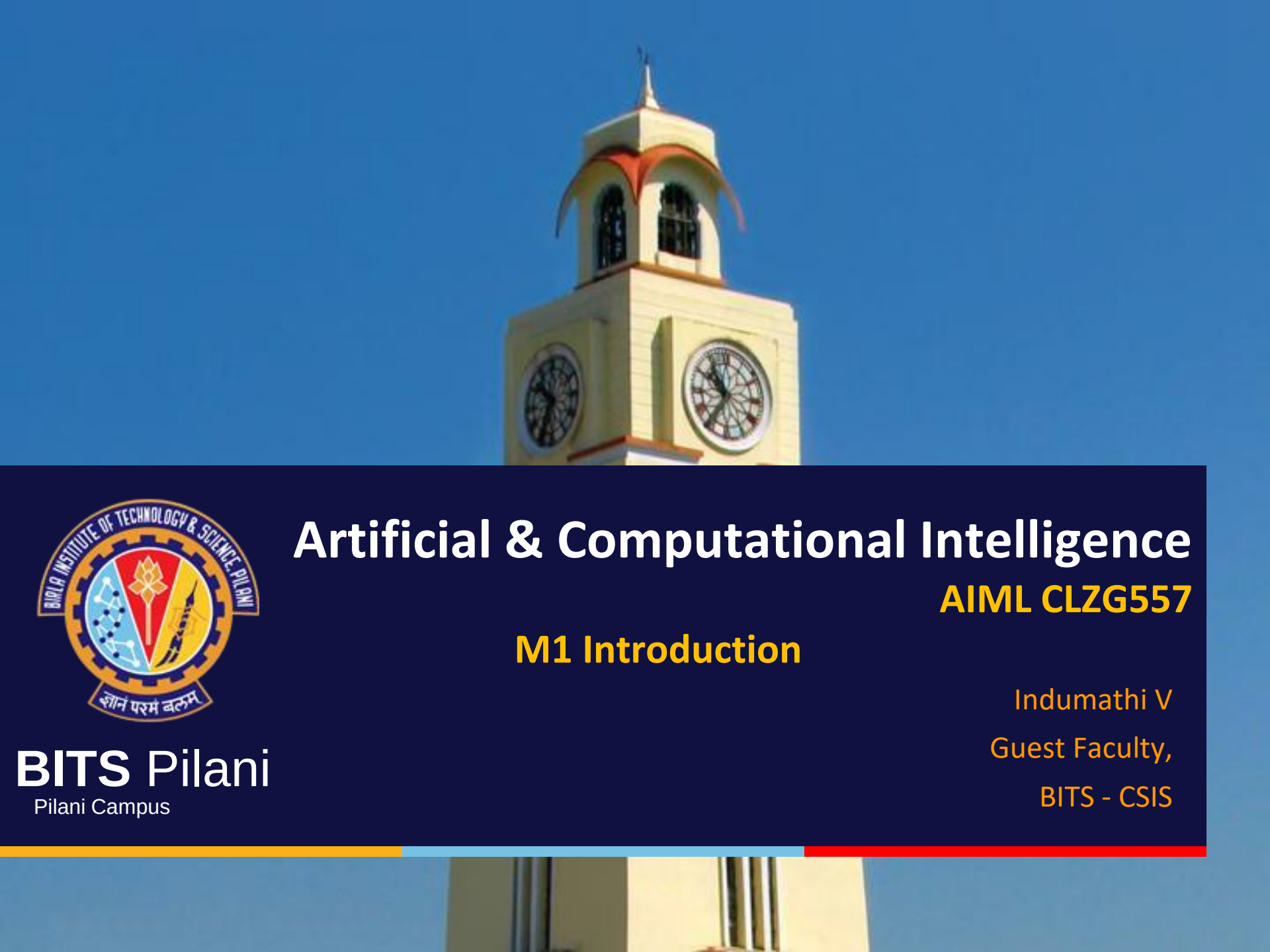


Artificial & Computational Intelligence

AIML CLZG557

M1 Introduction

Indumathi V

Guest Faculty,

BITS - CSIS

BITS Pilani

Pilani Campus

Agenda

- Course Administration
- Course Overview with example
- Getting Started (Module 1)

innovate achieve lead

Course Administration

About the course

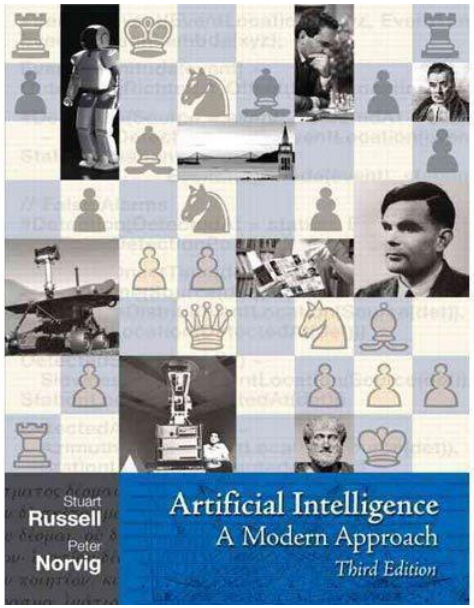
- Focus on
  - Principles of artificial intelligence
  - Concepts, algorithms involved in building rational **agents**
  - Topics covered like
    - (informed, uninformed & local) search & optimizations & applications
    - (logical & probabilistic) knowledge representation
    - (logical & probabilistic) Reasoning & applications
  - Topics not-covered like
    - Hardware aspect of the design
    - Machine Learning - Parallel CORE course - Machine Learning
    - Reinforcement Learning - There is a full fledged CORE course called "Deep Reinforcement Learning" for them in next semester.
    - Deeper applications & algorithms of application of AI in Computer Vision, Natural Language processing etc.

Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time
M7	Ethics in AI

About the course

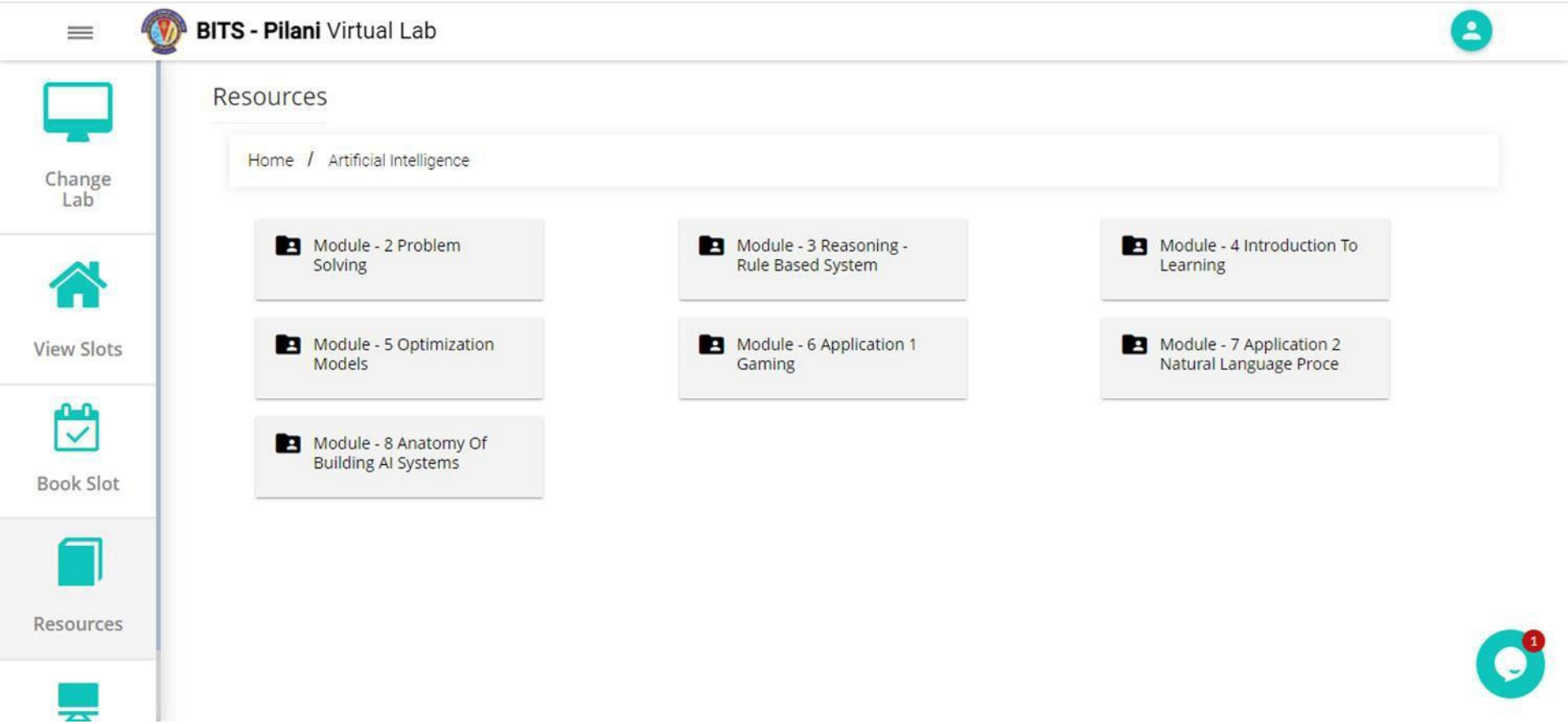
Text Book



Exercises : In Python & its libraries

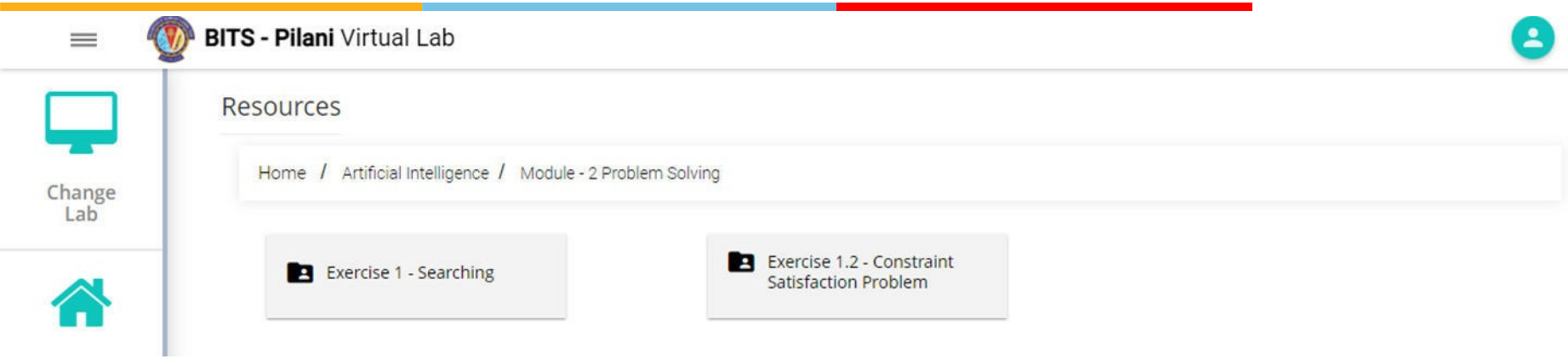
Evaluation : 25% Assignment + 5% Quiz + 30% Mid Semester Written Exam + 40% Comprehensive Written Exam

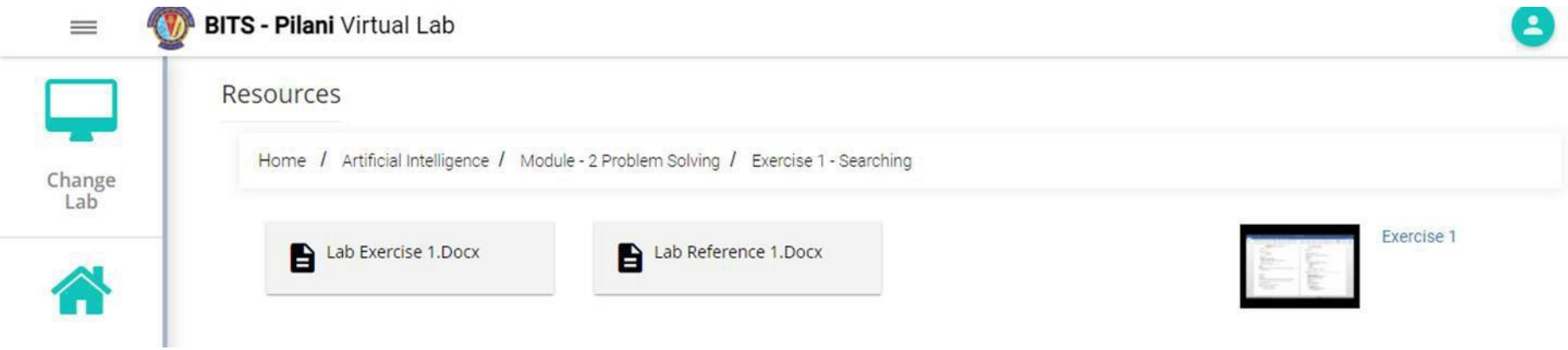
About Labs



Exercises : In Python & its libraries

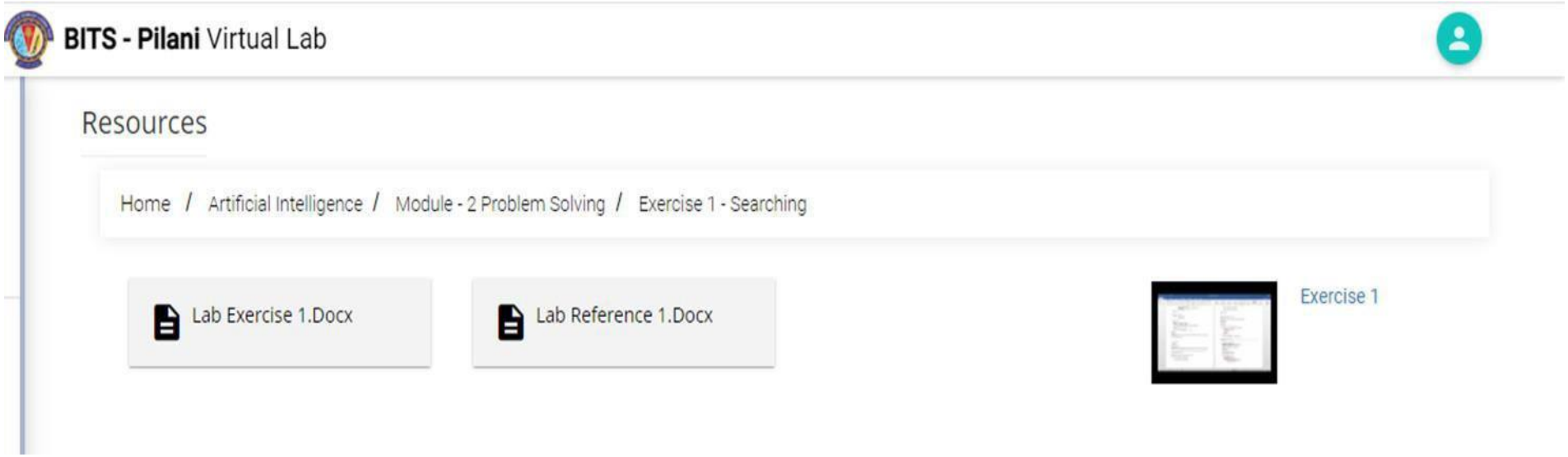
About Labs





Exercises : In Python & its libraries

About Labs



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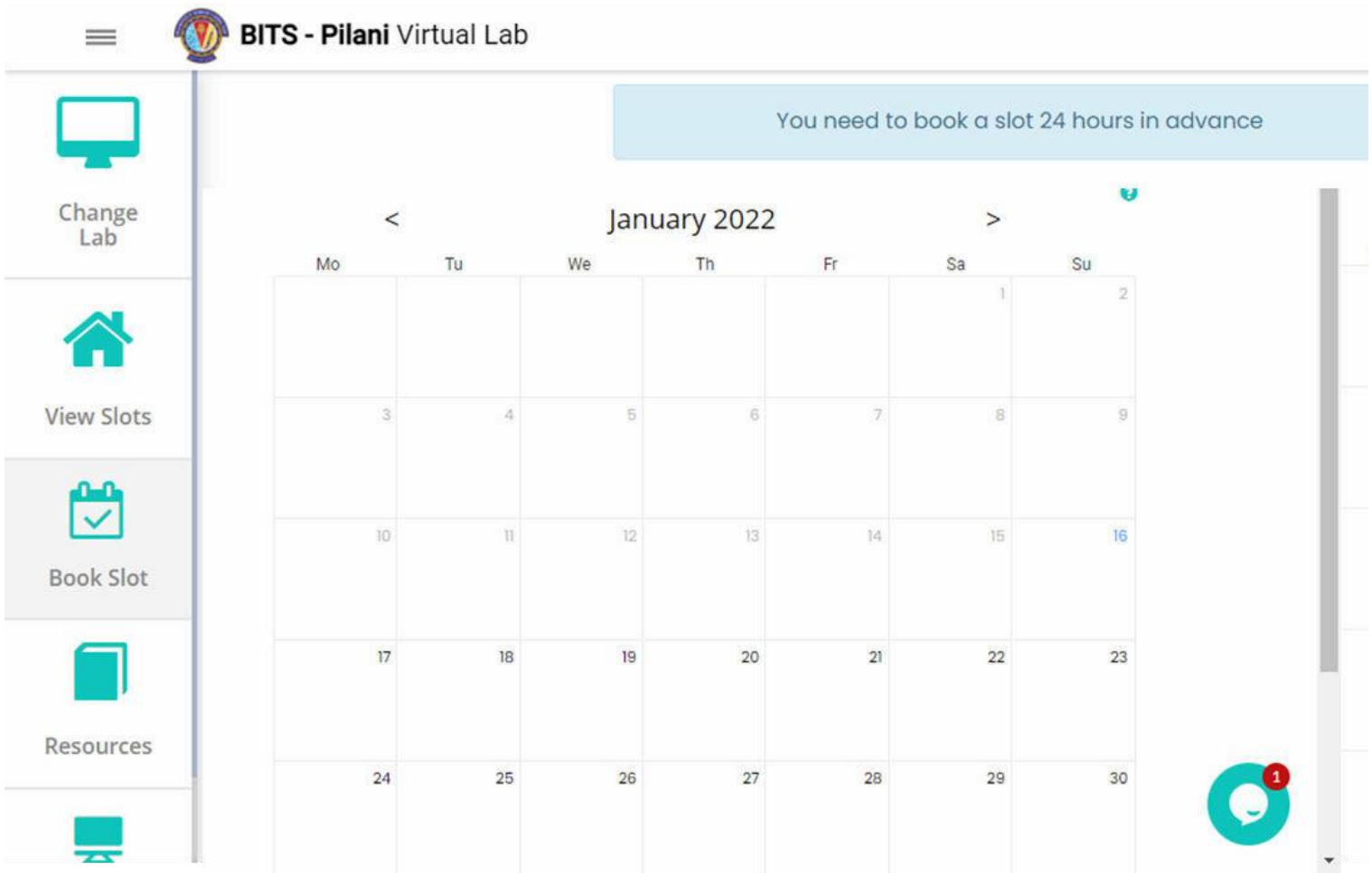
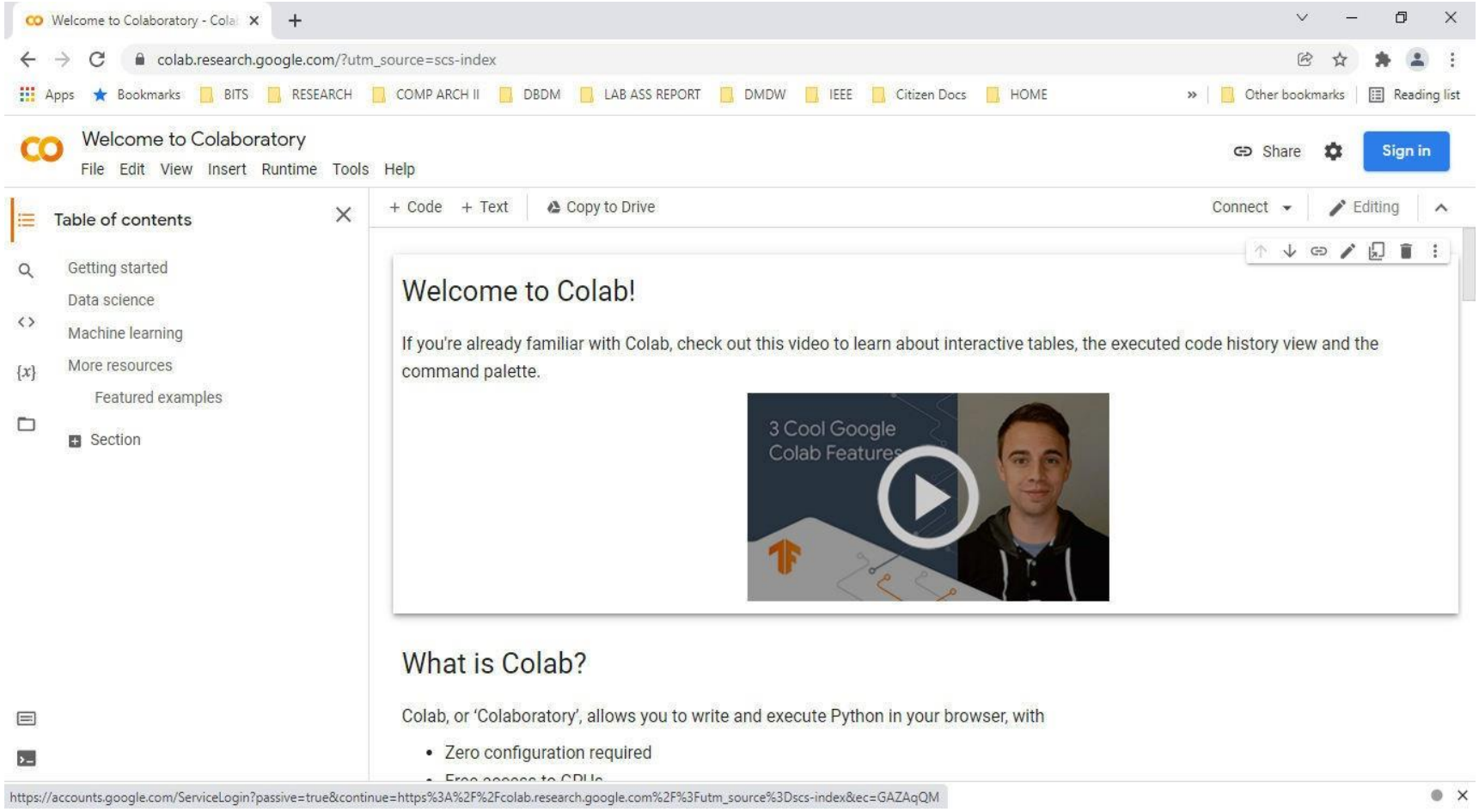


Exercises : In Python & its libraries

About Labs



Exercises : In Python & its libraries

Course Overview

Artificial Intelligence

- Term coined by, *John McCarthy* (1955) & *Dartmouth Summer Research Project on Artificial Intelligence* (1956)

On September 2, 1955, the project was formally proposed by McCarthy, Marvin Minsky, Nathaniel Rochester and Claude Shannon. The proposal is credited with introducing the term 'artificial intelligence'.

The Proposal states[7]

> We propose that a 2-month, 10-man study of artificial intelligence be carried out during the summer of 1956 at Dartmouth College in Hanover, New Hampshire. The study is to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it. An attempt will be made to find how to make machines use language, form abstractions and concepts, solve kinds of problems now reserved for humans, and improve themselves. We think that a significant advance can be made in one or more of these problems if a carefully selected group of scientists work on it together for a summer.

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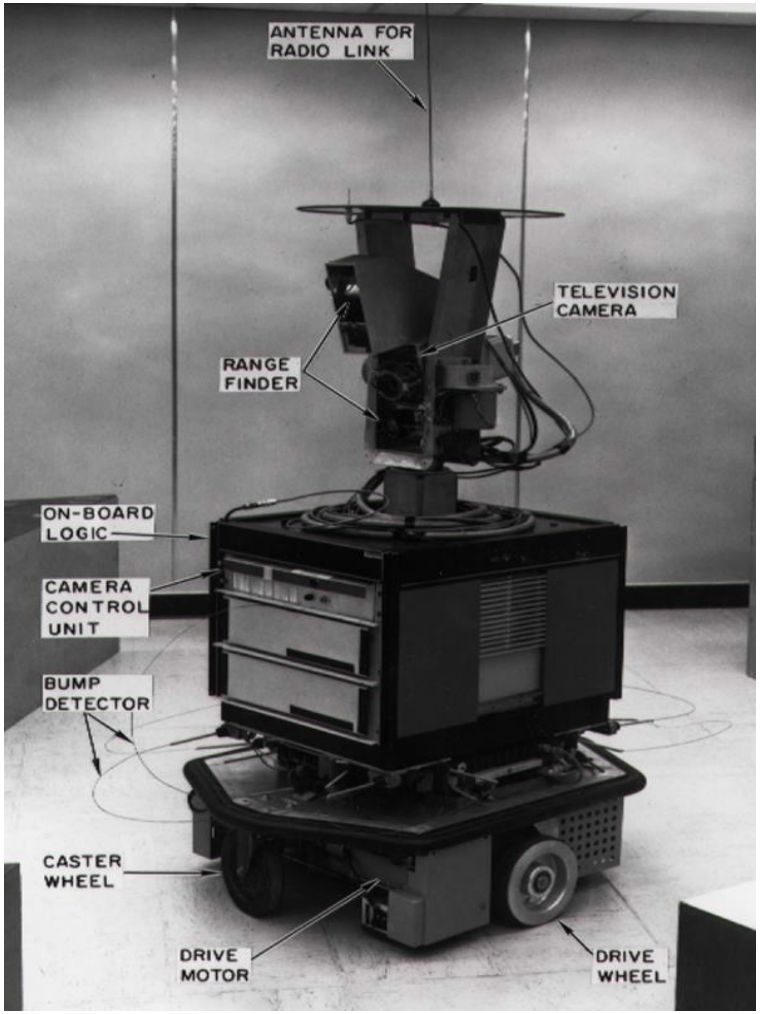
https://en.wikipedia.org/wiki/Dartmouth_workshop [01 June, 2019]

Larger Intent, Dream, Overconfidence ...

Some Early successes of Dartmouth

Many key projects were initiated after Dartmouth summer project.

Shakey robot - First mobile robot to perceive environment & reason about surroundings, actions - Introduced A* algorithm to find paths - Hough Transform for image analysis - Used Lisp for programming - visibility graph used for finding shortest paths in the presence of obstacles...

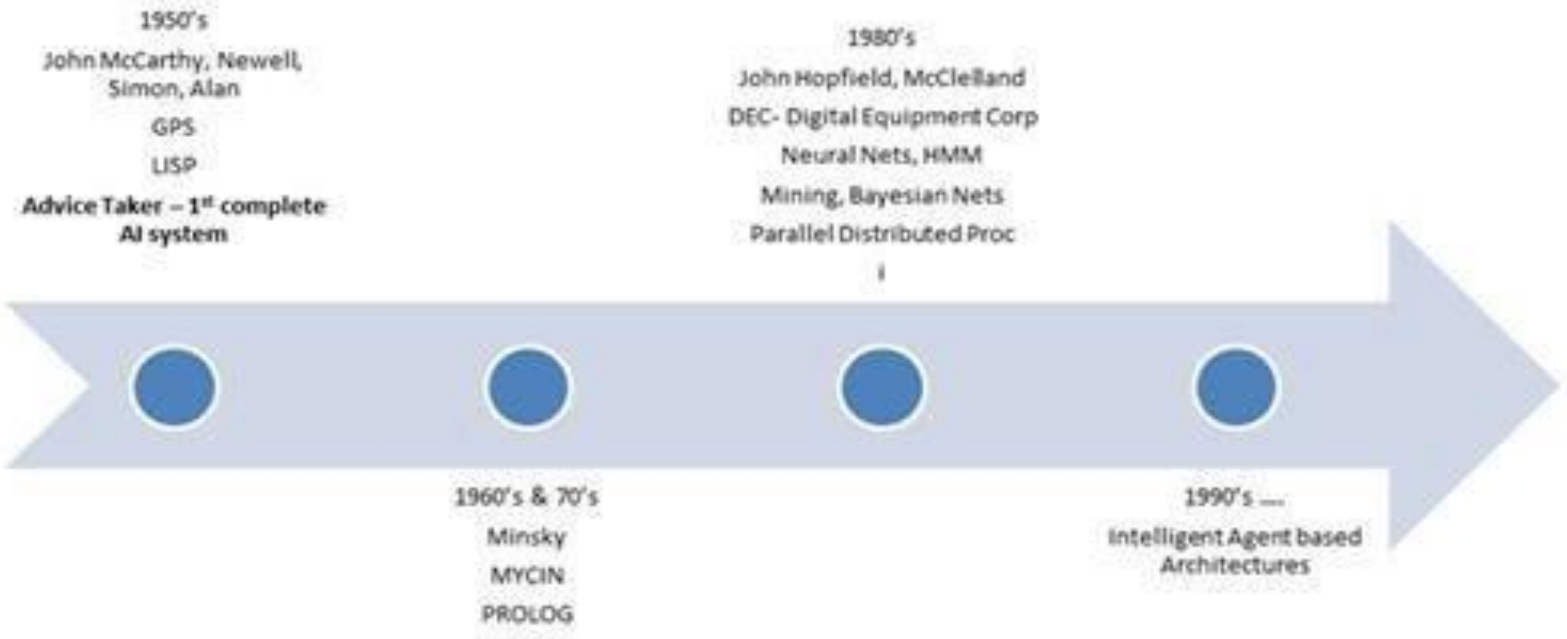


Some Early successes of Dartmouth

DENDRAL -
Attempted to encode the domain expertise in molecular biology as an expert system
Led to the creation of expert systems for various other domain, including medical.

A milestone worship in the history of AI !!!

A brief history of AI



Perspectives of AI

Definitions

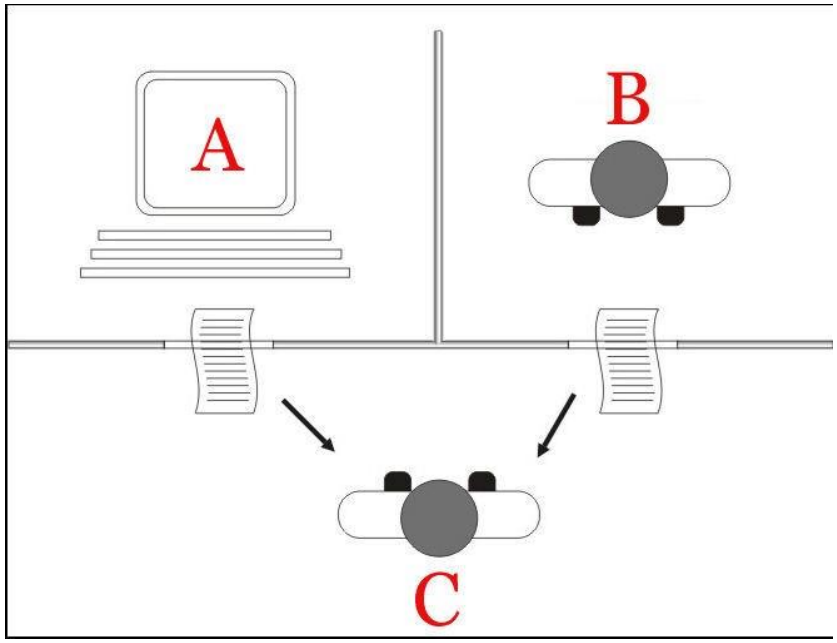
	Thought / Reasoning	Acting
Human Performance	THINKING HUMANLY "[The automation of] activities that we associate with human thinking, activities such as decision-making, problem solving, learning, ... " (Bellman, 1978)	ACTING HUMANLY "The art of creating machines that perform functions that require intelligence when performed by people" (Kurzweil, 1990)
Rational Performance	THINKING RATIONALLY "The study of computations that make it possible to perceive, reason, and act" (Winston, 1992)	ACTING RATIONALLY "Computational intelligence is the study of the design of intelligent agents" (Poole et al., 1998)

Acting Humanly

Turing Test Approach

- *Turing Test* & *Total Turing test*
  (operational test to determine an entity is intelligent / not) [50's]

- Skills necessary to pass these tests
  - NLP, Knowledge Representation, Automated Reasoning, ML + Computer Vision & Robotics(for total turing test)



Pictorial Representation of Turing Test from https://en.wikipedia.org/wiki/Turing_test

Passing the Turing Test

- 2014 - Royal Society (London) - Sixteenth Anniversary of Alan Turing -
- Chabot - Eugene Goostman - Pretended to be a thirteen-year-old Ukrainian boy
  - Passed the turing test for the first time
  - 10/30 Judges believed the response is from human

- *Turing predicted in 50 years time, computers can be programmed to play imitation game in which an average interrogator fails to identify the machine 70% time in a 5 mins questioning*

Passing the Turing Test

Transcript of a chat

EUGINE - a thirteen-year-old Ukrainian boy, chats

JUDGE: Hello.

EUGENE: Hello, I'm really glad to have the chance to chat with you! My guinea pig Bill sends his regards too!

JUDGE: Is Bill a male or a female?

EUGENE: Ask Bill personally, please.

JUDGE: Well I'd rather talk to you. What is your name?

EUGENE: Call me Eugene. I am glad to talk to you!

JUDGE: My name is Jane and I am female. How about you? What's your gender?

EUGENE: I'm a male. A “guy’ I'd say.

JUDGE: Pleased to meet you Eugene. What's the weather like where you are?

EUGENE: Let's get on with our conversation!

JUDGE: Don't you like talking about the weather?

EUGENE: All these talks about weather is a waste of time.

JUDGE: What would you like to discuss?

EUGENE: I don't know. Better tell me more about yourself!

Acting Humanly

Turing Test Approach

Some Definitions of AI:

"The art of creating machines that perform functions that require intelligence when performed by people." (Kurzweil, 1990)

"The study of how to make computers do things at which, at the moment, people are better." (Rich and Knight, 1991)

Thinking Humanly

Cognitive Modelling Approach

- How do we capture human thinking to implement?
  - Introspection
  - Psychological Experiments
  - Brain Imaging

- System : *"General Problem Solver"* (*Newell and Simon, 1961*)
  - Designed to work as a universal problem solver
  - Problems represented by horn clauses
  - First AI Machine which has KB + Inference separation
  - Authors focus on this is on comparing the trace of its reasoning steps to traces of human subjects solving the same problems

- Growth of Cognitive science and AI supports each other

Thinking Humanly

Cognitive Modelling Approach

Some Definitions of AI:

"The exciting new effort to make computers think . . . machines with minds, in the full and literal sense." (Haugeland, 1985)

"[The automation of] activities that we associate with human thinking, activities such as decision-making, problem solving, learning . . ." (Bellman, 1978)

Thinking Rationally

"Laws of Thought" Approach

- Invention of Formal Logic, Greek Philosopher **Aristotle**, Third century BC.
- Introduced syllogisms, providing argument structures

In all boring classes, students sleep It

is a boring class

Students sleep in this class [Are you ?
]

- Field of Logics gave rise to codifying rational thinking
    - When elements are '*things*', we reason about things

Hurdles to the idea :
(1) Not everything can be logically coded
(2) no provably correct action at a moment
(3) Exhaustive computational resources

Acting Rationally

The Rational Agent Approach

- An agent is an entity that perceives and acts

This course is about designing rational agents

- Abstractly, an agent is a function from percept histories to actions: [f: P* → A]
- For any given class of environments and tasks, we seek the agent (or class of agents) with the best performance
- Computational limitations make perfect rationality unachievable
- Design best program for given machine resources

Acting Rationally

The Rational Agent Approach

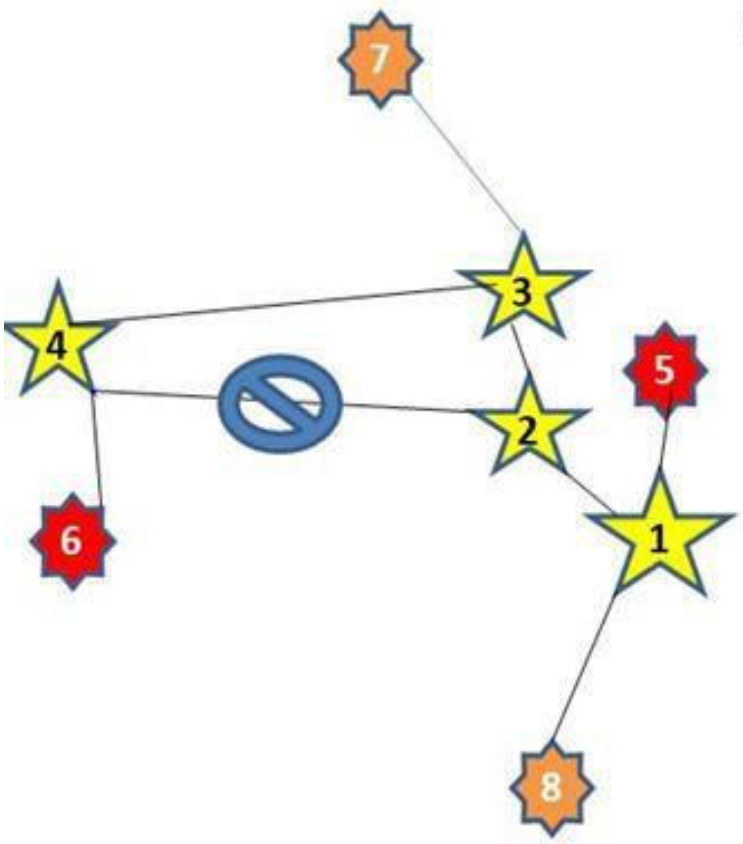
- Rational behaviour: doing the *right thing*
- The *right thing:* that which is expected to maximize goal achievement, given the available information
- Rational behaviour is not just about correct inference / thinking, skills needed to pass turing test etc.

(adv) : More General - Correct inference is just a thing

(adv) : More amenable for scientific developments, as the rational behaviour is better defined than human thinking and behaviour

Traveller's Problem

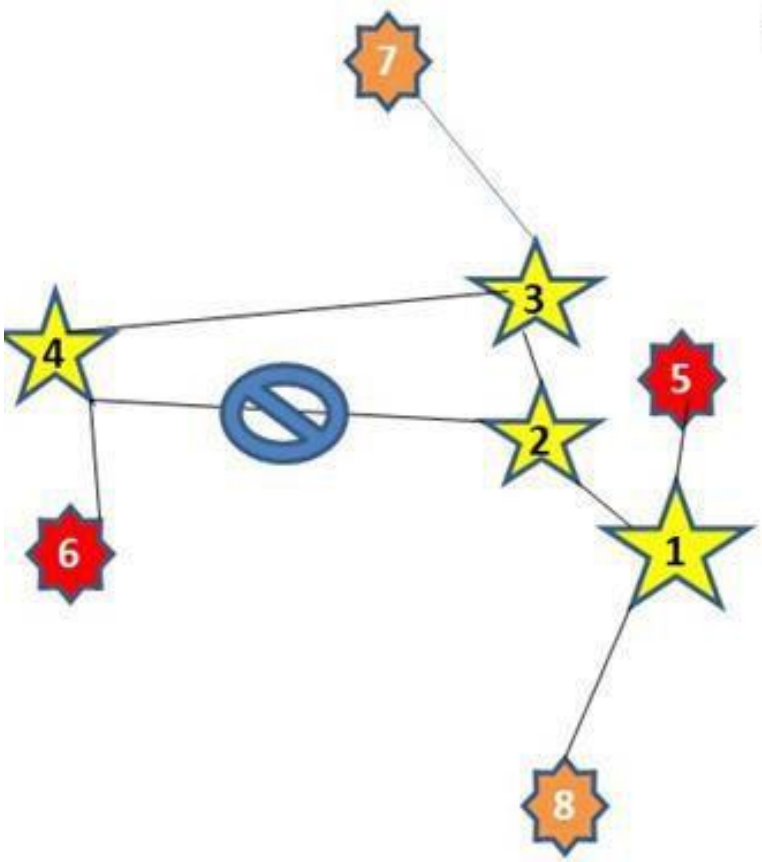
Traveller's Problem



Sensors → Environment → Actuators

Sketch the problem	Computer Vision
Searching Technique	
Path Finding	Planning - Constraint
Derive Solution/s	Automated Reasoning
Improve Solution	Optimization Problems
Suggest or Act	Natural Language Processing / Robotics

Traveller's Problem



Sensors → Environment → Actuators

Sketch the problem

Searching Technique

Path Finding

Derive Solution/s ← Knowledge Representation

Improve Solution

Machine Learning

Suggest or Act

AI in HealthCare

Lyrebird's Project Re-Voice

AI in Culinary Field

Spyce

AI in Transportation

AI in NLS

IBM Watson

Computer Vision
NLP
ML
Speech Recognition
Automation

Required Reading: AIMA - Chapter # 1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML **CLZG557**

M1 : Introduction
&
M2 : Problem Solving Agent using Search

Indumathi V

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Rational Agents

Rational Agent

	Thought / Reasoning	Acting
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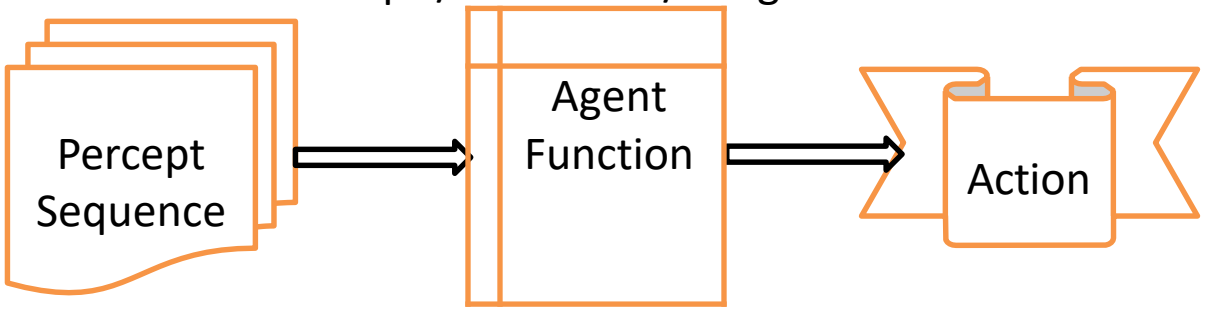
Properties of Rational Agent

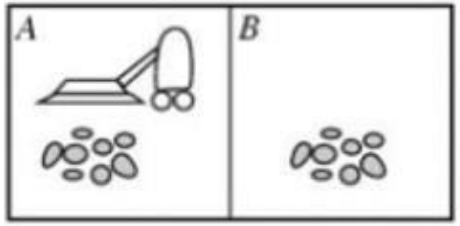
- Omniscience : Expected Vs Actual Performance
- Learning Capability : Apriori Knowledge
- Autonomous in decision making: An agent is autonomous if its behaviour is determined by its own experience (with ability to learn and adapt)

Intelligent Agent

Rational Agent is one that acts to achieve the best outcome or the best expected outcome even under uncertainty

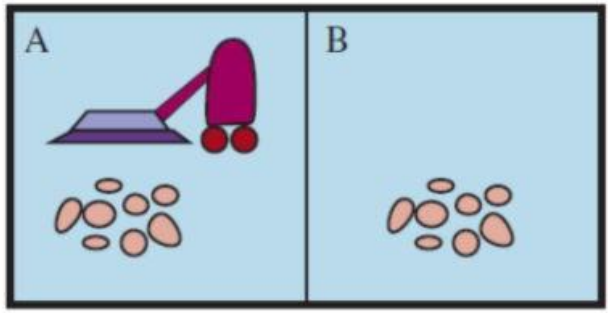
Maps / Tabulated / Programmed





Percept sequence	Action
[A,Clean]	Right
[A, Dirty]	Suck
[B, Clean]	Left
[B, Dirty]	Suck
[A, Clean],[A, Clean]	Right
[A, Clean],[A, Dirty]	Suck
...	...

Intelligent Agent



- Percepts: location and contents, e.g., [A , Dirty]
- Actions: *Left*, *Right*, *Suck*, *NoOp*

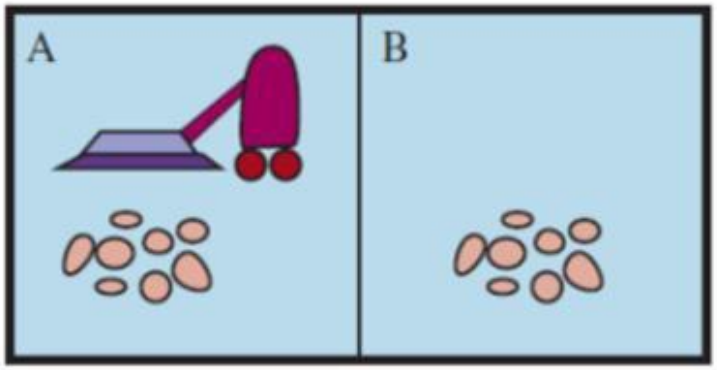
Performance measure: An objective criterion for success of an agent's behaviour

E.g., performance measure of a vacuum-cleaner agent
» amount of dirt cleaned up
» amount of time taken
» amount of electricity consumed
» amount of noise generated, etc.

PEAS Design

Intelligent Agent

Percept sequence	Action
[A, Clean]	Right
[A, Dirty]	Suck
[B, Clean]	Left
[B, Dirty]	Suck
[A, Clean], [A, Clean]	Right
[A, Clean], [A, Dirty]	Suck
⋮	⋮
[A, Clean], [A, Clean], [A, Clean]	Right
[A, Clean], [A, Clean], [A, Dirty]	Suck
⋮	⋮

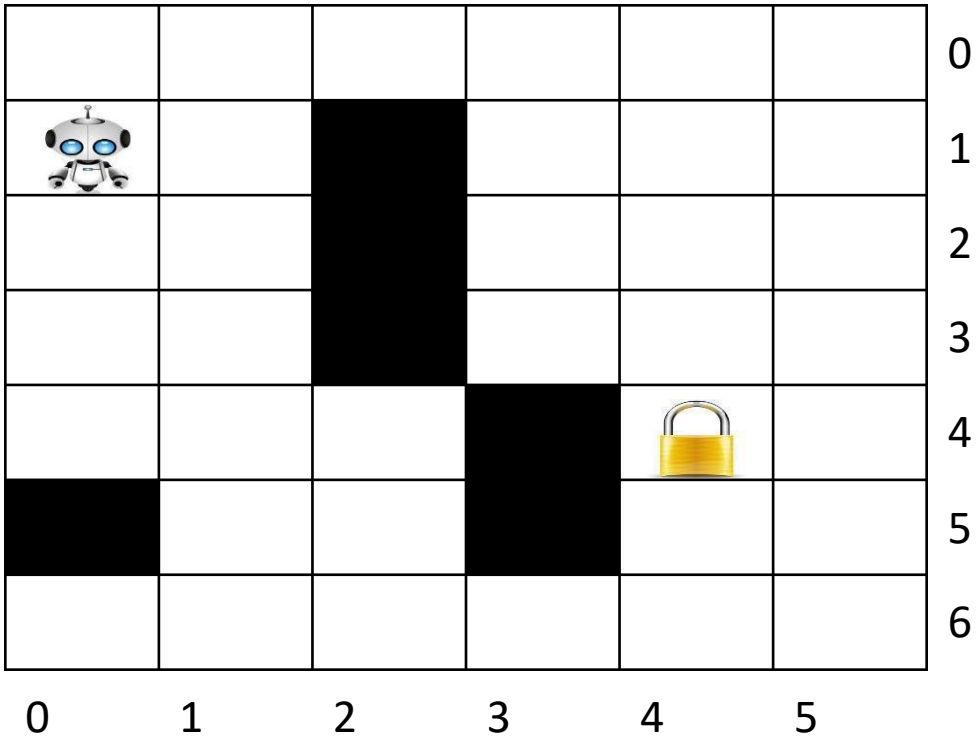


PEAS Environment

Design on what an application wants the agent to do in the environment

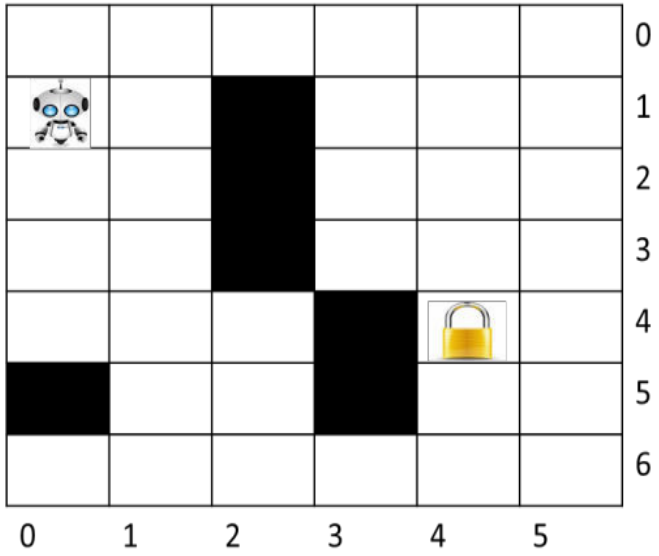
Agent	Performance	Environment	Sensors	Actuators
Medical diagnosis system	Healthy patient, reduced costs	Patient, hospital, staff	Keyboard entry of symptoms, findings, patient's answers	Display of questions, tests, diagnosis, treatments, referrals
Satellite Image analysis system	Correct image categorization	Downlink from orbiting satellite	Color pixel analysis	Display of scene categorization
Interactive English tutor	Student's score on test	Set of students, testing agency	Keyboard entry	Display of exercises, suggestions, corrections

Path finding Robot - Lab Example

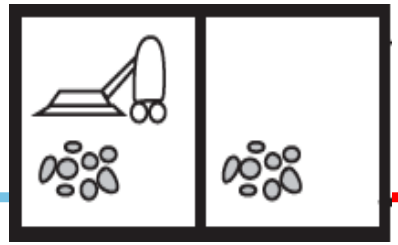


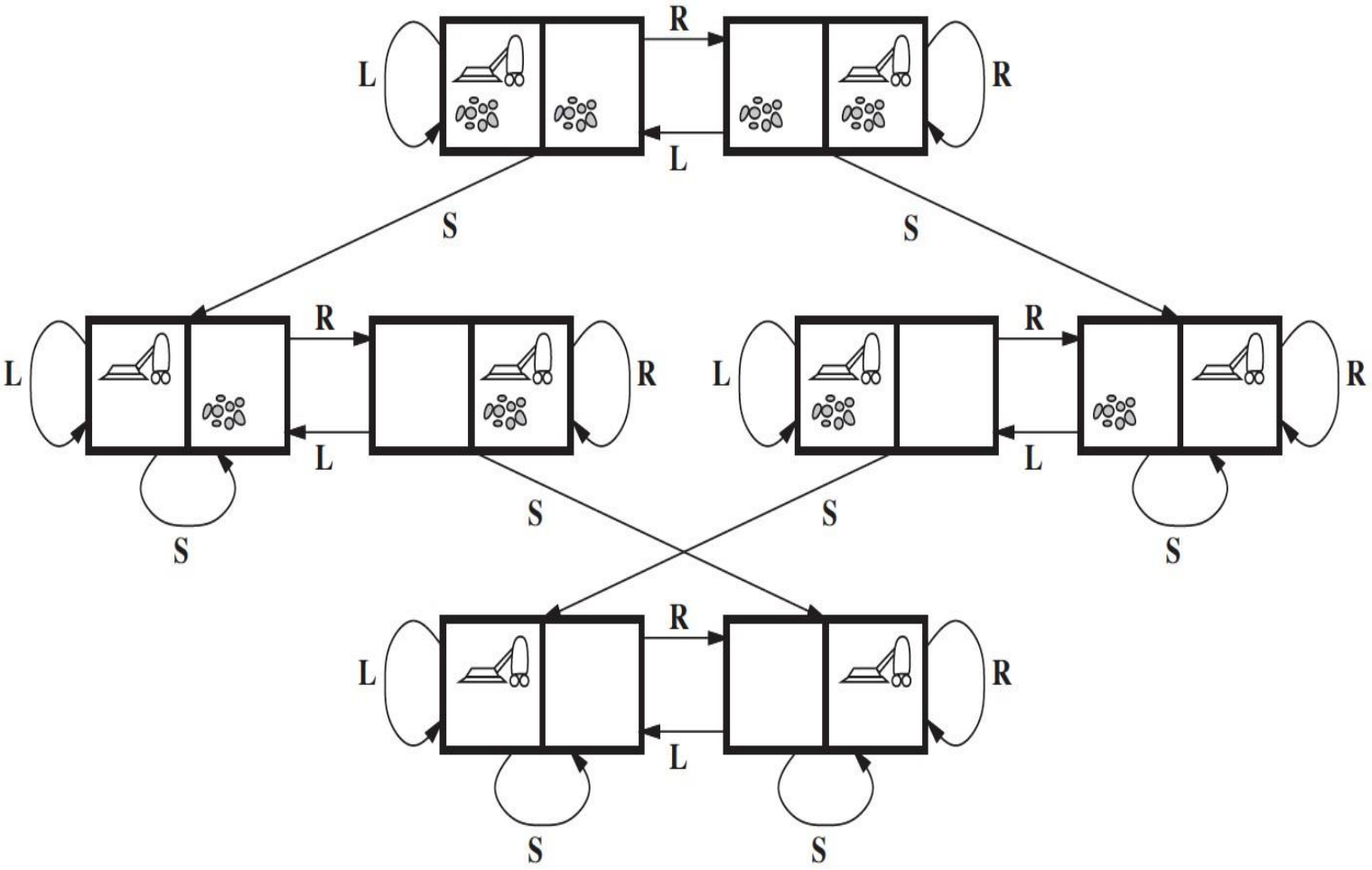
PEAS Environment

Agent
Performance
Environment
Sensors
Actuator



Vacuum World Problem





Dimensions of Task Environment

Sensor Based:

- Observability : Full Vs Partial

Action Based:

- Dependency : Episodic Vs Sequential

State Based:

- No.of.State : Discrete Vs Continuous

Agent Based:

> Cardinality : Single Vs MultiAgent

Action & State Based:

- State Determinism : Deterministic Vs Stochastic | Strategic
- Change in Time : Static Vs Dynamic

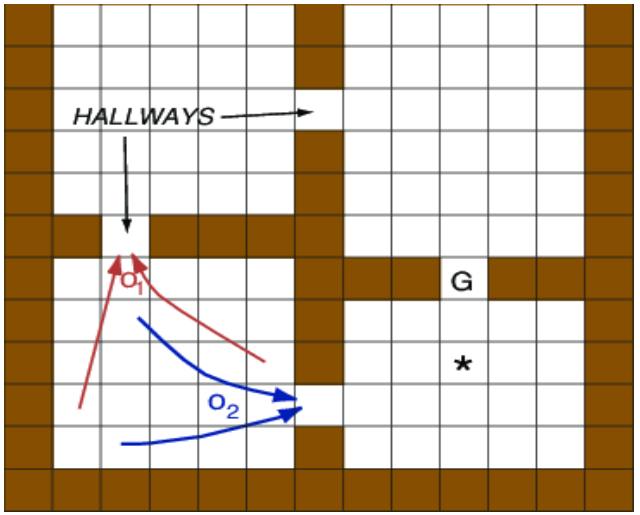
Task Environment

A rational agent is built to solve a specific task. Each such task would then have a
different environment which we refer to as Task Environment

Based on the applicability of each technique for agent implementation its task environment design is determined by multiple dimension

Sensor Based:

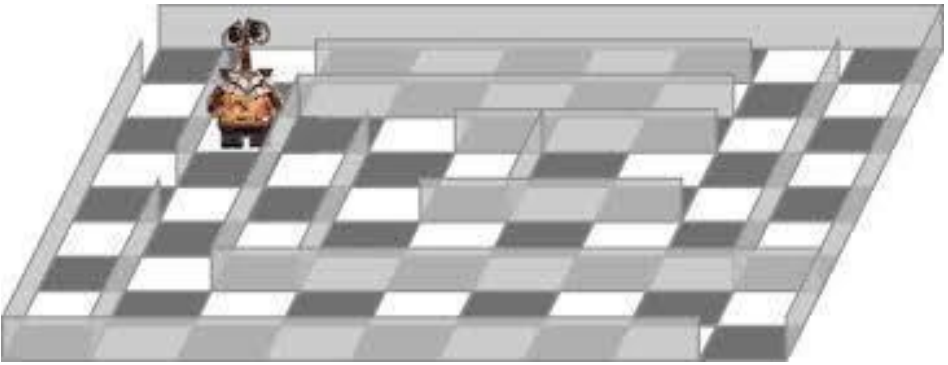
- Observability : Full Vs Partial



Task Environment

Action Based:

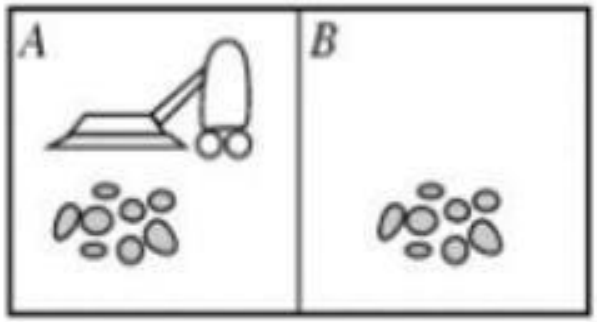
- Dependency : Episodic Vs Sequential

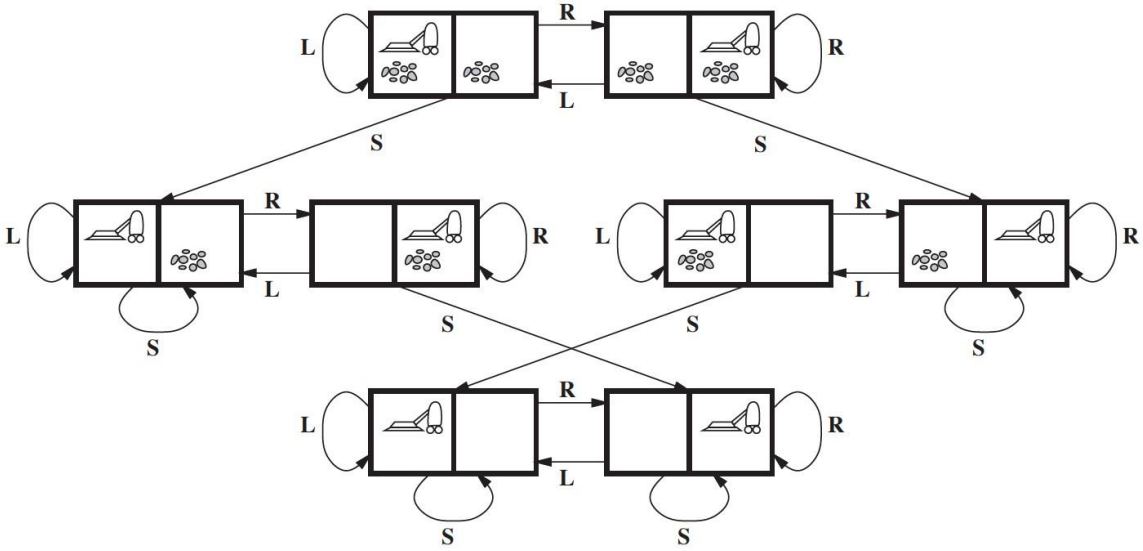


Task Environment

State Based:

- No.of.State : **Discrete** Vs Continuous





Task Environment

State Based:

- No.of.State : Discrete Vs Continuous

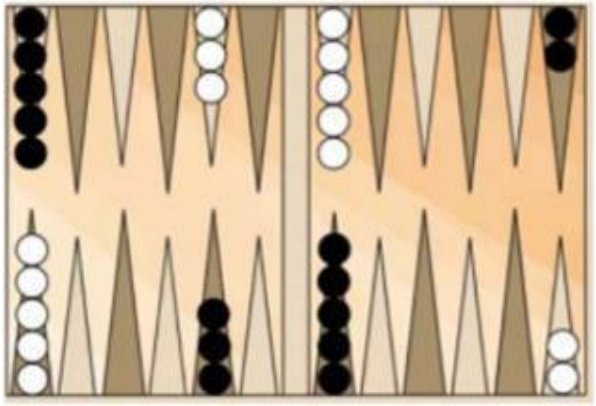
VS.

