Phys 512 : PS5

1) Computing the fit of the temperature fluctuations as a function of the multiple moment l, we have the following results:

$$\chi^2 = 15267.9372 \quad , \quad DOF = 2501$$

For a proper fit, at the very least the order of magnitudes of the χ^2 & the DOF must match, which isn't the case so the fit isn't acceptable.

↳ If we repeat the above w/ the new suggested parameters, we get:

$$\chi^2 = 3272.2054 \quad , \quad DOF = 2501$$

Although the plot looks fine, there is still a significant discrepancy between the χ^2 & the DOF, it is still not an acceptable fit.

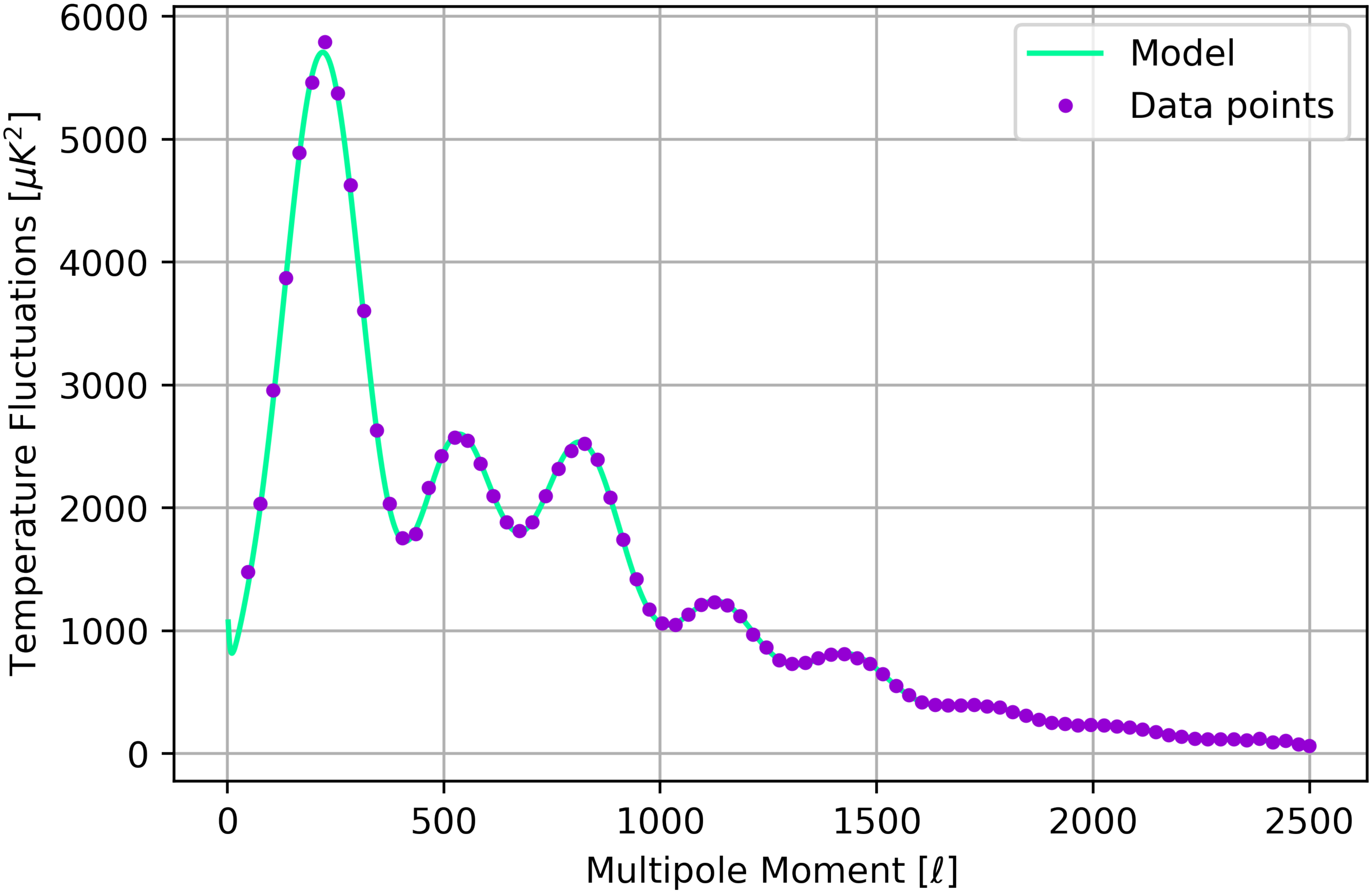
2) Using Newton's method & applying a 1-loop order correction to As, we get the parameters & uncertainties:

