A Game Theoretic Approach for Travel Mode Choice

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Dedicated to

Travel Mode Choice Modeling using Game theory

Abstract

Acknowledgment

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Introduction

The grow of private vehicle use causes congestion that eventually increases travel time, that pushes governments to motivate people towards public transport. However, more usage of public transport should be followed by an improvement of public transport services.

In order to determine a transport demand analysis, it is essential to make a travel mode behavior model, where a demand is the accumulation of individuals decisions. The most important element in modeling a transport system is the mode split model, which provides a mathematical framework of the choices a traveler can have of which mode of transport is more suitable.

The different objectives in travel mode choice leads to the urge of applying game theory for making decisions based on finding the equilibrium of the passenger choices. However, game theory is rarely used in the transport field with several obstacles appearing in the transport characteristics.

The purpose of this study is to describe how travelers adjust their mode of transport choice behaviors using an evolutionary game model. Chapter 1 describes the basic theories applied in this work, including game theory concepts and evolutionary dynamics. In Chapter 2, key studies from the literature regarding travel choice behavior are briefly examined. The third chapter describes the model used in this study. Limitations of the proposed modeling method and further research directions are discussed at the end.

Chapter 1

Game Theory and Evolutionary Dynamics

1.1 Game Theory

Game theory is a branch of applied mathematics that derives mathematical models to predict the outcome of competitive interactions between two or more rational decision makers. A game may involve:

- Common interest (coordination);
- Competing interests (rivalry);
- Rational behavior: players can do the best they can, in their own eyes;
- a rational decision in a game must be based on a prediction of others' responses;

1.1.1 Defining Games

A game is the interaction between rational players, where the decisions of some players changes the payoff of others. A Game consists of four parts : Players, Actions, strategies and Payoffs.

- Players are the decision makers and they can be : People, Governments or Companies;
- Actions are decisions that the players make;
- Strategies are composed of actions;
- Payoffs are the outcomes which players receive as a result of their decisions and those of their opponents;

Games can be represented using two methods : Normal forms and Extensive Forms:

1.1.1.1 Extensive Form

An extensive form game includes timing of moves. Players move sequentially, represented as a tree.

- Chess: white player moves, then black player can see white's move and react...

Keeps track of what each player knows when he or she makes a decision :

- Poker: bet sequentially - what can a given player see when they bet.

1.1.1.2 Normal Form Games

A normal form game is a strategic interaction in which each of n players chooses a strategy and the receives a payoff that depends on all agents choices of strategy. In other words, a normal form represents a list of what players get on function of their actions. Finite, n-person normal form game $\langle N, A, u\rangle$:

- Players: $N = 1, ..., n$ is a finite set of n, indexed by i.
- Actions set for player i A_i

  $a = (a_1, ..., a_n) \in A = A_1 * ... * A_n$ is an action profile.
- Utility function or Payoff function for player $i : u_i : A \to \mathbb{R}$

  $u = (u_1, ..., u_n)$, is a profile of utility functions.

1.1.1.3 Best Response and Nash Equilibrium

A Nash Equilibrium specifies that the optimal outcome of a game is one from which no player can benefit by changing his strategy if none of his opponents do so as well, for a_i to be a best response:

Definition 1 (Best Response)

$$a_i^* \in BR(a_{-i}) iff \forall a_i \in A_i, u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i}) \quad (1.1)$$

Definition 2 (Nash Equilibrium) *$a =< a_1, ..., a_n >$ is a "**pure strategy Nash equilibrium**" if $\forall i, a_i \in BR(a_{-i})$*

1.1.1.4 Dominant strategies

A strategy is dominant if regardless of what any other players do, the strategy earns the player a larger payoff than the others

let s_i and $s_i^{`}$ be two strategies for player i, and let S_{-i} be the set of all possible strategy profiles for other players.

- s_i **strictly dominates** $s_i^{`}$ if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s_i^{`}, s_{-i})$
- s_i **very weakly dominates** $s_i^{`}$ if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) \geq u_i(s_i^{`}, s_{-i})$
- A strategy is called **dominant** if it dominates all others.
- A strategy profile consisting of dominant strategies for every player must be a Nash Equilibrium.

1.1.2 Mixed Strategies and Nash Equilibrium

A mixed strategy consists of possible move and a probability distribution which corresponds to how frequently each move is to be played. A strategy s_i for agent i as any probability distribution over the actions A_i.

