Chromatic aberration

When the rays of white light (light having waves of different wavelengths) parallel to the principle axis are incident on a lens, the different colors are refracted by different amounts or are dispersed into various colors, and are focused at different distances from the lens as shown in Fig. 1 and 2.

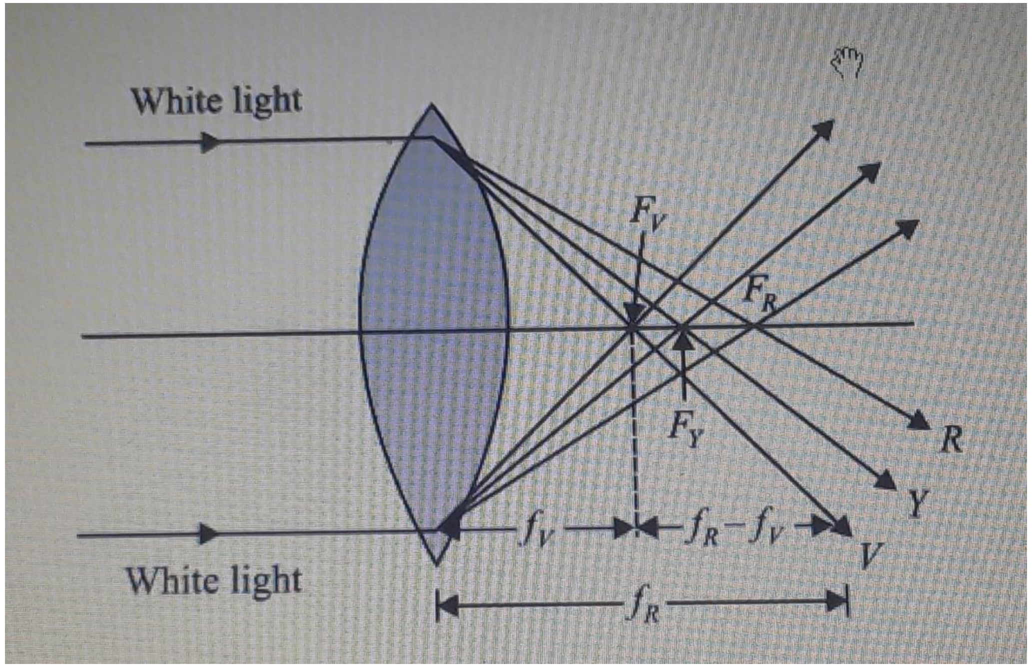


Fig. 1

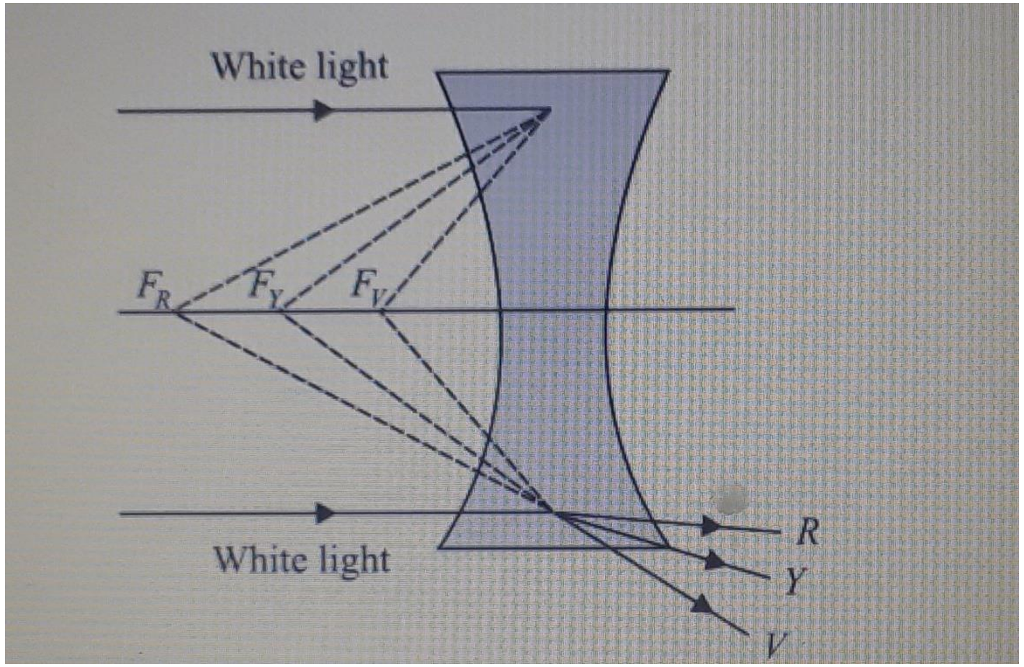
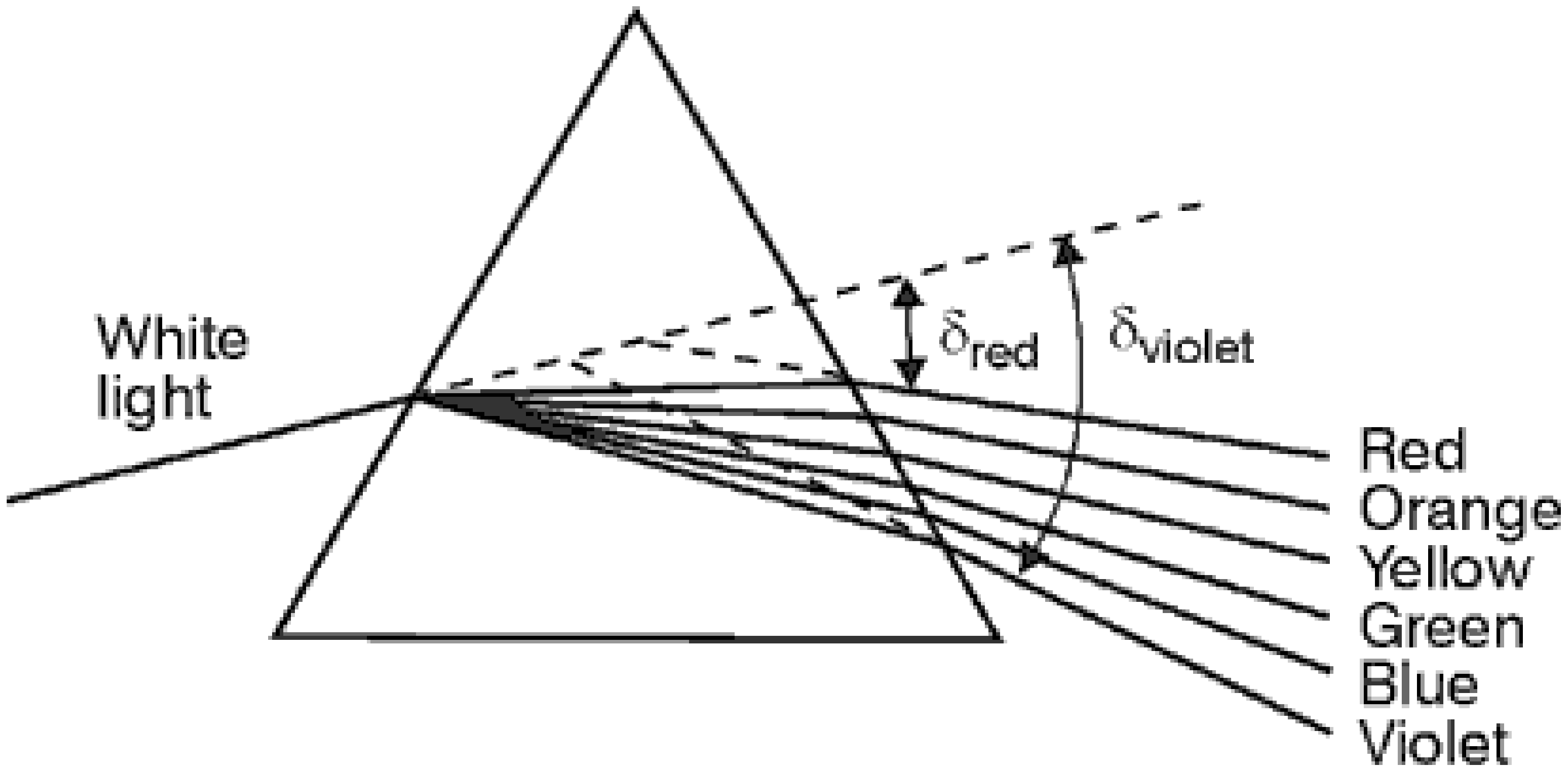


