

# Calcite growth kinetics: Modeling the effect of solution stoichiometry

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## Abstract

Until recently the influence of solution stoichiometry on calcite crystal growth kinetics has attracted little attention, despite the fact that in most aqueous environments calcite precipitates from non-stoichiometric solution. In order to account for the dependence of the calcite crystal growth rate on the cation to anion ratio in solution, we extend the growth model for binary symmetrical electrolyte crystals of Zhang and Nancollas (1998) by combining it with the surface complexation model for the chemical structure of the calcite–aqueous solution interface of Wolthers et al. (2008). To maintain crystal stoichiometry, the rate of attachment of calcium ions to step edges is assumed to equal the rate of attachment of carbonate plus bicarbonate ions. The model parameters are optimized by fitting the model to the step velocities obtained previously by atomic force microscopy (AFM, Teng et al., 2000; Stack and Grantham, 2010). A variable surface roughness factor is introduced in order to reconcile the new process-based growth model with bulk precipitation rates measured in seeded calcite growth experiments. For practical applications, we further present empirical parabolic rate equations fitted to bulk growth rates of calcite in common background electrolytes and in artificial seawater-type solutions. Both the process-based and empirical growth rate equations agree with measured calcite growth rates over broad ranges of ionic strength, pH, solution stoichiometry and degree of supersaturation. © 2011 Elsevier Ltd. All rights reserved.

## 1. INTRODUCTION

The effects of physico-chemical parameters, including temperature, pressure, pH, ionic strength, degree of supersaturation and the presence of inhibitors, on the precipitation of calcite have been studied extensively (see for example the review by Morse et al. (2007)). However, until recently the dependence of calcite growth rates on the solution stoichiometry has received limited attention

(Tai et al., 2005; Nehrke et al., 2007; Perdikouri et al., 2009; Larsen et al., 2010; Stack and Grantham, 2010). This is remarkable given that (a) the compositions of most natural waters are highly non-stoichiometric with respect to calcium carbonate, and (b) a large body of work has shown that growth and dissolution rates of ionic minerals are markedly affected by the anion to cation concentration ratio in solution (Davies and Jones, 1955; Davies and Nancollas, 1955; Christoffersen and Christoffersen, 1979; Stubičar et al., 1990; Verbeeck and Devenyns, 1990, 1992; Zhang and Nancollas, 1992; Burke and Nancollas, 1999; Chernov et al., 2006; Rashkovich et al., 2006; Kowacz et al., 2007).

In a recent study, Nehrke et al. (2007) conducted single calcite crystal growth experiments at fixed pH (10.2) and two degrees of supersaturation while varying the  $\text{Ca}^{2+}$  to

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$\text{CO}_3^{2-}$  solution activity ratio over several orders of magnitude, from values smaller to larger than one. Their results unambiguously show that the growth rate is highest when the solution activity ratio,  $r_{\text{aq}} = \{\text{Ca}^{2+}\}/\{\text{CO}_3^{2-}\}$ , is close to one, and decreases with increasing or decreasing values of  $r_{\text{aq}}$  (Fig. 1a). The observed dependence of the growth kinetics of calcite on  $r_{\text{aq}}$  was interpreted using the kink growth theory for non-Kossel crystals. However, to explain the near-symmetric dependence of the growth rate on  $r_{\text{aq}}$  observed in Fig. 1a, the attachment frequencies of calcium and carbonate must be assumed to be of comparable magnitude. A similar behavior of the growth kinetics with respect to the aqueous cation to anion ratio was observed for calcium and magnesium oxalate (Chernov et al., 2006). However, as remarked by Nehrke et al. (2007), the assumption of comparable attachment frequencies of cations and anions is counter-intuitive, given the much slower water exchange kinetics of cations compared to anions (Nielsen, 1984). More recently, Larsen et al. (2010) and Stack and Grantham (2010) investigated the effect of  $r_{\text{aq}}$  on the step velocity of a calcite growth hillock using atomic force microscopy (AFM). Both groups observed a strong dependency of the velocities on solution stoichiometry. In addition, they found that the step velocities of the obtuse and acute sides of growth spirals were differently affected by  $r_{\text{aq}}$ .

There is general agreement that above pH 4 the growth and dissolution of calcite are controlled by surface reactions (Plummer et al., 1978; Reddy et al., 1981). Several authors have linked calcite dissolution rates (Van Cappellen et al., 1993; Arakaki and Mucci, 1995; Pokrovsky and Schott, 2002; Pokrovsky et al., 2005) and growth rates (Nilsson and Sternbeck, 1999) to the chemical structure of the mineral–solution interface predicted by surface complexation models. In particular, Nilsson and Sternbeck (1999) applied a surface complexation approach to model the crystal growth kinetics of calcite.

The calcite growth rates measured by Nilsson and Sternbeck were obtained for solution stoichiometries in the range

$0.0003 < 1/r_{\text{aq}} < 0.5$ . While the model presented by Nilsson and Sternbeck is able to reproduce their own experimental growth rates, it fails to do so for the measured growth rates of Nehrke et al. (2007) (Fig. 2). In fact, the model of Nilsson and Sternbeck does not produce a rate maximum in the vicinity of  $r_{\text{aq}} = 1$ , as observed by Nehrke and coworkers (Fig. 1a). This calls into question the general applicability of the rate equation proposed by Nilsson and Sternbeck, in particular when the solution ratio  $r_{\text{aq}}$  is smaller than 1.

In this paper, we derive a new process-based growth model for calcite, by combining the surface structural model for divalent metal carbonates of Wolthers et al. (2008) (Table 1) with the growth model for binary electrolyte crystals of Zhang and Nancollas (1998). The latter authors showed that the growth rate of a binary crystal from aqueous solution cannot be defined solely in terms of the ion activity product, but is also a function of the solution activity ratio. In particular, their model (hereafter referred to as the Z&N model) yields a growth rate maximum at a value of the solution activity ratio that directly depends on the relative attachment frequencies of the lattice ions to growth sites at the crystal surface. The Z&N model is extended here by explicitly accounting for the different molecular configurations of surface sites at the carbonate mineral–aqueous solution interface.

## 2. PROCESS-BASED CALCITE GROWTH MODEL

### 2.1. Conceptual approach

As starting point, we consider the growth model developed by Kossel (1930) for crystals with a simple cubic lattice growing through the incorporation of a single type of (molecular) growth units at distinct surface sites, for example kink sites and step edges. The degree of supersaturation for a so-called Kossel crystal is given by the ratio of the activity of the growth unit in solution and the corresponding solubility equilibrium value. The growth of an

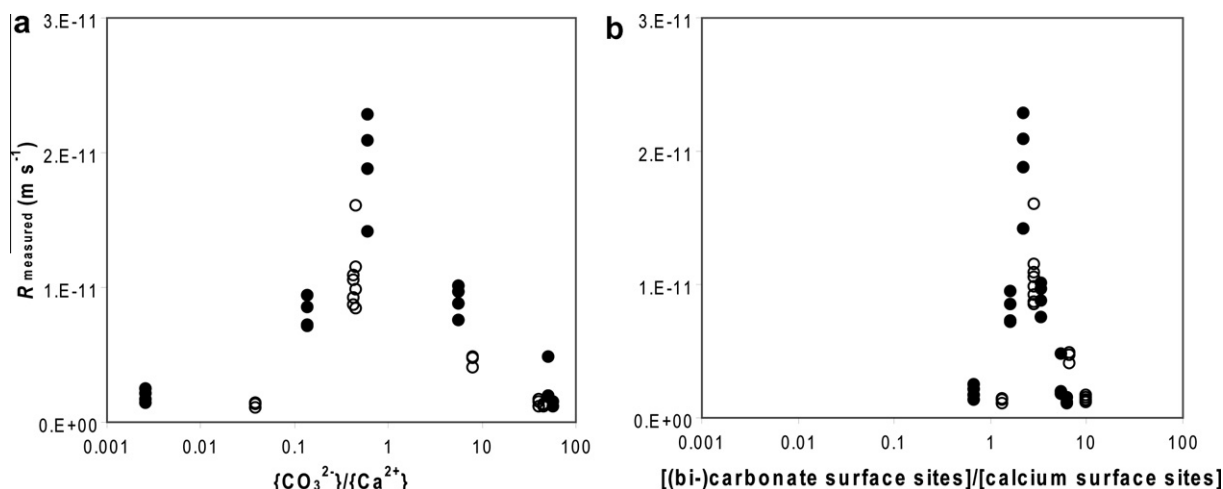


Fig. 1. Measured growth rates of single calcite crystals at  $\Omega = 5.5 \pm 0.5$  (○) and  $\Omega = 16 \pm 2$  (●) at pH  $10.15 \pm 0.05$ ,  $20 \pm 0.2$  °C and in the absence of a gas phase (Nehrke et al., 2007), plotted versus (a) the solution ion activity ratio and (b) the ratio of the surface density of carbonate plus bicarbonate sites over the surface density of calcium, sites calculated using the CD-MUSIC model (Table 1).