- **pure strategy:** only one action is played with positive probability;
- **mixed strategy:** more than one action is played with positive probability, these actions are called the support of the mixed strategy;
- Let the set of all strategies for i be S_i;
- let the set of all strategy profiles be $S = S_1 \times ... \times S_n$;

1.1.2.1 Utility in Mixed Strategies

In order to find the payoff if all the players follow mixed strategy profile $s \in S$ we can use the **expected utility** from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) P(a|s) \tag{1.2}$$

$$P(a|s) = \prod_{j \in N} s_j(a_j) \tag{1.3}$$

1.1.2.2 Best Response and Nash Equilibrium

The definitions of best response and Nash equilibrium using mixed strategy are generalized from actions to strategies.

Definition 3 (Best response)

$$s_i^* \in BR(s_{-i}) \ \textbf{\textit{if}} \ \forall s_i \in S_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \tag{1.4}$$

Definition 4 (Nash Equilibrium) *The expected payoff must be at least as large as that obtainable by any other strategy:*

$$s =< s_1, ..., s_n > \ \textbf{\textit{is a Nash Equilibrium if}} \ \forall i, s_i \in BR(s_{-i})$$

Theorem (Nash, 1950) Every finite game has a Nash equilibrium.

1.1.2.3 Computing Nash Equilibrium

For finding Nash Equilibrium, there exists two main algorithms:

- LCP(Linear Complimentary) (Lemke-Howson).
- Support Enumeration Method (Porter et al).

1.1.2.4 Complexity Analysis

Computing a Nash Equilibrium is classified as a **PPAD-complete**[1] , this has been proven for:

- for games ≥ 4 players;
- for games with 3 players;
- for games with 2 players;

[1] PPAD : Polynomial Parity Argument on Directed Graphs

1.1.2.5 Strictly Dominated Strategies

A strategy is strictly dominant if regardless of what any other players do, the strategy earns the player a strictly higher payoff than any other. If one strategy is strictly dominant, than all others are dominated.

1.1.2.6 Weakly Dominated Strategies

A strategy is weakly dominant if regardless of what other players do, the strategy earns a player a payoff at least as high as any other strategy, and the strategy earns a strictly higher payoff than other players.

1.1.2.7 Perfect information games

A game is called a perfect information game if only one player moves at a time and if each player knows every action of the players that moved before him at every point in the game. The extensive form is an alternative representation that makes the temporal structure explicit. A finite perfect information game in extensive form is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$ where:

- Players: N is a set of n players.
- Actions: A is set of actions.
- Choice nodes and labels for these nodes:
  - Choice nodes: H is a set of non-terminal choice nodes.
  - Action function: $\chi : H \to 2^A$ assigns to each choice a set of actions.
  - Player function: $\rho : H \to N$ assigns to each non-terminal node h a player $i \in N$ who chooses an action at h.
- Terminal nodes: Z is a set of terminal nodes, disjoint from H.
- Successor function: $\sigma : H \times A \to H \cup Z$ maps a choice node and an action to a new choice node or terminal node such that for all $h_1, h_2 \in H$ and $a_1, a_2 \in A$, if $\sigma(h_1, a_1) = \sigma(h_2, a_2)$ then $h_1 = h_2$ and $a_1 = a_2$
- Utility function: $u = (u_1, ..., u_n)$ where $u_i : Z \to R$

figure 1.1 shows a sharing game represented in the extensive form

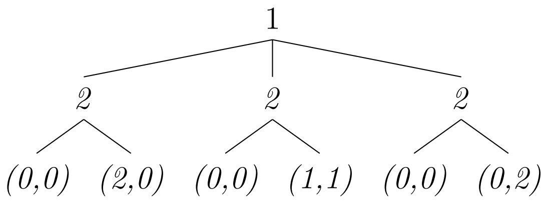

Figure 1.1: Sharing Game

1.1.2.8 Pure Strategies

A pure strategy for a player in a perfect-information game is a complete specification of which action to take at each node belonging to that player.

Definition Let $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ be a perfect-information extensive-form game. Then the pure strategies of player i consist of the cross product

$$\prod_{h \in H, \rho(h)=i} \chi(h) \tag{1.5}$$

Given our new definition of pure strategy, we can reuse our old definitions of mixed strategies and Nash equilibrium in 1.2.

1.1.2.9 Sub-game Perfection

A subgame Nash equilibrium is an equilibrium such that the strategies of players constitute a Nash equilibrium in each subgame of the game. It may be found by backwards induction.

