# The Delayed-Choice Quantum Eraser Leaves No Choice

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## Abstract

A realizable delayed-choice quantum eraser, using a modified Mach-Zehnder (MZ) interferometer and polarization entangled photons, is theoretically analyzed here. The signal photon goes through a modified MZ interferometer, and the polarization of the idler photon provides path information for the signal photon. The setup is very similar to the delayed-choice quantum eraser experimentally studied by the Vienna group. In the class of quantum erasers with discrete output states, it is easy to see that the delayed mode leaves no choice for the experimenter. The which-way information is always erased, and every detected signal photon fixes the polarization state of the idler, and thus gives information on precisely how the signal photon traversed the two paths. The analysis shows that the Vienna delayed-choice quantum eraser is the first experimental demonstration of the fact that the delayed mode leaves no choice for the experimenter, and the which-way information is always erased. Additionally it is shown that this argument holds even in a conventional two-slit quantum eraser. Every photon registered anywhere on the screen, fixes the state of the two-state which-way detector in a unique mutually unbiased basis. In the delayed-choice quantum eraser experiments, the role of mutually unbiased basis sets for the which-way detector, has been overlooked till now.

## 1 Introduction

The concept of wave-particle duality started out as a debate on the corpuscular nature versus wave nature of light. With the advent of quantum mechanics, a new language emerged which described this concept, namely the Bohr's principle of complementarity [1]. According to Bohr, the two natures of quantum objects, which we shall refer to as quantons here, the wave and particle natures complement each other. However, the two natures are also mutually exclusive so that an experiment which brings out one nature, necessarily hides the other. The two-slit interference experiment, where one additionally tries to probe which of the two slits the quanton went through, became a test-bed for the concept of wave-particle duality right from the time of its inception [2]. It soon became clear that if one tries to get

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the which-path or which-way information about the quanton, the interference is destroyed. Jaynes [3] came up with an interesting idea that there can be ways in which the acquired which-way information can be erased such that the destroyed interference can be brought back, in perfect harmony with the concept of wave-particle duality. The modern formulation of "quantum eraser" was proposed by Scully and Drühl [4]. What made their proposal more exciting was their suggestion that one may choose to erase the which-way information much after the quanton had registered on the screen, and the interference could still be recovered. This initiated a lively debate on the subject which continues till date [5–15]. The concept of quantum eraser implied that the experimenter could choose to retain the which-way information or erase it, and as a consequence, can force the quanton to behave either as a particle or a wave. The "delayed-choice" quantum eraser went a leap further by suggesting that the quanton which traveled the two paths and hit the screen, may be forced to behave like a particle or a wave by a choice made the experimenter after it has already hit the screen. This kind of thinking led to a talk of "retrocausality" in the delayed-choice quantum eraser experiment, which is still being hotly debated [10–14].

With the advances in experimental techniques the quantum eraser, with or without delayed-choice, was realized in various ways [16–27], and several other proposals were made [28–31]. It has also been demonstrated that the idea of quantum eraser should also work for three-path interference [32]. However, no experimental progress has been in that direction yet. It may be pertinent to mention a new class of delayed-choice experiments with a quantum twist, that were recently studied [33–38]. The idea in those experiment was to explore the possibility of a quantum superposition of wave and particle behavior.

The current debate mainly revolves around the interpretation of delayed-choice quantum eraser. A widely held view, due to Englert, Scully and Walther [5], is that the choice to retain or erase the information regarding which of the two paths the quanton followed, always rests with the experimenter. While this view is quite acceptable for the normal quantum eraser, it is hard to digest for many people when applied to the quantum eraser experiment carried out in the delayed mode. According to this view, even in the delayed mode, the experimenter chooses whether the quanton displays wave nature or particle nature. However, the authors of this view do not comment on how one should interpret the "actual behavior" of the quanton in such experiments. Questions like if the quanton shows particle nature in the delayed mode, does it actually follow only one of the two paths, are left unanswered. Here we re-investigate the delayed choice quantum eraser, by proposing a realizable experiment using entangled photons, and try to find answers to the questions which are under debate.

