

Weak values extraction from neutron interferograms

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- **Part 2:** Introduction to weak values

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- **Part 2:** Introduction to weak values
- **Part 3:** Weak values based description of a Mach-Zehnder interferometer

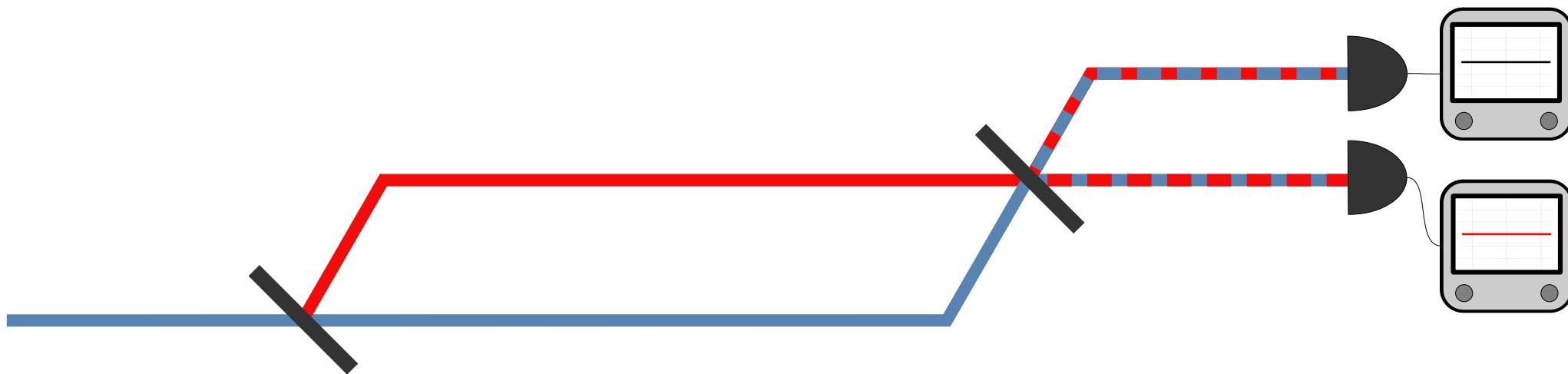
Content

- **Part 1:** Motivation and goal
- **Part 2:** Introduction to weak values
- **Part 3:** Weak values based description of a Mach-Zehnder interferometer
- **Part 4:** Experimental measurement of path weak values directly from interferograms

Part 1: Motivation and goal

Motivation

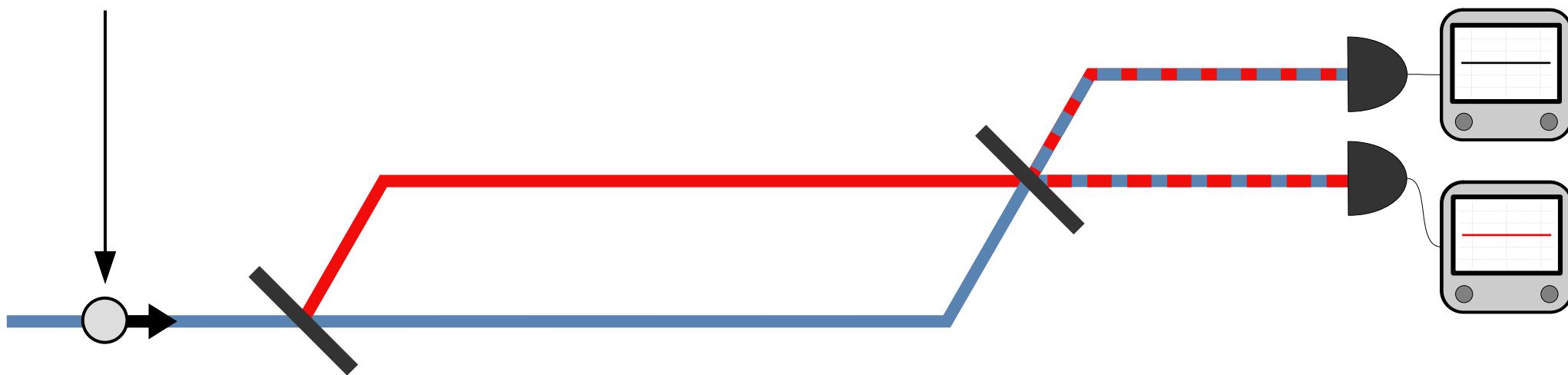
Mach-Zehnder interferometry:



Motivation

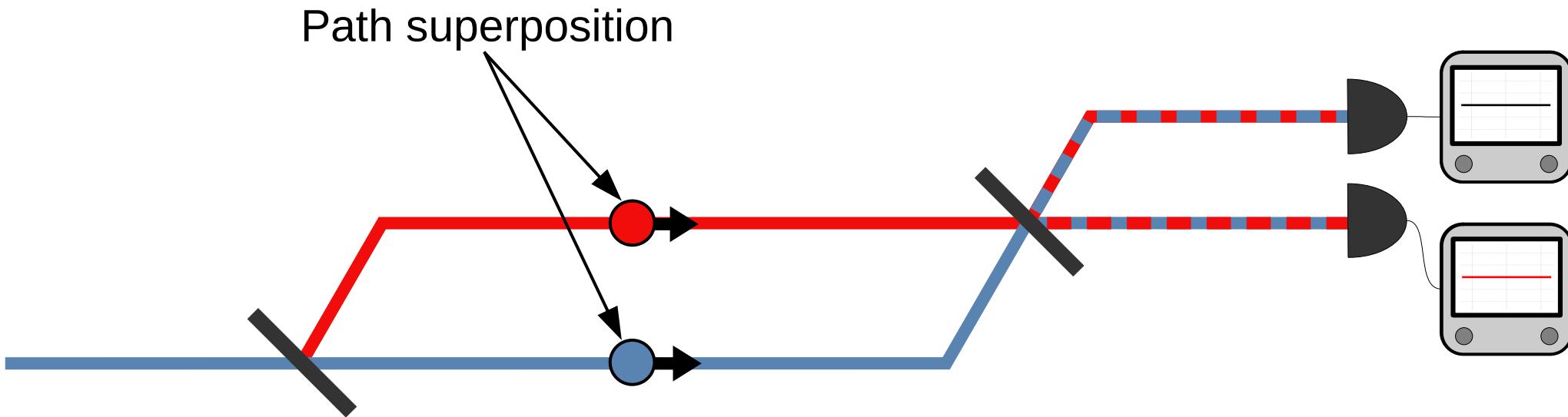
Mach-Zehnder interferometry:

Particle



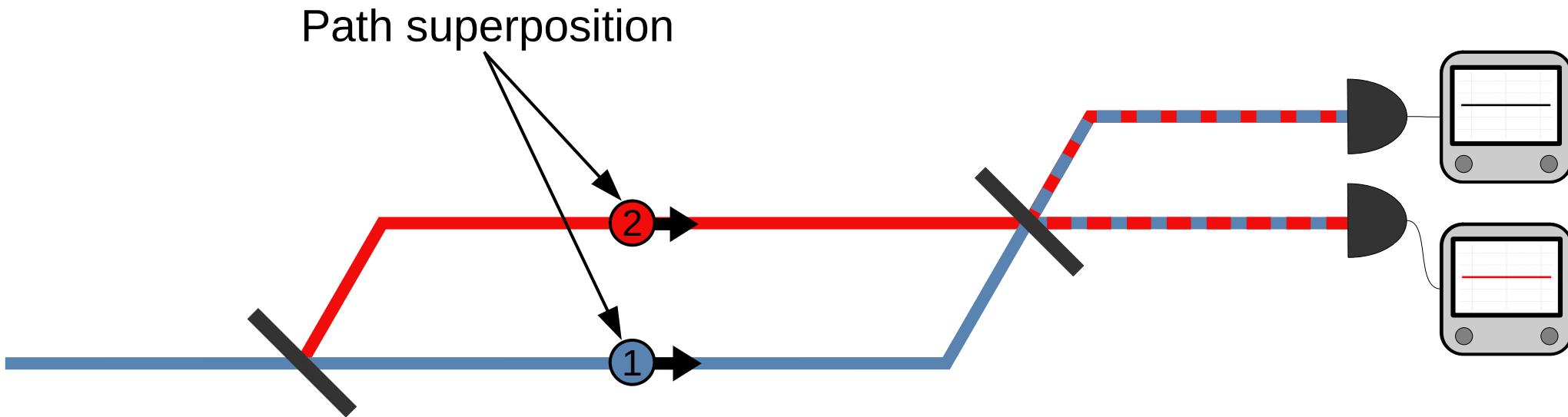
Motivation

Mach-Zehnder interferometry:



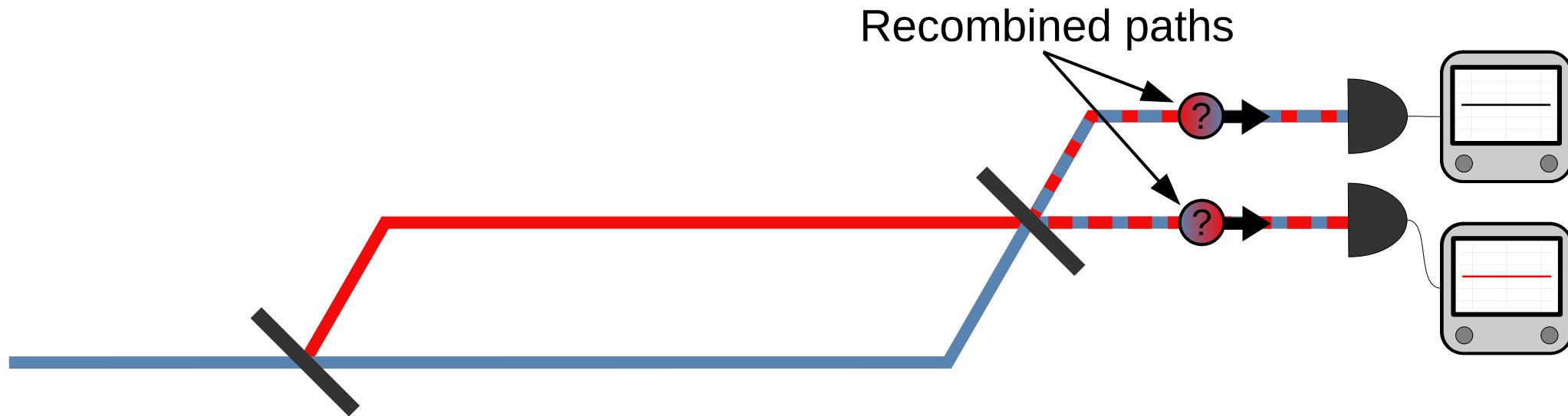
Motivation

Mach-Zehnder interferometry:



Motivation

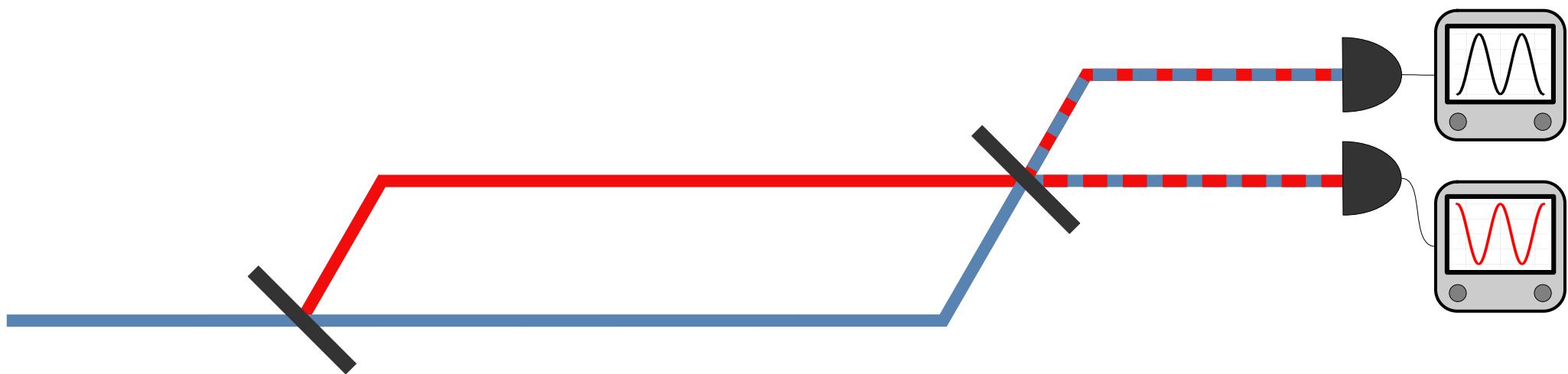
Mach-Zehnder interferometry:



Motivation

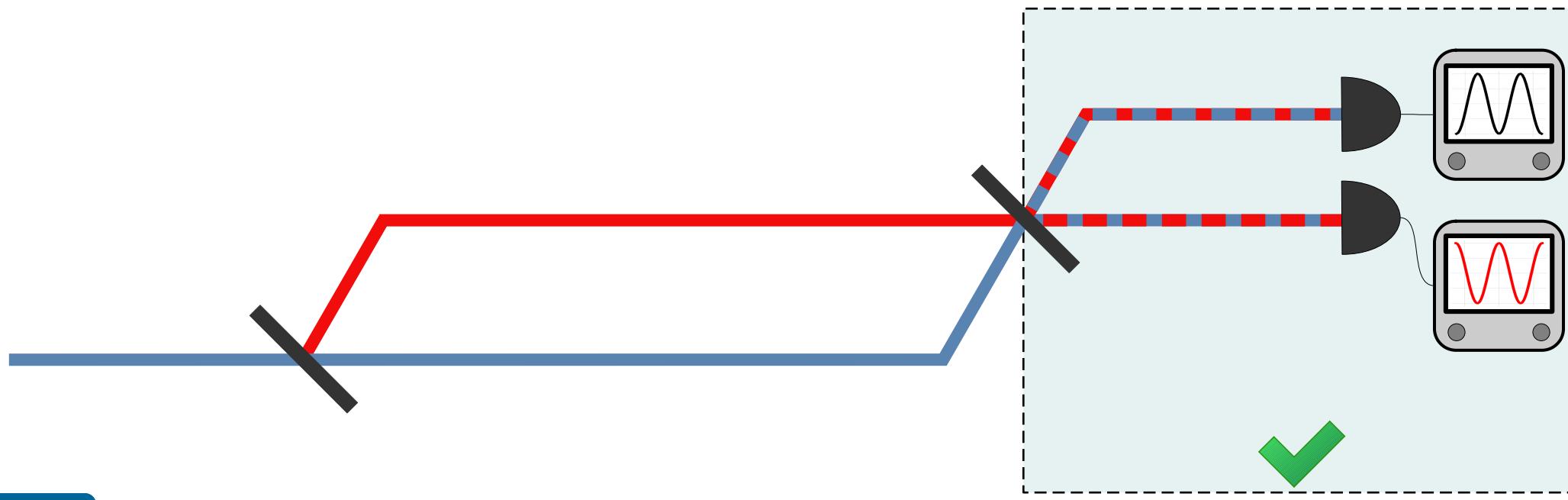
Mach-Zehnder interferometry:

Interference!



