Probability Course

Lecture 4

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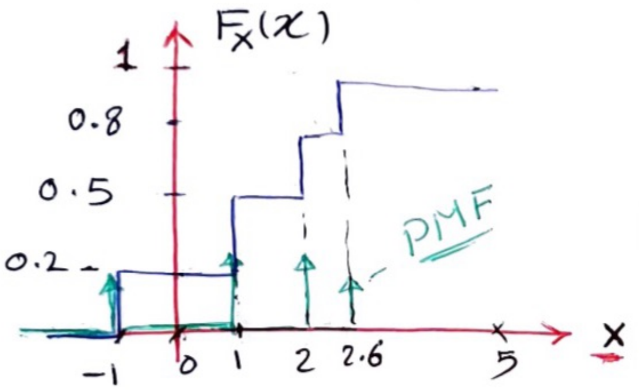
Session 4

we will discus :-

- continuous R.V.'s
- continuous probability distributions

Review:-

Review; Discrete Probability distributions



$P_X(0) = F_X(0^+) - F_X(0^-) = 0$

$P_X(1) = 0.5 - 0.2 = 0.3$

$P_X(5) = 0$

$P(2 < x \leq 5) = 1 - 0.8 = 0.2$

$P(0 < X) = 0.8$

$P(x \leq 0) = F_X(0) = 0.2$

$P(X < 0) = 0.2$

$P(x > 2) \neq P(X \geq 2)$

$P(X \geq 2) = P(X > 2) + P_X(2)$

$P_X(-1)$ = nonzero

$P_X(1)$ = nonzero

$P_X(2)$ = "

$P_X(2.6)$ = "

that's why it called discrete because specific points has values

So :-

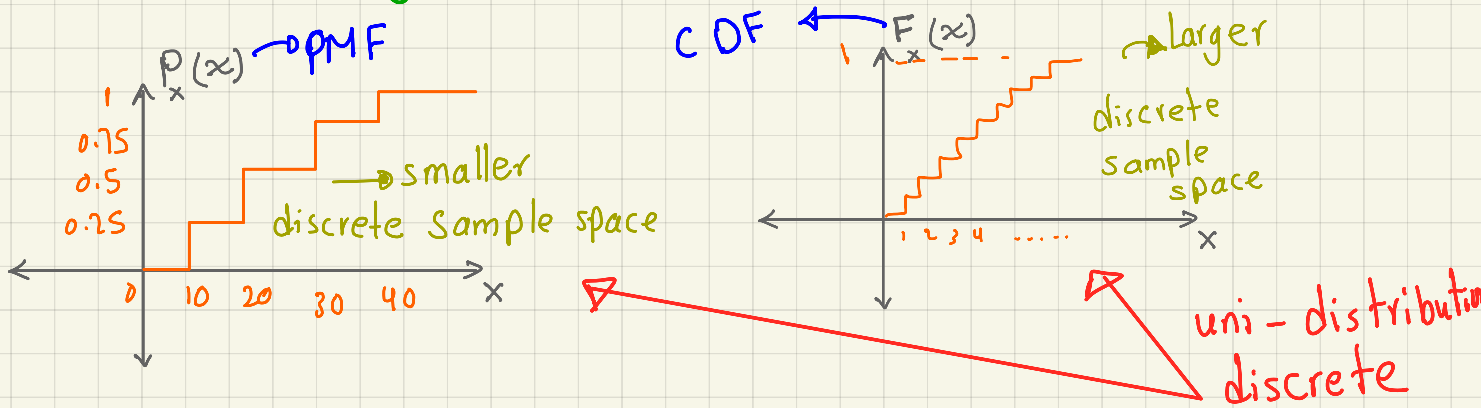
- Continuous Random value doesn't have value for a specific number

