3 зад

b_1, b_2, b_3, b_4 са базис на $\mathbb{Q}^4$ тстк не са ЛЗ.

Допускаме противното. b_1, b_2, b_3, b_4 са линейно зависими
и $\exists\, \lambda_1, \lambda_2, \lambda_3, \lambda_4 \neq$ така че:

$$\lambda_1 b_1 + \lambda_2 b_2 + \lambda_3 b_3 + \lambda_4 b_4 = \vec{O}$$

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 2 & -1 & -1 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 \end{array}\right) \begin{array}{l} R_{21}(-2) \\ R_{31}(-1) \\ R_{41}(-1) \\ \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & -5 & -7 & -1 & 0 \\ 0 & -1 & -4 & -1 & 0 \\ 0 & -3 & -3 & 0 & 0 \end{array}\right) \begin{array}{l} R_{23} \\ R_2(-1) \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & -5 & -7 & -1 & 0 \\ 0 & -3 & -3 & 0 & 0 \end{array}\right) \begin{array}{l} \\ R_{32}(5) \\ R_{42}(3) \\ \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 13 & 4 & 0 \\ 0 & 0 & 9 & 3 & 0 \end{array}\right) \begin{array}{l} R\left(\frac{1}{13}\right) \\ R_4\left(\frac{1}{3}\right) \\ \\ \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{13} & 0 \\ 0 & 0 & 3 & 1 & 0 \end{array}\right) \begin{array}{l} \\ R_{43}(-3) \\ R_4(13) \\ \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 0 \\ 0 & 1 & 4 & 1 & 0 \\ 0 & 0 & 1 & \frac{4}{13} & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right) \quad \begin{array}{l} \lambda_4 = 0 \\ \lambda_3 = -\frac{4}{13}\lambda_4 = 0 \\ \lambda_2 = -4\lambda_3 - \lambda_4 = 0 \\ \lambda_1 = -2\lambda_2 - 3\lambda_3 - \lambda_4 = 0 \end{array}$$

Противоречие на допуснатото. $\Rightarrow$
$\Rightarrow b_1, b_2, b_3, b_4$ са ЛНЗ $\Rightarrow b_1, b_2, b_3, b_4$ са базис на $\mathbb{Q}^4$

Коорд на $V(1,2,0,1)$ спрямо базиса.

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 2 & -1 & -1 & 1 & 2 \\ 1 & 1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 1 \end{array}\right) \begin{array}{l} R_{21}(-2) \\ R_{31}(-1) \\ R_{41}(-1) \end{array} \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 0 & -5 & -7 & -1 & 0 \\ 0 & -1 & -4 & -1 & -1 \\ 0 & -3 & -3 & 0 & 0 \end{array}\right) \begin{array}{l} R_{2,3} \\ R_2(-1) \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & -5 & -7 & -1 & 0 \\ 0 & -3 & -3 & 0 & 0 \end{array}\right) \begin{array}{l} R_{32}(5) \\ R_{42}(3) \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 0 & 13 & 4 & 5 \\ 0 & 0 & 9 & 3 & 3 \end{array}\right) \begin{array}{l} R_3\left(\frac{1}{13}\right) \\ R_4\left(\frac{1}{3}\right) \end{array}$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 0 & 1 & \frac{4}{13} & \frac{5}{13} \\ 0 & 0 & 3 & 1 & 1 \end{array}\right) \begin{array}{l} R_{43}(-3) \\ R_4(13) \end{array} \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 1 & 1 \\ 0 & 1 & 4 & 1 & 1 \\ 0 & 0 & 1 & \frac{4}{13} & \frac{5}{13} \\ 0 & 0 & 0 & 1 & -2 \end{array}\right)$$

$$\Rightarrow \left| \begin{array}{l} \lambda_4 = -2 \\ \lambda_3 = \frac{5}{13} + 2 \cdot \frac{4}{13} = \frac{13}{13} = 1 \\ \lambda_2 = 1 - (-2) - 4 = -1 \\ \lambda_1 = 1 - 2.(-1) - 3.1 - 1.(-2) = 1 + 2 - 3 + 2 = 2 \end{array} \right.$$

$$\Rightarrow 2b_1 - b_2 + b_3 - 2b_4 = V$$