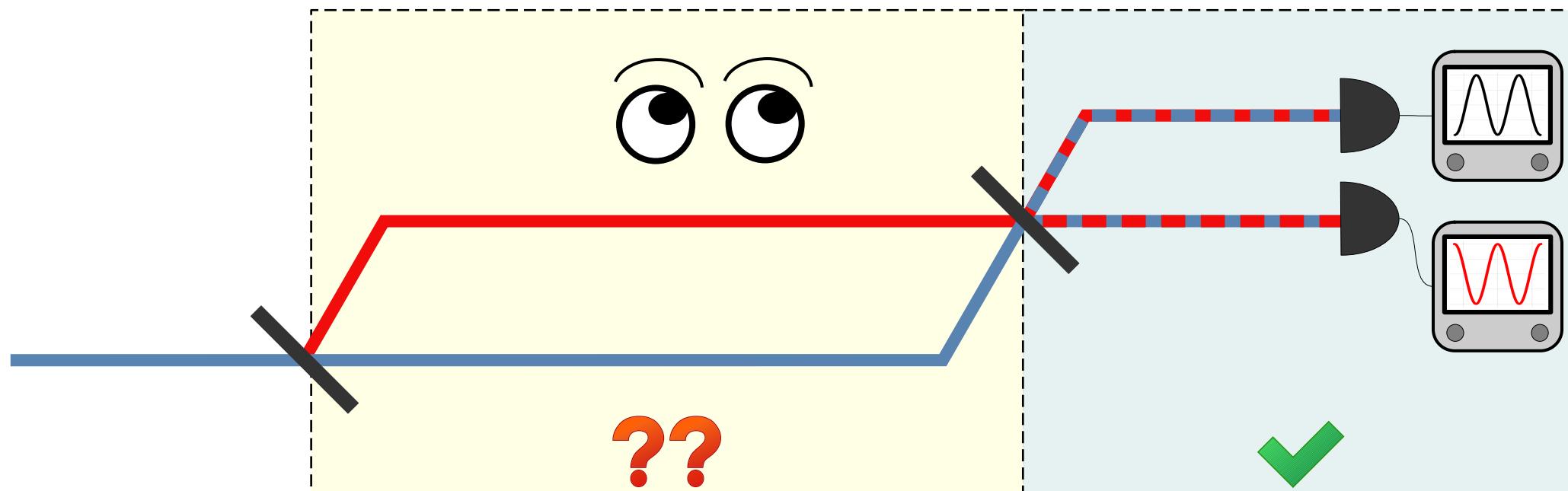
Motivation

Mach-Zehnder interferometry:



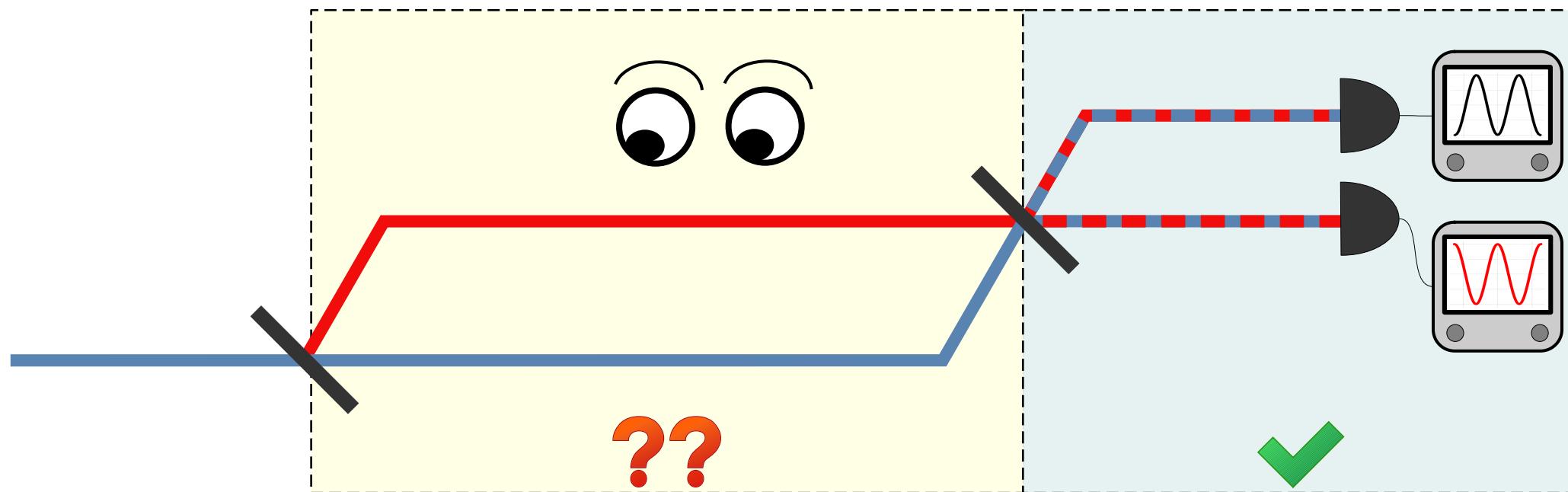
Motivation

Mach-Zehnder interferometry:



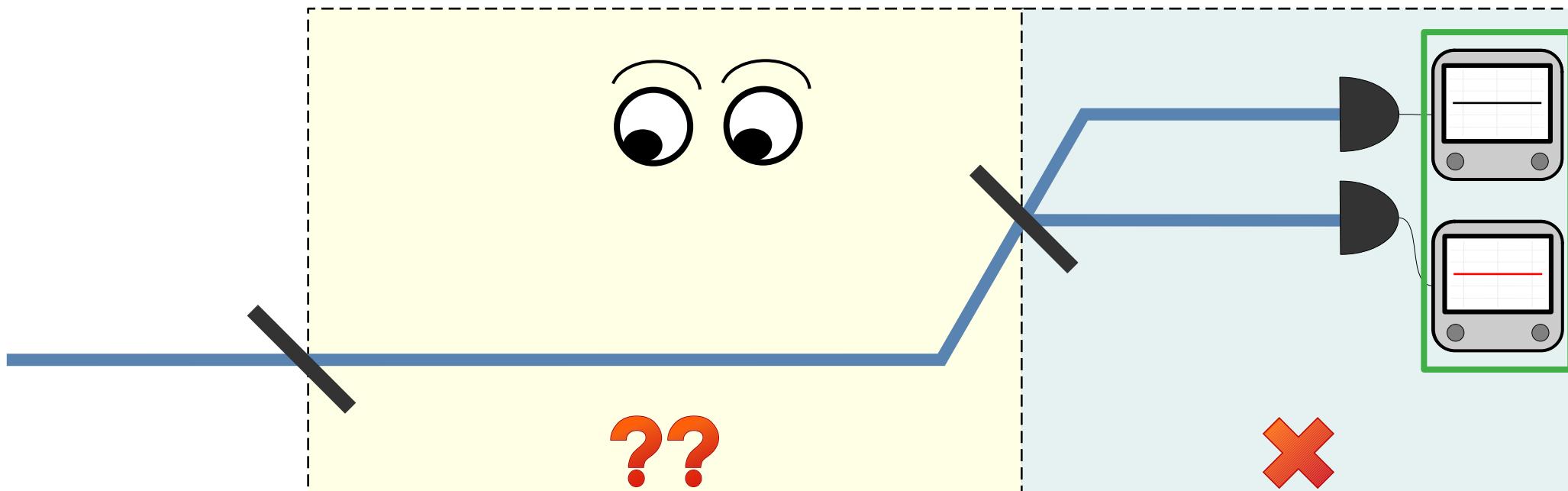
Motivation

Mach-Zehnder interferometry:



Motivation

Mach-Zehnder interferometry:



Motivation

Weak values carry information about an observable in between states
in the limit of minimum disturbance

Goal

Use weak values for a new perspective on interferometry and hopefully gain some new insights!

Part 2: Introduction to weak values

Weak value

Definition:

$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Weak value

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Weak value

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- Observable: \hat{A}
- Pre-selected state: $|\psi_{in}\rangle$
- Post-selected state: $|\psi_f\rangle$

Weak value

Definition:

$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Expectation value:

$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

- Observable: \hat{A}
- Pre-selected state: $|\psi_{in}\rangle$
- Post-selected state: $|\psi_f\rangle$

Weak value

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- Pre-selected state: $|\psi_{in}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Not bounded by eigenvalues

Weak value

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$$A_w = \frac{\langle \psi_f | \hat{A} | \psi_{in} \rangle}{\langle \psi_f | \psi_{in} \rangle}$$

Expectation value:

$$\langle \hat{A} \rangle = \frac{\langle \psi | \hat{A} | \psi \rangle}{\langle \psi | \psi \rangle}$$

- Observable: \hat{A}
- Pre-selected state: $|\psi_{in}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Not bounded by eigenvalues
- Complex number

Weak value

$$A_w = A_w^{\Re} + i A_w^{\Im}$$

↑ ↑
Real Imaginary

- Observable: \hat{A}
- Pre-selected state: $|\psi_{\text{in}}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Not bounded by eigenvalues
- Complex number

Weak value

$$A_w = A_w^{\Re} + i A_w^{\Im}$$

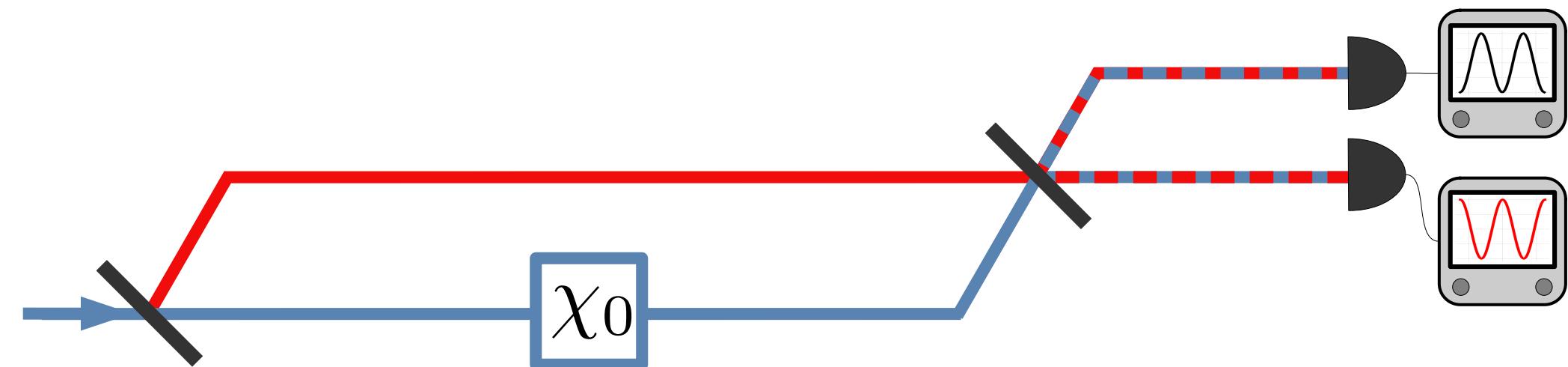
↑ ↑
Real Imaginary
↓

Describes the observable in the limit of minimal disturbance

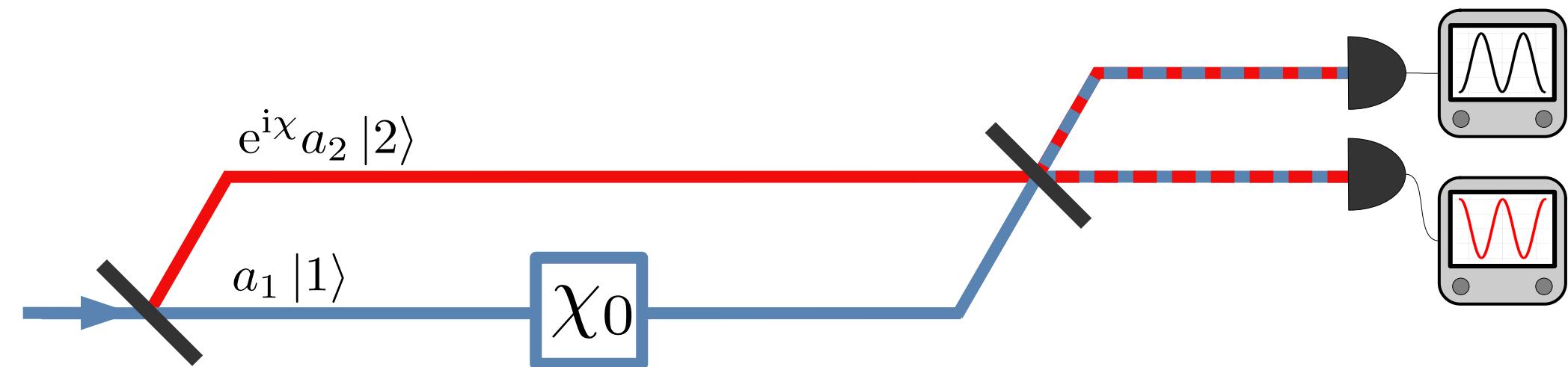
- Observable: \hat{A}
- Pre-selected state: $|\psi_{\text{in}}\rangle$
- Post-selected state: $|\psi_f\rangle$
- Not bounded by eigenvalues
- Complex number