Fig. 2



Angle of deviation (δ) is the angle between emergent ray and incident ray.

$\delta = (\mu - 1)A$, where n is the refractive index and A is the prism angle

$$\delta = \left(\frac{c}{v} - 1\right)A = \left(\frac{c}{n\lambda} - 1\right)A$$

This is due to the dependence of the refractive index of the material of the lens on the wavelength of light. The refractive index of the material of the lens (glass) is greater for violet rays than that of the red rays, i.e., $n_V > n_R$ and $f_V > f_R$ (as the focal length of the lens is inversely proportional to the refractive index). Since violet light gets refracted more than red light, the point at which the violet light would focus is nearer the lens than the point at which the red light would focus. Thus, the inability of a lens to form a single image of the white object is called *chromatic aberration*. The image of white object formed by the lens is usually colored and blurred.

Types of chromatic aberration

(i) Longitudinal or axial chromatic aberration

(ii) Lateral chromatic aberration

Longitudinal or axial chromatic aberration

The spreading of an image along the axis (principal axis), or the formation of images of different colors at different positions along the axis is called *axial or longitudinal chromatic aberration*. The axial distance between the positions of red and violet images is a measure of longitudinal or axial chromatic aberration as shown in Fig. 3. The quantity $(I_R - I_V)$ is the measure of this aberration.

