PHY 111 MADE EASY

(MECHANICS, THERMAL PHYSICS AND PROPERTIES OF MATTER)

(EXAMPLES $ EXERCISES SOLVED WITH PAST QUESTIONS)

FOR FULL TIME, PART TIME, JUPEB AND DIPLOMA

CHAPTER 1
MEASUREMENT AND UNITS

PHYSICAL QUANTITIES: They are quantities that have measurable properties. They consist of numerical values and unit and are classified into:

1. FUNDAMENTAL PHYSICAL QUANTITIES: They are basic quantities that provide the basic units of measurement. Below are examples with their S.I. unit in the bracket: Mass (Kg), Length (m), Time (s). Others are electric current (A, Ampere) Luminous intensity (candela), amount of substance (mole) and thermodynamic temperature (K).

2. DERIVED PHYSICAL QUANTITIES: They are obtained by combining two or more fundamental quantities. (If you do not understand this table, be patient, we will explain it later)

DERIVED QUANTITY	DERIVATION	BASIC INTERPRETATION
VELOCITY	$\frac{\text{Displacement}}{\text{time}}$	$\frac{L}{T} = LT^{-1}$
ACCELERATION	$\frac{\text{Increase in velocity}}{\text{time}}$	$\frac{LT^{-1}}{T} = LT^{-2}$
FORCE	Mass × acceleration	MLT^{-2}
WORK	force × distance	ML^2T^{-2}

DIMENSIONS

The dimensions of a physical quantity indicate the fundamental quantities: mass, length and time are represented by the letters M,L and T respectively can be used to relate the fundamental units of physical quantities. If velocity, $V = \frac{\text{displacement}}{\text{Time}}$

Recall, displacement

t is a measure of length :

$$V = \frac{L}{T} = LT^{-1}$$

APPLICATION OF DIMENSIONS

It is used for checking whether an equation that has been derived or is being used in solving a problem has the correct form **(DIMENSIONALLY CORRECT).**

NOTE 1:

Anywhere you find any quantity that has to do with length, distance, displacement, radius substitute L. anywhere you find mass substitute M and anywhere you find time substitute T.

Note: Constants (eg. 2,3,π…) are dimensionless quantities. They are omitted in calculations.

EXAMPLE 1 : find the dimensions of the following: (a) velocity (b) acceleration (c) force (d) gravitational constant (e) elastic modulus or material.

SOLUTION :

a. Velocity $= \frac{\text{displacement}}{\text{Time}}$

$$v = \frac{L}{T} = LT^{-1}$$

(see note 1 above)

b. Acceleration, $a = \frac{\text{velocity}}{\text{Time}}$

Recall, velocity is LT^{-1}

So, $a = LT^{-1} = \frac{LT^{-1}}{T}$

Using indices, $a = LT^{-2}$

c. Force = ma where m and a are mass(m) and acceleration (a).

$F = M \times LT^{-2}$

$F = MLT^{-2}$

Always write dimension in the form MLT and not MTL.

d. Gravitational constant G from Newton's law of universal gravitation,

e. $F = \frac{Gm_1m_2}{r^2}$

where m_1, m_2, r and f are mass (m), radius (L) and force (MLT^{-2}) respectively. Make G subject of the formula formula:

$$F = \frac{Gm_1m_2}{r^2}$$

cross multiply

$$Fr^2 = Gm_1m_2$$

$$\text{So, } G = \frac{Fr^2}{m_1m_2}$$

Substitute: $G = \frac{MLT^{-2} \times L^2}{M\times M}$

Always write in the form MLT not MTL.

$$G = \frac{\cancel{M}L^3T^{-2}}{\cancel{M}\times M} \quad , \quad G = \frac{L^3T^{-2}}{M} = M^{-1}L^3T^{-2}$$

f. Elastic modulus of material

$$E = \frac{\frac{\text{force}}{\text{area}}}{\frac{\text{extension}}{\text{length}}}$$

Recall: area = L × b (length and breath are all in meters, so dimension is "L".

So, area = L×L=L^2, extension is also in meters, so dimension "L" length is in meters, so dimension is "L", substitute:

$$E = \frac{\frac{\text{Force}}{\text{Area}}}{\frac{\text{Extension}}{\text{Length}}} = \frac{\frac{MLT^{-2}}{L^2}}{\frac{L}{L}}$$

$$E = \frac{MLT^{-2}}{L^2} \div \frac{L}{L} = \frac{M\cancel{L}T^{-2}}{L^{\cancel{2}}} \times \frac{\cancel{L}}{\cancel{L}} = \frac{MT^{-2}}{L}$$

Write in form **"MLT"**, $E = ML^{-1}T^{-2}$

EXAMPLE 2: Find the dimensions of angular momentum.

SOLUTION: Angular momentum = Iw

Where I and W are moment of inertia and angular velocity respectively

but $I = mr^2$ and

$w = 2\pi f$, $f = \frac{1}{T}$, where T= period(seconds)

Dimension of frequency = $\frac{1}{T}$

Dimension for moment of inertia = ML^2

Angular momentum = $Iw = mr^2 \times 2\pi f$

Substitute : $Iw = ML^2 \times \frac{1}{T} = ML^2T^{-1}$

EXAMPLE 3 : what is the dimension of coefficient of friction and viscosity?

SOLUTION : coefficient of friction is μ

$F_r = \mu R$, where F_r is the frictional force and R is the normal reaction. F_r has the same dimension as force (MLT^{-2}). But R = mg (i.e mass times acceleration). **The dimensions are :**

$a = LT^{-2}$, m = M , $F_r = MLT^{-2}$

thus, from $F_r = \mu R$, $\mu = \frac{F_r}{R} = \frac{\cancel{MLT^{-2}}}{\cancel{M\times LT^{-2}}} = 1$

Thus, coefficient of friction is dimentionless.

Now, let's do for coefficient of viscosity, η , (or viscosity)

$F = 6\pi\eta rV_T$ (**stokes's law),** where F, r and V_T are force radius and velocity respectively.

$$\eta = \frac{F}{6\pi rV_T} = \frac{M\cancel{L}T^{-\cancel{2}}}{L \times \cancel{L}\cancel{T}^{-1}} = ML^{-1}T^{-1}$$

EXAMPLE 11 : which of the following has the same dimension as time ? (a) $\frac{x}{a}$ (b) $\sqrt{\frac{2x}{a}}$ (c) $\sqrt{\frac{v}{x}}$ (d) vx (e) xa

SOLUTION : just check the dimensions of all the options the one that gives you **T (i.e the dimension of time is the answer)**

Let me just do only option B

$$\sqrt{\frac{2x}{a}} = \sqrt{\left(\frac{\cancel{L}}{\cancel{L}T^{-2}}\right)} = \sqrt{T^2} = T \text{ (\textbf{answer is B})}$$

you can try doing other options yourself

COVERSION FACTOR

These are used to convert the units of one system to the units of another. **Note that it can be done using your calculator.**

EXAMPLE 4 : convert 648km to miles

SOLUTION : 1km = 0.621mi

648km = x

just cross multiply X = 402mi

EXAMPLE 5 : express the velocity of 60mi/hr in ft/s.

SOLUTION : this is complex just use your calculator to do it . **answer : 88ft/s**

Note : 1 ton = 1000kg and 1rad/s = $\frac{60}{2\pi}$rev/min

EXAMPLE 5 : A car travels at a constant speed of 15m/s. how many miles does it travel in 1hr? **HINT : it means convert 15m/s to mi/hr, just use your calculator. Answer : 33.55mi/hr**

EXAMPLE 6 : The area of a floor is 25ft^2. How

Many m^2 are there in the floor? **Use your calculator. Answer : 2.322576m^2**

An equation is said to dimensionally correct if the dimensions of each term in the equation is the same.

EXAMPLE 7 : a professor puts two equations on the board. V = u + at and x = V/2a. show which of the is dimensionally correct or not.

SOLUTION : jus substitute their dimensions

V = u + at , $\frac{L}{T} = \frac{L}{T} + \frac{L \times \cancel{T}}{T^{\cancel{2}}}$

$\frac{L}{T} = \frac{L}{T} + \frac{L}{T}$ **(dimensionally correct)**

x = V/2a , $L = \frac{\cancel{L}T^{-1}}{\cancel{L}T^{-\cancel{2}}}$

L = 1/T **(not dimensionally correct)**

EXAMPLE 8 : if x refers to distance, u and v refer to velocities, a to acceleration and t to time. Which of the equations is dimensionally correct and which is not ?

SOLUTION : it's the same way

a. x = ut + at^3 , $L = \frac{L \times \cancel{T}}{\cancel{T}} + \frac{L \times T^{\cancel{3}}}{\cancel{T^2}}$

L = L + LT (**not correct)**

Now do the rest yourself.

b. v^2 = u^2 + 2at, **answer : (not correct)**

c. x = at + vt^2 **, answer : (not correct)**

d. v^2 = u^2 + 2ax. **Answer : (correct)**

EXAMPLE 9 : you are told that the volume of a sphere is given by **V = πd^3/4** , where v and d are volume and diameter of the sphere. Is the

Equation dimensionally correct?

SOLUTION : V = l × b × h = L^3, and d = L

Substitute : $L^3 = \frac{\pi L^3}{4}$, L^3 = L^3 (**correct)**

EXAMPLE 10 : if x = gt^2/2, where x is length and t is time, is the equation dimensionally correct?

SOLUTION : $x = gt^2/2$, $L = \frac{L\cancel{T^2} \times \cancel{T^2}}{2}$

The S.I units of g is same as that of **a i.e m/s²**

L = L (**correct)**

CHAPTER 2

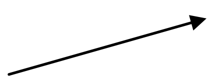
VECTORS

Scalars : a scalar quantity is a physical quantity that has magnitude only. E.g speed, distance, temperature, energy, density, pressure, mass, time e.t.c. they do not bear negative sign (-) as answers.

Vectors : a vector quantity is a physical quantity that has both magnitude and direction. E.g displacement, velocity, acceleration, force, momentum, electric field, magnetic field e.t.c

REPRESENTATION OF A VECTOR

a vector is represented by a given length of line. Its direction is represented with an arrow at the end of the line drawn.



ADDITION OF VECTORS BY VECTORIAL METHOD

Given two vectors : x = ai + bj + ck and

y = di + ej + fk, then the **resultant/sum**

x + y = (a + d)i + (b + e)j + (c + f)k

and the **magnitude** of x + y is written as

$$|x + y| = \sqrt{(a + d)^2 + (b + e)^2 + (c + f)^2}$$

EXAMPLE 1 : determine the magnitude of the resultant of two displacements A and B where

A = (5i – 2k)m , B = (-3i + 4j + 6k) m

SOLUTION : add them to get their resultant

A + B = (5 + -3)i + (0 + 4)j + (-2 + 6)k

A + B = 2i + 4j + 4k , thus the magnitude is

$|A + B| = \sqrt{2^2 + 4^2 + 4^2} = 6m$

EXAMPLE 2 : two force vectors

$F_1 = (3.0N)x - (3.0N)y$ and $F_2 = (-6.0N)x + (4.5N)y$

Are applied to a particle. What would force F_3 be, to make the net or resultant forces on the particle zero?

SOLUTION : $F_1 + F_2 + F_3 = 0$, $F_3 = - F_1 - F_2$

$F_3 = - \{(3.0N)x - (3.0N)y\} - \{(-6.0N)x + (4.5N)y\}$

$F_3 = (-3 + 6)x + (3 - 4.5)y = 3x - 1.5y$

Note that it can also be written as

$F_3 = 3i - 1.5j$

If you were told to find the **magnitude of F_3** ,

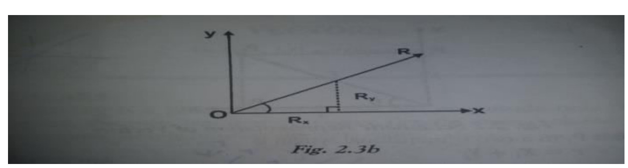
$|F_3| = \sqrt{3^2 + (-1.5)^2} = 3.4N$

If you were told to look for the direction of F_3

$\theta = \tan^{-1}\left(\frac{-1.5}{3}\right) = 27^0$ below the + x-axis

RSOLUTION OF VECTOR

It is the process of finding the effect of a resultant vector in its component direction.



