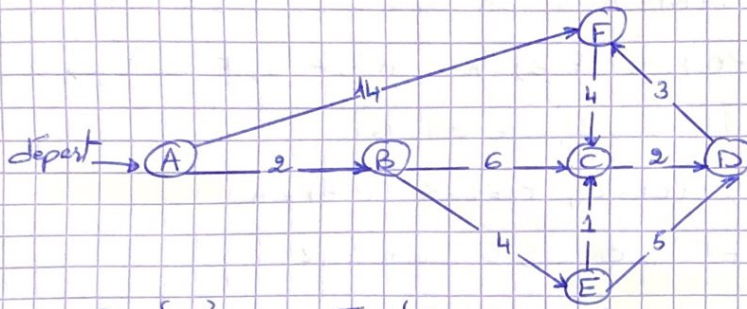


Exercice 1-1.

Saïdi Ismae



$$S = \{A\} \quad T = \{B, C, D, E, F\}$$

$$l = \begin{pmatrix} & A & B & C & D & E & F \\ A & 0 & 2 & \infty & \infty & \infty & 14 \end{pmatrix}$$

$$P = \begin{pmatrix} & A & B & C & D & E & F \\ A & - & A & N & N & N & A \end{pmatrix}$$

Etape 1: $j = B$.

$$P(C) = B \quad i = C \quad l(C) = \min(l(C), l(B) + l(B, C)) = \min(\infty, 2 + 6) = 8$$

$$P(E) = B \quad i = E \quad l(E) = \min(l(E), l(B) + l(B, E)) = \min(\infty, 2 + 4) = 6$$

$$S = \{A, B\} \quad T = \{C, D, E, F\}$$

$$l = \begin{pmatrix} & A & B & C & D & E & F \\ A & 0 & 2 & 8 & \infty & 6 & 14 \\ B & \infty & 0 & 6 & \infty & 4 & \infty \end{pmatrix} \quad P = \begin{pmatrix} & A & B & C & D & E & F \\ A & - & A & B & N & B & A \\ B & \infty & - & C & D & E & F \end{pmatrix}$$

Etape 2: $j = E$

$$P(C) = E \quad i = C \quad l(C) = \min(l(C), l(E) + l(E, C)) = \min(8, 6 + 1) = 7$$

$$P(D) = E \quad i = D \quad l(D) = \min(l(D), l(E) + l(E, D)) = \min(\infty, 6 + 5) = 11$$

$$S = \{A, B, E\} \quad T = \{C, D, F\}$$

$$l = \begin{pmatrix} & A & B & C & D & E & F \\ A & 0 & 2 & 7 & 11 & 6 & 14 \\ B & \infty & 0 & 6 & \infty & 4 & \infty \\ E & \infty & \infty & 0 & 5 & 0 & \infty \end{pmatrix}$$

Etape 3: $j = C$.

$$P = \begin{pmatrix} & A & B & C & D & E & F \\ A & - & A & E & E & B & A \\ B & \infty & - & C & D & E & F \\ E & \infty & \infty & - & C & D & F \end{pmatrix}$$

$$P(D) = C \quad i = D \quad l(D) = \min(l(D), l(C) + l(C, D)) = \min(11, 7 + 2) = 9$$

$$S = \{A, B, E, C\} \quad T = \{D, F\}$$

$$l = \begin{pmatrix} & A & B & C & D & E & F \\ A & 0 & 2 & 7 & 9 & 6 & 14 \\ B & \infty & 0 & 6 & \infty & 4 & \infty \\ E & \infty & \infty & 0 & 5 & 0 & \infty \\ C & \infty & \infty & \infty & 0 & \infty & \infty \end{pmatrix}$$

$$P = \begin{pmatrix} & A & B & C & D & E & F \\ A & - & A & E & C & B & A \\ B & \infty & - & C & D & E & F \\ E & \infty & \infty & - & C & D & F \\ C & \infty & \infty & \infty & - & D & F \end{pmatrix}$$