Task Environment

Action & State Based:

- State Determinism : Deterministic Vs Stochastic | Strategic

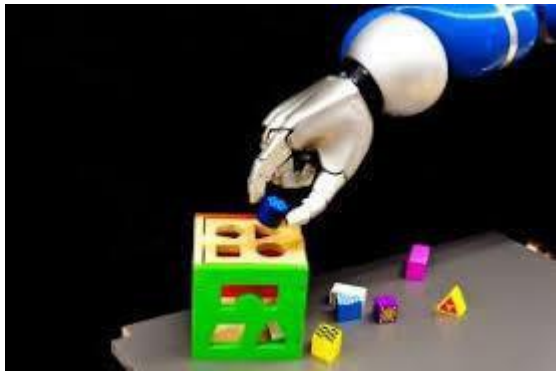
(If the environment is deterministic except for the actions of other agents, then the environment is strategic)



Task Environment

Agent Based:

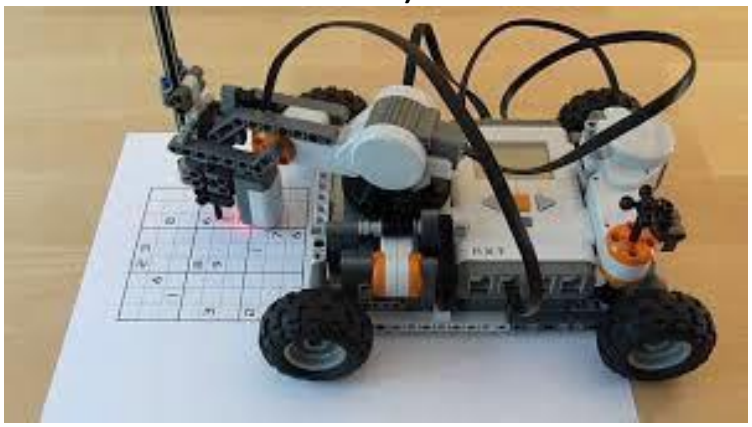
> Cardinality : Single Vs MultiAgent



Task Environment

Action & State Based:

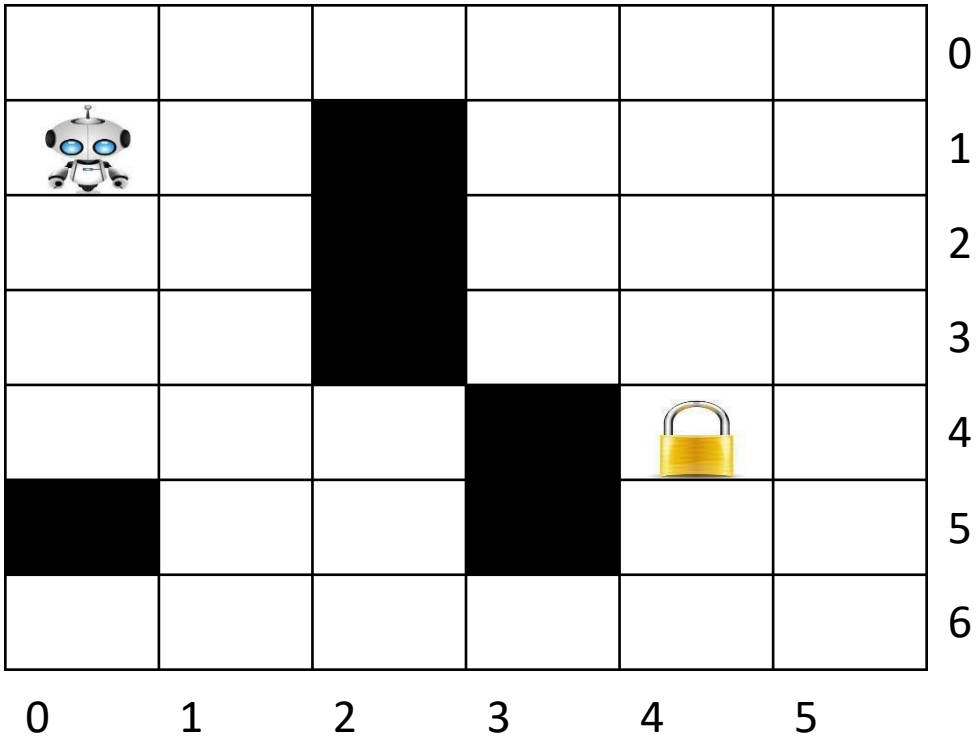
- Change in Time : Static Vs Dynamic
- (The environment is semi dynamic if the environment itself does not change with the passage of time but the agent's performance score does)



Task Environment

Task Environment	Fully vs Partially Observable	Single vs Multi-Agent	Deterministic vs Stochastic	Episodic vs Sequential	Static vs Dynamic	Discrete vs Continuous
Medical diagnosis system	Partially	Single	Stochastic	Sequential	Dynamic	Continuous
Satellite Image Analysis System	Fully	Single	Deterministic	Episodic	Static	Continuous
Interactive English tutor	Partially	Multi	Stochastic	Sequential	Dynamic	Discrete

Path finding Robot - Lab Example



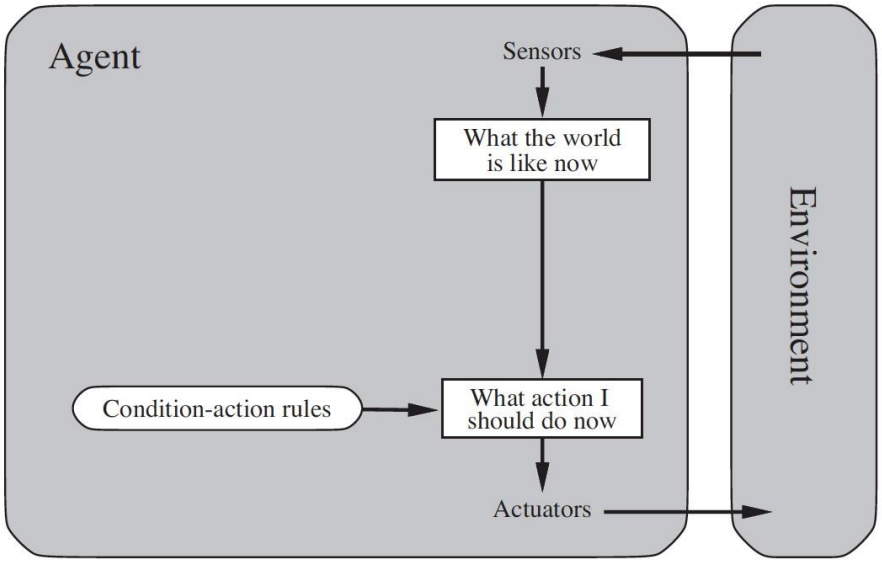
Agent
Observability
No.of.Agents
No.of.States
Determinism
Dynamicity
Output Dependency

Agent Architectures

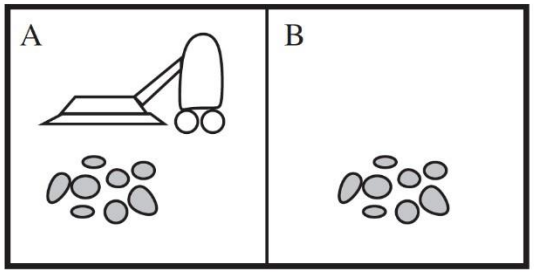
Reflex Agent

```
function SIMPLE-REFLEX-AGENT(percept) returns an action
    persistent: rules, a set of condition–action rules
    state←INTERPRET-INPUT(percept)
    rule←RULE-MATCH(state, rules)
    action ←rule.ACTION
    return action
```

```
function REFLEX-VACUUM-AGENT( [location,status]) returns an action
    if status = Dirty then return Suck
    else if location = A then return Right
    else if location = B then return Left
```



Simple Reflex Agents



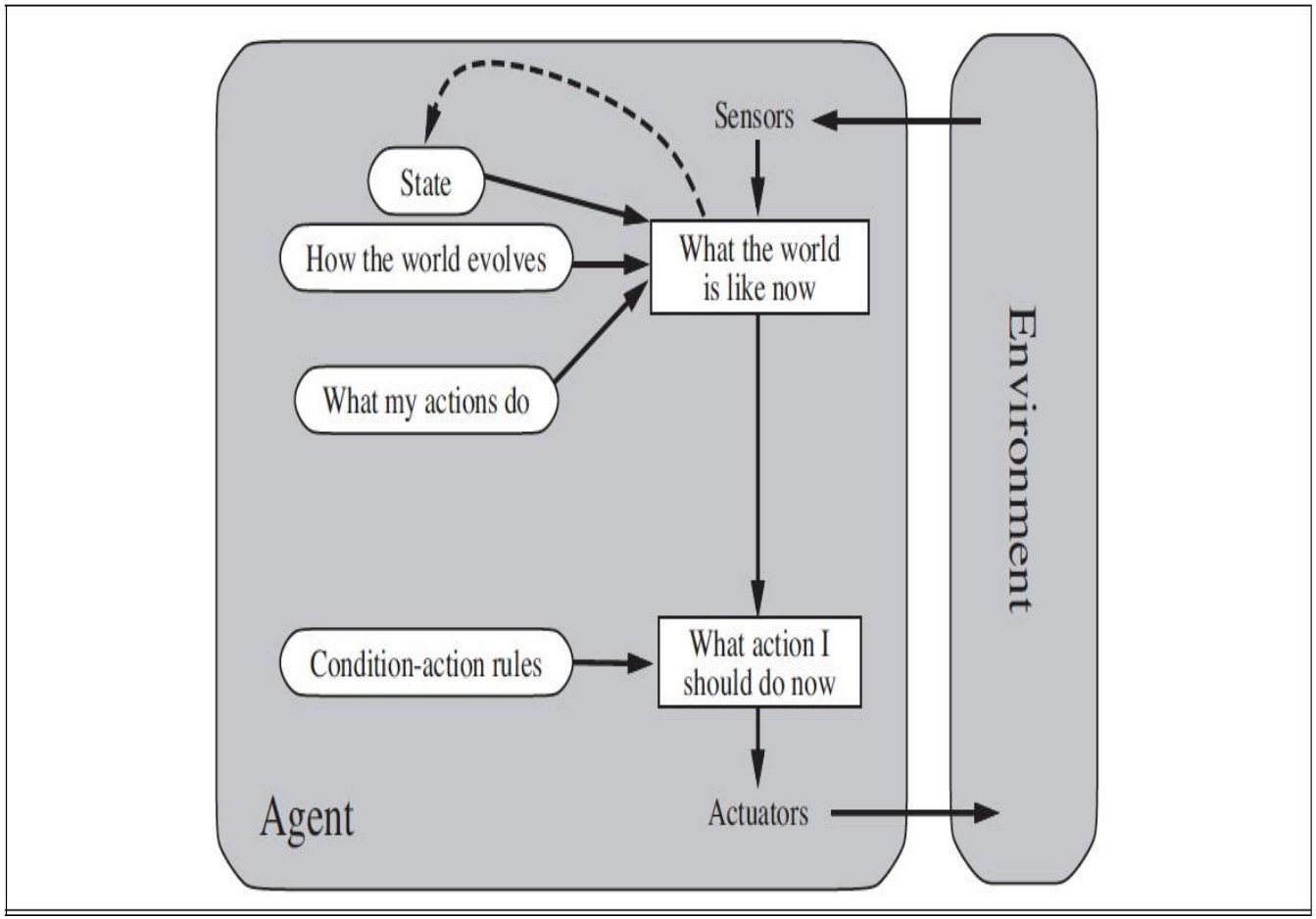
Agent Architectures

innovate achieve lead

Model based Agent

Simple Reflex Agents

Model Based Agents



Agent Architectures

innovate achieve lead

Model based Agent

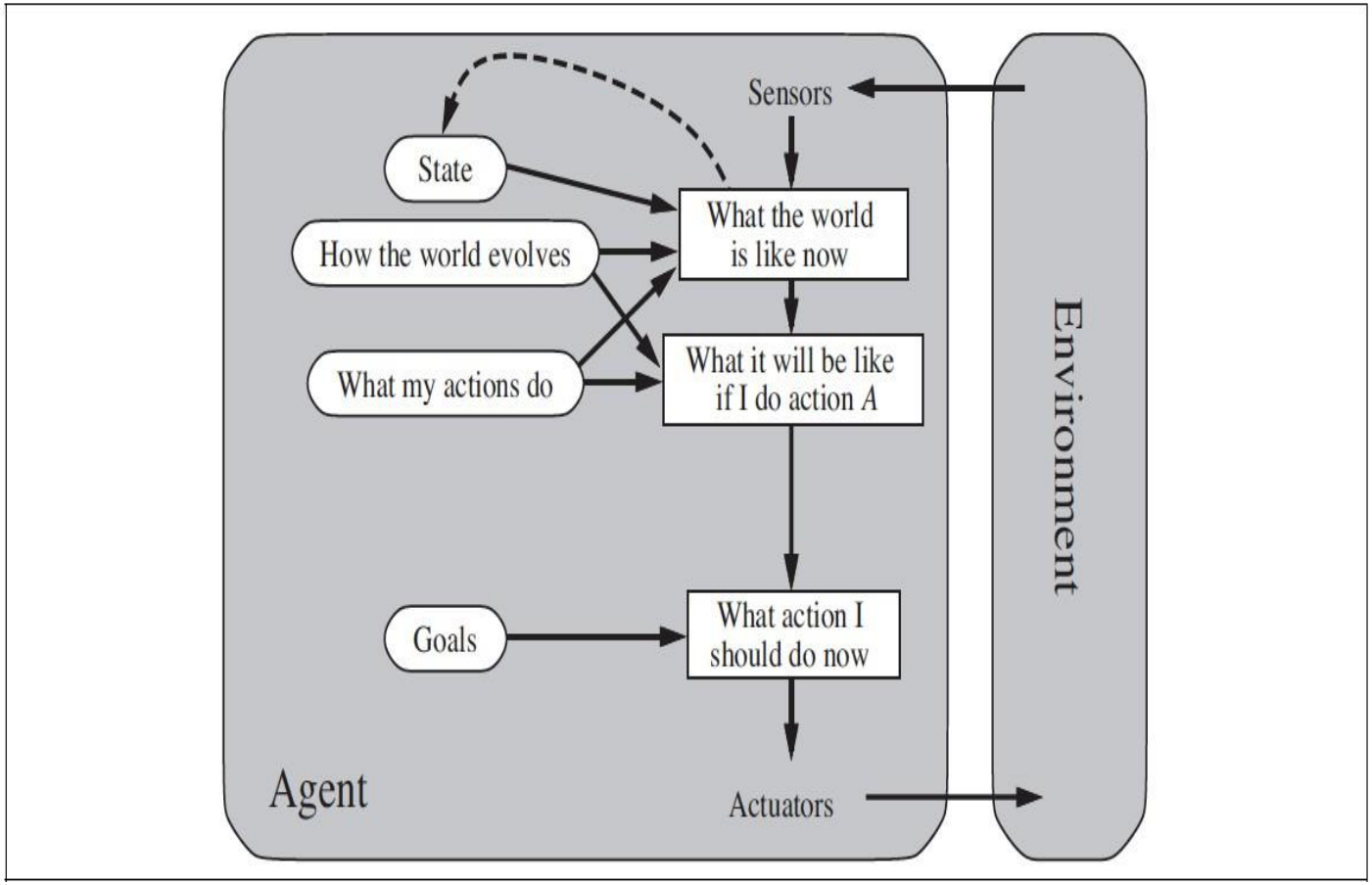
function MODEL-BASED-REFLEX-AGENT(*percept*) **returns** an action

persistent: *state*, the agent's current conception of the world state
transition model, a description of how the next state depends on the current state and action
sensor model , a description of how the current world state is reflected in the agent's percepts
rules, a set of condition–action rules
action, the most recent action, initially none

state←UPDATE-STATE(*state, action, percept, transition model, sensor model*)
rule←RULE-MATCH(*state, rules*)
action ←*rule*.ACTION
return *action*

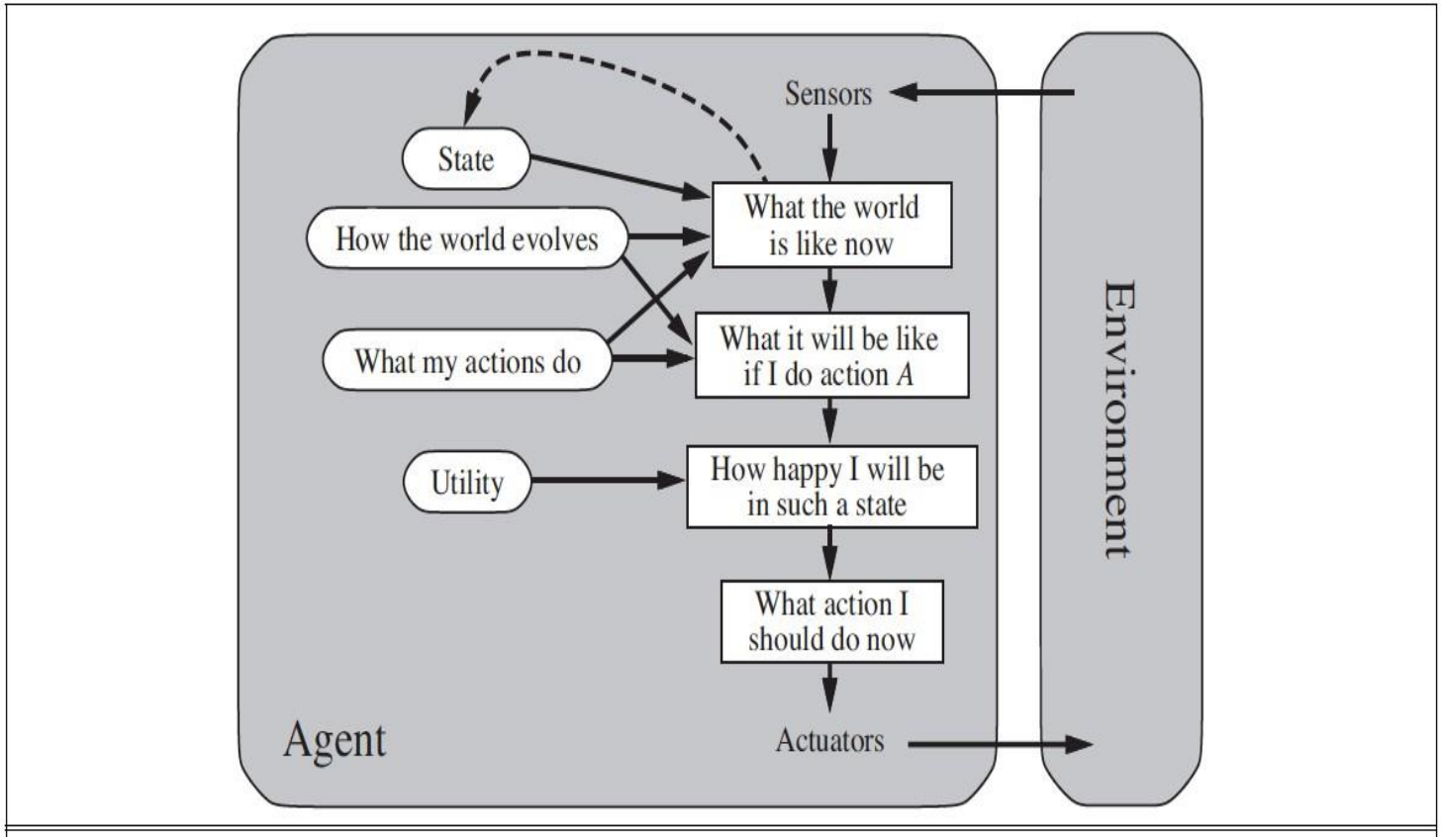
Agent Architectures

- Simple Reflex Agents
- Model Based Agents
- Goal Based Agents



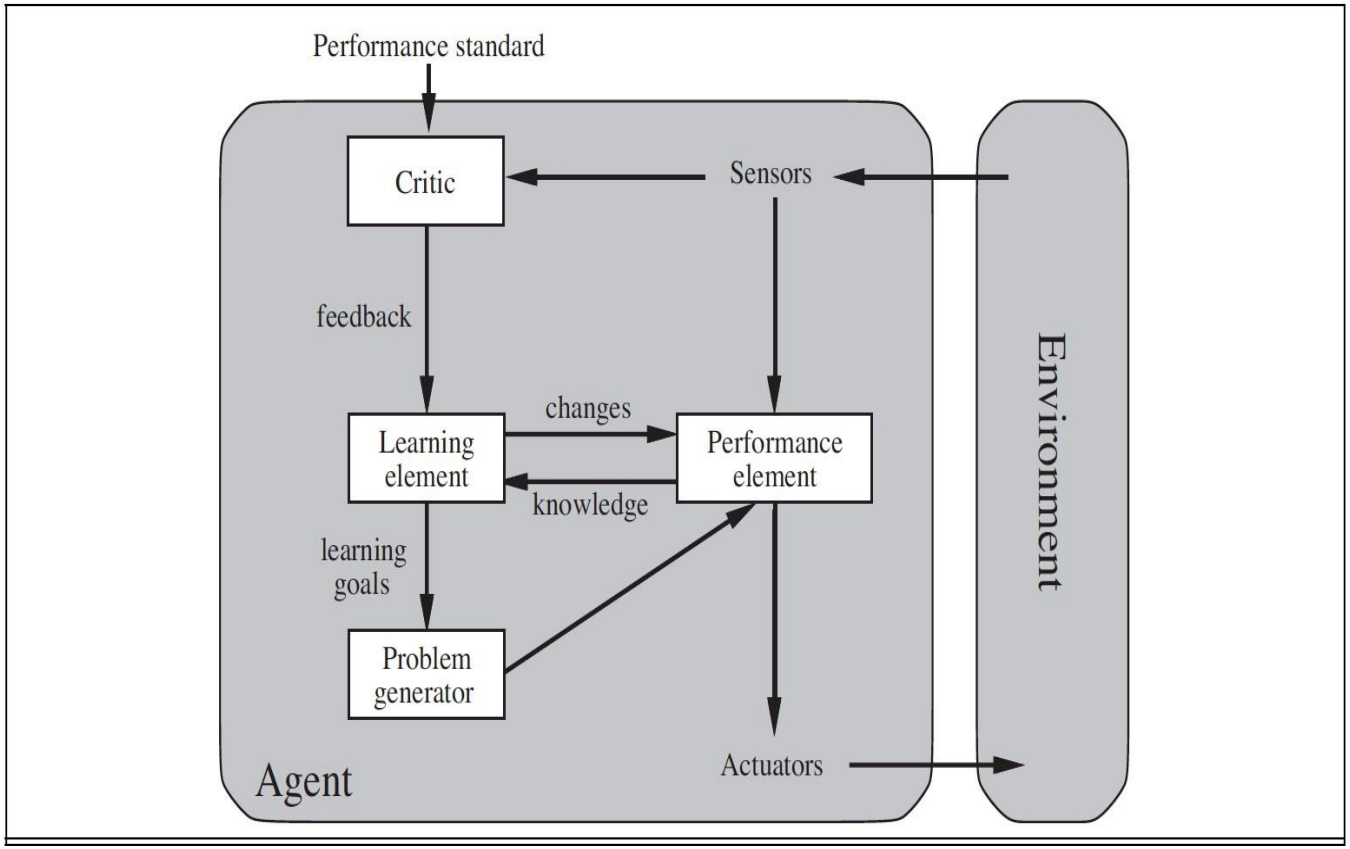
Agent Architectures

- Simple Reflex Agents
- Model Based Agents
- Goal Based Agents
- Utility Based Agents



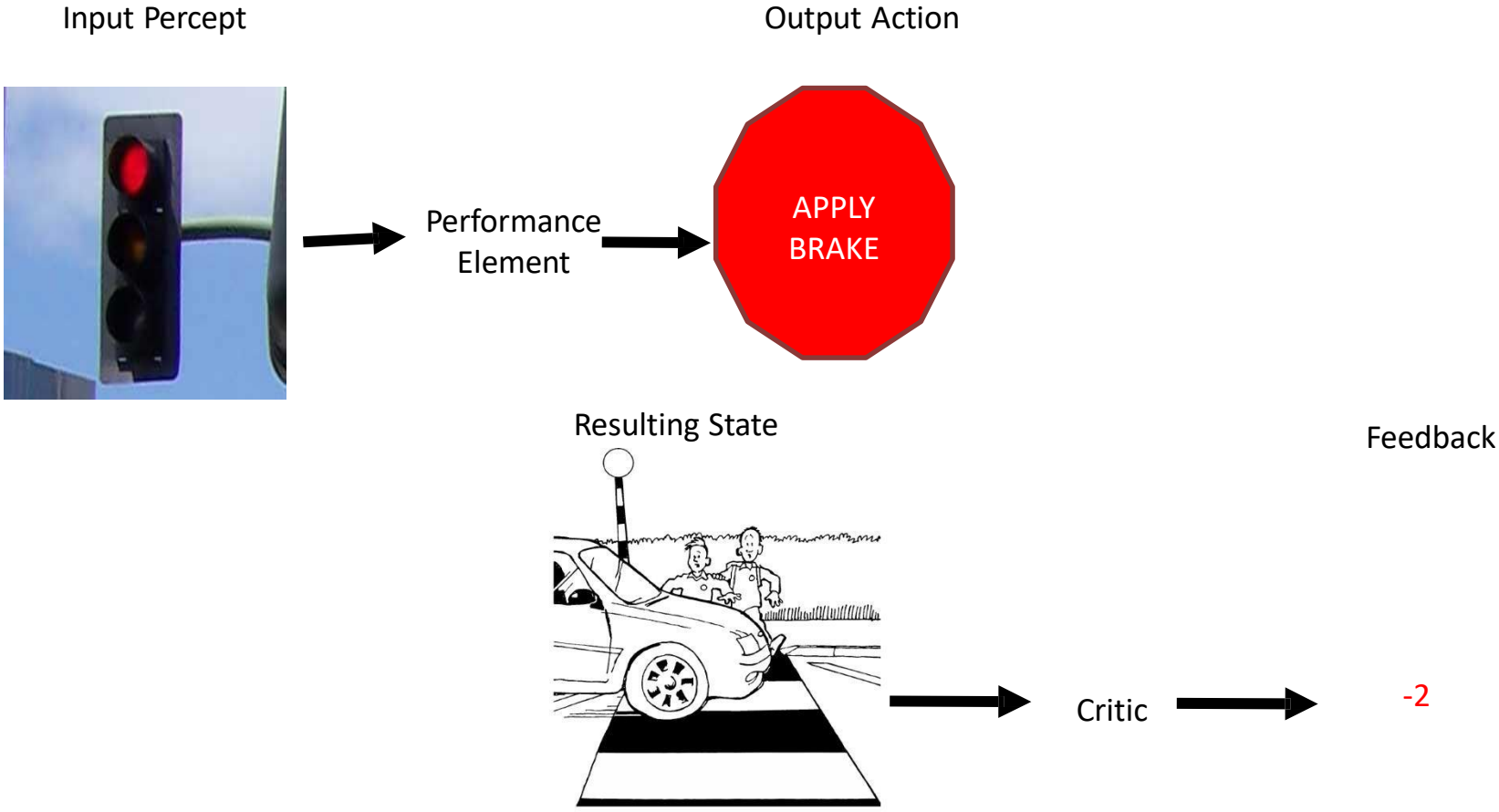
Agent Architectures

- Simple Reflex Agents
- Model Based Agents
- Goal Based Agents
- Utility Based Agents
- Learning Agents



Role of Learning

Agents that improve their performance by learning from their own experiences



Role of Learning

Input Percept	Possible Actions	Selected Action
	Brake Change Gear to Lower Change Gear to Higher Accelerate Steer left Steer right	Random → Change Gear to Lower

Role of Learning

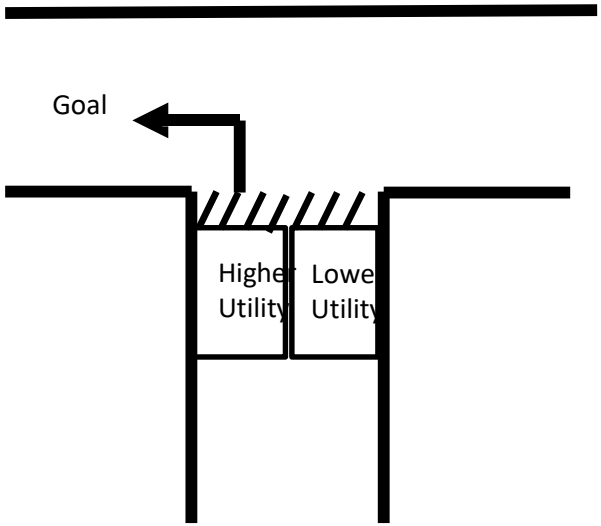
Performance Element – Takes decision on action based on percept

$$f\ (red\ signal,\ distance) = 15k\ N\ brake$$

$$distance \leftarrow f'\ (percept\ sequence)$$

$$f\ (percepts, distance, raining)$$

- $f(state_0, actionA) = 0.83,$
- $f(state_0, actionB) = 0.45$

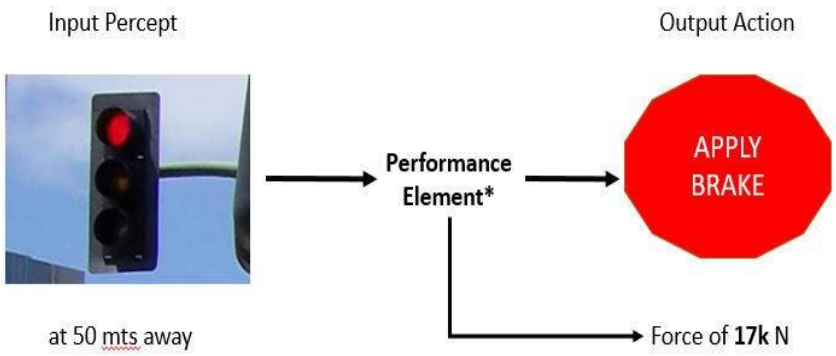


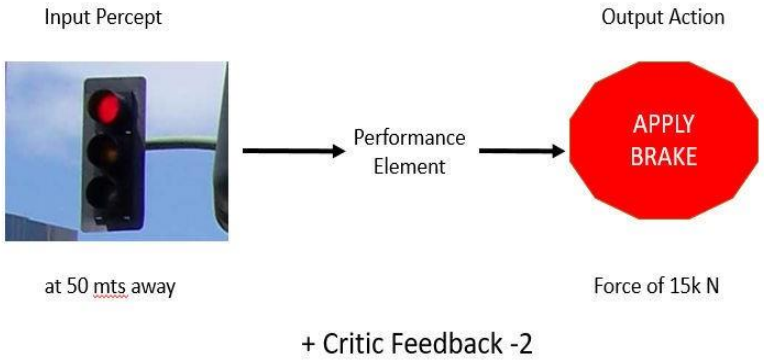
Role of Learning

Critic – Provides feedback on the actions taken

Learning :

Supervised Vs Unsupervised Vs Reinforcement

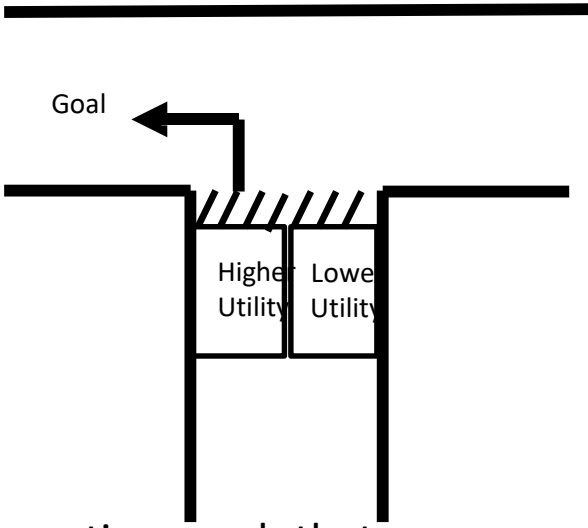




Role of Learning

Performance Element – Takes decision on action based on percept

$$f(red\ signal,\quad distance) = 15k\ N\ brake$$
$$distance = f'(percept\ sequence)$$
$$f(percepts, distance, raining)$$



( )

– $f(state_0, actionB) = 0.45$

Learning Element – Make the performance element select better actions such that the utility function is optimized

Critic – Provides feedback on the actions taken

Problem Generator – Make the Performance Element select sub-optimal actions such that you would learn from unseen actions

Problem Formulation

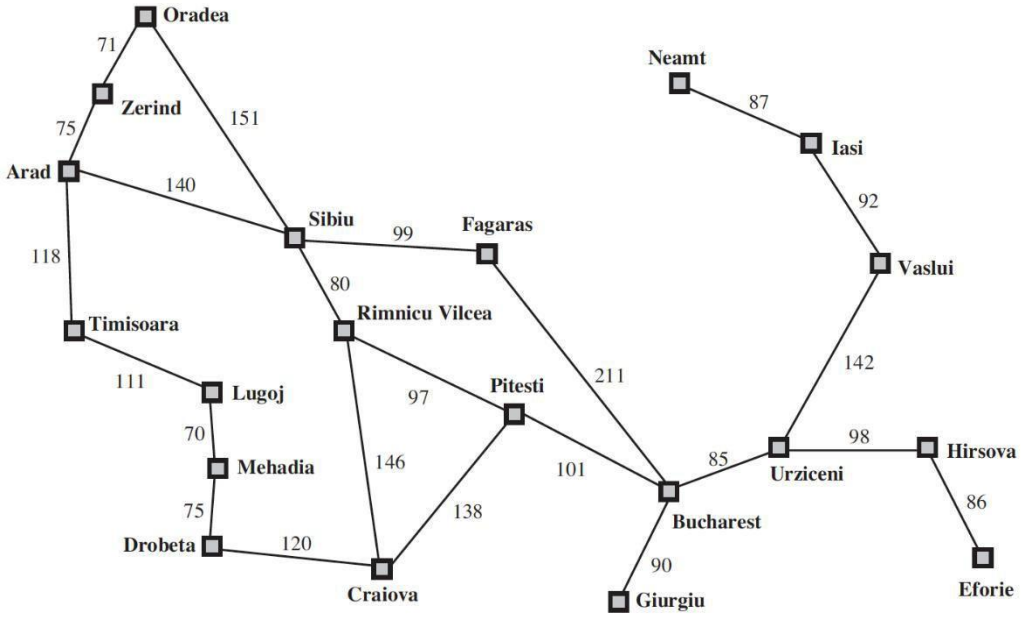
Problem Solving Agents

Goal based decision making agents which finds sequence of actions that leads to the desirable stated.

Phases of Solution Search by PSA

- Goal Formulation
- Problem Formulation
- Search Phase
- Execution Phase

Optimizes the Objective (Local | Global) Limits the Actions



Problem Solving Agents

Goal Formulation

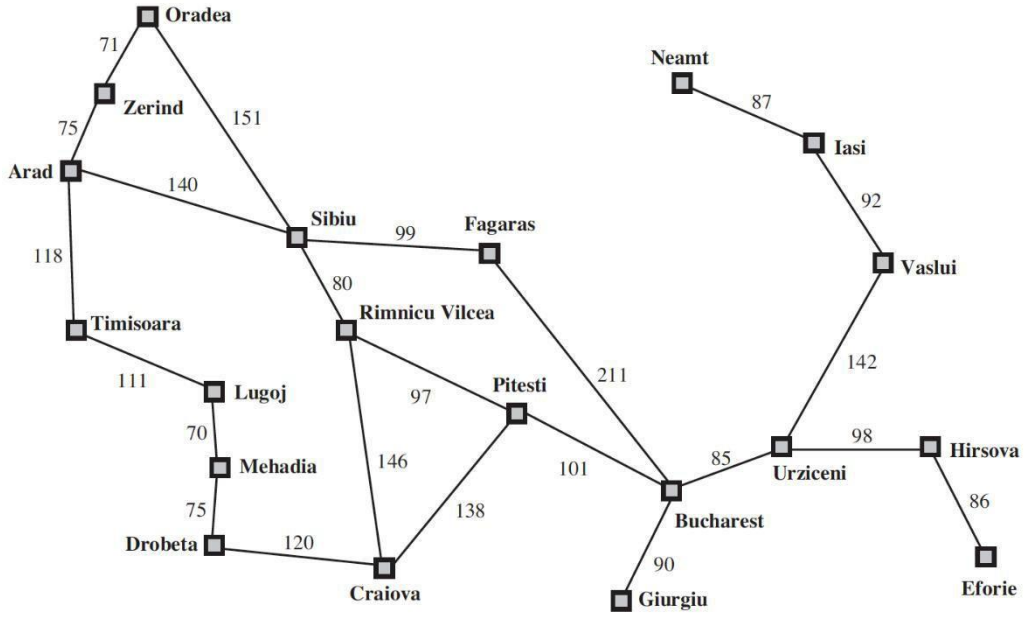
Problem Formulation

Search Phase

Execution Phase

Phases of Solution Search by PSA

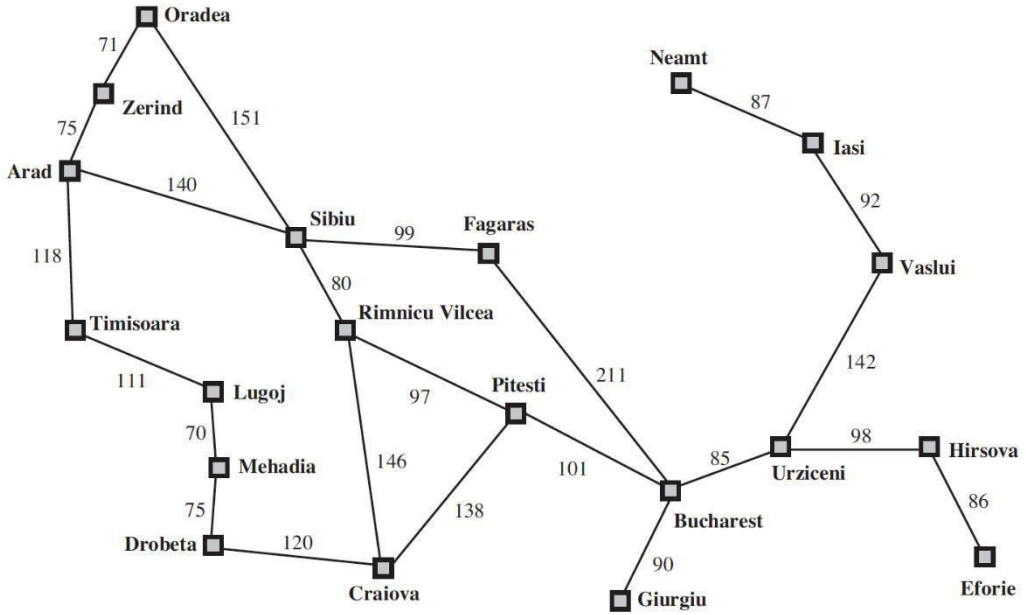
Assumptions – Environment :
Static
Observable Discrete
Deterministic



Problem Solving Agents

Phases of Solution Search

- Goal Formulation
- Problem Formulation
- Search Phase
- Execution Phase

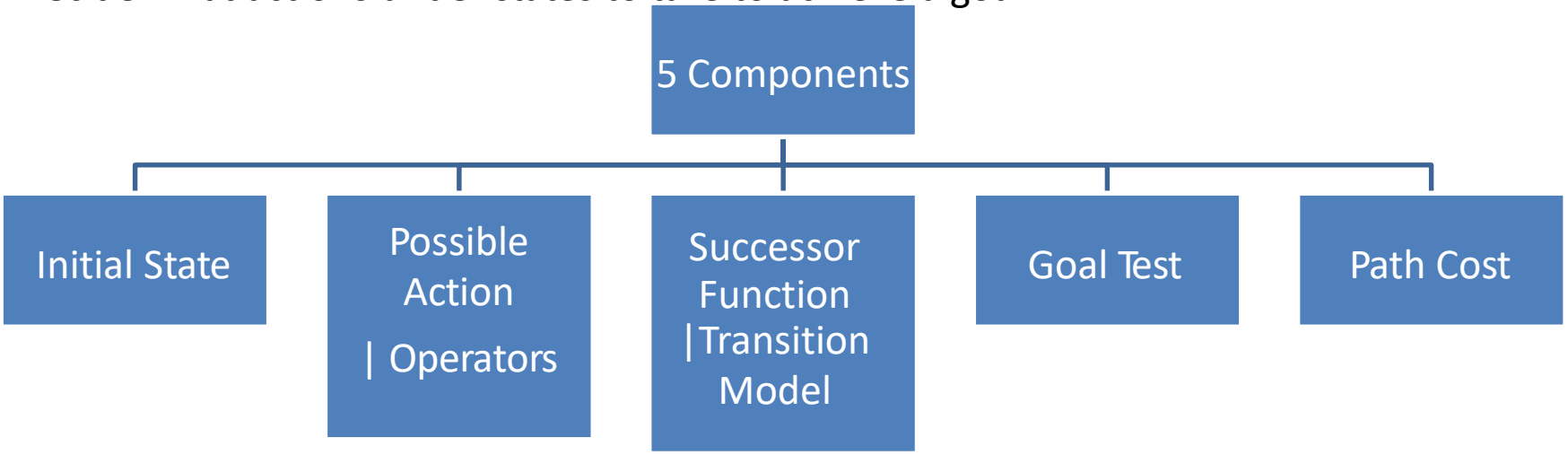


Examine all sequence
Choose best |
Optimal

Problem Solving Agents – Problem Formulation

Abstraction Representation

Decide what actions under states to take to achieve a goal

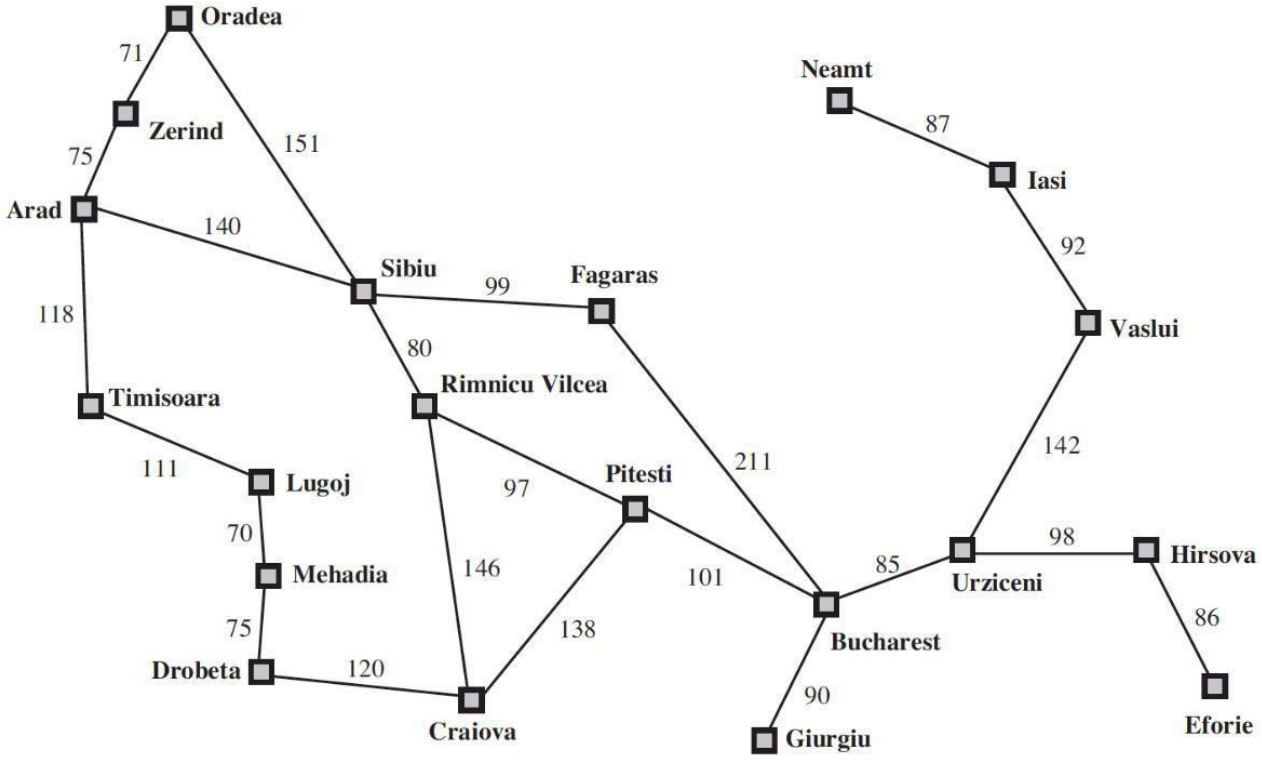


A function that assigns a numeric cost to each path. A path is a series of actions. Each action is given a cost depending on the problem.

Solution = Path Cost Function + Optimal Solution

Problem Solving Agents – Problem Formulation:

Book Example



Initial State –E.g., *In(Arad)*

Possible Actions – *ACTIONS(s)* □ *{Go(Sibiu), Go(Timisoara), Go(Zerind)}*

Transition Model – *RESULT(In(Arad), Go(Sibiu)) = In(Sibiu)*

Goal Test – *IsGoal(In(Bucharest)) = Yes*

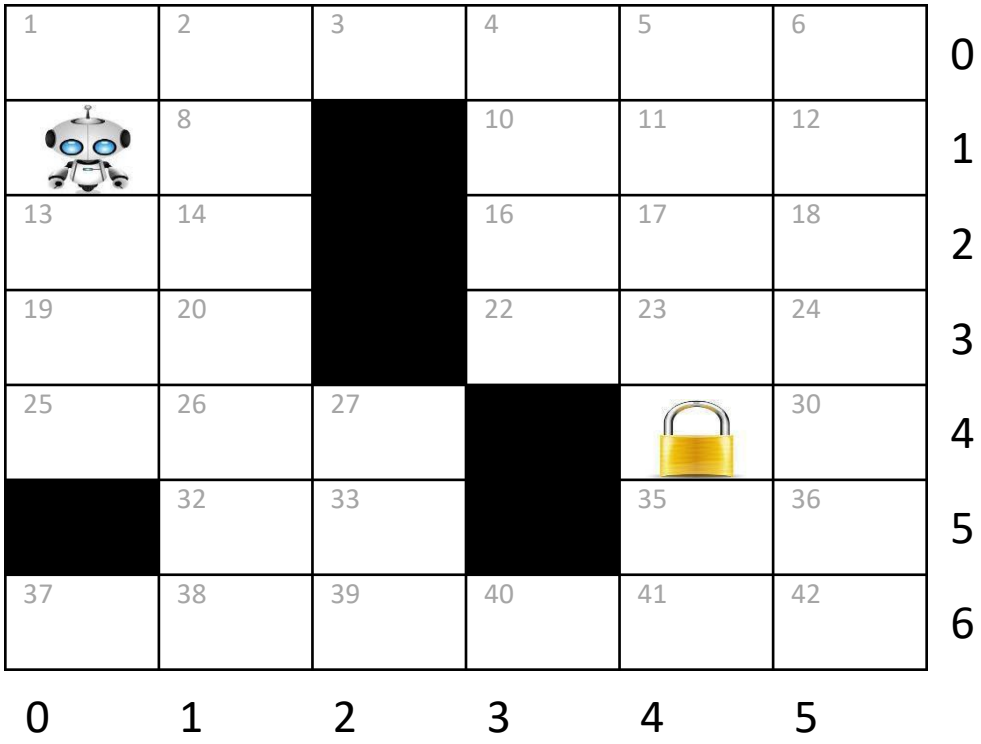
Path Cost – *cost(In(Arad), go(Sibiu)) = 140 kms*

Example Problem Formulation

	Vacuum World	Travelling Problem
Initial State	Any	Based on the problem
Possible Actions	[Move Left, Move Right, Suck]	Take a flight | Train | Shop
Transition Model	[A, ML] = [B , Dirty] [A, ML] = [B, Clean]	[A, Go(A->S)] = [S]
Goal Test	Is all room clean? [A, Clean] [B, Clean]	Is current = B (destination)
Path Cost	No of steps in path	Cost + Time + Quality

Path finding Robot

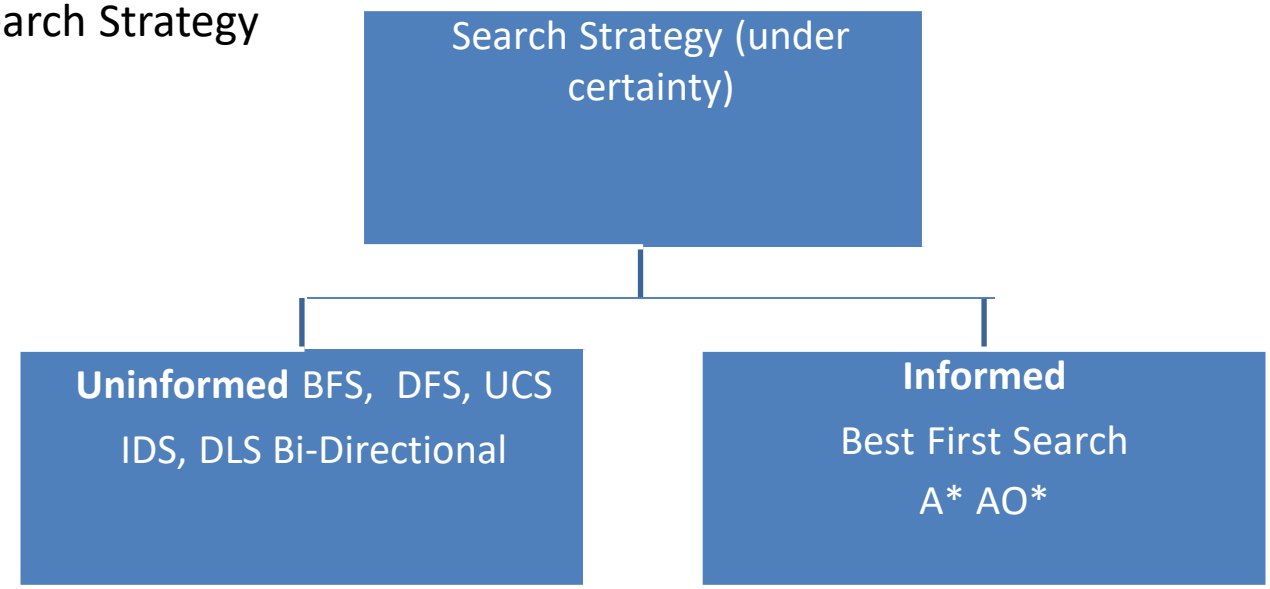
Successor Function Design



N-W-E-S

Searching for Solutions

Choosing the current state, testing possible successor function, expanding current state to generate new state is called Traversal. Choice of which state to expand – Search Strategy



Required Reading: AIMA - Chapter #1, 2, 3.1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML CLZG557

M1 : Introduction
&
M2 : Problem Solving Agent using Search

Indumathi V

Guest Faculty,

BITS - CSIS

BITS Pilani

Pilani Campus

Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time, Reinforcement Learning
M7	Ethics in AI

Rational Agents

Rational Agent

	Thought / Reasoning	Acting
Human Performance	THINKING HUMANLY "[The automation of] activities that we associate with human thinking, activities such as decision-making, problem solving, learning, ... " (Bellman, 1978)	ACTING HUMANLY "The art of creating machines that perform functions that require intelligence when performed by people" (Kurzweil, 1990)
Rational Performance	THINKING RATIONALLY "The study of computations that make it possible to perceive, reason, and act" (Winston, 1992)	ACTING RATIONALLY "Computational intelligence is the study of the design of intelligent agents" (Poole et al., 1998)

Acting Rationally

The Rational Agent Approach

- An agent is an entity that perceives and acts

This course is about designing rational agents

- Abstractly, an agent is a function from percept histories to actions: [f: P* → A]
- For any given class of environments and tasks, we seek the agent (or class of agents) with the best performance
- Computational limitations make perfect rationality unachievable
- Design best program for given machine resources

Acting Rationally

The Rational Agent Approach

- Rational behaviour: doing the *right thing*
- The *right thing:* that which is expected to maximize goal achievement, given the available information
- Rational behaviour is not just about correct inference / thinking, skills needed to pass turing test etc.

(adv) : More General - Correct inference is just a thing

(adv) : More amenable for scientific developments, as the rational behaviour is better defined than human thinking and behaviour

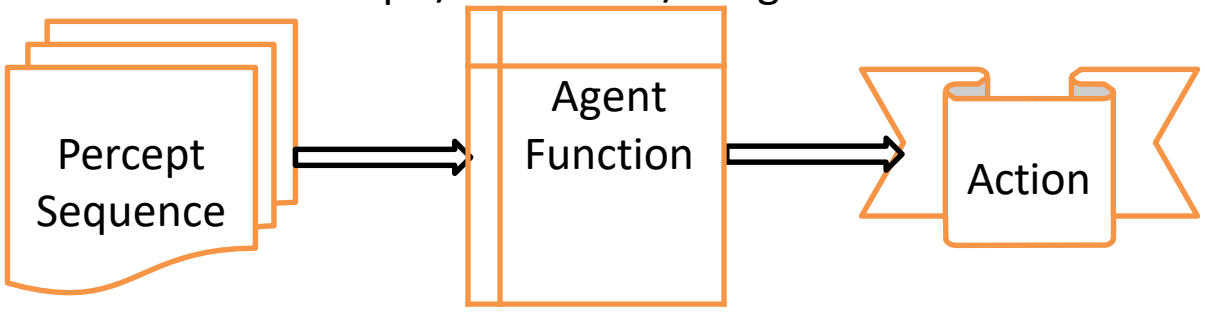
Properties of Rational Agent

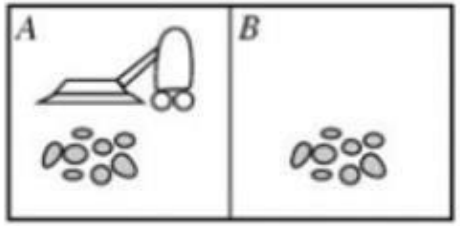
- Omniscience : Expected Vs Actual Performance
- Learning Capability : Apriori Knowledge
- Autonomous in decision making: An agent is autonomous if its behaviour is determined by its own experience (with ability to learn and adapt)

Intelligent Agent

Rational Agent is one that acts to achieve the best outcome or the best expected outcome even under uncertainty

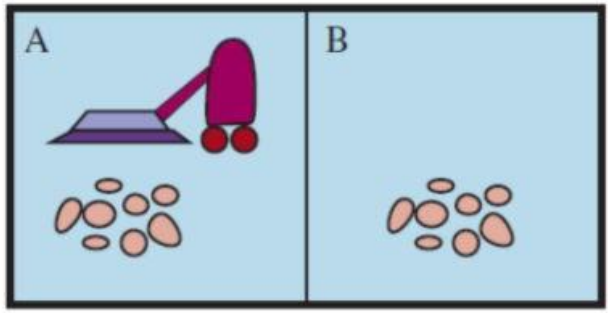
Maps / Tabulated / Programmed





Percept sequence	Action
[A,Clean]	Right
[A, Dirty]	Suck
[B, Clean]	Left
[B, Dirty]	Suck
[A, Clean],[A, Clean]	Right
[A, Clean],[A, Dirty]	Suck
...	...

Intelligent Agent



- Percepts: location and contents, e.g., [A , Dirty]
- Actions: *Left*, *Right*, *Suck*, *NoOp*

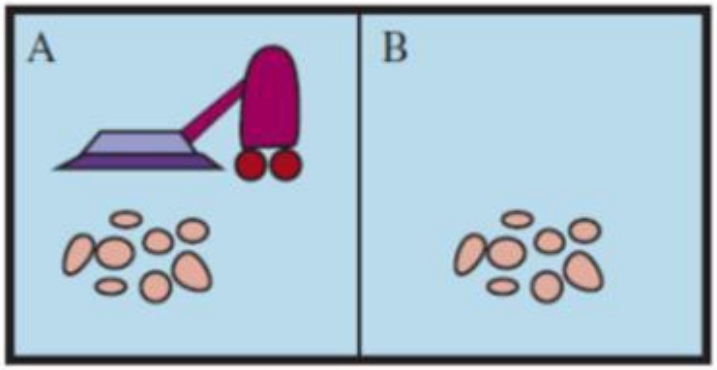
Performance measure: An objective criterion for success of an agent's behaviour

E.g., performance measure of a vacuum-cleaner agent
» amount of dirt cleaned up
» amount of time taken
» amount of electricity consumed
» amount of noise generated, etc.

PEAS Design

Intelligent Agent

Percept sequence	Action
[A, Clean]	Right
[A, Dirty]	Suck
[B, Clean]	Left
[B, Dirty]	Suck
[A, Clean], [A, Clean]	Right
[A, Clean], [A, Dirty]	Suck
⋮	⋮
[A, Clean], [A, Clean], [A, Clean]	Right
[A, Clean], [A, Clean], [A, Dirty]	Suck
⋮	⋮

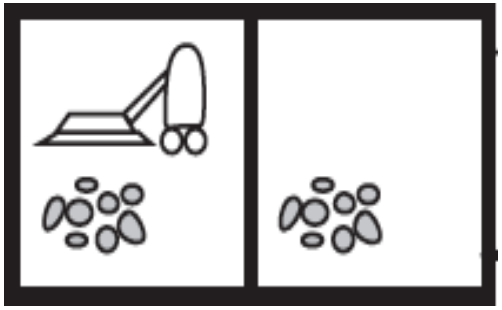


PEAS Environment

Design on what an application wants the agent to do in the environment

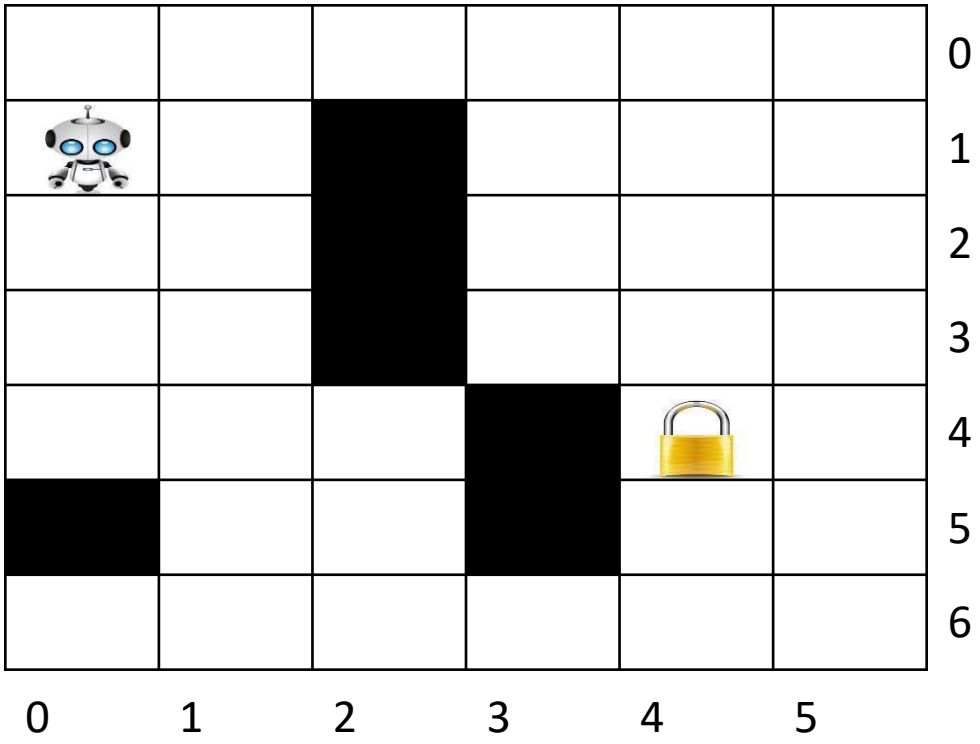
Agent	Performance	Environment	Sensors	Actuators
Medical diagnosis system	Healthy patient, reduced costs	Patient, hospital, staff	Keyboard entry of symptoms, findings, patient's answers	Display of questions, tests, diagnosis, treatments, referrals
Satellite Image analysis system	Correct image categorization	Downlink from orbiting satellite	Color pixel analysis	Display of scene categorization
Interactive English tutor	Student's score on test	Set of students, testing agency	Keyboard entry	Display of exercises, suggestions, corrections

Vacuum World Problem



Performance	» amount of dirt cleaned up » amount of time taken » amount of electricity consumed » amount of noise generated, etc
Environment	Dirt
Sensor	Presence and absence of dirt, Location of robot
Actuator	Physical Intelligent agent - Movement Sucking bump – Cleans the dirt

Path finding Robot - Lab Example



PEAS Environment

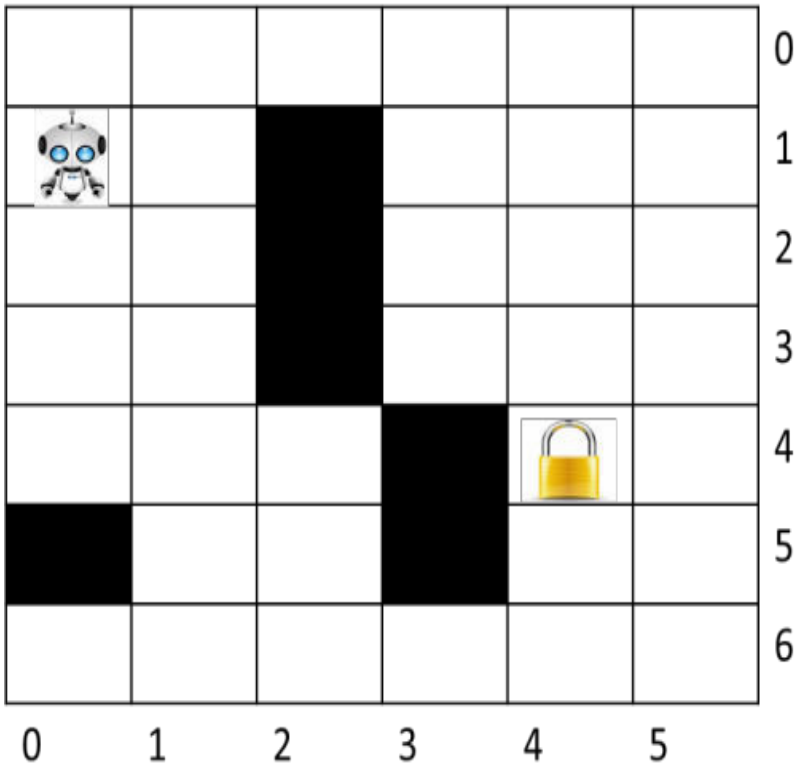
Agent

Performance

Environment

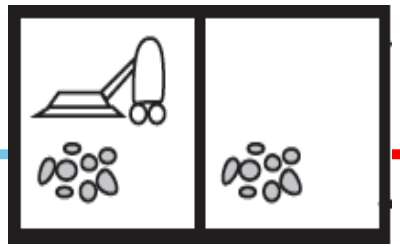
Sensors

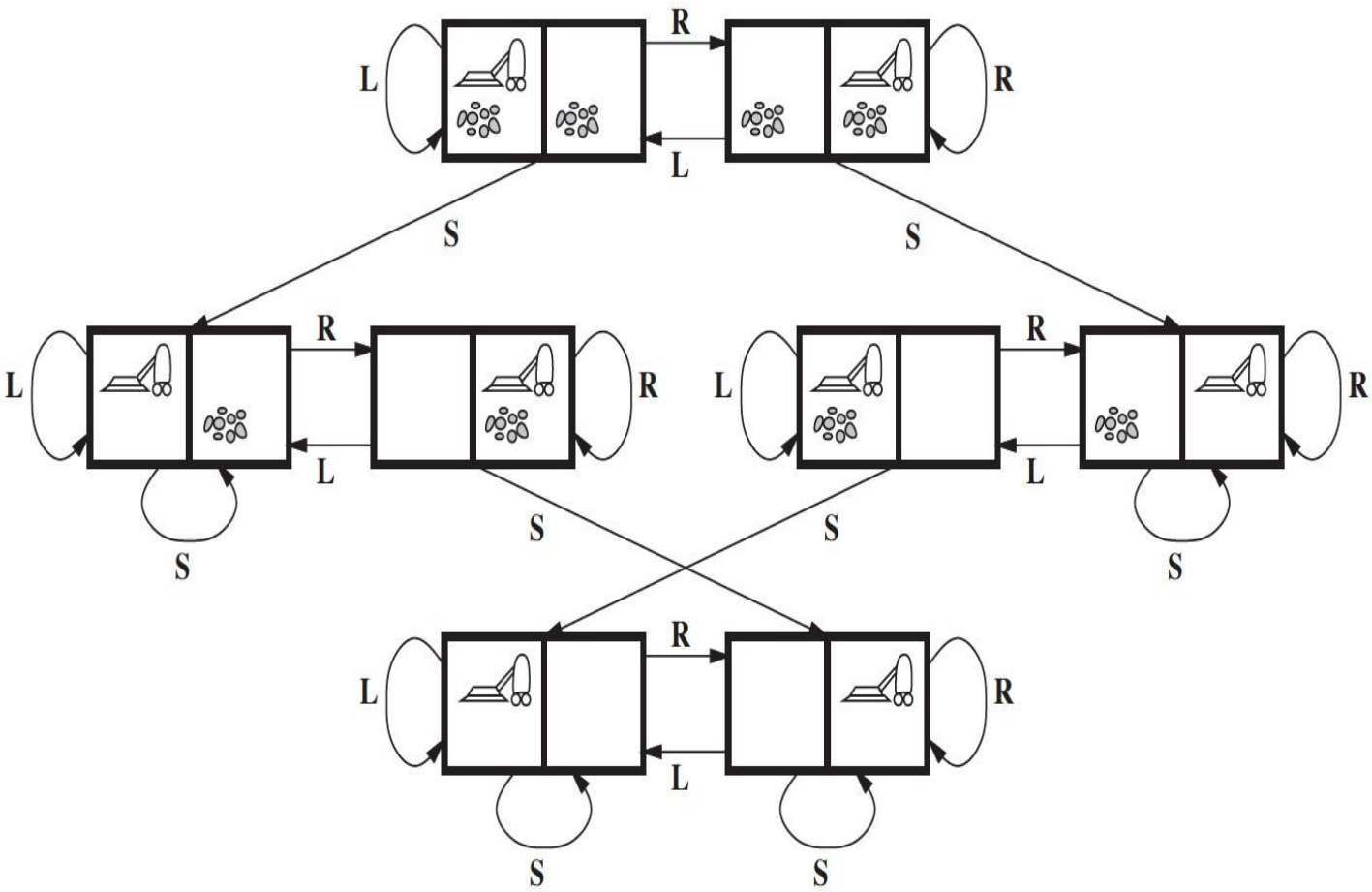
Actuator



Step 1	**Identification of problem statement**
Step 2	**Extraction of 4 important parameters PEAS**
Step 3	**?**

Vacuum World Problem





Dimensions of Task Environment

Sensor Based:

- Observability : Full Vs Partial

Action Based:

- Dependency : Episodic Vs Sequential

State Based:

- No.of.State : Discrete Vs Continuous

Agent Based:

> Cardinality : Single Vs MultiAgent

Action & State Based:

- State Determinism : Deterministic Vs Stochastic | Strategic
- Change in Time : Static Vs Dynamic

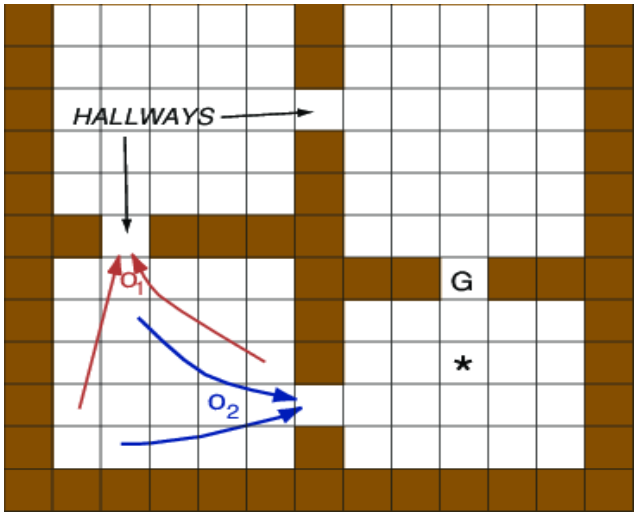
Task Environment

A rational agent is built to solve a specific task. Each such task would then have a
different environment which we refer to as Task Environment

Based on the applicability of each technique for agent implementation its task environment design is determined by multiple dimension

Sensor Based:

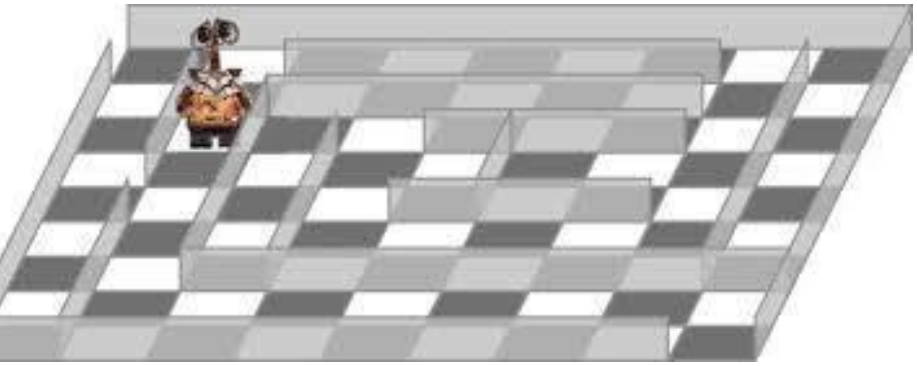
- Observability : Full Vs Partial



Task Environment

Action Based:

- Dependency : Episodic Vs Sequential

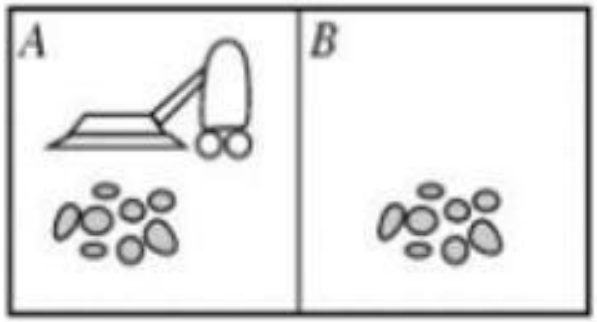


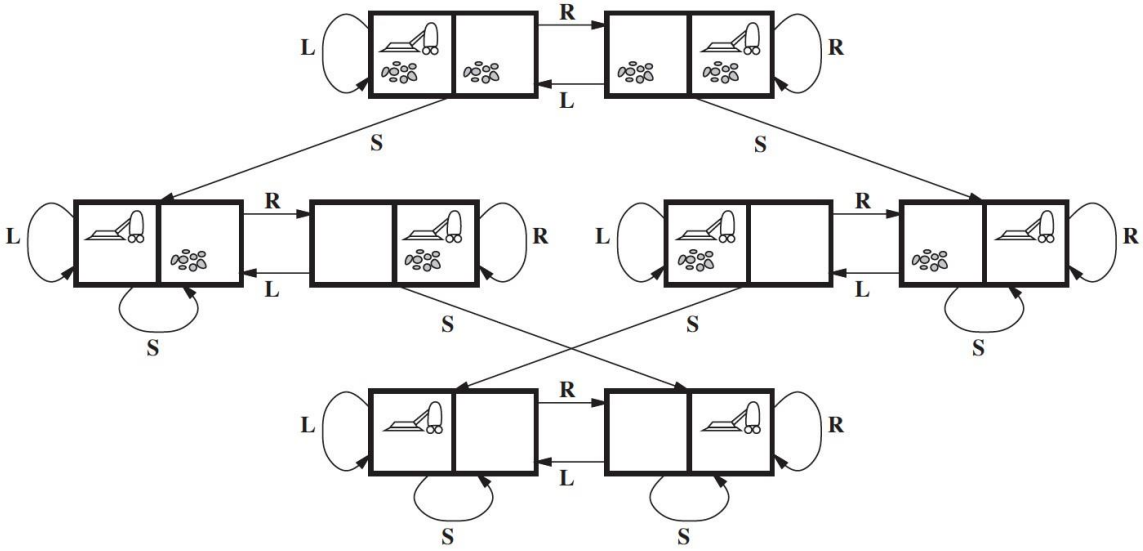
Fully / Partial
Episodic Vs Sequential

Task Environment

State Based:

- No.of.State : **Discrete** Vs Continuous





Task Environment

State Based:

- No.of.State : Discrete Vs Continuous

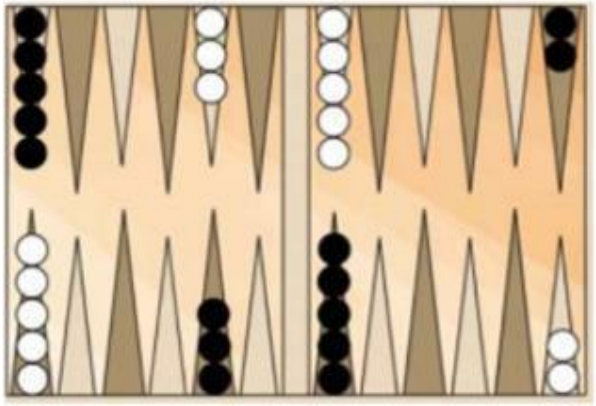
VS.