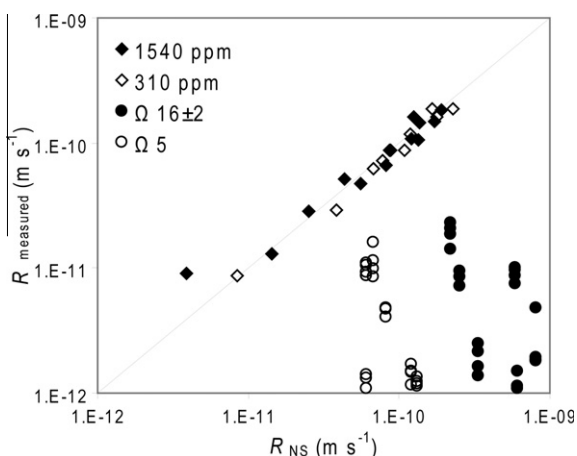


Fig. 2. Measured growth rates at 1540 ppm CO<sub>2</sub>,  $\Omega = 1.5$ –8.3 (■) and 310 ppm CO<sub>2</sub>,  $\Omega = 1.7$ –9.8 (□) (Nilsson and Sternbeck, 1999), and at  $\Omega = 5$  (●) and  $\Omega = 16 \pm 2$  (○) at pH  $10.15 \pm 0.05$ ,  $20 \pm 0.2$  °C and in the absence of a gas phase (Nehrke et al., 2007), plotted versus the model rates  $R_{NS}$  calculated using the growth model of Nilsson and Sternbeck (1999). For legibility, a solid line is plotted to indicate the  $x = y$  relationship between the measured and modeled rates.

ionic crystal, such as NaCl, requires the alternating incorporation of cations and anions into the surface lattice. In this case, the growth rate is not only dependent on the saturation ratio  $\Omega$  (with  $\Omega = IAP/K_S$ , where  $K_S$  is the solubility product and  $IAP$  is the activity product of

the constituent ions), but also on the relative abundances of cations and anions in solution. Hence, several authors have extended the original Kossel model to describe the growth of multicomponent, ionic crystals (Nielsen, 1981; Zhang and Nancollas, 1998; Chernov et al., 2006).

In their growth model for ionic AB crystals, Zhang and Nancollas (1998) assume (among others) identical detachment frequencies for the cation and anion, a relatively low kink density so that nearby kink site interactions can be neglected, and negligible detachment from non-kink sites in supersaturated solutions. Based on these assumptions, they derive analytical solutions for the kink density, the kink nucleation rate, the rate of step movement, and the resulting growth rate of ionic AB crystals. In the development of the process-based model for calcite growth, we closely follow the approach of Zhang and Nancollas (1998).

The kinetic growth model for binary crystals is coupled to a recently developed surface model for divalent metal carbonate minerals (Wolthers et al., 2008; Table 1), based on the Charge Distribution MultiSite Ion Complexation (CD-MUSIC) modeling approach of Hiemstra et al. (1996) and Hiemstra and Van Riemsdijk (1996). This model allows for variable proportions of face, edge and corner (or kink) sites exposed at the calcite surface. In addition, each of the structural sites can exhibit different chemical configurations that are described through the set of surface complexation reactions listed in Table 1. (Note that the set of reactions in Table 1 can be expanded when adsorbates other than the lattice ions are present in the aqueous medium.) In the CD-MUSIC model, charge across the electrical double layer (EDL) is distributed over three planes. This

Table 1

CD-MUSIC model tableau describing calcite surface chemistry (Wolthers et al., 2008). Subscript indexes c, e and f indicate corner, edge and face sites.  $\Delta z_0$  and  $\Delta z_1$  are Boltzmann parameters describing the charge distribution between the different planes across the mineral–solution interface (Hiemstra and Van Riemsdijk, 1996). Equilibrium constants obtained using a total of <sup>a</sup>2.2 and <sup>b</sup>2.4 accepting and donating hydrogen bridges (Wolthers et al., 2008); <sup>c</sup>from Van Cappellen et al. (1993).

	H <sup>+</sup>	OH <sup>−</sup>	Ca <sup>2+</sup>	CO <sub>3</sub> <sup>2−</sup>	$\equiv_{c,e,f}CO_3H$	$\equiv_{c,e,f}CaOH$	log <i>K</i>	$\Delta z_0$	$\Delta z_1$
<b>Corner sites</b>									
$\equiv_c CO_3^-$	−1				1		−3.58 <sup>a</sup>	−1	0
$\equiv_c CO_3 H^+$	1				1		−8.30 <sup>a</sup>	+1	0
$\equiv_c CO_3 Ca^+$	−1		1		1		−2.8 <sup>c</sup>	−1	2 <sup>c</sup>
$\equiv_c CaOH_2^+$	1					1	12.85 <sup>b</sup>	+1	0
$\equiv_c CaO^-$	−1					1	−24.73 <sup>b</sup>	−1	0
$\equiv_c CaHCO_3^0$	1	−1		1		1	10.15 <sup>c</sup>	0.6	−0.6
$\equiv_c CaHCO_3^-$		−1		1		1	1.55 <sup>c</sup>	0.6	−1.6
<b>Edge sites</b>									
$\equiv_e CO_3^{-2/3}$	−1				1		−3.58 <sup>a</sup>	−1	0
$\equiv_e CO_3 Ca^{+4/3}$	−1		1		1		−2.8 <sup>c</sup>	−1 <sup>c</sup>	2 <sup>c</sup>
$\equiv_e CaOH_2^{+2/3}$	1					1	12.85 <sup>b</sup>	+1	0
$\equiv_e CaO^{-4/3}$	−1					1	−24.73 <sup>b</sup>	−1	0
$\equiv_e CaHCO_3^{-1/3}$	1	−1		1		1	10.15 <sup>c</sup>	0.6	−0.6
$\equiv_e CaCO_3^{-4/3}$		−1		1		1	1.55 <sup>c</sup>	0.6	−1.6
<b>Face sites</b>									
$\equiv_f CO_3^{-1/3}$	−1				1		−3.58 <sup>a</sup>	−1	0
$\equiv_f CO_3 Ca^{+5/3}$	−1		1		1		−2.8 <sup>c</sup>	−1	2
$\equiv_f CaOH_2^{+1/3}$	1					1	12.85 <sup>b</sup>	+1	0
$\equiv_f CaO^{-5/3}$	−1					1	−24.73 <sup>b</sup>	−1	0
$\equiv_f CaHCO_3^{-2/3}$	1	−1		1		1	10.15 <sup>c</sup>	0.6	−0.6
$\equiv_f CaCO_3^{-5/3}$		−1		1		1	1.55 <sup>c</sup>	0.6	−1.6

Table 2

Proposed growth and dissolution reactions for the calcite surface, consistent with the thermodynamic calcite surface model of Table 1.  $k_{A1}$ ,  $k_{A2}$ ,  $k_{B1}$ ,  $k_{B2}$  are attachment frequencies;  $v_{A1}$ ,  $v_{A2}$ ,  $v_{B1}$ ,  $v_{B2}$  are detachment frequencies.

[3.1]	$\equiv_c \text{CaCO}_3^- + \text{Ca}^{2+}$	$\xrightleftharpoons[v_{A1}]{k_{A1}}$	$\equiv_c \text{CO}_3\text{Ca}^+$
[3.2]	$\equiv_c \text{CaHCO}_3^0 + \text{Ca}^{2+}$	$\xrightleftharpoons[v_{A2}]{k_{A2}}$	$\equiv_c \text{CO}_3\text{Ca}^+ + \text{H}^+$
[3.3]	$\equiv_c \text{CO}_3\text{Ca}^+ + \text{CO}_3^{2-}$	$\xrightleftharpoons[v_{B1}]{k_{B1}}$	$\equiv_c \text{CaCO}_3^-$
[3.4]	$\equiv_c \text{CO}_3\text{Ca}^+ + \text{HCO}_3^-$	$\xrightleftharpoons[v_{B2}]{k_{B2}}$	$\equiv_c \text{CaHCO}_3^0$

approach distinguishes between outer sphere complexes in the outermost plane and strongly bound surface ions.

The CD-MUSIC model for divalent metal carbonates has been calibrated against surface charge and potential measurements by adjustment of the number of hydrogen bridges between surface oxygen atoms and the number of water molecules in the interface (Wolthers et al., 2008). In the case of calcite, *a posteriori* testing against a large data set of different  $\zeta$ -potential measurements and a sensitivity analysis indicated that (1) the CD-MUSIC model requires the accurate knowledge of the surface-site bond distances and number of hydrogen bridges, and (2) the validation of the model must be carried out with experimental data for which equilibrium between (clean) calcite surfaces, solution and gas phase is clearly demonstrated. A detailed discussion of the application of the CD-MUSIC model to calcite and its sensitivity to different parameters and assumptions is given in Wolthers et al. (2008).

Within the framework of the CD-MUSIC model description of the calcite–water interface, growth corresponds to the transfer of calcium and (bi-)carbonate ions from the outer to the inner region of the EDL, where they bind to (bi-)carbonate or calcium surface groups, respectively. (In what follows (bi-)carbonate refers to “carbonate plus bicarbonate”.) In addition, the adsorbed ions preferentially incorporate into the lattice when binding to surface groups located in kink sites. Hence, calcite crystal growth can be represented by the four reactions given in Table 2. The reactions are assumed to be reversible and therefore account for both growth and dissolution of the mineral phase. It is important to note that the growth and dissolution reactions in Table 2 are internally consistent with the CD-MUSIC model, and that they are formulated so that during growth or dissolution the surface sites are continuously regenerated. Furthermore, the CD-MUSIC model is used to calculate the fraction,  $\chi$ , of sites that represents growth sites at the calcite surface (Section 2.3).

In the derivation of the equations for calcite growth, we adopt most assumptions made by Zhang and Nancollas (1998). The assumption that the overall detachment frequencies for anions and cations are the same, however, does not hold for calcite as the net detachment frequency of (bi-)carbonate is related to the solubility product, as will be shown in the next section. We do assume that the

theoretical approach for a cubic lattice is directly applicable to the rhombohedral calcite lattice. This assumption may be relaxed in future refinements of the model in order to account for AFM observations that show asymmetrical behavior of crystallographically distinct growth terraces (e.g., Teng et al., 2000; Stack and Grantham, 2010). A further implicit assumption is that the surface complexation reactions are rapid, relative to growth and dissolution. In other words, the chemical speciation of the calcite surface is assumed to remain at equilibrium.