Part 3: Weak values based description of interferometry

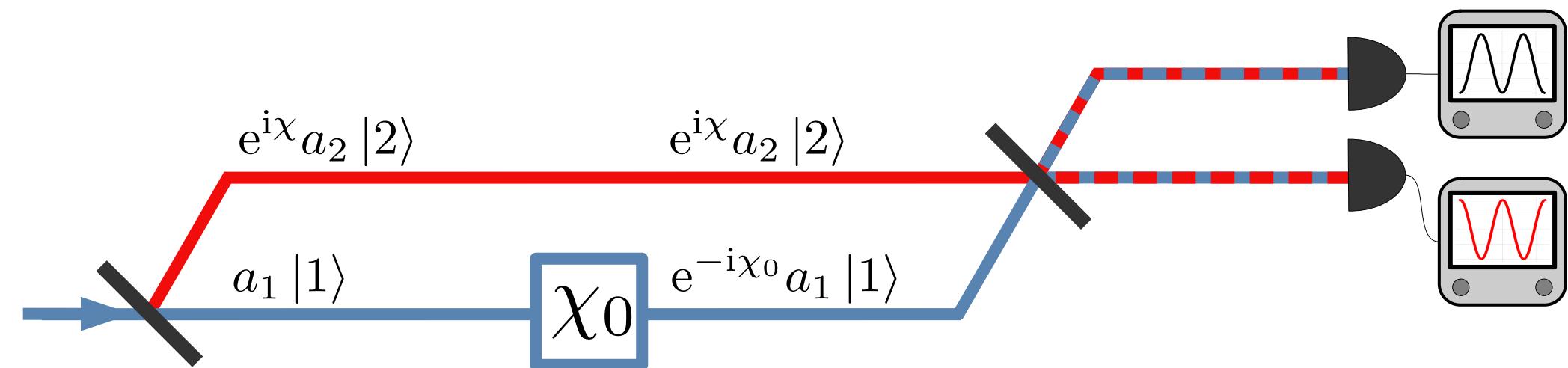
Standard interferometry formalism



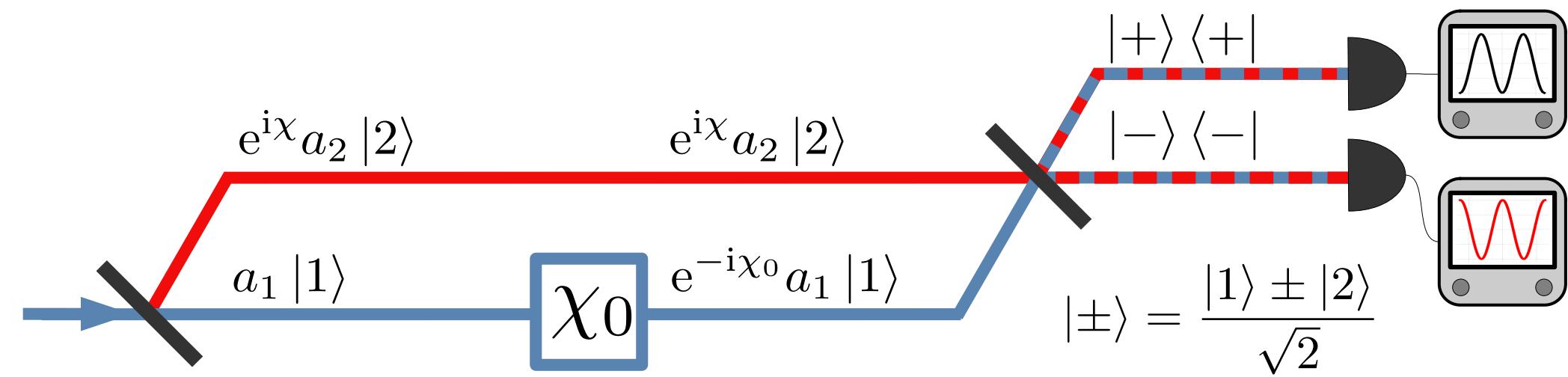
Standard interferometry formalism



Standard interferometry formalism



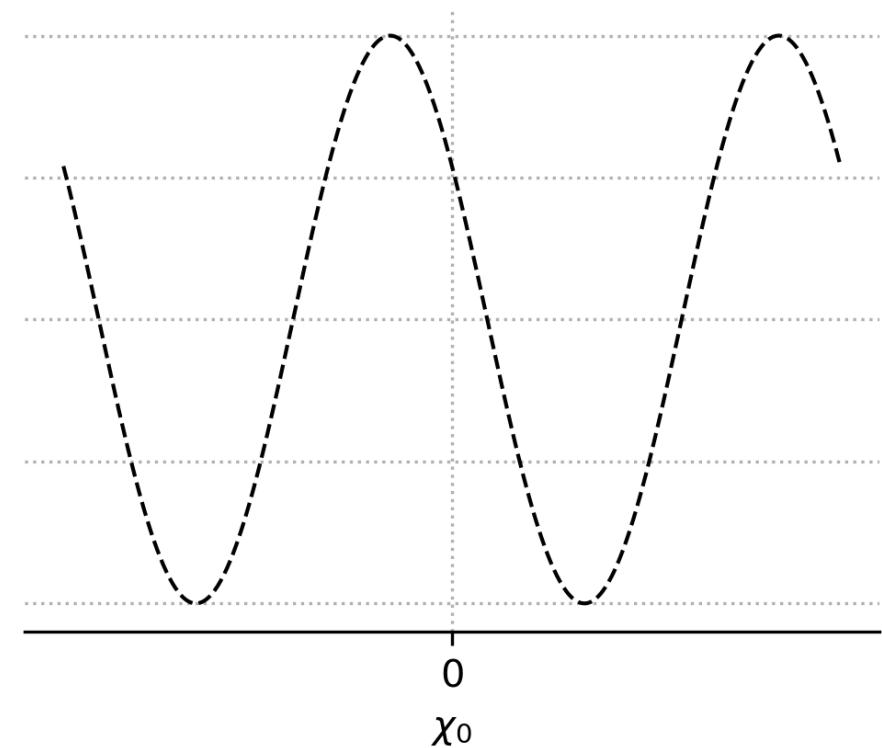
Standard interferometry formalism



Standard interferometry formalism

Measured intensity

$$I_{\pm,1}(\chi, \chi_0) = \frac{1}{2} \pm a_1 a_2 \cos(\chi + \chi_0)$$

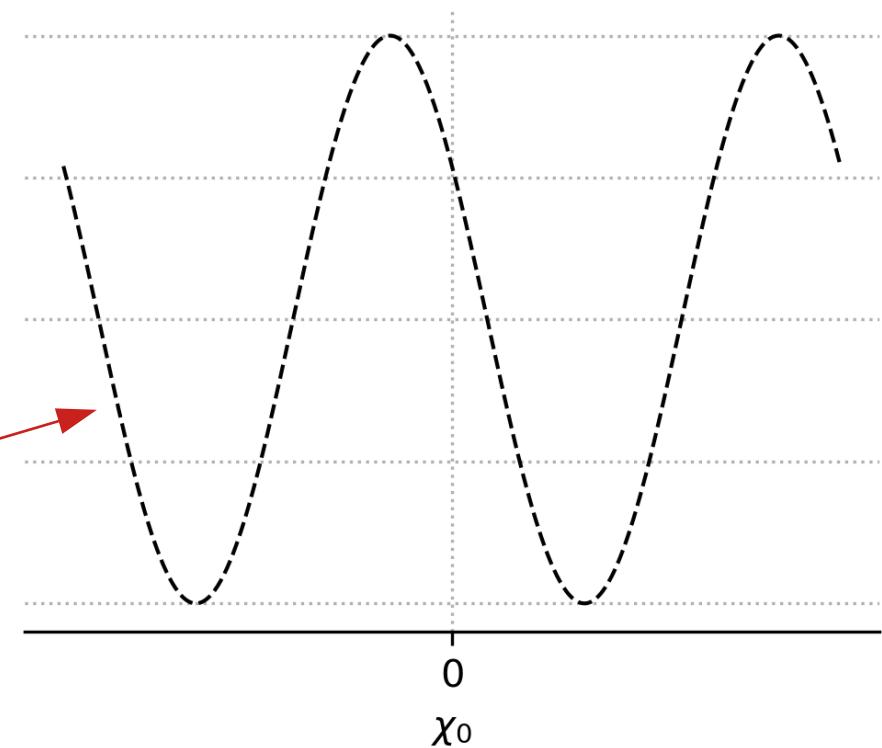


Standard interferometry formalism

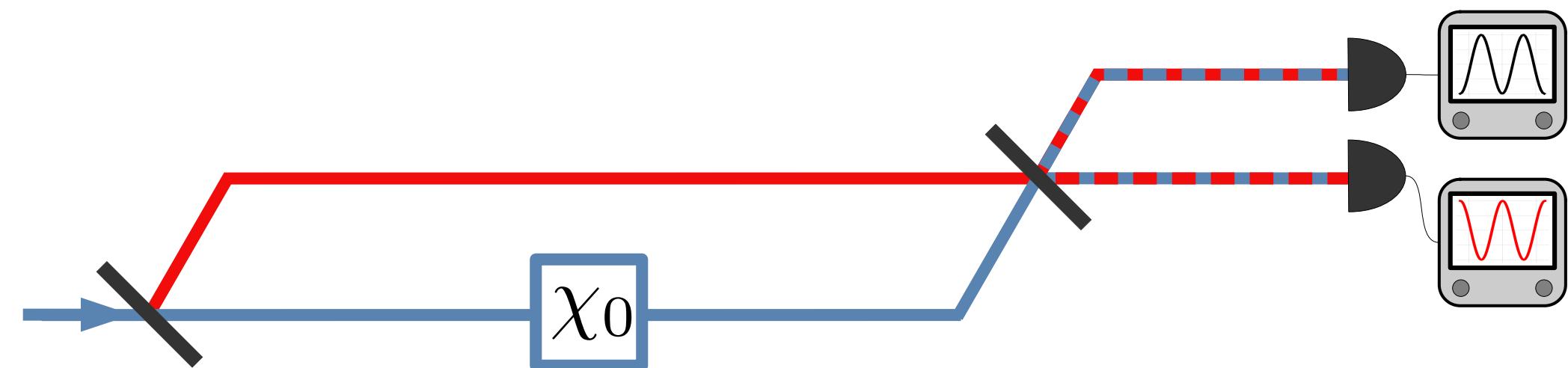
Measured intensity

$$I_{\pm,1}(\chi, \chi_0) = \frac{1}{2} \pm a_1 a_2 \cos(\chi + \chi_0)$$

Interferogram



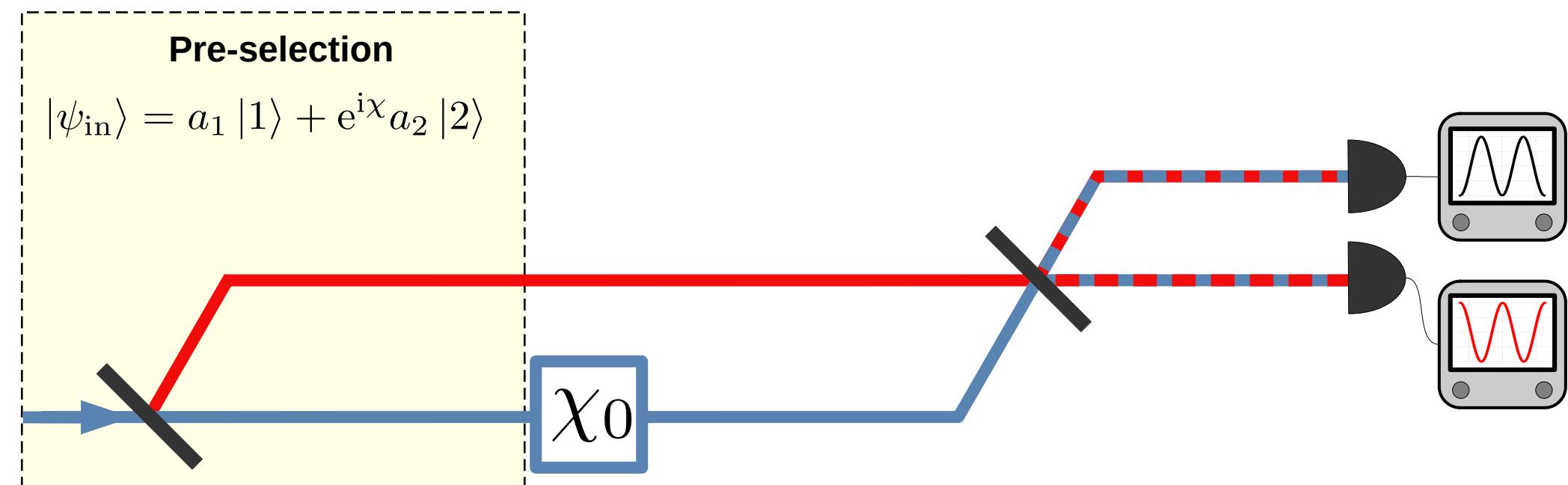
Weak value picture



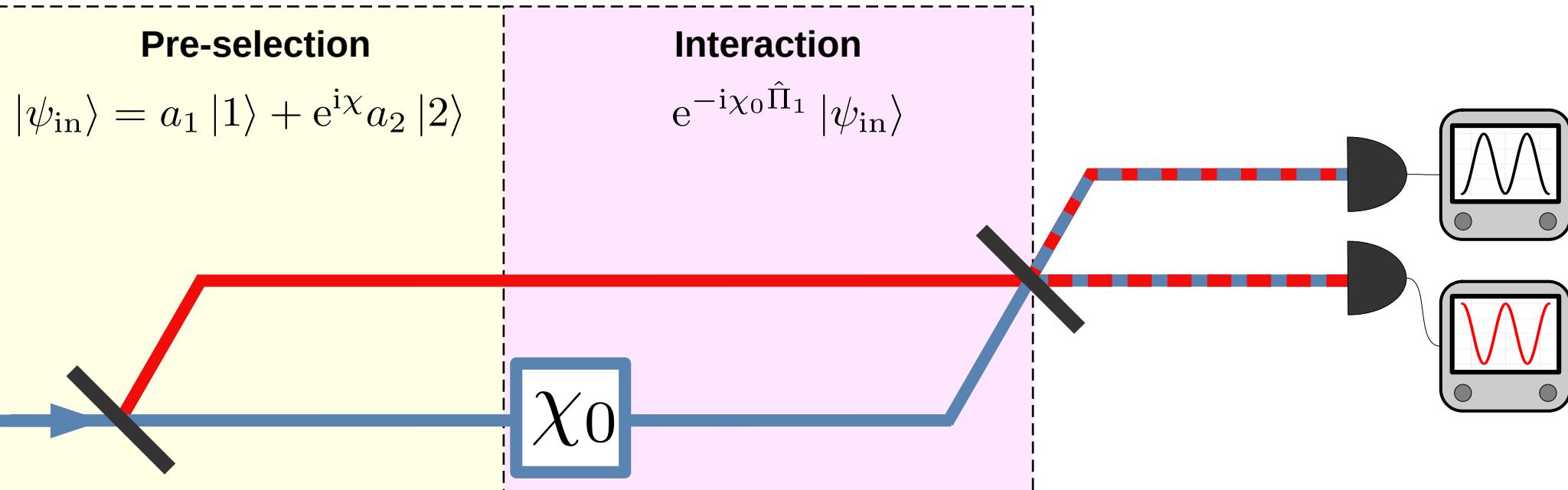
Weak value picture

Pre-selection

$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$



Weak value picture



Weak value picture

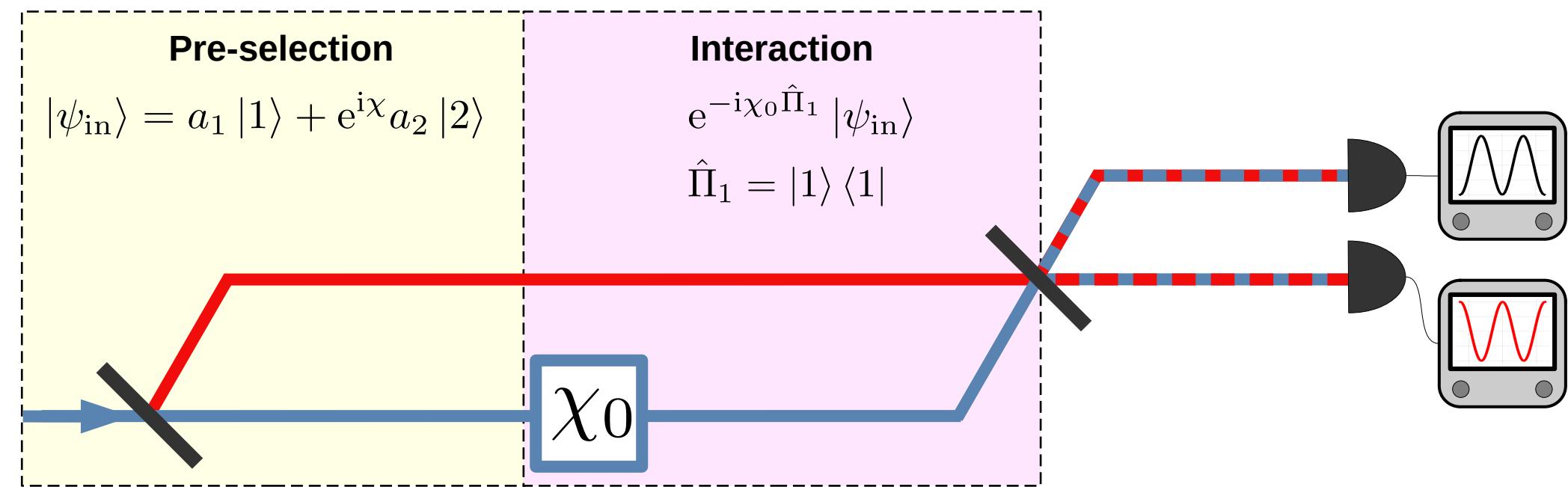
Pre-selection

$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$

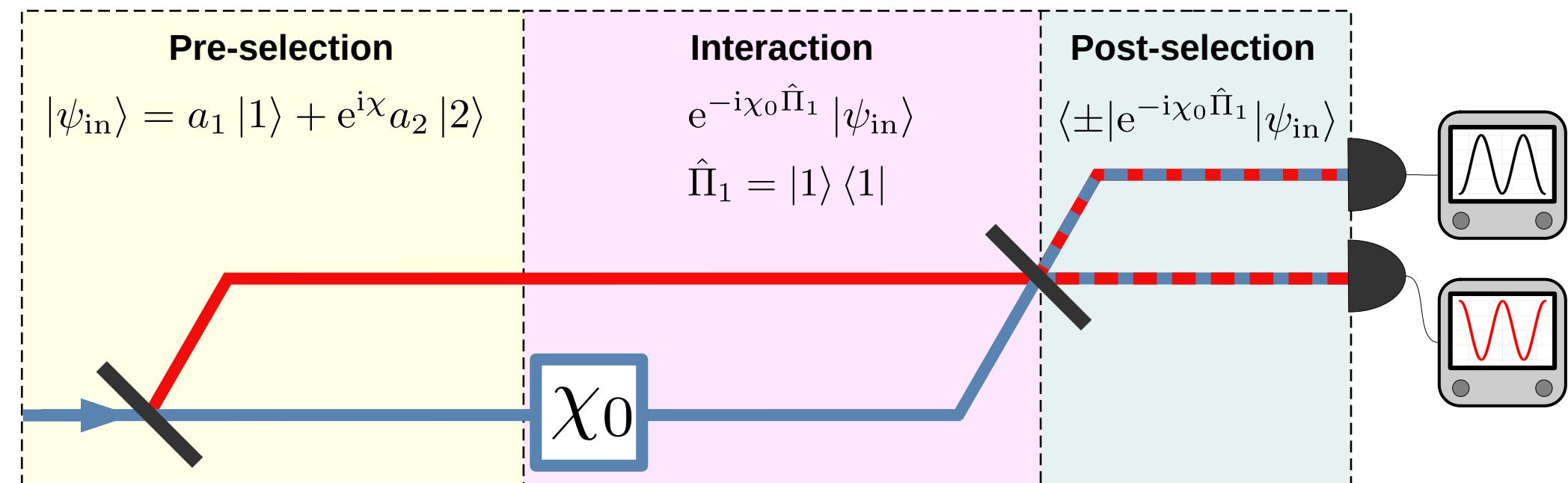
Interaction

$$e^{-i\chi_0 \hat{\Pi}_1} |\psi_{\text{in}}\rangle$$

$$\hat{\Pi}_1 = |1\rangle \langle 1|$$



Weak value picture



Weak value picture

Path weak value

$$w_{\pm,1} = \frac{\langle \pm | \hat{\Pi}_1 | \psi_{\text{in}} \rangle}{\langle \pm | \psi_{\text{in}} \rangle}$$

