Group No: 14

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Assignment No: 01

Instructor Name: Umair Umer

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QNo. 01

Sol:

Acc: to Question;

Output from;

Goods	Services	Purchased by
0.8	0.3	Services
0.2	0.7	Goods

Let;

P_g as amount output of goods sector.

P_s as amount output of services sector.

Equation for row one will be;

$$P_s = 0.8\, P_g + 0.3\, P_s$$

Equation for row two will be

$$P_g = 0.2 P_g + 0.7 P_s$$

To solve system of equations move all the unknown to left

$$-0.8\, P_g + 0.7 P_s = 0 \longrightarrow I$$

$$0.8 P_g + (-0.7) P_s = 0 \longrightarrow II$$

Augmented Matrix for eqn ① & ⑪ will be

$$\begin{bmatrix} -0.8 & 0.7 & : & 0 \\ 0.8 & -0.7 & : & 0 \end{bmatrix}$$

Adding R_2 by $8R_1$

$$\begin{bmatrix} -0.8 & 0.7 & : & 0 \\ 0 & 0 & : & 0 \end{bmatrix}$$

Dividing R_1 by -0.8

$$\begin{bmatrix} 1 & -0.875 & : & 0 \\ & 0 & : & 0 \end{bmatrix}$$

The general solution is $P_g = 0.875\ P_s$ and work. P_s is feed.
Equilibrium will remain on effected prices until the ratio $P_g = 0.875\ P_s$ -
Consider $P_s = 1000$. than $P_g = 0.875 \times 1000$
$P_g = 875$ -

Question # 02;

Sol:

Consider P_s = 300 million dollor
than $P_c = 0.94\ P_s$, $P_c = 0.85\ P_s$
$P_c = 282$, $P_c = 255$
No! then will be no change on equilibrium until and unless the ratio are not stasfied

Question # 03;

⇒ Sol:

Output from:			Purchased By
Chemicals	fuel	Machinery	
0.2	0.8	0.4	Chemicals
0.3	0.9	0.4	fuel
0.5	0.9	0.2	Machinery

let;
P_c is amount out for chemical sector
P_f is amount out for fuel sector
P_m is amount out for Machinery sector-

Equation for row 1 will be;
$P_c = 0.2P_c + 0.8P_f + 0.4P_m$

Equation for row 2 will be;
$P_f = 0.3P_c + 0.1P_f + 0.4P_m$

Equation for row 3 will be;
$P_m = 0.5P_c + 0.4P_f + 0.2P_m$

To solve system of all equations move all unknown to left;

$0.8P_c + -0.8P_f - 0.4P_m = 0$
$-0.3P_c + 0.9P_f - 0.4P_m = 0$
$-0.5P_c + 0.1P_f + 0.2P_m = 0$

Augmented Matrix will be;

$$\begin{bmatrix} 0.8 & -0.8 & -0.4 & : & 0 \\ -0.3 & 0.9 & -0.4 & : & 0 \\ -0.5 & 0.1 & 0.2 & : & 0 \end{bmatrix}$$

Multiply each Row by 10

$$\begin{bmatrix} 8 & -8 & -4 & : & 0 \\ -3 & 9 & -4 & : & 0 \\ -5 & 1 & 2 & : & 0 \end{bmatrix}$$

Divide R_1 by 8

$$\begin{bmatrix} 1 & -1 & -0.5 & : & 0 \\ -3 & 9 & -4 & : & 0 \\ -5 & -1 & 8 & : & 0 \end{bmatrix}$$

$R_2 + 3R_2$; $R_3 + (-5R_1)$

$$\begin{bmatrix} 1 & -1 & -0.5 & : & 0 \\ 0 & 6 & -5.5 & : & 0 \\ 0 & -6 & 5.5 & : & 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 0.5 & 0 \\ 0 & 1 & -0.917 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 + 6R_2$$

$$\left[\begin{array}{ccc:c} 1 & -1 & 0.5 & 0 \\ 0 & 1 & -0.917 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{ccc:c} 1 & 0 & -1.412 & 0 \\ 0 & 1 & -0.9417 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The General Solution is;