Definition 5 (Sub-game Perfection) *The set of sub-games of G is defined by the sub-games of G rooted at each of the nodes in G.*

Let s be a sub-game perfect equilibrium of G if for any sub-game G' of G, the restriction of s to G' is a Nash Equilibrium of G'. Since G is its own sub-game , every sub-game perfect is a Nash Equilibrium.

1.1.2.10 Backward Induction

Backward induction is an iterative process for solving finite extensive form games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next to last moving player is determined taking the last player's action as given. The process continues in this way backwards in time until the actions have been determined. Effectively, one determines the Nash Equilibrium of each subgame of the game, as shown in the next algorithm:

Algorithm 1 Backward Induction

```
u(h)
if h ∈ Z then
    return u(h)
end if
best − util ← −50
for all a ∈ ρ(h) do
    utc ← BACKWARDINDUCTION(σ(h, a))
    if utc_p(h) > best − util_p(h) then
        best − util ← utc
    end if
end for
return best − util
```

Backward Induction has been used in solving games since John von Neumann and Oskar Morgenstern published their book, Theory of Games and Economic Behaviors in 1944.

Denote *utc* is a utility vector for each player.

1.1.3 Repeated Games

So far we've payed attention to one stage games, that is, games in which players concerns do not extend than beyond the first stage interaction. However, games are often played with a futuristic mindset, and this can significantly change their outcomes and equilibrium strategies. The topic of this section is repeated games, that is, games in which players face similar situations on multiple occasions.

Definition 6 (Average Utilities) *Given an infinite sequence of payoffs* $(r_1, r_2, ...)$ *for player* i*, the average reward of* i *is*

$$\lim_{x \to +\infty} \sum_{j=1}^{k} \frac{r_j}{k} \tag{1.6}$$

Definition 7 (Discounted Utilities) *Given an infinite sequence of payoffs* $(r_1, r_2, ...)$ *for player* i *and a discount factor* β *with* $0 < \beta < 1$*, the corresponding future discount reward is:*

$$\sum_{j=1}^{+\infty} \beta^j r_j \tag{1.7}$$

In 1.7, the players care about the future j just as much as the present, but with a probability of $1 - \beta$ the game will end in any round.

Definition 8 (Equilibrium of Infinitely Repeated Games) *Consider any* n*-player game* $G = (N, A, u)$ *and a payoff vector* $r = (r_1, r2, ..., r_n)$

$$v_i = \min_{S_{-i} \in S_{-i}} \max_{S_i \in S} u_i(S_{-i}, S_i) \tag{1.8}$$

The minmax value of the player i in equation 1.8 is the amount of utility i can get when the other player $-i$ plays a minmax strategy against him.

Definition 9 (Enforceability) *A payoff profile* r *is enforceable if* $i_i \geq v_i$*.*

Definition 10 (Feasibility) *A payoff profile* r *is feasible if there exists a rational non negative value* α_a *for all* i*, we can express* r_i *as* $\sum a \in A(\alpha_a u_i(a)$ *with* $\sum a \in A(\alpha_a) = 1$*.*

1.1.3.1 Folk Theorem

Consider any n-player game G and any payoff vector $(r_1, r_2, ..., r_n)$:

- if r is the payoff in any Nash equilibrium of the infinitely repeated game G with average rewards, then for each player i, r_i is enforceable.
- if r is both feasible and enforceable, then r is the payoff in some Nash equilibrium of the infinitely repeated game with average rewards.

1.1.4 Population Games

Population games provide a simple and general framework for studying strategic interactions in large populations whose members play pure strategies. The simplest population games are generated by random matching in normal form games, but the population game framework allows for interactions of a more intricate nature.

We focus here on games played by a single population. All players in this game play equivalent roles. Suppose that there is a unit mass of players, each of whom chooses a pure strategy from the set $S = 1, ..., n$. The aggregate behavior of these players is described by a population state $x \in X$, with x_j representing the proportion of agents choosing pure strategy j. We identify a population game with a continuous vector valued payoff function $F : X \to R^n$. The scalar $F(x)$ represents the payoff to strategy i when the population state is x.

Population state x^* is a Nash Equilibrium of F if no player can improve his payoff by unilaterally switching strategies.