- $H_0 = 6.8247 \times 10^{1} \pm 5.9231 \times 10^{-2}$
- $\Omega_b h^2 = 2.2364 \times 10^{-2} \pm 1.5466 \times 10^{-5}$
- $\Omega_c h^2 = 1.1766 \times 10^{-1} \pm 1.2487 \times 10^{-4}$
- $\tau = 8.5434 \times 10^{-2} \pm 2.0807 \times 10^{-3}$
- $A_s = 2.2193 \times 10^{-9} \pm 1.1102 \times 10^{-11}$
- $n_s = 9.7307 \times 10^{-1} \pm 3.6262 \times 10^{-4}$

Moreover, we get $\chi^2 = 2576.1526$ & $DOF = 2501$

↳ As can be seen, the difference in χ^2 & DOF is much smaller (less than the variance & mean of χ^2) & is thus an adequate fit

↳ The fit looks like:



Model
Data points
Temperature Fluctuations [μK^2]
Multipole Moment [ℓ]
0
1000
2000
3000
4000
5000
6000
0
500
1000
1500
2000
2500

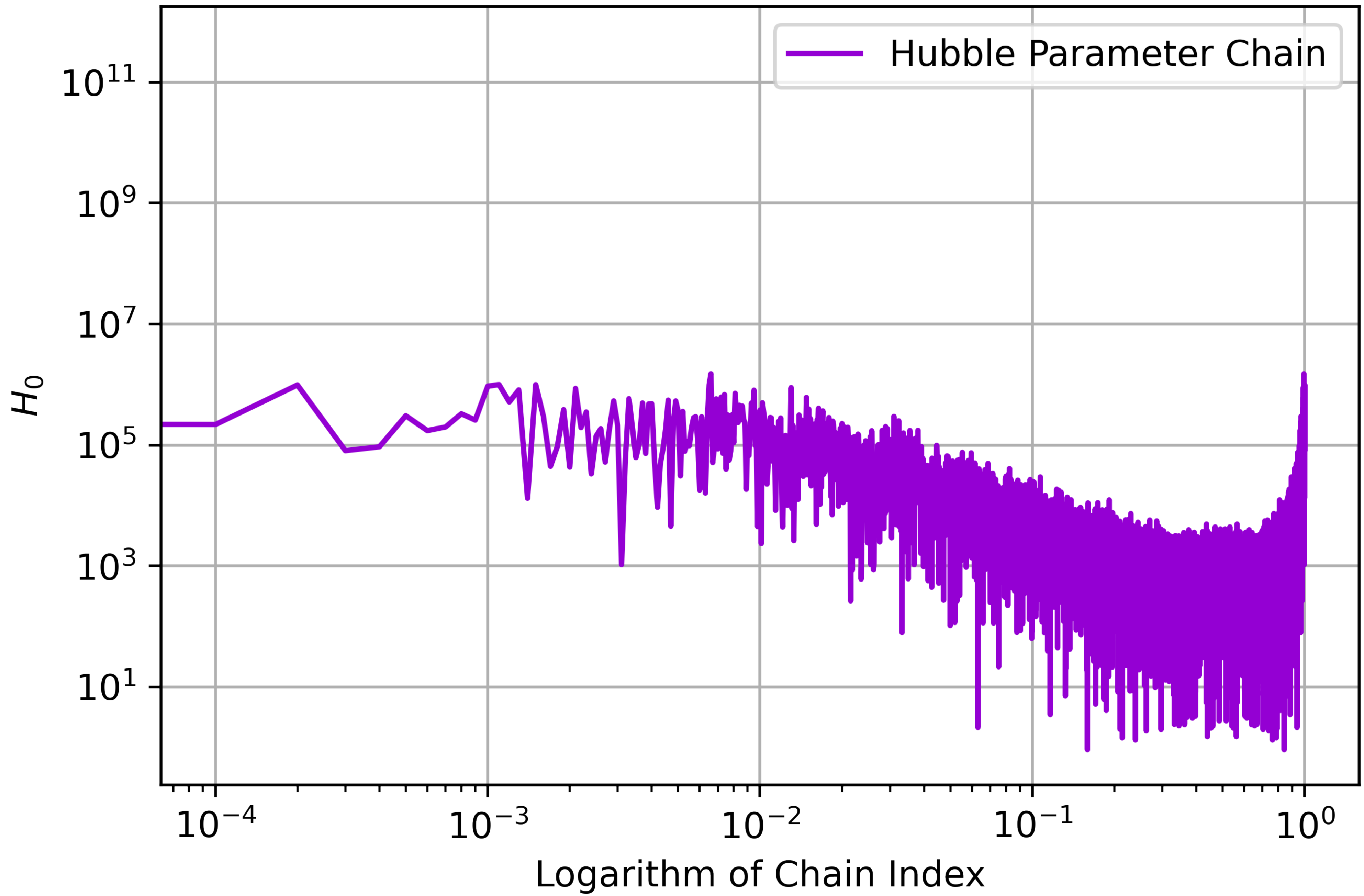
3) Now we have the daunting task of taking the fit w/ an MCMC (this took MANY hours to get right)

↳ As suggested by Jon our trial steps are based on the curvature matrix from the Newton fit in Q2.

