Interference experiment, anomalous weak value, and Leggett-Garg test of macrorealism

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Macrorealism is a classical world view asserting that the properties of macro-objects exist independently and irrespective of observation. One practical approach to test this view in quantum theory is to observe the quantum coherence for a macro-object in an interference experiment. An elegant and conceptually appealing approach for testing the notion of macrorealism in quantum theory is through the violation of Leggett-Garg inequality. However, a conclusive Leggett-Garg test hinges on how the noninvasive measurability criteria are guaranteed in an experiment and remains a debated issue to date. In this work, we connect the practical and the conceptual approaches for testing the macrorealism through the weak value. We argue that whenever a quantum effect is observed in an interference experiment there is an existence of anomalous weak value. Further, we demonstrate that whenever such weak value exists, one obtains the violation of a Leggett-Garg inequality in any interference experiment. Since in a path-only interference experiment effectively no prior measurement is performed, the Leggett-Garg test of macrorealism presented here is without assuming the noninvasive measurability. We provide a rigorous discussion about the assumptions involved in the Leggett-Garg scenario and how our scheme fits into it. Further, we provide a simple argument about the macrorealistic understanding of our results by using the recently developed approach involving quasiprobability.

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I. INTRODUCTION

Macrorealism is a belief that the properties of a macroscopic object in our everyday world must always possess a definite ontic state, even though the state is not precisely known. Since the inception of quantum mechanics (QM), it remains a debatable question as to how such a view can be reconciled with the formalism of QM. Historically, this issue was first raised by Schrödinger [1] through his famous thought experiment. Since then, quite a number of attempts have been made to pose the appropriate questions relevant to this issue and to answer those questions. An approach within the formalism of QM is the decoherence program [2], which explains how interaction between quantum systems and the environment leads to classical behavior, but does not by itself provide the desired "cut" (*à la* Heisenberg [3]). Even when the decoherence effect is made negligible, the quantum effect may disappear by coarsening the measurements [4]. An unified description of microscopic and macroscopic systems was also provided [5] by suitably modifying the dynamics of standard QM.

There is yet another approach to examine the macro-objectification problem through the realization of quantum coherence of the Schrödinger-cat-like states for large objects in an interference experiment [6,7]. This is in fact a practical approach to test the validity of QM for large object. Arndt *et al.* [6] first experimentally produced the double-slit interference of a C_{60} molecule having mass 720 amu. The largest molecule to date that exhibits the interference pattern is TPPF152, having a mass 5310 amu and size 500 nm [7]. An important question in this context is how such experiments shed light on the subtle conceptual notion of macrorealism.

However, the aforementioned attempts do not directly address the fundamental question whether macrorealism is, in principle, compatible with the formalism of QM. In 1985, Leggett and Garg [8] first provided a refined definition of macrorealism. Specifically, the notion of macrorealism advocated by them consists of the following two main assumptions [8,9]:

(i) Macrorealism *per se* (MRps): If a macroscopic system has two or more states available to it, it remains in one of them at any instant of time.

(ii) Noninvasive measurability (NIM): It is possible, at least in principle, to determine the state of the system without affecting the system itself and its subsequent dynamics.

Now, consider that the measurement of a dichotomic observable $\hat{M}$ is performed at three different times t_1, t_2, and t_3 ($t_3 \geqslant t_2 \geqslant t_1$). In the Heisenberg picture, this in turn implies the sequential measurement of the observables $\hat{M}_1$, $\hat{M}_2$, and $\hat{M}_3$ corresponding to t_1, t_2, and t_3, respectively. Using the above two assumptions, one can derive the standard Leggett-Garg inequality (LGI)

$$K_3 = 1 - m_1 m_2 \langle \hat{M}_1 \hat{M}_2 \rangle - m_2 m_3 \langle \hat{M}_2 \hat{M}_3 \rangle + m_1 m_3 \langle \hat{M}_1 \hat{M}_3 \rangle \geqslant 0, \tag{1}$$

where $m_i m_j = \pm 1$ and $\langle M_i M_j \rangle = \sum_{m_i, m_j = \pm 1} m_i m_j p(m_i, m_j)$ with $i, j = 1, 2, 3$ and $i < j$. This inequality is violated for certain states and measurements even for a qubit system. Such a violation is supposed to establish that a system is not behaving macrorealistically.

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We note here that there are two separate programs for the similar task. The purpose of the practical approach is to test macroscopic quantum coherence and the goal of the conceptual approach is to demonstrate the incompatibility between macrorealism and QM through the violation LGIs. While the former wishes to establish the quantumness of macro objects, the latter demonstrates the prohibition of classicality of macro objects. Hence, an important question would be to ask here whether one implies the other in some way. We demonstrate here that the interference experiments for testing macroscopic quantum coherence itself constitute a test of LGIs.

There is a common perception that LGI is a *temporal analog* of Bell's inequalities. This inference is motivated from the structural resemblance between Bell-Clauser-Horne-Shimony-Holt inequalities and four-time LGIs. However, the Leggett-Garg (LG) test shows serious pathology [10–20] in comparison to a Bell test. The no-signaling in space condition (the statistical version of the locality postulate) is always satisfied in any physical theory. Thus, the violation of a given Bell local realist inequality can unequivocally warrant a failure of local realism and not the locality alone. In contrast, the statistical version of the noninvasive measurability assumption, the operational noninvasiveness (well known as no signaling in time) is in general not satisfied in QM. Thus, rather than attributing the violation of LGIs as an incompatibility of macrorealism with QM, a stubborn macrorealist can always salvage the macrorealism in QM by simply abandoning noninvasive measurability. Hence to draw meaningful conclusions from a LG test one requires the satisfaction of the no-signaling-in-time condition in QM by adopting a suitable measurement scheme.

