Mikroelectronics 1

Fundamentals of Semiconductor Devices

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Chapter 1

Introduction to Semiconductors

1.1 Purpose of This Manual and Course Scope

This manual accompanies the course *Mikroelektronik 1* and provides a coherent, exam-oriented introduction to the physical principles of semiconductor materials and devices. The content follows the core ideas of Sze and Ng, focusing on Parts I–III (up to Chapter 7) and emphasizing concepts that are central to written and oral examinations.

> **Exam Focus.** Typical ME1 exam tasks emphasize precise definitions, correct physical interpretation, qualitative reasoning based on band concepts, and consistent use of a small set of key formulas.

1.2 What Makes Semiconductors Special?

Semiconductors are materials whose electrical conductivity can be controlled over many orders of magnitude through temperature, intentional doping, and external excitation such as electric fields or illumination.

A convenient classification is:

- **Metals:** high carrier density, Fermi level located inside an energy band, limited control via doping.

- **Insulators:** very large band gap, negligible free carrier density at room temperature.

- **Semiconductors:** moderate band gap, carrier density that is highly tunable, enabling active electronic devices.

1.3 Energy Units, Thermal Energy, and Typical Scales

In semiconductor physics, energy is commonly expressed in electronvolts:

> **Definition.**
> $$1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}.$$

The thermal energy scale at room temperature is

$$k_{\mathrm{B}}T \approx 25.9 \text{ meV} \quad \text{at } T = 300 \text{ K.}$$

This scale is essential when assessing carrier statistics, dopant ionization, and the validity of approximations.

Exam Focus. You should remember the order of magnitude: $k_{\mathrm{B}}T \approx 26$ meV at room temperature. It appears repeatedly in degeneracy criteria and carrier concentration formulas.

1.4 Electrons in Solids: From Atomic Levels to Bands

Isolated atoms possess discrete electronic energy levels. When many atoms are brought together to form a crystal, interactions between neighboring atoms cause these discrete levels to split into densely spaced states, forming continuous **energy bands**.

Two bands are of primary importance:

- **Valence band (VB):** typically filled at $T = 0$.

- **Conduction band (CB):** typically empty at $T = 0$.

Definition. The **band gap** E_{g} is defined as

$$E_{\mathrm{g}} = E_{\mathrm{C}} - E_{\mathrm{V}},$$

where E_{C} and E_{V} denote the conduction and valence band edges.

1.5 Band Gap Values and Material Examples

Typical room-temperature band gaps (order of magnitude) are:

- Silicon (Si): $E_{\mathrm{g}} \approx 1.1$ eV, indirect

- Germanium (Ge): $E_{\mathrm{g}} \approx 0.66$ eV, indirect

- Gallium arsenide (GaAs): $E_{\mathrm{g}} \approx 1.42$ eV, direct

Exam Focus. A standard exam fact: silicon is an indirect-gap semiconductor and therefore inefficient for light emission, whereas GaAs has a direct band gap and is suitable for optoelectronic applications.

1.6 The $E(k)$ Relationship and Direct vs Indirect Band Gaps

The electronic band structure is described by the energy E as a function of crystal momentum k.

Direct band gap

In a direct-gap semiconductor, the minimum of the conduction band and the maximum of the valence band occur at the same value of k. As a result, optical transitions can occur without a change in crystal momentum.

Indirect band gap

In an indirect-gap semiconductor, the extrema of the conduction and valence bands occur at different values of k. Optical transitions then require the additional participation of a phonon, making radiative recombination less probable.

Exam Focus. Typical reasoning task: Explain qualitatively why indirect-gap semiconductors exhibit weak light emission due to the need for phonon-assisted transitions.

1.7 Photon Energy and Optical Absorption

The energy of a photon is given by

$$E_\gamma = h\nu = \frac{hc}{\lambda}.$$

A first-order condition for band-to-band absorption is

$$E_\gamma \geq E_{\mathrm{g}},$$

with the understanding that indirect transitions often involve phonons.

Example. For silicon with $E_{\mathrm{g}} = 1.1$ eV, the corresponding wavelength is approximately
$$\lambda \approx \frac{1.24 \text{ eV} \cdot \mu\text{m}}{1.1 \text{ eV}} \approx 1.13 \ \mu\text{m}.$$
Silicon is therefore transparent to longer wavelengths.

1.8   Equilibrium, the Fermi Level, and Occupation Probability

At thermal equilibrium, electronic states are occupied according to the Fermi–Dirac distribution:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right)}.$$

Meaning of the Fermi level

The Fermi level E_{F} represents the chemical potential of electrons. In thermal equilibrium, a single Fermi level characterizes the entire system.

Boltzmann approximation

If the energy separation from the Fermi level satisfies $(E - E_{\mathrm{F}}) \gtrsim 3k_{\mathrm{B}}T$, the occupation probability may be approximated by

$$f(E) \approx \exp\left(-\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right).$$

Exam Focus. You must be able to state when the Boltzmann approximation is valid and explain the physical meaning of a non-degenerate semiconductor.

1.9   Carriers: Electrons and Holes

When an electron is excited from the valence band into the conduction band, it leaves behind an unoccupied state that behaves as a positively charged, mobile quasi-particle called a **hole**.

Definition. Electrons are carriers in the conduction band, and **holes** are carriers in the valence band. Both contribute to electrical current.

1.10   Basic Material Parameters Used Throughout ME1

A compact description of semiconductor devices relies on a small set of material parameters:

- Band gap E_{g}

- Effective density of states N_C and N_V

- Dielectric permittivity ε_s

- Carrier mobilities μ_n and μ_p

- Diffusion coefficients D_n and D_p

Exam Focus. Know which physical phenomena each parameter controls: ε_s for electrostatics, μ for drift transport, D for diffusion, and E_g for thermal and optical behavior.

1.11 A First Look at Transport Mechanisms

Carrier transport in semiconductors occurs primarily through:

- **Drift:** motion driven by an electric field.

- **Diffusion:** motion driven by concentration gradients.

The conductivity is given by
$$\sigma = q(n\mu_n + p\mu_p),$$
where n and p denote electron and hole concentrations.

1.12 Generation and Recombination: Qualitative Picture

Carrier populations change through:

- **Generation** (thermal excitation, optical excitation),

- **Recombination** (radiative, defect-assisted, Auger processes).

Direct-gap materials favor radiative recombination, whereas indirect-gap materials are dominated by non-radiative mechanisms.

1.13 Temperature Effects

Temperature influences nearly all semiconductor properties:

- intrinsic carrier concentration $n_i(T)$,

- dopant ionization,

- carrier mobility through phonon scattering,

- band gap narrowing with increasing temperature.

Exam Focus. You should be able to justify qualitatively: increasing temperature leads to a strong increase in n_i, while mobilities often decrease due to enhanced phonon scattering.

1.14   Device Landscape and Course Outlook

The physical principles developed in Chapters 1–7 form the basis for:

- the p–n junction,

- bipolar junction transistors (BJTs),

- MOS structures and MOSFETs.

In ME1, the p–n junction serves as the central device that integrates electrostatics, carrier statistics, and transport.

1.15   Chapter Roadmap

- Chapter 2: crystal structures and lattice concepts

- Chapter 3: energy bands and effective mass

- Chapters 4–5: intrinsic and extrinsic carrier concentrations

- Chapter 6: drift, diffusion, and continuity

- Chapter 7: electrostatics and operation of the p–n junction

1.16   Key Takeaways

- Semiconductors possess a finite band gap that enables controllable conductivity.

- Band structure determines electrical and optical properties.

- Fermi–Dirac statistics govern carrier occupation.

- Electrons and holes act as mobile charge carriers.

- Temperature strongly affects carrier density and transport.

Chapter 2

Crystal Structure of Semiconductors

2.1 Why Crystal Structure Matters in Microelectronics

Semiconductor devices are fabricated from crystalline solids. The periodic arrangement of atoms in a crystal determines the electronic band structure, effective masses, carrier mobility, and many optical properties.

A clear understanding of crystal structure is therefore essential for explaining material-dependent electronic behavior.

> **Exam Focus.** In ME1 examinations, crystal structure questions focus on the identification of lattice types, Miller indices, and qualitative consequences for electronic properties, rather than detailed crystallographic derivations.

2.2 Crystalline and Amorphous Solids

A **crystalline solid** exhibits long-range periodic order, meaning that the atomic arrangement repeats regularly in space over macroscopic distances.

An **amorphous solid** lacks long-range periodicity, although short-range atomic order may still be present, as in amorphous silicon.

> **Definition.** A **crystal** is a solid whose atoms are arranged in a three-dimensional periodic lattice.

High-performance semiconductor devices are almost exclusively fabricated from crystalline materials because periodicity enables well-defined electronic energy bands.

2.3 Lattice, Basis, and Crystal Structure

The atomic arrangement of a crystal is described using two elements:

- **Lattice**: a periodic array of points in space.

- **Basis**: the group of atoms associated with each lattice point.

Definition. The **crystal structure** is obtained by attaching an identical basis to every lattice point.

This distinction is fundamental: different materials may share the same lattice but differ in the atomic basis, leading to different physical properties.

2.4   Unit Cells

The **unit cell** is the smallest repeating volume that fully represents the crystal structure.

Primitive and Conventional Unit Cells

- **Primitive unit cell**: contains exactly one lattice point.

- **Conventional unit cell**: contains multiple lattice points and is chosen for symmetry and clarity.

In semiconductor physics, conventional cubic unit cells are often preferred because they make symmetry relations and directions easier to describe.

