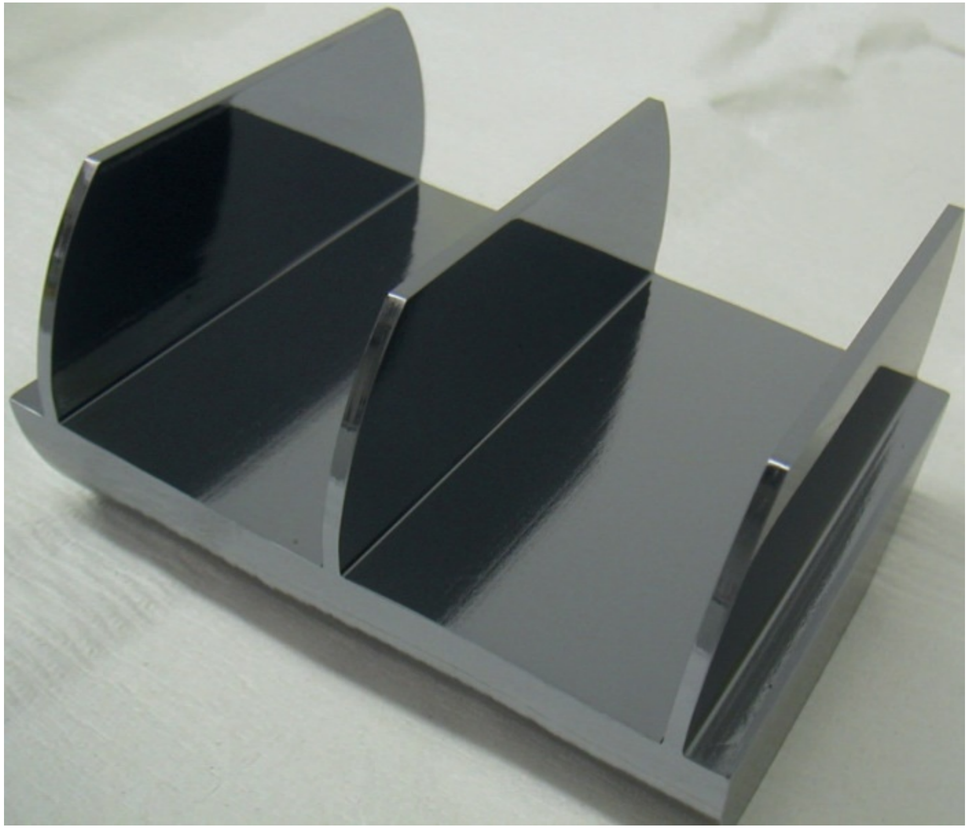
$$\frac{I_\pm(\chi,\frac{\pi}{2}) - I_\pm(\chi,\frac{3\pi}{2})}{4I_\pm(\chi,0)} = w^{\Im}_{\pm,1}$$ ✔

$$-\frac{I_\pm(\chi,\pi) - I_\pm(\chi,0)}{4I_\pm(\chi,0)} + |w_{\pm,1}|^2 = w^{\Re}_{\pm,1}$$ ✔

I am running
out of time, unexpectedly...

