Lecture 4

Chapter-02

Discrete-Event System Simulation

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Simulation of Queuing System

Single channel Queue

Simulation of Queuing Systems

- Example: Single-Channel Queue

Assumptions

- Only one checkout counter.
- Customers arrive at this checkout counter at random from 1 to 8 minutes apart. Each possible value of interarrival time has the same probability of occurrence, as shown in Table 2.6.
- The service times vary from 1 to 6 minutes with the probabilities shown in Table 2.7.
- The problem is to analyze the system by simulating the arrival and service of 20 customers.

Simulation of Queuing Systems

Table 2.6 Distribution of Time Between Arrivals

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.125	0.125	001–125
2	0.125	0.250	126–250
3	0.125	0.375	251–375
4	0.125	0.500	376–500
5	0.125	0.625	501–625
6	0.125	0.750	626–750
7	0.125	0.875	751–875
8	0.125	1.000	876–000

Table 2.7 Service-Time Distribution

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.10	0.10	01–10
2	0.20	0.30	11–30
3	0.30	0.60	31–60
4	0.25	0.85	61–85
5	0.10	0.95	86–95
6	0.05	1.00	96–00

Simulation of Queuing Systems

- Set of Random Digits

RD for IAT:	**0, 913, 727, 15, 948, 309, 922, 753, 235, 302, 109, 93, 607, 738, 359, 888, 106, 212, 493, 535**
RD for ST:	**84, 10, 74, 53, 17, 79, 91, 67, 89, 38, 32, 94, 79, 5, 79, 84, 52, 55, 30, 50**

Simulation of Queuing Systems

Student	RD for Arrival	Inter Arrival Time	Arrival Time	RD for Service	Service Time	Service Begin	Service End	Wait In Queue	Service Idle Time	Time Spend in System
1	0	0	**0**	84	**4**	**0**	**4**	0	0	4
2	913	8	**8**	10	**1**	**8**	**9**	0	4	1
3	727	6	**14**	74	**4**	**14**	**18**	0	5	4
4	15	1	**15**	53	**3**	**18**	**21**	3	0	6
5	948	8	**23**	17	**2**	**23**	**25**	0	2	2
6	309	3	**26**	79	**4**	**26**	**30**	0	1	4
7	922	8	**34**	91	**5**	**34**	**39**	0	4	5
8	753	7	**41**	67	**4**	**41**	**45**	0	2	4
9	235	2	**43**	89	**5**	**45**	**50**	2	0	7
10	302	3	**46**	38	**3**	**50**	**53**	4	0	7
11	109	1	**47**	32	**3**	**53**	**56**	6	0	9
12	93	1	**48**	94	**5**	**56**	**61**	8	0	13
13	607	5	**53**	79	**4**	**61**	**65**	8	0	12
14	738	6	**59**	5	**1**	**65**	**66**	6	0	7
15	359	3	**62**	79	**5**	**66**	**71**	4	0	9
16	888	8	**70**	84	**4**	**71**	**75**	1	0	5
17	106	1	**71**	52	**3**	**75**	**78**	4	0	7
18	212	2	**73**	55	**3**	**78**	**81**	5	0	8
19	493	4	**77**	30	**2**	**81**	**83**	4	0	6
20	535	5	**82**	50	**3**	**83**	**86**	1	0	4
Total		**82**			**68**			**56**	**18**	**124**

Simulation of Queuing Systems

- Example (Cont.)
  - The average waiting time for a customer : 2.8 minutes

$$average\ waitng\ time = \frac{total\ time\ customers\ wait\ in\ queue}{total\ numbers\ of\ customers} = \frac{56}{20} = 2.8\,(\mathrm{min})$$

  - The probability that a customer has to wait in the queue : 0.65

$$probability\,(wait) = \frac{number\ of\ customers\ who\ wait}{total\ numbers\ of\ customers} = \frac{13}{20} = 0.65$$