In their original paper, Leggett and Garg proposed ideal negative-result measurements to ensure the noninvasive measurability in QM which is adopted in recent experiments [21,22]. However, this approach is criticized by pointing out that collapse indeed occurs in the negative-result measurement procedure and eventually disturbs the system and its subsequent dynamics [23]. Another approach to tackle the noninvasive measurability in experiments is by using the technique of weak measurement [24]. In weak measurement the strength of the measurement is possible to adjust and, in principle, the back-action (the invasiveness) on the system can be reduced to an arbitrarily small amount [25–27]. An interesting approach to tackle this issue was provided in Ref. [13] by using four suitable quasiprobabilities in a two-time LG scenario which can be positive or negative. The positive quasiprobabilities provide four two-time LGIs. Interestingly, when they are negative, they provide the violation of LGIs. Another approach through the violation of an augmented set of LGIs using a noninvasive continuous-in-time velocity measurement has also been reported [28,29].

It is an ineluctable nonclassical feature of quantum measurement that it entails an interaction between the measuring apparatus and the observed system resulting in the state of the measured system to be necessarily entangled with the state of the observing apparatus. If the measurement is perfect then the system is disturbed the most. However, the nonclassicality of a system (including a macrosystem) can also be revealed even when no explicit apparatus is involved. The most famous example is the double-slit experiment (in fact, any path-only interference experiment) where the interference pattern produced by matter waves is truly quantum in nature. In such a scenario, there is no prior measurement to disturb the subsequent measurement. We argue here that such a path-only interference experiment provides an interesting route to implement the LG test of macrorealism without assuming the NIM condition.

In this work, we provide a hitherto unexplored connection between the two approaches for testing macrorealism: the practical approach of an interference experiment of a large object and the conceptual approach of a LG test satisfying noninvasive measurability. For this we demonstrate the following two theses. First, if a path-only interference experiment produces a quantum effect (destructive interference) then there is an existence of an anomalous weak value. Second, such existence of an anomalous weak value warrants the incompatibility between macrorealism and QM through the violation of LGIs. Since the system is not disturbed by the act of the measurement in a path-only interference experiment, the NIM assumption in the conventional LG test in QM has no significant role in our scheme. Hence, our scheme provides a decisive test of the notion of macrorealism *per se* provided by LG. We have also provided a rigorous discussion about the assumptions involved in the LG scenario and how our LG test of macrorealism is satisfactory for a macrorealist. The latter is discussed through a recently developed quasiprobability approach [13,15].

II. ANOMALOUS WEAK VALUE AND THE VIOLATION OF LEGGETT-GARG INEQUALITY

The anomalous weak value appears in a novel conditional measurement protocol first introduced in Ref. [24] widely known as weak measurement aided with postselection (in short, weak measurement). Such a conditional average value of an observable is experimentally measurable and can be beyond eigenvalue ranges of the concerned observable (for a review, see Ref. [30]). For simplicity, consider a two-dimensional system prepared in a state $|i\rangle$ and the measured observable is denoted by $\hat{A}$. Let one collect the measurement statistics through another measurement basis corresponding to an observable $B = |f\rangle\langle f| - |f'\rangle\langle f'|$ where $|f\rangle\langle f| + |f'\rangle\langle f'| = \mathbb{I}$ where $|f\rangle$ and $f'\rangle$ are the eigenstates of that observable B. Then the expectation value can be written as

$$\langle\hat{A}\rangle = \langle i|\hat{A}|i\rangle = \langle i|\hat{A}|f\rangle\langle f|i\rangle + \langle i|\hat{A}|f'\rangle\langle f'|i\rangle. \tag{2}$$

Further rearrangement provides us

$$\begin{aligned}\langle\hat{A}\rangle &= |\langle f|i\rangle|^2 \frac{\langle i|\hat{A}|f\rangle}{\langle i|f\rangle} + |\langle f'|i\rangle|^2 \frac{\langle i|\hat{A}|f'\rangle}{\langle i|f'\rangle} \\ &= p(f)(A)_w^f + p(f')(A)_w^{f'},\end{aligned} \tag{3}$$

where $(A)_w^f = \langle i|\hat{A}|f\rangle/\langle i|f\rangle$ is the well-known definition of weak value [24] and $p(f)$ is the postselection probability, and similarly for the other term in Eq. (3). Thus, the expectation value can be calculated in a different measurement arrangement through the evaluation of weak values which are also experimentally measurable quantities. Note that whenever $\langle i|f\rangle$ is sufficiently small the weak value $(A)_w^f$ becomes anomalous; i.e., it can be beyond the ranges of eigenvalues. However, the higher the weak value $(A)_w^f$ the lower the postselection probability $p(f)$. Note also that both $(A)_w^f$ and $(A)_w^{f'}$ cannot be anomalous together. Another way to understand the connection between the expectation value and weak value of $\hat{A}$ is that one wants to evaluate the expectation value through the suitable conditional subensemble statistics.