## 2 Quantum Eraser with a Mach-Zehnder Setup

Let us consider an experimental setup as shown in Fig. 1, where there is a spontaneous parametric down conversion (SPDC) source producing pairs of photons which are entangled in polarization such that the state is given by

$$|\Psi_0\rangle = \tfrac{1}{\sqrt{2}}(|V_s\rangle|H_i\rangle + |H_s\rangle|V_i\rangle)|\psi_0\rangle|\phi_0\rangle \qquad (1)$$

where $H$ and $V$ denotes the horizontal and vertical polarization states, the labels $s$, $i$ denote the signal and idler photons, respectively. The spatial states for the signal and idler photons are denoted by $|\psi_0\rangle|\phi_0\rangle$, respectively. A Mach-Zehnder interferometer can be easily

**Fig. 1** A schematic diagram of a quantum eraser setup using entangled photons, and a modified Mach-Zehnder interferometer. The two paths of the signal photon end up getting entangled with the polarization states of the idler photon
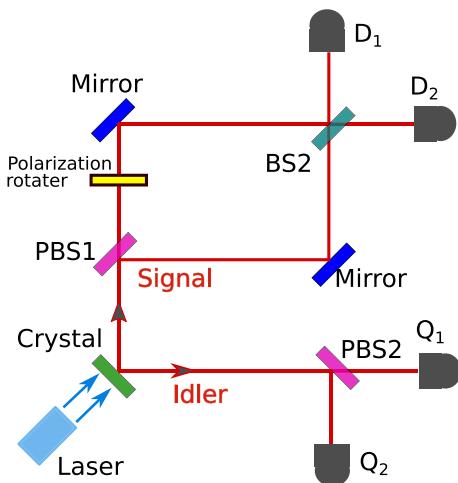


analyzed using quantum mechanics [39, 40]. After the signal photon passes through the polarizing beam-splitter PBS1, the state changes to

$$
\begin{aligned}
|\Psi_1\rangle &= U_{PBS1}|\psi_0\rangle \\
&= \tfrac{1}{\sqrt{2}}(|H_s\rangle|V_i\rangle|\psi_1\rangle + |V_s\rangle|H_i\rangle|\psi_2\rangle)|\phi_0\rangle, \quad (2)
\end{aligned}
$$

where the $|\psi_1\rangle$, $|\psi_2\rangle$ represent the states of the signal photon in the upper and lower path of the Mach-Zehnder interferometer, respectively. One would notice that the two paths of the signal photon are now entangled with the polarization states of the two photons. The signal photon in the upper path (path 1) passes through a polarization rotator, possibly a half-wave plate, which rotates the polarization by 90 degrees, flipping the $|H_s\rangle$ state to $|V_s\rangle$ state, so that the state now reads

$$
|\Psi_2\rangle = \tfrac{1}{\sqrt{2}}(|V_i\rangle|\psi_1\rangle + |H_i\rangle|\psi_2\rangle)|V_s\rangle|\phi_0\rangle. \quad (3)
$$

This process now entangled the two paths of the signal photon with the polarization states of the idler photon. Interestingly, the polarization of the signal photon is now disentangled from that of the idler. One may wonder how one can disentangle the polarization of the signal photon from the polarization of the idler by just a local operation on the signal photon. It is well known that entanglement cannot be changed by a local operation. The answer is that by this operation, the entanglement has not changed at all, but has been *transferred* from the polarization degree of freedom to the spatial degree of freedom of the signal photon. This is indeed possible, and similar methods have been employed before to create *hybrid entanglement* [41]. Separating out the horizontal and vertical components of the idler photon can yield information on which of the two paths the signal photon followed, simply because $\langle V_i|\Psi_2\rangle = |\psi_1\rangle|V_s\rangle|\phi_0\rangle$, and $\langle H_i|\Psi_2\rangle = |\psi_2\rangle|V_s\rangle|\phi_0\rangle$.