2.5   Common Crystal Lattices in Semiconductors

Several lattice types occur in solid-state materials.

2.5.1   Simple Cubic (SC)

Lattice points are located at the cube corners only. This lattice is rare in elemental semiconductors.

2.5.2   Body-Centered Cubic (BCC)

Lattice points are located at the cube corners and at the cube center. BCC structures are not typical for common semiconductor materials.

2.5.3   Face-Centered Cubic (FCC)

Lattice points are located at the cube corners and at the centers of all cube faces. The FCC lattice forms the underlying framework of several important semiconductor crystal structures.

2.6 Diamond Cubic Structure

Elemental semiconductors such as silicon (Si) and germanium (Ge) crystallize in the **diamond cubic structure**.

Construction

The diamond structure consists of:

- an FCC lattice,

- a basis of two identical atoms.

The second atom in the basis is displaced by

$$\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$$

relative to the first atom, expressed in units of the cubic lattice constant.

> **Exam Focus.** A frequent exam question: "Describe the crystal structure of silicon." Expected answer: diamond cubic structure, consisting of an FCC lattice with a two-atom basis.

Coordination Number

Each atom in the diamond structure has four nearest neighbors, forming tetrahedral bonds. This coordination geometry is closely related to sp^3 hybridization in covalent bonding.

2.7 Zincblende Structure

Compound semiconductors such as gallium arsenide (GaAs) crystallize in the **zincblende structure**.

Relation to the Diamond Structure

The zincblende structure is geometrically identical to the diamond structure, except that the two atoms in the basis are chemically different.

- Lattice: FCC
- Basis: two different atomic species

> **Exam Focus.** A key comparison for ME1: silicon (diamond) and GaAs (zincblende) share the same lattice geometry, but differ in their atomic basis, leading to different electronic properties.

2.8   Wurtzite Structure

Some compound semiconductors, such as GaN, crystallize in the **wurtzite structure**. This structure is characterized by:

- a hexagonal lattice,

- tetrahedral coordination,

- strong relevance for wide-bandgap semiconductors.

While ME1 emphasizes cubic structures, wurtzite materials play an important role in optoelectronic and power devices.

2.9   Lattice Constant and Atomic Density

The **lattice constant** a defines the size of the unit cell and sets the spatial scale of the crystal.

The atomic density depends on:

- the lattice type,

- the number of atoms per unit cell.

For diamond and zincblende structures, the conventional cubic unit cell contains eight atoms.

> **Exam Focus.** A common qualitative fact: diamond and zincblende structures contain eight atoms per conventional cubic unit cell.

2.10   Miller Indices: Planes and Directions

Crystallographic planes are specified using Miller indices (hkl).

Miller Indices for Planes

The procedure for determining (hkl) is:

1. Determine the intercepts of the plane with the crystal axes.

2. Take the reciprocals of these intercepts.

3. Reduce the result to the smallest set of integers.

Crystallographic Directions

Directions are denoted by $[uvw]$. Families of equivalent planes and directions are written as $\{hkl\}$ and $\langle uvw \rangle$, respectively.

Exam Focus. In exams, you should be able to interpret common indices such as (100), (110), and (111), and explain their relevance for silicon wafers.

2.11 Importance of Crystal Orientation in Devices

Crystal orientation influences several technological properties:

- surface atomic density,

- oxidation behavior,

- carrier mobility anisotropy (qualitatively).

In silicon technology, frequently used wafer orientations are:

- (100),

- (110),

- (111).

2.12 Crystal Defects

Real crystals are never perfectly periodic. Deviations from ideal periodicity are referred to as **crystal defects**.

2.12.1 Point Defects

Point defects include:

- vacancies,

- interstitial atoms,

- substitutional atoms, including dopants.

2.12.2 Line Defects

Line defects, such as dislocations, locally disturb the lattice and can affect both mechanical and electronic properties.

Exam Focus. In ME1, defects are treated qualitatively. You should understand their existence and general impact, not detailed defect models.

2.13 Connection to Electronic Properties

The periodicity of a crystal enables:

- formation of electronic energy bands,

- definition of crystal momentum k,

- introduction of the effective mass concept.

Crystal defects and disorder tend to:

- scatter charge carriers,

- reduce mobility,

- introduce localized energy states.

These concepts provide the foundation for the band theory developed in Chapter 3.

2.14 Chapter Summary

- Semiconductors used in devices are crystalline solids with long-range order.

- Crystal structure is defined by lattice and basis.

- Silicon and germanium crystallize in the diamond cubic structure.

- Compound semiconductors such as GaAs crystallize in the zincblende structure.

- Miller indices describe crystallographic planes and directions.

- Crystal structure strongly influences electronic behavior.

Chapter 3

Energy Bands in Solids

3.1 Why Energy Bands Are Central to Semiconductor Physics

The electrical and optical behavior of semiconductors is fundamentally determined by their electronic energy band structure. Band theory provides the conceptual link between:

- crystal structure (Chapter 2),

- carrier statistics (Chapters 4 and 5),

- transport phenomena (Chapter 6),

- and device operation (Chapter 7).

A consistent understanding of energy bands is therefore essential for analyzing semiconductor devices.

> **Exam Focus.** Most ME1 examinations include at least one conceptual question that directly or indirectly tests understanding of energy bands, band gaps, or effective mass.

3.2 Electrons in a Periodic Potential

In a crystalline solid, electrons move in the periodic electrostatic potential created by the lattice of positively charged ion cores. This periodicity fundamentally alters the allowed energy states compared to free electrons.

> **Definition.** A **periodic potential** satisfies
>
> $$V(\mathbf{r}) = V(\mathbf{r} + \mathbf{R}),$$
>
> where $\mathbf{R}$ is a lattice translation vector.

As a consequence, the solutions of the Schrödinger equation in a crystal are qualitatively different from those of particles in free space.

3.3   Bloch's Theorem

Bloch's theorem states that the eigenfunctions of an electron in a periodic potential can be written as

$$\psi_{n,\mathbf{k}}(\mathbf{r}) = u_{n,\mathbf{k}}(\mathbf{r})\, e^{i\mathbf{k}\cdot\mathbf{r}},$$

where $u_{n,\mathbf{k}}(\mathbf{r})$ has the same periodicity as the lattice.

- n denotes the band index.

- $\mathbf{k}$ is the crystal momentum.

Exam Focus. You are not required to derive Bloch's theorem in ME1, but you must understand the physical meaning of $\mathbf{k}$ and why electronic energy depends on it.

3.4   Formation of Allowed and Forbidden Energy Bands

When isolated atoms are brought together to form a crystal:

- discrete atomic energy levels split,

- the resulting closely spaced levels form continuous energy bands,

- energy intervals with no allowed states form band gaps.

Definition. An **energy band** is a continuous range of allowed electronic energies. A **band gap** is an energy interval containing no allowed electronic states.

The magnitude of the band gap depends on:

- the atomic species,

- the crystal structure,

- the interatomic spacing.

3.5   Free Electron Model and Its Limitations

In the free electron model, electrons are assumed to move without experiencing the periodic potential of the lattice. The energy–momentum relation is

$$E = \frac{\hbar^2 k^2}{2m_0}.$$

Although this model captures certain features of metallic behavior, it cannot explain the existence of forbidden energy gaps.

Exam Focus. A standard conceptual point: the free electron model fails for semiconductors because it cannot account for band gaps.

3.6 Reciprocal Space and Brillouin Zones

The reciprocal lattice provides a natural framework for describing electron states in periodic solids.

Definition. The **first Brillouin zone** is defined as the Wigner–Seitz cell of the reciprocal lattice.

Key properties:

- Energy bands are periodic in crystal momentum $\mathbf{k}$.

- All unique electronic states can be described within the first Brillouin zone.

3.7 Band Edges and Notation

Within a given band structure, several quantities are of central importance:

- E_C: conduction band edge,

- E_V: valence band edge,

- $E_g = E_C - E_V$: band gap energy.

These quantities determine carrier concentrations, optical transitions, and transport behavior.

Exam Focus. You should be able to explain the physical meaning of E_C, E_V, and E_g and how they relate to carrier excitation.

3.8 Direct and Indirect Band Gaps

Semiconductors are classified according to the relative positions of their band extrema in k-space:

- **Direct band gap:** conduction band minimum and valence band maximum occur at the same $\mathbf{k}$.

  • **Indirect band gap:** these extrema occur at different **k** values.

This distinction has important consequences for optical absorption and emission processes.

3.9   Effective Mass Approximation

Near extrema of an energy band, the energy–momentum relation can be approximated by a parabola:

$$E(k) \approx E_0 + \frac{\hbar^2 k^2}{2m^*}.$$

Definition. The **effective mass** m^* is defined as

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2}.$$

  • A small curvature corresponds to a large effective mass.

  • A large curvature corresponds to a small effective mass.

Exam Focus. A typical ME1 question: Explain the physical meaning of effective mass in terms of the curvature of the energy–momentum relation.

3.10   Electron and Hole Effective Masses

Electrons near the conduction band minimum and holes near the valence band maximum are characterized by different effective masses:

  • electron effective mass m_n^*,

  • hole effective mass m_p^*.

For holes, it is convenient to describe transport as the motion of positively charged quasi-particles with effective mass m_p^*.

3.11   Density of States: Qualitative Considerations

The **density of states** (DOS) specifies how many electronic states are available per unit energy interval.

Qualitative properties relevant for ME1:

- the DOS depends on effective mass,

- the DOS is zero inside the band gap,

- the DOS increases with energy within each band.

Exam Focus. You should be able to explain qualitatively why a larger effective mass leads to a higher density of states.