Surface speciation calculations were carried out with the Visual Minteq software, modified to account for the existence of multiple surface sites (editions 2.60  $\beta$  and 3.0; Gustafsson, 2008). Process-based growth model parameters and empirical rate equations were optimized using the MS Excel solver tool (Newton’s method).

## 2.2. Equilibrium structure of the calcite step edge

The equilibrium and growth morphology of calcite is dominated by the (10 $\bar{1}$ 4) cleavage plane with obtuse and acute step edges (e.g., Stipp and Hochella, 1991; Paquette and Reeder, 1995; de Leeuw and Parker, 1998). Crystal growth occurs preferentially through the incorporation of the constituent ions, calcium and (bi-)carbonate, into the surface lattice at kink (or corner) sites along step edges (Nielsen and Toft, 1984). The following derivation of the process-based growth model therefore focuses on the fluxes of growth units in and out of kink sites.

For calcite, equilibrium is defined by the solubility product,  $K_S$  (Plummer and Busenberg, 1982):

$$K_S = \{\text{Ca}^{2+}\}_e \{\text{CO}_3^{2-}\}_e = \{\text{A}\}_e \{\text{B}_1\}_e = 10^{-8.48} (25^\circ\text{C}, 1 \text{ bar}) \quad (1)$$

where  $\{\text{Ca}^{2+}\}_e$  and  $\{\text{A}\}_e$  represent the activity of aqueous calcium, and  $\{\text{CO}_3^{2-}\}_e$  and  $\{\text{B}_1\}_e$  represent the activity of aqueous carbonate at equilibrium (index e). Note that we use activities when deriving the growth rate equation in order to be consistent with the model derivations of Zhang and Nancollas (1998) and to account for non-ideality effects experienced by aqueous species. There is no general agreement about whether to express rate laws in terms of activities or concentrations (Lasaga, 1998). Nielsen (1981), for instance, uses concentrations rather than activities in his non-Kossel growth equations. The solubility product  $K_S$  can be rewritten to one expressed in terms of the activities of calcium and bicarbonate (e.g., Hofmann et al., 2010):

$$K_{\text{SH}} = \{\text{Ca}^{2+}\}_e \{\text{HCO}_3^-\}_e \{\text{H}^+\}^{-1} = \{\text{A}\}_e \{\text{B}_2\}_e 10^{-\text{pH}} \\ = 10^{-1.85} (25^\circ\text{C}, 1 \text{ bar}) \quad (2)$$

where  $\{\text{HCO}_3^-\}_e$  and  $\{\text{B}_2\}_e$  represent the equilibrium activity of aqueous bicarbonate at equilibrium.

At equilibrium, the flux of calcium or (bi-)carbonate ions entering a given type of kink sites must be balanced by a corresponding outward flux. Considering the reactions in Table 2, we thus have:

$$k_{A_1}\{A\}_e\rho_{B_1} = v_{A_1}\rho_A, \quad (3)$$

$$k_{A_2}\{A\}_e\rho_{B_2} = k_{A_2}K_{\equiv\text{CaHCO}_3^0}10^{-\text{pH}}\{A\}_e\rho_{B_1} = v_{A_2}\rho_A, \quad (4)$$

$$k_{B_1}\{B_1\}_e\rho_A = v_{B_1}\rho_{B_1}, \quad (5)$$

$$k_{B_2}\{B_2\}_e\rho_A = k_{B_2}K_{\text{HCO}_3^-}10^{-\text{pH}}\{B_1\}_e\rho_A = v_{B_2}K_{\equiv\text{CaHCO}_3^0}10^{-\text{pH}}\rho_{B_1}, \quad (6)$$

where  $\rho_A$ ,  $\rho_{B_1}$ , and  $\rho_{B_2}$  are the A-, B<sub>1</sub>- and B<sub>2</sub>-kink site densities,  $k_{A_1}$ ,  $k_{A_2}$ ,  $k_{B_1}$ , and  $k_{B_2}$  are the attachment frequencies of A, B<sub>1</sub> and B<sub>2</sub> ions, and  $v_{A_1}$ ,  $v_{A_2}$ ,  $v_{B_1}$ , and  $v_{B_2}$  are their detachment frequencies for the reactions defined in Table 2. The attachment of a calcium ion to a carbonate surface site is indicated by index A<sub>1</sub>, that of calcium to a bicarbonate surface site by index A<sub>2</sub>. Attachment of carbonate is indicated by index B<sub>1</sub> and attachment of bicarbonate is indicated by index B<sub>2</sub> (see also Fig. 3). The values of the equilibrium constants at 25 °C and 1 bar, are  $K_{\text{HCO}_3^-} = 10^{10.33}$  (Plummer and Busenitz, 1982) and  $K_{\equiv\text{CaHCO}_3^0} = 10^{8.6}$  (Table 1, Wolthers et al., 2008).

Combining Eqs. (3)–(6), leads to the following overall flux balances:

$$\bar{k}_A\{A\}_e\rho_{B_1} = \bar{v}_A\rho_A, \quad (7)$$

$$\bar{k}_B\{B_1\}_e\rho_A = \bar{v}_B\rho_{B_1}, \quad (8)$$

with

$$\bar{k}_A = k_{A_1} + K_{\equiv\text{CaHCO}_3^0}10^{-\text{pH}}k_{A_2}, \quad (9)$$

$$\bar{k}_B = k_{B_1} + K_{\text{HCO}_3^-}10^{-\text{pH}}k_{B_2}, \quad (10)$$

$$\bar{v}_A = v_{A_1} + v_{A_2}, \quad (11)$$

$$\bar{v}_B = v_{B_1} + K_{\equiv\text{CaHCO}_3^0}10^{-\text{pH}}v_{B_2}. \quad (12)$$

Eqs. (7) and (8) have exactly the same form as the flux balances derived by Zhang and Nancollas (1998, their equations [10] and [11]). Here, however, the pH dependency of the effective attachment and detachment frequencies ( $\bar{k}_i$ ,  $\bar{v}_j$ ) is explicitly accounted for via Eqs. (9)–(12).

Combining Eqs. (1), (7), and (8) leads to

$$K_S = \frac{\bar{v}_A\bar{v}_B}{\bar{k}_A\bar{k}_B}, \quad (13)$$

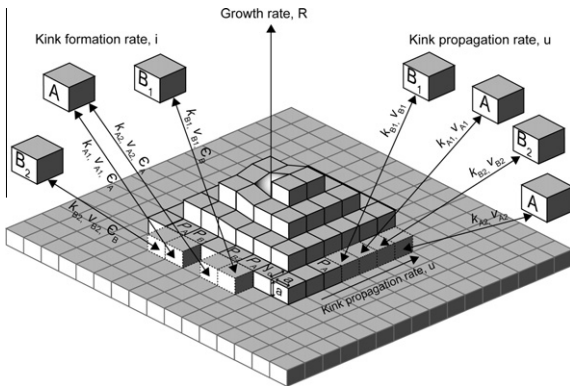


Fig. 3. Sketch to illustrate the concepts behind the process-based growth model.  $P_A$ ,  $P_{B_1}$ , and  $P_{B_2}$  are the calcium, carbonate and bicarbonate growth sites;  $k_{A_1}$ ,  $k_{A_2}$ ,  $k_{B_1}$ ,  $k_{B_2}$  are attachment frequencies;  $v_{A_1}$ ,  $v_{A_2}$ ,  $v_{B_1}$ ,  $v_{B_2}$  are detachment frequencies;  $\varepsilon_A$  and  $\varepsilon_B$  are the energies to form an A or B kink site;  $a$  is the closest spacing between  $\text{Ca}^{2+}$  and  $\text{CO}_3^{2-}$  (3.199 Å);  $v_0$  is the step spacing.

Thus, once three of the effective frequencies are known, the fourth one is fixed through Eq. (13). From Eqs. (7) and (8), the ratio of kink densities  $\rho_A$  and  $\rho_{B_1}$  at equilibrium can be derived as follows:

$$\rho_A/\rho_{B_1} = r_i^{1/2}, \quad (14)$$

where  $r_i$  represents the kinetic ionic ratio:

$$r_i = \frac{\bar{k}_A\bar{v}_B\{A\}}{\bar{k}_B\bar{v}\{B_1\}} \quad (15)$$

with, in the case of equilibrium,  $\{A\} = \{A\}_e$  and  $\{B_1\} = \{B_1\}_e$ .

### 2.3. Growth of the calcite surface

In this section, the process-based growth model for calcite is derived. A sketch illustrating the growth model is given in Fig. 3. A summary of the process-based model equations (subscript c) is given and compared to the corresponding Z&N model equations (subscript ZN) in Table 3; all symbols are defined in Appendix A. All equilibrium constant values are those for 25 °C and ambient pressure.