The three-time LGIs in Eq. (1) can be cast into two-time ones if the system is prepared in a particular initial state $|\psi_i\rangle = |+m_1\rangle$ where $\hat{M}_1|+m_1\rangle = |+m_1\rangle$, so that $m_1 = +1$. In that case we have

$$(K_3)_Q = 1 - m_2\langle\hat{M}_2\rangle_{+m_1\rangle} - m_2m_3\langle\hat{M}_2\hat{M}_3\rangle_{|+m_1\rangle} + m_3\langle\hat{M}_3\rangle_{|+m_1\rangle}. \tag{4}$$

In a macrorealistic model $K_3 \leqslant 0$. For different values of $m_2 = \pm 1$ and $m_3 = \pm 1$ we have four two-time LGIs that can be written as

$$K_{31} = 1 - \langle\hat{M}_2\rangle - \langle\hat{M}_2\hat{M}_3\rangle + \langle\hat{M}_3\rangle \geqslant 0, \tag{5a}$$
$$K_{32} = 1 + \langle\hat{M}_2\rangle + \langle\hat{M}_2\hat{M}_3\rangle + \langle\hat{M}_3\rangle \geqslant 0, \tag{5b}$$
$$K_{32} = 1 - \langle\hat{M}_2\rangle + \langle\hat{M}_2\hat{M}_3\rangle - \langle\hat{M}_3\rangle \geqslant 0, \tag{5c}$$
$$K_{34} = 1 + \langle\hat{M}_2\rangle - \langle\hat{M}_2\hat{M}_3\rangle - \langle\hat{M}_3\rangle \geqslant 0. \tag{5d}$$

As discussed in Refs. [13,15], inequalities (5a)–(5d) provide necessary and sufficient conditions for macrorealism in the two-time LG scenario that is being considered here. Also, in the three-time LG scenario, inequalities (5a)–(5d) along with eight more two-time LGIs provide necessary and sufficient conditions for a weaker form of macrorealism [15].

Now, the quantum values of the LG expressions in the two-time scenario given by Eqs. (5a)–(5d) are derived for the state $|+m_1\rangle$. This can be linked with the two weak values. We show that there is one-to-one correspondence with the weak value and the violation of two-time LGIs. Such a connection was first pointed out in Ref. [31]. But, there is an important conceptual difference between Ref. [31] and our scheme. This is due to the fact that in our scheme the system itself serves as the apparatus and no weak coupling is needed to obtain the weak value. It naturally appears in a path-only interference experiment exhibiting destructive interference.

By writing $\hat{M}_3 = 2|+m_3\rangle\langle+m_3| - \mathbb{I} = \mathbb{I} - 2|-m_3\rangle\langle-m_3|$, one can cast the left-hand sides of Eqs. (5a)–(5d) as

$$(K_{31})_Q = 2\,p(+m_3)\big[1 - (\hat{M}_2)_w^{|+m_3\rangle}\big], \tag{6a}$$
$$(K_{32})_Q = 2\,p(+m_3)\big[1 + (\hat{M}_2)_w^{|+m_3\rangle}\big], \tag{6b}$$
$$(K_{33})_Q = 2\,p(-m_3)\big[1 - (\hat{M}_2)_w^{|-m_3\rangle}\big], \tag{6c}$$
$$(K_{34})_Q = 2\,p(-m_3)\big[1 + (\hat{M}_2)_w^{|-m_3\rangle}\big], \tag{6d}$$

where

$$(\hat{M}_2)_w^{|\pm m_3\rangle} = \frac{\langle+m_1|\hat{M}_2|\pm m_3\rangle}{\langle+m_1|\pm m_3\rangle} \tag{7}$$

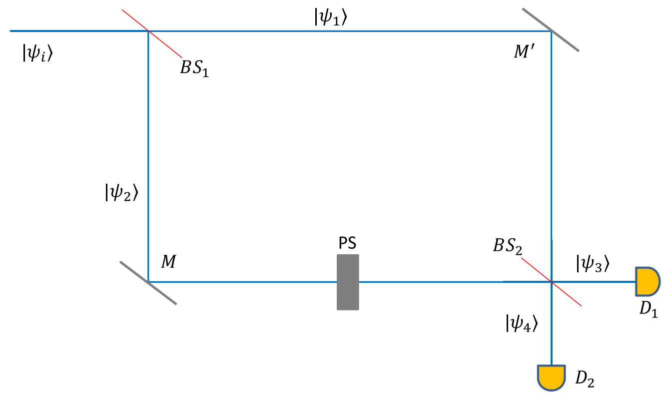

FIG. 1. The Mach-Zehender interferometer (see text for details).

is the weak value of $\hat{M}_2$ given the preselected and postselected states $|+m_1\rangle$ and $|\pm m_3\rangle$, respectively, and $p(\pm m_3) = |\langle+m_1|\pm m_3\rangle|^2$ is the postselection probability.