  - The fraction of idle time of the server : 0.21

$$probability\ of\ idle\ server = \frac{total\ idle\ time\ of\ server}{total\ run\ time\ of\ simulation} = \frac{18}{86} = 0.21$$

  - The probability of the server being busy: 0.79 (=1-0.21)

Simulation of Queuing Systems

- Example (Cont.)
  - The average service time : 3.4 minutes

$$average\ service\ time = \frac{total\ service\ time}{total\ numbers\ of\ customers} = \frac{68}{20} = 3.4\ (\text{min})$$

This result can be compared with the expected service time by finding the mean of the service-time distribution using the equation in table 2.7.

$$E(S) = \sum_{s=0}^{\infty} sp(s)$$

$$E(S) = 1(0.10) + 2(0.20) + 3(0.30) + 4(0.25) + 5(0.10) + 6(0.05) = 3.2\ (\text{min})$$

The expected service time is slightly lower than the average service time in the simulation. The longer the simulation, the closer the average will be to $E(S)$

Simulation of Queuing Systems

- Example (Cont.)
  - The average time between arrivals : 4.3 minutes

$$average\ time\ between\ arrivals = \frac{sum\ of\ all\ times\ between\ arrivals}{numbers\ of\ arrivals - 1} = \frac{82}{19} = 4.3\,(\text{min})$$

  - This result can be compared to the expected time between arrivals by finding the mean of the discrete uniform distribution whose endpoints are a=1 and b=8.

$$E(A) = \frac{a+b}{2} = \frac{1+8}{2} = 4.5\,(\text{min})$$

The longer the simulation, the closer the average will be to $E(A)$

  - The average waiting time of those who actually wait in the queue

$$average\ waiting\ time\ of\ those\ who\ wait = \frac{total\ time\ customers\ wait\ in\ queue}{total\ numbers\ of\ customers\ who\ wiat} = \frac{56}{13} = 4.3\,(\text{min})$$

Simulation of Queuing Systems

- Example (Cont.)
  - The average time a customer spends in the system : 6.2 minutes

$$average\ time\ customer\ spends\ in\ the\ system = \frac{total\ time\ customers\ spend\ in\ system}{total\ numbers\ of\ customers} = \frac{124}{20} = 6.2\ (\text{min})$$

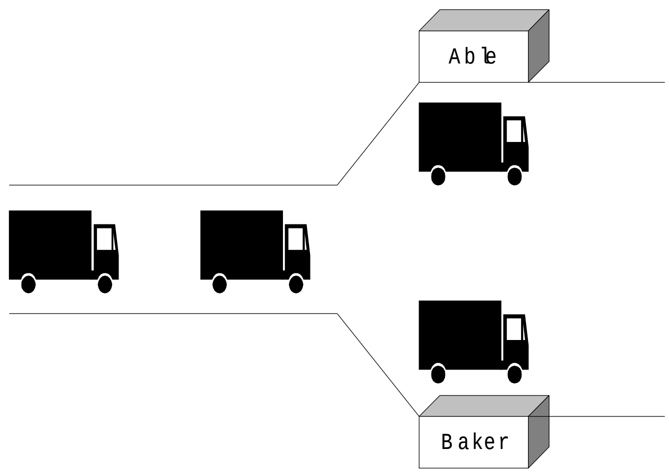
average time customer spends in the system	=	average time customer spends waiting in the queue	+	average time customer spends in service

$\therefore$ average time customer spends in the system = 2.8 + 3.4 = 6.2 (min)

End for Today

Simulation of Queuing Systems

- Example: The Able Baker Carhop Problem



- A drive-in restaurant where carhops take orders and bring food to the car.
- Assumptions
  - Cars arrive in the manner shown in Table 2.11.
  - Two carhops Able and Baker - Able is better able to do the job and works a bit faster than Baker.
  - The distribution of their service times is shown in Tables 2.12 and 2.13.

Simulation of Queuing Systems

- Example: (Cont.)
  - A simplifying rule is that Able gets the customer if both carhops are idle.
  - If both are busy, the customer begins service with the first server to become free.
  - To estimate the system measures of performance, a simulation of 1 hour of operation is made.
  - The problem is to find how well the current arrangement is working.