On the other hand, (3) can also be rewritten as

$$
\begin{aligned}
|\Psi_2\rangle = \tfrac{1}{2}\{&|R_i\rangle(|\psi_2\rangle - i|\psi_1\rangle) \\
&+ |L_i\rangle(|\psi_2\rangle + i|\psi_1\rangle)\}|V_s\rangle|\phi_0\rangle, \quad (4)
\end{aligned}
$$

where $|R_i\rangle = \tfrac{1}{\sqrt{2}}(|H_i\rangle + i|V_i\rangle)$, $|L_i\rangle = \tfrac{1}{\sqrt{2}}(|H_i\rangle - i|V_i\rangle)$ represent the left and right circular polarization states, respectively. If one measured the circular component of polarization of the idler photon, obtaining the state $|R_i\rangle$ would tell one that the state of the signal photon is

$\frac{1}{\sqrt{2}}(|\psi_2\rangle - i|\psi_1\rangle)$, and obtaining the state $|L_i\rangle$ would tell one that it is $\frac{1}{\sqrt{2}}(|\psi_2\rangle + i|\psi_1\rangle)$. With this, our arrangement for obtaining path information is fully in place. Obtaining the state $|R_i\rangle$ of the idler photon tells us that the signal photon followed both paths, *exactly as it would, if there were no path-detecting mechanism in place*, except for a phase difference of $-\pi/2$ between the two paths. Obtaining the state $|L_i\rangle$ of the idler tells us again that the signal photon followed both paths, but now with a phase difference of $\pi/2$ between the two paths. Quantum mechanics then implies that if one measures the polarization of the idler in the horizontal-vertical (linear polarization) basis, while the signal photon is still traveling, one can force it to follow one of the two MZ paths. On the other hand, by measuring the polarization of the idler in the circular basis, one can force the signal photon to follow both the paths. The choice lies with the experimenter.

The effect of the second beam-splitter, on the two components $|\psi_1\rangle, |\psi_2\rangle$, is the following [39]

$$\boldsymbol{U}_{BS2}|\psi_1\rangle = \frac{1}{\sqrt{2}}(|D_1\rangle + i|D_2\rangle)$$
$$\boldsymbol{U}_{BS2}|\psi_2\rangle = \frac{1}{\sqrt{2}}(i|D_1\rangle + |D_2\rangle), \tag{5}$$

where $\boldsymbol{U}_{BS2}$ represents the unitary evolution due to the mirrors and the second beam-splitter BS2, and $|D_1\rangle, |D_2\rangle$ are the states at the detectors $D_1, D_2$, respectively. If the state of the signal photon is $|\psi_1\rangle$ or $|\psi_2\rangle$, in both the situations it is equally likely to hit $D_1$ or $D_2$. This implies a loss of interference, resulting from extraction of which-path information by the idler. So, obtaining the horizontal or vertical state of the idler, destroys the interference of the signal photon, but yields its precise path information.

However, if one measured the circular polarization of the idler, and obtained the state $|R_i\rangle$, it would tell us that the state of the signal photon would be $\frac{1}{\sqrt{2}}(|\psi_2\rangle - i|\psi_1\rangle)$. The second beam-splitter BS2 would take this state only to $D_2$: $\boldsymbol{U}_{BS2}\frac{1}{\sqrt{2}}(|\psi_2\rangle - i|\psi_1\rangle) = |D_2\rangle$. This implies interference with the bright fringe at $D_2$ and the dark one at $D_1$. On the other hand, obtaining the state $|L_i\rangle$ would tell us that the state of the signal photon would be $\frac{1}{\sqrt{2}}(|\psi_2\rangle + i|\psi_1\rangle)$. The beam-splitter BS2 would take this state only to $D_1$: $\boldsymbol{U}_{BS2}\frac{1}{\sqrt{2}}(|\psi_2\rangle + i|\psi_1\rangle) = i|D_1\rangle$. This also implies interference, but with the bright fringe at $D_1$ and the dark one at $D_2$. Both these situations describe the phenomenon of quantum erasure, where the lost interference comes back if the which-path information is erased. However, the two interferences are mirror images of each other, and taken together, they cancel each other out.