Task Environment

Action & State Based:

- State Determinism : Deterministic Vs Stochastic | Strategic

(If the environment is deterministic except for the actions of other agents, then the environment is strategic)



Required Reading: AIMA - Chapter #1, 2, 3.1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML CLZG557

M1 : Introduction
&
M2 : Problem Solving Agent using Search

Indumathi V

Guest Faculty,

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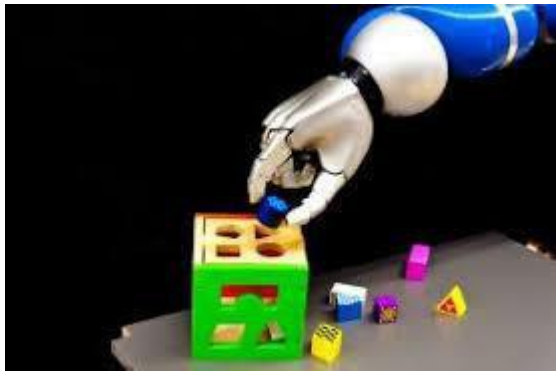
Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time, Reinforcement Learning
M7	Ethics in AI

Task Environment

Agent Based:

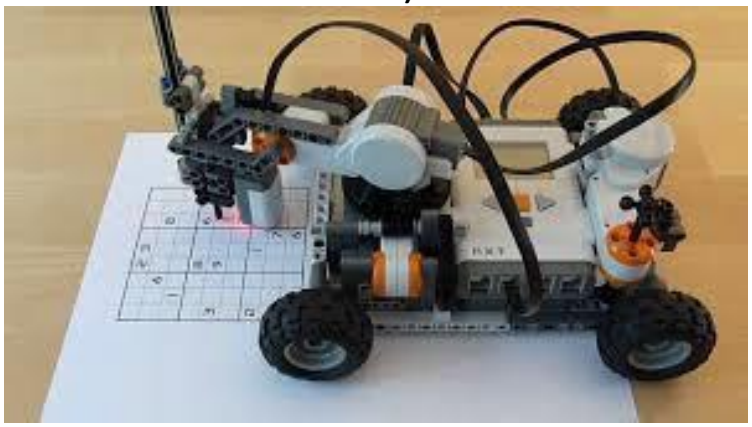
> Cardinality : Single Vs MultiAgent



Task Environment

Action & State Based:

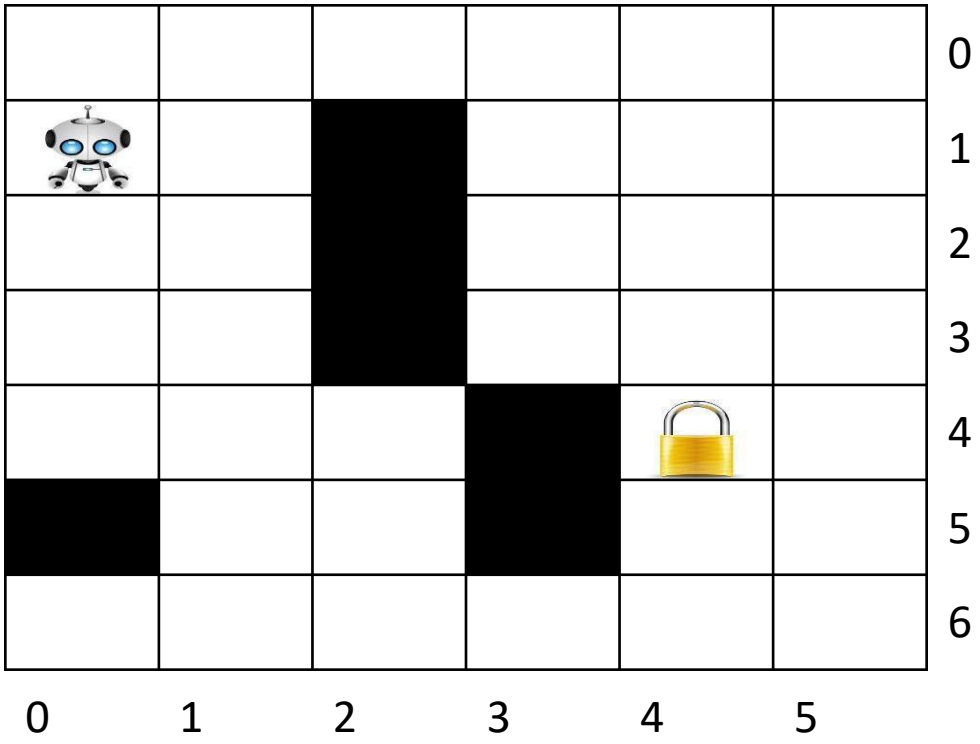
- Change in Time : Static Vs Dynamic
- (The environment is semi dynamic if the environment itself does not change with the passage of time but the agent's performance score does)



Task Environment

Task Environment	Fully vs Partially Observable	Single vs Multi-Agent	Deterministic vs Stochastic	Episodic vs Sequential	Static vs Dynamic	Discrete vs Continuous
Medical diagnosis system	Partially (cant see the patient, can get input from keyboard entry)	Single (Single chat bot system)	Stochastic P :---- C:---- P:---- C:---Recom1 :---Recom2	Sequential	Dynamic (Patient involed in Environment)	Continuous
Satellite Image Analysis System	Fully	Single	Deterministic	Episodic	Static	Continuous
Interactive English tutor	Partially	Multi (IM and Canvas)	Stochastic	Sequential	Dynamic	Discrete (only 10 question for evaluation)

Path finding Robot - Lab Example



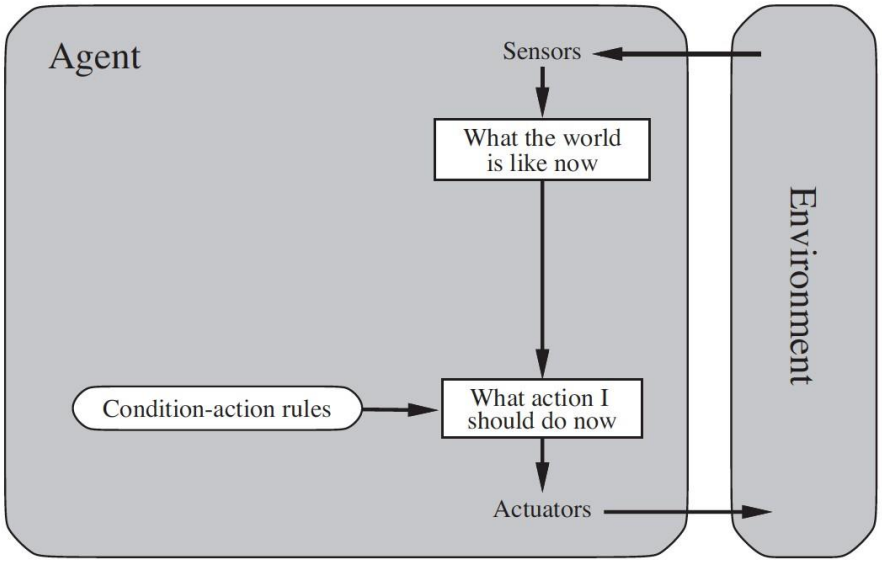
Agent
Observability
No.of.Agents
No.of.States
Determinism
Dynamicity
Output Dependency

Agent Architectures

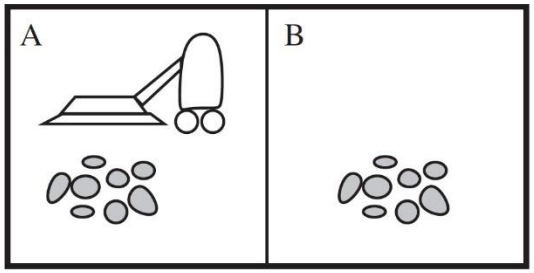
Reflex Agent

```
function SIMPLE-REFLEX-AGENT(percept) returns an action
    persistent: rules, a set of condition–action rules
    state←INTERPRET-INPUT(percept)
    rule←RULE-MATCH(state, rules)
    action ←rule.ACTION
    return action
```

```
function REFLEX-VACUUM-AGENT( [location,status]) returns an action
    if status = Dirty then return Suck
    else if location = A then return Right
    else if location = B then return Left
```



Simple Reflex Agents



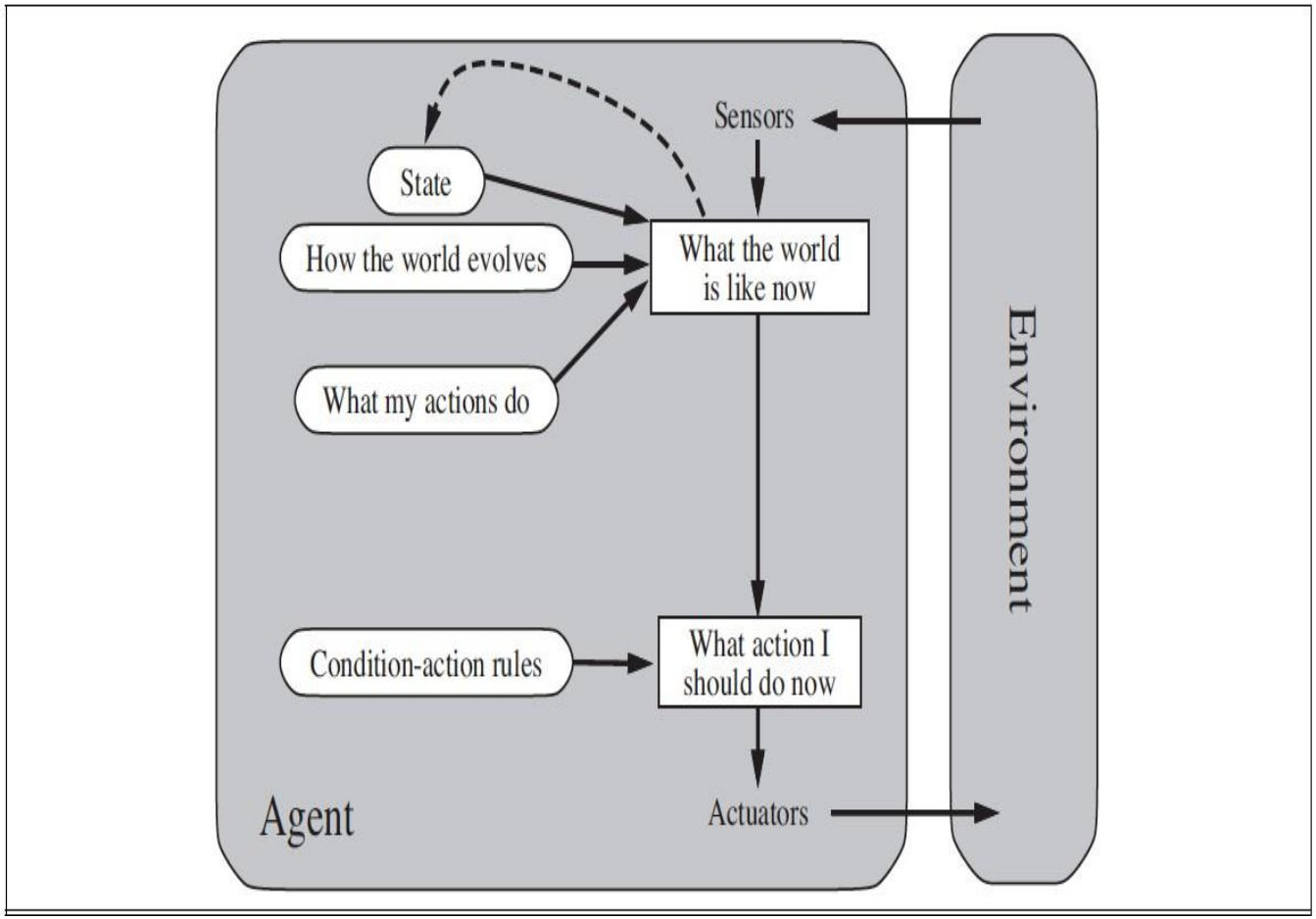
Agent Architectures

innovate achieve lead

Model based Agent

Simple Reflex Agents

Model Based Agents



Agent Architectures

innovate achieve lead

Model based Agent

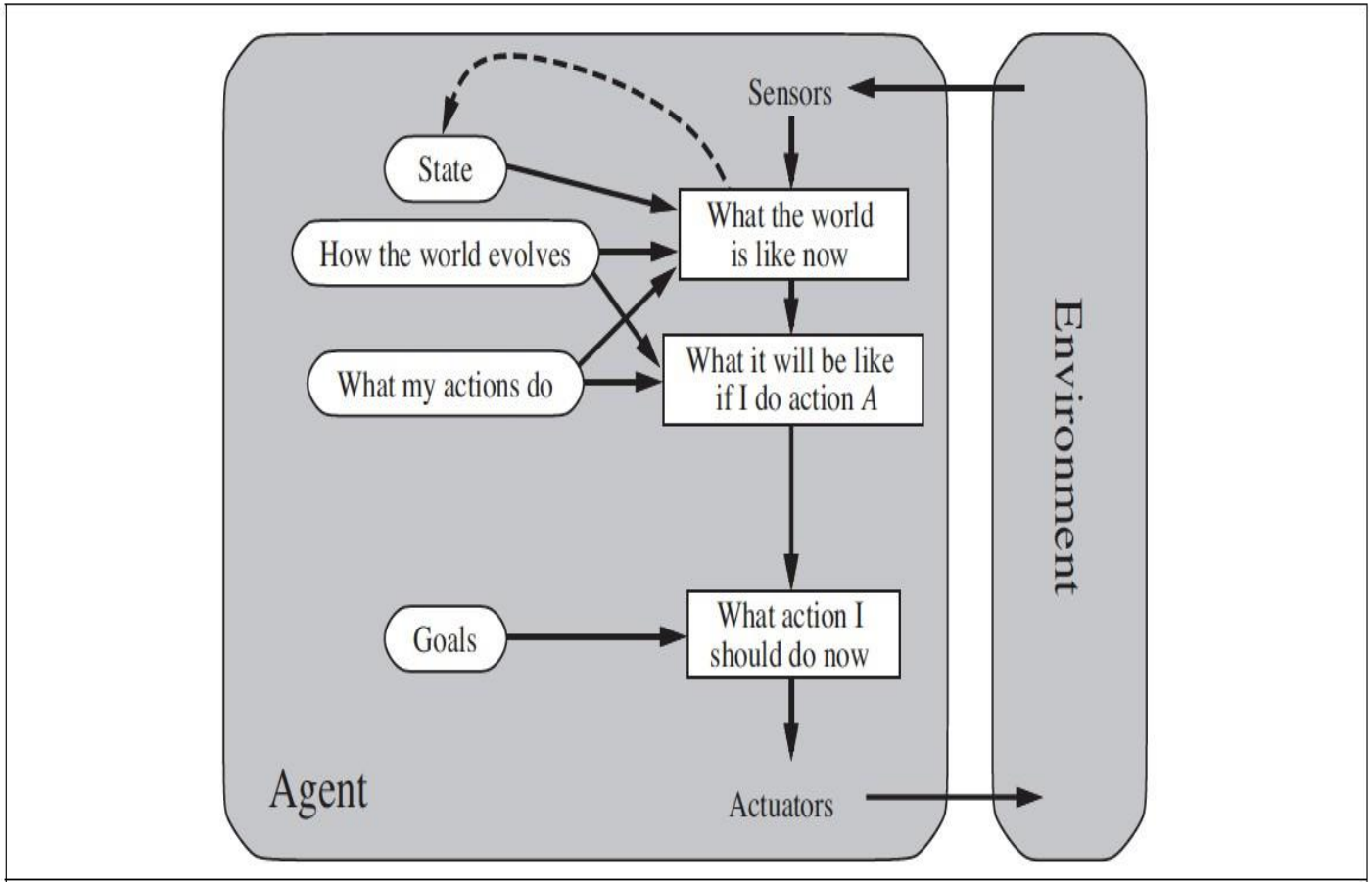
function MODEL-BASED-REFLEX-AGENT(*percept*) **returns** an action

persistent: *state*, the agent's current conception of the world state
transition model, a description of how the next state depends on the current state and action
sensor model , a description of how the current world state is reflected in the agent's percepts
rules, a set of condition–action rules
action, the most recent action, initially none

state←UPDATE-STATE(*state, action, percept, transition model, sensor model*)
rule←RULE-MATCH(*state, rules*)
action ←*rule*.ACTION
return *action*

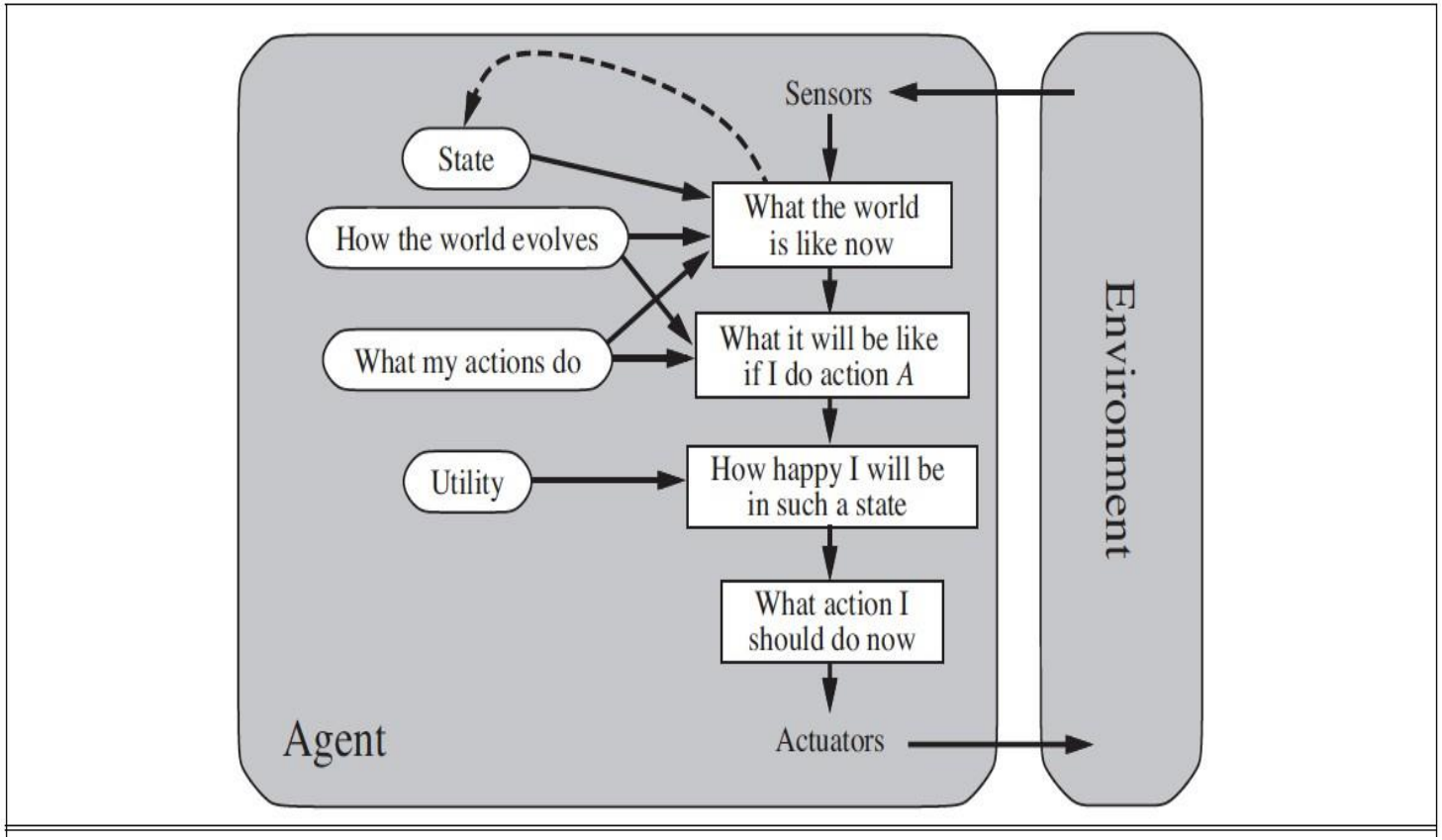
Agent Architectures

- Simple Reflex Agents
- Model Based Agents
- Goal Based Agents



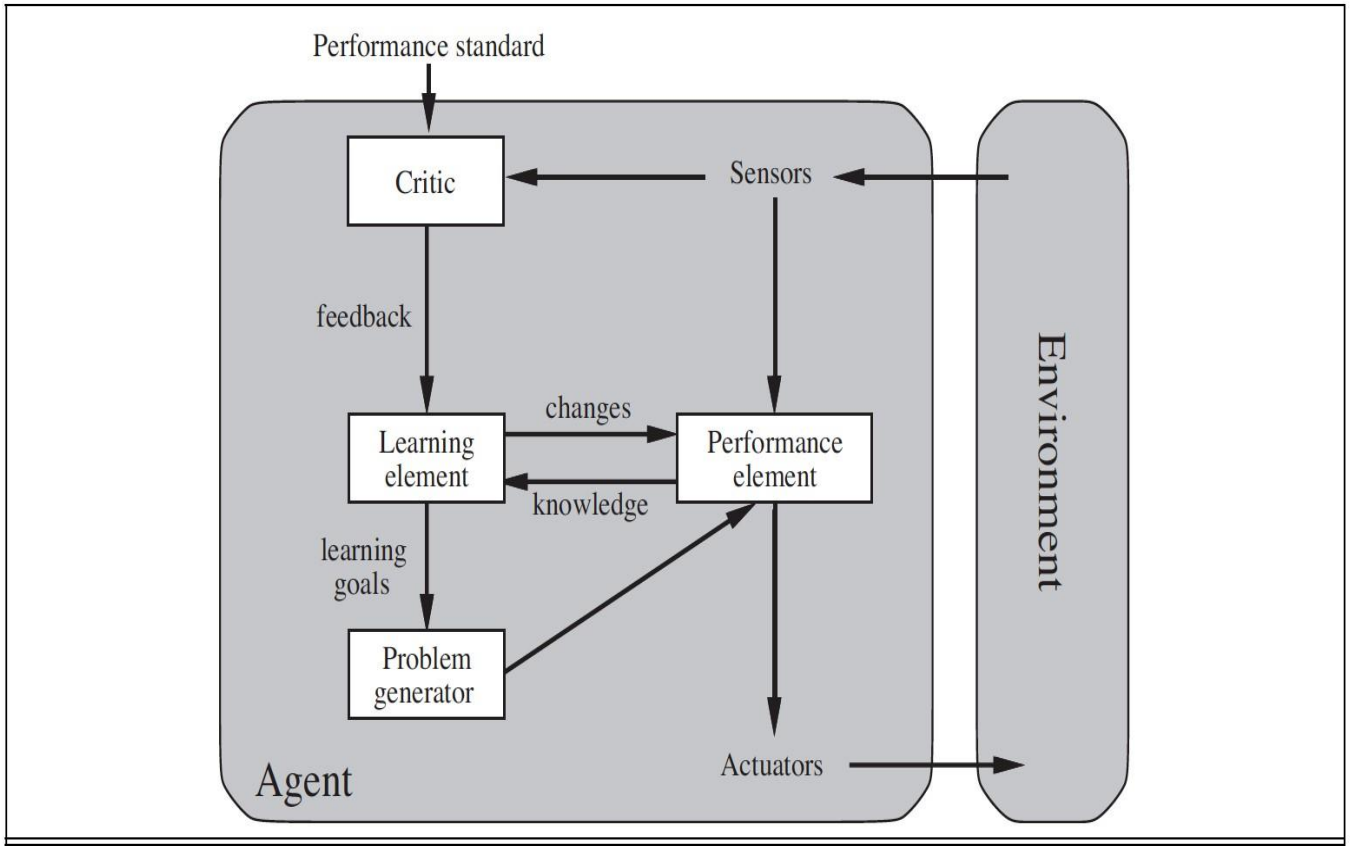
Agent Architectures

- Simple Reflex Agents
- Model Based Agents
- Goal Based Agents
- Utility Based Agents

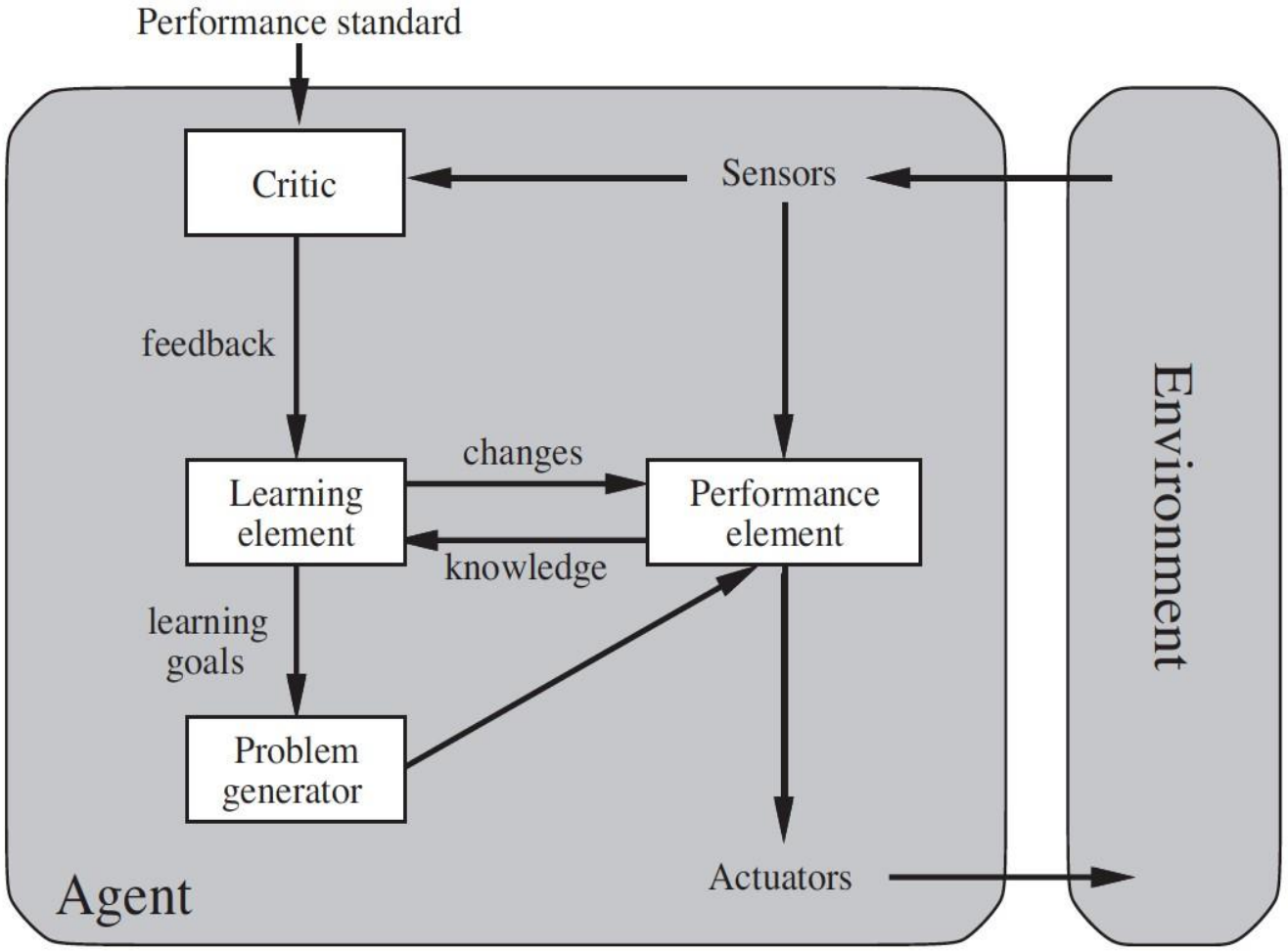


Agent Architectures

- Simple Reflex Agents
- Model Based Agents
- Goal Based Agents
- Utility Based Agents
- Learning Agents

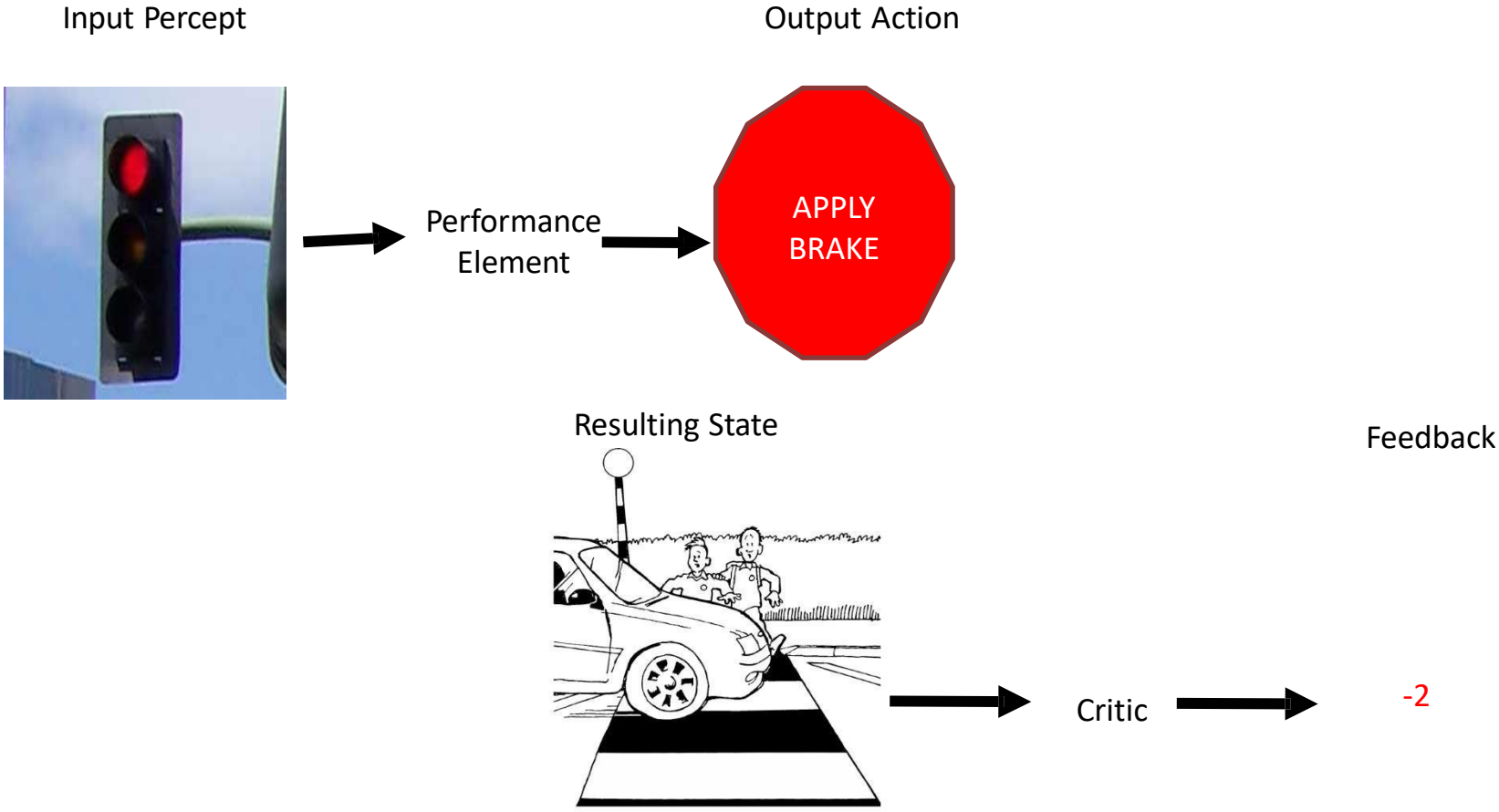


Agent Architectures



Role of Learning

Agents that improve their performance by learning from their own experiences

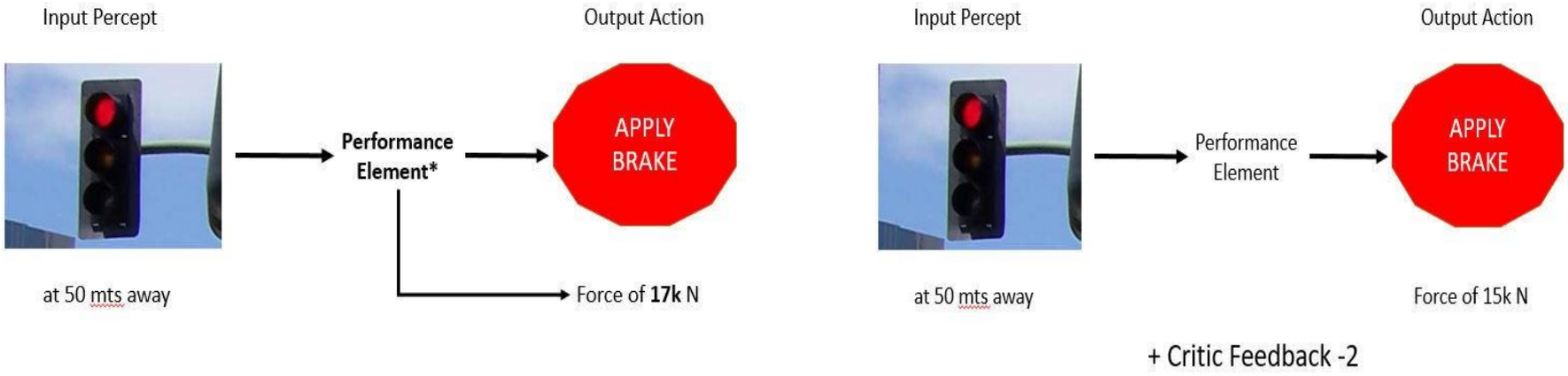


Role of Learning

Critic – Provides feedback on the actions taken

Learning :

Supervised Vs Unsupervised Vs Reinforcement



Role of Learning

Input Percept	Possible Actions	Selected Action
	Brake Change Gear to Lower Change Gear to Higher Accelerate Steer left Steer right	Random → Change Gear to Lower

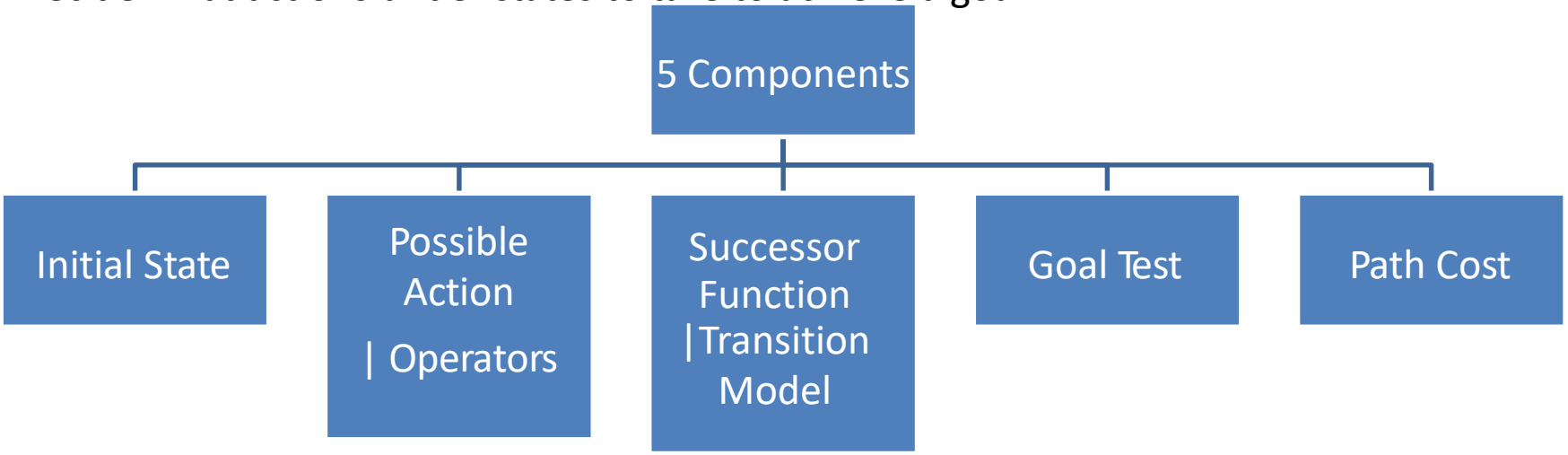
Step 1	**Identification of problem statement**
Step 2	**Extraction of 4 important parameters PEAS**
Step 3	**Identification of task environment**
Step 4	**?**

Problem Formulation

Problem Solving Agents – Problem Formulation

Abstraction Representation

Decide what actions under states to take to achieve a goal



A function that assigns a numeric cost to each path. A path is a series of actions. Each action is given a cost depending on the problem.

Solution = Path Cost Function + Optimal Solution

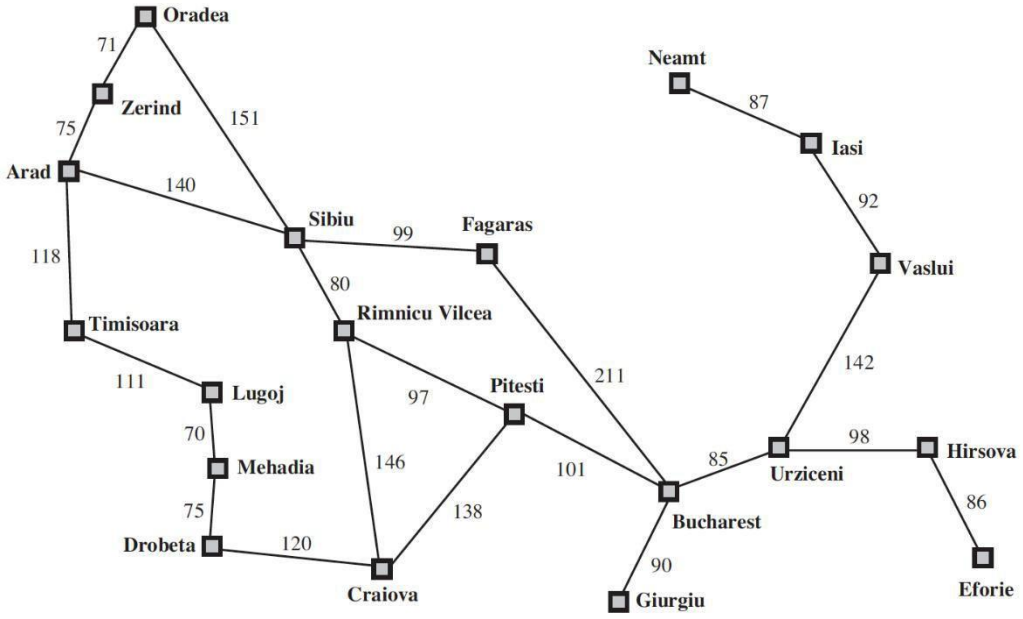
Problem Solving Agents

Goal based decision making agents which finds sequence of actions that leads to the desirable stated.

Phases of Solution Search by PSA

- Goal Formulation
- Problem Formulation
- Search Phase
- Execution Phase

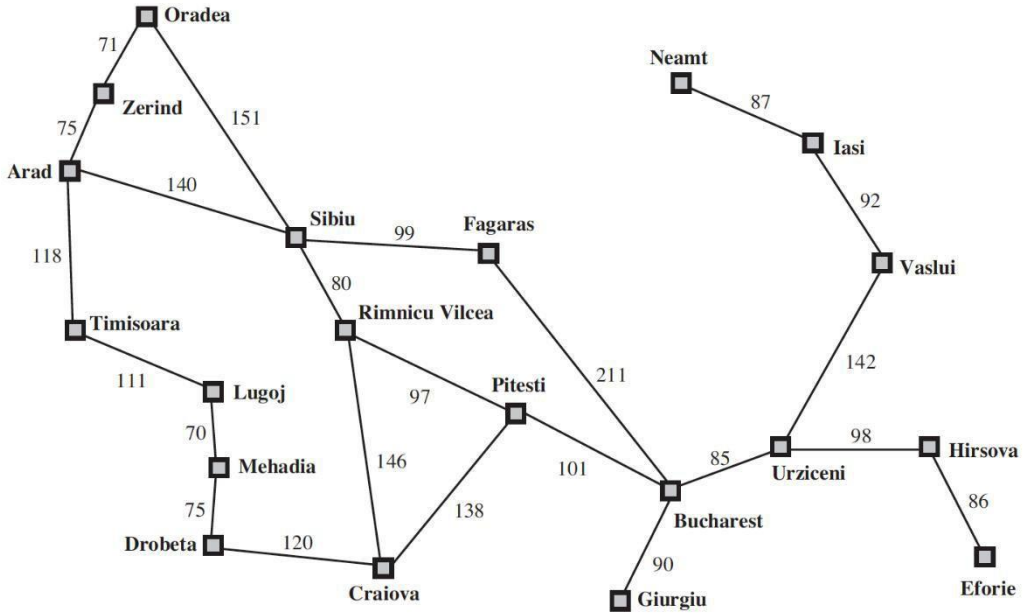
Optimizes the Objective (Local | Global) Limits the Actions



Problem Solving Agents

Phases of Solution Search

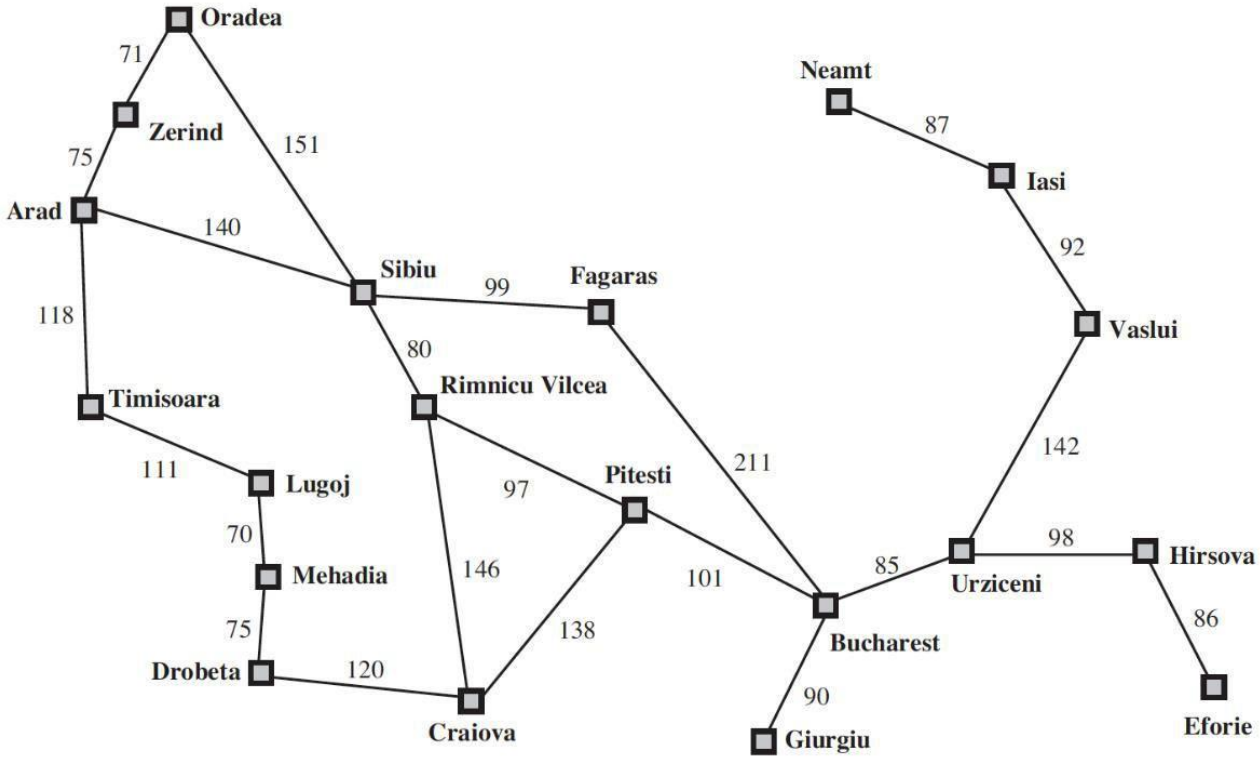
- Goal Formulation
- Problem Formulation
- Search Phase
- Execution Phase



Examine all sequence
Choose best |
Optimal

Problem Solving Agents – Problem Formulation:

Book Example



Initial State –E.g., *In(Arad)*

Possible Actions – *ACTIONS(s)* → *{Go(Sibiu), Go(Timisoara), Go(Zerind)}*

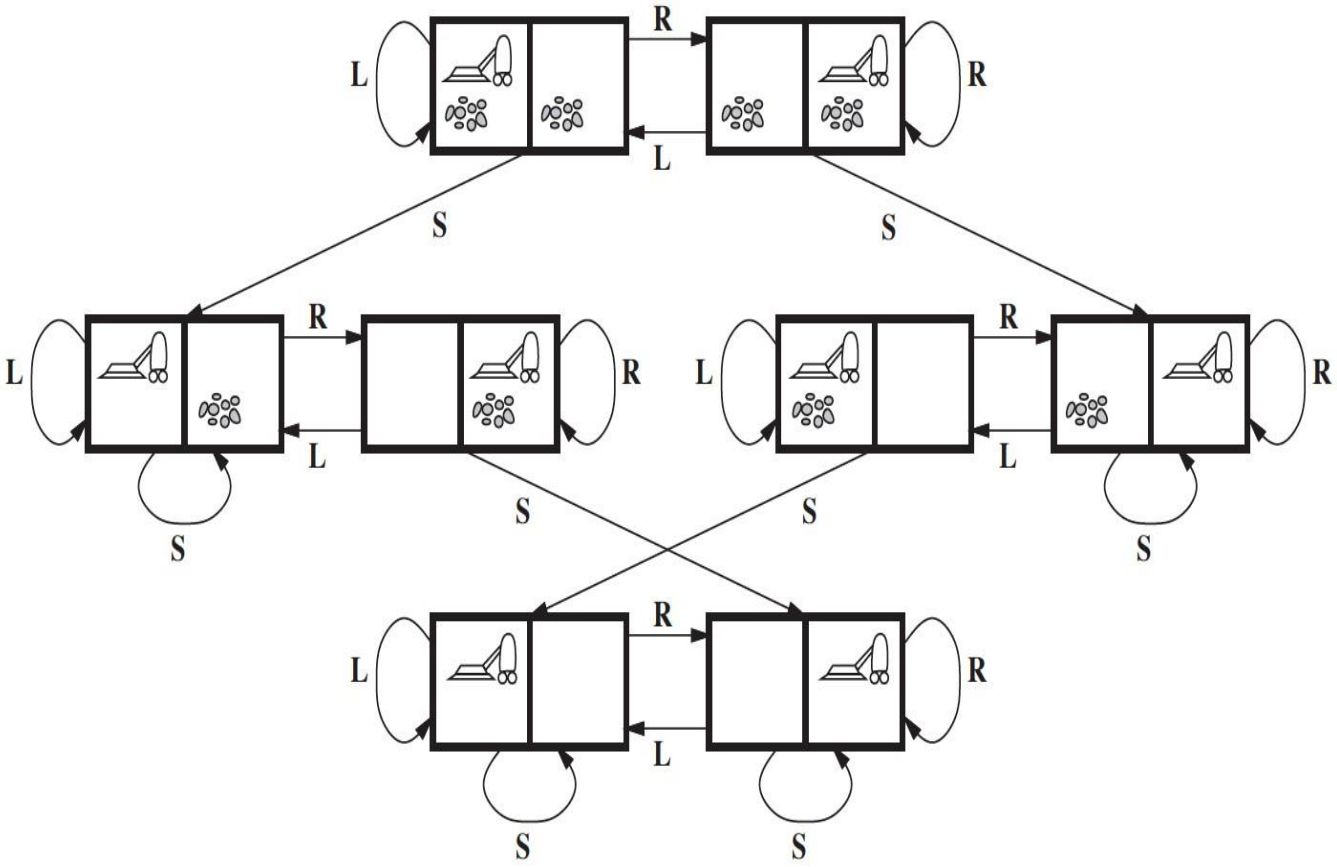
Transition Model – *RESULT(In(Arad), Go(Sibiu)) = In(Sibiu)*

Goal Test – *IsGoal(In(Bucharest)) = Yes*

Path Cost – *cost(In(Arad), go(Sibiu)) = 140 kms*

Example Problem Formulation

	Vacuum World	Travelling Problem
Initial State	Any of the state from S0 to S7	Based on the problem
Possible Actions	[Move Left, Move Right, Suck]	Go(B),Go(d) Based on the problem
Transition Model	[A, ML] = [B , Dirty] [A, ML] = [B, Clean]	[A, Go(A->S)] = [S]
Goal Test	Is all room clean? [A, Clean] [B, Clean]	Is current = B (destination)
Path Cost	No of steps in path +10 → one room to another room +50 → Same state	Cost + Time + Quality


R
L
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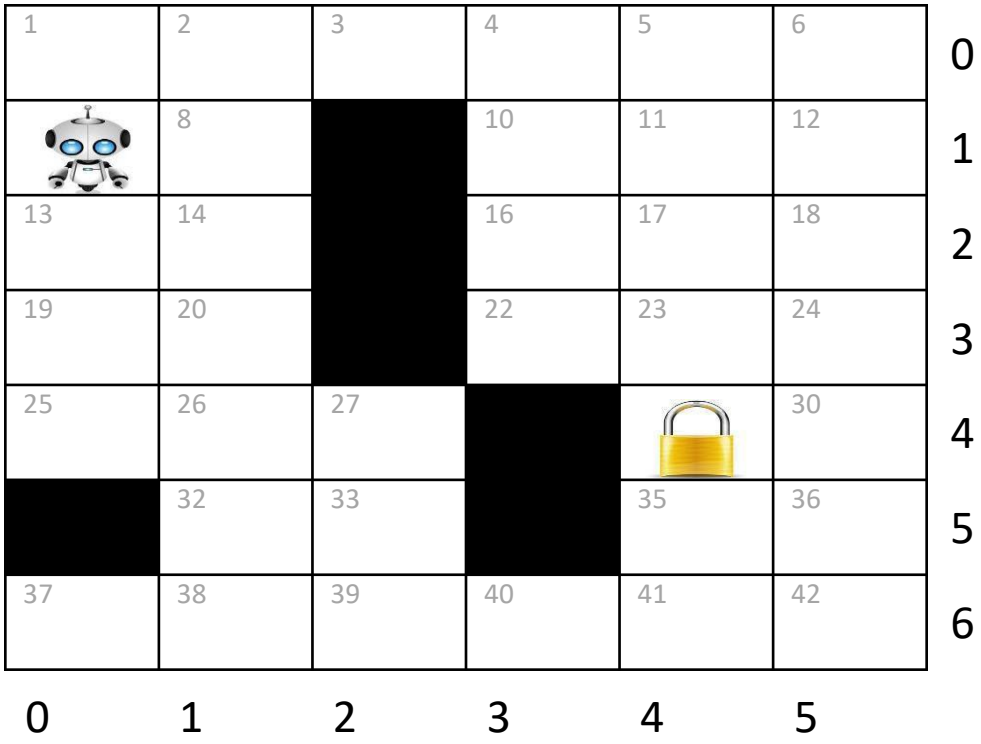
S
S

Path finding Robot

Successor Function Design

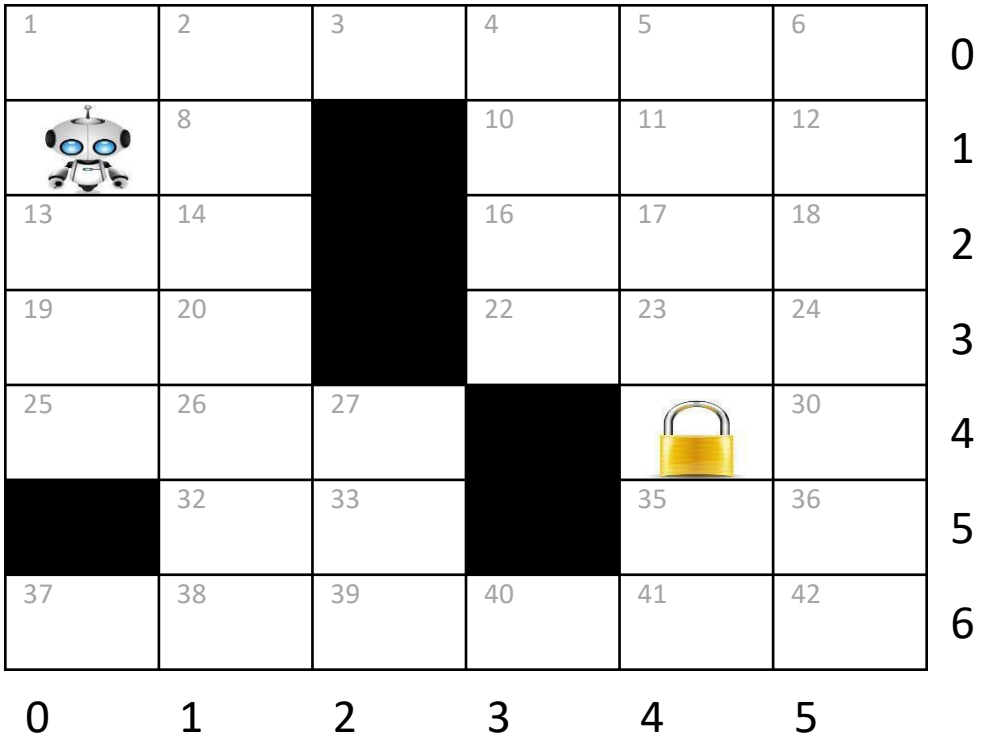


N-W-E-S

	Path finding Robot
Initial State	(1,0)
Possible Actions	[N W E S]
Transition Model	T [X,Y,Issafe,Isgoal] = ? T(1,0,1,0] ,N) = [0,0,1,0]
Goal Test	Is goal = No (0) Is goal = yes (1)
Path Cost	One state to another add some +K contant For eg +1 (unit cost)

Path finding Robot

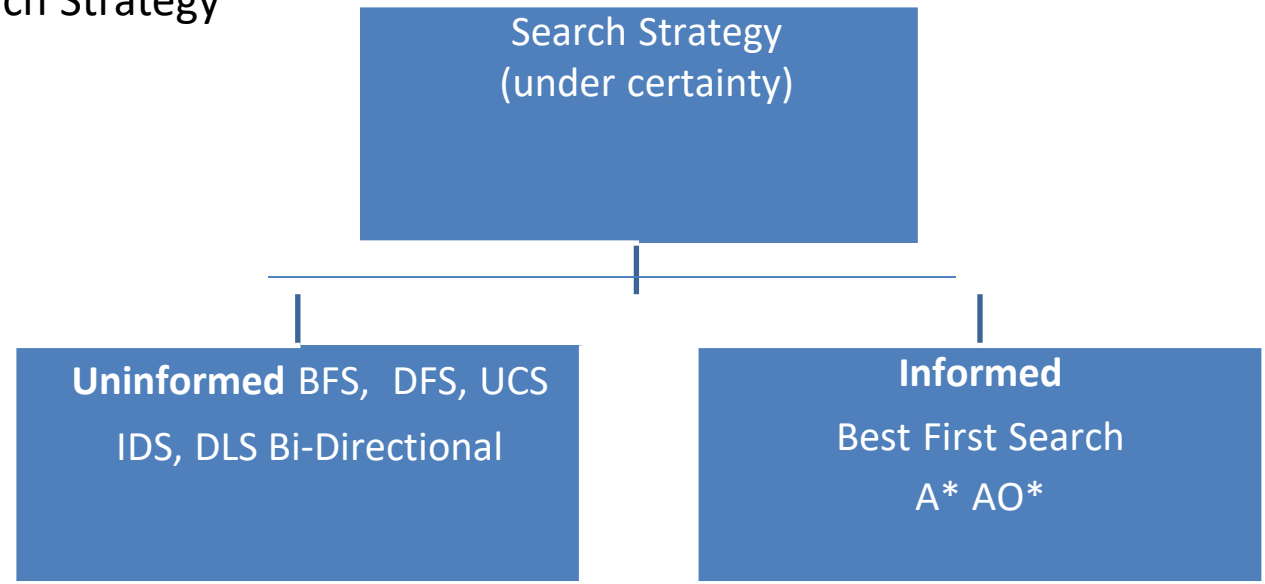
Successor Function Design



N-W-E-S

Searching for Solutions

Choosing the current state, testing possible successor function, expanding current state to generate new state is called Traversal. Choice of which state to expand – Search Strategy



