MATH 415: Assignment 4

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Question 1:

The standard equation form with slack variables:

$$\begin{aligned} \text{minimize} \quad & z = -x_1 - x_2 \\ \text{subject to} \quad & x_1 + 2x_2 + x_3 = 2, \\ & 2x_1 + 3x_2 + x_4 = 4, \\ & x_1, x_2 \leq 1, \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

All the steps with MATLAB codes and outputs presented below:

```
    % Given Information
A = [1 2 1 0;2 3 0 1]

A =

     1     2     1     0
     2     3     0     1

b = [2;4]

b =

     2
     4

c = [-1;-1;0;0]

c =

    -1
    -1
     0
     0

u = [1; 1; Inf; Inf]

u =

     1
     1
   Inf
   Inf

%Initial basis x3 and x4
basis = [3 4]

basis =

     3     4

nonbasis0 = [1 2]

nonbasis0 =
```

```
     1     2

nonbasisu = []

nonbasisu =

     []

basisupdateu

x =

     0
     0
     2
     4


z =

     0


reducedatzero =

    -1    -1
     1     2


reducedatupper =

     []

%x1 enters basis, by Dantzig's rule
enteringvar = 1

enteringvar =

     1


a = A(:,enteringvar)

a =

     1
     2

d = solveBxb(L, U, p, a)

d =

     1
     2


eps2 = 1.0e-4 ; eps3 = 1.0e-4
```

```
eps3 =

   1.0000e-04


[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

     1


leavingvar =

     0


leavingbound =

     1

%x1 reached it's upper limit. So, we will consider x1 as nonbasisu

basis = [3 4]

basis =

     3     4

nonbasis0= [2]

nonbasis0 =

     2

nonbasisu = [1]

nonbasisu =

     1


basisupdateu

x =

     1
     0
     1
     2


z =

    -1


reducedatzero =
```

```
    -1
     2


reducedatupper =

    -1
     1

%x2 enters, increasing
enteringvar=2

enteringvar =

     2


a = A(:,enteringvar)

a =

     2
     3

d = solveBxb(L, U, p, a)

d =

     2
     3


[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

    0.5000


leavingvar =

     3


leavingbound =

     0

%x2 increase all the way to 0.5 and x3 is the leaving variable goes to 0
basis = [2 4]

basis =

     2     4

nonbasisu = [1]
```

```
nonbasisu =

     1

nonbasis0 = [3]

nonbasis0 =

     3

basisupdateu

x =

    1.0000
    0.5000
         0
    0.5000


z =

   -1.5000


reducedatzero =

    0.5000
    3.0000


reducedatupper =

   -0.5000
    1.0000

% As we are getting positive entry for reducedatzero values and negative
% entry for reducedatupper value, the LP is in it's optimal solution.

%Checking:
A*x-b

ans =

     0
     0

%So, this LP is optimized.
```

We got an optimal solution for this LP where objective function $z = -1.5$ and we have to use $x_1 = 1$, $x_2 = 0.5$, $x_3 = 0$, $x_4 = 0.5$.

Question 2:

We will use the two-phase Simplex method to solve the given LP problem.

Phase-I

The standard equation form:

$$\begin{aligned}
\text{minimize} \quad & x_5 + x_6 \\
\text{subject to} \quad & x_1 + 2x_2 + x_3 - x_4 + x_5 = 7, \\
& -3x_1 + x_2 + x_3 + 3x_4 + x_6 = 9, \\
& 0 \leq x_1 \leq 1, \\
& 0 \leq x_2 \leq 5, \\
& 0 \leq x_3 \leq 8, \\
& 0 \leq x_4 \leq 1, \\
& x_5, x_6 \geq 0
\end{aligned}$$

```
    %Phase-1

% Given Information
A = [1 2 1 -1 1 0;-3 1 1 3 0 1]

A =

     1     2     1    -1     1     0
    -3     1     1     3     0     1

b = [7;9]

b =

     7
     9

c = [0;0;0;0;1;1]

c =

     0
     0
     0
     0
     1
     1

u = [1;5;8;1; Inf; Inf]

u =

     1
     5
     8
     1
   Inf
   Inf
```

```
%Initial basis x5 and x6
basis = [5 6]

basis =

     5     6

nonbasis0 = [1 2 3 4]

nonbasis0 =

     1     2     3     4

nonbasisu = []

nonbasisu =

     []

basisupdateu

x =

     0
     0
     0
     0
     7
     9


z =

    16


reducedatzero =

     2    -3    -2    -2
     1     2     3     4


reducedatupper =

     []

%x2 enters basis, by Dantzig's rule
enteringvar = 2

enteringvar =

     2


a = A(:,enteringvar)

a =
```

```
     2
     1

d = solveBxb(L, U, p, a)

d =

     2
     1


eps2 = 1.0e-4 ; eps3 = 1.0e-4

eps3 =

   1.0000e-04


[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

    3.5000


leavingvar =

     5


leavingbound =

     0

%x5 leaving the basis and leaving bound 0, so x5 entering to nonbasis0
%group
basis = [2 6]

basis =

     2     6

nonbasis0= [1 3 4 5]

nonbasis0 =

     1     3     4     5

nonbasisu = []

nonbasisu =

     []


basisupdateu
```

```
x =

         0
    3.5000
         0
         0
         0
    5.5000


z =

    5.5000


reducedatzero =

    3.5000   -0.5000   -3.5000    1.5000
    1.0000    3.0000    4.0000    5.0000


reducedatupper =

     []

%x4 enters basis, by Dantzig's rule
enteringvar = 4

enteringvar =

     4


a = A(:,enteringvar)

a =

    -1
     3

d = solveBxb(L, U, p, a)

d =

   -0.5000
    3.5000

[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

     1


leavingvar =

     0
```

```
leavingbound =

     1


% as non of the basis is becomem 0 if we increase x4 to it's highest limit.
% x4 enters the nonbasisu

basis = [2 6]

basis =

     2     6

nonbasis0= [1 3 5]

nonbasis0 =

     1     3     5

nonbasisu = [4]

nonbasisu =

     4


basisupdateu

x =

     0
     4
     0
     1
     0
     2


z =

     2


reducedatzero =

    3.5000   -0.5000    1.5000
    1.0000    3.0000    5.0000


reducedatupper =

   -3.5000
    4.0000

%x3 enters basis, by Dantzig's rule
enteringvar = 3
```

```
enteringvar =

     3


a = A(:,enteringvar)

a =

     1
     1

d = solveBxb(L, U, p, a)

d =

    0.5000
    0.5000



[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

     4


leavingvar =

     6


leavingbound =

     0

%x6 is the leaving variable and leavingbound 0, so would consider in
%nonbasis0 group

basis = [2 3]

basis =

     2     3

nonbasis0= [1 6 5]

nonbasis0 =

     1     6     5

nonbasisu = [4]

nonbasisu =

     4
```

```
basisupdateu

x =

     0
     2
     4
     1
     0
     0


z =

     0


reducedatzero =

     0     1     1
     1     6     5


reducedatupper =

     0
     4

% The LP is optimal and z = 0. So, we can use x2 and x3 as our initial
% basis for phase 2
```