Table 2.11 Interarrival Distribution of Cars

Time between Arrivals (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
1	0.25	0.25	01–25
2	0.40	0.65	26–65
3	0.20	0.85	66–85
4	0.15	1.00	86–00

Table 2.12 Service Distribution of Able

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
2	0.30	0.30	01–30
3	0.28	0.58	31–58
4	0.25	0.83	59–83
5	0.17	1.00	84–00

Table 2.13 Service Distribution of Baker

Service Time (Minutes)	*Probability*	*Cumulative Probability*	*Random-Digit Assignment*
3	0.35	0.35	01–35
4	0.25	0.60	36–60
5	0.20	0.80	61–80
6	0.20	1.00	81–00

Table 2.14 Simulation Table for Carhop Example

A	B	C	D	E	F	G	H	I	J	K	L
						Able			*Baker*		
Customer No.	*Random Digits for Arrival*	*Time between Arrivals*	*Clock Time of Arrival*	*Random Digits for Service*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time Service Begins*	*Service Time*	*Time Service Ends*	*Time in Queue*
1	–	–	0	95	0	5	5				0
2	26	2	2	21				2	3	5	0
3	98	4	6	51	6	3	9				0
4	90	4	10	92	10	5	15				0
5	26	2	12	89				12	6	18	0
6	42	2	14	38	15	3	18				1
7	74	3	17	13	18	2	20				1
8	80	3	20	61	20	4	24				0
9	68	3	23	50				23	4	27	0
10	22	1	24	49	24	3	27				0
11	48	2	26	39	27	3	30				1
12	34	2	28	53				28	4	32	0
13	45	2	30	88	30	5	35				0
14	24	1	31	01				32	3	35	1
15	34	2	33	81	35	4	39				2
16	63	2	35	53				35	4	39	0
17	38	2	37	81	39	4	43				2
18	80	3	40	64				40	5	45	0
19	42	2	42	01	43	2	45				1
20	56	2	44	67	45	4	49				1
21	89	4	48	01				48	3	51	0
22	18	1	49	47	49	3	52				0
23	51	2	51	75				51	5	56	0
24	71	3	54	57	54	3	57				0
25	16	1	55	87				56	6	62	1
26	92	4	59	47	59	3	62				0
						56			43		11

