A Travel Mode Choice Model Using Game Theory

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Dedicated to

Travel Mode Choice Modeling using Game theory

Abstract

Acknowledgment

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Introduction

The grow of private vehicle use causes congestion that eventually increases travel time, this increase pushes governments to motivate people towards public transport. However, more usage of public transport should be followed by an improvement of public transport services.

With the intention of making a transport demand analysis, it is essential to understand the traveler's mode choice behavior, where a demand is the accumulation of individuals decisions. The most important element in modeling a transport system is the mode split model, which provides a mathematical framework of the choices a traveler can have of which mode of transport is more suitable.

The different objectives in travel mode choice leads to the urge of applying game theory for making decisions based on finding the equilibrium of the passenger choices. However, game theory is rarely used in the transport field with several obstacles appearing in the transport characteristics. But, it is possible to predict the behaviors of travelers? this question has been asked over time, but we still do not have clear answers. Despite the common knowledge that human actions are random and unpredictable, human mobility follows certain patterns.

The purpose of this study is to describe how travelers adjust their mode of transport choice behaviors using an evolutionary game model. In evolutionary game theory, a dynamic process is set to describe how players adjust their choices overtime as they learn from the game and also from other players. The present document is organized into three chapters. Chapter 1 describes the basic theories applied in this work, including traditional game theory concepts and evolutionary dynamics. In Chapter 2, key studies from the literature regarding travel choice behavior are briefly examined. The third chapter describes the model used in this study. Limitations of the proposed modeling method and further research directions are discussed at the end.

Chapter 1

Game Theory and Evolutionary Dynamics

1.1 Game Theory

Game theory is a branch of applied mathematics that derives mathematical models to predict the outcome of competitive interactions between two or more rational decision makers. A game may involve:

- Common interest (coordination);
- Competing interests (rivalry);
- Rational behavior: players can do the best they can, in their own eyes;
- a rational decision in a game must be based on a prediction of others' responses;

1.1.1 Defining Games

A game is the interaction between rational players, where the decisions of some players changes the payoff of others. A game consists of four parts : Players, Actions, strategies and Payoffs.

- Players are the decision makers and they can be : People, Governments or Companies;
- Actions are decisions that the players make;
- Strategies are composed of actions;
- Payoffs are the outcomes which players receive as a result of their decisions and those of their opponents;

Games can be represented using two methods : Normal forms and Extensive Form.

1.1.1.1 Extensive Form

An extensive form game includes timing of moves. Players move sequentially, represented as a tree. These are examples of games that can be represented in the extensive form:

- Chess: white player moves, then black player can see white's move and react...

Keeps track of what each player knows when he or she makes a decision :

- Poker: bet sequentially - what can a given player see when they bet.

1.1.1.2 Normal Form Games

A normal form game is a strategic interaction in which each of n players chooses a strategy and the receives a payoff that depends on all agents choices of strategy. In other words, a normal form represents a list of what players get on function of their actions (Jackson, Leyton-Brown and Shoham; 2013) Finite, n-person normal form game $\langle N, A, u\rangle$:

- Players: $N = 1, ..., n$ is a finite set of n, indexed by i.
- Actions set for player i A_i

  $a = (a_1, ..., a_n) \in A = A_1 * ... * A_n$ is an action profile.
- Utility function or Payoff function for player $i : u_i : A \to \mathbb{R}$

  $u = (u_1, ..., u_n)$, is a profile of utility functions.

1.1.1.3 Nash Equilibrium

A Nash Equilibrium specifies that the optimal outcome of a game is one from which no player can benefit by changing his strategy if none of his opponents do so as well. Nash Equilibrium is reached over time, in most cases. However, these different choices over time before reaching an equilibrium is often played out in the business world when two firms are determining prices for products.

Definition 1 (Nash Equilibrium) *Nash equilibrium is the profile of actions such that each action is a best response to the other actions,* $a =< a_1, ..., a_n >$ *is a pure strategy Nash equilibrium if* $\forall i, a_i \in BR(a_{-i})$.

1.1.1.4 Dominant strategies

A strategy is called dominant if regardless of what any other players do, the strategy earns the player a larger payoff than the others. Consider s_i and $s_i^{`}$ to be two strategies for player i, and S_{-i} be the set of all possible strategy profiles for other players (Jackson, Leyton-Brown and Shoham; 2013).

- A strategy is strictly dominant if regardless of what any other players do, the strategy earns the player a strictly higher payoff than any other. If one strategy is strictly dominant, than all others are dominated. s_i strictly dominates s_i' if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$;

- A strategy is weakly dominant if regardless of what other players do, the strategy earns a player a payoff at least as high as any other strategy, and the strategy earns a strictly higher payoff than other players;

- A strategy profile consisting of dominant strategies for every player must be a Nash Equilibrium;

1.1.2 Mixed Strategies and Nash Equilibrium

A mixed strategy consists of possible move and a probability distribution which corresponds to how frequently each move is to be played. A strategy s_i for agent i as any probability distribution over the actions A_i.

- **pure strategy:** only one action is played with positive probability;

- **mixed strategy:** more than one action is played with positive probability, these actions are called the support of the mixed strategy;

1.1.2.1 Utility in Mixed Strategies

In order to find the payoff if all the players follow mixed strategy profile $s \in S$ we can use the expected utility from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) P(a|s) \tag{1.1}$$

$$P(a|s) = \prod_{j \in N} s_j(a_j) \tag{1.2}$$

1.1.2.2 Best Response and Nash Equilibrium

The definitions of best response and Nash equilibrium using mixed strategy are generalized from actions to strategies.

