

Direct extraction of path weak values from interferograms without auxiliary qubits

Content

Content

- **Part 1:** Introduction to weak values and weak measurements

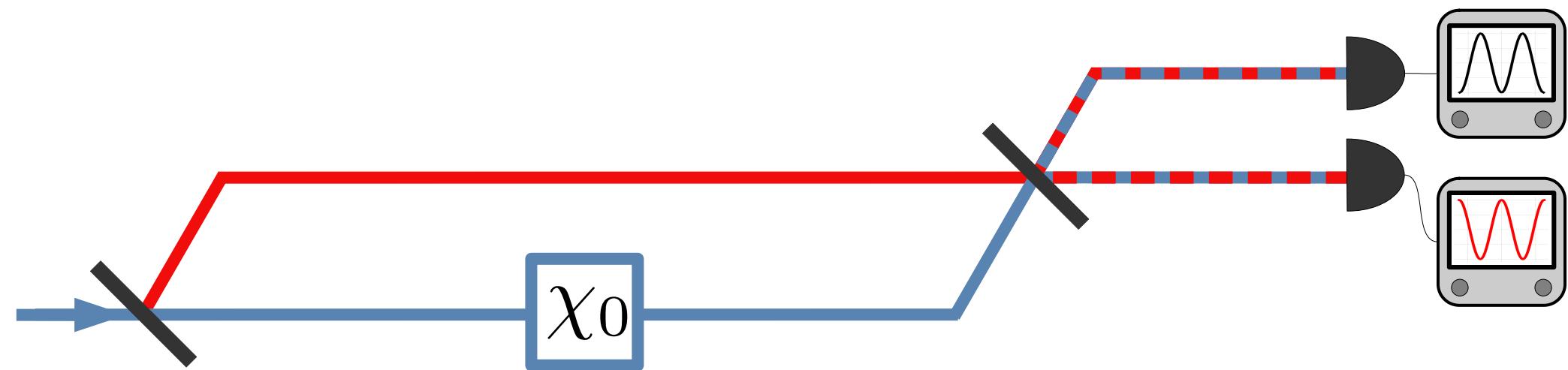
Content

- **Part 1:** Introduction to weak values and weak measurements
- **Part 2:** Weak values based description of a Mach-Zehnder interferometer

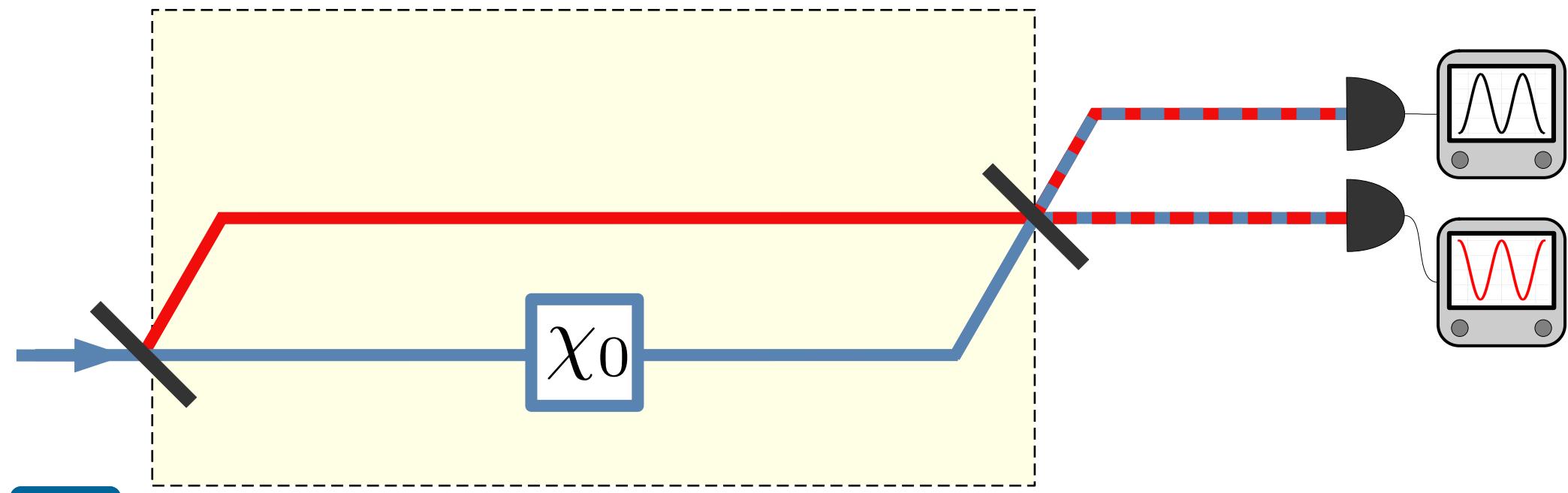
Content

- **Part 1:** Introduction to weak values and weak measurements
- **Part 2:** Weak values based description of a Mach-Zehnder interferometer
- **Part 3:** Experimental measurement of path weak values directly from interferograms

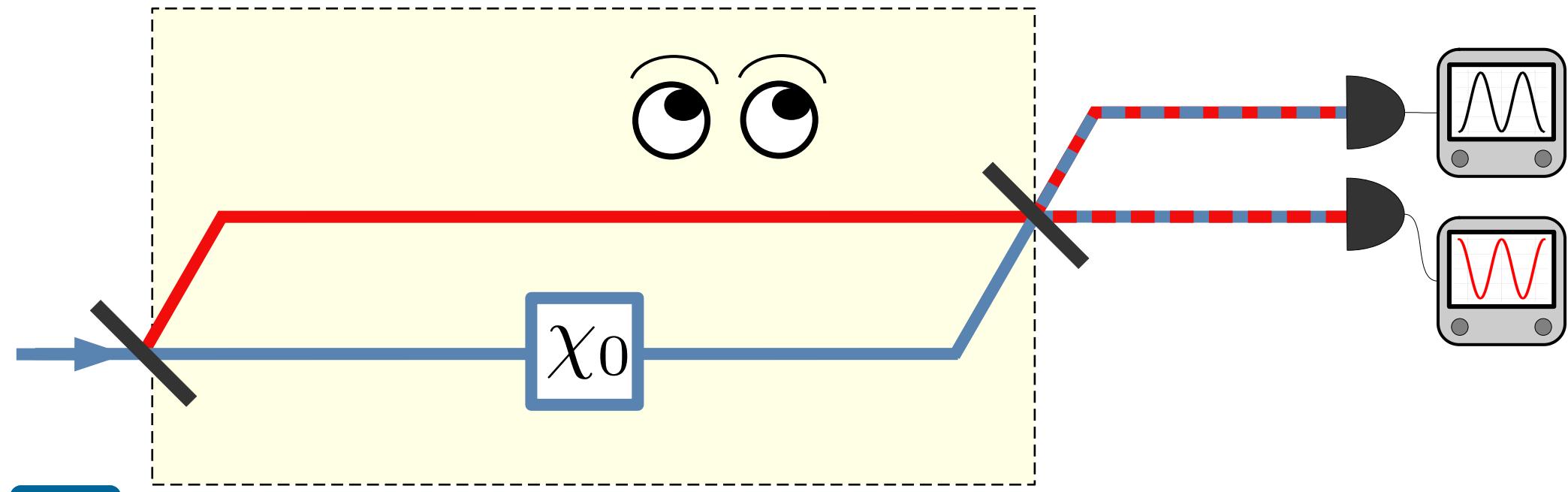
Motivation



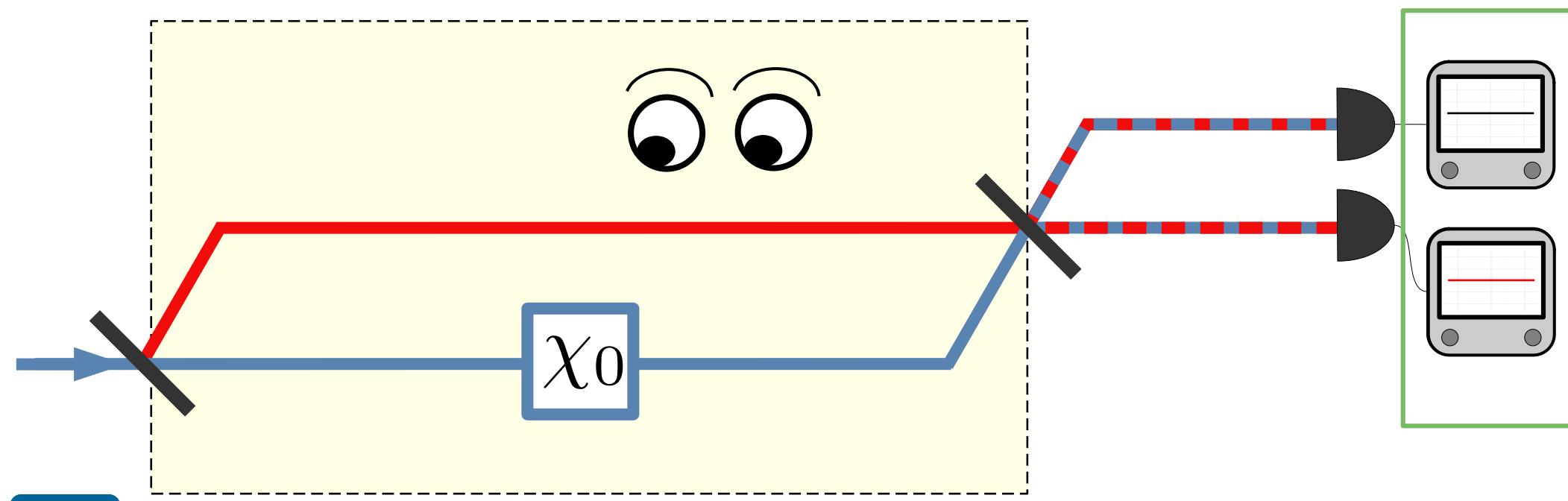
Motivation



Motivation



Motivation



Goal

Use weak values for a new perspective on interferometry, and hopefully some new insight!

Part 1: Introduction to weak values and weak measurement

VOLUME 60, NUMBER 14

PHYSICAL REVIEW LETTERS

4 APRIL 1988

How the Result of a Measurement of a Component of the Spin of a Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100

Yakir Aharonov, David Z. Albert, and Lev Vaidman

PHYSICAL REVIEW A 76, 044103 (2007)

Complex weak values in quantum measurement

Richard Jozsa

PHYSICAL REVIEW A 85, 012107 (2012)

Significance of the imaginary part of the weak value

J. Dressel and A. N. Jordan

Weak value

Definition:

$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Weak value

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Weak value

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- Observable: \hat{A}
- Pre-selected state: $|\psi_{in}\rangle$
- Post-selected state: $|\psi_f\rangle$

Weak value

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Expectation value:

$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

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- Complex number

Weak value

$$A_w = A_w^{\Re} + i A_w^{\Im}$$

↑ ↑
Real Imaginary

- Observable: \hat{A}
- Pre-selected state: $|\psi_{\text{in}}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Complex number

Weak value

$$A_w = A_w^{\Re} + i A_w^{\Im}$$

↑ ↑
Real Imaginary
↓

Observable in the limit of minimal disturbance

- Observable: \hat{A}
- Pre-selected state: $|\psi_{\text{in}}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Complex number

Weak value

Definition:

$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Expectation value:

$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

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Weak value

Definition:

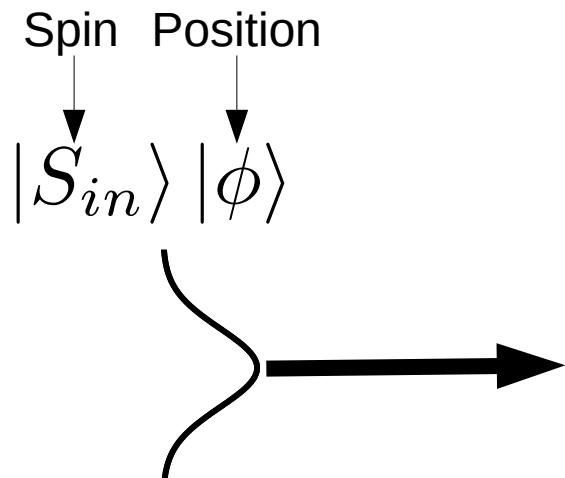
$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Expectation value:

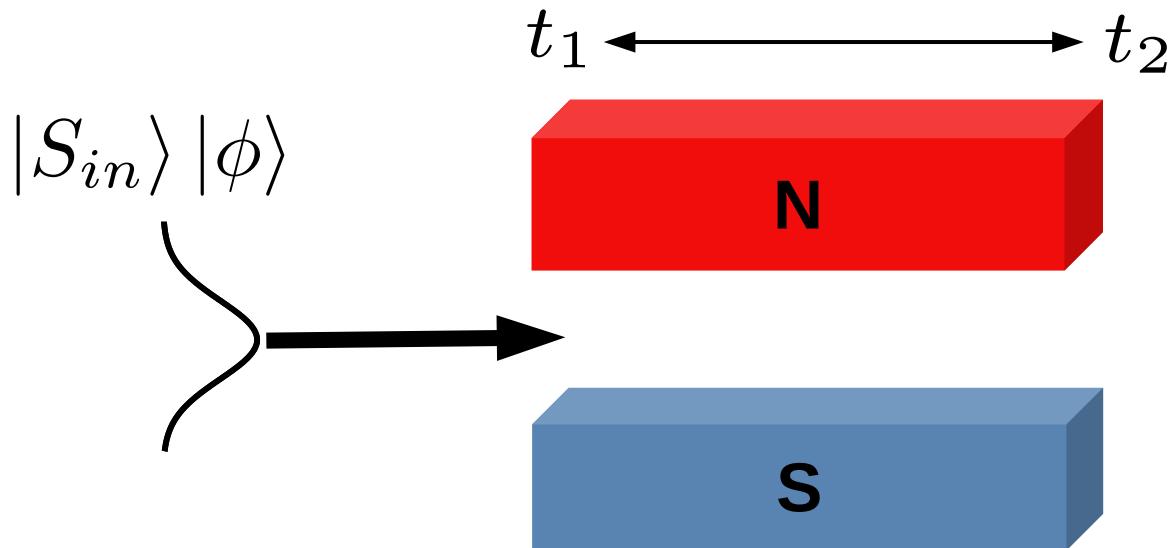
$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

- Observable: \hat{A}
- Pre-selected state: $|\psi_{in}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Complex number
- Not bounded by eigenvalues