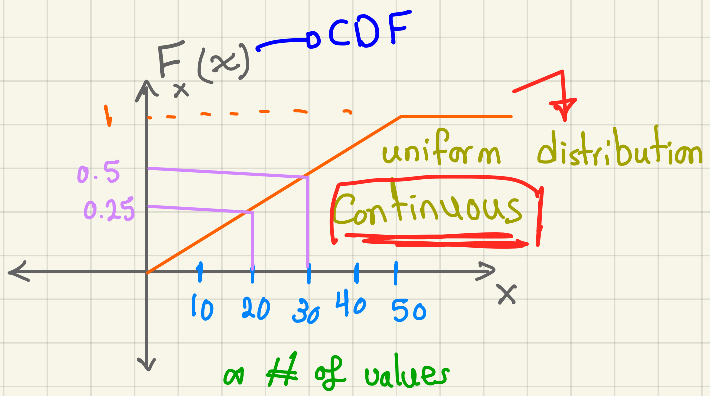
- probability for value = Zero (approximately because we have infinate # of possible outcomes)

* when we increase the sample space the probability of specific value dicreases.

* so in continuous samples you have infinte number of posible outcomes because it's a range

* so the probability maybe assumed zero (reflects how small is it) that's because it's like dividing by ∞ which is approximately zero so we calculate it for ranges only





$P_x(30) = 0$

$P_x(10 < x < 30) = 0.5 - 0$
$= 0.5$

$P_x(20 < x < 30)$
$= 0.5 - 0.25 = 0.25$

→ same as $(20 < x \leq 30)$

Some general concepts

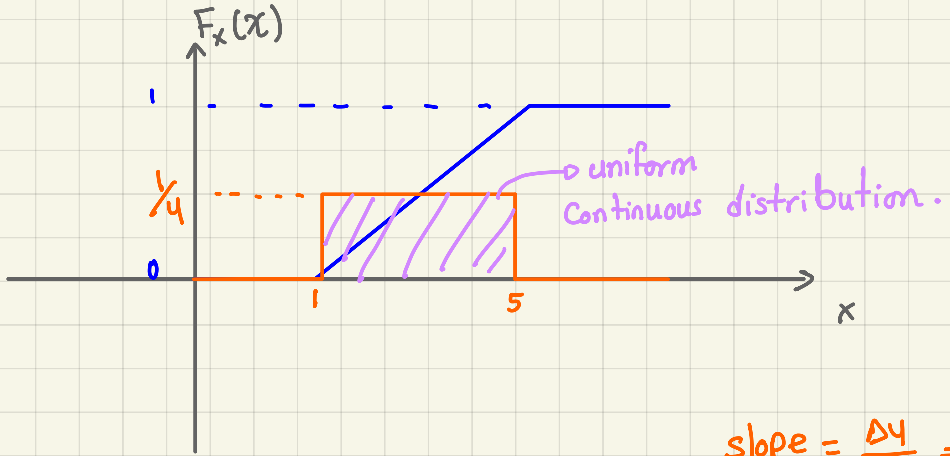
- التكامل هو جمع حاجات كتير infinate summission
- التفاضل هو طرح لـ range كبير

PDF ⇒ probability density function

PMF ⇒ probability mass function

CDF ⇒ comulative distribution function

"Probability density function" PDF



$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{1}{4}$

- called $f_X(x) = \frac{d}{dx} F_X(x)$

  or $p_X(x)$

∴ PDF is the derivative of CDF

$P(2 < X < 4) = F_X(4) - F_X(2) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$ → from CDF

$= \int_2^4 f_X(x)\,dx = F_X(x)\Big|_2^4 = \underbrace{F_X(4) - F_X(2)}$

from PDF (same as CDF)

area under the curve

$= \frac{1}{4} \times (4-2) = \frac{1}{2}$

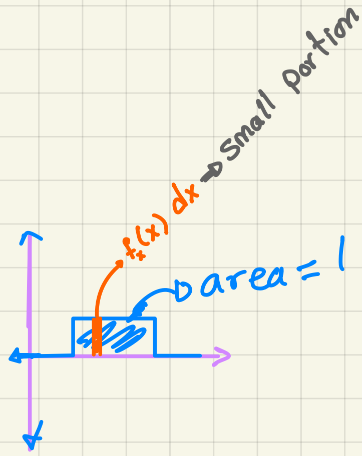
or $\int_2^4 f_X(x)\,dx = \int_2^4 \frac{1}{4}\,dx = \frac{1}{4}x\Big|_2^4 = 1 - \frac{1}{2} = \frac{1}{2}$ # same