3.12   Band Structure and Carrier Dynamics

The energy band structure determines key dynamical properties of carriers:

- carrier velocity,

- carrier response to electric fields,

- scattering behavior and mobility.

The group velocity of a carrier is given by

$$v = \frac{1}{\hbar}\frac{dE}{dk}.$$

3.13   Negative Curvature and the Hole Concept

Near the top of the valence band, the curvature of $E(k)$ is negative. Instead of describing electron motion with a negative effective mass, transport is described in terms of holes with positive charge and positive effective mass.

Exam Focus. The introduction of holes as charge carriers is a key conceptual step and is frequently tested qualitatively in ME1.

3.14   Temperature Effects on Band Structure

Temperature influences band structure primarily through:

- thermal lattice expansion,

- electron–phonon interactions.

As a result, the band gap typically decreases with increasing temperature. A commonly used empirical relation is

$$E_{\mathrm{g}}(T) = E_{\mathrm{g}}(0) - \alpha \frac{T^2}{T + \beta},$$

where α and β are material-dependent constants.

> **Exam Focus.** You are not required to memorize this expression, but you should know that E_{g} decreases as temperature increases.

3.15   Relation to Carrier Statistics

Energy band structure defines the allowed electronic states to which Fermi–Dirac statistics are applied. This framework enables the calculation of carrier concentrations, as developed in Chapters 4 and 5.

3.16   Validity and Limitations of Simple Band Models

The parabolic band approximation:

- is accurate near band extrema,

- becomes inaccurate at higher energies.

More advanced band models are used in research and advanced courses, but ME1 relies primarily on the effective mass approximation.

3.17   Chapter Summary

- A periodic crystal potential leads to the formation of energy bands and gaps.

- Bloch's theorem governs electron states in crystalline solids.

- Band gaps distinguish metals, semiconductors, and insulators.

- Effective mass connects band curvature to carrier dynamics.

- Band structure underlies carrier statistics, transport, and device operation.

Chapter 4

Intrinsic Semiconductors

4.1 Scope and Learning Goals

This chapter establishes the quantitative foundation for equilibrium carrier concentrations in semiconductors. Using band concepts (Chapter 3) and statistical occupation (Chapter 1), we derive the central results:

- equilibrium electron and hole concentrations n and p,

- effective densities of states N_C and N_V,

- intrinsic carrier concentration $n_i(T)$ and intrinsic level E_i,

- intrinsic conductivity and its temperature dependence.

> **Exam Focus.** Typical ME1 tasks from this chapter include: (i) apply $n = N_C \exp\left(-\frac{E_C - E_F}{k_B T}\right)$ and $p = N_V \exp\left(-\frac{E_F - E_V}{k_B T}\right)$, (ii) compute n_i and/or the equilibrium Fermi-level position, (iii) derive or use the mass-action law $np = n_i^2$, (iv) justify qualitative temperature trends (carrier density and conductivity).

4.2 Equilibrium Carrier Concentrations

In thermal equilibrium, the occupation probability of an electronic state of energy E is given by the Fermi–Dirac distribution:

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}.$$

Carrier concentrations follow from integrating the product of:

- the density of states in the band,

- the appropriate occupation factor: $f(E)$ for electrons and $1 - f(E)$ for holes.

4.2.1 Definitions of n and p

Definition. The electron concentration in the conduction band is

$$n = \int_{E_\mathrm{C}}^{\infty} g_C(E)\, f(E)\, dE.$$

The hole concentration in the valence band is

$$p = \int_{-\infty}^{E_\mathrm{V}} g_V(E) \left[1 - f(E) \right] dE.$$

4.3   Density of States in 3D (Parabolic Band Approximation)

For ME1 we use the effective-mass approximation near the band edges:

$$E(\mathbf{k}) \approx E_\mathrm{C} + \frac{\hbar^2 k^2}{2m_n^*} \quad \text{(conduction band)},$$

$$E(\mathbf{k}) \approx E_\mathrm{V} - \frac{\hbar^2 k^2}{2m_p^*} \quad \text{(valence band)}.$$

Under this assumption, the 3D density of states per unit volume is:

Definition. Conduction band DOS:

$$g_C(E) = \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2} \right)^{3/2} \sqrt{E - E_\mathrm{C}}, \quad E \geq E_\mathrm{C}.$$

Valence band DOS:

$$g_V(E) = \frac{1}{2\pi^2} \left(\frac{2m_p^*}{\hbar^2} \right)^{3/2} \sqrt{E_\mathrm{V} - E}, \quad E \leq E_\mathrm{V}.$$

Exam Focus. If asked about the origin of the square-root dependence: it arises from 3D state counting in k-space combined with the parabolic relation between E and k.

4.4   Non-Degenerate Approximation (Boltzmann Limit)

In many ME1 applications, semiconductors are non-degenerate:

$$E_\mathrm{C} - E_\mathrm{F} \gtrsim 3k_\mathrm{B}T \quad \text{and/or} \quad E_\mathrm{F} - E_\mathrm{V} \gtrsim 3k_\mathrm{B}T.$$

Then the occupation factors can be approximated as

$$f(E) \approx \exp\left(-\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right), \qquad 1 - f(E) \approx \exp\left(-\frac{E_{\mathrm{F}} - E}{k_{\mathrm{B}}T}\right).$$

Exam Focus. A common exam step is to justify the Boltzmann limit by checking that E_{F} is sufficiently far from the band edge compared to $k_{\mathrm{B}}T$.

4.5 Electron Concentration in the Conduction Band

Starting from

$$n = \int_{E_{\mathrm{C}}}^{\infty} g_C(E)\, f(E)\, dE \approx \int_{E_{\mathrm{C}}}^{\infty} g_C(E)\, \exp\left(-\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right) dE,$$

insert the DOS expression:

$$n \approx \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{3/2} \int_{E_{\mathrm{C}}}^{\infty} \sqrt{E - E_{\mathrm{C}}}\, \exp\left(-\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right) dE.$$

Let $x = \frac{E - E_{\mathrm{C}}}{k_{\mathrm{B}}T}$, so that $E - E_{\mathrm{C}} = x k_{\mathrm{B}}T$ and $dE = k_{\mathrm{B}}T\, dx$. Then

$$n \approx \frac{1}{2\pi^2} \left(\frac{2m_n^*}{\hbar^2}\right)^{3/2} (k_{\mathrm{B}}T)^{3/2} \exp\left(-\frac{E_{\mathrm{C}} - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right) \int_0^{\infty} \sqrt{x}\, e^{-x}\, dx.$$

The integral equals $\frac{\sqrt{\pi}}{2}$, yielding:

Definition.

$$n = N_C \exp\left(-\frac{E_{\mathrm{C}} - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right),$$

with the **effective density of states** in the conduction band

$$N_C = 2\left(\frac{2\pi m_n^* k_{\mathrm{B}}T}{h^2}\right)^{3/2}.$$

4.6 Hole Concentration in the Valence Band

Similarly,

$$p = \int_{-\infty}^{E_{\mathrm{V}}} g_V(E)\, [1 - f(E)]\, dE \approx \int_{-\infty}^{E_{\mathrm{V}}} g_V(E)\, \exp\left(-\frac{E_{\mathrm{F}} - E}{k_{\mathrm{B}}T}\right) dE.$$

Using the substitution $x = \frac{E_{\mathrm{V}} - E}{k_{\mathrm{B}}T}$ gives:

Definition.

$$p = N_V \exp\left(-\frac{E_F - E_V}{k_B T}\right),$$

with the **effective density of states** in the valence band

$$N_V = 2\left(\frac{2\pi m_p^* k_B T}{h^2}\right)^{3/2}.$$

Exam Focus. Memorize the structure: n depends exponentially on $(E_C - E_F)$ and p depends exponentially on $(E_F - E_V)$. Also memorize the scaling: $N_C, N_V \propto T^{3/2}$.

4.7 Intrinsic Semiconductor

An intrinsic semiconductor is chemically pure and undoped. In equilibrium, each thermally excited conduction-band electron corresponds to a valence-band hole. Therefore:

Definition. For an intrinsic semiconductor in equilibrium:

$$n = p = n_i.$$

4.8 Mass-Action Law and Intrinsic Carrier Concentration

Multiplying the equilibrium expressions for n and p yields:

$$np = N_C N_V \exp\left(-\frac{E_C - E_F}{k_B T}\right) \exp\left(-\frac{E_F - E_V}{k_B T}\right) = N_C N_V \exp\left(-\frac{E_C - E_V}{k_B T}\right).$$

Since $E_C - E_V = E_g$:

Definition.

$$np = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right).$$

For intrinsic material, $n = p = n_i$, so:

Definition.

$$n_i^2 = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right), \qquad n_i = \sqrt{N_C N_V}\, \exp\left(-\frac{E_g}{2 k_B T}\right).$$

> **Exam Focus.** This is one of the most important ME1 results: n_i depends exponentially on $-E_g/(2k_BT)$, explaining the strong temperature sensitivity of intrinsic carrier density.

4.9 Intrinsic Fermi Level E_i

For intrinsic material, $n = p = n_i$. From

$$n_i = N_C \exp\left(-\frac{E_C - E_F}{k_BT}\right), \qquad n_i = N_V \exp\left(-\frac{E_F - E_V}{k_BT}\right),$$

solve for the equilibrium Fermi level in the intrinsic case (denoted E_i). Equating the two expressions leads to:

> **Definition.**
> $$E_i = \frac{E_C + E_V}{2} + \frac{k_BT}{2}\ln\left(\frac{N_V}{N_C}\right).$$

Because $N_C \propto (m_n^*)^{3/2}$ and $N_V \propto (m_p^*)^{3/2}$, E_i lies near midgap but is exactly midgap only if $m_n^* = m_p^*$.

