Spherical Aberration

Consider a point source O of monochromatic light placed on the axis of a large aperture convex lens as shown in Fig.1.

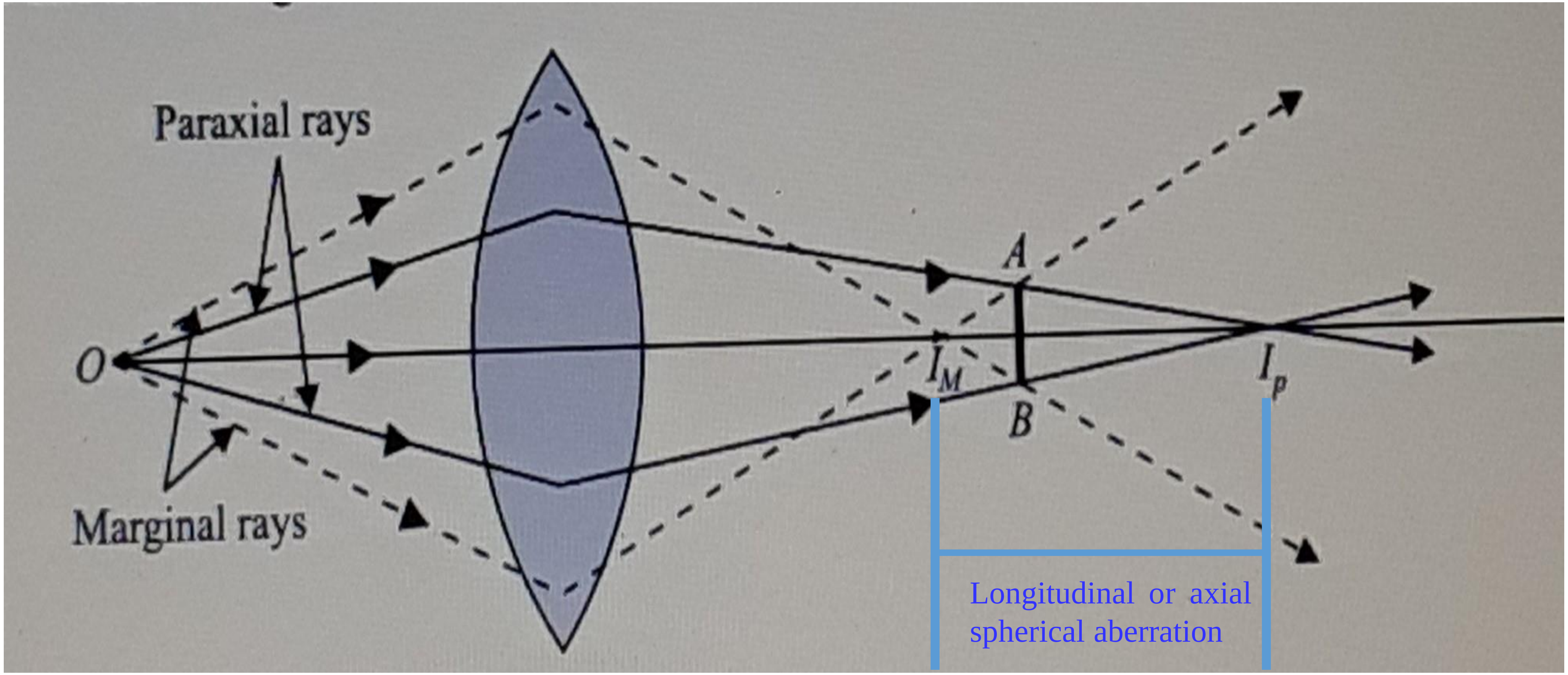

Fig. 1: Spherical aberration

The rays, which are incident near the axis (called paraxial rays) come to focus at point I_P, while the rays incident near the rim of the lens (called marginal or peripheral rays) come to focus at I_M. The intermediate rays are brought to focus between I_M and I_P. It is clear from Fig. 1, that the paraxial rays from the image focus at a point distance than marginal rays. Thus, the image is not sharp at any point on the axis. The failure or inability of the lens to form a point image of an axial point object is called ***spherical aberration***.

