Assignment 1: Labour supply and children

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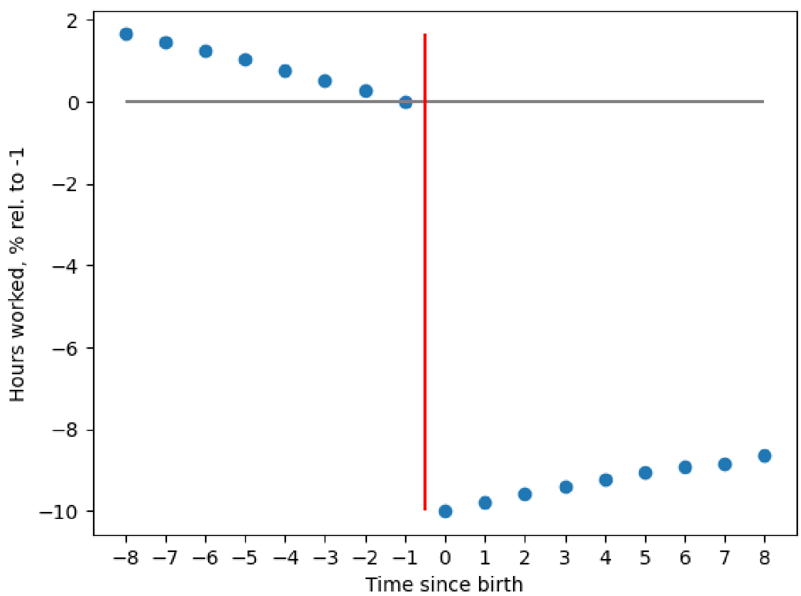
1 Calibration of β_1

I combine the code for structural estimation with the code for calculating the child penalty to set up an automatic estimation of β_1 targeting a relative penalty in hours of 10 pct. I inititalize the model, using the parameter settings from the notebook for lecture five. In lecture five, we visualised the *absolute* labour supply reduction when having a child, but Kleven, Landais, and Søgaard (2019) find a *relative* difference of 10 pct. I take this into account when estimating the true β_1, which I use for the remainder of the assignment. The estimated β_1 is 0.0523, and the child penalty is visualised in figure 1.

Figure 1: Child penalty in hours with $\beta_1 = 0.052992$



2 Marshall elasticity

The standard definition of the Marshall elasticity is the relative labour supply response to a 1 pct. change in after tax wages:

$$e_{M,t} \equiv \frac{\partial \log h_t}{\partial \log(1-\tau)}$$

If we were to be exact, we should thus look at changing the tax rate to 10.9 pct., as this would yield a 1 pct. decrease in $(1-\tau)$. Given the formulation of the question, however, I assume that we are asked to find the relative hours response to a 1pp increase in the tax rate, τ. This approximately gives a 1 pct. reduction in after tax wages (for any given level of wages i.e., $(t-\tau)$. I thus calculate the relative change in hours from changing the tax rate from 10 pct. to 11 pct.

The aggregate Marshall elasticity over the full period is -0.228 pct. This implies that people increase their labour supply in response to an increase in taxes. In other words, the income effect dominates the substitution effect. This response varies somewhat over the lifetime, although it is negative for all ages. In the last three periods there is more variablility in the estimated elasticities, and the relationship between age and Marshall elasticity is markedly non-monotonic. The age contingent elasticities are shown in figure 2