> **Exam Focus.** If $m_n^* = m_p^*$, then $N_C = N_V$ and E_i equals the midgap energy. Otherwise, E_i shifts toward the band with the larger effective density of states.

4.10 Temperature Dependence of N_C, N_V, and n_i

From the definitions,
$$N_C \propto T^{3/2}, \qquad N_V \propto T^{3/2},$$

so
$$\sqrt{N_C N_V} \propto T^{3/2}.$$

Therefore,
$$n_i(T) \propto T^{3/2} \exp\left(-\frac{E_g}{2k_BT}\right).$$

Effect of Band-Gap Shrinkage

In real materials, E_g typically decreases as temperature increases, which further increases n_i. For ME1 it is sufficient to know the trend:

$$T \uparrow \Rightarrow E_g \downarrow \Rightarrow n_i \uparrow.$$

> **Exam Focus.** A common qualitative reasoning task: if E_{g} is treated constant, $\ln(n_i)$ depends approximately linearly on $1/T$, with slope $\approx -E_{\mathrm{g}}/(2k_{\mathrm{B}})$.

4.11   Intrinsic Conductivity

The conductivity is

$$\sigma = q\,(n\mu_n + p\mu_p).$$

For an intrinsic semiconductor, $n = p = n_i$, hence:

> **Definition.**
> $$\sigma_i = q\,n_i(\mu_n + \mu_p).$$

The resistivity is $\rho_i = 1/\sigma_i$.

Temperature Dependence

Two competing trends occur:

- $n_i(T)$ increases strongly with temperature (dominant exponential effect),

- mobilities μ_n and μ_p typically decrease with temperature due to increased phonon scattering.

At sufficiently high temperature, the growth of n_i dominates:

$$T \uparrow \Rightarrow \sigma_i \uparrow \quad \text{strongly.}$$

> **Exam Focus.** If asked whether intrinsic conductivity increases or decreases with T: it increases strongly because n_i rises exponentially, overriding the mobility decrease.

4.12   Intrinsic and Extrinsic Regimes (Context)

Even in doped semiconductors, intrinsic carriers dominate at sufficiently high temperature. This motivates the standard regime classification:

- **freeze-out (low T):** dopants not fully ionized,

- **extrinsic (moderate T):** dopants dominate carrier density,

- **intrinsic (high T):** $n \approx p \approx n_i$ dominates.

Chapter 5 develops the quantitative treatment for doped semiconductors.

4.13 Worked Example (Symbolic)

Assume N_C, N_V, and E_{g} are known at temperature T.

Example. Intrinsic carrier concentration:

$$n_i = \sqrt{N_C N_V} \, \exp\!\left(-\frac{E_{\mathrm{g}}}{2k_{\mathrm{B}}T}\right).$$

Intrinsic conductivity:

$$\sigma_i = q \, n_i (\mu_n + \mu_p).$$

Exam Focus. A typical problem-solving pipeline: $E_{\mathrm{g}}, T \Rightarrow n_i \Rightarrow \sigma_i$ (when mobilities are given).

4.14 Key Takeaways

- Under non-degenerate conditions:

$$n = N_C \exp\!\left(-\frac{E_{\mathrm{C}} - E_{\mathrm{F}}}{k_{\mathrm{B}}T}\right), \qquad p = N_V \exp\!\left(-\frac{E_{\mathrm{F}} - E_{\mathrm{V}}}{k_{\mathrm{B}}T}\right).$$

- Effective densities of states:

$$N_C = 2\left(\frac{2\pi m_n^* k_{\mathrm{B}}T}{h^2}\right)^{3/2}, \qquad N_V = 2\left(\frac{2\pi m_p^* k_{\mathrm{B}}T}{h^2}\right)^{3/2}.$$

- Mass-action law and intrinsic carrier concentration:

$$np = n_i^2 = N_C N_V \exp\!\left(-\frac{E_{\mathrm{g}}}{k_{\mathrm{B}}T}\right), \qquad n_i = \sqrt{N_C N_V} \, \exp\!\left(-\frac{E_{\mathrm{g}}}{2k_{\mathrm{B}}T}\right).$$

- Intrinsic level:

$$E_i = \frac{E_{\mathrm{C}} + E_{\mathrm{V}}}{2} + \frac{k_{\mathrm{B}}T}{2} \ln\!\left(\frac{N_V}{N_C}\right).$$

- Intrinsic conductivity increases strongly with temperature because $n_i(T)$ increases exponentially.

Chapter 5

Extrinsic Semiconductors (Doped Semi-conductors)

5.1 Scope and Learning Goals

In practice, semiconductors are intentionally doped to control equilibrium carrier concentrations. This chapter develops the carrier statistics of **extrinsic** (doped) semiconductors. Building on Chapter 4, we show how dopants shift the Fermi level and determine n and p through charge neutrality and the mass-action law.

Key outcomes:

- donor and acceptor concepts, ionization, and dopant energy levels (E_D, E_A),

- charge neutrality and equilibrium relations for n and p,

- majority/minority carriers and the mass-action law $np = n_i^2$,

- temperature regimes: freeze-out, extrinsic, intrinsic,

- conditions for degeneracy (limits of the Boltzmann approximation),

- practical solution strategies for typical ME1 problems.

> **Exam Focus.** Typical ME1 tasks include: (i) determine n-type vs p-type from dopants, (ii) compute n, p, and E_F for given N_D, N_A, and T, (iii) apply charge neutrality with full ionization (most common), (iv) justify temperature regimes qualitatively, (v) decide whether the non-degenerate approximation is valid.

5.2 Doping: Donors and Acceptors

Doping introduces impurity atoms into a semiconductor lattice. These dopants create localized energy levels near the band edges and can donate or accept electrons.

5.2.1 Donors (n-type doping)

Donor atoms have one more valence electron than the host semiconductor (e.g., P in Si). They introduce a donor level E_D slightly below the conduction band edge E_C. At

sufficiently high temperature, donors ionize and contribute free electrons:

$$D^0 \to D^+ + e^-.$$

5.2.2 Acceptors (p-type doping)

Acceptor atoms have one fewer valence electron than the host (e.g., B in Si). They introduce an acceptor level E_A slightly above the valence band edge E_V. At sufficiently high temperature, acceptors ionize and generate holes in the valence band. A convenient representation is:

$$A^0 + e^- \to A^-,$$
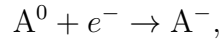
meaning that an electron is captured and a mobile hole remains in the valence band.

> **Definition. n-type semiconductor:** donors dominate, electrons are majority carriers.
> **p-type semiconductor:** acceptors dominate, holes are majority carriers.

> **Exam Focus.** Conceptual exam point: n-type doping shifts E_F upward toward E_C; p-type doping shifts E_F downward toward E_V (equilibrium, bulk material).

5.3 Dopant Energy Levels and Ionization Energies

Donor and acceptor levels lie close to the band edges:

$$E_D \lesssim E_C, \qquad E_A \gtrsim E_V.$$

The ionization energies are

$$\Delta E_D = E_C - E_D, \qquad \Delta E_A = E_A - E_V.$$

For moderate doping at room temperature, dopants are usually nearly fully ionized.

> **Exam Focus.** Unless a problem explicitly targets freeze-out or very low temperature, ME1 calculations typically assume full ionization.

5.4 Carrier Concentrations in the Non-Degenerate Limit

Under non-degenerate conditions (Boltzmann limit), from Chapter 4:

$$n = N_C \exp\left(-\frac{E_C - E_F}{k_B T}\right), \qquad p = N_V \exp\left(-\frac{E_F - E_V}{k_B T}\right).$$

5.4.1 Mass-Action Law

Multiplying these expressions gives

$$np = N_C N_V \exp\left(-\frac{E_C - E_V}{k_B T}\right) = N_C N_V \exp\left(-\frac{E_g}{k_B T}\right) = n_i^2.$$

Definition.

$$np = n_i^2 \qquad \text{(thermal equilibrium, non-degenerate model).}$$

Exam Focus. In equilibrium and non-degenerate conditions, $np = n_i^2$ is a key shortcut: after determining the majority carrier concentration, the minority carrier concentration follows immediately.

5.5 Ionized Dopant Concentrations: N_D^+ and N_A^-

Charge neutrality involves the **ionized** dopant concentrations:

- N_D^+: ionized donors (positive charge),

- N_A^-: ionized acceptors (negative charge).

Under full ionization:
$$N_D^+ \approx N_D, \qquad N_A^- \approx N_A.$$

Under partial ionization, the ionized fractions depend on the Fermi-level position. A standard model gives:

$$N_D^+ = \frac{N_D}{1 + g_D \exp\left(\frac{E_F - E_D}{k_B T}\right)}, \qquad N_A^- = \frac{N_A}{1 + g_A \exp\left(\frac{E_A - E_F}{k_B T}\right)},$$

where g_D and g_A are degeneracy factors (often $g_D \approx 2$, $g_A \approx 4$).

Exam Focus. In ME1, partial ionization is mainly a qualitative topic (freeze-out). Most numerical tasks assume full ionization unless stated otherwise.