A step at the calcite surface advances or retreats through the addition or loss of calcium and (bi-)carbonate ions at growth sites. If  $P_A$ ,  $P_{B_1}$ , and  $P_{B_2}$  are the probabilities of a kink site to be a calcium site  $\equiv\text{CO}_3\text{Ca}^+$ , a carbonate site  $\equiv\text{CaCO}_3^-$  or a bicarbonate site  $\equiv\text{CaHCO}_3^0$ , then the net rates at which suitable ions are incorporated are

$$u_{A_1} = k_{A_1}\{A\}P_{B_1} - v_{A_1}P_A, \quad (16)$$

$$u_{A_2} = k_{A_2}\{A\}P_{B_2} - v_{A_2}P_A, \quad (17)$$

$$u_{B_1} = k_{B_1}\{B_1\}P_A - v_{B_1}P_{B_1}, \quad (18)$$

$$u_{B_2} = k_{B_2}\{B_2\}P_A - v_{B_2}P_{B_2}. \quad (19)$$

Combining Eqs. (16)–(19) with (9)–(12), and realizing that  $P_{B_2} = 10^{8.6} \cdot 10^{-\text{pH}} \cdot P_{B_1}$  (Table 1), gives

$$u_A = \bar{k}_A\{A\}P_{B_1} - \bar{v}_AP_A, \quad (20)$$

$$u_B = \bar{k}_B\{B_1\}P_A - \bar{v}_BP_{B_1}. \quad (21)$$

In order to maintain crystal stoichiometry, the net rates at which calcium and (bi-)carbonate ions are incorporated at kink sites must be equal. This means that the net rate of kink propagation ( $u_c$ ) can be expressed as

$$u_A = u_B = \frac{1}{2}u_c. \quad (22)$$

If we define the fraction of potential growth sites as  $\chi = (P_A + P_{B_1} + P_{B_2})$ , from which it follows that  $P_A = \chi - (1 + 10^{-\text{pH}}K_{\equiv\text{CaHCO}_3^0}P_{B_1} = \chi - \theta P_{B_1}$ , then the propagation rate of a single kink can be derived by combining Eqs. (20)–(22) with (15):

$$u_c = 2\chi\bar{v}_A\bar{v}_B(S^2 - 1) \frac{1}{\bar{v}_A(Sr_i^{1/2} + \theta) + \bar{v}_B(\theta Sr_i^{-1/2} + 1)} \quad (\text{in s}^{-1}) \quad (23)$$

where  $S$  is the saturation ratio for calcite:

$$S = \left[ \frac{\{A\}\{B_1\}}{K_S} \right]^{1/2} = \left[ \frac{\bar{k}_A\bar{k}_B\{A\}\{B_1\}}{\bar{v}_A\bar{v}_B} \right]^{1/2}, \quad (24)$$



Table 3

Summary of the key equations of the process-based growth model for calcite (index c), compared to the corresponding equations of the binary crystal growth model of Zhang and Nancollas (index ZN).

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Process-based growth model summary

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Kink propagation rate (in  $s^{-1}$ )

$$u_c = 2\chi\bar{v}_A\bar{v}_B(S^2 - 1) \frac{1}{\bar{v}_A(Sr_i^{1/2} + \theta) + \bar{v}_B(\theta Sr_i^{-1/2} + 1)}$$

$$u_{ZN} = v(S^2 - 1) \frac{1}{S(r_i^{1/2} + r_i^{-1/2})/2 + 1}$$

Kink formation rate (in  $s^{-1}$ )

$$i_c = \chi\bar{v}_A\bar{v}_B S(S^2 - 1) \exp(-2\epsilon/kT) \left( \frac{1}{\bar{v}_A r_i^{1/2} + \bar{v}_B S} + \frac{1}{\bar{v}_B r_i^{-1/2} + \bar{v}_A S} \right)$$

$$i_{ZN} = vS(S^2 - 1) \exp(-2\epsilon/kT) \left( \frac{1}{r_i^{1/2} + S} + \frac{1}{r_i^{-1/2} + S} \right)$$

Kink density

$$\rho_c = S^{1/2} \exp(-\epsilon/kT) \left[ \left( \frac{1}{\bar{v}_A S r_i^{1/2} + \bar{v}_B S} + \frac{1}{\bar{v}_B r_i^{-1/2} + \bar{v}_A S} \right) / \left( \frac{1}{\bar{v}_A (S r_i^{1/2} + \theta) + \bar{v}_B (\theta S r_i^{-1/2} + 1)} \right) \right]^{1/2}$$

$$\rho_{ZN} = 2S^{1/2}(S + 1) \exp \left[ \left( \frac{1}{r_i^{1/2} + S} + \frac{1}{r_i^{-1/2} + S} \right) / \left( \frac{1}{S(r_i^{1/2} + r_i^{-1/2})/2 + 1} \right) \right]^{1/2}$$

Growth rate (in  $s^{-1}$ )

$$R_c = \chi k_{in} (K_S \bar{k}_A \bar{k}_B)^{1/2} (S^2 - 1) S^{1/2} \ln S \left[ \left( \frac{1}{\bar{v}_A r_i^{1/2} + \bar{v}_B S} + \frac{1}{\bar{v}_B r_i^{-1/2} + \bar{v}_A S} \right) \left( \frac{1}{\bar{v}_A (S r_i^{1/2} + \theta) + \bar{v}_B (\theta S r_i^{-1/2} + 1)} \right) \right]^{1/2}$$

$$R_{ZN} = k_{in} (S^2 - 1) S^{-1/2} \ln S \left[ \left( \frac{1}{r_i^{1/2} + S} + \frac{1}{r_i^{-1/2} + S} \right) \left( \frac{1}{S(r_i^{1/2} + r_i^{-1/2})/2 + 1} \right) \right]^{1/2}$$

With

$$S = \left[ \frac{\{A\}\{B_1\}}{K_S} \right]^{1/2} = \left[ \frac{\bar{k}_A \bar{k}_B \{A\}\{B_1\}}{\bar{v}_A \bar{v}_B} \right] \quad K_S = \frac{\bar{v}_A \bar{v}_B}{\bar{k}_A \bar{k}_B}$$

$$r_i = \frac{\bar{k}_A \bar{v}_B \{A\}}{\bar{k}_B \bar{v}_A \{B_1\}} \quad k_{in} = \frac{2aK_S^{1/2} (\bar{k}_A \bar{k}_B)^{1/2} \exp(-\epsilon/kT)}{19\gamma/kT}$$

$$\chi = P_A + P_{B_1} + P_{B_2} \quad \theta = (1 + 10^{-pH} K_{\equiv CaHCO_3})$$


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and  $r_i$  is expressed in terms of the activities of calcium and carbonate in the growth solution (Eq. (15)). If the effective detachment frequencies are equal,  $\bar{v}_A = \bar{v}_B = v$ , and all kink sites are suitable growth sites, and therefore  $\chi = 1$ , then Eq. (23) reduces to the kink propagation rate for a binary crystal obtained by Zhang and Nancollas (Table 3).

In order to use Eq. (23) to calculate growth rates, the density of growth sites needs to be known. Under supersaturated conditions, new kinks, or 1D clusters, are created mainly by addition of ions at non-kink positions along a step (Fig. 3). If a sufficiently elongated 1D cluster is formed by the attachment of additional ions, the cluster will be stable and two new kinks appear. Logically, both calcium and (bi-)carbonate sites can initiate a 1D cluster and overall stoichiometry during growth should be maintained. Following the approach of Zhang and Nancollas (1998), the rates of kink formation initiated on B kink sites ( $i_A$ ) and A kink sites ( $i_B$ ) are given by:

$$i_A = 2\chi \exp(-2\epsilon_A/kT) (S^2 - 1) \frac{\bar{k}_A \bar{v}_B \{A\}}{\bar{k}_A \{A\} + \bar{v}_B}, \quad (25)$$

$$i_B = 2\chi \exp(-2\epsilon_B/kT) (S^2 - 1) \frac{\bar{k}_B \bar{v}_A \{B_1\}}{\bar{k}_B \{B_1\} + \bar{v}_A}. \quad (26)$$

where  $\epsilon$  is the energy to form an A ( $\epsilon_A$ ) or B ( $\epsilon_B$ ) kink site. As the overall rate of kink formation equals  $i_c = (i_A + i_B)/2$ , Eqs. (25) and (26), combined with Eqs. (15), (24), and the assumption  $\epsilon_A = \epsilon_{B_1} = \epsilon_{B_2} = \epsilon$ , lead to the following expression for the 1D nucleation rate:

$$i_c = \chi \bar{v}_A \bar{v}_B S \exp(-2\epsilon/kT) (S^2 - 1) \left( \frac{1}{\bar{v}_A r_i^{1/2} + \bar{v}_B S} + \frac{1}{\bar{v}_B r_i^{-1/2} + \bar{v}_A S} \right). \quad (27)$$

The rate of step advancement,  $V_c$ , can now be calculated using the net propagation rate of a single kink ( $u_c$ , Eq. (23)), and the steady state kink density ( $\rho_c$ ) under supersaturated ( $S > 1$ ) conditions (Zhang and Nancollas, 1998):

$$\rho_c = 2(i_c/2u_c)^{1/2} \quad (28)$$

$$V_c = \rho_c u_c \text{ (in m s}^{-1}\text{)} \quad (29)$$

where  $a$  is the closest spacing between calcium and carbonate ions in the lattice (3.199 Å). Further assuming that, for spiral growth, the surface is covered by parallel steps of equal spacing,  $y_0$ , the crystal growth rate,  $R_c$ , normal to the crystal surface is given by

$$R_c = aV_c/y_0 \text{ (in m s}^{-1}\text{)} \quad (30)$$

where, according to the Burton–Cabrera–Frank (BCF) theory of spiral growth,

$$y_0 = \frac{19a\gamma}{kT \ln S} \text{ (in m)} \quad (31)$$

and  $\gamma$  represents the edge work (Burton et al., 1951).

### 3. MODEL PARAMETERS

The model parameters that must be specified in order to apply the process-based growth model are identified in Table 4. In this section, we derive values of the parameters

Table 4

Parameters of the process-based growth model. The literature ranges<sup>a</sup> are used as parameter constraints in the model calibration, see Section 3 in text for detailed discussion.