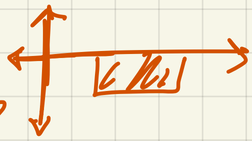
Important Note

PDF $\leftarrow$ $f_X(x) = \frac{d}{dx} \underbrace{F_X(x)}$ $\rightarrow$ CDF

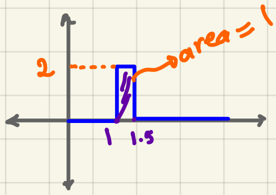
$$F_X(x) = \int_{-\infty}^{x} f_X(x)\, dx$$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1$$



✓ PDF can be > 1 but not (-ve)  ✗

not like CDF and PMF



because Range is too small

Exponential distribution

$$F_X(x) = (1-e^{-\lambda x})\,\underbrace{u(x)}_{\text{Heaviside unit step function}} = \begin{cases} 1-e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$

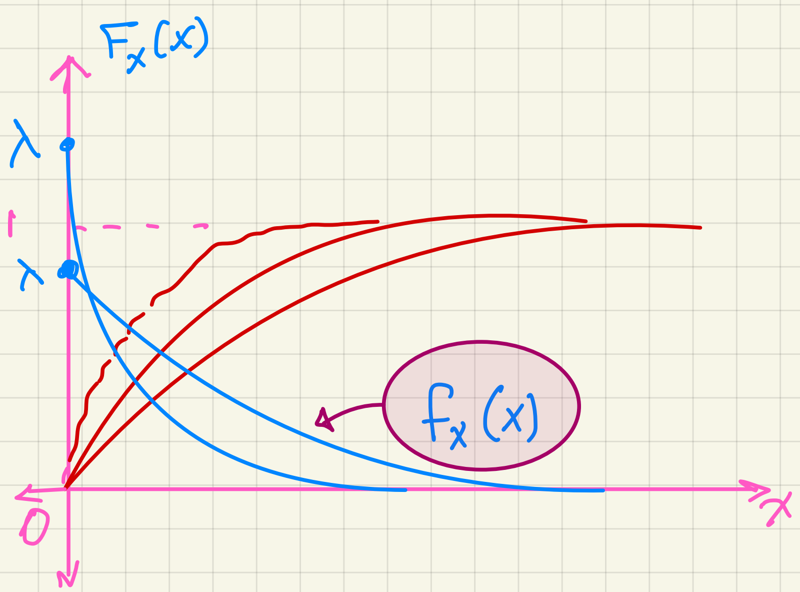
Heaviside unit step function

- if you multiplied any −ve valeus by it ⇒ the result will be 0
- if you multiplied any +ve " " " ⇒ " " " " 1

λ was in possion distribution

- poisson distribution is used to calculate Number of arrivals of some case that's why it's a discrete

- the difference in time/distance of the arrival of each 2 is exponential (continuous random variable) (with same lambda "λ" value)



$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & ; x \geq 0 \\ 0 & ; x < 0 \end{cases}$$ Single param. distro. "λ" only

λ can be more than 1 but the area under the $f_X(x)$ need to equal 1

poisson vs exponential Distributions

imagine that Senario :-

when I told you that there's a road full of cars

road 1	30 Car	15 Car	25 Car	10 Car
road 2	2 Car	2.5 Car	1.5 Car	2 Car

\# of cars ~ possion dist.

