Contents

1 Modelling challenge

1.1 Neural firing in the mammalian brain

1.1.1 Experimental Data

1.1.1.1 Regular firing

Neurons in the mammalian neocortex process information by responding to an input current (I) and producing a rhythmic output known as an action potential. Fig. 1.1.1 shows the experimentally observed behaviour of a nerve cell membrane voltage (v) in an experimental protocol where the input current (I) is varied in a step-like fashion.

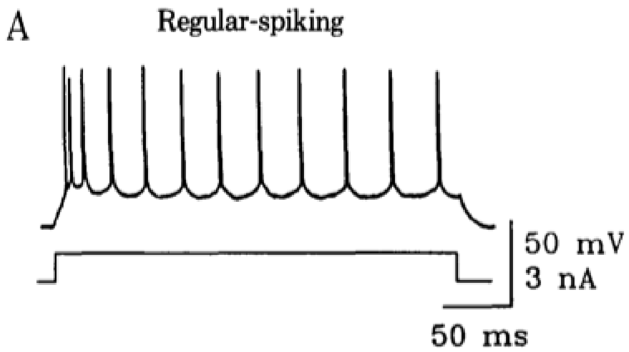

Figure 1.1.1: Trace of action potential in a regular spiking neuron. (Bottom line: applied current. Top line: measured voltage.) Taken from Connors (1990) [1]

1.1.1.2 Burst firing

Not all neurons are alike. A second type of neurons in the mammalian neocortex predominantly responds to current input with a unique distinct firing pattern, see Fig. 1.1.2.

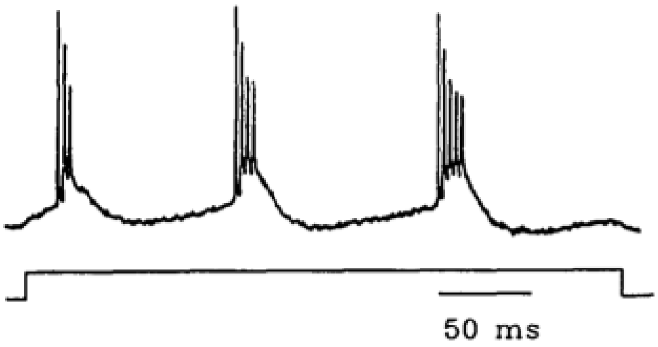

Figure 1.1.2: Trace of action potential in a burst firing neuron. (Bottom line: applied current. Top line: measured voltage.) Taken from Conners (1990) [1]

1.1.2 Models

1.1.2.1 The FitzHugh-Nagamo model of neural spiking

Hodgkin, Huxley and Katz derived a model of action potentials in neurons based on detailed measurements of the ionic current under different experimental conditions (voltage clamp). Work by FitzHugh and Nagumo developed this work leading to a simplfied model which captured the essential behaviour of the system but used only two differential equations:

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w + I$$
$$\frac{dw}{dt} = \frac{1}{\tau}(v + a - bw)$$

where v represents the membrane voltage, w represents a relaxation or recovery variable, I represents an input current and a, b and τ are parameters of the model. The suggested values for the parameters, taken from fits to experimental measurements were $a = 0.7$, $b = 0.8$ and $\tau = 12.5$.

1.1.2.2 A modification to the FitzHugh-Nagamo model

Rinzel (1985) proposed a modification which changed the behaviour of the model. This adds a third variable:

$$
\begin{aligned}
\frac{dv}{dt} &= v - v^3/3 - w + z + I \\
\frac{dw}{dt} &= \frac{1}{\tau}(v + a - bw) \\
\frac{dz}{dt} &= \epsilon(-v + c - z)
\end{aligned}
$$

The added equation for dz/dt has a similar form to the equation for the recovery variable dw/dt, but a key difference is that Rinzel assigns a very small value to the parameter ϵ. This means the variable z changes much more slowly in comparison to variables v and w. Note that the first equation is modified to include feedback from z.
The parameters suggested by Rinzel for the model are $\epsilon = 0.0001$ and $c = -0.775$.

1.1.3 Suggested Investigation

It is suggested that you investigate the behaviour of the FitzHugh-Nagamo model by coding the systems in MATLAB and exploring their behaviour over time. You should:

- Investgate the equibrium state of the system when no current is applied. (Hint: to do this you could run the ode solver with no applied current and find the steady state reached, or find the analytical solution by using dsolve on the symbolic equations).
- Explore the behaviour of the FitzHugh-Nagamo model with an applied current. (Hint. Try to determine the range of input current levels which produce neural firing).
- Try adjusting the parameters of the model (a, b and τ) and try to describe how they affect the behaviour ofthe model.
- Run your model with the same current conditions used in 1.1.1. (Hint. In this case you need to use an if command to set the current to turn on and off as time progresses). Compare and contrast the behviour of your model to the data. Can you improve the match by adjusting the parameters?
- In the same way explore the behaviour of the modified model, (Hint you should be able to produce burst firing with this model).

Note: If you are familiar with phase space plots you may find it useful to make phase plots of v versus w . (This type of plot is covered briefly in M1.3 Section 1.6 and its associated mathematical background material).

1.1.3.1 Optional extension

Adjust your code for the original model so the current is set slightly below the threshold of neural firing. Next modify your code so that the current includes noise, (i.e. add a randomly fluctuating component). Explore the behaviour of this model at different noise levels, and discuss the biological relevance of the modification.

Bibliography

[1] BW Connors and MJ Gutnick. Intrinsic firing patterns of diverse neocortical neurons. *Trends in Neurosciences*, 13(3):99–104, 1990.