$P_c = 1.1412\ P_m$

$P_f = 0.9417\ P_m$

$P_m =$ free variable

If $P_m = 100$

$P_c = 1.147\ (100) = 114.7$

$P_f = 0.9417\ (100) = 94.17$

$P_m = 100$

Question - 04

Output				Purchased By
A	E	M	T	
0.65	0.30	0.30	0.20	A
0.10	0.10	0.15	0.10	E
0.25	0.36	0.15	0.30	M
0	0.25	0.40	0.40	T

Denoting total annual outcome of sector by P_a, P_e, P_m & P_t

Equation for row 1:

$$P_a = 0.65P_a + 0.3P_e + 0.30P_m + 0.20P_t$$

Eqn for row: 2, 3, 4 respectively are:

$$P_e = 0.10P_a + 0.10P_e + 0.15P_m + 0.10P_t$$

$$P_m = 0.25P_a + 0.35P_e + 0.15P_m + 0.30P_t$$

$$P_t = 0.25P_a + 0.40P_m + 0.40P_t$$

Augmented Matrix will be;

$$\begin{bmatrix} 0.65 & 0.3 & 0.30 & 0.20 : 0 \\ 0.10 & 0.10 & 0.15 & 0.10 : 0 \\ 0.25 & 0.35 & 0.15 & 0.30 : 0 \\ 0.25 & 0.40 & 0.40 & 0 : 0 \end{bmatrix}$$

Dividing r_1 by 0.35

$$\begin{bmatrix} 1 & -0.86 & -0.86 & 0.52 : 0 \\ 0 & 0.81 & -2.4 & -1.6 : 0 \\ 0 & 0 & 1.0 & -1.17 : 0 \\ 0 & 0 & 0 & 0 : 0 \end{bmatrix}$$

Dividing r_a by 0.81

$$\left[\begin{array}{cccc:c} 1 & -0.86 & -0.86 & 0.57 & 0 \\ 0 & 1 & -2.96 & 1.98 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 + 0.86\, R_2 \rightarrow R_1$

$$\left[\begin{array}{cccc:c} 1 & 0 & 2.55 & 0.97 & 0 \\ 0 & 1 & -2.96 & 1.98 & 0 \\ 0 & 0 & 1.0 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_2 + 2.96\, R_3$ & $R_1 - 2.55\, R_3$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & -0.97 & 0 \\ 0 & 1 & 0 & -1.48 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General Solution;

$P_a = 2.03\, pt$, $P_e = 0.53\, pt$, $P_m = 1.17\, pt$

& pt = free variable

if $pt = 100$ than

$P_a = 203$; $P_e = 053$; $P_m = 117$

Question # 05

Sol:

$$B_2S_3\begin{bmatrix}2\\3\\0\\0\end{bmatrix}, \quad H_2O\begin{bmatrix}0\\0\\2\\1\end{bmatrix}, \quad H_3BO_3\begin{bmatrix}1\\0\\3\\3\end{bmatrix}, \quad H_2S\begin{bmatrix}0\\1\\2\\0\end{bmatrix}$$

- boron (B)
- Sulphur (S)
- Hydrogen (H)
- Oxygen (O)

The co-efficient of equation

$$x_1 B_2S_3 + x_2 H_2O \rightarrow x_3 H_3BO_3 + x_4 H_2S$$

$$x_1\begin{bmatrix}2\\3\\0\\2\end{bmatrix} + \begin{bmatrix}0\\3\\2\\0\end{bmatrix} \rightarrow \begin{bmatrix}1\\0\\3\\3\end{bmatrix} + \begin{bmatrix}0\\1\\2\\0\end{bmatrix}$$

Using the right term to the left side; and writing in augmented form;

$$\begin{bmatrix}2 & 0 & -1 & 0 & : & 0\\ 3 & 0 & 0 & -1 & : & 0\\ 0 & 2 & -3 & -2 & : & 0\\ 0 & 1 & -3 & 0 & : & 0\end{bmatrix}$$

$R_1 + 2$; $2R_2 - 3R_1$

$$\begin{bmatrix}1 & 0 & -1/2 & 0 & : & 0\\ 0 & 0 & 3/2 & -1 & : & 0\\ 0 & 2 & -3 & -2 & : & 0\\ 0 & 1 & -3 & 0 & : & 0\end{bmatrix}$$