5.6 Charge Neutrality

In a homogeneous semiconductor, net charge density is zero at equilibrium. Positive charge comes from holes and ionized donors; negative charge comes from electrons and ionized acceptors. Therefore,

Definition.

$$p + N_D^+ = n + N_A^-.$$

With full ionization:

$$p + N_D = n + N_A.$$

5.7   Net Doping and Semiconductor Type

Define the net doping concentration:

$$N_{\text{net}} = N_D - N_A.$$

- If $N_{\text{net}} > 0$, the semiconductor is n-type.

- If $N_{\text{net}} < 0$, the semiconductor is p-type.

- If $N_D \approx N_A$, the semiconductor is compensated.

Exam Focus. Compensation is frequently tested: use $N_D - N_A$ consistently (do not ignore the smaller dopant concentration).

5.8   Standard ME1 Solution Strategy (Full Ionization, Non-Degenerate)

This is the most common calculation case. Assume

$$N_D^+ = N_D, \qquad N_A^- = N_A,$$

and use

$$p + N_D = n + N_A, \qquad np = n_i^2.$$

From charge neutrality:

$$n - p = N_D - N_A \equiv N_{\text{net}}.$$

Together with $np = n_i^2$, this yields a quadratic equation.

5.8.1   General Solution for n and p

Using $p = \frac{n_i^2}{n}$ in $n - p = N_{\text{net}}$ gives:

$$n - \frac{n_i^2}{n} = N_{\text{net}} \quad \Rightarrow \quad n^2 - N_{\text{net}}n - n_i^2 = 0.$$

Solving:

Definition.

$$n = \frac{N_\text{net}}{2} + \sqrt{\left(\frac{N_\text{net}}{2}\right)^2 + n_i^2}, \qquad p = \frac{n_i^2}{n}.$$

This expression is valid for both n-type and p-type material, as long as full ionization and the non-degenerate model hold (N_net may be negative).

Exam Focus. This quadratic solution is a robust ME1 tool: it remains valid even when the semiconductor is near the intrinsic transition, provided full ionization and non-degenerate statistics apply.

5.8.2 Extrinsic Approximation (Majority Carrier Dominance)

If $|N_\text{net}| \gg n_i$, the majority carrier concentration is approximately equal to the net doping:

- n-type: $n \approx N_\text{net}, \quad p \approx \frac{n_i^2}{N_\text{net}}$,

- p-type: $p \approx |N_\text{net}|, \quad n \approx \frac{n_i^2}{|N_\text{net}|}$.

Exam Focus. A standard ME1 shortcut: first show $|N_\text{net}| \gg n_i$, then set majority carriers equal to net doping.

5.9 Fermi Level Position in Doped Semiconductors

From the electron expression,

$$n = N_C \exp\left(-\frac{E_\text{C} - E_\text{F}}{k_\text{B}T}\right) \;\Rightarrow\; E_\text{F} = E_\text{C} - k_\text{B}T \ln\left(\frac{N_C}{n}\right).$$

Definition.

$$E_\text{F} = E_\text{C} - k_\text{B}T \ln\left(\frac{N_C}{n}\right).$$

From the hole expression,

$$p = N_V \exp\left(-\frac{E_\text{F} - E_\text{V}}{k_\text{B}T}\right) \;\Rightarrow\; E_\text{F} = E_\text{V} + k_\text{B}T \ln\left(\frac{N_V}{p}\right).$$

Definition.

$$E_\text{F} = E_\text{V} + k_\text{B}T \ln\left(\frac{N_V}{p}\right).$$

Qualitative Shift of E_F with Doping

- n-type doping increases n and shifts E_F upward toward E_C,

- p-type doping increases p and shifts E_F downward toward E_V.

Exam Focus. Conceptual point: doping changes the equilibrium position of E_F relative to E_C and E_V in the bulk material. (The band edges are material properties and do not move due to doping in a uniform region.)

5.10   Degeneracy and Breakdown of the Boltzmann Model

The non-degenerate approximation fails when E_F approaches a band edge within a few $k_\mathrm{B}T$.

Definition. Degenerate conditions (rule of thumb):

$$E_\mathrm{C}-E_\mathrm{F} \lesssim 3k_\mathrm{B}T \quad \text{(n-type degeneracy)}, \qquad E_\mathrm{F}-E_\mathrm{V} \lesssim 3k_\mathrm{B}T \quad \text{(p-type degeneracy)}.$$

In degenerate conditions:

- Fermi–Dirac integrals are required,

- the simple exponential expressions for n and p are not accurate,

- the simple form of $np = n_i^2$ is not generally valid.

Exam Focus. In ME1, the typical task is to *judge* degeneracy, not to compute full Fermi–Dirac integrals.

5.11   Temperature Regimes in Doped Semiconductors

A doped semiconductor exhibits three characteristic regimes:

5.11.1   Freeze-Out (Low T)

- dopants are not fully ionized,

- carrier concentration is much lower than N_D or N_A,

- ionization energies ΔE_D and ΔE_A are important.

5.11.2 Extrinsic Regime (Intermediate T)

- dopants are (nearly) fully ionized,

- majority carrier concentration is approximately the net doping,

- minority carrier concentration follows from $np = n_i^2$.

5.11.3 Intrinsic Regime (High T)

- intrinsic carriers dominate: $n \approx p \approx n_i$,

- doping becomes relatively unimportant.

Exam Focus. A common qualitative reasoning task: explain how the dominant source of carriers changes from dopants (extrinsic regime) to thermal generation (intrinsic regime) as temperature increases.

5.12 Compensated Semiconductors

If both donors and acceptors are present,

$$N_{\text{net}} = N_D - N_A.$$

If $N_D \approx N_A$, the majority carrier concentration can be small even when both dopant concentrations are large, which can increase resistivity and place E_{F} closer to midgap.

Exam Focus. Compensation is a frequent source of mistakes: always use $N_D - N_A$ in charge neutrality.

5.13 Worked Example (Symbolic Method)

Assume full ionization and non-degenerate conditions.

Example. Given N_D, N_A, and n_i at temperature T:

1. Compute $N_{\text{net}} = N_D - N_A$.

2. Compute
$$n = \frac{N_{\text{net}}}{2} + \sqrt{\left(\frac{N_{\text{net}}}{2}\right)^2 + n_i^2}.$$

3. Compute
$$p = \frac{n_i^2}{n}.$$

4. Compute E_{F} using

$$E_{\mathrm{F}} = E_{\mathrm{C}} - k_{\mathrm{B}}T \ln\left(\frac{N_C}{n}\right) \quad \text{or} \quad E_{\mathrm{F}} = E_{\mathrm{V}} + k_{\mathrm{B}}T \ln\left(\frac{N_V}{p}\right).$$

5. Check degeneracy by comparing $E_{\mathrm{C}} - E_{\mathrm{F}}$ (or $E_{\mathrm{F}} - E_{\mathrm{V}}$) with $3k_{\mathrm{B}}T$.

5.14   Key Takeaways

- Donors produce electron majority carriers (n-type); acceptors produce hole majority carriers (p-type).

- Charge neutrality:

$$p + N_D^+ = n + N_A^-, \qquad \text{and under full ionization: } p + N_D = n + N_A.$$

- Mass-action law (equilibrium, non-degenerate):

$$np = n_i^2.$$

- Full-ionization solution:

$$n = \frac{N_{\mathrm{net}}}{2} + \sqrt{\left(\frac{N_{\mathrm{net}}}{2}\right)^2 + n_i^2}, \qquad p = \frac{n_i^2}{n}.$$

- Fermi level relations:

$$E_{\mathrm{F}} = E_{\mathrm{C}} - k_{\mathrm{B}}T \ln\left(\frac{N_C}{n}\right), \qquad E_{\mathrm{F}} = E_{\mathrm{V}} + k_{\mathrm{B}}T \ln\left(\frac{N_V}{p}\right).$$

- Temperature regimes: freeze-out $\to$ extrinsic $\to$ intrinsic.

- Degeneracy occurs when E_{F} approaches a band edge within a few $k_{\mathrm{B}}T$.

Chapter 6

Carrier Transport in Semiconductors

6.1 Scope and Learning Goals

This chapter develops the physical laws governing the motion of charge carriers in semiconductors under non-equilibrium conditions. Carrier transport links material properties (Chapters 3–5) to device operation (Chapter 7).

After this chapter, you should be able to:

- distinguish drift and diffusion transport mechanisms,

- derive and apply electron and hole current density equations,

- interpret mobility, scattering mechanisms, and temperature trends,

- apply the Einstein relation,

- use continuity equations and interpret generation/recombination terms,

- understand quasi-Fermi levels as a non-equilibrium concept.

Exam Focus. Chapter 6 is among the most calculation-heavy parts of ME1. Typical tasks involve: identifying drift vs diffusion contributions, computing current densities, using the Einstein relation, and reasoning about continuity equations qualitatively.

6.2 Equilibrium and Non-Equilibrium

In thermal equilibrium:

- there is no net current flow,

- a single Fermi level E_{F} describes the system,

- drift and diffusion currents cancel exactly.

In non-equilibrium:

- external influences (fields, illumination, gradients) disturb equilibrium,

- carrier concentrations may vary in space and/or time,

- net currents can flow,

- separate quasi-Fermi levels can be defined for electrons and holes.

Exam Focus. Common conceptual question: "Why is the net current zero at equilibrium although carriers are in motion?" Answer: drift and diffusion contributions cancel exactly.

6.3 Drift Transport

Drift transport arises when an electric field $\mathbf{E}$ acts on charged carriers.

6.3.1 Microscopic Picture

An electron experiences the force

$$\mathbf{F} = -q\mathbf{E}.$$

Between scattering events, carriers accelerate; frequent scattering with phonons, impurities, and defects leads to a steady average velocity.