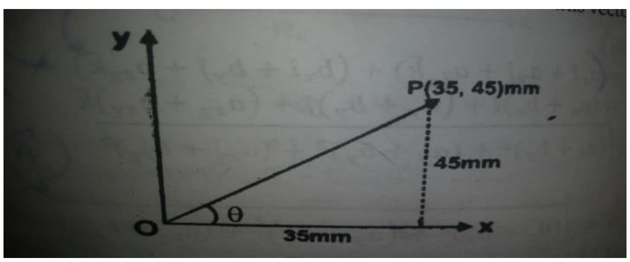
the vector R can be resolved along the oy direction to give $R_y = R\sin\theta$ and along the ox direction to give $R_x = R\cos\theta$

the magnitude of the resultant vector is

$|R| = \sqrt{(R_x^2 + R_y^2)}$ while the **direction, θ is**

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

EXAMPLE 3 : an ant craws on a table top. A position vector R locates the ant at a point with coordinates x = 35 and y = 45m relative to the origin of the coordinate system below. Determine the magnitude and direction of the vector.



SOLUTION : $R_x = 35mm$, $R_y = 45mm$

$|R| = \sqrt{(R_x^2 + R_y^2)} = = \sqrt{(35^2 + 45^2)} = 57mm$

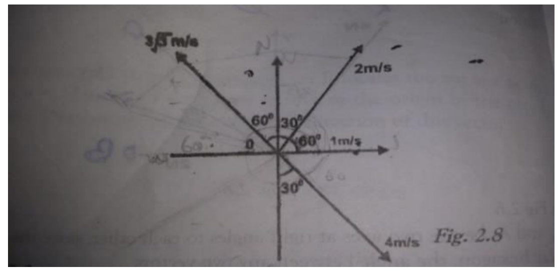
The direction, $\theta = \tan^{-1}(R_y/R_x)$

$\theta = \tan^{-1}(45/35) = \tan^{-1}(1.2857) = 52^o$

Note : in the following examples, we resolve along the x and y axis respectively. When resolving along the y axis we use *sine* but when resolving along the x axis we use *cosine. Also note that the angle, θ we will be using must be measured anticlocwise from the positive x-axis to the force we are resolving.*

EXAMPLE 4 : a particle has velocities 1m/s, 2m/s, $3\sqrt{3}$m/s, 4m/s inclined at angles 0^o, 60^o, 150^o and 300^o respectively to the x-axis. Find the resultant velocity in magnitude and direction

SOLUTION :



$R_x = 1\cos0 + 2\cos60 + 3\sqrt{3}\cos150 + 4\cos300$

$R_x = 1 + 1 - 9/2 + 2 = \frac{-1}{2}$ m/s

To find R_y , simply change the **cos** to **sine**

$R_y = 1\sin0 + 2\sin60 + 3\sqrt{3}\sin150 + 4\sin300$

$R_y = 0 + \sqrt{3} + \frac{3\sqrt{3}}{2} + 2\sqrt{3} = \frac{\sqrt{3}}{2}$ m/s

$|R| = \sqrt{(R_x^2 + R_y^2)} = = \sqrt{(\frac{-1}{2}^2 + \frac{\sqrt{3}}{2}^2)}$

$|R| = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1m/s$

Direction $\theta = \tan^{-1}(R_y/R_x)$

$= \tan^{-1}(\frac{\sqrt{3}}{2} \div \frac{-1}{2}) = \tan^{-1}(\frac{\sqrt{3}}{2} \times \frac{2}{-1}) = \tan^{-1}\sqrt{3} = -60^0$

anytime the an angle is negative add 180^o the angle, if it is still negative add 180^o to it again

thus, $\theta = -60 + 180 =$ **120^o**

EXAMPLE 5 : a particle of mass 10,000g is acted upon by forces 2N, $4\sqrt{2}$N, 6N and 8N inclined at angles of 30^o, 45^o, 60^o and 120^orespectively to a given direction. The magnitude of the resultant force and the acceleration are?

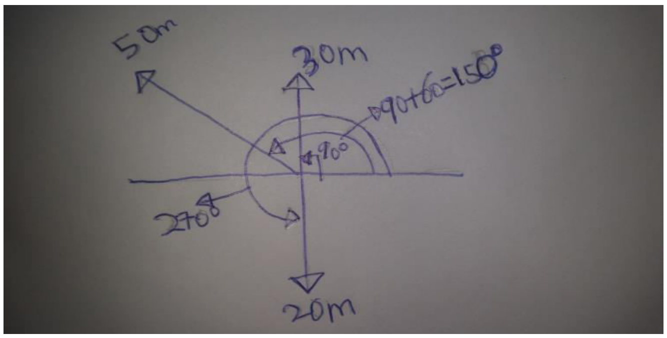
Answers : R = 17.77N, a = 1.777m/s^2

Hint: after finding R, the use $a = R/m$ to find a, where m is in kilogram, kg.

EXAMPLE 6 : a boy starts from his room and walked 20m south, then turns and walk 50m in the direction 30^{o}N of W, finally, he walked

30m directly north. Find the magnitude of the resultant displacement from his room.

SOLUTION:



$R_x = 3\cos90 + 50\cos150 + 20\cos270 = -25\sqrt{3}$m

$R_y = 3\sin90 + 50\sin150 + 20\sin270 = 35$m

$|R| = \sqrt{(R^2_x + R^2_y)} = \sqrt{((-25\sqrt{3})^2 + 35^2)} = \sqrt{300}$

$|R| = 55.68$m

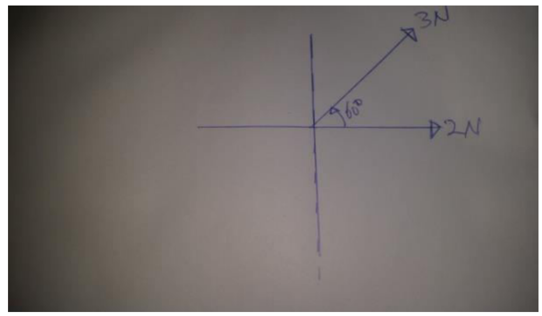
EXAMPLE 7 : a lady drove her car northwest for a distance 10km, then east for 40km and then south for 60km. calculate the overall displacement of the car from the starting point. **Answer: 62.3km, at 58.1^o S of E**

EXAMPLE 8 : four forces 8N, 6N, 2N and 4N act at a point O in the direction North, East, South and West respectively, find the magnitude and direction of their resultant. **Answer : 6.32N, 71.6^o**

Note : anytime you are not given the exact position(directions) of the forces in the the question, *assume that the first force is acting along the positive x-axis and begin to measure others taking the first force as the reference position.*

EXAMPLE 9 : if two forces of magnitude 2N and 3N acting at a point are inclined at an angle 60^o to each other, find the magnitude and direction of the resultant.

SOLUTION :



$R_x = 2\cos0 + 3\cos60 = 3.5$N

$R_x = 2\sin0 + 3\sin60 = \frac{3\sqrt{3}}{2}$ N

$|R| = \sqrt{(R^2_x + R^2_y)} = \sqrt{((3.5^2 + \frac{3\sqrt{3}}{2}^2)} = \sqrt{19} = 4.36$N

try to find the direction θ. answer: $\theta = 36.6^o$

EXAMPLE 10 : determine the resultant of two vectors each of magnitude 10m if the angle between them is 120^o. **answer : 10m, 60^o**

EXAMPLE 11 : three forces of magnitude 15p, 10p, 5p act on a particle in directions which makes 120^o with one another. Find their resultant.

SOLUTION :

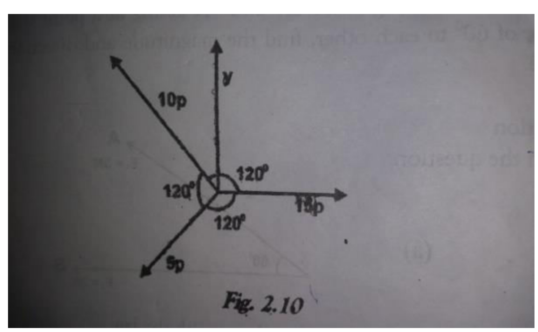


Fig. 2.10

$R_x = 15p\cos 0 + 10p\cos 120 + 5p\cos 240 = \frac{15p}{2}$

$R_x = 15p\sin 0 + 10p\sin 120 + 5p\sin 240 = \frac{5\sqrt{3}\,p}{2}$

$|R| = \sqrt{(R^2_x + R^2_y)} = \sqrt{((\frac{15p}{2})^2 + (\frac{5\sqrt{3}\,p}{2})^2)} = 5p\sqrt{3}$

$\theta = \tan^{-1}(\frac{5\sqrt{3}\,p}{2} \div \frac{15p}{2})$ **use fraction** $\theta = 30^o$

EXAMPLE 12 : (a) find the resultant of the force vectors in the figure below **(b)** if F_1 in the figure were at an angle 27^o instead of 37^o with the + x-axis, what would be the resultant.

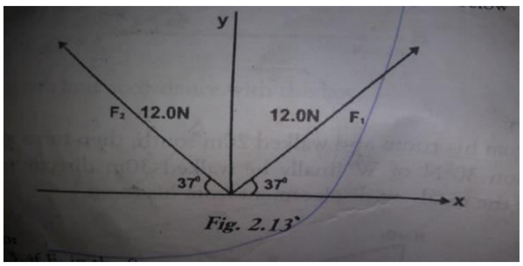


Fig. 2.13

Answers : (a) 3.8N (b) 122.2N

CHAPTER 3

MECHANICS

It is the study of the relationship between **force, matter and motion.**

Kinematics : it is the part of mechanics that deals with the description of the positions and motions of objects in space as a function of time **without consideration to the cause of the motion .** e.g the study of velocity or acceleration.

Dynamics : *it is the part of mechanics that deals with the* ***cause of motion.***

SPEED

It is the rate of change of distance with time.

speed = $\frac{distance\ (d)}{time\ (t)}$ **its unit is m/s**

VELOCITY (V)

It is the rate of change of displacement with time in a specified direction.

V = $\frac{displacement}{time}$ **its unit is m/s**

EXAMPLE 1 : suppose that john runs along a cycle of radius 100m in 50sec, what is the average speed.

SOLUTION : t = 50sec , r = 100m

Since it is a circle, the total distance = circumference of the circle ($\pi = 22/7$)

$d = 2\pi r = 2\pi \times 100 = 628.3m$

Speed = $\frac{d}{t} = \frac{628.3}{50} = 12.57m/s$

EXAMPLE 2 : a J5 bus traveling from port Harcourt to Aba maintains an average speed of 100km per hour. It left port Harcourt at 10:00 am and arrived its destination at exactly 10:48am, what distance was covered by the bus ?

SOLUTION : d = ?, convert speed to m/s

$V = 100km/hr = \frac{100 \times 1000}{3600} = 27.78m/s$

10:00am to 10:48am (that is 48mins),

$t = 48mins = 48 \times 60 = 2800s$

$v = \frac{d}{t}$, $27.78 = \frac{d}{2800}$, cross multiply, d = 80006.4m

NEWTON'S EQUATIONS OF UNIFORM MOTION

$$V = u \pm at,\ \ S = ut \pm \frac{1}{2}at^2,\ V^2 = u^2 \pm 2as$$

Take note of his equation: $S = \frac{(V+u)t}{2}$

1. when an object starts from rest, u = 0

2. when an object goes to rest, V = 0

3.when an object moves with a uniform or constant velocity, a = 0

STOPPING DISTANCE FOR MOVING OBJECTS

EXAMPLE 3 : a driver of a saloon car traveling at 120km/hr suddenly sights an obstacle 60m ahead of him. Calculate the minimum acceleration required to stop the car from hitting the obstacle.

