Chapter-09

(1) Rectangular window:-

$$w_R(n) = \begin{cases} 1 & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

(2) Hamming window:-

$$w_H(n) = \begin{cases} 0.54 - 0.46 \cos \dfrac{2\pi n}{N-1} ; & 0 \leq n \leq N-1 \\ 0 & \text{otherwise} \end{cases}$$

(3) Hanning window:-

$$W_{hn}(n) = \begin{cases} 0.5 - 0.5 \cos \dfrac{2\pi n}{N-1} : & 0 \leq n \leq N-1 \\ 0 ; & \text{otherwise} \end{cases}$$

Filter: [frequency response and impulse response]

(1) Low-pass filter:- (frequency and impulse response)

$$H_d(w) = \begin{cases} e^{-j\alpha w} ; & -w_c \le w \le w_c \\ 0 ; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w) e^{jwn} dw$$

$$= \frac{\sin w_c(n-\alpha)}{\pi(n-\alpha)}$$

(2) High pass filter:

$$H_d(w) = \begin{cases} e^{-j\alpha w} ; & -\pi \le w \le -w_c \\ e^{-j\alpha w} ; & w_c \le w \le \pi \\ 0 ; & -w_c \le w < w_c \end{cases}$$

$$h_d(n) = \frac{\sin(n-\alpha)\pi - \sin w_c(n-\alpha)}{\pi(n-\alpha)}$$

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(3) Band-pass filter:-

$$H_d(w) = \begin{cases} e^{-j\alpha w} ; & -w_{c_2} \leq w \leq -w_{c_1} \\ e^{-j\alpha w} ; & w_{c_1} \leq w \leq w_{c_2} \\ \cancel{e^{-j\alpha w}}\ 0 ; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{\sin w_{c_2}(n-\alpha) - \sin w_{c_1}(n-\alpha)}{\pi(n-\alpha)}$$

(4) Band-stop filter:-

$$H_d(w) = \begin{cases} e^{-jw\alpha} ; & -\pi \leq w \leq -w_{c_2} \\ e^{-jw\alpha} ; & -w_{c_1} \leq w \leq w_{c_1} \\ e^{-jw\alpha} ; & w_{c_2} \leq w \leq \pi \\ 0 ; & \text{otherwise} \end{cases}$$

$$h_d(n) = \frac{\sin w_{c_1}(n-\alpha) + \sin \pi(n-\alpha) - \sin w_{c_2}(n-\alpha)}{\pi(n-\alpha)}$$

Example: 9.7 (page-699):- A low pass filter is to be designed with the following desired $\boxed{\omega_c = \pi/4}$ frequency response:-

$$H_d(e^{j\omega}) = \begin{cases} e^{-j2\omega} ; & -\pi/4 \leq \omega \leq \pi/4 \\ 0 & \pi/4 \leq |\omega| \leq \pi . \end{cases}$$

Determine the $\boxed{\text{filter co-efficient}}$ $h(n)$ if the window function is defined as:-

$$w(n) = \left.\begin{cases} 1 ; & 0 \leq n \leq 4 \\ 0 ; & \text{otherwise} \end{cases}\right\} \text{rectangular}$$

and determine the $\boxed{\text{frequency response}}$ $H(e^{j\omega})$ of the designed filter.

Soln:-

The filter coefficients are given by

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-j2\omega} \cdot e^{j\omega n}\, d\omega .$$

$$= \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} e^{jw(n-2)}\,dw$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-2)}}{j(n-2)}\right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{\pi(n-2)}\left[\frac{e^{j(n-2)\pi/4} - e^{-j(n-2)\pi/4}}{2j}\right]$$

$$h_d(n) = \frac{1}{\pi(n-2)} \cdot \sin(n-2)\pi/4 \qquad \left[\frac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta\right]$$

For $n=2$ we apply L' Hospitals rule:

$$h_d(2) = \lim_{n\to 2} \frac{1}{\pi} \cdot \boxed{\frac{\sin(n-2)\frac{\pi}{4}}{(n-2)}}$$

$$= \frac{1}{\pi}\cdot\frac{\pi}{4} = \frac{1}{4} \qquad \left[\text{since } \lim_{\theta\to 0}\frac{\sin(\theta-1)}{(\theta-1)} = 1\right]$$

$$\therefore h_d(0) = \frac{1}{\pi(-2)} \cdot \sin(-2)\frac{\pi}{4} = \frac{1}{-2\pi}\left(-\sin\frac{\pi}{2}\right)$$

$$= \frac{1}{-2\pi}(-1) = \frac{1}{2\pi}$$

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$$h_d(1) = \frac{1}{\pi(1-2)} \cdot \sin(1-2)\pi/4 =$$

$$= \frac{1}{-\pi}\left(-\sin\frac{\pi}{4}\right) = \frac{1}{\pi} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}\pi}$$

$$h_d(3) = \frac{1}{\pi(3-2)} \cdot \sin(3-2)\pi/4 = \frac{1}{\pi}\sin\frac{\pi}{4}$$

$$= \frac{1}{\pi}\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}\pi}$$

$$h_d(4) = \frac{1}{\pi(4-2)} \cdot \sin(4-2)\pi/4 = \frac{1}{2\pi} \cdot \sin 2\pi/2 = \frac{1}{2\pi}$$

The filter co-efficient, $\boxed{h(n) = h_d(n) \cdot w(n)}$

$\therefore h(0) = \frac{1}{2\pi} = h(4);$

$\therefore h(1) = \frac{1}{\sqrt{2}\pi} = h_d(3);$

$\therefore h(2) = \frac{1}{4}$

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The frequency response $H(\omega)$
∴ The ~~real~~ realizable ~~digital filter~~:

$$H(\omega) = \sum_{n=0}^{4} h(n) e^{-j\omega n}$$

$$= h(0) + h(1) e^{-j\omega} + h(2) \boxed{e^{-2j\omega}} + h(3) e^{-3j\omega} + h(4) e^{-4j\omega}$$

$$= e^{-2j\omega} \left[\boxed{h(0)} e^{2j\omega} + h(1) e^{j\omega} + h(2) + h(3) e^{-j\omega} + \boxed{h(4)} e^{-2j\omega} \right]$$

$$= e^{-2j\omega} \left[\frac{1}{2\pi} \left\{ e^{2j\omega} + e^{-2j\omega} \right\} + \frac{1}{\sqrt{2}\pi} \left\{ e^{j\omega} + e^{-j\omega} \right\} + \frac{1}{4} \right]$$

$$= e^{-2j\omega} \left[\frac{1}{4} + \frac{1}{2\pi} \cdot 2\cos 2\omega + \frac{1}{\sqrt{2}\pi} \cdot 2\cos\omega \right]$$

$$\therefore H(\omega) = e^{-2j\omega} \left[\frac{1}{4} + \pi\cos 2\omega + \frac{\sqrt{2}}{\pi}\cos\omega \right]$$

$$\boxed{\cos\theta = \frac{e^{i\theta} + e^{-i\theta}}{2}}$$

Ar

$$2 \left\{ \frac{e^{2j\omega} + e^{-2j\omega}}{2} \right\}$$

$$= 2 \times \cos 2\omega$$

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The realizable digital filter is:-

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) z^{0} + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}$$

Ans

$$= \frac{1}{2\pi} + \frac{1}{\sqrt{2}\pi} z^{-1} + \frac{1}{4} z^{-2} + \frac{1}{\sqrt{2}\pi} z^{-3} + \frac{1}{2\pi} z^{-4}$$

$$= z^{-2}\left[\frac{1}{4} + \frac{1}{2\pi}\left(z^{2} + z^{-2}\right) + \frac{1}{\sqrt{2}\pi}\left(z + z^{-1}\right)\right]$$

Ans.