So $\lambda \cong 20$ Car / km

So $\lambda \equiv 0.5$ car / km

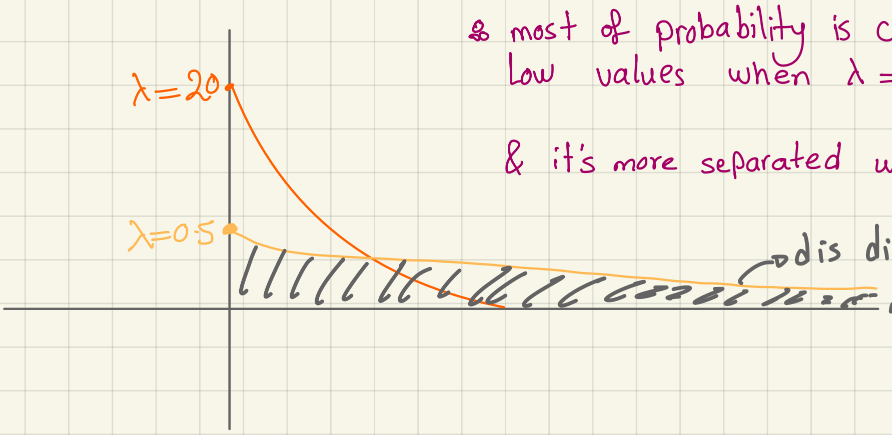
"each rectangle is 1km"

Side Note :-

$\mu = \frac{1}{\lambda}$

* most of probability is crowded in the low values when $\lambda = 20$

& it's more separated when $\lambda = 0.5$



dis distro. called "heavy tailed distribution"

⇒ the λ of 20 will decay quickely with low values and λ of 0.5 will stays more , why?

Let me ask a question :-

→ what is the propability to get 1 car in the distance $0.5 < x < 1.5$ km it will be very small value because you may find it or not because the road is too crowded "not normally distributed" you may find it but with low probability value

⇒ what about when $\lambda = 0.5$ "0.5 car / km" ⇒ car per 2km in avg. you have more possibility to find it so the probability is higher values and so on.

⇒ Note that the λ is average value & $f_x(x)$ is the density

تدريب على التكامل

get $F(x)$ from $f(x)$

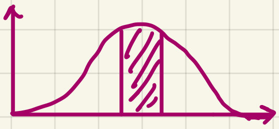
$$F_X(x) = \int_{-\infty}^{x} f_X(\alpha)\, d\alpha \quad \text{for } x<0 \implies F_X(x) = 0 \checkmark$$

$$F_X(x) = \int_{0}^{x} \lambda\, e^{-\lambda\alpha}\, d\alpha$$

$$= \cancel{\lambda}\, \frac{e^{-\lambda x}}{-\cancel{\lambda}} \Big|_{0}^{x}$$

$$= -e^{-\lambda x} - \left(-e^{-\lambda 0}\right) = 1 - e^{-\lambda x}$$

$f_X(x)$ is probability density function.



Gaussian "Normal" Distribution

"التوزيع الطبيعي"

Everything thing in life follows the Gaussian distribution (approximately)

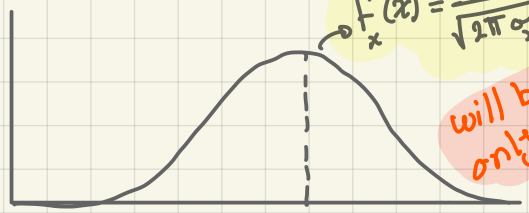
$F_x(x) = ?$

$p(x_1 < x < x_2) = ?$

it's hard to do $\int$ here so some tables are used to predict the probability

$$f_x(x) = \frac{1}{\sqrt{2\pi \sigma_x^2}} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}$$

will be zero only in $x = \infty$



μ_x → mean value

Side note:

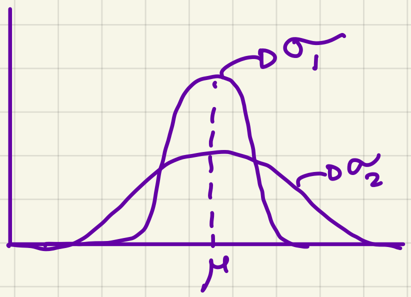
- distribution has many types that is the probability it self but the PDF is about the density and where it is more located which can be normally distributed or biased anywhere