Part 3: Experimental measurement of path weak value from interferograms

Neutron interferometer



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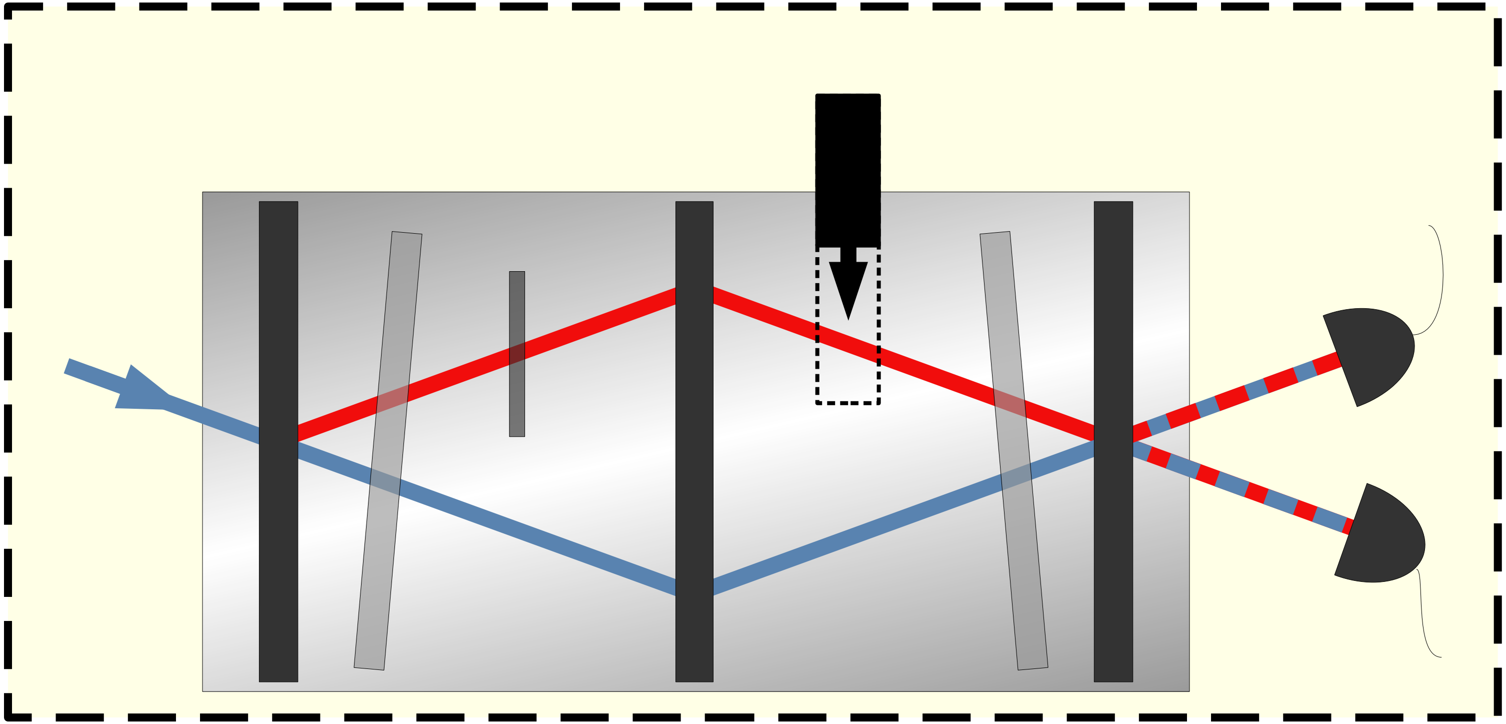
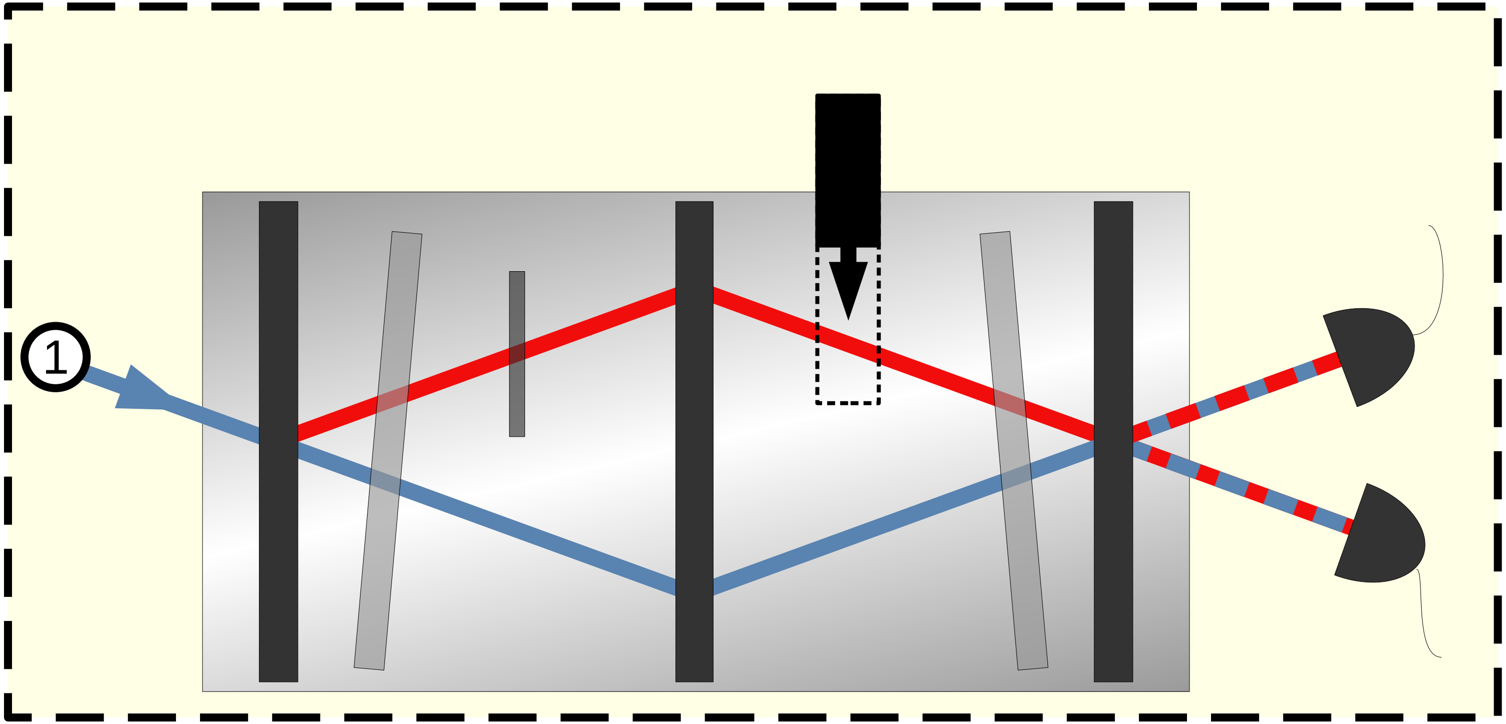
Setup



Setup

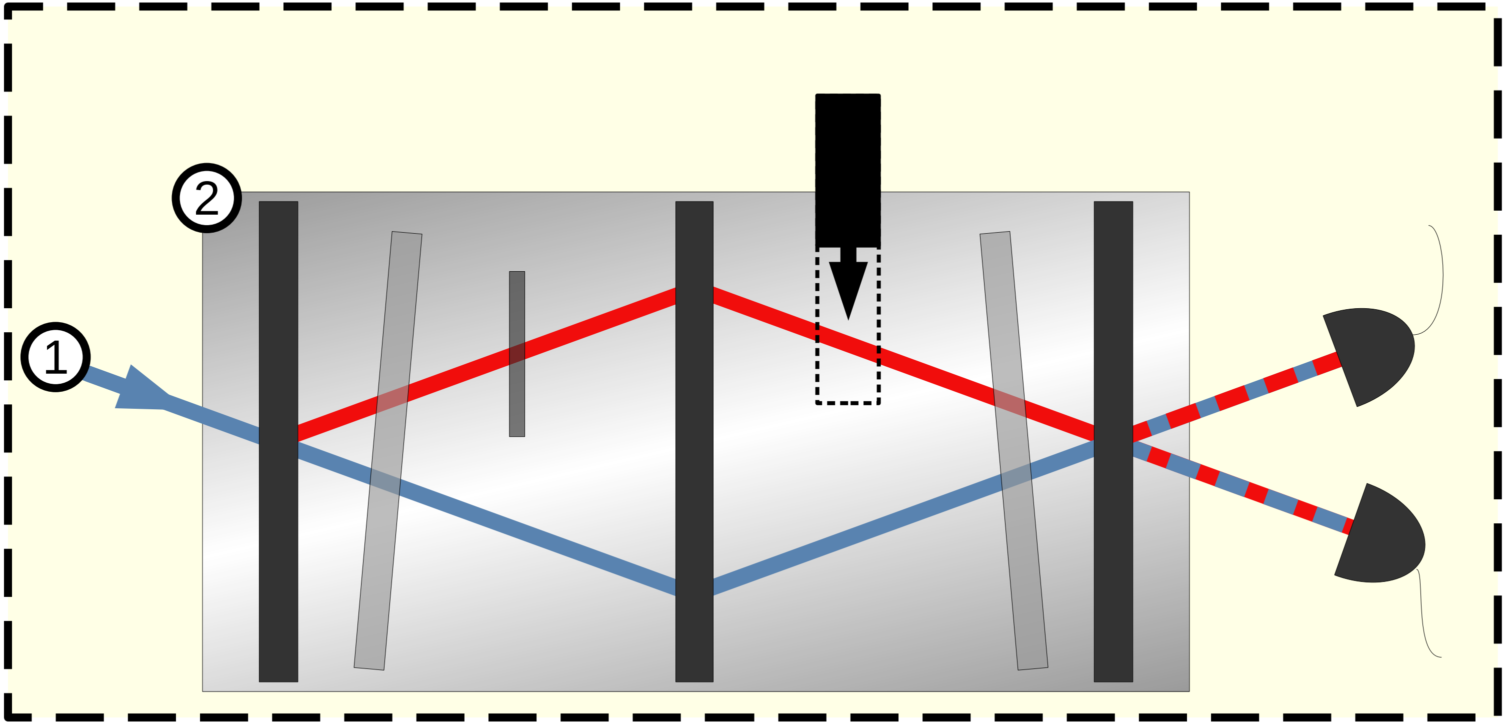
1 Unpolarised neutron beam
$\lambda = 1.92Å$
$\delta\lambda/\lambda \approx 0.02$