↳ Using MCMC, we get the following parameters & their respective uncertainties:

- $H_0 = 6.8194 \times 10^{1} \pm 1.1494 \times 10^{0}$
- $\Omega_b h^2 = 2.2356 \times 10^{-2} \pm 2.4999 \times 10^{-4}$
- $\Omega_c h^2 = 1.1783 \times 10^{-1} \pm 2.5978 \times 10^{-3}$
- $\tau = 8.2724 \times 10^{-2} \pm 2.6254 \times 10^{-3}$
- $A_s = 2.2107 \times 10^{-9} \pm 1.1023 \times 10^{-10}$
- $n_s = 9.7272 \times 10^{-1} \pm 6.9779 \times 10^{-3}$

To see that the chains have converged, consider the chain plot for H_0 for example: (FFT of the chain):



Hubble Parameter Chain
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
10^{1}
H_0
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
Logarithm of Chain Index

As we can see, the left half of the FFT chain flattens off and thus implies convergence

↳ The same holds for all the other parameters.

For a spatially flat universe the following holds:

$$\Omega_b + \Omega_c + \Omega_\Lambda = 1 \Rightarrow \Omega_\Lambda = 1 - \frac{1}{h^2}\left(\Omega_b h^2 + \Omega_c h^2\right)$$

Furthermore, using the fact that $h = H_0/100$, the error in h is $\sigma_h = \sigma_{H_0}/100$

↳ Moreover $\sigma_{\Omega_{b/c}} = |\Omega_{b/c}| \sqrt{\left(\frac{\sigma_h}{h}\right)^2 + \left(\frac{\sigma_{\Omega_{b/c}h^2}}{\Omega_{b/c}h^2}\right)^2}$

Thus $\sigma_{\Omega_\Lambda} = \sqrt{\Omega_b^2 + \Omega_c^2}$

Using the above, our estimate for the vacuum energy & its uncertainty is:

- $\Omega_\Lambda = 6.9856 \times 10^{-1} \pm 1.6509 \times 10^{-3}$

4) Now we repeat what we've done in Q3 except with the polarization constraint:

$$\tau = 0.0540 \pm 0.0074$$

First we want to see how much the best fit parameters via importance sampling

↳ Looking at the output Chisquarevector, the first 110 samples are highly divergent from the mean so we cut out those samples [111 TO 19]

↳ We use this to compute a new likelihood & compute the new chain weights

↳ Using importance sampling we get the parameters:

- $H_0 = 6.7777 \times 10^{1} \pm 9.5632 \times 10^{-1}$
- $\Omega_b h^2 = 2.2300 \times 10^{-2} \pm 2.0577 \times 10^{-4}$
- $\Omega_c h^2 = 1.1871 \times 10^{-1} \pm 2.1837 \times 10^{-3}$
- $\tau = 5.5856 \times 10^{-2} \pm 6.7166 \times 10^{-4}$
- $A_s = 2.0976 \times 10^{-9} \pm 3.0033 \times 10^{-11}$
- $n_s = 9.7063 \times 10^{-1} \pm 5.4312 \times 10^{-3}$

We see that:

$$\tau = 8.2724 \times 10^{-2} \pm 2.6254 \times 10^{-3} \longrightarrow 5.5856 \times 10^{-2} \pm 6.7166 \times 10^{-3}$$

We see a non-trivial shift in $|\tau|$ & $|\sigma_\tau|$ has reduced by a new order of magnitude