SOLUTION : convert u to m/s

u = $\frac{120 \times 1000}{3600}$ = 33.33m/s, v = 0 (stop the car)

$V^2 = u^2 - 2as$, $0^2 = 33.33^2 - 2 \times a \times 60$

0 = 1110.89 – 12a, 120a = 1110.89

a = 1110.89/120 = 9.26m/s^2

EXAMPLE 4 : the speed limit in a school zone is 40km/hr. a driver travelling at this speed sees a child run into the road 13m ahead of his car. He applies the breaks, and the car decelerates at a uniform rate of 0.8m/s^2. If the driver's reaction time is 0.25s, will the car stop before hitting the child?

SOLUTION : u = 40km/hr = 11.11m/s

a = 0.8, t = 0.25s. to know if the car will hit the child of not, we have to first find the distance the car moves through.

$S = ut - \frac{1}{2}at^2 = 11.11 \times 0.25 - \frac{1}{2} \times 0.8 \times 0.25^2$

S = 2.778 – 0.025 = 2.75m

The car stopped after travelling only 2.75m. but the distance between the car and the child is 13m. **thus, we will say : YES, the cat will stop before hitting the child. The car will stop at a distance of (13 – 2.75) = 10.25m.**

CALCULUS (DIFFERENTIATION, $\frac{dy}{dx}$) IN MECHANICS

If you differentiate the displacement (y of x) **once** you will get velocity **(V or $\frac{dy}{dx}$).** If you

Differentiate the velocity you just got you will get **acceleration (a or d^2y/dx^2)** .

Here, you need a little knowledge of calculus

EXAMPLE 5 : the displacement of a body in the positive x-direction is $y = 3t^4+2t^3+4t^2+5t$. calculate the velocity and acceleration after 0.5 sec.

SOLUTION : $y = 3t^4+2t^3+4t^2+5t$. differentiate

$V = 12t^3+6t^2+8t+5$ at t = 0.5s

$V = 12(0.5)^3+6(0.5)^2+8(0.5)+5$ = 12m/s

We get the acceleration by differentiating the velocity we got. $V = 12t^3+6t^2+8t+5$ thus,

$a = 36t^2+12t+8$ at t = 0.5s

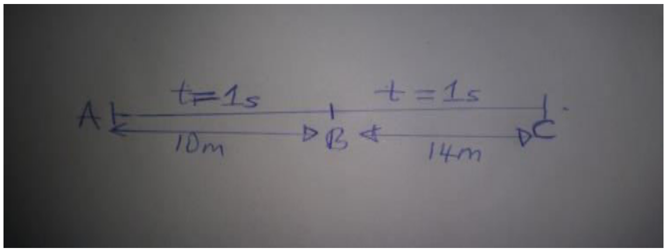
$a = 36(0.5)^2+12(0.5)+8$ = 23m/s^2

EXAMPLE 6 : the displacement of a body in the positive x-direction is given as $x = 3t^3+2t^2+4t+ 5$. Find the velocity and acceleration of the body after 5s. **do it the same way . Answers : 249m/s, 94m/s^2**

Let's consider what I call : horizontal case

EXAMPLE 7 : A body is moving in a straight line with constant acceleration one second after passing point A. it is 10m from A in the next second it travels further 14m. find the velocity of passing A and the acceleration on passing through point A.

SOLUTION :



The body starts from A. **firstly, always consider the distance from the starting point(A) to next point(B)**

$t = 1s$, $s = 10m$

$S = ut + \frac{1}{2}at^2$, $10 = u \times 1 - \frac{1}{2} \times a \times 1^2$

$10 = u + 0.5a$ ....(1)

Secondly, always consider the distance from the starting point (A) to the last point(C).

$t = 1 + 1 = 2s$, $S = 10 + 14 = 24m$

$S = ut + \frac{1}{2}at^2$, $24 = u \times 2 - \frac{1}{2} \times a \times 2^2$

$24 = 2u + 2a$ ....(2)

Solving the two equations simultaneously, using your calculator

$u = 8m/s$ and $a = 4m/s^2$

let's consider what I call: vertical case

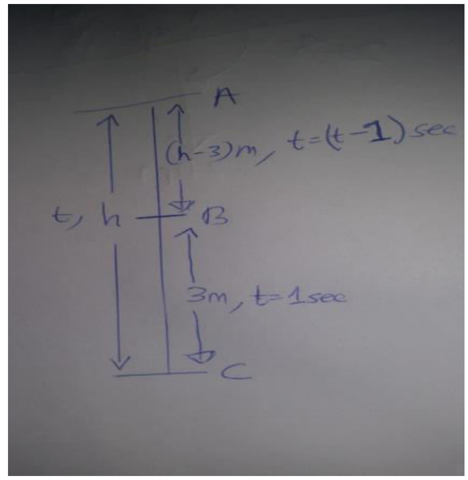
the only difference between this case and the horizontal case is that in this case the body is moving vertically. **Note that the calculations are done the same way.**

EXAMPLE 8 : if a ball falling from a height travelled the last 3m in 1s. determine

a. the height above the ground

b. the total distance traveled before the last 1s

SOLUTION : $u = 0m/s$ (starts from rest)



As we did before, consider the distance from A to B. $t = (t-1)sec$, $h = (h-3)m$

$h = ut + \frac{1}{2}gt^2$, $(h-3) = 0\times(t-1) - \frac{1}{2} \times 9.8\times(t-1)^2$

$h-3 = 4.9(t-1)^2$ .... (1)

also, consider the distance from A to C

height = h, time = t

$h = ut + \frac{1}{2}gt^2$, $h = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$

$h = 4.9t^2$ ....(2)

put $h = 4.9t^2$ into equation (1)

$h-3 = 4.9(t-1)^2$, $4.9t^2 -3 = 4.9(t-1)^2$ expand

$4.9t^2 -3 = 4.9(t^2 - 2t + 1)$

$\cancel{4.9t^2} -3 = \cancel{4.9t^2} - 9.8t + 4.9$, $9.8t - 4.9 -3 = 0$

$9.8t = 7.9$, $t = 9.8/7.9 = 0.806s$

a. put $t = 0.806s$ into equation (1) to find h

$h= 4.9t^2 = 4.9 \times 0.806^2$, $h = 3.2m$

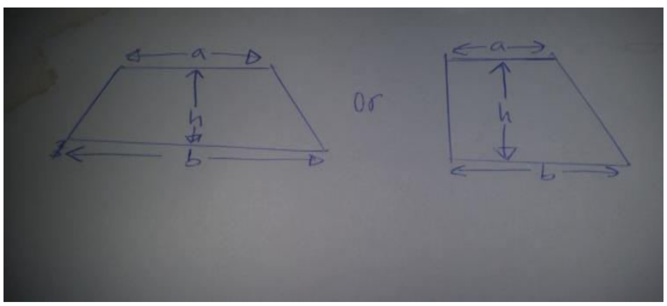
b. distance traveled before the last one second = $(h-3) = 3.2 - 2 = 0.2m$

EXAMPLE 9 : in a movie the FBI is investigating an assassination attempt on the life of the president. The setting is a parade in New York, and an amateur photographer has made a videotape of the passing motorcade. A careful examination of the tape shows in the background a falling object that turns out to be a binoculars used by the would-be assassin. From the tape, the FBI is able to determine the binoculars fell the last 12m before hitting the ground in 0.38s. it is important for them to know the height and hence, the building floor, from which the binoculars were dropped. Can this be determined from the given info? If so, from what height were the binoculars dropped?

$g = 9.8 m/s^2$

ANSWERS : YES, h = 57m

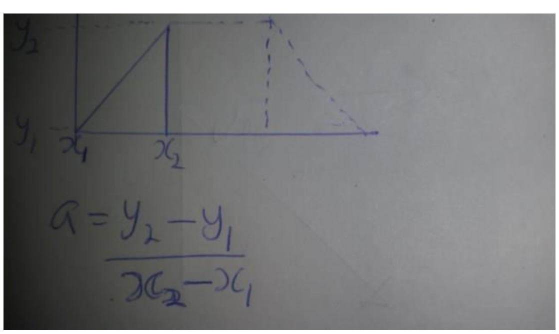
DISPLACEMENT – TIME GRAPH



Consider the trapezium. **Area, A = $\frac{1}{2}$(a + b)h**

This area is what we call **distance** in motion.

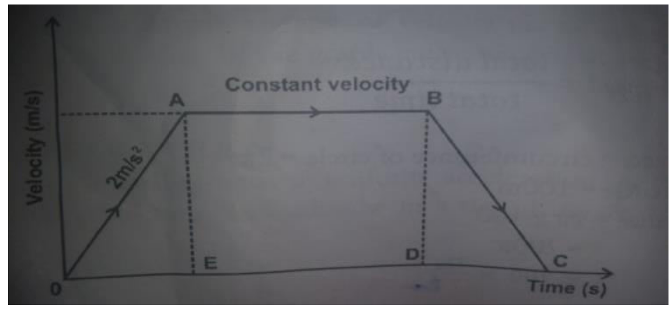
Average speed = $\frac{distance}{time}$. The acceleration is gotten from the place where the graph is increasing



EXAMPLE 10 : a train starts from rest and accelerates uniformly at a rate of $2m/s^2$ until it attains a maximum velocity in 20s. it maintains this velocity for 60s and then retards uniformly to rest after a further 20s.

a. draw a graph of the motion (b) use the graph to determine : (i) the maximum velocity reached (ii) the retardation (ii) the distance covered

SOLUTION :



Using the acceleration formula above

b. (i) $2 = \frac{v-0}{20-0}$ **, v = 2** $\times$ 20 = 40m/s

using the same formula :

ii. $a = \frac{40-0}{100-80} = \frac{40}{20} = 2m/s^2$

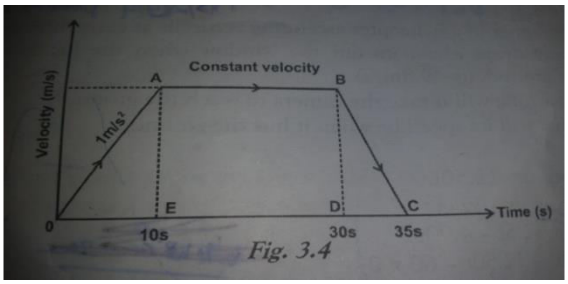
iii. A = $\frac{1}{2}$(a + b)h **=** $\frac{1}{2}$(60 + 100)40= **3200m**

EXAMPLE 11 : a car starts from rest and accelerates for $4m/s^2$ **for 5s.** then maintain that velocity for 10s and then decelerates at the rate of $2m/s^2$ for 4s. what is the final speed of the car? **Answer : 12m/s**

EXAMPLE 12 : a train passes another on a parallel track. The first train is running at a uniform speed of 60km/h and the second is running at a speed of 15km/h, with an acceleration of $0.15m/s^2$. How long will it be before the second train catches the first again and how far will the train run in the interval. **Answer : s = 2 $\frac{7}{9}$ km**

EXAMPLE 13 : a car starts from rest and accelerates at $1m/s^2$ for 10s. it then continues at a steady speed for a further 20s and decelerates to rest in 5s. find : (a) the distance traveled in m (b) the average speed in m/s (c) the time to cover half the distance.

