After Study

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Find Yourself to Be what you want

Question 1 .... Constant ثابت

1) int t = 0

```
for(int i=1; i<=n; i++)                  -> n or n+1
   for(int j=0; j*j <= 4*n; j++)         -> j^2 <= 4n ; j <= 2√n
      for(int k=1; k*k <= 9*n; k++)      -> k^2 <= 9n ; k <= 3√n
         t++
```

$\Rightarrow n(2\sqrt{n})(3\sqrt{n}) = n^2$

or

$(n+1)(2\sqrt{n})(3\sqrt{n})$

$= (n+1)6n = 6n^2 + 6n = n^2$

$O(n^2)$

2) int z = 0;

```
int x = 0;
for(int i=1; i<=n; i=i*3)
{   z = z+5
    z++
    x = 2*x
}
```

مضاعفات

ثلاثة ثلاثة كأنه

في حالة فرق i=i*2

1

$1*3=3$

$3*3=9=3^2$

$9*3=27$

continue

3^k

will repeat end when $i=n$

$i = 3^k$

$3^k = n$

$k = \log_3 n \Rightarrow O(\log n)$

3) int x=0;

```
for(int i=1; i<=n; i=i*3)   -> log3 n from Perviouse Question
{
   if(i%2 != 0)              -> odd number     رقم دايما هيكون فردي كدا ممكن
      for(int j=0; j<i; j++)   -> n           مضاعفات ثلاثة    هتدخل جوا Condation
         x++;
}
```

$O(\cancel{n\log n})$ → $O(n)$ عشان هو بيجمع ~~مش~~ كأن بعضه مش ضرب

EL-SHINAWY

- 1st loop i = 1
- j from 0 to 0
- x incremented by 1
- x become 1

- 2nd loop i = 3
- j from 0 to 2
- x incremented by 3
- x become 4

- 3rd loop i = 9
- " " 0 - 8
- x
- x

اذا بتلوف قيمة i بتلف كام لفة وبنقسم
نجمع قيم n كعدد لفات حلقة اللي هو n

دي بطلع Series في الاخر بتجيبي Formula

← Geometric Serie

---

4)
```
int fun (int n)
{ int count = 0;
    for (i = n; i > 0; i /= 2)           --> n/2
        for (int j = 0; j < i; j++)      --> n
            count += 1
return count
}
```

$O(n \log n)$

$n, \frac{n}{2}, \frac{n}{4}, \frac{n}{8}, \ldots$

$\frac{n}{2^k} < i$ condition

$n = 2^k$

$k = \log_2 n$

---

5)
```
int n, rev;
rev = 0;
while (n > 0)
{
    rev = rev * 10 + n % 10;
    n = n / 10      -->
}
```

شبه for loop دي بس
بيقسم هنا هيزود ولا
هيتقسم ولا
ايه هيحصل

$\frac{n}{2}$ شبه $\frac{n}{10}$

$\Rightarrow O(\log n)$ يعني $(\log_{10} n)$

6) int Pent(int n)
{
int i, j, k, P, q = 0;
for (i = 1; i < n; ++i) $\longrightarrow n$
{
P = 0;
for (j = n; j > 1; j = j/2) $\longrightarrow \frac{n}{2} \Rightarrow \log_2 n$
++P
for (k = 1; k < P; k = k*2) $\Rightarrow \log_2 n$
++q
}
return q;
}

$O(n \log_2 n \log_2 n)$

7) int i = 1, z = 0;
while (z < n * (n+1)/2) $\longrightarrow \frac{n(n+1)}{2}$
{ z += i;
i++;
}

natural number مجموع

z will stop at $\frac{i(i+1)}{2}$

$\frac{i(i+1)}{2} > \frac{n(n+1)}{2}$

$i(i+1) > n(n+1)$

$i^2 + i > n^2 + n$

$i^2 > n^2$

$i > n$

$O(n)$

Question 2

1) $T(n) = T(n-1) + 5 \rightarrow I$ $T(1) = 1$ and $n > 1$

$T(n-1) = T(n-2) + 5 \rightarrow II$

$T(n-2) = T(n-3) + 5 \rightarrow III$

Substitute (II) in (I)

$T(n) = [T(n-2) + 5] + 5$

$T(n) = T(n-2) + 10 \rightarrow (IV)$

Substitute (III) in (IV)