Figure 2: Age contingent Marshall elasticities

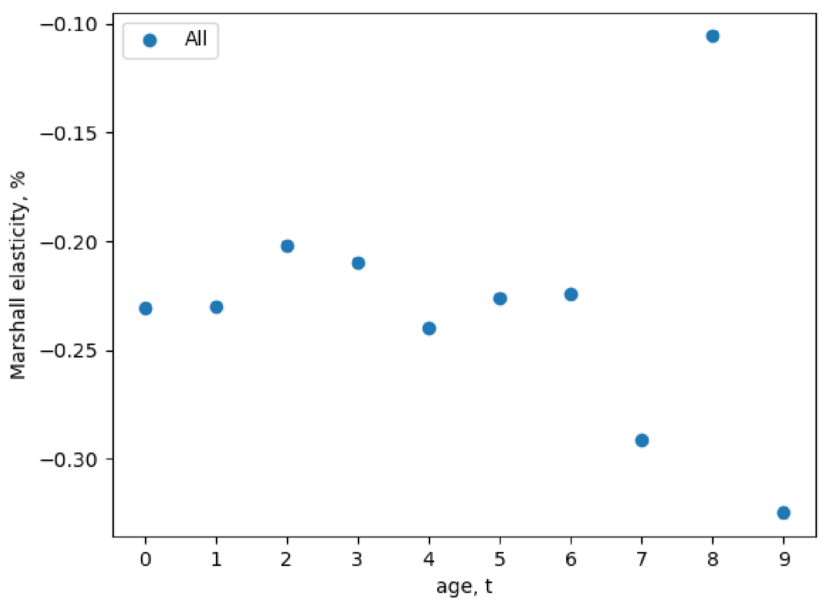
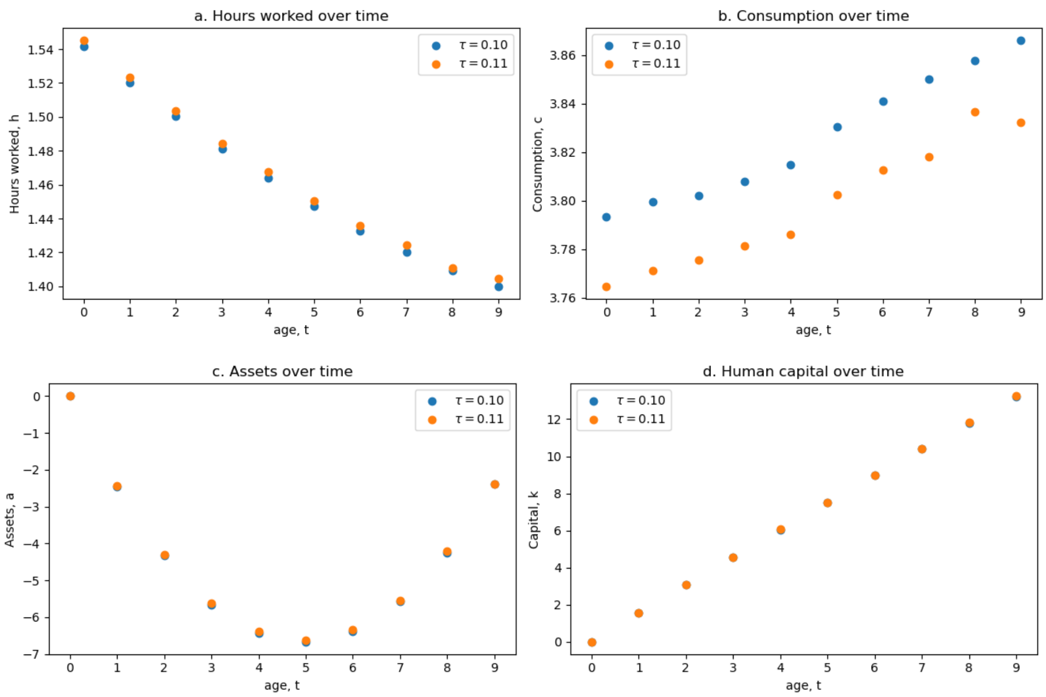


Figure 3: Simulated behaviour



The simulated behaviour might shed light on why this is so. Average consumption is approximately linear for the $\tau = 0.10$ case but seems to kink in the penultimate period for the $\tau = 0.11$ case. This is also reflected in simulated hours, which generates the variablility in estimated elasticities. I have not been able to trace any non-negligible errors in either solution or simulation of these results. The simulated behaviour is shown in figure 3.