Etape 4: $j = D$

$$P(F) = D \quad i = F \quad l(F) = \min(l(F), l(D) + l(D, F)) = \min(14, 9 + 3) = 12$$

$$S = \{A, B, E, C, D\} \quad T = \{F\}$$

$$l = \begin{pmatrix} & A & B & C & D & E & F \\ A & 0 & 2 & 7 & 9 & 6 & 12 \\ B & \infty & 0 & 6 & \infty & 4 & \infty \\ E & \infty & \infty & 0 & 5 & 0 & \infty \\ C & \infty & \infty & \infty & 0 & \infty & \infty \\ D & \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix}$$

$$P = \begin{pmatrix} & A & B & C & D & E & F \\ A & - & A & E & C & B & D \\ B & \infty & - & C & D & E & F \\ E & \infty & \infty & - & C & D & F \\ C & \infty & \infty & \infty & - & D & F \\ D & \infty & \infty & \infty & \infty & - & F \end{pmatrix}$$

Etape 5: $j = F$ pas de voisins.

$$S = \{A, B, E, C, D, F\} \quad T = \emptyset$$

$$l = \begin{pmatrix} & A & B & C & D & E & F \\ A & 0 & 2 & 7 & 9 & 6 & 12 \\ B & \infty & 0 & 6 & \infty & 4 & \infty \\ E & \infty & \infty & 0 & 5 & 0 & \infty \\ C & \infty & \infty & \infty & 0 & \infty & \infty \\ D & \infty & \infty & \infty & \infty & \infty & \infty \\ F & \infty & \infty & \infty & \infty & \infty & \infty \end{pmatrix}$$

$$P = \begin{pmatrix} & A & B & C & D & E & F \\ A & - & A & E & C & B & D \\ B & \infty & - & C & D & E & F \\ E & \infty & \infty & - & C & D & F \\ C & \infty & \infty & \infty & - & D & F \\ D & \infty & \infty & \infty & \infty & - & F \\ F & \infty & \infty & \infty & \infty & \infty & - \end{pmatrix}$$

	A	B	C	D	E	F
P	-	A	E	C	B	D
l	0	2	7	9	6	12

• A et B : $P(B)=A$: $A \rightarrow B : 2$

• A et C : $P(C)=E$, $P(E)=B$, $P(B)=A$: $A \rightarrow B \rightarrow E \rightarrow C : 7$

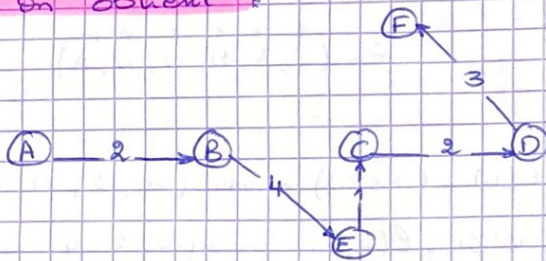
• A et D : $P(D)=C$, $P(C)=E$, $P(E)=B$, $P(B)=A$: $A \rightarrow B \rightarrow E \rightarrow C \rightarrow D : 9$

• A et E : $P(E)=B$, $P(B)=A$: $A \rightarrow B \rightarrow E : 6$

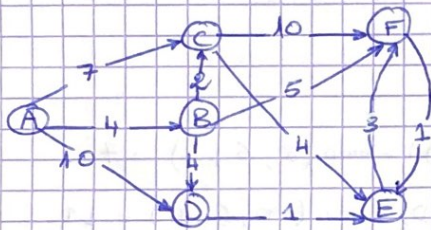
• A et F : $P(F)=D$, $P(D)=C$, $P(C)=E$, $P(E)=B$, $P(B)=A$

$A \rightarrow B \rightarrow E \rightarrow C \rightarrow D \rightarrow F : 12$

Alors on obtient :



Exercice 1-2 :



$S = \{A\}$ et $T = \{B, C, D, E, F\}$

$\ell = \begin{pmatrix} A & B & C & D & E & F \\ 0 & 4 & 7 & 10 & \infty & \infty \end{pmatrix}$

$P = \begin{pmatrix} A & B & C & D & E & F \\ - & A & A & A & N & N \end{pmatrix}$