Stern-Gerlach

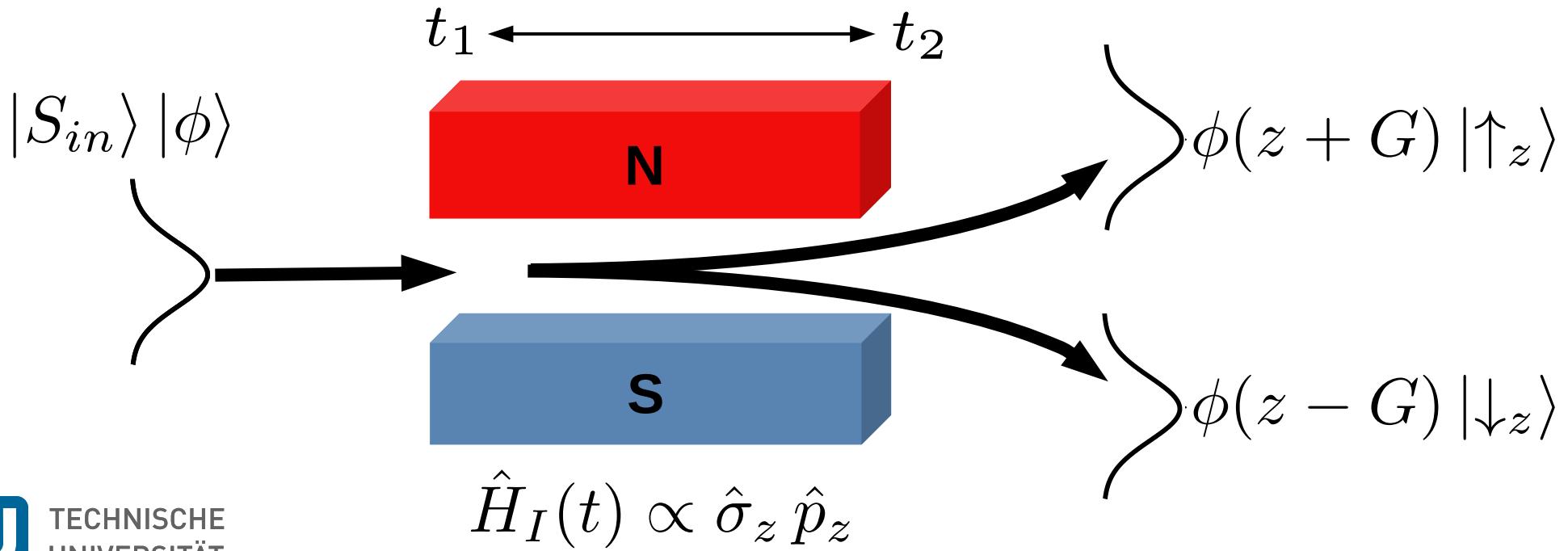


Stern-Gerlach



$$\hat{H}_I(t) \propto \hat{\sigma}_z \hat{p}_z$$

Stern-Gerlach



Example of standard measurement (Von Neumann scheme)

Example of standard measurement (Von Neumann scheme)

Two quantum states

$$|\psi_{\text{in}}\rangle_s |\phi\rangle_m$$



System Meter / Auxiliary

Example of standard measurement (Von Neumann scheme)

Two quantum states

$$|\psi_{\text{in}}\rangle_s |\phi\rangle_m$$

System observable \hat{A}_s

Example of standard measurement (Von Neumann scheme)

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$$\hat{A}_s |a\rangle_s = a |a\rangle_s , \quad \{|a\rangle_s\}$$

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$$|\psi_{\text{in}}\rangle_s |\phi\rangle_m$$

System observable \hat{A}_s

$$\hat{A}_s |a\rangle_s = a |a\rangle_s , \quad \{|a\rangle_s\}$$

Interaction Hamiltonian

$$\hat{H}_I(t) \propto \hat{A}_s \hat{p}_m$$

Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$e^{-i \int_{t_1}^{t_2} \hat{H}_I(t) dt} = e^{-iG\hat{A}_s \hat{p}_m}$$

Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$e^{-i \int_{t_1}^{t_2} \hat{H}_I(t) dt} = e^{-i G \hat{A}_s \hat{p}_m}$$



Interaction strength

Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$e^{-i \int_{t_1}^{t_2} \hat{H}_I(t) dt} = e^{-i G \hat{A}_s \hat{p}_m}$$

$$\hat{\mathbb{I}} = \sum_a |a\rangle_s \langle a|_s$$

Example of standard measurement (Von Neumann scheme)

Effect of the interaction Hamiltonian

$$\begin{aligned} e^{-i \int_{t_1}^{t_2} \hat{H}_I(t) dt} &= e^{-i G \hat{A}_s \hat{p}_m} \\ &= \sum_a e^{-i G \hat{A}_s \hat{p}_m} |a\rangle_s \langle a|_s \end{aligned}$$

Example of standard measurement (Von Neumann scheme)

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Translation operator

Example of standard measurement (Von Neumann scheme)

Overall effect

Example of standard measurement (Von Neumann scheme)

Overall effect

$$\phi_m(x) \rightarrow \phi_m(x + G\alpha)$$

Eigenvalue



Interaction strength

Weak measurement

Effect of the interaction Hamiltonian

$$e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m$$

Weak measurement

Effect of the interaction Hamiltonian

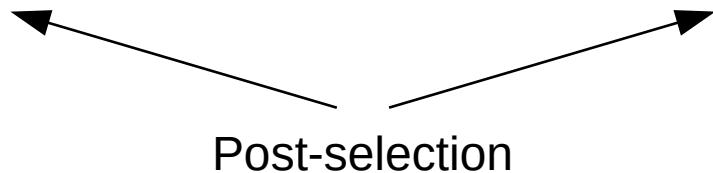
$$e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m \approx \left(\hat{\mathbb{I}} - iG\hat{A}_s \hat{p}_m \right) |\psi_{in}\rangle_s |\phi\rangle_m$$

\uparrow
 $G \ll 1$

Weak measurement

Effect of the interaction Hamiltonian

$$\langle \psi_f |_s e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m \approx \langle \psi_f |_s (\hat{\mathbb{I}} - iG\hat{A}_s \hat{p}_m) |\psi_{in}\rangle_s |\phi\rangle_m$$



Weak measurement

Effect of the interaction Hamiltonian

$$\begin{aligned} \langle \psi_f |_s e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m &\approx \langle \psi_f |_s \left(\hat{\mathbb{I}} - iG\hat{A}_s \hat{p}_m \right) |\psi_{in}\rangle_s |\phi\rangle_m \\ &= \langle \psi_f | \psi_{in} \rangle \left(\hat{\mathbb{I}} - iG \frac{\langle \psi_f | \hat{A}_s | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle} \hat{p}_m \right) |\phi\rangle_m \end{aligned}$$

Weak measurement

Effect of the interaction Hamiltonian

$$\begin{aligned} \langle \psi_f |_s e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m &\approx \langle \psi_f |_s \left(\hat{\mathbb{I}} - iG\hat{A}_s \hat{p}_m \right) |\psi_{in}\rangle_s |\phi\rangle_m \\ &= \langle \psi_f | \psi_{in} \rangle \left(\hat{\mathbb{I}} - iG \frac{\langle \psi_f | \hat{A}_s | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle} \hat{p}_m \right) |\phi\rangle_m \end{aligned}$$

Weak value of \hat{A}_s

Weak measurement

Effect of the interaction Hamiltonian

$$\begin{aligned} \langle \psi_f |_s e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m &\approx \langle \psi_f |_s \left(\hat{\mathbb{I}} - iG\hat{A}_s \hat{p}_m \right) |\psi_{in}\rangle_s |\phi\rangle_m \\ &= \langle \psi_f | \psi_{in} \rangle \left(\hat{\mathbb{I}} - iGA_w \hat{p}_m \right) |\phi\rangle_m \end{aligned}$$

Weak measurement

Effect of the interaction Hamiltonian

$$\langle \psi_f |_s e^{-iG\hat{A}_s \hat{p}_m} |\psi_{in}\rangle_s |\phi\rangle_m \approx \langle \psi_f |_s (\hat{\mathbb{I}} - iG\hat{A}_s \hat{p}_m) |\psi_{in}\rangle_s |\phi\rangle_m$$

$$= \langle \psi_f | \psi_{in} \rangle \left(\hat{\mathbb{I}} - iG A_w \hat{p}_m \right) |\phi\rangle_m$$
$$\approx \langle \psi_f | \psi_{in} \rangle \left(e^{-iG A_w \hat{p}_m} \right) |\phi\rangle_m$$

Translation operator

Weak measurement

Overall effect

Weak measurement

Overall effect

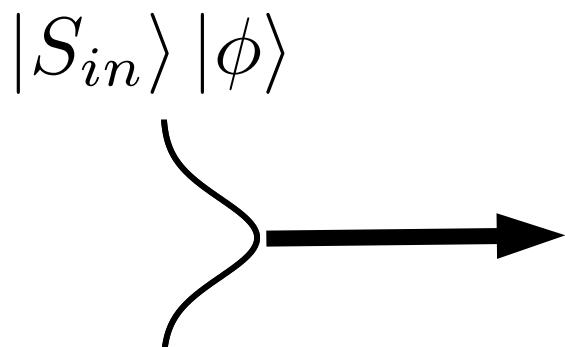
$$\phi(x) \rightarrow \phi(x + GA_w)$$

Weak value

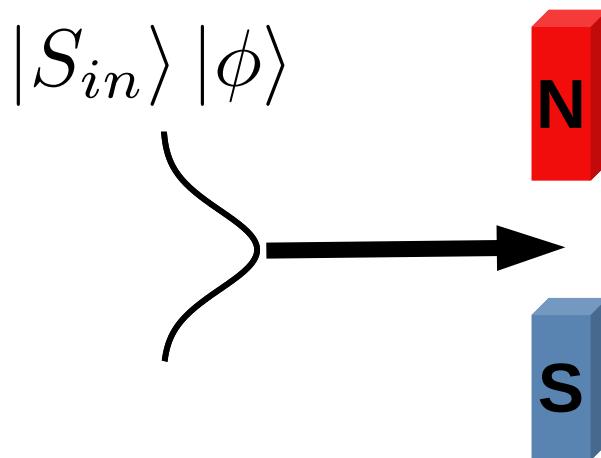


Interaction strength

Stern-Gerlach weak measurement

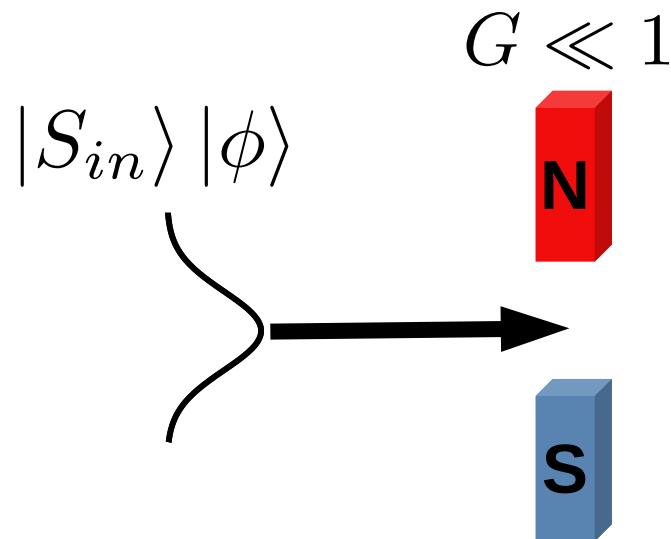


Stern-Gerlach weak measurement



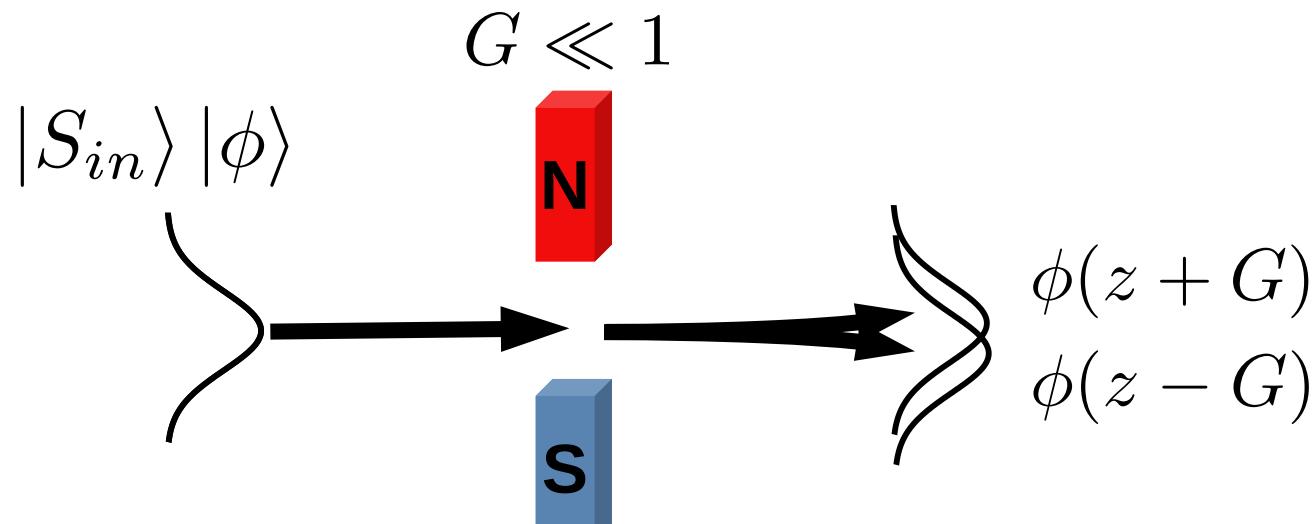
$$\hat{H}_I(t) \propto \hat{\sigma}_z \hat{p}_z$$

Stern-Gerlach weak measurement



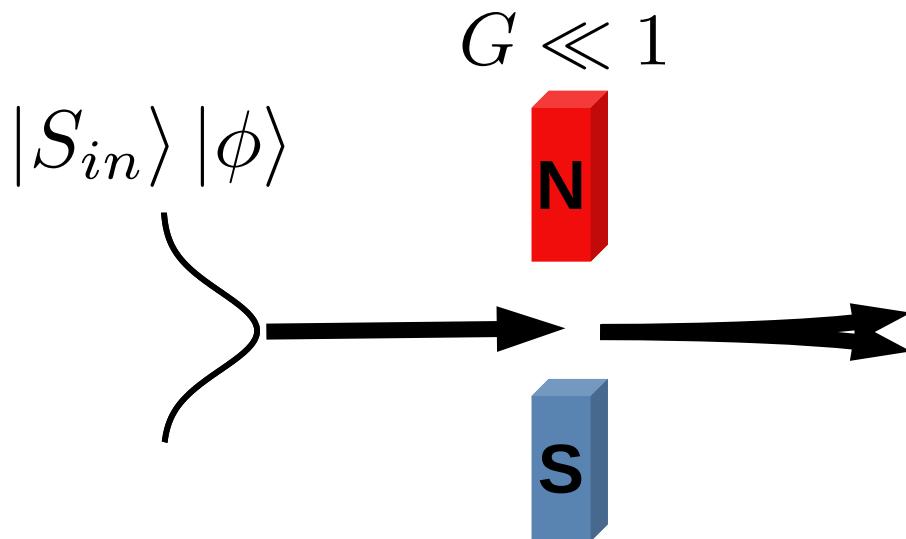
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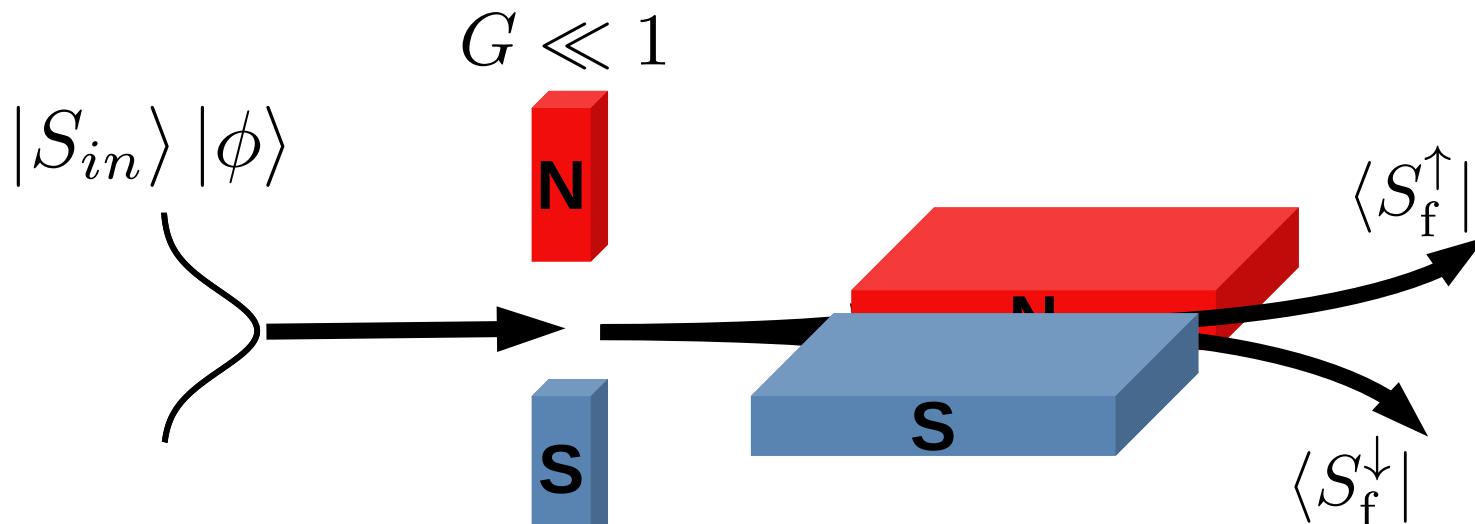
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Stern-Gerlach weak measurement



