Зад 1.

$$\lambda_1 \begin{pmatrix} 1 \\ 3 \\ -1 \\ 2 \end{pmatrix} + \lambda_2 \begin{pmatrix} 2 \\ 1 \\ -1 \\ -3 \end{pmatrix} + \lambda_3 \begin{pmatrix} 1 \\ 2 \\ -1 \\ -2 \end{pmatrix} + \lambda_4 \begin{pmatrix} p \\ 4 \\ -1 \\ 3 \end{pmatrix} = \vec{O}$$

$$\left|\begin{array}{cccc|c} 1 & 2 & 1 & p & 0 \\ 3 & 1 & 2 & 4 & 0 \\ -1 & -1 & -1 & -1 & 0 \\ 2 & -3 & -2 & 3 & 0 \end{array}\right| \begin{array}{l} R_3(-1) \\ R_{1,3} \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 2 & 4 & 0 \\ 1 & 2 & 1 & p & 0 \\ 2 & -3 & -2 & 3 & 0 \end{array}\right) \begin{array}{l} R_{31}(-1) \\ R_{21}(-3) \\ R_2(-1) \\ R_{41}(-2) \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 1 & -1 & 0 \\ 0 & 1 & 0 & p-1 & 0 \\ 0 & -5 & -4 & 1 & 0 \end{array}\right) \begin{array}{l} \\ R_2(\frac{1}{2}) \\ R_{3,2}(-1) \\ R_{42}(5) \end{array} \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & p-\frac{1}{2} & 0 \\ 0 & 0 & -\frac{3}{2} & -\frac{3}{2} & 0 \end{array}\right) \begin{array}{l} R_3(-2) \\ R_4(-\frac{2}{3}) \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 1-2p & 0 \\ 0 & 0 & 1 & 1 & 0 \end{array}\right) R_{43}(-1) \sim \left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & 1-2p & 0 \\ 0 & 0 & 0 & 2p & 0 \end{array}\right)$$

$\lambda_4 . 2p = 0$ но за да бъде линейната комбинация линейно зависима трябва $\lambda_4 \neq 0 \Rightarrow p = 0$
и за λ_4 получаваме което и да е $\lambda_4 \in \mathbb{R} \setminus \{0\}$

$$\left| \begin{array}{l} \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0 \\ \lambda_2 + \frac{\lambda_3}{2} - \frac{\lambda_4}{2} = 0 \\ \lambda_3 + \lambda_4 + 2.0 = 0 \Rightarrow \lambda_3 = -\lambda_4 \\ \lambda_4 = \lambda_4 \end{array} \right.$$

$$\left|\begin{array}{l} \lambda_1 = -\lambda_2 - (-\lambda_4) - \lambda_4 = -\lambda_2 \\ \lambda_2 = \frac{\lambda_4}{2} - \left(\frac{-\lambda_4}{2}\right) = \lambda_4 \\ \lambda_3 = -\lambda_4 \\ \lambda_4 = \lambda_4 \end{array}\right.$$

$$\left|\begin{array}{l} \lambda_1 = -\lambda_2 \\ \lambda_2 = \lambda_4 \\ \lambda_3 = -\lambda_4 \\ \lambda_4 = \lambda_4 \end{array}\right. \Rightarrow \begin{array}{l} \lambda_1 a_1 + \lambda_2 a_2 + \lambda_3 a_3 + \lambda_4 a_4 = \vec{O} \\ -\lambda_4 a_1 + \lambda_4 a_2 - \lambda_4 a_3 + \lambda_4 a_4 = \vec{O} \end{array}$$

Нека за $\lambda_4 = 4$

$$-4\begin{pmatrix} 1 \\ 3 \\ -1 \\ 2 \end{pmatrix} + 4\begin{pmatrix} 2 \\ 1 \\ -1 \\ -3 \end{pmatrix} - 4\begin{pmatrix} 1 \\ 2 \\ -1 \\ -2 \end{pmatrix} + 4\begin{pmatrix} 0 \\ 4 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left|\begin{array}{l} -4 \cdot 1 + 4 \cdot 2 - 4 \cdot 1 + 4 \cdot 0 = -4 + 8 - 4 + 0 = 0 \\ -4 \cdot 3 + 4 \cdot 1 - 4 \cdot 2 + 4 \cdot 4 = -12 + 4 - 8 + 16 = 0 \\ -4 \cdot (-1) + 4 \cdot (-1) - 4 \cdot (-1) + 4 \cdot (-1) = 4 - 4 + 4 - 4 = 0 \\ -4 \cdot 2 + 4 \cdot (-3) - 4 \cdot (-2) + 4 \cdot 3 = -8 - 12 + 8 + 12 = 0 \end{array}\right.$$