Etape 1 : $j = B$

$P(C)=B$ $i=C$ $\ell(C) = \min(\ell(C), \ell(B) + \ell(B, C)) = (7, 4 + 2) = 6$

$P(D)=B$ $i=D$ $\ell(D) = \min(\ell(D), \ell(B) + \ell(B, D)) = (10, 4 + 4) = 8$

$P(F)=B$ $i=F$ $\ell(F) = \min(\ell(F), \ell(B) + \ell(B, F)) = (\infty, 4 + 5) = 9$

$S = \{A, B\}$ et $T = \{C, D, E, F\}$

$\ell = \begin{pmatrix} A & B & C & D & E & F \\ 0 & 4 & 6 & 8 & \infty & 9 \end{pmatrix}$

$P = \begin{pmatrix} A & B & C & D & E & F \\ - & A & B & B & N & B \end{pmatrix}$

Etape 2 : $j = C$

$P(E)=C$ $i=E$ $\ell(E) = \min(\ell(E), \ell(C) + \ell(C, E)) = (\infty, 6 + 4) = 10$

$i=F$ $\ell(F) = \min(\ell(F), \ell(C) + \ell(C, F)) = (9, 6 + 10) = 9$

pas de changement !

$S = \{A, B, C\}$ $T = \{D, E, F\}$

$\ell = \begin{pmatrix} A & B & C & D & E & F \\ 0 & 4 & 6 & 8 & 10 & 9 \end{pmatrix}$

$P = \begin{pmatrix} A & B & C & D & E & F \\ - & A & B & B & C & B \end{pmatrix}$

Etape 3: $j = D$

$P(E) = D$ $i = E$ $l(E) = \min(l(E), l(D) + l(D, E)) = \min(10, 8 + 1) = 9$

$S = \{A, B, C, D\}$ $T = \{E, F\}$

$l = \begin{pmatrix} A & B & C & D & E & F \\ \textcircled{0} & \textcircled{4} & \textcircled{6} & \textcircled{8} & \textcircled{9} & \textcircled{9} \end{pmatrix}$

$P = \begin{pmatrix} A & B & C & D & E & F \\ - & A & B & B & D & B \end{pmatrix}$

Etape 4: $j = E$

$i = F$ $l(F) = \min(l(F), l(E) + l(E, F)) = \min(9, 9 + 3) = 9$
pas de changement !

$S = \{A, B, C, D, E\}$ $T = \{F\}$

$l = \begin{pmatrix} A & B & C & D & E & F \\ \textcircled{0} & \textcircled{4} & \textcircled{6} & \textcircled{8} & \textcircled{9} & \textcircled{9} \end{pmatrix}$

$P = \begin{pmatrix} A & B & C & D & E & F \\ - & A & B & B & D & B \end{pmatrix}$

Etape 5: $j = F$ pas de voisins

$S = \{A, B, C, D, E, F\}$ $T = \{\emptyset\}$

$l = \begin{pmatrix} A & B & C & D & E & F \\ \textcircled{0} & \textcircled{4} & \textcircled{6} & \textcircled{8} & \textcircled{9} & \textcircled{9} \end{pmatrix}$

$P = \begin{pmatrix} A & B & C & D & E & F \\ - & A & B & B & D & B \end{pmatrix}$

	A	B	C	D	E	F
P	-	A	B	B	D	B
l	0	4	6	8	9	9

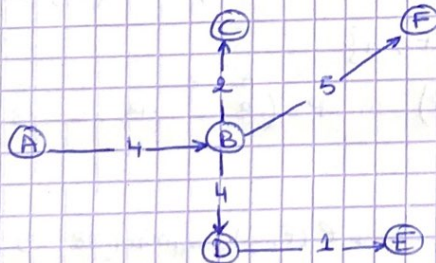
A et B: $P(B) = A$ $A \rightarrow B : 4$

A et C: $P(C) = B$ et $P(B) = A$ $A \rightarrow B \rightarrow C : 6$

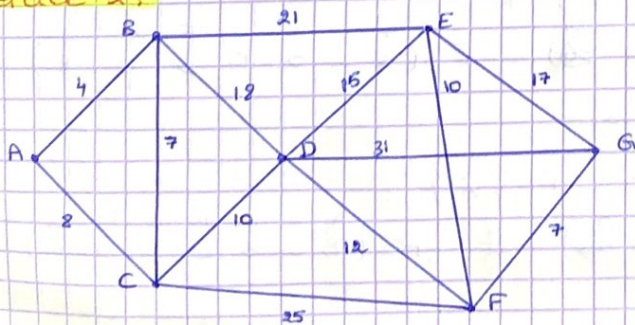
A et D: $P(D) = B$ et $P(B) = A$ $A \rightarrow B \rightarrow D : 8$