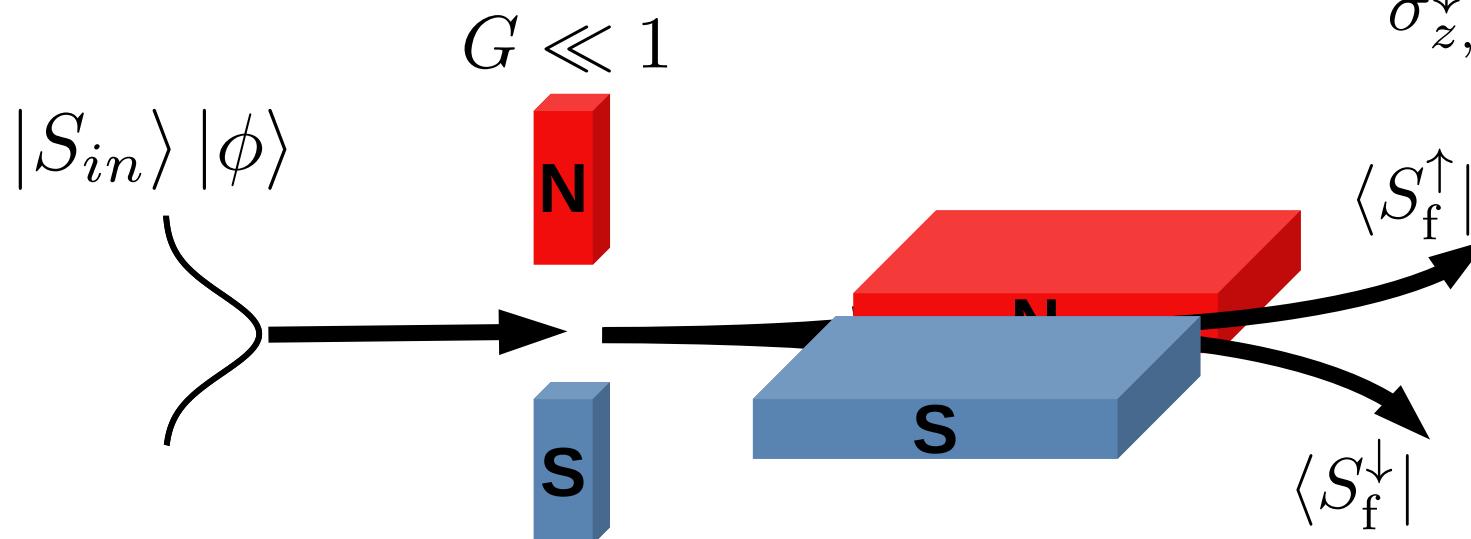
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Stern-Gerlach weak measurement



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Stern-Gerlach weak measurement

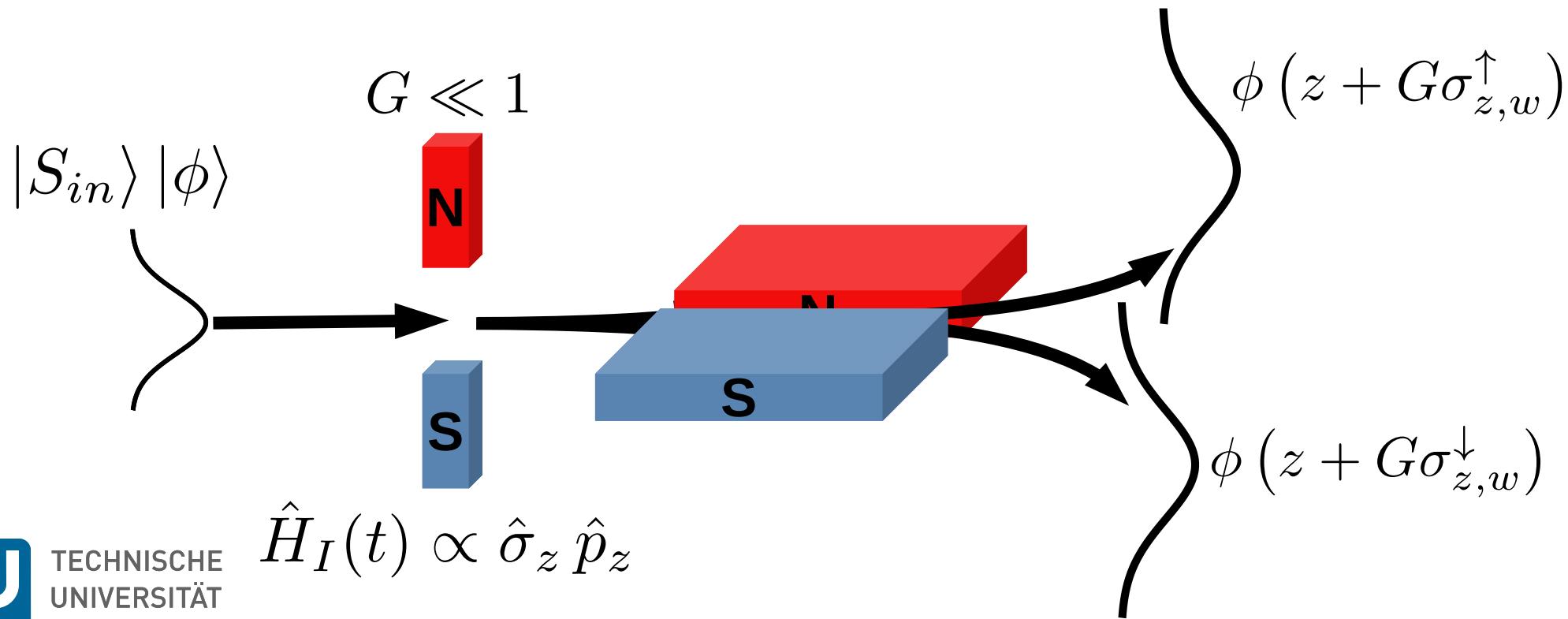


Spin weak value:

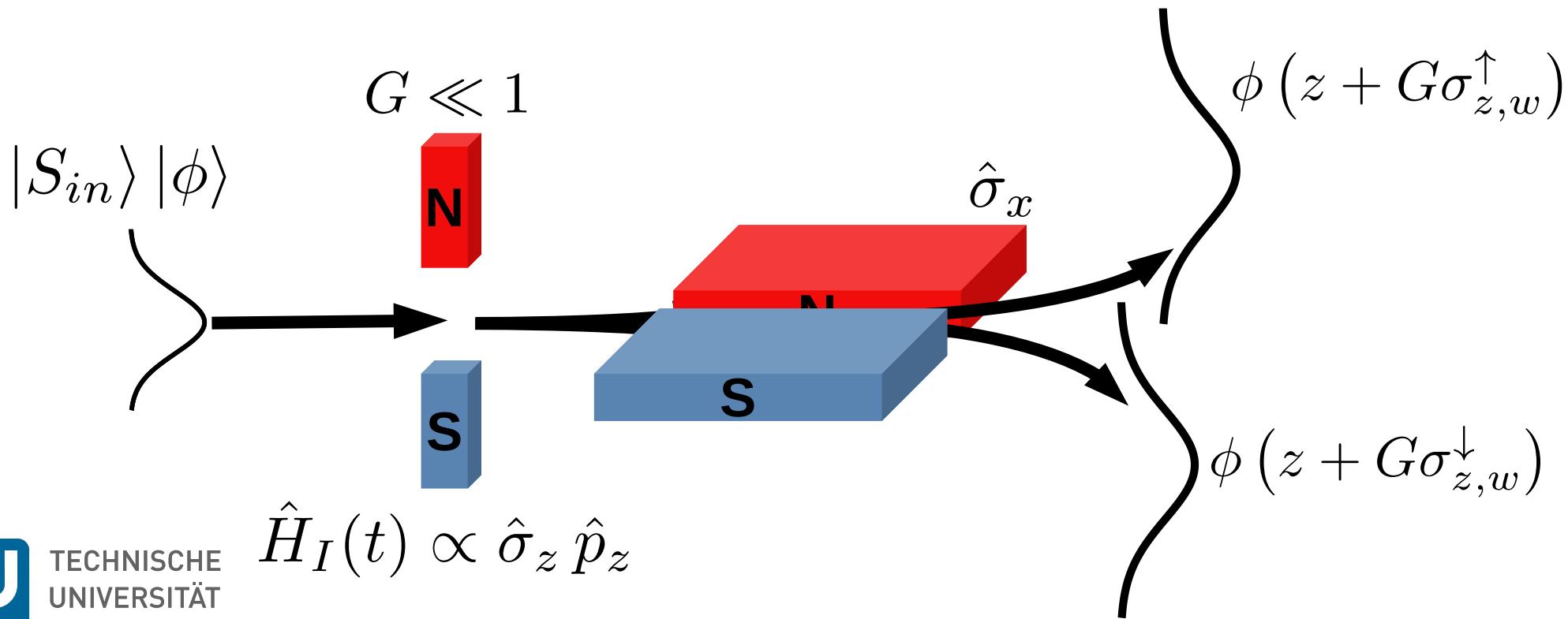
$$\sigma_{z,w}^{\uparrow\downarrow} = \frac{\langle S_f^\downarrow | \hat{\sigma}_z | S_{in} \rangle}{\langle S_f^\uparrow | S_{in} \rangle}$$

$$\hat{H}_I(t) \propto \hat{\sigma}_z \hat{p}_z$$

Stern-Gerlach weak measurement



Stern-Gerlach weak measurement



First paper on weak values

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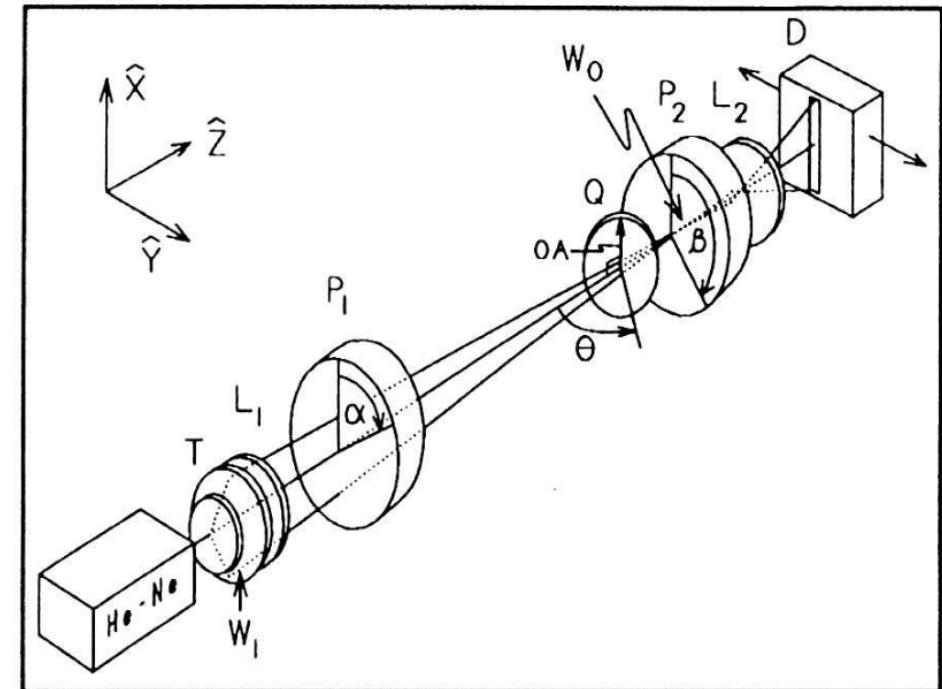
**How the Result of a Measurement of a Component of the Spin of a
Spin- $\frac{1}{2}$ Particle Can Turn Out to be 100**

Yakir Aharonov, David Z. Albert, and Lev Vaidman

First measurement of weak values

Measurement of a Weak Value*

R. G. Hulet, N. W. M. Ritchie, and J. G. Story
Physics Department, Rice University, Houston, TX 77251, USA
Z. Naturforsch. **52a**, 31–33 (1997)



What are they good for?

Theory and experiment

- Quantum amplification
- Quantum paradoxes
- Negative quasi-probability distributions
- Uncertainty relations

Theory and experiment

- Quantum amplification
- Quantum paradoxes
- Negative quasi-probability distributions
- Uncertainty relations
- And more...

J. Dressel. Weak values as interference phenomena. *Phys. Rev. A*, 91:032116.

J. Dressel, M. Malik, F. M. Miatto, A. N. Jordan, and R. W. Boyd. Colloquium: Understanding quantum weak values: Basics and applications. *Rev. Mod. Phys.*, 86:307–316.

