

# SFP

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## 1 Game theory applied to transportation systems in smart cities

**The game** The car pooling system can be modelled as a game in which the organisation and car sharers are competing to maximize their benefits.

**Players** Player 1 is the organisation and car sharers are player 2.

### Strategies

- Player 1:  $X = S_R, S_P$  where  $S_R$  is the reward strategy and  $S_P$  is the punishment strategy.
- Player 2:  $Y = S_C, S_N$  where  $S_C$  is the cooperative strategy and  $S_N$  is the defection strategy.

**Nash Equilibrium using pure strategies**  $S_C$  is a best response for player 2, player 1's best response is  $S_R$ .  $A = (S_R, S_C)$  is a Nash Equilibrium.

**Nash Equilibrium using mixed strategies** Due to the unpredictable behavior of the players in reality, Nash Equilibrium is achieved mixing the strategies  $A = (S_R, S_P)$  with  $P = (p, 1 - p)$  and  $B = (S_C, S_N)$  with  $P = (q, 1 - q)$ .

**Evolutionary stability in games** Evolutionary game theory has set two conditions for a strategy  $S$  to be evolutionary stable, where  $E(S, T)$  represents the payoff for playing strategy  $S$  against strategy  $T$ .

- Strict Nash Equilibrium:  $E(S, S) > E(T, S)$
- Maynard Smith's second condition:  $E(S, S) = E(T, S)$  and  $E(S, T) > E(T, T)$

## 2 Mode Choice Models

Popular Models:

- Probit Models
- Binary Logit Models
- Multinomial Logit Models
- Nested Logit Models
- Ordered Logit Models

### 2.1 Logit Models

Logit models are used in mode choice models in transportation. Logit models are classified into two main categories namely binary and multinomial logit model.

#### 2.1.1 Binary Logit Models

Simple and Nested Binary Logit models are used when only two choices are available. The examples are presented in figures 1,2.

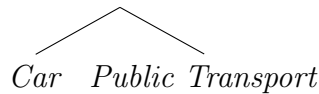


Figure 1: Binary Logit Model

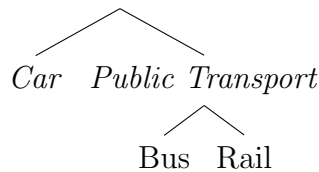


Figure 2: Nested Binary Logit Model

### 2.1.2 Multinomial Logit Models

Instead of a limited number of choices, multinomial logit models imply a larger set of alternatives. Multinomial Logit Models are categorised into simple and nested multinomial logit models. The examples are presented in figures 3,4.

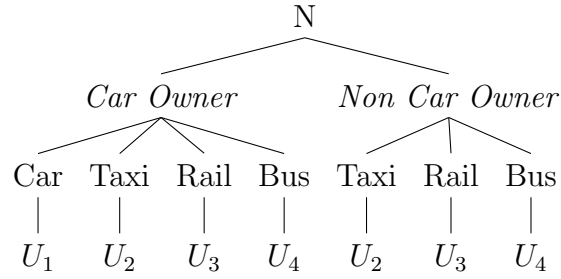


Figure 3: MTO Game