Discrete Time Signal and Systems

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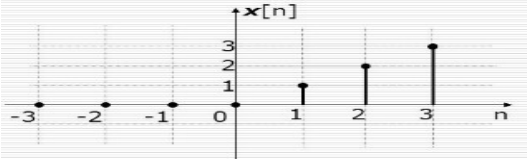
Discrete Time Signal

- Signals that are discrete in time but continuous in amplitude are referred to as **discrete-time signals.**
- A discrete time signal x(n) is a function of an independent variable that is an integer.
- We can represent a discrete time signal in following ways :-

1. Graphical representation:



2. Tabular representation:

n	…	-2	-1	0	1	2	3	4	5	…
$x(n)$	…	0	0	0	1	4	1	0	0	…

3. Functional representation:

$$x(n) = \begin{cases} 1 \quad for & n = 1,3 \\ 6 \quad for & n = 0,7 \\ 0 & elsewhere \end{cases}$$

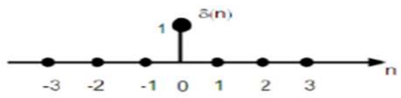
4. Sequential representation:

$x(n)$ ={...0,0,1,4,1,1,0,0,...}
↑

Some Elementary Discrete Time Signal

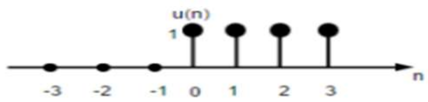
- **Unit Sample Signal:**
  It is denoted as δ(n) in discrete time domain and can be defined as-

$$\delta(n) = \begin{cases} 1, & for \quad n = 0 \\ 0, & Otherwise \end{cases}$$



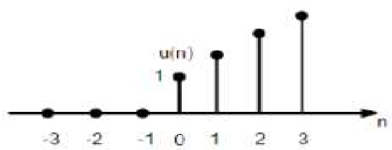
- **Unit Step Signal:** Discrete time unit step signal is defined as-

$$U(n) = \begin{cases} 1, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$



- **Unit Ramp Signal:**
  A discrete unit ramp function can be defined as –

$$r(n) = \begin{cases} n, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$



Classification of Discrete Time Signal

- **Even and Odd Signals**
  - A signal is said to be even if it satisfies the following condition;
    $$x(-t)=x(t)$$
  - A signal is said to be odd, if it satisfies the following condition
    $$x(-t)=-x(t)$$
- **Periodic and Non-Periodic Signals:**
  - Periodic signal repeats itself after certain interval of time. We can show this in equation form as –
    $$x(t)=x(t)\pm nT$$
    Where, n = an integer 1,2,3……
  - Non-periodic signals do not follow a certain format. therefore, no particular mathematical equation can describe them.

Classification of Discrete Time Signal

- **Energy and Power Signals:**

A signal is said to be an Energy signal, if and only if, the total energy contained is finite and nonzero $0<E<\infty$. Therefore, For any energy type signal, The total normalized signal is finite and non-zero.

For any finite signal x(t) the energy can be symbolized as E and is written as:

$$E = \int_{-\infty}^{+\infty} x^2(t)dt$$

A signal is said to be power type signal, if and only if, normalized average power is finite and non-zero i.e. $0<p<\infty$.

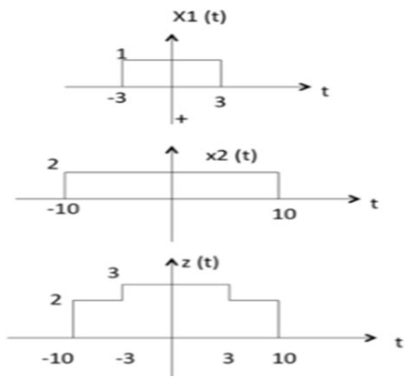
In mathematical form, the power of a signal x(t) can be written as;

$$P = \lim_{T\to\infty} 1/T \int_{-T/2}^{+T/2} x^2(t)dt$$

Simple Manipulations Of Discrete Time Signal

- **Addition:** Addition of two signals is nothing but addition of their corresponding amplitudes. This can be best explained by using the following example:

Addition of signals is illustrated in the diagram below, where $X_1(t)$ and $X_2(t)$ are two time dependent signals, performing the additional operation on them we get, $z(t)=x1(t)+x2(t)$