Parameter	Symbol	Process-based model value	Literature range <sup>a</sup>	References
Kink formation energy	$\epsilon, \epsilon_{A,B}$	$0.67 \times 10^{-20} \text{ J}$	0.62– $12 \times 10^{-20} \text{ J}$	Zhang and Nancollas (1998) and Liang and Baer (1997)
Edge work	$\gamma$	$1.2 \times 10^{-19} \text{ J}$	$1.0\text{--}13 \times 10^{-20} \text{ J}$	Söhnel and Mullin (1982), Nielsen (1984), and Teng et al. (1998)
Attachment frequencies	$k_{A_1}$	$8.5 \times 10^7 \text{ s}^{-1}$	$\sim 10^5\text{--}10^8 \text{ s}^{-1}$	Nielsen (1984) and Christoffersen and Christoffersen (1990)
	$k_{A_2}$	$\approx k_{A_1}$		This study
	$k_{B_1}$	$= 2k_{A_1} \frac{1+10^{8.6}10^{-\text{pH}}}{1+10^{10.33}10^{-\text{pH}}}$		This study
	$k_{B_2}$	$\approx k_{B_1}$		This study
Detachment frequencies	$v_{A_1}$	$2.0 \times 10^3 \text{ s}^{-1}$	$< k_{A_1}$	This study
	$v_{A_2}$	$\approx v_{A_1}$		This study
	$v_{B_1}$	$= \frac{K_S \bar{k}_A \bar{k}_B}{\bar{v}(1+10^{8.6}10^{-\text{pH}})}$		This study
	$v_{B_2}$	$\approx \log v_{B_1}$		This study

by combining available constraints on the parameters with an optimization of the model step advancement rate ( $V_c$ , Eq. (29)) to AFM step velocities determined by Teng et al. (2000) and Stack and Grantham (2010). These data sets were selected because the solution conditions are well-defined. For most other published AFM data sets, including some experiments performed by Stack and Grantham (2010), we are only able to reproduce the solution chemistry after arbitrarily adjusting the pH or  $p\text{CO}_2$  value. Any uncertainties in the actual pH or  $p\text{CO}_2$  values during the AFM measurements affect the calculated values of  $S$  and  $\chi$ , which result in large uncertainties in the model-calculated rates. The data of Teng et al. (2000) were collected at constant pH (8.5), ionic ratios  $r_{\text{aq}}$  close to 1, in NaCl background electrolyte and at variable degrees of supersaturation  $\Omega$  (1–3.6). Those of Stack and Grantham (2010) were collected at  $7 < \text{pH} < 9$ ,  $10^{-3} < r_{\text{aq}} < 10^3$ , in NaCl background electrolyte and at approximately constant degree of supersaturation  $\Omega$  (2.2–2.7).

The kink formation energy ( $\epsilon$ ) is often assumed to have a value of  $2kT = 6.2 \times 10^{-21} \text{ J}$  in theoretical crystal growth rate calculations (Zhang and Nancollas, 1998). Burke and Nancollas (1999) used  $\epsilon$  as a fitting parameter in their model and obtained values in the range  $8.6\text{--}8.7 \times 10^{-20} \text{ J}$  for growth of octacalcium phosphate, and  $14.6\text{--}26.0 \times 10^{-20} \text{ J}$  for dissolution of the same mineral phase. Liang and Baer (1997) calculated single kink formation energies from AFM data on a dissolving calcite surface and proposed a value of  $9.8 \pm 1.8 \times 10^{-20} \text{ J}$ . Here, we use the theoretical minimum of  $6.2 \times 10^{-21} \text{ J}$  and assume that  $\epsilon_A = \epsilon_B$ . In the sensitivity analysis discussed below the value for  $\epsilon$  is varied between the theoretical value of  $6.2 \times 10^{-21} \text{ J}$  and the maximum value of  $11.6 \times 10^{-20} \text{ J}$  reported by Liang and Baer (1997).

The edge work  $\gamma$  can be estimated using (Nielsen, 1984):

$$\gamma = kT[2.82 - 0.272 \ln(C_s)] \quad (32)$$

where  $C_s$  is the mineral solubility at equilibrium. For calcite, this implies that  $\gamma$  is slightly dependent on pH and  $p\text{CO}_2$ : for a pH range of 5–13 in a closed system,  $\gamma = 1.5 \pm 0.2 \times 10^{-20} \text{ J}$ , while for the same pH range in a system in equilibrium with the atmosphere,  $\gamma = 0.9 \pm 0.3 \times 10^{-20} \text{ J}$ . A slightly higher

average value was reported by Kristensen et al. (2004) from molecular dynamic simulations of a calcite surface in vacuum:  $1.85 \times 10^{-10} \text{ J m}^{-1}$  or  $5.9 \times 10^{-20} \text{ J}$  (using the mean ionic diameter to recalculate to Joules). Söhnel and Mullin (1982) obtained a value of  $1.0 \times 10^{-20} \text{ J}$  from experimental calcite nucleation data. Teng et al. (1998) reported the highest values estimated from *in situ* growth experiments in an AFM fluid cell for obtuse and acute steps. Their average recalculated value is  $13 \times 10^{-20} \text{ J}$ . Here, we use the value obtained from Eq. (32) as initial estimate, while the lowest and highest reported values are used in the sensitivity analysis (see below).

According to Nielsen (1984), the attachment frequency of an ion is approximately  $10^{-3}$  times its dehydration frequency when, following dehydration, the ion still has to diffuse from solution to a kink site. In our process-based model, however, the incorporation of a constituent ion corresponds to the transformation of an outer-sphere adsorbed calcium or (bi-)carbonate ion into an inner sphere complex with the accompanying removal of a water molecule. It is therefore expected that the attachment frequency is comparable to the dehydration frequency of the aqueous ion (about  $10^8 \text{ s}^{-1}$  for calcium). Christoffersen and Christoffersen (1990) further noted that the surface lattice cations surrounding a kink site must be dehydrated for the anion to enter the site. If dehydration of surface cations is frequency determining for the attachment of anions, then the attachment frequency of the anions could be expected to be of the same order of magnitude as that of the cations.

The growth rate data of Nehrke et al. (2007; Fig. 1a) provide a further clue as to the relative attachment frequencies of cations and anions. For the conditions of each of the growth experiments, we computed the calcite surface speciation according to the calcite surface model presented in Table 1. As can be seen in Fig. 1b, the measured growth rates reach a maximum when the concentration ratio of (bi-) carbonate to calcium surface sites (Table 1) is approximately two. This suggests that under optimum growth conditions there are about twice as many attachment sites for calcium than for (bi-)carbonate ions, which further suggests a two-times slower attachment rate of calcium than

(bi-)carbonate. In what follows, the default assumption is therefore  $\bar{k}_B = 2\bar{k}_A$ .

Deprotonation of aqueous bicarbonate is very fast compared to the estimated attachment frequencies ( $6 \times 10^9 \text{ M}^{-1} \text{ s}^{-1}$ ; Jolley, 1984). Thus, it is reasonable to assume that deprotonation of surface bicarbonate ions does not have a significant effect on the attachment and detachment frequencies, that is,  $k_{A1} \approx k_{A2}$ ,  $k_{B1} \approx k_{B2}$ ,  $v_{A1} \approx v_{A2}$ , and  $v_{B1} \approx v_{B2}$ . To the authors' knowledge, no previously published growth model studies report detachment frequencies. It is often assumed that detachment frequencies are very small compared to attachment frequencies during mineral growth (Zhang and Nancollas, 1990). Here, the detachment frequency  $v_{A1}$  is used as the only free fitting parameter, the other attachment and detachment frequencies are related as given in Table 4.

From the calcite surface model (Table 1, Wolthers et al., 2008), the amount of potential growth sites can be derived. For the experimental conditions under which Teng and coworkers and Stack and Grantham measured step velocities and Nehrke and coworkers measured single calcite crystal growth rates, the surface model yields growth site fractions  $\chi$  on the order of  $10^{-2}$ . In the rate calculations that follow, we impose the values of  $\chi$  predicted by the CD-MUSIC model for the given experimental conditions. Note that this approach differs from that of Zhang and Nancollas (1998), who assume that all kink sites are potential growth sites (i.e.,  $\chi = 1$ ).

With the above constraints and initial parameter estimates, the process-based kinetic model is fitted to the step velocities of Teng et al. (2000) and Stack and Grantham (2010). (Note: some of the measured step velocities are

excluded from the fit, due to uncertainties associated with the corresponding chemical compositions of the aqueous phase.) In the fitting procedure, model step velocities are computed for each individual set of experimental conditions, and the Residual Sum of Squares (RSS) between modeled and measured rates is minimized by adjusting model parameters  $\gamma$ ,  $\epsilon$  and  $k_{A1}$ , within the respective ranges, as discussed above and given in Table 4, and by freely varying the value for detachment frequency  $v_{A1}$ . The resulting fit is shown in Fig. 4a and the corresponding set of parameter values is given in Table 4. As can be seen, the model is able to accurately reproduce the obtuse step velocities of Teng and coworkers and most of the step velocities reported by Stack and Grantham (2010). However, because the process-based growth model does not distinguish between different crystallographic orientations, it cannot account for the systematic differences in obtuse and acute step velocities measured with AFM (Fig. 4a). Further model developments aimed at reproducing the growing body of data on AFM step velocities will therefore have to separately represent the creation, densities and propagation of kink sites along acute and obtuse edges.