$T(n) = [T(n-3) + 5] + 10$

$T(n) = T(n-3) + 15$

$T(n) = T(n-3) + (5 \times 3)$

Continue To k

$T(n) = T(n-k) + 5k$

assume $\rightarrow n - k = 1 \rightarrow \boxed{k = n-1}$

$T(n) = T(n - (n-1)) + 5(n-1)$

$= T(1) + 5n - 5$

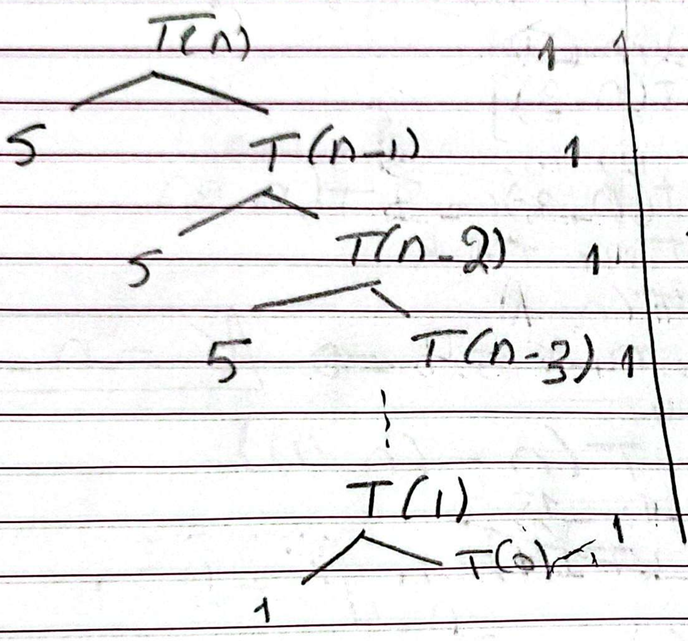
$= 1 + 5n - 5$

$= 5n - 4$

$O(n)$

$$T(n) = T(n-1) + 5$$

$$T(n) = \begin{cases} 1 & n=1 \\ T(n-1)+5 & n>1 \end{cases}$$



$f(n) = n-1$

$O(n)$

2) $T(n) = 3T(n-1) \rightarrow I$, $T(1) = 4$, and $n \geq 1$

$T(n-1) = 3T(n-2) \rightarrow II$

$T(n-2) = 3T(n-3) \rightarrow III$

Subsitute (II) in (I)

$T(n) = 3[3T(n-2)] \rightarrow IV$

$T(n) = 9\,T(n-2)$

Subitute (III) in (IV)

$T(n) = 9[3T(n-3)]$

$T(n) = 27T(n-3) = 3^3 T(n-3)$

continue to K

$T(n) = 3^k T(n-K)$

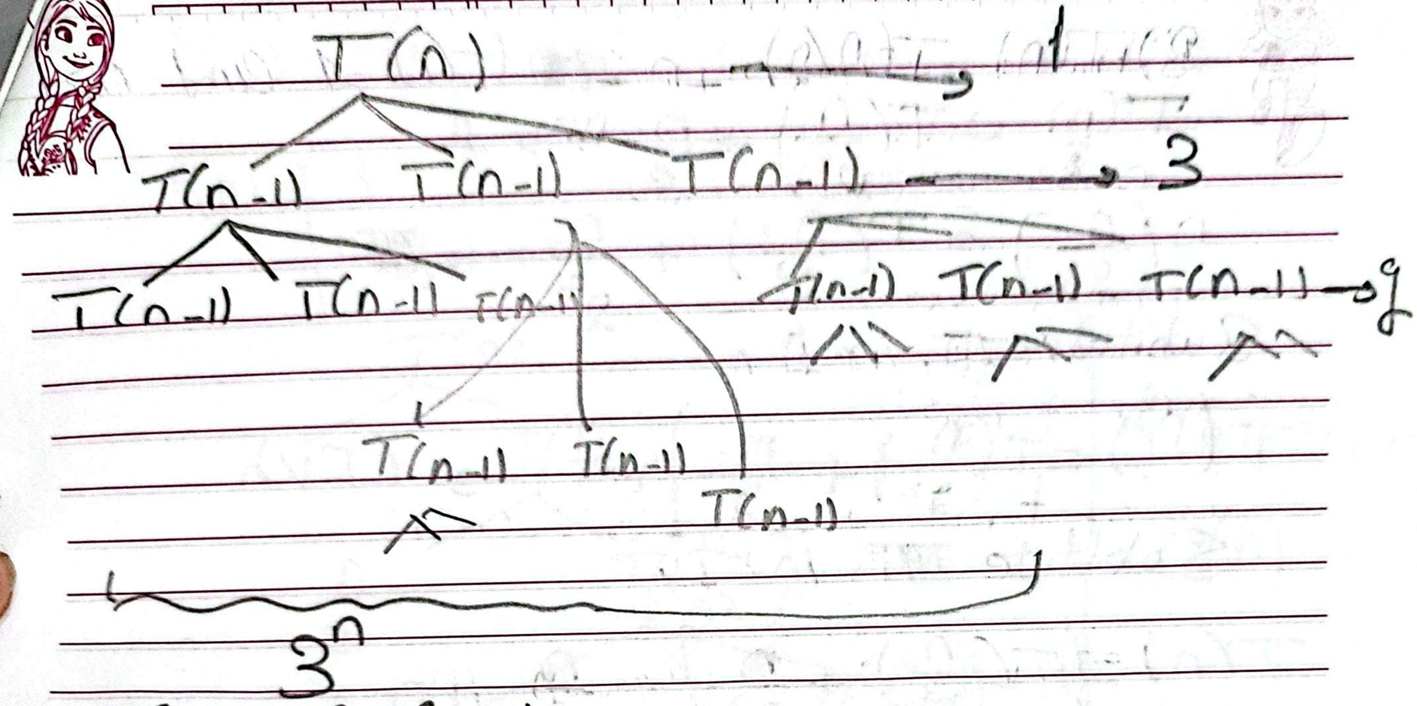
assume $n-K = 4 \Rightarrow \boxed{K = n-4}$