Definition 2 (Best response) *The best response for a player is the strategy of that generate the greatest payoff.*

$$s_i^* \in BR(s_{-i}) \ \textbf{\textit{if}} \ \forall s_i \in S_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \tag{1.3}$$

Definition 3 (Nash Equilibrium) *The expected payoff must be at least as large as that obtainable by any other strategy:*

$$s =< s_1, ..., s_n > \ \textbf{\textit{is a Nash Equilibrium if}} \ \forall i, s_i \in BR(s_{-i})$$

Theorem (Nash, 1950) Every finite game has a Nash equilibrium.

1.1.2.3 Computing Nash Equilibrium

Computing a Nash equilibrium is classified to be a PPAD-complete [1]. The first algorithm is known to be the complementary pivot algorithm as a solution to the linear complementary problem[2], which was developed by Lemke and Howson. The algorithm was later generalized for noncooperative n-person games by Rosenmuller. The idea of the Lemke-Howson algorithm is to perform pivoting steps between the vertices of a polytope related to the game until a Nash equilibrium is found. The issue with these algorithms is that their worst case running time is exponential to the number of players and the size of the strategy sets.

[1] PPAD : Polynomial Parity Argument on Directed Graphs

[2] Linear complementary algorithm is a general problem that unifies linear programming, quadratic programming and bi-matrix games

1.1.2.4 Perfect information games

A game is called a perfect information game if only one player moves at a time and if each player knows every action of the players that moved before him at every point in the game. The extensive form is an alternative representation that makes the temporal structure explicit. A finite perfect information game in extensive form is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$ where:

- Players: N is a set of n players.
- Actions: A is a set of actions.
- Choice nodes and labels for these nodes:
  - Choice nodes: H is a set of non-terminal choice nodes.
  - Action function: $\chi : H \to 2^A$ assigns to each choice a set of actions.
  - Player function: $\rho : H \to N$ assigns to each non-terminal node h a player $i \in N$ who chooses an action at h.
- Z is a set of terminal nodes[3], disjoint from H.
- Successor function: $\sigma : H \times A \to H \cup Z$ maps a choice node and an action to a new choice node or terminal node such that for all $h_1, h_2 \in H$ and $a_1, a_2 \in A$, if $\sigma(h_1, a_1) = \sigma(h_2, a_2)$ then $h_1 = h_2$ and $a_1 = a_2$
- Utility function: $u = (u_1, ..., u_n)$ where $u_i : Z \to R$.

An example of a game that can be represented in extensive form is the sharing game, where two players try to decide how they can split 2$ in between them as shown in figure 1.1.

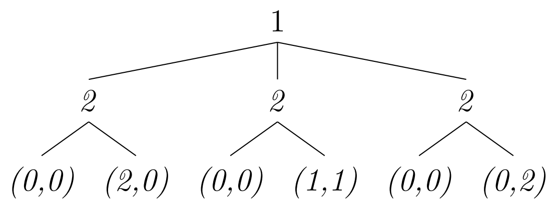

Figure 1.1: Sharing Game

[3]Terminal node: a node of a tree data structure that has no child nodes.

1.1.2.5 Pure Strategies

A pure strategy for a player in a perfect-information game is a complete specification of which action to take at each node belonging to that player.

Definition Let $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ be a perfect-information extensive-form game. Then the pure strategies of player i consist of the cross product

$$\prod_{h \in H, \rho(h)=i} \chi(h) \tag{1.4}$$

Given our new definition of pure strategy, we can reuse our old definitions of mixed strategies and Nash equilibrium defined in 1.1 through 1.3.

1.1.2.6 Sub-game Perfection

A subgame Nash equilibrium is an equilibrium such that the strategies of players constitute a Nash equilibrium in each subgame of the game. It may be found by backwards induction.

Definition 4 (Sub-game Perfection) *The set of sub-games of G is defined by the sub-games of G rooted at each of the nodes in G.*

Let s be a sub-game perfect equilibrium of G if for any sub-game G' of G, the restriction of s to G' is a Nash equilibrium of G'. Since G is its own sub-game , every sub-game perfect is a Nash equilibrium.

1.1.2.7 Backward Induction

Backward induction is an iterative process for solving finite extensive form games. First, one determines the optimal strategy of the player who makes the last move of the game. Then, the optimal action of the next to last moving player is determined taking the last player's action as given. The process continues in this way backwards in time until the actions have been determined.

Backward Induction has been used in solving games since John von Neumann and Oskar Morgenstern published their book, Theory of Games and Economic Behavior in 1944 (Von Neumann and Morgenstern; 1944). note that *utc* or "utility at child" is a utility vector for each player (Jackson, Leyton-Brown and Shoham, 2013). Effectively, one determines the Nash Equilibrium of each subgame of the game, as shown in algorithm 1.

Algorithm 1 Backward Induction

Input : node h
Output : $u(h)$
if $h \in Z$ **then**
  return $u(h)$ {return the payoff vector if the node h is a leaf node}
end if
best-util $\leftarrow -\infty$ {best-util is a payoff vector associated with each agent}
for all $a \in \rho(h)$ **do**
  $utc \leftarrow BACKWARDINDUCTION(\sigma(h,a))$ {look at all actions available from node h, where $\sigma(h,a)$ is the child node arrived at by taking action a }
  if $utc_{\rho(h)} > best - util_{\rho(h)}$ **then**
    best-util $\leftarrow utc$ {update the best utility vector if the utility at the child node is better}
  end if
end for
return best-util

1.1.3 Repeated Games

So far we've payed attention to one stage games, that is, games in which players concerns do not extend than beyond the first stage interaction. However, games are often played with a futuristic mindset, and this can significantly change their outcomes and equilibrium strategies. The topic of this section is repeated games, that is, games in which players face similar situations on multiple occasions.