- Exchanging R_2 & R_3

$$\begin{bmatrix}1 & 0 & 1/2 & 0 & : & 0\\ 0 & 2 & -3 & -2 & : & 0\\ 0 & 0 & 3/2 & -1 & : & 0\\ 0 & 1 & -3 & 0 & : & 0\end{bmatrix}$$

Dividing R_2 by 2 ~~$R_2 \times 2/3$~~

$$\begin{bmatrix}1 & 0 & -1/2 & 0 & : & 0\\ 0 & 1 & -3/2 & -2 & : & 0\\ 0 & 0 & 3/2 & -1 & : & 0\\ 0 & 1 & -3 & 0 & : & 0\end{bmatrix}$$

$R_2 + R_3$: $R_2 \times 2/3$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & : & 0 \\ 0 & 1 & 0 & -2 & : & 0 \\ 0 & 0 & 1 & -2/3 & : & 0 \\ 0 & 0 & 3 & -2 & : & 0 \end{bmatrix}$$

$1/2\ R_3 + R_1$: $R_2 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & -1/3 & : & 0 \\ 0 & 1 & 0 & -2 & : & 0 \\ 0 & 0 & 1 & -2/3 & : & 0 \\ 0 & 0 & 3 & -2 & : & 0 \end{bmatrix}$$

$3R_3 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 1/3 & : & 0 \\ 0 & 1 & 0 & -2 & : & 0 \\ 0 & 0 & 1 & -2/3 & : & 0 \\ 0 & 0 & 0 & 0 & : & 0 \end{bmatrix}$$

The General Solution is;

$x_1 = 1/3\ x_4$

$x_2 = 2x_4$

$x_3 = 2/3\ x_4$

with x_4 = free

let $x_4 = 3$

than:

$x_1 = 1$; $x_2 = 6$; $x_3 = 1$

The Balance Equation will be;

$$B_2S_2 + 6H_2O \rightarrow 2H_3BO_3 + 3H_2S$$

Question # 6

$$Al_2O_3 + C \longrightarrow Al + CO_2$$

$$(x_1)\,Al_2O_3 + (x_2)\,C \longrightarrow (x_3)\,Al + (x_4)\,CO_2$$

$Al_2O_3 \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $C \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $Al \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $CO_2 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$ - Aluminium, - Oxygen, - Carbon

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

$$2R_2 + (-3)(R_1) \Rightarrow \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 3 & -4 & 0 \\ 0 & 1 & 0 & -1 & 0 \end{bmatrix}$$

interchange : $R_2 \Leftrightarrow R_3 \Rightarrow$ $\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 3 & -4 & 0 \end{bmatrix}$

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Scale R_3 by $1/3 \Rightarrow$ $\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$

$R_1 + 1(R_3) \Rightarrow$ $\begin{bmatrix} 2 & 0 & 0 & -4/3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & -4/3 & 0 \end{bmatrix}$

$R_3 + (-6)R_4 \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 0 & -6 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

$R_2 + 2(R_4) \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & 0 & -2 & 0 \\ 0 & 0 & -6 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

Scale R_3 by $(-1/6) \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & 0 & -2 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

$R_2 + 2(R_3) \Rightarrow$ $\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 8 & 0 & 0 & -8/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

Scale R_2 by $(1/8) \Rightarrow$ $\begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1/3 & 0 \\ 0 & 0 & 1 & 0 & -1/3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

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General Solution:

$x_1 = x_5$, $x_1 = 1/3\, x_5$

$x_3 = 1/3\, x_5$, $x_4 = x_5$

x_5 is free variable:

Take $x_5 = 3$, so, $x_1 = 3$, $x_2 = 1$

$x_3 = 1$, $x_4 = 3$

putting values in eq (1)

$$3NaHCO_3 + H_3C_6H_5O_7 \longrightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$$

balanced - Ans!

$$x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} + x_3\begin{bmatrix}-3\\-5\\-6\\-7\end{bmatrix} + x_4\begin{bmatrix}0\\-2\\0\\-1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\-1\\-2\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 1 & 8 & -5 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0\end{bmatrix}$$

$R_2 + (-1)(R_1) \Rightarrow$

$$\begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 0 & 8 & -2 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0\end{bmatrix}$$