$T(n) = 3^{n-4}\, T(n-(n-4))$

$= 3^{n-4}\, \cancel{T(4)}\ 1$

$= 3^{n-4}(1) = 3^{n-4}$

$\Theta(3^n)$



$$T(n) = 3T(n-1)$$

$$T(n) = \begin{cases} 4 & n = 1 \\ 3T(n-1) & n > 1 \end{cases}$$

3) $T(n) = T(n/2) + n \rightarrow (I)$ $T(1) = 1$ and $n > 1$

$T(\frac{n}{2}) = T(\frac{n}{2^2}) + \frac{n}{2} \rightarrow II$

$T(\frac{n}{2^2}) = T(\frac{n}{2^3}) + \frac{n}{2^2} \rightarrow III$

Subitute II in I

$T(n) = \left[T(\frac{n}{2^2}) + \frac{n}{2}\right] + n \rightarrow (IV)$

Subitute III in IV

$T(n) = \left[T(\frac{n}{2^3}) + \frac{n}{2^2}\right] + \frac{n}{2} + n$

Continue for k

$T(n) = T(\frac{n}{2^k}) + \frac{n}{2^{k-1}} + \frac{n}{2^{k-2}} + n$

assume

$\frac{n}{2^k} = 1 \Rightarrow n = 2^k \Rightarrow \boxed{k = \log_2 n}$

$T(n) = \underbrace{T(\frac{n}{2^{\log n}})}_{T(1) \rightarrow 1} + \underbrace{n}_{\text{Common}} \left[\frac{1}{2^{\log_2 n - 1}} + \frac{1}{2^{\log_2 n - 2}} \cdots \frac{1}{2} + 1\right]$

$O(n)$

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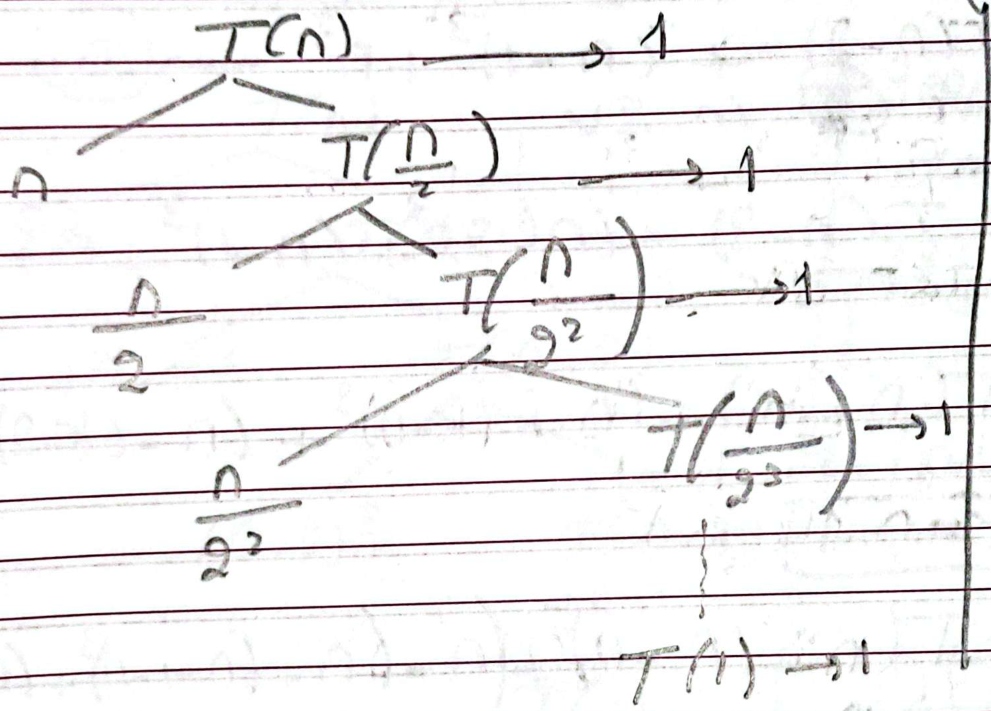
$f(n) = T(\frac{n}{2}) + n \quad T(1) = 1 \quad \text{and } n > 1$