It can be seen that inequalities (5a)–(5d) can only be violated if the corresponding weak values are anomalous, i.e., beyond ± 1. The violations of inequalities (5a) and (5b) require $(\hat{M}_2)_w^{|+m_3\rangle} > 1$ and $(\hat{M}_2)_w^{|+m_3\rangle} < -1$, respectively. Similarly, inequalities (5c) and (5d) are violated if $(\hat{M}_2)_w^{|-m_3\rangle} > 1$ and $(\hat{M}_2)_w^{|-m_3\rangle} < -1$, respectively. However, both $(\hat{M}_2)_w^{|+m_3\rangle}$ and $(\hat{M}_2)_w^{|-m_3\rangle}$ cannot be anomalous simultaneously and hence only one of inequalities (5a)–(5d) can be violated. In the following we demonstrate that in any path interference experiment, exhibiting destructive interference implies that there exists an associated anomalous weak value. This, in turn, provides the violation of one of the two-time LGIs given by inequalities (5a)–(5d) in an interference experiment.

III. INTERFERENCE EXPERIMENT, ANOMALOUS WEAK VALUE, AND LGI

While our argument is valid for any path interferometric experiment, in this work, we consider the archetypical example of the Mach-Zehender (MZ) setup (Fig. 1). The MZ setup consists of two 50:50 beam splitters (BS_1 and BS_2), two mirrors (M and M′), a phase shifter (PS), and two detectors (D_1 and D_2). The system having state $|\psi_i\rangle$ is incident on BS_1 where $|\psi_i\rangle = \alpha|\psi_1\rangle + \beta|\psi_2\rangle$. For simplicity, we take α and β as real, satisfying $\alpha^2 + \beta^2 = 1$. After BS_1, the state becomes $|\psi_{BS_1}\rangle = \alpha|\psi_1\rangle + i\beta|\psi_2\rangle$ which is incident on the phase shifter (PS) and then BS_2. The state after BS_2 can be written as $|\psi_{BS_2}\rangle = [(\alpha+\beta)|\psi_3\rangle + i(\alpha-\beta)|\psi_4\rangle]/\sqrt{2}$, where $|\psi_3\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$ and $|\psi_4\rangle = (|\psi_1\rangle - |\psi_2\rangle)/\sqrt{2}$.

The probability of detecting the particles in the path $|\psi_3\rangle$ (detected at D_1) and in the path $|\psi_4\rangle$ (detected at D_2) are respectively given by

$$p(\psi_3) = \frac{(\alpha+\beta)^2}{2}, \quad p(\psi_4) = \frac{(\alpha-\beta)^2}{2}. \tag{8}$$

We shortly show that when there is a destructive interference (small detection probability in one of the detectors) there exists an anomalous weak value. For example, a small detection probability in the path $|\psi_4\rangle$ (or $|\psi_3\rangle$) indicates the existence of a very large weak value.

There is a close resemblance between the weak measurement procedure and standard path interference experiment. Both of them require a three-step procedure as follows. A preselection procedure is used to prepare the state of the system $|\psi_i\rangle$, interaction of the system with the interferometric setup (state passes through BS_1), and the postselection of particles in suitable states ($|\psi_4\rangle$ and $|\psi_3\rangle$ by BS_2). Here, the first beam splitter (BS_1) corresponds to the measurement of the dichotomic path observable $M_2 = |\psi_1\rangle\langle\psi_1| - |\psi_2\rangle\langle\psi_2|$ and second beam splitter (BS_2) along with the phase shifter (PS) implements the measurement of $M_3 = |\psi_4\rangle\langle\psi_4| - |\psi_3\rangle\langle\psi_3|$.

Now, the weak values of path observable M_2 corresponding to the postselected states $|\psi_3\rangle$ and $|\psi_4\rangle$ are respectively given by

$$(M_2)_w^{|\psi_3\rangle} = \frac{(\alpha - \beta)}{(\alpha + \beta)}, \quad (M_2)_w^{|\psi_4\rangle} = \frac{(\alpha + \beta)}{(\alpha - \beta)}, \tag{9}$$

with respective postselected probabilities given by Eq. (8).

It is straightforward to check that whenever $\alpha \neq \beta$ either $(M_2)_w^{|\psi_3\rangle}$ or $(M_2)_w^{|\psi_4\rangle}$ becomes anomalous. Thus the appearance of the destructive interference implies the existence of an anomalous weak value. However, $(M_2)_w^{\psi_3}$ and $(M_2)_w^{\psi_4}$ both cannot be anomalous together. In the extreme condition, when $\beta = 0(1)$ and $\alpha = 1(0)$, there is no destructive interference and both the weak values $(M_2)_w^{|\psi_3\rangle} = (M_2)_w^{|\psi_4\rangle} = 1$ with same postselection probability $p(\psi_3) = p(\psi_4) = 1/2$. There is an important difference between the path weak value considered here and the standard weak value [24]. In our scheme, no additional apparatus is involved and the system itself acts as an apparatus. If the preselected state of the system is taken to be a pure state, it remains in a pure state after the interaction with BS_1. So, no explicit weak coupling for the measurement at BS_1 is needed to be ensured.

We note here that there is an intense debate [32,33] whether weak values are inherently quantum or rather a purely statistical feature of pre- and postselection with disturbance. The view that classical probabilities with suitable noise mimic weak value has been criticized by Qin *et al.* [33] through the Stern-Gerlach setup for spin measurements. It is demonstrated by Pusey [34] that anomalous weak values are proof of contextuality, which is experimentally verified in Ref. [35]. Dressel [36] argued that weak values arise due to a quantum interference effect.