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2 (|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$



Amplitude
square

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2 (|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

Real part

Amplitude
square

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos \chi_0) + 2w_{\pm,1}^{\Im} \sin \chi_0]$$

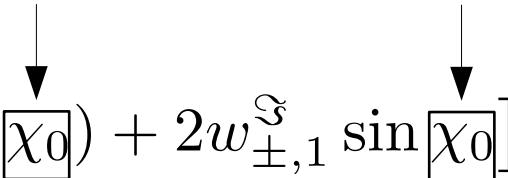
Amplitude
square

Real part

Imaginary
part

Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos[\boxed{\chi_0}]) + 2w_{\pm,1}^{\Im} \sin[\boxed{\chi_0}]]$$


Weak value picture

Measured intensity

$$I_{\pm}(\chi, \chi_0) = |\langle \pm | \psi_{\text{in}} \rangle|^2 [1 + 2(|w_{\pm,1}|^2 - w_{\pm,1}^{\Re}) (1 - \cos[\chi_0]) + 2w_{\pm,1}^{\Im} \sin[\chi_0]]$$

$$I_{\pm}(\chi, 0) = |\langle \pm | \psi_{\text{in}} \rangle|^2$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Weak value picture

Almost there...

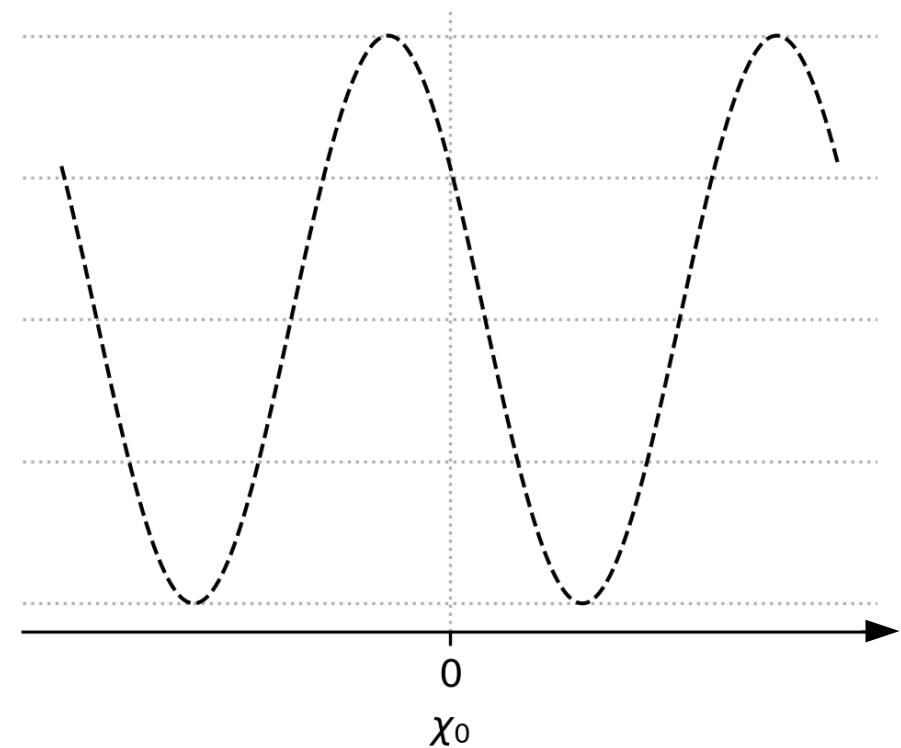
$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Interferogram

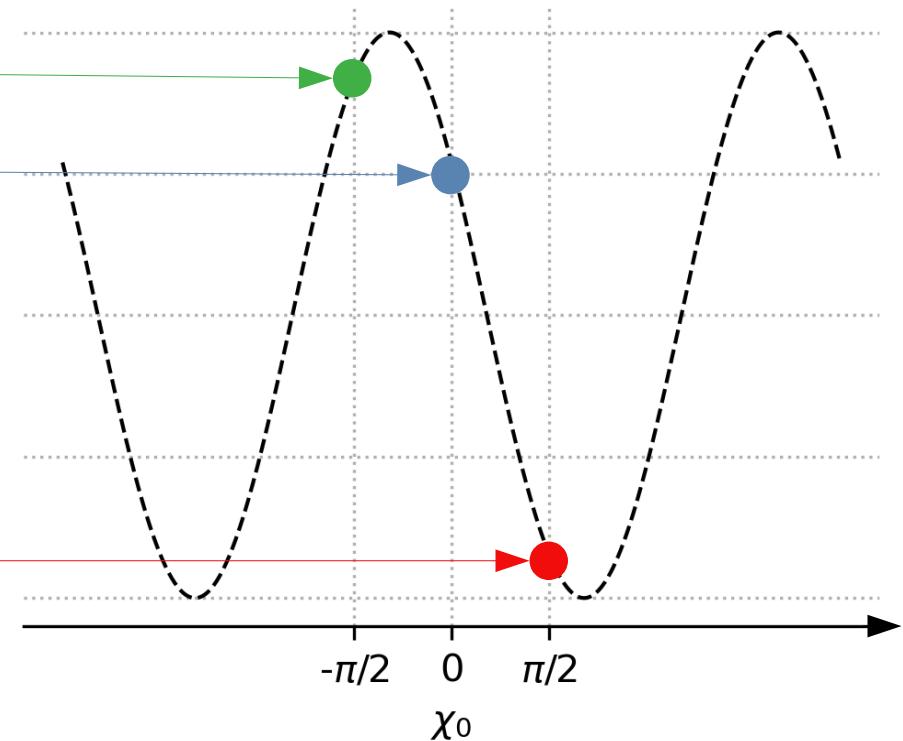


Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

Interferogram



Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

$|w_{\pm,1}|^2 = \sqrt{w_{\pm,1}^{\Re}^2 + w_{\pm,1}^{\Im}^2}$

Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re} \quad \leftarrow \text{Quadratic equation}$$