SOLUTION : v = 10m/s



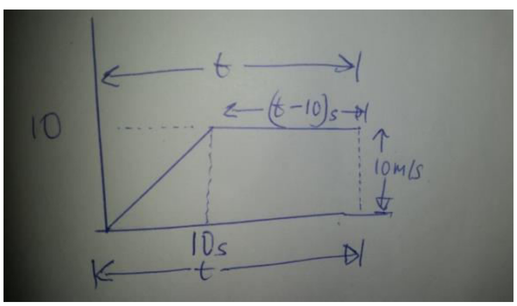
a. $a = \frac{v}{t}$, v = at = 1(10) = 10m/s^2

h = 10m, a = 20sec, b = 35sec

$A = \frac{1}{2}(a + b)h = \frac{1}{2}(20 + 35)10 = 275m$

b. $V = \frac{d}{t} = \frac{275}{35} = 7.86m/s$

c. half the distance = $\frac{1}{2} \times 275 = 137.5m$



from the graph, a = t-10 , b = t , h = 10m/s

$s = \frac{1}{2}(a + b)h$, $137.5 = \frac{1}{\cancel{2}}(\{t - 10\} + t)\cancel{10}$

137.5 = (2t – 10) 5, 137.5 = 10t – 5, t = 18.75s

EXAMPLE 14 : a car starts from rest, accelerates at 2m/s^2, for 15s, then continues at a steady speed for further 25s and decelerates to rest in 5s. Find : (a) the distance travelled in m (b) the maximum velocity (c) the average velocity (d) the time taken to cover two third of the distance. **Answers : 1050m, 30m/s, 23.33m/s, 30.84s**

FURTHER QUESTIONS

EXAMPLE 15 : a man of mass 50kg is dropped from a height of 45m. what is his velocity when he is at a height 15m from the ground level?

SOLUTION : u = 0, V = ? , h=45-15 =30m(because he has only fallen through 30m)

$V^2 = u^2 + 2gh = 0^2 + 2\times 9.8 \times 30 = 600$

$V = \sqrt{600} = 24.5m/s$

EXAMPLE 16 : a ball is thrown vertically upwards return to the thrower 4s later. Determine the speed with which the ball is

Thrown. g = 10m/s^2.

SOLUTION : if total time T = 4s, time taken to reach maximum height, t = 4/2 = 2s,

At maximum height, u = ?

V = u + at, 0 = u + 10× 2, 0 = u – 20, u = 20m/s

EXAMPLE 17 : what is the effective take off velocity of a ball that bounces to a maximum height of 4m(g = 10m/s^2)

SOLUTION : h = 4m, at maximum height V=0

$V^2 = u^2 - 2gh$, $0^2 = u^2 - 2\times10\times4$,

$0 = u^2 - 80$, u=8.4m/s

EXAMPLE 18: if the speed of a truck is reduced from 26.7m/s to 6.7m/s within a distance of 800m. find (a) how long were the brakes applied (b) how much longer would it take before coming to rest ?

SOLUTION : u = 26.7m/s, V = 6.7m/s, s = 800m

a. $V^2 = u^2 + 2as$, $6.7^2 = 26.7^2 + 2 \times a \times 800$

$44.89 = 712.89 – 1600a$, $160a = 668$, $a = 0.42 m/s^2$

Then , $V = u – at$, $6.7 = 26.7 – 0.42 \times t$

$0.42t = 26.7 – 6.7$, 0.42

$t = 20$, $t = 48s$

Or use only **$S = \frac{(V+u)t}{2}$ and you won't have to solve twice**

b. o come to rest, $V = 0$,

$V = u – at$, $0 = 26.7 – 0.42 \times t$,

$0.4t = 26.7$, $t = 63.6s$

But the questions says **how much longer..** thus, $63.6 - 48 = 16s$

NO JESUS, NO SUCCESS,

Now , if you are not saved and you want to be forgiven of all your sins , say these words after me(say it from your heart): Lord Jesus, have mercy on me and forgive me of all my sins, even as I forgive everyone that has trespassed against me. I believe that you shed your blood and died for me and on the third day you resurrected that I may live. Lord Jesus , from this day, I accept you as my Lord and personal savior, in Jesus name, Amen. If you have just said those words, ***Congratulations****,* ***welcome to the body of Christ.*** *Look for a bible believing church and start attending, study your bible and pray daily*

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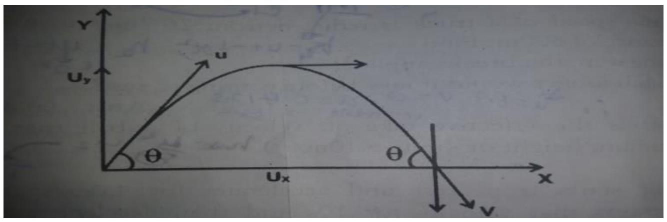
IN GOD WE TRUST

CHAPTER 4

PROJECTILE MOTION

A projectile is an object that is launched into space which takes a parabolic path called a **trajectory.** Every projectile motions consists of two independent motions : (i) the constant motion with constant acceleration, that is the velocity changes uniformly. (ii) the horizontal motion with constant velocity, that is acceleration is zero on the plane.



TIME OF FLIGHT (T)

It is the time required for a projectile to return to the same level from which it was projected.

T = 2t, where t is the time taken to reach the maximum height.

$t = \frac{u\sin\theta}{g}$, **thus,** $T = \frac{2u\sin\theta}{g}$

MAXIMUM HEIGHT (H_{max})

It is the highest distance travelled on the vertical plane.

$$H_{max} = \frac{u^2\sin^2\theta}{2g}$$

HORIZONTAL RANGE (R)

It is the total horizontal distance moved from the point of projection to the point where the projectile hits the plain of projection.

$$R = \frac{u^2\sin2\theta}{g}$$

note : maximum range occurs when θ =45°

$$R_{max} = \frac{u^2}{g}$$

EXAMPLE 1 : if a particle is projected with speed 30m/s at an angle $\tan^{-1}2$, find the greatest height and the corresponding horizontal distance. Find also the time of flight

SOLUTION : $\theta = \tan^{-1}2 = 63.43^o$, u = 30m/s

Note : $\sin^2\theta$ is same as $(\sin\theta)^2$.

$$H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{30^2(\sin63.43)^2}{2\times10} = 35.997\text{m}$$

$\sin2\theta$ is same as $\sin(2\times\theta)$ i.e multiply θ by 2

$$R = \frac{u^2\sin2\theta}{g} = \frac{30^2\sin(2\times63.43)}{10} = \frac{30^2\sin126.86}{10}$$

R = 720.19/10 = 72m

$$T = \frac{2u\sin\theta}{g} = \frac{2\times30\sin63.64}{10} = 5.36\text{s}$$

If you were told to find the maximum range use **$R_{max} = u^2/g$**

EXAMPLE 2 : when a stone is projected, its horizontal range is 24m and greatest height 6m. find its velocity of projection.

Answer : 17.32m/s

EQUATIONS OF MOTION FOR A PROJECTILE

They are similar to the equations of motion.

From the diagram above : $u_x = u\cos\theta$, $u_y = u\sin\theta$

Where u_x and u_y are horizontal and vertical velocities respectively

Equations for Y-plane

$V_y = u\sin\theta - gt$, $S_y = u(\sin\theta)t - \frac{1}{2}gt^2$

$V^2_y = u^2\sin^2\theta - 2gS_y$

Equations for x-plane

Here , $g = 0m/s^2$

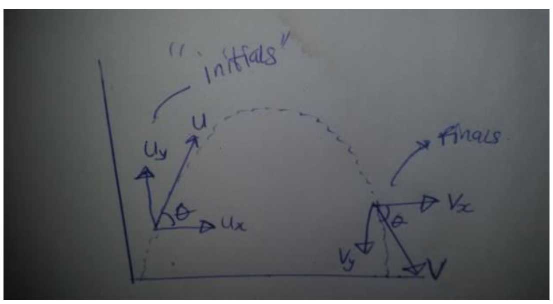
$V_x = u\cos\theta$, $S_x = u(\cos\theta)t$, $V^2_x = u^2\cos^2\theta$

The magnitude of the velocity (V) at any time t is $V = \sqrt{(V^2_x + V^2_y)}$

and the direction is $\theta = \tan^{-1}(V_y/V_x)$

the distance from the origin at an point is

$S = \sqrt{(S^2_x + S^2_y)}$



Where V is the velocity the object uses in **hitting the ground (i.e speed at touch down).**

V_y is the velocity the object uses in **reaching the ground.**

EXAMPLE 3 : a particle is projected with speed 20m/s at an angle of elevation 60°. find:

a. its greatest height (b) the range on a horizontal plane through the point of projection (c) the total time of flight (d)the velocity in magnitude and direction after 0.75s

SOLUTION : u = 20m/s, $\theta = 60^o$

a. $H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{20^2(\sin60)^2}{2\times9.8} = 15.31m$

b. $R = \frac{u^2\sin2\theta}{g} = \frac{20^2\sin(2\times60)}{9.8} = \frac{30^2\sin120}{9.8}$

R = 35.35m

c. $T = \frac{2u\sin\theta}{g} = \frac{2\times20\sin 60}{9.8} = 3.5s$

d. V = ?, at t = 0.7s. since time is given

use $V_x = u\cos\theta = 20\cos60 = 10m/s$

$V_y = u\sin\theta - gt = 20\sin60 - 9.8(0.75) = 9.97m/s$

$V = \sqrt{(V^2_x + V^2_y)} = \sqrt{(10^2 + 9.97^2)} = 14.1m/s$

$\theta = \tan^{-1}(V_y/V_x) = \theta = \tan^{-1}(9.97/10) = 45^o$

Thus, the particle makes an angle of 45° with the horizontal after 0.75s

If a body is projected horizontally $\theta = 0^o$

EXAMPLE 4 : a body projected horizontally with a velocity 40m/s from the top of a cliff hits the ground after 3s. calculate **the speed at touch down.**

SOLUTION : $\theta = 0^o$ (horizontally), u= 40m/s

$V_x = u\cos\theta = 40\cos0 = 40m/s$

$V_y = u\sin\theta + gt = 40\sin0 + 10(3) = 30m/s$

$V = \sqrt{(V^2_x + V^2_y)} = \sqrt{(40^2 + 30^2)} = 50m/s$

EXAMPLE 5 : referring to example 4 above, what are the vertical and horizontal displacement at touch down?

SOLUTION :

$S_y = u(\sin\theta)t - \frac{1}{2}gt^2 = 40\sin0 + \frac{1}{2}\times10\times3^2 = 45m$

$S_x = u(\cos\theta)t = 40(\cos0)\times3 = 120m$

EXAMPLE 6 : a body is projected horizontally from the top of a cliff with a velocity 30m/s. if the body hits the ground with a velocity of 50m/s, calculate the time of flight.

SOLUTION : **$\theta = 0^o$ (horizontally),** u= 30m/s

V = 50m/s

$V_x = u\cos\theta = 30\cos 0 = 30m/s$

$V = \sqrt{(V^2_x + V^2_y)}$, $50 = \sqrt{(30^2 + V^2_y)}$

Square both sides : $50^2 = 30^2 + V^2_y$

$25000 = 900 + V^2_y$, $V^2_y = \sqrt{1600} = 40m/s$

But : $V_y = u\sin\theta + gt$, $40 = 30\sin 0 + 10t$

$40 = 0 + 10t$, $t = 40/10 = 4s$

EXAMPLE 7 : referring to example 6 above, calculate the height of the cliff

HINT: $S_y = u(\sin\theta)t + \frac{1}{2}gt^2$, **answer: 80m**

EXAMPLE 8 : a body projected horizontally with a speed of 30m/s from the top of a cliff is observed to touch the ground with a speed of 50m/s. calculate : (a) the height of the cliff (b) the horizontal distance covered from the cliff

SOLUTION : u = 30m/s, V = 50m/s

Since time is not given : $V^2_x = u^2\cos^2\theta$

$V^2_x = 30^2\cos^2(0) = 900$, $V_x = \sqrt{900} = 30m/s$

$V = \sqrt{(V^2_x + V^2_y)}$, $50 = \sqrt{(30^2 + V^2_y)}$

Square both sides : $50^2 = 30^2 + V^2_y$

$25000 = 900 + V^2_y$, $V^2_y = \sqrt{1600} = 40m/s$

$V^2_y = u^2\sin^2\theta - 2gS_y$, $40^2 = 30^2\sin^2 0 - 2\times 10\times S_y$

1600 $= 0 + 20S_y$, $S_y = 1600/20 = 80m$

When a body is fired vertically upward, $\theta = 90^o$

EXAMPLE 9 : a rocket is fired vertically upwards with a velocity of 70m/s. calculate the time taken to reach a height of 12000cm on the upwards journey.

