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Black to white hole tunneling: An exact classical solution

Hal M. Haggard
Physics Program, Bard College, 30 Campus Rd, Annandale-on-Hudson, NY 12504, USA
hhaggard@bard.edu

Carlo Rovelli
Aix-Marseille Université and Université de Toulon,
CPT-CNRS, Luminy, F-13288 Marseille, France
rovelli@cpt.univ-mrs.fr

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We present a metric that describes conventional matter collapsing into a black hole, bouncing and emerging from a white hole, and that satisfies the vacuum Einstein equations everywhere, *including in the interior of the black hole and the subsequent white hole*, except for a small compact 4d "quantum tunneling" zone. This shows that a black hole can tunnel into a white hole without violating classical general relativity where this can be trusted. We observe that quantum gravity can affect the metric in a region outside the horizon without violating causality because small quantum effects might pile up over time. We study how quantum theory can determines the bouncing time.

Black holes are now normal astrophysical objects, but we know surprising little about what happens *inside* them. General relativity (GR) appears to describe well the region surrounding the horizon,[1] and this is also likely true of a substantial region inside the horizon. But classical GR must fails to describe Nature at small radii because nothing prevents quantum mechanics from affecting the high curvature zone, and classical GR becomes ill-defined at $r = 0$ anyway. The current tentative quantum gravity theories, such as loops and strings, are not sufficiently understood to convincingly predict what happens at small radii, so we are quite in the dark: what does really happens to gravitationally collapsing matter?

Here we explore the possibility that when matter reaches Planckian density quantum gravity generates sufficient pressure to counterbalance weight, the collapse ends, and matter bounces out. This is similar to the way the wave packet representing a collapsing universe tunnels into an expanding universe in loop cosmology.[2] A collapsing star might similarly avoid sinking into $r = 0$, much as a quantum electron in a Coulomb potential does. Such "Planck star"[3] phenomenology has been considered before.[4–14] The picture is like Giddings's remnants,[15] with a macroscopic remnant developing into a white hole.[16] In particular, here we study whether this picture is compatible with a realistic metric *satisfying the Einstein equations where classical GR can be trusted, which includes regions inside the horizon.*

Surprisingly, we find that such a metric exists: it is an exact solution of the Einstein equations everywhere including a portion inside the Schwarzschild radius r_s, except for a *finite* — small, as we shall see — region, surrounding the points where the classical Einstein equations are likely to fail. It describes conventional in-falling and then out-coming matter. The reason this metric has so far escaped notice is that it is locally isometric to the Kruskal solution (outside the quantum region), but it is *not* a portion of the Kruskal solution. Rather, it is a portion of a double cover of the Kruskal solution, in the sense that there are *distinct* regions isomorphic to *the same* Kruskal region.

A number of indications make this scenario plausible. Hájíček and Kiefer[17,18] have studied the quantum dynamics of a null spherical shell coupled to gravity and shown that an in-falling wave packet can tunnel ("bounce") into an expanding one and Ambrus and Hájíček[19] have attempted a calculation of the bouncing time. Here we show that the Hájíček–Kiefer external solution can be extended to include a classical portion of the *interior* of the black hole, as well as a *later* portion of the *interior* of the white hole. The existence of this solution of the Einstein equtions shows that it is possible to have a black hole bouncing out into a white hole *without affecting spacetime in the regions where we expect the classical theory to be good.*

A distant observer sees a dimming, frozen star that reemerges, bouncing out after a very long time, determined by the star's mass and Planck's constant. The importance of studying this scenario cannot be underestimated, because, if realized, it could yield directly observable quantum gravitational phenomena.[20,21]

For this scenario to be possible, genuine quantum gravitational effects should appear outside the horizon. These are suppressed in the approximation provided by local quantum field theory on a curved geometry. Their possibility, however, cannot be ruled out in a non-perturbative quantum theory of gravity, and is increasingly considered plausible by a number of author, on the basis of diverse considerations.[22–25] These converge in suggesting that local quantum field theory might fail to account for quantum gravity phenomena. In the following, we give further arguments in this direction, we construct the metric describing this physics explicitly, then we discuss how quantum mechanics can describe the bounce across the non-classical region. We also estimate the radius at which these effects are stronger (which can relevant for Ref. 22) and the order of magnitude of the bouncing time, obtaining a result more realistic than the one in Ref. 19.