$|w_{\pm,1}|^2$
 $w_{\pm,1}^{\Re} + w_{\pm,1}^{\Im}$

Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = |w_{\pm,1}|^2 - w_{\pm,1}^{\Re}$$

$|w_{\pm,1}|^2$

$$w_{\pm,1}^{\Re}{}^2 + w_{\pm,1}^{\Im}{}^2$$

Quadratic equation

↓

2 solutions

Weak value picture

Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im} \quad \checkmark$$

$$\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} = \boxed{|w_{\pm,1}|^2} - w_{\pm,1}^{\Re}$$

Weak value picture

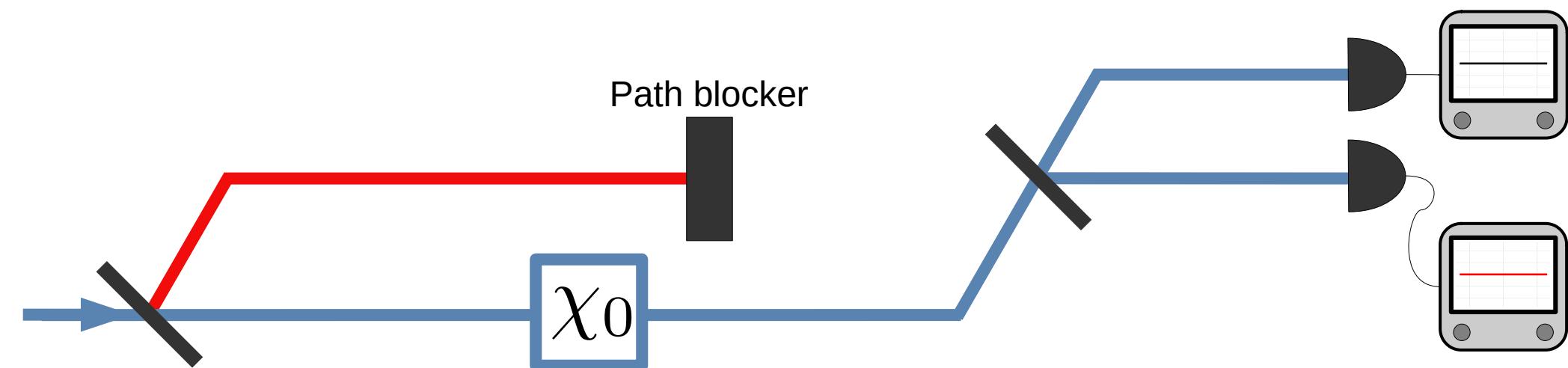
Almost there...

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



$$-\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} + |w_{\pm,1}|^2 = w_{\pm,1}^{\Re}$$

Weak value picture



Weak value picture

$$\langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} |\psi_{\text{in}}\rangle$$

Pre-selection

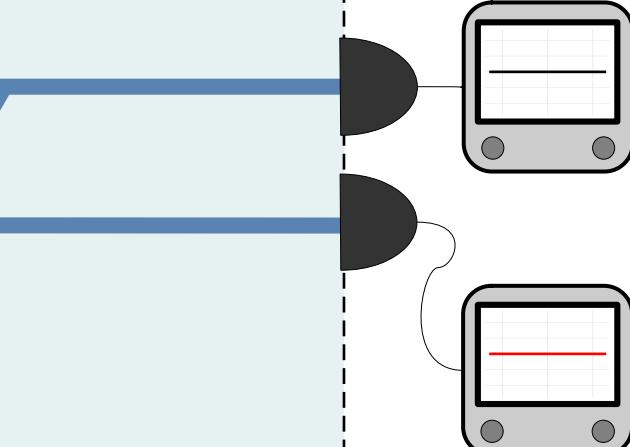
$$|\psi_{\text{in}}\rangle = a_1 |1\rangle + e^{i\chi} a_2 |2\rangle$$

Interaction

$$e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} |\psi_{\text{in}}\rangle$$

Path blocker

Post-selection



Weak value picture

Measured intensity

$$\begin{aligned} I_{\pm}^{Bl.\ 2} &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= |\langle \pm | \psi_{\text{in}} \rangle|^2 |w_{\pm,1}|^2 \end{aligned}$$

Weak value picture

Measured intensity

$$\begin{aligned} I_{\pm}^{Bl.\ 2} &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} \lim_{\alpha \rightarrow \infty} e^{-\alpha \hat{\Pi}_2} | \psi_{\text{in}} \rangle \right|^2 \\ &= \left| \langle \pm | e^{-i\chi_0 \hat{\Pi}_1} | \psi_{\text{in}} \rangle \right|^2 = \left| \langle \pm | \hat{\Pi}_1 | \psi_{\text{in}} \rangle \right|^2 \\ &= |\langle \pm | \psi_{\text{in}} \rangle|^2 |w_{\pm,1}|^2 \end{aligned}$$

Weak value picture

Measured intensity

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Weak value picture

We got there!

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$

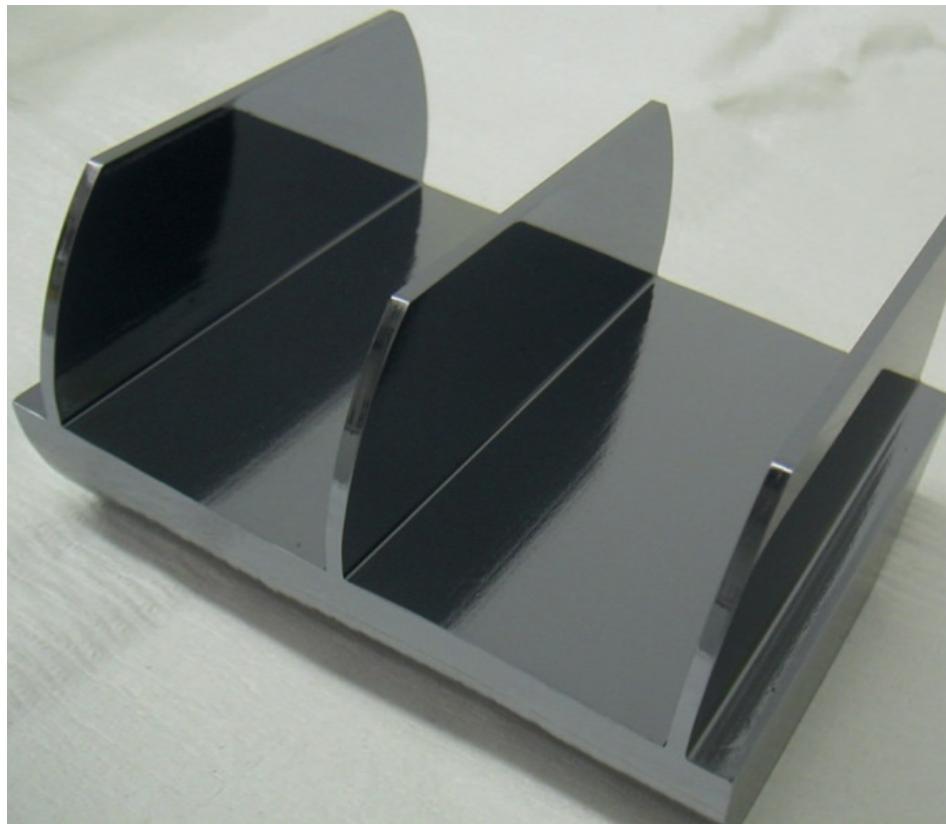


$$-\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} + |w_{\pm,1}|^2 = w_{\pm,1}^{\Re}$$