1.2 Evolutionary Game Theory

Evolutionary Game Theory was introduced by John Maynard Smith in Evolution and The Theory of Games. The Theory was formulated to understand the behavior of animals in game theoretic situations. But it can be applied to modeling human behavior.

After the emergence of traditional game theory, biologists realized the potential of game theory to formally study adaptation and convolution of biological populations, especially in contexts where the fitness of a phenotype depends on the composition of the population (Hamilton, 1967). The main assumption of evolutionary game theory was that strategies with greater payoffs at a particular time would tend to spread more and thus have better chances of being present in the future.

The most important concept of evolutionary thinking that was introduced by Manynard Smith and Price (1973) is the notion of **Evolutionary Stable Strategy**(ESS), for 2-player symmetric games played by individuals belonging to the same population. Furthermore, a strategy s is an ESS if and only if, when adopted by all members of a population, meaning that any other strategy i that could enter the population in a low percentage would obtain a strictly lower expected payoff in the population than the s strategy.

The basic ideas behind Evolutionary game theory is that strategies with greater payoffs tend to spread more, and that fitness is frequency dependent soon transcended the borders of biology and started to spread through many other disciplines. In economic context, it was understood that natural selection would derive from competition among entities for small resources or market shares. In social contexts, evolution was often understood as cultural evolution, and it referred to dynamic changes in behavior or ideas over time (**Nelson and Winter**; **1982**)(**Boyd and Richerson**; **1985**).

In order to extend this understanding further, let's consider this example: Suppose that a small group of mutants choosing a strategy different from δ^* to enter the population.

- Denote the fraction of mutants in the population by ε and assume that the mutant adopts the strategy δ.
- The expected payoff of a mutant is : $(1-\varepsilon)u(\delta, \delta*) + \varepsilon u(\delta*, \delta)$
- The expected payoff of a mutant that adopts the strategy is : $(1-\varepsilon)u(\delta*, \delta*) + \varepsilon u(\delta*, \delta)$
- For any mutation to be driven out of the population we need the expected payoff of any mutant to be less than the expected payoff of normal organism :

$$(1-\varepsilon)u(\delta*, \delta*) + \varepsilon u(\delta*, \delta) > (1-\varepsilon)u(\delta, \delta*) + \varepsilon u(\delta*, \delta) \tag{1.9}$$

1.2.1 Static Notions of Evolutionary Stability

Maynard Smith offered a stability concept for populations of animals sharing a common behavioral trait, that of player a mixed strategy in the game. Maynard defines such a population as stable if it is resistant to invasion by a small group of mutants carrying a different strategy(Sandholm, 2017).

Suppose that a large population is randomly matched to play the symmetric normal form game A. We call a mixed strategy $x \in X$ an **evolutionarily stable strategy** (ESS) if

$$x'A((1-\epsilon)x + \epsilon y) > y'A((1-\epsilon)x + \epsilon y) \tag{1.10}$$

$$\forall \epsilon \leq \epsilon(y) \text{ and } y \neq x.$$

In order to explain condition 1.10, let's consider a population programmed to play mixed strategy x is invaded by a small group of mutants programmed to play the alternative mixed strategy y. Equation **??** requires that regardless of the choice of y, an incumbent's expected payoff from a random match in the post entry population exceeds that of a mutant so long as the size of the invading group is sufficiently small.

The definition of ESS above can also be expressed as a combination of two conditions:

$$x'Ax \geq y'Ax \, \forall \, y \in X \tag{1.11}$$

For all $y \neq x$.

$$[x'Ax = y'Ax] \implies [x'Ay > y'Ay] \tag{1.12}$$

Condition 1.11 requires that the incumbent strategy x be a best response to itself. Condition 1.12 requires that if a mutant strategy y is an alternative best response against the incumbent strategy x then the incumbent ears a higher payoff against the mutant than the mutant earns against itself.

Maynard Smith's notion of ESS attempts to capture the dynamic process of natural selection using a static definition.

Chapter 2

Literature Survey and Methodology

The current approach to mode choice behavior in the perspective of expected utility theory or random utility theory. However, travelers evaluate the alternative modes by individual experience and attitude which are not considered in the expected utility theory or random utility theory models. Therefore, many alternative theories have been proposed, for example, prospect theory, cumulative prospect theory and regret theory. Among them, cumulative prospect theory draws the most attention because it describes the bounded rational behaviors under various conditions.