Here, we argue that the converse also holds; i.e., for any quantum interference experiment there is an associated anomalous weak value of the path observable. However, there is an important difference. In Refs. [33,36], the interference of the postselected apparatus (not the system) states plays a key role in contrast to our scheme where no separate apparatus is involved.

The arguments provided up to now thus enable us to demonstrate that if the path interference experiment exhibits destructive interference then there is a violation of LGIs. In other words, the interference experiment itself constitutes a proof of incompatibility between QM and realism through the violation of LGIs. In order to explicitly show this, by identifying $|-m_3\rangle = |\psi_3\rangle$, $|+m_3\rangle = |\psi_4\rangle$, and $|+m_1\rangle = |\psi_i\rangle$, we can write Eqs. (6a)–(6d) as

$$(K_{31})_Q = 2\beta(\beta - \alpha), \quad (K_{32})_Q = 2\alpha(\alpha - \beta), \tag{10a}$$

$$(K_{33})_Q = 2\beta(\alpha + \beta), \quad (K_{34})_Q = 2\alpha(\alpha + \beta). \tag{10b}$$

For the values $\alpha > \beta$, from Eq. (6a), we have $(K_{31})_Q < 0$ implying the violation of LGI in Eq. (5a) and consequently $(M_2)_w^{|\psi_4\rangle} > 1$ (i.e., anomalous weak value). If $\beta > \alpha$, we have the violation of LGI in Eq. (5b) and $(M_2)_w^{|\psi_4\rangle} < -1$. In both the above cases, we have destructive interference at the detector D_2. Similar arguments can be made for Eq. (10b) which requires anomalous values of $(M_2)_w^{|\psi_3\rangle}$. Hence, the existence of an anomalous weak value in a path-only interference experiment warrants the violation of two-time LGIs. One may then argue that the interference experiments for large molecules [6,7] have already tested the quantum violation of macrorealism proposed by Leggett and Garg.

IV. NONINVASIVE MEASURABILITY, TWO-TIME LGIS, AND MACROREALISM

Since the introduction of LGIs it remains a debatable issue regarding what specific notion of macrorealism is tested in the LG scenario. The roots of the assumptions made in LGIs are explained by Leggett in many of his writings (for example, see Ref. [9]). This is significantly motivated from Bohr's "solution" of the quantum measurement problem (denying definite properties to microscopic objects in the absence of observation but asserting macroscopic objects to have such properties at all times irrespective of observation) and Schrödinger's criticism to such insistence (there should not be any logical reason to insist that an electron and an everyday object behave differently in quantum theory unless a well-defined criterion is set by theory itself). He argued that it is not always necessary to have a quantum description of everyday objects but, in principle, it should be legitimate to ask for a quantum mechanical account for them.

In the LG scenario, there are two subtle issues regarding the assumptions used to derive LGIs. For an excellent review with enticing discussion, we refer to Ref. [23]. First, what exactly the "state" implies in the MRps assumption in a macrorealistic theory is unclear. A macrorealist would have desired ontic states. The macrorealistic model advocated by Leggett and Garg strongly suggests that its background framework is in fact quantum theory along with the superselection rule for denying the linear superposition in our everyday world. If macrorealism is understood as the realism of the macroscopic system, then a one-to-one correspondence between macroscopically distinct states and definite values of macro objects is needed to be ensured, irrespective of observation. It is the fundamental feature of quantum theory that the states do not directly correspond to the properties unless it is a classical mixture of eigenstates of the observables.

Note that the ontological model of quantum theory put forward by Bohm [37], being statistically equivalent to quantum theory, ensures the deterministic properties of the system of an undivided world by keeping the linear superposition. However, there is a subtle difference between deterministic and definite value within the Bohmian model. One may argue that determinism is a weaker notion than definite value as the latter implies the former but the converse is not true. As argued by Vaidman [38], the Bohmian positions (the ontic states) of a given ensemble are fixed by the modulus of the wave function and the outcomes of measurements of the observables are predetermined, but may not be value definite. This is due to the fact that the Bohmian model is inherently contextual as different experimental setups for measurements of the same observable may lead to different observed values. Such a contextuality is not similar to the Kochen-Specker [39] or Spekkens [40] forms. Then the Bohmian model is deterministic but contextual value definite. It may then be assumed that the "state" in the MRps assumption involved in the LG scenario refers to the ontic state providing noncontextual definite value.

A few more comments would be helpful before concluding this part of discussion. An elegant and generalized framework of the ontological model was proposed by Spekkens [40,41] that corresponds to any operational theory and without reference to quantum theory. For example, the notion of noncontextuality introduced in Ref. [40] is not based on quantum theory in contrast to the Kochen-Specker version. But, the original LG formulation is so tied up with quantum theory that it is difficult to provide a model-independent formulation by separating it out from the conceptual influence of the quantum framework.

Now we come to the second issue, i.e., the noninvasiveness measurability (NIM) assumption in a macrorealistic model. This issue remains debatable and is most discussed in the literature. Leggett's original view regarding NIM is based on the ideal negative-result measurement in QM and he argued that it is a natural corollary of MRps. Note that the statistical version of the NIM condition—the operational noninvasiveness (also well known as the no-signaling-in-time condition)—implies that prior measurements do not influence the statistics of future measurements. In two-time and three-time LG scenarios they imply LGIs [11,12,15,23]. Mathematically,

$$p(m_j) = \sum_{m_i} p(m_i, m_j), \tag{11}$$

meaning that the prior measurement does not disturb the subsequent measurement when $i < j$. In Refs. [11,12], Clemente and Kofler argued that a suitable set of such operational noninvasiveness conditions provides the necessary and sufficient condition (NSC) for macrorealism for the three-time LG scenario, while three-time LGIs do not. Interestingly, in the two-time LG scenario, the four inequalities (5a)–(5d) provide the NSC for macrorealism.