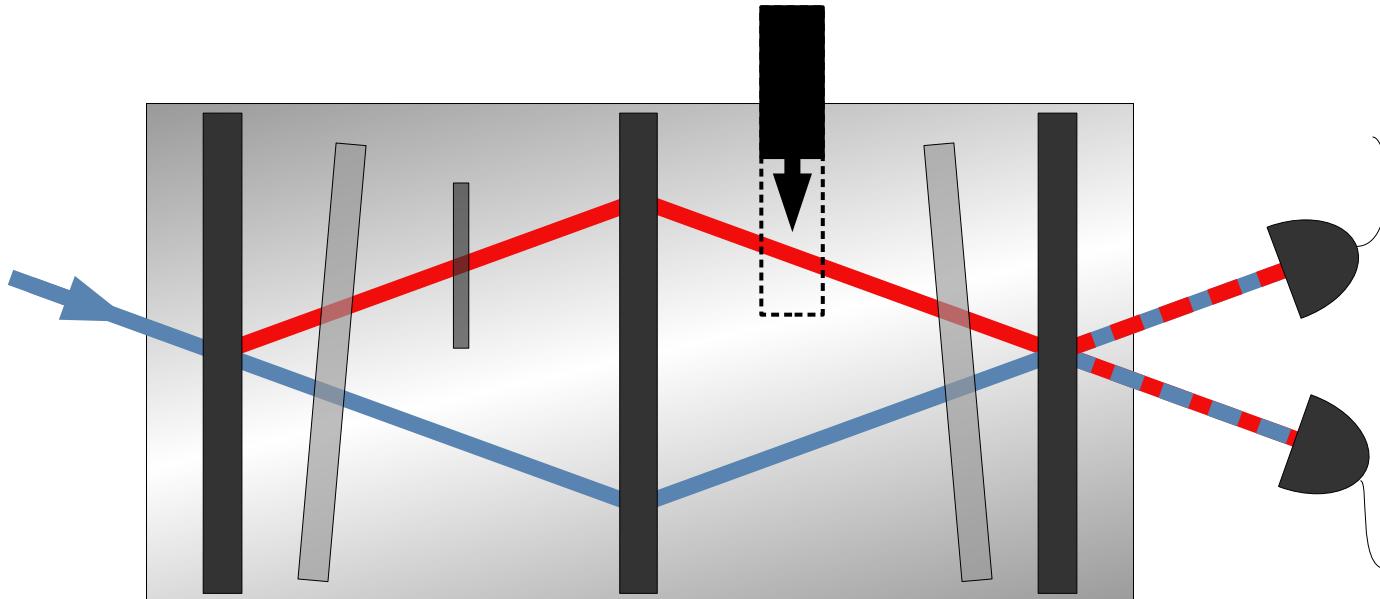


Part 4: Experimental measurement of path weak value from interferograms

Neutron interferometer

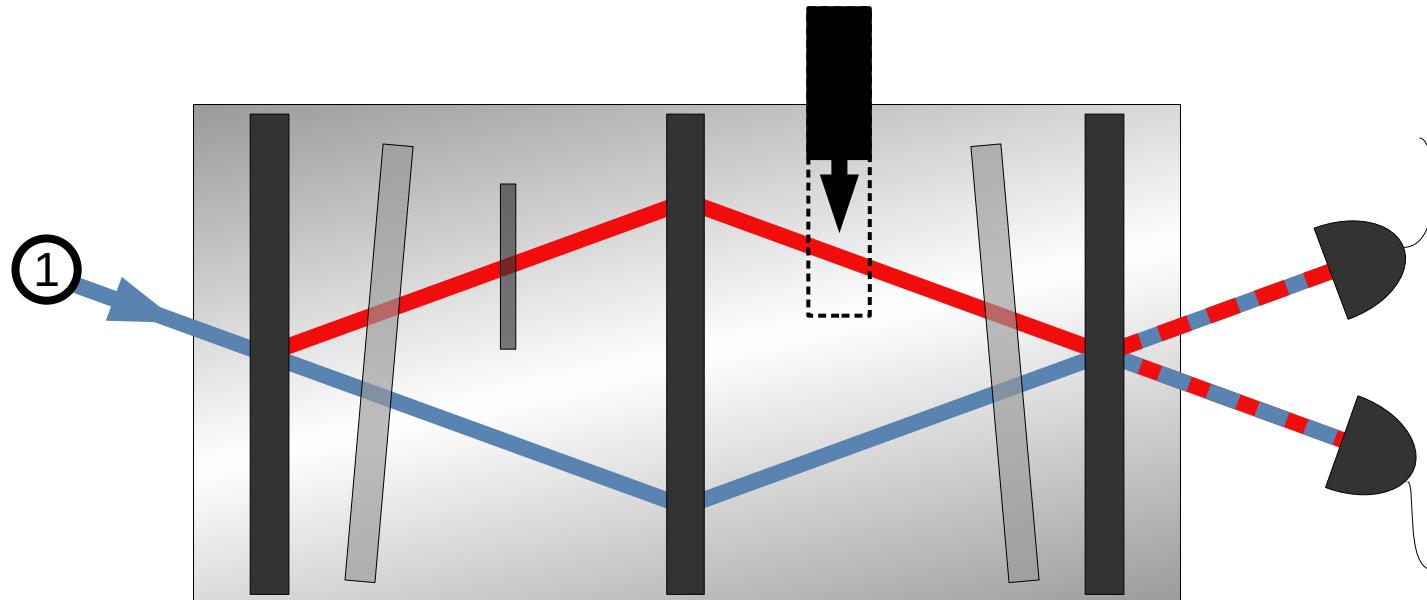


Setup



Setup

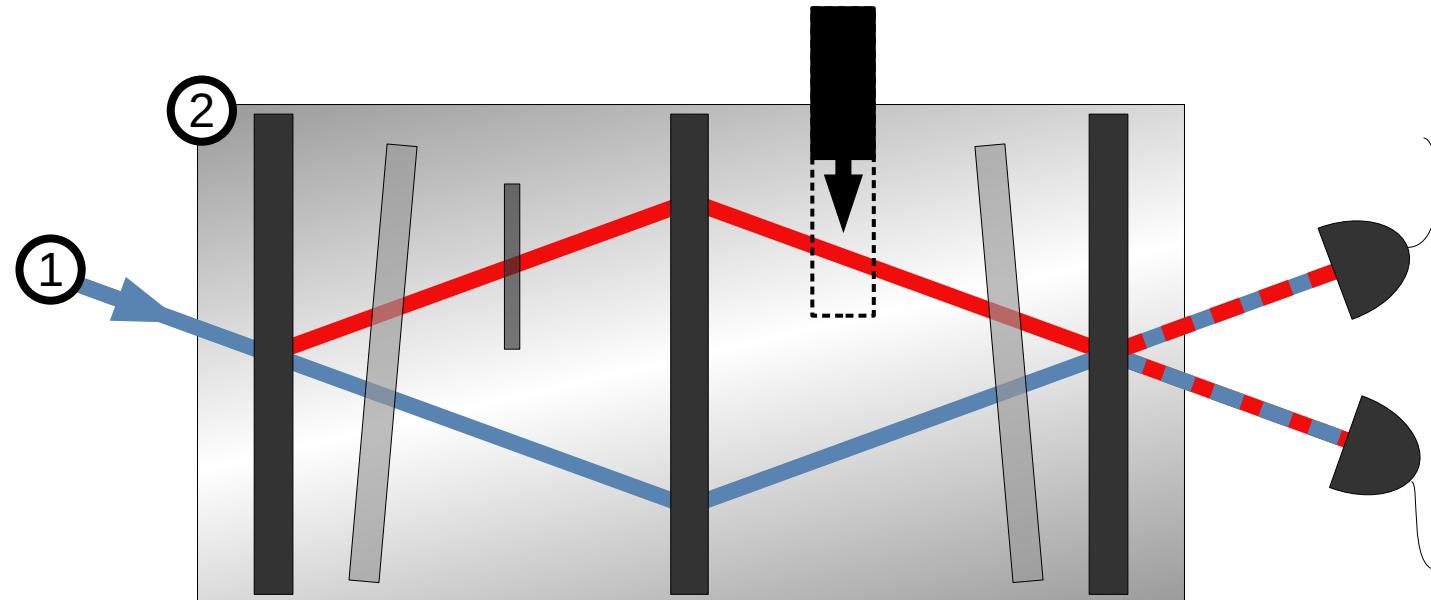
① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$



Setup

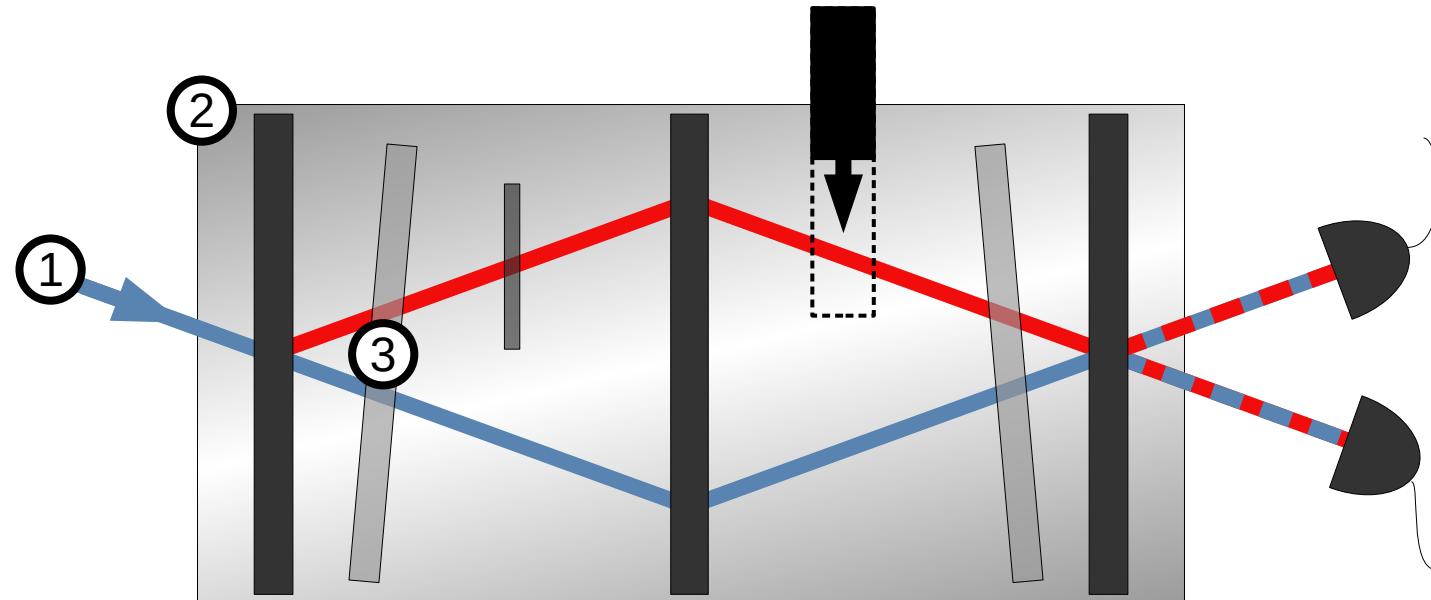
① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$

② Neutron interferometer



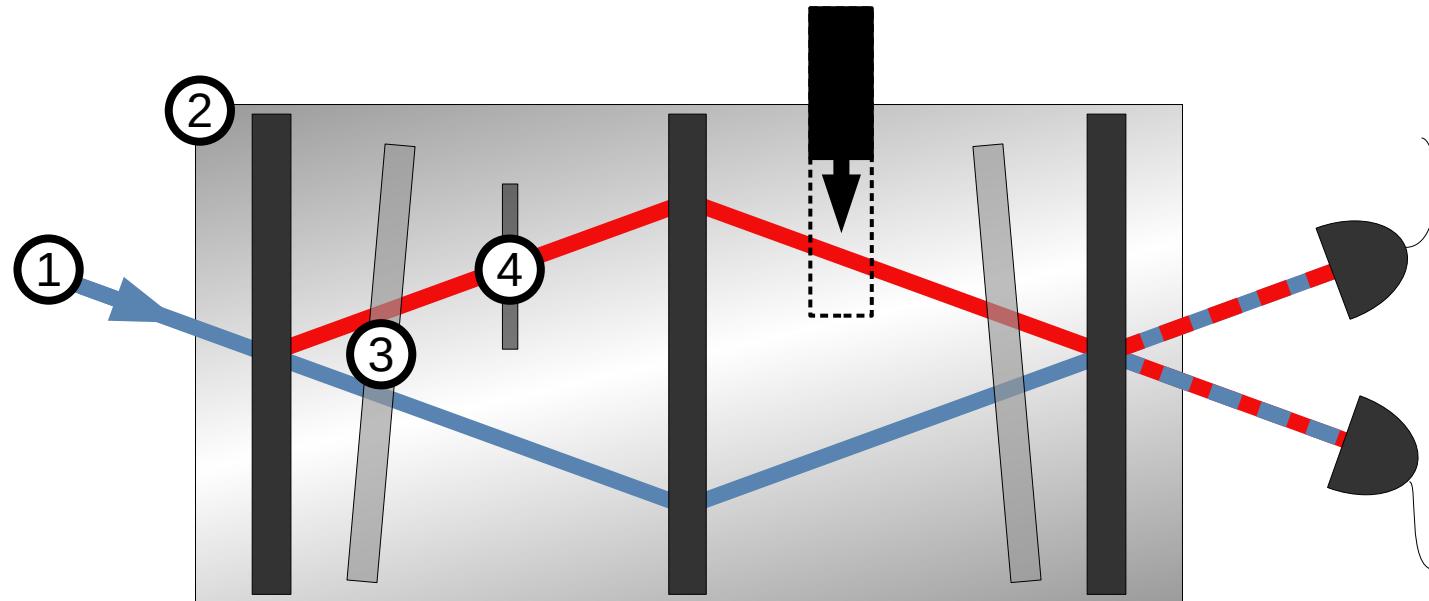
Setup

- ① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- ② Neutron interferometer
- ③ Phase-shifter χ



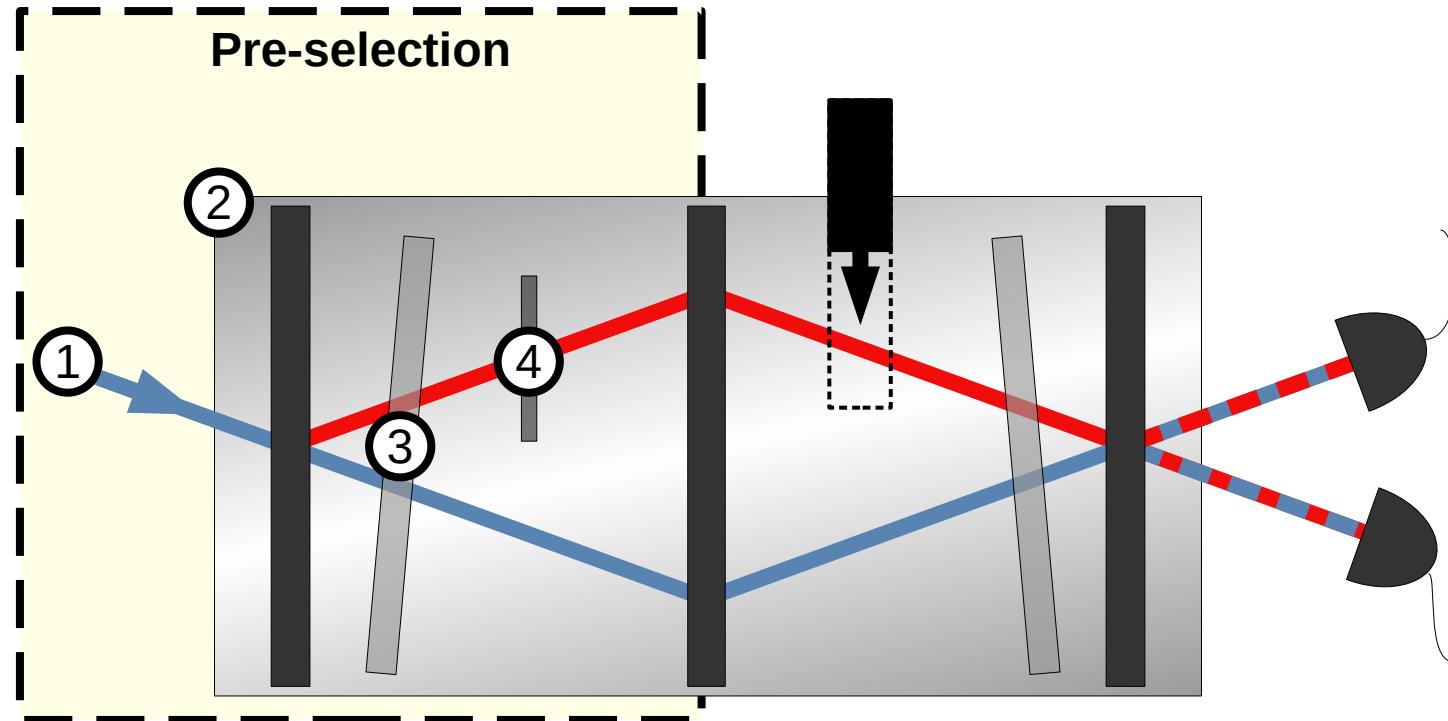
Setup

- ① Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- ② Neutron interferometer
- ③ Phase-shifter χ
- ④ Indium foils to adjust path amplitudes
 $a_2/a_1 \approx 0.59$