We search for the metric of a bouncing star by gluing a collapsing region with its time reversal.[26] This describes an elastic process that disregards dissipative effects, which are not time-reversal symmetric. In particular, we disregard Hawking radiation. As we shall see, the process we are studying could be faster than Hawking's (extremely long) evaporation time, so that this can be disregarded in a first approximation (see Refs. 27 and 28).

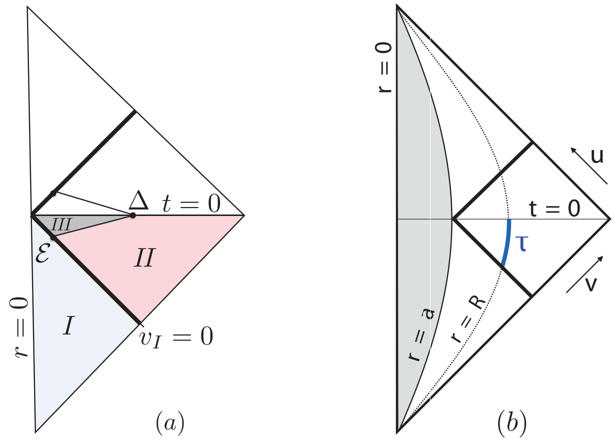

Fig. 1. (a) The spacetime of a bouncing star. (b) The mirrored ball is in grey, the thick lines represent the bouncing shell of light. The dotted line is the observer and the bounce time τ is indicated.

We start with a spherically symmetric spacetime with an in-falling spherical shell of (for simplicity) null matter. The shell moves in from past null infinity, enters its own Schwarzschild radius r_s, keeps ingoing, enters the quantum region, bounces, and then exits r_s and moves outwards to infinity. We assume that there are no event horizons: the causal structure is that of Minkowski spacetime, and we demand that the quantum process is quasilocal, i.e. confined to a finite region of spacetime.

Because of spherical symmetry, we can use coordinates (u, v, θ, ϕ) with u and v null coordinates in the r-t plane. The metric is then determined by two functions of u and v: $ds^2 = -F(u,v)dudv + r^2(u,v)(d\theta^2 + \sin^2\theta d\phi^2)$. We take the $t = v - u = 0$ hyperplane as the surface of reflection under time reversal and represent it in a conformal diagram by an horizontal line, see Fig. 1(a). By symmetry, the bounce must be at $t = 0$. For simplicity, we assume that it is at $r = 0$. The two shells are represented by the two thick lines in Fig. 1(a). Two significant points, Δ and $\mathcal{E}$ lie on the boundary of the quantum region. The point Δ has $t = 0$ and is the maximal extension in space of the region where the Einstein equations are violated. Point $\mathcal{E}$ is where the shell enters the quantum region.

Thanks to time-reversal symmetry, it is sufficient to construct the metric below $t = 0$. The spacetime splits into three regions. See Fig. 1(a): Region I, inside the shell, must be flat by Birkhoff's theorem. Region II, again by Birkhoff's theorem, must be a portion of the Kruskal–Szekeres metric of a mass m. Finally, region III is where quantum gravity becomes non-negligible. The only requirement we impose about the (effective) metric here is to join smoothly the rest of the spacetime.

Region I is easy: the Minkoswki metric in null coordinates (u_I, v_I) is given by $F(u_I, v_I) = 1$ and $r_I(u_I, v_I) = \frac{v_I - u_I}{2}$. It is bounded by the past light cone of the origin, that is, by $v_I = 0$.

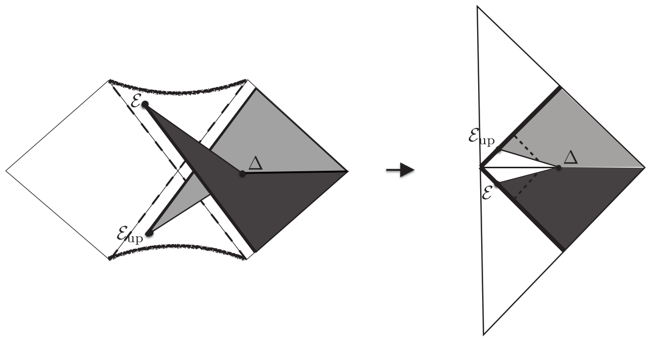

Fig. 2. The portion of a classical black hole spacetime which is reproduced in the quantum case. The contours $r = 2m$ are indicated in both panels by dashed lines.

