Travel Mode Choice Modeling using Game theory

FADELI Ismail

A thesis presented for the degree of
Masters in Transport Engineering

Department of Transport and Logistics Engineering
National High School of Technology
ENST
Algeria
2020

Travel Mode Choice Modeling using Game theory

Thesis Subtitle

FADELI Ismail

Abstract

Contents

List of Figures

List of Tables

Introduction

The purpose of this study is to describe how travelers adjust their mode of transportation choice behaviors under the influence of traffic information. Chapter 1 describes the basic theories applied in this work, including game theory concepts and evolutionary dynamics. In Chapter 2, key studies from the literature regarding travel choice behavior are briefly examined. The third chapter describes the database and model used in this study. Limitations of the proposed modeling method and further research directions are discussed at the end of this chapter.

Chapter 1

Concepts

1.1 Game Theory

Game Theory is the study of rational behavior in situations involving interdependence as it may involve:

- Common interest (coordination);
- Competing interests (rivalry);
- Rational behavior: players can do the best they can, in their own eyes;
- Because of the players' interdependence, a rational decision in a game must be based on a prediction of others' responses;

1.1.1 The parts of a Game

A Game consists of three parts : Players, Actions and Payoffs

Players

Players are the decision makers and they can be : People, Governments or Companies.

Actions

What can the players do ? Decide when to sell a stock, decide how to vote or enter a bid in an auction...

Payoffs

Payoffs can represent the motivation of the players, for example : Do they care about profit ? or Do they care about other players ?

1.1.2 Defining Games

Games can be represented in two methods : Normal forms and Extensive Forms.

1.1.3 Extensive Form

An extensive form game includes timing of moves. Players move sequentially, represented as a tree.

- Chess: white player moves, then black player can see white's move and react...

Keeps track of what each player knows when he or she makes a decision :

- Poker: bet sequentially - what can a given player see when they bet.

1.1.4 Normal Form

A normal form represents a list of what players get on function of their actions. Finite, n-person normal form game $\langle N, A, u \rangle$:

- Players: $N = 1, ..., n$ is a finite set of n, indexed by i.
- Actions set for player i A_i

  $a = (a_1, ..., a_n) \in A = A_1 * ... * A_n$ is an action profile.
- Utility function or Payoff function for player $i : u_i : A \to \mathbb{R}$

  $u = (u_1, ..., u_n)$, is a profile of utility functions.

1.1.5 Best Response and Nash Equilibrium

Best Response :

$$a_i^* \in BR(a_{-i}) iff \forall a_i \in A_i, u_i(a_i^*, a_{-i}) \geq u_i(a_i, a_{-i}) \tag{1.1}$$

Nash Equilibrium (Definition): $a =< a_1, ..., a_n >$ is a "**pure strategy**" if $\forall i, a_i \in BR(a_{-i})$

1.1.6 Dominant strategies

let s_i and $s_i^{`}$ be two strategies for player i, and let S_{-i} be the set of all possible strategy profiles for other players.

- s_i **strictly dominates** $s_i^{`}$ if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) > u_i(s_i^{`}, s_{-i})$
- s_i **very weakly dominates** $s_i^{`}$ if $\forall s_{-i} \in S_{-i}, u_i(s_i, s_{-i}) \geq u_i(s_i^{`}, s_{-i})$
- A strategy is called **dominant** if it dominates all others.
- A strategy profile consisting of dominant strategies for every player must be a Nash Equilibrium.

1.1.7 Pareto Optimal

An outcome o^* is **Pareto-optimal** if there is no other outcome that Pareto-dominates it.

1.2 Mixed Strategies and Nash Equilibrium

A strategy s_i for agent i as any probability distribution over the actions A_i.

- **pure strategy:** only one action is played with positive probability
- **mixed strategy:** more than one action is played with positive probability

  these actions are called the support of the mixed strategy.
- Let the set of all strategies for i be S_i
- let the set of all strategy profiles be $S = S_1 \times ... \times S_n$

1.2.1 Utility in Mixed Strategies

In order to find the payoff if all the players follow mixed strategy profile $s \in S$ we can use the **expected utility** from decision theory:

$$u_i(s) = \sum_{a \in A} u_i(a) P(a|s) \tag{1.2}$$

$$P(a|s) = \prod_{j \in N} s_j(a_j) \tag{1.3}$$

1.2.2 Best Response and Nash Equilibrium

The definitions of best response and Nash equilibrium are generalized from actions to strategies.