Additional published step velocities measured by AFM are compared in Fig. 4b to model-predicted step velocities. The figure shows that the model step velocities compare rather well with step velocities measured by Larsen et al. (2010), whose experimental conditions were similar to those of Teng et al. (2000) (see caption to Fig. 4) except for the background electrolyte, which was 0.1 M KCl rather than NaCl. The model overestimates the step velocities measured by Perdikouri et al. (2009) and Ruiz-Agudo et al. (2010) who worked at variable pH (7–11) and slightly higher

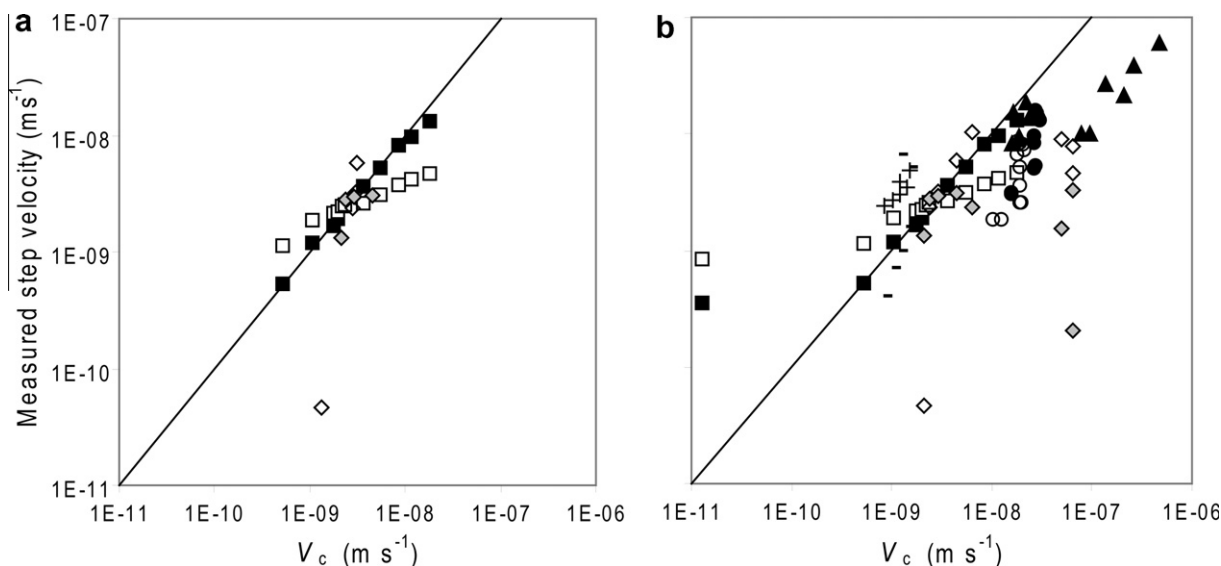


Fig. 4. (a) Model step velocities ( $V_c$ ) calculated for the experimental conditions during measurements and optimized to fit the measured ( $\blacksquare$ ,  $\blacklozenge$ ) obtuse and ( $\square$ ,  $\diamond$ ) acute step velocities on growing calcite surfaces observed with AFM at  $r_{\text{aq}} \approx 1.0$ , pH 8.5 at  $1 < \Omega < 3.6$  (squares, Teng et al., 2000) and  $7 < \text{pH} < 9$ ,  $10^{-3} < r_{\text{aq}} < 10^3$ , in NaCl background electrolyte and  $2.2 < \Omega < 2.7$  (diamonds, Stack and Grantham, 2010). Solid line indicates  $x = y$ . RSS is  $1.1 \times 10^{-19} \text{ m s}^{-1}$ . Corresponding model parameters are given in Table 4. (b) Model step velocities ( $V_c$ ) calculated for the experimental conditions during measurements of step velocities on growing calcite surfaces observed with AFM: for  $2 \times 10^{-4} < r_{\text{aq}} < 2 \times 10^{-4}$  and pH 9.1–11.1 at ( $\bullet$ )  $\Omega = 6.4$  and ( $\circ$ )  $\Omega = 5.2$  (Perdikouri et al., 2009); for  $0.01 < r_{\text{aq}} < 10$  and pH 8.5 at  $\Omega = 4$  for (–) acute and (+) obtuse step (Larsen et al., 2010); ( $\blacktriangle$ )  $r_{\text{aq}} = 1$ ,  $\Omega = 6.5$  and varying pH (Ruiz-Agudo et al., 2010). Model step velocities were calculated using the parameters listed in Table 4. Solid line indicates  $x = y$ .



degrees of supersaturation in 0.1 M NaCl. Still, the trends in Fig. 4b tend to parallel the 1:1 line, implying that the model can account, at least in part, for the relative changes in step velocities under a wide range of chemical conditions.

### 3.1. Model sensitivity

The model-derived step velocities ( $V_c$ ) and growth rates ( $R_c$ ) are more sensitive to the kink formation energy ( $\epsilon$ ) than to the edge work ( $\gamma$ ). When using the maximum value of  $\epsilon$  ( $11.6 \times 10^{-20}$  J), together with the minimum or maximum values of  $\gamma$ , the step velocities shown in Fig. 4a can not be fitted unless unrealistically high attachment frequencies are used ( $>> 10^{30} \text{ s}^{-1}$ ). In contrast, the minimum value for the kink formation energy ( $6.2 \times 10^{-21}$  J) yields similarly good fits to the step velocities with both the minimum and maximum value for  $\gamma$  and realistic attachment frequencies.

As expected, the position of the growth rate optimum is sensitive to the ratio of  $\bar{k}_B$  to  $\bar{k}_A$  (Fig. 5). While the shape of the  $R_c$  versus  $1/r_{\text{aq}}$  curve is unaffected by changes in the  $\bar{k}_B : \bar{k}_A$  ratio, varying  $\bar{k}_B : \bar{k}_A$  by one order of magnitude shifts the growth rate optimum by more than two orders of magnitude with respect to  $1/r_{\text{aq}}$ .

## 4. SOLUTION STOICHIOMETRY AND GROWTH RATES

The dependence of model-derived calcite crystal growth rates ( $R_c$ ) on solution stoichiometry is illustrated in Fig. 6. The rates are calculated with the process-based growth model using the parameter values listed in Table 4, which were obtained from calibration on single step velocities measured at  $7 < \text{pH} < 9$ ,  $10^{-3} < r_{\text{aq}} < 10^3$ , and  $1 < \Omega < 3.6$ . The results in Fig. 6a correspond to solution chemical conditions close to those in the single crystal growth experiments of Nehrke et al. (2007). The model yields rate maxima for solution activity ratios  $r_{\text{aq}} \approx 1$ , and near-symmetrical rate distributions that are only slightly dependent on the solution supersaturation. These model predictions are consistent with the observations of Nehrke et al. (compare Figs. 1a and 6a). Furthermore, the maximum growth rates predicted by the process-based model ( $7.2 \times 10^{-11} \text{ m s}^{-1}$  for  $\Omega = 16 \pm 2$  and  $1.3 \times 10^{-11} \text{ m s}^{-1}$  for  $\Omega = 5.5 \pm 0.5$ ) are within order-of-

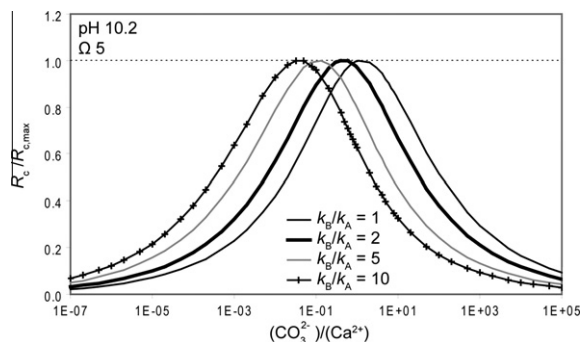


Fig. 5. Normalized process-based model rates at pH 10.2 and  $\Omega = 5$  plotted against the solution activity ratio, for different ratios of attachment frequencies  $\bar{k}_B$  versus  $\bar{k}_A$ .

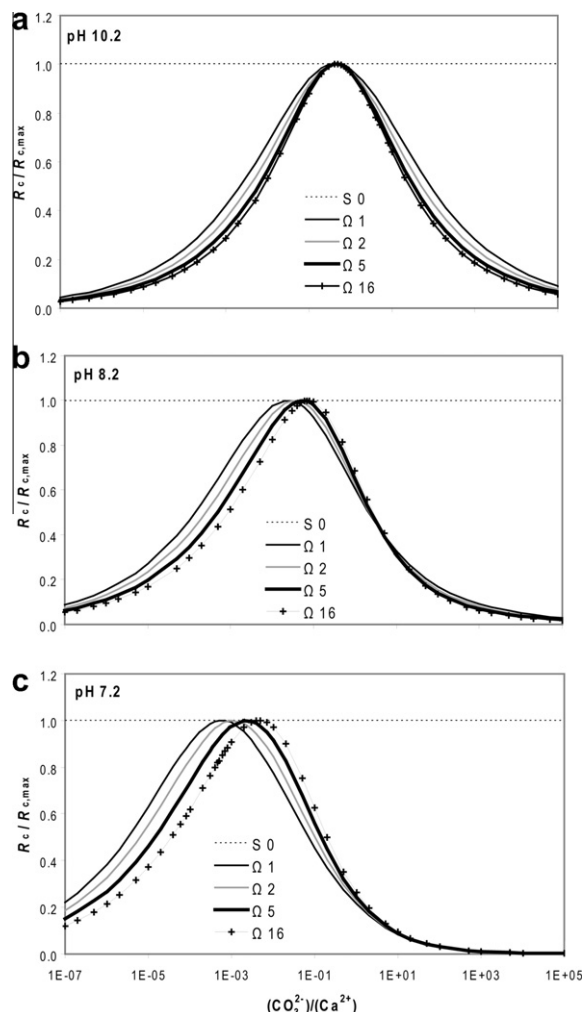


Fig. 6. Process-based model rates ( $R_c$ ), normalized to maximum model rates ( $R_{c,\text{max}}$ ), plotted for different degrees of supersaturation and (a) pH 10.2, (b) pH 8.2, (c) pH 7.2. Model rates for  $\Omega = 16$  and 5 at pH 10.2 were calculated at the experimental conditions of Nehrke et al. (2007). Model rates were calculated using the parameter values listed in Table 4.

magnitude agreement with the measured  $R_c$  values ( $1.9 \pm 0.4 \times 10^{-11} \text{ m s}^{-1}$  for  $\Omega = 16 \pm 2$  and  $1.1 \pm 0.2 \times 10^{-11} \text{ m s}^{-1}$  for  $\Omega = 5.5 \pm 0.5$ ). Taken together, these findings are quite remarkable, given that no model fitting to the growth rate measurements of Nehrke and coworkers is involved. That is, the model is able to scale from single step velocities up to single crystal growth rates.