Normally distributed

σ_x^2 : variance

σ_x : standard deviation

} represent the spread of data



$\sigma_2 > \sigma_1$

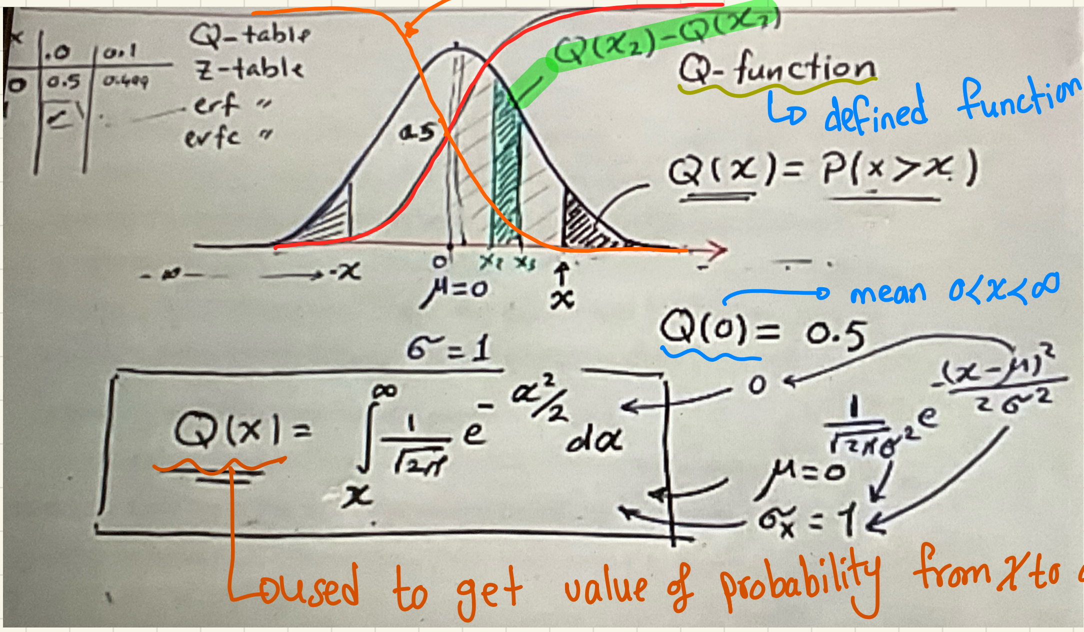
same μ different σ

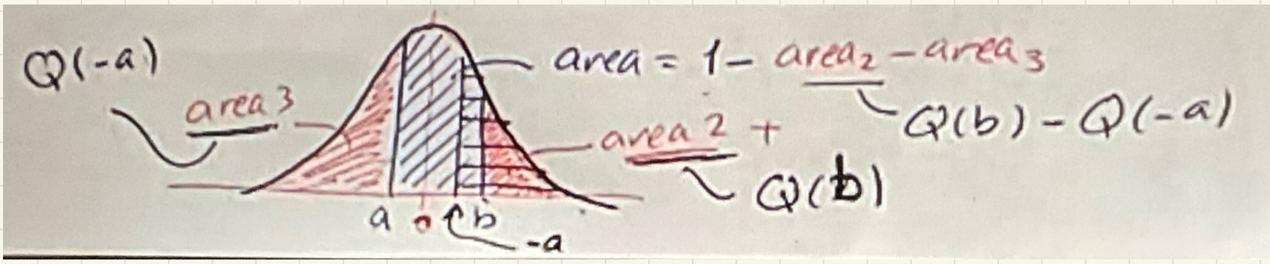
Note

this curve usually goes from $-\infty$ to ∞ but in a certain point the probability will be very small value

Normal distribution is known by 3 ways:- "you need μ & σ"

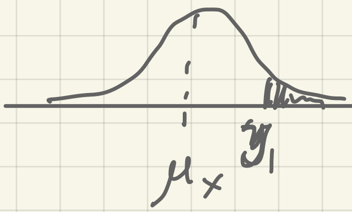
- → Coding functions
- → tables
- → rough estimations "later discussed"