2.1 Travel Choice Models

Many models are available for analyzing data of travel mode choice. However, three main models have been dominant: logit models, probit models, and discriminant models. These simple choice models are described first. Mode-use models are different from other mode choice models in their dependent variables and model structure. In the third part of this section, we discuss some studies that have used psychological scaling models to probe more deeply into the nature of mode choice process. This is followed by a discussion of reliability and validity analysis in mode choice models.

2.1.1 Simple Choice Models

Three simple-choice models are usually discussed in the context of utility theory. According to this understanding, the utility U_i of alternative mode i is expressed as the sum of a deterministic component V_i and a random component ϵ_i(**Richard**; **David**; **1982**):

$$U_i = V_i + \epsilon_i \tag{2.1}$$

The probability of choosing the ith mode from a set of n alternatives is thus:

$$P_i = P_r[U_i > U_j](j = i) \tag{2.2}$$

Alternatively,

$$P_i = P_r[\epsilon_j < V_i - V_j + \epsilon_i](j = i) \tag{2.3}$$

If the cumulative density function of the error $\epsilon = (\epsilon_1, ..., \epsilon_n)$ is $F(t_1, ..., t_n)$, and the partial of the cumulative density function with respect to variable i is $F_i(t_1, ..., t_n)$, then equation 2.3 becomes:

$$P_i = \int_{-\infty}^{+\infty} F_i(..., t + V_i - V_j, ...)df \tag{2.4}$$

If the error terms are independent identically distributed Gumbel variate, then Equation 2.4 is a multivariate logit model. If the error terms have a joint multivariate normal distribution, then 2.4 defines a multinomial probit model.

The third simple-choice model, discriminant analysis, was originally developed for taxonomic purposes. However, discriminant analysus has been avoided in mode choice analysis because it lacks the probabilistic theory that is possessed by other behavioral-choice models. In recent decades, logit models have been the most used when it comes to travel mode choice analysis.

It is also important to note that probit models are always associated with maximum-likelihood procedures, and discriminant models are always associated with least-squares procedures.

2.1.2 Mode Use Models

These models seek to explain the degree of actual or anticipated use for a given mode. Models of this type do not fit into the travel-demand models of planners as well as mode choice models, but they are legitimate means of investigating the behavioral determinants and relations of mode selection.

Mode use models vary in complexity from single equation models that explain the frequency of mode use or customer satisfaction with a particular mode, to more complex multi-equation models that investigate the structure of the mode choice process. An example would be the study by Dobson, Dunbarn Smith, Reibstein, and Lovelock (1978) that used structural equations on cross-sectional data to try to determine the casual relations between transportation attitudes and behavioral responses. Another study done by Tischer and Philips(1979), have used quasi-experimental designs employing time series data to measure the patterns of causality.

2.1.3 Scaling Models

Although the psychological models of Juce (1959) and Thurstone(1927) are often used to justify the use of multinomial logit model, these individual choice models are rarely used to investigate the mode choices of individuals. The reason for the absence of psychological models in transportation may be the modest results that were reported in early studies in which deterministic vector models were used to analyze subjects preferences.

Mode use and scaling models have expanded our knowledge of the mode choice process. The simple-choice, mode-use, and scaling models utilize different types of data to explain mode choice at different levels of analysis. Models developed for psychological stimuli cannot just be taken off the shelf and applied to complex situations like mode choice without modification.

2.1.4 Reliability and Validity

Reliability and validity testing is very important in mode choice models. Early applications of logit analysis were largely descriptive in nature. Later applications became more sophisticated in their use of statistical procedures.

2.2 Travel Demand Management

One of the most important socio-economic problems in recent decades has been the optimization of an urban transport system. Furthermore, This type of problem mainly occurs in developing countries, and the reason behind it is the increasing rate of car ownership.[1]. Which urges cities to realize transport strategies combating this effect and also to decrease the negative impacts of transportation on the environment [2].

2.3 Choice Decision Elements

The framework for the choice process is that the individual determines the available alternatives(modes), next, evaluates the attributes of each alternative, and then, uses a decision rule to select an alternative from among the available alternatives (Ben-Akiva and Lerman, 1985). Further in this section, we see that the elements of a choice process are : the individuals, travel modes, the attributes of modes and the decision rule.

2.4 Mode Characteristics

The travel mode choice (TMC) is an important step of the transportation forecasting (**Litman**; **2011**). The main modes for travelers are private cars or public transportation. TMC is usually mathematically represented by logit functions, due to its consideration of particular qualities of travelers(**Bravo et al**; **2009**).