Let us now look at the delayed mode where no measurement is made on the idler photon, the path of the idler being much longer, and the signal photon reached the detectors. For example, in one experiment performed by the Vienna group [17], the idler photon travels a distance of 144 kilometers before it reaches the analyzing detectors, whereas the MZ paths are of the order of 2 meters. In our setup, the final state of the two photons, just before the signal photon hits the detectors, is given by

$$|\Psi_3\rangle = \boldsymbol{U}_{BS2}|\Psi_2\rangle = \frac{1}{\sqrt{2}}\boldsymbol{U}_{BS2}(|V_i\rangle|\psi_1\rangle + |H_i\rangle|\psi_2\rangle)|V_s\rangle|\phi_0\rangle$$
$$= \frac{1}{2}[|V_i\rangle(|D_1\rangle + i|D_2\rangle) + |H_i\rangle(i|D_1\rangle + |D_2\rangle)]|V_s\rangle|\phi_0\rangle. \tag{6}$$

This state indicates that $D_1$ and $D_2$ are equally likely to register the signal photon, as $|\langle D_1|\Psi_3\rangle|^2 = |\langle D_2|\Psi_3\rangle|^2 = 1/2$, which in turn implies no interference.

An interesting scenario emerges if one rewrites the state (6) as

$$|\Psi_3\rangle = \frac{1}{\sqrt{2}}(i|D_1\rangle|L_i\rangle + |D_2\rangle|R_i\rangle)|V_s\rangle|\phi_0\rangle. \tag{7}$$

This state indicates that if the signal photon registers at $D_2$, it fixes the polarization state of the idler to the right circular state $|R_i\rangle$, and if it registers at $D_1$, it fixes polarization state of the idler to the left circular state $|L_i\rangle$. But the states $|R_i\rangle$, $|L_i\rangle$ correspond to the erased which-path information, and tell us that the signal photon followed both the paths, and not one of the two. In the delayed mode, the experimenter no longer has the choice to seek either which-path information or quantum eraser. This runs counter to the widely accepted notion that the choice of which-path information or quantum eraser, lies with the experimenter in the delayed mode [5]. Not only does the registered signal photon tell us the that the which-path information is erased, it tells us precisely how the signal photon traversed the two paths, and the phase difference between the two paths, by virtue of (4). This correlation, of course, can also be used to recover the lost interference, constituting the usual quantum eraser.

More interesting is the fact that the correlation between the left-right circularly polarized states of the idler, and the detectors $D_1$ and $D_2$ of the modified MZ setup has already been experimentally observed in the Vienna delayed choice quantum eraser experiment [17]. However, its implication was not recognized for want of an analysis similar to the one presented here. The equivalence between the setup studied here and the one implemented by the Vienna group can be easily seen. They used, what they call, a 'hybrid entangler' to achieve entanglement between the two paths of one photon and the polarization states of a causally disconnected photon. Although the authors go an extra step by varying the position of PBS1, and observing the counts of each detector (coincident with remote photon), there is a central position of PBS1 for which one detector gives maximum counts, and the other one gives almost zero (see figure 3D of Ref. [17]). Figure 2 shows simulated data of our suggested experiment, and is closely similar to the Vienna experiment results. The maximum and minimum counts at the central position are the equivalent of the bright and dark fringe of the traditional two-slit experiment. At this position of PBS1, registering of a photon at a particular detector, fixes the polarization state of the other photon which is 144 km away. In the experiment, this emerges as the prefect correlation between the two observations.

The prevalent belief [5] says that even in the delayed mode, observing the idler in the horizontal-vertical basis, gives one the path-information about the signal photon. The preceding analysis shows that this is incorrect. For example, if the signal photon registers at
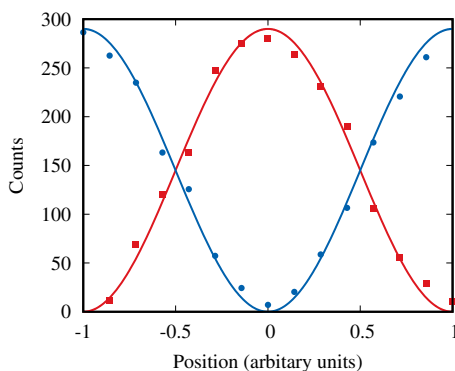