SOLUTION : $\theta = 90^o$, $S_y = 12000 = 120m$,

$V^2_y = u^2\sin^2\theta - 2gS_y = 70^2\sin^2 90 - 2\times 10\times 120$

$V^2_y = 2500$, $V_y = \sqrt{2500} = 50m/s$

$V_y = u\sin\theta - gt$, $50 = 70\sin 90 - 10t$

$10t = 70 - 50$, $t = 20/10 = 2s$

Or just **$S = \frac{(V+u)t}{2}$, u = 70m/s, V = 50m/s**

EXAMPLE 10 : during a football match, the ball kicked at 45^o angle of elevation went just over the post, height 2.4m. assuming the goal post is the greatest height, calculate : (a) the speed at which the ball was projected (b) the time taken to reach the greatest height (c) the horizontal distance between the point of kick and the foot of the goal post bar(neglect the thickness of the bar)

SOLUTION : H_{max} = 2.4m, $\theta = 45$,

a. $H_{max} = \frac{u^2\sin^2\theta}{2g}$, $2.4 = \frac{u^2(\sin 45)^2}{2\times 10}$

$\frac{2.4}{1} = \frac{u^2(0.49999)}{20}$, $u^2 = \frac{48}{0.49999}$, $u^2 = 96.001$

$u = \sqrt{96.001} = 9.798m/s$

b. $t = \frac{u\sin\theta}{g} = \frac{9.798\sin 45}{10} = 0.69s$

c. $R = \frac{u^2\sin 2\theta}{g} = \frac{9.798^2\sin(2\times 45)}{10}$

$R = \dfrac{9.798^2 \sin 90}{10} = 9.6m$

but we need is half of range = 9.6/2=4.8m

RELATIVE VELOCITY

Here, you are required to think deeply

EXAMPLE 11 : A person riding in the back of a pickup truck travelling at 70km/h on a straight, level road throws a ball with a speed of 15km/h relative to the truck in the direction opposite its motion. What is the velocity of

The ball : (a) relative to a stationary observer by the side of the road (b) relative to a driver of a car moving in the same direction as the truck at a speed of 90km/h.

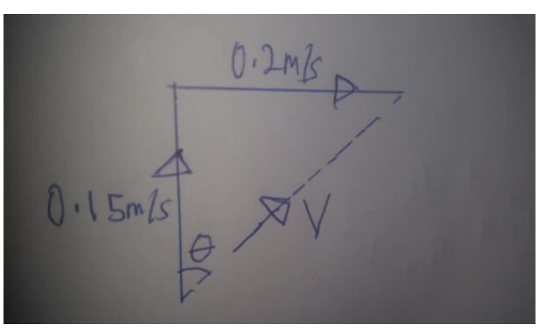
SOLUTION:

a. because the velocity of the car does not increase or decrease the velocity of the ball, thus, the observer sees the ball moving with its speed . **answer = 15m/s**

b. 90 + 15 = 105km/hr

EXAMPLE 12 : a swimmer swims north across a river that flows at 0.2m/s from west to east. If the speed of the swimmer is 0.15m/s relative to still water. What is the swimmer's velocity relative to the river bank?

SOLUTION :



Using Pythagoras theorem in mathematics

$V^2 = \sqrt{0.2^2 + 0.15^2} = 0.0625$

$V = \sqrt{0.0625} = 0.25m/s$

Direction is : $\tan\theta = \frac{opposite}{adjacent} = \frac{0.21}{0.15} = 1.4$

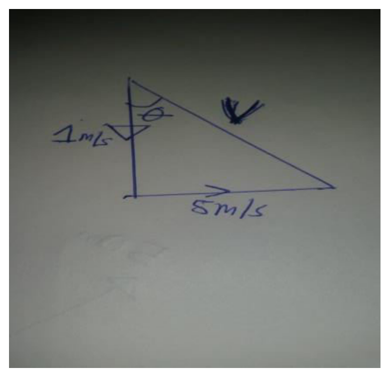
$\theta = \tan^{-1}(1.4) = 53.1^o$ i.e N53.1^{o}E

EXAMPLE 13 : a river has a current with a velocity of 1m/s south. A boat whose speed in still water is 5m/s is directed east across the 100m wide river. (a) how long does it take the boat to reach the opposite shore? (b) how far downstream will the boat land? (c) what is the velocity of the boat relative to the shore?

SOLUTION : (a)$V = \frac{d}{t}$, $5 = \frac{100}{t}$, thus, t = 20m/s

b. d = Vt =1(20) = 20m

c.



$V = \sqrt{5^2 + 1^2} = 5.1m/s$

$\theta = \tan^{-1}(5/1) = 79^o$

EXAMPLE 14 : if a person on the boat in example 13 above want to travel directly across the river: (a) what angle upstream must the boat be directed? (b) with what speed will the boat cross the river ? (c) how long will it take the boat to reach the opposite shore?

SOLUTION :

Using trigonometry, $\theta = \sin^{-1}(1/5) = 12^o$ upstream from a straight line across the the river

b. $5 = \sqrt{V^2 + 1^2}$ square both sides

$25 = V^2 + 1$, $V^2 = 24$, $V = 4.9m/s$

c. $t = d/v = 100/4.9 = 20s$

take note : $\mathbf{gT^2 - 2u\sin\theta + 2(h_f - h_i) = 0}$**,** in physics

CHAPTER 5

FORCE AND NEWTON'S LAWS OF MOTION

Forces are classifies as :

1. contact force : they involve direct contact between two bodies. E.g tension, frictional force, fluid resistance

2. field or long-range force : they act without the bodies involved getting in contact. E.g a stone released from a height will fall to the ground because of force of gravity (gravitational force) , magnetic force, electric force.

FUNDAMENTAL FORCES

There are four fundamental forces in nature :

1. gravitational interaction : it has dominant effect in the motion of planets and in the internal structure of stars.

2. electromagnetic interaction : it comprises both electric and magnetic forces and are both respectively strong. **It is responsible for holding atoms and molecules together and for the structure of matter.**

3. strong interaction of nuclear force : for large number of like charges (proton-proton) to be confined to such a small portion, an attraction force stronger than the electrostatic(columbic) repulsion is required, this force is very strong. This force is responsible for **holding constituents of an atomic nucleus (the nucleon-proton and the neutrons) together.**

4. weak interaction : it is responsible for the common form of radioactivity called beta decay.

NEWTON'S LAWS OF MOTION

Newton's laws fail to apply to :

1. quantum mechanics : the motion of very small objects(quantum particle of the atom and nucleus)

2. relativity : the motion of very fast moving objects (objects moving at the speed close to the speed of light.

Newton' first law : it is also called the law of **inertia**. *Inertia is the tendency of bodies to remain in their state of rest or of uniform linear motion in the absence of applied forces.* **Newton's first law states** *that every object continue in its state of rest or of uniform motion in a straight line unless acted upon by an external force.* **This law explains why passengers should belt up in a vehicle.**

Newton's second law : *it states that the rate of change of momentum is proportional to the impressed force and takes place in the direction of that force.*

$F = \frac{m(v-u)}{t}$ *thus* $\boldsymbol{F = ma}$

Ft = m(v – u)

thus, change in momentum =impulse

MASS AND WEIGHT

Mass is the quantity of matter present in a body. It is a measure of inertia. **Mass is measured with a chemical balance or a beam malance.** Weight is the product of its mass and gravitational acceleration. **W = mg. weight is measured with a spring balance.**

Newton's third law : *it state that to every action there is an equal and opposite reaction.*

SOME APPLICATIONS OF NEWTON'S LAWS

1. A BODY IN A LIFT (ELEVATOR) : two forces act on a man in a lift – the man's true weight (W = mg) and the reaction (R) of the ground on the man acting upwards.

Case 1 : *when the elevator is stationary or moves with a constant velocity (a = 0)*

W = R = mg. hence, there is no net force on the man. If the man is standing on a spring scale, the scale will read his true weight.

Case 2 : *when the lift accelerates upwards with an acceleration (a).* the unbalanced force is given by F = ma = R – mg, thus **R = m(g + a)**

Thus, man's apparent weight recorded by the scale will be more than the true weight of the man

Case 3 : *if the elevator accelerates downwards with an acceleration, a.* the unbalanced force on the man is F = ma = mg-R, thus **R = (g – a)**

Hence, the apparent weight of the man will be less than his true weight.

Case 4 : *elevator falling freely .* if the elevator descends with an acceleration **a = g,** we say the lift is falling freely. This happens when the cable is cut. From case 4 above, we see that

R = m(g – a) = 0, **thus , the man appears to have no weight – weightlessness.** The man and the floor are exerting no force on each other, **thus, the scale reads zero (0).**

EXAMPLE 1 : a 10kg box rests on a scale which is placed on the floor of an elevator. Find the apparent weight of the box if the elevator is : (a) moving with a constant velocity of 2m/s (b) acceleration of $5m/s^2$ downward (c) acceleration of $5m/s^2$ upwards.

SOLUTION : m = 10kg, $g = 10m/s^2$

a. for constant velocity,

R = mg = 10(10) = 100N

b. downwards, R = m(g – a)

R = 10(10 – 5) = 50N

c. upwards, R = m(g + a) = 10(10+5) = 150N

EXAMPLE 2 : an upward force of 1.8×10^4N acts on an elevator of mass 2.1×10^3kg. calculate the elevator's acceleration.

SOLUTION : $g = 10m/s^2$, $R = F = 1.8\times 10^4$N

a = ?, $m = 2.1 \times 10^3$kg, for upward movement,

R = m(g + a) **answer : -**$1.43m/s^2$

EXAMPLE 3 : a body hangs from a spring balance supported from the roof of an elevator. if the elevator has an upward acceleration of $3m/s^2$ and the balance reads 50N, what is the true weight of the body?

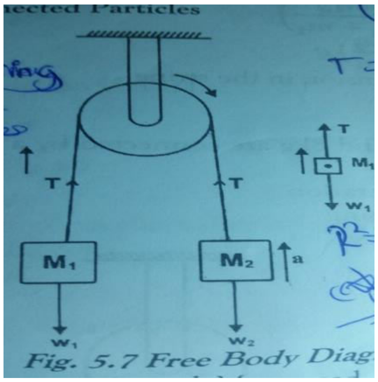
SOLUTION : R =m(g + a)

50 = m(10 + 3), 50 = 13m, m = 50/13 3.846kg

W = mg = 3.846(10) = 38.46N

2. MOTION OF CONNECTED PERTICLES : here, we assume that the string is light and inextensible.

Case 1 : *string passed over a pulley.*



T is the tension in the string

$$a = \frac{(m_1 - m_2)\,g}{m_1 + m_2}$$

$$T = \frac{2m_1 m_2\, g}{m_1 + m_2}$$

the force on the pulley, F = 2T

EXAMPLE 4 : two bodies of masses 5kg and 8kg are connected by a light string passing over a smooth pulley. Find (a) their common acceleration (b) the tension in the string (c) the force on the pulley

SOLUTION :

a. $a = \frac{(m_1 - m_2)\,g}{m_1 + m_2} = \frac{(8-5)10}{5+8} = \frac{3 \times 10}{13} = 2.3\text{m/s}^2$

b. $T = \frac{2m_1 m_2\, g}{m_1 + m_2} = \frac{2 \times 5 \times 8 \times 10}{5+8} = 61.54\text{N}$

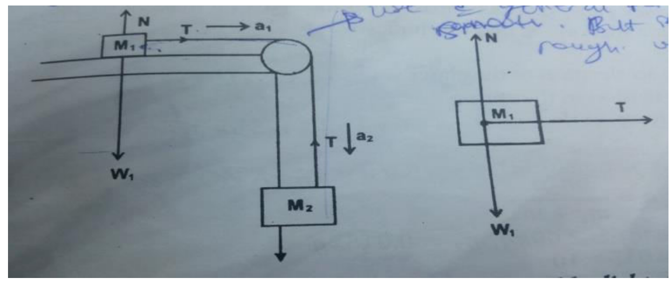
c. $F = 2T = 2 \times 61.54 = 123.08\text{N}$

now, do the following, its same way.