$$f(n) = \begin{cases} 1 & n = 1 \\ T(\frac{n}{2}) + n & n > 1 \end{cases}$$



$f(n) = T(\frac{n}{2}) + n$

$O(n)$

4) $T(n) = T(n-1) + n^3$ ① $T(1)=1$ and $n>1$

$T(n-1) = T(n-2) + (n-1)^3 \longrightarrow$ II

$T(n-2) = T(n-3) + (n-2)^3 \longrightarrow$ III

Subitute II in I

$T(n) = T(n-2) + (n-1)^3 + n^3 \longrightarrow$ IV

Subitute III in IV

$T(n) = T(n-3) + (n-2)^3 + (n-1)^3 + n^3$

continous To K

$T(n) = T(n-K) + (n-(K-1))^3 + (n-(K-2))^3 + n^3$

assum $n-K=1$

$\boxed{K=n-1}$

$T(n) = T(n-(n-1)) + (n-(n-(n-1-1))^3 + (n-(n-1-2))^3 + n$

$T(n) = T(1) + 0^3 + 1^3 + 2^3 + 3^3 + \dots n^3$

$= \left(n\left(\frac{n+1}{2}\right)\right)^2$

$\Theta(n^4)$

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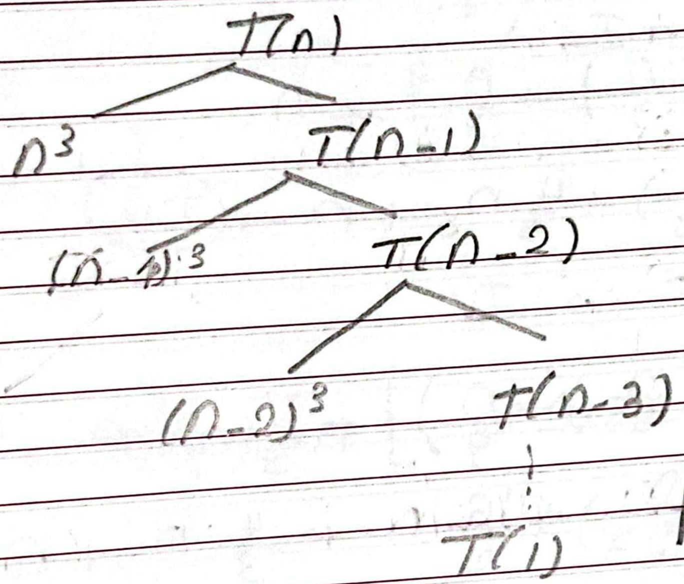
$T(n) = T(n-1) + n^3 \quad T(1) = 1 \quad n > 1$

$$T(n) = \begin{cases} 1 & n = 1 \\ T(n-1) + n^3 & n > 1 \end{cases}$$



Sequence
Sum of cubes

$1^3 + 2^3 + 3^3 + \dots n^3$

$\left(\frac{n(n+1)}{2}\right)^2$

$O(n^4)$

5) $T(n) = 4T(\frac{n}{3}) + n \rightarrow I$ $T(1) = 1$ and $n > 1$

$T(\frac{n}{3}) = 4T(\frac{n}{9}) + \frac{n}{3} \rightarrow II$

$T(\frac{n}{9}) = 4T(\frac{n}{27}) + \frac{n}{9} \rightarrow III$

Subsitue II in I

$T(n) = 4[4T(\frac{n}{9}) + \frac{n}{3}] + n$

$= 16T(\frac{n}{9}) + \frac{4}{3}n + n \rightarrow IV$

Subsitute III in IV

$T(n) = 16[4T(\frac{n}{27}) + \frac{n}{9})] + \frac{4}{3}n + n$

$= 64T(\frac{n}{27}) + \frac{16}{9}n + \frac{4}{3}n + n$

$= 4^3 T(\frac{n}{3^3}) + \frac{4^1}{3^2}n + \frac{2^2}{3^1}n + n$

Continue to K

$= 4^k T(\frac{n}{3^k}) + \frac{4^{k-2}}{3^{k-1}}n + \frac{2^{k-1}}{3^{k-2}}n + n$

assume $\frac{n}{3^k} = 1 \Rightarrow n = 3^k$

$\boxed{K = \log_3 n}$

$= 4^k T(\frac{n}{3^{\log_3 n}}) + \frac{4^{\log n - 2}}{3^{\log n - 1}}n + \frac{2^{\log n - 1}}{3^{\log n - 2}}n + n$

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$$4^{\log_3 n}\, T\left(\frac{n}{3^{\log_3 n}}\right) = 1$$

$$4^{\log_3 n} = 1$$

$$4^{\log_3 n} = \boxed{n^{\log_3 4}}$$

$$O\left(n^{\log_3 4}\right)$$

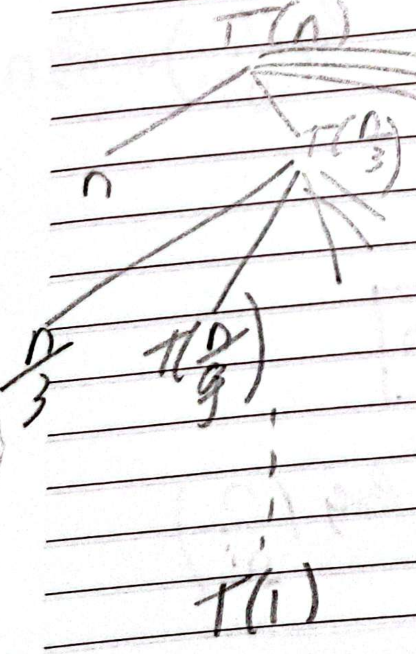
Note

$$a^{\log_b c} = c^{\log_b a}$$

Tree

$T(n) = 4T\left(\frac{n}{3}\right) + n \qquad T(1) = 1$ and $n > 1$

$$T(n) = \begin{cases} 1 & n = 1 \\ 4T\left(\frac{n}{3}\right) + n & n > 1 \end{cases}$$



$\longrightarrow n \rightarrow n$

$\longrightarrow 4\left(\frac{n}{3}\right) \rightarrow n$

$\longrightarrow 4\left(\frac{n}{9}\right) \rightarrow n$

- Number of Subproblems $= 4^K$
- each Subproblem Size $= \frac{n}{3^K}$
- we can say $= 4^K\left(\frac{n}{3^K}\right) \Rightarrow \infty(n)$ ← level
- $\frac{n}{3^K} = 1 \Rightarrow$ Stop Condition