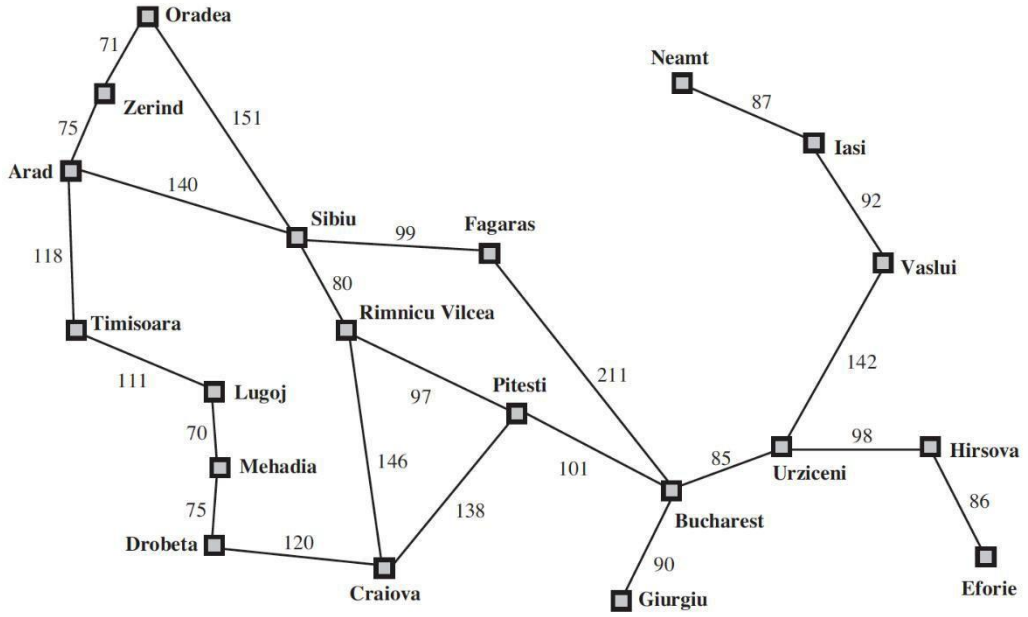
Problem Solving Agents

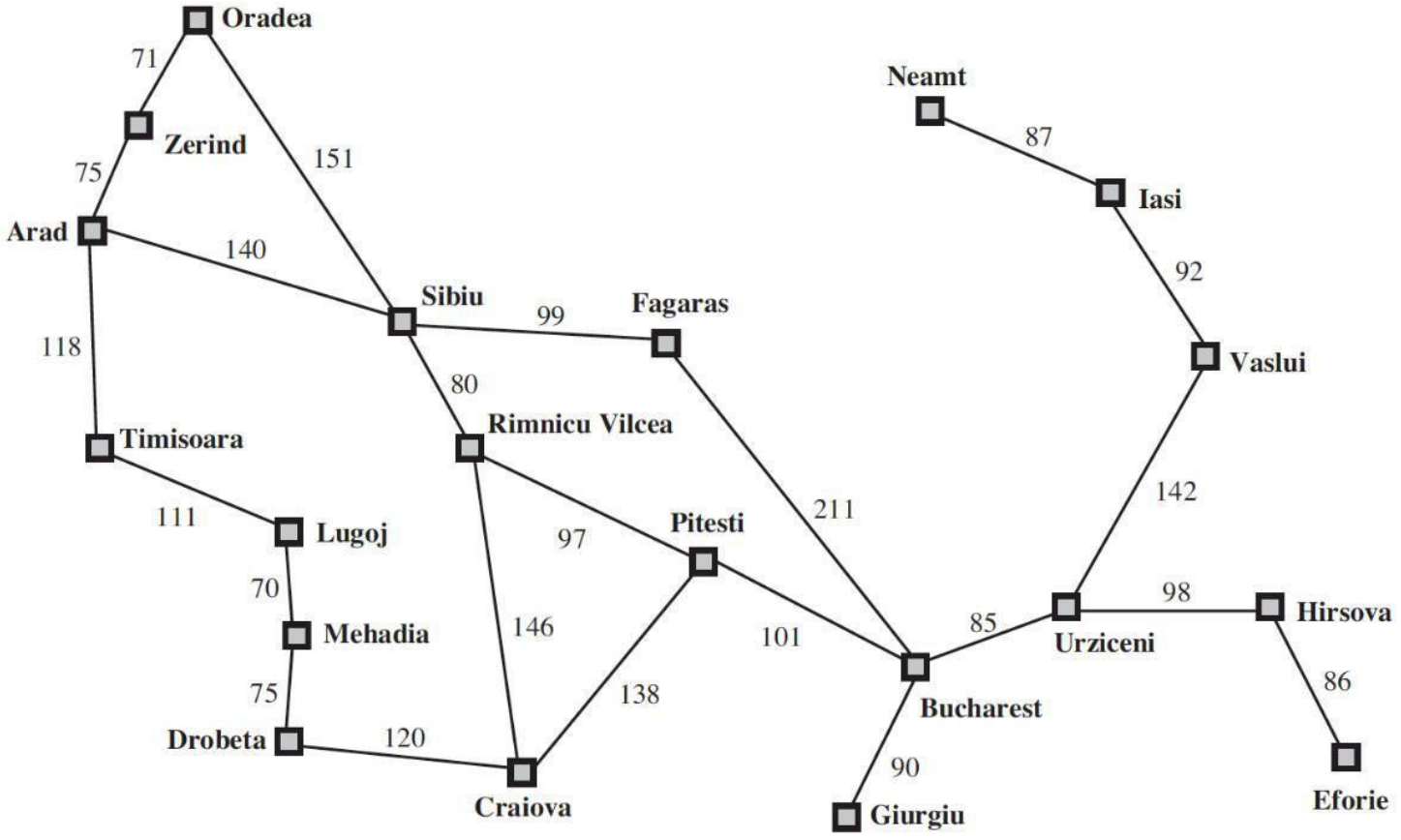
- Goal Formulation
- Problem Formulation
- Search Phase
- Execution Phase

Phases of Solution Search by PSA

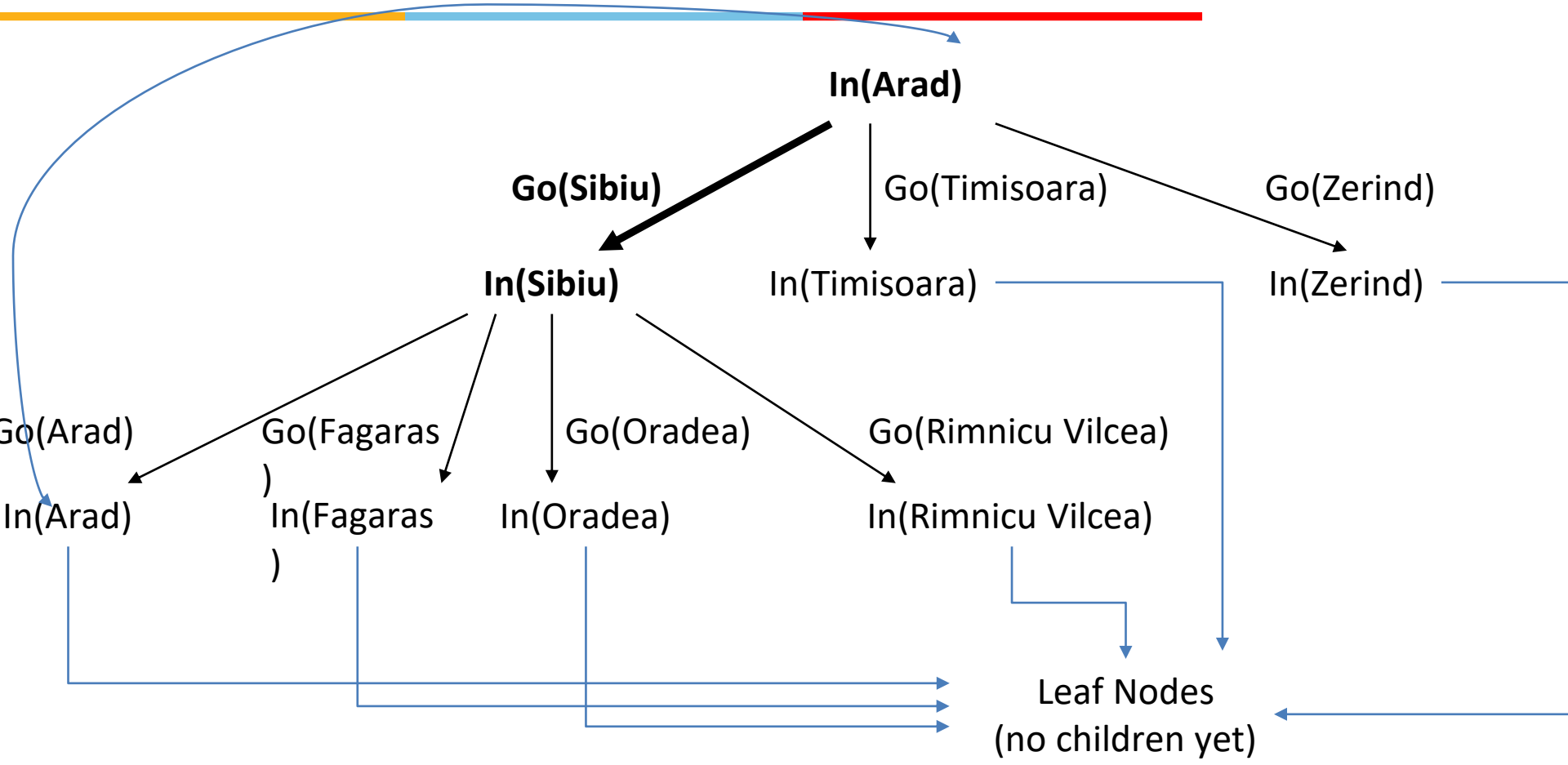
Assumptions – Environment :

Static
Fully Observable
Discrete
Deterministic




Oradea
71
Zerind
151
75
Arad
140
Sibiu
99
Fagaras
118
80
Rimnicu Vilcea
Timisoara
111
Lugoj
97
Pitesti
211
70
Mehadia
146
101
75
138
Drobeta
120
Craiova
Neamt
87
Iasi
92
Vaslui
142
98
Hirsova
85
Urziceni
86
Bucharest
90
Giurgiu
Eforie

Partial Search tree



Terminologies

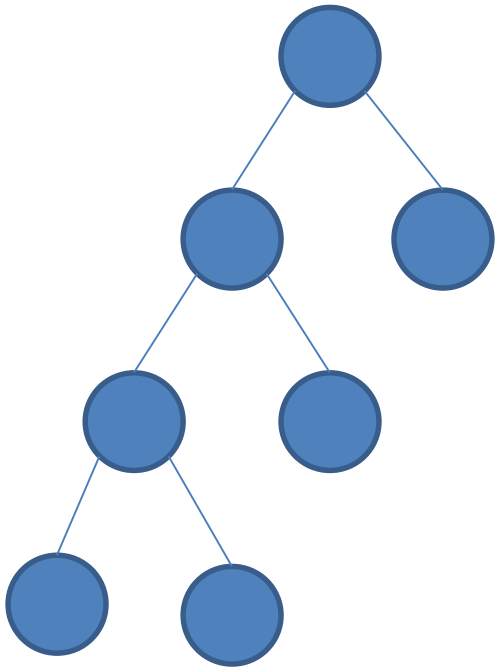
- Nodes
- States
- Frontier | Fringes
- Search Strategy : LIFO | FIFO | Priority Queue
- Performance Metrics
  - Completeness
  - Optimality
  - Time Complexity
  - Space Complexity
- Algorithm Terminology

- d Depth of a node	- m – maximum
- b Branching factor	- C* - Optimal Cost
- n – nodes	- E – least Cost
- l – level of a node	- N –total node generated

Application

Breadth First Search

- Finding path in a graph (many solutions)
- Finding the Bipartitions in a graph



Depth First Search

- Find the Connectedness in a graph
- Topological Sorting

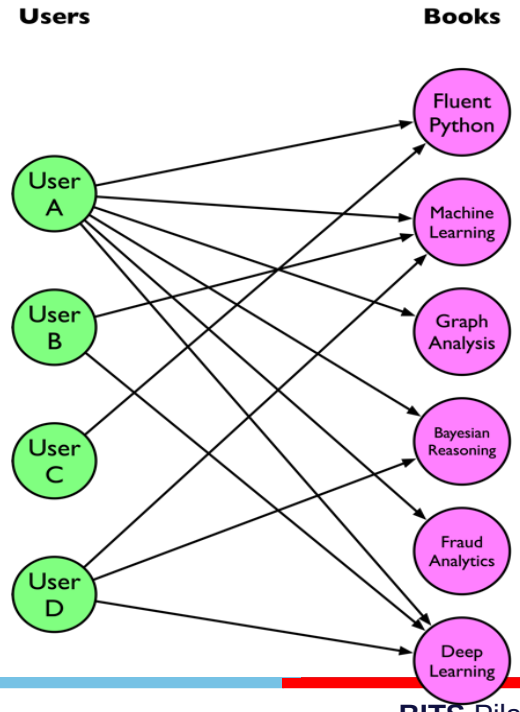
Application

Breadth First Search

- Finding path in a graph (many solutions)
- Finding the Bipartitions in a graph

Depth First Search

- Find the Connectedness in a graph
- Topological Sorting

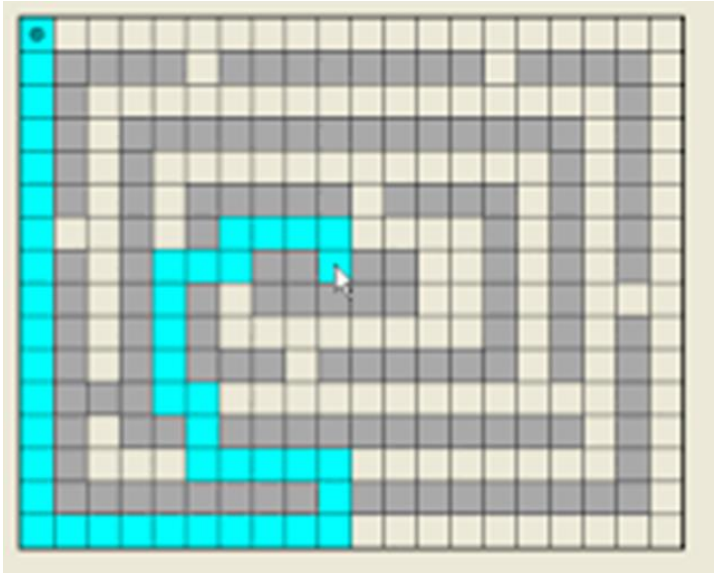


Application

Breadth First Search

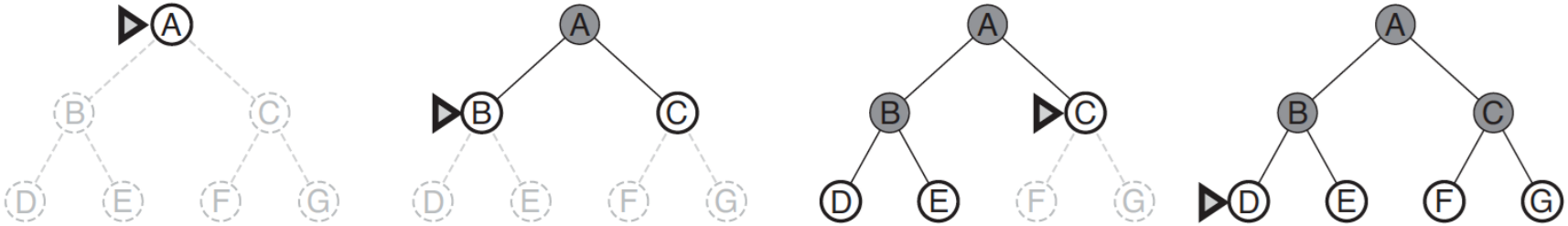
- Finding path in a graph (many solutions)
- Finding the Bipartitions in a graph

Depth First Search



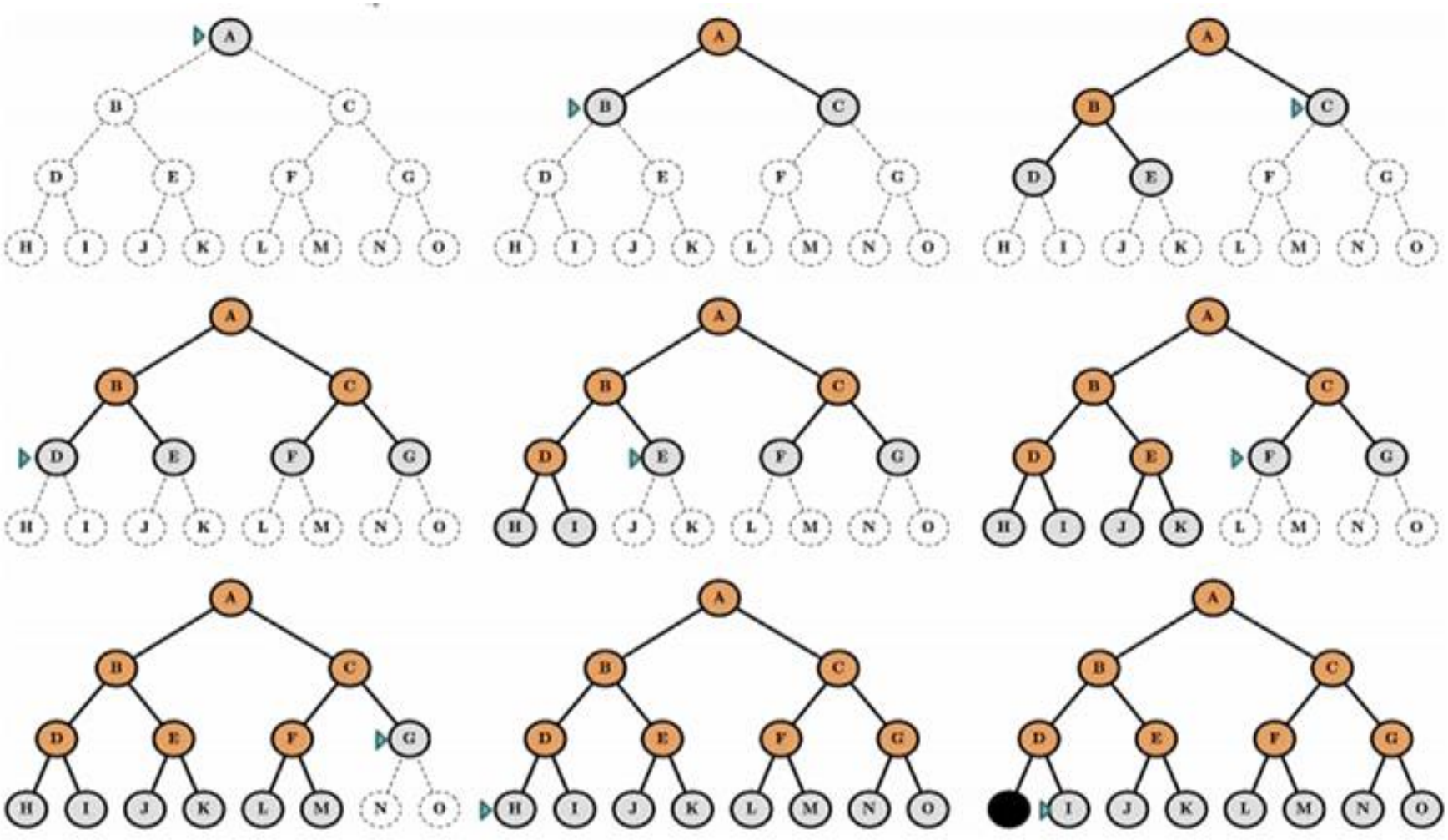
- Find the Connectedness in a graph
- Topological Sorting

Breadth First Search (BFS)

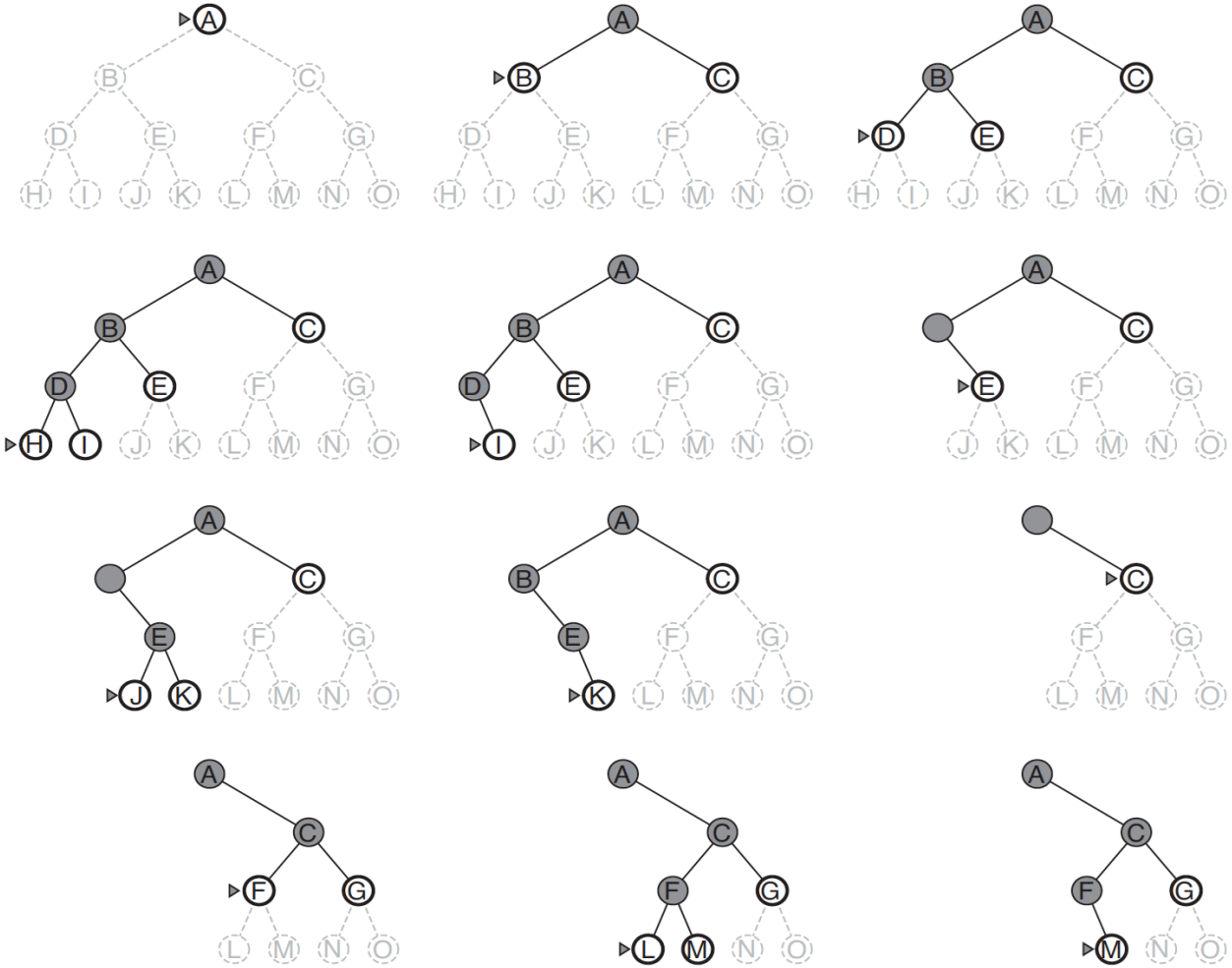


Generate all nodes at a given depth
before proceeding to deeper nodes

Breadth First Search (BFS)



Depth First Search (DFS)



Required Reading: AIMA - Chapter #1, 2, 3.1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML CLZG557

M2 : Problem Solving Agent using Search

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BITS Pilani

Pilani Campus

Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time, Reinforcement Learning
M7	Ethics in AI

Learning Objective

At the end of this class , students Should be able to:

1. Create search tree for given problem
2. Apply uninformed search algorithms to the given problem
3. Compare performance of given algorithms in terms of completeness, optimality, time and space complexity
4. Differentiate for which scenario appropriate uninformed search technique is suitable and justify
5. Differentiate between Tree and Graph search

Module 2 : Problem Solving Agent using Search

A. Uninformed Search

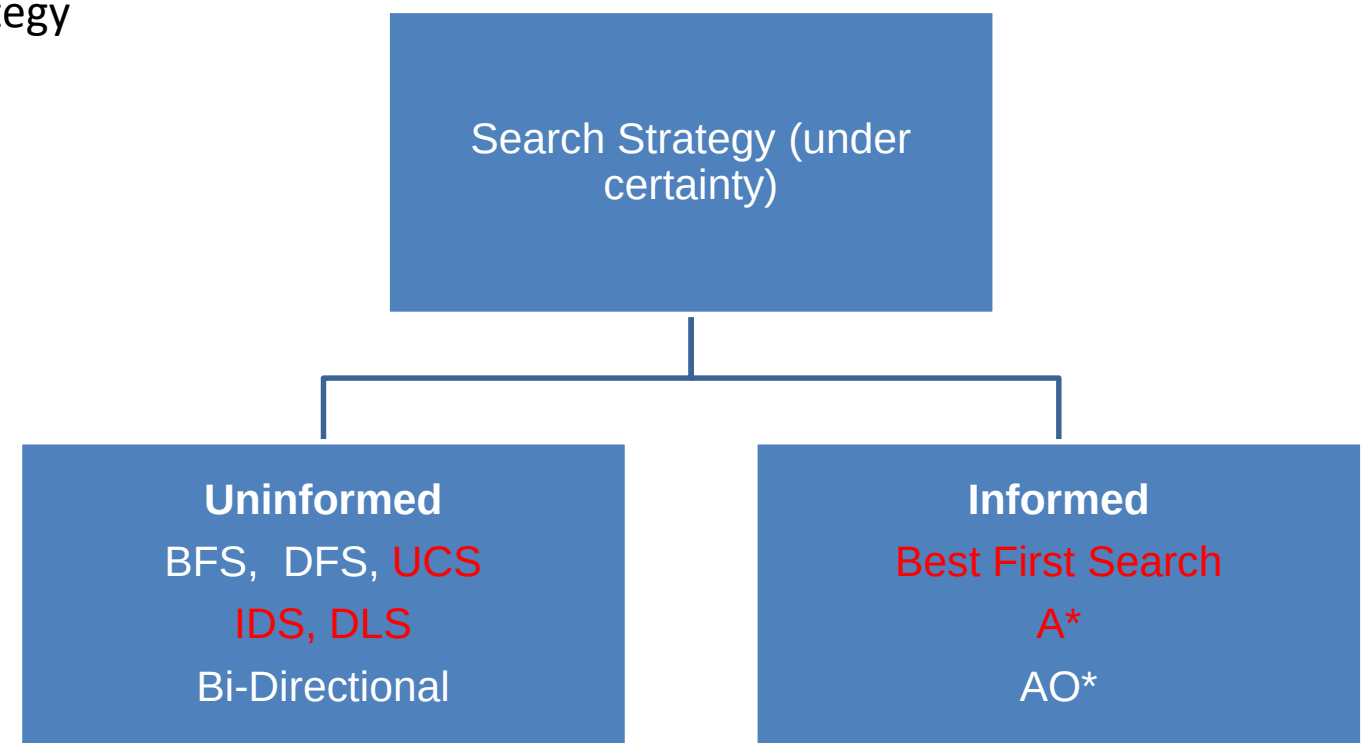
B. Informed Search

C. Heuristic Functions

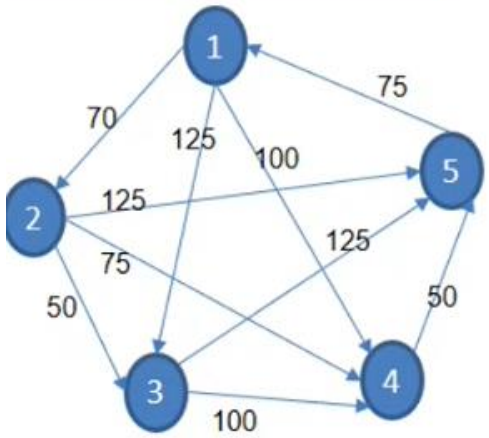
D. Local Search Algorithms & Optimization Problems

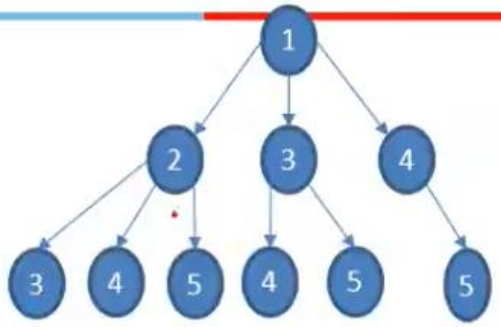
Searching for Solutions

Choosing the current state, testing possible successor function, expanding current state to generate new state is called Traversal. Choice of which state to expand – Search Strategy



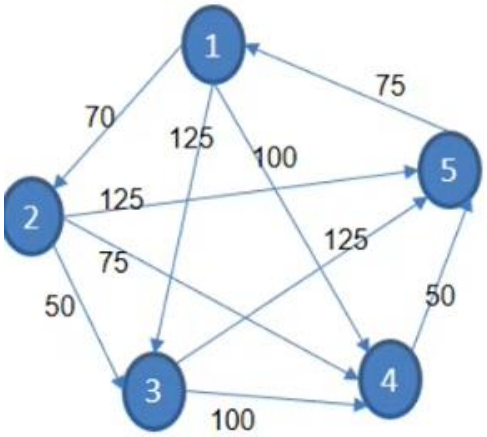
Breath First Search (DFS)

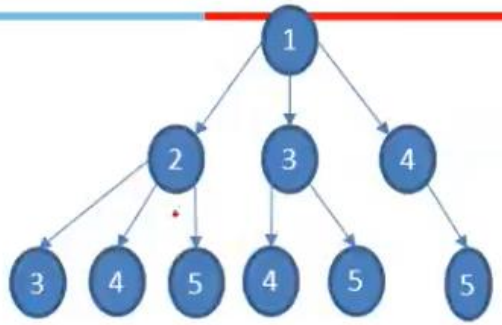




C(1-2-5) = 70 + 125 =195
Expanded : 4
Generated : 10
Max Queue Length : 6

Breath First Search (DFS)

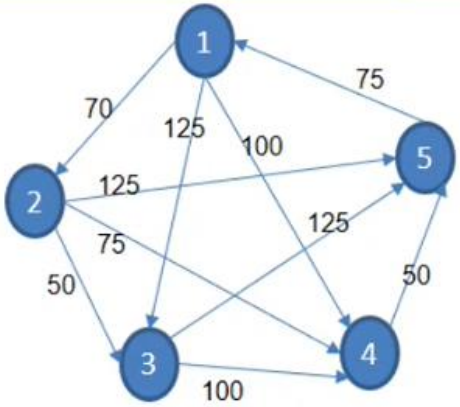


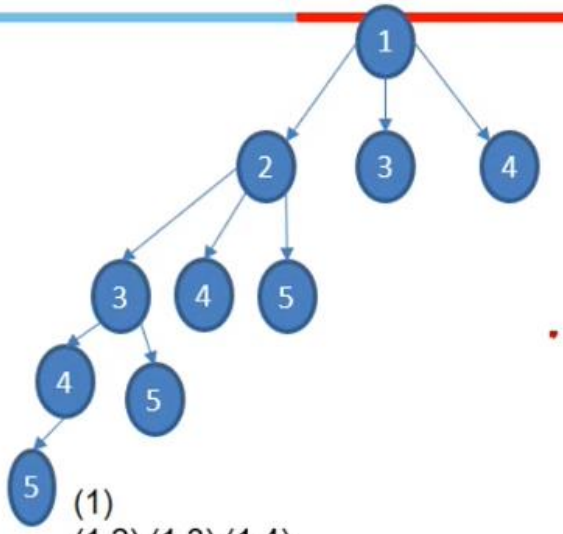


Iter	Open List / Frontiers / Fringes	Closed List	Goal Test (5)
1.	(1)		Fail on (1)
2.	(1 2), (1 3), (1 4)	(1)	Fail on (1 2)
3.			

C(1-2-5) = 70 + 125 =195
Expanded : 4
Generated : 10
Max Queue Length : 6

Depth First Search (DFS)

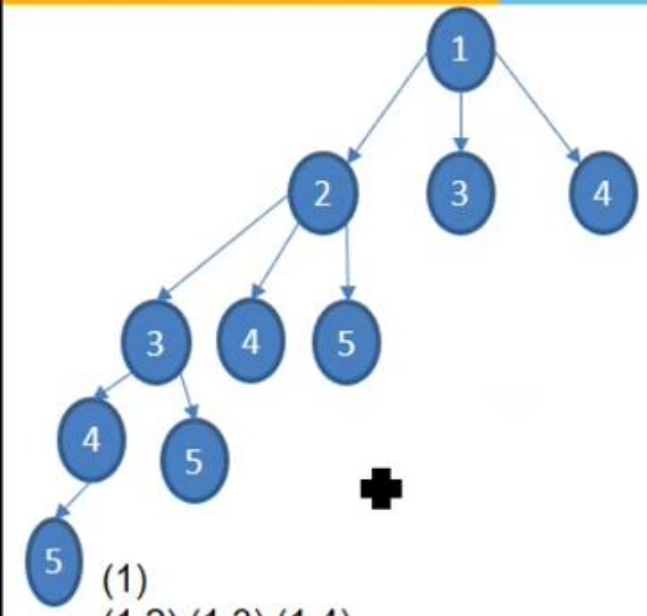




(1)
(1 2) (1 3) (1 4)
(1 2 3) (1 2 4) (1 2 5) (1 3) (1 4)
(1 2 3 4) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)
(1 2 3 4 5) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)

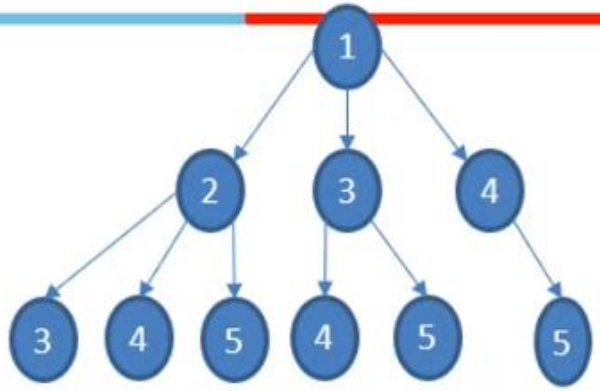
C(1-2-3-4-5) = 70 + 50 + 100 + 50 =270
Expanded : 4
Generated : 10
Max Queue Length : 6

Search Algorithm – Uninformed Example - 2



(1)
(1 2) (1 3) (1 4)
(1 2 3) (1 2 4) (1 2 5) (1 3) (1 4)
(1 2 3 4) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)
(1 2 3 4 5) (1 2 3 5) (1 2 4) (1 2 5) (1 3) (1 4)

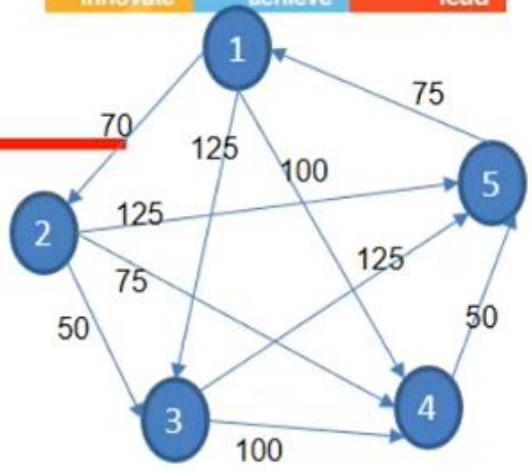
C(1-2-3-4-5) = 70 + 50 + 100 + 50 =270
Expanded : 4
Generated : 10
Max Queue Length : 6



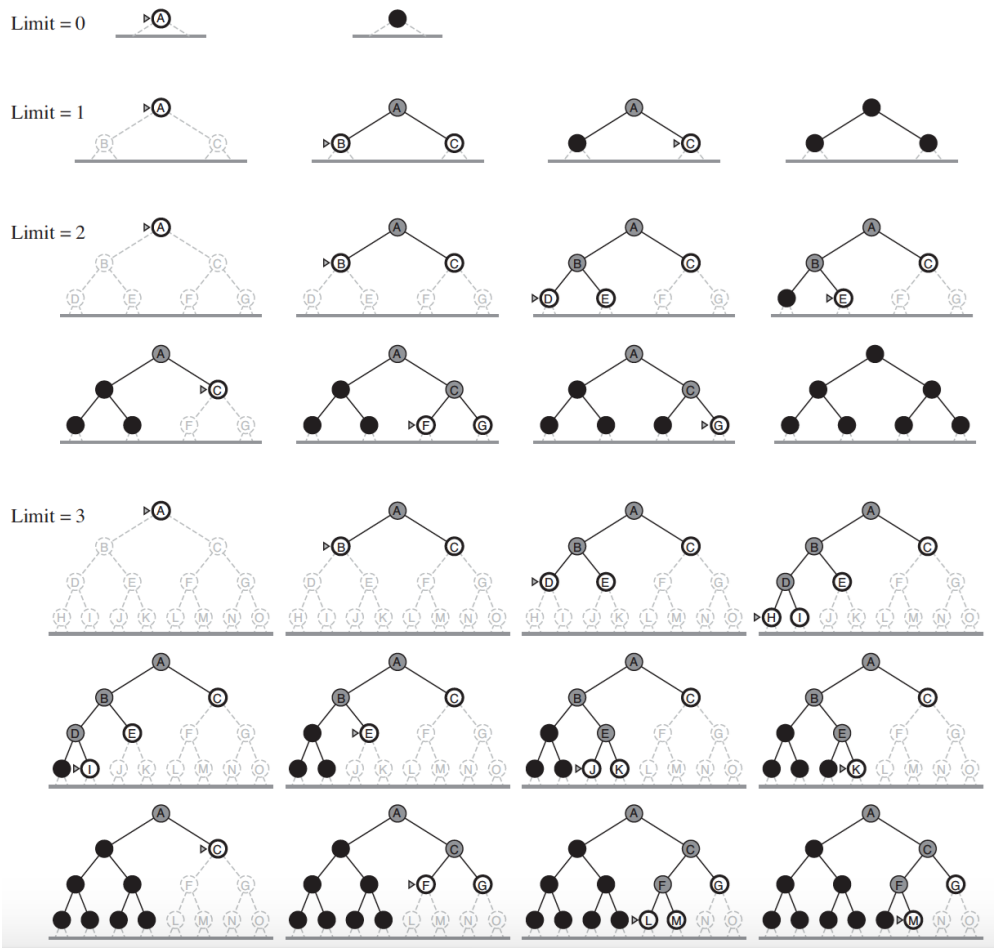
(1)
(1 2) (1 3) (1 4)
TEST FAILED

(1 3) (1 4) (1 2 3) (1 2 4) (1 2 5)
(1 2 3) (1 2 4) (1 2 5) (1 3 4) (1 3 5) (1 4 5)
TEST PASSED

C(1-2-5) = 70 + 125 =195
Expanded : 4
Generated : 10
Max Queue Length : 6



Iterative Deepening Depth First Search (IDS)



Iterative Deepening Depth First Search (IDS)

Run Depth Limited Search (DLS) by gradually increasing the limit l

- First with l=1, then l=2, l=3 and so on – until goal is found

Its is a combination of Depth First Search + Breadth First Search

Like DFS, memory requirement is a modest $\mathcal{O}(bd)$ where d is the depth of shallowest goal

Like BFS

- Complete when branching factor is finite
- Optimal when path cost is non decreasing function of depth

Iterative Deepening Depth First Search (IDS)

innovate achieve lead

Time Complexity:

- Appears that IDS is generating a lot of nodes multiple times
- However, most of nodes are present in the lower levels which are not repeated often
- Generation of nodes
  - At level 1 - b nodes generated d times – (d)b
  - At level 2 – b^2 nodes generated d-1 times – (d-1)b^2
  - At level d – b^d nodes generated once – (1) b^d
- Time Complexity N(IDS) = $\mathcal{O}(b^d)$ same as BFS

IDS is the preferred uninformed search method when search space is large and depth is unknown

Uniform Cost Search

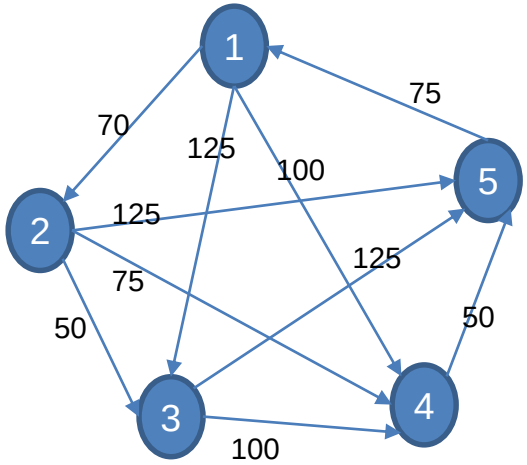
Instead of expanding the shallowest node, Uniform-Cost search expands the node n with the lowest path cost g(n)

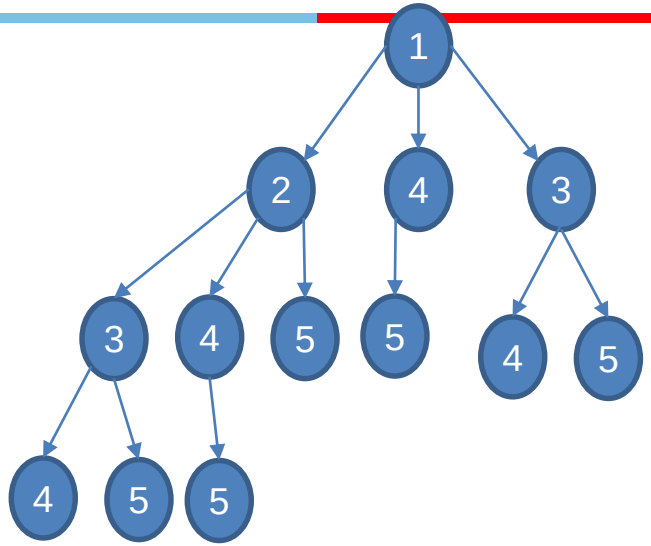
Sorting the Frontier as a priority queue ordered by g(n)

Goal test is applied during expansion

- The goal node if generated may not be on the optimal path
- Find a better path to a node on the Frontier

UCS





(1)
(1 2 : 70) (1 4 : 100) (1 3 : 125)
TEST-F
(1 4 : 100) (1 2 3 :120)(1 3 : 125) (1 2 4 : 145) (1 2 5 : 195)
TEST-F
(1 2 3 :120)(1 3 : 125) (1 2 4 : 145) (1 4 5: 150)(1 2 5 : 195)
TEST-F
(1 3 : 125) (1 2 4 : 145) (1 4 5: 150) (1 2 3 4:170) (1 2 5 : 195) (1 2 3 5 : 245)
TEST-F
(1 2 4 : 145) (1 4 5: 150) (1 2 3 4:170) (1 2 5 : 195) (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)
TEST-F
(1 4 5: 150) (1 2 3 4:170) (1 2 4 5 : 195) (1 2 5 : 195) (1 3 4 : 225) (1 2 3 5 : 245) (1 3 5 : 250)
TEST - P

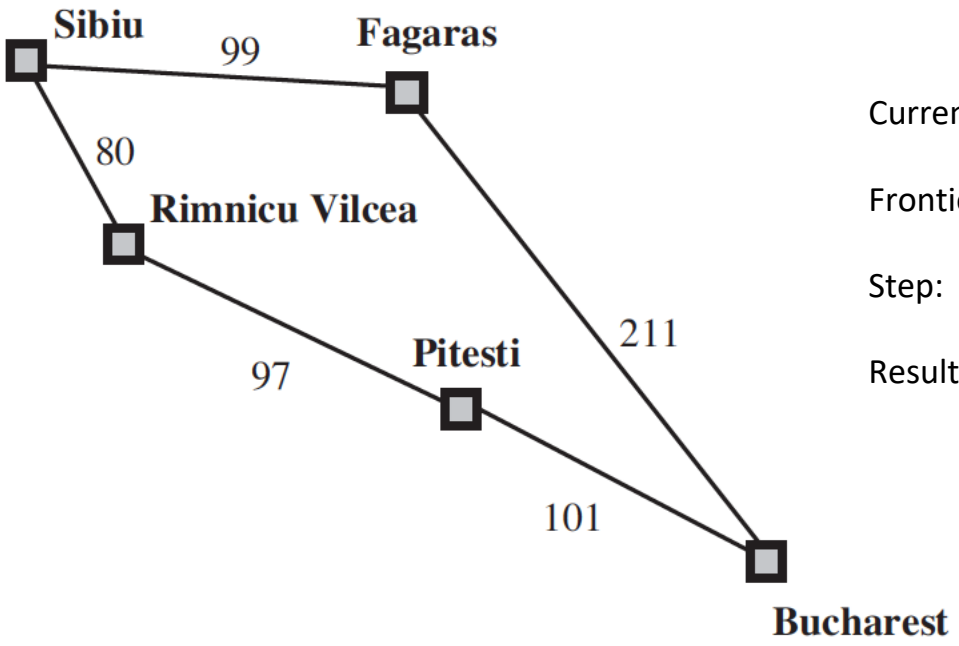
Uniform Cost Search – Evaluation

Completeness – It is complete if the cost of every step > small +ve constant ∈
- It will stuck in infinite loop if there is a path with infinite sequence of zero cost actions

Optimal – It is Optimal. Whenever it selects a node, it is an optimal path to that node.

Time and Space complexity – Uniform cost search is guided by path costs not depth or branching factor.
- If C* is the cost of optimal solution and ∈ is the min. action cost
- Worst case complexity = $\mathcal{O}(b^{1+\frac{C^*}{\epsilon}})$,
- When all action costs are equal → $\mathcal{O}(b^{d+1})$, the BFS would perform better
  - As Goal test is applied during expansion, Uniform Cost search would do extra work

Uniform Cost Search



Current State: Sibiu

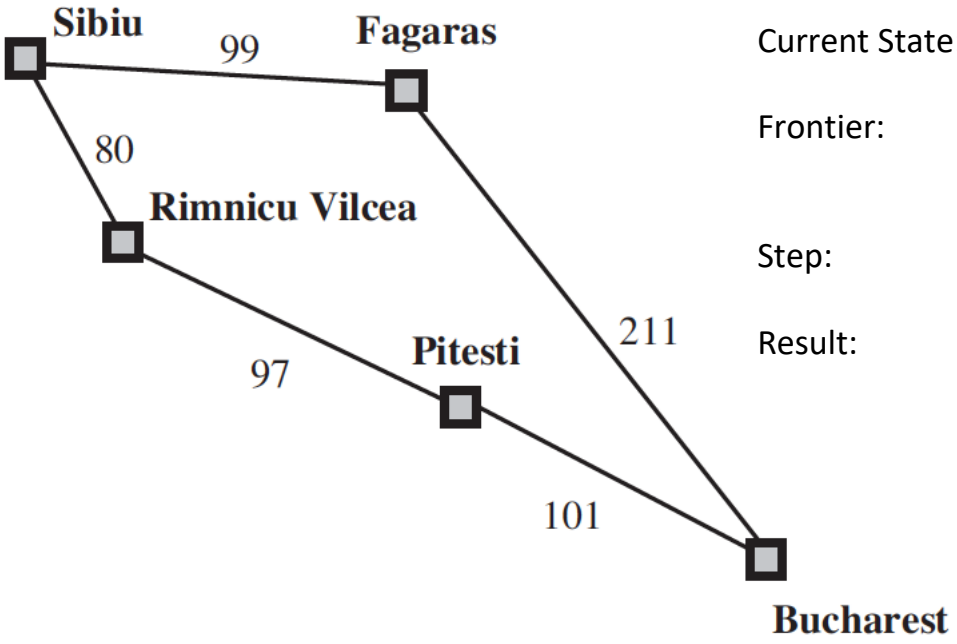
Frontier: []

Step: Expand Sibiu

Result: Generates (“Rimnicu Vilcea” 80)
(“Fagaras”, 99)
Add to Frontier

Initial State: Sibiu
Goal State: Bucharest

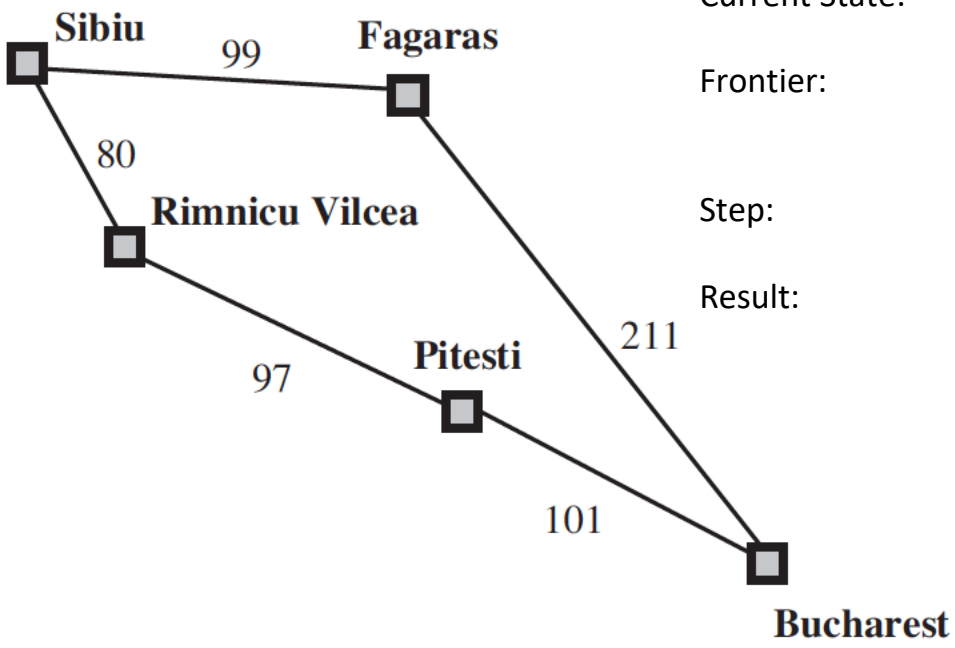
Uniform Cost Search



Current State:	Sibiu
Frontier:	[(“Rimnicu Vilcea” 80) (“Fagaras”, 99)]
Step:	Expand “Rimnicu Vilcea” (least cost)
Result:	Generates (“Pitesti”, 177) Add to Frontier

Initial State:	Sibiu
Goal State:	Bucharest

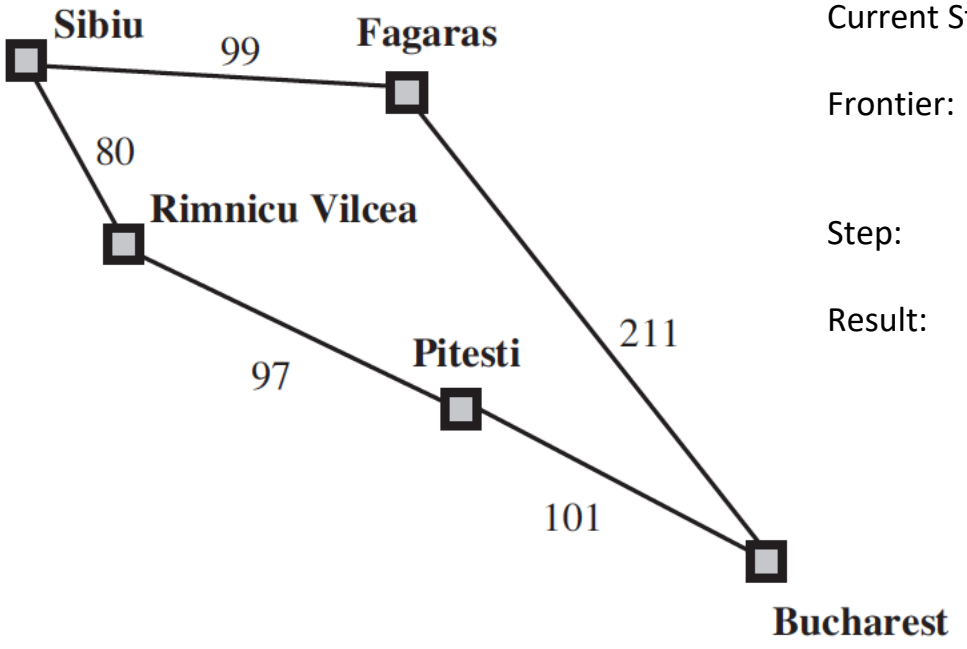
Uniform Cost Search



Current State:	Rimnicu Vilcea (not a Goal state)
Frontier:	[(“Fagaras”, 99) (“Pitesti”, 177)]
Step:	Expand “Fagaras” (least cost)
Result:	Generates (“Bucharest”, 310) Add to Frontier (It’s a Goal State but we won’t test during generation)

Initial State:	Sibiu
Goal State:	Bucharest

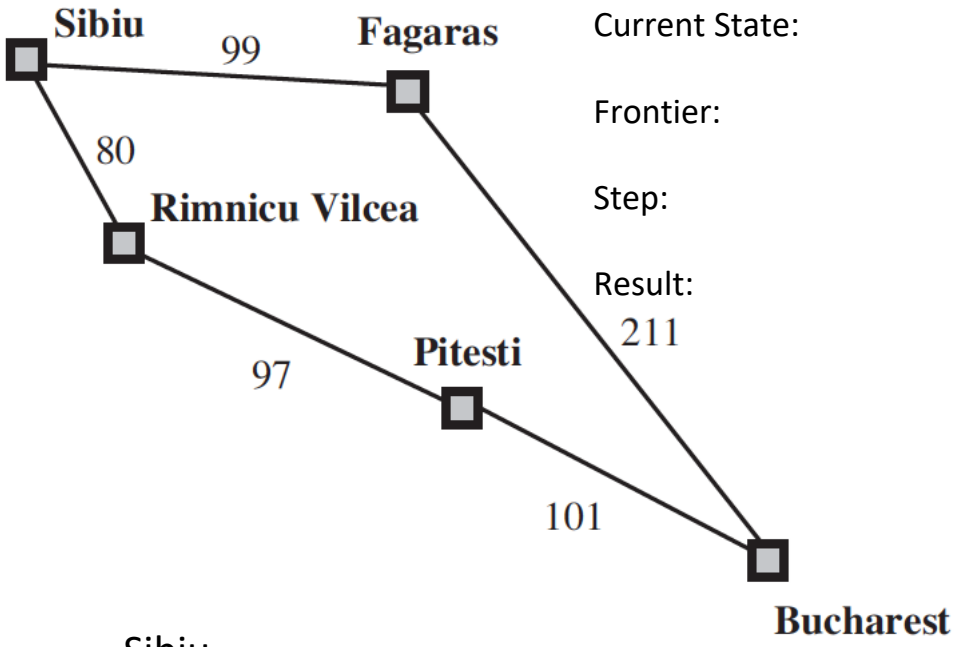
Uniform Cost Search



Current State:	Fagaras (not a goal state)
Frontier:	[(“Pitesti”, 177) (“Bucharest”, 310)]
Step:	Expand “Pitesti” (least cost)
Result:	Generates (“Bucharest”, 278) Replace in Frontier (It’s a Goal State but we won’t test during generation)

Initial State:	Sibiu
Goal State:	Bucharest

Uniform Cost Search



Current State: Pitesti (not a goal state)

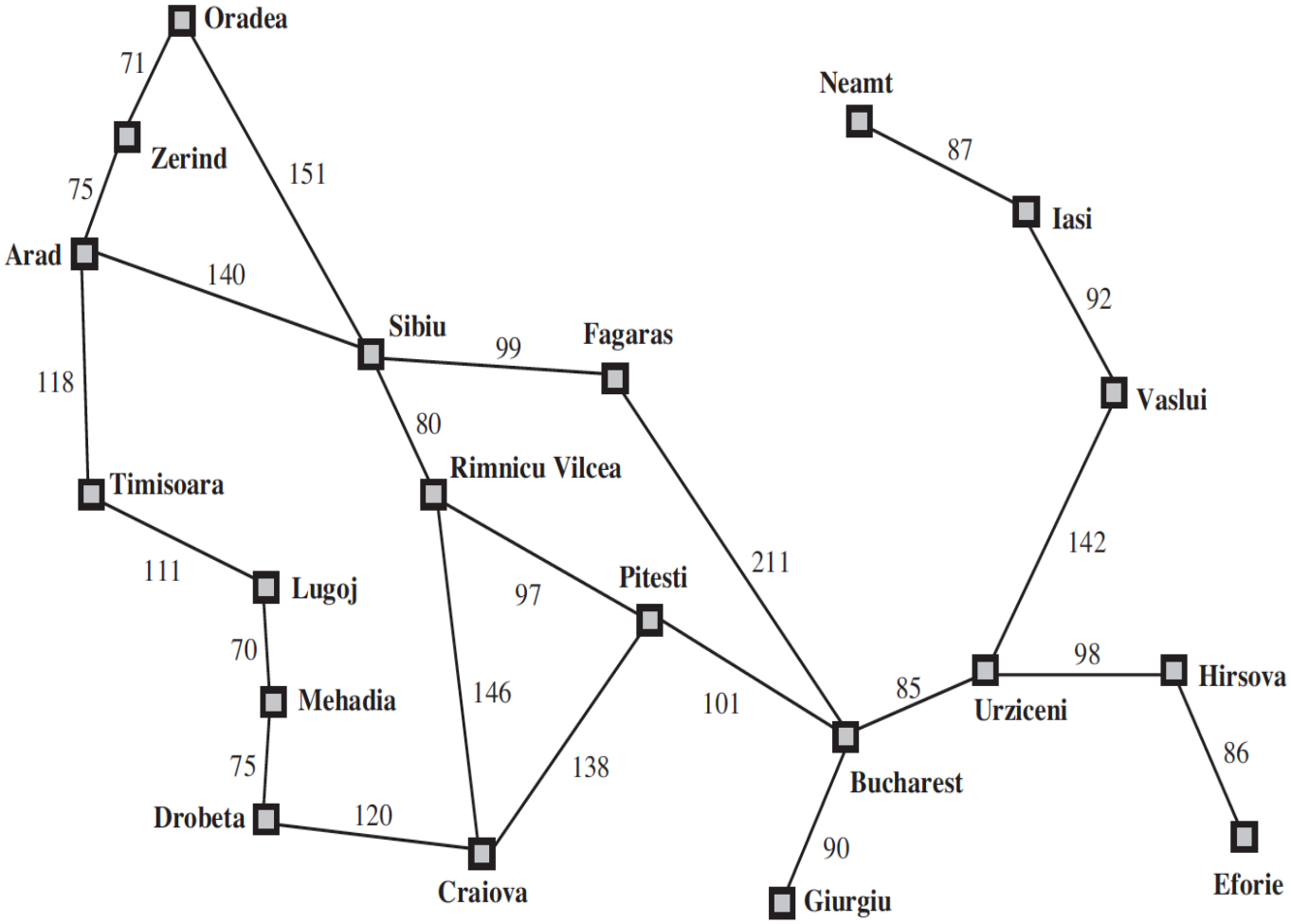
Frontier: [(“Bucharest”, 278)]

Step: Expand “Bucharest”

Result: No further generation as Goal Test satisfied Return Solution

Initial State: Sibiu
Goal State: Bucharest

Tree Search Vs Graph Search


Oradea
71
Zerind
151
75
Arad
140
Sibiu
99
Fagaras
118
80
Rimnicu Vilcea
Timisoara
111
Lugoj
97
Pitesti
211
70
Mehadia
146
101
75
138
Drobeta
120
Craiova
Neamt
87
Iasi
92
Vaslui
142
98
Hirsova
85
Urziceni
86
Bucharest
90
Giurgiu
Eforie

Coding Aspects

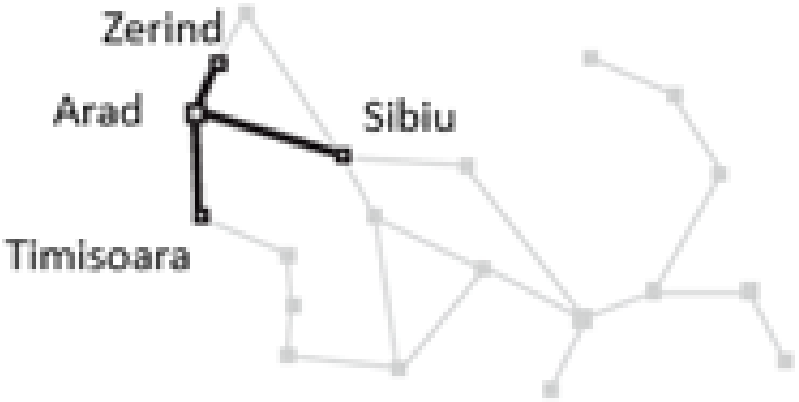
For each node n of the tree,

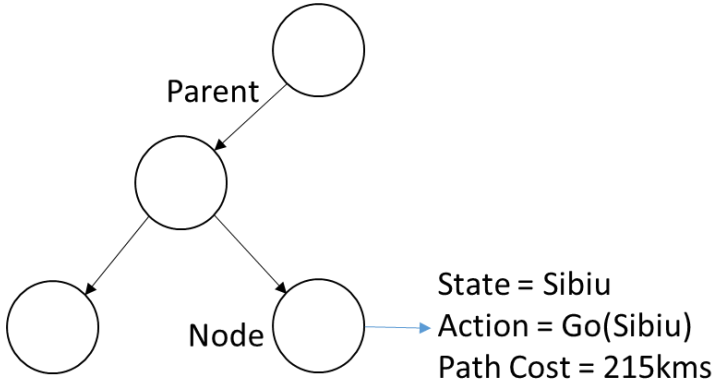
n.STATE : the state in the state space to which node corresponds

n.PARENT : the node in the search tree that generated this node

n.ACTION : the action that was applied to parent to generate the node

n.PATH-COST : the cost, denoted by g(n), of the path from initial state to node





Tree Search Algorithms

```
function Tree-Search (problem, strategy) returns a solution, or failure
        initialize the search tree using the initial state of problems
        loop do
                if there are no candidate for expansion
                        then return failure
                choose: leaf node for expansion according to strategy
                if the node contains a goal state
                        then return the corresponding solution
                else
                        Expand the node
                        Add the resulting nodes to the search tree
        end
```

Graph Search

Coding Aspects

For each node n of the tree,

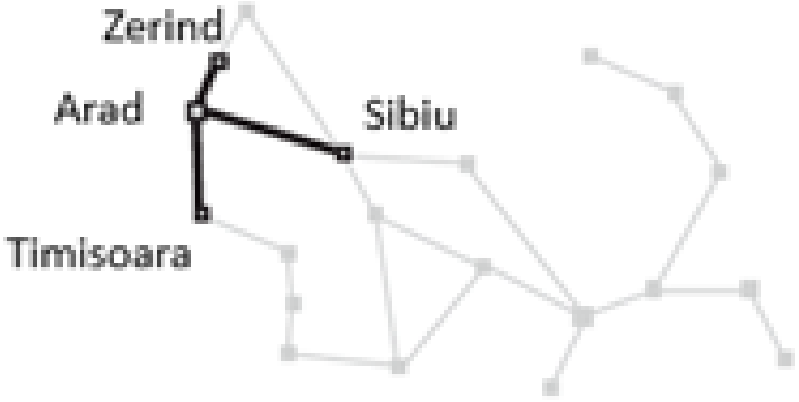
n.STATE : the state in the state space to which node corresponds

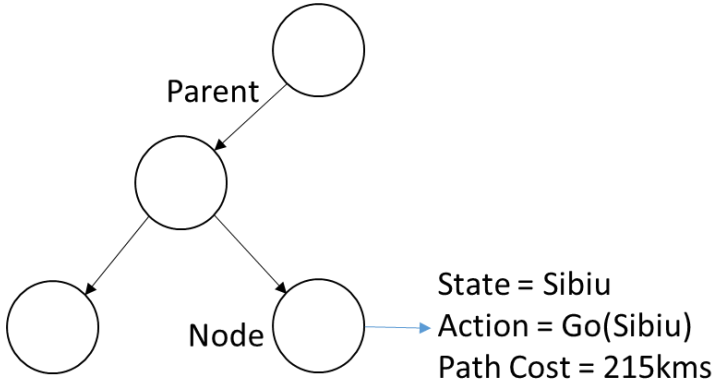
n.PARENT : the node in the search tree that generated this node

n.ACTION : the action that was applied to parent to generate the node

n.PATH-COST : the cost, denoted by g(n), of the path from initial state to node

n.VISITED **:** the boolean indicating if the node is already visited and tested (**or)** a global SET of visited nodes





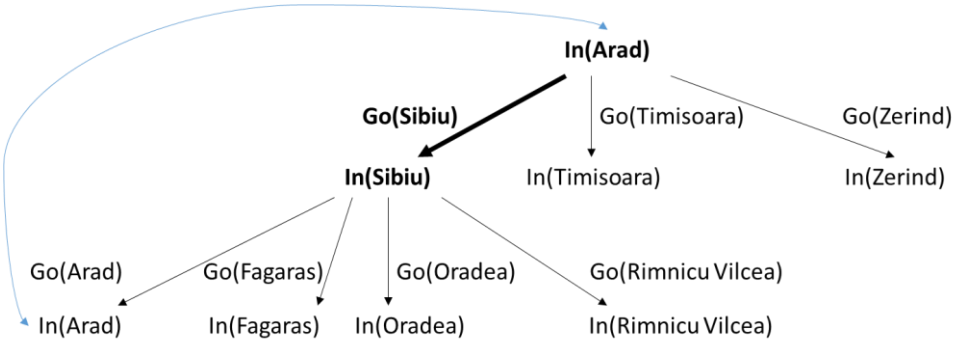
Tree Search Vs Graph Search Algorithms

innovate achieve lead

Coding Aspects

Graph-Search Algorithm

Augments the Tree-Search algorithm to solve redundancy by keeping track of states that are already visited called as Explored Set. Only one copy of each state is maintained/stored.