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$R_4 + (-3)R_1 \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 6 & -3 & 0 & -1 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}$

$4R_3 + (-3)(R_2) \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 0 & -6 & 6 & -4 & 0 \\ 0 & 7 & 2 & -1 & -2 & 0 \end{bmatrix}$

$8R_4 + (-7)(R_2) \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 0 & -6 & 6 & -4 & 0 \\ 0 & 0 & 30 & 6 & -16 & 0 \end{bmatrix}$

$R_4 + 5R_3 \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 \\ 0 & 8 & -2 & -2 & 0 & 0 \\ 0 & 0 & -6 & 6 & -4 & 0 \\ 0 & 0 & 0 & 36 & -36 & 0 \end{bmatrix}$

Scale R_4 by $(1/36) \Rightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 0 \\ 0 & 8 & -2 & 0 & 0 \\ 0 & 0 & -6 & -4 & 0 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$

Question #07

Sol:

$NaHCO_3$: $\begin{bmatrix}1\\1\\1\\3\end{bmatrix}$, $H_3C_6H_5O_7$: $\begin{bmatrix}0\\8\\6\\7\end{bmatrix}$, $Na_3C_6H_5O_7$: $\begin{bmatrix}3\\5\\6\\7\end{bmatrix}$

H_2O : $\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$, CO_2 : $\begin{bmatrix}0\\0\\1\\2\end{bmatrix}$

Sodium (s)
hydrogen (h)
Carbon (c)
Oxygen (O)

The Co-efficient of equation

$x_1 \cdot NaHCO_3 + x_2 \cdot H_3C_6H_5O_7 \rightarrow x_3 \cdot Na_3C_6H_5O_7 + x_4 H_2O + x_5 \cdot CO_2$

The co-efficient x_1, x_2, x_3, x_4, x_5 satisfy the vector equation;

$$x_1\begin{bmatrix}1\\1\\1\\3\end{bmatrix} + x_2\begin{bmatrix}0\\8\\6\\7\end{bmatrix} = x_3\begin{bmatrix}3\\5\\6\\7\end{bmatrix} + x_4\begin{bmatrix}0\\2\\0\\1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\1\\2\end{bmatrix}$$

Using the Eight term to the left side; & reducing the augmented Matrix

$$\begin{bmatrix}1 & 0 & -3 & 0 & 0 & 0\\ 1 & 8 & -5 & -2 & 0 & 0\\ 1 & 6 & -6 & 0 & -1 & 0\\ 3 & 7 & -7 & -1 & -2 & 0\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 0 & 0 & -1 & :0\\ 0 & 1 & 0 & 0 & -1/3 & :0\\ 0 & 0 & 1 & 0 & -1/3 & :0\\ 0 & 0 & 0 & 1 & -1 & :0\end{bmatrix}$$

The General solution is;

$x_1 = x_5$

$x_2 = 1/3\, x_5$

$x_3 = 1/3\, x_5$

$x_4 = x_5$

x_5 = free variable

The balanced eqi. is;

$$3NaHCO_3 + H_3C_6H_5O_7 \rightarrow Na_3C_6H_5O_7 + 3H_2O + 3CO_2$$

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Question # 8

$$H_3O + CaCO_3 \longrightarrow H_2O + Ca + CO_2$$

$$(x_1)H_3O + (x_2)CaCO_3 \Rightarrow (x_3)H_2O + (x_4)Ca + x_5(CO_2) \quad -(1)$$

$$H_3O\begin{bmatrix}3\\1\\0\\0\end{bmatrix}, CaCO_3\begin{bmatrix}0\\3\\1\\1\end{bmatrix} + H_2O\begin{bmatrix}2\\1\\0\\0\end{bmatrix}, Ca\begin{bmatrix}0\\0\\1\\0\end{bmatrix}, CO_2\begin{bmatrix}0\\2\\0\\1\end{bmatrix}$$

$$x_1\begin{bmatrix}3\\1\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\3\\1\\1\end{bmatrix} + x_3\begin{bmatrix}-2\\-1\\0\\0\end{bmatrix} + x_4\begin{bmatrix}0\\0\\-1\\0\end{bmatrix} + x_5\begin{bmatrix}0\\-2\\0\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}3 & 0 & -2 & 0 & 0 & 0\\ 1 & 3 & -1 & 0 & -2 & 0\\ 0 & 1 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\end{bmatrix}$$