$n = 3^K \Rightarrow K = \log_3 n \longrightarrow$ all levels

$$O\left(n^{\log_3 n}\right)$$

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c) $T(n) = 2T(n/2) + \log n \quad \text{(I)} \quad T(1) = 1, \text{ and } n > 1$

$T(\frac{n}{2}) = 2\,T(\frac{n}{4}) + \log(\frac{n}{2}) \longrightarrow \text{(II)}$

$T(\frac{n}{4}) = 2T(\frac{n}{8}) + \log(\frac{n}{4}) \quad \text{(III)}$

Subsitude II in I

$$T(n) = 2\left[2T(\frac{n}{4}) + \log(\frac{n}{2})\right] + \log n$$

$$= 4T(\frac{n}{4}) + 2\log(\frac{n}{2}) + \log(n) \rightarrow \text{IV}$$

Subsiute I in IV

$$T(n) = 4\left[2T(\frac{n}{8}) + \log(\frac{n}{4})\right] + 2\log(\frac{n}{2}) + \log n$$

$$= \overset{2^3}{8}\,T(\frac{n}{8}) + \overset{2^2}{4}\log(\frac{n}{4}) + 2\log(\frac{n}{2}) + \log n$$

Continous for k

$$= 2^k\,T(\frac{n}{2^k}) + 2^{k-1}\log(\frac{n}{2^{k-1}}) + 2^{k-2}\log(\frac{n}{2^{k-2}}) + \log n$$

Assume $\frac{n}{2^k} = 1$

$n = 2^k \qquad \boxed{k = \log_2 n}$

$$T(n) = 2^{\log_2 n}\,T(1) + \underbrace{\sum_{i=0}^{\log_2 n - 1} 2^i \log(\frac{n}{2^i})}_{n \text{ is dominant}}$$

$\Theta(n)$

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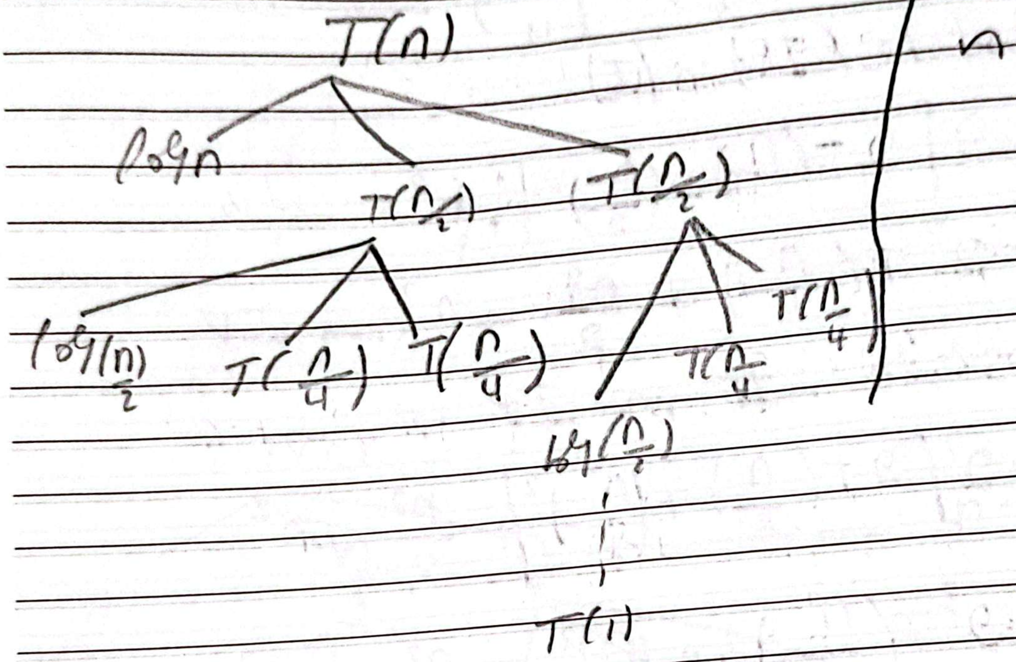
$T(n) = 2T(n/2) + \log n \quad T(1) = 1 \quad \text{and } n > 1$

$$T(n) = \begin{cases} 1 & n = 1 \\ 2T(n/2) + \log n & n > 1 \end{cases}$$



7) $T(n) = 2T(\frac{n}{2}) + n^2 \rightarrow I$ $T(1) = 1$ and $n > 1$

$T(\frac{n}{2}) = 2T(\frac{n}{4}) + (\frac{n}{2})^2 \rightarrow II$

$T(\frac{n}{4}) = 2T(\frac{n}{8}) + (\frac{n}{4})^2 \rightarrow III$

Subsitute (II) in (I)