Example : 9.8

$$H_d(e^{jw}) = \begin{cases} 0; & -\pi/2 \le w \le \pi/2 \\ e^{-j2w}; & \pi/2 \le |w| \le \pi \end{cases}$$

$$w(n) = \begin{cases} 1; & 0 \le n < 4 \\ 0; & \text{otherwise} \end{cases} \Big\} \text{ rectangular}$$

Find filter co-efficient $h(n)$ and frequency response $H(e^{jw})$.

Soln :- Filter co-efficients are:

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(w) \cdot e^{jwn} \, dw$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\pi/2} e^{-j2w} \cdot e^{jwn} \, dw + \frac{1}{2\pi} \int_{\pi/2}^{\pi} e^{-jw2} \cdot e^{jwn} \, dw$$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\pi/2} e^{jw(n-2)} \, dw + \frac{1}{2\pi} \int_{+\pi/2}^{\pi} e^{jw(n-2)} \, dw.$$

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$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-2)}}{jw(n-2)}\right]_{-\pi}^{-\pi/2} + \frac{1}{2\pi}\left[\frac{e^{jw(n-2)}}{j(n-2)}\right]_{\pi/2}^{\pi}$$

$$= \frac{1}{2\pi j(n-2)}\left[e^{-j(n-2)\pi/2} - e^{-j(n-2)\pi} + e^{j(n-2)\pi} - e^{j(n-2)\pi/2}\right]$$

$$= \frac{1}{\pi(n-2)}\left\{\left[\frac{e^{j(n-2)\pi} - e^{-j(n-2)\pi}}{2j}\right] - \left[\frac{e^{j(n-2)\pi/2} - e^{-j(n-2)\pi/2}}{2j}\right]\right\}$$

$$h_d(n) = \frac{1}{\pi(n-2)}\left[\sin(n-2)\pi - \sin(n-2)\pi/2\right]$$

For $n=2$ we apply L Hospitals rule

$$h_d(2) = \lim_{n\to 2} \frac{1}{\pi}\left[\frac{\sin (n-2)\pi}{(n-2)} - \frac{\sin (n-2)\pi/2}{(n-2)}\right]$$

$$= \frac{1}{\pi}\left[\pi - \pi/2\right] = \frac{1}{\pi}\cdot\frac{\pi}{2} = \frac{1}{2}$$

$$\therefore h_d(0) = \frac{1}{\pi(-2)}\left[\sin(-2)\pi - \sin(-2)\pi/2\right]$$

$$= \frac{1}{-2\pi}\left[-\sin 2\pi + \cancel{2}\sin \pi/\cancel{2}\right]$$

$$= 0 \quad [\because \sin n\pi = \sin n\pi = 0]$$

$$h_d(1) = \frac{1}{\pi(-1)}\left[\sin(-1)\pi - \sin(-1)\pi/2\right]$$

$$= \frac{1}{-\pi}\left[-\underbrace{\sin \pi}_{0} + \underbrace{\sin \pi/2}_{1}\right] = -\frac{1}{\pi}$$

$$h_d(3) = \frac{1}{\pi}\left[\underbrace{\sin(\pi)}_{0} - \underbrace{\sin \pi/2}_{-1}\right] = -\frac{1}{\pi}$$

$$h_d(4) = \frac{1}{2\pi}\left[\sin 2\pi - \sin 2\cdot\pi/2\right] = 0$$

$h(n) = h_d(n) \cdot W(n)$

$\therefore h(0) = 0 = h(4)$

$\therefore h(1) = -\frac{1}{\pi} = h(3)$

$\therefore h(2) = \frac{1}{2}$

$\therefore$ Filter transfer function:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}$$

$$= 0 + \frac{-1}{\pi} z^{-1} + \frac{1}{2} z^{-2} + \frac{-1}{\pi} z^{-3} + 0$$

$$= z^{-2} \left[\frac{1}{2} - \frac{1}{\pi} \left\{ z + z^{-1} \right\} \right]$$

Ans.

$\therefore$ frequency response $H(w)$

$$H(w) = \sum_{n=0}^{4} h(n)\, e^{-jwn}$$

$$= h(0) + h(1)\, e^{-jw} + h(2)\, e^{-2jw} + h(3)\, e^{-3jw} + h(4)\, e^{-4jw}$$

$$= e^{-j2w}\left[h(0)\, e^{2jw} + h(1)\, e^{jw} + h(2) + h(3)\, e^{-jw} + h(4)\, e^{-2jw}\right]$$

$$= e^{-j2w}\left[\frac{1}{2} + 0 - \frac{1}{\pi} 2\cos w\right]$$

$$= e^{-2jw}\left[\frac{1}{2} - \frac{2}{\pi}\cos w\right]$$

Ans

example: 9.9 :- Using Hamming window ; $N=7$

$$H_d(e^{jw}) = \begin{cases} e^{-3jw} ; & -\pi/4 \le w \le \pi/4 \\ 0 ; & \pi/4 \le |w| \le \pi \end{cases}$$

Sol^n: Filter co-efficients are:

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(w) \cdot e^{jwn} \, dw$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-3jw} \cdot e^{jwn} \, dw$$

$$= \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{jw(n-3)} \, dw$$

$$= \frac{1}{2\pi} \left[\frac{e^{jw(n-3)}}{j(n-3)} \right]_{-\pi/4}^{\pi/4}$$

$$= \frac{1}{\pi(n-3)}\left[\frac{e^{j(n-3)\frac{\pi}{4}} - e^{-j(n-3)\pi/4}}{2j}\right]$$

$$\therefore h_d(n) = \frac{1}{\pi(n-3)} \cdot \sin(n-3)\frac{\pi}{4}$$

For $n=3$, applying L'Hospital rules;

$$h_d(3) = \lim_{n \to 3} \frac{1}{\pi} \cdot \frac{\sin(n-3)\pi/4}{(n-3)}$$

$$= \frac{1}{\pi} \cdot \frac{\pi}{4} = \frac{1}{4}$$

$$\therefore h_d(0) = \frac{1}{\pi(-3)} \cdot \sin(-3)\frac{\pi}{4}$$

$$= \frac{1}{-3\pi}\left(-\sin\frac{3\pi}{4}\right)$$

$$= \frac{1}{3\pi} \cdot \frac{\sqrt{2}}{2} \quad [\text{Rad mod on}]$$

$$= \frac{0.707}{3\pi};$$

$$h_d(1) = \frac{1}{\pi(-2)} \cdot \sin(-2)\frac{\pi}{4}$$

$$= \frac{1}{2\pi} \sin\frac{\pi}{2} = \frac{1}{2\pi}$$

$$h_d(2) = \frac{1}{\pi(-1)} \sin(-1)\frac{\pi}{4} = \frac{1}{\pi} \sin\frac{\pi}{4} = \frac{1}{\pi} \cdot \frac{1}{\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}\pi} = \frac{0.707}{\pi}$$