Definition. Drift velocities:

$$\mathbf{v}_{d,n} = -\mu_n\mathbf{E} \quad \text{(electrons)}, \qquad \mathbf{v}_{d,p} = \mu_p\mathbf{E} \quad \text{(holes)}.$$

6.4 Mobility

Definition. The mobility μ is the proportionality constant between drift velocity and electric field magnitude:
$$|\mathbf{v}_d| = \mu|\mathbf{E}|.$$

Mobility depends on:

- temperature (phonon scattering),

- doping level (ionized impurity scattering),

- crystal quality and defects.

Exam Focus. Qualitative trends to know: at higher temperature, phonon scattering increases and mobility often decreases; at higher doping, ionized impurity scattering increases and mobility decreases.

6.5 Drift Current Density

Current density is charge flow per unit area per unit time.

6.5.1 Electron Drift Current

Electron charge density is $-qn$ and electron drift velocity is $\mathbf{v}_{d,n}$:

$$\mathbf{J}_{n,\mathrm{drift}} = (-qn)\mathbf{v}_{d,n} = qn\mu_n\mathbf{E}.$$

6.5.2 Hole Drift Current

Hole charge density is $+qp$ and hole drift velocity is $\mathbf{v}_{d,p}$:

$$\mathbf{J}_{p,\mathrm{drift}} = (qp)\mathbf{v}_{d,p} = qp\mu_p\mathbf{E}.$$

Definition. Total drift current density:

$$\mathbf{J}_{\mathrm{drift}} = q(n\mu_n + p\mu_p)\mathbf{E}.$$

Exam Focus. A frequent mistake is sign confusion. A reliable method is always:
current density = charge density $\times$ velocity.

6.6 Ohmic Conduction and Conductivity

Comparing $\mathbf{J} = \sigma\mathbf{E}$ with the drift current expression yields

$$\sigma = q(n\mu_n + p\mu_p).$$

For an intrinsic semiconductor $(n = p = n_i)$,

$$\sigma_i = qn_i(\mu_n + \mu_p).$$

6.7 Diffusion Transport

Diffusion arises from spatial gradients in carrier concentration.

6.7.1   Physical Origin

Carriers undergo random thermal motion. If a concentration gradient exists, the net flux is from regions of higher concentration to lower concentration, producing a diffusion current.

6.8   Diffusion Current Density (1D)

In one spatial dimension x:

6.8.1   Electron Diffusion

Electron particle flux is proportional to $-dn/dx$. Because electrons carry negative charge, the resulting current density is

$$J_{n,\text{diff}} = qD_n\frac{dn}{dx}.$$

6.8.2   Hole Diffusion

Hole particle flux is proportional to $-dp/dx$. Because holes carry positive charge, the diffusion current density is

$$J_{p,\text{diff}} = -qD_p\frac{dp}{dx}.$$

Definition. Total current densities in 1D:

$$J_n = qn\mu_n E + qD_n\frac{dn}{dx}, \qquad J_p = qp\mu_p E - qD_p\frac{dp}{dx}.$$

Exam Focus. Sign check: electron diffusion current has the same sign as dn/dx because the electron charge is negative. Hole diffusion current has the opposite sign of dp/dx.

6.9   Einstein Relation

Mobility and diffusion coefficient are related.

Definition. Einstein relation (non-degenerate, near-equilibrium):

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_{\text{B}}T}{q}.$$

This relation holds under:

- non-degenerate (Boltzmann) statistics,

- conditions close to thermal equilibrium.

Exam Focus. Common calculation task: given μ at temperature T, compute D using $D = \mu \, k_{\mathrm{B}} T / q$.

6.10 Drift–Diffusion Balance at Equilibrium

At equilibrium, net current vanishes:

$$J_n = 0, \qquad J_p = 0.$$

For electrons:

$$q n \mu_n E + q D_n \frac{dn}{dx} = 0.$$

Using the Einstein relation gives

$$E = -\frac{k_{\mathrm{B}} T}{q} \frac{1}{n} \frac{dn}{dx}.$$

This relation is central when deriving built-in fields in inhomogeneous systems (e.g., p–n junctions in Chapter 7).

6.11 Generation and Recombination

Carrier densities change through:

- **generation** (thermal, optical),

- **recombination** (electron–hole annihilation).

6.11.1 Recombination Mechanisms (Qualitative)

Important mechanisms include:

- radiative recombination (more relevant in direct-gap materials),

- Shockley–Read–Hall (defect-assisted recombination),

- Auger recombination (important at high carrier densities).

Exam Focus. ME1 typically tests the qualitative meaning of these mechanisms, not detailed recombination rate models.

6.12   Continuity Equations

Carrier conservation relates time variation of carrier density to current flow and generation/recombination.

6.12.1   Electron Continuity Equation

Definition.
$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n.$$

6.12.2   Hole Continuity Equation

Definition.
$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p.$$

Exam Focus. Even when not solving continuity equations, you must interpret every term correctly, including signs and physical meaning.

6.13   Steady-State Conditions

In steady state:
$$\frac{\partial n}{\partial t} = 0, \qquad \frac{\partial p}{\partial t} = 0.$$

Thus, the divergence of current balances net generation and recombination.

6.14   Quasi-Fermi Levels

Under non-equilibrium conditions, electrons and holes may no longer share a single Fermi level.

Definition. Quasi-Fermi levels E_{Fn} and E_{Fp} are defined by

$$n = N_C \exp\!\left(-\frac{E_\mathrm{C} - E_{Fn}}{k_\mathrm{B}T}\right), \qquad p = N_V \exp\!\left(-\frac{E_{Fp} - E_\mathrm{V}}{k_\mathrm{B}T}\right).$$

- At equilibrium: $E_{Fn} = E_{Fp} = E_{\mathrm{F}}$.

- Under bias or illumination: $E_{Fn} \neq E_{Fp}$.

Exam Focus. ME1 typically requires only conceptual understanding: splitting of quasi-Fermi levels indicates non-equilibrium carrier populations and is associated with current flow or net generation.

6.15 Summary of Core Transport Equations (1D)

- Drift:
$$J_{n,\mathrm{drift}} = qn\mu_n E, \qquad J_{p,\mathrm{drift}} = qp\mu_p E.$$

- Diffusion:
$$J_{n,\mathrm{diff}} = qD_n \frac{dn}{dx}, \qquad J_{p,\mathrm{diff}} = -qD_p \frac{dp}{dx}.$$

- Total current:
$$J_n = qn\mu_n E + qD_n \frac{dn}{dx}, \qquad J_p = qp\mu_p E - qD_p \frac{dp}{dx}.$$

- Einstein relation:
$$\frac{D}{\mu} = \frac{k_{\mathrm{B}}T}{q}.$$

6.16 Chapter Summary

- Carrier transport occurs through drift (field-driven) and diffusion (gradient-driven).

- Mobility connects microscopic scattering to macroscopic current response.

- Drift and diffusion cancel at equilibrium, yielding zero net current.

- The Einstein relation connects diffusion coefficients and mobilities in the non-degenerate limit.

- Continuity equations express carrier conservation including generation and re-combination.

- Quasi-Fermi levels provide a useful description of non-equilibrium carrier populations.

Chapter 7

The p–n Junction (Abrupt Junction)

7.1 Scope and Learning Goals

The p–n junction is the central device structure in ME1. It combines carrier statistics (Chapters 4–5), electrostatics (Poisson equation), and transport concepts (Chapter 6) into a single physical model.

After this chapter, you should be able to:

- explain how a p–n junction forms and reaches equilibrium,

- describe the equilibrium energy-barrier picture in terms of E_C, E_V, and E_F, and explain qualitatively what changes under bias,

- derive the built-in potential V_{bi},

- apply the depletion approximation to obtain $\rho(x)$, $E(x)$, and $\phi(x)$, as well as x_n, x_p, and the total depletion width W,

- compute the maximum electric field $E_{\max}$,

- derive the depletion (junction) capacitance and its voltage dependence,

- explain forward/reverse bias operation and the physical origin of rectification,

- connect applied bias to quasi-Fermi levels conceptually (ME1 level).

Exam Focus. Common ME1 tasks for Chapter 7: (i) compute V_{bi} from dopings and n_i, (ii) compute depletion widths and $E_{\max}$ under a given bias, (iii) describe the equilibrium and biased energy-barrier picture using E_C, E_V, E_F, (iv) explain rectification physically, (v) compute depletion capacitance and its voltage dependence.

7.2 Physical Formation of the p–n Junction

Consider a p-type semiconductor (acceptor-doped) joined to an n-type semiconductor (donor-doped). Far from the junction, each side is quasi-neutral but has different majority carriers:

- p-side: holes majority, electrons minority,

- n-side: electrons majority, holes minority.

7.2.1 Diffusion of Majority Carriers

Due to concentration gradients:

- electrons diffuse from n to p,

- holes diffuse from p to n.

When electrons leave the n-side, they expose positively charged ionized donors (D^+). When holes leave the p-side, they expose negatively charged ionized acceptors (A^-).

7.2.2 Space-Charge (Depletion) Region

The region near the metallurgical junction becomes depleted of mobile carriers and contains fixed ionized dopants.

Definition. The **depletion region** (space-charge region) is the region around the p–n junction that is depleted of free carriers and contains fixed charged ions.

The fixed charges create an electric field directed from the positively charged donor region toward the negatively charged acceptor region, opposing further diffusion.

7.2.3 Equilibrium Condition

At equilibrium:

- diffusion current is exactly balanced by drift current,

- the net current is zero,

- the Fermi level is constant throughout the entire structure.