$T(n) = 2\left[2T(\frac{n}{4}) + (\frac{n}{2})^2\right] + n^2$

$T(n) = 2^2 T(\frac{n}{4}) + \frac{n^2}{2} + n^2 \rightarrow IV$

Subsitute III in IV

$T(n) = 2^2\left[2T(\frac{n}{8}) + (\frac{n}{4})^2\right] + \frac{n^2}{2} + n^2$

$T(n) = 2^3 T(\frac{n}{2^3}) + \frac{(n)^2}{2^2} + \frac{n^2}{2} + n^2$

Continous To K

$T(n) = 2^K T(\frac{n}{2^K}) + \frac{n^2}{2^{K-1}} + \frac{n^2}{2^{K-2}} + n^2$

assume

$\frac{n}{2^K} = 1$

$1 = 2^K$

$\boxed{K = \log n}$

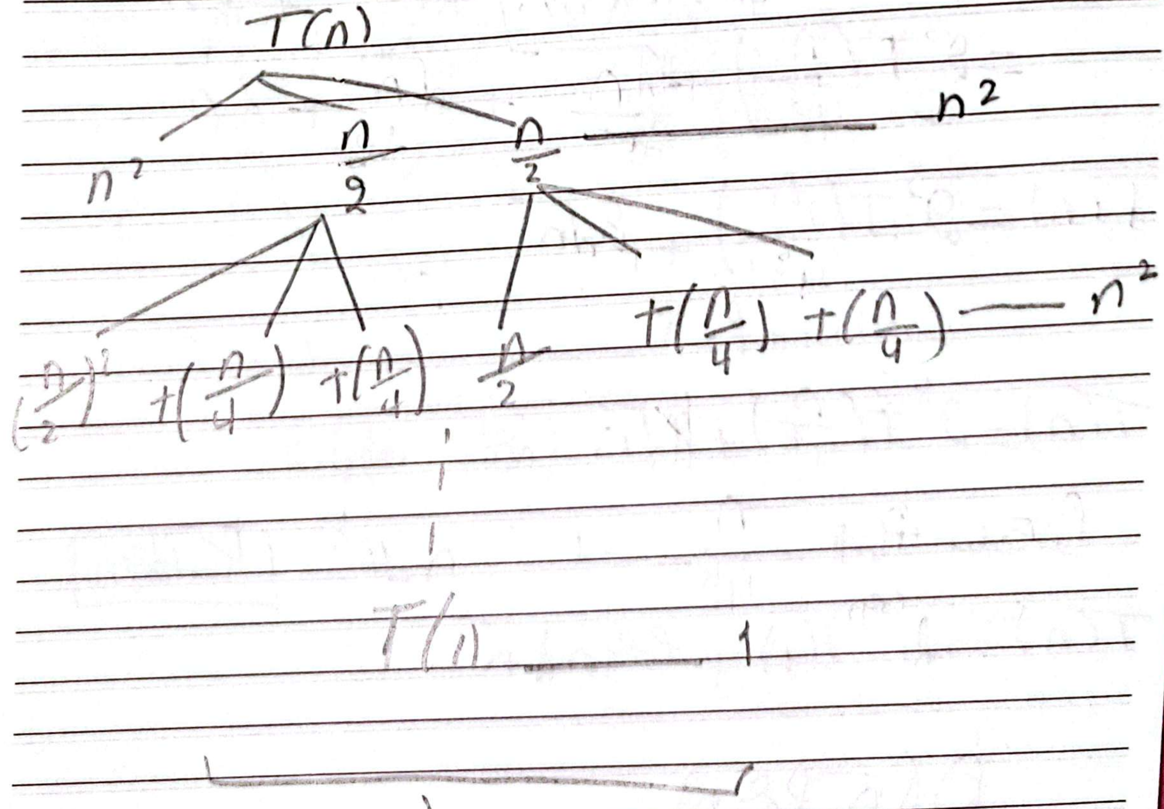
$$T(n) = 2^{\log n} T(1) + n^2 \left[\frac{1}{2^{\log n - 1}} + \frac{1}{2^{\log n - 2}} + 1 \right]$$

$$\Theta(n^2)$$

tree

$$f(n) = 2T\left(\frac{n}{2}\right) + n^2$$

$$T(n) = \begin{cases} 1 & 1 \\ 2T\left(\frac{n}{2}\right) + n^2, & n > 1 \end{cases}$$



$$O(n^2)$$

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Find Yourself to Be what you want

8) $T(n) = 2T(\frac{n}{4}) + \sqrt{n}$, $T(1) = 1$, and $n > 1$ $\rightarrow$ I

$T(\frac{n}{4}) = 2T(\frac{n}{4^2}) + \sqrt{\frac{n}{4}} \longrightarrow$ II

$T(\frac{n}{4^2}) = 2T(\frac{n}{4^3}) + \sqrt{\frac{n}{4^2}} \longrightarrow$ III

Substitue II in I

$T(n) = 2\left[2T(\frac{n}{4^2}) + \sqrt{\frac{n}{4}}\right] + \sqrt{n}$

$T(n) = 2^2 T(\frac{n}{4^2}) + \sqrt{n} + \sqrt{n} \longrightarrow$ IV

Substitue III in IV

$T(n) = 2^2\left[2T(\frac{n}{4^3}) + \sqrt{\frac{n}{4^2}}\right] + \sqrt{n} + \sqrt{n}$