Setup

1 Unpolarised neutron beam
$\lambda = 1.92 Å$
$\delta\lambda/\lambda \approx 0.02$

2 Neutron interferometer

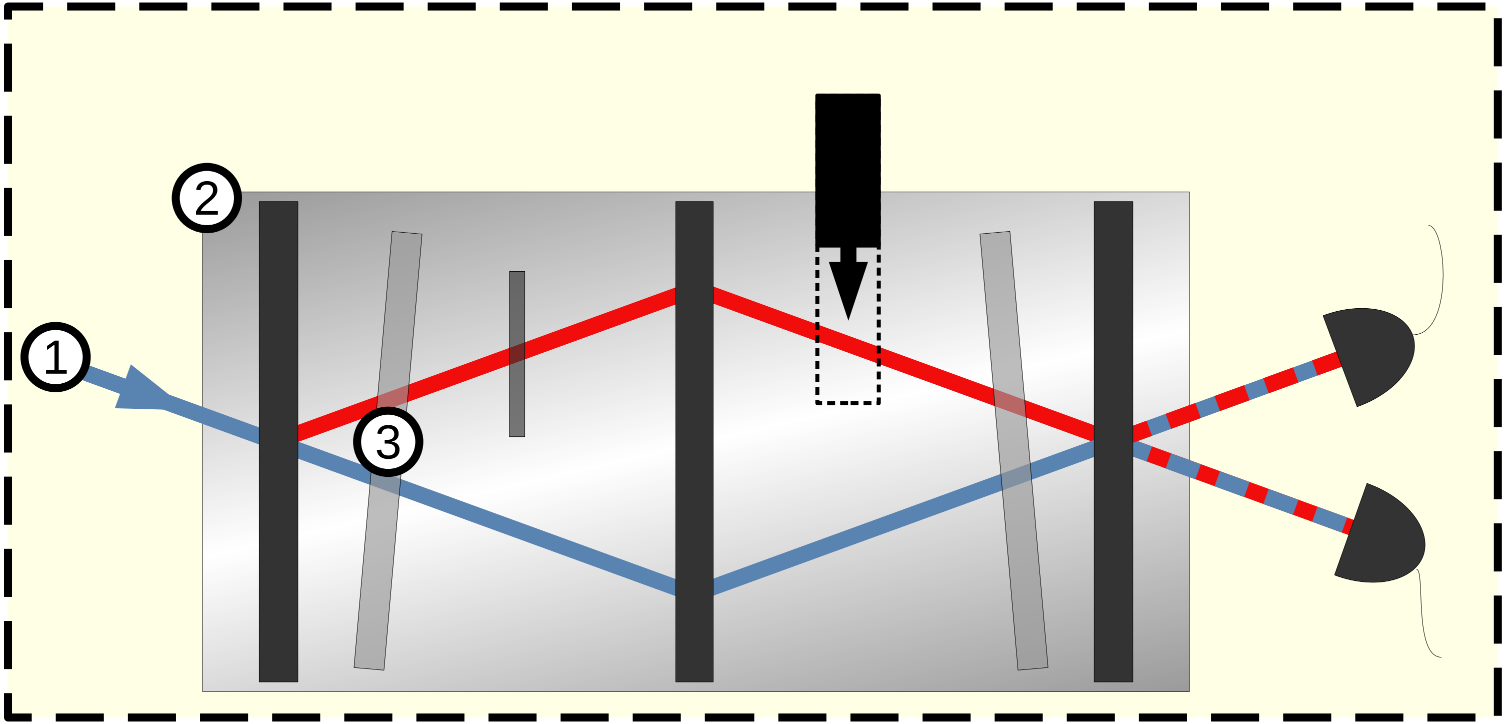


Setup

1 Unpolarised neutron beam
$\lambda = 1.92Å$
$\delta\lambda/\lambda \approx 0.02$

2 Neutron interferometer

3 Phase-shifter
χ



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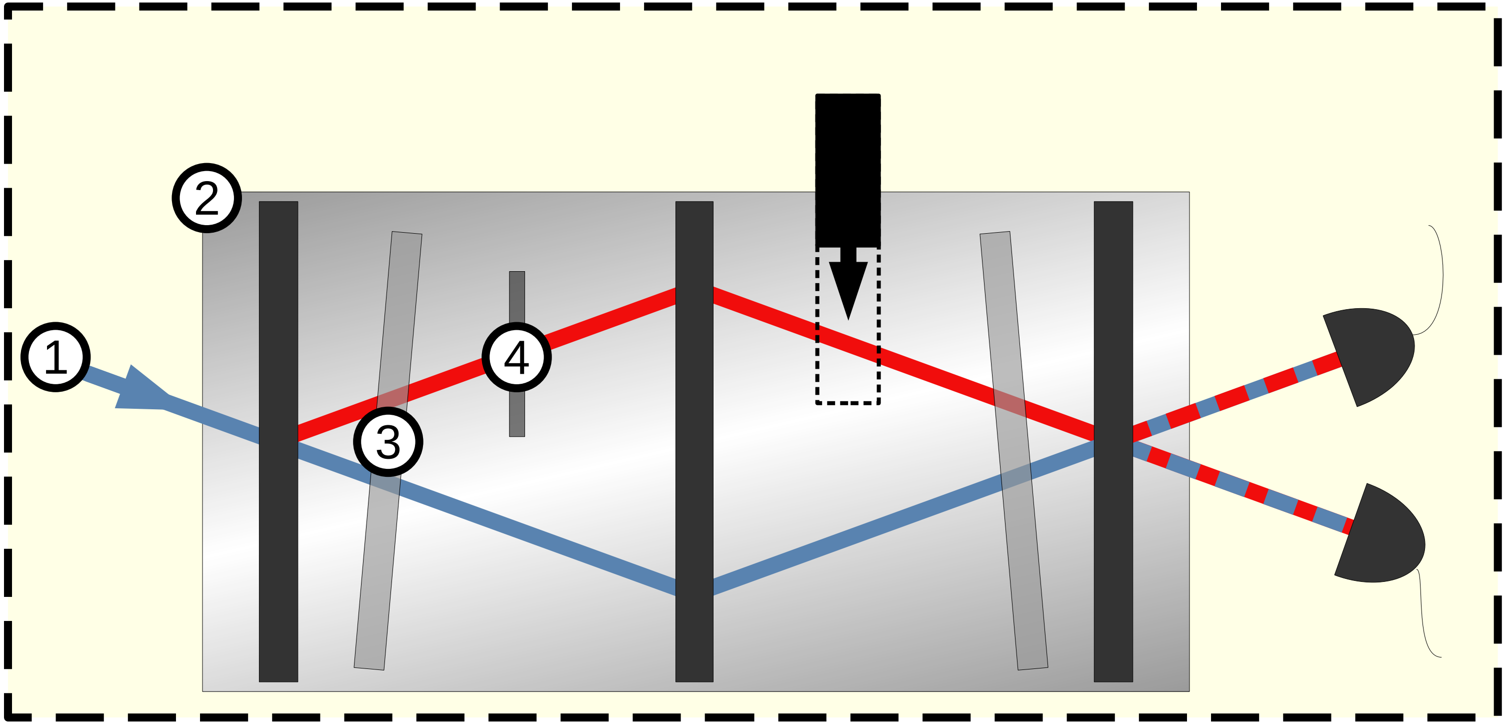
Setup