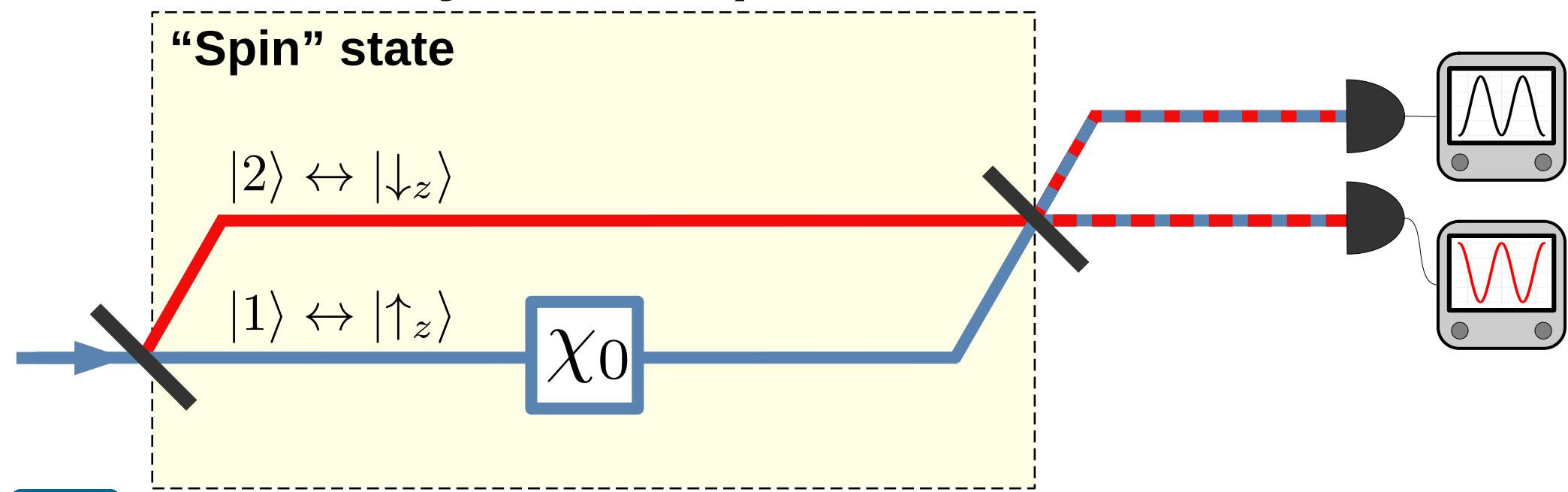
M. J. W. Hall. Prior information: How to circumvent the standard joint-measurement uncertainty relation. *Phys. Rev. A*, 69:052113.

“In between” evolution of a system

- Minimal disturbance
- Non-commuting observables

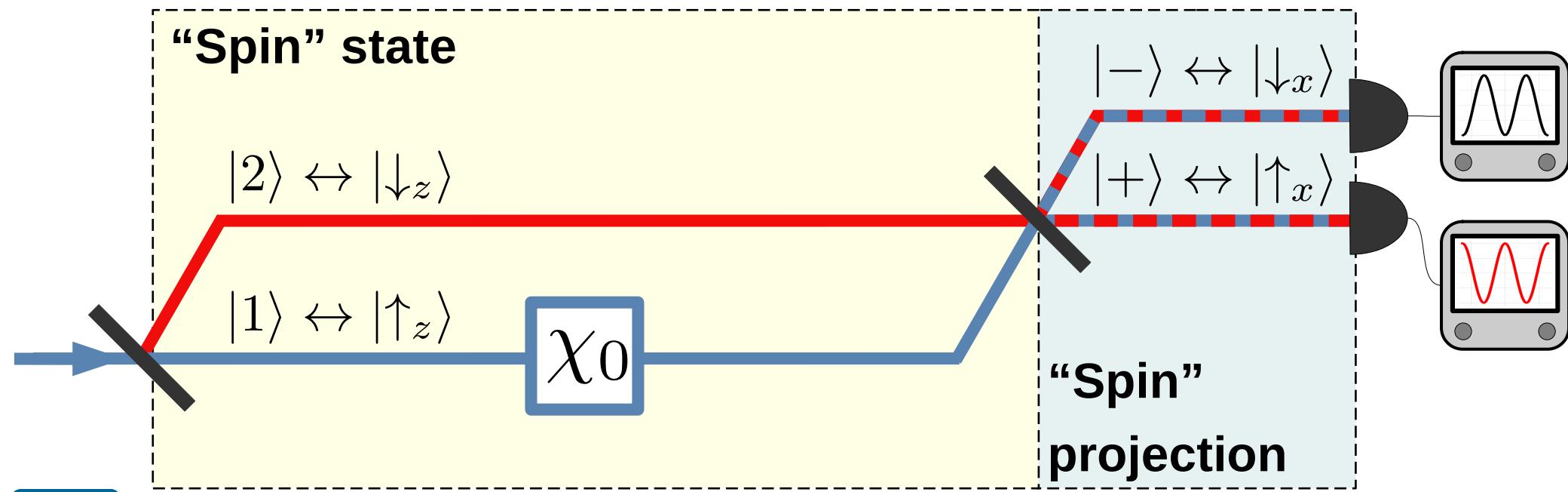
Interferometry

Two level system = spinor



Interferometry

Two level system = spinor



Do we actually need a weak measurement?

Weak value

Definition:

$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Weak value

Definition:

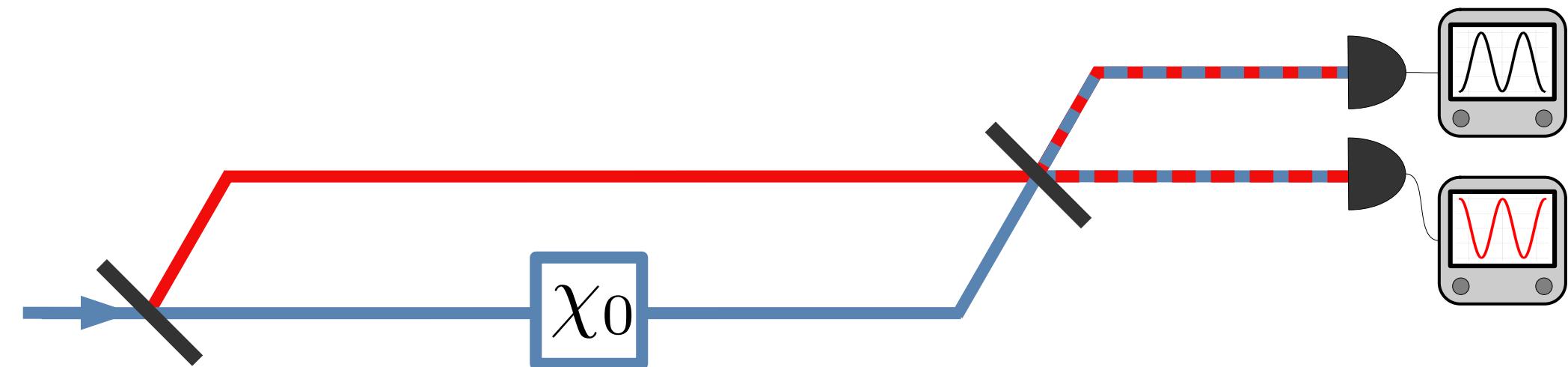
$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

- No mention of interaction strength
- No mention of meter/auxiliary state

Part 2: Weak values based description of interferometry

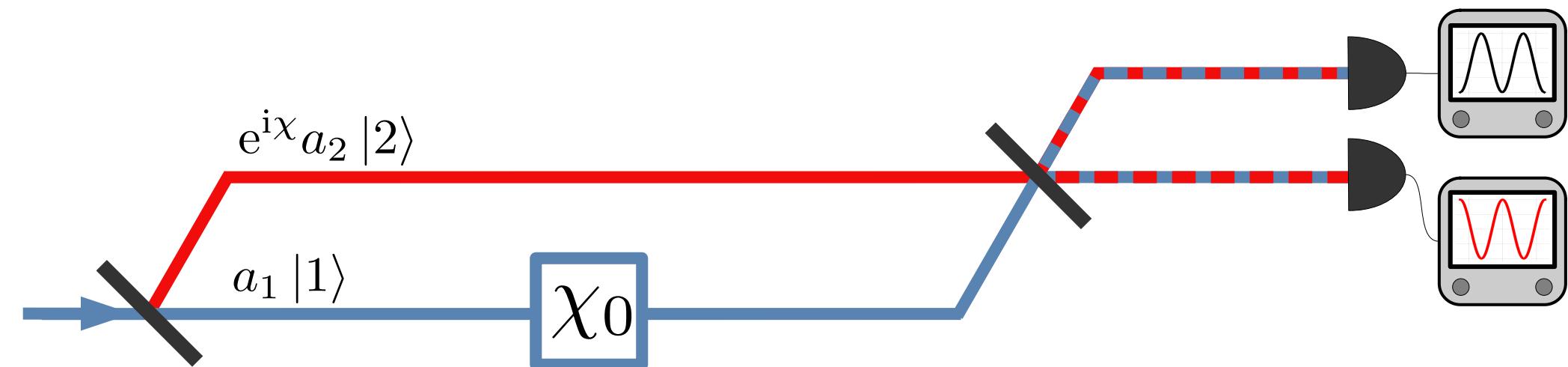
Weak values and interferometry

Standard interferometry formalism



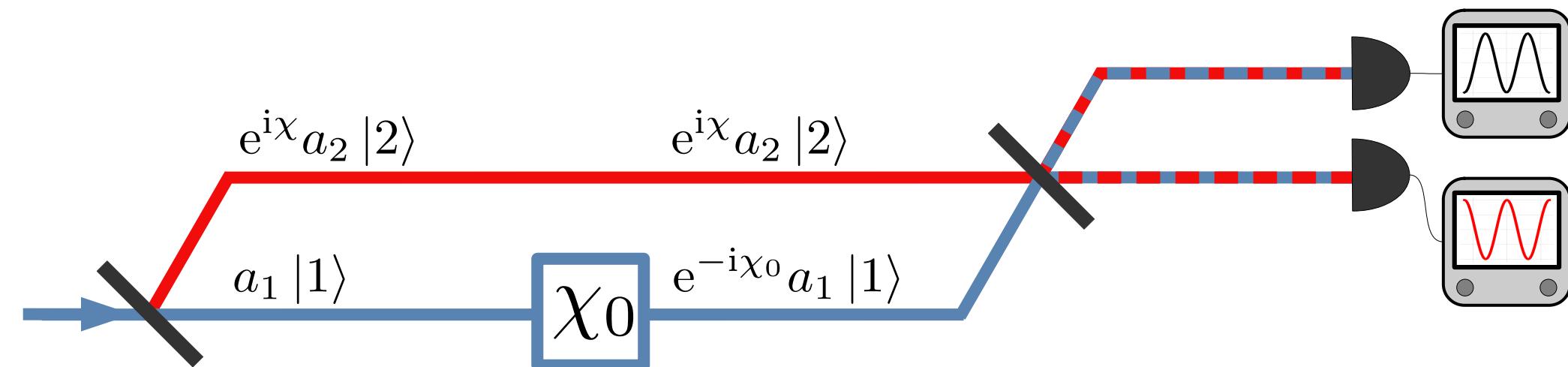
Weak values and interferometry

Standard interferometry formalism



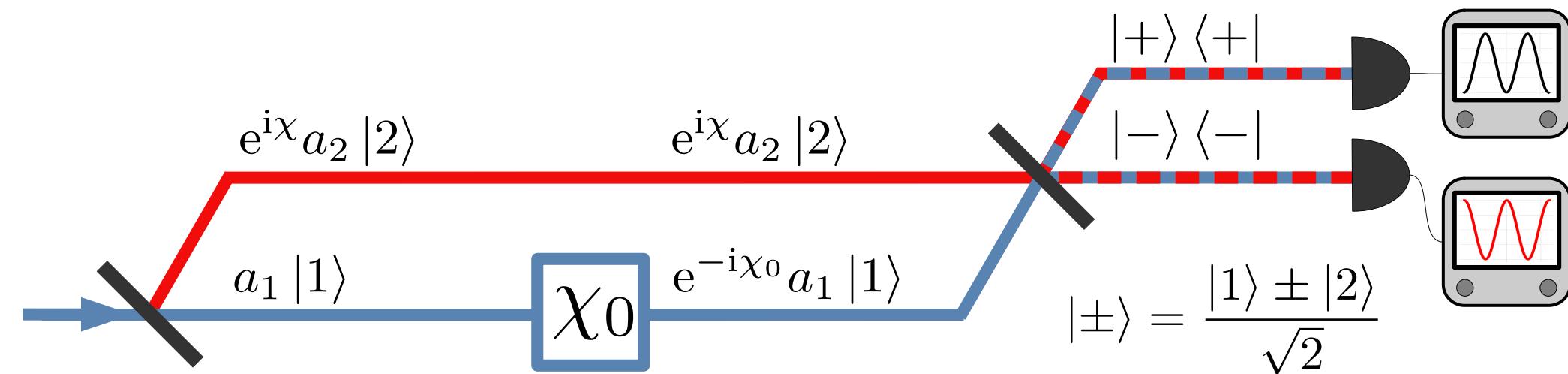
Weak values and interferometry

Standard interferometry formalism



Weak values and interferometry

Standard interferometry formalism



Weak values and interferometry

Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi, \chi_0) = (e^{-i\chi_0} a_1 |1\rangle + e^{i\chi} a_2 |2\rangle)$$

Weak values and interferometry

Standard interferometry formalism

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Weak values and interferometry

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$$I_{\pm,1}(\chi, \chi_0) = \left| \langle \pm | (e^{-i\chi_0} a_1 |1\rangle + e^{i\chi} a_2 |2\rangle) \right|^2$$

Weak values and interferometry

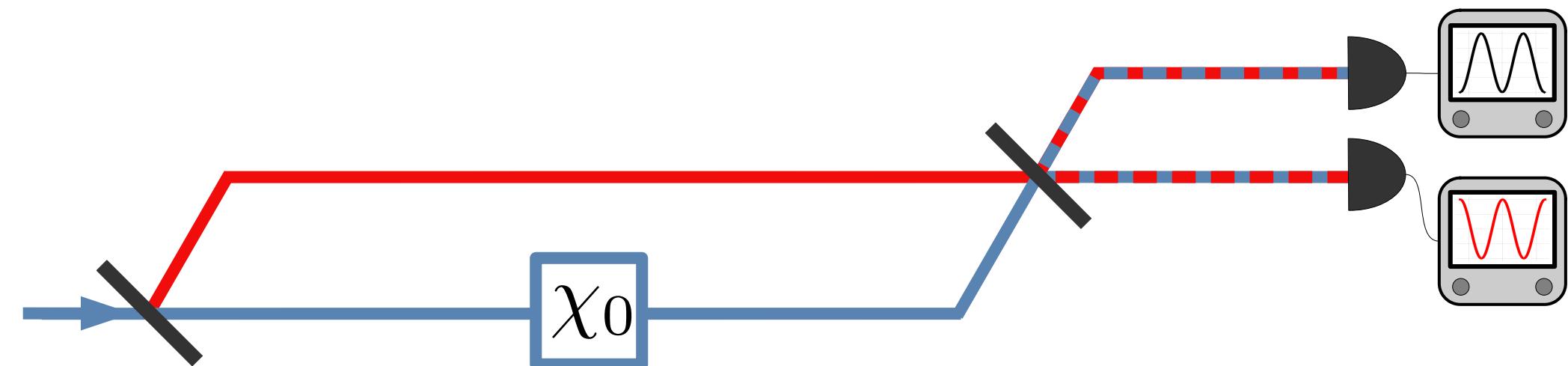
Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi, \chi_0) = \left| \langle \pm | (e^{-i\chi_0} a_1 |1\rangle + e^{i\chi} a_2 |2\rangle) \right|^2 = \frac{1}{2} \pm a_1 a_2 \cos(\chi + \chi_0)$$