In general, the operational noninvasiveness condition is not satisfied in QM. However, it was argued in Ref. [23] that the NIM is an independent condition from MRps in contrast to Leggett's view. Thus, what the violation of LG signifies is unclear. A macrorealist may claim that it is simply due to the violation of operational noninvasiveness and nothing can be said about the violation of macrorealism *per se*. Note that the NIM condition can also be assumed as ontic noninvasiveness as in the Spekkens [40] formulation. This is a stronger reading of the NIM condition than operational noninvasiveness. The former implies the latter but the converse does not hold [23].

Even in such a case, to provide the NIM condition the similar status of the locality in the Bell scenario, the operational noninvasiveness in QM, still needs to be satisfied. As mentioned earlier, there have been a few proposals to achieve this goal. One particular approach is by using quasiprobability proposed by Halliwell [13]. In our scheme, there is effectively no prior measurement to disturb the future measurements. Hence, the operational noninvasiveness which is demanded to be satisfied by a macrorealist in the conventional LG test in QM does not play any role in our proposed LG test of macrorealism. Using the quasiprobability approach we explain below how the violation of LGIs in our scheme can satisfy a macrorealist.

Note that the LG test requires the joint sequential probabilities of two noncommuting observables and there is no unique prescription is available in QM. In Ref. [13], it is proposed that instead of $p(m_i, m_j)$ one can use suitably defined quasiprobabilities $q(m_i, m_j)$. Importantly, they correctly reproduce the sequential correlation and, crucially, satisfy the operational noninvasiveness in QM. Such quasiprobabilities are defined as

$$q_Q(m_i, m_j) = \tfrac{1}{2}\mathrm{Tr}\big[\{P_{m_j}P_{m_i} + P_{m_i}P_{m_j}\}\rho\big], \tag{12}$$

satisfying $\sum_{m_i,m_j} q_Q(m_i, m_j) = 1$ where $P(m_{i(j)}) = (\mathbb{I} + m_{i(j)}M_{i(j)})/2$. Due to the symmetry in order, they satisfy operational noninvasiveness in QM,

$$p(m_j) = \sum_{m_i} q(m_i, m_j) = \mathrm{Tr}[P(m_j)\rho], \tag{13a}$$

$$p(m_i) = \sum_{m_j} q(m_i, m_j) = \mathrm{Tr}[P(m_i)\rho]. \tag{13b}$$

This means that $p(m_j)$ remains independent of the fact if a prior measurement of M_i is performed. Importantly, the correlation remains the same as

$$\langle M_i M_j\rangle = \sum_{m_i,m_j} m_i m_j\, q(m_i, m_j) = \sum_{m_i,m_j} m_i m_j\, p(m_i, m_j). \tag{14}$$

The macrorealistic reading of the quasiprobabilities is given by

$$q(m_i, m_j) = \tfrac{1}{4}(1 + m_i\langle M_i\rangle + m_j\langle M_j\rangle + m_i m_j\langle M_i M_j\rangle). \tag{15}$$

The quasiprobabilities $q(m_i, m_j)$ can be negative or positive. Importantly, $q(m_i, m_j) \geqslant 0$ implies the four two-time LGIs (with a multiplicative factor 4) and also provides the NSC for macrorealism in the two-time LG scenario. As argued in Refs. [13,15], in the three-time LG scenario when $i, j = 1, 2, 3$ with $j > i$, twelve such quasiprobabilities provide the NSC for a weaker form of macrorealism. Thus, in our case $q(m_i, m_j) < 0$ implies the violation of two-time LGIs. But, $q(m_i, m_j) \geqslant 0$ does not necessarily imply the operational nondisturbance remains satisfied for $p(m_i, m_j)$ as explained in Ref. [15]. One still requires to use a standard noninvasive measurement technique to ensure the operational noninvasiveness of $p(m_i, m_j)$ in QM. However, in our scheme the operational noninvasiveness does not play any role as there is no effective measurement performed before the second beam splitter and hence the assumption of NIM is avoided.

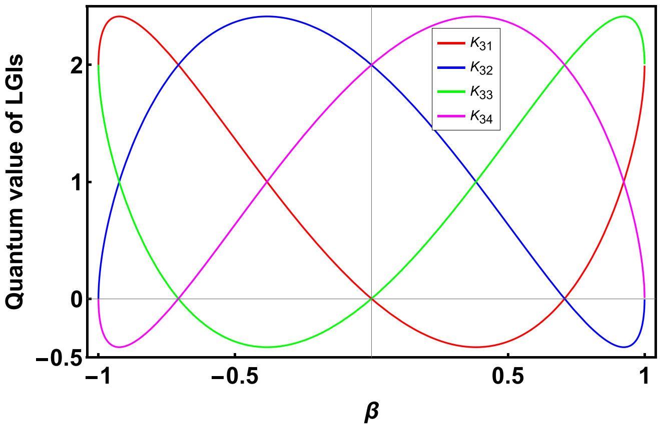

FIG. 2. Quantum values of four LG expressions given in Eqs. (10a) and (10b) are plotted against β. Curves demonstrate that one of the four LGIs will be violated for any given value of β, except for $\beta = 0$ or ± 1.

