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Dear Editor of New Journal of Physics,

please find enclosed a revised version of our manuscript entitled "Simultaneous determination of two path weak-values with time-dependent phase manipulation in neutron interferometry", reference: NJP-118200. The updated contents are colored in blue to differentiate from contents in the original manuscript.

We would like to thank the Reviewers for their comments which contributed to the improvement of the manuscript. In the following, we provide an answer to the Reviewers and describe the changes implemented in the manuscript according to their comments. We list the comments as numbered in the report and provide an answer to each in italic font.

Please do not hesitate to contact us if any further information is required.

Best Regards,
Ismaele Vincent Masiello*, Andreas Dvorak, Hartmut Lemmel, Armin Danner, and Yuji Hasegawa
(*corresponding author)

Answer to Referee 2

We would like to thank the Referee for the contribution. Please note that during the review we decided to explicitly add Eq.(9) in Sec. 2.1 to better fit the new structure of the paper. Therefore, the numbers of some equations are shifted. The updated contents are colored in blue to differentiate with contents in the original manuscript. In what follows, we list the comments as numbered in the report and provide an answer to each in italic font.

1. In addition to the schematic drawing in Fig. 3, a photo of the physical experimental setup assembled for the experiment is suggested to be attached.

   Due to its structure, it is quite challenging to accurately depict the experimental setup in its entirety through a photo. However, we selected 2 photos that represent the elements of interest (interferometer station and coils) and added them to Fig. 3.

2. The authors mention that the phase shifter is placed inside the array, but how much rotation is applied, and what is the adjustment process to have the desired relative phase in this experiment? A more detailed explanation of this process would be helpful.

   More details about the phase shifter and how the relative phase is extracted have been added in the first paragraph of Sec. 3.1. Plots of the intensity vs phase shifter rotation in degrees, i.e. an interferograms, are now shown in Fig. 4 and 6.

3. What were the configuration characteristics in the Helmholtz coil that guarantee an oscillating magnetic field parallel to the spin?

   We have added a reference to a paper that describes the general features of coils for spin manipulation [30] to the third paragraph of Sec. 3.1. In the same paragraph, two methods that have been used to guarantee an oscillating magnetic field parallel to the spin are now mentioned. Moreover, a photo of the coils has been added in Fig. 3.

4. In Sec. 3.1. The setup. I suggest adding a continuous time graph, as mentioned in the measurements performed for 7 to 10 hours. This will help to corroborate and observe the stability and non-significant variations in the phase.

   Measurements to corroborate and observe the phase stability are now provided in Fig. 4.

5. What were the characteristics of the beam detectors used in the experiment?

   A description of the characteristics of the detector used for the time-resolved measurement has been added in the last paragraph of Sec. 3.1.

6. The authors mention that the frequencies used in the experiment were 2 and 3 KHz for each path. How was this frequency value selected? Can these oscillation values change in this experiment?

   We have included an explanation for the choice of frequencies in Sec. 3.2. For completeness, we also indicated the oscillation periods of the fields and the width of the time bins.

7. Why were only these four positions presented in Figure 4? Were these the ones that achieved the best theoretical-experimental fit? Also, did you find the maximum phase value at which the most prominent time-dependent oscillations occurred?

   Both questions have been addressed in the last paragraph of Sec. 3.2.

8. In Sec. 3.3. measured neutron counts, the authors describe the process to obtain the time-averaged intensity here experimentally; it is mentioned that the average of the measurements allows obtaining

an estimate of the proportionality of the constant A, where they were able to observe an offset in the signal, as well as the percentage of interfering C components, to support and highlight these results more thoroughly, I suggest adding a graph where the description of this phenomenon and the characteristics mentioned in the final paragraph of this section can be observed.

A plot describing how the time-averaged intensity is obtained and explicitly showing the oscillations allowing for the extraction of "A" and "C" is now presented in Fig. 6.

9. What was the variation between the two extraction methods (G(t) function and Fourier analysis) to be accounted for the affirmation: The two methods of extraction presented in the previous section yield similar outputs?

