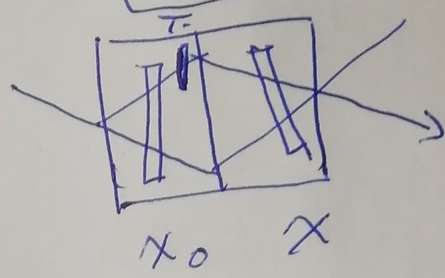
15.2.2024 (1)



$$|\psi^0_{in}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + \sqrt{T}\, e^{ix_0}|2\rangle\right)$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$$

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + e^{ix}\sqrt{T}\, e^{ix_0}|2\rangle\right)$$

where $\hat{\pi}_j = |j\rangle\langle j|$

$$\langle +|\psi_{in}\rangle = \langle +|\hat{\pi}_1|\psi^0_{in}\rangle + e^{ix}\langle +|\hat{\pi}_2|\psi^0_{in}\rangle$$

$$= \langle +|\psi^0_{in}\rangle \cdot \Big(\underbrace{w_{1+}}_{(1-w_{2+})} + e^{ix} w_{2+}\Big)$$

where $w_{j+} = \dfrac{\langle +|\hat{\pi}_j|\psi^0_{in}\rangle}{\langle +|\psi^0_{in}\rangle}$

$$= \langle +|\psi^0_{in}\rangle \left\{1 + (e^{ix}-1)\, w_{2+}\right\}$$

$$\Rightarrow I^{Tot}(x; x_0) = |\langle +|\psi_{in}\rangle|^2$$

$$= I(x_0)\cdot \underbrace{|1+(e^{ix}-1)\, w_{2+}|^2}_{K(x)} \quad \text{— (1)}$$

where $I(x_0) = |\langle +|\psi^0_{in}\rangle|^2$

$$K(x) = |1 + (\cos x + i\sin x - 1)(R + i\,Im)|^2$$

where $w_{2+} = R + i\,Im = A e^{i\phi}$

$$= \{1 + R(\cos x - 1) - Im \sin x\}^2 + \{Im(\cos x - 1) + R\sin x\}^2$$

$$= 1 + (R^2 + Im^2)(\cos x - 1)^2 + (R^2 + Im^2)\sin^2 x + 2R(\cos x - 1) - 2\,Im \sin x \cancel{- 2R\,Im(\cos x - 1)\sin x} \cancel{+ 2R\,Im(\cos x - 1)\sin x}$$

$$K(x) = 1 + A^2 \underbrace{\{(\cos x - 1)^2 + \sin^2 x\}}_{2(1-\cos x)}$$
$$+ 2R(\cos x - 1)$$
$$- 2 I_m \sin x$$

Real)

$$K(x) = \frac{I^{tot}(x; x_0)}{I(x)}$$

$$I^{tot}(x_0; x) = 1 + T + 2\sqrt{T}\cos(x + x_0)$$

$$I(x) = 1 + T + 2\sqrt{T}\cos x_0$$

- $K(0) = 1$. (trivial)
- $K(\pi) = 1 + 4A^2 - 4R = 1 + 4(R^2 + I^2) - 4R$
  $= (1-2R)^2 + 4I_m^2$ — 2
- $K(\pm\pi/2) = 1 + 2A^2 - 2R \mp 2I_m$

$$K(-\pi/2) - K(\pi/2) = 4I_m \quad \text{— 3}$$

<Proof> $K(-\pi/2) - K(\pi/2)$

$$= \frac{(1 + T + 2\sqrt{T}\sin x_0) - (1 + T - 2\sqrt{T}\sin x_0)}{1 + T + 2\sqrt{T}\cos x_0}$$

$$= 4 \cdot \frac{\sqrt{T}\sin x_0}{1 + T + 2\sqrt{T}\cos x_0} = \underline{\underline{4I_m}}$$