$$h_d(4) = \frac{1}{\pi} \sin\frac{\pi}{4} = \frac{1}{\pi} \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}\pi} = \frac{0.707}{\pi}$$

$$h_d(5) = \frac{1}{2\pi} \sin\frac{\pi}{2} = \frac{1}{2\pi}.$$

$$h_d(6) = \frac{1}{3\pi} \sin(3)\frac{\pi}{4} = \frac{0.707}{3\pi}$$

$$\cancel{h_d(7) = \frac{1}{4\pi} \sin(4)\frac{\pi}{4} = \frac{1}{4} \sin\pi}$$

$\therefore h_d(0) = \frac{0.707}{3\pi} = h_d(6)$

$\therefore h_d(1) = \frac{1}{2\pi} = h_d(5)$

$\therefore h_d(2) = \frac{0.707}{\pi} = h_d(4)$

$\therefore h_d(3) = \frac{1}{4};$

$\therefore \cancel{h_d(4) =}$

For Hamming window we know

$$w(n) = \begin{cases} 0.54 - 0.46 \cos \frac{2\pi n}{N-1} & 0 \leq n \leq N-1 \\ 0; & \text{otherwise} \end{cases}$$

$\therefore w(0) = 0.54 - 0.46 \cos 0 = 0.08$

$\therefore W(1) = 0.54 - 0.46 \cos \frac{2\pi \cdot 1}{6} = 0.31$

$\therefore W(2) = 0.54 - 0.46 \cos \frac{2\pi \cdot 2}{6} = 0.77$

$\therefore W(3) = 0.54 - 0.46 \cos \frac{2\pi \cdot 3}{6} = 1$

$\therefore W(4) = 0.54 - 0.46 \cos \frac{2\pi \cdot 4}{6} = 0.77$

$\therefore W(5) = 0.54 - 0.46 \cos \frac{2\pi \cdot 5}{6} = 0.31$

$\therefore W(6) = 0.54 - .46 \cos \frac{2\pi \cdot 6}{6} = 0.08$

The filter co-efficient;

$h(n) = h_d(n) \cdot W(n)$

$\therefore h(0) = h_d(0) \cdot W(0) = \frac{0.707}{3\pi} \times 0.08 = 0.006$

$\therefore h(1) = h_d(1) \times W(1) = \frac{1}{2\pi} \times 0.31 = 0.049$

$\therefore h(2) = h_d(2) \times W(2) = \frac{0.707}{\pi} \times 0.77 = 0.173$

$\therefore h(3) = \frac{1}{4} \times 1 = \frac{1}{4} = 0.25$

$\therefore h(4) = \frac{0.707}{\pi} \times .77 = 0.173$

$h(5) = \frac{1}{2\pi} \times 0.31 = 0.049$

$h(6) = \frac{0.707}{3\pi} \times 0.08 = 0.006$

∴ Frequency response:

$$H(w) = \sum_{n=0}^{6} h(n) e^{-jwn}$$

$$= h(0)e^{0} + h(1)e^{-jw} + h(2)e^{-2jw} + h(3)e^{-3jw}$$

$$+ h(4)e^{-4jw} + h(5)e^{-5jw} + h(6)e^{-6jw}$$

$$= e^{-3jw}\Big[h(3) + \{h(0)e^{3jw} + h(6)e^{-3jw}\}$$

$$+ \{h(1)e^{2jw} + h(5)e^{-2jw}\} + \{h(2)e^{jw} + h(4)e^{-jw}\}\Big]$$

$$= e^{-3jw}\Big[0.25 + (2\times 0.006)\cos 3w +$$

$$(2\times 0.049)\cos 2w + (2\times 0.173)\cos w\Big]$$

$$= e^{-3jw}\Big[0.25 + 0.012\cos 3w + 0.098\cos 2w$$

$$+ 0.346\cos w\Big]$$

Ans

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The transfer function:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4} +$$
$$h(5) z^{-5} + h(6) z^{-6}$$

$$= z^{-3} \left[h(3) + h(0)(z^{3} + z^{-3}) + h(1)(z^{2} + z^{-2}) + h(2)(z + z^{-1}) \right]$$

$$= z^{-3} \left[0.25 + 0.006(z^{3} + z^{-3}) + 0.049(z^{2} + z^{-2}) + 0.173(z + z^{-1}) \right]$$

$$= 0.006 + 0.049 z^{-1} + 0.173 z^{-2} + 0.25 z^{-3} + 0.173 z^{-4} + 0.049 z^{-5} + 0.006 z^{-6}$$

Ans

Ex-9.10: Low-pass filter using rectangular window taking 9 samples and cut-off frequency 1.2 rad/sec

Soln:– For low pass filter.

$$H_d(w) = \begin{cases} e^{-j\alpha w} & -w_c \le w \le w_c \\ 0 & \text{otherwise} \end{cases}$$

$$\therefore h_d(w) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{-jw\alpha} \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-w_c}^{w_c} e^{jw(n-\alpha)}\, dw$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{-w_c}^{w_c}$$

$$= \frac{1}{\pi(\cancel{\pi}(n-\alpha))}\left[\frac{e^{jw_c(n-\alpha)} - e^{-jw_c(n-\alpha)}}{2j}\right]$$

$$h_d(n) = \frac{1}{\pi \cdot \cancel{\pi}(n-\alpha)} \cdot \cancel{\cos w_c(n-\alpha)}$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)} \cdot \cancel{\cos} \sin w_c(n-\alpha) \quad \text{--- (1)}$$

For $n=\alpha$; using L. Hospital rule;

$$h_d(\alpha) = \lim_{n \to \alpha} \frac{1}{\pi(n-\alpha)} \cdot \sin w_c(n-\alpha)$$

$$h_d(\alpha) = \frac{w_c}{\pi}$$

For rectangular window

$$w_R(n) = \begin{cases} 1 & \text{for } 0 \le n \le N-1 \\ 0 & \text{otherwise} \end{cases}$$

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$w_c = 1.2$ rad/sec and $\alpha = \frac{N-1}{2} = \frac{9-1}{2} = 4$.

$\therefore h(0) = \frac{\sin(0-4)\times 1.2}{\pi(0-4)} = -0.0793$

$\therefore h(1) = \frac{\sin(1-4)\times 1.2}{\pi(1-4)} = -0.0470$

$\therefore h(2) = \frac{\sin(2-4)\times 1.2}{\pi(2-4)} = 0.1075$

$\therefore h(3) = \frac{\sin(3-4)\times 1.2}{\pi(3-4)} = 0.2967$

$\therefore h(4) = \frac{\cancel{\sin}\; w_c}{\pi} = \frac{1.2}{\pi} = 0.382$

$\therefore h(5) = \frac{\sin(5-4)\times 1.2}{\pi(5-4)} = 0.2967$

$\therefore h(6) = \frac{\sin(6-4)\times 1.2}{\pi(6-4)} = 0.1075$

$\therefore h(7) = \frac{\sin(7-4)\, 1.2}{\pi(7-4)} = -0.047$

$\therefore h(8) = \frac{\sin(8-4)}{\pi(8-4)} = 0.0793$

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$\therefore$ Transfer function:

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n}$$

$$= h(0) + h(1) z^{-1} + h(2) z^{-2} + h(3) z^{-3} + h(4) z^{-4}$$
$$+ h(5) z^{-5} + h(6) z^{-6} + h(7) z^{-7} + h(8) z^{-8}$$

$$= z^{-4} \left[h(4) + h(0) \left\{ z^{4} + z^{-4} \right\} + h(1) \left\{ z^{3} + z^{-3} \right\} \right.$$
$$\left. + h(2) \left\{ z^{2} + z^{-2} \right\} + h(3) \left\{ z + z^{-1} \right\} \right]$$

$$= z^{-4} \left[0.382 + (-0.0793)(z^{+4} + z^{-4}) + \right.$$
$$(-0.0970)(z^{3} + z^{-3}) + 0.1075 (z^{2} + z^{-2}) +$$
$$\left. 0.2967 (z + z^{-1}) \right]$$

Ans

Frequency response.

$$H(w)=\sum_{n=0}^{8} h(n)e^{-jwn}$$

$$= h(0)+h(1)e^{-jw}+h(2)e^{-2jw}+h(3)e^{-3jw}$$

$$+h(4)e^{-4jw}+h(5)e^{-5jw}+h(7)e^{-7jw}$$

$$+h(8)e^{-8jw}+h(6)e^{-6jw}$$

$$= e^{-4jw}\Big[h(4)+h(0)\big[e^{4jw}+e^{-4jw}\big]+$$

$$h(1)\{e^{3jw}+e^{-3jw}\}+h(2)\{e^{2jw}+e^{-2jw}$$

$$+h(3)\{e^{jw}+e^{-jw}\}\Big]$$

$$= e^{-4jw}\Big[0.382+2\times(-0.0793)\cos 4w+$$

$$2\times(-0.0470)\cos 3w+2\times(0.1075)\cos 2w$$

$$+2(0.2967)\cos w\Big]$$

Ans

EX-9.11: high pass filter using Hamming Windo[w]
cutoff frequency 1.2 rad/sec and N=9

Soln:- For high pass filter

$$H_d(w) = \begin{cases} e^{-jw\alpha} ; & \cancel{w_c \le |w| \le w_c \pi} \\ & -\pi \le w \le -w_c \text{ and } w_c \le w \le \pi \\ 0 ; & \text{otherwise} \end{cases}$$

$$\therefore h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{-w_c} e^{-jw\alpha} \cdot e^{jwn}\, dw + \frac{1}{2\pi}\int_{w_c}^{\pi} e^{-jw\alpha}\, e^{+jwn}\, d[w]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-w_c} e^{jw(n-\alpha)}\, dw\right] + \frac{1}{2\pi}\int_{+w_c}^{\pi} e^{jw(n-\alpha)}\, d[w]$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-w_c} + \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{w_c}^{\pi}$$

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$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j(n-\alpha)w_c} - e^{-j(n-\alpha)\pi}}{2j}\right] +$$

$$\frac{1}{\pi(n-\alpha)}\left[\frac{e^{j(n-\alpha)\pi} - e^{j(n-\alpha)w_c}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j(n-\alpha)\pi} - e^{-j(n-\alpha)\pi}}{2j} - \frac{e^{j(n-\alpha)w_c} - e^{-j(n-\alpha)w_c}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\sin(n-\alpha)\pi - \sin(n-\alpha)w_c\right]$$

If $n=\alpha$ then using L'Hospital's Law:

$$h_d(\alpha) = \lim_{n\to\alpha} \frac{\sin(n-\alpha)\pi - \sin(n-\alpha)w_c}{\pi(n-\alpha)}$$

$$= \lim_{n\to\alpha}\left[\frac{\sin(n-\alpha)\pi}{\pi(n-\alpha)} - \frac{\sin(n-\alpha)w_c}{\pi(n-\alpha)}\right]$$

$$= \left[\frac{\pi}{\pi} - \frac{w_c}{\pi}\right] = 1 - \frac{w_c}{\pi}$$

π

w_c

$\therefore \cancel{h_d(0) =} \; \alpha = \frac{N-1}{2} = \frac{9-1}{2} = 4 \; ; \; \omega_c = 1.2$

$$\therefore h_d(0) = \frac{\sin(0-4)\pi - \sin(0-4) \cdot 1.2}{\pi(0-4)} = 0.0792$$

$$\therefore h_d(1) = \frac{\sin(1-4)\pi - \sin(1-4) \times 1.2}{\pi(1-4)} = 0.0469$$

$$\therefore h_d(2) = \frac{\sin(2-4)\pi - \sin(2-4) 1.2}{\pi(2-4)} = -.1075$$

$$\therefore h_d(3) = \frac{\sin(3-4)\pi - \sin(3-4) 1.2}{\pi(3-4)} = -0.2966$$

$$\therefore h_d(4) = 1 - \frac{1.2}{\pi} = 0.618$$

$$\therefore h_d(5) = \frac{\sin(5-4)\pi - \sin(5-4) 1.2}{\pi(5-4)} = -0.29$$

$$\therefore h_d(6) = \frac{\sin(6-4)\pi - \sin(6-4) 1.2}{\pi(6-4)} = -0.1075$$

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$$\therefore h_d(7) = \frac{\sin(7-4)\pi - \sin(7-4)1\cdot2}{\pi(7-4)} = 0.0469$$

$$\therefore h_d(8) = \frac{\sin(8-4)\pi - \sin(8-4)1\cdot2}{\pi(8-4)} = 0.0792$$

For hamming window:

$$W(n) = 0.54 - 0.46 \cos \frac{2\pi n}{N-1};$$

$$\therefore W(0) = 0.54 - 0.46 \cos \frac{2\pi\cdot0}{8} = 0.08$$

$$\therefore W(1) = 0.54 - 0.46 \cos \frac{2\pi\cdot1}{8} = 0.2147$$

$$\therefore W(2) = 0.54 - 0.46 \cos \frac{2\pi\cdot2}{8} = 0.54$$

$$\therefore W(3) = 0.865$$

$$\therefore W(4) = 1$$

$$\therefore W(5) = 0.865$$

$$\therefore W(6) = 0.54$$

$$\therefore W(7) = 0.2147$$

$$\therefore W(8) = 0.08$$

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$\therefore h(0) = 0.0792 \times 0.18 = 0.0063$

$\therefore h(1) = 0.0469 \times 0.2147 = 0.0100$

$\therefore h(2) = -0.1075 \times 0.54 = -0.0580$

$\therefore h(3) = -0.2966 \times 0.865 = -0.2566$

$\therefore h(4) = 0.618 \times 1 = 0.618$

$\therefore h(5) = -0.2966 \times 0.865 = -0.2566$

$\therefore h(6) = -0.1075 \times 0.54 = -0.0580$

$\therefore h(7) = 0.0469 \times 0.2147 = 0.0100$

$\therefore h(8) = 0.0792 \times 0.08 = 0.0063$

$\therefore$ Frequency response:

$$H(w) = \sum_{n=0}^{8} h(n) e^{-jwn}$$

$$= h(0) + h(1) e^{-jw} + h(2) e^{-2jw} + h(3) e^{-3jw} + h(4) e^{-4jw} + h(5) e^{-5jw} + h(6) e^{-6jw} + h(7) e^{-7jw} + h(8) e^{-8jw}$$

$$= e^{-4jw}\Big[h(4) + 2h(3)\cos w + 2h(2)\cos 2w + 2h(1)\cos 3w + 2h(0)\cos 4w\Big]$$

$$= e^{-4jw}\Big[0.618 - 0.5732\cos w - 0.116\cos 2w + 0.020\cos 3w + 0.012\cos 4w\Big]$$

