2. Write neatly on paper the expression of repulsive forces for the following two objects: 1) repulsion upward from the Workspace plane, which we assume to be parallel to the $x_0 - y_0$ plane, and have a z_0 value of 32 mm; 2) Repulsion from a cylinder of finite length. The bottom of the cylinder lies on the $x_0 - y_0$ plane and the height of the cylinder is a parameter h.

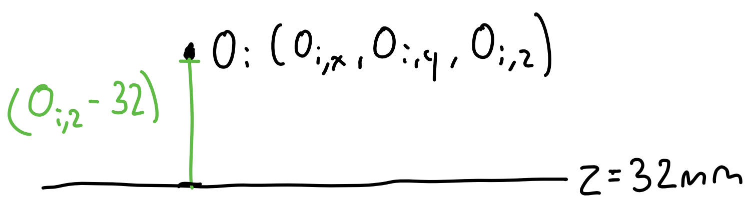
$$F_{rep,i}(q) = \eta_i \left(\frac{1}{\rho(o_i(q))} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2(o_i(q))} \nabla \rho(o_i(q))$$

$\rho(o_i(q))$: distance in workspace from origin of DH frame i to the rigid obstacle

$$\rho(o_i(q)) = \min_{x \in \mathcal{O}} \| o_i(q) - x \|$$

$$\nabla \rho(x) \Big|_{x = o_i(q)} = \frac{o_i(q) - b}{\| o_i(q) - b \|}$$

1) Repulsion from $x_0 - y_0$ plane with $z_0 = 32\,mm$



$$\boxed{\| O_i - b \| = O_{i,z} - 32}$$

$$\boxed{O_i - b = \| O_i - b \| \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$$

→ as it is an $x_0 - y_0$ plane direction is always z_0

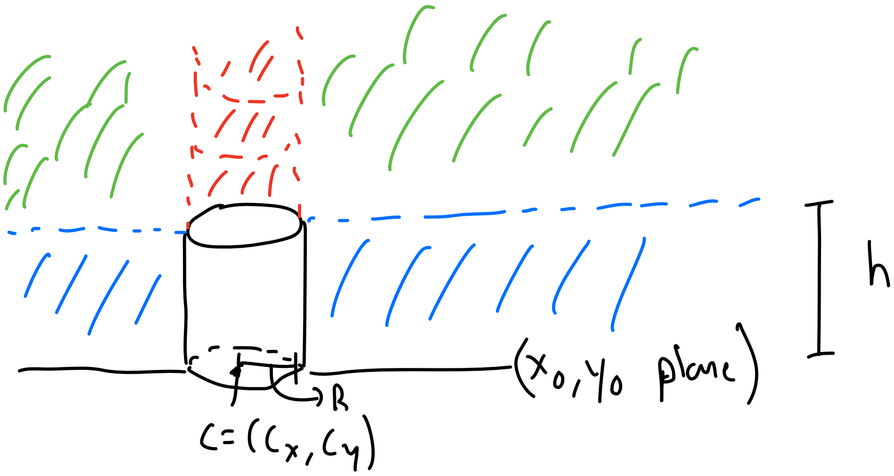
$$\boxed{F_{rep,i}(q) = \eta_i \left(\frac{1}{\| O_i - b \|} - \frac{1}{\rho_0} \right) \frac{1}{(\| O_i - b \|)^2} \frac{O_i - b}{\| O_i - b \|}}$$

→ η : design parameter

→ ρ_0 : property of obstacle

★ Note: in above equation we assume robot can not be below this working plane.

2) Repulsion From cylinder of finite length. Bottom of cylinder on x_0-y_0 plane and height of cylinder is a parameter h



→ 3 cases: a) $O_{i,z} < h$ (O_i to the side of cylinder)

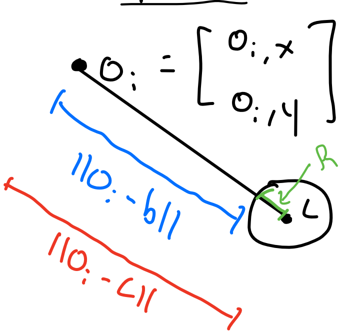
b) $\|(O_{i,x}, O_{i,y}) - (C_x, C_y)\| < R$ (O_i directly above cylinder)

c) Else

Case a)

→ From Lab 3 prelab:

Top View:



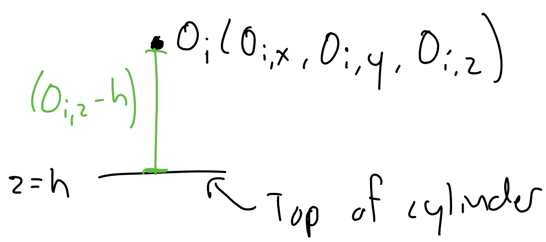
$$\|O_i - b\| = \sqrt{(O_{i,x} - C_x)^2 + (O_{i,y} - C_y)^2} - R$$

$$O_i - b = \left(\begin{bmatrix} O_{i,x} - C_x \\ O_{i,y} - C_y \\ 0 \end{bmatrix} \right) \left(1 - \frac{R}{\sqrt{(O_{i,x} - C_x)^2 + (O_{i,y} - C_y)^2}} \right)$$

$$F_{rep,i}(q) = \eta_i \left(\frac{1}{\|O_i - b\|} - \frac{1}{\rho_0} \right) \frac{1}{(\|O_i - b\|)^2} \frac{O_i - b}{\|O_i - b\|}$$

Case b) → Similar to plane

Side View:

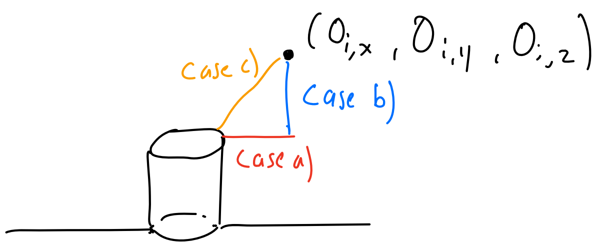


$$||O_i - b|| = O_{i,z} - h$$

$$O_i - b = (O_{i,z} - h) \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$F_{rep,i}(q) = \eta_i \left(\frac{1}{||O_i - b||} - \frac{1}{\rho_0} \right) \frac{1}{(||O_i - b||)^2} \frac{O_i - b}{||O_i - b||}$$

Case c) → Combination of case a, case b)



Using pythagorean theorem:

case a: $||O_i - b|| = \sqrt{(O_{i,x} - C_x)^2 + (O_{i,y} - C_y)^2}$

case b: $||O_i - b|| = O_{i,z} - h$

Case c) $||O_i - b|| = \sqrt{||O_i - b||_a^2 + ||O_i - b||_b^2}$

$$\Rightarrow ||O_i - b||_c = \sqrt{\left(\sqrt{(O_{i,x} - C_x)^2 + (O_{i,y} - C_y)^2} - R\right)^2 + (O_{i,z} - h)^2}$$

$$(O_i - b)_c = (O_i - b)_a + (O_i - b)_b$$

$$= \begin{bmatrix} O_{i,x} - C_x \\ O_{i,y} - C_y \\ 0 \end{bmatrix} \left(1 - \frac{R}{\sqrt{(O_{i,x} - C_x)^2 + (O_{i,y} - C_y)^2}} \right) + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} (O_{i,z} - h)$$

Combining all 3 cases

- Let's define two distances: ① Radial distance (d_{rad})

  → distance to cylinder if infinite height

  ② Vertical distance (d_z)

  → height with respect to top of cylinder

$$\boxed{d_{rad} = \max\left(0, \sqrt{(O_{i,x} - C_x)^2 + (O_{i,y} - C_y)^2} - R\right)}$$

$$\boxed{d_z = \max(0, O_{i,z} - h)}$$

$$\boxed{\|O_i - b\| = \sqrt{d_{rad}^{\,2} + d_z^{\,2}}}$$

Case 1) $d_{rad} = 0$:

$$O_i - b = \begin{bmatrix} 0 \\ 0 \\ d_z \end{bmatrix}$$

Case 2) $d_{rad} \neq 0$

$$O_i - b = \begin{bmatrix} O_{i,x} - C_x \\ O_{i,y} - C_y \\ 0 \end{bmatrix}\left(1 - \frac{R}{d_{rad} + R}\right) + \begin{bmatrix} 0 \\ 0 \\ d_z \end{bmatrix}$$

$$\boxed{F_{rep,i}(q) = \eta_i \left(\frac{1}{\|O_i - b\|} - \frac{1}{\rho_0}\right) \frac{1}{(\|O_i - b\|)^2} \frac{O_i - b}{\|O_i - b\|}}$$