1. Unpolarised neutron beam
$\lambda = 1.92 Å$
$\delta\lambda/\lambda \approx 0.02$

2. Neutron interferometer

3. Phase-shifter
χ

4. Indium foils to adjust path amplitudes
$a_2/a_1 \approx 0.59$



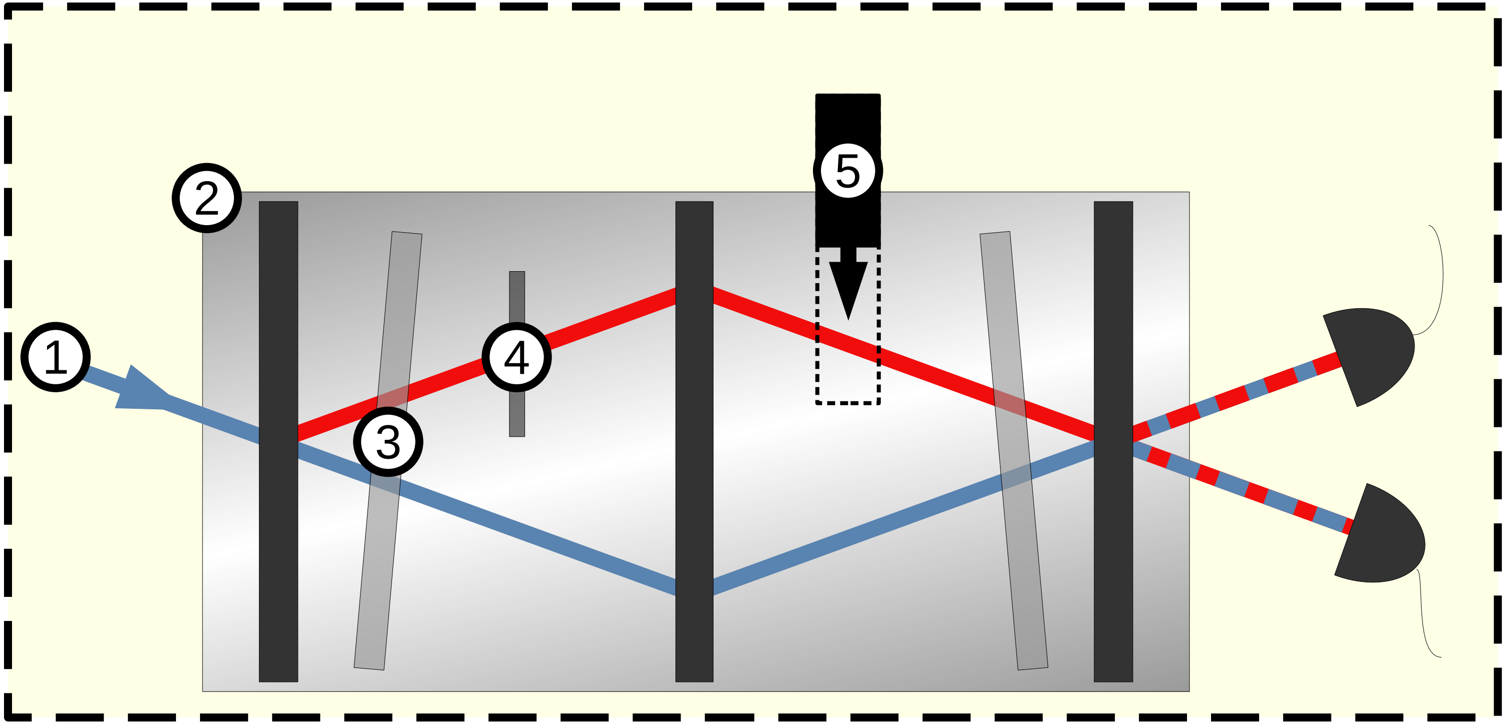
Setup

1 Unpolarised neutron beam
$\lambda = 1.92 Å$
$\delta\lambda/\lambda \approx 0.02$

2 Neutron interferometer

3 Phase-shifter
χ

4 Indium foils to adjust path amplitudes
$a_2/a_1 \approx 0.59$



5 Cadmium path blocker

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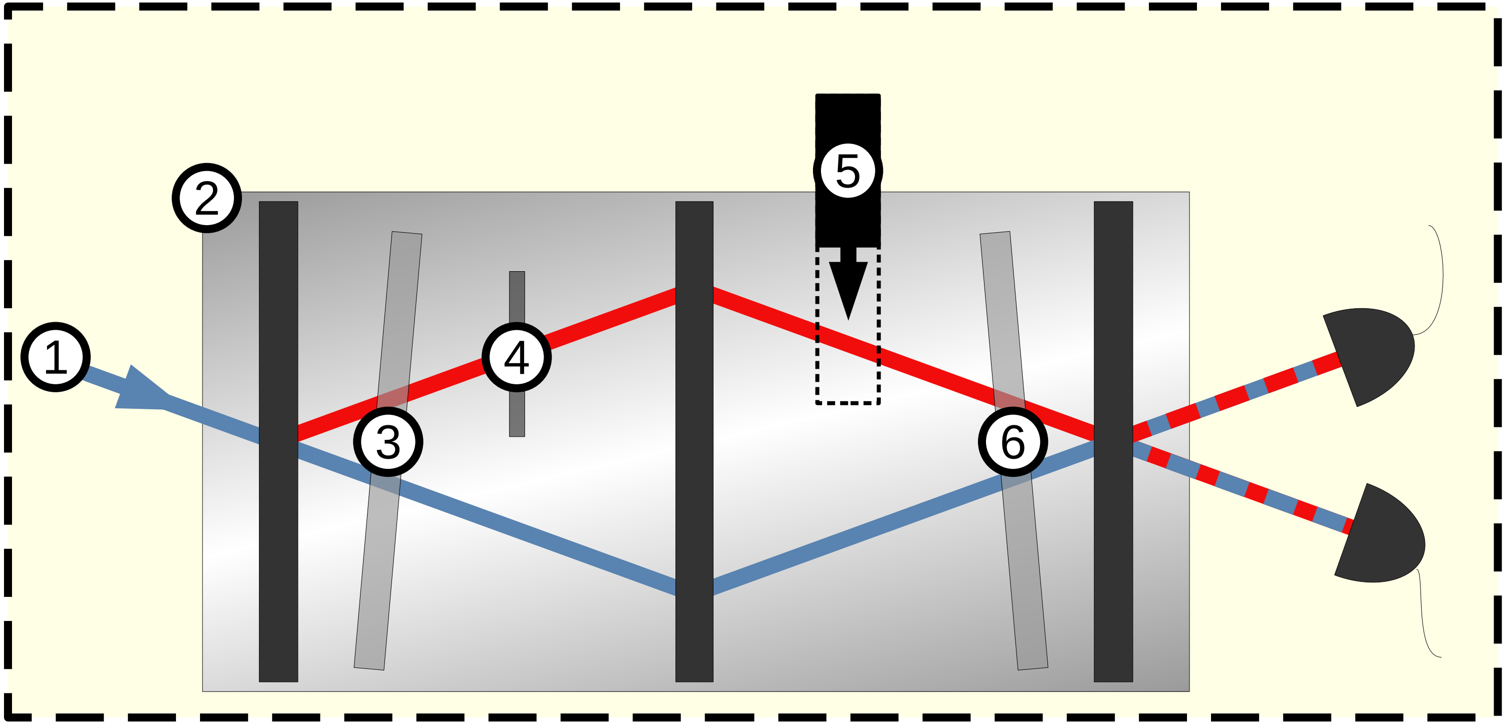
Setup