Exam Focus. Common conceptual point: diffusion does not "stop" at equilibrium; it is balanced by drift due to the built-in field.

7.3 Built-In Potential V_{bi}

The built-in potential is the electrostatic potential difference across the depletion region at equilibrium.

7.3.1 Derivation from Equilibrium Carrier Concentrations

In equilibrium, far from the junction (full ionization assumed):

$$n_n \approx N_D \quad \text{(n-region)}, \qquad p_p \approx N_A \quad \text{(p-region)}.$$

Minority carrier concentrations follow from the mass-action law:

$$p_n = \frac{n_i^2}{n_n} \approx \frac{n_i^2}{N_D}, \qquad n_p = \frac{n_i^2}{p_p} \approx \frac{n_i^2}{N_A}.$$

Aligning a single Fermi level across the entire junction requires a potential difference that corresponds to the built-in barrier. The result is:

Definition.

$$V_{bi} = \frac{k_{\mathrm{B}} T}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right).$$

Exam Focus. Exam checklist: (i) use net dopings if compensated, (ii) keep units consistent (cm^{-3} vs m^{-3}), (iii) confirm full ionization is consistent with the stated temperature.

7.4 Equilibrium Energy-Barrier Picture (No-Figure Description)

At equilibrium:

- E_{F} is constant across p-region, depletion region, and n-region,

- the electrostatic potential varies across the depletion region, which shifts the band edges E_{C} and E_{V} as a function of position,

- the energy barrier opposing majority-carrier diffusion corresponds to qV_{bi}.

Exam Focus. You should be able to describe (in words and symbols) the equilibrium situation: constant E_{F}, position-dependent E_{C} and E_{V} across the depletion region, and a barrier of height qV_{bi} for majority carriers.

7.5 Depletion Approximation

To derive tractable formulas, we apply the depletion approximation:

- inside the depletion region, mobile carriers are negligible ($n \approx 0$, $p \approx 0$), so charge density is due only to fixed ionized dopants,

- outside the depletion region, the semiconductor is quasi-neutral and $E \approx 0$.

Let $x = 0$ at the metallurgical junction, with:

- p-side depletion from $x = -x_p$ to $x = 0$,

- n-side depletion from $x = 0$ to $x = x_n$.

7.5.1 Space Charge Density $\rho(x)$

Within the depletion region:

$$\rho(x) = \begin{cases} -qN_A, & -x_p \leq x \leq 0, \\ +qN_D, & 0 \leq x \leq x_n, \\ 0, & x < -x_p \text{ or } x > x_n. \end{cases}$$

Exam Focus. Sign convention checkpoint: uncovered acceptors are negative $(-qN_A)$ on the p-side, uncovered donors are positive $(+qN_D)$ on the n-side.

7.6   Poisson Equation and Electric Field

Electrostatics is governed by Poisson's equation:

$$\frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\varepsilon_s},$$

where $\phi(x)$ is the electrostatic potential and ε_s is the semiconductor permittivity. The electric field is

$$E(x) = -\frac{d\phi}{dx}.$$

Equivalently,

$$\frac{dE}{dx} = \frac{\rho(x)}{\varepsilon_s}.$$

7.6.1 Electric Field in the Depletion Region

p-side $(-x_p \leq x \leq 0)$:

$$\frac{dE}{dx} = \frac{-qN_A}{\varepsilon_s} \quad \Rightarrow \quad E(x) = -\frac{qN_A}{\varepsilon_s}(x + x_p),$$

using $E(-x_p) = 0$.

n-side $(0 \leq x \leq x_n)$:

$$\frac{dE}{dx} = \frac{+qN_D}{\varepsilon_s} \quad \Rightarrow \quad E(x) = \frac{qN_D}{\varepsilon_s}(x - x_n),$$

using $E(x_n) = 0$.

7.6.2 Maximum Electric Field and Charge Balance

The maximum field magnitude occurs at the metallurgical junction $(x = 0)$:

$$E_{\max} = |E(0)|.$$

From the p-side and n-side expressions:

$$E(0^-) = -\frac{qN_A x_p}{\varepsilon_s}, \qquad E(0^+) = -\frac{qN_D x_n}{\varepsilon_s}.$$

Field continuity implies

$$N_A x_p = N_D x_n.$$

Definition. Charge balance:
$$N_A x_p = N_D x_n.$$

Maximum field magnitude:

$$E_{\max} = \frac{qN_A x_p}{\varepsilon_s} = \frac{qN_D x_n}{\varepsilon_s}.$$

Exam Focus. Core exam step: use $N_A x_p = N_D x_n$ to relate depletion widths.

7.7 Electrostatic Potential Across the Depletion Region

The potential difference across the depletion region is

$$V = \phi(x_n) - \phi(-x_p) = \int_{-x_p}^{x_n} -E(x)\,dx.$$

Evaluating with the piecewise-linear field gives

$$V = \frac{q}{2\varepsilon_s}\left(N_A x_p^2 + N_D x_n^2\right).$$

At equilibrium, $V = V_{bi}$. Under applied bias V_a (defined positive for forward bias), the barrier becomes

$$V = V_{bi} - V_a.$$

Definition.

$$V_{bi} - V_a = \frac{q}{2\varepsilon_s} \left(N_A x_p^2 + N_D x_n^2 \right).$$

7.8   Depletion Widths x_n, x_p, and Total Width W

Using

$$N_A x_p = N_D x_n, \qquad W = x_p + x_n, \qquad V_{bi} - V_a = \frac{q}{2\varepsilon_s} \left(N_A x_p^2 + N_D x_n^2 \right),$$

the standard abrupt-junction results are obtained:

Definition. Total depletion width:

$$W = \sqrt{\frac{2\varepsilon_s}{q} \left(\frac{N_A + N_D}{N_A N_D} \right) (V_{bi} - V_a)}.$$

Partition into each side:

$$x_n = \frac{N_A}{N_A + N_D} W, \qquad x_p = \frac{N_D}{N_A + N_D} W.$$

Exam Focus. Key trend: reverse bias ($V_a < 0$) increases ($V_{bi} - V_a$) and therefore increases W; forward bias decreases W.

7.9   Junction Capacitance (Depletion Capacitance)

The depletion region behaves like a capacitor: separated charge with dielectric permittivity ε_s.

Charge per unit area on either side is

$$Q = q N_D x_n = q N_A x_p.$$

The depletion capacitance per unit area is

$$C' = \frac{dQ}{dV_a}.$$

Because $W \propto \sqrt{V_{bi} - V_a}$, one obtains

Definition.

$$C' = \frac{\varepsilon_s}{W} = \sqrt{\frac{q \varepsilon_s N_A N_D}{2(N_A + N_D)(V_{bi} - V_a)}}.$$

Thus,

$$C' \propto \frac{1}{\sqrt{V_{bi} - V_a}}.$$

| **Exam Focus.** Reverse bias increases W and therefore decreases C' as $1/\sqrt{V}$.

7.10 Forward and Reverse Bias: Qualitative Operation

Define applied voltage V_a as:

- $V_a > 0$: forward bias (reduces the barrier),

- $V_a < 0$: reverse bias (increases the barrier).

7.10.1 Forward Bias

Forward bias reduces the effective barrier:

$$V = V_{bi} - V_a.$$

Majority carriers cross the junction more easily: electrons are injected from n to p and holes from p to n. Injected carriers become minority carriers on the opposite side and diffuse away, recombining over characteristic lengths.

7.10.2 Reverse Bias

Reverse bias increases the barrier and widens the depletion region. Majority-carrier injection is strongly suppressed. A small current remains due to minority carriers (reverse saturation current), whose origin is thermally generated minority-carrier transport.

| **Exam Focus.** Rectification in words: forward bias lowers the barrier and enhances injection; reverse bias raises the barrier and suppresses injection.

7.11 Energy Picture Under Bias (No-Figure Description)

Under bias, the system is not in global equilibrium:

- the barrier height changes by qV_a,

- carrier injection changes minority carrier concentrations near the junction,

- quasi-Fermi levels can split ($E_{Fn} \neq E_{Fp}$) under steady-state current flow.

Forward bias reduces band-edge separation across the depletion region and narrows W, whereas reverse bias increases the separation and widens W.

> **Exam Focus.** You should be able to state, without drawings: forward bias $\Rightarrow$ barrier decreases and W decreases; reverse bias $\Rightarrow$ barrier increases and W increases.

7.12   Current Flow Mechanism (ME1-Level)

In steady state, forward current is dominated by diffusion of injected minority carriers in the quasi-neutral regions. Reverse current is dominated by the transport of thermally generated minority carriers toward the depletion region.

A full derivation of the diode equation

$$I = I_S \left(e^{qV_a/k_{\mathrm{B}}T} - 1 \right)$$

may be covered in later material, but the physical origin should be understood here: barrier reduction increases injected carrier concentrations exponentially.

> **Exam Focus.** Even without deriving the diode equation, know the reason for exponential forward current: the barrier reduction changes carrier injection exponentially with $qV_a/k_{\mathrm{B}}T$.

7.13   Special Case: One-Sided Junction

If one side is much more heavily doped than the other, e.g.,

$$N_D \gg N_A,$$

then most of the depletion region extends into the more lightly doped side:

$$x_p \approx W, \qquad x_n \ll W.$$

Similarly, if $N_A \gg N_D$, then $x_n \approx W$.

> **Exam Focus.** One-sided approximation: the depletion region extends primarily into the **more lightly doped** side.