Reason

The Lens can be supposed to be divided into circular zones. It can be proved mathematically that the focal lengths slightly vary with the radius of the zone, i.e., different zones have different focal lengths. The focal length of the marginal rays is lesser than the paraxial zone, hence the marginal rays are focused first. The spherical aberration can also be explained by saying that the marginal rays suffer greater deviation than the paraxial rays, because they are incident at a greater height than the later. The distance between I_M and I_P is the measure of ***longitudinal or axial spherical aberration.***

Spherical aberration due to spherical surface

Let AB be the spherical surface of radius R. Let an incident ray parallel to principal axis meet the spherical surface at a height h from the principal axis. The refracted ray intersects the axis at a point F_h(Fig. 1). Here OF_h is equal to the focal length f_h for rays in zone of height h.

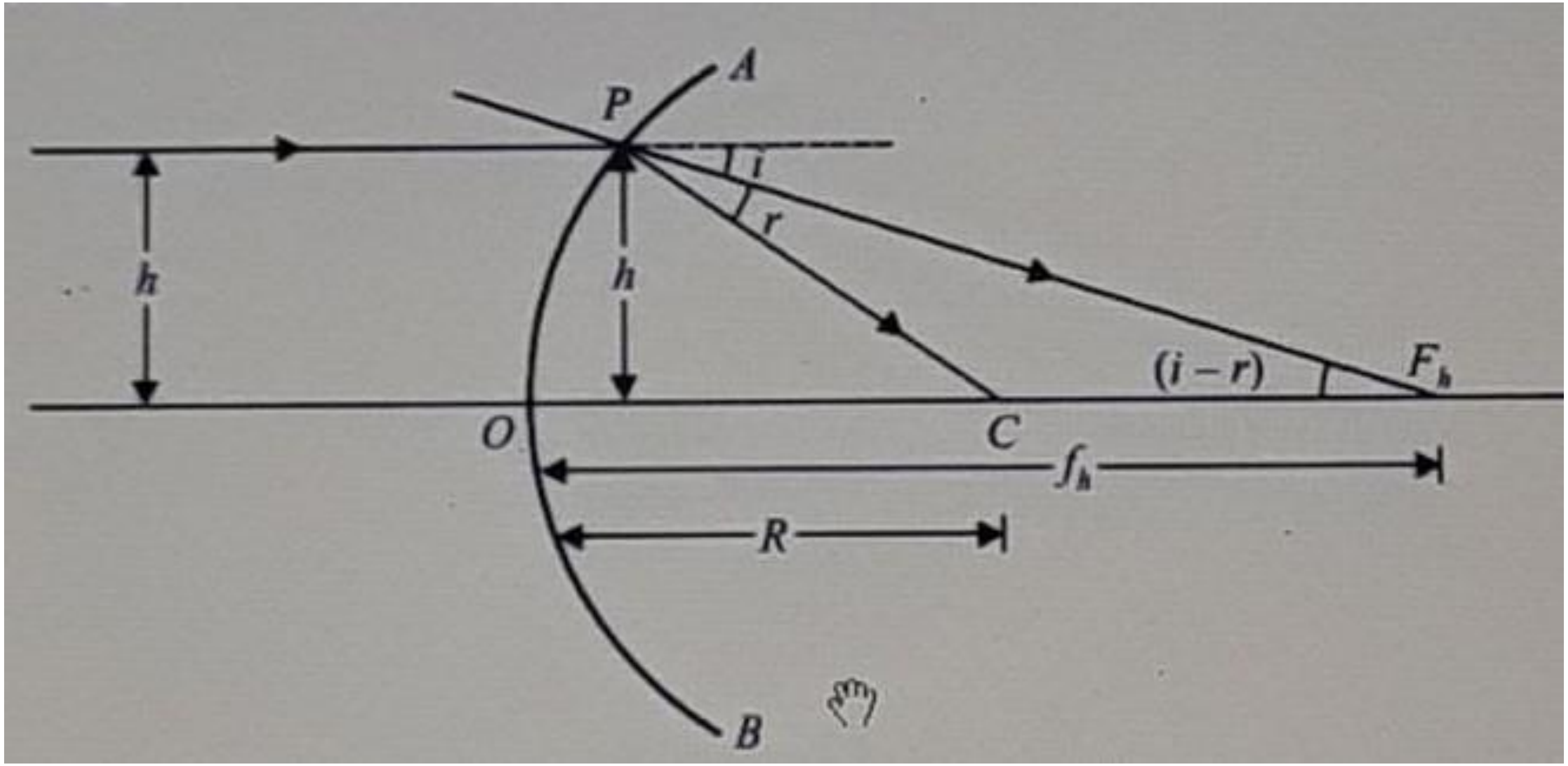

Fig.1. Spherical aberration due to spherical surface

From Fig. 1 $$f_h = R + CF_h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Now, in ΔCPF_h, we have $$\frac{CF_h}{sinr} = \frac{R}{sin(i-r)}$$