The results in Fig. 6 show that the position of the growth optimum and the shape of the rate distribution with respect to the solution activity ratio depend on pH. As pH decreases, aqueous carbonate ions become less abundant and growth increasingly relies on the attachment of bicarbonate ions (reaction 3.4 in Table 2). As a result, the growth optimum shifts to lower  $1/r_{\text{aq}}$  values and the rate distribution becomes progressively more asymmetric. These trends are accentuated as the solution supersaturation decreases. Hence, from alkaline to neutral conditions, the functional dependence of calcite growth rates becomes

increasingly dependent on the degree of supersaturation. That is, the behavior of  $R_c$  with respect to solution stoichiometry becomes more complex at less alkaline conditions.

## 5. BULK CALCITE GROWTH RATES

In this section we assess to what extent the process-based growth model is able to reproduce bulk rates of calcite precipitation in seeded growth experiments. The sources of the published data sets used are identified in the caption of Fig. 7. The data comprises calcite growth rates measured at saturation ratios,  $\Omega = IAP/K_s$ , ranging from 1 to 33, pH values between 7.5 and 10, temperatures from 10 to 40 °C, in systems without or with a gas phase and, in the latter case, with  $p\text{CO}_2$  values up to almost 800 times the atmospheric level and background electrolyte solution concentrations varying from uncontrolled ion strength to 0.001–0.7 M NaCl or KCl. Only rates measured in systems for which the chemical composition is completely known are used, because the aqueous and surface speciation are required to calculate the model growth rates. Additionally, only rates obtained in common background electrolyte solutions are used, since the current version of the CD-MUSIC model of the calcite surface cannot account for sorption of impurity ions. The model parameters are fixed to the values given in Table 4. It is important to note that these parameter values are independent of the measured rate data in Fig. 7: the only direct experimental rate data used to constrain the parameter values are the AFM step advancement rates of Teng et al. (2000) and Stack and Grantham (2010) as discussed in Section 3.

In Fig. 7a, the measured bulk calcite growth rates ( $R_c$ ) are compared to the rates predicted by the process-based growth model. Although the plot exhibits significant scatter, most individual sets of measured rates fall either systematically above or below the 1:1 line. In addition, the trends defined by the data from the individual studies generally tend to parallel the 1:1 line, implying that the model reproduces the relative changes in growth rates under variable chemical conditions. It must be emphasized that the experimental growth rates were obtained under widely different experimental conditions, by different researchers using a variety of calcite seed materials and experimental approaches. The consistency of the trends suggests that the scatter in Fig. 7a reflects systematic differences among the individual rate data sets rather than a fundamental shortcoming of the process-based growth model.

The surface kink densities on which the model-calculated growth rates ( $R_c$ ) in Fig. 7a are based are determined by the line density ( $\rho_c$ ) and the step edge spacing ( $y_0$ ) given by Eqs. (28) and (31), respectively. These equations are rooted in the classic Burton–Cabrera–Frank (BCF) crystal growth model and its derivatives (for a complete discussion, see Zhang and Nancollas, 1998). The calcite–water interface, however, is highly dynamic (Stipp and Hochella, 1991; Paquette and Reeder, 1995). Actual surface densities of growth sites could conceivably be lower, for example, when sites are blocked by impurities or step edges interact destructively, or higher, for example, when surface nucleation creates additional growth edges on the crystal surface (Kowacz et al., 2007) or when electrolyte ions interact with

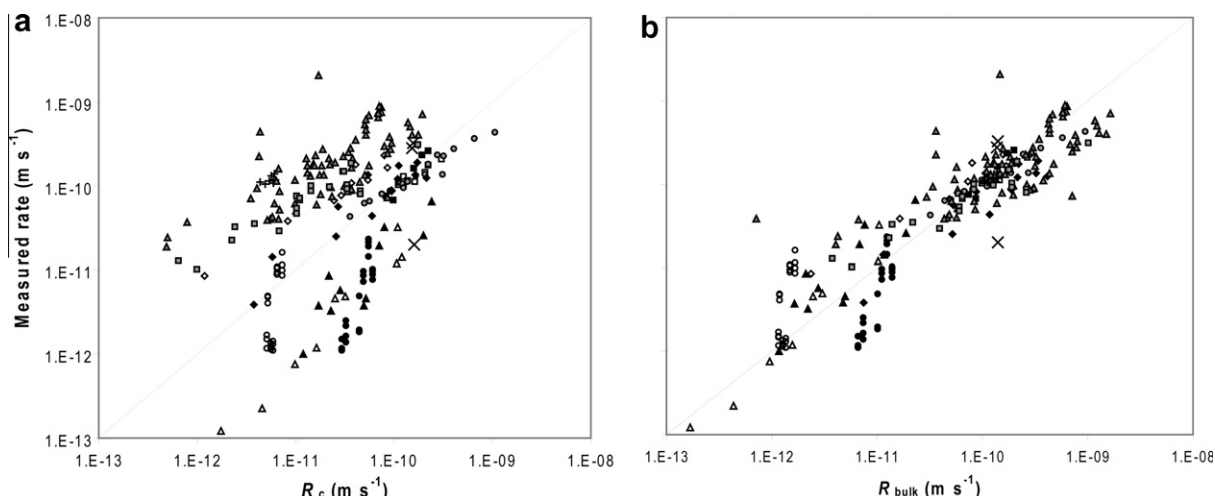


Fig. 7. (a) Measured bulk growth rate for calcite plotted versus process-based model rates calculated for the corresponding experimental conditions ( $\text{RSS} = 1.2 \times 10^{-17} \text{ m s}^{-1}$ ). Measured growth rates: (●)  $\Omega 16 \pm 2$  and (○)  $\Omega 5\text{--}6$  at pH  $10.15 \pm 0.05$ , no gas phase,  $I = 0.1 \text{ M NaCl}$  and  $20 \pm 0.2^\circ \text{C}$  (Nehrke et al., 2007); (+) at  $\Omega 4$ ,  $I = 0.018 \text{ M NaCl}$  and pH 9.5 (Tai et al., 2005); (◆) 1540 ppm  $\text{CO}_2$ ,  $\Omega 1.5\text{--}8.3$  and (◇) 310 ppm  $\text{CO}_2$ ,  $\Omega 1.7\text{--}9.8$  ( $I = 0.07 \text{ M NaCl}$ ; Nilsson and Sternbeck, 1999); (▲) 30,500 Pa  $p\text{CO}_2$  and (△) 2040 Pa  $p\text{CO}_2$  ( $I = \text{variable NaCl}$ ; Zuddas and Mucci, 1994); (△) variable  $\Omega$  and  $p\text{CO}_2$ , pH 7.5–8.9 ( $I = 0.7 \text{ M KCl}$ ; van der Weijden et al., 1997); (□) pH 8.3–10 and  $T = 37^\circ \text{C}$  ( $I = \text{variable}$ ; Christoffersen and Christoffersen, 1990); (■) pH 8.2–8.7 at variable  $p\text{CO}_2$ , (○) pH 8.4 and variable ionic strength and (×) pH 8.4 and variable  $T$  ( $I = \text{variable KCl}$ ; Inskeep and Bloom, 1985). The solid line indicates  $x = y$ . (b) Measured bulk growth rate for calcite plotted versus  $\alpha R_c$  (Eq. (33)) calculated for the corresponding experimental conditions ( $\text{RSS} = 1.1 \times 10^{-17} \text{ m s}^{-1}$ ). Symbols as in (a). Optimized values for  $\log \alpha$  are: (●) and (○)  $-0.64$ ; (+) 1.36; (◆) and (◇) 0.30; (▲) and (△)  $-1.02$ ; (△) 0.93; (□) 0.76; (■), (○) and (×)  $-0.05$ .

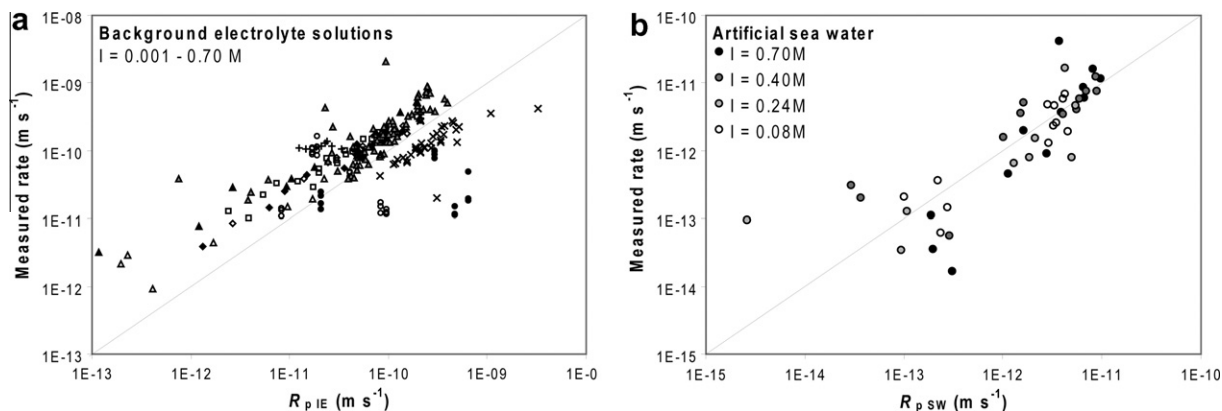


Fig. 8. (a) Measured bulk growth rate for calcite plotted versus rates calculated for the corresponding experimental conditions using:  $R_{p\text{ BE}} = I^{-0.004} \text{pH}^{-10.71} r_{\text{aq}}^{-0.35} (S-1)^2$  with  $r_{\text{aq}} = \{\text{Ca}^{2+}\}/\{\text{CO}_3^{2-}\}$ ; RSS is  $1.0 \times 10^{-16} \text{ m s}^{-1}$ . For symbol explanation see Fig. 7a. (b) Bulk growth rate for calcite measured in (diluted) artificial sea water (Zhong and Mucci, 1989) plotted versus rates calculated at exact experimental conditions using the parabolic rate law:  $R_{p\text{ SW}} = I^{0.36} \text{pH}^{-10.99} r_{\text{aq}}^{0.71} (S-2)^2$ ; RSS is  $1.8 \times 10^{-21} \text{ m s}^{-1}$ . Solid lines indicate  $x = y$ .

the mineral surface (Fenter et al., 2010) or with dissolved constituent ions (Di Tommaso and de Leeuw, 2010).