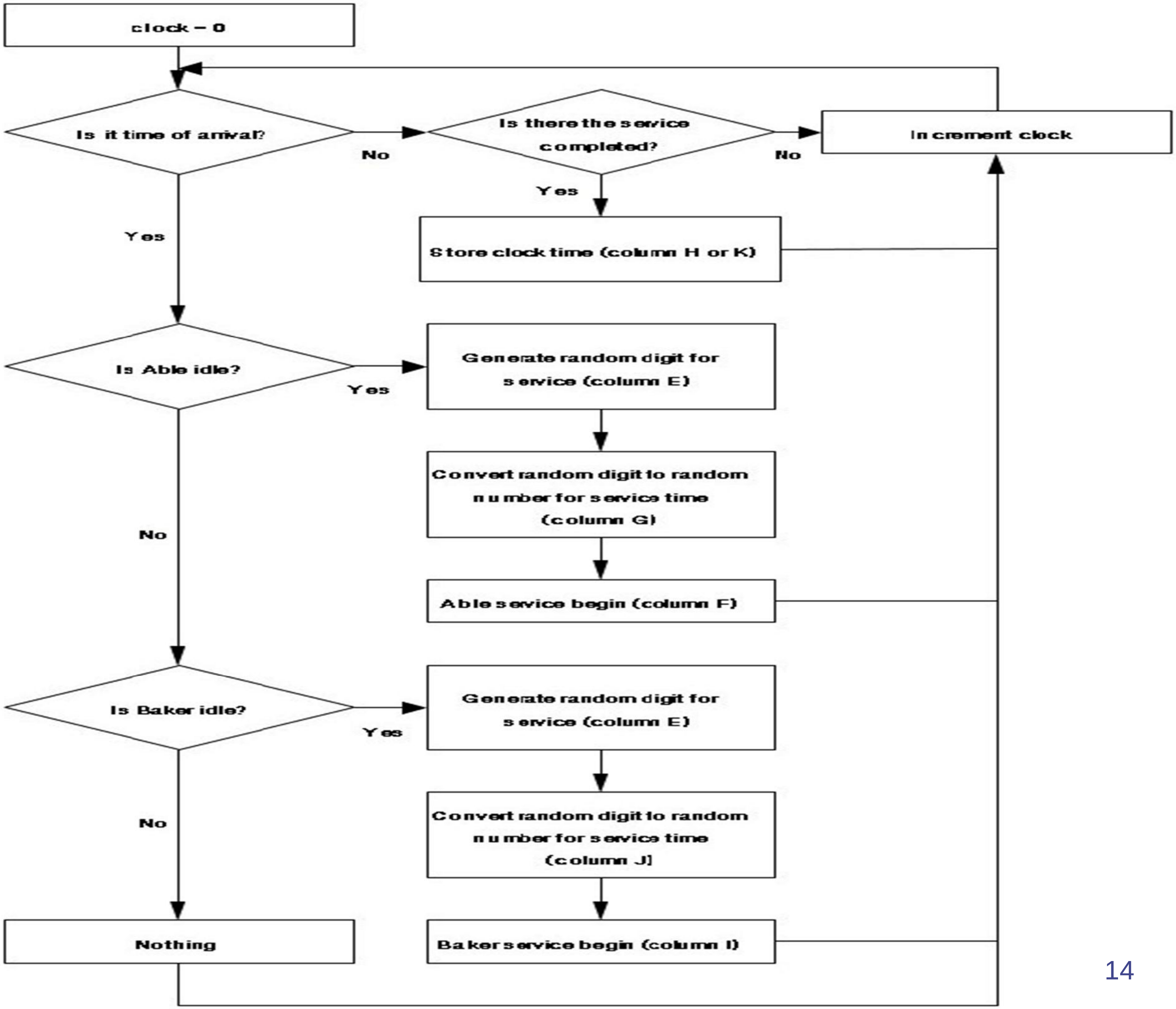
Simulation of Queuing Systems

- Example 2.2 (cont.)
  - The row for the first customer is filled in manually, with the random-number function RAND() in case of Excel or another random function replacing the random digits.
  - After the first customer, the cells for the other customers must be based on logic and formulas. For example, the "Clock Time of Arrival" (column D) in the row for the second customer is computed as follows:

D2 = D1 + C2

  - The logic to computer who gets a given customer can use the Excel macro function IF(), which returns one of two values depending on whether a condition is true or false.

IF(condition, value if true, value if false)


clock = 0
Is it time of arrival?
No
Is there the service completed?
No
Increment clock
Yes
Store clock time (column H or K)
Yes
Is Able idle?
Yes
Generate random digit for service (column E)
Convert random digit to random number for service time (column G)
No
Able service begin (column F)
Is Baker idle?
Yes
Generate random digit for service (column E)
Convert random digit to random number for service time (column J)
No
Nothing
Baker service begin (column I)

Simulation of Queuing Systems

- The analysis of Table 2.14 results in the following:
  - Over the 62-minute period Able was busy 90% of the time.
  - Baker was busy only 69% of the time. The seniority rule keeps Baker less busy (and gives Able more tips).
  - Nine of the 26 arrivals (about 35%) had to wait. The average waiting time for all customers was only about 0.42 minute (25 seconds), which is very small.
  - Those nine who did have to wait only waited an average of 1.22 minutes, which is quite low.
  - In summary, this system seems well balanced. One server cannot handle all the diners, and three servers would probably be too many. Adding an additional server would surely reduce the waiting time to nearly zero. However, the cost of waiting would have to be quite high to justify an additional server.