$$\Rightarrow \quad CF_h = \frac{Rsinr}{sin(i-r)} = \frac{Rsinr}{sinicosr - cosisinr}$$

$$= \frac{Rsinr}{nsinrcosr - cosisinr} \left[\because n = \frac{sini}{sinr}\right]$$

$$= \frac{R\times sinr}{sinr(ncosr - cosi)} = \frac{R}{ncosr - cosi} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting the value of CF_h from equation (2) in equation (1), we get

$$f_h = R + \frac{R}{ncosr - cosi} = R\left[1 + \frac{1}{ncosr - cosi}\right]$$

$$f_h = R + \frac{R}{ncosr - cosi} = R\left[\frac{ncosr - cosi + 1}{ncosr - cosi}\right] \ldots\ldots\ldots\ldots (3)$$

For paraxial rays

In the limiting case, when $h \to 0,\ cosi$ and $cosr$ tend to 1

In this case, $$f_h = f_p = \frac{nR}{n-1} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

which is equal to $f_h = f_p$. i.e., the focal length of paraxial rays

The change in focal length for h zone as compared to axial zone is given as

$$\Delta f_h = f_p - f_h = \frac{nR}{n-1} - R\left[1 + \frac{1}{ncosr - cosi}\right]$$

$$= R\left[\frac{n}{n-1} - 1 - \frac{1}{ncosr - cosi}\right] = R\left[\left(\frac{n}{n-1} - 1\right) - \frac{1}{ncosr - cosi}\right]$$

$$\Delta f_h = R\left[\frac{1}{n-1} - \frac{1}{ncosr - cosi}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

In general, this represents the longitudinal spherical aberration. The approximate value of spherical aberration can be calculated as follows:

From Fig. 1, we have $sini = \frac{h}{R}$

Or $sinr = \frac{h}{nR}$ $\left[\because n = \frac{sini}{sinr}\right]$

Now,

$$cosi = \sqrt{1 - sin^2 i} = \sqrt{1 - \frac{h^2}{R^2}} = \left(1 - \frac{h^2}{R^2}\right)^{1/2} = 1 - \frac{h^2}{2R^2}$$ (By binomial expansion)

and

$$cosr = \sqrt{1 - sin^2 r} = \sqrt{1 - \frac{h^2}{n^2R^2}} = \left(1 - \frac{h^2}{n^2R^2}\right)^{1/2} = 1 - \frac{h^2}{2n^2R^2}$$ (By binomial expansion)

Substituting these values in equation (5), we get

$$\Delta f_h = R\left[\frac{1}{n-1} - \frac{1}{n\left(1-\frac{h^2}{2n^2R^2}\right)-\left(1-\frac{h^2}{2R^2}\right)}\right] = R\left[\frac{1}{n-1} - \frac{1}{n-\frac{h^2}{2nR^2}-1+\frac{h^2}{2R^2}}\right] =$$

$$R\left[\frac{1}{n-1} - \frac{1}{(n-1)+\frac{h^2}{2R^2}\left(1-\frac{1}{n}\right)}\right] = R\left[\frac{1}{n-1} - \frac{1}{(n-1)+\frac{h^2}{2nR^2}(n-1)}\right]$$

Solving it, we get $$\Delta f_h = \frac{h^2}{2(n-1)^2 f_p} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(6)}$$

With $f_p = \frac{nR}{n-1}$

Equation (6) gives the approximate value of the spherical aberration due to spherical surface

Minimization of spherical aberration

The following are the methods for minimizing the spherical aberration

(i) **By means of stops**: We know that the spherical aberration is due to different focal lengths of different zones. The spherical aberration can be minimized by using stops (Fig. 1). The stops may be of such a nature that they permit only the axial rays and stop the marginal rays or permits the marginal rays and stop the axial rays. This method is not generally used, since the intensity of the image is very much reduced.