EXAMPLE 5 : two particles of masses 7kg and 9kg are connected by a light inextensible string passing over a smooth fixed pulley. find the force on the pulley. $g = 10\text{m/s}^2$ **T =154.35N**

Case 2 : *block and cord system (air-track glider)*

NOTE : The mass on the table is the air-track glider



$$a = \frac{m_2 g}{m_1 + m_2}$$

$$T = \frac{m_1 m_2 g}{m_1 + m_2}$$

$F = T\sqrt{2} = 1.414T$

EXAMPLE 7 : if an air-track glider of mass 0.015kg with a string over a frictionless, massless pulley, find (a) the acceleration of the glider (b) the tension in the string (c) the force on the pulley..

SOLUTION :

a. $a = \frac{m_2 g}{m_1 + m_2} = \frac{0.015 \times 10}{0.015 + 2} = 0.074\text{m/s}^2$

b. $T = \frac{m_1 m_2 g}{m_1 + m_2} = \frac{2 \times 0.015 \times 10}{2 + 0.015} = 0.15\text{N}$

c. $F = T\sqrt{2} = 0.15\sqrt{2} = 0.12\text{N}$

now, do the following. It's the same way

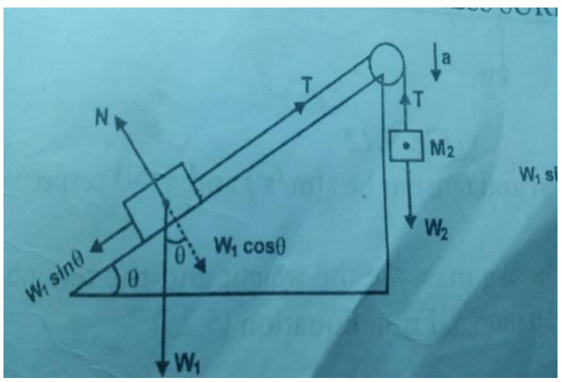
EXAMPLE 8 : an air-track glider of mass 2kg is connected to a mass of 5kg with a string over a frictionless, massless pulley. what is the force on the pulley. $g = 10m/s^2$ **T = 20.2N**

If the table is rough, coefficient of friction is involved, then we will have to resolve and solve simultaneously:

$$W_2 - T = m_2 a$$

$$T - \mu R = m_1 a$$

Case 3 : *a mass on an inclined plane and another hanging freely.*



$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2}$$

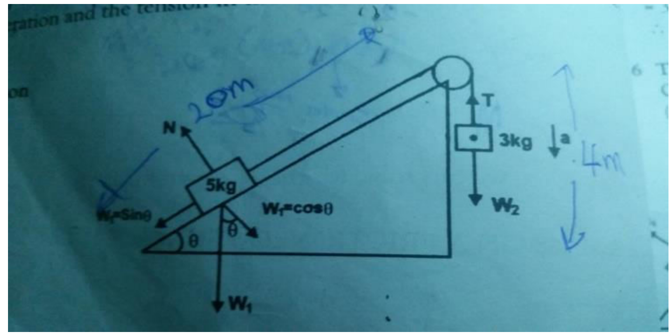
$$T = \frac{m_1 m_2 (1 + \sin\theta)g}{m_1 + m_2}$$

or $T = 3g - 3a$

in this case 3, dimensions are always given

EXAMPLE 9 : a particle of mass 5kg is placed on a smooth plane whose height is 4m and length 20m. the particle is connected by a light string passing over a smooth pulley at the top of the plane to a mass 3kg hanging freely. Find the common acceleration and the tension in the string.

SOLUTION :



dimensions (length and height) given will help us to find the angle

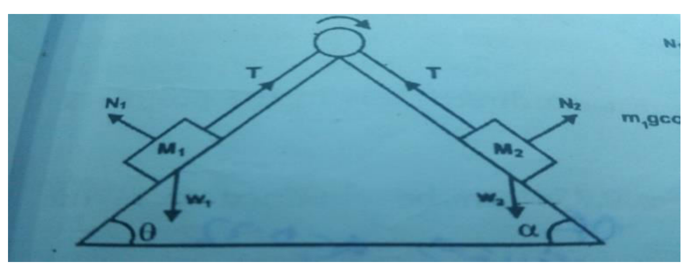
$\sin\theta = \frac{4}{20} = 0.2$, $\theta = \sin^{-1}(0.2) = 11.54^o$

$$a = \frac{(m_2 - m_1 \sin\theta)g}{m_1 + m_2} = \frac{\{3 - 5\sin(11.54)\}10}{5 + 3} = 2.5m/s^2$$

$T = 3g - 3a = 3\times 10 - 3\times 2.5 = 22.5N$

EXAMPLE 10 : A particle of mass 2500g is placed on a smooth plane of height 4m and length 20m. the particle is connected by a light string passing over a smooth pulley at the top of a the plane to a mass of 1500g hanging freely. Calculate the common acceleration and tension of the string. **Do not forget to convert the masses to kg. $a = 2.45m/s^2$, T = 22.5N**

Case 4 : *two bodies resting on different inclined plane connected by a light string passing through a pulley.*



$$a = \frac{(m_2 \sin\theta_2 - m_1 \sin\theta_1)g}{m_1 + m_2}$$

the distance covered at a time t is

$$S = ut + \frac{1}{2}at^2$$

In this case 4, two angles are given

EXAMPLE 11 : two blocks at rest on a frictionless inclined surface with mass 3kg on a side inclined at 42^o to the horizontal and mass 4.5kg on a side inclined at 30^o to the horizontal. If the two bodies are attached together by an inextensible string passed through a pulley. (a) determine the acceleration and the direction the blocks will move (b) how far do the blocks move in 0.75s after release?

SOLUTION :

$$a = \frac{(m_2\sin\theta_2 - m_1\sin\theta_1)g}{m_1 + m_2} = \frac{(4.5\sin30 - 3\sin42)10}{3 + 4.5}$$

$a = 0.32m/s^2$

b. t = 0.75s, u = 0 (from rest)

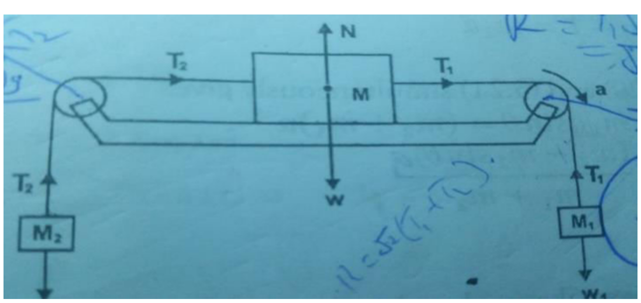
$$S = ut + \frac{1}{2}at^2 = 0 \times 0.75 + \frac{1}{2} \times 0.32 \times 0.75^2 = 0.09m$$

EXAMPLE 12 : two blocks at rest on an inclined frictionless plane connected by light

String passed through a pulley. find the acceleration of the system if the first mass is 5kg at an angle 30^o and the second mass is 8kg at an angle 60^o. **$a = 3.41m/s^2$**

Case 5 : *three masses*

Where $m_2 > m_1$ and m_3 is the mass on the table (i.e the glider)



$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + m_3}$$

here, we have two tensions

$T_2 = m_2(g - a)$ the tension of the bigger mass

$T_1 = m_1(g + a)$ tension of the smaller mass

EXAMPLE 13 : a 0.84kg glider on a level air truck is joined by a string to two hanging masses m_1 = 4.85kg, and m_2 = 3.62kg. the string has negligible mass and passed over a light , frictionless pulley (a) find the acceleration of the masses (b) the tensions in the strings. ($g = 9.8m/s^2$)

SOLUTION :

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + m_3} = \frac{(4.85 - 3.62)9.8}{4.85 + 3.62 + 0.84} = 1.3m/s^2$$

to the right

b. $T_2 = m_2(g - a) = 4.85(9.8 - 1.3) = 41.3N$

$T_1 = m_1(g + a) = 3.62(9.8 + 1.3) = 40.2N$

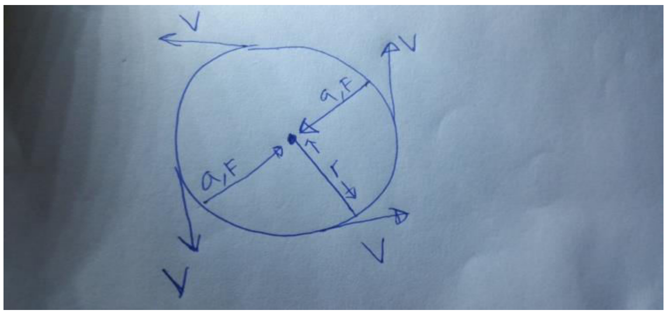
CHAPTER 6

CIRCULAR MOTION AND GRAVITATION

The motion of objects in a circular path is with **constant velocity** is called **uniform circular motion.** These objects are accelerating because their velocity is changing in either magnitude or direction hence, unbalanced force must act on them to keep them moving in a circular path with a constant **speed.**

UNIFORM CIRCULAR MOTION AND CENTREPETAL ACCELERATION(a_c)

A body moving in a circular path with a constant speed is said to be moving with uniform circular motion. The velocity of the object is changing because its direction is changing, but its speed remains constant. Therefore the object is acceleration even though its magnitude is not acceleration. **The acceleration and force are acting towards the centre of the circle.**

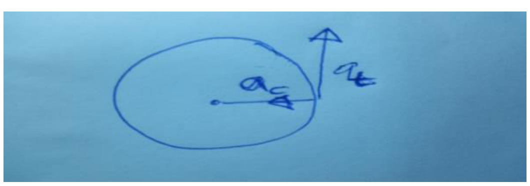


$$a_c = \frac{V^2}{r} = Vw = w^2r$$

V is he linear velocity or the tangential velocity. *It is the velocity with which the object will fly out when the string cuts.*

$V = wr$, $w = 2\pi f$, $T = \frac{2\pi}{w}$ $a_t = \alpha r$

a_t and α are tangential acceleration and angular acceleration respectively



EXAMPLE 1 : a racing car travels round a circular racetrack that has a radius 100m. if the car travels with a constant speed of 100km/h, find the magnitude of its centripetal acceleration.

SOLUTION: r = 100m, V = 100km/h =27.28m/s

a = ?,

$$a_c = \frac{V^2}{r} = \frac{27.28^2}{100} = 7.7m/s^2$$

EXAMPLE 2 : a communication satellite in a circular orbit moves round the earth at an altitude of 5×10^2km. the satellite makes one complete revolution every 90min. (a) what is its tangential velocity (b) what is its centripetal acceleration

SOLUTION : radius of the earth, $R_e = 6400$km

$h = 5 \times 10^2$km, since it is above the earth, then

$r = 5 \times 10^2 + 6400 = 6.9 \times 10^3 km = 6.9 \times 10^6 m$

$f = 1/90min = 1/(90 \times 60) = 1.851\times 10^{-4}Hz$

$w = 2\pi f = 2\pi \times 1.851\times 10^{-4} = 1.16\times 10^{-3}rad/s$

a. $V=wr = 1.16\times 10^{-3} \times 6.9 \times 10^6 = 8028.5m/s$

b. $$a_c = \frac{V^2}{r} = \frac{8028.5^2}{6.9 \times 10^6} = 9.63m/s^2$$

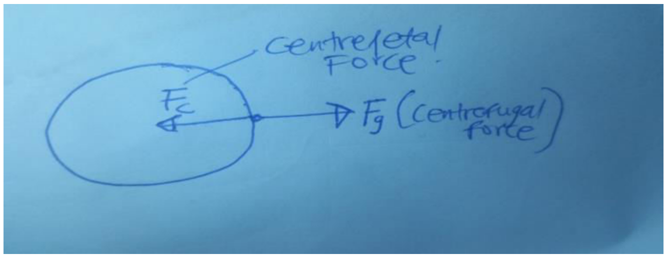
EXAMPLE 3 : if a body of mass 0.5kg is whirled in a horizontal circle at the rate of 1000 revolution per minute. Determine the angular velocity. **W = 104.72rad/sec**

HINT : do not use the mass, it is not needed

CENTRIPETAL FORCE (F)

It is an inward force required to keep an object with a constant speed in a circular path. Thus, anyone or the combination of the forces : **friction, tension in a string, gravity, magnetic force, or electric force,** may act as a centripetal force. *Since the centripetal force must be applied to a body to restrain or keep it moving in a circular path, there must be according to Newton's third law of motion an* ***equal and oppositely directed force. This***

force is called CENTRIFUGAL FORCE.



$$\mathbf{F = \frac{mV^2}{r} = mw^2r}$$

anytime coefficient of friction is involved in the question, then, $F = \mu R$ where $R = mg$

EXAMPLE 4 : find the centripetal force required by 2000N racing car that makes a turn of radius 90m at a speed of 10km/h. also calculate the minimum coefficient of friction that must be present if the car is to make the turn without skidding .