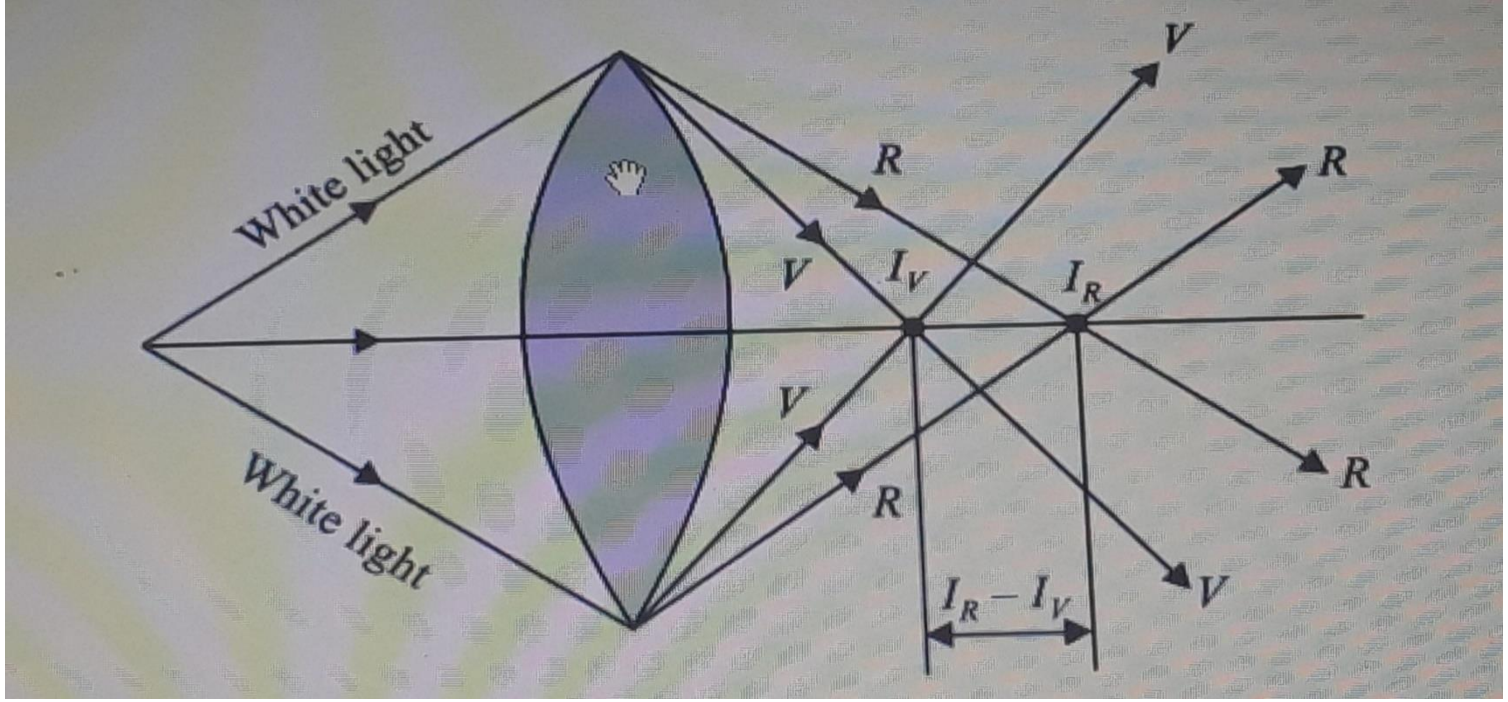


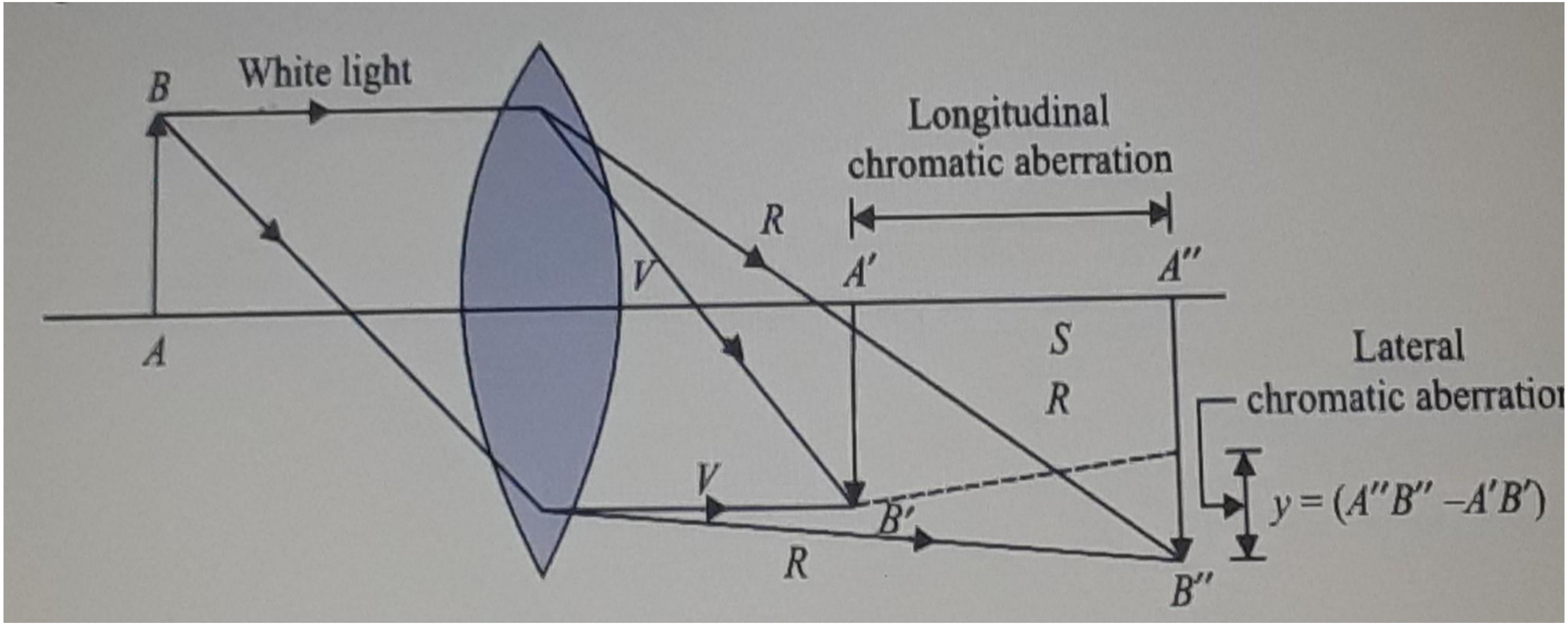
Fig. 3

If an object is situated at infinity, then the longitudinal chromatic aberration becomes equal to the difference in the focal lengths for red and violet colors, i.e., equal to $(f_R - f_V)$ as shown in Fig. 1 and 2. The longitudinal chromatic aberration is positive for a convex lens and is negative for a concave lens because of sign convention.

Lateral chromatic aberration

The magnification $[m = \frac{f}{(u+f)}]$ produced by a lens depends on the focal length of the lens. Therefore, when the white object (finite size) is placed on the axis of the lens, the images of different colors do not focus at different sizes as shown in Fig. 4.

Thus, the formation of images of different sizes for different wavelengths (colors) due to a variation of the lateral magnification with the wavelength is called *lateral chromatic aberration*. The sizes of violet and red images are $A'B'$ and $A''B''$ respectively as shown in Fig. 4. The distance $y = A''B'' - A'B'$ is the measure of the lateral chromatic aberration.



B
White light
Longitudinal
chromatic aberration
R
A′
A″
V
A
S
R
Lateral
chromatic aberration
V
$y = (A''B'' - A'B')$
B′
R
B″

Fig. 4

Calculation of longitudinal chromatic aberration of thin lens

Let us calculate the longitudinal chromatic aberration for thin lens in two situations:

(i) For an object at infinity

(ii) For an object an finite distance

(i) For an object at infinity: If the white object is situated at infinity, the images of different colors are focused at different focal points on the principal axis of the lens. The focal length of a thin lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{1}$$