1 Unpolarised neutron beam
$\lambda = 1.92\text{Å}$
$\delta\lambda/\lambda \approx 0.02$

2 Neutron interferometer

3 Phase-shifter
χ

4 Indium foils to adjust path amplitudes
$a_2/a_1 \approx 0.59$



5 Cadmium path blocker

6 Phase-shifter
χ_0

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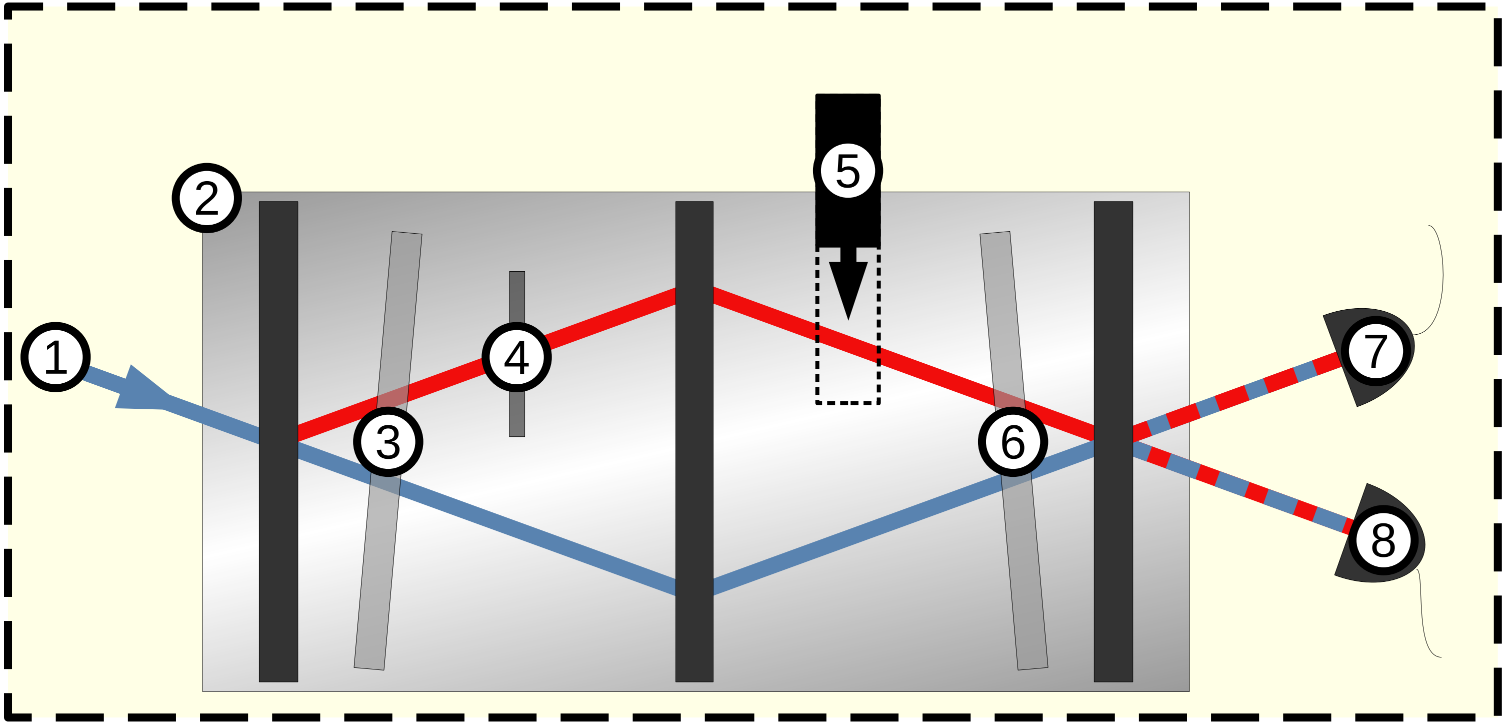
Setup