Interchange $R_1 \leftrightarrow R_2$ =

$$\begin{bmatrix}1 & 3 & -1 & 0 & -2 & 0\\ 3 & 0 & -2 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\end{bmatrix}$$

$$R_2 + (-3)R_1 \Rightarrow \begin{bmatrix}1 & 3 & -1 & 0 & -2 & 0\\ 0 & -9 & 1 & 0 & 6 & 0\\ 0 & 1 & 0 & -1 & 0 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\end{bmatrix}$$

$$9R_3 + R_2 \Rightarrow \begin{bmatrix}1 & 3 & -1 & 0 & -2 & 0\\ 0 & -9 & 1 & 0 & 6 & 0\\ 0 & 0 & 1 & -9 & 6 & 0\\ 0 & 1 & 0 & 0 & -1 & 0\end{bmatrix}$$

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$9R_4+R_2 = \begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \end{bmatrix}$

$R_4+(-1)R_3 = \begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 0 & 0 & 9 & -9 & 0 \end{bmatrix}$

Scale R_4 by $(1/9) \Rightarrow \begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & -9 & 6 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

$R_3+9(R_4) \Rightarrow \begin{bmatrix} 1 & 3 & -1 & 0 & -2 & 0 \\ 0 & -9 & 1 & 0 & 6 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

$R_1+1(R_3) \Rightarrow \begin{bmatrix} 1 & 3 & 0 & 0 & -5 & 0 \\ 0 & -9 & 0 & 0 & 9 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

Scale R_2 by $(-1/9) \Rightarrow \begin{bmatrix} 1 & 3 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

$R_1+(-3)R_2 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}$

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General Solution:

$x_1 = 2x_5$, $x_2 = x_5$

$x_3 = 3x_5$, $x_4 = x_5$

x_5 is free variable:

Take $x_5 = 1$, so

$x_1 = 2$, $x_2 = 1$, $x_3 = 3$, $x_4 = 1$

putting values in (1)

$$2H_3O + CaCO_3 \longrightarrow 3H_2O + Ca + CO_2$$

↓

balanced

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Question No 9:

$$B_2S_3 + H_2O \longrightarrow H_3BO_3 + H_2S$$

$$(x_1)B_2S_3 + (x_2)H_2O \rightarrow (x_3)H_3BO_3 + (x_4)H_2S \quad -(1)$$

$$B_2S_3 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}, \quad H_2O \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \quad H_3BO_3 \begin{bmatrix} 1 \\ 0 \\ 3 \\ 3 \end{bmatrix}, \quad H_2S \begin{bmatrix} 0 \\ 1 \\ 2 \\ 0 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 2 \\ 3 \\ 0 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 0 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ -3 \\ -3 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ -1 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & -1 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix} \Rightarrow 2R_2 + (-3)R_1 = \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

Interchange: $R_2 \Leftrightarrow R_3 \Rightarrow$

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 1 & -3 & 0 & 0 \end{bmatrix}$$

$2R_4 + (-1)(R_2) \Rightarrow$

$$\begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & -3 & 2 & 0 \end{bmatrix}$$

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$R_4 + 1(R_3) \Rightarrow \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Scale R_3 by $(1/3) \Rightarrow \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & -3 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_2 + 3R_3 \Rightarrow \begin{bmatrix} 2 & 0 & -1 & 0 & 0 \\ 0 & 2 & 0 & -4 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$R_1 + 1(R_3) \Rightarrow \begin{bmatrix} 2 & 0 & 0 & -2/3 & 0 \\ 0 & 2 & 0 & -4 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Scale R_2 by $1/2 \Rightarrow \begin{bmatrix} 2 & 0 & 0 & -2/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Scale R_1 by $(1/2) \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -1/3 & 0 \\ 0 & 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -2/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