Where n is the refractive index of the material of the lens and R_1 and R_2 are the radii of the curvature of the two surfaces of the lens.

If f_V, f_R and f_Y are the focal length of the lens for violet, red and yellow colors respectively and n_V, n_R and n_Y are their refractive indices for the material of the lens, then

$$\frac{1}{f_V} = (n_V - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (2)$$

$$\frac{1}{f_R} = (n_R - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (3)$$

$$\frac{1}{f_Y} = (n_Y - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad (4)$$

Subtracting Eq. (3) from Eq. (2), we get

$$\frac{1}{f_V} - \frac{1}{f_R} = [(n_V - 1) - (n_R - 1)]\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_V} - \frac{1}{f_R} = [n_V - 1 - n_R + 1]\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_V} - \frac{1}{f_R} = (n_V - n_R)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{f_R - f_V}{f_V f_R} = (n_V - n_R)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad (5)$$

From Eq. (4), we get $$\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{(n_Y - 1)f_Y}$$

Substituting this value of $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ in Eq. (5), wet get

$$\frac{f_R - f_V}{f_V f_R} = \frac{(n_V - n_R)}{(n_Y - 1)f_Y} \quad \Rightarrow \quad \frac{f_R - f_V}{f_Y^2} = \frac{\omega}{f_Y}$$

$$[\because f_V f_R = f_Y^2 \text{ and } \omega = \frac{(n_V - n_R)}{(n_Y - 1)}]$$

$$f_R - f_V = \omega f_Y \qquad (6)$$

Where ω is the dispersive power of the material of the lens and $f_R - f_V$ is the measure of the longitudinal chromatic aberration. Thus, the longitudinal or axial chromatic aberration for parallel rays is equal to the product of the dispersive power and the mean focal length of the lens.