The standarization:-

$$P(Y>y_1) = Q\left(\frac{y_1-\mu_y}{\sigma_y}\right)$$



$y_1-\mu_y \Rightarrow$ 0 is the center

$\frac{y_1-\mu_y}{\sigma_y} \Rightarrow$ bell shape

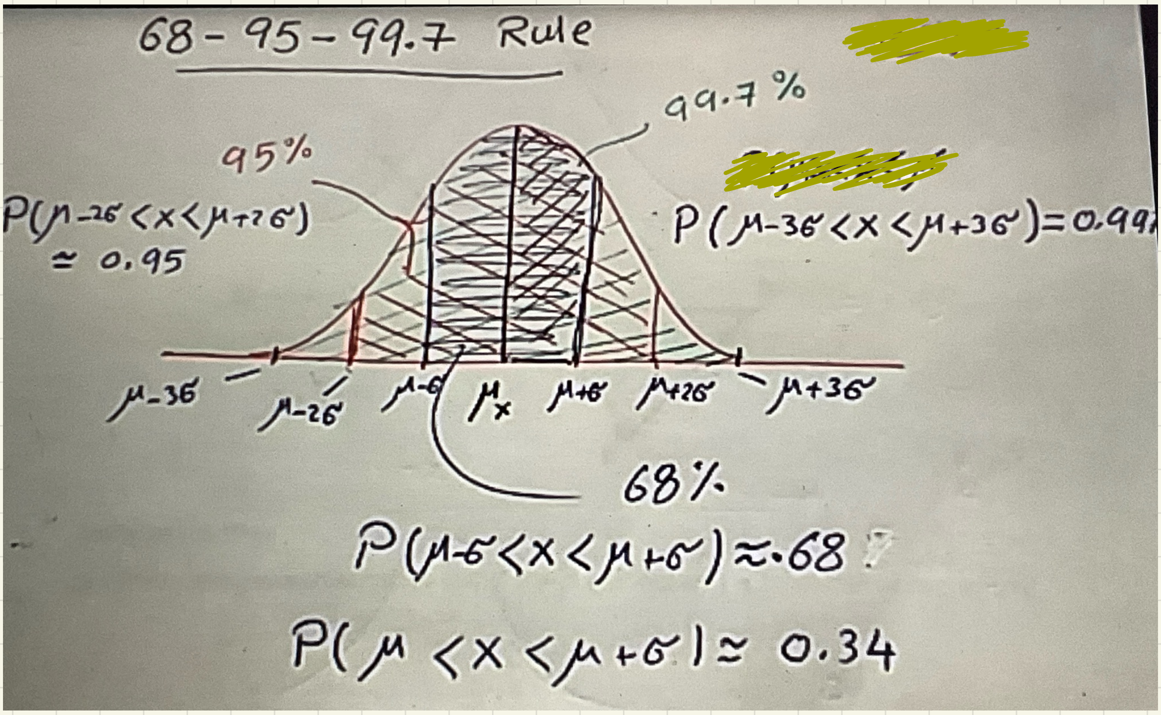
Search

Quality Control

_Six-sigma

Another simple method:-

68 – 95 – 99.7 Rule



with gaussian assumption.

Note:-

exponential distribution ⇒ one param defined "λ"

Normal distribution ⇒ 2 param. "start & end"

Gausian " ⇒ 2 param. "μ & σ"

so calculating mean value is easier in exponential distro.

μ & σ^2

Continuous

$$\mu_X \equiv \bar{X} \equiv \int_{-\infty}^{\infty} x\, f_X(x)\, dx$$

P.D.F.

$$\overline{x^2} = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$\sigma_X^2 = \int_{-\infty}^{\infty} (x-\bar{x})^2 f_X(x)\, dx$$

Disrete distributions

$$\mu_X \equiv \bar{X} = \sum_{i=1}^{n} x_i\, P_X(x_i)$$

$$\sigma_X^2 = \overline{x^2} - \bar{x}^2$$

$$\overline{x^2} = \sum_{i=1}^{n} x_i^2\, P_X(x_i)$$

$$\sigma_X^2 = \overline{(x_i - \bar{x})^2} = \sum_{i} (x_i - \bar{x})^2\, P_X(x)$$

why most of distributions follow
the gaussian distro.?
hy it's called Normal dist. ?

CLT "centeral limit theorem"

has alot of cases we will study only 2

CLT

Central Limit theorem

X is an R.V.