It will be interesting to analyze the macrorealistic reading of our results. For this, let us first analyze the argument of the quasiprobability approach from a quantum mechanical point of view.

In our interferometric experiment, if we identify $|+m_2\rangle \equiv |\psi_1\rangle$, $|-m_2\rangle \equiv |\psi_2\rangle$ $|+m_3\rangle \equiv |\psi_3\rangle$, and $|-m_3\rangle \equiv |\psi_4\rangle$; then by using Eqs. (13a) and (13b) the probabilities $p(\psi_3)$ and $p(\psi_4)$ can be written as

$$p(\psi_3) = q(\psi_1, \psi_3) + q(\psi_2, \psi_3), \tag{16a}$$

$$p(\psi_4) = q(\psi_1, \psi_4) + q(\psi_2, \psi_4), \tag{16b}$$

where $q(\psi_1, \psi_3)$ is one of the four quasiprobabilities. It is simple to check that in QM, the values of $p(\psi_3)$ and $p(\psi_4)$ match with Eq. (8). As mentioned, the four conditions $q(\psi_1, \psi_3) \geqslant 0$ (with a multiplicative factor 4) are just the four two-time LGIs in Eqs. (5a)–(5d). It is already shown (see also Fig. 2) that one of the LGIs given in Eq. (5a)–(5d) will be violated whenever destructive interference occurs; i.e., at least one of the quasiprobabilities is negative. The above quantum mechanical argument thus rules out the possibility that while $p(\psi_3)$ [or $p(\psi_4)$] is very small or zero (destructive interference), the quasiprobabilities are also zero.

A simple understanding of the above quantum mechanical argument of destructive interference (enabling the violation of LGIs) from the point of view of a macrorealist could be the following. For a macrorealist, the particle takes the upper path with probability $|\alpha|^2$ and the lower one with probability $|\beta|^2$. At the second beam splitter, BS_2, the probabilities are equally divided between the two possible subsequent paths in both the cases. This means that $p(\psi_3) = p(\psi_4) = 1/2$ along with all four paths having nonzero probability. But, in QM such a case does not arise unless $\alpha = 0$ (or $\beta = 0$). As already discussed, in this particular choice, there is no destructive interference, no anomalous weak value, and no violation of LGIs. Thus, a macrorealistic argument is not compatible with the QM for any other value of $\alpha(\beta)$ and our scheme provides an elegant test of macrorealism in QM.

V. SUMMARY AND CONCLUSIONS

In sum, we have provided a hitherto unexplored link between the interference experiment and the incompatibility of macrorealism in QM through the violation of the LGI. The interference experiment for large objects [6,7] is a practical approach to test the macroscopic quantum coherence. On the other hand, the LG formulation is a conceptual approach for testing the incompatibility of the notion of macrorealism in QM in our everyday world. In this work, we demonstrated that whenever there is destructive interference there exists an anomalous weak value that enables one to demonstrate the quantum violation of LGIs. Further, we have provided a detailed discussion regarding the assumptions involved in the LG framework and how our scheme fits into that framework.

The LGIs are derived based on two key assumptions: macrorealism *per se* and noninvasive measurability. Since LGI involves the measurements of noncommuting observables, then a prior measurement in general disturbs the future measurements and hence the statistical version of noninvasive measurability (the no-signaling-in-time condition) is not in general satisfied in QM. In such a case, the quantum violation of LGIs may not convince a macrorealist who may claim to salvage the macrorealism *per se* by simply abandoning the noninvasive measurability. Hence, unless this loophole is closed in experiment the LG test of macrorealism is *not* as conclusive as the test of local realism through Bell's inequalities.

There have been a few interesting proposals in the literature regarding how to close this loophole. In their original work, Leggett and Garg [9] advocated the negative-result measurement of QM to validate the noninvasive measurability which was later tested in experiment [26]. Proposals using weak measurement [27], quasiprobabilities [13], and continuous velocity measurements [28,29] have also been reported. But, a string of criticisms [23] has also been made regarding the viability of some of the approaches. It is argued that in negative-result measurement the collapse from distance occurs and subsequent dynamics of the state is disturbed. In the LGI test through the weak measurement of spin or polarization observables, it is assumed that the system and apparatus is minimally entangled but in principle invasiveness still occurs due to this entanglement.

In contrast, our scheme does not involve measurement apparatus (the root of causing disturbance by entangling the system) and hence effectively there is no prior measurement to disturb the future measurement. This means that the assumption of noninvasive measurability has no active role in our scheme. In order to explicitly explain it, we use an interesting approach involving quasiprobabilities. Such probabilities mimic the sequential LG correlation and crucially satisfy the no-signaling-in-time condition in our two-time LG scenario. Moreover, the quasiprobabilities, while positive, are proportional to four two-time LGIs providing the necessary and sufficient condition for macrorealism in the two-time LG scenario. It is shown that, unless the quantum superposition exists, one of the quasiprobabilities is negative, implying the violation of LGIs and consequently the weak value becomes anomalous. Our proposed scheme thus provides a conclusive test of the LG formulation of macrorealism. We have also provided a simple argument for macrorealistic understanding of our argument. However, there is a scope for analyzing the present results through a more sophisticated macrorealistic model. This could be an interesting study.