∴ Transfer function:

$$H(z) = \sum_{0}^{8} h(n)z^{-n}$$

$$= h(0) + h(1)z^{-1} + h(2)z^{-2} + h(3)z^{-3} + h(4)z^{-4} + h(5)z^{-5} + h(6)z^{-6} + h(7)z^{-7} + h(8)z^{-8}$$

$$= z^{-4}\Big[h(4) + \cdots\Big]$$

Ans

বাকি → ২নং প্রশ্ন

DSP

Chap-09

9.12:- Band Pass filter using Hanning window.
Frequency range 1 to 2 rad/sec and $N=5$

Soln: For Band Pass filter:

$$H_d(\omega) = \begin{cases} e^{-j\omega\alpha} & ; -\omega_{c_2} \leq \omega \leq -\omega_{c_1} \\ e^{-j\omega\alpha} & ; \omega_{c_1} \leq \omega \leq \omega_{c_2} \\ 0 & ; \text{otherwise} \end{cases}$$

$$\therefore h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(\omega)\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_{c_2}}^{-\omega_{c_1}} e^{-j\omega\alpha} \cdot e^{j\omega n}\, d\omega +$$

$$\frac{1}{2\pi}\int_{\omega_{c_1}}^{\omega_{c_2}} e^{-j\omega\alpha}\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-w_{c_2}}^{-w_{c_1}} e^{jw(n-\alpha)}\, dw + \frac{1}{2\pi}\int_{+w_{c_1}}^{w_{c_2}} e^{jw(n-\alpha)}\, dw$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{-w_{c_2}}^{-w_{c_1}} + \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{w_{c_1}}^{w_{c_2}}$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j(n-\alpha)w_{c_1}} - e^{-j(n-\alpha)w_{c_2}}}{2j}\right] +$$

$$\frac{1}{\pi(n-\alpha)}\left[\frac{e^{j(n-\alpha)w_{c_2}} - e^{j(n-\alpha)w_{c_1}}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j(n-\alpha)w_{c_2}} - e^{-j(n-\alpha)w_{c_2}} - e^{j(n-\alpha)w_{c_1}} + e^{-j(n-\alpha)w_{c_1}}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\sin(n-\alpha)w_{c_2} - \sin(n-\alpha)w_{c_1}\right]$$

$$\Rightarrow h_d(n) = \frac{\sin(n-\alpha)\cdot 2 - \sin(n-\alpha)}{\pi(n-\alpha)}.$$

For $n = \alpha$;

$$h_d(\alpha) = \lim_{n \to \alpha} \frac{\sin(n-\alpha)2 - \sin(n-\alpha)}{\pi(n-\alpha)}$$

$$= \frac{2-1}{\pi} = \frac{1}{\pi}$$

$$\therefore \alpha = \frac{N-1}{2} = \frac{5-1}{2} = 2;$$

$$\therefore h_d(0) = \frac{\sin(0-2)\cdot 2 - \sin(0-2)}{\pi(0-2)} = -0.2651$$

$\therefore h_d(1) = 0.0215$

$\therefore h_d(2) = 0.3183$

$\therefore h_d(3) = 0.0215$

$\therefore h_d(4) = -0.2651$

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For hanning window;

$$W(n) = \begin{cases} 0.5 - 0.5 \cos \frac{2\pi n}{N-1}; & 0 \le n \le N-1 \\ 0; & \text{otherwise} \end{cases}$$

$\therefore W(0) = 0.5 - 0.5 \cos \frac{2\pi \cdot 0}{4} = 0$

$\therefore W(1) = 0.5$

$\therefore W(2) = 1$

$\therefore W(3) = 0.5$

$\therefore W(4) = 0$

$\therefore h(0) = h_d(0) \times W(0) = (-0.2651) \times (0) = 0$

$\therefore h(1) =$ $h(2) =$ $h(3) =$ $h(4) =$

Using same method:

Frequency response; $H(w) = \sum_{n=0}^{4} h(n) e^{-jwn}$. Ans.

Transfer function $H(z) = \sum_{n=0}^{N-1} h(n) e^{-jwn}$. Ans

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9.13: Band-stop filter using rectangular. frequency 1 to 2 and $N=7$.

Solⁿ: For Band-stop filter,

$$H_d(w) = \begin{cases} e^{-jw\alpha} ; & -\pi \le w \le w_{c_2} \\ e^{-jw\alpha} ; & -w_{c_1} \le w \le w_{c_1} \\ e^{-jw\alpha} ; & -w_{c_2} \le w \le \pi \\ 0 ; & \text{otherwise} \end{cases}$$

$$\therefore h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(w) \cdot e^{jwn} \cdot dw$$

$$= \frac{1}{2\pi}\int_{-\pi}^{w_{c_2}} e^{jw(n-\alpha)} dw + \frac{1}{2\pi}\int_{-w_{c_1}}^{w_{c_1}} e^{jw(n-\alpha)} dw$$

$$+ \frac{1}{2\pi}\int_{w_{c_2}}^{\pi} e^{jw(n-\alpha)} dw.$$

$$= \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{-\pi}^{-w_{c_2}} + \frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{-w_{c_1}}^{w_{c_1}} +$$

$$\frac{1}{2\pi}\left[\frac{e^{jw(n-\alpha)}}{j(n-\alpha)}\right]_{w_{c_2}}^{\pi}$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{-j(n-\alpha)w_{c_2}} - e^{-j(n-\alpha)\pi} + e^{j(n-\alpha)w_{c_1}} - e^{-j(n-\alpha)w_{c_1}} + e^{j(n-\alpha)\pi} - e^{+j(n-\alpha)w_{c_2}}}{2j}\right]$$

$$= \frac{1}{\pi(n-\alpha)}\left[\frac{e^{j(n-\alpha)\pi} - e^{-j(n-\alpha)\pi}}{2j} + \frac{e^{j(n-\alpha)w_{c_1}} - e^{-j(n-\alpha)w_{c_1}}}{2j} - \frac{e^{j(n-\alpha)w_{c_2}} - e^{-j(n-\alpha)w_{c_2}}}{2j}\right]$$

$$= \frac{\sin(n-\alpha)\pi + \sin(n-\alpha)w_{c_1} - \sin(n-\alpha)w_{c_2}}{\pi(n-\alpha)}$$

$$\therefore \alpha = \frac{N-1}{2} = \frac{6}{2} = 3$$

$w_{c_2} = 2;\quad w_{c_1} = 1$

For ~~n~~ $n = \alpha$

$$h_d(\alpha) = \lim_{n \to \alpha} \frac{\sin(n-\alpha)\pi + \sin(n-\alpha) - \sin(n-\alpha)\cdot 2}{\pi(n-\alpha)}$$

$$= \frac{\pi + 1 - 2}{\pi} = \frac{\pi - 1}{\pi} = 0.6816$$

$\therefore h_d(0) = \cancel{0.0462}$ 0.0446 $\quad \therefore h_d(4) = -0.0216$

$\therefore h_d(1) = 0.2651 \quad \therefore h_d(5) = 0.2651$

$\therefore h_d(2) = -0.0216 \quad \therefore h_d(6) = 0.0446$

$\therefore h_d(3) = 0.6816$

$\therefore h(n) = w(n)\, h_d(n);$

$\therefore h(n) = h_d(n);$

$\therefore$ Frequency response $= H(w) = \sum_{n=0}^{6} h(n) e^{-jwn}$.