Definition (Best Response):

$$s_i^* \in BR(s_{-i}) \textbf{ if } \forall s_i \in S_i, u_i(s_i^*, s_{-i}) \geq u_i(s_i, s_{-i}) \tag{1.4}$$

Definition (Nash Equilibrium)

$$s =< s_1, ..., s_n > \textbf{ is a Nash Equilibrium if } \forall i, s_i \in BR(s_{-i})$$

Theorem (Nash, 1950) Every finite game has a Nash equilibrium.

1.2.3 Computing Nash Equilibrium

Two algorithms for finding NE

- LCP(Linear Complimentary) [Lemke-Howson].
- Support Enumeration Method [Porter et al].

1.2.4 Complexity Analysis

Computing a Nash Equilibrium is a **PPAD-complete**[1] , this theorem has been proven for:

- for games $\geq$ 4 players;
- for games with 3 players;
- for games with 2 players;

1.2.5 Summary of mixed strategies

- Some games have mixed strategy Nash Equilibria.
- A player must be indifferent between the actions he or she randomizes over.
- Randomization happen in business interactions, society, sports...

[1] PPAD : Polynomial Parity Argument on Directed Graphs

1.2.6 Strictly Dominated Strategies

Definition a strategy $a_i \in A_i$ is strictly dominated by $a'_i \in A_i$ if

$$u_i(a_i, a_{-i}) < u_i(a'_i, a_{-i}), \forall a_{-i} \in A_{-i} \tag{1.5}$$

1.2.7 Weakly Dominated Strategies

Definition a strategy $a_i \in A_i$ is weakly dominated by $a'_i \in A_i$ if

$$u_i(a_i, a_{-i}) \leq u_i(a'_i, a_{-i}), \forall a_{-i} \in A_{-i} \tag{1.6}$$

and

$$u_i(a_i, a_{-i}) < u_i(a'_i, a_{-i}), \exists a_{-i} \in A_{-i} \tag{1.7}$$

1.3 Perfect inf

The extensive form is an alternative representation that makes the temporal structure explicit.

- Perfect information extensive form games.
- Imperfect information extensive form games.

A finite perfect information game in extensive form is defined by the tuple $(N, A, H, Z, \chi, \rho, \sigma, u)$ where:

- Players: N is a set of n players.
- Actions: A is set of actions.
- Choice nodes and labels for these nodes:
  - Choice nodes: H is a set of non-terminal choice nodes.
  - Action function: $\chi : H \to 2^A$ assigns to each choice a set of actions.
  - Player function: $\rho : H \to N$ assigns to each non-terminal node h a player $i \in N$ who chooses an action at h.
- Terminal nodes: Z is a set of terminal nodes, disjoint from H.
- Successor function: $\sigma : H \times A \to H \cup Z$ maps a choice node and an action to a new choice node or terminal node such that for all $h_1, h_2 \in H$ and $a_1, a_2 \in A$, if $\sigma(h_1, a_1) = \sigma(h_2, a_2)$ then $h_1 = h_2$ and $a_1 = a_2$
- Utility function: $u = (u_1, ..., u_n)$ where $u_i : Z \to R$

figure 1.1 shows a sharing game represented in the extensive form

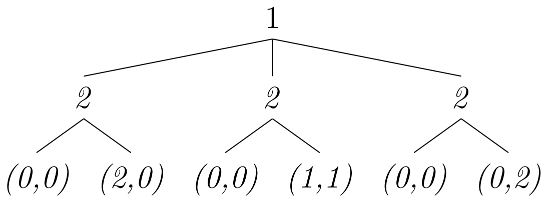

Figure 1.1: Sharing Game

1.3.1 Pure Strategies

A pure strategy for a player in a perfect-information game is a complete specification of which action to take at each node belonging to that player.

Definition Let $G = (N, A, H, Z, \chi, \rho, \sigma, u)$ be a perfect-information extensive-form game. Then the pure strategies of player i consist of the cross product

$$\prod_{h \in H, \rho(h)=i} \chi(h)$$

Given our new definition of pure strategy, we can reuse our old definitions of mixed strategies and Nash equilibrium in 1.1.

1.3.2 Sub-game Perfection

Definition 1 (Sub-game Perfection) *The set of sub-games of G is defined by the sub-games of G rooted at each of the nodes in G.*

Let s be a sub-game perfect equilibrium of G if for any sub-game G' of G, the restriction of s to G' is a Nash Equilibrium of G'. Since G is its own sub-game , every sub-game perfect is a Nash Equilibrium.

1.3.3 Backward Induction

Backward Induction has been used in solving games since John von Neumann and Oskar Morgenstern published their book, Theory of Games and Economic Behaviors in 1944.