$= 2^3 T(\frac{n}{4^3}) + \frac{4\sqrt{n}}{4} + \sqrt{n} + \sqrt{n}$

$T(n) = 2^3 T(\frac{n}{4^3}) + 3\sqrt{n}$

Continous for K

$T(n) = 2^k T(\frac{n}{4^k}) + K\sqrt{n}$

assume that $\frac{n}{4^k} = 1$ $\quad n = 4^k$ $\quad \boxed{K = \log n}$

$T(n) = 2^{\log n} t(1) + \log n \sqrt{n}$

$O(\sqrt{n} \log n)$

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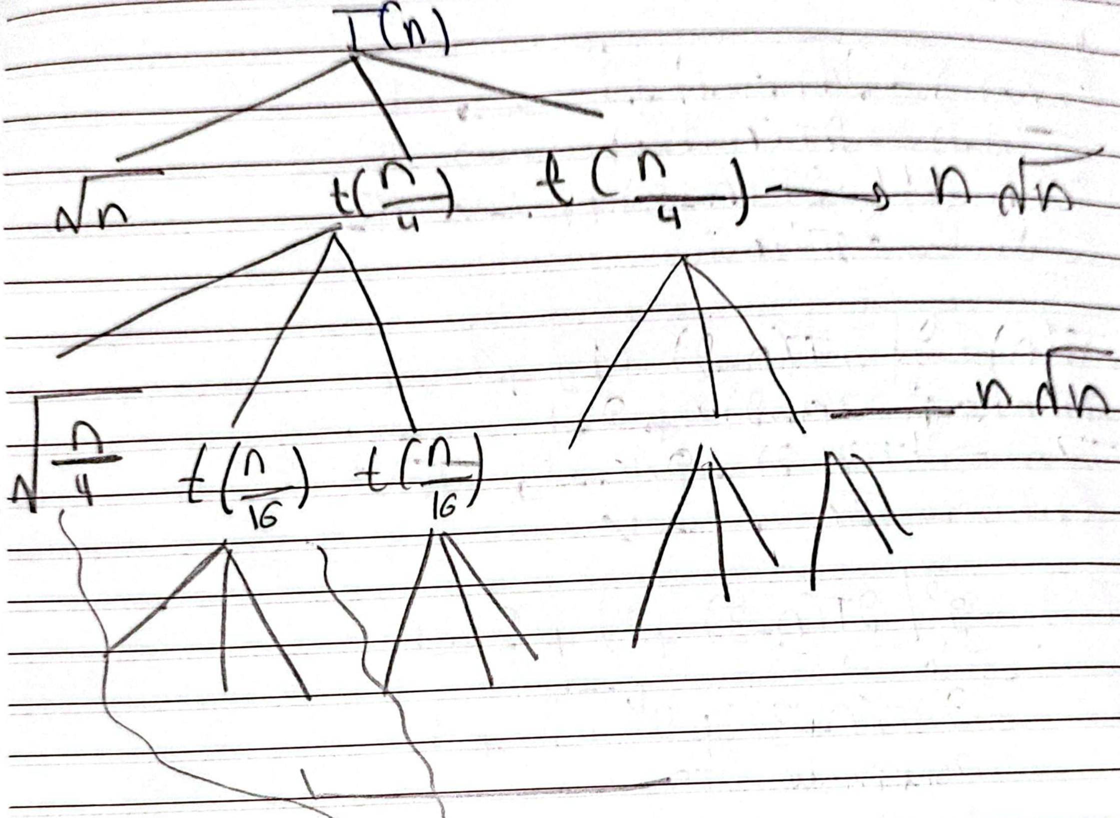
Pree

$T(n) = 9T\left(\frac{n}{4}\right) + \sqrt{n}$

$$T(n) = \begin{cases} 1 & n=1 \\ 9T\left(\frac{n}{4}\right) + \sqrt{n} & n>1 \end{cases}$$



$O(\sqrt{n}\ \log n)$

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Q3) 1) Public static int f(int n){
if (n==1){
return 1; $\longrightarrow 1$
} else {
return f(n-1) + f(n-1); $\longrightarrow 2T(n-1)$
}
}

$T(n) = 2T(n-1) + 1 \longrightarrow I$

$T(n-1) = 2T(n-2) + 1 \longrightarrow II$

$T(n-2) = 2T(n-3) + 1 \longrightarrow III$

Subtitue II in I

$T(n) = 2[2T(n-2) + 1] + 1$

$T(n) = 2^2 T(n-2) + 2 + 1$

$T(n) = 2^2 T(n-2) + 2 + 1 \longrightarrow IV$

Subtitue III in IV

$T(n) = 2^2[2T(n-3) + 1) + 2 + 1$

$= 2^3 T(n-3) + 2^2 + 2 + 1$

Continous To

$= 2^k T(n-k) + 2^{k-1} + 2^{k-2} + 1$

base

$n-k=1$ $\boxed{k=n-1}$

$= 2^{n-1} \cancel{T(n-n+1)} + 2^{n-1-1} + 2^{n-1-2} + 1$

$= 2^{n-1} (1) + 2^n + 2^{n-3} + 1$

$O(2^n)$

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2) Public static int f(int n)
{ f(n==1)
{ return 1; }
else
{ int y = f(n/2) + f(n/2) → $2T(n/2)$
for(int i=0; i<n; i++) → n
{
for (int j=0; j*j<n; j++) → $j^2 < n$, $j < \sqrt{n}$
{
y = y+1; → 1
}
}
return y;
}
}