Required Reading: AIMA - Chapter # 3.1, 3.2, # 3.3,

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML CLZG557

M2 : Problem Solving Agent using Search

Indumathi V

Guest Faculty,

BITS - CSIS

BITS Pilani

Pilani Campus

Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time, Reinforcement Learning
M7	Ethics in AI

Learning Objective

At the end of this class , students Should be able to:

1. Create Search tree for given problem
2. Design and compare heuristics apt for given problem
3. Apply UCS, GBFS & A* algorithms to the given problem
4. Compare performance of given algorithms in terms of completeness, optimality, time and space complexity
5. Differentiate between uninformed and informed search requirements
6. Prove if the given heuristics are admissible and consistent

Module 2 : Problem Solving Agent using Search

A. Uninformed Search

B. Informed Search

C. Heuristic Functions

D. Local Search Algorithms & Optimization Problems

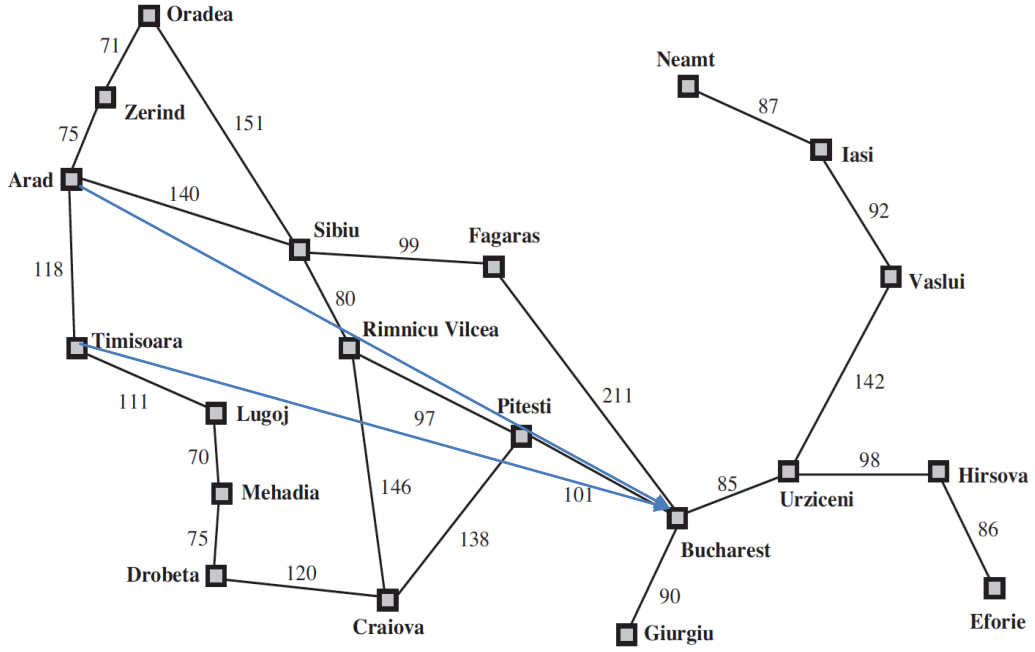
Informed Search
Greedy Best First
A*

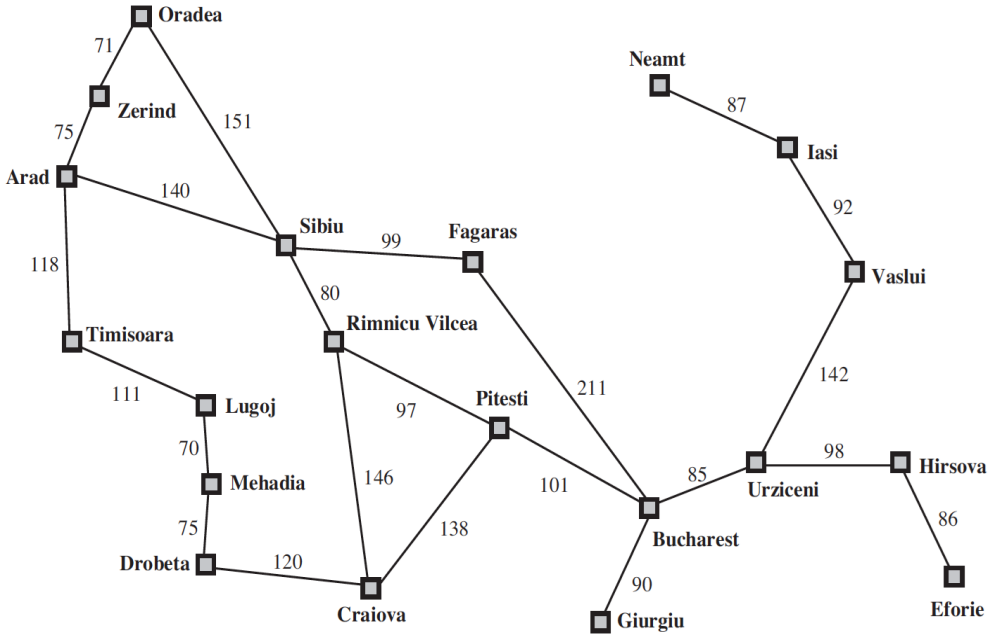
Informed /Heuristic Search

Strategies that know if one non-goal state is more promising than another non-goal state

h(Arad) = 366

h(Timisoara) = 329





Sample Heuristic

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
Drobeta	242	Pitesti	100
Eforie	161	Rimnicu Vilcea	193
Fagaras	176	Sibiu	253
Giurgiu	77	Timisoara	329
Hirsova	151	Urziceni	80
Iasi	226	Vaslui	199
Lugoj	244	Zerind	374

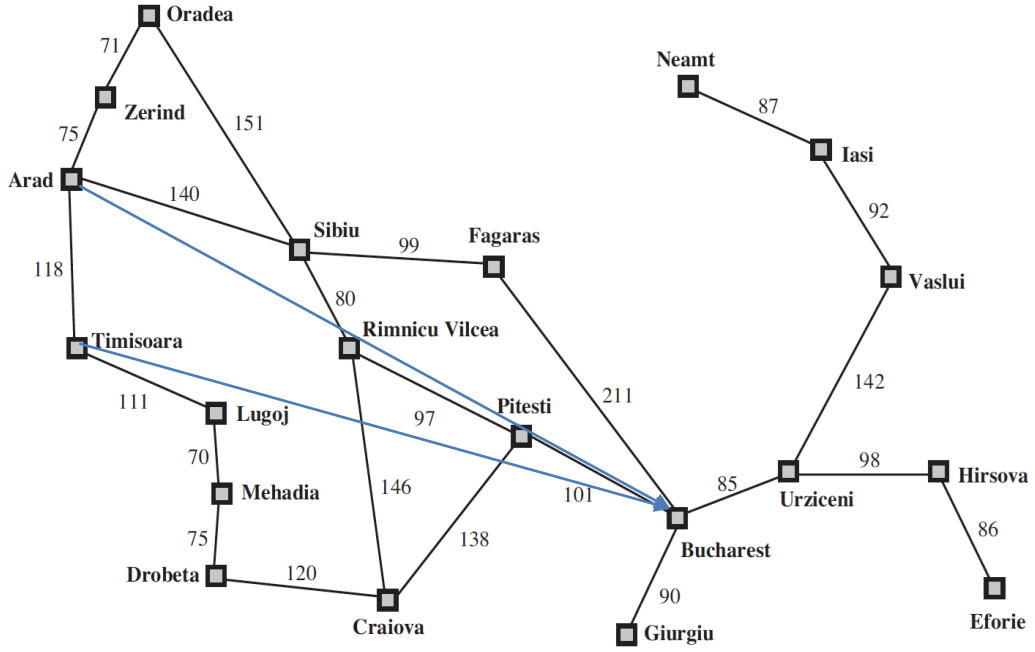
A	B	C	D	E
0	10	50	40	50
10	5	0	15	20

Informed /Heuristic Search

Strategies that know if one non-goal state is more promising than another non-goal state

h(Arad) = 366

h(Timisoara) = 329



Greedy Best First Search

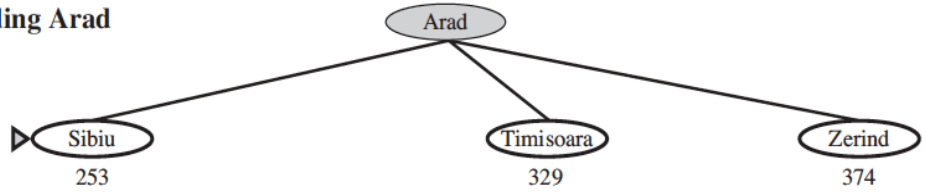
Expands the node that is closest to the goal

Thus, f(n) = h(n)

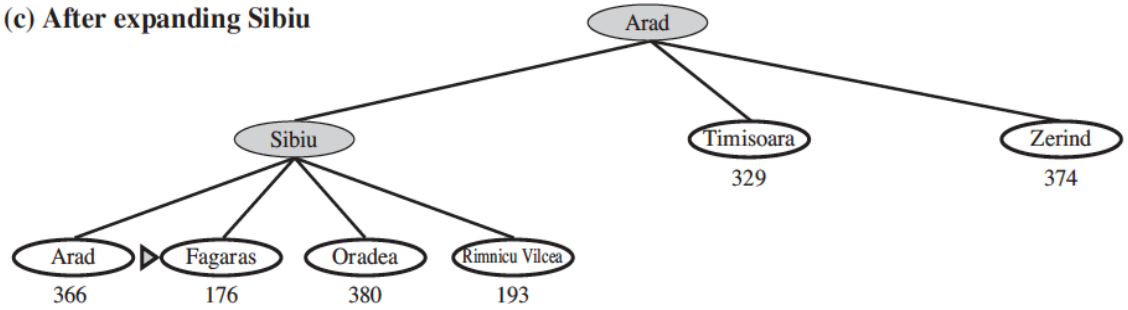
Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

(a) The initial state

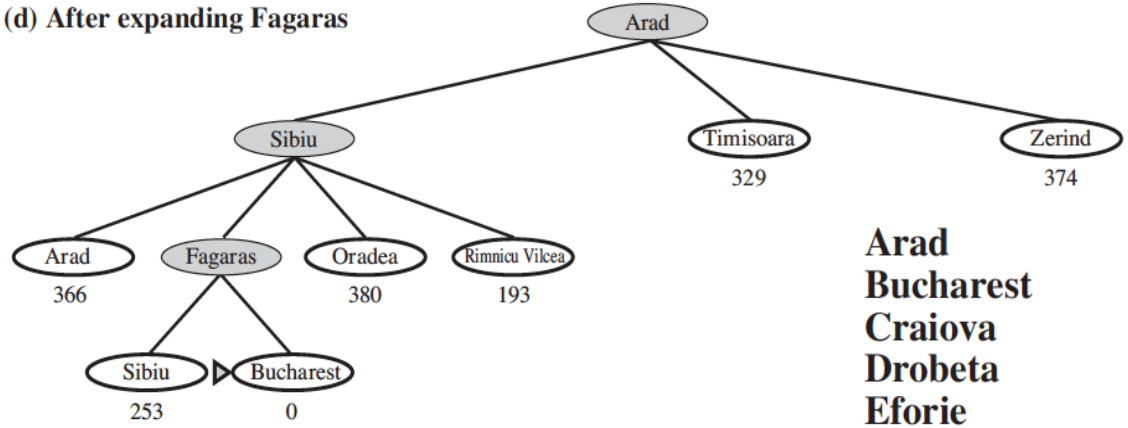
(b) After expanding Arad



(c) After expanding Sibiu

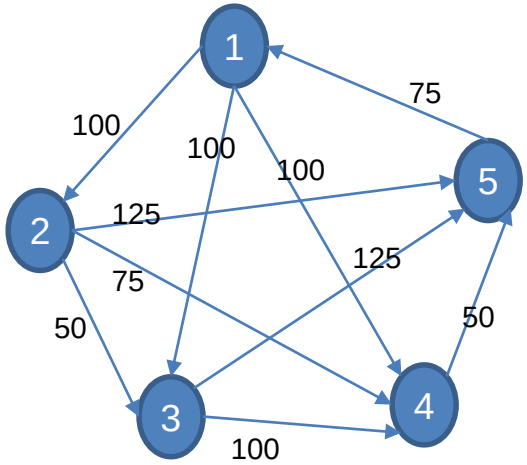


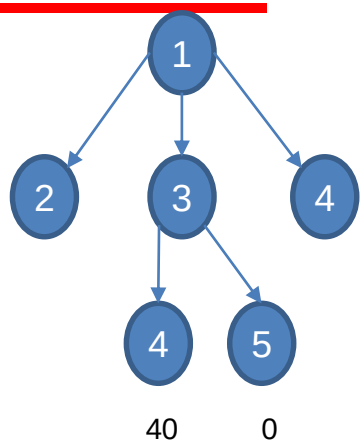
(d) After expanding Fagaras



Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy Best First Search

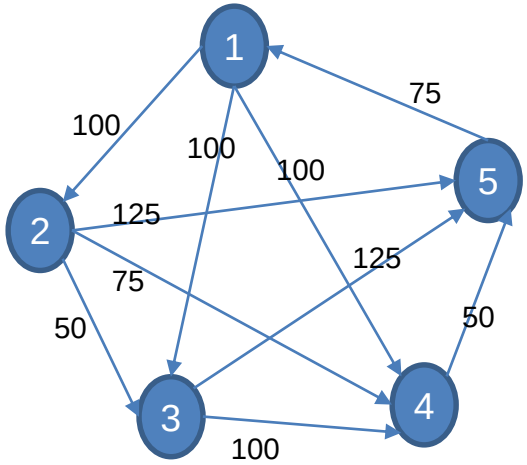


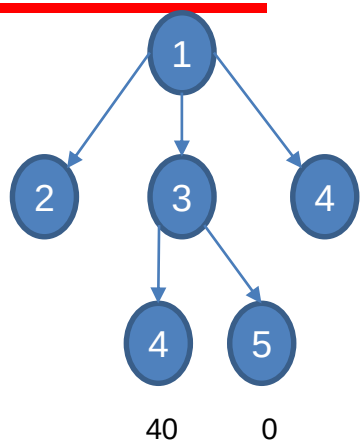


n	h(n)
1	60
2	120
3	30
4	40
5	0

(1)
(1 3) (1 4) (1 2)
(1 3 5) (1 3 4)

Greedy Best First Search



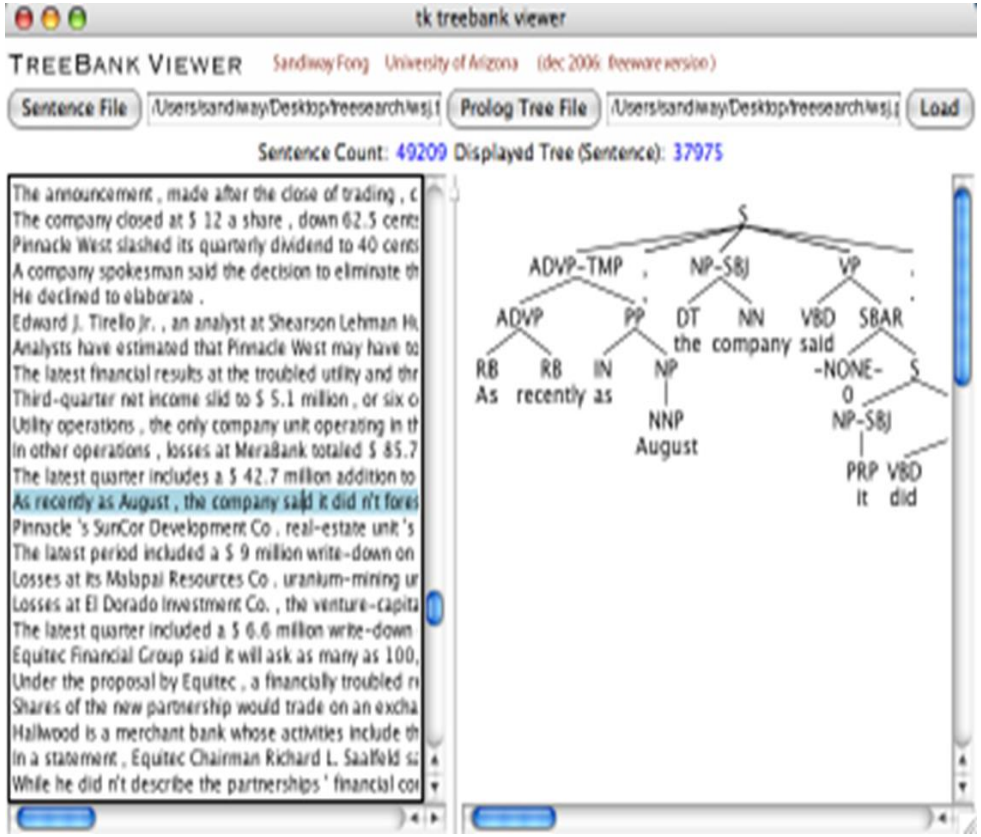


n	h(n)
1	60
2	120
3	30
4	40
5	0

(1)
(1 3) (1 4) (1 2)
(1 3 5) (1 3 4)

C(1-3-5) = 100 + 125 =225
Expanded : 2
Generated : 6
Max Queue Length : 3
Idea: Optimize DFS. Choose next nearest to goal in the same hill.

Case Study – Search in Treebanks

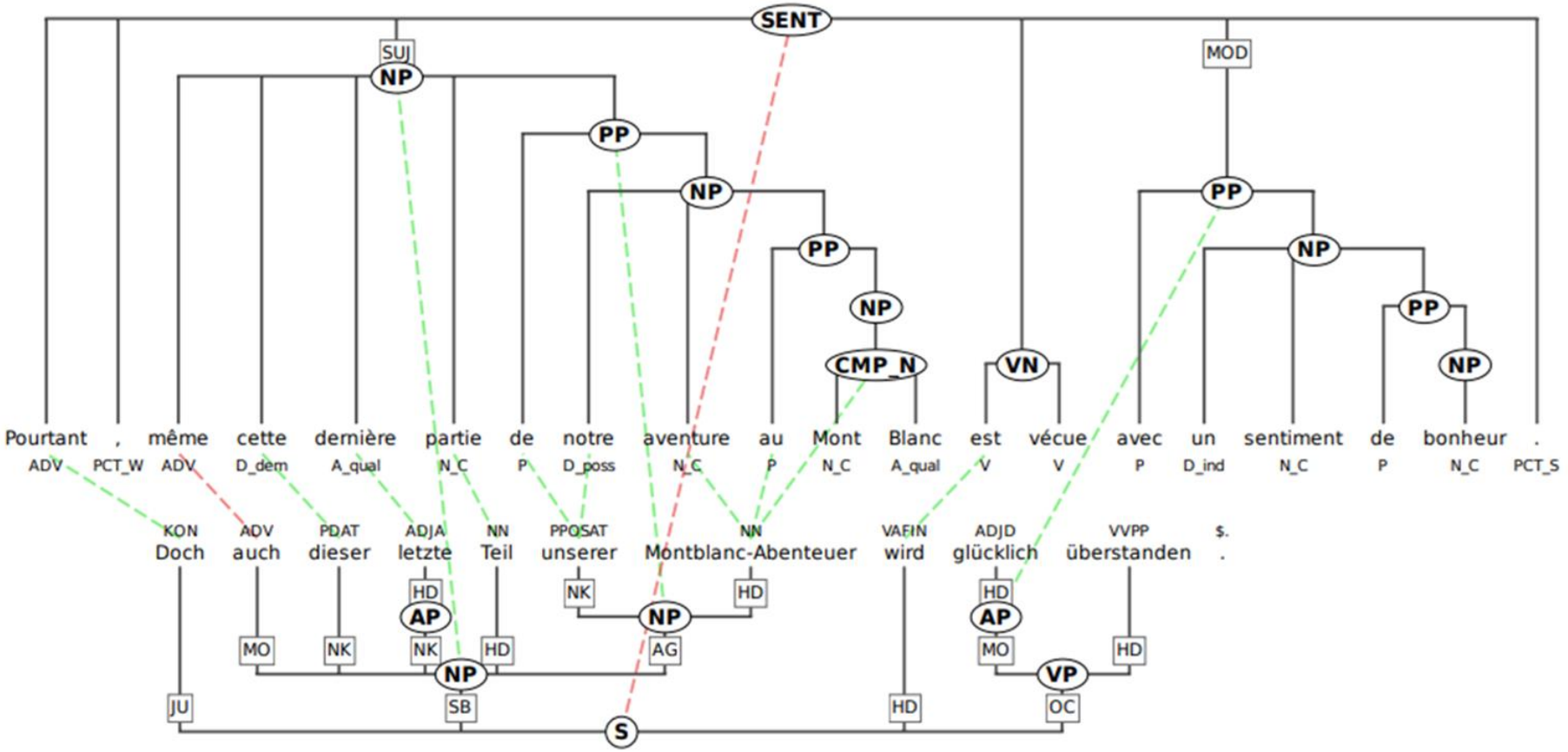


Source Credit :

https://catalog.ldc.upenn.edu/docs/LDC95T7/cl93.html

https://ufal.mff.cuni.cz/pdt3.5

Case Study – Search in Treebanks



Source Credit :

https://catalog.ldc.upenn.edu/docs/LDC95T7/cl93.html

https://ufal.mff.cuni.cz/pdt3.5

Greedy Best First Search

Not Optimal

- Because the algorithm is greedy
- It only optimizes for the current action

Not Complete

- Often ends up in state with a dead end as the heuristic doesn't guarantee a path but is only an approximation

Time and Space Complexity - $\mathcal{O}(b^m)$ where m – max depth of search tree

A* Search

Expands the node which lies in the closest path (estimated cheapest path) to the goal

Evaluation function **f(n) = g(n) + h(n)**

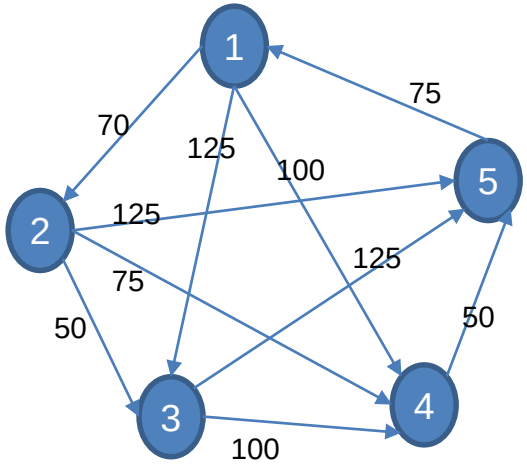
g(n) – the cost to reach the node

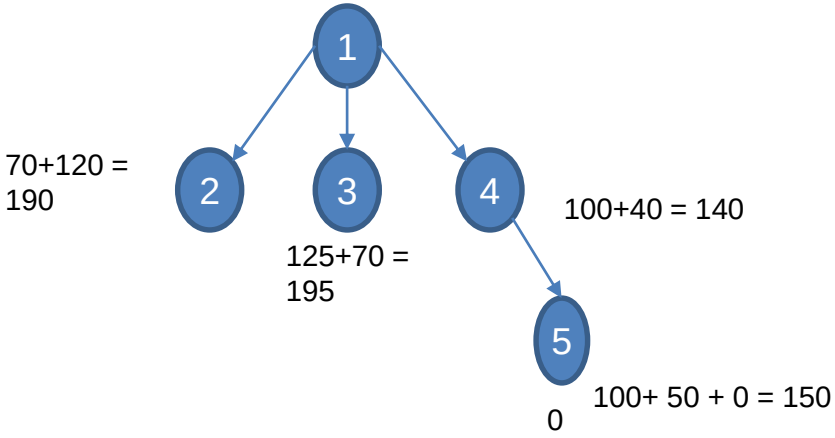
h(n) – the expected cost to go from node to goal

f(n) – estimated cost of cheapest path through node n

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

A* Search



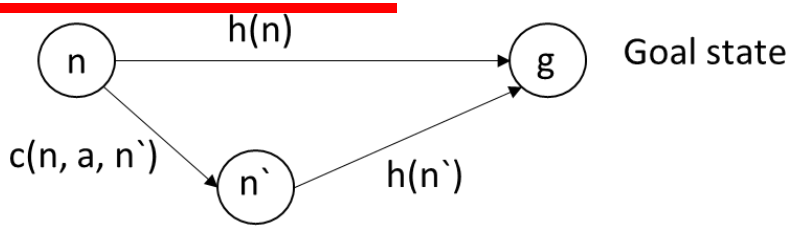


n	h(n)
1	60
2	120
3	70
4	40
5	0

(1)
(1 4) (1 2) (1 3)
(1 4 5) (1 2) (1 3)

C(1-4-5) = 100 + 150 =150
Expanded : 2
Generated : 5
Max Queue Length : 3

A* Search



Optimal on condition

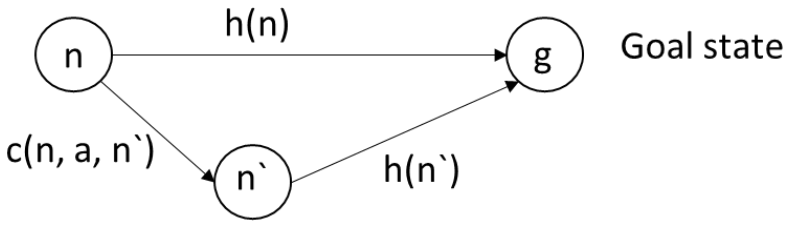
h(n) must satisfies two conditions:

- Admissible Heuristic – one that never overestimates the cost to reach the goal
- Consistency – A heuristic is consistent if for every node n and every successor node n` of n generated by action a, h(n) <= c(n, a, n`) + h(n`)

Complete

- If the number of nodes with cost <=C* is finite
- If the branching factor is finite
- A* expands no nodes with f(n) > C*, known as pruning

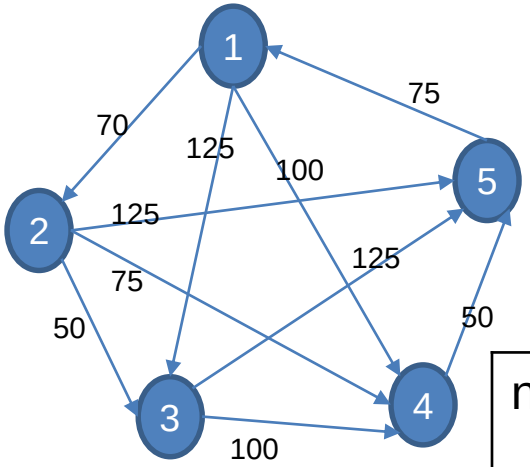
Time Complexity - $\mathcal{O}(b^{\Delta})$ where the absolute error $\Delta = h^* - h$


h(n)
n
g
Goal state
c(n, a, n`)
n`
h(n`)

A* Search

innovate achieve lead

Is the heuristic designed leads to optimal solution?



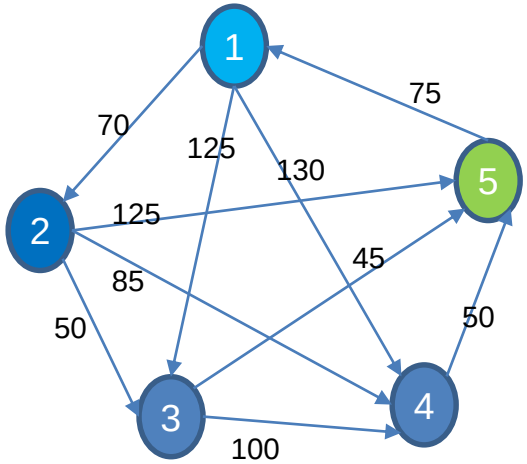
Assuming node 3 as goal, taking only sample edges per node below is checked for consistency

n	h(n)	Is Admissible? h(n) <= h*(n)	Is Consistent? For every arc (i,j): h(i) <= g(i,j) + h(j)
1	80	Y	N (5→1) : 190<=155
2	60	N	Y (1→2) : 80<=130
3	0	Y	
4	200	Y	Y (1→4) : 80<=300 Y (2→4) : 60<=275
5	190	Y	Y (2→5) : 60<=315 Y (4→ 5) : 200<=240

Variations of A*

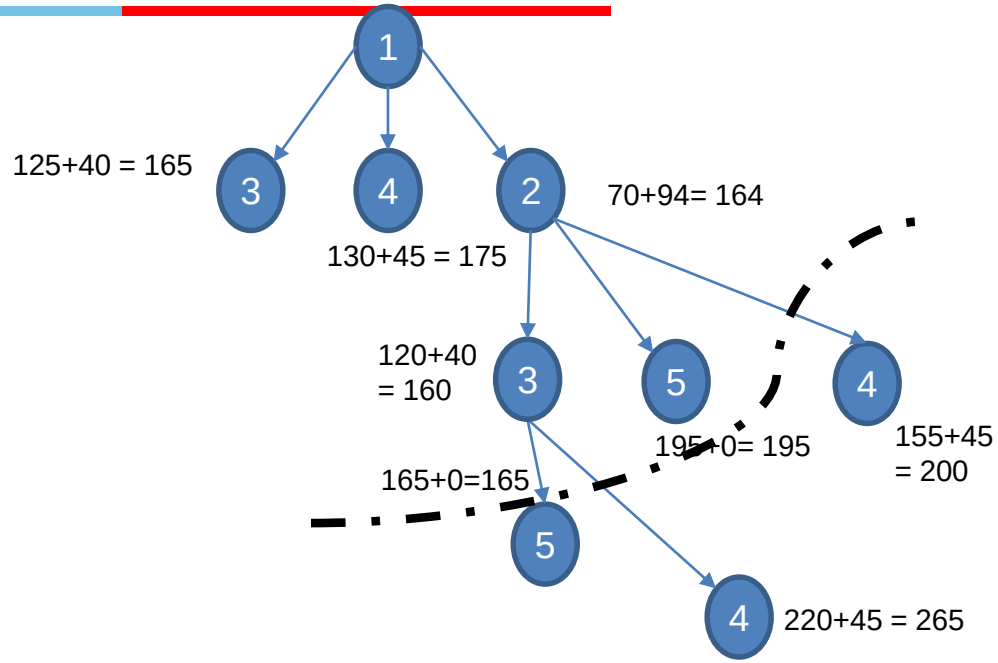
Memory Bounded Heuristics

Iterative Deepening A*



n	h(n)
1	60
2	94
3	40
4	45
5	0

Set limit for f(n)

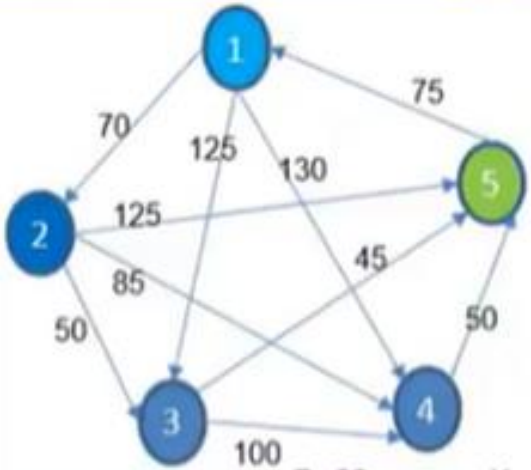


Cut off value is the smallest of f-cost of any node that exceeds the cutoff on previous iterations

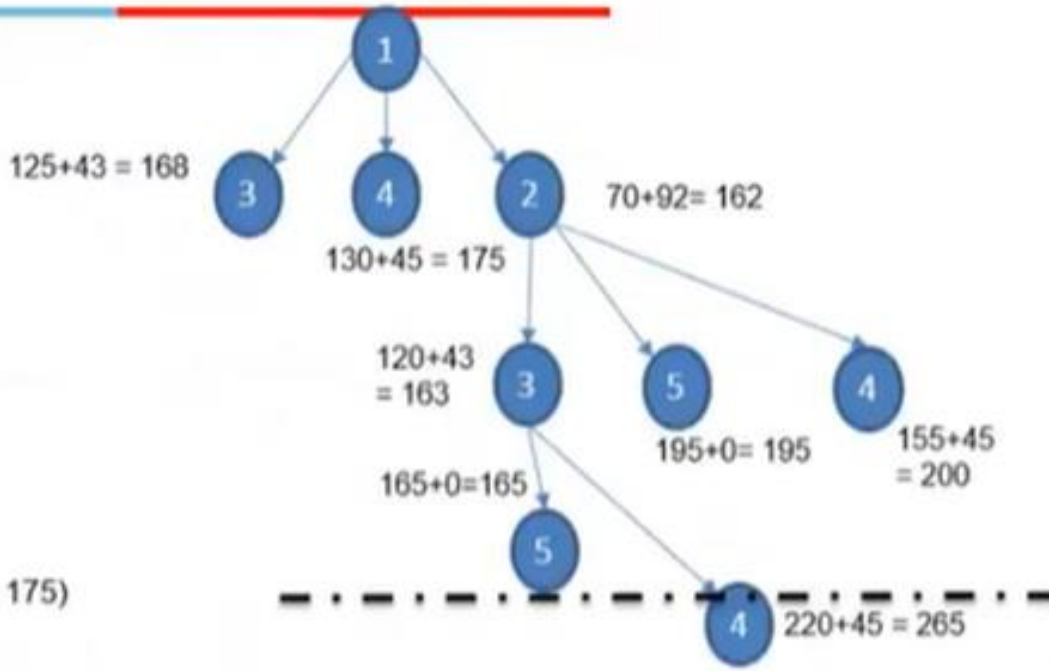
Iterative Limit : Eg

f(n) = 180
f(n) = 195
f(n) = 200
.
.
.
.

Iterative Deepening A*



Set limit for f(n)



n	h(n)
1	60
2	92
3	43
4	45
5	0

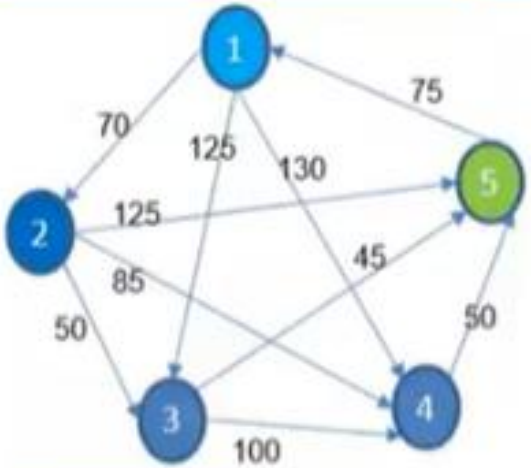
B=60
(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)

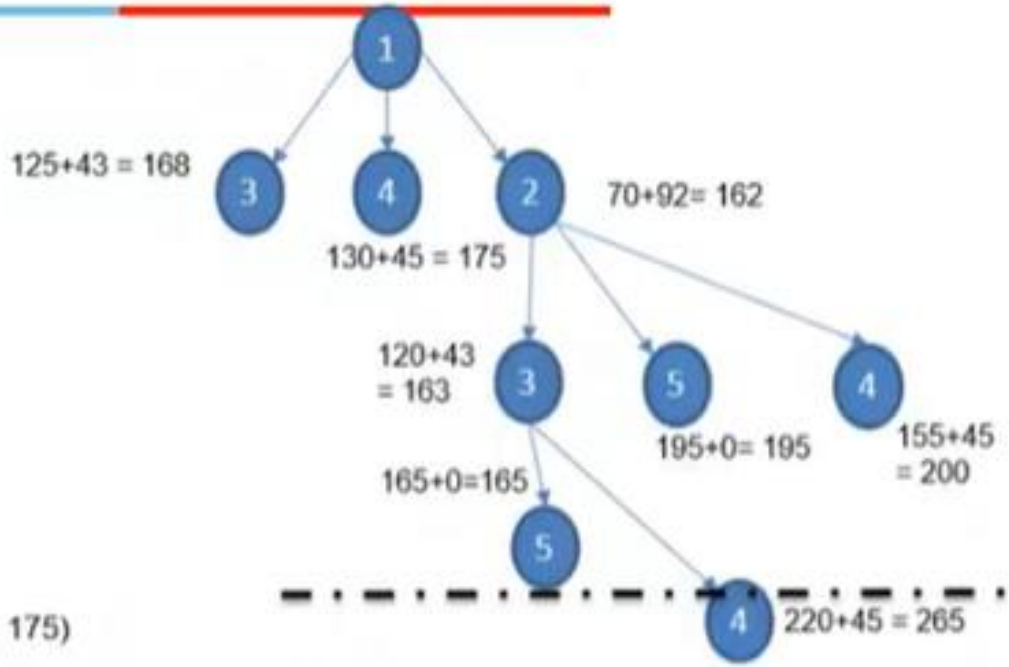
B=162
(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)

B=163
(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)
TEST-F

Iterative Deepening A*

Set limit for f(n)





n	h(n)
1	60
2	92
3	43
4	45
5	0

B=163

(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3 5: 165) (1 2 3 4: 265) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)

B=165

(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3 5: 165) (1 2 3 4: 265) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)

Required Reading: AIMA - Chapter # 3.1, 3.2, # 3.3,

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML CLZG557

M2 : Problem Solving Agent using Search

V Indumathi

Guest Faculty,

BITS

BITS Pilani

Pilani Campus

Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time, Reinforcement Learning
M7	Ethics in AI

Learning Objective

At the end of this class , students Should be able to:

1. Apply A* variations algorithms to the given problem
2. Compare given heuristics for a problem and analyze which is the best fit
3. Design relaxed problem with appropriate heuristic design
4. Prove the designed relaxed problem heuristic is admissible
5. Understand the notion of Local Search algorithms

Module 2 : Problem Solving Agent using Search

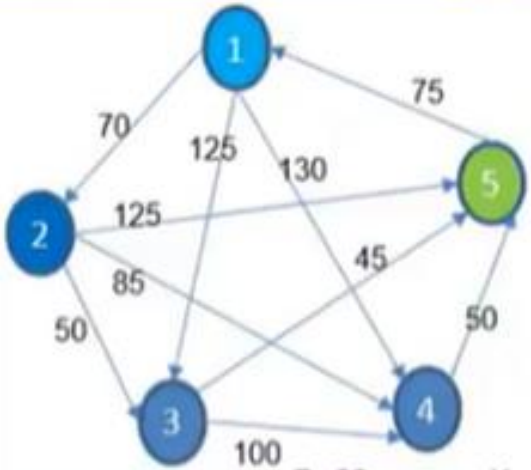
A. Uninformed Search

B. Informed Search

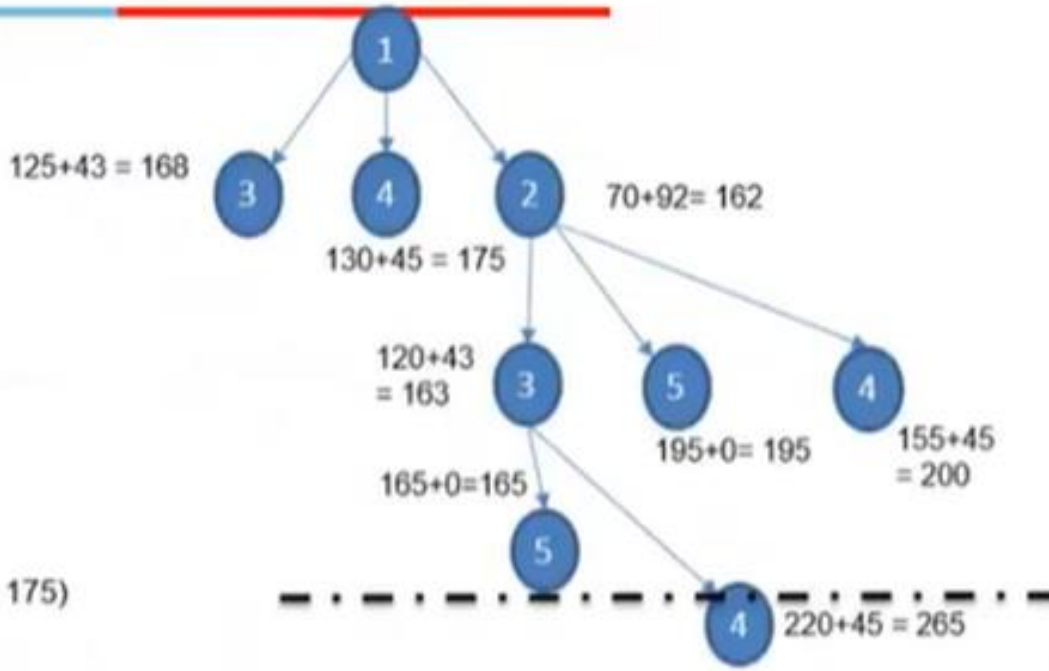
C. Heuristic Functions

D. Local Search Algorithms & Optimization Problems

Iterative Deepening A*



Set limit for f(n)



n	h(n)
1	60
2	92
3	43
4	45
5	0

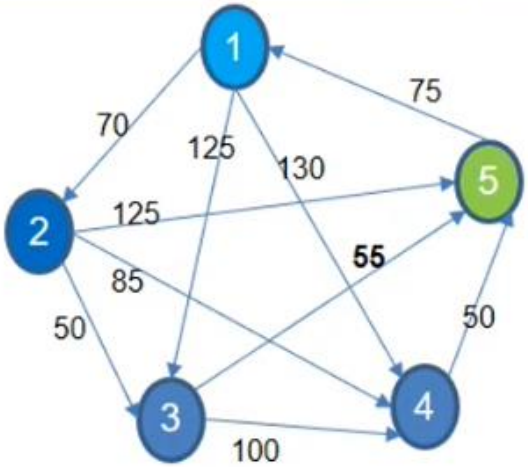
B=60
(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)

B=162
(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)

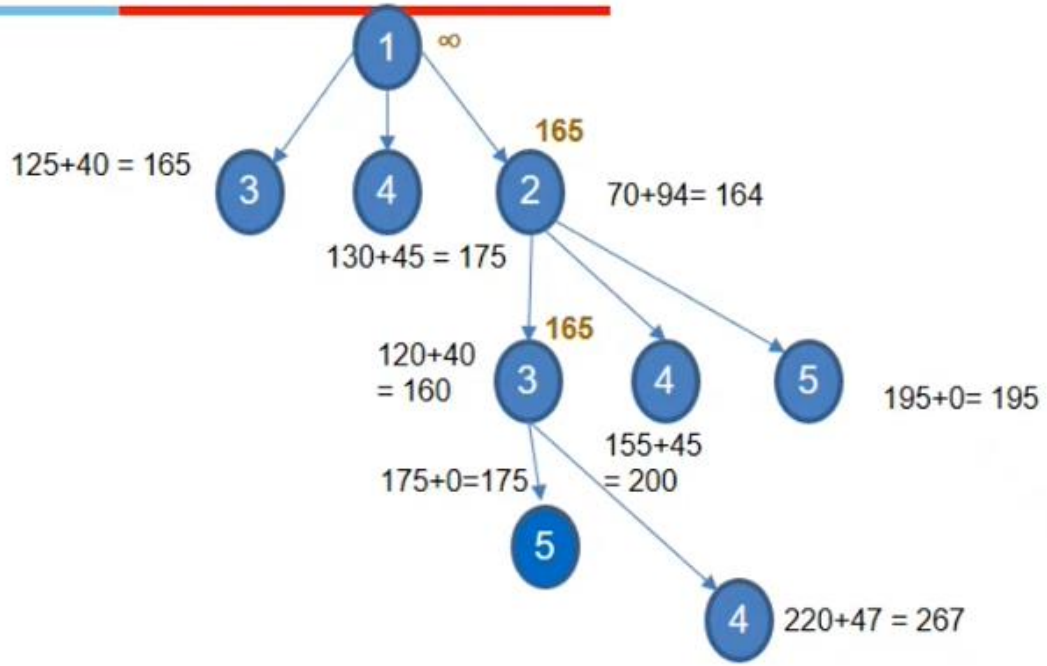
B=163
(1: 60)
TEST-F
(1 2: 162) (1 3: 168) (1 4: 175)
TEST-F
(1 2 3: 163) (1 2 4: 200) (1 2 5: 195) (1 3: 168) (1 4: 175)
TEST-F

Recursive Best First Search A*

Remember the next best alternative f-Cost to regenerate



n	h(n)
1	60
2	94
3	40
4	45
5	0



(1, 60)
(1 2 | 164) (1 3 | 165) (1 4 | 175)

(1 2 | 175) (1 4 | 175) (1 3 | 180)

(1 2 3 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200)

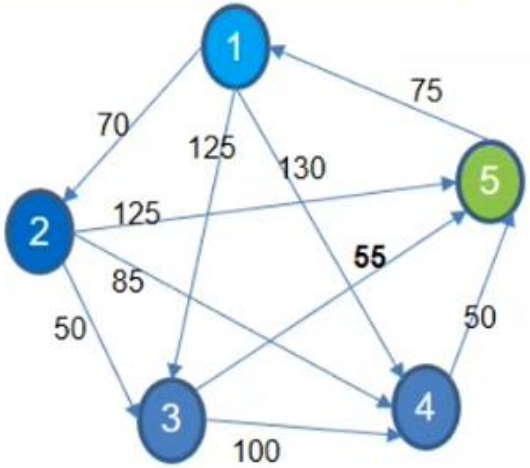
(1 2 3 5 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200) (1 2 3 4 | 267)

PASS

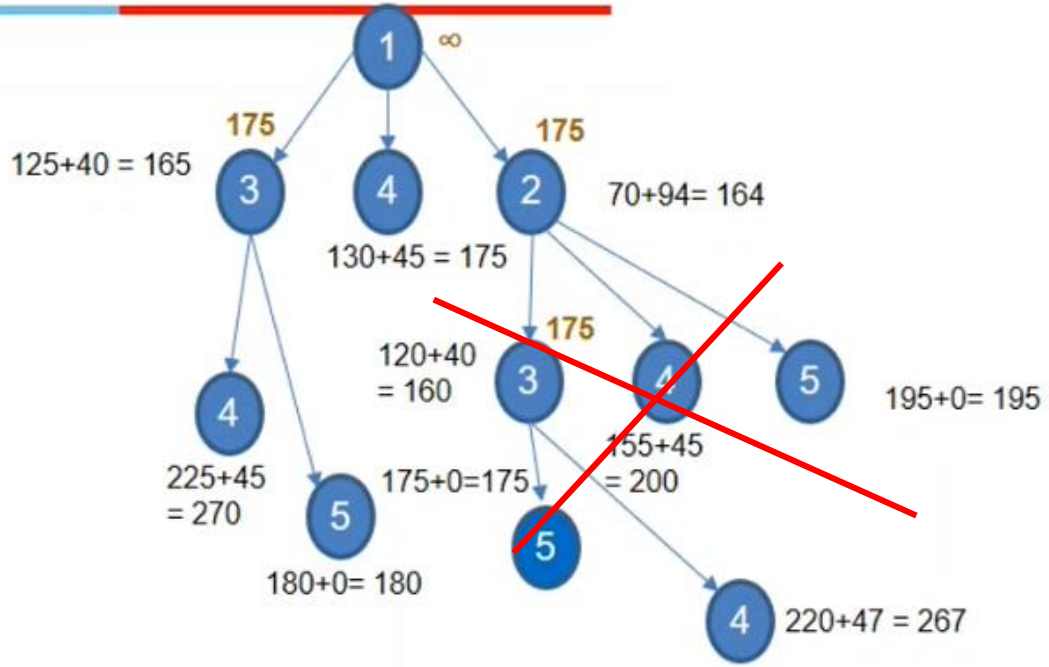
Recursive Best First Search A*

innovate achieve lead

Remember the next best alternative f-Cost to regenerate



n	h(n)
1	60
2	94
3	40
4	45
5	0



(1, 60)
(1 2 | 164) (1 3 | 165) (1 4 | 175)

(1 2 | 175) (1 4 | 175) (1 3 | 180)

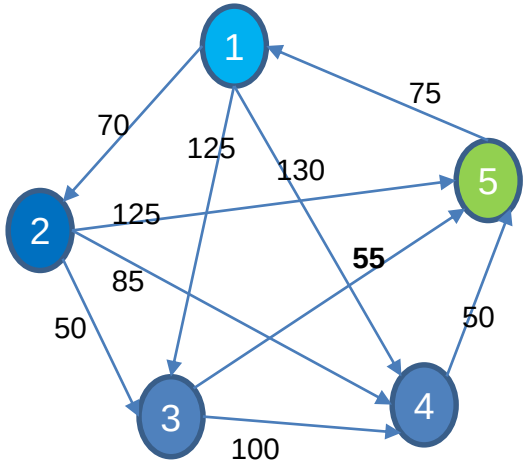
(1 2 3 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200)

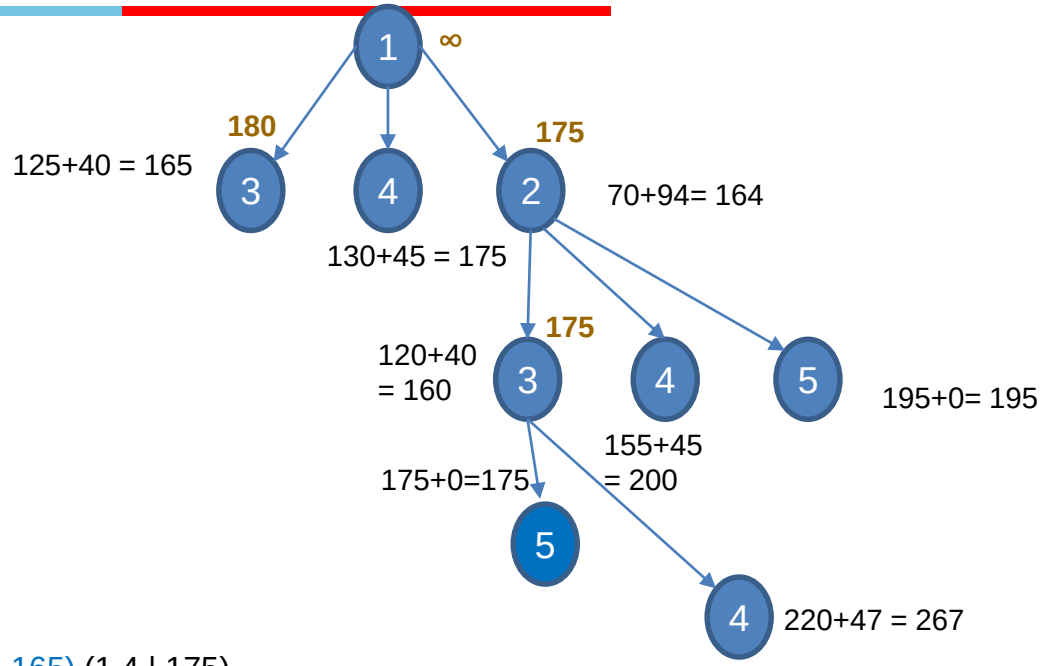
(1 2 3 5 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200) (1 2 3 4 | 267)

PASS

Recursive Best First Search A*

Remember the next best alternative f-Cost to regenerate





n	h(n)
1	60
2	94
3	40
4	45
5	0

(1, 60)
(1 2 | 164) (1 3 | 165) (1 4 | 175)
……
……

(1 2 | 175) (1 4 | 175) (1 3 | 180)

(1 2 3 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200)

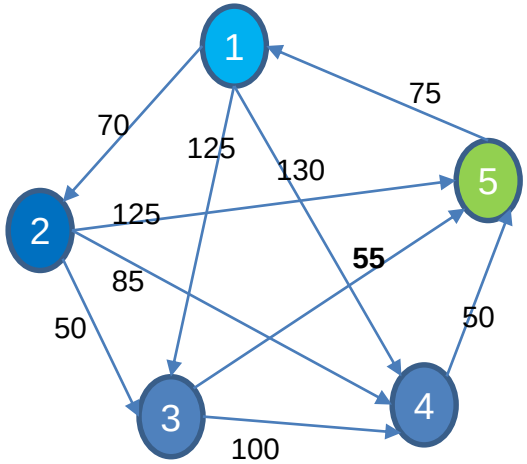
(1 2 3 5 | 175) (1 4 | 175) (1 3 | 180) (1 2 5 | 195) (1 2 4 | 200) (1 2 3 4 | 267)

PASS

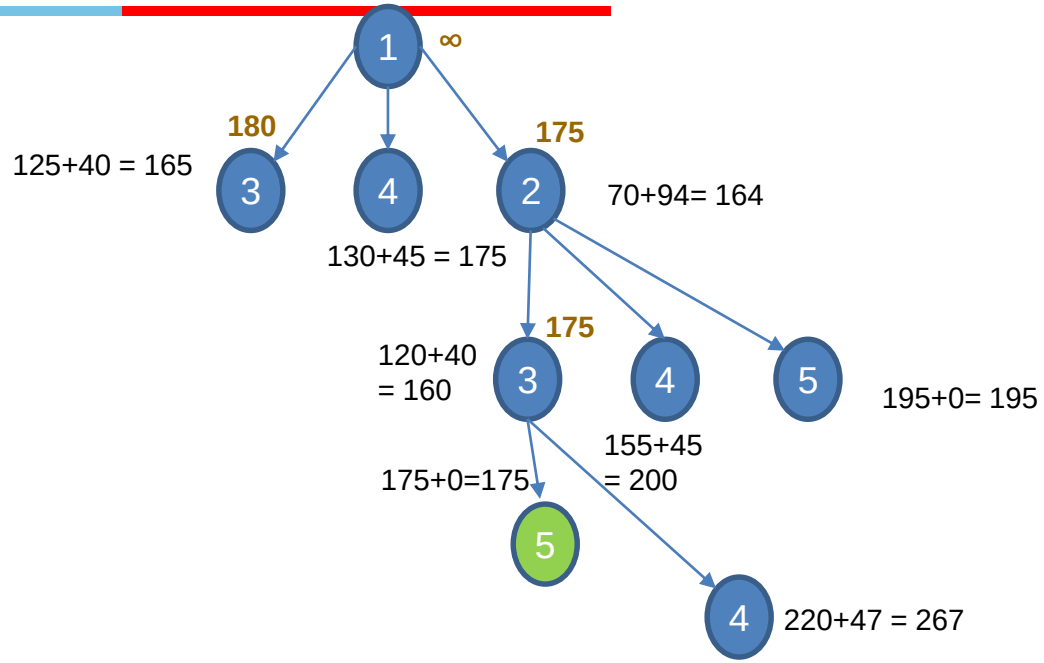
Recursive Best First Search A*

innovate achieve lead

Remember the next best alternative f-Cost to regenerate



n	h(n)
1	60
2	94
3	40
4	45
5	0



If the current best leaf value > best alternative path

Best leaf value of the forgotten subtree is backed up to the ancestors

Recursion unwinds

Else

Continue expansion

Space Usage = O(bd) very less

Design of Heuristics

Heuristic Design

- **Effective Branching Factor**
- Good Heuristics
- Notion of Relaxed Problems
- Generating Admissible Heuristics

Effective branching factor (b^*):

If the algorithm generates N number of nodes and the solution is found at depth d, then

$$N + 1 = 1 + (b^*) + (b^*)^2 + (b^*)^3 + \ldots + (b^*)^d$$

Heuristic Design

- Effective Branching Factor
- Good Heuristics
- **Notion of Relaxed Problems**
- Generating Admissible Heuristics

Simplify the problem

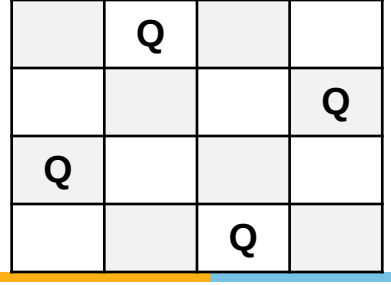
Assume no constraints

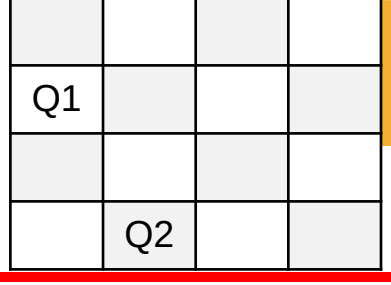
Cost of optimal solution to relaxed problem ≤ Cost of optimal solution for real problem

BITS Pilani, Deemed to be University under Section 3 of UGC Act, 1956

Design of Heuristics

N-Queen

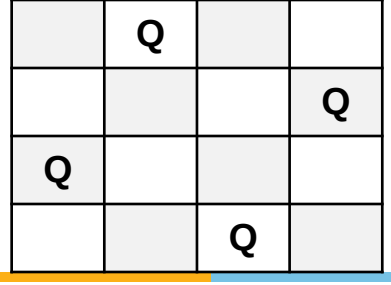


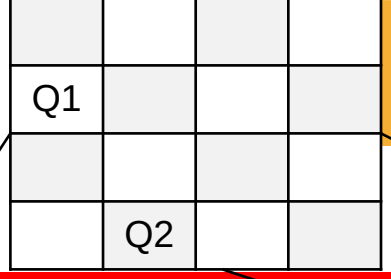


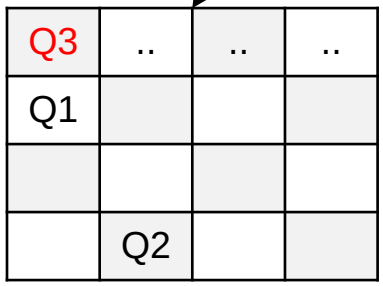
innovate achieve lead

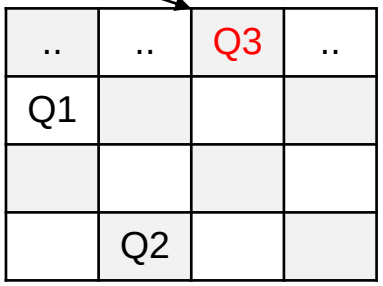
Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
< Xi , Yi >	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	n!

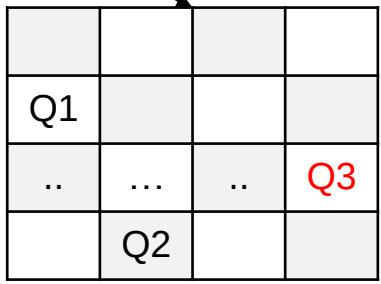
N-Queen





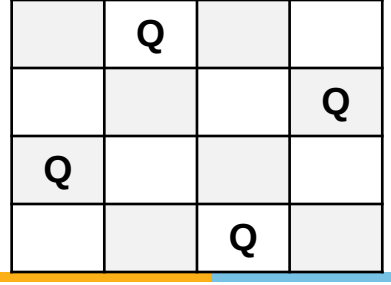


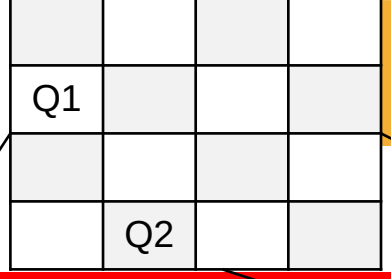


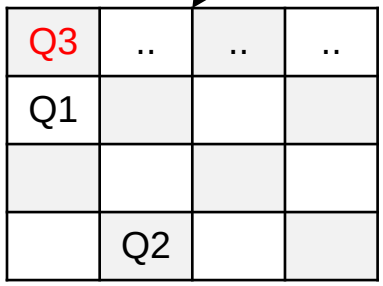


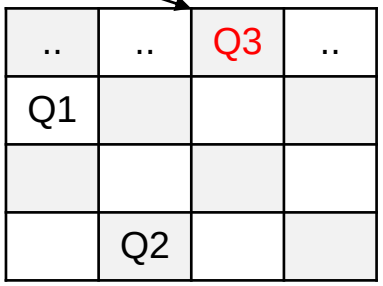
Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
< Xi , Yi >	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	n!

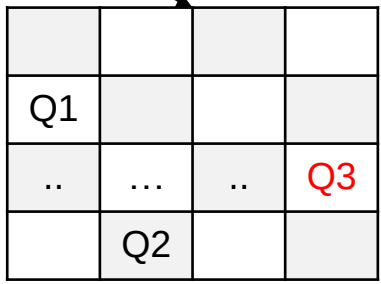
N-Queen





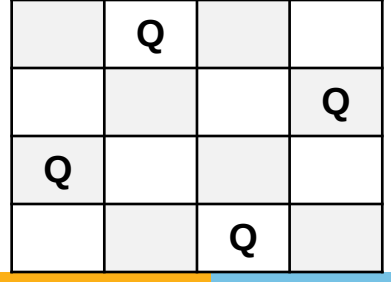


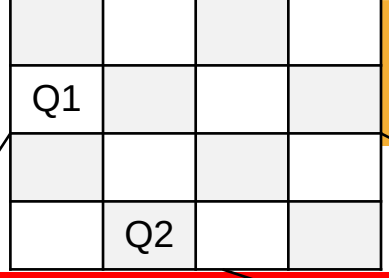




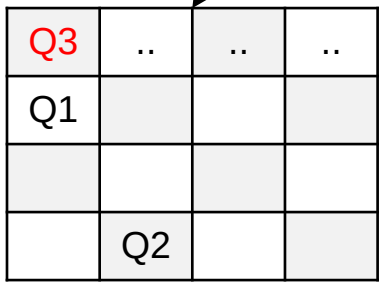
Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
< Xi , Yi >	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	n!

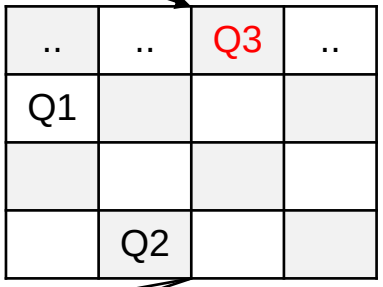
N-Queen

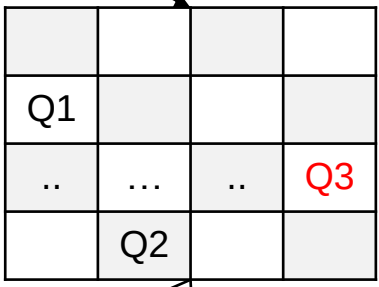


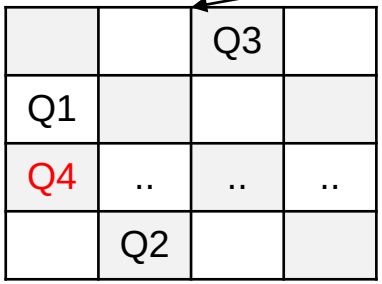


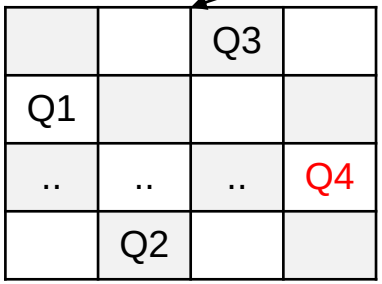
- Construct the search tree by considering one row of the board at a time
- State space graph of relaxed problem is a super graph of original state space because of removal of restrictions

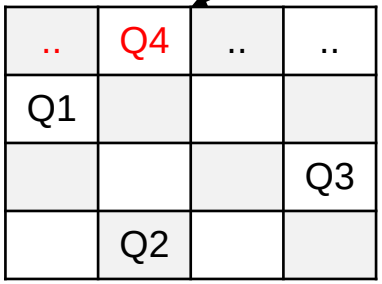


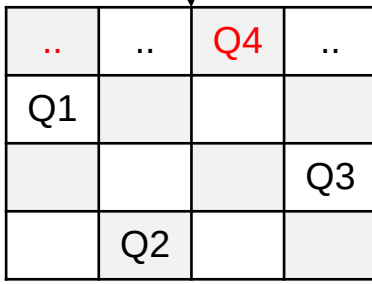












Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
< Xi , Yi >	Place in any non-occupied row in board		isValid Non-Attacking	Transition + Valid Queens	n!

N-Tile

-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
<LOC, ID>	Move Empty to near by Tile		ID=LOC-1	Transition + Positional + Distance+ Other approaches	9!