Definition 5 (Average Utilities) *Given an infinite sequence of payoffs* $(r_1, r_2, ...)$ *for player* i*, the average reward of* i *is*

$$\lim_{x \to +\infty} \sum_{j=1}^{k} \frac{r_j}{k} \tag{1.5}$$

Definition 6 (Discounted Utilities) *Given an infinite sequence of payoffs* $(r_1, r_2, ...)$ *for player* i *and a discount factor* β *with* $0 < \beta < 1$*, the corresponding future discount reward is:*

$$\sum_{j=1}^{+\infty} \beta^j r_j \tag{1.6}$$

In 1.6, the players care about the future j just as much as the present, but with a probability of $1 - \beta$ the game will end in any round.

Definition 7 (Equilibrium of Infinitely Repeated Games) *Consider any n-player game $G = (N, A, u)$ and a payoff vector $r = (r_1, r_2, ..., r_n)$*

$$v_i = \min_{S_{-i} \in S_{-i}} \max_{S_i \in S} u_i(S_{-i}, S_i) \tag{1.7}$$

The minmax value of the player i in equation 1.7 is the amount of utility i can get when the other player $-i$ plays a minmax strategy against him. For example, the minmax payoff in a Prisoner's Dilemma is (1,1) as shown in table 1.1.

1/2	w	s
w	2,2	0,3
s	3,0	1,1

Table 1.1: Prisoner's Dilemma

Definition 8 (Folk Theorem) *Consider a finite normal form game $G = (N, A, u)$, and $a = (a_1, ..., a_n)$ to be a Nash equilibrium of the stage game G. If $a' = (a'_1, ..., a'_n)$ is such that $u_i(a') > u_i(a)$ for all i, then there exists a discount factor $\beta < 1$, such that if $\beta_i \geq \beta$ for all i, then there exists a subgame perfect Nash equilibrium of the infinite repetition of G that has a' played in every period on the equilibrium path.*

1.1.4 Population Games

Population games provide a simple and general framework for studying strategic interactions in large populations whose members play pure strategies. The simplest population games are generated by random matching in normal form games, but the population game framework allows for interactions of a more intricate nature.

We focus here on games played by a single population. All players in this game play equivalent roles. Suppose that there is a unit mass of players, each of whom chooses a pure strategy from the set $S = 1, ..., n$. The aggregate behavior of these players is described by a population state $x \in X$, with x_j representing the proportion of agents choosing pure strategy j. We identify a population game with a continuous vector valued payoff function $F : X \to R^n$. The scalar $F(x)$ represents the payoff to strategy i when the population state is x.

Population state x^* is a Nash equilibrium of F if no player can improve his payoff by unilaterally switching strategies.

1.1.5 The lack of dynamics in traditional game theory

Von Neuman and Morgenstern mentioned that the theory of games is static, and a dynamic theory would be more fitted and preferable (The theory of games, V.Neuman and Morgenstern; 1953). At the time of the publication, evolutionary game theory was unknown.

In order to capture the dynamics of a decision making process, it is preferable to represent the game in the extensive form rather than normal form for games of a normal complexity. However, extensive form games tend to get complex and difficult to manage. Traditional game theory represents a set of strategies for a player at each stage of the game. This representation lacks the element of learning in players when encountering similar choices which leads to better decision making. This shows the incompetence of traditional game theory in the dynamics model of decision making. Therefore, evolutionary game theory incorporates dynamic factors.

1.2 Evolutionary Game Theory

Evolutionary Game Theory was introduced by John Maynard Smith in Evolution and The Theory of Games in 1982. The theory was formulated to understand the behavior of animals in game theoretic situations. But it can be applied to modeling human behavior.

After the emergence of traditional game theory, biologists realized the potential of game theory to formally study adaptation of biological populations, especially in contexts where the fitness of a phenotype depends on the composition of the population (Hamilton, 1967). The main assumption of evolutionary game theory was that strategies with greater payoffs at a particular time would tend to spread more and thus have better chances of being present in the future.

The most important concept of evolutionary thinking that was introduced by Manynard Smith and Price (1973) is the notion of Evolutionary Stable Strategy (ESS), for 2-player symmetric games played by individuals belonging to the same population. Furthermore, a strategy s is an ESS if and only if, when adopted by all members of a population, meaning that any other strategy i that could enter the population in a low percentage would obtain a strictly lower expected payoff in the population than the s strategy.

The basic ideas behind Evolutionary game theory is that strategies with greater payoffs tend to spread more, and that fitness is frequency dependent soon transcended the borders of biology and started to spread through many other disciplines. In economic context, it was understood that natural selection would derive from competition among entities for small resources or market shares. In social contexts, evolution was often understood as cultural evolution, and it referred to dynamic changes in behavior or ideas over time (Nelson and Winter, 1982)(Boyd and Richerson, 1985).

In order to extend this understanding further, let's consider this example: Suppose that a small group of mutants choosing a strategy different from δ^* to enter the population.

- Denote the fraction of mutants in the population by ε and assume that the mutant adopts the strategy δ.
- The expected payoff of a mutant is : $(1-\varepsilon)u(\delta,\delta*)+\varepsilon u(\delta*,\delta)$
- The expected payoff of a mutant that adopts the strategy is : $(1-\varepsilon)u(\delta*,\delta*)+\varepsilon u(\delta*,\delta)$
- For any mutation to be driven out of the population we need the expected payoff of any mutant to be less than the expected payoff of normal organism :

$$(1-\varepsilon)u(\delta*,\delta*)+\varepsilon u(\delta*,\delta) > (1-\varepsilon)u(\delta,\delta*)+\varepsilon u(\delta*,\delta) \tag{1.8}$$

1.2.1 Static Notions of Evolutionary Stability

Maynard Smith offered a stability concept for populations of animals sharing a common behavioral trait, that of player a mixed strategy in the game. Maynard defines such a population as stable if it is resistant to invasion by a small group of mutants carrying a different strategy(Sandholm, 2017).