From 2 $(1-2R)^2 = K(\pi) - 4I_m^2$ (≥ 0?) — ※

$$1 - 2R = \sqrt{K(\pi) - 4I_m^2}$$

Then [redacted] $2R = 1 - \sqrt{K(\pi) - 4I^2}$ — 4

<Proof>

$$K(\pi) - 4I_m^2 = \frac{1+T-2\sqrt{T}\cos x_0}{1+T+2\sqrt{T}\cos x_0} - 4\left(\frac{\sqrt{T}\sin x_0}{1+T+2\sqrt{T}\cos x_0}\right)^2$$

$$= \frac{(1+T+2\sqrt{T}\cos x_0)(1+T-2\sqrt{T}\cos x_0) - 4T\overbrace{\sin^2 x_0}^{(1-\cos^2 x_0)}}{(1+T+2\sqrt{T}\cos x_0)^2}$$

$$= \frac{(1+T)^2 \cancel{- 4T\cos^2 x_0} - 4T(1 \cancel{- \cos^2 x_0})}{(1+T+2\sqrt{T}\cos x_0)^2}$$

$$= \frac{(1-T)^2}{(1+T+2\sqrt{T}\cos x_0)^2}$$

Then $1 - \sqrt{K(\pi) - 4I^2}$

$$= 1 - \frac{1-T}{1+T+2\sqrt{T}\cos x_0}$$

$$= \frac{(\cancel{1}+T+2\sqrt{T}\cos x_0) - (\cancel{1}-T)}{1+T+2\sqrt{T}\cos x_0}$$

$$= \frac{2(T+\sqrt{T}\cos x_0)}{1+T+2\sqrt{T}\cos x_0} = \underline{\underline{2R}}$$

By using 3 4

$$Im(x) = \{k(-\pi/2) - k(\pi/2)\} \times \frac{1}{4}$$

$$= \left\{ \frac{I^{tot}(x_0; -\pi/2) - I^{tot}(x_0; \pi/2)}{\underset{\uparrow\, I^{tot}(x_0;0)}{I(x_0)}} \right\} \times \frac{1}{4}$$

$$R(x) = \{1 \mp \sqrt{k(\pi) - 4Im(x)}\} \times \frac{1}{2}$$

$$= \{1 - \sqrt{k(\pi) + k(\pi/2) - k(-\pi/2)}\} \times \frac{1}{2}$$

$$= \left\{ 1 - \sqrt{\frac{I^{tot}(x_0; \pi) + I^{tot}(x_0; \pi/2) - I^{tot}(x_0; -\pi/2)}{\underset{\uparrow\, I^{tot}(x_0;0)}{I(x_0)}}} \right\} \times \frac{1}{2}$$

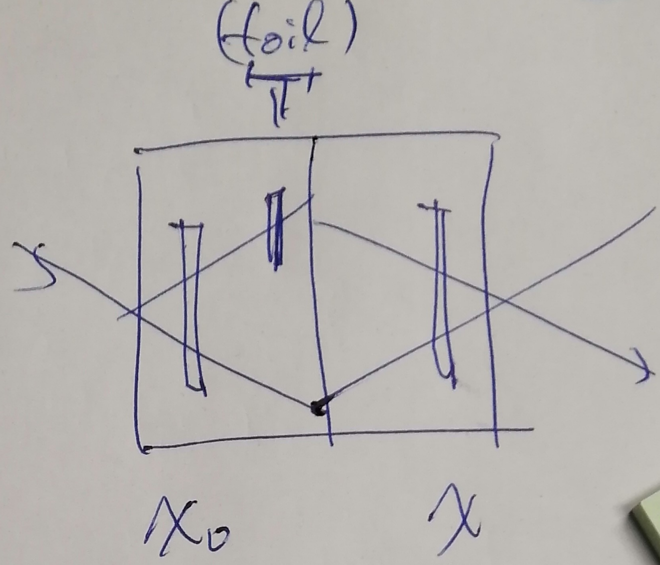
<u>Remark for (*)</u>

$$I^{tot}(x_0; \pi) + I^{tot}(x_0; \pi/2) - I^{tot}(x_0; -\pi/2)$$

$$= (1+T) - 2\sqrt{T}\cos x_0 \cancel{- 2\sqrt{T}\sin x_0 + 2\sqrt{T}\sin x_0}$$

$$= (1+T) - 2\sqrt{T}\cos x_0 \underline{\underline{\geq 0}}$$

Measurement



- Set two phase-shifters (χ_0, χ)
- Put absorber (T)

× At each phase shifter Position of χ_0

set $\chi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$\rightarrow I(\chi_0; 0), I(\chi_0; \pi/2), I(\chi_0; \pi), I(\chi_0; -\pi/2)$

These values give $R(\chi_0)$ & $Im(\chi_0)$

Rem: $(\chi_0 + \chi)$ can be realized by ONE phaseshifter,

so long as $\chi = 0, \pi/2, \pi, 3\pi/2$ can be done

If one is allowed to use a fit-curve $A + B\cos(\chi + C)$ for interferogram, this gives (not descrete) prediction of the weak-value.