Region II is a portion of Kruskal spacetime. Consider the darker portion of Fig. 2 bounded on the left by an ingoing null shell. The point Δ is a generic point in the region outside the horizon; we take it on the $t = 0$ surface, so that the gluing with the future is immediate. Crucially, $\mathcal{E}$ must be inside the horizon, because when the shell enters the horizon the physics is still classical. Therefore the region II is isometric to the darkly shaded region of Kruskal spacetime depicted in Fig. 2, which includes a portion of the black hole's interior. In null Kruskal–Szekeres coordinates (u, v) the metric of the Kruskal spacetime is given by $F(u, v) = \frac{32m^3}{r} e^{-\frac{r}{2m}}$ with r the function of (u, v) defined by $\left(1 - \frac{r}{2m}\right) e^{\frac{r}{2m}} = uv$. The region of interest is bounded by a constant $v = v_o$ null line. The constant v_o cannot vanish, because $v = 0$ is an horizon, which is not the case for the in-falling shell. Therefore v_o is a constant that determines our metric.

Matching regions I and II is key to what follows. The v coordinates match simply by identifying $v_I = 0$ with $v = v_o$. The matching of the u coordinate is determined by the requirement that the radius must be equal across the matching, that is $r_I(u_I, v_I) = r(u, v)$. This gives $u(u_I) = \frac{1}{v_o}\left(1 + \frac{u_I}{4m}\right) e^{-\frac{u_I}{4m}}$. If the induced 3-metrics on the boundaries agree it turns out that it is not necessary to impose further conditions:[29–31] the extrinsic geometries also match.

The matching condition between the region II and its time reversal along the $t = 0$ surface is immediate. But note that the ensemble of these two regions is *not* a portion of Kruskal space, but rather a portion of a double cover of it, as in Fig. 2: the bouncing metric is obtained by "opening up" the two overlapping flaps in the figure and inserting a quantum region in between.

Finally, it is easy to give an ansatz for the metric in the quantum region III. We do not do it here explicitly because a credible effective metric for this region, including a determination of the points $\mathcal{E}$ and Δ and the line connecting them, requires a better understanding of quantum gravity. We take $\mathcal{E}$ to be the point that has (u_I, v_I) coordinates $(-2\epsilon, 0)$ and Δ the point that has Schwarzschild radius $r_\Delta = 2m + \delta$ and lies on the symmetry line $u + v = 0$. The value ϵ is determined by the onset of quantum effects on the collapsing shell. The point Δ must be outside the point where the two shells cross in the left panel of Fig. 2. Therefore δ is positive and its lowest possible value is determined by v_0.

The parameter v_o codes the time it takes the full process to happen. This can be seen as follows. Consider an observer at fixed radius $R > r_\Delta$. A straightforward calculation gives the proper time of this observer between its encounter with the ingoing shell and $t = 0$

$$\tau_R = \sqrt{1 - \frac{2m}{R}} \left(R + 2m \ln \frac{R - 2m}{2m} - 4m \ln v_o \right) . \qquad (1)$$

If v_o is very small, the last term dominates all other scales in the problem, and for a distant observer ($R \gg 2m$) we can write an "asymptotic bounce time" $\tau = -4m \ln v_o$.

Remarkably, the duration of the process seen by an outside observer is arbitrarily long in comparison to the one measured by an observer inside the collapsing shell: the proper time between the two encounters with the shell, measured in the interior Minkowski metric, in the frame of the shell, is only $2R$. Therefore the process we are describing can be very fast seen from the interior of the star, but very slow, due to relativistic time dilatation, viewed from the outside. An astrophysical black hole could be a bouncing star seen in extremely slow motion because of its colossal relativistic time dilation. A metric describing the quantum bounce of a star is thus defined. According to an external observer, the entire process is characterized by two numbers: m and v_0, or, equivalently, m and the bounce time τ.