Suppose that a large population is randomly matched to play the symmetric normal form game A. We call a mixed strategy $x \in X$ an **evolutionarily stable strategy** (ESS) if

$$x'A((1-\epsilon)x+\epsilon y) > y'A((1-\epsilon)x+\epsilon y) \tag{1.9}$$

$$\forall \epsilon \leq \epsilon(y) \text{ and } y \neq x.$$

In order to explain condition 1.9, let's consider a population programmed to play mixed strategy x is invaded by a small group of mutants programmed to play the alternative mixed strategy y. Equation 1.9 requires that regardless of the choice of y, an incumbent's expected payoff from a random match in the post entry population exceeds that of a mutant so long as the size of the invading group is sufficiently small.

The definition of ESS above can also be expressed as a combination of two conditions:

$$x'Ax \geq y'Ax \,\forall\, y \in X \tag{1.10}$$

For all $y \neq x$.

$$[x'Ax = y'Ax] \implies [x'Ay > y'Ay] \tag{1.11}$$

Condition 1.10 requires that the incumbent strategy x be a best response to itself. Condition 1.11 requires that if a mutant strategy y is an alternative best response against the incumbent strategy x then the incumbent ears a higher payoff against the mutant than the mutant earns against itself.

Maynard Smith's notion of ESS attempts to capture the dynamic process of natural selection using a static definition.

Chapter 2

Literature Survey and Methodology

Choice models are constantly evolving, and it is necessary to understand where the literature is. In this Chapter, we will discuss various travel choice models as well as the decision maker's different attributes, then give an overview of the applications of game theory in engineering and transportation problems.

2.1 Travel Choice Models

Many models are available for analyzing data of travel mode choice. However, three main models have been dominant: logit models, probit models, and discriminant models. These simple choice models are described first. Mode-use models are different from other mode choice models in their dependent variables and model structure. In the third part of this section, we discuss some studies that have used psychological scaling models to probe more deeply into the nature of mode choice process. This is followed by a discussion of reliability and validity analysis in mode choice models.

2.1.1 Simple Choice Models

Three simple-choice models are usually discussed in the context of utility theory. According to this understanding, the utility U_i of alternative mode i is expressed as the sum of a deterministic component V_i and a random component ϵ_i capturing the uncertainty:

$$U_i = V_i + \epsilon_i \tag{2.1}$$

The probability of choosing the ith mode from a set of n alternatives is:

$$P_i = P_r[U_i > U_j](j = i) \tag{2.2}$$

Alternatively,

$$P_i = P_r[\epsilon_j < V_i - V_j + \epsilon_i](j = i) \tag{2.3}$$

If the cumulative density function of the error $\epsilon = (\epsilon_1, ..., \epsilon_n)$ is $F(t_1, ..., t_n)$, and the partial of the cumulative density function with respect to variable i is $F_i(t_1, ..., t_n)$, then equation 2.3 becomes:

$$P_i = \int_{-\infty}^{+\infty} F_i(..., t + V_i - V_j, ...)df \tag{2.4}$$

If the error terms are independent identically distributed Gumbel variate, then Equation 2.4 is a multivariate logit model. If the error terms have a joint multivariate normal distribution, then 2.4 defines a multinomial probit model.

The third simple-choice model, discriminant analysis, was originally developed for taxonomic purposes. However, discriminant analysis has been avoided in mode choice analysis because it lacks the probabilistic theory that is possessed by other behavioral-choice models. In recent decades, logit models have been the most used when it comes to travel mode choice analysis.

It is also important to note that probit models are always associated with maximum-likelihood procedures, and discriminant models are always associated with least-squares procedures.

2.1.2 Mode Use Models

These models seek to explain the degree of actual or anticipated use for a given mode. Models of this type do not fit into the travel-demand models of planners as well as mode choice models, but they are legitimate means of investigating the behavioral determinants and relations of mode selection.

Mode use models vary in complexity from single equation models that explain the frequency of mode use or customer satisfaction with a particular mode, to more complex multi-equation models that investigate the structure of the mode choice process. An example would be the study by Dobson, Dunbarn Smith, Reibstein, and Lovelock (1978) that used structural equations on cross-sectional data to try to determine the casual relations between transportation attitudes and behavioral responses. Another study done by Tischer and Philips(1979), have used quasi-experimental designs employing time series data to measure the patterns of causality.

2.1.3 Scaling Models

Although the psychological models of Juce (1959) and Thurstone(1927) are often used to justify the use of multinomial logit model, these individual choice models are rarely used to investigate the mode choices of individuals. The reason for the absence of psychological models in transportation may be the modest results that were reported in early studies in which deterministic vector models were used to analyze subjects preferences.

Mode use and scaling models have expanded our knowledge of the mode choice process. The simple-choice, mode-use, and scaling models utilize different types of data to explain mode choice at different levels of analysis. Models developed for psychological stimuli cannot just be taken off the shelf and applied to complex situations like mode choice without modification.

2.1.4 Reliability and Validity

Reliability and validity testing has been critical in constructing mode choice models. Early applications of logit analysis were largely descriptive in nature. Later applications became more sophisticated in their use of statistical procedures. Studies have been classified in table 2.1 shows the use of reliability and validity in past studies on mode choice.