General Solution: $x_1 = 1/3\, x_4$, $x_2 = 2x_4$

$x_3 = 2/3\, x_4$

x_4 is free variable

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Take

$x_4 = 3$, $x_1 = 1$, $x_2 = 6$, $x_3 = 2$

Putting values in eq (1)

$$B_2S_3 + 6H_2O \longrightarrow 2H_3BO_3 + 3H_2S$$

Balanced

Question NO 10

$$PbN_6 + CrM_2O_8 \longrightarrow PB_3O_4 + Cr_2O_3 + MnO_2 + NO$$

$$(x_1)PbN_6 + (x_2)CrM_2O_8 \longrightarrow (x_3)PB_3O_4 + (x_4)Cr_2O_3 + (x_5)MnO_2 + (x_6)NO \longrightarrow eq(1)$$

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix} = x_3\begin{bmatrix}3\\0\\0\\0\\4\end{bmatrix} + x_4\begin{bmatrix}0\\0\\0\\2\\0\\3\end{bmatrix} + x_5\begin{bmatrix}0\\0\\0\\1\\2\end{bmatrix} + x_6\begin{bmatrix}0\\1\\0\\0\\1\end{bmatrix}$$

$$x_1\begin{bmatrix}1\\6\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\1\\2\\8\end{bmatrix} + x_3\begin{bmatrix}-3\\0\\0\\0\\-4\end{bmatrix} + x_4\begin{bmatrix}0\\0\\-2\\0\\-3\end{bmatrix} + x_5\begin{bmatrix}0\\0\\0\\-1\\-2\end{bmatrix} + x_6\begin{bmatrix}0\\-1\\0\\0\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}$$

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$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$$

$R_2 + (-6)R_1 \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$

Interchange $R_2 \leftrightarrow R_3 \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$

$R_4 + (-2)(R_2) \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +4 & -1 & 0 & 0 \\ 0 & 8 & -4 & -3 & -2 & -1 & 0 \end{bmatrix}$

$R_5 + (-8)R_2 \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & -4 & 13 & -2 & -1 & 0 \end{bmatrix}$

$9R_5 + (-2)R_3 \Rightarrow$ $\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 4 & -1 & 0 & 0 \\ 0 & 0 & 0 & 117 & -18 & -1 & 0 \end{bmatrix}$

Scale R_4 by $1/4 \Rightarrow$

$$\begin{bmatrix} 1 & 0 & -3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & -18 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1/4 & 0 & 0 \\ 0 & 0 & 0 & 117 & -18 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1/6 & 0 \\ 0 & 1 & 0 & 0 & 0 & -22/45 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/18 & 0 \\ 0 & 0 & 0 & 1 & 0 & -11/45 & 0 \\ 0 & 0 & 0 & 0 & 1 & -44/45 & 0 \end{bmatrix}$$

General Solution:

$x_1 = (1/6)x_6$, $x_2 = (22/45)x_6$

$x_3 = (1/18)x_6$, $x_4 = (11/45)x_6$, $x_5 = (44/45)x_6$

x_6 is free variable : $x_6 = 90$

$x_1 = 15$, $x_2 = 44$, $x_3 = 5$

$x_4 = 22$, $x_5 = 88$ putting back in eq (1)

$$15PbN_6 + 44CrMnO_8 \longrightarrow 5Pb_3O_4 + 22Cr_2O_3 + 88MnO_2 + 90NO$$

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Question NO 11

$$MnS + As_2Cr_{10}O_{35} + H_2SO_4 \rightarrow HMnO_4 + AsH_3 + CrS_3O_{12} + H_2O$$

$$(x_1)MnS + (x_2)As_2Cr_{10}O_{35} + (x_3)H_2SO_4 \rightarrow (x_4)HMnO_4 + (x_5)AsH_3 + (x_6)CrS_3O_{12} + (x_7)H_2O \quad -(1)$$

$$x_1\begin{bmatrix}1\\1\\0\\0\\0\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\\10\\35\\0\end{bmatrix} + x_3\begin{bmatrix}0\\1\\0\\0\\4\\2\end{bmatrix} + x_4\begin{bmatrix}-1\\0\\0\\0\\-4\\-1\end{bmatrix} + x_5\begin{bmatrix}0\\0\\-1\\0\\0\\-3\end{bmatrix} + x_6\begin{bmatrix}0\\-3\\0\\-1\\-12\\0\end{bmatrix} + x_7\begin{bmatrix}0\\0\\0\\0\\-1\\-2\end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}1&0&0&-1&0&0&0&0\\1&0&1&0&0&-3&0&0\\0&2&0&0&-1&0&0&0\\0&10&0&0&0&-1&0&0\\0&35&4&-4&0&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix}$$