7.14   Worked Example (Symbolic Pipeline)

Example. Given N_A, N_D, T, n_i, ε_s, and applied bias V_a:

1. Built-in potential:
$$V_{bi} = \frac{k_\mathrm{B}T}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right).$$

2. Total depletion width:
$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_a)}.$$

3. Partition:
$$x_n = \frac{N_A}{N_A + N_D}W, \qquad x_p = \frac{N_D}{N_A + N_D}W.$$

4. Maximum field:
$$E_\mathrm{max} = \frac{q N_D x_n}{\varepsilon_s} = \frac{q N_A x_p}{\varepsilon_s}.$$

5. Depletion capacitance per unit area:
$$C' = \frac{\varepsilon_s}{W}.$$

7.15   Key Takeaways

- The p–n junction forms by diffusion of majority carriers, leaving fixed ionized dopants.

- The depletion region contains space charge and a built-in electric field that balances diffusion at equilibrium.

- Built-in potential:
$$V_{bi} = \frac{k_\mathrm{B}T}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right).$$

- Depletion approximation yields:
$$N_A x_p = N_D x_n, \qquad W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_a)}.$$

- Reverse bias widens depletion and reduces capacitance; forward bias narrows depletion and enhances injection.

- Rectification arises because applied bias changes the energy barrier for majority carriers.

Chapter 8

Microelectronics 1 Formula Sheet

This appendix collects the core formulas used throughout *Mikroelektronik 1*. All formulas assume thermal equilibrium unless explicitly stated otherwise and follow the non-degenerate (Boltzmann) approximation unless noted.

> **Exam Focus.** This appendix is designed to be a **last-day-before-the-exam reference**. You should be able to recognize when and how to use each formula, not just memorize them.

8.1 Physical Constants

$$q = 1.602 \times 10^{-19}\,\mathrm{C}$$
$$k_\mathrm{B} = 1.381 \times 10^{-23}\,\mathrm{J/K}$$
$$k_\mathrm{B}T \approx 25.9\,\mathrm{meV} \quad (T = 300\,\mathrm{K})$$
$$h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$$
$$\hbar = \frac{h}{2\pi}$$
$$\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{F/m}$$

Energy unit:
$$1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}.$$

8.2 Band Structure Quantities

$$E_\mathrm{g} = E_\mathrm{C} - E_\mathrm{V}$$

Effective mass (parabolic band):
$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2}$$

Carrier velocity:
$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

8.3   Density of States (3D, Parabolic Bands)

Conduction band:

$$g_C(E) = \frac{1}{2\pi^2}\left(\frac{2m_n^*}{\hbar^2}\right)^{3/2}\sqrt{E - E_{\mathrm{C}}} \quad (E \geq E_{\mathrm{C}})$$

Valence band:

$$g_V(E) = \frac{1}{2\pi^2}\left(\frac{2m_p^*}{\hbar^2}\right)^{3/2}\sqrt{E_{\mathrm{V}} - E} \quad (E \leq E_{\mathrm{V}})$$

8.4   Effective Density of States

$$N_C = 2\left(\frac{2\pi m_n^* k_{\mathrm{B}} T}{h^2}\right)^{3/2}, \qquad N_V = 2\left(\frac{2\pi m_p^* k_{\mathrm{B}} T}{h^2}\right)^{3/2}$$

Scaling:

$$N_C,\ N_V \propto T^{3/2}$$

8.5   Carrier Statistics (Non-Degenerate)

Electron concentration:

$$n = N_C \exp\!\left(-\frac{E_{\mathrm{C}} - E_{\mathrm{F}}}{k_{\mathrm{B}} T}\right)$$

Hole concentration:

$$p = N_V \exp\!\left(-\frac{E_{\mathrm{F}} - E_{\mathrm{V}}}{k_{\mathrm{B}} T}\right)$$

Fermi–Dirac distribution:

$$f(E) = \frac{1}{1 + \exp\!\left(\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}} T}\right)}$$

Boltzmann approximation (valid if $|E - E_{\mathrm{F}}| \gtrsim 3k_{\mathrm{B}}T$):

$$f(E) \approx \exp\!\left(-\frac{E - E_{\mathrm{F}}}{k_{\mathrm{B}} T}\right)$$

8.6   Intrinsic Semiconductor

Intrinsic condition:

$$n = p = n_i$$

Intrinsic carrier concentration:

$$n_i = \sqrt{N_C N_V} \, \exp\left(-\frac{E_\mathrm{g}}{2k_\mathrm{B}T}\right)$$

Mass-action law:

$$np = n_i^2$$

Intrinsic Fermi level:

$$E_i = \frac{E_\mathrm{C} + E_\mathrm{V}}{2} + \frac{k_\mathrm{B}T}{2} \ln\left(\frac{N_V}{N_C}\right)$$

Intrinsic conductivity:

$$\sigma_i = qn_i(\mu_n + \mu_p)$$

8.7 Extrinsic Semiconductor (Full Ionization)

Charge neutrality:

$$p + N_D = n + N_A$$

Net doping:

$$N_\mathrm{net} = N_D - N_A$$

General solution:

$$n = \frac{N_\mathrm{net}}{2} + \sqrt{\left(\frac{N_\mathrm{net}}{2}\right)^2 + n_i^2}, \qquad p = \frac{n_i^2}{n}$$

Extrinsic approximation ($|N_\mathrm{net}| \gg n_i$):

$$\text{n-type:} \quad n \approx N_\mathrm{net}, \quad p \approx \frac{n_i^2}{N_\mathrm{net}}$$

$$\text{p-type:} \quad p \approx |N_\mathrm{net}|, \quad n \approx \frac{n_i^2}{|N_\mathrm{net}|}$$

Fermi level:

$$E_\mathrm{F} = E_\mathrm{C} - k_\mathrm{B}T \ln\left(\frac{N_C}{n}\right) \quad \text{or} \quad E_\mathrm{F} = E_\mathrm{V} + k_\mathrm{B}T \ln\left(\frac{N_V}{p}\right)$$

Degeneracy criterion:

$$E_\mathrm{C} - E_\mathrm{F} \lesssim 3k_\mathrm{B}T \quad \text{or} \quad E_\mathrm{F} - E_\mathrm{V} \lesssim 3k_\mathrm{B}T$$

8.8   Drift and Diffusion Transport

Drift velocity:

$$\mathbf{v}_{d,n} = -\mu_n\mathbf{E}, \qquad \mathbf{v}_{d,p} = \mu_p\mathbf{E}$$

Drift current densities:

$$\mathbf{J}_{n,\text{drift}} = qn\mu_n\mathbf{E}, \qquad \mathbf{J}_{p,\text{drift}} = qp\mu_p\mathbf{E}$$

Diffusion current densities (1D):

$$J_{n,\text{diff}} = qD_n\frac{dn}{dx}, \qquad J_{p,\text{diff}} = -qD_p\frac{dp}{dx}$$

Total current:

$$J_n = qn\mu_nE + qD_n\frac{dn}{dx}, \qquad J_p = qp\mu_pE - qD_p\frac{dp}{dx}$$

Einstein relation:

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_{\text{B}}T}{q}$$

Conductivity:

$$\sigma = q(n\mu_n + p\mu_p)$$

8.9   Continuity Equations (1D)

Electrons:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - R_n$$

Holes:

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} + G_p - R_p$$

Steady state:

$$\frac{\partial n}{\partial t} = \frac{\partial p}{\partial t} = 0$$

8.10   p–n Junction (Abrupt Junction)

Built-in potential:

$$V_{bi} = \frac{k_{\text{B}}T}{q}\ln\left(\frac{N_AN_D}{n_i^2}\right)$$

Charge balance in depletion region:

$$N_A x_p = N_D x_n$$

Total depletion width:

$$W = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{N_A + N_D}{N_A N_D}\right)(V_{bi} - V_a)}$$

Depletion widths:

$$x_n = \frac{N_A}{N_A + N_D}W, \qquad x_p = \frac{N_D}{N_A + N_D}W$$

Maximum electric field:

$$E_{\max} = \frac{qN_D x_n}{\varepsilon_s} = \frac{qN_A x_p}{\varepsilon_s}$$

Depletion capacitance per unit area:

$$C' = \frac{\varepsilon_s}{W} = \sqrt{\frac{q\varepsilon_s N_A N_D}{2(N_A + N_D)(V_{bi} - V_a)}}$$

Voltage dependence:

$$C' \propto \frac{1}{\sqrt{V_{bi} - V_a}}$$

8.11 Very Common Qualitative Rules

- Intrinsic: $n = p = n_i$.

- Doping shifts E_{F}; band edges remain material properties.

- Diffusion and drift cancel at equilibrium.

- Depletion region extends mostly into the lightly doped side.

- Reverse bias $\uparrow \Rightarrow W \uparrow$, $C' \downarrow$.

- Forward bias $\uparrow \Rightarrow W \downarrow$, carrier injection increases.

Bibliography

[1] S. M. Sze and K. K. Ng, *Physics of Semiconductor Devices*, 3rd ed., John Wiley & Sons, Hoboken, NJ, USA, 2007.

[2] R. F. Pierret, *Semiconductor Device Fundamentals*, Addison–Wesley, Reading, MA, USA, 1996.

[3] R. S. Muller, T. I. Kamins, and M. Chan, *Device Electronics for Integrated Circuits*, 3rd ed., John Wiley & Sons, Hoboken, NJ, USA, 2010.

[4] I. Kallfass, *Mikroelektronik 1 – Lecture Notes*, Bachelor Course in Electrical Engineering, Institute of Robust Power Semiconductor Systems, University of Stuttgart, Germany.