Table 2.1: Summary of charateristics of selected studies on travel mode choice

	Type of mode	Estimation method	R/V	No of modes
Warner (1962)	R/L/P/D	M/L	Y	3
Beesley (1965)	O	O	N	3
Ben-Akiva & Richards (1976)	L	M	Y	2
Lerman & Ben-Akiva (1976)	L	M	Y	6
Tischer & Philips (1979)	O	L	Y	3

[1] D = discriminant analysis, L = logit, P = probit, R = regression, O = other.
[2] M = maximum likelihood, L = least squares, O = other.
[3] R = reliability, V = validity, Y = yes, N = no.

2.2 Choice Decision Elements

The framework for the choice process is that the individual determines the available alternatives(modes), next, evaluates the attributes of each alternative, and then, uses a decision rule to select an alternative from among the available alternatives (Ben-Akiva and Lerman, 1985). Further in this section, we see that the elements of a choice process are : the individuals, travel modes, the attributes of modes and the decision rule.

2.2.1 Mode Characteristics

The travel mode choice is an important step of the transportation forecasting (Litman,2011). The main modes for travelers are private cars or public transportation. TMC is usually mathematically represented by logit functions, due to its consideration of particular qualities of travelers(Bravo et al, 2009). However, comfort, safety ,and reliability have been included in mode choice models.

2.2.1.1 Time and Cost

Travel time is probably the most important among all other attributes. Mode choice models often assume that travelers are experienced with network conditions, therefore, are able to estimate travel times. Since the implementation of Intelligent Transportation Systems, new models have been developed. Travel time and expenses are the two most commonly investigated determinants of travel-mode choice. Studies done by Lisco (1967) and Quarmby (1967) used travel time and travel cost differences as two independent variables in their models. Another method was used by Warner (1962), who used travel time and cost as ratios.

Watson (1974) believed that the difference formulation is most appropriate for between city trips, but when intercity trips are being analyzed other factors may be in order. On longer intercity trips, it is difficult to say whether a traveler would base their mode choice on time, whereas the preference for faster modes is a reasonable assumption on a short commuting trip.

Many studies have made the specification of the time and cost variables between overall travel time and excess travel time. This distinction is based on the assumption that time spent in different ways while traveling may be valued differently. A study by Quarmby (1967) divided travel time into "travel time" and "excess travel time", mentioning that the excess out of vehicle time on a journey may be greater for bus than car users. An important assumption made by Ben-Akiva and Richards (1976), that in vehicle time is generally viewed the same for all modes, whereas out of vehicle time tends to be mode specific.

Travel cost has been discussed in detail by Gillen (1977), who notes that many mode choice studies gave added the cost of parking to automobile running costs (Williams, 1978). Gillen found that parking cost is a crucial variable if the study aims to obtain unbiased estimates of operating costs on mode choice.

It is still unknown of which costs are relevant to mode choice decision. The microeconomic theory that underlies the specification of these models suggests that "marginal operating costs" are the relevant costs. However, from the consumer's perspective, the total cost of ownership including purchase price and maintenance costs may be the more important consideration.

2.2.2 Consumer Characteristics

Mode choice models have implemented the characteristics related to the traveler, either as independent variables or as bases for segmentation (Richard Barff and David Mackay, 1982).

Car ownership was used as an independent variable in models such as Beesly's (1965) and Williams (1978). These studies found that car ownership was not only an important variable, but a significant determinant of mode choice.

Geographic location and income are also found to be determinants of mode choice. The availability of some modes of transportation is directly related to the location of the consumer.

2.3 Evolutionary Game Theory and Engineering

Many Engineering Infrastructures are becoming increasingly complex to manage due to large scale distributed nature and the nonlinear interdependence between their components (Quijano et al, 2017).Including transportation systems, communication networks, data networks, and teams of anonymous vehicles.

Controlling large scale distributed systems requires the implementation of decision rules for interconnected components that grantee the accomplishment of a collective objective in an environment that is often dynamic and uncertain. In order to achieve this goal, traditional control theory is often of little use, since distributed systems generally lack a central entity with access or control over all components (Marden and Shamma, 2015).

2.4 Game Theory applications in Transportation

One of the first studies was done by Fisk(1984), where he discussed the applications of Stackelberg and Nash games in transportation systems planning and operations. Fisk found out that Wardrop's user equilibrium principle in road traffic research is essentially the condition for a Nash equilibrium, mentioning that no driver can reduce his or her travel time by switching to a different route choice. Another study by Reyniers (1992) on a game between the railway operators who sets the capacities for different fare classes and the passengers who chooses which class to use. A recent research by Padma and Bakshi (2016) on the optimization of parking lot area in smart cities using game theory. Another study utilized evolutionary game theory on taxi service mode choice by Zhu et al (2018).

Mode choice models are becoming more behavioral, despite having economic characteristics such as time, costs, and income. These models tend to explain the presents attitudes in mode choice and lack the changing side of behavior. Game theory has the potential of developing a model that uses the dynamic information of the mode choice process. These studies provides us with a basis for the construction of a model capable of making predictions.

Chapter 3

Model and Analysis

The contribution of this study on travel mode choice is the research on modeling choice behaviors in travel mode selection using game theory concepts, and using an evolutionary analysis to determine the behavior of the travelers.

3.1 Evolutionary Game Theory and Travel Mode Choice

Evolutionary game theory is used as a vehicle for discussing travel mode choice based on the following apparent similarities:

A group can be a substitute for an individual as a participant in evolutionary game theory, and the proportions of the individuals choosing different pure strategies in the group can substitute for mixed strategy. The results of travel mode choice are group behavior within the travel mode subsystems, and the only proportions of individuals choosing each travel mode are meaningful for management and study.

