MATH 415: Exam 2 Prep

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Question 1:

LU Factorization

All the steps with MATLAB codes and outputs presented below:

```
% Question 1.

A = [1 5 16;0 4 5;2 2 4]

A =

     1     5    16
     0     4     5
     2     2     4

I3 = eye(3)

I3 =

     1     0     0
     0     1     0
     0     0     1


P1 = I3([3 2 1],:)

P1 =

     0     0     1
     0     1     0
     1     0     0

P1*A

ans =

     2     2     4
     0     4     5
     1     5    16

L1 = I3; L1(2,1)=0; L1(3,1) = -.5

L1 =

    1.0000         0         0
         0    1.0000         0
   -0.5000         0    1.0000

L1*P1*A

ans =

     2     2     4
     0     4     5
     0     4    14


P2 = I3
```

```
P2 =

     1     0     0
     0     1     0
     0     0     1

P2*L1*P1*A

ans =

     2     2     4
     0     4     5
     0     4    14

L2 = I3; L2(3,2) = -1

L2 =

     1     0     0
     0     1     0
     0    -1     1

L2*P2*L1*P1*A

ans =

     2     2     4
     0     4     5
     0     0     9


U = ans

U =

     2     2     4
     0     4     5
     0     0     9

P = P2*P1

P =

     0     0     1
     0     1     0
     1     0     0

L = P*inv(P1)*inv(L1)*inv(P2)*inv(L2)

L =

    1.0000         0         0
         0    1.0000         0
    0.5000    1.0000    1.0000
```

```
P*A-L*U

ans =

     0     0     0
     0     0     0
     0     0     0


%Looks a correct solution!
```

Question 2:

We will use the two-phase Simplex method to solve the given LP problem.

Phase-I

```
    %Question 3:

%Phase-1

% Given Information
A = [1 -3 -1 2 -1 0;1 2 0 -1 0 1]

A =

     1    -3    -1     2    -1     0
     1     2     0    -1     0     1

b = [-1;2]

b =

    -1
     2

c = [0;0;0;0;1;1]

c =

     0
     0
     0
     0
     1
     1

u = [1;1;1;1; Inf; Inf]

u =

     1
     1
     1
     1
   Inf
   Inf



%Initial basis x5 and x6
basis = [5 6]

basis =

     5     6
```

```
nonbasis0 = [1 2 3 4]

nonbasis0 =

     1     2     3     4

nonbasisu = []

nonbasisu =

     []

basisupdateu

x =

     0
     0
     0
     0
     1
     2


z =

     3


reducedatzero =

     0    -5    -1     3
     1     2     3     4


reducedatupper =

     []



%x2 enters basis, by Dantzig's rule
enteringvar = 2

enteringvar =

     2


a = A(:,enteringvar)

a =

    -3
     2

d = solveBxb(L, U, p, a)
```

```
d =

     3
     2


eps2 = 1.0e-4 ; eps3 = 1.0e-4

eps3 =

   1.0000e-04


[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

    0.3333


leavingvar =

     5


leavingbound =

     0


%x5 leaving the basis and leaving bound 0, so x5 entering to nonbasis0
%group

basis = [2 6]

basis =

     2     6

nonbasis0= [1 3 4 5]

nonbasis0 =

     1     3     4     5

nonbasisu = []

nonbasisu =

     []


basisupdateu

x =

         0
    0.3333
```

```
         0
         0
         0
    1.3333


z =

    1.3333


reducedatzero =

   -1.6667    0.6667   -0.3333    1.6667
    1.0000    3.0000    4.0000    5.0000


reducedatupper =

     []


%x1 enters basis, by Dantzig's rule
enteringvar = 1

enteringvar =

     1


a = A(:,enteringvar)

a =

     1
     1

d = solveBxb(L, U, p, a)

d =

   -0.3333
    1.6667



[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

    0.8000


leavingvar =

     6
```

```
leavingbound =

     0


%x6 leaving the basis and leaving bound 0, so x1 entering to nonbasis0
%group

basis = [1 2]

basis =

     1     2

nonbasis0= [1 3 4 5 6]

nonbasis0 =

     1     3     4     5     6

nonbasisu = []

nonbasisu =

     []

basisupdateu

x =

    0.8000
    0.6000
         0
         0
         0
         0


z =

     0


reducedatzero =

     0     0     0     1     1
     1     3     4     5     6


reducedatupper =

     []


% The LP is optimal and z = 0. So, we can use x1 and x2 as our initial
% basis for phase 2
```

We found optimal solution for phase-1 where $z = 0$. So, we have to use x_1, x_2 as the starting basis for phase-2.

Phase-II

The standard form with slack variable is:

```
    % Phase 2:


% Given Information
A = [1 -3 -1 2;1 2 0 -1]

A =

     1    -3    -1     2
     1     2     0    -1

b = [-1;2]

b =

    -1
     2

c = [1;1;0;0]

c =

     1
     1
     0
     0

u = [1;1;1;1]

u =

     1
     1
     1
     1



%Initial basis x1 and x2
basis = [1 2]

basis =

     1     2

nonbasis0 = [3 4]

nonbasis0 =

     3     4

nonbasisu = []

nonbasisu =
```

```
     []

basisupdateu

x =

    0.8000
    0.6000
         0
         0


z =

    1.4000


reducedatzero =

    0.2000    0.4000
    3.0000    4.0000


reducedatupper =

     []
```