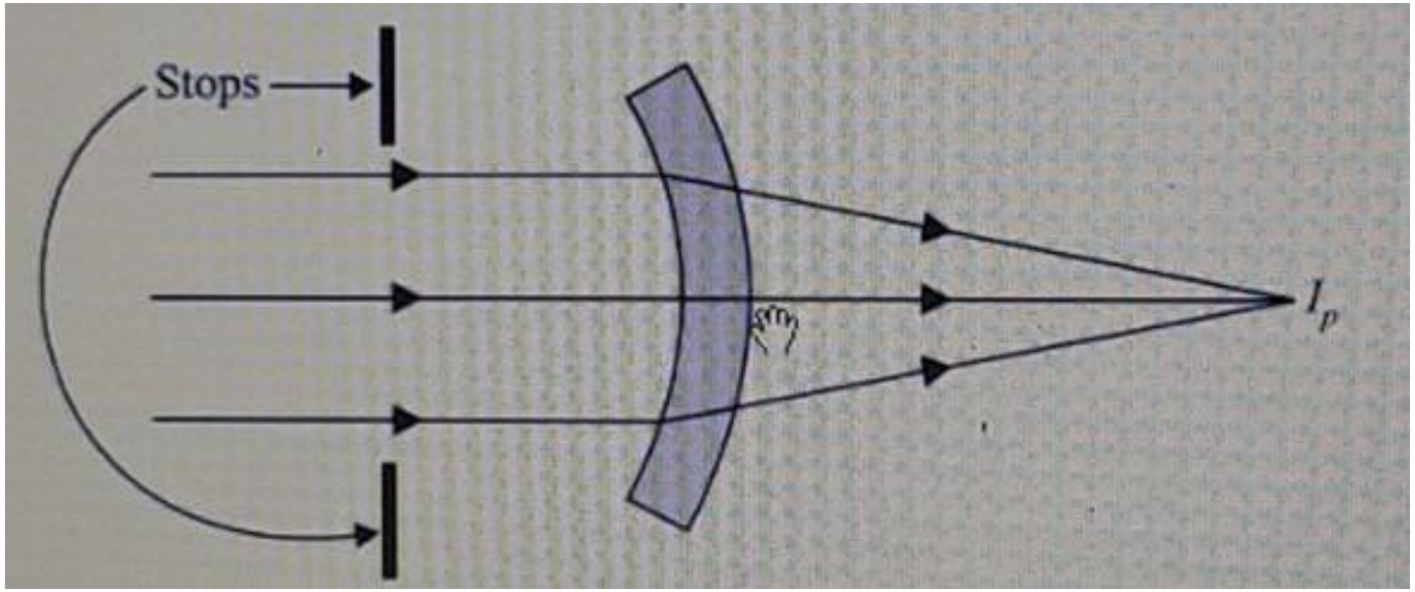

Fig.1. Use of stops

(ii) **By using two suitable lenses in contact**: It has been observed that in the case of a convex lens, the marginal image I_M lies towards the left of paraxial image I_P, while in case of a concave lens, the marginal image I_M lies towards the right of the paraxial image I_P. Thus, by a suitable combination of two lenses, the spherical aberration may be minimized. The difficulty with this combination is that it works only for a particular pair of object and image for which it is designed.

(iii) **Using planoconvex lens**: Spherical aberration can also be made minimum by using two planoconvex lenses separated by a distance equal to the difference in their focal lengths. In this case, the total deviation produced, is equally divided among the deviations, which are produced by two lenses. *This condition is for minimum spherical aberration.*

Let f_1 and f_2 be the focal lengths of two lenses and d, the separation between two lenses as shown in Fig. 1.