$$R_2+(-1)(R_1) \Rightarrow \begin{bmatrix}1&0&0&-1&0&0&0&0\\0&0&1&1&0&-3&0&0\\0&2&0&0&-1&0&0&0\\0&10&0&0&0&-1&0&0\\0&35&4&-4&0&-12&-1&0\\0&0&2&-1&-3&0&-2&0\end{bmatrix}$$

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Interchange : $R_2 \Leftrightarrow R_3 \Rightarrow$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & -3 & 0 & 0 \\ 0 & 10 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 35 & 4 & -4 & 0 & -12 & -1 & 0 \\ 0 & 0 & 2 & -1 & -3 & 0 & -2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -26/327 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -13/327 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -374/327 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -16/327 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -26/327 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -130/327 & 0 \end{bmatrix}$$

General Solution:

$x_1 = (26/327)x_7$, $x_2 = (13/327)x_7$

$x_3 = (374/327)x_7$, $x_4 = (16/327)x_7$, $x_5 = (26/327)x_7$

$x_6 = (130/327)x_7$

x_7 is a free variable

Take $x_7 = 327$

$x_1 = 26$, $x_2 = 13$, $x_3 = 374$, $x_4 = 16$

$x_5 = 26$, $x_6 = 130$ putting back in (1)

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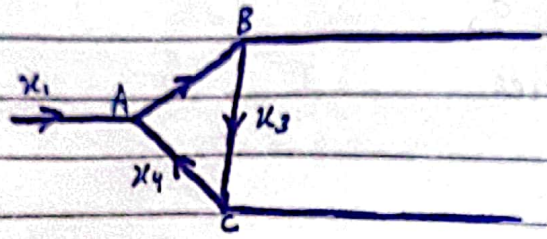
$16\,MnS + 13AsS_2Cr_{10}O_{35} + 374H_2SO_4 \rightarrow 16HMnO_4$

$+ 26AsH_3 + 130\,CrS_3O_{12} + 327HO$

Question NO # 12



Intersection	Flow in		Flow out
A	$x_1 + x_4$	=	x_2
B	x_2	=	$x_3 + 100$
C	$x_3 + 80$	=	x_4

$$x_1 - x_2 + x_4 = 0$$
$$x_2 - x_3 = 100$$
$$x_3 - x_4 = -80$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -80 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & -1 & 20 \\ 0 & 0 & 1 & -1 & 80 \end{bmatrix}$$

(a) General Sdution is

$$\begin{cases} x_1 = 80 - x_5 \\ x_2 = x_4 + x_5 - 180 \\ x_3 = x_4 + x_5 - 90 \\ x_4 \text{ is free} \\ x_5 \text{ is free} \end{cases}$$

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(b) Flow if x_5 closed?