2.4.1 Time and Costs

Travel and costs are the two most commonly investigated determinants of travel-mode choice. Studies done by Lisco (1967) and Quarmby (1967) used travel time and travel cost differences as two independent variables in their models. Travel time and cost differences are discussed in the next section. Another method was used by Warner (1962), who used travel time and cost as ratios.

Watson (1974) believed that the difference formulation is most appropriate for between city trips, but when intercity trips are being analyzed other factors may be in order. On longer intercity trips, it is difficult to say whether a traveler would base

[1]Khovako, 2014,.
[2]World Bank, 2011

their mode choice on time, whereas the preference for faster modes is a reasonable assumption on a short commuting trip.

Many studies have made the specification of the time and cost variables between overall travel time and excess travel time. This distinction arose on the assumption that time spent in different ways while traveling may be valued differently. A study by Quarmby (1967) divided travel time into "travel time" and "excess travel time", mentioning that the excess out of vehicle time on a journey may be greater for bus than car users. An important assumption made by Ben-Akiva and Richards (1976), that in vehicle time is generally viewed the same for all modes, whereas out of vehicle time tends to be mode specific.

Travel cost has been discussed in detail by Gillen (1977), who notes that many mode choice studies gave added the cost of parking to automobile running costs (Williams, 1978). Gillen found that parking cost is a crucial variable if the study aims to obtain unbiased estimates of operating costs on mode choice.

It is still unknown of which costs are relevant to mode choice decision. The microeconomic theory that underlies the specification of these models suggests that "marginal operating costs" are the relevant costs. However, from the consumer's perspective, the total cost of ownership including purchase price and maintenance costs may be the more important consideration.

The next function computes the probability of car mode choice, which depends on the difference between the travel time σ_t and the travel costs σ_c:

$$p^*(\sigma_t, \sigma_c) = \frac{1}{1 + exp(a_0 + a_i c^c + a_c \sigma_c)} \tag{2.5}$$

The objective function based on logit function may is presented in the following equation(**Hollander et al**; **2006**):

$$G(p) = (p - p^*(\sigma_t, \sigma_c))* = (p - \frac{1}{1 + exp(a_0 + a_i c^c + a_c \sigma_c)})^2 \rightarrow min_p. \tag{2.6}$$

The difference of the travel times between modes (private and public transport) for traveling between A and B :

$$\sigma t_{a,b} = T_a^r + T_b^r + T_b^p - T_a^t - T_b^t - t_w \tag{2.7}$$

where t_w is the waiting time in public transport.

The difference between travel costs could be :

$$\sigma c_{a,b} = c^c(t_b^r + t_b^r - t_b^p) - c_b^p - c^t \tag{2.8}$$

2.5 Evolutionary Game Theory and Engineering

Many Engineering Infrastructures are becoming increasingly complex to manage due to large scale distributed nature and the nonlinear interdependence between their components (**Quijano et al**; **2017**).Including transportation systems, communication networks, data networks, and teams of anonymous vehicles. Controlling these large scale distributed systems requires the implementation of decision rules for interconnected components that grantee the accomplishment of a collective objective in an environment that is often dynamic and uncertain. In order to achieve this goal, traditional control theory is often of little use, since distributed systems generally lack a central entity with access or control over all components(**Marden and Shamma**; **2015**).

Chapter 3

Model and Analysis

The contribution of this study on travel mode choice is the research on modeling choices in travel mode selection using game theory concepts, and using an evolutionary analysis to determine the behavior of the travelers.

3.1 Evolutionary Game Theory and Travel Mode Choice

Evolutionary game theory is used in this paper as a vehicle for discussing travel mode choice based on the following apparent similarities:

A group can be a substitute for an individual as a participant in evolutionary game theory, and the proportions of the individuals choosing different pure strategies in the group can substitute for mixed strategy. The results of travel mode choice are group behavior within the travel mode subsystems, and the only proportions of individuals choosing each travel mode are meaningful for management and study.

Group Nash equilibrium means that the frequency of the adopted strategies makes the strategy payoffs exactly equal with no one desiring a change in strategy, then the percentage of individuals choosing each different strategy remains stable and reaches equilibrium. In the stable travel context, a travel mode choice will tend to be stable, the Nash equilibrium of the evolutionary game will be changed by the means of traffic control, the construction, and the structure of the transportation system.