The idea behind Backward Induction is to identify the equilibrium in the buttom trees, and adopt these as one moves up the tree as the next algorithm shows.

Algorithm 1 Backward Induction

return $u(h)$
if $h \in Z$ **then**
  return $u(h)$
end if
$best - util \leftarrow -50$
for all $a \in \rho(h)$ **do**
  $util - at - child \leftarrow BACKWARDINDUCTION(\sigma(h, a))$
  if $util - at - child_p(h) > best - util_p(h)$ **then**
    $best - util \leftarrow util - at - child$
  end if
end for
return $best - util$

Denote $util_a t_c hild$ is a utility vector for each player.

1.4 Evolutionary Game Theory

Evolution and Game Theory was introduced by John Maynard Smith in Evolution and The Theory of Games. The Theory was formulated to understand the behavior of animals in game theoretic situations. But it can be applied to modeling human behavior.

After the emergence of traditional game theory, biologists realized the potential of game theory to formally study adaptation and convolution of biological populations, especially in contexts where the fitness of a phenotype depends on the composition of the population (Hamilton, 1967). The main assumption of evolutionary game theory was that strategies with greater payoffs at a particular time would tend to spread more and thus have better chances of being present in the future.

The most important concept of evolutionary thinking that was introduced by Manynard Smith and Price (1973) is the notion of **Evolutionary Stable Strategy**(ESS), for 2-player symmetric games played by individuals belonging to the same population. Furthermore, a strategy s is an ESS if and only if, when adopted by all members of a population, meaning that any other strategy i that could enter the population in a low percentage would obtain a strictly lower expected payoff in the population than the s strategy.

The basic ideas behind Evolutionary game theory is that strategies with greater payoffs tend to spread more, and that fitness is frequency dependent soon transcended the borders of biology and started to spread through many other disciplines. In economic context, it was understood that natural selection would derive from competition among entities for small resources or market shares. In social contexts, evolution was often understood as cultural evolution, and it referred to dynamic changes in behavior or ideas over time (**Nelson and Winter**; **1982**)(**Boyd and Richerson**; **1985**).

In order to extend this understanding further, let's consider this example: Suppose that a small group of mutants choosing a strategy different from δ^* to enter the population.

- Denote the fraction of mutants in the population by ε and assume that the mutant adopts the strategy δ.
- The expected payoff of a mutant is : $(1-\varepsilon)u(\delta, \delta*) + \varepsilon u(\delta*, \delta)$
- The expected payoff of a mutant that adopts the strategy is : $(1-\varepsilon)u(\delta*, \delta*) + \varepsilon u(\delta*, \delta)$
- For any mutation to be driven out of the population we need the expected payoff of any mutant to be less than the expected payoff of normal organism :

$$(1-\varepsilon)u(\delta*, \delta*) + \varepsilon u(\delta*, \delta) > (1-\varepsilon)u(\delta, \delta*) + \varepsilon u(\delta*, \delta) \tag{1.8}$$

Chapter 2

Literature Review

The current approach to mode choice behavior in the perspective of expected utility theory or random utility theory. However, travelers evaluate the alternative modes by individual experience and attitude which are not considered in the expected utility theory or random utility theory models. Therefore, many alternative theories have been proposed, for example, prospect theory, cumulative prospect theory and regret theory. Among them, cumulative prospect theory draws the most attention because it describes the bounded rational behaviors under various conditions.

2.1 Travel Demand Management

One of the most important socio-economic problems in recent decades has been the optimization of an urban transport system. Furthermore, This type of problem mainly occurs in developing countries, and the reason behind it is the increasing rate of car ownership.[1]. Which urges cities to realize transport strategies combating this effect and also to decrease the negative impacts of transportation on the environment [2].

2.2 Simple Choice Models

Three simple-choice models are usually discussed in the context of utility theory. According to this understanding, the utility U_i of alternative mode i is expressed as the sum of a deterministic component V_i and a random component ϵ_i(**Richard**; **David**; **1982**):

$$U_i = V_i + \epsilon_i \tag{2.1}$$

The probability of choosing the ith mode from a set of n alternatives is thus:

$$P_i = P_r[U_i > U_j](j = i) \tag{2.2}$$

Alternatively,

$$P_i = P_r[\epsilon_j < V_i - V_j + \epsilon_i](j = i) \tag{2.3}$$

If the cumulative density function of the error $\epsilon = (\epsilon_1, ..., \epsilon_n)$ is $F(t_1, ..., t_n)$, and the partial of the cumulative density function with respect to variable i is $F_i(t_1, ..., t_n)$, then equation 2.3 becomes:

$$P_i = \int_{-\infty}^{+\infty} F_i(..., t + V_i - V_j, ...)df \tag{2.4}$$

If the error terms are independent identically distributed Gumbel variate, then Equation 2.4 is a multivariate logit model. If the error terms have a joint multivariate normal distribution, then 2.4 defines a multinomial probit model.