If $x_5 = 0$, then general Sol =

$x_1 = 80$

$x_2 = x_4 - 180$

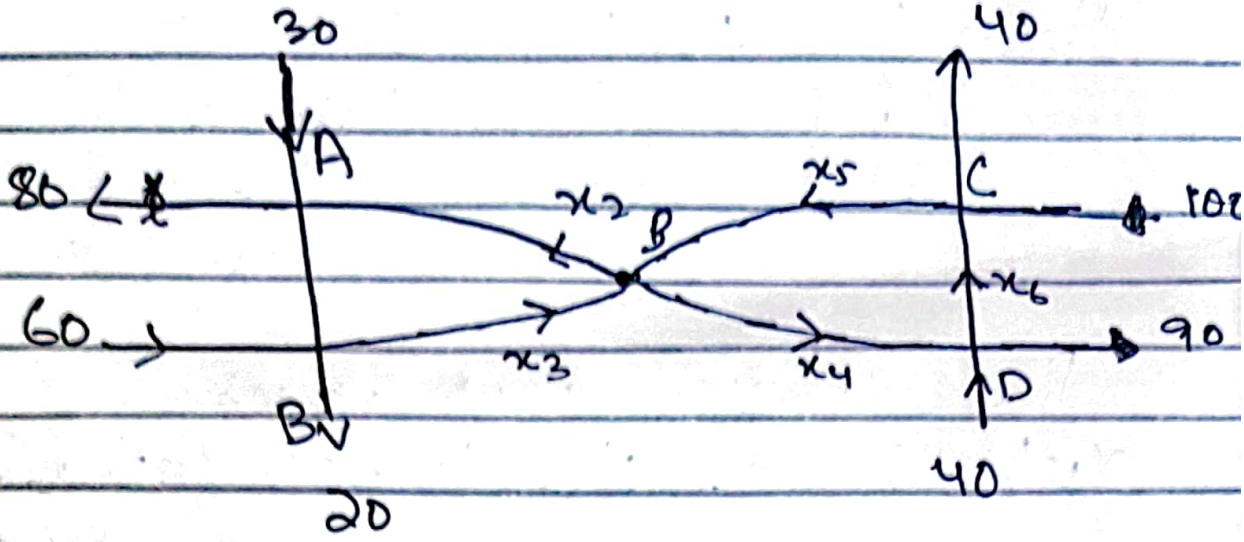
$x_3 = x_4 - 90$

x_4 is free

(c) When $x_5 = 0$, what is min value of x_4?

Since x_2 cannot be -ve, the min value of x_4 when $x_5 = 0$ is 180

Question No- #013



A: $x_2 + 30 = x_1 + 80$

B: $x_3 + x_5 = x_2 + x_4$

C: $x_6 + 100 = x_5 + 40$

D: $x_4 + 40 = x_6 + 90$

E: $x_1 + 60 = x_3 + 20$

Re-arrange the equations;

$x_1 - x_2 = -50$

$x_2 - x_3 + x_4 - x_5 = 0$

$x_5 - x_6 = 60$

$x_4 - x_6 = 50$

$x_1 - x_3 = -40$

Reduce Augmented Matrix

$$\left[\begin{array}{cccccc:c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & -1 & 0 & 0 & 0 & 0 & -40 \end{array}\right]$$

$$\left[\begin{array}{cccccc:c} 1 & -1 & 0 & 0 & 0 & 0 & -50 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccccc:c} 1 & 0 & -1 & 0 & 0 & 0 & -40 \\ 0 & 1 & -1 & 0 & 0 & 0 & 10 \\ 0 & 0 & 0 & 1 & 0 & -1 & 50 \\ 0 & 0 & 0 & 0 & 1 & -1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General Solution is;

$x_1 = x_3 - 40$

$x_2 = x_3 + 10$

x_3 is free variable

$x_4 = x_6 + 50$

$x_5 = x_6 + 60$

x_6 free variable.

Question # (14)

A: $x_1 = x_2 + 100$

B: $x_2 + 50 = x_3$

C: $x_3 = x_4 + 120$

D: $x_4 + 150 = x_5$

E: $x_5 = x_6 + 80$

F: $x_6 + 100 = x_1$

Re-arrange the eqn;

$x_1 - x_2 = 100$

$x_2 - x_3 = -50$

$x_3 - x_4 = 120$

$x_4 - x_5 = -150$

$x_5 - x_6 = 80$

$-x_1 + x_6 = -100$

Reduce the Augmented Matrix:

$$\left[\begin{array}{cccccc|c} 1 & -1 & 0 & 0 & 0 & 0 & 100 \\ 0 & 1 & -1 & 0 & 0 & 0 & -50 \\ 0 & 0 & 1 & -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 & -1 & 0 & -150 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ -1 & 0 & 0 & 0 & 0 & 1 & 100 \end{array}\right]$$

$$\left[\begin{array}{ccccc|cc} 1 & 0 & 0 & 0 & 0 & -1 & 100 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 50 \\ 0 & 0 & 0 & 1 & 0 & -1 & -70 \\ 0 & 0 & 0 & 0 & 1 & -1 & 80 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

General solution is;

$x_1 = 100 + x_6$

$x_2 = x_6$

$x_3 = 50 + x_6$

$x_4 = -70 + x_6$

$x_5 = 80 + x_6$

x_6 is free variable

Since;

x_4 is not -ve the value of x_6 is 70 -