**Fig. 2** Simulated results of the quantum eraser setup using entangled photons, and a modified Mach-Zehnder interferometer (see Fig. 1). Points represent coincident counts of $D_1$ (red squares) and $D_2$ (blue circles) with one particular state of the idler, as a function of the position of BS1. Solid lines are the corresponding theoretical curves. Compare with figure 3D of Ref. [17]. The peak will be shifted if the coincidence is done with a state of a different mutually unbiased basis of the polarization of the idler photon

$D_2$, (7) tells us that the polarization state of the idler is $|R_i\rangle$. Since $|R_i\rangle = \frac{1}{\sqrt{2}}(|H_i\rangle + i|V_i\rangle)$, if one insists on measuring the polarization in the horizontal-vertical basis, one will get the two results with equal probability, and hence no interference to speak of. However, in this case, getting a $|H_i\rangle$ or $|V_i\rangle$ does not give one any path information. This is simply because there is a correlation between $|D_1\rangle$, $|D_2\rangle$ and $|R_i\rangle$, $|L_i\rangle$ (by virtue of (7)), and getting a (say) $|D_1\rangle$ destroys the possibility of using (3) to infer path information [15]. So, the loss of interference is not because of obtaining any path information by looking at $|H_i\rangle$ and $|V_i\rangle$ states. The interference is lost anyway, unless one correlates with the $|R_i\rangle$, $|L_i\rangle$ states.

## 3 Movable Beam-Splitter

An objection can be raised that the preceding analysis holds only for certain fixed locations of BS1 or BS2, as only for those locations one of the detectors $D_1$, $D_2$ will show zero count (destructive interference). In the following we will show that that is not the case, and this argument can be made quite general. Let us suppose that the beam-splitter BS1 is movable, so that its position leads to a phase factor of $e^{2\pi i x/\lambda}$ for the upper path, where $\lambda$ is the wavelength of the light used in the experiment. For $x = 0$ the two path lengths are the same, and all the preceding arguments go through. For an arbitrary $x$, the final state of the two photons, just before the signal photon hits the detectors, instead of (6), is now given by

$$|\Psi_3'\rangle = \frac{1}{\sqrt{2}} U_{BS2}(|H_i\rangle|\psi_2\rangle e^{\frac{2\pi i x}{\lambda}} + |V_i\rangle|\psi_1\rangle)|V_s\rangle|\phi_0\rangle$$
$$= \frac{1}{2}[|H_i\rangle e^{\frac{2\pi i x}{\lambda}}(i|D_1\rangle + |D_2\rangle) + |V_i\rangle(|D_1\rangle + i|D_2\rangle)]|V_s\rangle|\phi_0\rangle. \qquad (8)$$

In terms of the states $|R_i\rangle$, $|L_i\rangle$, the above can be written as

$$|\Psi_3'\rangle = \frac{1}{2}\Big[i|D_1\rangle\big\{(e^{\frac{2\pi i x}{\lambda}} - 1)|R_i\rangle + (e^{\frac{2\pi i x}{\lambda}} + 1)|L_i\rangle\big\}$$
$$+ |D_2\rangle\big\{(e^{\frac{2\pi i x}{\lambda}} + 1)|R_i\rangle + (e^{\frac{2\pi i x}{\lambda}} - 1)|L_i\rangle\big\}\Big]|V_s\rangle|\phi_0\rangle. \qquad (9)$$

Now there is no correlation between the states $|R_i\rangle$, $|L_i\rangle$ and the detector states $|D_1\rangle$, $|D_2\rangle$. So a signal photon registered at the detectors cannot not tell us if the state of the idler will be $|R_i\rangle$ or $|L_i\rangle$.