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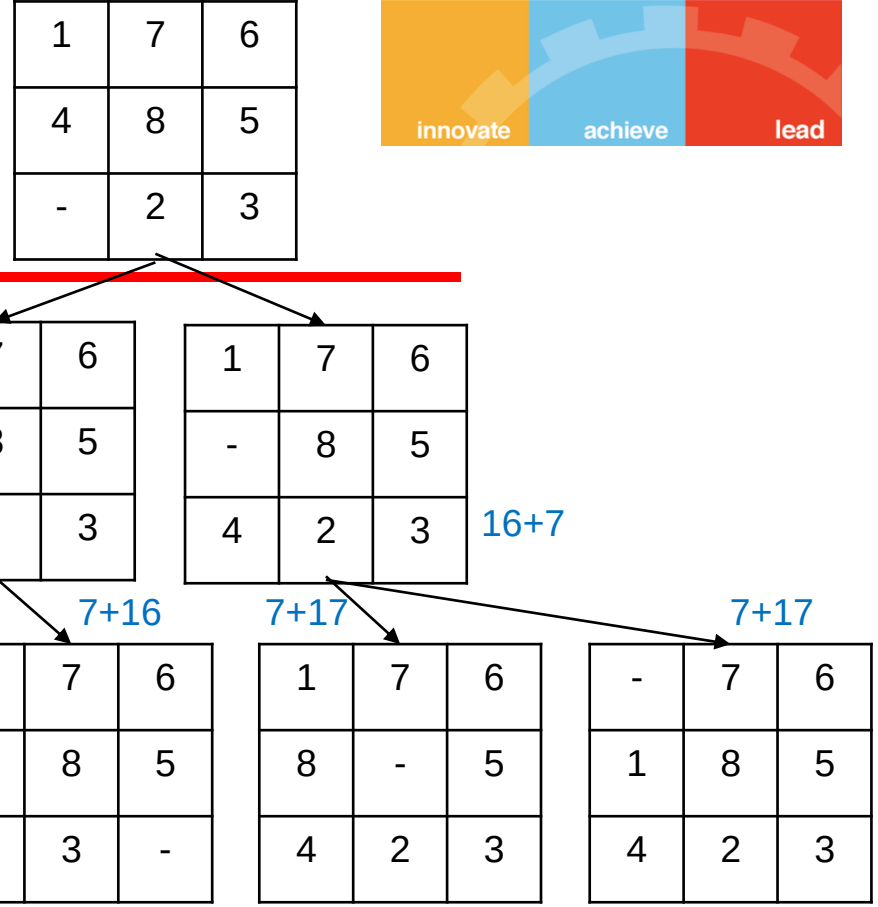
N-Tile

-	1	2
3	4	5
6	7	8



- Effective Branching Factor

: ~3

: Avg.cost = 18

: No.of.States = ~ 3^{18}

: Graph states : 9!/2 = 181, 440 states

Initial State	Possible Actions	Transition Model	Goal Test	Path Cost	No.Of.States
<LOC, ID>	Move Empty to near by Tile		ID=LOC-1	Transition + Positional + Distance+ Other approaches	9!

Learn from experience

Trail / Puzzle	X1(n) : No.of.Misplaced Tiles	X2(n): Pair of adjacent tiles that are not in goal	X3(n): Position of the empty tile	……h`(n)
Example 1	7	10	7	……
Example 2	5	6	6	……
……	..	..	..	……

-	1	2
3	4	5
6	7	8

1	7	6
4	8	5
-	2	3

Create a suitable model:
h(n) = c1*X1(n) + c2*X2(n) + ……..

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Local Search & Optimization

Local Search

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Optimization Problem

Goal : Navigate through a state space for a given problem such that an optimal solution can be found

Objective : Minimize or Maximize the objective evaluation function value

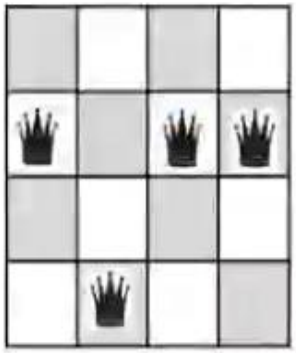
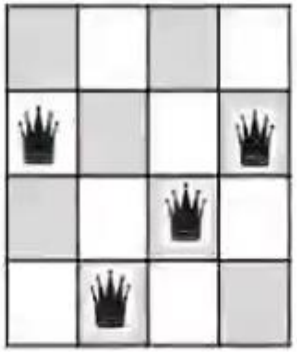
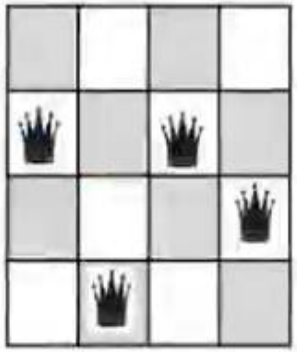
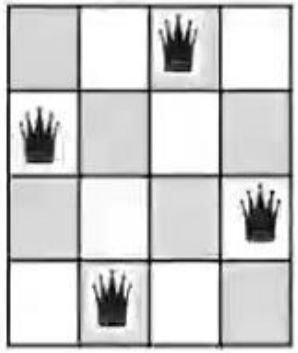
Scope : Local

Objective Function : Fitness Value evaluates the goodness of current solution

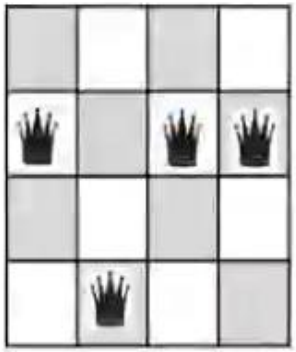
Local Search : Search in the state-space in the neighbourhood of current position until an optimal solution is found

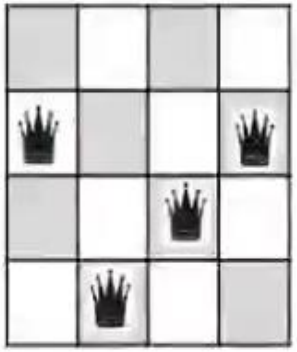
Single Instance Based	Multiple Instance Based
Hill Climbing	Genetic Algorithm
Simulated Annealing	Particle Swarm Optimization
Local Beam Search	Ant Colony Optimization
Tabu Search	

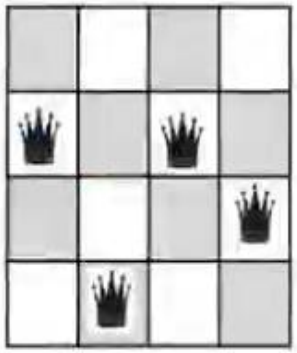
Local Search : Search in the state-space in the neighbourhood of current position until an optimal solution is found

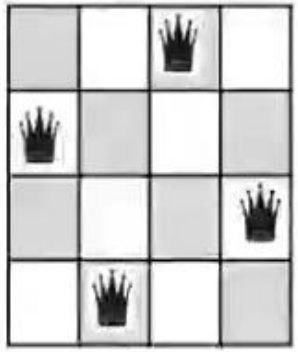





Local Search : Search in the state-space in the neighbourhood of current position until an optimal solution is found









Feasible State/Solution **Neighboring States** **Optimal Solution**

Fitness Value:

h(n) = 4 h(n) = 4 h(n) = 2 h(n) = 0

Above is an example of h(n) = No.of.Conflicting **pairs** of queens

h(n) = 0 h(n) = 0 h(n) = 1 h(n) = 0

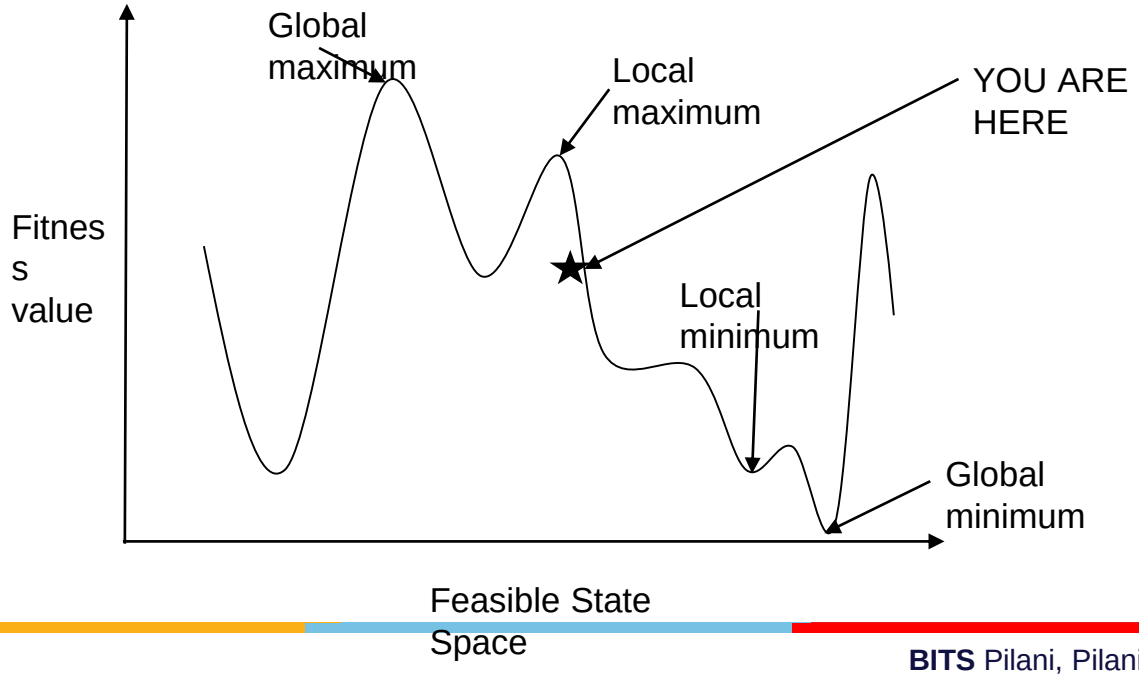
Local Search

Terminology

Local Search : Search in the state-space in the neighbourhood of current position until an optimal solution is found

Algorithms:

- Choice of Neighbor
- Looping Condition
- Termination Condition



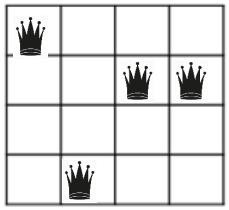
Hill Climbing

Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

h(n) = No.of non-conflicting pairs of queens in the board.

Q1-Q2
Q1-Q3
Q1-Q4
Q2-Q3
Q2-Q4
Q3-Q4

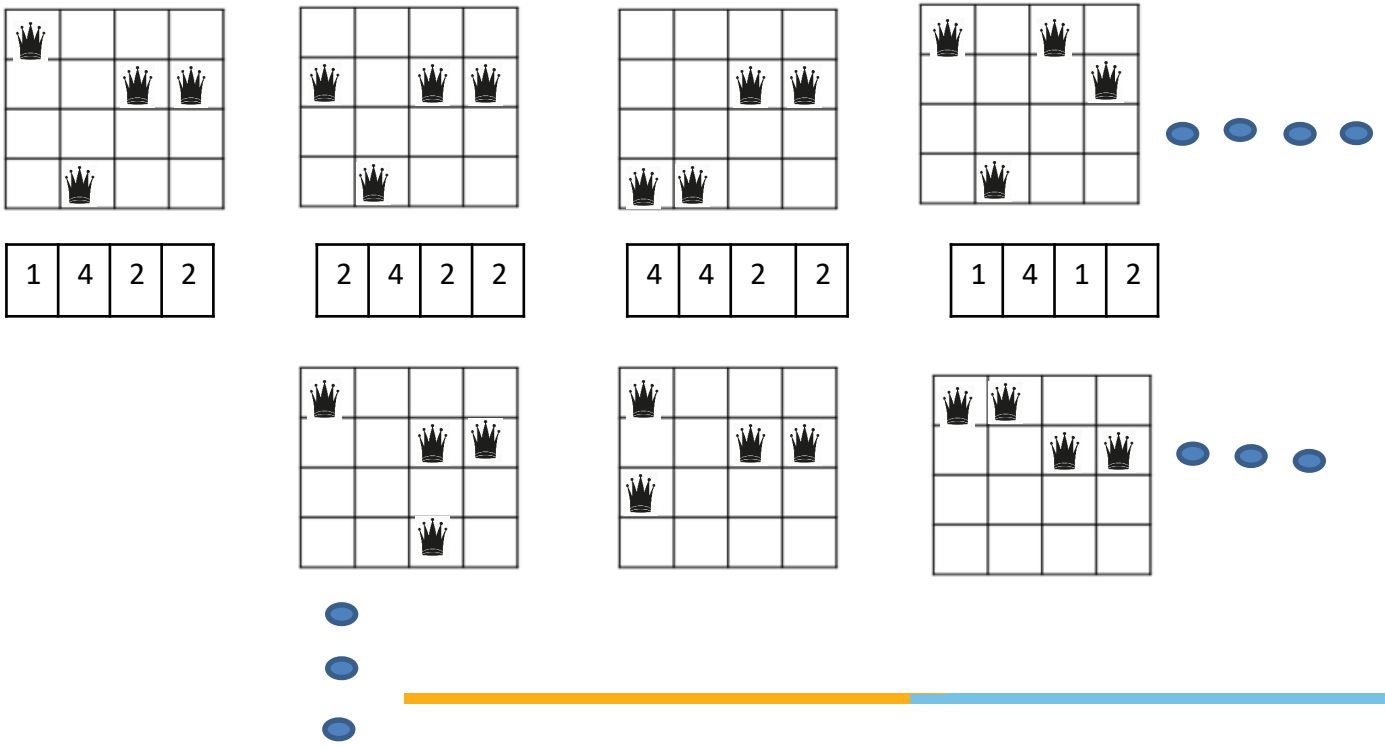


1	4	2	2	4

Note : Steps 3 & 4 in the above algorithm will be a part of variation of Hill climbing

Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2



Hill Climbing

1. Select a random state
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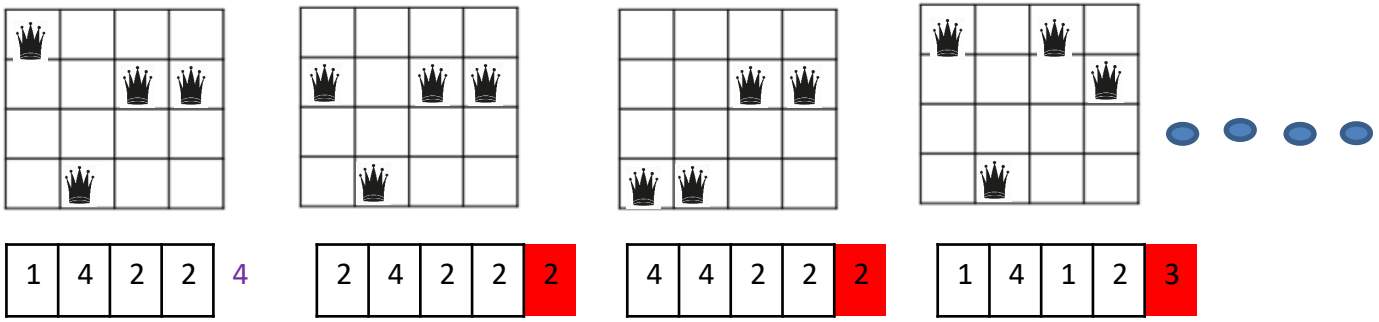
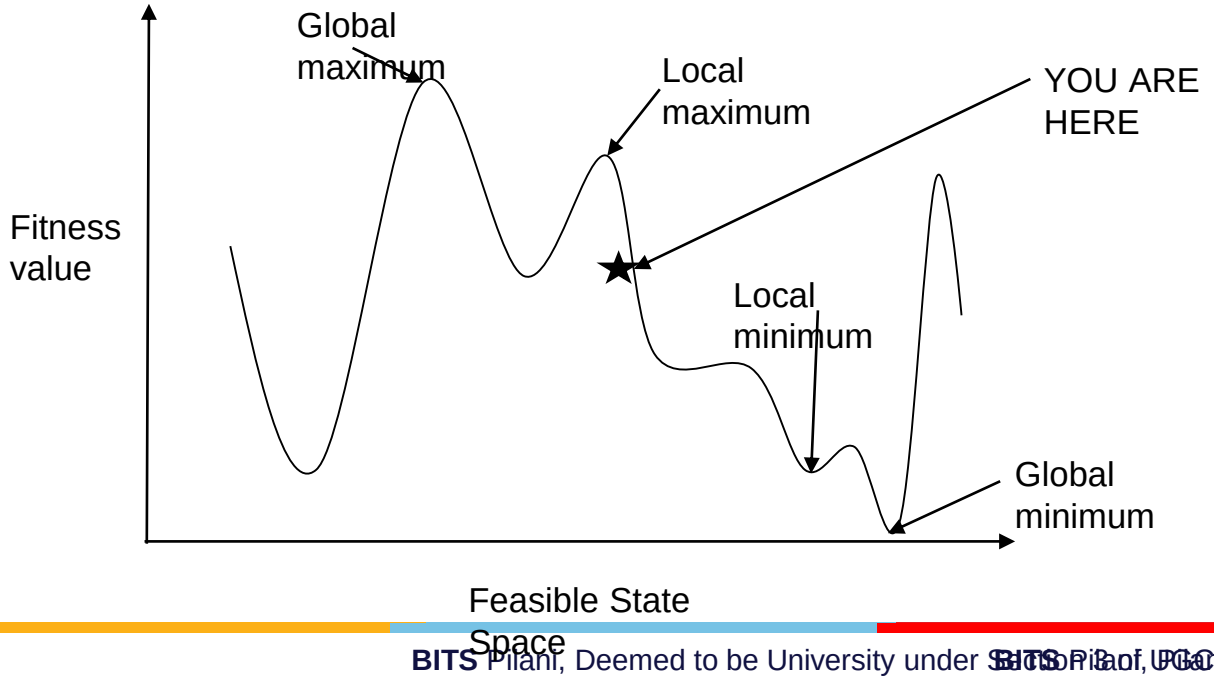


Hill Climbing



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Required Reading: AIMA - Chapter # 4.1, #4.2

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

Artificial & Computational Intelligence

AIML CLZG557

M3 : Game Playing

Indumathi V

Guest Faculty,

BITS - WILP

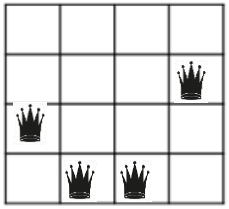
BITS Pilani

Pilani Campus

Hill Climbing

Random Restart

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2



3	4	4	2	3

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

current ← MAKE-NODE(*problem*.INITIAL-STATE)
loop do
neighbor ← a highest-valued successor of *current*
if neighbor.VALUE ≤ current.VALUE **then return** *current*.STATE
current ← *neighbor*

Hill Climbing

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Random Restart

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
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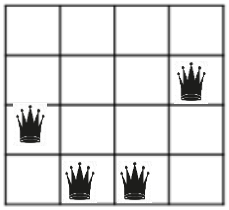

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  current ← *neighbor*

Hill Climbing

Random Restart

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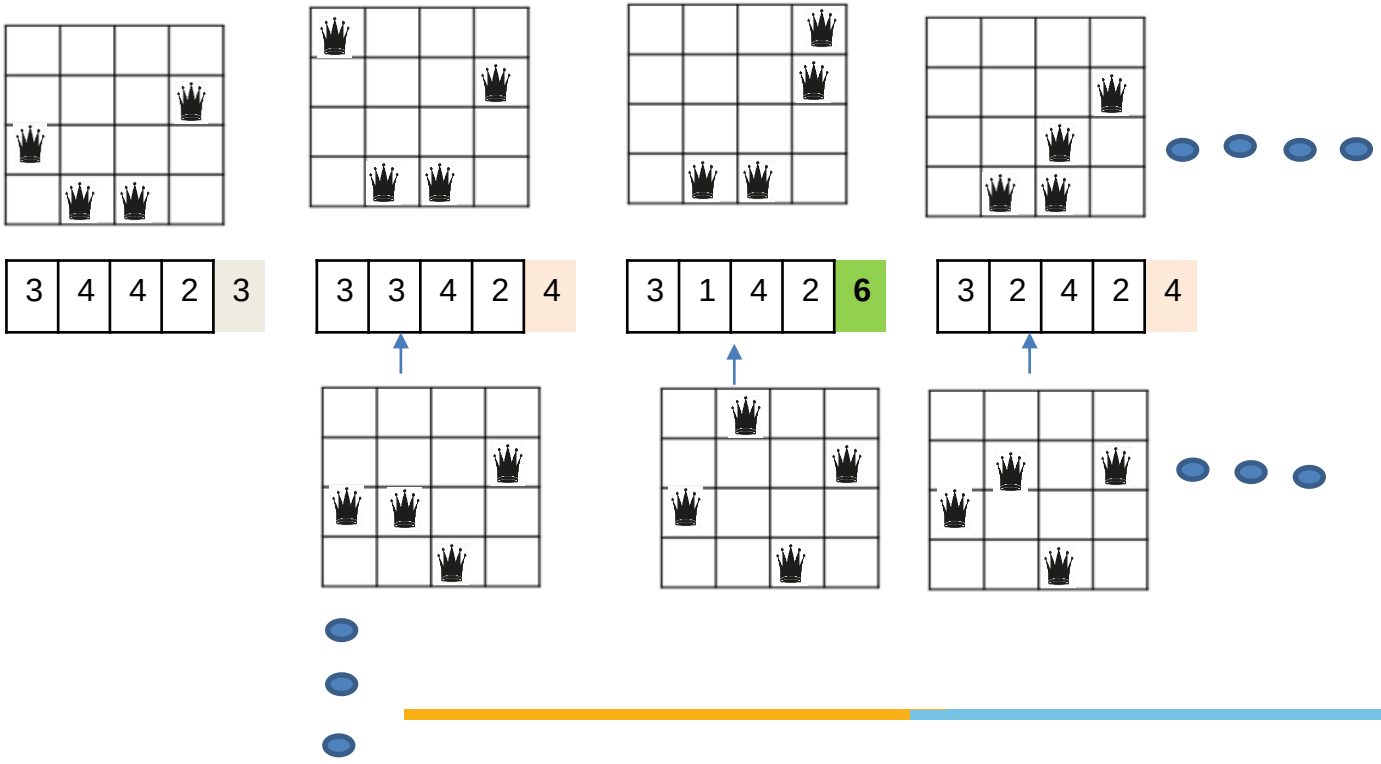
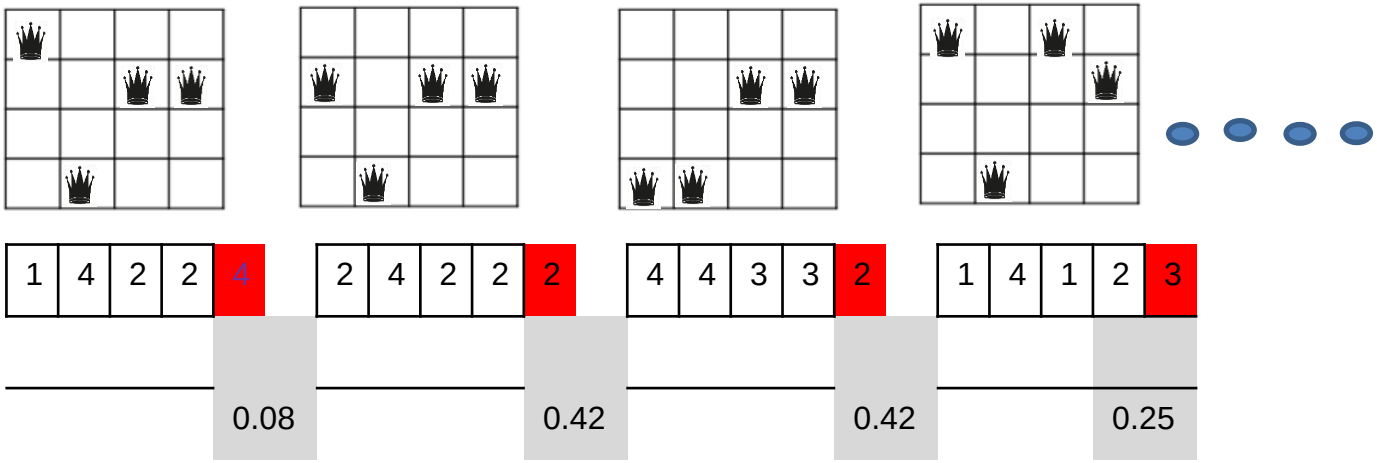


Stochastic Hill Climbing

1. Select a random state
2. Evaluate the fitness scores for all the successors of the state
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. Repeat from Step 2

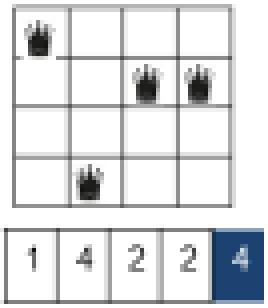


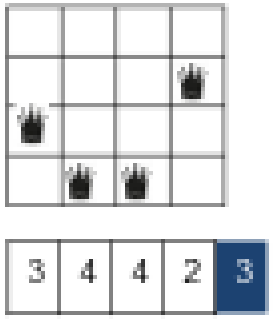
12 N = {4,2,2,3,3,2,2,0,2,1,3,0}

Local Beam Search

Beam Search

1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next 'k' states randomly based on the probability
6. Repeat from Step 2





Beam Search

1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next ‘k’ states randomly based on the probability
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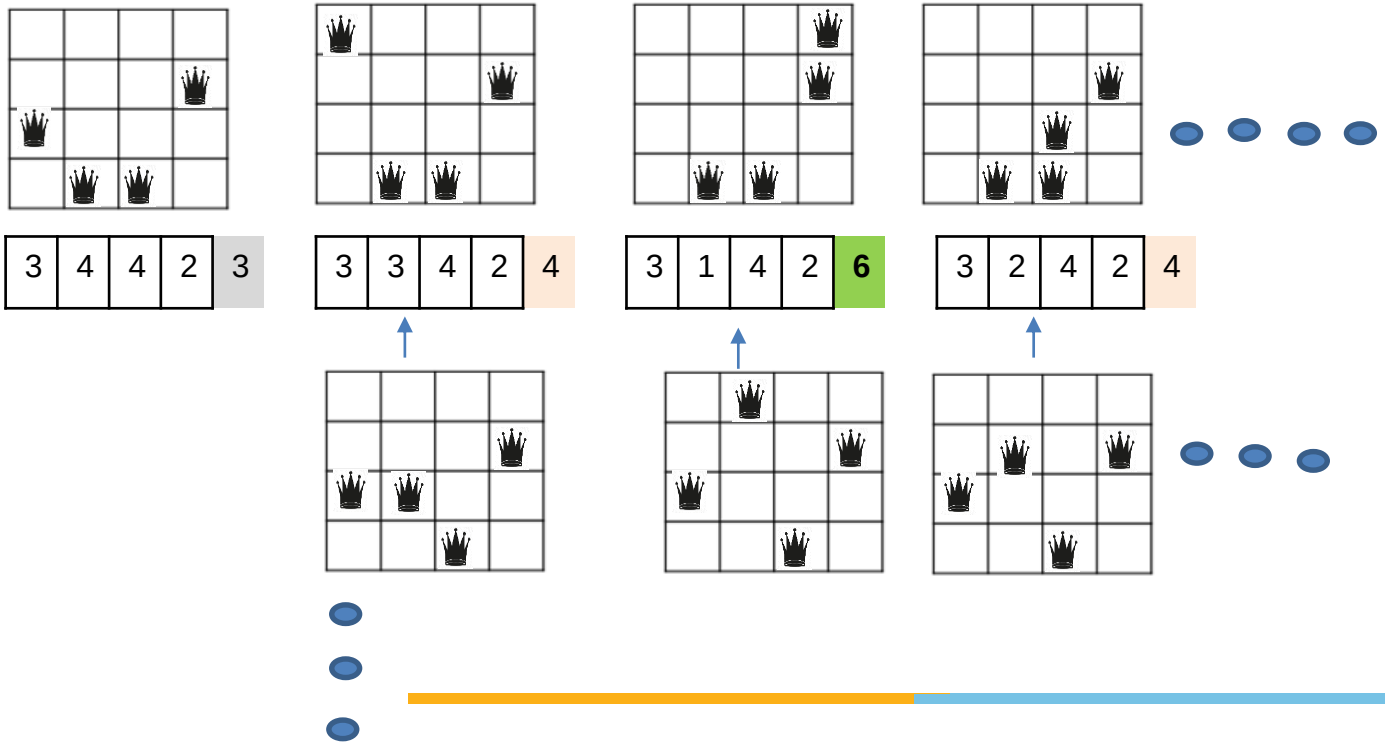
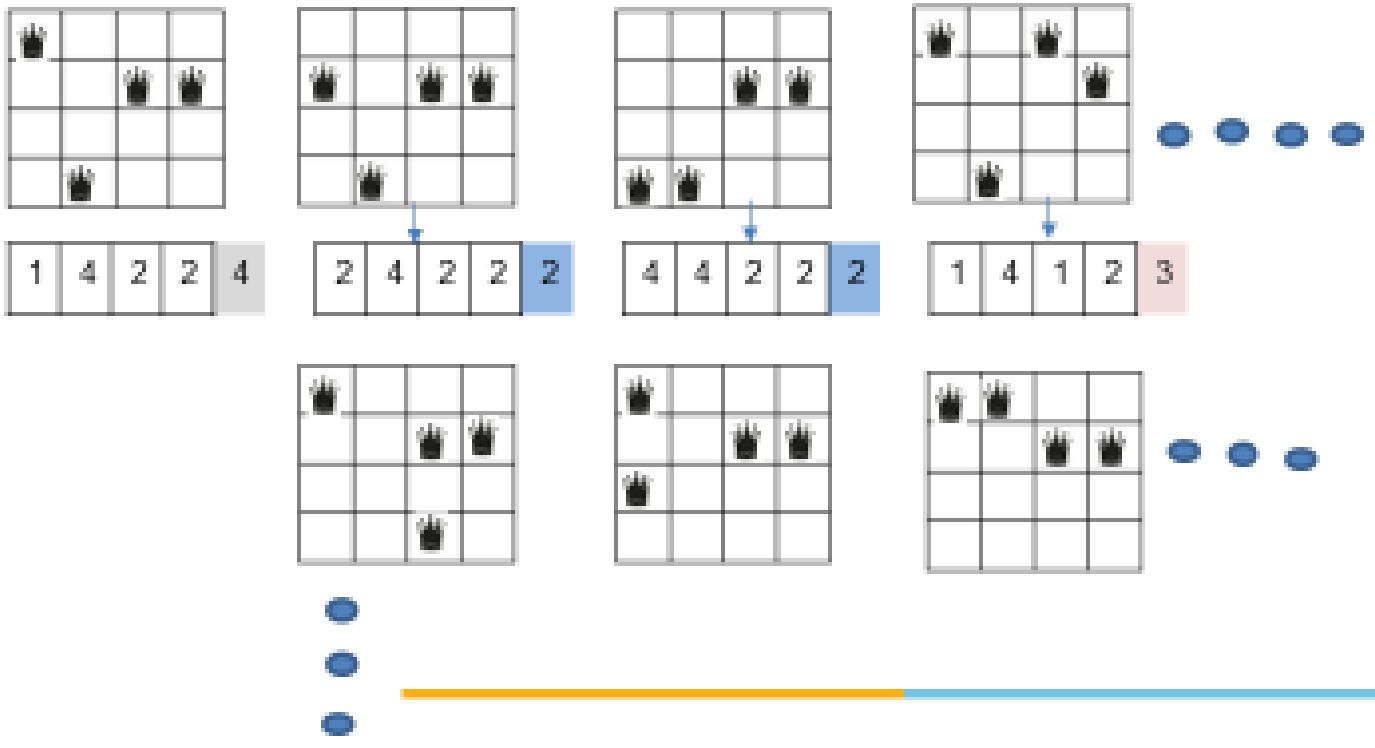


Beam Search

1st State

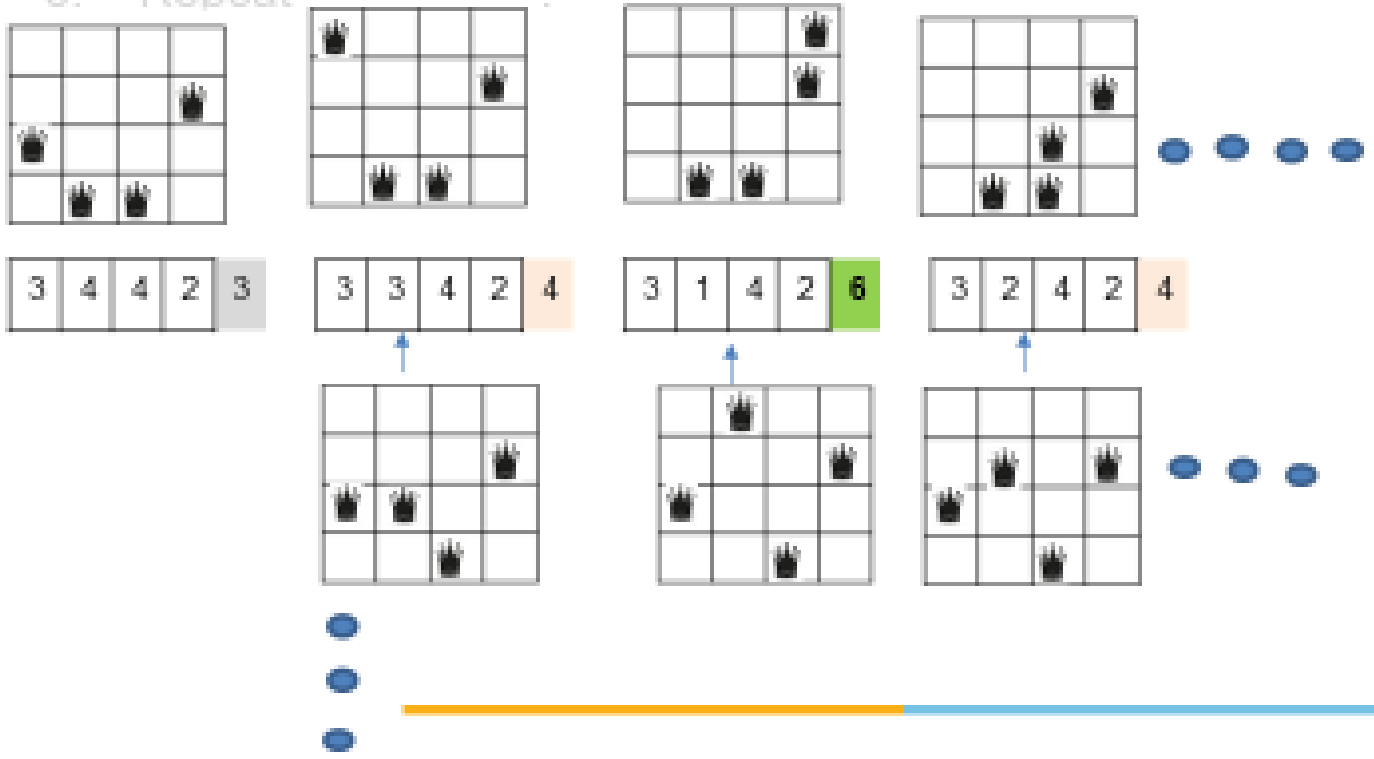
1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next 'k' states randomly based on the probability
6. Repeat from Step 2



Beam Search

2nd State

1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next 'k' states randomly based on the probability
6. Repeat from Step 2



Stochastic Beam Search

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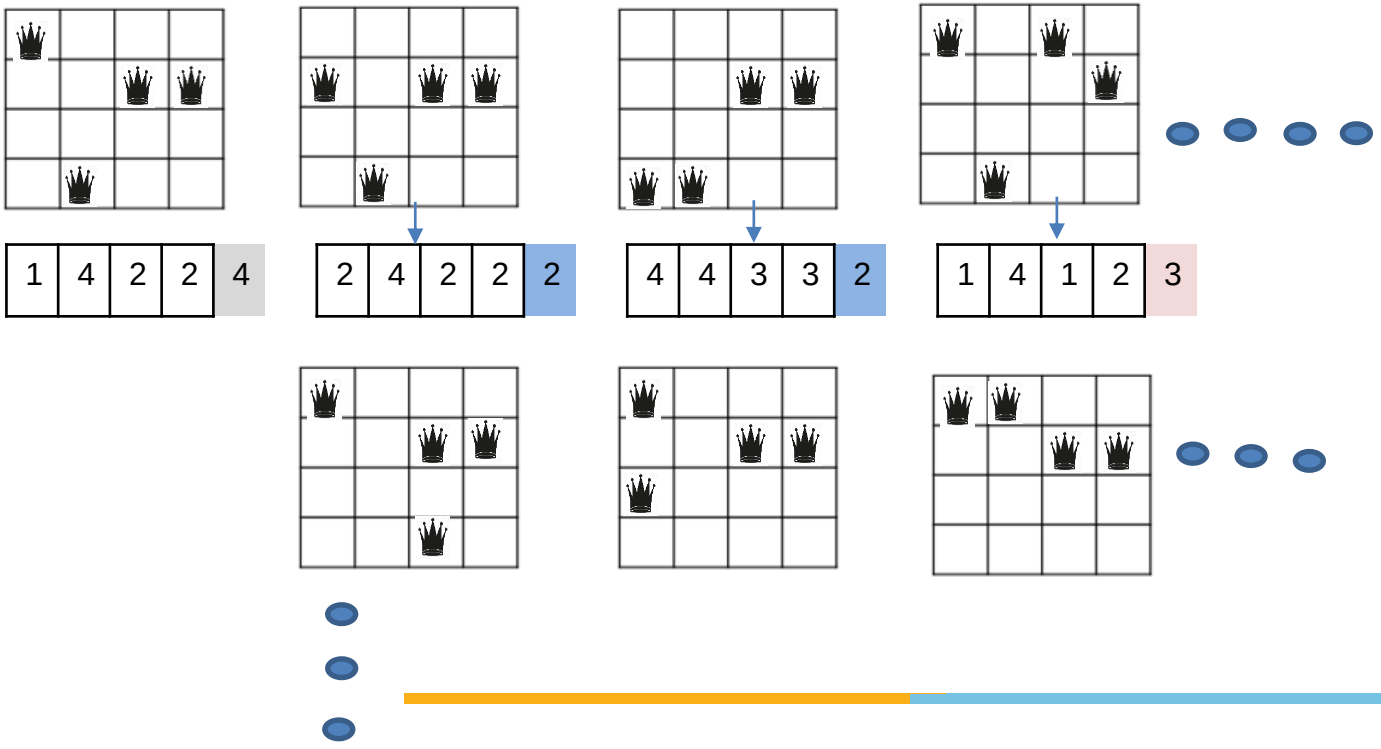
Sample from 1st State

1. Initialize k random state
2. Evaluate the fitness scores for all the successors of the k states
3. Calculate the probability of selecting a successor based on fitness score
4. Select the next state based on the highest probability
5. If the goal is not found, Select the next ‘k’ states randomly based on the probability
6. Repeat from Step 2



Genetic Algorithm

Genetic Algorithm

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states
3. If anyone of the state’s has achieved the threshold fitness value or threshold new states or no change is seen than previous iteration then the algorithm stops
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5. Pairs selected produces new state (successor) by crossover
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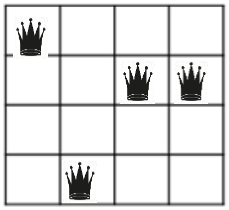
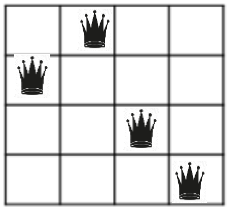
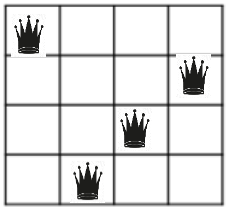
7. Repeat from Step 2



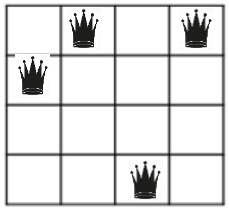
1	4	2	2



2	1	3	4



1	4	3	2



2	1	4	1

Genetic Algorithm

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens Threshold = 6
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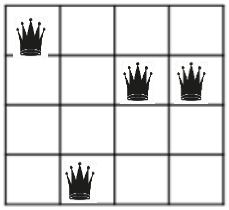
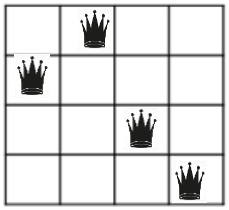
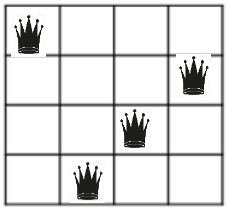


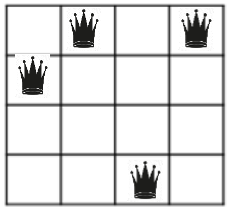
1	4	2	2	4



2	1	3	4	4



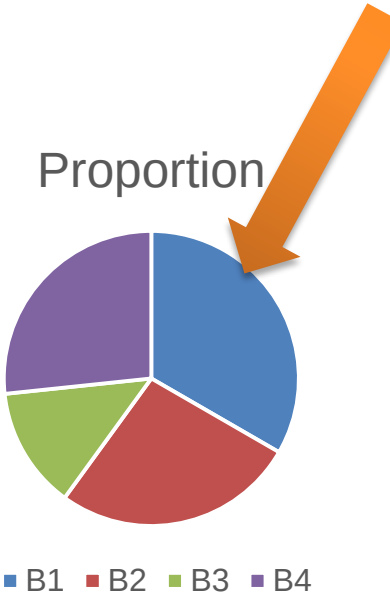
1	4	3	2	2

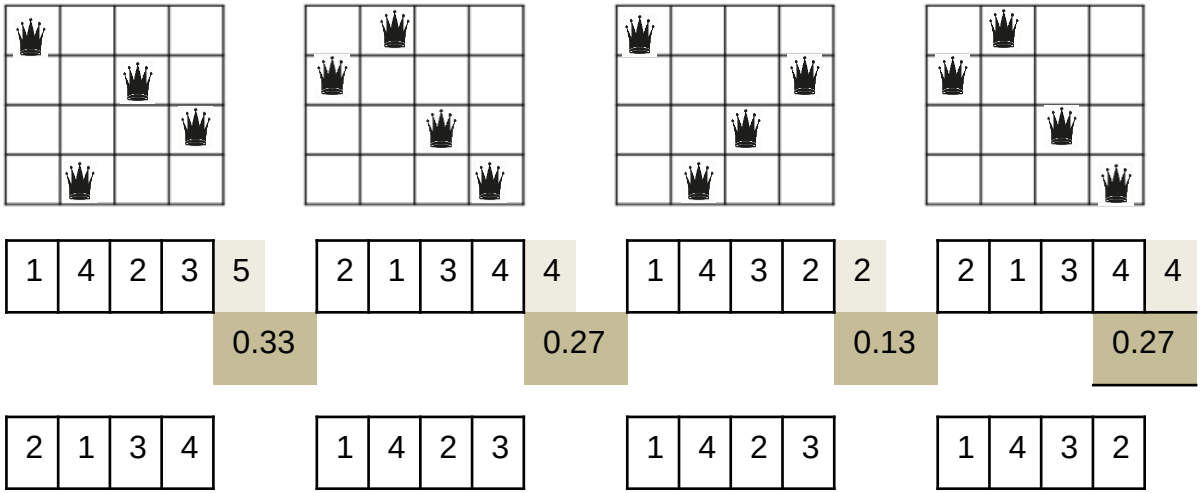


2	1	4	1	3

Genetic Algorithm

Eg., use roulette wheel mechanism to select pair/s





Sample winners of game -1 ,2,3,4 : B4, B1, B1, B3

Genetic Algorithm

1. Select ‘k’ random states – **Initialization : k=4**
2. Evaluate the fitness value all states : Maximizing function : No.of.Non-attacking pairs Queens Threshold = 6
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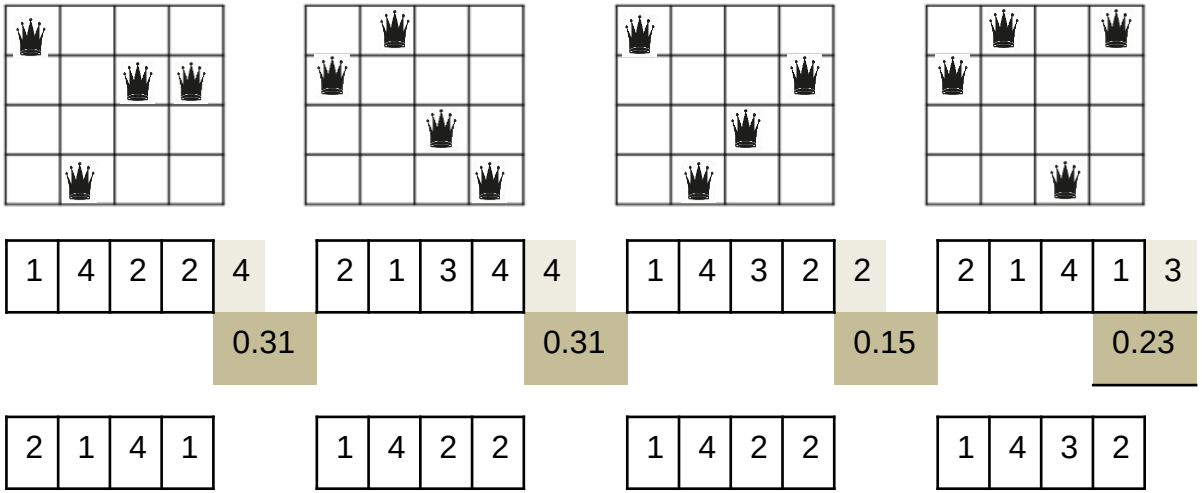

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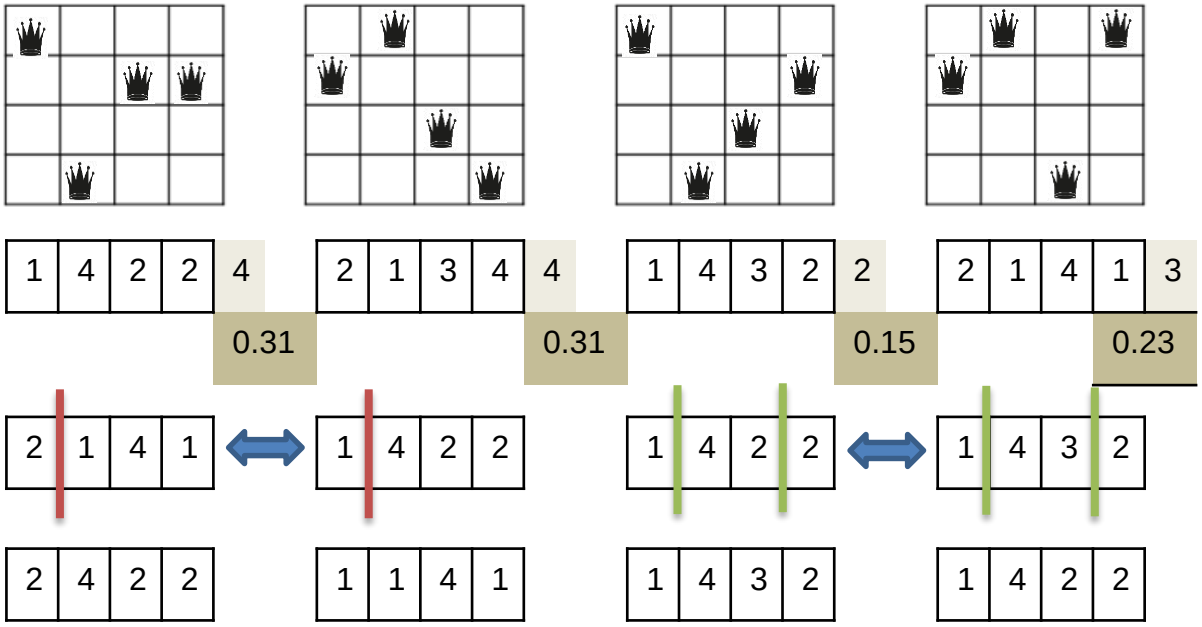

Genetic Algorithm

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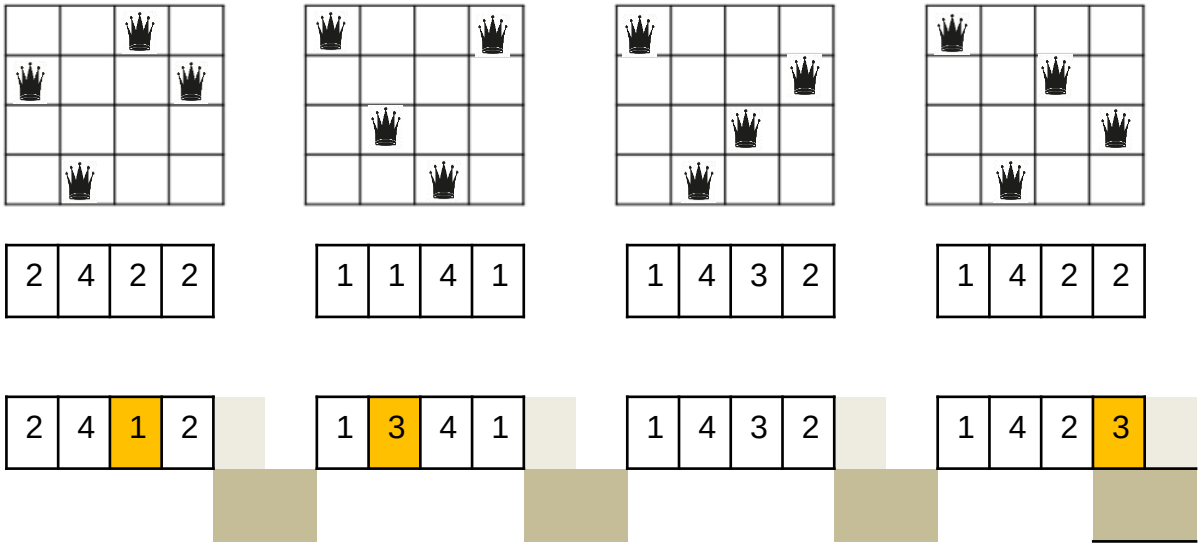


Genetic Algorithm

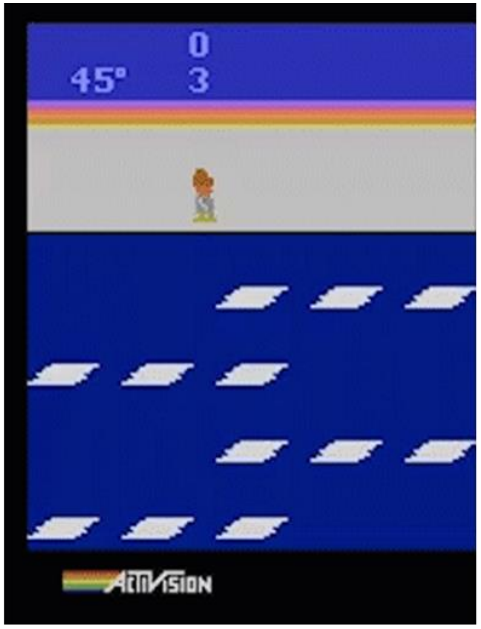
Techniques:

1. Design of the fitness function
2. Diversity in the population to be accounted
3. Randomization

Application:

- Creative tasks
- Exploratory in nature
- Planning problem
- Static Applications

Genetic Algorithm - Application in Games



Source Credit:

https://ai.googleblog.com/2018/03/using-evolutionary-automl-to-discover.html

https://eng.uber.com/deep-neuroevolution/

Course Plan

M1	Introduction to AI
M2	Problem Solving Agent using Search
M3	Game Playing
M4	Knowledge Representation using Logics
M5	Probabilistic Representation and Reasoning
M6	Reasoning over time, Reinforcement Learning
M7	Ethics in AI

Learning Objective

At the end of this class , students Should be able to:

1. Convert a given problem into adversarial search problem
2. Formulate the problem solving agent components
3. Design static evaluation function value for a problem
4. Construct a Game tree
5. Apply Min-Max
6. Apply and list nodes pruned by alpha pruning and nodes pruned by beta pruning

Module 3 : Searching to play games

A. Minimax Algorithm

B. Alpha-Beta Pruning

C. Making imperfect real time decisions

Task Environment

- Goal Formulation
- Problem Formulation
- Search Phase
- Execution Phase

Phases of Solution Search by PSA

Assumptions – Environment :

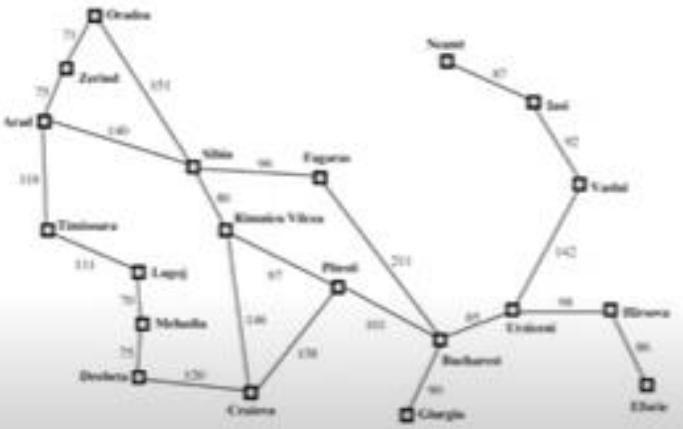
Static (4.5)

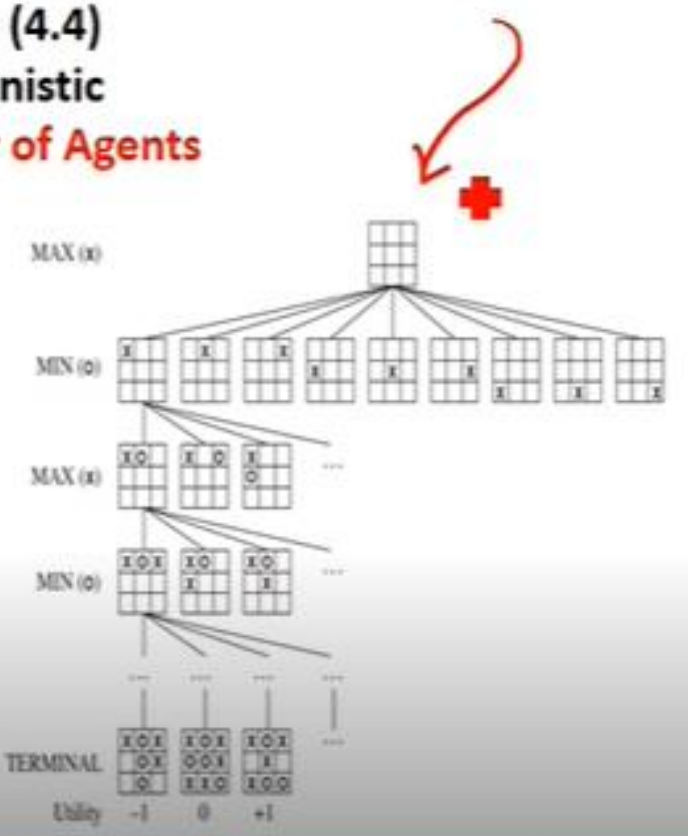
Observable

Discrete (4.4)

Deterministic

Number of Agents



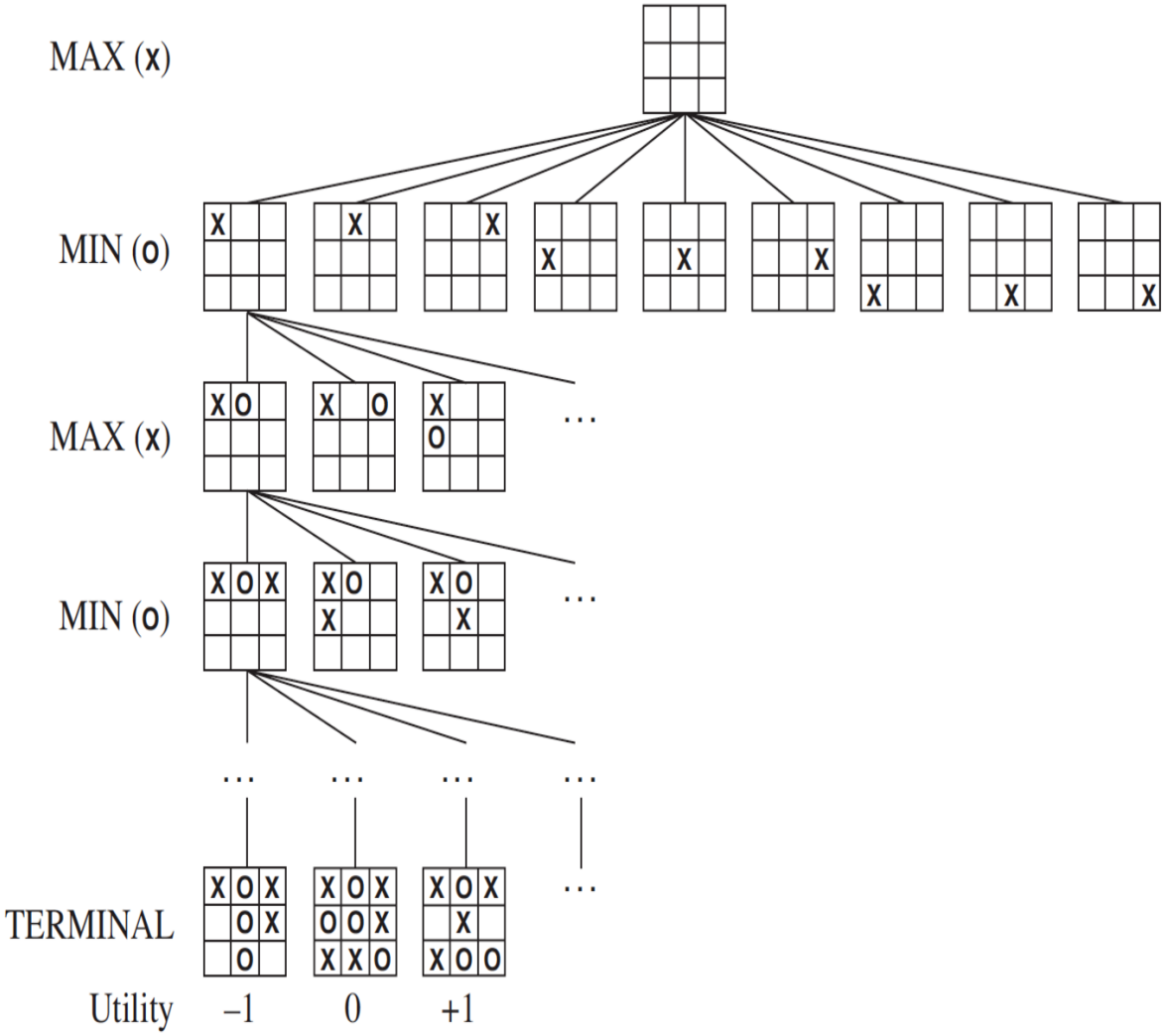


Games as Search Problem

PSA : Representation of Game:

- INITIAL STATE: S0
- PLAYER(s)
- ACTIONS(s)
- RESULT(s, a)
- TERMINAL-TEST(s)
- UTILITY(s, p)

Eg., Tic Tac Toe



Game Problem

Study & design of games enables the computers to model ways in which humans think & act hence simulating human intelligence.

AI for Gaming:

- Interesting & Challenging Problem
- Larger Search Space Vs Smaller Solutions
- Explore to better the Human Computer Interaction

Characteristics of Games:

- Observability
- Stochasticity
- Time granularity
- Number of players

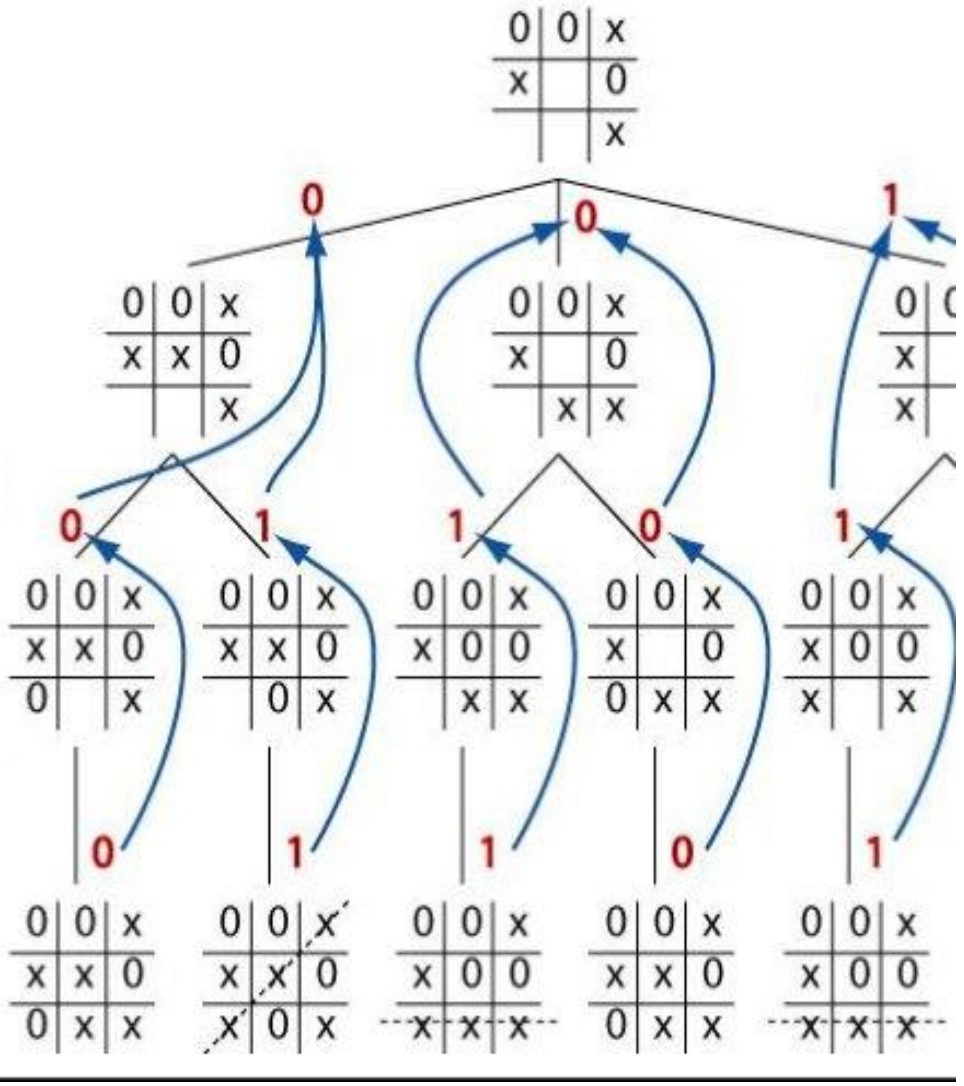
Adversarial Games:

Goals of agents are in conflict where one's optimized step would reduce the utility value of the other.