Y is an R.V.

indep.

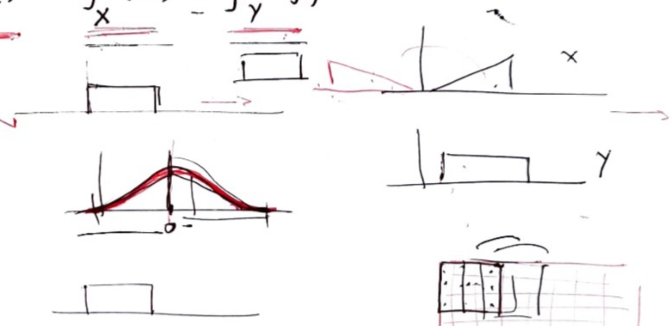
$Z = X + Y$

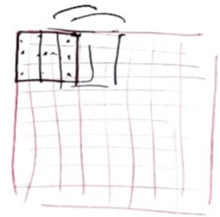
convolution!

CNN
convolutional

$f_Z(z) = f_X(x) * f_Y(y)$

1





CLT

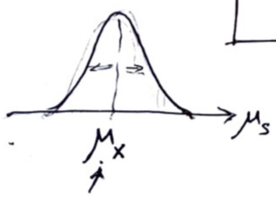
2 sample mean

sample #1

$\mu_{s_1} = \sim 168$

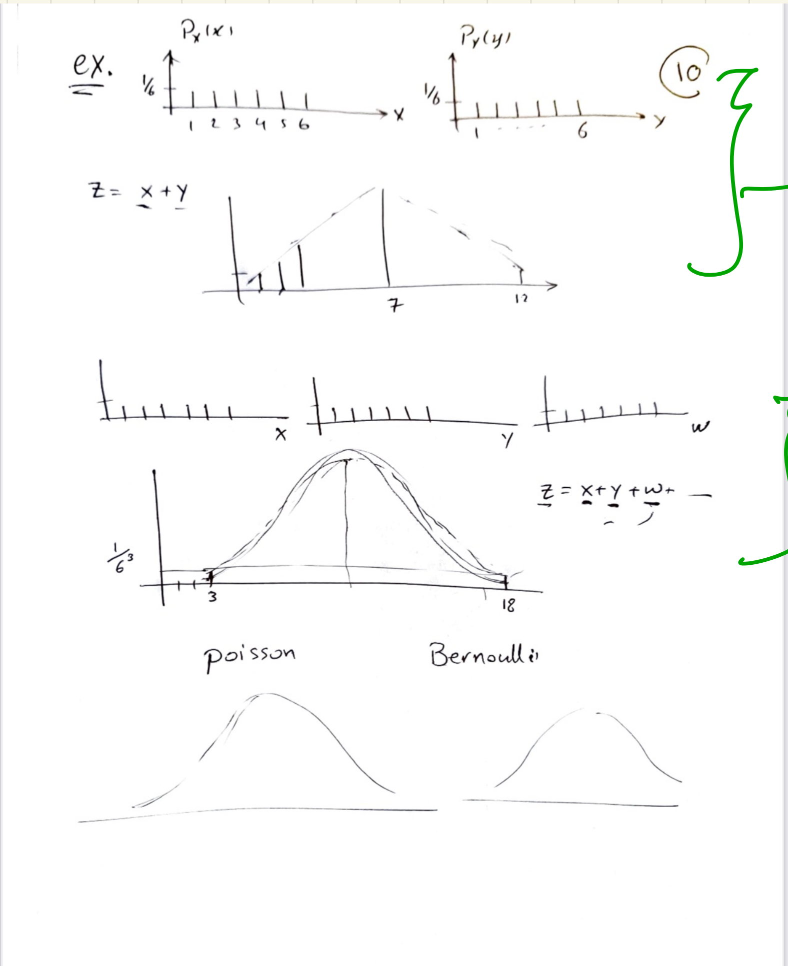
$\mu_{s_2} = 171$

CLT



Search for uniform & discrete distributions

for geometric distribution



throw 2

throw 3