However, one should realize that there is nothing sacred about the basis $|R_i\rangle$, $|L_i\rangle$ chosen for the idler photon. Given the polarization states $|H_i\rangle$, $|V_i\rangle$, there exist an infinite number of *mutually unbiased* basis states which can be used for the purpose. The basis defined by $|R_i\rangle$, $|L_i\rangle$ happens to be just one such basis which is unbiased with respect to $|H_i\rangle$, $|V_i\rangle$. One might as well choose the following basis states for the polarization of the idler photon

$$|P_i\rangle = \frac{1}{\sqrt{2}}(e^{i\theta}|H_i\rangle + i|V_i\rangle),$$
$$|Q_i\rangle = \frac{1}{\sqrt{2}}(e^{i\theta}|H_i\rangle - i|V_i\rangle), \qquad (10)$$

where $\theta$ is an arbitrary phase factor. The state of the two photons, just before the signal photon enters BS2

$$|\Psi_2'\rangle = \frac{1}{\sqrt{2}}(e^{\frac{2\pi i x}{\lambda}}|H_i\rangle|\psi_2\rangle + |V_i\rangle|\psi_1\rangle)|V_s\rangle|\phi_0\rangle, \qquad (11)$$

can be written in terms of this new basis as

$$|\Psi_2'\rangle = \tfrac{1}{2}\{|P_i\rangle(e^{i(\frac{2\pi x}{\lambda}-\theta)}|\psi_2\rangle - i|\psi_1\rangle)$$
$$+|Q_i\rangle(e^{i(\frac{2\pi x}{\lambda}-\theta)}|\psi_2\rangle + i|\psi_1\rangle)\}|V_s\rangle|\phi_0\rangle. \quad (12)$$

Now, if one chooses the basis such that $\theta = \frac{2\pi x}{\lambda}$, the state of the two photons, before the signal photon enters BS2, is given by

$$|\Psi_2'\rangle = \tfrac{1}{2}\{|P_i\rangle(|\psi_2\rangle - i|\psi_1\rangle) + |Q_i\rangle(|\psi_2\rangle + i|\psi_1\rangle)\}|V_s\rangle|\phi_0\rangle. \quad (13)$$

It shows that for every position of the beam-splitter BS1, there exists a basis (10) for the idler, the states of which get correlated to $\frac{1}{\sqrt{2}}(|\psi_2\rangle + i|\psi_1\rangle)$ and $\frac{1}{\sqrt{2}}(|\psi_2\rangle - i|\psi_1\rangle)$, exactly as $|R_i\rangle$, $|L_i\rangle$ did in (4), and hence are indicators of the signal photon following both paths. After the signal photon passes through BS2 and is about to hit the detectors $D_1$, $D_2$, the combined state of the two is given by

$$|\Psi_3'\rangle = \tfrac{1}{\sqrt{2}}\left(i|D_1\rangle|Q_i\rangle + |D_2\rangle|P_i\rangle\right)|V_s\rangle|\phi_0\rangle. \quad (14)$$

This means that every signal photon registered at the detectors, fixes the polarization state of the idler in this particular basis. So, for every position of the beam-splitter BS1, there exists a basis (10) for the idler, the states of which are perfectly correlated with two detectors of the signal photon. While it is true that (9) also indicates that every signal photon registered at (say) $D_1$, fixes the state of the idler, but there is no way to know, a priori, how that state would be read. In the present case, looking at the position of the beam-splitter BS1, one can choose the basis (10) in which to measure the polarization of the idler so that the results of the two are perfectly correlated. Choosing the basis may amount to choosing the angle by which the polarization of the idler has to be rotated, or something equally straightforward. Each detected signal photon tells one whether the state of the idler is $|P_i\rangle$ or $|Q_i\rangle$, and consequently also tells that the signal photon followed both paths, and not one of the two. Since this experiment can be performed, and the correlation measured, at least in principle, it tells us that in the delayed mode of the quantum eraser, which-way information is *always* erased.