1 Unpolarised neutron beam
$\lambda = 1.92 \mathring{A}$
$\delta\lambda/\lambda \approx 0.02$

2 Neutron interferometer

3 Phase-shifter
χ

4 Indium foils to adjust path amplitudes
$a_2/a_1 \approx 0.59$



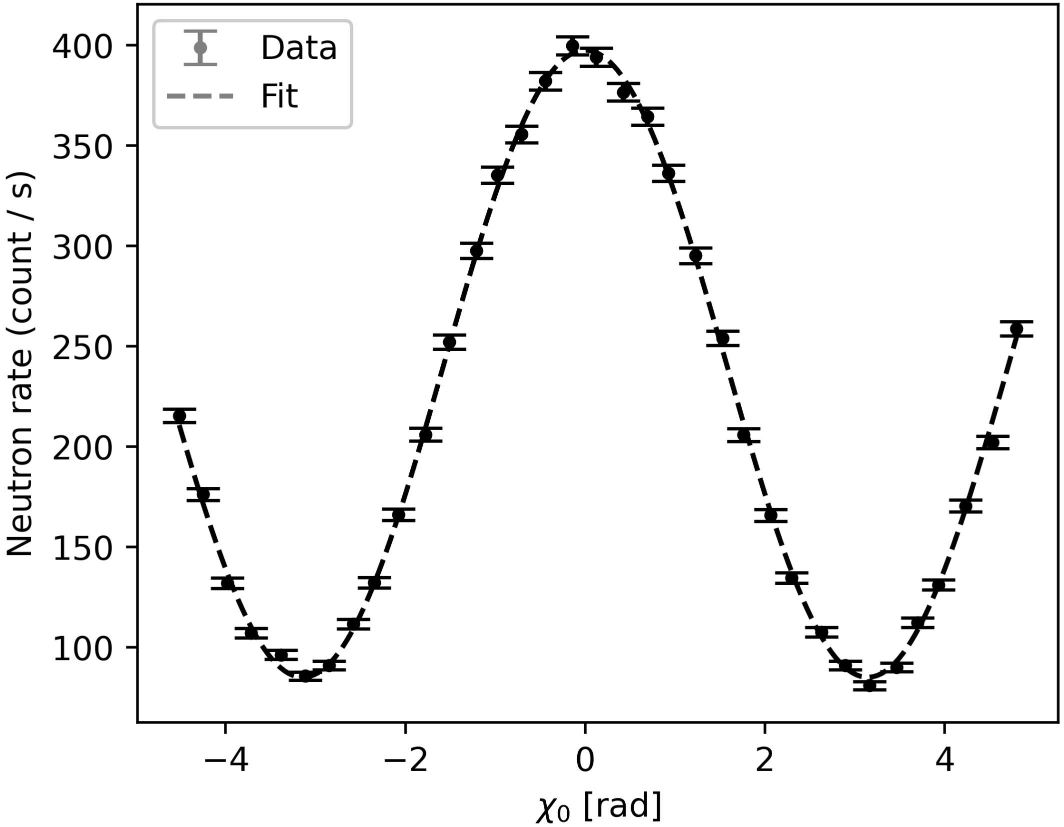
5 Cadmium path blocker

6 Phase-shifter
χ_0

8 $|+\rangle$-detector

7 $|-\rangle$-detector