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[1] E. Schroedinger, Naturwissenschaften **23**, 807 (1935).
[2] H. D. Zeh, Found. Phys. **1**, 69 (1970); W. H. Zurek, Phys. Rev. D **26**, 1862 (1982).
[3] W. Heisenberg, Z. Phys. **33**, 879 (1925).
[4] J. Kofler and C. Brukner, Phys. Rev. Lett. **99**, 180403 (2007).
[5] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986); A. Bassi and G. Ghirardi, Phys. Rep. **379**, 257 (2003).
[6] M. Arndt *et al.*, Nature (London) **401**, 680 (1999).
[7] S. Gerlich *et al.*, Nat. Commun. **2**, 263 (2011).
[8] A. J. Leggett and A. Garg, Phys. Rev. Lett. **54**, 857 (1985).
[9] A. J. Leggett, J. Phys.: Condens. Matter **14**, R415 (2002).
[10] M. Wild and A. Mizel, Found. Phys. **42**, 256 (2012).
[11] L. Clemente and J. Kofler, Phys. Rev. A **91**, 062103 (2015).
[12] L. Clemente and J. Kofler, Phys. Rev. Lett. **116**, 150401 (2016).
[13] J. J. Halliwell, Phys. Rev. A **93**, 022123 (2016).
[14] S. Kumari and A. K. Pan, Europhys. Lett. **118**, 50002 (2017).
[15] J. J. Halliwell, Phys. Rev. A **96**, 012121 (2017).
[16] S. Kumari and A. K. Pan, Phys. Rev. A **96**, 042107 (2017).
[17] A. Kumari, Md. Qutubuddin, and A. K. Pan, Phys. Rev. A **98**, 042135 (2018).
[18] A. K. Pan, Md. Qutubuddin, and S. Kumari, Phys. Rev. A **98**, 062115 (2018).
[19] J. J. Halliwell, Phys. Rev. A **99**, 022119 (2019).
[20] J. J. Halliwell and C. Mawby, Phys. Rev. A **100**, 042103 (2019)
[21] G. C. Knee *et al.*, Nat. Commun. **3**, 606 (2012)
[22] C. Robens, W. Alt, D. Meschede, C. Emary, and A. Alberti, Phys. Rev. X **5**, 011003 (2015).
[23] O. Maroney and C. Timpson, arXiv:1412.6139.
[24] Y. Aharonov, D. Z. Albert, and L. Vaidmann, Phys Rev. Lett. **60**, 1351 (1988).
[25] A. Palacios-Laloy *et al.*, Nat. Phys. **6**, 442 (2010).
[26] M. E. Goggin *et al.*, Proc. Natl. Acad. Sci. USA **108**, 1256 (2011).
[27] A. Avella *et al.*, Phys. Rev. A **96**, 052123 (2017).
[28] J. J. Halliwell, Phys. Rev. A **94**, 052114 (2016).
[29] S. S. Majidy, H. Katiyar, G. Anikeeva, J. Halliwell, and R. Laflamme, Phys. Rev. A **100**, 042325 (2019).
[30] J. Dressel, M. Malik, F. M. Miatto, A. N. Jordan, and R. W. Boyd, Rev. Mod. Phys. **86**, 307 (2014).
[31] N. S. Williams and A. N. Jordan, Phys. Rev. Lett. **100**, 026804 (2008).
[32] C. Ferrie and J. Combes, Phys. Rev. Lett. **113**, 120404 (2014); L. Vaidman, arXiv:1409.5386; Y. Aharonov and D. Rohrlich, arXiv:1410.0381; A. Brodutch, Phys. Rev. Lett. **114**, 118901 (2015); C. Ferrie and J. Combes, *ibid.* **114**, 118902 (2015); A. Romito, A. N. Jordan, Y. Aharonov, and Y. Gefen, Quantum Stud.: Math. Found. **3**, 1 (2016); H. F. Hofmann, M. Iinuma, Y. Shikano, arXiv:1410.7126; L. Vaidman, Philos. Trans. R. Soc. A **375**, 20160395 (2017); A. C. Ipsen, Phys. Rev. A **91**, 062120 (2015); D. Sokolovski, Phys. Lett. A **379**, 1097 (2015).
[33] L. Qin, W. Feng, and X.-Q. Li, Sci. Rep. **6**, 20286 (2016).
[34] M. Pusey, Phys. Rev. Lett. **113**, 200401 (2014).
[35] F. Piacentini *et al.*, Phys. Rev. Lett. **116**, 180401 (2016).
[36] J. Dressel, Phys. Rev. A **91**, 032116 (2015).
[37] D. Bohm, Phys. Rev. **85**, 166 (1952); **85**, 180 (1952).
[38] L. Vaidman, Quantum Stud.: Math. Found. **1**, 5 (2014).
[39] S. Kochen and E. P. Specker, J. Math. Mech. **17**, 59 (1967).
[40] R. W. Spekkens, Phys. Rev. A **71**, 052108 (2005).
[41] N. Harrigan and R. Spekkens, Found. Phys. **40**, 125 (2010).