3 Introducing a spouse

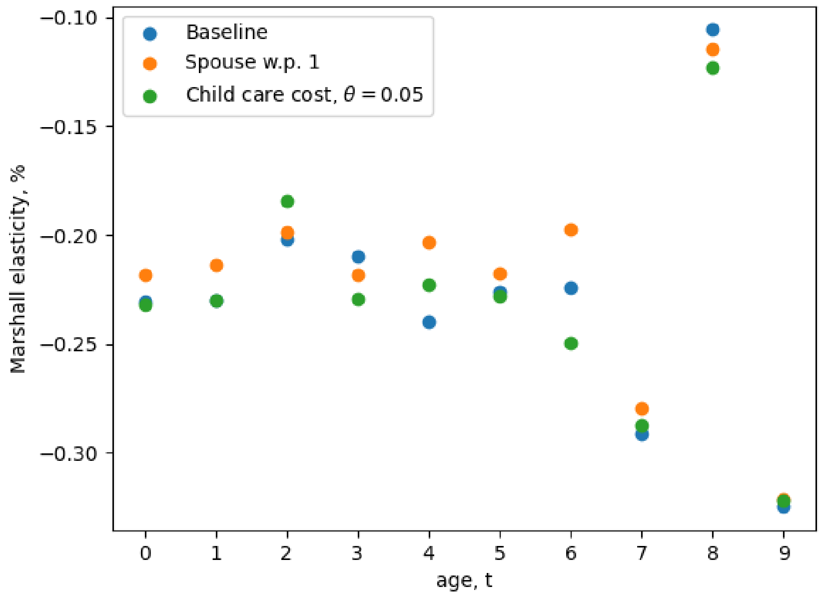
In this section, I include a spouse for all women in the model. This is equivalent to a pure income effect for the households, as the spouse contributes with a certain income, y_t, which increases with age. This means that the income of the household and thus the asset accumulation equation change to

$$income_t = w\,(1 + \alpha k_t)\,h_t + y_t \tag{1}$$

$$a_{t+1} = (1 + r)(a_t + income_t - c_t) \tag{2}$$

$$y_t = 0.1 + 0.01t \tag{3}$$

Figure 4: Age contingent Marshall elasticities, alternative models



This implies a decrease in hours of the woman, and an increase in consumption. The child penalty in hours increases to around -10.1 pct, and the average Marshall elasticity is reduced to -0.218 pct. i.e., by 0.01pp. This small reduction is intuitive, as the household is now less dependent on the woman's earnings, so the responsiveness of her hours to changes in the tax rate is smaller. The age contingent elasticities have variation similar to the baseline case. These are shown in figure 4.

4 Introducing a child care cost

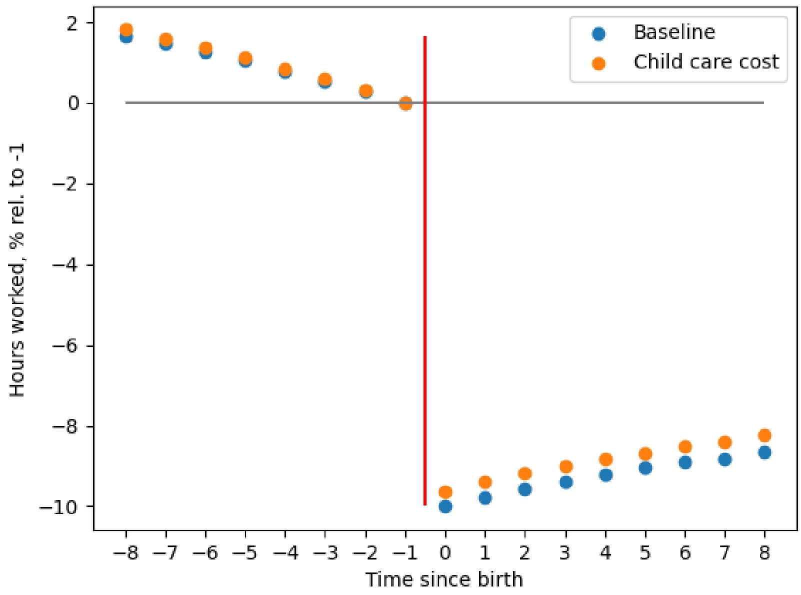
In this section, I include a cost of child care for all mothers in the model. Since fertility is exogenous in the model, this implies a negative income effect which all agents expect, and which is realised for all women when they become mothers. The income equation is the same as in the baseline case, but the asset accumulation equation changes to

$$income_t = w\,(1+\alpha k_t)\,h_t \tag{4}$$

$$a_{t+1} = (1+r)(a_t + income_t - c_t - n_t\theta) \tag{5}$$

The terminal condition is changed trivially. Since all mothers have to pay the extra cost, they will reduce their labour supply less than in the baseline case to balance the extra cost

Figure 5: Child penalty in hours, alternative models



of having a child with the extra utility cost of working more. The immediate child penalty in hours falls to -9.6 pct. as shown in 5. To smooth consumption better, women increase their labour supply slightly in anticipation of having a child.