SOLUTION : r = 90m, V = 10km/h = 2.78m/s

W =2000N

W =mg , 2000N = m × 10 , m = 200kg

a. $F = \frac{mV^2}{r} = \frac{200 \times 2.78^2}{90} = 17.2N$

b. $F = \mu R$, $\mu = \frac{F}{R} = \frac{F}{mg} = \frac{17.2}{200 \times 10} = 8.6 \times 10^{-3}$

EXAMPLE 5 : calculate the force necessary to Keep a particle of mass 0.2kg moving in a horizontal circle of radius 0.5m with a period of 0.5sec. what is the direction of the force.

SOLUTION : T = 0.5s, m = 0.2kg, r = 0.5m

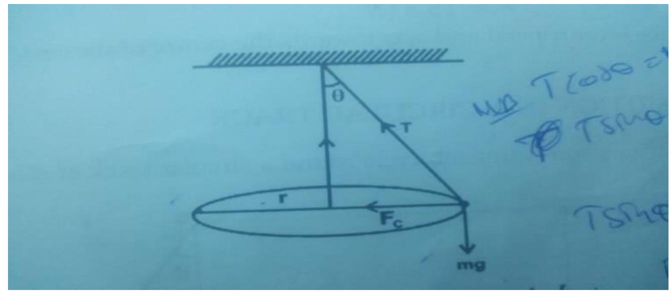
$T = \frac{2\pi}{w}$, $0.5 = \frac{2\pi}{w}$ cross multiply, w = 12.567rad/s

$F = mw^2r = 0.2 \times 12.567^2 \times 0.5 = 15.8N$

The direction : ***towards the centre of the circle***.

CONICAL PENDULUM

(horizontal circle)



$\mathbf{T\cos\theta = mg}$

$$\mathbf{T\sin\theta = \frac{mV^2}{r}}$$

$\boldsymbol{\theta = \tan^{-1}(V^2/gr)}$ this is the formula used when looking for the angle.

$\mathbf{r = L\sin\theta}$, $\mathbf{w = \sqrt{\frac{g}{L\cos\theta}}}$, $\mathbf{T = \frac{2\pi}{w}}$

EXAMPLE 6 : an object of mass 10kg is whirled round a horizontal circle of radius 4m by a revolving string inclined to the vertical. If the uniform speed of the object is 5m/s. calculate (a) the angle of inclinaton of the string to the vertical. (b) the tension in the string.

SOLUTION : V = 5m/s, m = 10kg, r = 4m

a. $\theta = \tan^{-1}(V^2/gr) = \tan^{-1}(5^2/10 \times 4) = 32^o$

b. $T\cos\theta = mg$, Tcos32 = 10 × 10,

T = 100/cos32 = 117.9N

EXAMPLE 7 : a body of mass 0.2kg is whirled round in a horizontal circle of radius 0.444m by a string inclined 30^o to the vertical. Find (i) the tension in the string (ii) the speed of the body in the circle. **T = 2.26N, V = 2.57m/s**

MOTION IN A CIRCULAR TRACK

Consider a cyclist ridding a bicycle round a circular track of radius, r. the cyclist leans inward.

$\theta = \tan^{-1}(V^2/gr)$, θ is the cyclist lean from the vertical. **θ increases if : (a) the speed V is higher (b) the radius is greater**

If **$\theta = \tan^{-1}(V^2/gr)$ skidding will not occur**

If $\theta > \tan^{-1}(V^2/gr)$ skidding will occur

MOTION OF A CAR IN A BANKED ROAD

If a car of mass m moving with a constant speed round a track banked at an angle θ**.** If R_1 and R_2 are the normal reactions of the wheels A and B, the centripetal force is provided largely by the normal reaction.

The formulae here is similar to those of the conical pendulum, the difference its just that anywhere you have *T in the conical pendulum put ($R_1 + R_2$)*

$$(R_1 + R_2)\cos\theta = mg$$

$$(R_1 + R_2)\sin\theta = \frac{mV^2}{r}$$

$\theta = \tan^{-1}(V^2/gr)$ this formula is used to find the angle

EXAMPLE 8 : to avoid the likelihood of skids, highway, curves are often banked so that the road tilts inward. Find the proper banking angle for a car making a turn of radius 100m at the speed of 15m/s

SOLUTION : r = 100m, V = 15m/s

$\theta = \tan^{-1}(V^2/gr) = \tan^{-1}(15^2/10 \times 100) = 12.7^o$

EXAMPLE 9 : a racing car of mass 1000kgmoves round a banked track at constant speed of 30m/s. if the total reaction at the horizontal radius of the track is 100m. calculate the angle of inclination of the track to the horizontal ($g=10m/s^2$) **$\theta = 42^o$**

GRAVITATION

Case 1 : Kepler's Laws. The laws state

1. the orbit of every planet is an ellipse with the sun at one of the two.

2. the line joining the sun and the planet sweeps out equal area in equal times.

3. the squares of the periods of revolution of

the planets are proportional to the cubes of their mean distances from the sun **$T^2 \alpha r^3$**

Case 3 : Newton's Universal Law

Gravitation is a universal interaction that occurs between all bodies by virtue of their possession of mass. *The law states : every objects in the universe attracts every other object with a force directly proportional to the product of their masses and inversely proportional to the square of the distance between them.*

$$F \alpha \frac{m_1 m_2}{r^2}$$

$$F = \frac{Gm_1m_2}{r^2}$$

G is the universal gravitational constant

$G = 6.67 \times 10^{-11}Nm^2kg^{-2}$ or $6.67 \times 10^{-11}m^3s^{-2}kg$

$$g = \frac{Gm}{r^2}$$

EXAMPLE 10 : two particles of masses 4kg and 8kg are separated by 2m. what is the gravitational interaction between them.

SOLUTION :

$$F = \frac{Gm_1m_2}{r^2} = \frac{6.67 \times 10^{-11} \times 4 \times 8}{2^2} = 5.3\times10^{-10}N$$

EXAMPLE 11 : from example 10 above, determine the approximate mass of the earth.

SOLUTION : recall : R_e = 6400km = 6.4×10^6m

$g_e = 10m/s^2$

$$g = \frac{Gm}{r^2} , \quad 10 = \frac{6.67 \times 10^{-11} \times m_e}{2^2}$$

cross multiply and proceeds, $\mathbf{m_e = 6 \times 10^{24}kg}$

objects weight decreases with altitude(h) because F α $1/r^2$. At the sea level, h = 0

Gravitational intensity(I_g) : it is also called **gravitational field strength**. It is the force per unit mass. $\mathbf{I_g = F/m}$

Satellite Motion : one natural satellite is the moon. Others are artificial

$\mathbf{V = \sqrt{(Gm_e/r)}}$, V is velocity of an object(e.g satellite) in an orbit or orbital velocity

NOTE: m_e is mass of the earth = 5.98×10^{24}kg

Radius of the earth, R_e = 6400km= 6.4×10^6m

the formula above implies that all satellite in circular orbits at a given altitude **must therefore have the same speed.**

Sometimes satellites move in elliptical orbits. The point of closest approach to the earth is called the ***perigee****, and the point of farthest from the earth is called the* ***apogee****. As the earth satellite moves in an elliptical orbit, it's orbital speed varies, it is greater at perigee and least at apogee.* ***When a continuous retarding force causes a satellite movement to orbits which have lower perigees, the satellite is said to undergo ORBITAL DECAY.***

$\mathbf{V = \sqrt{(R_eg)}}$ V is velocity of an object(e.g satellite) in an orbit or orbital velocity

Escape Velocity (V_e) : it is the minimum

velocity required for an object (e.g satellite or rocket) to just escape or leave the gravitational influence or field of an astronomical body (e.g the earth) permanently

$\mathbf{V_e = \sqrt{(2Gm_e/r)}}$ **and** $\mathbf{V_e = \sqrt{(2R_eg)}}$

If we put R_e = 6400km = 6.4×10^6m and g = 10

into $\mathbf{V_e}$ = $\sqrt{(2R_eg)}$ V_e will be **11km/s.**

Let V be the velocity at which the satellite is projected; for :

$V < V_e$ = 11km/s, object falls back to the earth

7.9 < V < 11km/s, satellite's orbit is elliptical

$V = V_e$ = 11km/s, satellite's orbit is circular

$V > V_e$, satellite escapes from the surface of the earth.

Orbital Period (T) : it is the time taken for a satellite to complete one revolution in its orbit

$\mathbf{T = 2\pi\sqrt{(r^3/Gm_e)}}$

Synchronous or Parking or Stationary Orbit : this occurs when a communication satellite is to be placed in an orbit so that it appears to be stationary over one point on the earth's surface. **It is called synchronous because the period of the satellite is equal to the period of the earth (T = 24hrs = *8.64×10^4seconds*)**

EXAMPLE 12 : what is the speed of a satellite in a circular orbit just above the surface of the earth if the radius of the earth is 6.4×10^6m, and g = $9.8m/s^2$.

SOLUTION :

$V = \sqrt{(R_e g)} = \sqrt{(6.4 \times 10^6 \times 9.8)} = 7.92 \times 10^3 m/s$

EXAMPLE 13 : an artificial earth satellite is placed in a circular orbit 630km above the earth's surface. What orbital speed is required of this attitude so that the satellite remains in a stable circular orbit? Hence find it's period.

HINT : h = 630km = 630000m, $R_e = 6.4 \times 10^6$m,

$r = R_e + h = 6.4 \times 10^6 + 630000 = 7030000$m

$m_e = 5.98 \times 10^{24}$kg, **use** $V = \sqrt{(Gm_e/r)}$ **and**

$T = 2\pi\sqrt{(r^3/Gm_e)}$, **V = 7.54×10^3m/s, T =5864s**

This question is very simple, just do it

EXAMPLE 14 : a communication satellite is to be placed in an orbit so that it appears stationary over one point on the earth's surface. What is the altitude required for this type of satellite?($m_e = 5.98 \times 10^{24}$kg, $R_e = 6.36 \times 10^6$m)

HINT : if it appears stationary, $T = 8.64 \times 10^4$s

Use $T = 2\pi\sqrt{(r^3/Gm_e)}$ to find r. then,

Use **$r = R_e + h$ find h.** **$h = 3.59 \times 10^4$m**

EXAMPLE 15 : if the period of the moon about the earth is 2.75days. calculate the acceleration of the moon if the radius of the circular path is 3.8×10^5km.