Understanding what happens in region III requires a quantum theory of gravity. To begin with, this must determine ϵ, the radius where the shell starts to be affected by quantum theory. Conventional arguments suggest that this happens when the curvature becomes Planckian; this gives $\epsilon \sim (m/m_p)^{\frac{1}{3}} l_p$,[3] where l_P is the Planck length and m_P the Planck mass (we use $c = G = 1$ units). Note that for a macroscopic m this radius is much larger than the Planck length.[3]

More importantly, since the initial data know only about m (by time translation invariance of Minkowski), the quantum theory must establish the (probabilistic) dependence of τ on m. How? The quantum region is bounded by a hypersurface whose (intrinsic and extrinsic) geometry depends on m and v_o. Given a classical boundary geometry, we can in principle compute its associated quantum transition amplitude. This amplitude is a function $A(m, \tau)$ of the parameters that determine the geometry. The parameter region where the corresponding probability becomes non-negligible, $|A(m, \tau)|^2 \sim 1$, determines a relation $\tau = \tau(m)$ which predicts the (mean) decay time for a black hole.

Since there is no classical solution that matches the in and out geometries of this region, this calculation is conceptually somewhat similar to a tunneling calculation in quantum mechanics. Formally, this is the path integral over internal geometries, contracted with the boundary state representing the boundary geometry. Concretely, this is precisely the form of the problem that is adapted for a calculation in a theory like covariant loop quantum gravity,[32] which is defined to give amplitudes associated to boundary states of the geometry.

This calculation has not yet been performed. But we can still estimate $\tau = \tau(m)$ using a simple model that mimics many features of the external metric of a bouncing hole, and to which we now turn. This model allows us to address also a key conceptual point: the possibility that quantum theory affects a region outside r_s.

Consider a ball of radius a with a reflective surface and negligible mass, at rest in flat space.[19] A shell of light, with total energy m centered on this mirrored ball, comes in from infinity, and reflects off it (Fig. 1(b)). An observer sitting at a reference radius R will measure a proper time $2\tau_R$ between the shell's inward and outgoing passes. For $a \gg 2m$, relativistic effects are negligible, and $\tau_R \sim R - a$: the time it takes light to reach the mirror. As a approaches $2m$, we enter a general relativistic regime. The metric outside the shell is Schwarzschild and a straightforward calculation gives[19]

$$\tau_R = \sqrt{1 - \frac{2m}{R}}\left(R - a - 2m \ln \frac{a - 2m}{R - 2m}\right), \tag{2}$$

which can be compared with (1). For an observer at large radius $R \gg 2m$, this expression simplifies, to $\tau_R \sim (R - a) - 2m \ln \frac{a-2m}{R}$. In addition to the non-relativistic light-travel-time $(R - a)$, there is a logarithmic, relativistic correction. When $a \to 2m$ *the bounce time becomes arbitrarily large*: $\tau_R \underset{a \to 2m}{\longrightarrow} +\infty$. This divergence is realized for any fixed value of $R > 2m$, hence observers agree that the bouncing time becomes arbitrarily long.

Notice the strong effect of general relativistic time dilatation: near the bounce, $R \sim a$, the bouncing proper time is short; the shell reaches the mirrored ball and bounces out always moving at the speed of light. Seen locally the process is fast. But since the bounce happens close to r_{s}, the slowing of the local time with respect to an observer far away is huge. This, we stress, is standard classical GR: classical GR is compatible with an incoming null shell that then expands out at a much later time.

The classical theory predicts that when a reaches $2m$ the bounce time becomes infinite: the light remains trapped forever and a singularity forms. But this picture disregards quantum theory. When and how does quantum theory enter the game, as a approaches $2m$?

By analogy with conventional quantum tunneling one may be tempted to guess an exponential dependence of the bouncing time τ on $m^2/\hbar$ as the onset of quantum phenomena. This possibility cannot be ruled out, but a number of argument makes it less plausible. First, the exponential weight comes from a saddle point approximation, where, however, it is balanced by a measure factor. In the case of black holes, this is the same factor that measures the entropy, which is exponential in the mass squared as well. As argued by S. Mathur,[24] we expect these two exponential scalings to cancel.