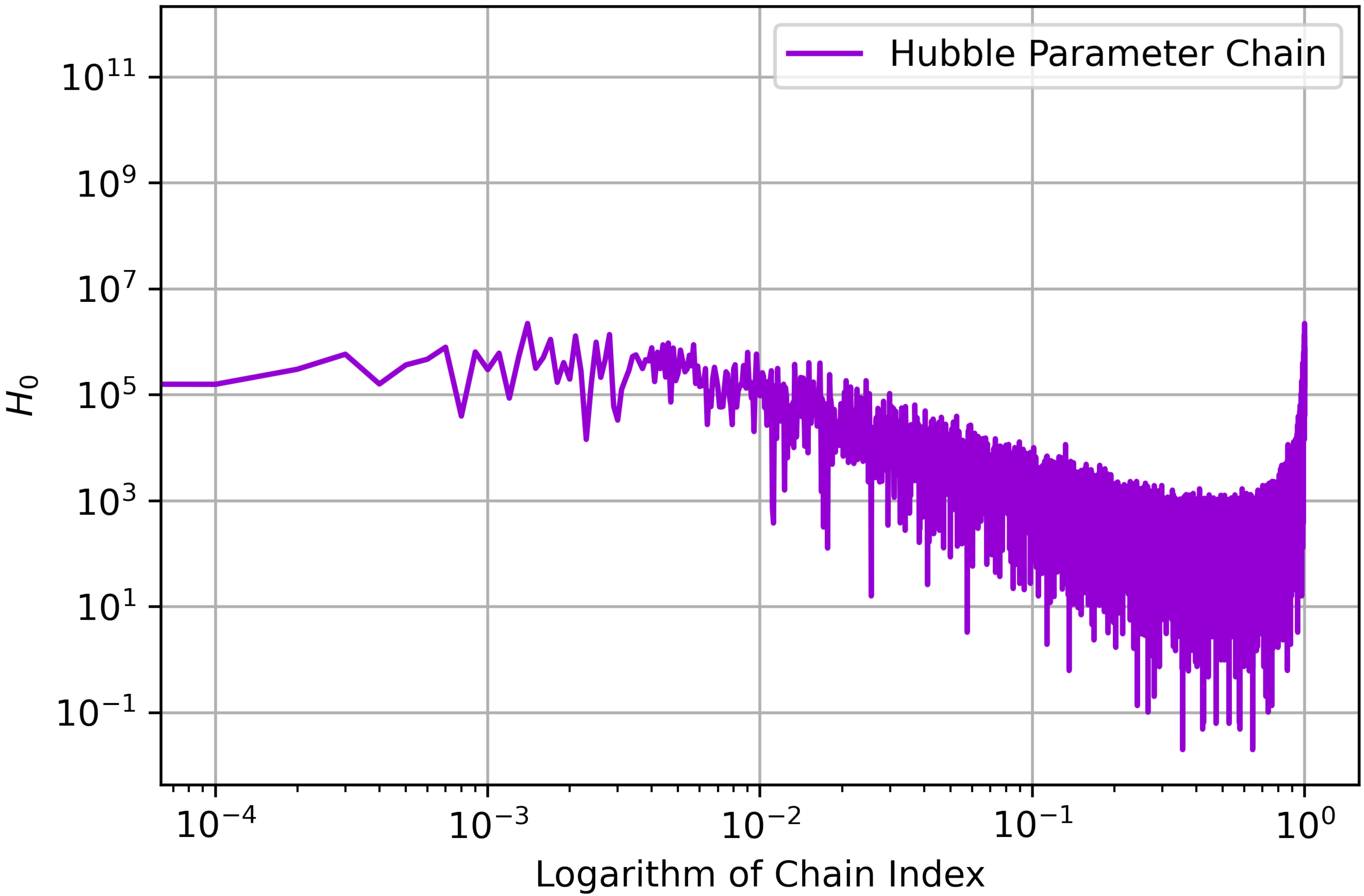
↳ We shall use this to run a new MCMC (with a new covariance matrix from the chain)

Using importance sampling & recomputing the covariance matrix to determine our new step sizes, we use MCMC to get the new best fit parameters & their uncertainties:

- $H_0 = 6.7855 \times 10^{1} \pm 1.0274 \times 10^{0}$ • $\Omega_b h^2 = 2.2280 \times 10^{-2} \pm 2.1116 \times 10^{-4}$
- $\Omega_c h^2 = 1.1845 \times 10^{-1} \pm 2.3110 \times 10^{-4}$ • $\tau = 5.5999 \times 10^{-2} \pm 7.2519 \times 10^{-3}$
- $A_s = 2.0973 \times 10^{-9} \pm 3.2011 \times 10^{-11}$ • $n_s = 9.7098 \times 10^{-1} \pm 5.3484 \times 10^{-3}$

These results don't differ too much from the importance sampling parameters, with the main difference being the order of magnitudes for σ_{H_0} & $\sigma_{\Omega_c h^2}$.

↳ Moreover, we know the chains converged from the FFT plots, for example the FFT chain for H_0:



Hubble Parameter Chain
H_0
10^{11}
10^{9}
10^{7}
10^{5}
10^{3}
10^{1}
10^{-1}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
Logarithm of Chain Index

We can see that the flattening on the left implies converge, & the same holds for the other parameters.

Since the parameters & errors from the importance sampling & the τ-constrained MCMC, we conclude the use of an MCMC wasn't required for Q4.