Weak values and interferometry

Weak value picture

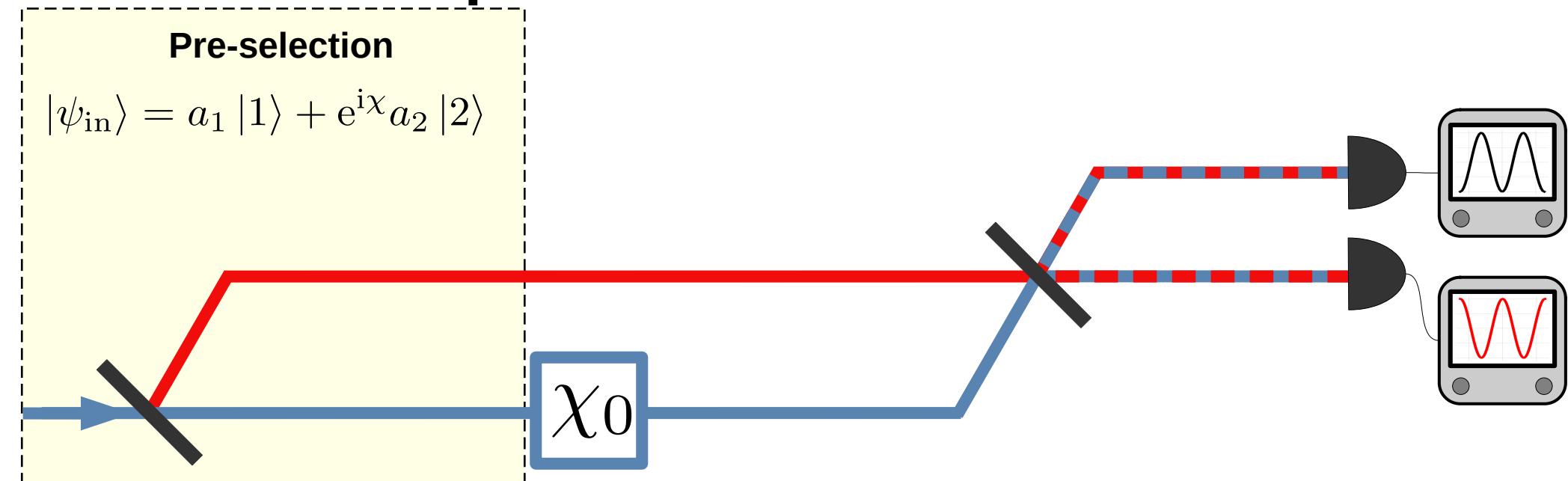


Weak values and interferometry

Weak value picture

Pre-selection

$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$



Weak values and interferometry

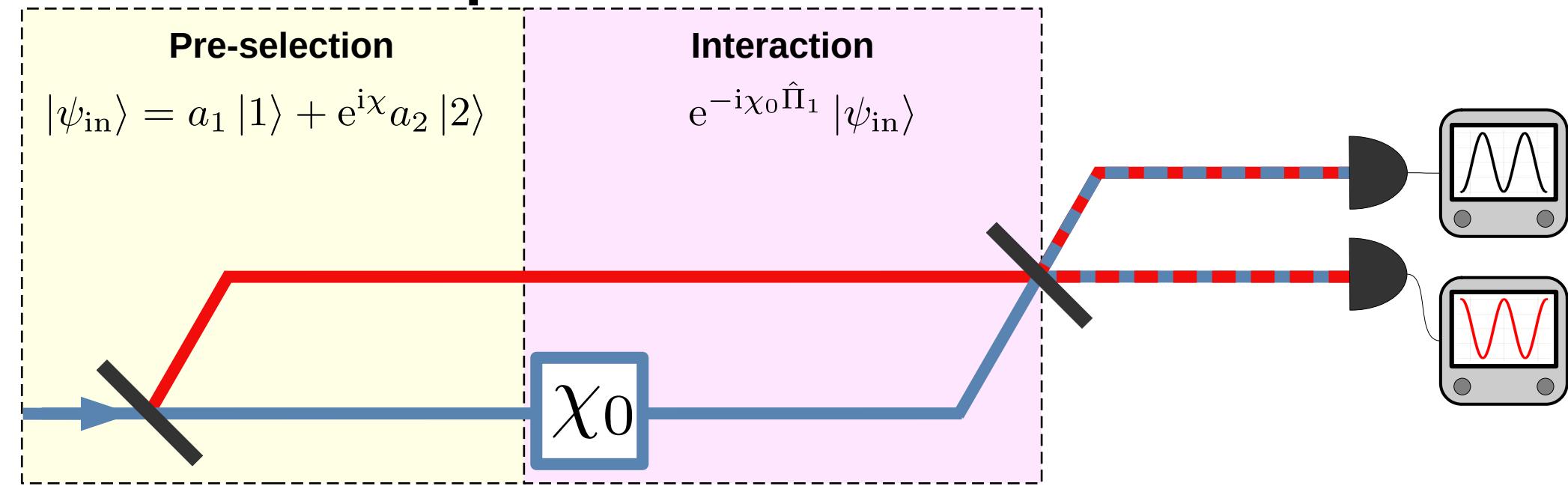
Weak value picture

Pre-selection

$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$

Interaction

$$e^{-i\chi_0 \hat{\Pi}_1} |\psi_{\text{in}}\rangle$$



Weak values and interferometry

Weak value picture

Pre-selection

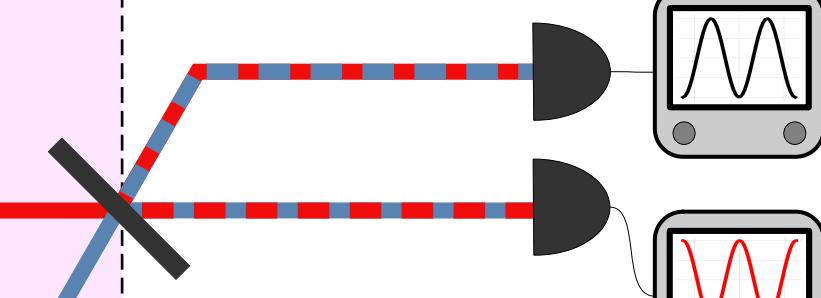
$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$

Interaction

$$e^{-i\chi_0 \hat{\Pi}_1} |\psi_{\text{in}}\rangle$$

$$\hat{\Pi}_1 = |1\rangle \langle 1|$$

χ_0



Weak values and interferometry

Weak value picture

Pre-selection

$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$

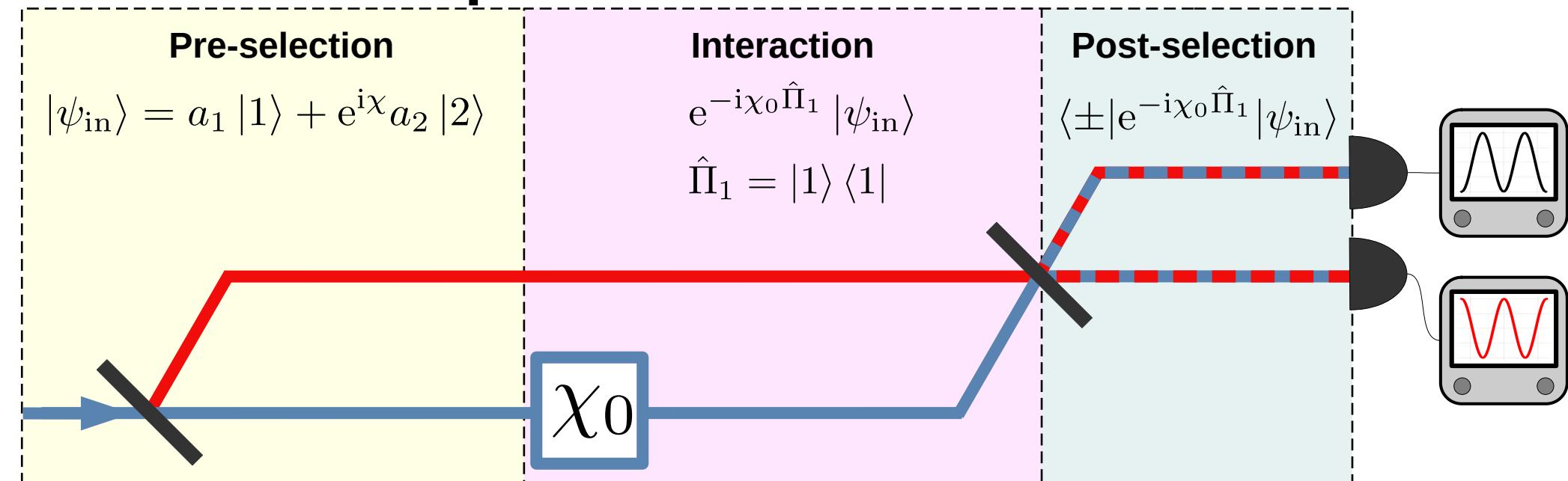
Interaction

$$e^{-i\chi_0 \hat{\Pi}_1} |\psi_{\text{in}}\rangle$$

$$\hat{\Pi}_1 = |1\rangle \langle 1|$$

Post-selection

$$\langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle$$



Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | 1 + (e^{-i\chi_0} - 1) \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2$$

Weak values and interferometry

Weak value picture

Measured intensity

$$\begin{aligned} I_{\pm}(\chi, \chi_0) &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | 1 + (e^{-i\chi_0} - 1) \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | \psi \rangle \left[1 + (e^{-i\chi_0} - 1) \frac{\langle \pm | \hat{\Pi}_1 | \psi \rangle}{\langle \pm | \psi \rangle} \right] \right|^2 \end{aligned}$$

Weak values and interferometry

Weak value picture

Measured intensity

$$\begin{aligned} I_{\pm}(\chi, \chi_0) &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | 1 + (e^{-i\chi_0} - 1) \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | \psi \rangle \left[1 + (e^{-i\chi_0} - 1) \frac{\langle \pm | \hat{\Pi}_1 | \psi \rangle}{\langle \pm | \psi \rangle} \right] \right|^2 \end{aligned}$$

Path weak value $w_{\pm,1}$

Weak values and interferometry

Weak value picture

Measured intensity

$$\begin{aligned} I_{\pm}(\chi, \chi_0) &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | 1 + (e^{-i\chi_0} - 1) \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | \psi \rangle [1 + (e^{-i\chi_0} - 1) w_{\pm,1}] \right|^2 \end{aligned}$$

Weak values and interferometry

Weak value picture

Measured intensity

$$\begin{aligned} I_{\pm}(\chi, \chi_0) &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | 1 + (e^{-i\chi_0} - 1) \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | \psi \rangle [1 + (e^{-i\chi_0} - 1) w_{\pm,1}] \right|^2 \\ &= |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0] \end{aligned}$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2 (|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Amplitude
square

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2 (|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Real part

Amplitude
square

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Amplitude
square

Real part

Imaginary
part

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

$$I_{\pm}(\chi, 0) = |\langle \pm | \psi_{\text{in}} \rangle|^2$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

$$I_{\pm}(\chi, 0) = |\langle \pm | \psi_{\text{in}} \rangle|^2$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

Weak values and interferometry

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

$$I_{\pm}(\chi, 0) = |\langle \pm | \psi_{\text{in}} \rangle|^2$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Weak values and interferometry

Almost there...

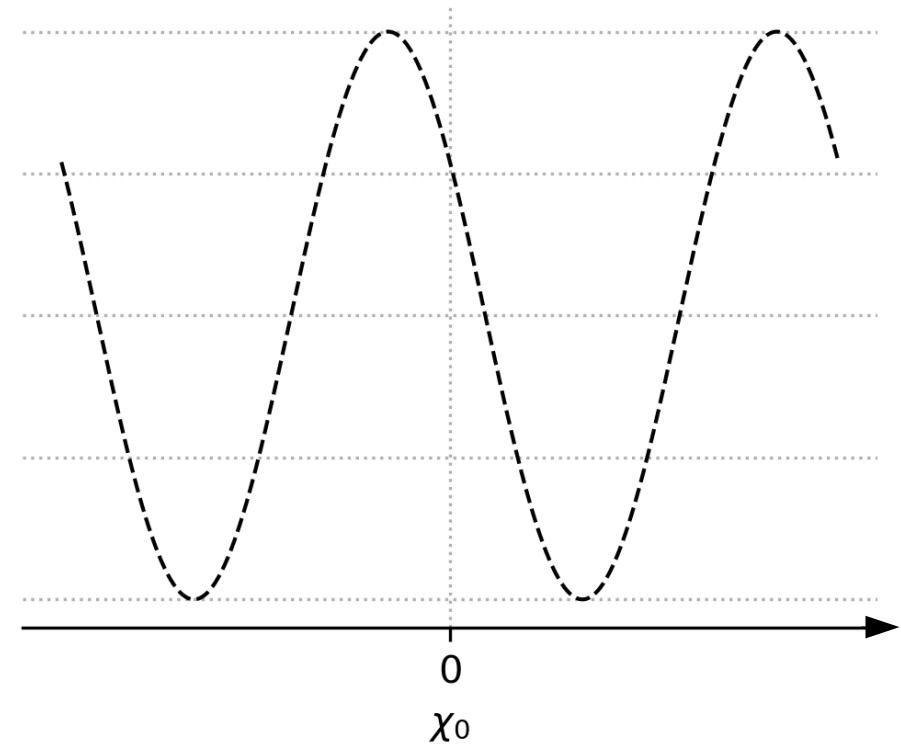
$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Interferogram

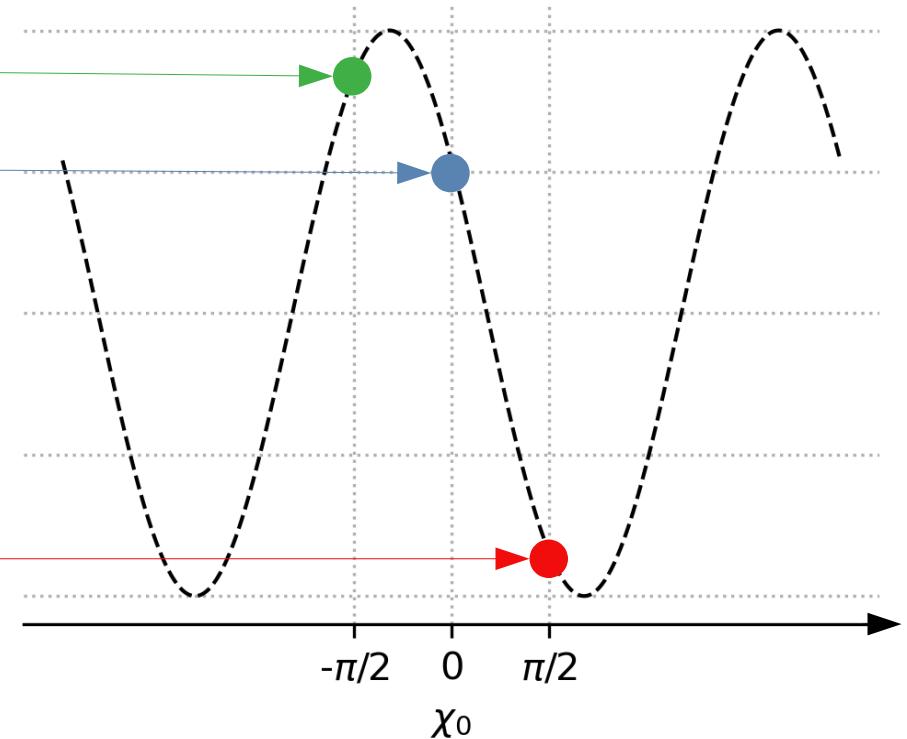


Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Interferogram



Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$
$$w_{\pm,1}^{\Re}{}^2 + w_{\pm,1}^{\Im}{}^2$$

Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re} \quad \leftarrow \text{2}^{\text{nd}} \text{ order equation}$$

$|w_{\pm,1}|^2 = w_{\pm,1}^{\Re} + w_{\pm,1}^{\Im}$

Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re} \quad \leftarrow \text{2}^{\text{nd}} \text{ order equation}$$