The nature of group strategies acts is that only a bounded rationality human gets closer to Nash equilibrium by summarizing their experience and adjusting their strategies rather than by using a perfectly rational method and Nash equilibrium analysis as reasoning. The players can obtain information such as travel time, travel cost, and traffic information, and they can observe the historical results and the strategies adopted by others. The inhabitants observe and experience the service provided by trip modes during their frequent travels and finally determine their strategies. Although they are bounded by rationality, constantly repeated travel leads the structure of travel mode choice by modes to reach excellent stability.

3.1.1 Evolutionary Game Model

The extensive form of the sequential, the game displayed in Figure 1, describes the process of travel mode choice. Two conditions have been implemented in this model.

- Travelers can be divided into two main categories: car owners and noncar owners. As mentioned before, travel is a two stage process, first, every player chooses whether they own a car or not; then, the car owners will select from one of four modes: car, taxi, bus, or rail, and the noncar owners will only select from taxi, bus, or rail.

- The payoff function that inhabitants must contribute is independent of the proportion of inhabitants traveling in a particular mode.

As mentioned in Chapter 1, the extensive form is defined by three main objects.

- the set of players $N = 1, ..., n$, all travelers are players.
- The strategy sets of the players are $S_1 = Carowneer, Noncarowner$ and $S_2 = Travelbycar, Trav$
- The payoff functions of the players are $f_{car} = u_1$, $f_{taxi} = u_2$, $f_{bus} = u_3$, and $f_{rail} = u_4$

Players use mixed strategy, because it is impossible for them to travel using the same pure strategy mode multiple times with certainty.

As explained in Chapter 1, the mixed strategy happens when an individual plays one of the pure strategies of a game with a continuous probability p between 0 and 1. As a result, the payoff the of the individual using mixed strategy depends on the probabilities of the mixed strategy.

Figure 3.1 shows the game model of travel mode choice, we note that in stage 1 of the game p_c and p_n are the probabilities of car owners and non car owners respectively. In stage 2, p_c^c, p_t^c, p_b^c, and p_r^c are the respective probabilities of car owner traveling by car, taxi, bus or rail. The probabilities of the noncar owner traveling by taxi, bus or rail are p_t^n, p_b^n, and p_r^n.

The payoff function of the players are the following:

$$f_{car} = T_{car} C_{car} \tag{3.1}$$

$$f_{taxi} = T_{taxi} C_{taxi} \tag{3.2}$$

$$f_{bus} = T_{bus} C_{bus} \tag{3.3}$$

$$f_{rail} = T_{rail} C_{rail} \tag{3.4}$$

where travel time averages for car, taxi, bus, and rail are T_{car}, T_{taxi}, T_{bus}, T_{rail}, and their average travel costs are C_{car}, C_{taxi}, C_{bus}, C_{rail} respectively.

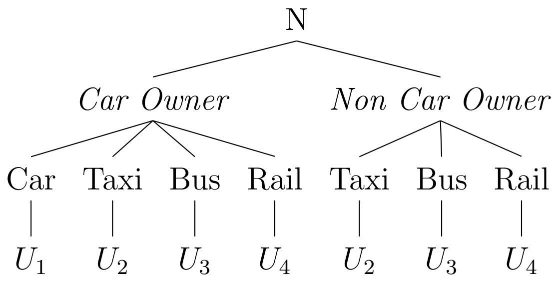

Figure 3.1: Travel mode choice game

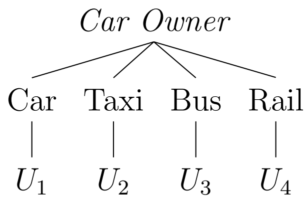

Figure 3.2: Car owner subgame

3.1.2 Nash Equilibrium of Travel Mode Choice Game

According to the Folk Theorem mentioned in Chapter 1, any payoff vector satisfying individual rationality can be obtained through a set of specific subgame perfect equilibriums in an infinitely repeated game. Tn travel mode choice, there are two subgame: Car owner subgame and noncar owner subgame, as shown in figure 3.1. If Nash equilibrium is reached in each of the subgames, the Nash equilibrium of the game is subgame perfect Nash equilibrium. As mentioned in the previous section, backward induction is the solution method for obtaining the Nash equilibrium of this game.