We can get a better insight by asking when do we expect the quantum theory to fail. Consider an observer at a small radius R not much larger than a. The curvature at the observer's position is constant in time after the shell has moved in, and is of the order of $\mathcal{R} \sim m/R^3$. (The Kretschmann invariant is $\mathcal{R}^2 = R^{abcd}R_{abcd} = \frac{48m^2}{r^6}$.) We expect local quantum gravity effects to be small, but non-vanishing, in a small curvature region. An estimate of the magnitude of these effects can be obtained from the quantum corrections to the classical equations of motion. These are proportional to $\hbar$. After a proper time τ the resulting relative quantum effect is of the order $q = l_P \mathcal{R}\tau$, which implies that they can drive the classical solution substantially ($q \sim 1$) away from the classical solution in a proper time $\tau \sim (l_P\mathcal{R})^{-1} \sim m^2/l_P$. Thus, a *cumulative* quantum effect, can be relevant also in a region of small curvature, provided that is has enough time to act: *there is no reason to trust the classical theory outside the horizon for arbitrarily long times, sufficiently close to* r_s.

Let us be more general and more precise. Assume the classicality parameter is $q = l_P^{2-b}\mathcal{R}\tau^b$, with b reasonably taken in the range $b \in \left[\frac{1}{2}, 2\right]$. A straightforward calculation in the Schwarzschild metric shows that the maximum value of q occurs at $R_q = 2m(1 + b/6) + O(1/\ln(a - 2m))$, which is a finite distance, but not much, outside r_s. (Notice the nice separation of scales: the result R_q is independent of a in the $a \to 2m$ limit.) Quantum effects can appear at a reasonable location in space: a macroscopic distance from the Schwarzschild radius, necessary for the long bounce time, but close to it, so that the curvature is still reasonably large. Requiring $q \sim 1$ at this radius gives $\tau \approx \left(2{l_p}^{1-\frac{2}{b}}k^b\right)m^{\frac{2}{b}}$, with $k = 27(4b)^{\frac{b}{2}}/(b+6)^{3+\frac{b}{2}}$. In the likely case $b = 1$ the quantum effects appear at a distance $R = \frac{7}{6}2m$ after an asymptotic time $\tau = 2k\frac{m^2}{l_P}$. That is, quite independent of the value of b, it is possible that quantum gravity affects the *exterior* of the Schwarzschild radius already at a time of order m^2. Note that this effect has nothing to do with the $r = 0$ singularity: there is no singularity, nor a horizon in the physics considered in the mirrored ball model. The common argument according to which there cannot be quantum gravity effects outside the horizon, since this region is causally disconnected from the interior of the horizon, is wrong. There is room for quantum gravity even if there is no interior of the horizon at all.

Let us return to our full bounce metric. Since $\Delta > 2m$ the classical equations fail (during a finite time) also outside the horizon: this is why the black hole horizon is not an event horizon. As we have just seen, this is not a sufficient reason for excluding quantum gravity effects in this region, as is often done. There is no causality violation, because the quantum transition outside the horizon is driven by the long time there, and not by physics from the vicinity of $r = 0$.

In conclusion, we have found a classical metric describing (the non-quantum region of) a black hole that tunnels into a white hole. Indirect arguments point to an asymptotic bounce time of the order of $\tau \sim m^2/l_P$. This is very long for a macroscopic black hole (about 10^{32} seconds for a solar mass), but is much shorter than the Hawking evaporation time, which is of order m^3. Clearly this result affects the discussion on the black hole information puzzle. In this regard, the firewall argument[33] shows that under a certain number of assumptions "something strange" appears to have to happen at the horizon of a macroscopic black hole. Here we point out that indeed it does, *independently from the Hawking process*. But, it is a less dramatic phenomenon than expected: the spacetime can quantum tunnels out of the black hole and this can happen without violating causality because over a long stretch of time quantum gravitational effects can accumulate outside the horizon.

More interesting, this phenomenon could open a novel window on quantum gravity phenomenology.[20,21]

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