We found optimal solution for phase-1 where $z = 0$. So, we have to use x_2, x_3 as the starting basis for phase-2.

Phase-II

The standard form with slack variable is:

$$\begin{aligned} \text{minimize} \quad & x_1 - 3x_2 - 6x_3 + x_4 \\ \text{subject to} \quad & x_1 + 2x_2 + x_3 - x_4 = 7, \\ & -3x_1 + x_2 + x_3 + 3x_4 = 9, \\ & 0 \le x_1 \le 1, \\ & 0 \le x_2 \le 5, \\ & 0 \le x_3 \le 8, \\ & 0 \le x_4 \le 1 \end{aligned}$$

```
    % Given Information
A = [1 2 1 -1;-3 1 1 3]

A =

     1     2     1    -1
    -3     1     1     3
```

```
b = [7;9]

b =

     7
     9

c = [1;-3;-6;1]

c =

     1
    -3
    -6
     1

u = [1;5;8;1]

u =

     1
     5
     8
     1



%Initial basis x2 and x3
basis = [2 3]

basis =

     2     3

nonbasis0 = [1]

nonbasis0 =

     1

nonbasisu = [4]

nonbasisu =

     4

basisupdateu

x =

     0
     2
     4
     1


z =
```

```
   -29


reducedatzero =

   -29
     1


reducedatupper =

    31
     4

%x1 enters basis, by Dantzig's rule
enteringvar = 1

enteringvar =

     1


a = A(:,enteringvar)

a =

     1
    -3

d = solveBxb(L, U, p, a)

d =

     4
    -7



[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,d,u(enteringvar),eps2,eps3)

tlimit =

    0.5000


leavingvar =

     2


leavingbound =

     0

%x2 leaving variable and leaving bound 0, so x2 will consider as nonbasis0

basis = [1 3]
```

```
basis =

     1     3

nonbasis0 = [2]

nonbasis0 =

     2

nonbasisu = [4]

nonbasisu =

     4

basisupdateu

x =

    0.5000
         0
    7.5000
    1.0000


z =

  -43.5000


reducedatzero =

    7.2500
    2.0000


reducedatupper =

     2
     4

%The sign of x4 for reducedatupper is positive, so if we reduce x4 then objective function
%should decrease.

enteringvar = 4

enteringvar =

     4


a = A(:,enteringvar)

a =

    -1
     3
```

```
d = solveBxb(L, U, p, a)

d =

   -1.0000
   -0.0000


[tlimit,leavingvar,leavingbound] = ratiotestu(basis,xb,ub,-d,u(enteringvar),eps2,eps3)

tlimit =

    0.5000


leavingvar =

     1


leavingbound =

     0

%x1 is the leaving variable and leaving at 0

basis = [4 3]

basis =

     4     3

nonbasis0 = [1 2]

nonbasis0 =

     1     2

nonbasisu = []

nonbasisu =

     []

basisupdateu

x =

         0
         0
    7.5000
    0.5000


z =
```

```
  -44.5000


reducedatzero =

    2.0000    7.7500
    1.0000    2.0000


reducedatupper =

     []

% So, our LP is optimal.
```

We got an optimal solution for this LP where objective function $z = -44.5$ and we have to use $x_1 = 0$, $x_2 = 0$, $x_3 = 7.5$, $x_4 = 0.5$.