3.1.2.1 Nash Equilibrium of Car Owner Subgame

The key feature of mixed strategy Nash equilibrium is that the expectations of the pure strategies are equal, that is, in car owner subgame of figure 3.2, the products of the travel mode's payoffs and its probabilities are equal and the sum of their probabilities is 1, so that

$$\mu_1 r_c^c = \mu_2 r_t^c = \mu_3 r_b^c = \mu_4 r_r^c \tag{3.5}$$

$$\mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c = 1 \tag{3.6}$$

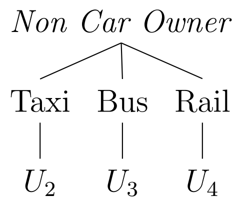

Figure 3.3: Non car owner subgame

Solving 3.5 and 3.6

$$r_c^c = \frac{1}{1 + (\mu_1/\mu_2) + (\mu_1/\mu_3) + (\mu_1/\mu_4)} \tag{3.7}$$

$$r_t^c = \frac{1}{1 + (\mu_2/\mu_1) + (\mu_2/\mu_3) + (\mu_2/\mu_4)} \tag{3.8}$$

$$r_b^c = \frac{1}{1 + (\mu_3/\mu_1) + (\mu_3/\mu_2) + (\mu_3/\mu_4)} \tag{3.9}$$

$$r_r^c = \frac{1}{1 + (\mu_4/\mu_1) + (\mu_4/\mu_2) + (\mu_4/\mu_3)} \tag{3.10}$$

3.1.2.2 Nash Equilibrium for Noncar Owners Subgame

Using backward induction properties as explained in Chapter 1 to solve the subgame shown in Figure 3.3, the products of the travel mode's payoffs and its probabilities are equal, and the sum of their probabilities is 1, resulting in:

$$\mu_2 r_t^n = \mu_3 r_b^n = \mu_4 r_{rail}^n \tag{3.11}$$

$$\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n = 1 \tag{3.12}$$

Solving 3.11 and 3.12

$$r_t^n = \frac{1}{1 + (\mu_2/\mu_3) + (\mu_2/\mu_4)} \tag{3.13}$$

$$r_b^n = \frac{1}{1 + (\mu_3/\mu_2) + (\mu_3/\mu_4)} \tag{3.14}$$

$$r_r^n = \frac{1}{1 + (\mu_4/\mu_2) + (\mu_4/\mu_3)} \tag{3.15}$$

3.1.2.3 Nash Equilibrium of Travel Mode Choice Game

The payoffs of the car owner and the non car owner are their overall expectations. Using backward induction, the products of the payoffs and probabilities are equal and the sum of their probabilities is one:

$$r_n(\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n) = r_c(r_c^c + r_t^c + r_b^c + r_r^c) \tag{3.16}$$

$$p_c + p_n = 1 \tag{3.17}$$

Solving 3.16 and 3.17

$$p_c = \frac{\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n}{\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n + \mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c} \tag{3.18}$$

$$p_n = \frac{\mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c}{\mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c + \mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n} \tag{3.19}$$

The proportion of travel by car for the traveler is the product of its probability and the probability of car owners traveling by car:

$$\gamma_{car} = p_c r_c^c \tag{3.20}$$

$$\gamma_{taxi} = p_c r_t^c + p_n r_t^n \tag{3.21}$$

$$\gamma_{bus} = p_c r_b^c + p_n r_b^n \tag{3.22}$$

$$\gamma_{rail} = p_c r_r^c + p_n r_r^n \tag{3.23}$$

Substituting equations 3.7 to 3.20

$$p_c = \frac{\frac{3}{(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}}{\frac{4}{(1/\mu_1)+(1/\mu_2)+(1/\mu_3)+(1/\mu_4)} + \frac{3}{(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}} \tag{3.24}$$

$$p_n = \frac{\frac{4}{(1/\mu_1)+(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}}{\frac{4}{(1/\mu_1)+(1/\mu_2)+(1/\mu_3)+(1/\mu_4)} + \frac{3}{(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}} \tag{3.25}$$

The equations above represent the Nash Equilibrium of the travel mode choice game. Looking through equations 3.20 to 3.23 we can see that there is a relationship between the individual's payoff and his proportion. That is, as its payoff is increasing, the proportion is decreasing. However in the last four equations there is a relationship between the proportion and the payoffs of all the travel modes.

3.2 Model Analysis

The essence of evolutionary analysis is to discuss how the probability changes when one side of the game changes. The learning ability of players(travelers), which is usually reflected by the tendency dynamic characteristics, in order to determine the change rate.

Conclusion

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