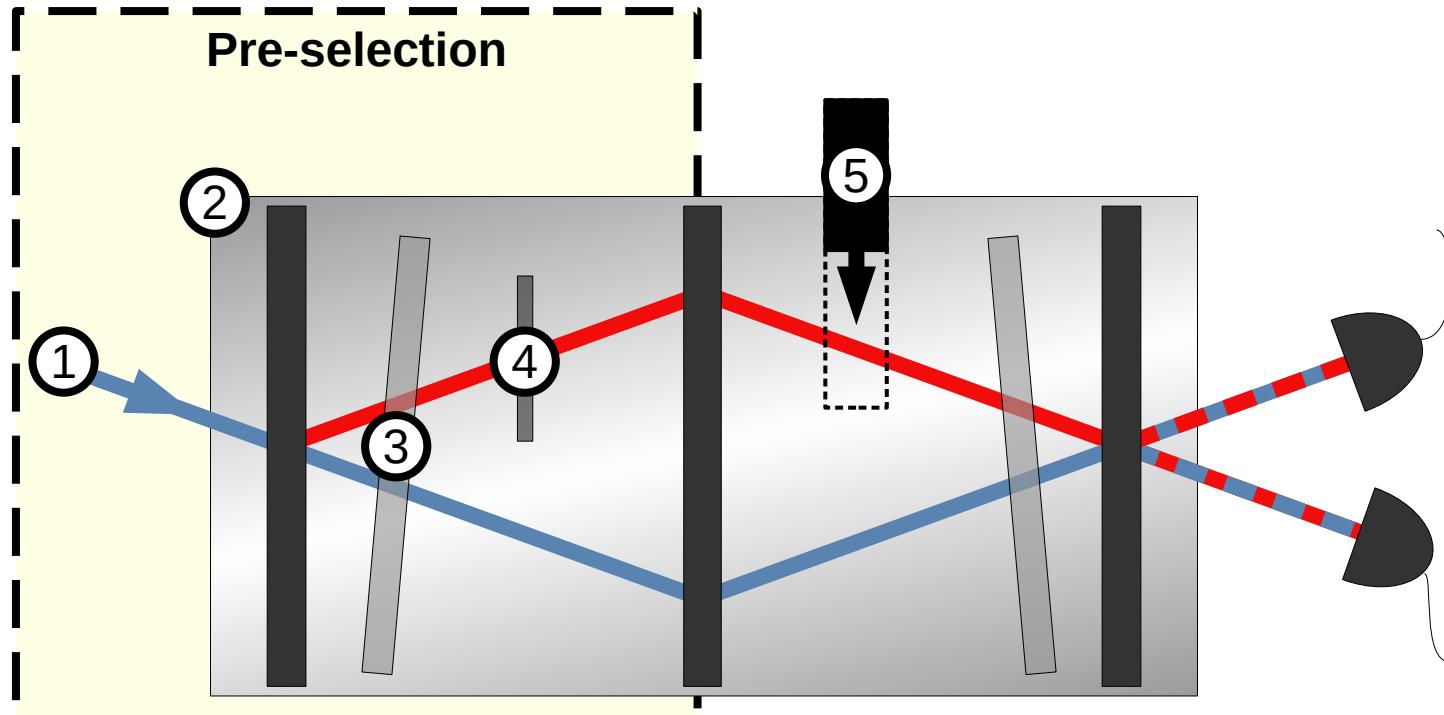
Setup

- 1 Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- 2 Neutron interferometer
- 3 Phase-shifter
 χ
- 4 Indium foils to adjust path amplitudes
 $a_2/a_1 \approx 0.59$



Setup

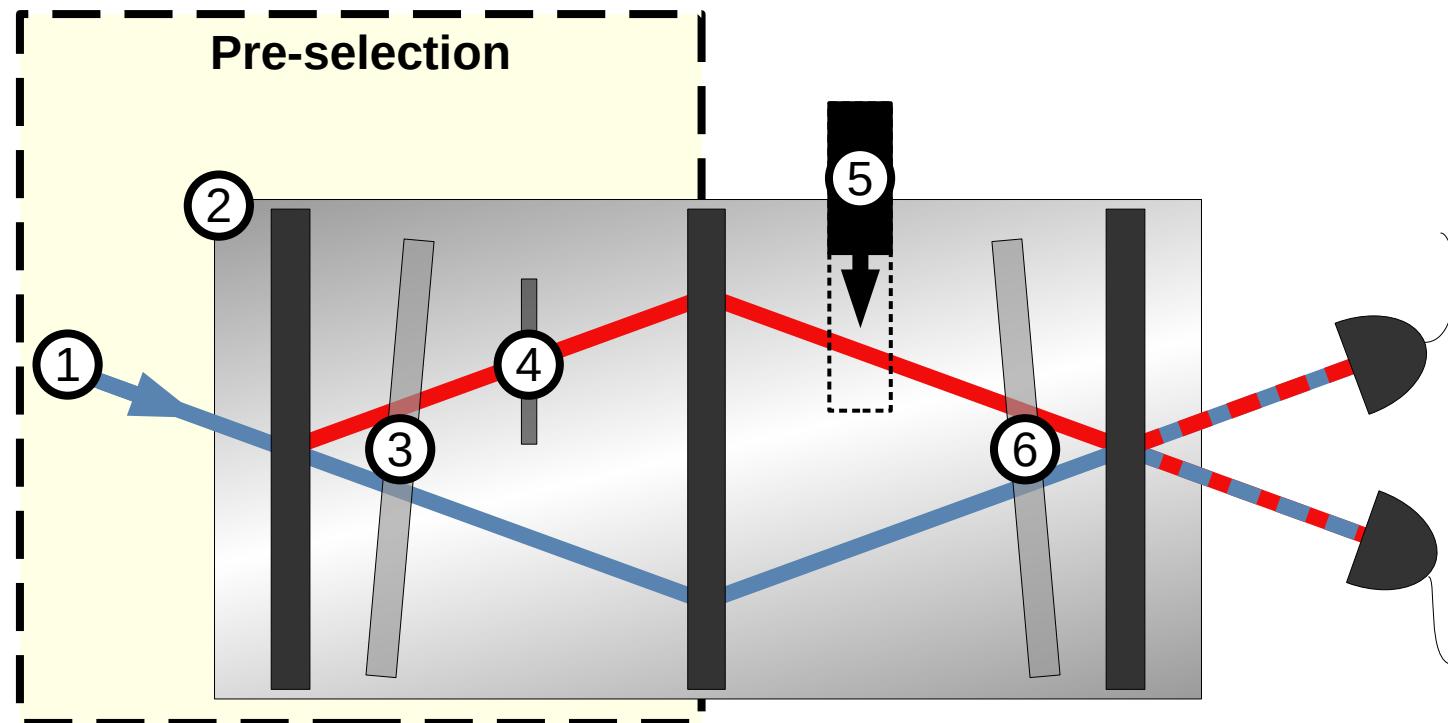
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- 5 Cadmium path blocker

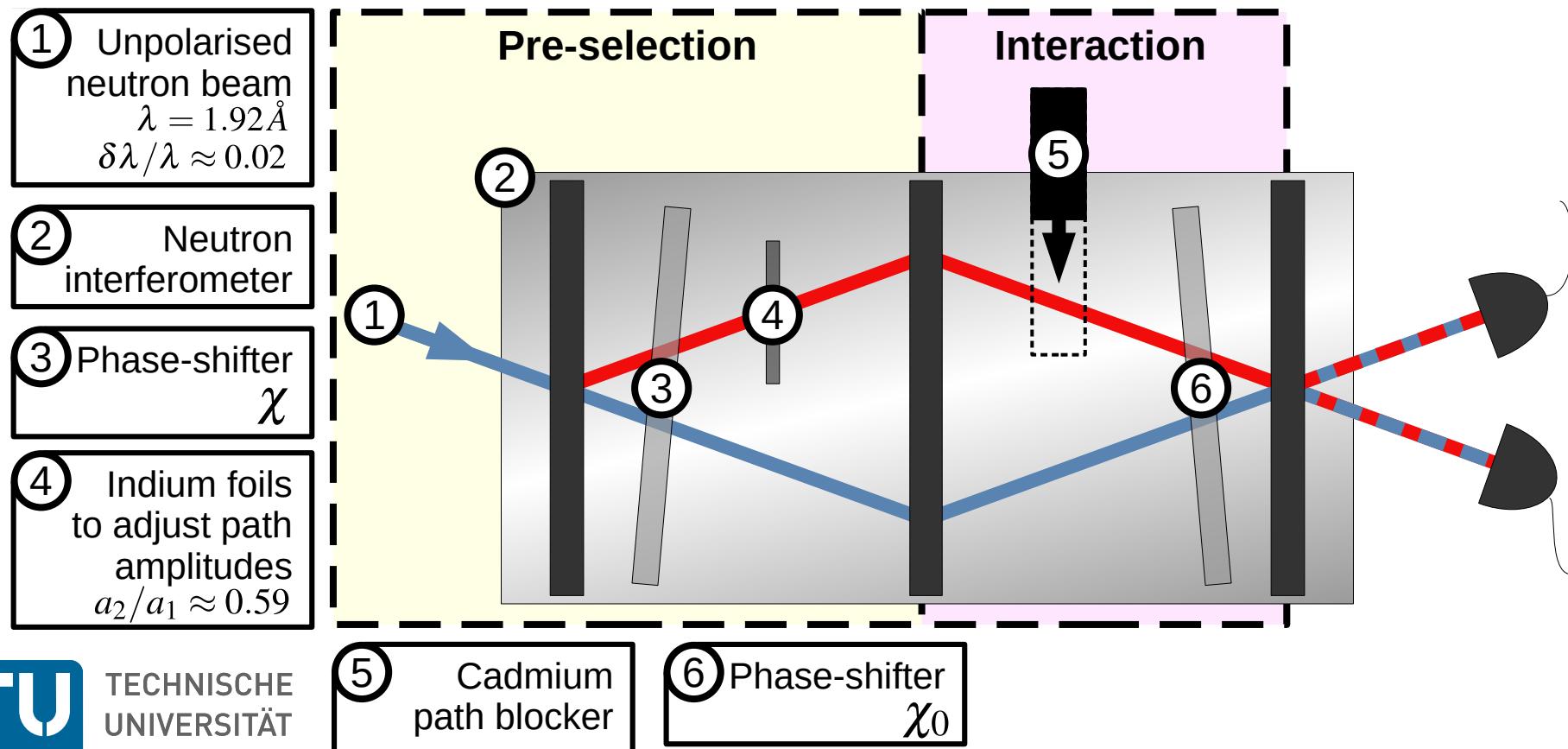
Setup

- 1 Unpolarised neutron beam
 $\lambda = 1.92\text{\AA}$
 $\delta\lambda/\lambda \approx 0.02$
- 2 Neutron interferometer
- 3 Phase-shifter χ
- 4 Indium foils to adjust path amplitudes
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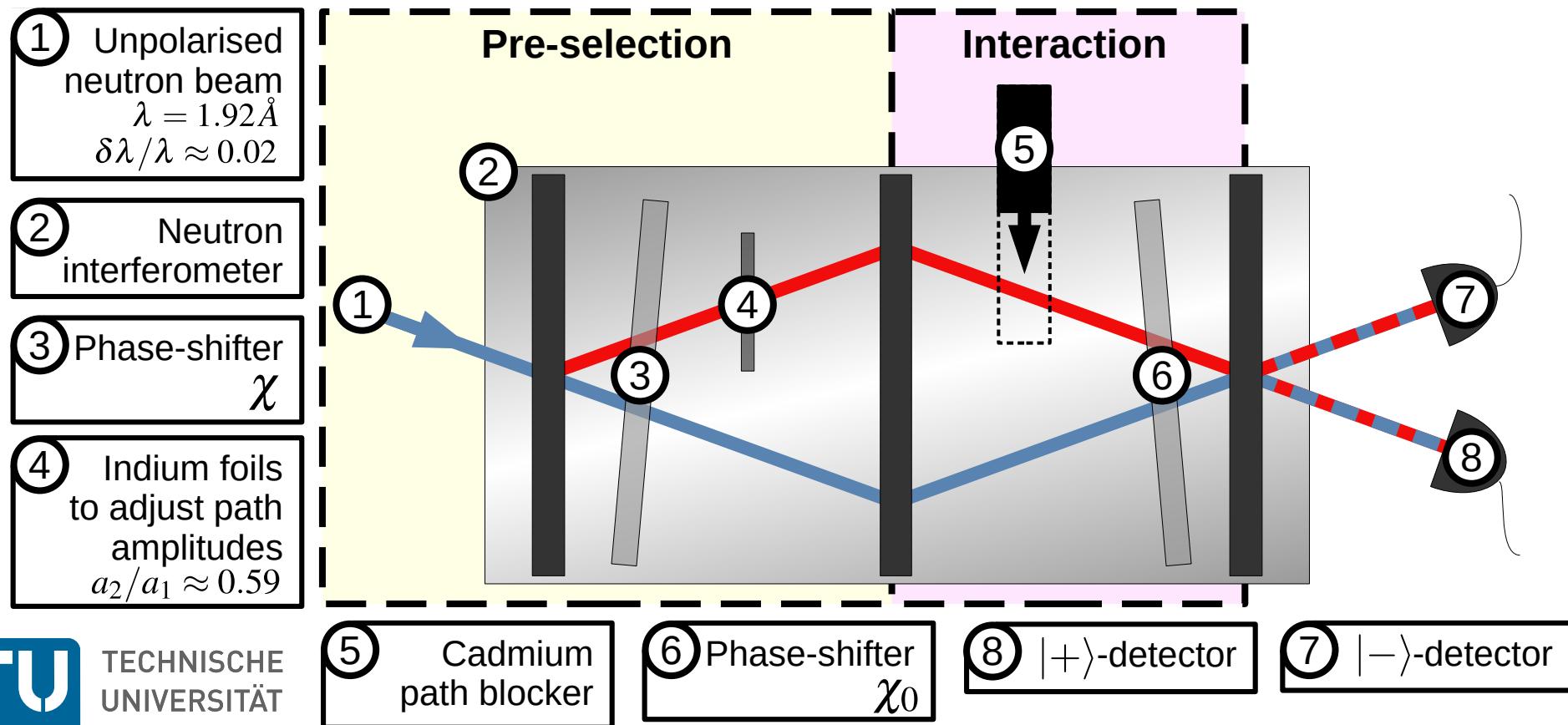