Example of interferogram



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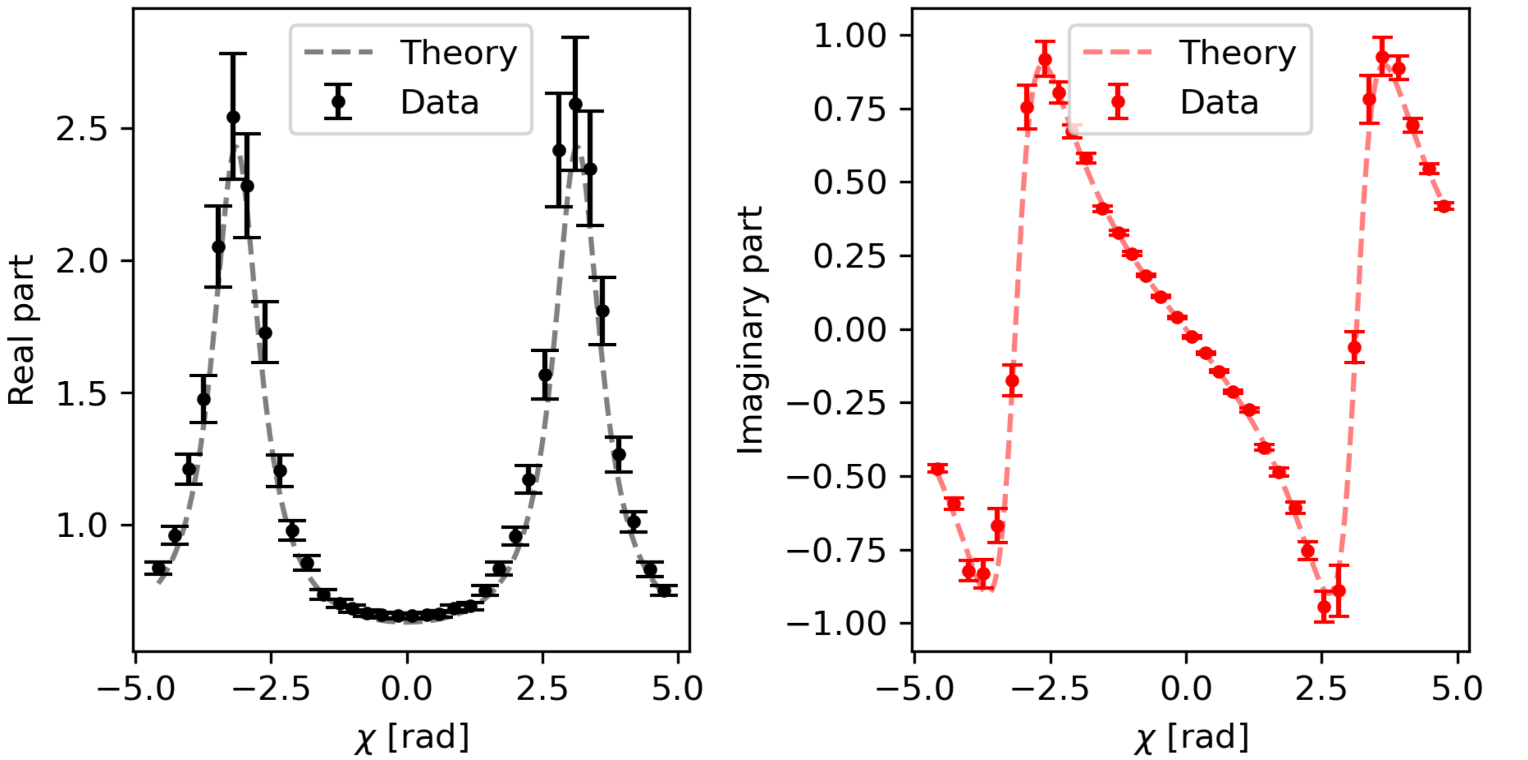
Results: Weak value of path 1



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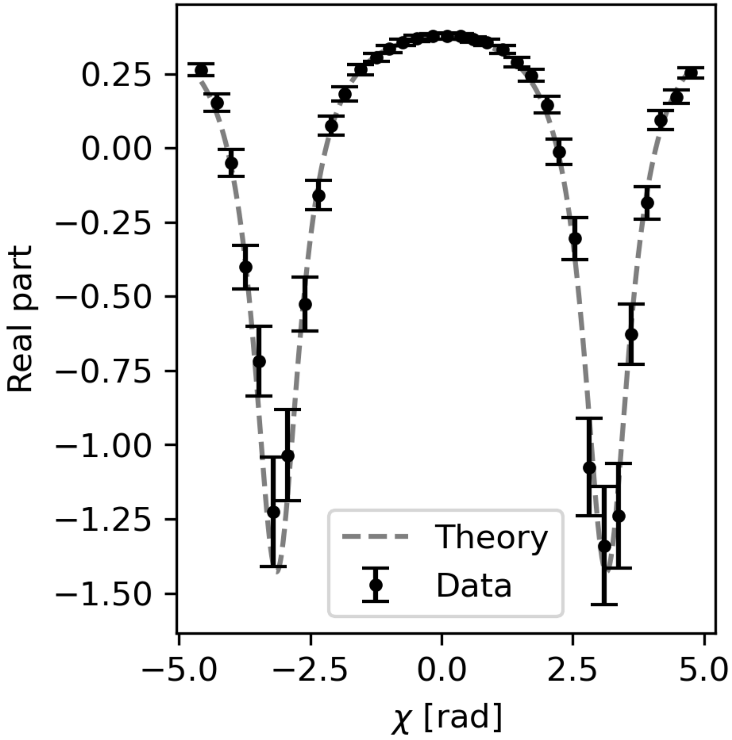
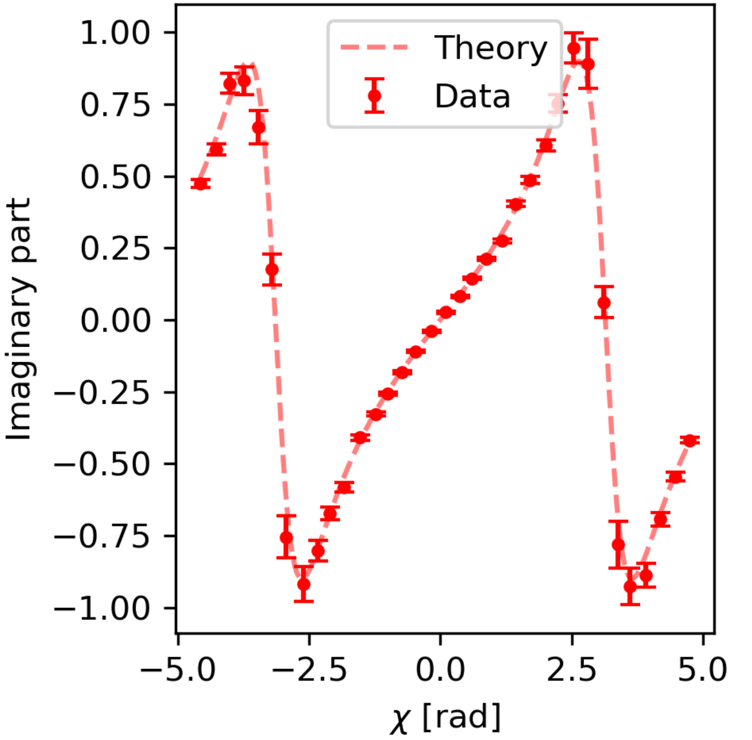
Results: Weak value of path 2