$\therefore$ Transfer function $H(z) = \sum_{n=0}^{6} h(n) z^{-n}$.

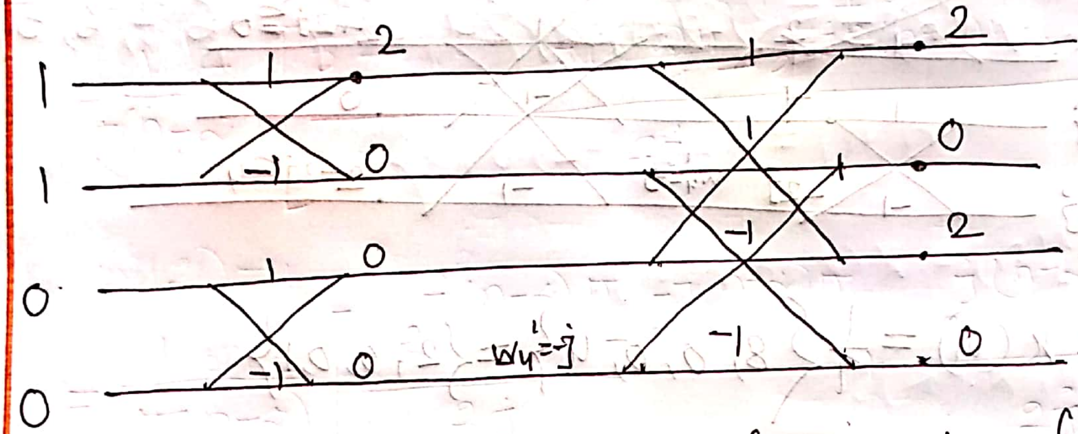
Ans

Question

2017

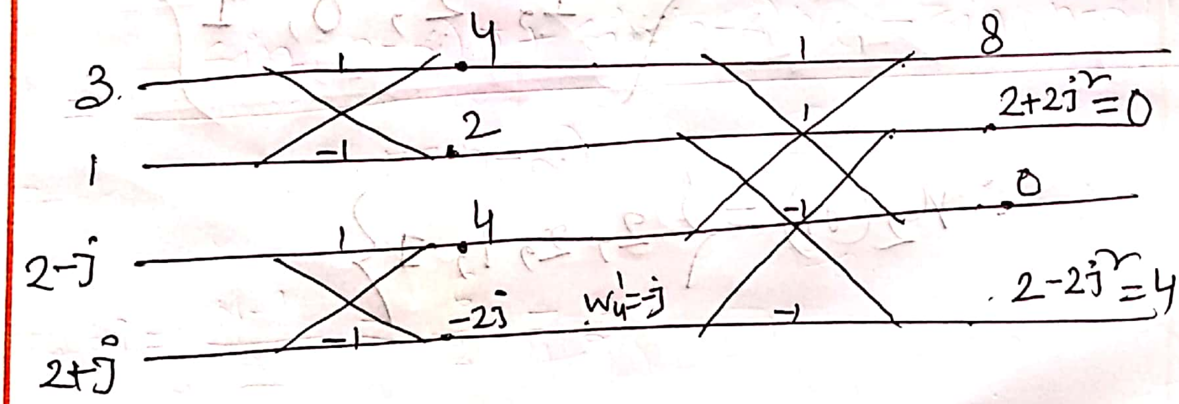
3(c): Find IDFT using DIT FFT

$$X(K) = \{1, 0, 1, 0\} \Rightarrow X^*(K) = \{1, 0, 1, 0\}$$



$$\therefore x(n) = \frac{1}{4}\{2, 0, 2, 0\} = \{\frac{1}{2}, 0, \frac{1}{2}, 0\}$$

$$X(K) = \{3, 2+j, 1, 2-j\} \Rightarrow X^*(K) = \{3, 2-j, 1, 2+j\}$$



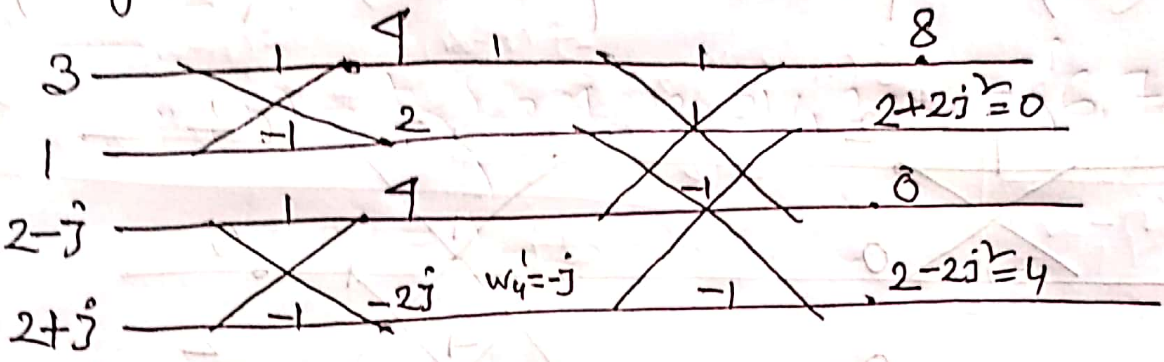
$$\therefore x(n) = \frac{1}{4}\{8, 0, 0, 4\} = \{2, 0, 0, 1\}$$ Ans

2016

6(b) $X(K) = \{3, 2+j, 1, 2-j\}$ Find IDFT using ~~DI~~

$$X^*(K) = \{3, 2-j, 1, 2+j\}$$

using DIT:



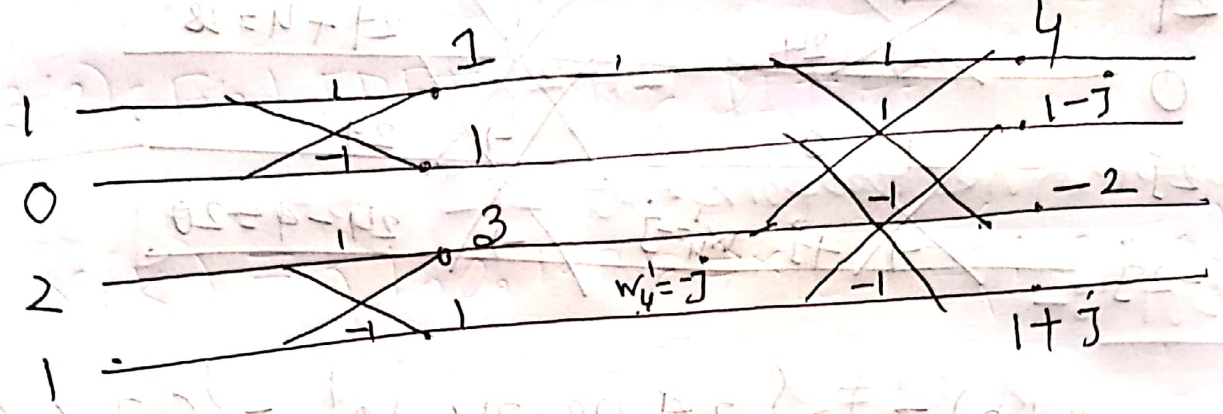
$$\therefore x(n) = \frac{1}{4}\{8, 0, 0, 4\} = \{2, 0, 0, 1\}$$