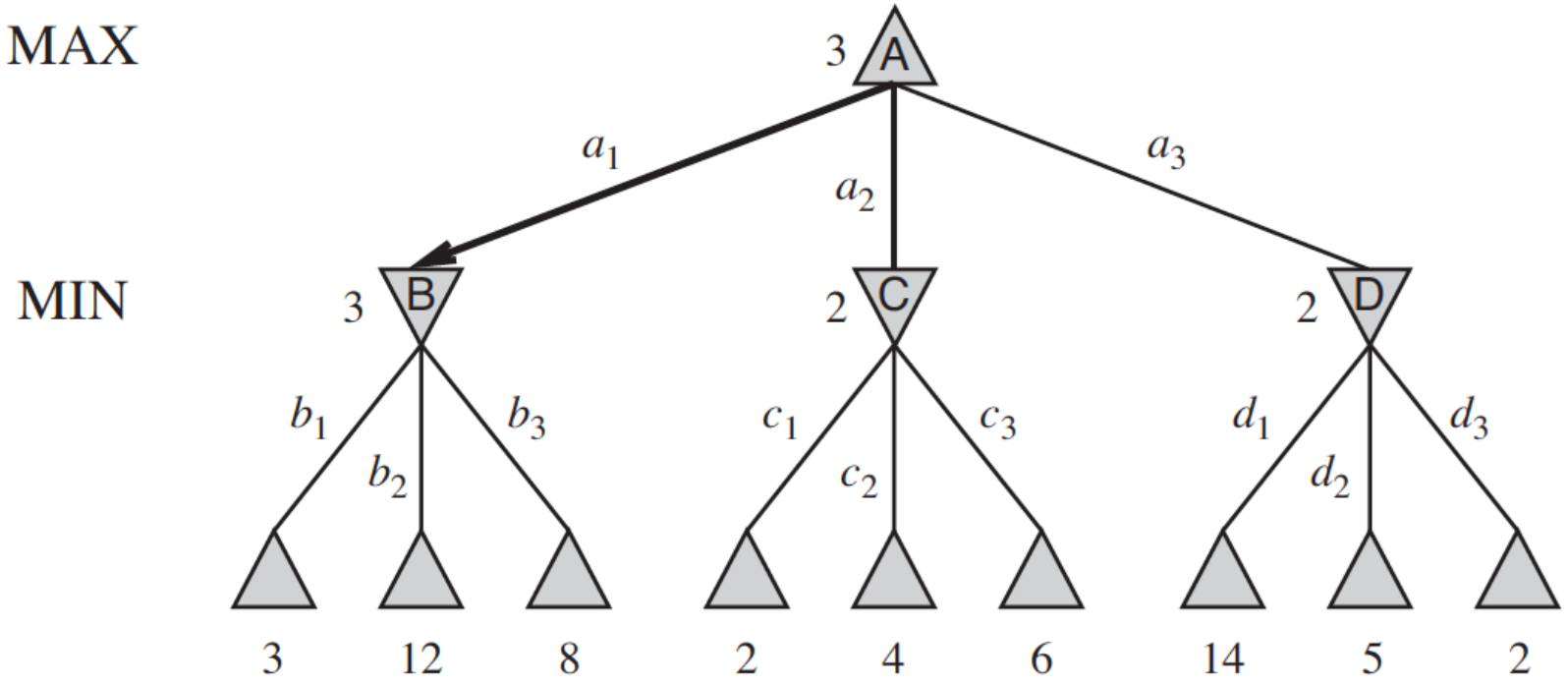
Min-Max Algorithm

Idea: Uses Depth – First search exploration to decide the move

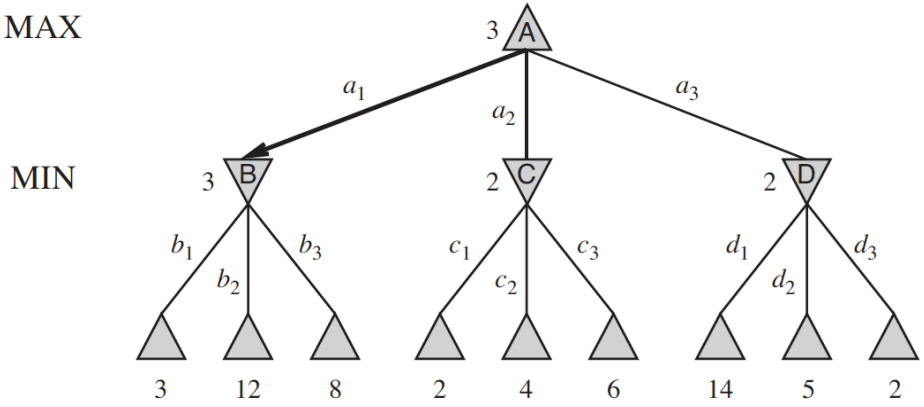
Let
start Player = MAX
Depth m =3



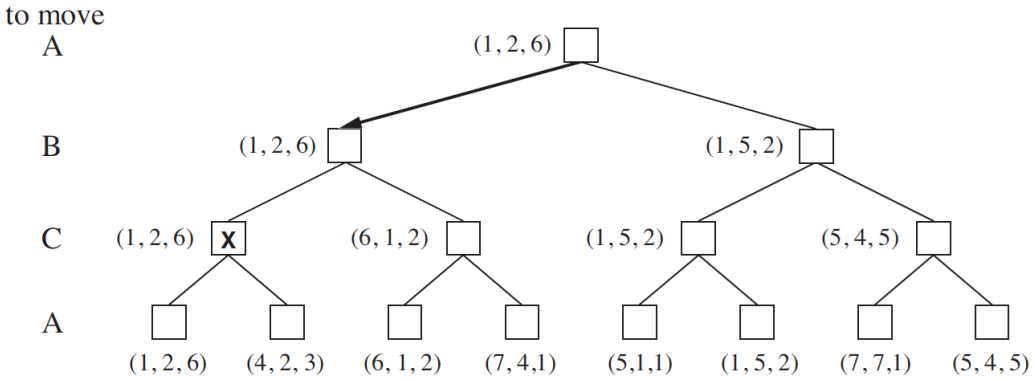
Book Example



Multiplayer Games



Two Player Game : 1-Ply Game



Multiplayer Game

Min-Max Algorithm

```
function MINIMAX-DECISION(state) returns an action
  return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
```

```
function MAX-VALUE(state) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s, a)))
  return v
```

```
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  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← ∞
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  return v
```
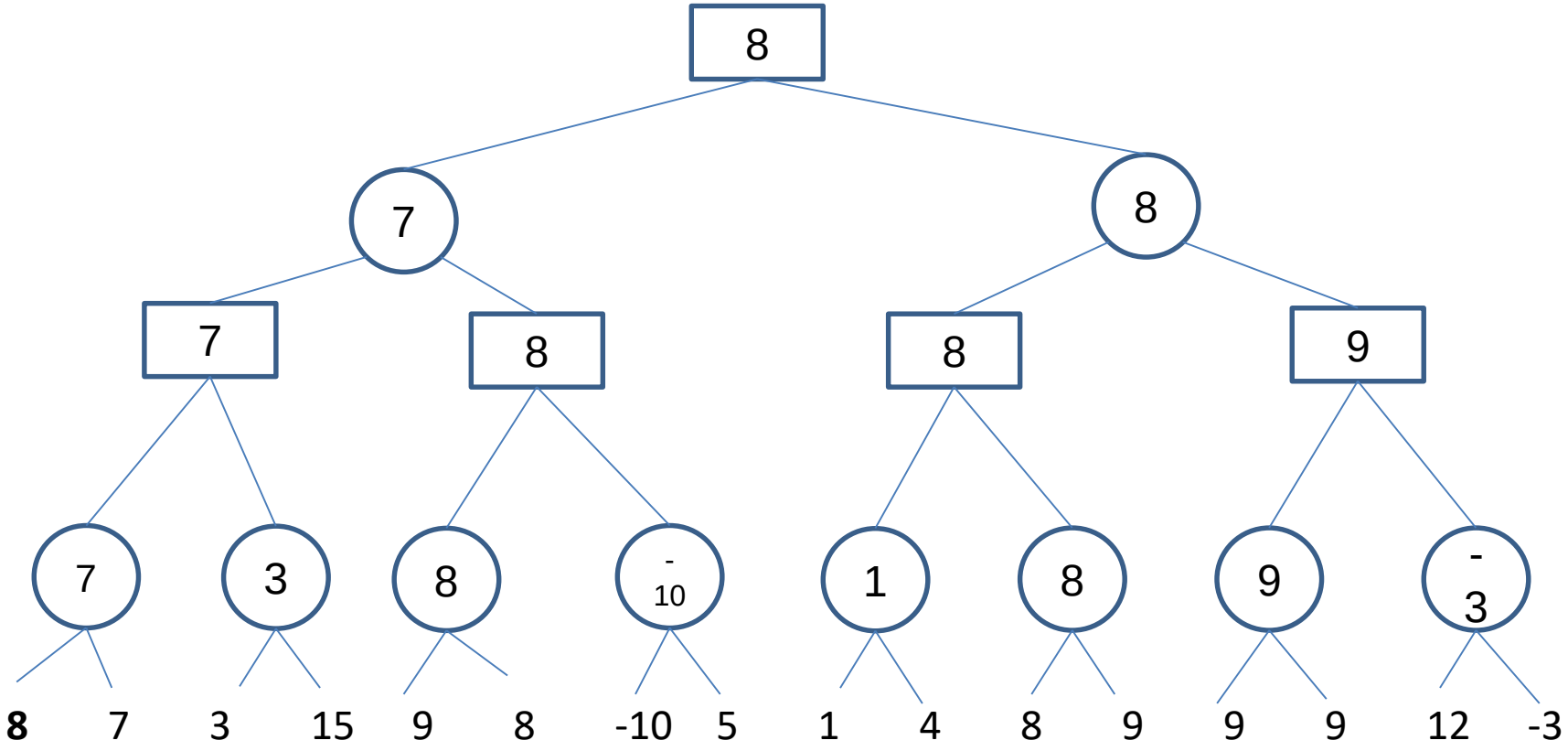

Min-Max Algorithm – Example -1

Squares represent MAX nodes

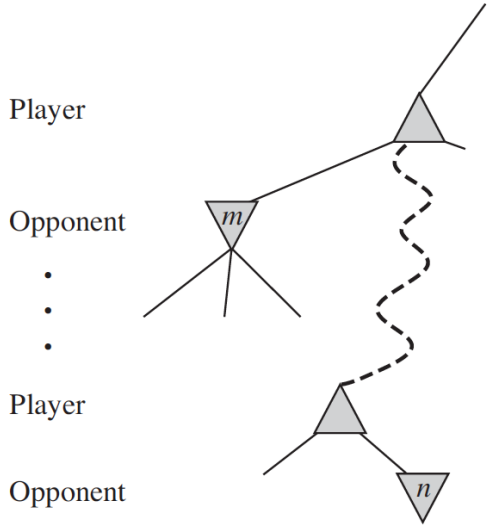
Circles represent MIN nodes

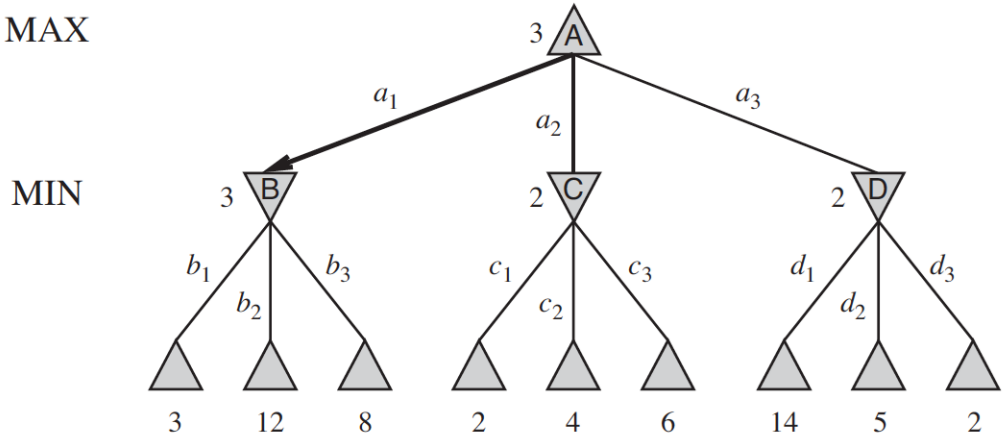


Alpha – beta Pruning

General Principle:

At a node ***n*** if a player has better option at the parent of ***n*** or further up, then ***n*** node will never be reached .Hence the entire subtree pruned

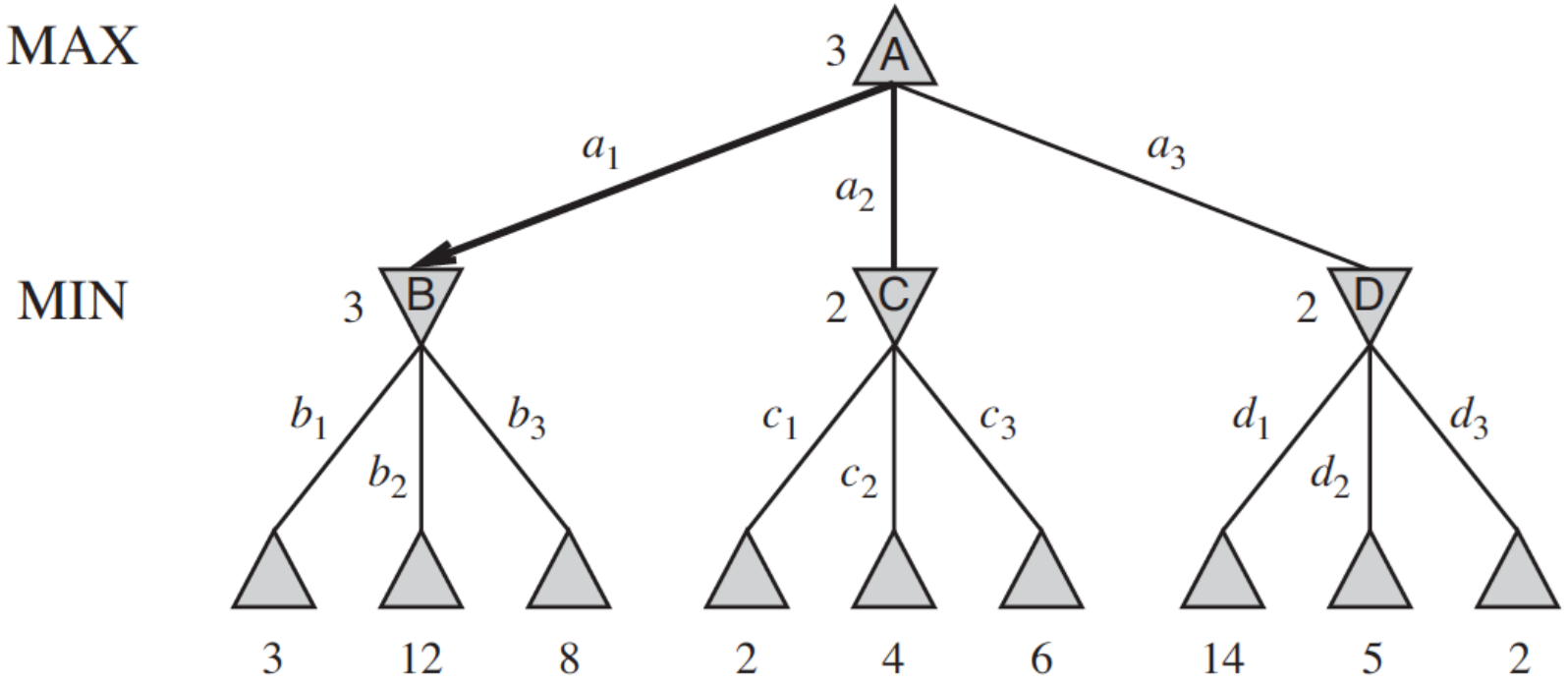




$$\begin{aligned}\text{MINIMAX}(root) &= \max(\min(3,12,8),\min(2,x,y),\min(14,5,2)) \\ &= \max(3,\min(2,x,y),2) \\ &= \max(3,z,2) \qquad \text{where } z = \min(2,x,y) \le 2 \\ &= 3.\end{aligned}$$

Alpha Beta Pruning

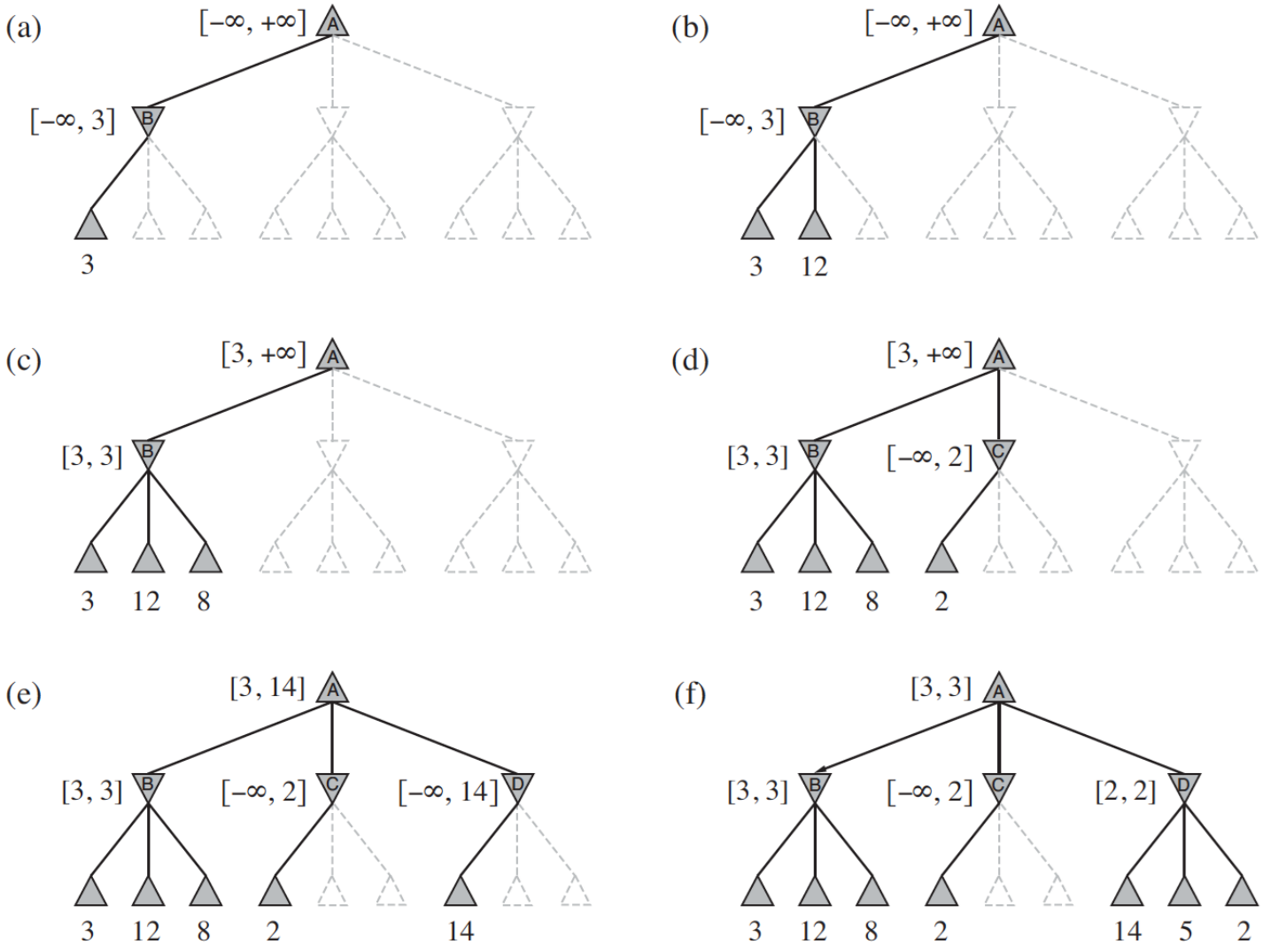
Book Example



Alpha Beta Pruning

innovate achieve lead

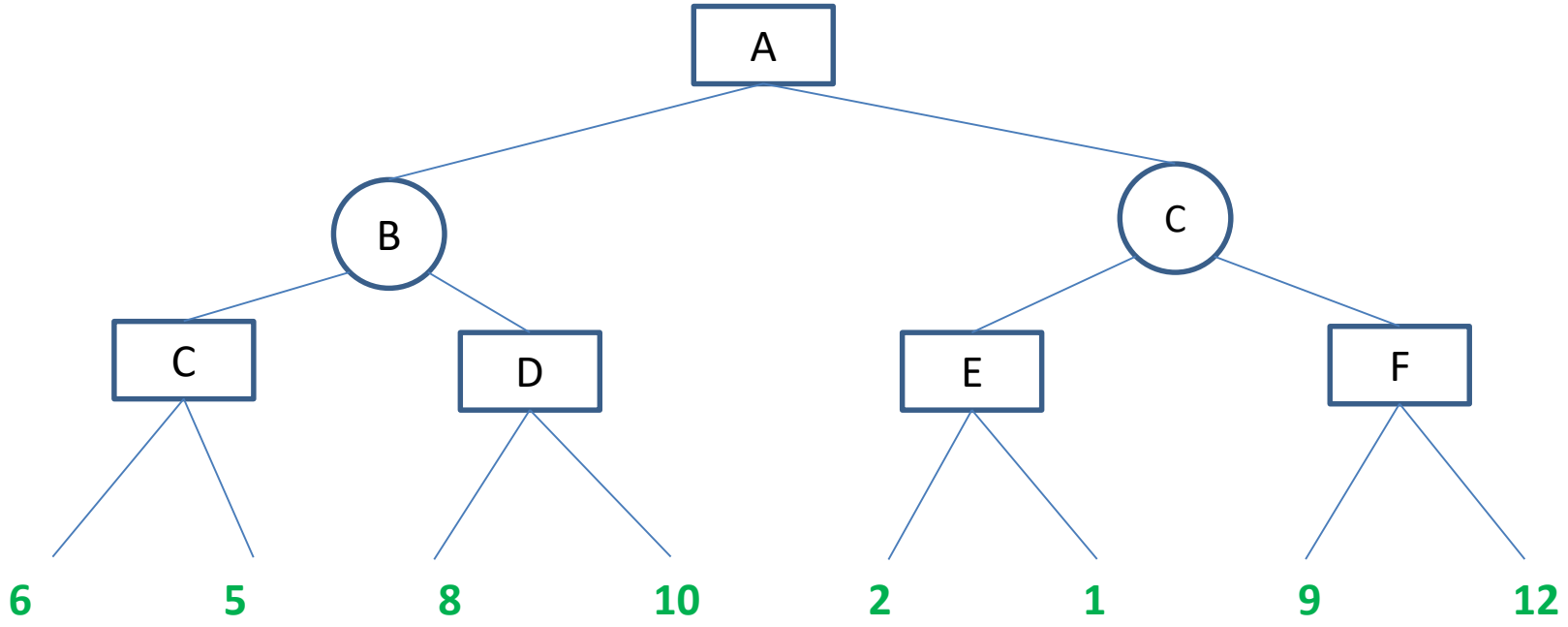
Book Example



Alpha Beta Pruning - Another Example

innovate achieve lead

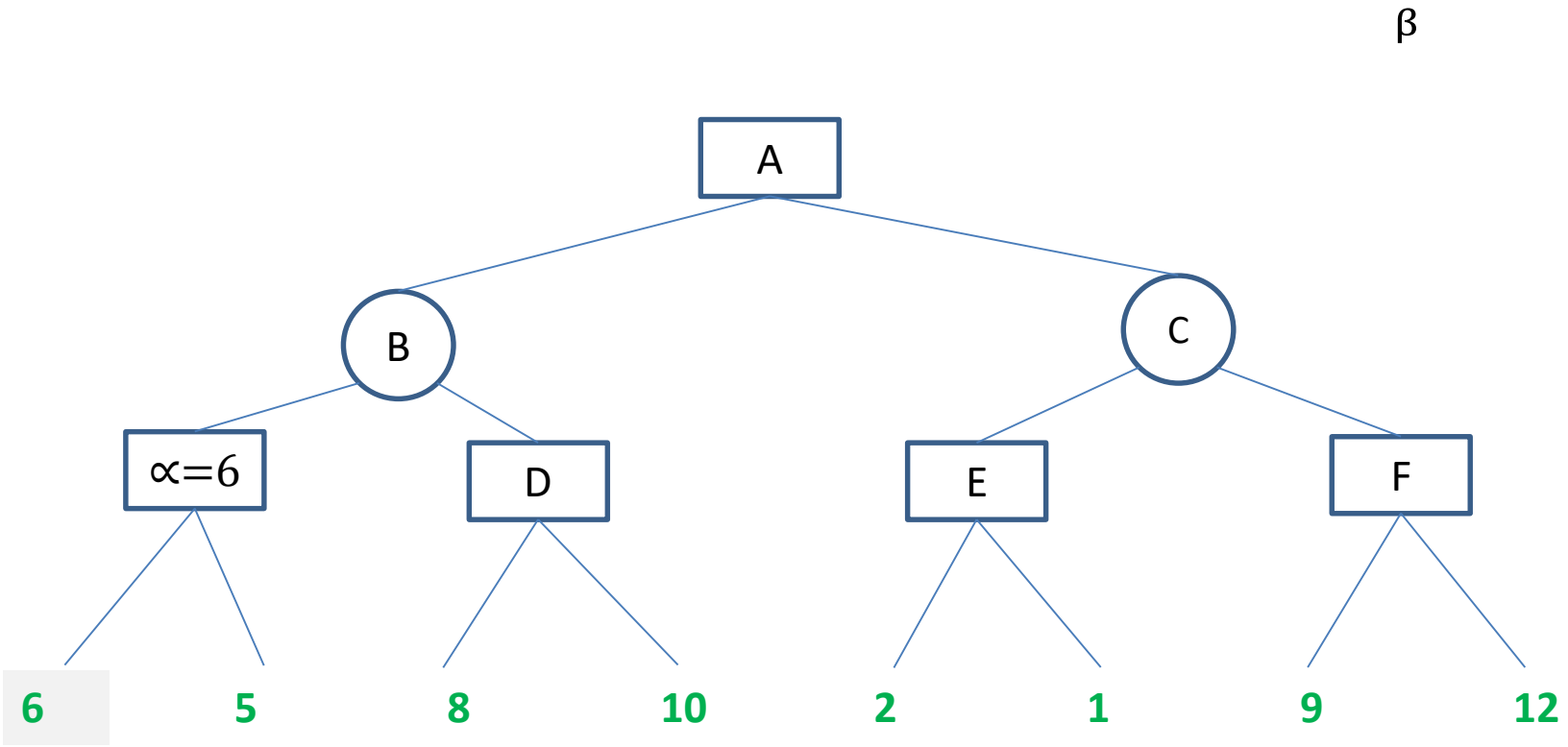
Idea –Pruning



Alpha Beta Pruning

innovate achieve lead

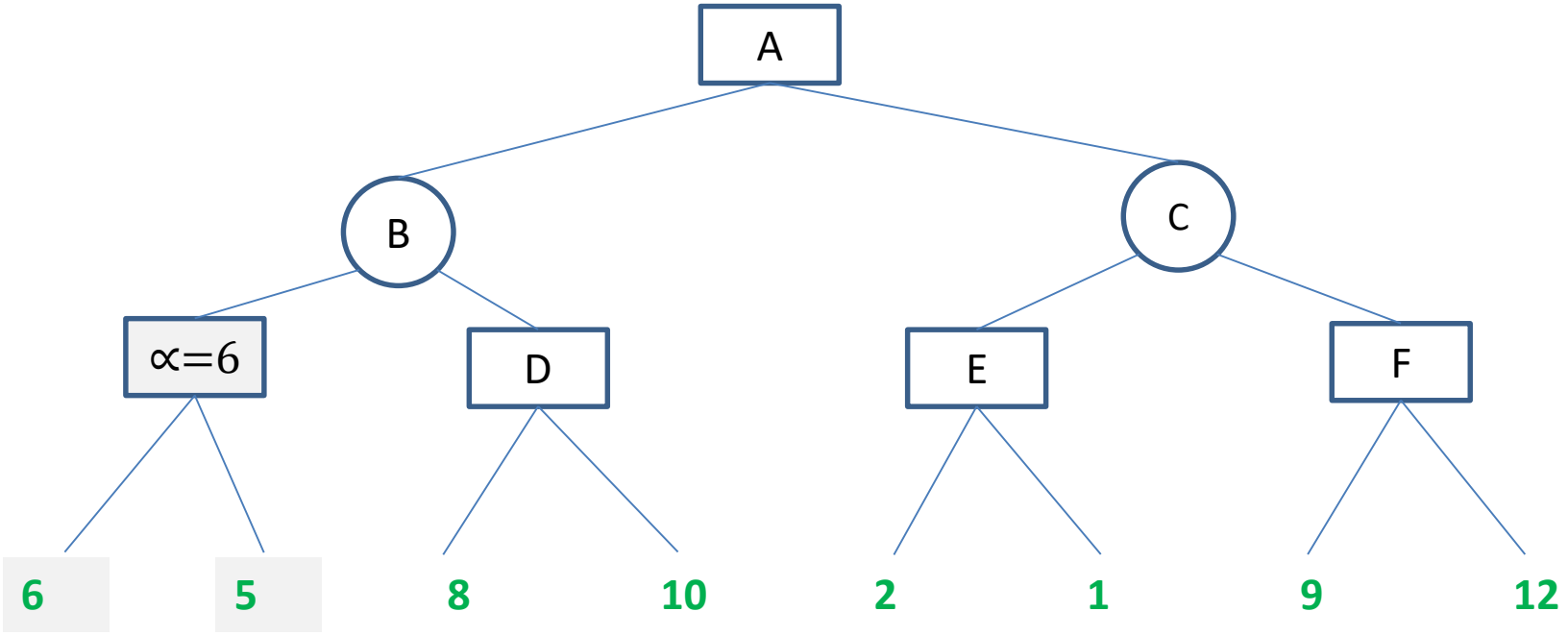
Idea –Pruning



Alpha Beta Pruning

innovate achieve lead

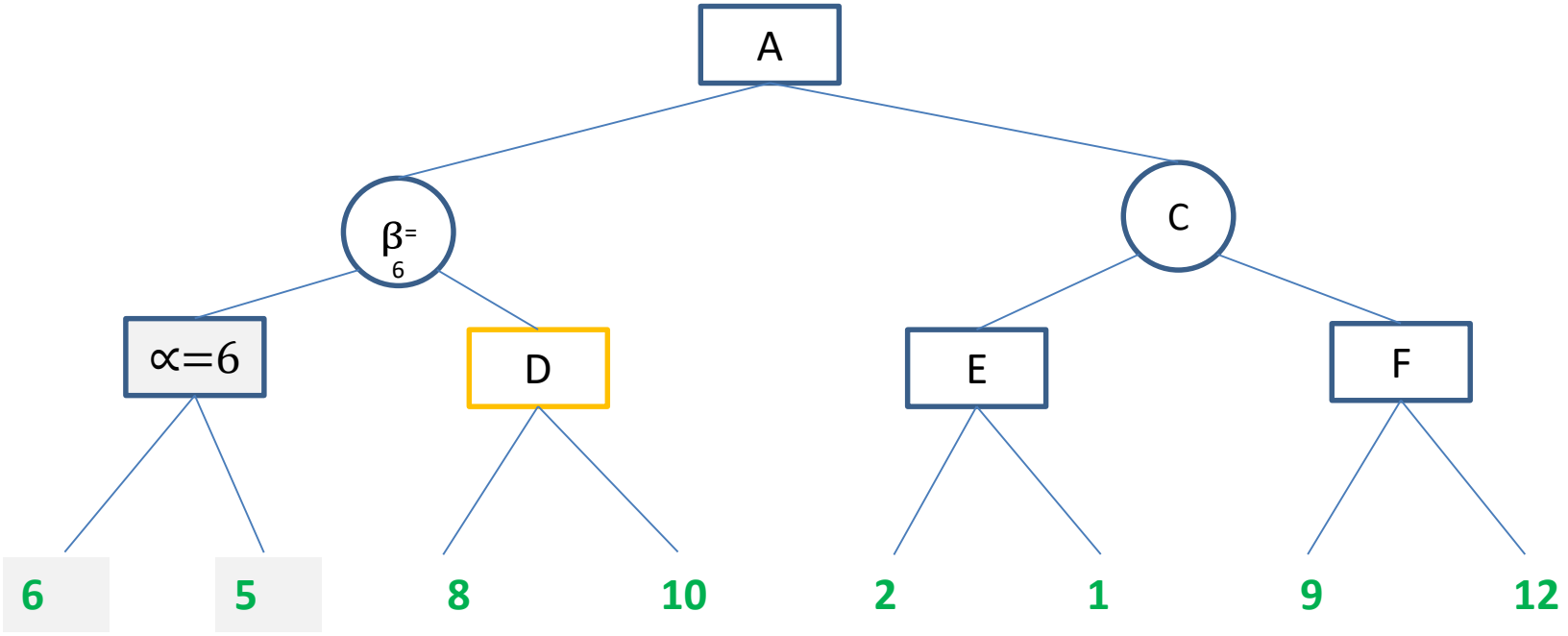
Idea –Pruning



Alpha Beta Pruning

innovate achieve lead

Idea –Pruning



Alpha Beta Pruning

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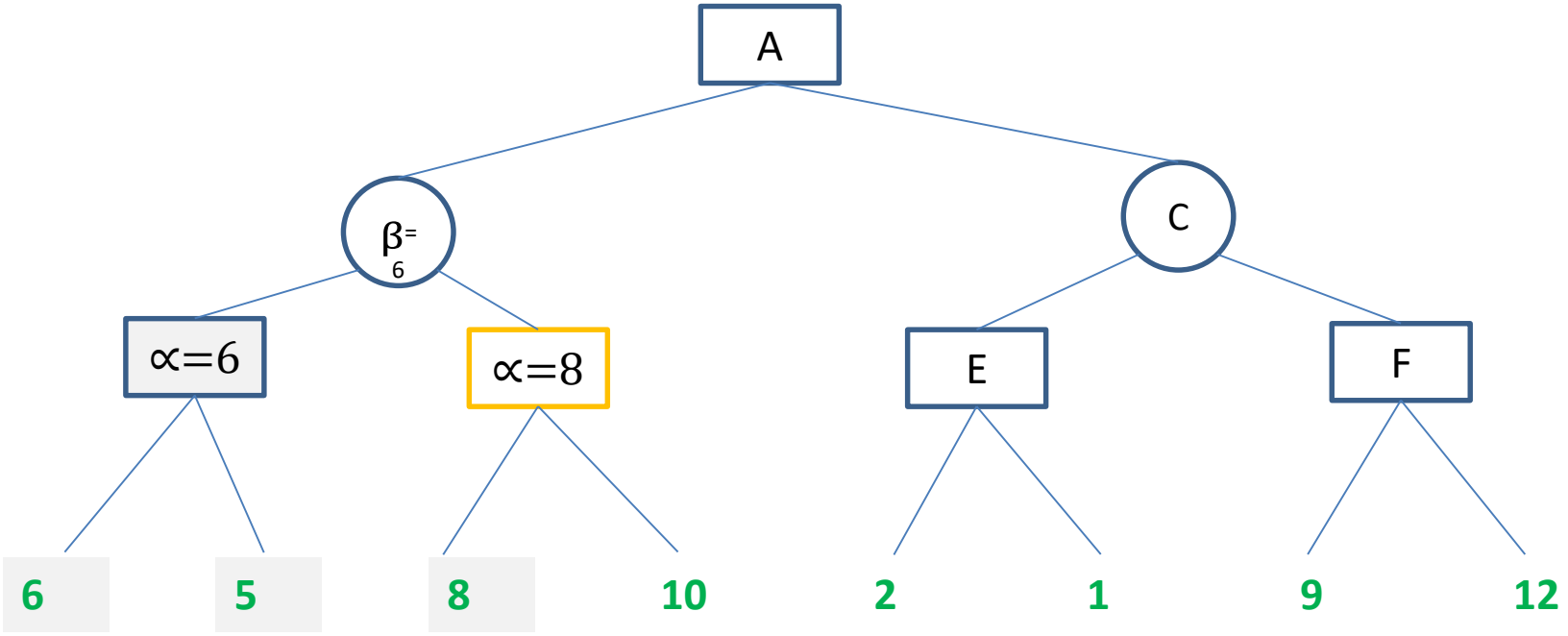

Alpha Beta Pruning

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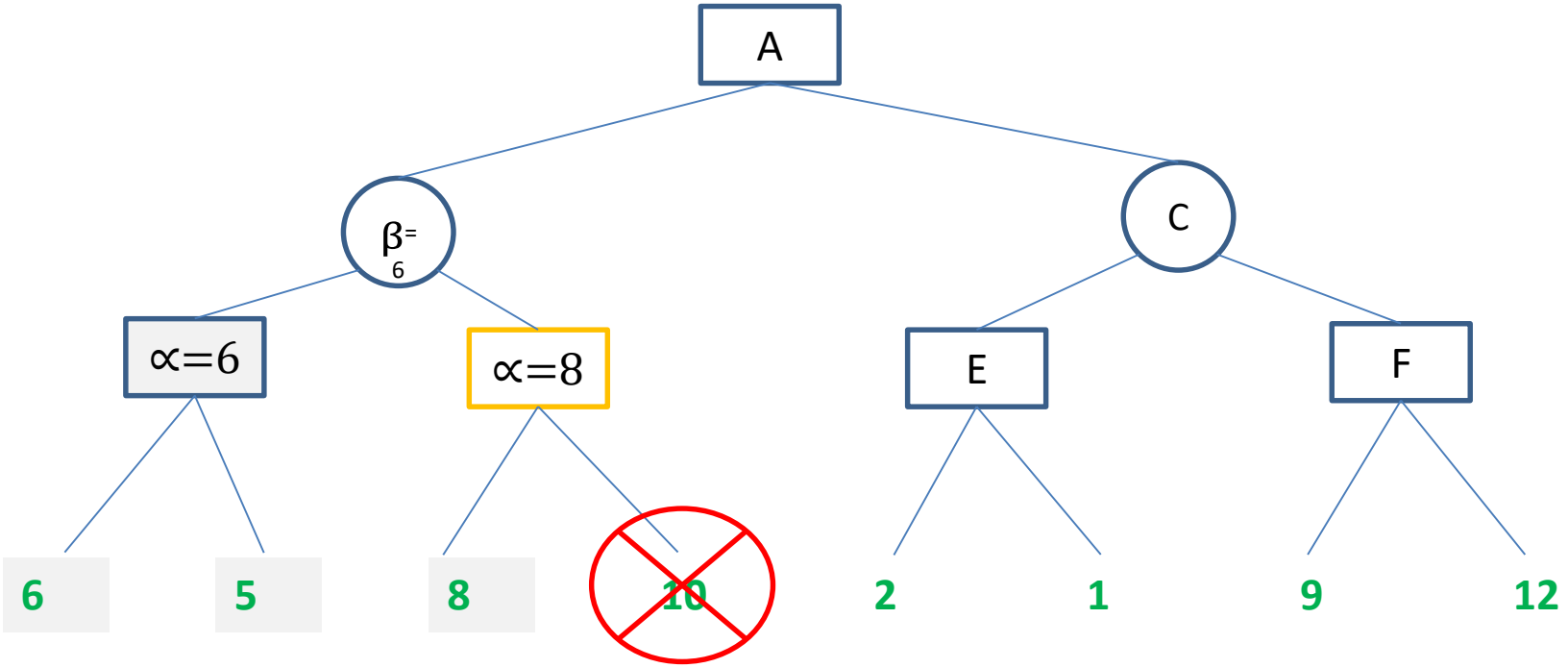
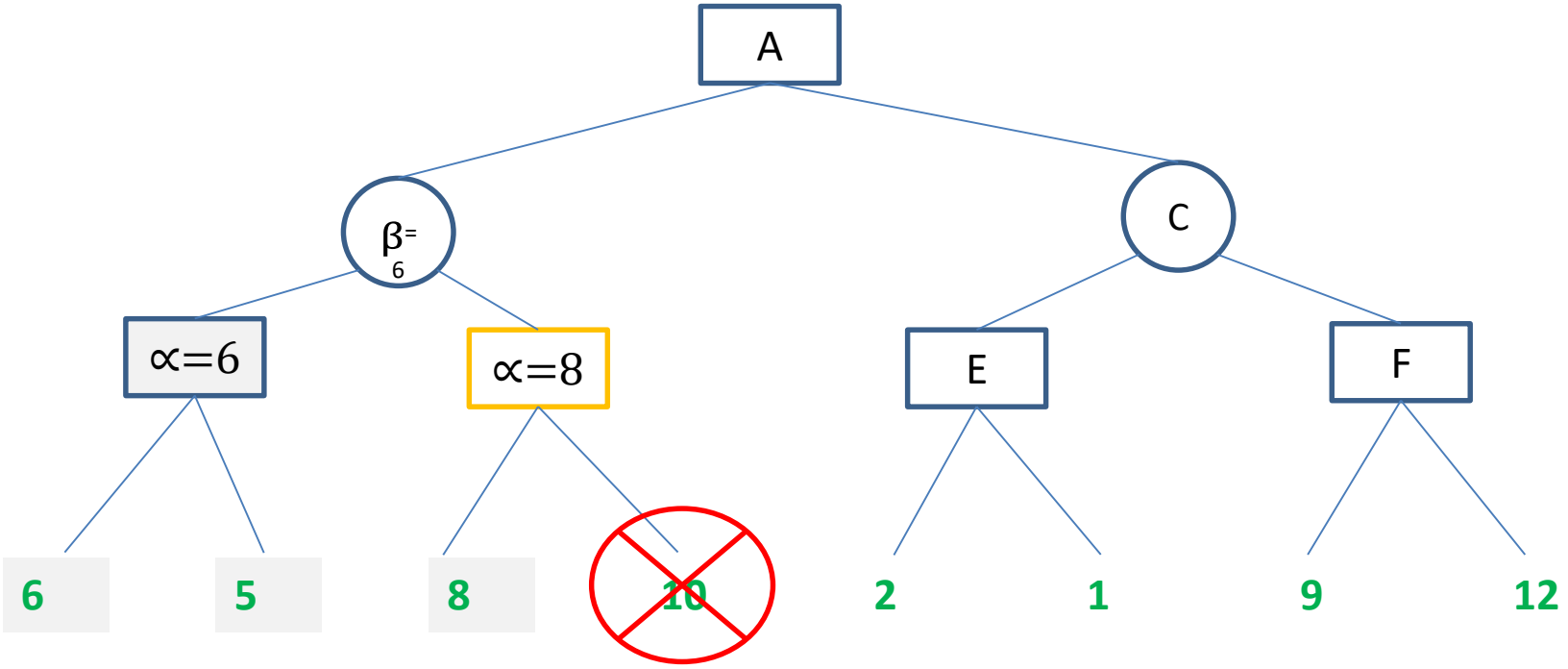


Alpha Beta Pruning

Idea –Pruning



Alpha Beta Pruning

innovate achieve lead

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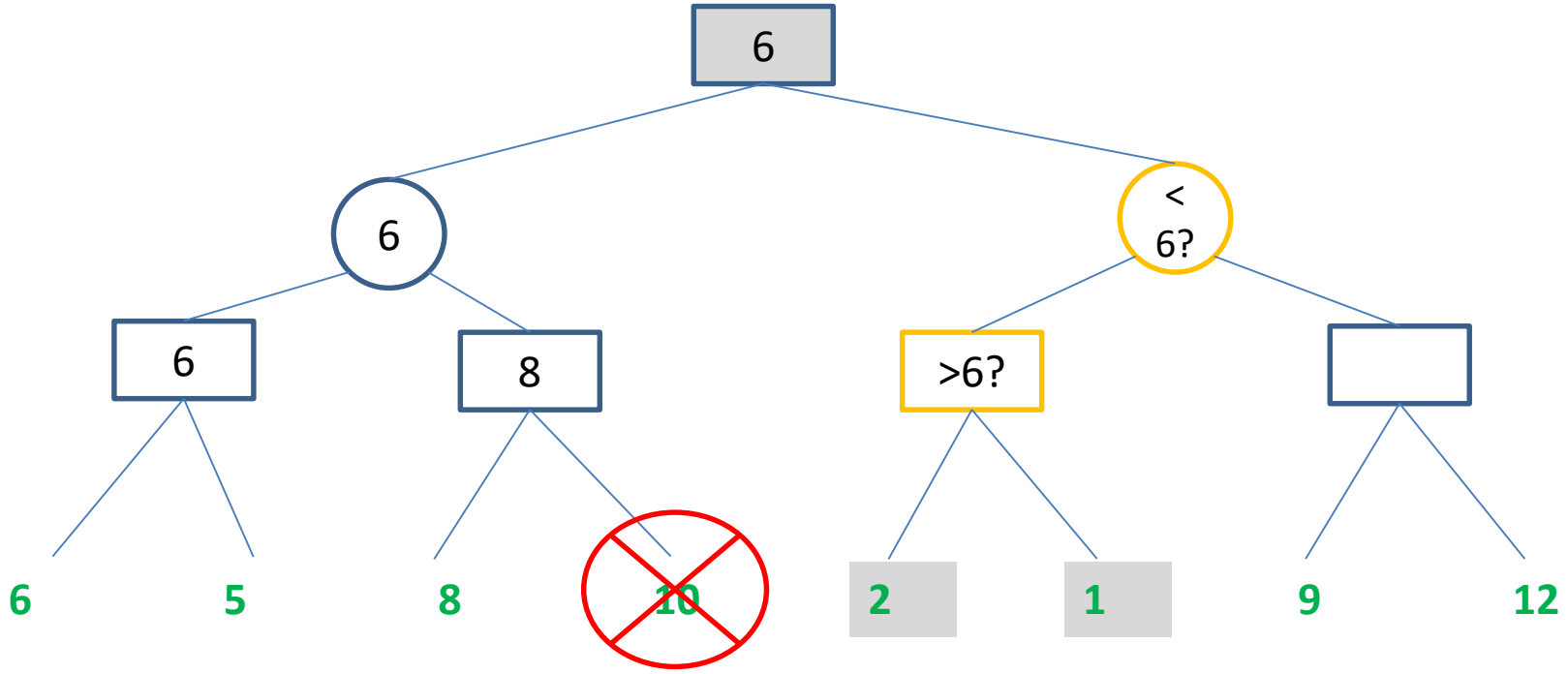

Alpha Beta Pruning

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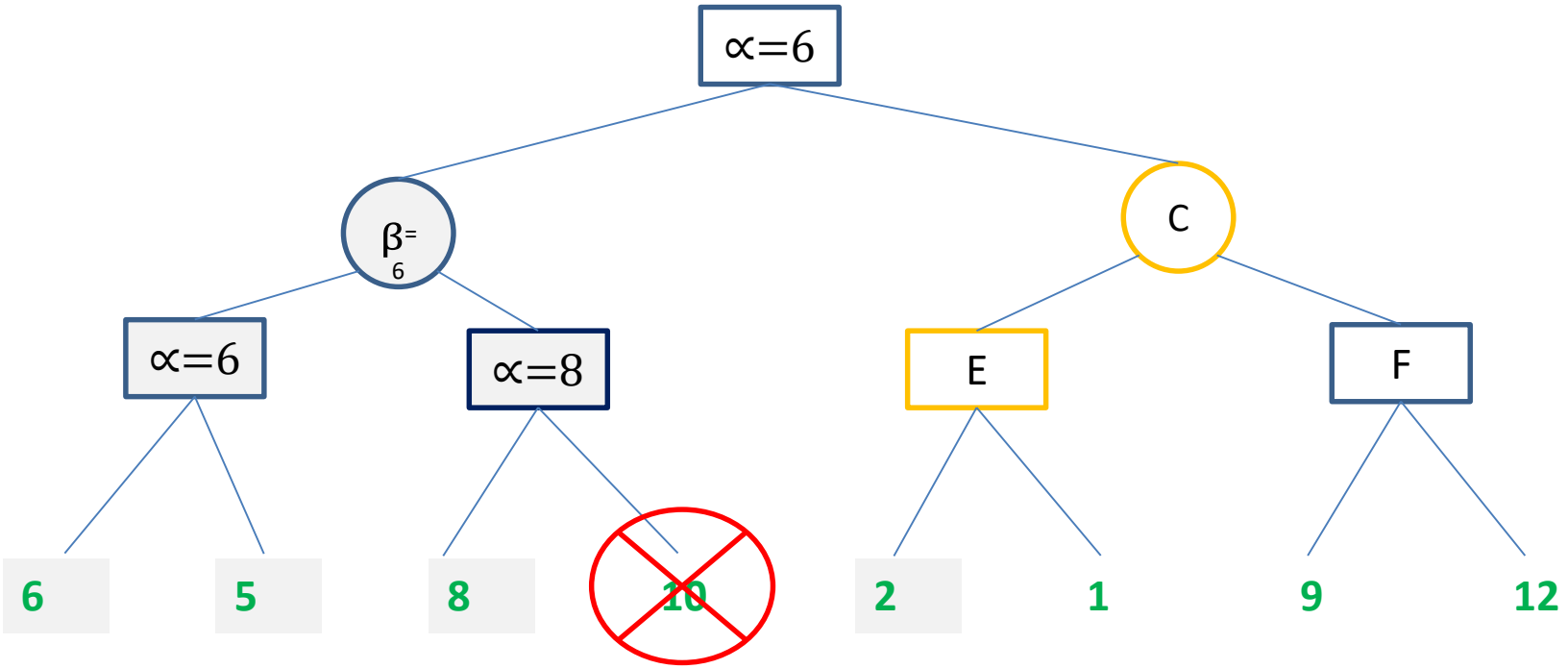

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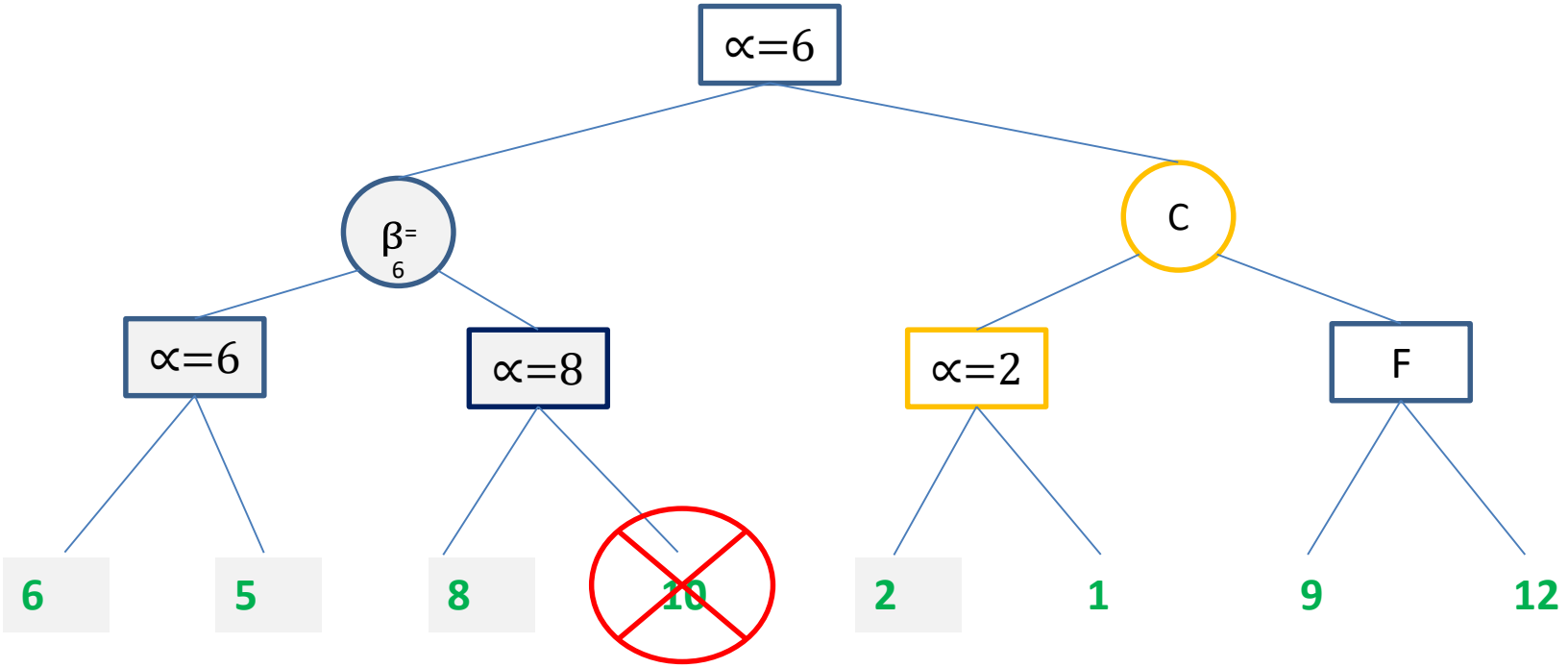
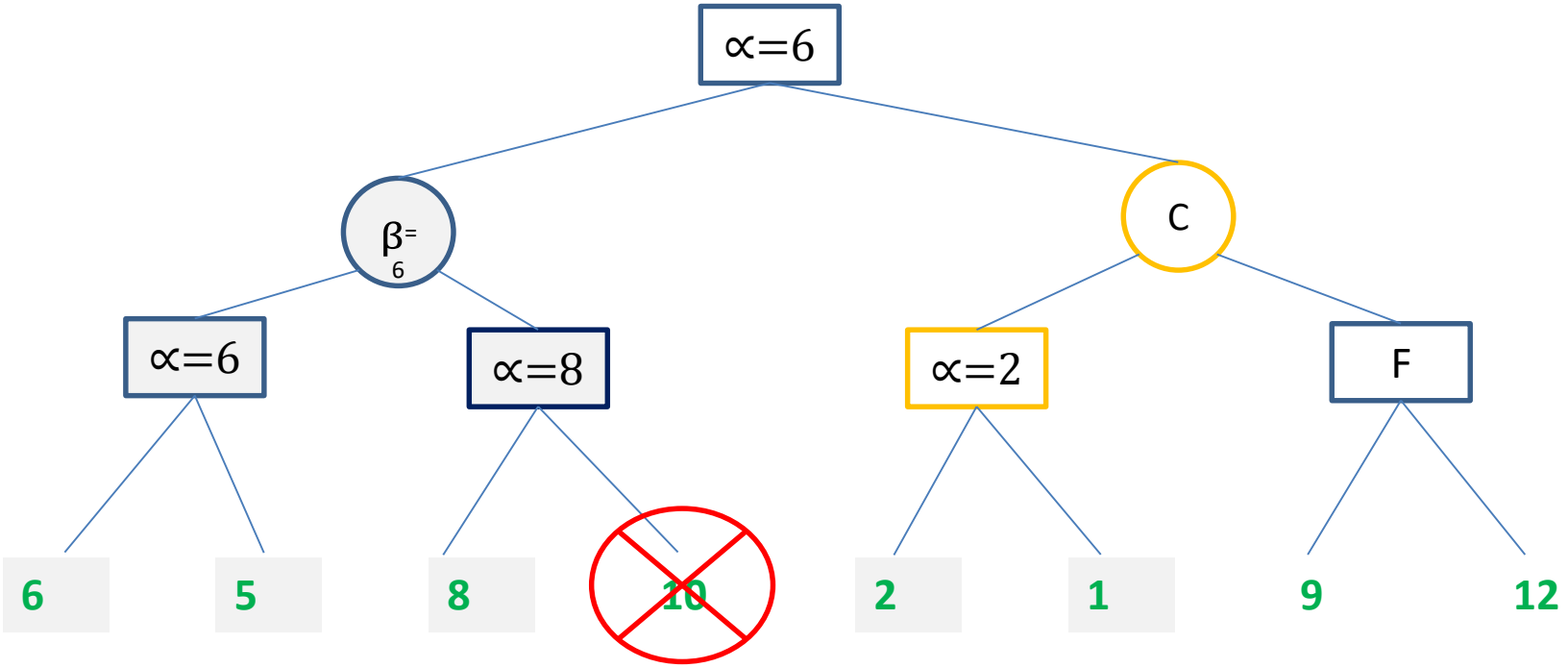


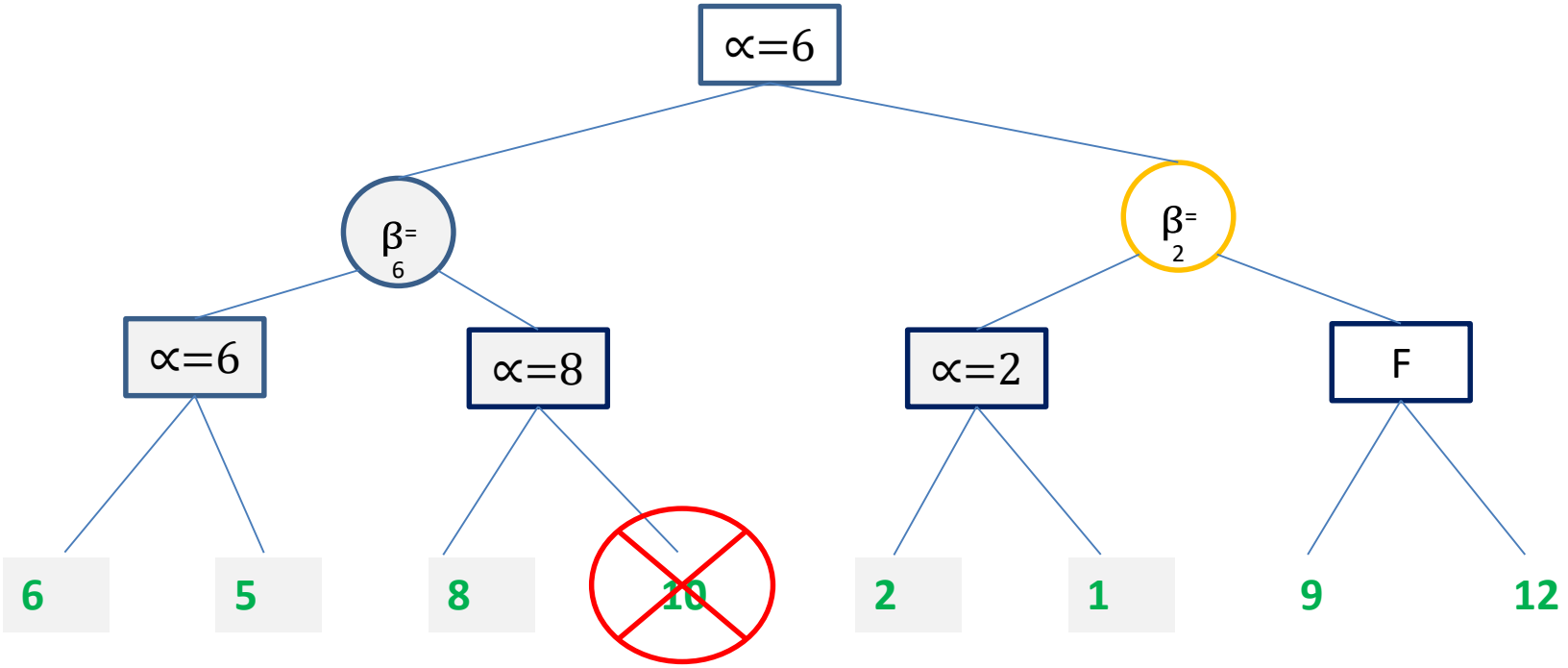
Alpha Beta Pruning

Idea –Pruning

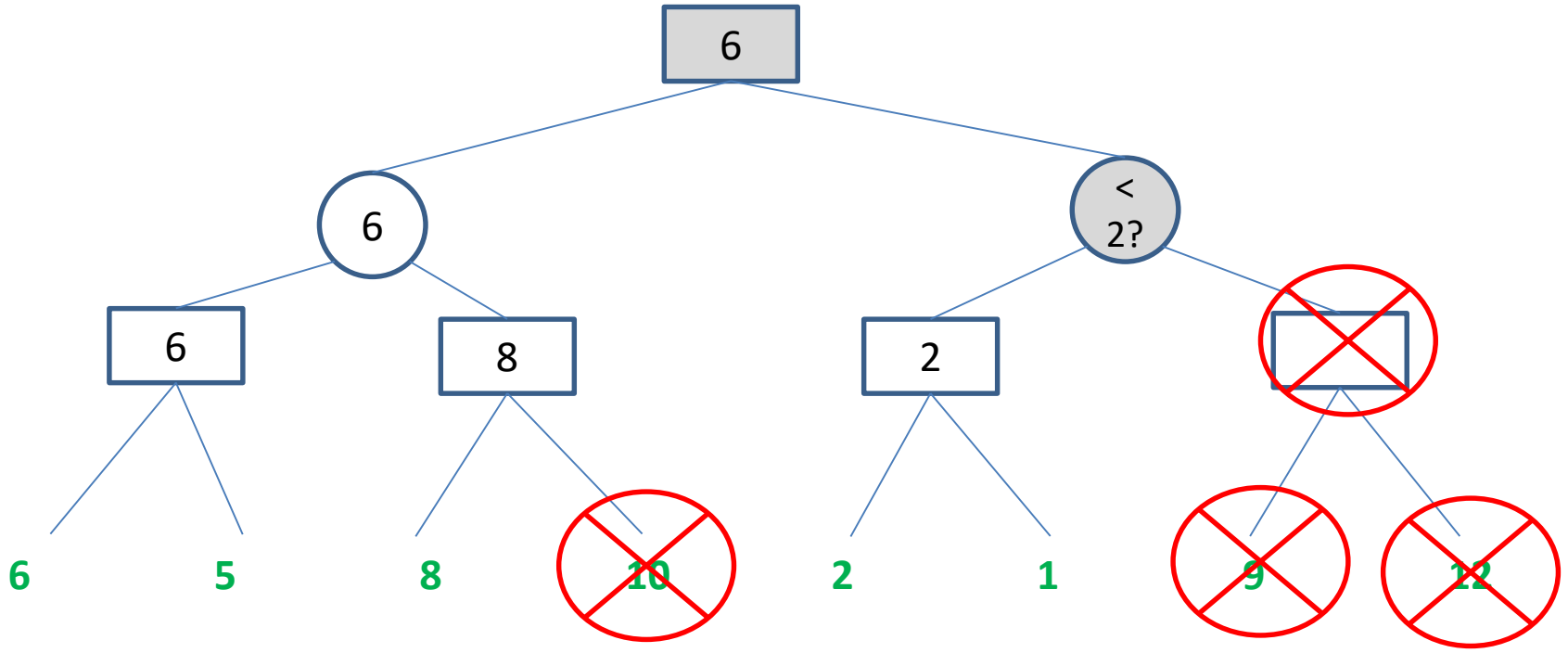


Alpha Beta Pruning

Idea –Pruning



Idea – Alpha Pruning



Alpha – Lower bound of Maximizer's value. Perceived value that Maximizer hopes to against a competitive Minimizer

Min-Max Algorithm

Alpha beta Modifications

```
function ALPHA-BETA-SEARCH(state) returns an action
  v ← MAX-VALUE(state, −∞, +∞)
  return the action in ACTIONS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← −∞
  for each a in ACTIONS(state) do
    v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
    if v ≥ β then return v
    α ← MAX(α, v)
  return v
```

```
function MIN-VALUE(state, α, β) returns a utility value
  if TERMINAL-TEST(state) then return UTILITY(state)
  v ← +∞
  for each a in ACTIONS(state) do
    v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
    if v ≤ α then return v
    β ← MIN(β, v)
  return v
```

Is it possible to compute the minimax decision for a node without looking at every successor node?

C. Alpha – beta Pruning

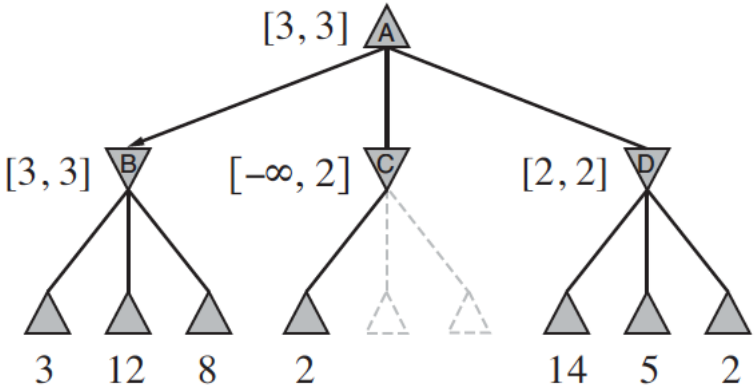
Steps in Alpha – Beta Pruning:

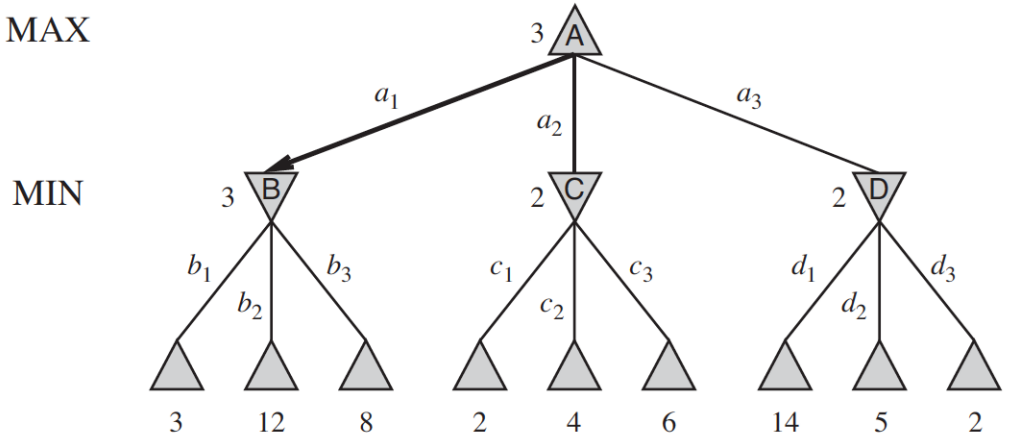
1. At root initialize alpha = -∞ and beta = + ∞. This is to set the worst case boundary to start the algorithm which aims to increase alpha and decrease beta as much as optimally possible
2. Navigate till the depth / limit specified and get the static evaluated numeric value.
3. For every value VAL being analyzed : Loop till all the leaf/terminal/specified state level nodes are analyzed & accounted for OR until **beta <= alpha.**
    1. If the player is MAX :
        1. If VAL > alpha
        2. then reset alpha = VAL
        3. also check **if** beta <=alpha **then** tag the path as unpromising (TO BE AVOIDED) **and** prune the branch from game tree. Rest of their siblings are not considered for analysis
    2. Else if the player is MIN:
        1. If VAL < beta
        2. then reset beta = VAL
        3. also check **if** beta <=alpha **then** tag the path as unpromising (TO BE AVOIDED) **and** prune the branch from game tree. Rest of their siblings are not considered for analysis

Gaming
(Imperfect Decisions)

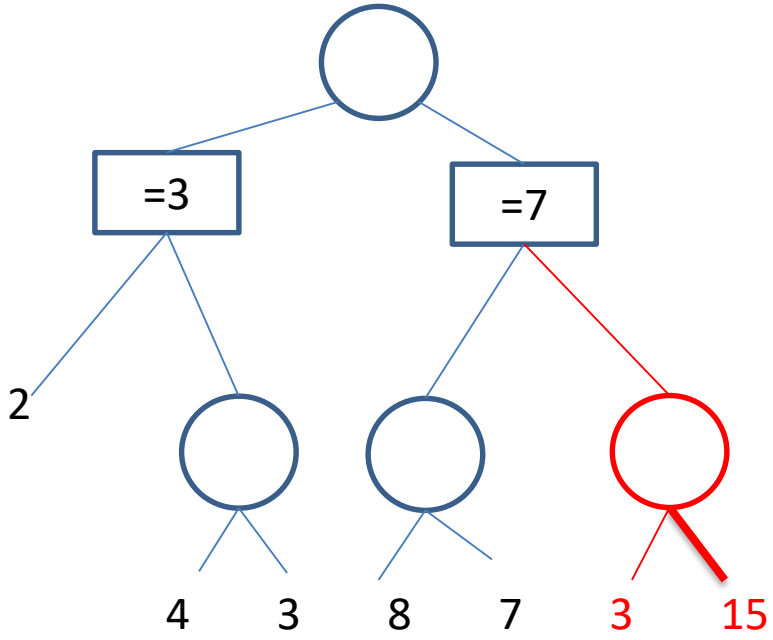
Computational Efficiency

How to reduce the move generations better along while doing Alpha-Beta Pruning?