A et E: $P(E) = D$ $P(D) = B$ et $P(B) = A$ $A \rightarrow B \rightarrow D \rightarrow E : 9$

A et F: $P(F) = B$ et $P(B) = A$ $A \rightarrow B \rightarrow F : 9$



Exercice 2:



$$S = \{B\} \quad \text{et} \quad T = \{A, C, D, E, F, G\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 12 & 21 & \infty & \infty \end{pmatrix}$$

$$P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & B & B & N & N \end{pmatrix}$$

Etape 1: $j = A$

$$i = C \quad l(C) = \min(l(C), l(A) + l(A, C)) = \min(7, 4 + 8) = 7$$

pas de changement !

$$S = \{B, A\} \quad \text{et} \quad T = \{C, D, E, F, G\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 12 & 21 & \infty & \infty \end{pmatrix}$$

$$P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & B & B & N & N \end{pmatrix}$$

Etape 2: $j = C$

$$P(D) = C \quad i = D \quad l(D) = \min(l(D), l(C) + l(C, D)) = \min(12, 7 + 10) = 17$$

$$P(F) = C \quad i = F \quad l(F) = \min(l(F), l(C) + l(C, F)) = \min(\infty, 7 + 25) = 32$$

$$S = \{B, A, C\} \quad \text{et} \quad T = \{D, E, F, G\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 17 & 21 & 32 & \infty \end{pmatrix}$$

$$P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & C & B & C & N \end{pmatrix}$$

Etape 3: $j = D$

$$i = E \quad l(E) = \min(l(E), l(D) + l(D, E)) = \min(21, 17 + 15) = 21$$

pas de changement !

$$P(F) = D \quad i = F \quad l(F) = \min(l(F), l(D) + l(D, F)) = \min(32, 17 + 12) = 29$$

$$P(G) = D \quad i = G \quad l(G) = \min(l(G), l(D) + l(D, G)) = \min(\infty, 17 + 31) = 48$$

$$S = \{B, A, C, D\} \quad \text{et} \quad T = \{E, F, G\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 17 & 21 & 29 & 48 \end{pmatrix}$$

$$P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & C & B & D & D \end{pmatrix}$$

Etape 4: $j = E$

$$P(G) = E \quad i = G \quad l(G) = \min(l(G), l(E) + l(E, G)) = \min(48, 21 + 17) = 38$$

$$S = \{B, A, C, D, E\} \quad \text{et} \quad T = \{F, G\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 17 & 21 & 29 & 38 \end{pmatrix}$$

$$P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & C & B & D & E \end{pmatrix}$$

Etape 5: $j = F$

$$P(G) = F \quad i = G \quad l(G) = \min(l(G), l(F) + l(F, G)) = \min(38, 29 + 7) = 36$$

$$S = \{B, A, C, D, E, F\} \quad \text{et} \quad T = \{G\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 17 & 21 & 29 & 36 \end{pmatrix}$$

$$P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & C & B & D & F \end{pmatrix}$$

Etape 6: $j = G$ pas de voisins

$$S = \{B, A, C, D, E, F, G\} \quad T = \{\emptyset\}$$

$$l = \begin{pmatrix} A & B & C & D & E & F & G \\ 4 & 0 & 7 & 17 & 21 & 29 & 36 \end{pmatrix}$$

$$\text{et} \quad P = \begin{pmatrix} A & B & C & D & E & F & G \\ B & - & B & C & B & D & F \end{pmatrix}$$

	A	B	C	D	E	F	G
P	B	-	B	C	B	D	F
L	4	0	7	17	21	29	36

B et G : $P(G) = F$ et $P(F) = D$ et $P(D) = C$ et $P(C) = B$

on obtient : $B \longrightarrow C \longrightarrow D \longrightarrow F \longrightarrow G \dots 36$