6(c): $x_1(n) = \delta(n) + 2\delta(n-1) + \delta(n-3)$ $\Rightarrow n-1=0, n=1$

$$\Rightarrow x_1(n) = \{1, 2, 0, 1\}$$

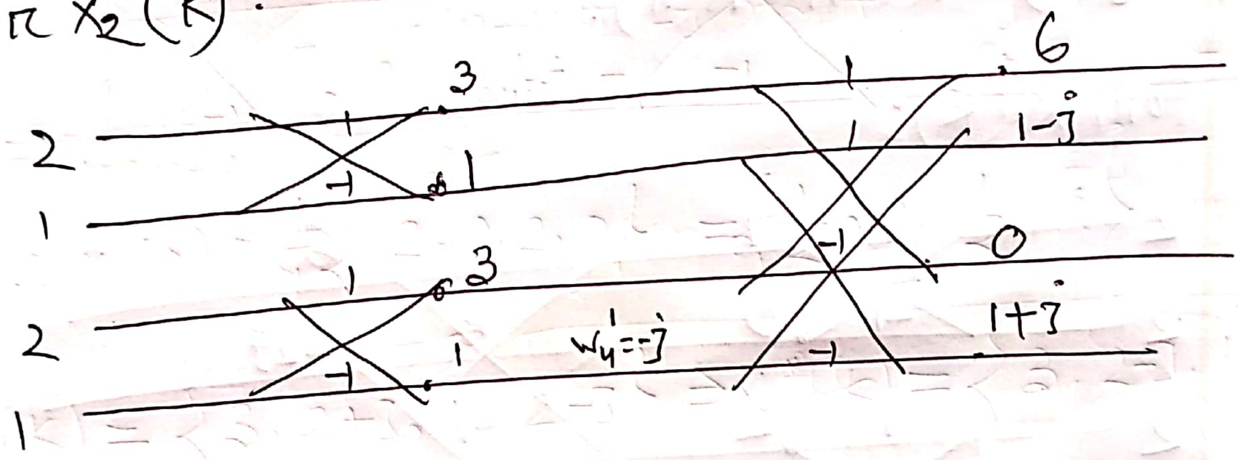
$$\therefore x_2(n) = \{2, 2, 1, 1\}$$

For $X_1(K)$: using DIT



$\therefore X_1(K) = \{4, 1-j, -2, 1+j\}$

For $X_2(K)$:



$X_2(K) = \{6, 1-j, 0, 1+j\}$

$\therefore X(K) = X_1(K) * X_2(K)$

$= \{24, -2j, 0, 2j\}$

$(1-j)(1-j)$

$= 1-j-j+j^2$

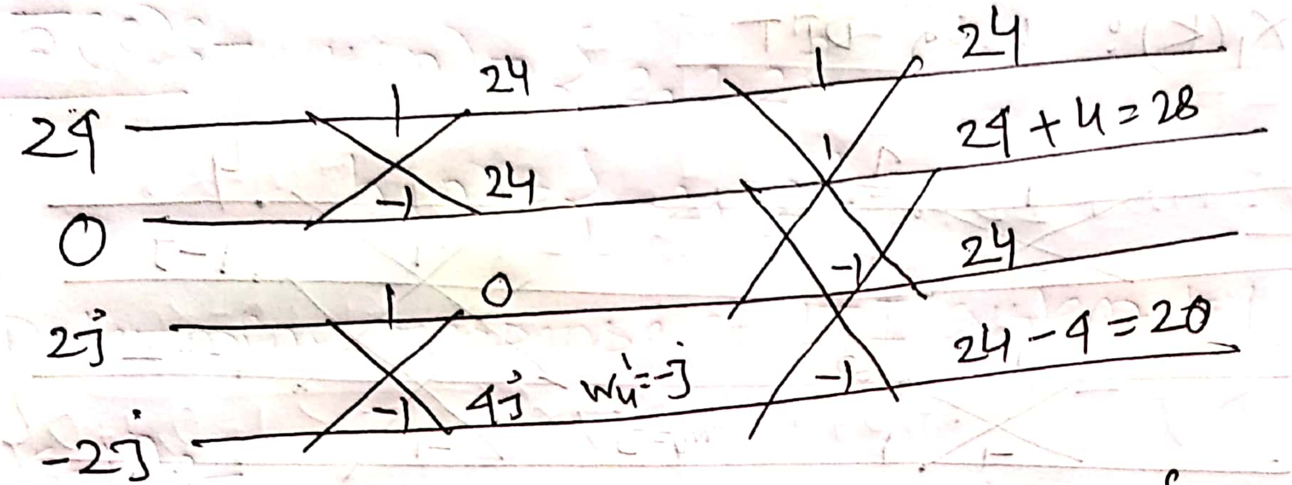
$= -2j$

$(1+j)(1+j)$

$= 1+j+j+j^2$

$= 2j$

$\therefore X^*(K) = \{24, 2j, 0, -2j\}$

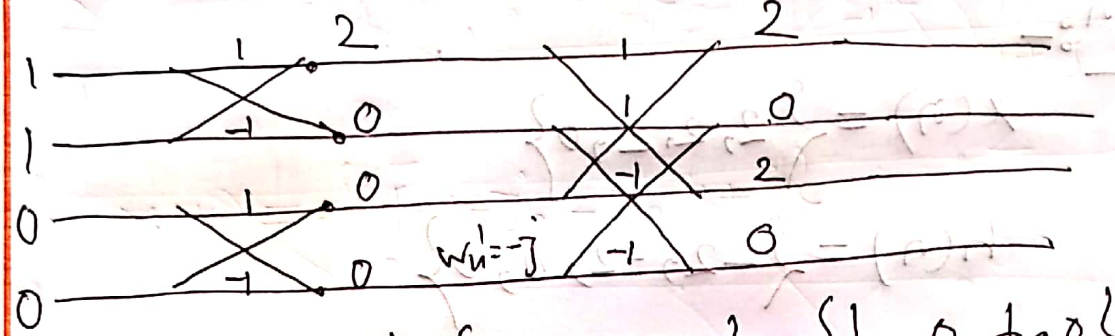


$$\therefore x(n) = \frac{1}{4}\{24, 28, 24, 20\} = \{6, 7, 6, 5\}$$ Ans

7(b):- IDFT: by DIT FFT

$X(K) = \{1, 0, 1, 0\}$

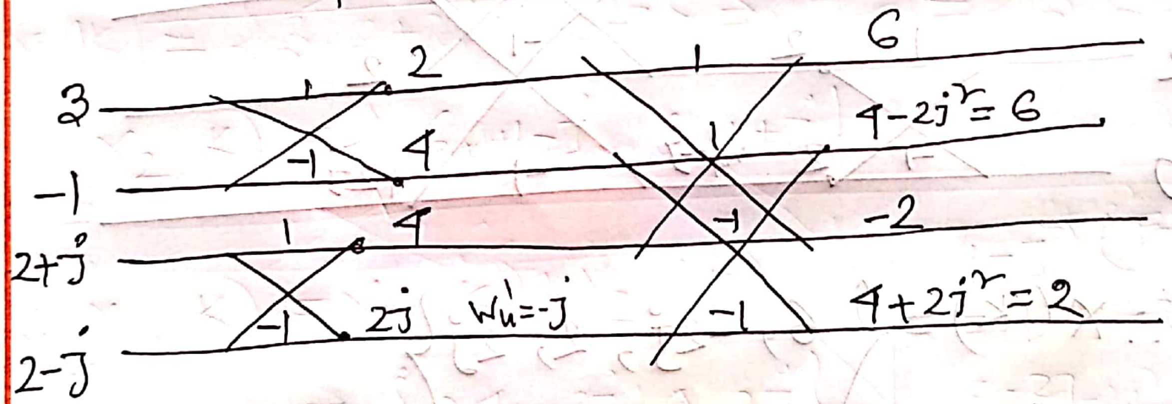
$\therefore X^*(K) = \{1, 0, 1, 0\}$



$\therefore x(n) = \frac{1}{4}\{2, 0, 2, 0\} = \{\frac{1}{2}, 0, \frac{1}{2}, 0\}$