$T(n) = 2T(\frac{n}{2}) + n\sqrt{n} + 1$ → I ← constant ثابت

$T(\frac{n}{2}) = 2T(\frac{n}{4}) + \frac{n}{2}\sqrt{\frac{n}{2}}$ → II

$T(\frac{n}{4}) = 2T(\frac{n}{8}) + \frac{n}{4}\sqrt{\frac{n}{4}}$ → III

Substitue II in I

$T(n) = 2\left[2T(\frac{n}{4}) + \frac{n}{2}\sqrt{\frac{n}{2}}\right] + n\sqrt{n}$

$= 2^2 T(\frac{n}{4}) + n\sqrt{\frac{n}{2}} + n\sqrt{n}$ → IV

Substitue III in IV

$T(n) = 2^2\left[2T(\frac{n}{8}) + \frac{n}{4}\sqrt{\frac{n}{4}}\right] + n\sqrt{\frac{n}{2}} + n\sqrt{n}$

$= 2^3 T(\frac{n}{2^3}) + n\sqrt{\frac{n}{2^2}} + n\sqrt{\frac{n}{2^1}} + n\sqrt{n}$

Continous To K

$= 2^k T(\frac{n}{2^k}) + n\sqrt{\frac{n}{2^{k-1}}}$ EL SHNAWY $\sqrt{\frac{n}{2^{k-2}}} + n\sqrt{n}$

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assume $\frac{n}{2^k} = 1 \rightarrow n = 2^k \Rightarrow \boxed{k = \log n}$

$$= 2^{\log n} T\left(\frac{n}{2^{\log n}}\right) + n\sqrt{\frac{n}{2^{\log n - 1}}} + n\sqrt{\frac{n}{2^{\log n - 2}}} + n\sqrt{n}$$

$O(n\sqrt{n})$

```
Public static int f(int n)
{ int result;
  if (n == 1)            → Base Case
  { result = 1; }
  else
  { result = 0;
   for (int i = 0; i < n; i++)     → n
   { result = result + f(n-1)       → T(n-1)
   }
  }
}
```

$T(n) = nT(n-1) + 1 \rightarrow I$

$T(n-1) = (n-1)T(n-2) + 1 \rightarrow II$

$T(n-2) = (n-2)T(n-3) + 1 \rightarrow III$

Sublitue II in I

$T(n) = n(n-1)T(n-2) + 1 + 1 \rightarrow IV$

Substu III in IV

$T(n) = n(n-1)(n-2)T(n-3) + \overbrace{1+1+1}^{3}$

Continous to k

$= n(n-(k-2))(n-(k-1)) \ldots T(n-k) + k$

assum $n - k = 1 \rightarrow k = n - 1$

$= n(n-(n-1-2))(n-(n-1-1)) \longrightarrow 1 + n - 1$

$(n-2)(n-3)$

Factorial

$O(n!)$

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To Be what you want

Master Theorem (Question 2)

3) $T(n) = T(n/2) + n$ $T(1) = 1$ and $n > 1$

$a = 1$, $b = 2$ $f(n) = n \Rightarrow O(n^K \log_n P)$

$\log_b a$ $K = 1$

$\log_2 1 = 0$ $P = 0$ no log factor

$\log_b a < 1 \leftarrow K$ $O(n^1 \log n^0)$

$P = 0$

$P \geq 0 \Rightarrow O(n^K) \Rightarrow O(n^1)$

Same as back substitution

5) $T(n) = 4T(n/3) + n$ $T(1) = 1$ and $n > 1$

$a = 4$ $b = 3$ $f(n) = n \Rightarrow O(n^K \log_n P)$

$\log_b a$ $K = 1$

$\log_3 4 = 1.26$ $P = 0$

$\log_b a > 1 \leftarrow K$

$O(n^{\log_b a}) = O(n^{\log_3 4})$

6) $f(n) = 2T(n/2) + \log n$ $T(1) = 1$ and $n>1$
$a=2$ $b=2$

$\log_b a$ $f(n) = \log n \Rightarrow O(n^k \log n^p)$

$\log_2 2 = 1$ $k=0$

$p=1$

$\log_b a > k$

$O(n^{\log_b a}) = O(n^{\cancel{\log_2 2}^{1}}) = O(n)$

7) $T(n) = 2T(n/2) + n^2$ $T(1) = 1$ and $n>1$
$a=2$
$b=2$ $f(n) = n^2 \Rightarrow O(n^2 \log n^0)$
$\log_b a$ $k=2$
$p=0$
$\log_2 2 = 1$

$\log_b a < k$

$p \geq 0 \rightarrow O(n^k) \rightarrow O(n^2)$

8) $T(n) = 2T(n/4) + \sqrt{n}$
$a=2$, $b=4$ $f(n) = \sqrt{n} \Rightarrow O(n^k \log n^p)$
$\log_b a$

$\log_4 2 = 0.5$ $k = \frac{1}{2}$ $\Rightarrow O(n^{\frac{1}{2}} \log n^0)$

$p=0$

$\log_b a = k$

$p > -1 \rightarrow O(n^k \log \log n) \Rightarrow$

 $O(n^{\frac{1}{2}} \log \log n)$