5 Endogenous fertility

If instead fertility were endogenous, households would take a child care cost into account. The alternative cost of having a child would increase, meaning that women would likely reduce their fertility slightly.

In general, women would likely choose low fertility early in their lives in order to save for parenthood (by accumulating both human capital and assets).

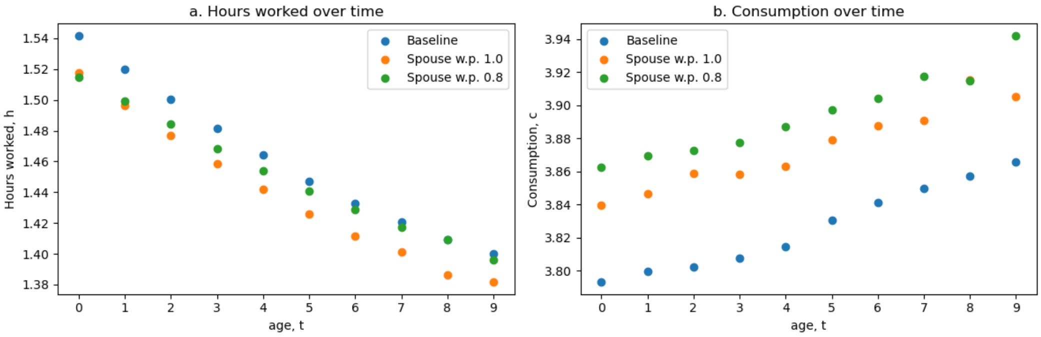
6 Shifty spouse

In this section i expand the model from question 3 to make it uncertain whether a spouse will be present in each period. This changes the income and the Bellman equation:

$$V_t(n_t, a_t, k_t) = \max_{c_t, h_t} \frac{c_t^{1+\eta}}{1+\eta} - \beta(n_t)\frac{h_t^{1+\gamma}}{1+\gamma} + \rho\mathbb{E}_t[V_{t+1}(n_{t+1}, a_{t+1}, k_{t+1}, y_{t+1})] \tag{6}$$

$$income_t = w\,(1+\alpha k_t)\,h_t + y_t \tag{7}$$

Figure 6: Simulated behaviour, models with a spouse



Note: I have no idea where the non-linearity in period $t = 8$ comes from, the behaviour in that period looks super wrong, and I suspect it is a mistake, which also generates the strange pattern of the age contingent Marshall elasticities.

Here, y_t is zero, if there is no spouse and follows the age contingent path in the presence of a spouse:

$$y_t = \begin{cases} 0 & \text{with probability } 0.2 \\ 0.1 + 0.01t & \text{with probability } 0.8 \end{cases} \tag{8}$$

Furthermore, as it requires a spouse to get a child, this change negatively affects the probability of becoming a mother. There is in other words a compositional effect by which fewer simulated women will be mothers. For women without a child, the probability of having one is now affected by the probability of having a child

$$n_{t+1} = \begin{cases} n_t + 1 & \text{with probability } p_{birth} * p_{spouse} = 0.1 * 0.8 = 0.08 \\ n_t & \text{with probability } 1 - p_{birth} * p_{spouse} = 0.92 \end{cases} \tag{9}$$

In each period, 80 pct. of households have a higher income than in the baseline case. This income effect shifts up consumption and pushes down hours worked, as shown in figure 6. The income effect is lower than in the model with a certain spouse, but since women are less likely to have a child, they will have fewer precautionary savings (they will borrow more early in life). This is why aggregate consumption is higher despite a lower income effect.

In addition, fewer women will face a child penalty each period, which means that average hours fall less each period than when a spouse is certain. In the first period, hours are slightly lower than in the certain spouse case, but in the terminal period, hours are almost as high as in the baseline case.

Table 1: Main estimates for each model

	e_M	Hours Penalty
Baseline	-0.2282	-10.00%
Spouse	-0.2181	-10.08%
Child care cost	-0.2306	-9.63%
Spouse w.p. 0.8	-0.2174	-9.71%

References

Kleven, Henrik, Camille Landais, and Jakob Egholt Søgaard (2019). "Children and Gender Inequality: Evidence from Denmark". In: *American economic journal. Applied economics* 11.4, pp. 181–209. ISSN: 1945-7782. DOI: 10.1257/app.20180010.