The third simple-choice model, discriminant analysis, was originally developed for taxonomic purposes. However, discriminant analysus has been avoided in mode choice analysis because it lacks the probabilistic theory that is possessed by other behavioral-choice models. In recent decades, logit models have been the most used when it comes to travel mode choice analysis.

[1] Khovako, 2014,.
[2] World Bank, 2011

2.3 Choice Decision Elements

The travel mode choice is an important step of the transportation forecasting(**Litman**; **2011**). The main modes for travelers are private cars or public transportation.

The framework for the choice process is that the individual determines the available alternatives(modes), next, evaluates the attributes of each alternative, and then, uses a decision rule to select an alternative from among the available alternatives (Ben-Akiva and Lerman, 1985, Chapter 3).

Further in this section, we see that the elements of a choice process are : the decision maker, the alternatives, the attributes of alternatives and the decision rule.

Travel mode choice is usually mathematically represented by logit functions, due to its consideration of particular qualities of travelers(**Bravo et al**; **2009**).

The next function computes the probability of car mode choice, which depends on the difference between the travel time σ_t and the travel costs σ_c:

$$p^*(\sigma_t, \sigma_c) = \frac{1}{1 + exp(a_0 + a_i c^c + a_c \sigma_c)} \tag{2.5}$$

The objective function based on logit function may is presented in the following equation(**Hollander et al**; **2006**):

$$G(p) = (p - p^*(\sigma_t, \sigma_c))* = (p - \frac{1}{1 + exp(a_0 + a_i c^c + a_c \sigma_c)})^2 \rightarrow min_p. \tag{2.6}$$

The difference of the travel times between modes (private and public transport) for traveling between A and B :

$$\sigma t_{a,b} = T_a^r + T_b^r + T_b^p - T_a^t - T_b^t - t_w \tag{2.7}$$

where t_w is the waiting time in public transport.

The difference between travel costs could be :

$$\sigma c_{a,b} = c^c(t_b^r + t_b^r - t_b^p) - c_b^p - c^t \tag{2.8}$$

2.4 Evolutionary Game Theory and Engineering

Many Engineering Infrastructures are becoming increasingly complex to manage due to large scale distributed nature and the nonlinear interdependence between their components (**Quijano et al**; **2017**).Including transportation systems, communication networks, data networks, and teams of anonymous vehicles. Controlling these large scale distributed systems requires the implementation of decision rules for interconnected components that grantee the accomplishment of a collective objective in an environment that is often dynamic and uncertain. In order to achieve this goal, traditional control theory is often of little use, since distributed systems generally lack a central entity with access or control over all components(**Marden and Shamma**; **2015**).

Works Cited

[1] Boyd, R. and Richerson, P. J. (1985). Culture and the Evolutionary Process. University of Chicago Press.

[2] Maynard Smith, J. (1982). Evolution and the Theory of Games. Cambridge University Press, Cambridge.

[3] Maynard Smith, J. and Price, G. R. (1973). The logic of animal conflict. Nature, 246:15–18.

[4] Marden, J. R. and Shamma, J. S. (2015). Game theory and distributed control. In Young, H.P. and Zamir, S., editors, Handbook of Game Theory with Economic Applications, volume 4, chapter 16, pages 861–899. Elsevier, Amsterdam.

[5] Nelson, R. R. and Winter, S. G. (1982). An Evolutionary Theory of Economic Change. Harvard University Press.

[6] Quijano, N., Ocampo-Martinez, C., Barreiro-Gomez, J., Obando, G., Pantoja, A., and Mojica-Nava, E. (2017). The role of population games and evolutionary dynamics in distributed control systems: The advantages of evolutionary game theory. IEEE Control Systems Magazine, 37(1):70–97.

[7] Litman, T. (2011). Why and how to reduce the amount of land paved for roads and parking facilities. Environmental Practice. 13(1), pp. 38-46.

[8] Bravo et al, (2009) An integrated behavioral model of the land-use and transport systems with network congestion and location externalities. Transportation Research Part B. 44(4), pp. 584-596. https://doi.org/10.1016/j.trb.2009.08.002

[9] Richard, B. David, M. and Richard, W(1982). A Selective Review of Travel-Mode Choice Models, pp. 370-375.