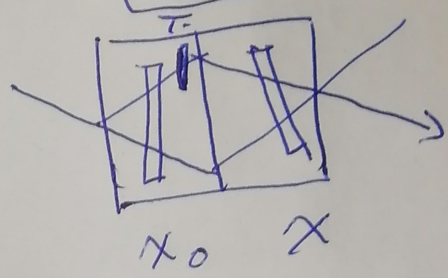
15.2.2024 (1)



$$|\psi^0_{in}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + \sqrt{T}\, e^{ix_0}|2\rangle\right)$$

$$|+\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + |2\rangle\right)$$

$$|\psi_{in}\rangle = \frac{1}{\sqrt{2}}\left(|1\rangle + e^{ix}\sqrt{T}\, e^{ix_0}|2\rangle\right)$$

where $\hat{\pi}_j = |j\rangle\langle j|$

$$\langle +|\psi_{in}\rangle = \langle +|\hat{\pi}_1|\psi^0_{in}\rangle + e^{ix}\langle +|\hat{\pi}_2|\psi^0_{in}\rangle$$

$$= \langle +|\psi^0_{in}\rangle \cdot \left(\underbrace{w_{1+}}_{(1-w_{2+})} + e^{ix} w_{2+}\right)$$

where $w_{j+} = \frac{\langle +|\hat{\pi}_j|\psi^0_{in}\rangle}{\langle +|\psi^0_{in}\rangle}$

$$= \langle +|\psi^0_{in}\rangle \left\{1 + (e^{ix}-1) w_{2+}\right\}$$

$$\Rightarrow I^{Tot}(x; x_0) = |\langle +|\psi_{in}\rangle|^2$$

$$= I(x_0)\cdot \underbrace{|1 + (e^{ix}-1) w_{2+}|^2}_{K(x).} \quad \text{— (1)}$$

where $I(x_0) = |\langle +|\psi^0_{in}\rangle|^2$

$$K(x) = |1 + (\cos x + i\sin x - 1)(R + i I_m)|^2$$

where $w_{2+} = R + i I_m$
$= A e^{i\phi}$

$$= \{1 + R(\cos x - 1) - I_m \sin x\}^2$$
$$+ \{I_m(\cos x - 1) + R\sin x\}^2$$

$$= 1 + (R^2 + I_m^2)(\cos x - 1)^2 + (R^2 + I_m^2)\sin^2 x$$
$$+ 2R(\cos x - 1) - 2 I_m \sin x$$
$$\cancel{- 2 R I_m (\cos x - 1)\sin x}$$
$$\cancel{+ 2 R I_m (\cos x - 1)\sin x}$$

<Proof>

$$K(\pi) - 4I_m^2 = \frac{1+T-2\sqrt{T}\cos x_0}{1+T+2\sqrt{T}\cos x_0} - 4\left(\frac{\sqrt{T}\sin x_0}{1+T+2\sqrt{T}\cos x_0}\right)^2$$

$$= \frac{(1+T+2\sqrt{T}\cos x_0)(1+T-2\sqrt{T}\cos x_0) - 4T\overbrace{\sin^2 x_0}^{(1-\cos^2 x_0)}}{(1+T+2\sqrt{T}\cos x_0)^2}$$

$$= \frac{(1+T)^2 - \cancel{4T\cos^2 x_0} - 4T(1 - \cancel{\cos^2 x_0})}{(1+T+2\sqrt{T}\cos x_0)^2}$$

$$= \frac{(1-T)^2}{(1+T+2\sqrt{T}\cos x_0)^2}$$

Then $1 - \sqrt{K(\pi) - 4I^2}$

$$= 1 - \frac{1-T}{1+T+2\sqrt{T}\cos x_0}$$

$$= \frac{(\cancel{1}+T+2\sqrt{T}\cos x_0) - (\cancel{1}-T)}{1+T+2\sqrt{T}\cos x_0}$$

$$= \frac{2(T+\sqrt{T}\cos x_0)}{1+T+2\sqrt{T}\cos x_0} = \underline{\underline{2R}}$$

By using ③ ④

$$\mathrm{Im}(x) = \left\{ k(-\pi/2) - k(\pi/2) \right\} \times \frac{1}{4}$$

$$= \left\{ \frac{I^{tot}(x_0; -\pi/2) - I^{tot}(x_0; \pi/2)}{I(x_0)} \right\} \times \frac{1}{4}$$

$$I(x_0) \leftarrow I^{tot}(x_0; 0)$$

$$R(x) = \left\{ 1 - \sqrt{k(\pi) - 4\,\mathrm{Im}(x)} \right\} \times \frac{1}{2}$$

$$= \left\{ 1 - \sqrt{k(\pi) + k(\pi/2) - k(-\pi/2)} \right\} \times \frac{1}{2}$$

$$= \left\{ 1 - \sqrt{\frac{I^{tot}(x_0; \pi) + I^{tot}(x_0; \pi/2) - I^{tot}(x_0; -\pi/2)}{I(x_0)}} \right\} \times \frac{1}{2}$$

$$I(x_0) \leftarrow I^{tot}(x_0; 0)$$

Remark for ⊛

$$I^{tot}(x_0; \pi) + I^{tot}(x_0; \pi/2) - I^{tot}(x_0; -\pi/2)$$

$$= (1+T) - 2\sqrt{T}\cos x_0 \cancel{- 2\sqrt{T}\sin x_0 + 2\sqrt{T}\sin x_0}$$

$$= (1+T) - 2\sqrt{T}\cos x_0 \geq 0$$