## 4 The Two-Slit Which-Way Experiment

Various other delayed choice quantum eraser experiments have been performed using conventional double-slit interference. One might wonder if the arguments presented in the preceding discussion hold for the two-slit delayed choice quantum eraser experiments too. This is the question we address in the following analysis. Consider a two-slit interference experiment with a two-state which-way detector, as shown in Fig. 3. Without specifying the nature of the which-way detector, we assume that its effect is to entangle the two photon paths with the two states of the which-way detector, such that the combined state, when the photon reaches the screen, is given by

$$\Psi(x) = \tfrac{1}{\sqrt{2}}(\psi_1(x)|d_1\rangle + \psi_2(x)|d_2\rangle), \quad (15)$$

where $|d_1\rangle$, $|d_2\rangle$ are orthonormal states of the which-way detector. One can now define a mutually unbiased basis by the states $|d_\pm^\theta\rangle = \frac{1}{\sqrt{2}}(e^{i\theta}|d_1\rangle \pm |d_2\rangle)$. The state of the photon and which-way detector may be rewritten in the new basis

$$\Psi(x) = \tfrac{1}{2}\left([e^{-i\theta}\psi_1(x) + \psi_2(x)]|d_+^\theta\rangle + [e^{-i\theta}\psi_1(x) - \psi_2(x)]|d_-^\theta\rangle\right). \quad (16)$$

**Fig. 3** A schematic diagram of a two-slit interference experiment with which-way detection. The two paths of the photon get entangled with the two states of the which-way detector
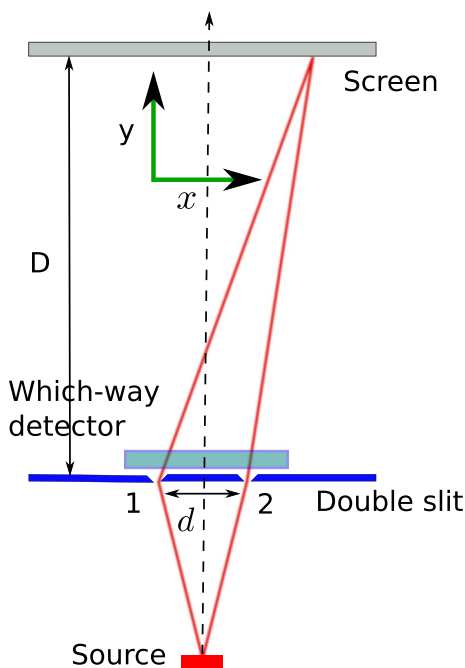


The two states of the photon, corresponding to the which-way detector states $|d_{\pm}^{\theta}\rangle$ are given by $\psi_{\pm}(x) = \frac{1}{\sqrt{2}}[e^{-i\theta}\psi_1(x) \pm \psi_2(x)]$, respectively. One can do a rigorous wave-packet analysis of the dynamics of the photon, and find the two states to have the following typical form [2]

$$\psi_{\pm}(x) = A(x)[1 \pm \cos\left(\tfrac{2\pi xd}{\lambda D} - \theta\right)], \tag{17}$$

where $d$ is the separation between the two slits, $D$ is the distance between the slits and the screen, and $A(x)$ is an envelope function. For $\theta = 0$, $\psi_+(x)$ represents an interference pattern with a central peak at $x = 0$. On the other hand, $\psi_-(x)$ represents a similar, but shifted interference pattern, with a *minimum* at $x = 0$ (see Fig. 4). One can see that if a photon is detected at $x = 0$, it can only belong to $\psi_+(x)$, because $|\psi_-(x = 0)|^2 = 0$. Then,

**Fig. 4** Typical recovered two-slit interference because of quantum erasure. Recovered interference $|\psi_+(x)|^2$ for $\theta = 0$ (solid blue line), $|\psi_-(x)|^2$ for $\theta = 0$ (dotted green line), and $|\psi_+(x)|^2$ for $\theta \neq 0$ (dot-dashed red line)
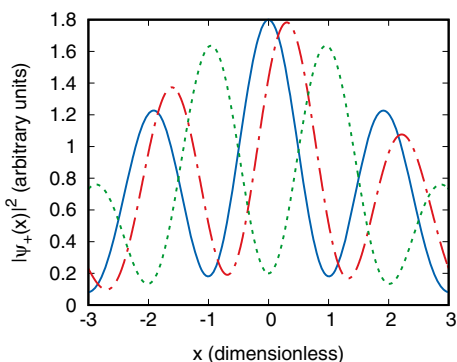
from (16) one infers that the state of the which-way detector is $|d_+^0\rangle$, and not $|d_-^0\rangle$. One can then conclude that the photon traveled both the paths, and not one of the two. The same argument can be made for all values of $x$ where $\psi_+(x)$ has a peak.