$|w_{\pm,1}|^2 = w_{\pm,1}^{\Re} + w_{\pm,1}^{\Im}$

2 solutions

Weak values and interferometry

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

Weak values and interferometry

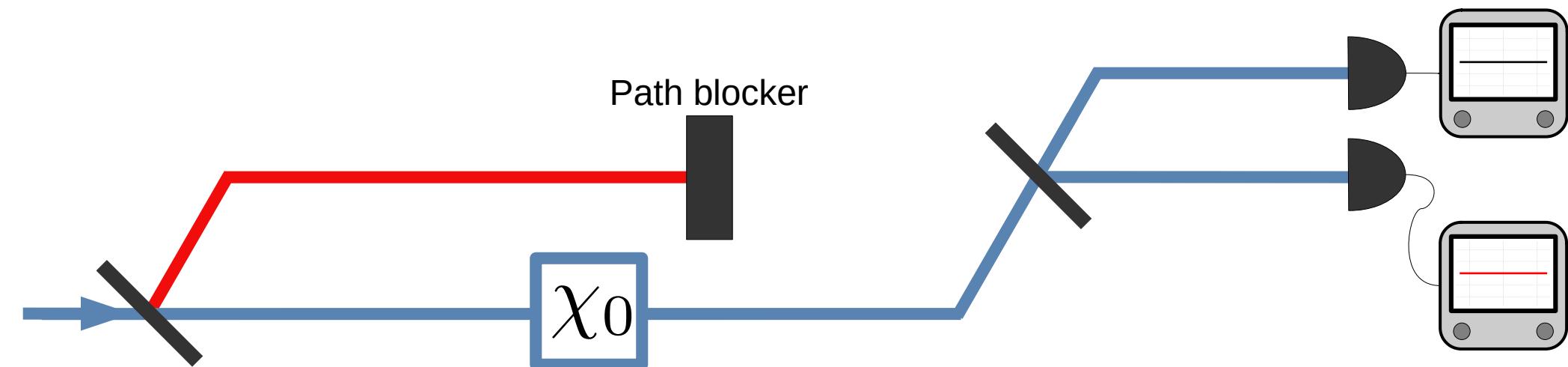
Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



$$-\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} + |w_{\pm,1}|^2 = w_{\pm,1}^{\Re}$$

Weak values and interferometry

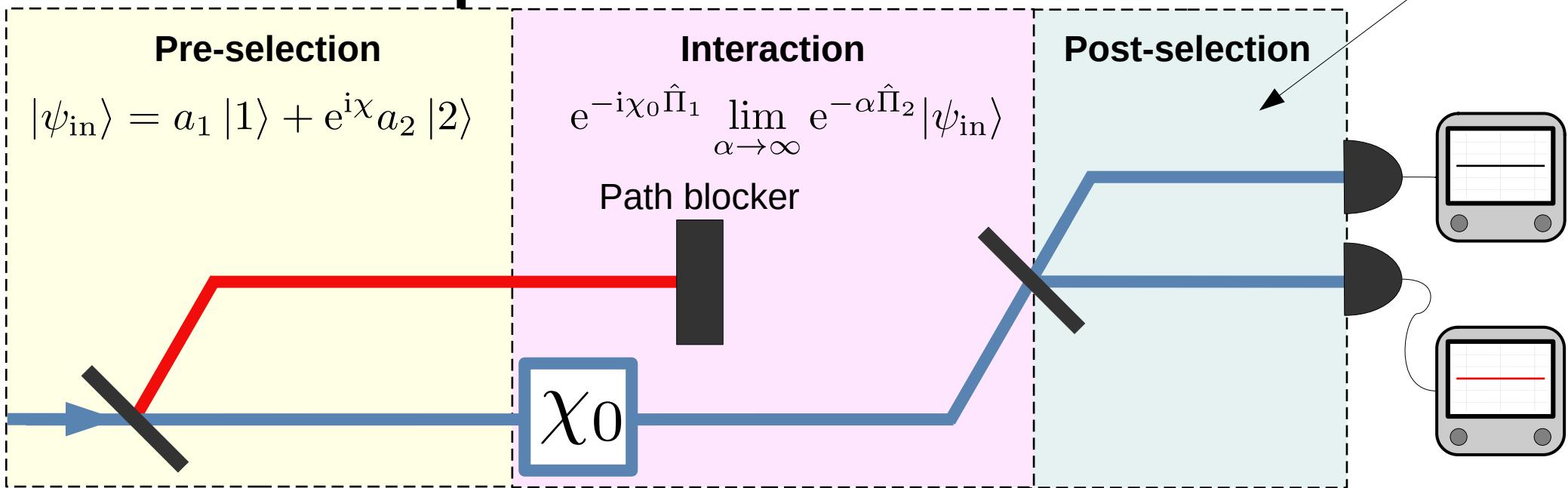


Weak values and interferometry

Weak value picture

Pre-selection

$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$



Weak values and interferometry

Measured intensity

$$\begin{aligned} I_{\pm}^{Bl.\,2} &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= |\langle \pm | \psi_{\text{in}} \rangle|^2 |w_{\pm,1}|^2 \end{aligned}$$

Weak values and interferometry

Measured intensity

$$\begin{aligned} I_{\pm}^{Bl. 2} &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= |\langle \pm | \psi_{\text{in}} \rangle|^2 |w_{\pm,1}|^2 \end{aligned}$$

Weak values and interferometry

Measured intensity

$$\begin{aligned} I_{\pm}^{Bl. 2} &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= \boxed{\left| \langle \pm | \psi_{\text{in}} \rangle \right|^2} \boxed{|w_{\pm,1}|^2} \end{aligned}$$

Weak values and interferometry

We got there!

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



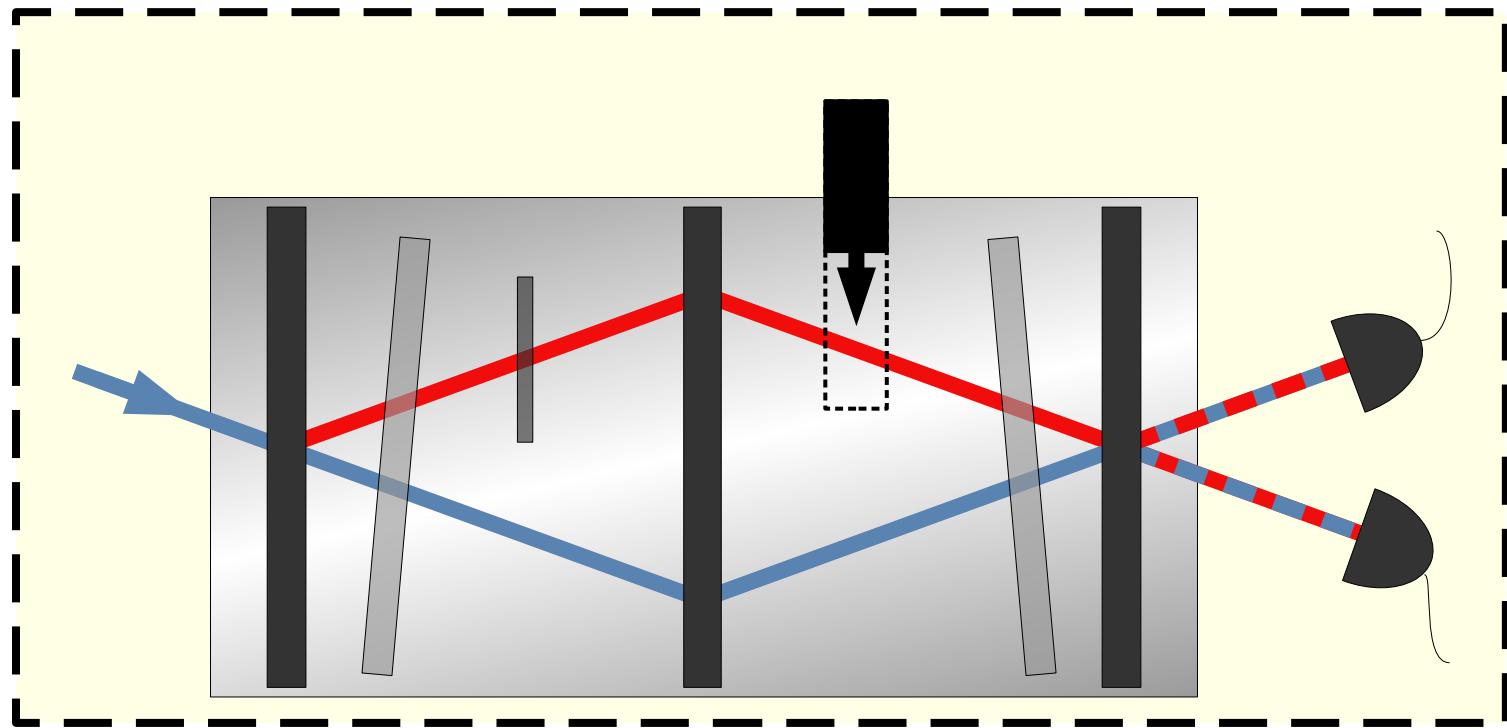
$$-\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} + |w_{\pm,1}|^2 = w_{\pm,1}^{\Re}$$



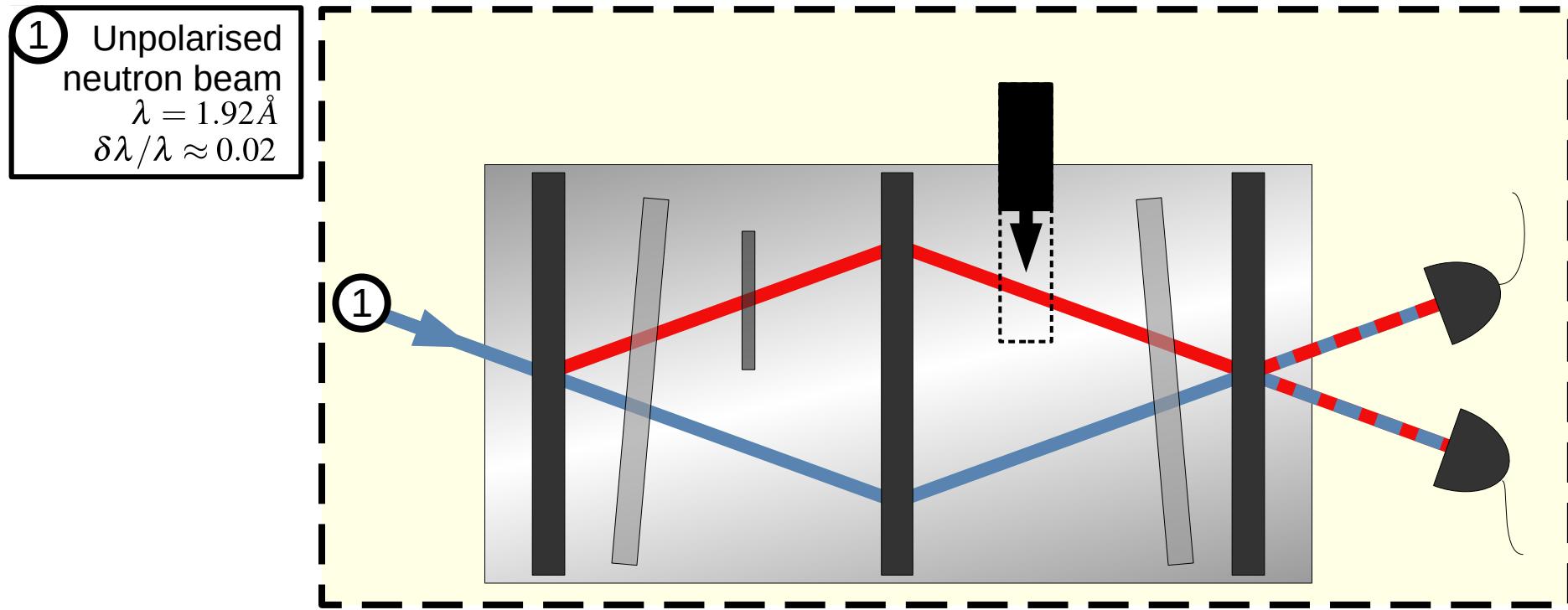
*I am unexpectedly running
out of time...*

