SECTION-A

Question numbers 1 to 6 carry 1 mark each

1. If A=$\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$,find α satisfying $0<\alpha<\frac{\pi}{2}$when $A+A^T$=$\sqrt{2}I_2$;where A^T is transpose of A.

2. For what values of k,the system of linear equations
   $x + y + z = 2$
   $2x + y - z = 3$
   $3x + 2y + kz = 4$
   has a unique solution?

3. If A is a 3×3 matrix and $|3A| = k|A|$, then write the value of k.

4. Write the sum of intercepts cut of by the plane $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) - 5 = 0$ on the three axes

5. Find λ and μ if
   $(\hat{i} + 3\hat{j} + 9\hat{k})\cdot(3\hat{i} - \lambda\hat{j} + \mu\hat{k})$=0

6. If $\vec{a}$=$4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$,then find a unit vector parallel vecter $\vec{a} = \vec{b}$

SECTION-B

Question numbers 7 to 19 carry 4 marks each.

7. Slove for $x : \tan^{-1}(x-1) + \tan^{-1} x + \tan^{-1}(x+1) = \tan^{-1} 3x$

OR

   Prove that $\tan^{-1}(\frac{6x-8x^3}{1-12x^2}) - \tan^{-1}(\frac{4x}{1-4x^2})$=$\tan^{-1} 2x; |2x|<\frac{1}{\sqrt{3}}$.

8. A typist charges ₹145 for typing 10 english and 3 hindi pages while charges 3 english and 10 hindi pages are ₹180 Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

9. If f(x) = $\begin{cases} \frac{\sin(a+1)x+2\sin x}{x}, & x<0 \\ 2, & x=0 \\ \frac{\sqrt{1+bx}-1}{x}, & x>0 \end{cases}$ is continuous at $x = 0$, then find the values of a and b.

10. If $x \cos a + y = \cos y$ then prove that $\frac{dy}{dx}$=[$\frac{\cos^2(a+y)}{\sin a}$].
hence show that $\sin a \frac{d^2y}{dx^2} + \sin 2(a+y)\frac{dy}{dx} = 0$

OR

Find $\frac{dy}{dx}$ if $y : wq = \sin^{-1}[\frac{6x-4\sqrt{1-4x^2}}{5}]$

11. Find the equation of tangents to the curve $y = x^3 + 2x - 4$ which perpendicular to line $x + 14y + 3 = 0$

12. Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}$dx.

OR

Find : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}$dx.

13. Evaluate : $\int_{-2}^{2} \frac{x^2}{1+5^x} dx$.

14. Find : $\int (x+3)\sqrt{3-4x-x^2} dx$

15. Find the particuler solution for differential equation : $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$ given that $y = 1$ when $x = 0$

16. Find the particular solution of thedifferential equation $2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$
given that $x = 0$ when $y = 1$.

17. Show that the four points $A(4,5,1)B(0,-1,-1)C(3,9,4)$and $D(-4,4,4)$ are coplanar.

18. Find the coordinates of the foot of perpendicular drawn from the point $A(-1,8,4)$ to the line joining the points $B(0,1,3)$ and $C(2,3,1)$. Hence find the image of the point A in the line BC.

19. A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

SECTION-C

Question numbers 20 to 26 carry 6 marks each.

20. Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.

21. $A = R \times R$ and * be a binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$
Show that is commutative and associative. Find the identity element for * on A. Also find the inverse of every element $(a, b) \epsilon A$.

22. Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ on $[0, \frac{\pi}{2}]$

OR

Shoe that semi-vertical angle of cone of maximum volume and given slant hight is $\cos^{-1}(\frac{1}{\sqrt{3}})$

23. Using the methode of integration. Find the area of the trianguler region whose vecters are $((2, -2)(4, 3)$ and$(1, 2)$

24. Find the euation of the plane which contains the line of intersection of the plane
$\vec{r} \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$
$\vec{r} \cdot (-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$
and whose intercept on x-axis is equal tothat of on y-axis.

25. A retired person wants to invest an amount of ₹$50,000$. His broker recommends investing in two type of bonds $'A'$ and $'B'$ yielding 10% and 9% return respectively on the invested amount. He decides to invest at least₹$20,000$ in bond $'A'$ and at least ₹$10,000$ in bond $'B'$. He also wants to invest at least as much in bond $'A'$ as in bond $'B'$. Solve this linear programming problem graphically to maximise his returns.

26. Using properties of determinant prove that
$$\begin{vmatrix} (x+y)^2 & zx & zy \\ zx & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz(x+y+z)^3$$

OR

27. If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$.and $A^3 - 6A^2 + 7A + kI_3 = 0$ find K.