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Interpretation of the results

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Interpretation of the results

Summary

- Introduction to weak values and weak measurements
- Weak values based description of a Mach-Zehnder interferometer
- Experimental measurement of path weak values directly from interferograms

Summary

- To be continued...

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Aknowledgements

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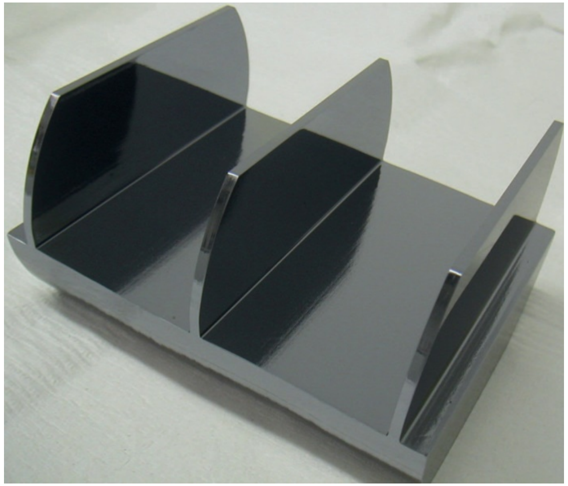
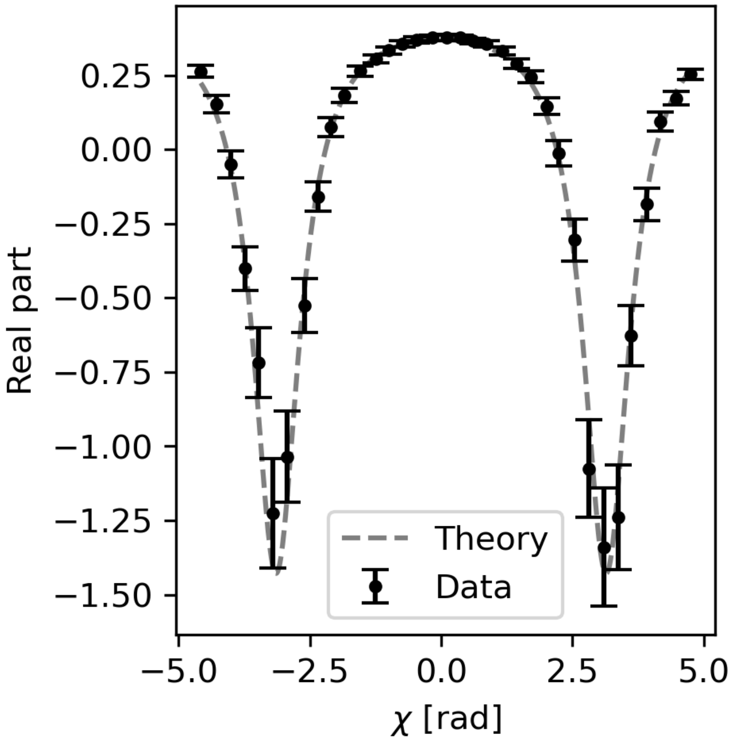
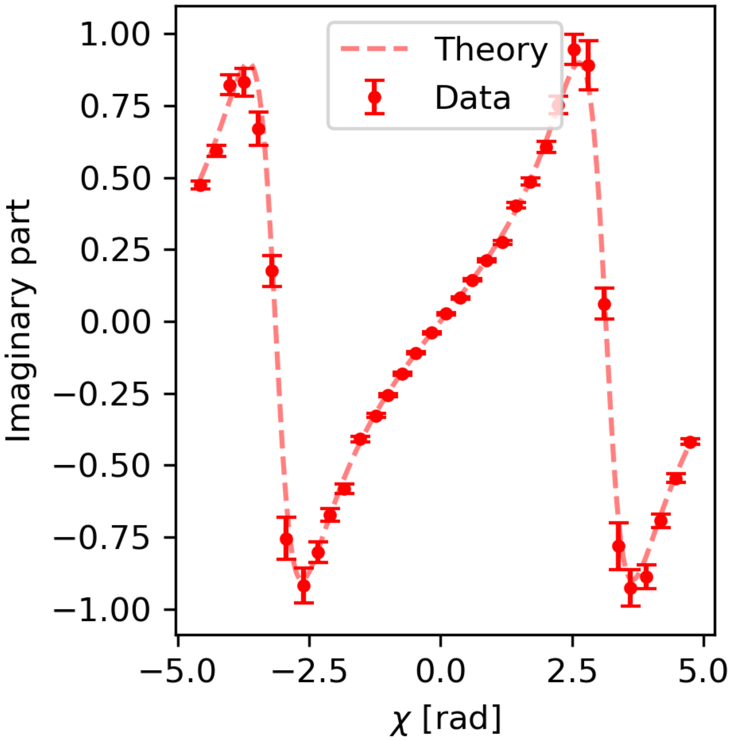
Thanks for your time!







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