In addition, the experimental linear growth rates plotted in Fig. 7a are obtained by dividing the measured mass of calcite precipitated per unit time by the surface area of the seed crystals. Specific surface areas are estimated based on experimental data (e.g., BET isotherms) or geometrical considerations. These approaches only incompletely account for variations in the surface microtopography of mineral surfaces (e.g., Dzombak and Morel, 1990; Dixit and Van Cappellen, 2002; Wolthers et al., 2003). Hence, we suspect that variations in the microscopic surface roughness of calcite seed crystals may be a major source of the scatter between the individual studies observed in Fig. 7a.

In order to account for possible deviations from the theoretical kink surface densities and the uncertainties associated with estimates of the specific surface areas of the calcite seed crystals, we introduce a non-dimensional surface roughness correction factor,  $\alpha$ , so that

$$R_{\text{measured}} = \alpha \cdot R_c \quad (33)$$

where for any given experimental growth study the value of  $\alpha$  is fixed, while it is allowed to vary from one study to another. As can be seen in Fig. 7b, using  $\alpha$  as the only adjustable parameter, it is possible to reconcile the large body of measured bulk growth rates of calcite with the process-based model.

The values of  $\log \alpha$  obtained by fitting the measured bulk growth rates to Eq. (33) range from  $-1.02$  to  $1.36$  (Fig. 7b). Further interpretation of this large variation in  $\alpha$  will require detailed characterization of the surfaces of calcite seed crystals. That such large differences in  $\alpha$  occur even among seeded growth experiments carried out under controlled laboratory conditions in common background electrolyte solutions point to the inherent difficulties associated with extrapolating laboratory rates of calcite growth to natural environments. This reactive-surface-area problem represents a recurrent challenge in geochemical mineral–water interactions (e.g., Velbel, 1985; White and Peterson, 1990;

Van Cappellen, 1996; White and Brantley, 2003; Jourabchi et al., 2008; Pallud et al., 2010).

## 6. EMPIRICAL GROWTH RATE EQUATIONS

The previous section shows that a diverse set of bulk calcite growth rates, obtained independently for a wide range of chemical conditions, can be successfully described by the process-based model. Nevertheless, the implementation of the model for routine geochemical modeling may be quite cumbersome. Furthermore, due to the introduction of the empirical correction factor  $\alpha$ , the applicability of the process-based growth model as a predictive tool for calcite growth rates is limited as long as the value for  $\alpha$  for a particular system is unknown. For practical applications, we therefore derive an empirical growth rate expression using the same set of bulk growth rate data as in Fig. 7. Additionally, we derive an empirical growth rate expression for growth in seawater-type solutions. Both expressions assume a parabolic dependency of the calcite growth rate on the degree of supersaturation (Nielsen, 1981) and explicitly accounts for additional effects of ionic strength, solution stoichiometry and pH.

The following parabolic rate law describes the calcite growth rate observed in experiments with common background electrolyte solutions ( $I = 0.001\text{--}0.7 \text{ M}$ ):

$$R_{p\text{ BE}} = I^{-0.004} \text{pH}^{-10.71} r_{\text{aq}}^{-0.35} (S-1)^2 \quad (34)$$

where  $I$  is the ionic strength,  $r_{\text{aq}} = \{\text{Ca}^{2+}\}/\{\text{CO}_3^{2-}\}$  activity ratio,  $S$  is the saturation ratio ( $\Omega^{1/2}$ ), and  $R_{p\text{ BE}}$  is expressed in  $\text{m s}^{-1}$ . (Note that linear growth rates can be converted to rates in units of mass per unit surface area and per unit time via the molecular density of calcite,  $2.71 \times 10^4 \text{ mol m}^{-3}$ .) As can be seen in Fig. 8a, the rate equation provides a reasonably good description of the measured rates.

A second parabolic rate law is derived for calcite growth in seawater-type solutions. It is based on the calcite growth rates of Zhong and Mucci (1989) measured in undiluted and diluted artificial seawater solutions ( $I = 0.08\text{--}0.7 \text{ M}$ ). These growth rates are orders of magnitude lower than

rates obtained at similar degrees of supersaturation in background electrolyte solutions. The rates are described by the following parabolic rate law (Fig. 8b):

$$R_{p\text{ SW}} = I^{0.36} \text{pH}^{-10.99} r_{aq}^{0.71} (S - 2)^2 \quad (35)$$

where  $R_{p\text{ SW}}$  is expressed in  $\text{m s}^{-1}$ . Comparison of Eqs. (34) and (35) implies a stronger dependency of calcite growth rate on ionic strength in seawater-type solutions. Mainly this reflects the presence of growth inhibiting ions such as magnesium, sulfate and phosphate in the artificial seawater solutions.

## 7. CONCLUSIONS

A new, process-based rate model for the crystal growth of calcite from aqueous solution is presented. The model combines kinetic descriptions of the incorporation of calcium, carbonate and bicarbonate ions at kink sites along step edges, with equilibrium calculations of the surface chemical structure of the calcite–solution interface under variable aqueous chemical conditions. Key model parameters are calibrated by fitting the model to a selected set of step velocities on calcite surfaces measured by AFM. Without any further adjustment of the parameter values, the model reproduces the observed dependence of the macroscopic growth rate of single calcite crystals on the solution activity ratio of calcium and carbonate ions ( $r_{aq} = \{\text{Ca}^{2+}\}/\{\text{CO}_3^{2-}\}$ ) under alkaline conditions. In order to reconcile the model with bulk precipitation rates measured in seeded growth experiments a surface roughness factor is introduced. The latter accounts for deviations of the calcite kink density from the theoretical density based on the classical BCF theory of spiral growth, but also for uncertainties associated with specific surface area estimations. Constraining the effective concentration of growth sites remains one of the main obstacles to the extrapolation of molecular-scale crystal growth models to mineral precipitation rates in geochemical field systems.

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A	(index for) calcium ion
{A}	activity of calcium ion
A <sub>1</sub> , A <sub>2</sub>	index for a calcium ion reacting with a carbonate or bicarbonate surface site, respectively
a	mean ionic diameter (3.199 Å for calcite)
B	(index for) carbonate and bicarbonate ions (see Eqs. (10) and (12))
B <sub>1</sub>	(index for) carbonate ion
{B <sub>1</sub> }	activity of carbonate ion
B <sub>2</sub>	(index for) bicarbonate ion
{B <sub>2</sub> }	activity of bicarbonate ion

BE	index for parabolic rate law in common background electrolyte solutions
c	index for process-based calcite growth model
C <sub>s</sub>	mineral solubility at equilibrium (Eq. (32) only)
e	index for equilibrium
IAP	Activity product of the constituent ions, $\{\text{Ca}^{2+}\}\{\text{CO}_3^{2-}\}$ for calcite
K <sub>S</sub>	solubility product of calcite (Eq. (1))
K <sub>SH</sub>	solubility product of calcite in terms of $\{\text{Ca}^{2+}\}$ and $\{\text{HCO}_3^-\}$ (Eq. (2))
k	Boltzmann constant
k <sub>A1</sub> , k <sub>A2</sub> , k <sub>B1</sub> , k <sub>B2</sub>	attachment frequencies for A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> and B <sub>2</sub> ions
$\frac{k_{B2}}{k_{A1} + k_{B1}}$	overall attachment frequencies for calcium and (bi-) carbonate
NS	(index) for Nilsson and Sternbeck (1999)
P	probability
p	index for parabolic rate law
R	overall growth rate
r <sub>aq</sub>	ionic activity ratio $\{\text{Ca}^{2+}\}/\{\text{CO}_3^{2-}\}$
r <sub>i</sub>	kinetic ionic ratio for integration (Eq. (15))
RSS	residual sum of squares
S	saturation ratio $(IAP/K_S)^{1/2} = \Omega^{1/2}$
S <sub>tot</sub>	total number of surface sites
SW	index for parabolic rate law in seawater type solutions
T	temperature
u	kink propagation rate
V	rate of step movement
ZN	(index for) Zhang and Nancollas (1998)
y <sub>0</sub>	step spacing in a spiral
α	surface roughness factor (Eq. (33))
γ	edge work
Δz <sub>0</sub> , Δz <sub>1</sub>	Boltzmann parameters for surface reactions (Table 1 only)
ε	kink formation energy
θ	term for speciation between carbonate and bicarbonate
v <sub>A1</sub> , v <sub>A2</sub> , v <sub>B1</sub> , v <sub>B2</sub>	detachment frequencies for A, B <sub>1</sub> and B <sub>2</sub> ions
$\frac{v_{B2}}{v_{A1} + v_{B1}}$	overall detachment frequencies for calcium and (bi-) carbonate
ξ	$=\exp(-2\varepsilon/kT)$
ρ	kink density
χ	fraction of growth sites at the surface
Ω	saturation ratio $(IAP/K_S)$
≡ <sub>c</sub> , ≡ <sub>e</sub> , ≡ <sub>f</sub>	corner site, edge site or face site at the calcite surface

## APPENDIX A. NOMENCLATURE

See also Table 4.

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