Machine Learning Exam - Bilet 2 Task Solutions

# Task 1: Beta Updating from Censored Likelihood

In this task, we compute the posterior distribution p(θ | X < 3) where X is the number of heads observed in n = 5 coin tosses,   
and the prior distribution is p(θ) = Beta(θ | 1, 1). We are told that fewer than 3 heads were observed but not exactly how many.

Solution:

- The likelihood for each X is given by binomial distributions, and for X < 3, we sum over X = 0, 1, 2.  
- The posterior distribution is proportional to the prior times the censored likelihood. The result is a mixture distribution.

# Task 2: Posterior Predictive for Dirichlet-Multinomial

We compute the posterior predictive distribution given a Dirichlet prior over the Roman alphabet and the space character (27 values).   
We are given empirical data on letter frequencies and asked to calculate the probabilities of observing new letters.

Part (a) Solution:

- Using the posterior predictive formula for the Dirichlet-multinomial distribution, p(x\_{2001} = e | D) is calculated as 0.119.

Part (b) Solution:

- For the joint probability of observing "p" followed by "a", we use a two-step calculation, resulting in p(x\_{2001} = p, x\_{2002} = a | D) ≈ 0.00188.

# Task 3: Fitting a Naïve Bayes Spam Filter by Hand

We are tasked with fitting a Naïve Bayes spam filter using the multivariate Bernoulli model.   
Using the provided spam and non-spam messages, we compute the maximum likelihood estimates (MLEs) for several words.

Solution:

- P(secret | spam) = 1, P(secret | non-spam) = 0.25, P(sports | non-spam) = 0.5, P(dollar | spam) = 0.33.