$X(K) = \{3, 2-j, -1, 2+j\}$

$X^*(K) = \{3, 2+j, -1, 2-j\}$



$\therefore x(n) = \frac{1}{4}\{6, 6, -2, 2\}$

$= \{\frac{3}{2}, \frac{3}{2}, -\frac{1}{2}, \frac{1}{2}\}$

5(c):- $x(n) = \{2, 2, 2\}$

$h(n) = \{-2, -2\}$

Determine response of LTI system by DIT;

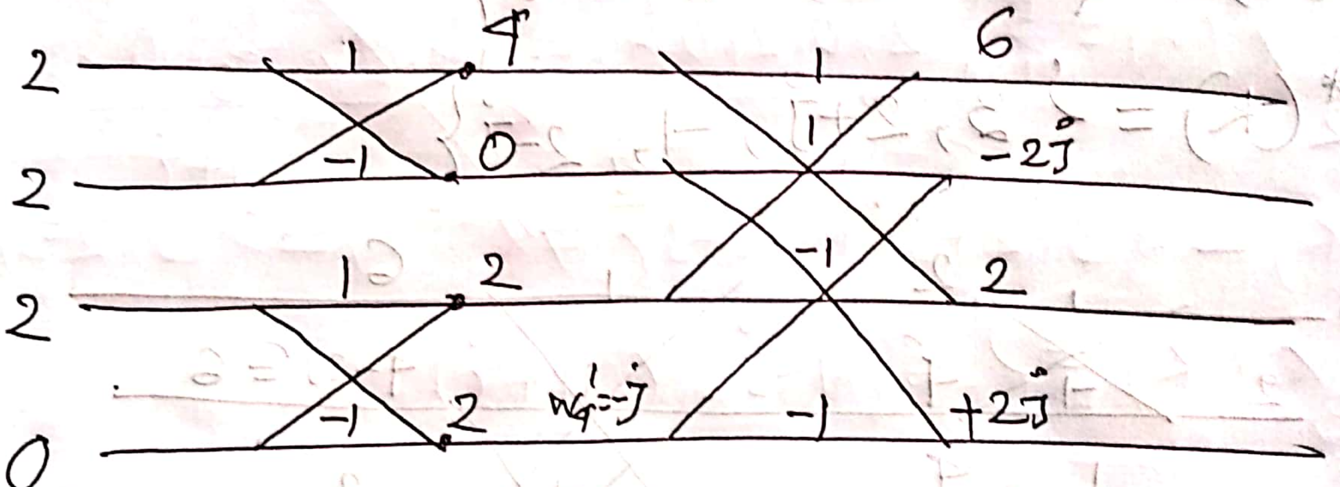
Soln:-

$x(n) = \{2, 2, 2, 0\}$

$h(n) = \{-2, -2, 0, 0\}$

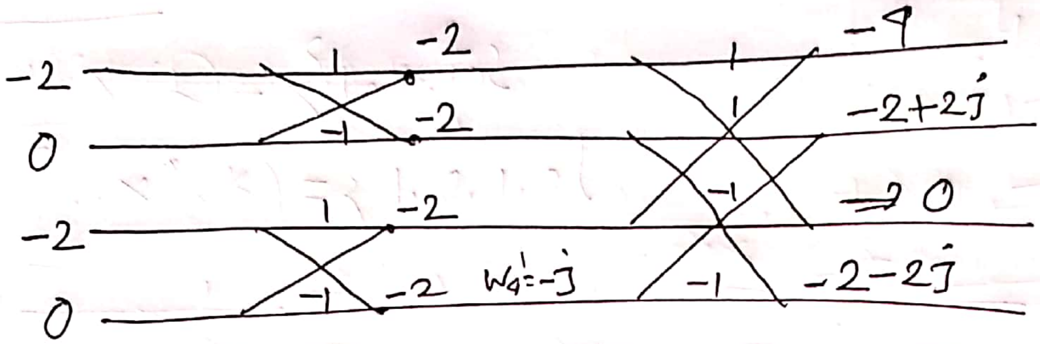
$y(n) = x(n) * h(n).$

For, $X(K)$:



$\therefore X(K) = \{6, -2j, 2, 2j\}$

For H(K):

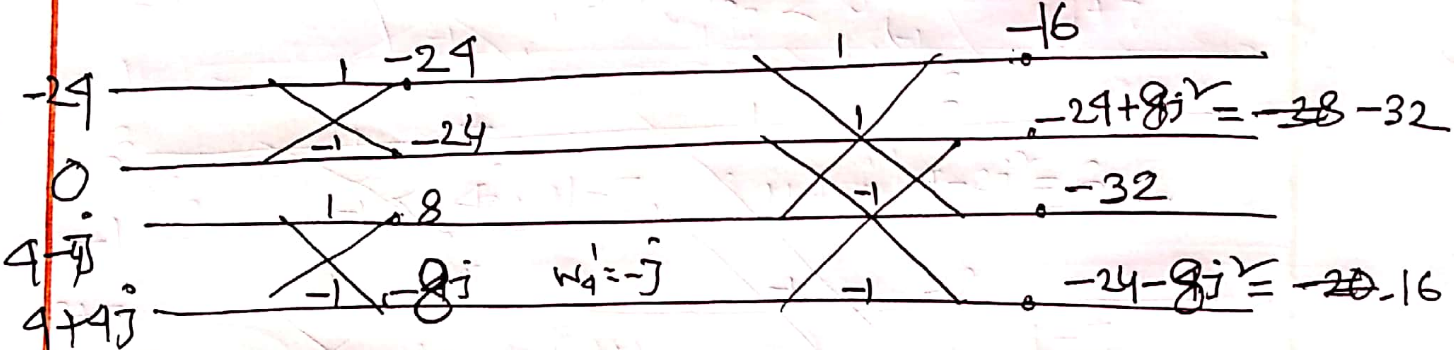


$\therefore H(K) = \{-4, -2+2j, 0, -2-2j\}$

$\therefore Y(K) = X(K) * H(K)$

$= \{-24, +4j+4, 0, -4j+4\}$

$\therefore Y^*(K) = \{-24, 4-4j, 0, 4+4j\}$



$\therefore y(n) = \frac{1}{4}\{-16, \cancel{-28}, -32, -\cancel{20}\,16\}$

$= \cancel{\{-4, -8, -8, -4\}}$

$= \{-4, -8, -8, -4\}$ Ans