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ASSIGNMENT #02

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Properties of Determinent:

Determinant:

It is a scalar value that is calculated from the elements of a square matrix.

In matrix, the vertical lines are columns and the horizontal lines are rows.

"n" order of the determinent has "n" number of rows and columns.

Determinent of 2x2 matrix:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

Determinant of 3x3 matrix:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \Rightarrow \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} =$$

$$a(ei-fh)-b(di-fg)-c(dh-eg)$$

Properties of a Determinent:

→ Change in the Value of determinant will not occur if rows and columns are interchanged but sign will be changed.

- If two rows and columns in determinent have same value, the determinent will be zero.
- If any variable says "K" is multiplied by rows and columns then it's value is also multipied by "K"
- If sum of all the elements of rows and colums are expressed as the sum of two or more terms, the the determinent can be expressed interms of two or more determinents

Reflection Property:

value of determineit is unchanged by interchanging rows and columns.

→

Case I	Case II
$\|M\| = 2(0-20)-(-3)(-4^2-4)+5(30-0)$	$= 2(-20)-6(2)(-25)+1(-12)$
$\|M\| = -40-138+150$	$= -40+24-12$
$= -28$	$= -28$

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$$M = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix} \rightarrow |M| = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix} = [$$

→ Switching Property:

If any two of rows or colums are interchanged the sign of determinent is changed. e.g;

$$A = \begin{bmatrix} 4 & 5 & 6 \\ 7 & 3 & 2 \\ 6 & 4 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4 & 5 & 6 \\ 7 & 3 & 2 \\ 6 & 4 & 3 \end{bmatrix}$$

Case 1:

$$|A| = 4(9-8) - 5(21-12) + 6(28-18)$$

$$|A| = 4 - 45 + 60$$

$$|A| = 19$$

Case:

$$\begin{bmatrix} 4 & 5 & 6 \\ 6 & 4 & 3 \\ 7 & 3 & 2 \end{bmatrix}$$

$$|A| = -4(8-9) - 5(12-21) + 6(18-28)$$

$$= 4(-1) - 5(-9) + 6(-10)$$

$$= -4 + 45 - 60$$

$$= -(19)$$

$$= -19 \text{ Ans}$$

Hence, sign of det is changed in Case 2.

→ All zero property:

If all the elements of any row or column are in matrix, than it's determinant is zero.

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 0 & 0 & 0 \\ 6 & 7 & 8 \end{bmatrix} = |A| = 0$$

Repetition of Proportionality:

If two rows and colums in a matrix are (zero) same then determinent of that matrix will be zero. e.g;

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 3 & 4 & 5 \\ 1 & 1 & 1 \end{bmatrix} = |A| = 0$$

Scalar Multiplication:

If elements of a raw or column of a determinant are multiplied by any non-zero constant the determinant also gets multiplied by the same constant.

e.g;

$$\begin{bmatrix} 12 & 9 & 6 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 3(4) & 3(3) & 3(2) \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{bmatrix} =$$

$$3 \times \begin{bmatrix} 4 & 3 & 2 \\ 13 & 4 & 5 \\ 6 & 8 & 9 \end{bmatrix}$$

So here, $K = 3$ "which is constant value."

- Sum property:

If elements of rows or columns of a determinant are expressed as sum of two or more terms then determinent can be expressed as a sum of two or more determinants.

e.g;

$$\begin{bmatrix} a & b & c \\ a+3q & b+7y & c+2z \\ x & y & z \end{bmatrix} = \begin{bmatrix} a & b & c \\ a & b & c \\ x & y & z \end{bmatrix} = \begin{bmatrix} a & b & c \\ 3q & 7y & 2z \\ x & y & z \end{bmatrix}$$

- Invariance Property:

Suppose any scalar multiples of corresponding elements of other two rows or columns are added to every element of any row or column of a determinat. In this case, the value of determinent stays the same.

e.g;

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$$= \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} a_1+kb_1 & + a_2+kb_2 & + a_3+kb_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

- Factor Property:

If determinant Δ becomes zero when we insert $x = \alpha$ then $(x-\alpha)$ is factor of Δ

e.g;

$$\Delta = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \Delta_1 = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$$

Here x_{ij} denoted the cofactor of then element a_{ij} of A matrix.

- Triangle property:

If the element below and above the diagonal are zero than the determinant is the product of diagonal elements.

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e.g;

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{bmatrix} \qquad |A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 4 & 0 \\ 3 & 5 & 6 \end{vmatrix}$$

So,

$$|A| = 0$$

Same if;

$$A = \begin{bmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 4 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 8 & 7 & 6 \\ 0 & 3 & 5 \\ 0 & 0 & 6 \end{vmatrix}$$

$$|A| = 0$$

- Deteminent of the co-factor matrix:

$$\sigma = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad \text{and} \quad \sigma 1 = \begin{vmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{vmatrix}$$

Here,

(σx^{x}) $\sigma 1$ represents the cofactors of the elements of a_{ij} in σ