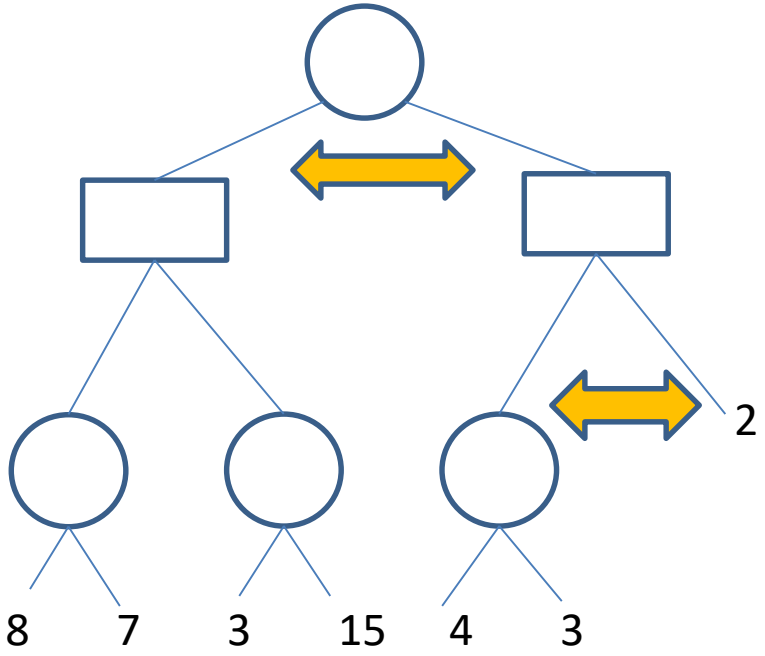




After Move Ordering

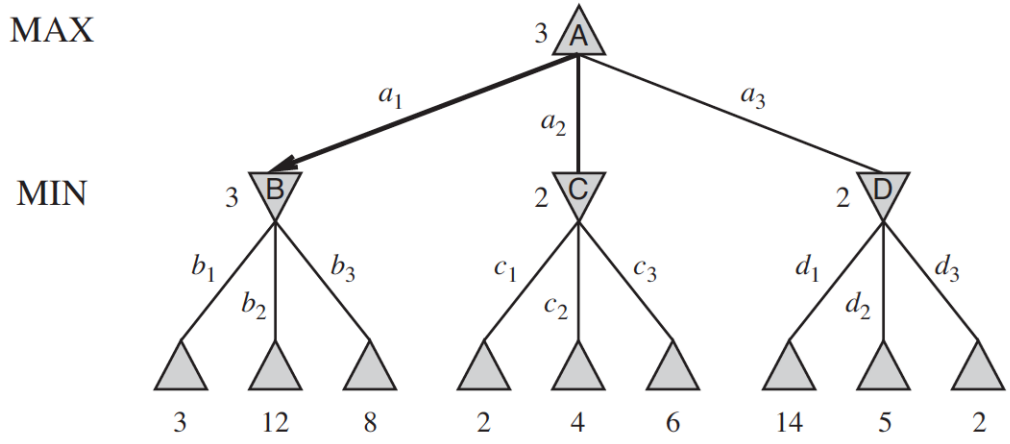


Before Move Ordering



Computational Efficiency

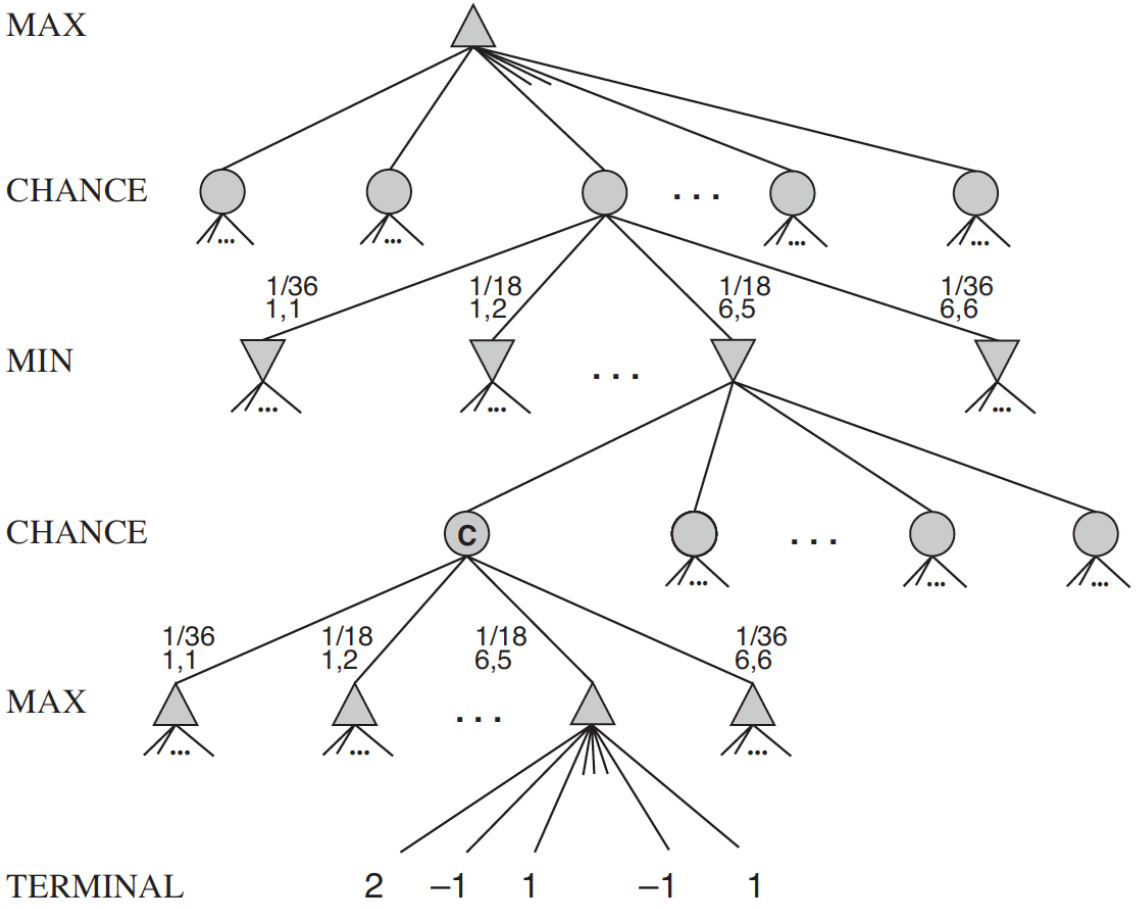
How games can be designed to handle imperfect decisions in real-time?



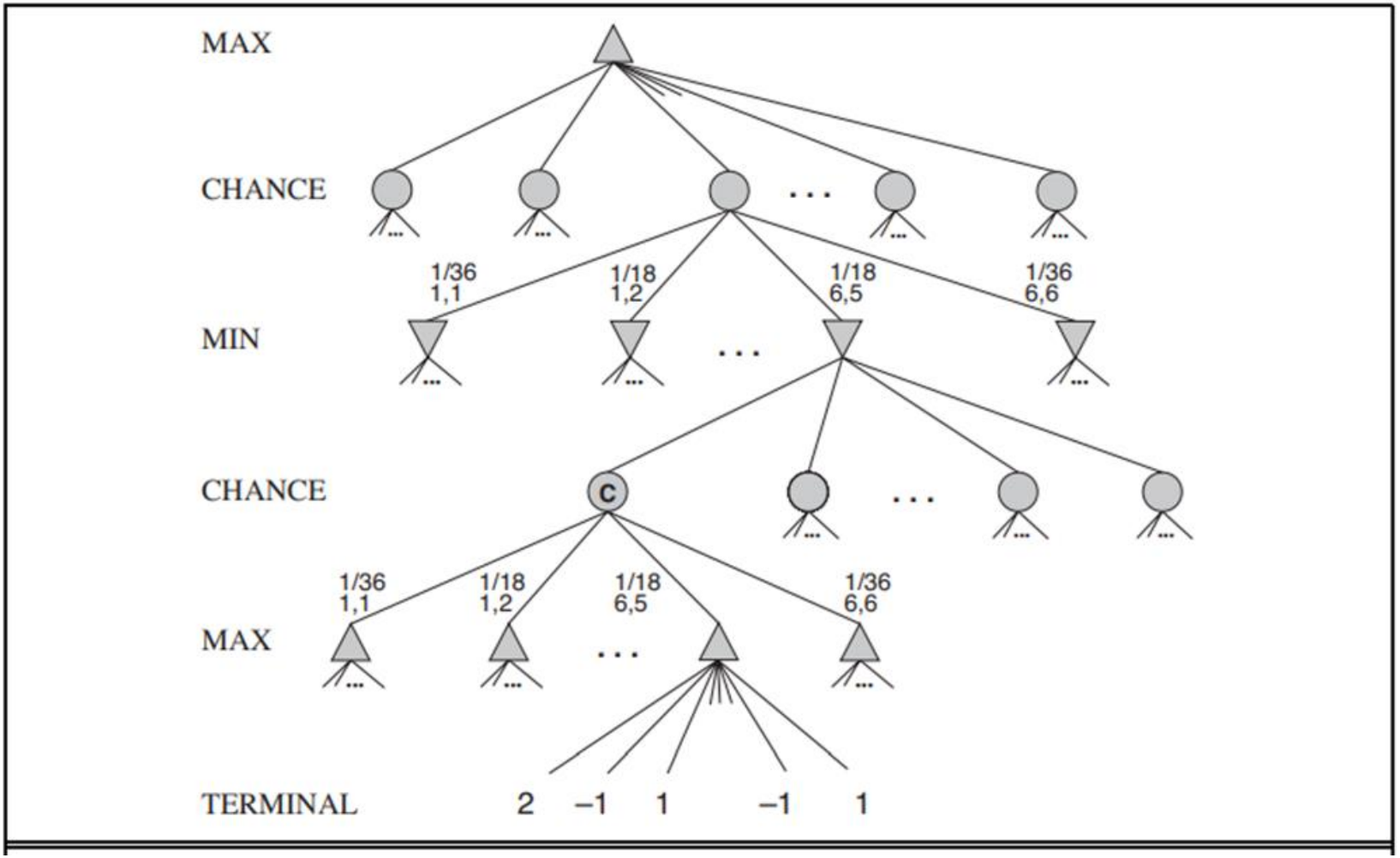
Computational Efficiency

Idea : Chance Node:

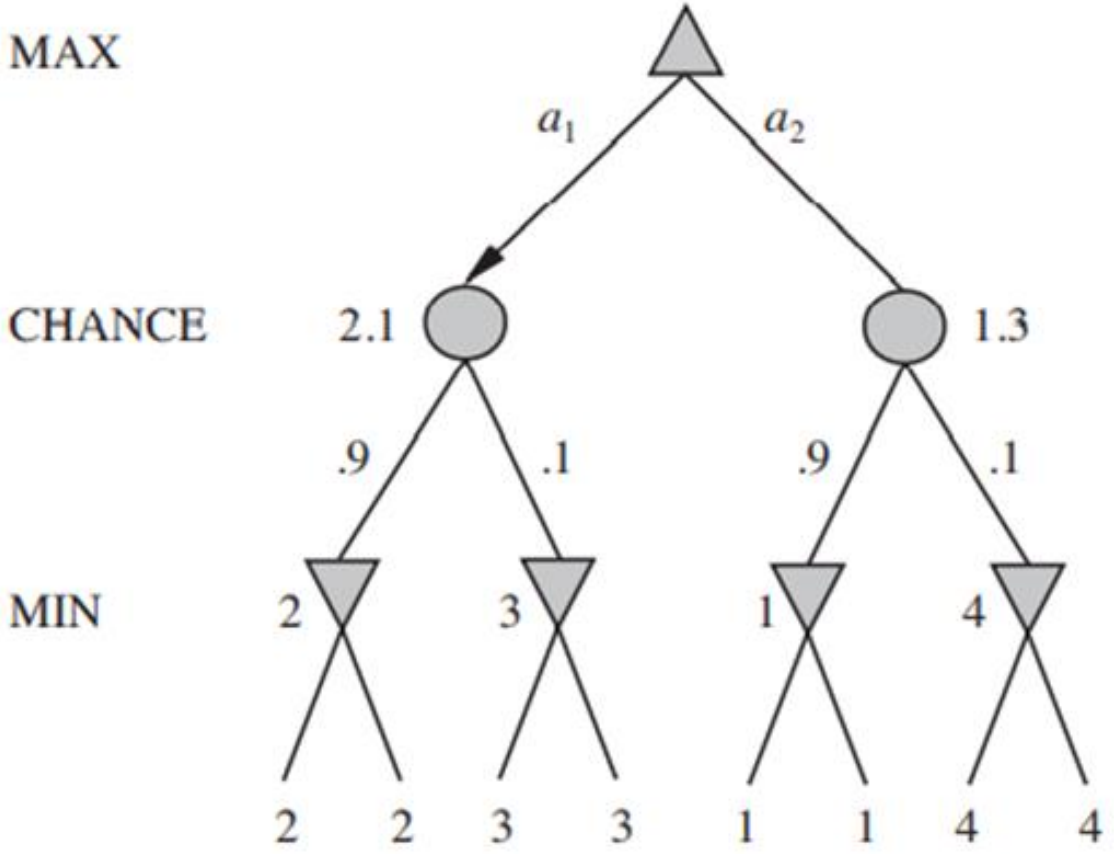
Holds the expected values that are computed as a sum of all outcomes weighted by their probability (of dice roll)

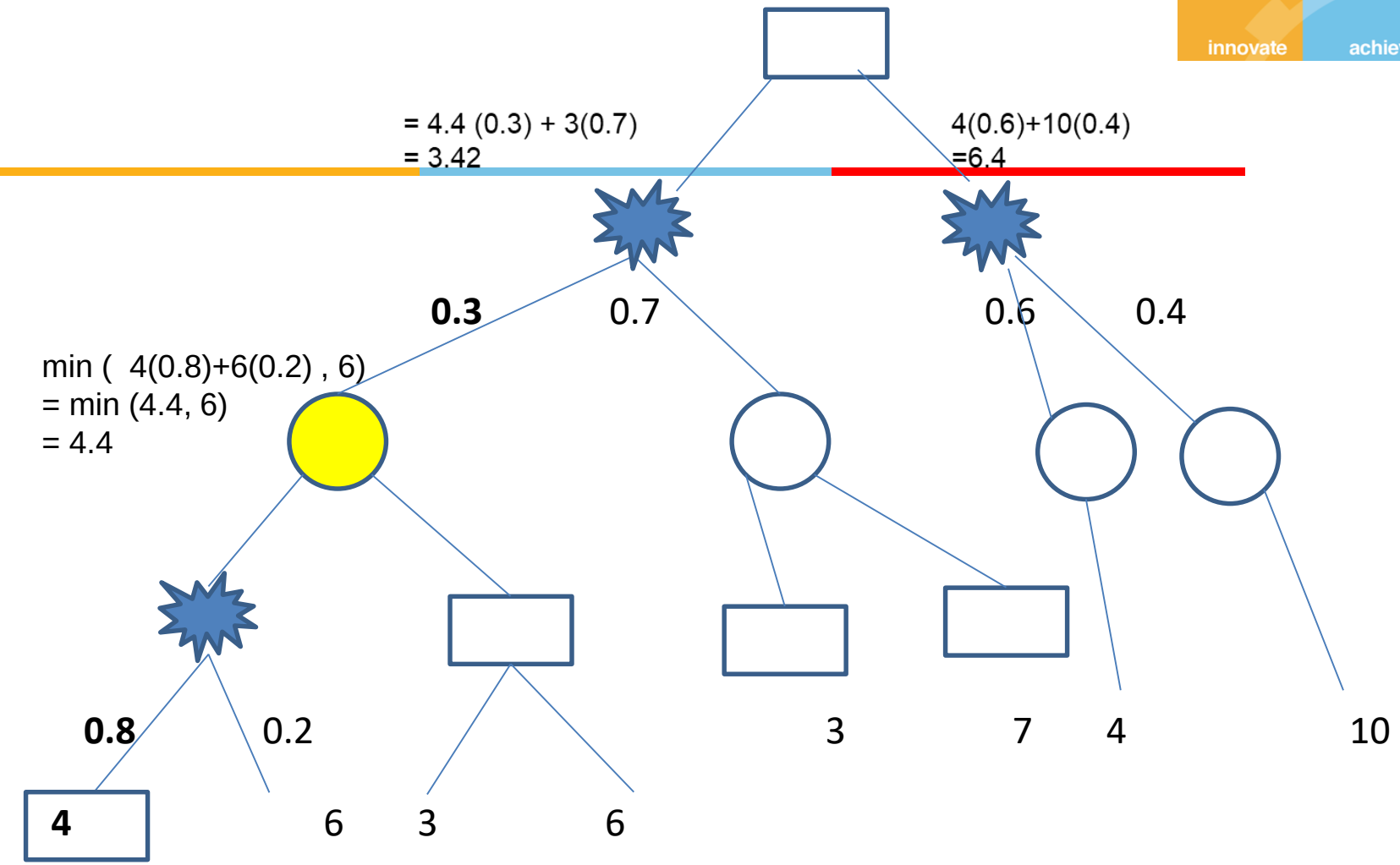


Expecti Mini Max Algorithm



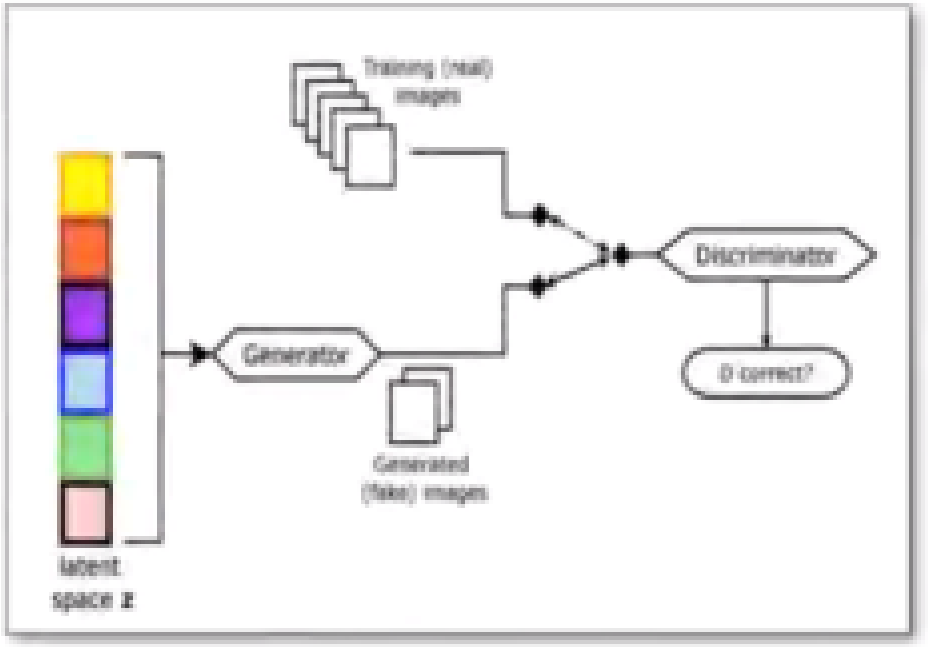
Expecti Mini Max Algorithm




= 4.4 (0.3) + 3(0.7)
= 3.42
4(0.6)+10(0.4)
=6.4
0.3
0.7
0.6
0.4
min (4(0.8)+6(0.2) , 6)
= min (4.4, 6)
= 4.4
0.8
0.2
4
6
3
6
3
7
4
10

Application of Games

Games in Image Processing

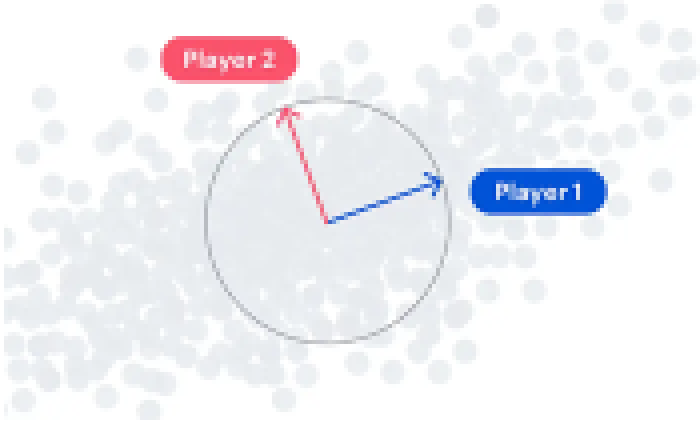


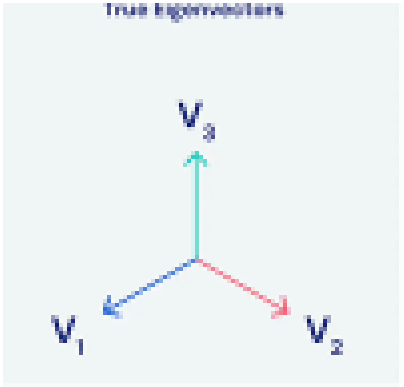
Source Credit:

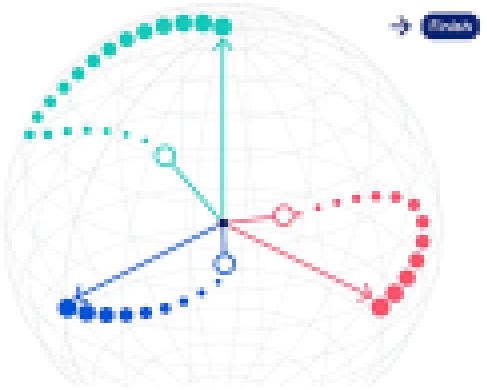
2019 - Analyzing and Improving the Image Quality of StyleGAN

Tero Karras, Samuli Laine, Miika Aittala, Janne Hellsten, Jaakko Lehtinen, Timo Aila

Games in Feature Engineering







Source Credit:

https://deepmind.com/blog/article/EigenGame

2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

Games in Feature Engineering

$$\mathbf{Utility}(v_i \mid v_{j<i}) = \mathbf{Var}(v_i) - \sum_{j<i} \mathbf{Align}(v_i, v_j)$$

Source Credit:

https://deepmind.com/blog/article/EigenGame

2021 - EigenGame: PCA as a Nash Equilibrium , Ian Gemp, Brian McWilliams, Claire Vernade, Thore Graepel

Required Reading: AIMA - Chapter # 4.1, #4.2, #5.1

Thank You for all your Attention

Note : Some of the slides are adopted from AIMA TB materials

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CROSSOVER OPERATORS IN GENETIC ALGORITHMS: A REVIEW

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Abstract

The performance of Genetic Algorithm (GA) depends on various operators. Crossover operator is one of them. Crossover operators are mainly classified as application dependent crossover operators and application independent crossover operators. Effect of crossover operators in GA is application as well as encoding dependent. This paper will help researchers in selecting appropriate crossover operator for better results. The paper contains description about classical standard crossover operators, binary crossover operators, and application dependant crossover operators. Each crossover operator has its own advantages and disadvantages under various circumstances. This paper reviews the crossover operators proposed and experimented by various researchers.

Keywords:

Evolutionary Algorithm, Genetic Algorithm, Crossover, Genetic Operators

1. INTRODUCTION

Genetic algorithm is a method of searching. It searches a result equal to or close to the answer of a given problem. New generation of solutions is created from solutions in previous generation. Basic strategy used in GA to create the best solutions/offspring is to crossover the parent genes. Various crossover techniques are built to get the optimum solution as early as possible in minimum generations. The selection of crossover operator has more impact on the performance of GA. The premature convergence [38] in GA can be avoided by selecting appropriate breeding operators. In this paper, the crossover operators are classified in three categories such as standard crossovers, binary crossovers and real/tree crossover s which are application dependant.

The Section 2 explains standard crossovers, which are application independent. Section 3 explains the binary crossovers with some modified crossovers to improve performance of GA. Section 4 explains the application dependant crossovers (real/tress). Section 5 discusses the findings of this review work.

2. STANDARD CROSSOVERS

2.1 1-POINT CROSSOVER

It is one of the simple crossover technique used for random GA applications. This crossover uses the single point fragmentation of the parents and then combines the parents at the crossover point to create the offspring or child.

1-Point crossover first selects two parents used for crossover and then randomly selects any crossover point p_i (i = 0 to n-1). Two offspring are created by combining the parents at crossover point. A simple example is shown below which performs one point crossover and creates two parents.

Parent 1: 1 0 1 0 | 1 0 0 1 0

Parent 2: 1 0 1 1 | 1 0 1 1 0

Offspring 1: 1 0 1 0 | 1 0 1 1 0

Offspring 2: 1 0 1 1 | 1 0 0 1 0

In above example, point between 4th and 5th gene is selected as crossover point.

2.2 K-POINT CROSSOVER

It uses the random crossover point to combine the parents same as per 1-Point crossover. To provide the great combination of parents it selects more than one crossover points to create the offspring or child [1].

K-Point Crossover first selects the two parents used for crossover and then randomly select K crossover points P_{1i} to $P_{k\text{-}1i}$ (i = 0 to n – 1). Two offspring are created by combining the parents at crossover point. A simple example is shown below which performs one point crossover and creates two parents.

Parent 1: 1 0 | 1 0 | 1 0 0 | 1 0

Parent 2: 1 1 | 0 0 | 1 0 1 | 1 0

Offspring 1: 1 0 | 0 0 | 1 0 0 | 1 0

Offspring 2: 1 1 | 1 0 | 1 0 1 | 1 0

In above example, the points between 2nd and 3rd, 4th and 5th and 7th and 8th gene are selected as crossover points.

2.3 SHUFFLE CROSSOVER

Shuffle Crossover helps in creation of offspring which have independent of crossover point in their parents. It uses the same 1-Point Crossover technique in addition to shuffle.

Shuffle Crossover selects the two parents for crossover. It firstly randomly shuffles the genes in the both parents but in the same way. Then it applies the 1-Point crossover technique by randomly selecting a point as crossover point and then combines both parents to create two offspring. After performing 1-point crossover the genes in offspring are then unshuffled in same way as they have been shuffled.

Select Shuffle Points

Parent 1: 1 1 1 0 1 0 0 1 0

Parent 2: 1 0 0 0 1 0 1 1 0

Shuffle genes as Shuffle Points

Parent 1: 0 1 0 1 1 0 1 1 0

Parent 2: 0 0 1 1 1 0 0 1 0

Select 1- Point Crossover Point

Parent 1: 0 1 0 1 | 1 0 1 1 0

Parent 2: 0 0 1 1 | 1 0 0 1 0

Perform 1-Point Crossover Point

Offspring 1: 0 1 0 1 | 1 0 0 1 0

Offspring 2: 0 0 1 1 | 1 0 1 1 0

Select unshuffled points same as shuffled points

Offspring 1: 0 1 0 1 1 0 0 1 0

Offspring 2: 0 0 1 1 1 0 1 1 0

Unshuffled the genes in Offspring

Offspring 1: 1 1 0 0 1 0 0 1 0

Offspring 2: 1 0 1 0 1 0 1 1 0

2.4 REDUCED SURROGATE CROSSOVER

Reduced Surrogate Crossover minimizes the unwanted crossover operations in case of the parents having same genes. In these cases the Reduced Surrogate Crossover first checks for the individual genes in the parents. It creates list of all possible crossover points where the genes of the both parents are different.

After performing this check, if no crossover point is there then no action is taken. But in case, if parents are differing in more than 1 gene then it keeps the list of all crossover points. It then randomly selects one crossover point from the list and performs 1-point crossover to create the offspring.

2.5 UNIFORM CROSSOVER

Uniform crossover provides the uniformity in combining the bits of both parents. It performs this operation of swapping bits in the parents to be included in the offspring by choosing a uniform random real number u (between 0 to 1).

Uniform crossover selects the two parents for crossover. It creates two child offspring of n genes selected from both of the parents uniformly. The random real number decides whether the first child select the i^{th} genes from first or second parent [1].

Parent 1: 1 1 1 0 1 0 0 1 0

Parent 2: 1 0 0 0 1 0 1 1 0

Offspring 1: 1 1 0 0 1 0 1 1 0

Offspring 2: 1 0 1 0 1 0 0 1 0

2.6 AVERAGE CROSSOVER (AX)

Average Crossover is the value based crossover technique. It uses two parents to perform crossover and creates only one offspring [1]. Average Crossover creates one offspring from taking average of the two parents. It selects two parents as X and Y and generate the child Z as follows: each gene in a child is taken by averaging genes from both parents.

Parent 1: 5 3 3 2 3 9 7 6 5

Parent 2: 5 4 7 6 5 2 6 1 3

Offspring 1: 5 3 5 4 4 5 6 3 4

2.7 DISCRETE CROSSOVER (DC)

Discrete Crossover uses the random real number to create one child from two parents.

Unlike the uniform crossover only one child is generated in Discrete Crossover. It selects two parents as X and Y and generate the child Z such that it select genes of both the parents uniformly. The random real number decides from which parent to take the genes for child. [1]

Parent 1: 1 1 1 0 1 0 0 1 0

Parent 2: 1 0 0 0 1 0 1 1 0

Offspring 1: 1 1 0 0 1 0 1 1 0

2.8 FLAT CROSSOVER (FC)

Flat Crossover uses the random real number to create one child from two parents.

Same as of Discrete Crossover it selects the genes from parent based on uniform random real number. But the selected random real number should be a subset of set having the minimum and maximum of the genes of the both parents. It selects two parents as X and Y and generate the child Z such that it selects random real number which is either min or max from genes in both parents and then assign this real number in child gene.

2.9 HEURISTIC CROSSOVER/INTERMEDIATE CROSSOVER (HC/IC)

Heuristic Crossover creates one child offspring from two parents. It uses the $\alpha\ \varepsilon < 0, 1 >$ assuming $x_i <= y_i$. For each gene in the child it select the uniform random real number α. And the child gene is calculated from above equation.

$$x_i\ (t+1) = x_i(t) + \alpha\ (y_i(t) - x_i(t))$$

Parameter α may be of constant value equal to 0.5 or may be selected by a draw from interval <0,1 (row: 5).

2.10 STATISTICS-BASED ADAPTIVE NON-UNIFORM CROSSOVER (SANUX)

It is based on the concepts of intrinsic attribute and extrinsic tendency of valuing allele for a gene locus. In optimal solution (encoded in binary string) of a given problem, for a gene locus if its allele is 1 it is called 1-intrinsic, if its allele is 0 it is called 0-intrinsic, otherwise if its allele either 0 or 1 it is called neutral. During the running of a GA, for a gene locus, if the frequency of 1's in its alleles over, the population tends to increase with time (generation), it called 1-inclined; if the frequency of 1's tends to decrease, it is called 0-inclined; otherwise it is called non-inclined. [13]

Usually and hopefully as the GA progresses, the gene loci which are 1-intrinsic will appear to be 1-inclined. SANUX makes use of this convergence information as feedback information to direct the crossover by adjusting the swapping probability of each locus.

Now during the evolution of the GA, after generation of new population the distribution of 1's $f_1(i, t)$ from each locus over the population is calculated. Then SANUX operation is performed. After applying SANUX, the mask is generated bit by bit by

flipping a coin biased to generate "1" with probability $p_s(i, t)$. Finally, the generated mask is used to guide the crossover in the same way it guides the traditional uniform crossover.

1's Frq. in loci:	0.4	0.2	0.6	0.9	0.9	0.2
Calculating:	|	|	|	|	|	|
Swapping prob:	0.4	0.2	0.4	0.1	0.1	0.2
biased flipping:	|	|	|	|	|	|
Created mask:	1	0	1	0	0	0
Applying mask:	|		|			
Parent P1:	0	1	0	1	1	1
Parent P2:	1	1	1	1	1	0
Swapping:	|		|			
Child C1:	1	1	1	1	1	1
Child C2:	0	1	0	1	1	0

SANUX

3. BINARY CROSSOVERS

3.1 RANDOM RESPECTFUL CROSSOVER (RRC)

It selects two parents for crossover and offspring is generated based on the similarity vector of the parents. It first creates similarity vector $S_{ab} = (S_{1ab},\ldots,S_{nab})$ such that if both genes of parents have the same values then the similarity vector contains the values of the parent else similarity vector contain null value for that gene [1].

After creation of similarity vector *S*, two children are created according to the values of the similarity vector. If similarity vector contains 1 then gene of both children is set to 1 and if it contains 9 then genes of both children are set to 0. Apart from this if similarity vector contains null value for any gene then the child gene is selected by taking a uniform random real number. If this number is < 0.5 then 1 is stored otherwise 0 is stored.

Parent 1: 1 1 1 0 1 0 0 1 0

Parent 2: 1 0 0 0 1 0 1 1 0

Offspring 1: 1 1 0 0 1 0 1 1 0

Offspring 1: 1 0 0 0 1 0 0 1 0

This algorithm duplicates genes of parents in an offspring at every position wherever they are identical.

3.2 MASKED CROSSOVER (MX)

The MX operator uses a mask vector to determine which bits of which parent are inherited by the offspring. The first step is the duplication of the bits of the parents. The bits of the first parent are copied to the first offspring and, accordingly, of the second parent to the second offspring. In the second step, the offspring exchange bits among each other at those positions where the mask vectors of the parent were equal to 1, indicated domination of that parent at that position and the mask vectors of the other parent were equal to 0.

The mask vectors are initiated in $P(0)$ randomly. During every iteration of GA, the mask vectors are inherited by each offspring from its parent. Then the mask vectors of the offspring as well as the parents undergo modification. The modification process is based on comparison of fitness of offspring and the parents. If good offspring are created, the masks of the parents do not need to be modified and the masks of the offspring may be very similar to those of the parents. In a situation where bad offspring were created the masks of the parents as well as of the offspring need to be modified.

3.3 1- BIT ADAPTATION CROSSOVER (1BX)

In the 1 BX method, the last bit of the solution vector is reserved for the code of one of the two of the applied crossover operators. Assuming "0" corresponds to Uniform Crossover (UX) operator and "1" corresponds to 2-Point Crossover (2-PX) operator, the choice of one of them is made according to the rule: if the last bit of the parents is off the same value then choose the operator indicated by this bit. Otherwise chooses the operator through the selection by a draw i.e. select the uniform random real number from 0 to 1. If this value is <0.5, then Uniform Crossover is performed otherwise 2-Point Crossover is used.

Application of the described crossover scheme combines the choice of the operator with the solution vector. Moreover, this choice is carried out separately for each parent pair; hence this scheme is called local adaptation. Global adaptation version has also presented, but as it was emphasized by the author, significantly worse results were obtained by its application.

3.4 MULTIVARIATE CROSSOVER (MC)

MC divides the whole parent string into the q substrings. The crossover is performed based on random value selected for each substring. If this value is $< P_c$ then crossover is performed other only parent genes are copied into child offspring. It performs the standard 1-Point Crossover for the substring when the condition is satisfied [1].

The most fundamental difference between the MC operator and other operators using variable-to-variable recombination is that the answer to the question "whether to crossover" is checked in the MC method separately for each substring. As for other operators, the answer to that question refers to parent vector as a whole.

3.5 HOMOLOGOUS CROSSOVER (HX)

The HX operator is based on the standard K-Point Crossover operator. Introduced modification relies on the fact that only strings of bits which are at least of a certain length or of an admissible degree of similarity are allowed to participate in crossover. Determination of the degree of similarity is based on the XOR operator.

This strategy is aimed at transferring strings with specified parameters to the next generation. In HX, the value of *o* and *r* is determined a priori as constant or dynamically changed in the GA run.

3.6 COUNT PRESERVING CROSSOVER (CPC)

The CPC operator carries out its task by assuming that the number of bits equal to "1" in every chromosome in the initial population $P(0)$ is the same.

CPC may guarantee the preservation of the constant number of bits equal to "1" due to application of two lists noting the differences between the parents. List L_{up} includes positions of those bits, on which there are differences between the parents, but the first parent at a given position holds a bit equal to "1" and the second equal to "0". List L down similarly notes the positions of differences, but the first parent at a given position holds a bit equal to "0" and the second equal to "1". The offspring creation process which makes use of those lists is based on the exchange of bits between the offspring at those positions which, are indicated by subsequent element pairs from lists L_{up} and L_{down}.

Number of elements in L_{up} and in L_{down} is the same, which is a direct result of the assumption, that the number of bits equal to "1" is constant for all chromosomes in P(0).

3.7 ELITIST CROSSOVER (EX)

In the standard genetic algorithm, the selection process is always preceded by the crossover process. In the EX method, both processes are integrated. During the first step of the entire population is randomly shuffled. Then from each successive pair of parental vectors, two new vectors are created by crossover. From a 'family' created, two best vectors are singled out and implemented as offspring to the next population.

Application of elitist selection in the traditional way that is on the level of the entire population may often be the reason for the premature convergence of the algorithm. An EX elitist selection applied on the "family" level eliminates this danger according to the authors.

3.8 SCANNING CROSSOVER (SCAN), UNIFORM SCANNING CROSSOVER (U-SCAN), OCCURRENCE BASED SCANNING CROSSOVER (OB-SCAN) FITNESS BASED SCANNING CROSSOVER (FB-SCAN)

Depending on the heuristics applied to scanning crossover, three variations are: uniform scanning crossover (U-Scan), occurrence based scanning crossover (OB-Scan), and fitness based scanning crossover (FB-Scan).

Increasing the number of parents in OB-Scan leads to a higher probability for the major alleles in the population to exist in every offspring, which will cause the population to concentrate on a certain region and thus lose the diversity rapidly [18], [19]. This situation is called premature convergence, a result of unbalanced exploitation and exploration.

3.9 SELF-ADAPTIVE SIMULATED BINARY CROSSOVER (SBX)

A self-adaptive procedure for updating the distribution index used in the simulated binary crossover or SBX operator which is a commonly-used real-parameter recombination operator. This crossover is also good for multi-objective optimization problem.

3.10 OTHER BINARY CROSSOVER

Some binary crossover is proposed by researchers are - Circ1e-ring crossover, Sufficient Exchanging crossover [12], Adaptive crossovers [12], Diagonal crossover [21, 22], Best combinatorial crossover (BCX) and Hybridization crossover (HX) [14].

4. APPLICATION DEPEDANT CROSSOVERS (REAL/TREE)

4.1 CROSSOVER FOR TSP PROBLEMS

4.1.1 Order Based Crossover (OBX):

The order based crossover operator selects at random several positions in one of the parent tours, and the order of the cities in the selected positions of this parent is imposed on the other parent to produce one child. The other child is generated in an analogous manner for the other parent [1].

4.1.2 Modified Order Crossover (MOC):

A randomly chosen crossover point divides the parent strings in left and right substrings. The right substrings of the parent s1 and s2 are selected. After selection of cities the process is the same as the order crossover. Only difference is that instead of selecting random several positions in a parent tour all the positions to the right of the randomly chosen crossover point are selected [1].

For example with the following parents and crossover point

s1 = (1 2 3 4 | 6 9 8 5 7) and

s2 = (2 1 9 8 |5 6 3 7 4)

After position selection

s1 = (1 2 * * * 9 8 * *) and

s2 = (2 1 * * * * 3 * 4)

Now obtain the generated pair of children as

b1 = (1 2 5 6 3 9 8 7 4) and

b2 = (2 1 6 9 8 5 3 7 4)

Clearly this method allows only the generation of valid strings.

4.1.3 Partially-Mapped Crossover (PMX):

It transmits ordering and values information from the parent strings to the offspring. A portion of one parent string is mapped onto a portion of the other parent string and the remaining information is exchanged. Consider, for example, the following two parents: (1 2 3 4 5 6 7 8) and (3 7 5 1 6 8 2 4). The PMX operator creates an offspring in the following way. It begins by selecting uniformly at random two cut points along the strings, which represent the parents. Suppose, for example, that the first cut point is selected between the third and the fourth string element, and the second one between the sixth and the seventh string element. Hence, (1 2 3 | 4 5 6 | 7 8) and (3 7 5 | 1 6 8 I 2 4). The substrings between the cut points are called the mapping sections. In our example, they define the mappings 4 <-> 1, 5 <-> 6, and 6 <-> 8. Now the mapping section of the first parent is copied into the second offspring, and the mapping section of the second parent is copied into the first offspring: offspring 1: (x x x | l 6 8 | x x) and offspring 2: (x x x | 4 5 6 | x x). Then offspring i (i = 1, 2) is filled up by copying the elements of the i^{th} parent. In case, a number is already present in the offspring it is replaced according to the mappings [2].

For example, the first element of offspring 1 would be a 1, like the first element of the first parent. However, there is already a 1 present in offspring 1. Hence, because of the mapping 1 <-> 4 choose the first element of offspring 1 to be a 4. The second, third and seventh elements of offspring 1 can be taken from the first parent. However, the last element of offspring 1 would be an 8, which is already present. Because of the mappings 8 <-> 6, and 6 <-> 5, it is chosen to be a 5. Hence, offspring 1: (4 2 3 | 1 6 8 | 7 5). Analogously, we find offspring 2: (3 7 8 | 4 5 6 | 2 1). The absolute positions of some elements of both parents are preserved.

4.1.4 Modified Partially-Mapped Crossover (MPMX):

Modified PMX (MPMX) crossover operator was proposed (independently) by Brown [39] in the late 80's. The MPMX operator initially partitions the parents' solution strings and the offspring strings into three sections (left, middle and right). These sections are randomly created through the selection of two random crossover points that will be used for both the parents and offspring for this instance of the crossover in the GA. Stage two provides the offspring with the middle section of its solution string. This is the donated middle section of parent 1. The third stage, is the insertion of elements into the left and right sections of the offspring. This is accomplished using parent 2 as the donator. Corresponding positions in the parents donate elements to the offspring, provided they have not already been donated by parent 1. The final stage is to complete the offspring using a random permutation of the elements not yet allocated to the offspring over the previous stages. The following example illustrates the:

Parent 1 - (0 8 | 4 5 6 7 | 1 2 3 9)

Parent 2 - (6 7 | 1 2 4 8 | 3 5 9 0)

Offspring stage1- (- - | - - - - | - - - -)

Offspring stage2- (- - | 4 5 6 7 | - - - -)

Offspring stage3- (- - | 4 5 6 7 | 3 - 9 0)

Offspring stage4- (8 1 4 5 6 7 3 2 9 0)

4.1.5 Cycle Crossover (CX) [3]:

It attempts to create an offspring from the parents where every position is occupied by a corresponding element from one of the parents. For example, consider again the parents (1 2 3 4 5 6 7 8) and (2 4 6 8 7 5 3 1). Now choose the first element of the offspring equal to either the first element of the first parent string or the first element of the second parent string. Hence, the first element of the offspring has to be a 1 or a 2. Suppose choose it to be 1, (1 * * * * * * *). Now consider the last element of the offspring. Since this element has to be chosen from one of the parents, it can only be an 8 or a 1. If a 1 were selected, the offspring would not represent a legal individual. Therefore, an 8 is chosen, (1 * * * * * * 8). It finds that the fourth and the second element of the offspring also have to be selected from the first parent, which results in (1 2 * 4 * * * 8). The positions of the elements chosen up to now are said to be a cycle. Now consider the third element of the offspring. This element it may choose from any of the parents. Suppose that we select it to be from parent 2. This implies that the fifth, sixth and seventh elements of the offspring also have to be chosen from the second parent, as they form another cycle. Thus, we find the following offspring: (1 2 6 4 7 5 3 8). The absolute positions, of on average half the elements of both parents are preserved.

4.1.6 Order Crossover Operator (0X1):

It constructs an offspring by choosing a substring of one parent and preserving the relative order of the elements of the other parent. For example, consider the following two parent strings: (1 2 3 4 5 6 7 8) and (2 4 6 8 7 5 3 l), and suppose that we select a first cut point between the second and the third bit and a second one between the fifth and the sixth bit. Hence, (1 2 | 3 4 5 | 6 7 8) and (2 4 | 6 8 7 | 5 3 1). The offspring are created in the following way. Firstly, the string segments between the cut point are copied into the offspring, which give (* * | 3 4 5 | * * *) and (* * | 6 8 7 | * * *). Next, starting from the second cut point of one parent, the rest of the elements are copied in the order in which they appear in the other parent, also starting from the second cut point and omitting the elements that are already present. When the end of the parent string is reached, we continue from its first position. In our example, this gives the following children: (8 7 | 3 4 5 | 1 2 6) and (4 5 | 6 8 7 | 1 2 3) [4].

4.1.7 Order-based Crossover (OX2):

OX2 was suggested in connection with schedule problems, is a modification of the OX1 operator. The OX2 operator selects at random several positions in a parent string, and the order of the elements in the selected positions of this parent is imposed on the other parent. For example, consider again the parents (1 2 3 4 5 6 7 8) and (2 4 6 8 7 5 3 I), and suppose that in the second parent in the second, third and sixth positions are selected. The elements in these positions are 4, 6 and 5 respectively. In the first parent, these elements are present at the fourth, fifth and sixth positions. Now the offspring are equal to parent 1 except in the fourth, fifth and sixth positions: (1 2 3 * * * 7 8). We add the missing elements to the offspring in the same order in which they appear in the second parent. This results in (1 2 3 4 6 5 7 8). Exchanging the role of the first parent and the second parent gives, using the same selected positions, (2 4 3 8 7 5 6 1) [5].

The position-based crossover operator (POS), which was also suggested in connection with schedule problems, is a second modification of the OX1 operator. It also starts with selecting a random set of positions in the parent strings. However, this operator imposes the position of the selected elements on the corresponding elements of the other parent. For example, consider the parents (1 2 3 4 5 6 7 8) and (2 4 6 8 7 5 3 l), and suppose that the second, third and sixth positions are selected. This leads to the following offspring: (1 4 6 2 3 5 7 8) and (4 2 3 8 7 6 5 1).

4.1.8 Voting Recombination Crossover operator (VR):

It can be seen as a P-sexual crossover operator, where p is a natural number greater than, or equal to, 2. It starts by defining a threshold, which is a natural number smaller than, or equal to p. Next, for every; $i \in \{l, 2, \ldots N\}$ the set of i^{th} elements of all the parents is considered. If in this set an element occurs at least the threshold number of times, it is copied into the offspring. For example, if we consider the parents (p = 4) (1 4 3 5 2 6), (1 2 4 3 5 6), (3 2 1 5 4 6), (1 2 3 4 5 6) and we define the threshold to be equal to 3 we find (1 2 x x x 6). The remaining positions of the offspring are filled with mutations. Hence, our example might result in (1 2 4 5 3 6) [6].

4.1.9 Maximal Preservation crossover (MPX):

The MPX operator was developed by Gorges-Schleuter and Mülhelenbein [42] in 1988 specifically for the TSP. It is closely related to the PMX crossover operator [48]. MPX operates by initially selecting a random substring (the TSP this is a subtler) from the first parent (called the donor). This subtour is usually defined as being a tour with string length less than or equal to the TSP problem size *n* divided by 2. A minimum subtour length is also set, typically at 10 elements (unless the TSP problem size is very small), as substrings that are very short are ineffective and substrings that are too large do not allow for meaningful variation. Selecting appropriate sized substrings provides a suitable means for parents to transmit significant loci information to the offspring. The second stage of MPX is to remove the elements currently in the offspring from the second parent. Then the remaining elements are inserted into the offspring, the first parent's substring having been placed at the start of the offspring and the remaining free elements of the offspring being filled by the clean parent 2 strings. This three stage operation of MPX is illustrated in following example:

Parent 1 - (1 4 3 5 2 6)

Parent 2 - (1 2 4 3 5 6)

Offspring (1 4 3 x x x)

Cleaned Parent 2 - (- 2 - - 5 6)

Offspring (1 4 3 2 5 6)

With regard to the MPX and its application to the TSP, although the MPX prevents invalid tour generation in the offspring, they are liable to be produced with few building blocks being inherited from both parents due to the cleaning of the second parent's string prior to completing the offspring strings.

4.1.10 Masked crossover (MkX):

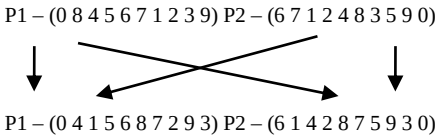
The Masked Crossover (MkX) technique was first proposed by Louis and Rawlins in 1991 [43] as a crossover operator which would efficiently operate in the combinatorial logic design problem area rather than as a combinatorial optimization technique. MkX [48] attempts to impart loci information from parent to offspring in a more effective manner than previous crossover methods. Louis and Rawlins state that MkX tries to preserve schemas identified by the masks and they identify this as one of their key goals [43]. The MkX operator assigns each parent a mask that biases crossover. Once these masks have been positioned then the operation is as following:

1. Copy Parent1 to Offspring1 and Parent2 to Offspring2
2. For (i from 1 to string-length)
   if Mask2i = 1 and Mask1i = 0
3. Copy the i^{th} bit from Parent2 to Offspring1
   if Mask1i = 1 and Mask2i = 0
4. Copy the i^{th} bit from Parent1 to Offspring2

The offspring of MkX also require masks, should they be selected to be parents in another generation. The masks are normally provided to the offspring by the parents. Typically the parent that is designated the dominant parent is called Parent1 the dominant parent with respect to Offspring1 as Offspring1 inherits Parent1's bits unless Parent2 feels strongly (Mask2i = 1) and Parent1 does not (Mask1i = 0). A number of mask rules are also defined by Louis and Rawlins. Two of which are used when the simple rule of assigning masks from dominant parent to offspring don't apply. Chan [47] notes that the MkX is an ineffective crossover operator for the TSP as it fails to preserve the ordering of the solutions. Validity of solution is problematic and (in conjunction with the selected mutation operator) typically involves a repair or penalty function.

4.1.11 Position crossover (PX):

The Position Crossover (PX) operator was developed by Syswerda in 1991 [5], [48]. PX was later evaluated by Barbulescu [44] where she examined and compared PX's operation to similar operators for scheduling problems. This crossover technique is closely related to OX and OX2 crossover techniques. PX operates by selecting several random locations along the parent strings. The elements are then inherited by the offspring in the order that they occur in the first parent (P). The remaining elements required to complete the offspring (O) are donated by the second parent (with the elements donated by the first parent omitted) in the order that they appear in the second parent. Each step of the operation is illustrated as follows:



4.1.12 Complete Subtour Exchange Crossover:

The complete subtour exchange crossover (CSEX) operator is designed to operate with the path representation. CSEX was proposed by Katayama and Narihisa [45] to be used specifically for permutation problems (such as the TSP). The philosophy behind CSEX [48] is to encourage the offspring to inherit as many good traits (substance) from the parents as possible. CSEX enumerates substance that have the same direction (or reversed direction) on two permutations as common substance.

Parent 1 - (0 1 2 3 4 5 6 7 8 9)

Parent 2 - (4 9 7 6 5 0 8 2 1 3)

Offspring 1- (0 2 1 3 4 5 6 7 8 9)

Offspring 2- (0 1 2 3 4 7 6 5 8 9)

Offspring 3- (0 2 1 3 4 7 6 5 8 9)

Offspring 4- (4 9 5 6 7 0 8 2 1 3)

Offspring 5- (4 9 7 6 5 0 8 1 2 3)

Offspring 6- (4 9 5 6 7 0 8 1 2 3)

In above see an example of CSEX in operation. The common subtours are 12 and 5 6 7 in parent 1, and 7 6 5 and 2 1 in parent 2. It should be noted that CSEX does not include the subtour 1 2 3 from parent 1 and 2 1 3 from parent 2 as they are not the same or symmetrical. CSEX by allowing only the same (or symmetrical) subtours can enumerate all the common subtours with $O(n)$ time. Having selected the common subtours, the offspring are produced by inverting the common subtours from the parent. In the example, parent 1 produces offspring 1, 2 and 3 by inverting a common subtour for each offspring. This is then repeated for parent 2 which produces offspring 4, 5 and 6. Once all the offspring are produced they are evaluated for fitness and the two fittest offspring survive to the next generation.

4.1.13 Heuristic crossover (HX):

It is worth noting that the previous crossover operators did not exploit the distances between the cities (i.e. the length of the edges). In fact, it is a characteristic of the genetic approach to avoid any heuristic information about a specific application domain, apart from the overall evaluation or fitness of each chromosome. This characteristic explains the robustness of the genetic search and its wide applicability [9] [10] [48].

However, some researchers departed from this line of thinking and introduced domain-dependent heuristics into the genetic search, to create "hybrid" genetic algorithms. They have sacrificed robustness over a wide class of problems, for better performance on a specific problem. The heuristic crossover HX is an example of this approach and can be described as follows:

1. Pick a random starting city at one of the two parents.
2. Compare the edges leaving the current city in both parents and select the shorter edge.
3. If the shorter parental edge introduces a cycle in the partial tour, then extend the tour with a random edge that does not introduce a cycle.
4. Repeat Steps 2 and 3 until all cities are included in the tour.

4.1.14 Edge Recombination crossover (ER):

Quite often, the alternate edge operator introduces many random edges in the offspring, particularly the last edges, when the choices for extending the tour are limited. Since the offspring must inherit as many edges as possible from the parents, the introduction of random edges should be minimized. The edge recombination operator reduces the myopic behavior of the alternate edge approach with a special data structure called the "edge map".

Basically, the edge map maintains the list of edges that are incident to each city in the parent tours, and lead to cities not yet included in the offspring. Hence, these edges are still available for extending the tour, and are said to be active. The strategy is to extend the tour by selecting the edge that leads to the city with the minimum number of active edges. In case of equality between two or more cities, one of these cities is selected at random. With this strategy, the approach is less likely to get trapped in a "dead end", namely, a city with no remaining active edges that require the selection of a random edge [8] [48].

For tours of 13564287 and 14236578 (path representation), the initial edge map is shown below:

City 1 has edges to: 3 4 7 8
City 2 has edges to: 3 4 8
City 3 has edges to: 1 2 5 6
City 4 has edges to: 1 2 6
City 5 has edges to: 3 6 7
City 6 has edges to: 3 4 5
City 7 has edges to: 1 5 8
City 8 has edges to: 1 2 7

Fig.1. Edge Map

Let us assume that city 1 is selected as the starting city. Accordingly, all edges incident to city 1 must be deleted from the initial edge map. From city 1, we can go to city 3, 4, 7 or 8. City 3 has three active edges, while cities 4, 7 and 8 have two active edges, as shown by the edge map (a) in Fig.1. Hence, a random choice is made between cities 4, 7 and 8. We assume that city 8 is selected. At 8, we can go to cities 2 and 7. As indicated in the edge map (b), city 2 has two active edges and city 7 only one, so the latter is selected. From 7, there is no choice, but to go to city 5. From this point, edge map (d) offers a choice between cities 3 and 6 with two active edges. Let us assume that city 6 is randomly selected. From city 6, we can go to cities 3 and 4, and edge map (e) indicates that both cities have one active edge. We assume that city 4 is randomly selected. Finally, from city 4 we can only go to city 2, and from city 2 we must go to the city 3.

Some modified edge recombination crossover are Edge Exchange Crossover (EXX) and Edge Assembly Crossover (EAX) [31].

4.1.15 Alternate Edges Crossover:

It is a good introduction to other edge-preserving operators. Here, a starting edge (*i*, *j*) is selected at random in one parent. Then, the tour is extended by selecting the edge (*j*, *k*) in the other parent. The tour is progressively extended in this way by alternatively selecting edges from the two parents. When an edge introduces a cycle, the new edge is selected at random (and is not inherited from the parents) [9] [48].

In following example, an offspring is generated from two parent chromosomes that encode the tours 13564287 and 14236578, respectively, using the adjacency representation. Here, edge (1, 4) is first selected in parent 2, and city 4 in position 1 of parent 2 is copied at the same position in the offspring. Then, the edges (4, 2) in parent 1, (2, 3) in parent 2, (3, 5) in parent 1 and (5, 7) in parent 2 are selected and inserted in the offspring. Then, edge (7, 1) is selected in parent 1. However, this edge introduces a cycle and a new edge incident to 7 and to a city not yet visited is selected at random. Let us assume that (7, 6) is chosen. Then, edge (6, 5) is selected in parent 2, but it also introduces a cycle. At this point, (6, 8) is the only selection that does not introduce a cycle. Finally, the tour is completed with edge (8, 1).

parent 1: 3 8 5 2 6 4 1 7

parent 2: 4 3 6 2 7 5 8 1

offspring: 4 3 5 2 7 8 6 1

The Alternate Edges Crossover

The final offspring encodes the tour 14235768, and all edges in the offspring are inherited from the parents, apart from the edges (7, 6) and (6, 8). In the above description, an implicit orientation of the parent tours is assumed. For symmetric problems, the two edges that are incident to a given city can be considered. In the above example, when we get to city 7 and select the next edge in parent 1, edges

(7, 1) and (7, 8) can both be considered. Since (7, 1) introduces a cycle, edge (7, 8) is selected. Finally, edges (8, 6) and (6, 1) complete the tour.

parent 1: 3 8 5 2 6 4 1 7

parent 2: 4 3 6 2 7 5 8 1

offspring: 4 3 5 2 7 1 8 6

It is alternate edges crossover.

4.1.16 Greedy Subtour Crossover:

New crossover operator named `Greedy Subtour Crossover (GSX)' that acquires the longest possible sequence of parents' subtours. Using GSX the solution can pop up from local minima more effectively than by using simulated annealing (SA) methods.

In the GSX, we use the path representation for a genetic coding. For example, chromosome g = (*D*, *H*, *B*, *A*, *C*, *F*, *G*, *E*) means that the salesperson visits towns D, H, B, A,.., E, successively, and returns to town D.

Suppose that chromosomes of parents are ga = (*D*, *H*, *B*, *A*, *C*, *F*, *G*, *E*) and gb = (*B*, *C*, *D*, *G*, *H*, *F*, *E*, *A*). First, choose one town at random. In this example, town C is chosen. Then, $x = 4$ and $y = 1$ because a_4 = C and b_1 = C respectively. Now the child g is (*C*).

Next, pick up towns from the parents alternately. Begin with a3 (town A) because x <- 4 <- 1 = 3, and next is b2 (town D) because y <- 1<- 1 = 2. The child becomes g = (*A*, *C*, *D*).

In the same way, add a_2 (town B), b_3 (town G), a_1 (town H), and the child becomes g = (*H*, *B*, *A*, *C*, *D*, *G*). Now the next town is b_4 = H and town H has already appeared in the child (remember the salesperson may not visit the same town twice), so we can't add any more towns from parent g_b. Therefore we add towns from parent g_a. The next town is $a_0 = D$, but *D* is already used. Thus we can't add towns from parent g_a, either. Then, we add the rest of the towns, i.e., E and F, to the child in the random order. Finally the child is g = (*H*, *B*, *A*, *C*, *D*, *G*, *F*, *E*) [11] [48].

4.1.17 Edge Assembly Crossover:

EAX [48] has two important features—preserving parents' edges using a novel technique and adding new edges by a greedy method, analogous to a minimal spanning tree. Several issues, including the selection mechanism and heuristic methods, which affect the performance of EAX have been considered.

4.2 CROSSOVER FOR OBJECT CLASSIFICATION PROBLEMS

- **Object classification problems:** Looseness control crossover (LCC) and Headless chicken crossover (HCC) [32]
- **Crossover for Sudoku problem:** Product Geometric Crossover (PMX) [33]
- **Crossover for grouping, graph, sequence and glude space applications:** Quotient Geometric Crossovers [34] and Merge Crossover (MX) [46].
- **Crossover for Graph Partitioning Problems:** Geometric Crossover (GX) & Geometric Crossover Labeling-independent (LI) GX Crossover and Landscape of Labeling-independent Crossover [36].
- **Crossover for Graph Colouring Problem (for Parallel GA):** Conflict elimination crossover (CEX), Greedy partition crossover (GPX), Union Independent set crossover (UISX) and Sum product crossover (SPPX) [37].
- **Multi-Parent Crossover/ Multi-Parent feature Crossover (MFX), Multicut crossover (MX) and Seed crossover (SX) [14],[15]and[16]:** The increase of parents brings about a more comprehensive survey for determining the offspring genes and leads to a stronger tendency towards exploitation or exploration or both [18], [19], [20].
- **Center of mass crossover (CMX):** These multi-parent crossovers can lead to better performance although the performance is problem-dependent [14, 15].
- **Unimodal normal distribution crossover (UNDX):** Multiple parents into unimodal normal distribution crossover (UNDX) to enhance the diversity of offspring. This multi-parent extension of UNDX exhibits its improvement in search ability on highly epistatic problems [24].
- **Simplex crossover (SPX):** SPX performs well with three or four parents for multimodal and epistatic problems [17].

4.3 CROSSOVER FOR SUDOKU PROBLEM

Knowledge-Based Nonuniform Crossover [26], Strong Context Preserving Crossover (SCPC) Weak Context Preserving Crossover (SCPC) [27], Hierarchical Crossover [28], Selective crossover [29], Rank & proximity Based Crossover (RPC) [30], Depth-Dependent Crossover (DDX) [35], Alternating-Position Crossover (AP) [7], Circle Ring Crossover [12] and Sufficient Exchanging [12].

5. CONCLUSION

Many crossover operators are present in GA that are used in applications. The encoding type used in GA is the major criteria for selecting the crossover. The global convergence and search space must be considered while selecting the crossover operators. Effect of crossover operators in GA is application as well as encoding dependent. Many researchers say that the value of crossover probability is in between 0.6 and 1.0 and it also depends on the type of crossover used. Increasing crossover probability increases the opportunity for recombination but also may disrupt in good combination. Depending on encoding, standard crossover can have high chance to produce illegal offspring (it is application dependent e.g. TSP). The application to be solved must consider for all the crossover operators available along with possible encoding methods to get good results.

Many new applications are solved by existing crossover operators by considering their effectiveness in related applications. Most of time new crossover operators use old crossover operators along with additional changes. To get the proper crossover operators for a new problem it is recommended that, to see similar types of problems solved using GA along with various crossover operators used. It is recommended that for solving any problem it is important to overview the search space with modality extremes. Continuity (nature of search space), study existing crossover operators and then select or create a new crossover (by mixing).

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