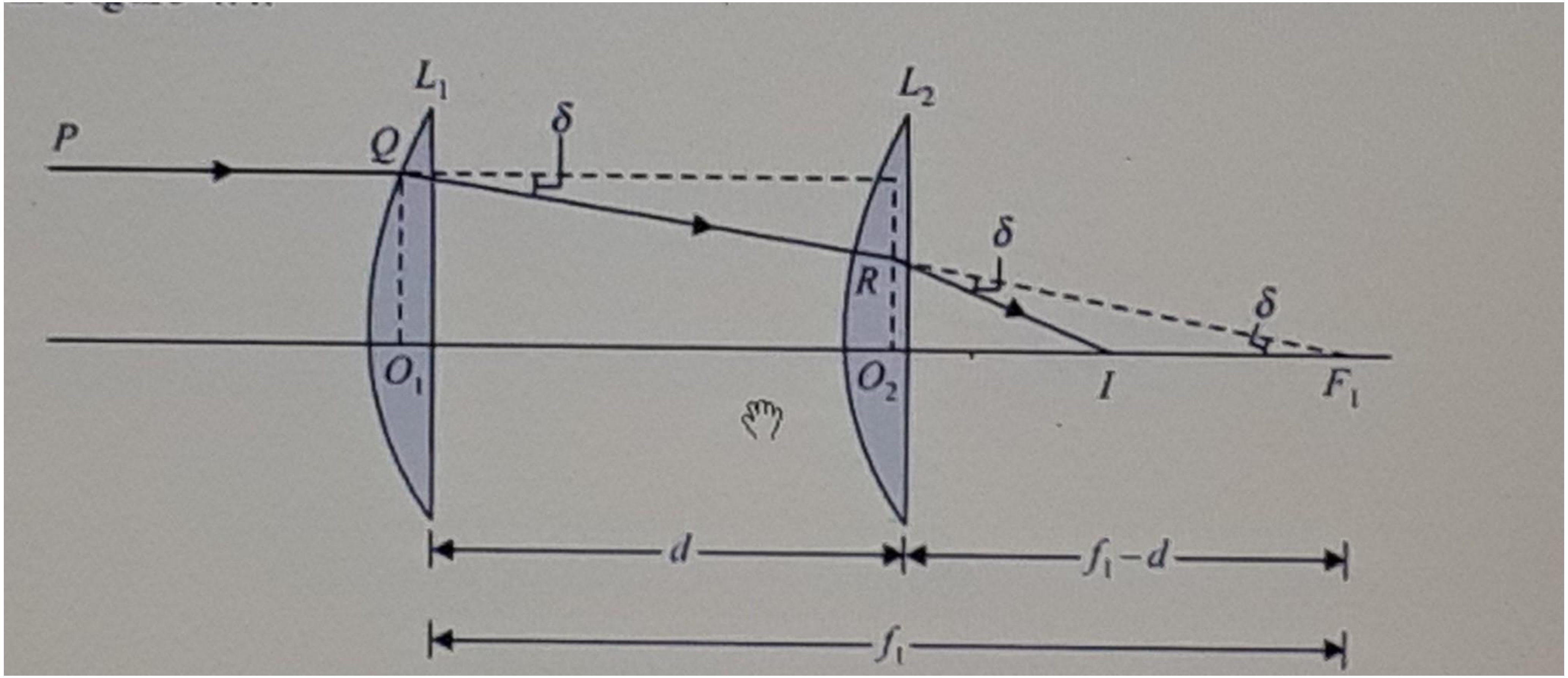

Fig. 1. Illustration for minimum spherical aberration by using planoconvex lens.

The deviations produced by the two lenses are equal to δ. A ray of light PQ parallel to the principal axis is incident on the first lens L_1 and after refraction, it deviates through an angle δ. The refracted ray from L_1 is incident on the second lens L_2 and after refraction, it deviates through the same angle δ and intersects the axis at I. From Fig. 1,

$$< RF_1I = \delta = < IRF_1$$

and from ΔRIF_1 $$RI = IF_1$$

or $$O_2I = IF_1\,(approximately)$$

$$\therefore\ O_2I = \frac{1}{2}O_2F_1 = \frac{1}{2}(f_1 - d) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

For the second lens, F_1 is the virtual object and I is the real image. Therefore,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

or $$\frac{1}{O_2I} - \frac{1}{O_2F_1} = \frac{1}{f_2} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Substituting the values of equation (1) in equation (2), we get

$$\frac{1}{(f_1-d)/2} - \frac{1}{f_1-d} = \frac{1}{f_2} \qquad \text{or} \qquad \frac{2}{f_1-d} - \frac{1}{f_1-d} = \frac{1}{f_2}$$

or $$\frac{1}{f_1-d} = \frac{1}{f_2} \qquad \text{or} \qquad f_2 = f_1 - d \qquad \text{or} \qquad f_1 - f_2 = d \qquad \ldots\ldots\ldots\ldots (2)$$

Thus, it is the ***condition for spherical aberration to be minimum***. This property is used in eyepieces.

Exercise: Two thin lenses of focal lengths 16 cm and 12 cm form a combination which is corrected for spherical aberration. Find the distance between the principal points of the combination.