- 5 Cadmium path blocker
- 6 Phase-shifter χ_0

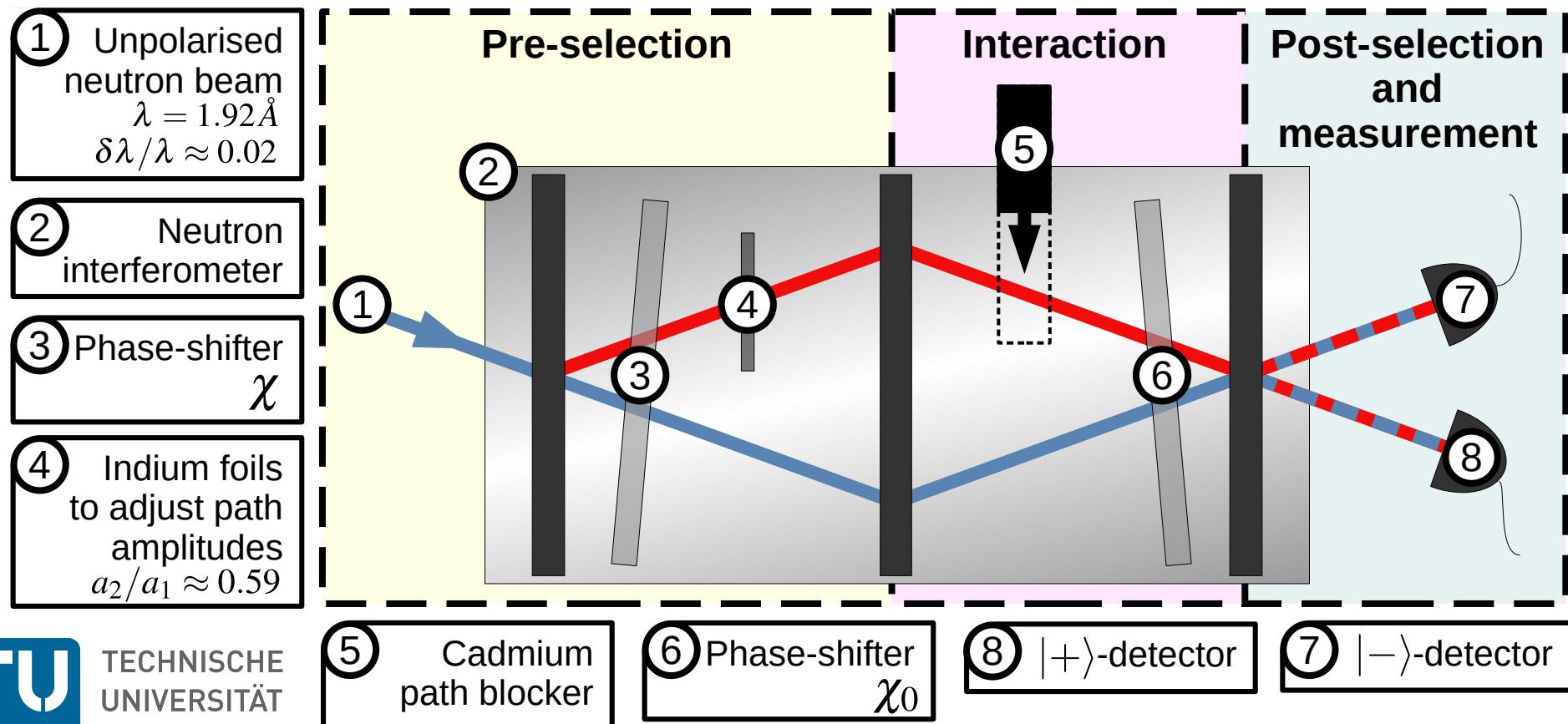
Setup



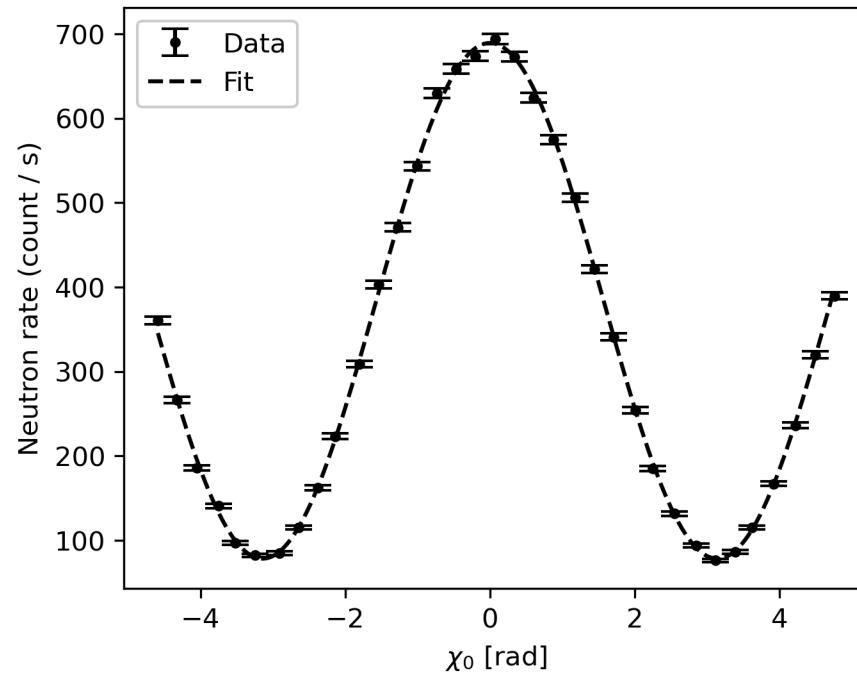
Setup



Setup



Example of interferogram



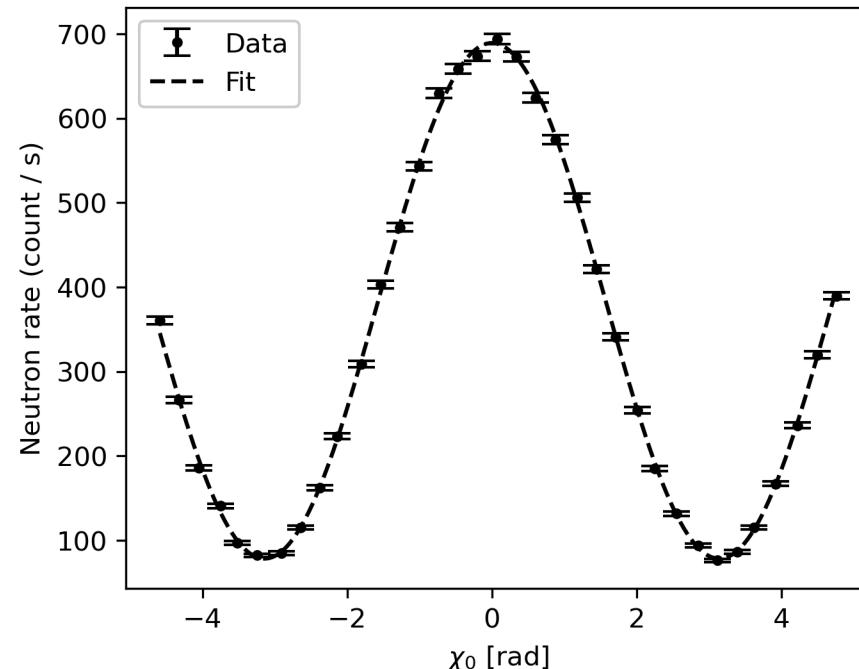
Example of interferogram

Real part

$$-\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} + |w_{\pm,1}|^2 = w_{\pm,1}^{\Re}$$

Imaginary part

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



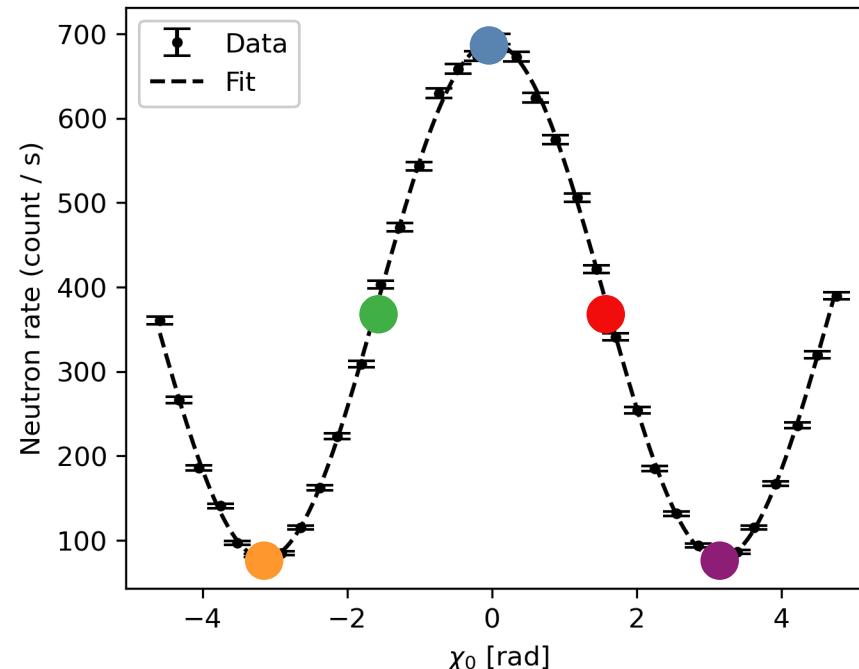
Example of interferogram

Real part

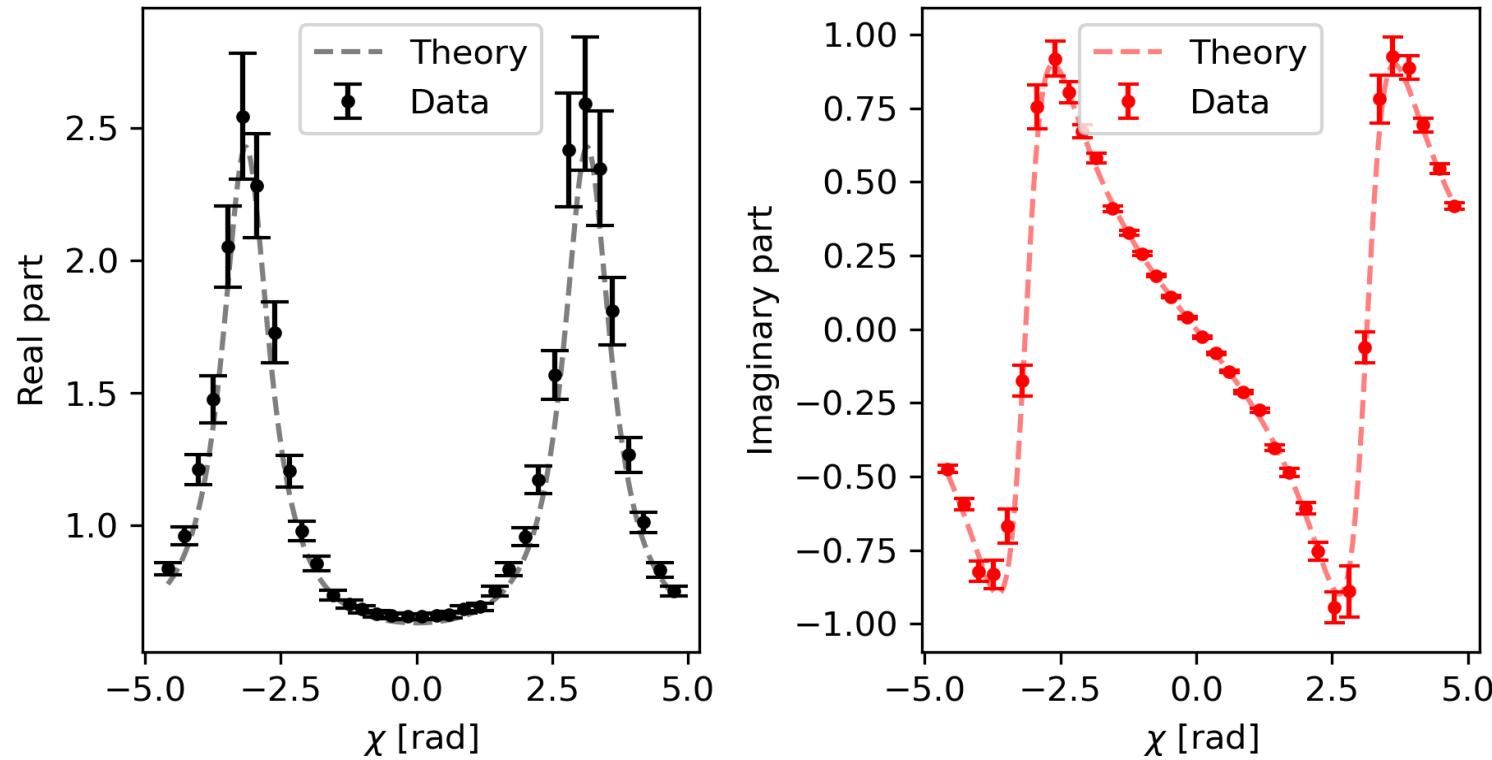
$$-\frac{I_{\pm}(\chi, \pi) - I_{\pm}(\chi, 0)}{4I_{\pm}(\chi, 0)} + |w_{\pm,1}|^2 = w_{\pm,1}^{\Re}$$

Imaginary part

$$\frac{I_{\pm}(\chi, \frac{\pi}{2}) - I_{\pm}(\chi, \frac{3\pi}{2})}{4I_{\pm}(\chi, 0)} = w_{\pm,1}^{\Im}$$



Results: Weak value of path 1



Interpretation of the results

Interpretation of the results



Acknowledgements

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Thanks for your time!