Part 3: Experimental measurement of path weak value from interferograms

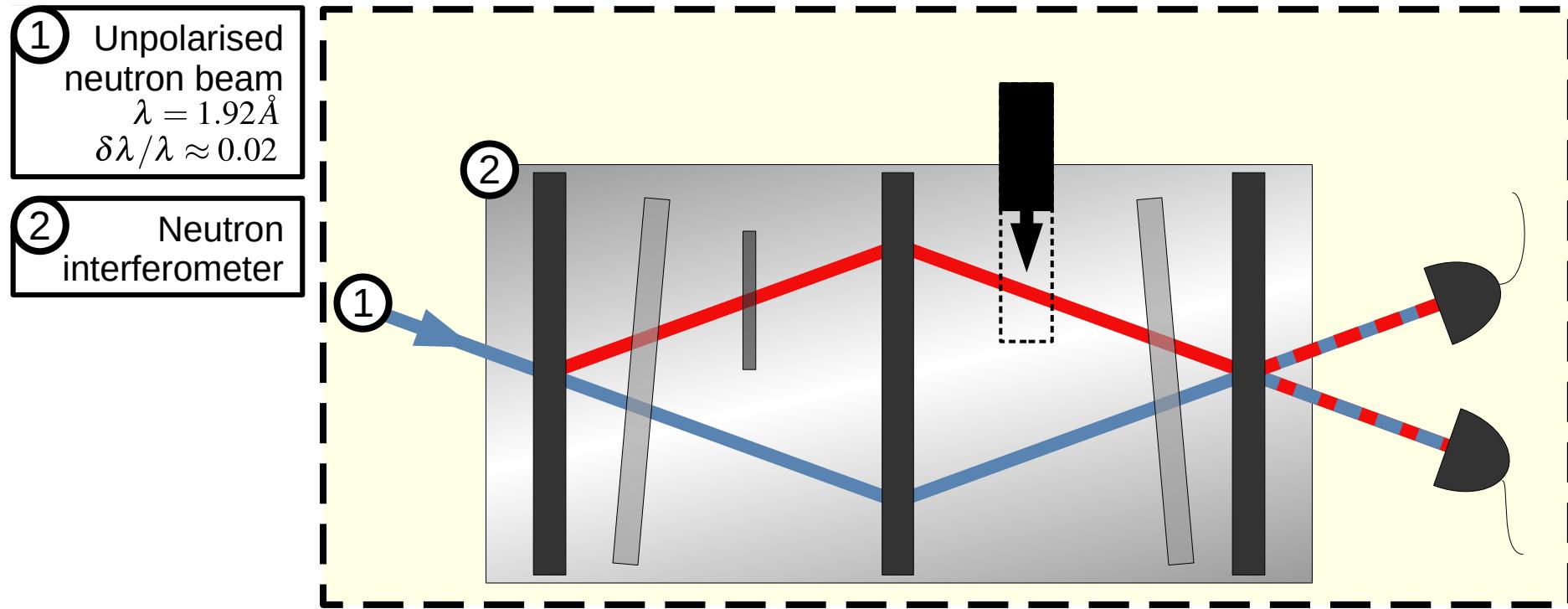
Setup



Setup

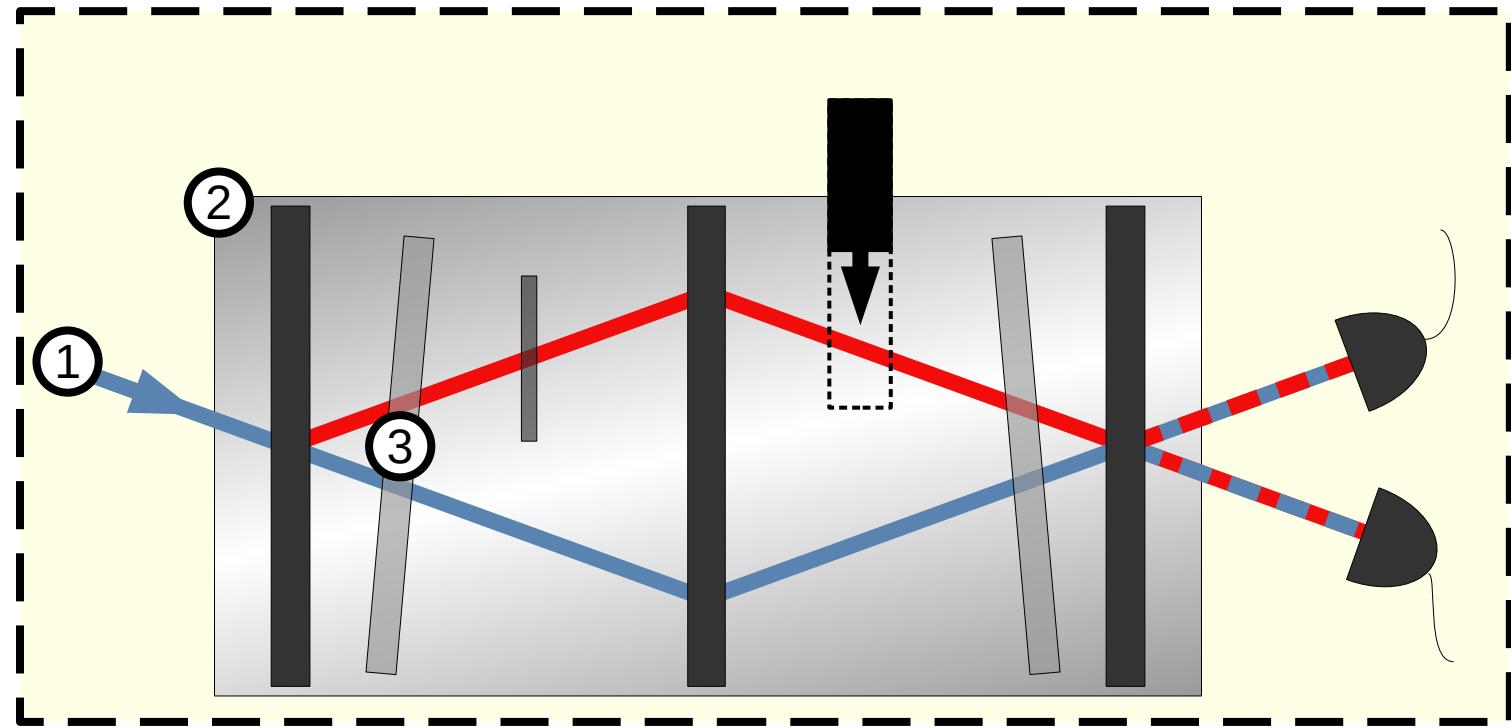


Setup



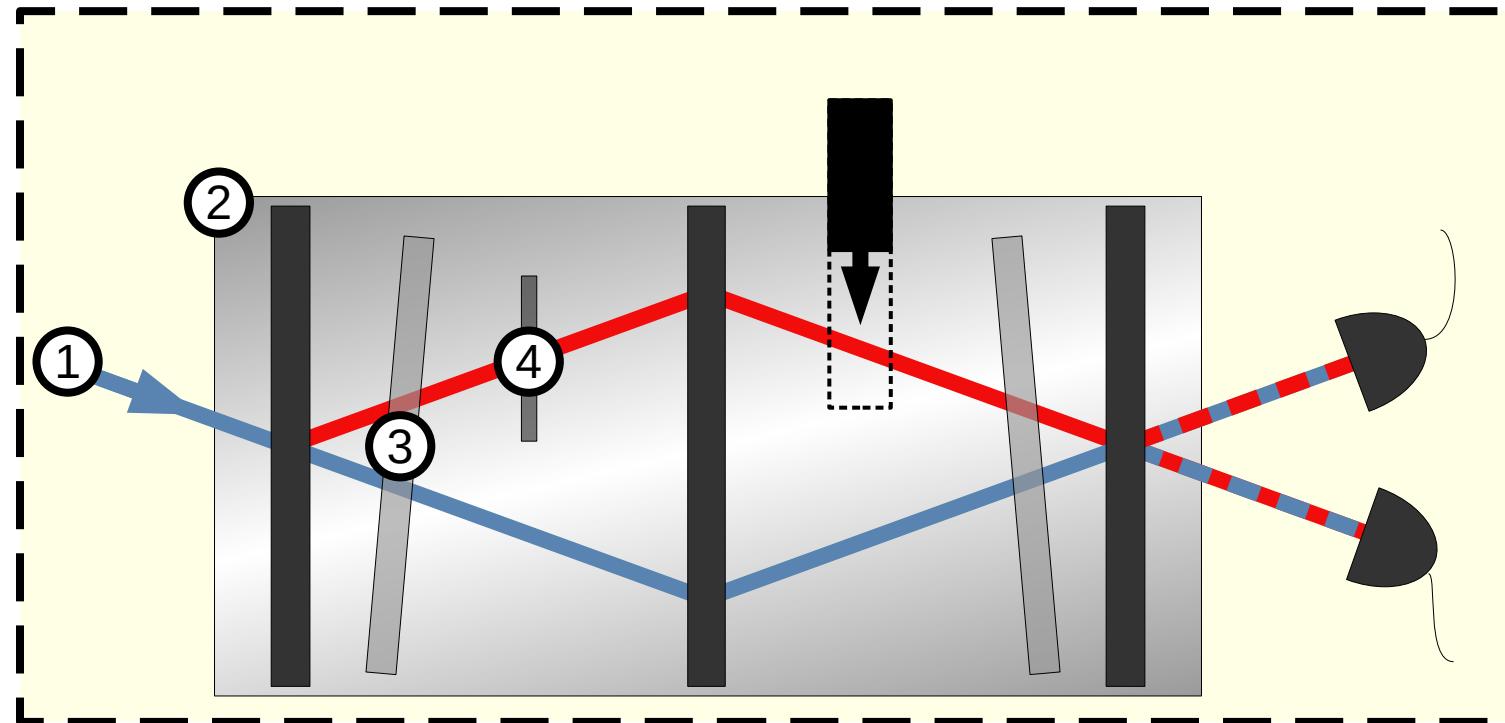
Setup

- ① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- ② Neutron interferometer
- ③ Phase-shifter χ



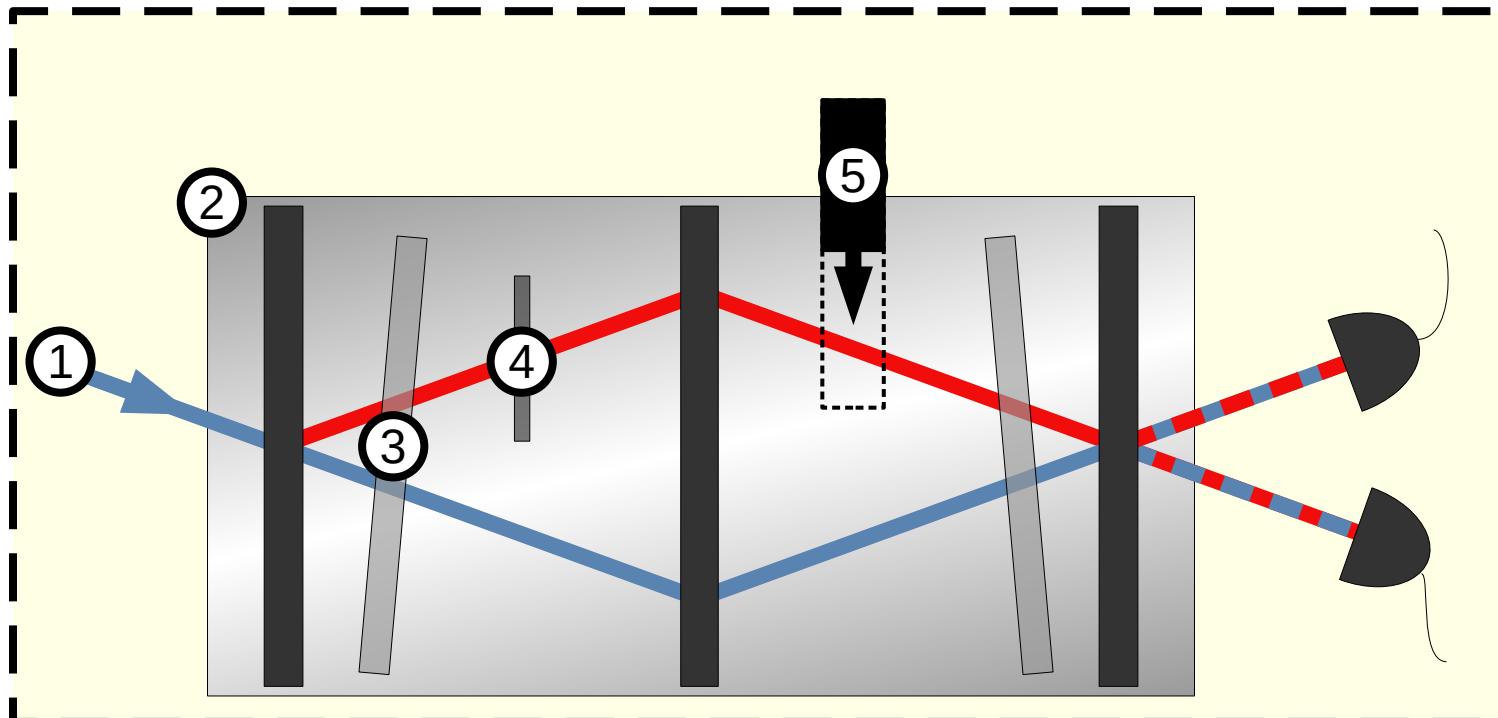
Setup

- ① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- ② Neutron interferometer
- ③ Phase-shifter
 χ
- ④ Indium foils to adjust path amplitudes
 $a_2/a_1 \approx 0.59$



Setup

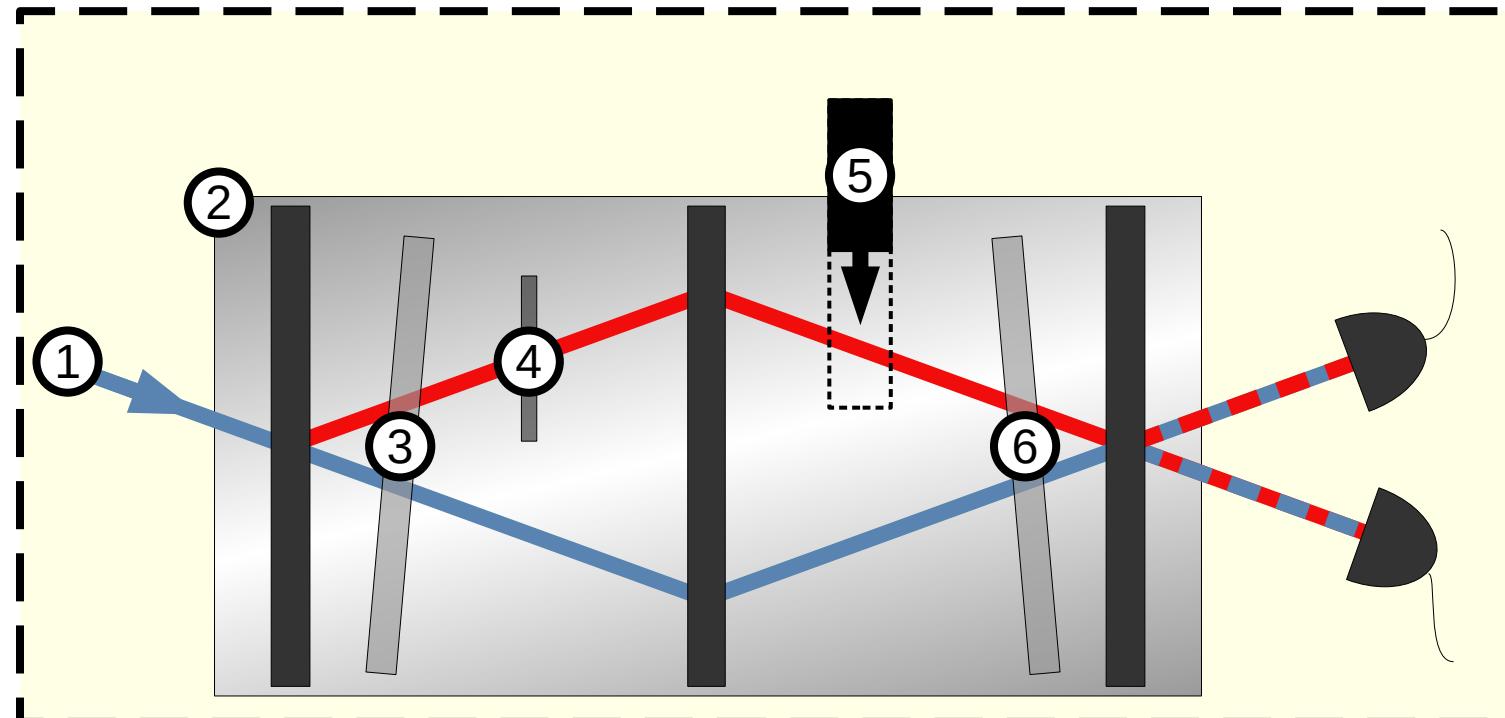
- ① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- ② Neutron interferometer
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 χ
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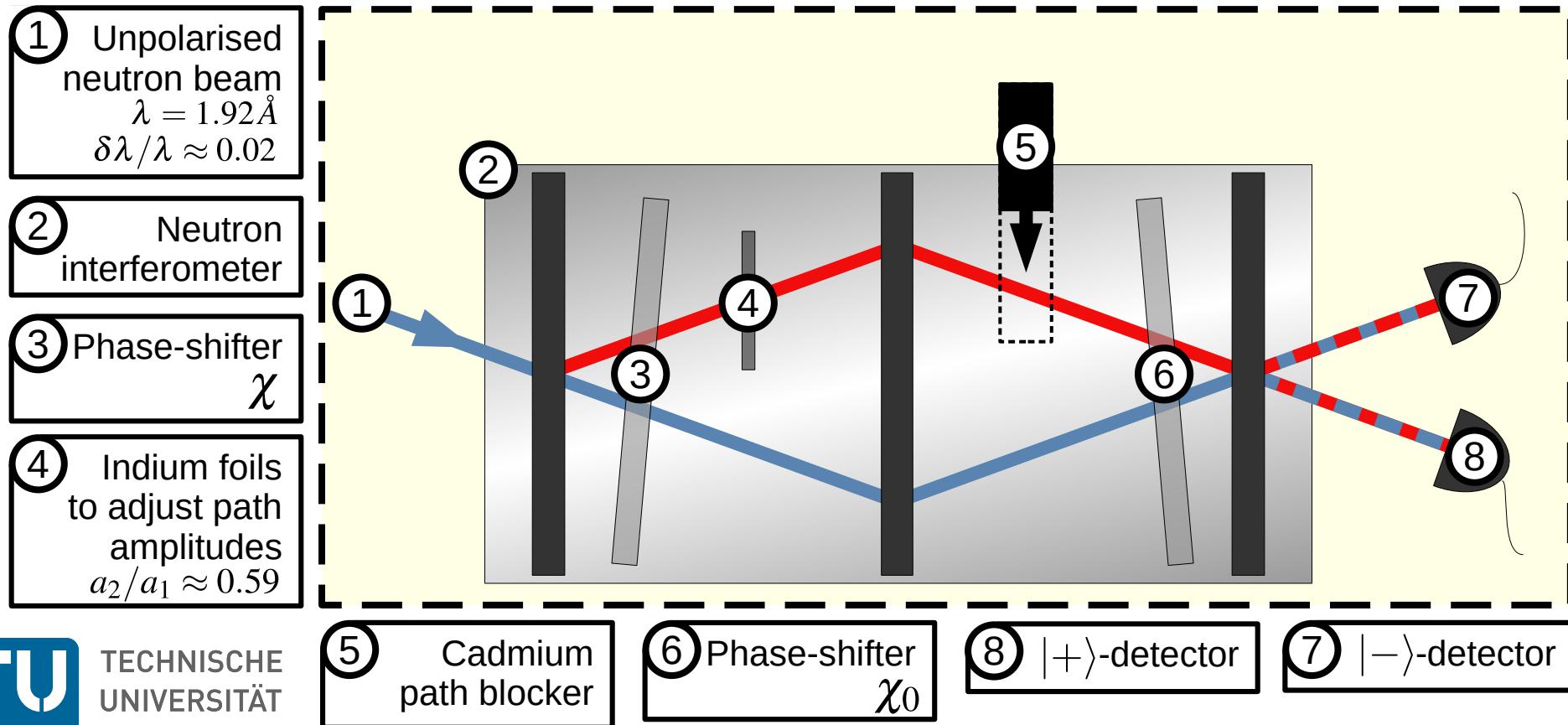
- ⑤ Cadmium path blocker