Nash equilibrium means that the frequency of the adopted strategies makes the strategy payoffs exactly equal with no one desiring a change in strategy, then the percentage of individuals choosing each different strategy remains stable and reaches equilibrium. In the stable travel context, a travel mode choice will tend to be stable, the Nash equilibrium of the evolutionary game will be changed by the means of traffic control, the construction, and the structure of the transportation system.

3.1.1 Evolutionary Game Model

Inspired by the nested binary logit model used to define mode choice and Zhu et al (2018) study, the multi-agent based mode choice game is represented in extensive form in Figure 3.1. Players in the travel mode choice game are be divided into two main categories: car owners and noncar owners. First, every player chooses whether they own a car or not; then, the car owners will select from one of four modes: car, taxi, bus, or rail, and the noncar owners will only select from taxi, bus, or rail. Furthermore, two sub-games are apparent, a car owners sub-game and non car owners sub-game.

As mentioned in Chapter 1, the extensive form is defined by the set of players, the strategy sets , and the payoff function.

- the set of players $N = 1, ..., n$, all travelers are players.
- The strategy sets of the players are $S_1 = Carowneer, Noncarowner$ and $S_2 =$(Travel by car, Travel by taxi, Travel by bus, Travel by rail).
- The payoff functions of the players are $f_{car} = u_1$, $f_{taxi} = u_2$, $f_{bus} = u_3$, and $f_{rail} = u_4$

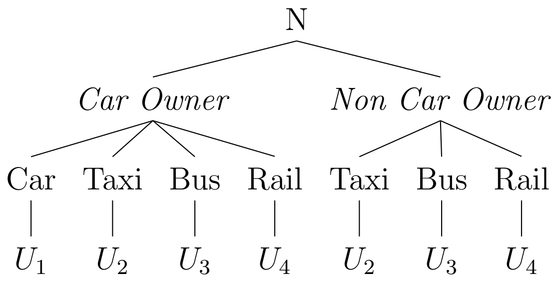

Figure 3.1: Travel mode choice game

Players use mixed strategy, because it is impossible for them to travel using the same pure strategy mode multiple times with certainty.

As explained in Chapter 1, the mixed strategy happens when an individual plays one of the pure strategies of a game with a continuous probability p between 0 and 1. As a result, the payoff the of the individual using mixed strategy depends on the probabilities of the mixed strategy.

Figure 3.1 shows the game model of travel mode choice, we note that in stage 1 of the game p_c and p_n are the probabilities of car owners and non car owners respectively. In stage 2, r^c_c, r^c_t, r^c_b, and r^c_r are the respective probabilities of car owner traveling by car, taxi, bus or rail. The probabilities of the noncar owner traveling by taxi, bus or rail are r^n_t, r^n_b, and r^n_r.

The payoff function of the players are the following:

$$f_{car} = T_{car} C_{car} \tag{3.1}$$

$$f_{taxi} = T_{taxi} C_{taxi} \tag{3.2}$$

$$f_{bus} = T_{bus} C_{bus} \tag{3.3}$$

$$f_{rail} = T_{rail} C_{rail} \tag{3.4}$$

where travel time averages for car, taxi, bus, and rail are T_{car}, T_{taxi}, T_{bus}, T_{rail}, and their average travel costs are C_{car}, C_{taxi}, C_{bus}, C_{rail} respectively.

3.1.2 Nash Equilibrium of Travel Mode Choice Game

According to the Folk Theorem mentioned in Chapter 1, any payoff vector satisfying individual rationality can be obtained through a set of specific subgame perfect equilibriums in an infinitely repeated game. In Figure 3.1 there are two subgame: Car owner subgame and noncar owner subgame, as shown in figures 3.2 and 3.3.The Nash equilibrium of the game is a subgame perfect Nash equilibrium of each sub-game. As mentioned in Chapter 1, backward induction is the method for solving extensive form games and obtaining the Nash equilibrium.

3.1.2.1 Nash Equilibrium of Car Owner Subgame

The key feature of mixed strategy Nash equilibrium is that the expectations of the pure strategies are equal, that is, in car owner subgame of figure 3.2.

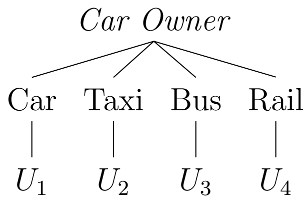

Figure 3.2: Car owner subgame

The products of the travel mode's payoffs and its probabilities are equal and the sum of their probabilities is 1:

$$\mu_1 r_c^c = \mu_2 r_t^c = \mu_3 r_b^c = \mu_4 r_r^c \tag{3.5}$$

$$\mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c = 1 \tag{3.6}$$

Solving 3.5 and 3.6

$$r_c^c = \frac{1}{1 + (\mu_1/\mu_2) + (\mu_1/\mu_3) + (\mu_1/\mu_4)} \tag{3.7}$$

$$r_t^c = \frac{1}{1 + (\mu_2/\mu_1) + (\mu_2/\mu_3) + (\mu_2/\mu_4)} \tag{3.8}$$

$$r_b^c = \frac{1}{1 + (\mu_3/\mu_1) + (\mu_3/\mu_2) + (\mu_3/\mu_4)} \tag{3.9}$$

$$r_r^c = \frac{1}{1 + (\mu_4/\mu_1) + (\mu_4/\mu_2) + (\mu_4/\mu_3)} \tag{3.10}$$

3.1.2.2 Nash Equilibrium for Noncar Owners Subgame

Using backward induction properties as explained in Chapter 1 to solve the subgame shown in Figure 3.3.