But what about the photons which land at positions where $\psi_+(x)$ does *not* have a peak? In that case one can choose a different basis for the which-way detector states such that $\theta = \frac{2\pi x d}{\lambda D}$. Remember that the interference patterns are obtained *only in coincidence* with the which-way detector states, and in coincidence with $|d_\pm^\theta\rangle$, the interference patterns will be shifted (see Fig. 4). They will be shifted in such a way that $|\psi_-(x)|^2 = 0$ for that particular $x$. One can then logically conclude that the state of the which-way detector is $|d_+^\theta\rangle$, and not $|d_-^\theta\rangle$, and the which-way information is erased. This again indicates that the photon followed both the paths. This correlation can be experimentally seen, as looking at the values of $x$ at which the photon is registered, one can choose a mutually unbiased basis of the which-way detector states $|d_\pm^\theta\rangle$, specified by $\theta = \frac{2\pi x d}{\lambda D}$. Thus, for every photon registered on the screen, one can choose a mutually unbiased basis of the which-way detector states, whose measurement result is predicted by the registered photon. So, even in a two-slit delayed-choice quantum eraser, every registered photon fixes the state of the which-way detector in a knowable basis, and thus always erases the which-way information.

Some comments on how one can choose a different mutually unbiased basis for the which-way detector in delayed-choice quantum eraser experiments. In the experiment of Kim et.al. [21], the idler photon, after traversing two paths, is recombined using a beam-splitter. Changing the relative lengths of the two paths would amount to choosing a different mutually unbiased basis for the idler. The recombining beam-splitter may be moved in synchrony with the movable detector for the signal photon [21]. In the experiment of Scarcelli et.al. [25], quantum erasing is achieved by letting the idler pass through a fixed narrow slit, and observing the signal photon in coincidence with it. Here, changing the basis of the path-detecting photon (idler) can be achieved by changing the position of the narrow slit through which the idler passes.

## 5 Conclusions

The correlation in (7) emerges in a straightforward fashion in a MZ like setup where there are only two discrete output states. The confusion prevailing in the literature regarding the delayed choice quantum eraser experiments, can be resolved by recognizing that the quantum eraser in a MZ like setup may be summarized by writing the state (7) in two complementary bases, $i|D_1\rangle|L_i\rangle + |D_2\rangle|R_i\rangle = |\psi_1'\rangle|V_i\rangle + |\psi_2'\rangle|H_i\rangle$, (ignoring normalization) where $|\psi_1'\rangle, |\psi_2'\rangle$ represent a complementary basis for the signal, and are proxies for the two paths. The idler is prepared in a basis depending on the basis in which the signal is measured, and vice-versa. Thus the states $|V_i\rangle, |H_i\rangle$ of the idler, provide path information of the signal photon, and $|L_i\rangle, |R_i\rangle$ represent erased path information. This makes it easy to see that the delayed mode, when the signal photon ends up in $|D_1\rangle$ or $|D_2\rangle$, can only result in erased path information. There is no choice left. As the Vienna experiment [17] is the first one to implement a delayed-choice quantum eraser in a MZ setup, it is also the first one to experimentally demonstrate that in the delayed mode, which-way information is always erased, and the photon always follows both the paths. In the traditional two-slit experiment, this effect is hidden because there is a continuous set of positions on the screen where the photon can register. However, we have shown that even in the two-slit delayed choice quantum eraser, for every photon detected on the screen, there exists a basis in which the state of the which-way detector gets fixed by the act of photon hitting the screen. This basis can be

known from the position of the photon, and the corresponding measurement can be made on the which-way detector to test the correlation. In the light of this analysis, and the results of the Vienna experiment, the long held notion that in the delayed mode, the experimenter has a choice between reading the which-way information or erasing it, should be given up. In the delayed mode, the which-way information is always erased. This takes the mystery out of the delayed-choice quantum eraser, and renders irrelevant any talk of retrocausality.

**Author Contributions**  The author is the sole contributor to this work.

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