Setup

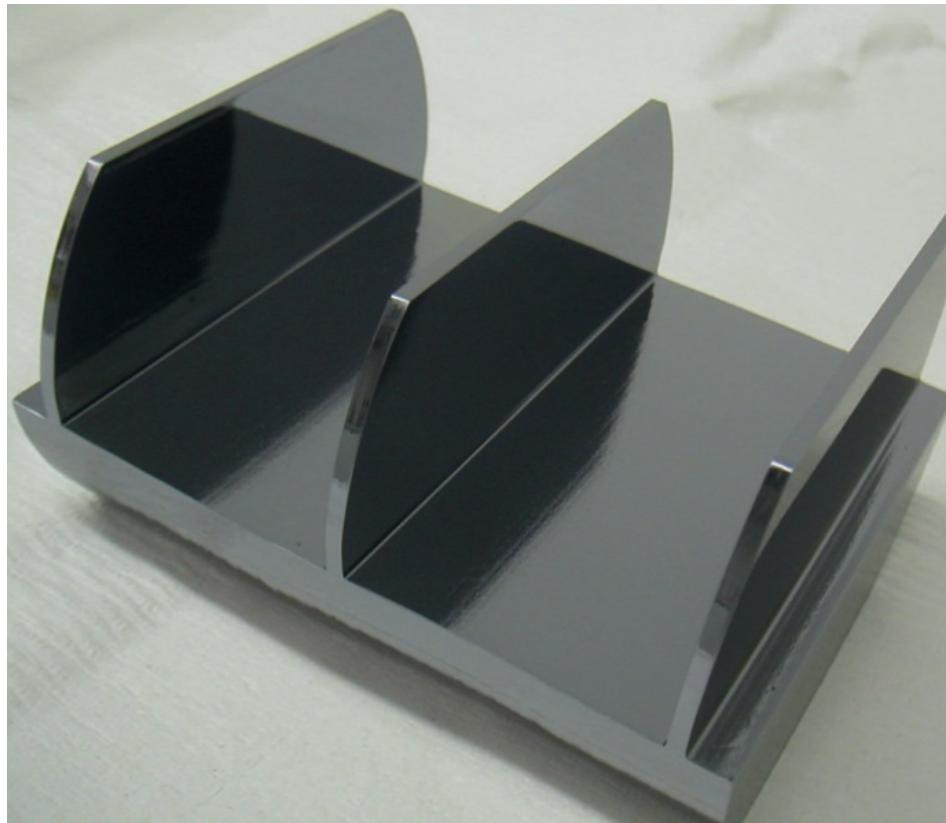
- ① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- ② Neutron interferometer
- ③ Phase-shifter χ
- ④ Indium foils to adjust path amplitudes
 $a_2/a_1 \approx 0.59$



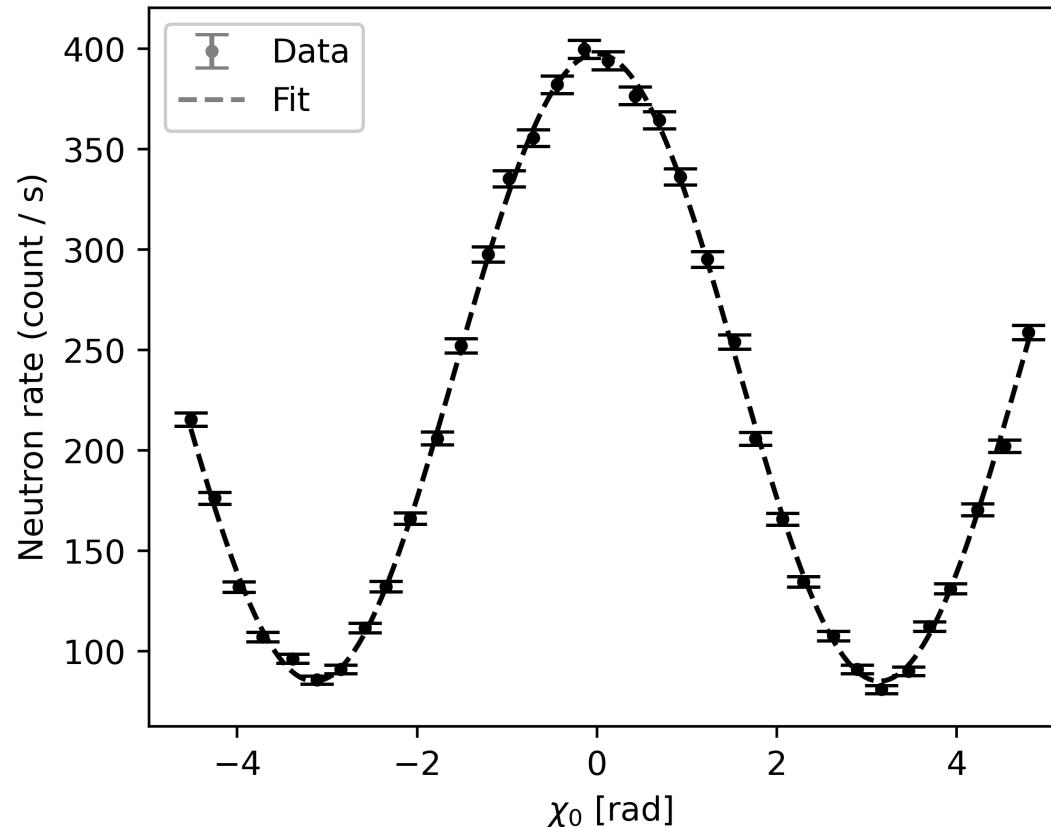
Setup



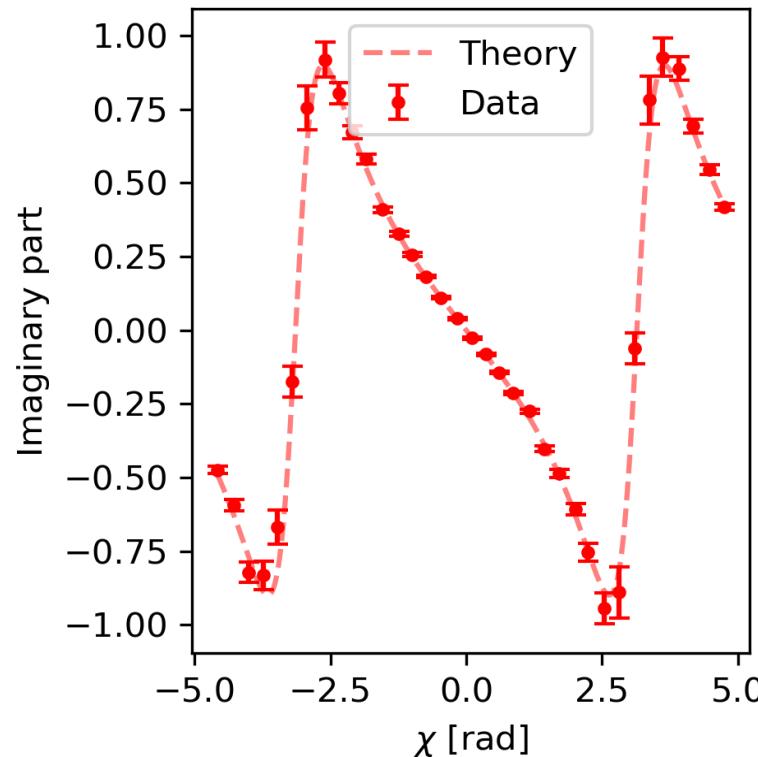
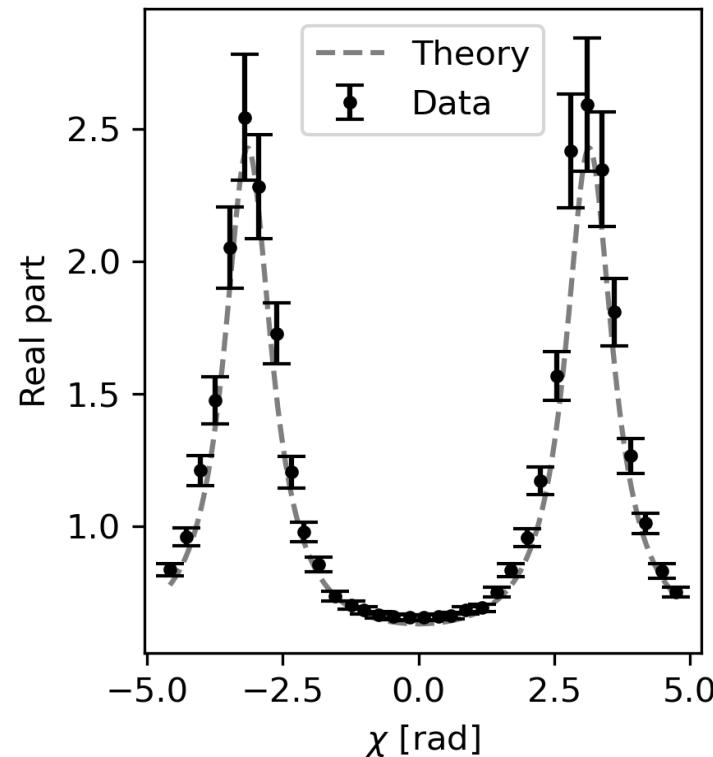
Neutron interferometer



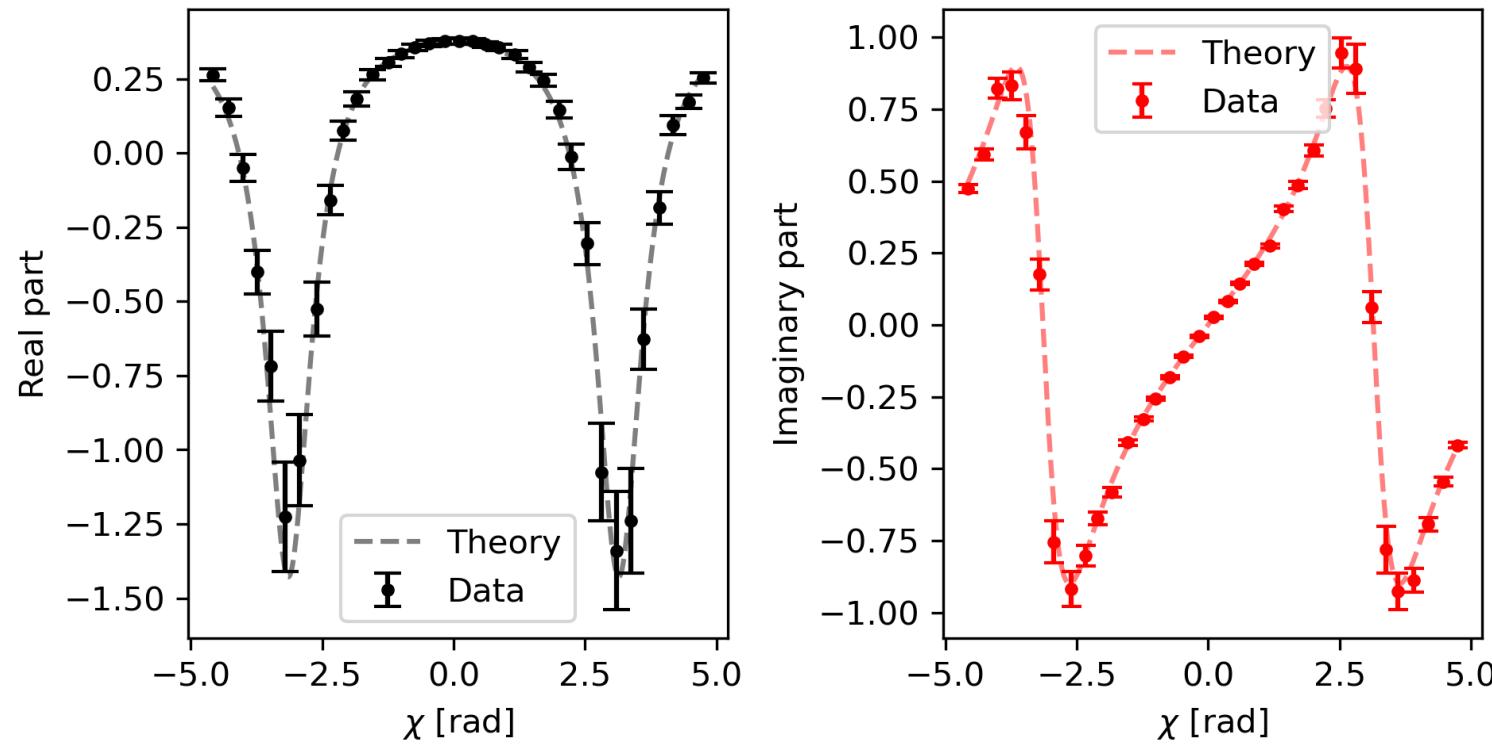
Example of interferogram



Results: Weak value of path 1



Results: Weak value of path 2



Interpretation of the results

Interpretation of the results



Summary

- Introduction to weak values and weak measurements
- Weak values based description of a Mach-Zehnder interferometer
- Experimental measurement of path weak values directly from interferograms

Summary

- To be continued...

Acknowledgements

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FWF Austrian
Science Fund

Thanks for your time!