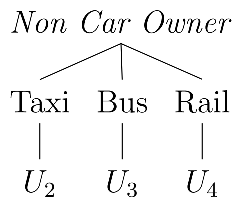

Figure 3.3: Non car owner subgame

The products of the travel mode's payoffs and its probabilities are equal, and the sum of their probabilities is 1, resulting in:

$$\mu_2 r_t^n = \mu_3 r_b^n = \mu_4 r_r^n \tag{3.11}$$

$$\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n = 1 \tag{3.12}$$

Solving 3.11 and 3.12

$$r_t^n = \frac{1}{1 + (\mu_2/\mu_3) + (\mu_2/\mu_4)} \tag{3.13}$$

$$r_b^n = \frac{1}{1 + (\mu_3/\mu_2) + (\mu_3/\mu_4)} \tag{3.14}$$

$$r_r^n = \frac{1}{1 + (\mu_4/\mu_2) + (\mu_4/\mu_3)} \tag{3.15}$$

3.1.2.3 Nash Equilibrium of Travel Mode Choice Game

The payoffs of the car owner and the non car owner are their overall expectations. Using backward induction, the products of the payoffs and probabilities are equal and the sum of their probabilities is one:

$$r_n(\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n) = r_c(r_c^c + r_t^c + r_b^c + r_r^c) \tag{3.16}$$

$$p_c + p_n = 1 \tag{3.17}$$

Solving 3.16 and 3.17

$$p_c = \frac{\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n}{\mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n + \mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c} \tag{3.18}$$

$$p_n = \frac{\mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c}{\mu_1 r_c^c + \mu_2 r_t^c + \mu_3 r_b^c + \mu_4 r_r^c + \mu_2 r_t^n + \mu_3 r_b^n + \mu_4 r_r^n} \tag{3.19}$$

The proportion of travel by car for the traveler is the product of its probability and the probability of car owners traveling by car, and the same goes through other modes:

$$\gamma_{car} = p_c r_c^c \tag{3.20}$$

$$\gamma_{taxi} = p_c r_t^c + p_n r_t^n \tag{3.21}$$

$$\gamma_{bus} = p_c r_b^c + p_n r_b^n \tag{3.22}$$

$$\gamma_{rail} = p_c r_r^c + p_n r_r^n \tag{3.23}$$

Substituting equations 3.7 to 3.20

$$p_c = \frac{\frac{3}{(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}}{\frac{4}{(1/\mu_1)+(1/\mu_2)+(1/\mu_3)+(1/\mu_4)} + \frac{3}{(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}} \tag{3.24}$$

$$p_n = \frac{\frac{4}{(1/\mu_1)+(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}}{\frac{4}{(1/\mu_1)+(1/\mu_2)+(1/\mu_3)+(1/\mu_4)} + \frac{3}{(1/\mu_2)+(1/\mu_3)+(1/\mu_4)}} \tag{3.25}$$

The equations above represent the Nash Equilibrium of the travel mode choice game. Looking through equations 3.20 to 3.23 we can see that there is a relationship between the individual's payoff and their proportion. That is, as its payoff is increasing, the proportion is decreasing. However, in the last four equations there is a relationship between the proportion and the payoffs of all the travel modes. The essence of evolutionary analysis is to discuss how the probability changes when one side of the game changes. The learning ability of travelers, which is usually reflected by the tendency dynamic characteristics, in order to determine the change rate.

3.2 Constructing the model

This model is an agent based model of artificial agents playing the travel mode choice game. Each agent occupies a single place in the game and could interact with other neighbor agents. Agents in this game will update their state based on their game choices. An agent's state can be either a car owner or a non car owner which makes him a public transport user. If a non car owner switches their state to a car owner, they have to go through "purchasing a car" according to the probability function. This probability to buy a car function is adopted from an agent based computational approach (Epstein, 2002).

3.3 Model Analysis

3.3.1 Travel Cost

The cost of traveling is an important factor in TMC, although, travel time can be an affecting factor. Measures like public transport fares, car utility, fuel and parking costs have been implemented to estimate travel cost.

3.3.2 Travel Time

Travel time is the time consumed when a traveler moves between two places in a network and is applicable in all transport modes. Two main differences exist in travel time. For bus, taxi, and rail travel time is divided into walking time, waiting time, in vehicle time, and transfer time. For car owners, travel time is transfer time.

3.3.3 Payoff

Travelers optimize between the time and cost of travel. The payoff of travel can be analyzed through the product of average travel cost and travel time in each mode, as shown in equations 3.1 to 3.4.

3.4 Experiments and results

For this experiment we are using Netlogo[1], the artificial environment is adequate for modeling complex systems which evolve over time. We are going to run the simulation during 100 tick.

In this experiment, we want to test all components available for modifying the passenger's behavior. As we want to remain the characteristics of each mode. Using Behavior Space tool in Netlogo that allows to make multiple simulations. It runs the model several times, with the ability to change the parameters of the model and record the results after each run.

To find the Nash equilibrium, we can get it by averaging the travel time and cost then use the Nash equilibrium equations from 3.20 to 3.23 and calculate the payoff in equations from 3.1 to 3.4. The results are shown in figure 3.1.

When the structure of travel mode choice reaches Nash equilibrium values in Table 3.1, travel mode will remain stationary unless the payoffs of one or more mode parameters are changed. If a mode changes condition, a small perturbation appears. Then Nash equilibrium will be reached again after self adjusting.