   Fig. 8 has been added to show an example of results obtained with both methods. The plot also shows the variation between methods as the difference between the obtained data points and between the obtained error-bars. Thanks to this Referee's comment, we have been able to spot and fix a minor mistake in the numerical computation of the error-bars in the Fourier method. The plots of the results have been updated with the correct error-bars, the data points were not affected.

Finally, we thank once more the Referee for the helpful comments and hope that we have now produced a more balanced and better account of our work. We hope that the revised manuscript is acceptable for publication in New Journal of Physics.

Answer to Referee 3

We would like to thank the Referee for the contribution. Please note that during the review we decided to explicitly add Eq.(9) in Sec. 2.1 to better fit the new structure of the paper. Therefore, the numbers of some equations are shifted. The updated contents are colored in blue to differentiate with contents in the original manuscript. In what follows, we list the comments as numbered in the report and provide an answer to each in italic font.

1. The main problem is that what the authors are really reporting are technical results on time-dependent intensity measurements when oscillating magnetic fields are present in the arms of the interferometer. No genuine weak measurements are made –the imaginary part of the weak value appears by rewriting the intensities at the output ports in a form (eq 12) that appears ad-hoc because there is motivation on the context. There is absolutely no discussion of why this form of extracting weak values is particularly interesting (eg, is it more efficient, more precise etc relative to previous methods?) and to which specific problems it could be applied (eg, are there problems in which deducing complex weak values from time dependent intensity measurements would be crucial?)

   We addressed this comment in Sec. 1 and Sec. 2.2. In the second paragraph of Sec. 1, after citing different experiments which utilize the imaginary part of the weak value, we highlighted the fact that in order to exploit its experimental potential effective methods of extraction are needed. Then, in the third paragraph, we emphasized the main advantages of the method: it allows for the simultaneous extraction, it does not require the measurement of an auxiliary state, it can be extended to an arbitrary number of paths, and the working principles of this experiment are transferable to any other kind of interferometry in which alike time-dependent phases are feasible. For comparison, citations of alternative methods of extraction which do not present these features have been provided[15, 16, 17, 18, 19], including the references suggested by the Referee in comments 2 and 3. The advantages are also more clearly mentioned in the abstract. In the second paragraph in Sec. 2.2 we further expanded on the method not presenting the usual formalism of the weak measurements.

2. The authors have published a recent work on the same topic, not cited in the Bibliography, https://doi.org/10.1038/s41598-024-76167-6. A discussion of the differences/similarities with that work would be expected.

   A discussion is now present in the third paragraph of Sec. 1.

3. As an interest to the readers, the authors should at least mention (and optionally briefly discuss) the situation for similar experiments in quantum optics, in which genuine weak measurements along both arms can be undertaken simultaneously, eg https://www.nature.com/articles/s42005-023-01317-7 and https://www.nature.com/articles/s41534-020-00350-6

   The papers have been cited in the third paragraph of Sec. 1.

4. It would be nice if the authors could clarify when the weakness of the couplings is needed (up to eq 11 the treatment seems to be valid for any strength) and what is the weakness parameter – from the analysis done in Sec. 3.4 the effective weakness parameter seems to be alpha, so it would be useful to say so in the theory part.

   In the second paragraph in Sec. 2.2, we now specify that the weakness of the coupling ensures that the state is minimally disturbed during the interaction and that the parameter indicating the interaction strength is α. This kind of weak coupling is in line with the treatment of the other mentioned experiments.

5. I further note that alpha is about 0.2 in the experiment and this is not really small: a comment is in order.

> *At the end of Sec. 3.4, we now mention that the fidelity between the initial state and the state after the interaction is* $\gtrsim 0.96$ *for the selected values of* α_i*, confirming the weakness of the interaction.*

In conclusion, we once again thank the Referee for the feedback, which has contributed to improving the quality of the manuscript. We hope that all the comments of Reviewer 3 have been duly addressed and that the revised manuscript will be recommended for publication.