3 зад

$$A=\begin{pmatrix}1 & -1 & 2\\ 2 & 2 & -4\\ 3 & 1 & -2\end{pmatrix},\quad B=\begin{pmatrix}-1 & -2 & 1 & 1\\ -2 & 0 & 2 & -2\\ -3 & -2 & 3 & -1\end{pmatrix}$$

$A \cdot X = B$

Проверка за същ. на A^{-1}

$$\begin{vmatrix}1 & -1 & 2\\ 2 & 2 & -4\\ 3 & 1 & -2\end{vmatrix} = -4+12+4-12-4+4=0$$

$\Rightarrow A$ не е обратима

$A_{3\times3}$ $\quad B = 3\times4 \Rightarrow X_{3\times4}$

$$\begin{pmatrix}1 & -1 & 2\\ 2 & 2 & -4\\ 3 & 1 & 2\end{pmatrix}\cdot\begin{pmatrix}x_{11} & x_{12} & x_{13} & x_{14}\\ x_{21} & x_{22} & x_{23} & x_{24}\\ x_{31} & x_{32} & x_{33} & x_{34}\end{pmatrix}=\begin{pmatrix}-1 & -2 & 1 & 1\\ -2 & 0 & 2 & -2\\ -3 & -2 & 3 & -1\end{pmatrix}$$

r_1: $x_{11}-x_{21}+2x_{31}=-1$
r_2: $2x_{11}+2x_{21}-4x_{31}=-2$
r_3: $3x_{11}+x_{21}+2x_{33}=-3$

r_1+r_3: $4x_{11}=-4 \Rightarrow x_{11}=-1$
r_2: $2x_{21}=4x_{31}$
Пол $x_{31}=p$, $p\in\mathbb{Q} \Rightarrow x_{21}=2p$

r_4: $x_{12}-x_{22}+2x_{32}=-2$
r_5: $2x_{12}+2x_{22}-4x_{32}=0$
r_6: $3x_{12}+x_{22}+2x_{32}=-2$

r_4+r_6: $4x_{12}=-4 \Rightarrow x_{12}=-1$
r_5: $2x_{22}=4x_{32}+2$
Пол $x_{32}=q$ $q\in\mathbb{Q} \Rightarrow x_{22}=2q+1$

r_7: $x_{13}-x_{23}+2x_{33}=1$
r_8: $2x_{13}+2x_{23}-4x_{33}=2$
r_9: $3x_{13}+x_{23}+2x_{33}=3$

r_7+r_9: $4x_{13}=4 \quad x_{13}=1$
r_8: $2x_{23}=4x_{33}$
Пол $x_{33}=t$, $t\in\mathbb{Q} \Rightarrow x_{23}=2t$

r_{10}: $x_{14}-x_{24}+2x_{34}=1$
r_{11}: $2x_{14}+2x_{24}-4x_{34}=-2$
r_{12}: $3x_{14}+x_{24}+2x_{34}=-1$

$r_{10}+r_{12}$: $4x_{14}=0 \quad x_{14}=0$
r_{11}: $2x_{24}=4x_{34}-2$
Пол $x_{34}=s$, $s\in\mathbb{Q}$
$\Rightarrow x_{24}=2s-1$

$$X = \begin{pmatrix} -1 & -1 & 1 & 0 \\ 2p & 2q+1 & 2t & 2s-1 \\ p & q & t & s \end{pmatrix}$$