[1]Netlog is a multi-agent programmable modeling environment

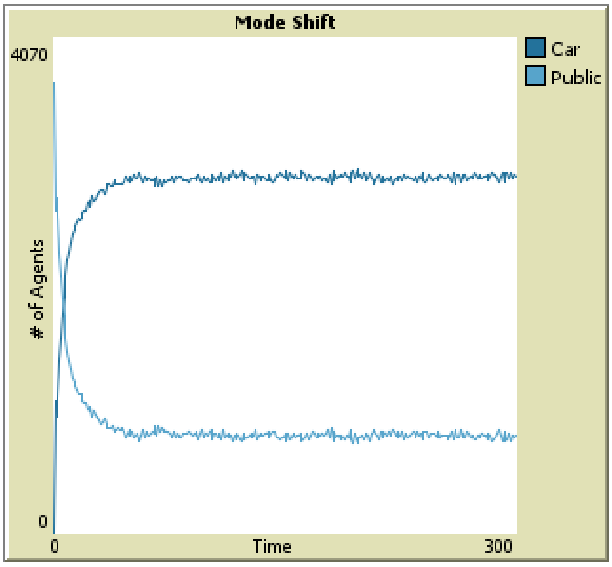

Figure 3.4: Mode Shift

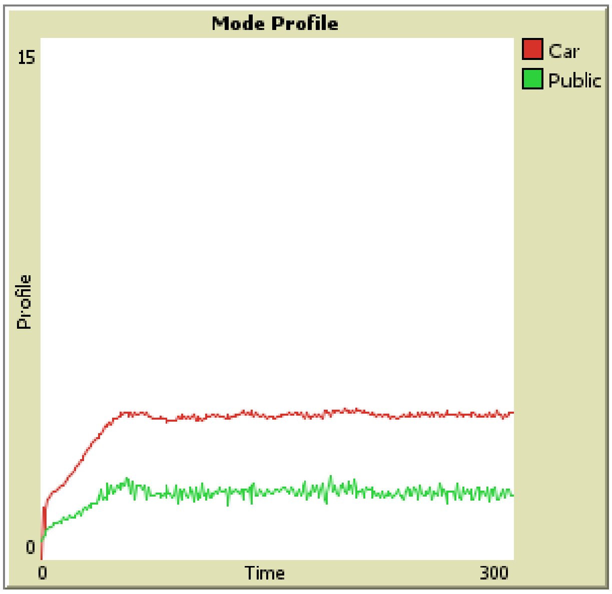

Figure 3.5: Mode Profile

It is important to note that in reality, traffic and weather conditions affect the payoff of travel by increasing the time of travel. Travelers tend to choose rail mode

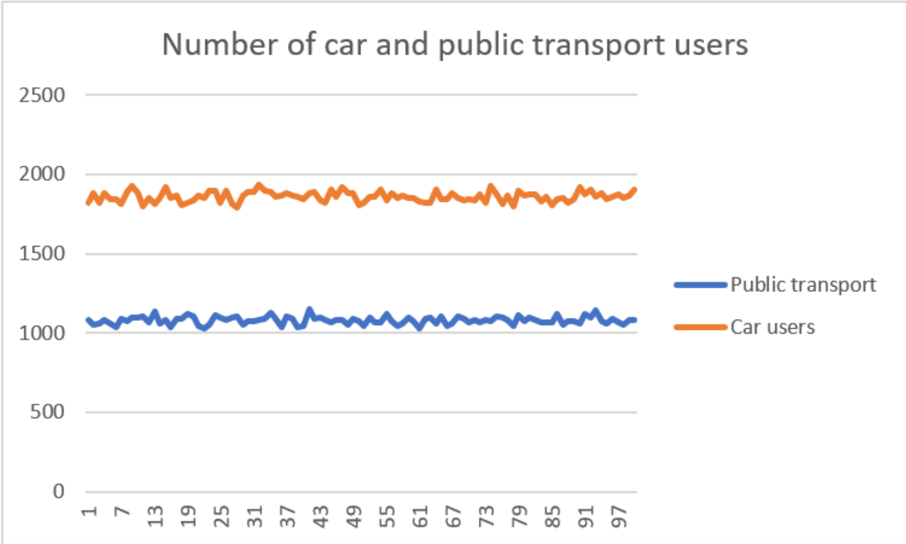

Figure 3.6: Mode Profile

Travel mode	Average travel time	Average travel cost	Payoff	Nash Equilibrium
Car	29.3	10	293	0.23
Taxi	50	25	1250	0.10
Bus	46	3	138	0.31
Rail	59.8	4.5	269.1	0.36

Table 3.1: Nash Equilibrium of TCM

because of its relatively stable travel time in weather conditions.

Conclusion

The goal of this project was to build a travel mode choice model capable of embedding game theory, the simulation model can be used in further research as a starting point. The model assumes that the travelers use mixed strategy when making choices, hence, the probability of a mode being chosen is dynamic.

The simulations used in this work modeling a set of travelers in making choices of four modes : car, bus, taxi, and rail. This model is based on a split mode framework often used to describe the choice behavior of travelers.

Game theory has proven a usefulness in modeling the relationship between travel mode choices and their payoffs through Nash Equilibrium. A decrease in a mode's payoff can increase its proportion. The evolution part of this study is the change in Nash Equilibrium of travel mode choice.

Although people often use multimodal transport, combining two or more travel modes in the same trip, which this model lacks. This mechanism is often used to avoid traffic or lack of coverage in some areas.

Finally, it would be interesting to add more components besides the one that this travel mode choice model have, which can be done by further research. Studying the dynamics of travel mode choice behavior could extract new patterns that can be out of the scope of this model.

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