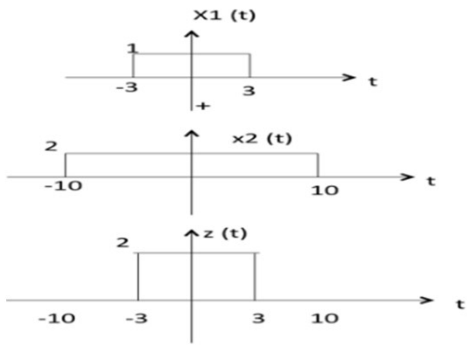


As seen from the diagram,
$-10 < t < -3$ amplitude of $z(t) = x1(t) + x2(t) = 0 + 2 = 2$
$-3 < t < 3$ amplitude of $z(t) = x1(t) + x2(t) = 1 + 2 = 3$
$3 < t < 10$ amplitude of $z(t) = x1(t) + x2(t) = 0 + 2 = 2$

Simple Manipulations Of Discrete Time Signal

- **Multiplication:** Multiplication of two signals is nothing but multiplication of their corresponding amplitudes. For example:

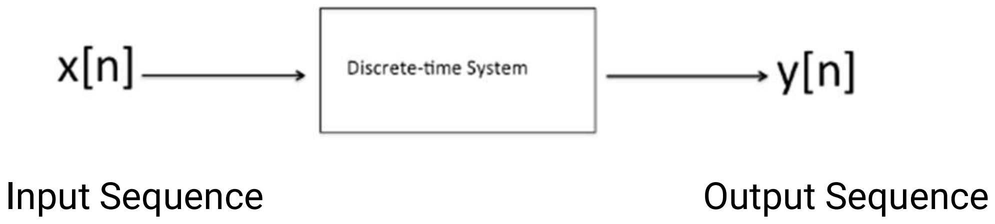
  Multiplication of signals is illustrated in the diagram below, where X1(t) and X2(t) are two time dependent signals, performing the multiplication operation on them we get, z(t)=x1(t).x2(t)



As seen from the diagram,
-10 < t < -3 amplitude of z (t) = x1(t) × x2(t) = 0 ×2 = 0
-3 < t < 3 amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2
3 < t < 10 amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0

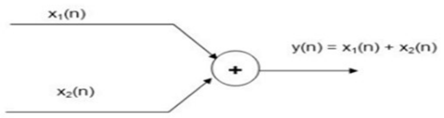
Discrete Time Systems

A **discrete-time system** is a device or algorithm that, according to some well-defined rule, operates on a **discrete-time** signal called the input signal or excitation to produce another **discrete-time** signal called the output signal or response.



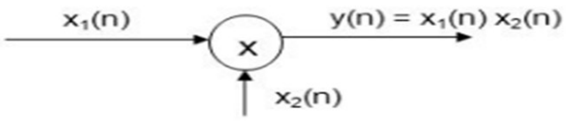
Block Diagram Representation of a Discrete Time System

- **An Adder:**

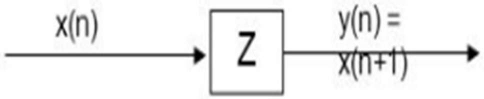


- **A Constant Multiplier:**

- **A Signal Multiplier:**



- **A Unit Advance Element:**



Classification Of Discrete Time System

- **Static and Dynamic Systems:**
  The system is said to be static if its output depends only on the present input. On the other hand, The system is said to be dynamic.

- **Causal and Non-Causal Systems:**
  If the output of the system depends on the past and presents input only, the system is said to be a causal system.On the other hand, if the output of the system depends on future inputs also, The system is known as a non-causal system.

- **Time Invariant and Time Variant Systems:**
  A system is said to be time variant if it is varied with respect to time, Otherwise it is said to be time invariant.

- **Linear and Non-Linear Systems:**
  If a system satisfies the principle of homogeneity and superposition, The system is said to be linear otherwise it is said to be nonlinear.

The Convolution Sum

- Convolution sum is a mathematical way of combining two signals to form a third signal.
- If h[n] is the impulse response of a stable LTI system, Its output y[n] can be computed by means of the convolution sum,

$$y[n] = \sum_{k} x[k]h[n-k]$$

Where x[k] is the input. The Z-transform of y[n] is the product

$$Y(z)=H(z)X(z)$$

Properties of Convolution and The Interconnection of LTI Systems

- **Commutative Property :** This states that the order in which signals are convolved can be exchanged.

$$a[n] * b[n] = b[n] * a[n]$$

- **Associative Property:** The associative property of convolution describes how three or more signals are convolved.

$$(a[n] * b[n]) * c[n] = a[n] * (b[n] * c[n])$$

- **Distributive Property:** This property of convolution describes how parallel systems are analyzed.

$$a[n] * b[n] + a[n] * c[n] = a[n] * (b[n] + c[n])$$

THANK YOU!

Any queries?