SOLUTION : $T = 2.75 \times 5 \times 24 \times 3600 = 2.376 \times 10^6$s, $r = 3.8 \times 10^5km = 3.8 \times 10^8$m

$W = \frac{2\pi}{T} = 2\pi/(2.376 \times 10^6) = 2.64 \times 10^{-6}$rad/s

$a = w^2 r = (2.64 \times 10^{-6})^2 \times (3.8 \times 10^8) = 2.66 \times 10^{-3} m/s^2$

EXAMPLE 16 : at what altitude on the earth's surface will the acceleration due to gravity be $4.9m/s^2$ (take the mean radius of the earth to be 6.4×10^6m, and g = $10m/s^2$)

HINT : $g_e = \frac{Gm_e}{r_e^2}$ WHERE $g_e = 9.8m/s^2$

use the formula above to find $\mathbf{r_e}$ **.** the use $g = Gm/r^2$ to find **r.** lastly, use **$r = r_e + h$** to find h

answer : $h = 2.65 \times 10^6$m

EXAMPLE 17 : a satellite is said to be at a distance where it revolves about the earth with angular velocity equal to that at which the earth rotates, so that it always remains above the same point on earth. Find the radius of the orbit if the mass of the earth is 5.98×10^{24}kg)

SOLUTION : from $T = \frac{2\pi}{w}$, if their angular velocities are equal, them it means their periods are equal. And if it remains above the same point on earth, then **$T = 8.64 \times 10^4$s**

Use $T = 2\pi\sqrt{(r^3/Gm_e)}$. **ans: $r = 4.22 \times 10^7$m**

CHAPTER 7

WORK ENERGY AND POWER

Work : it is the product of force and displacement. **W = Fd, W = mgh.**

If an object of mass m is displaced a distance d , by a force F, inclined to the horizontal at an angle θ, then the work done **$W = Fd\cos\theta$**

Work done by a variable force

$W = \frac{1}{2}kx^2 = \frac{(F_1 + F_2)x}{2}$

Note : work done on a graph is the area of the shape in that graph.

Power : it is the rate of work done. It is the measure of how far work is done by a force.

$P = \frac{work\ or\ energy}{time} = \frac{W}{t} = \frac{F \times d}{t} = Fv,$

V is velocity

EXAMPLE 1 : n a force of 300N acts on an object for 30s, causing it to move 50m in the direction of the applied force. find the power expended by the force.

SOLUTION : $p = \frac{F \times d}{t} = \frac{300 \times 50}{30} = \mathbf{500W}$

EXAMPLE 2 : a tugboat tows a ship at a

constant speed of 50km/h. if the tension in the t**ow** rope remains at a steady value of 200N, find the power output of the tugboat's motor in : (i) kilio watts (ii) Horsepower

SOLUTION : V = 50km/h = 13.89m/s

P = Fv = 200 $\times$ 13.89 = 2778W divide by 1000 to convert to kilo watts, **thus p = 2.778W**

ii. **1hp = 746W**

x = 2778W , x = 3.7hp

EXAMPLE 3 : a Chicago marathon runner with a mass 50kg runs up the stairs t the top of a 443m tall tower. In order to lift herself to the top in 15min, what must be her average power output in watts ? in hoursepower**?**

Answer : 241W, 0.323hp.

EXAMPLE 4 : A woman weighing 600N steps on a bathroom scale containing a stiff spring. In equilibrium, the spring is compressed at 1cm under the weight. Find the force constant of the spring and the total work done on it during the compression.

HINT : F = kx, and $W = \frac{1}{2}kx^2$, 60000N/m, 3j

A case involving Newton's equation(motion)

EXAMPLE 5 : A man throws a ball that leaves his hand at a speed 32m/s. the mass of the ball is 0.25kg. ignore air resistance. How much work has the man done on the ball in throwing it **?**

SOLUTION : V = 32m/s, u = 0 (throws a ball)

$V^2 = u^2 + 2gh$, $32^2 = 0^2 + 2 \times \mathbf{9.8} \times h$, h = 52.2m

W = mgh = 0.25 $\times$ **9.8** $\times$ **52.2 = 128j**

EXAMPLE 6 : a baseball of mass 0.145kg is

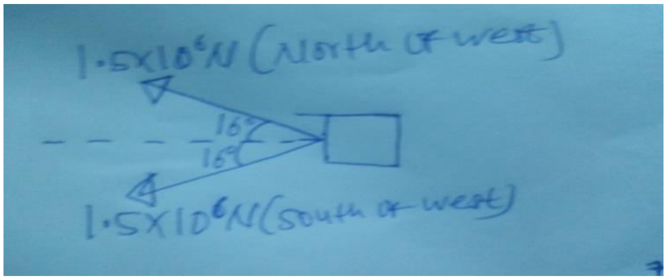
Thrown straight up in the air, giving it an initial upward velocity of magnitude 20m/s. use conservation of energy to find how high it goes.

SOLUTION : how high it goes, i.e maximum height

$V^2 = u^2 - 2gh$, $0^2 = 20^2 - 2 \times \mathbf{9.8} \times h$, h = 20.4m

EXAMPLE 7 : two tugboats pull a disabled supertanker. Each tug exerts a constant force of 1.5×10^6N, one 16^o north of west and the other 16^o south of west, as they pull the tanker 0.65km towards the west. What is the total work they do on the supertanker.

SOLUTION : d = 0.65km = 650m



Resolve each force along the x-axis the we get:

Force at N of W=$1.5\times10^{6}\cos16 = 1.442 \times 10^{6}$N

Force at S of W = $1.5\times10^{6}\cos16=1.442 \times 10^{6}$N

Total force F = $1.442 \times 10^{6} + 1.442 \times 10^{6}$

F = 2884000N

W = Fd = 2884000 $\times$ **650 =** 1.87×10^{9}j

EXAMPLE 8 : to compress a spring 4cm from its unstretched length , 12j of work must be done. How much work must be done to stretch the same spring 3cm from its unstretched length?

HINT : since it is the same spring then ,

$K_1 = K_2$ (their spring constant are equal)

Fins K_1 using $W_1 = \frac{1}{2}k_1{x_1}^2$ and put the value you get as k_2 in $W_2 = \frac{1}{2}k_2{x_2}^2$ to find w_2

Ans : 6.75j

WORK AND ENERGY

Energy is the capacity to do work. It can be classified into :

1. kinetic energy : it is the energy an object possess by virtue of its motion.

K.E = $\frac{1}{2}mv^2$

EXAMPLE 9 : find the k.E of a person of mass 80kg running with a speed of 5m/s

SOLUTION : use the formula above. **Ans : 1000j**

EXAMPLE 10 : a 2000kg pickup truck is moving at a speed of 90km/hr. how fast does a 1000kg sport car has to be going in order to have the same K.E as the pickup truck.

SOLUTION : m_1 = 2000kg, v_1 = 90km/h=25m/s

M_2 = 1000kg, v_2 =?

$\cancel{\frac{1}{2}}m_1{v_1}^2 = \cancel{\frac{1}{2}}m_2{v_2}^2$ (same K.E)

$2000 \times \mathbf{25^2} = \mathbf{1000} \times \mathbf{{v_2}^2}$, ${v_2}^2 = \sqrt{\frac{1000}{1280}}$ =35.4m/s

2. potential energy : it I the amount of energy possessed by a body by virtue of its position(height)

P.E = mgh

EXAMPLE 11 : a pile driver is slowly raised to a height of 1.5m above the ground. If the work

done by the raising mechanism is 115kj, find the mass of the pile driver.

SOLUTION : p.E = 115kj = 115×10^{3}j

P.E = mgh, 115×10^{3} = m $\times$ **10** $\times$ **1.5,**

h = 7.67×10^{3}kg.

CHAPTER 8

MOMENTUM (P)

it is the product of a body's mass and its velocity. Its unit is kgm/s of Ns.

P = mv

EXAMPLE 1 : what is the magnitude of the momentum of a 10000kg truck whose speed is 15m/s?

SOLUTION: P=mv=10000 × 15= 1.5 × 10^5kg/s

NOTE :

1. net momentum, $p_{net} = p_1 + p_2$ (i.e the sum of momenta {plural of momentum})

2. change in momentum $\Delta p = mv - mu = m(v-u)$

EXAMPLE 2 : a baseball of mass 0.145kg is moving in the +ve x-direction with a speed of 3.4m/s, and a tennis ball of mass 0.057kg is moving in the –ve x-direction with a speed of 6.2m/s. what are the magnitude and direction of the total momentum of the system consisting of the two balls?

SOLUTION : $p_b = m_b v_b$ = 0.145 × 5.4 = +0.493kgm/s

V_t = -6.2m/s(negative x-direction)

$P_t = m_t v_t$ = 0.057 × (-6.2) = -0.3534kgm/s

Total momentum = $p_b + p_t$ = 0.493+(-0.14)=0.14kgm/s

The direction of the total momentum of the system is the + x-direction.

MOMENTUM AND IMPULSE

Impulse is the product of the average force acting on a particle and the time during which it acts. **I = Ft.** it has the same unit as momentum. It is a vector

F = $\frac{m(v-u)}{t}$**,** if we cross multiply,

Ft = m(v – u) *therefore,*

Impulse (Ft) = change in momentum{m(v-u)}

EXAMPLE 3 : a car with **a mass** of 1350kg travelling at a speed of 80km/h is braked to a quick emergency stop in 4.5sec. find (a) the impulse (b) the average force the seat belt exerts on the 80kg driver.

SOLUTION : u = 80km/hr = 22.2m/s, v= 0(stop)

t = 4.5s, since Ft = m(v-u) then

I =m(v – u) = 1350(0 – 22.2) = -3000Ns

b. m = 80kg, f = ?

F = $\frac{m(v-u)}{t}$ = $\frac{80(0-22.2)}{4.5}$ = -39.5N

EXAMPLE 4 : High-speed photography reveals that when a bat strikes a baseball, a typical collision time is about 0.25s. if the speed of 35m/s is imparted to a ball of mass 0.325kg, what average force is exerted by the bat?

USE F = mv/t ……….. Answer : 45.5N

EXAMPLE 5 : (a) what is the magnitude of the momentum of a 10000kg truck whose speed is 15m/s? **(b)** what speed must a 5000kg truck have in order to have :

(i) the same momentum (ii) same **K.E**

HINT : (a) p = mv answer : 1.5 × 10^5kgm/s
(b) : (i) $m_1v_1 = m_2v_2$ answer : 30m/s

ii. $\cancel{\frac{1}{2}}m_1v_1 = \cancel{\frac{1}{2}}m_2v_2$ substitute. Ans: 21.2m/s

EXAMPLE 6 : a ball of mass 0.4kg is thrown against a brick wall. It hits the wall moving horizontally to the left at 30m/s and rebounds to the right at 20m/s. (i) find the impulse of the net force on the ball during its collision with the wall. (ii) if the ball is in contact with the wall for 0.01s, find the average horizontal force that the ball exerts on the wall during the impact.

SOLUTION : m = 0.4kg, u = -30m/s (left)

v = 20m/s, we know that the momentum before and after collision is p_1 and p_2 respectively

P_1 = mu =0.4 × -30 = -12kgm/s

$P_2 = mv = 0.4 \times 20 = 8$kgm/s

Change in momentum : $\Delta p = p_2 - p_1 = 8 - (-12) = 20$kgm/s . recall $I = \Delta p = 20$kgm/s

ii. $F = \frac{m(v-u)}{t} = \frac{20}{0.01} = 2000$N

EXAMPLE 7 : a 2kg ball of ice is moving on a frictionless horizontal surface. At t = 0, the block is moving to the right with a velocity of magnitude 3m/s. Calculate the velocity of the block (magnitude and direction) after a force of 5N directed to the right has been applied for 4s. use $F = \frac{m(v-u)}{t}$ **ans: 13m/s , right**

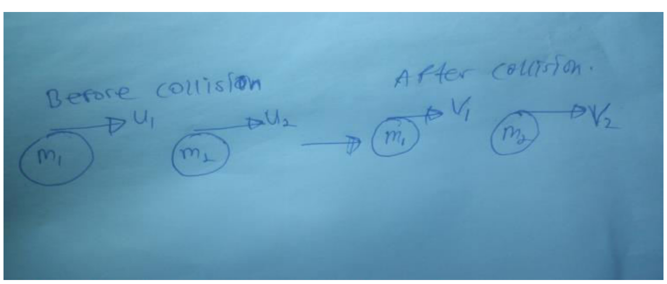
CONSERVATION OF MOMENTUM

The principle of conservation of linear momentum states that if two or more bodies collide in a closed system, the total momentum before collision is equal to the total momentum after collision. It was derived from Newton's 2nd and 3rd laws

COLLISIONS

It is classified into categories :

1. Elastic Collision : here, the total momentum and total K.E is the same before and after collision. **Momentum and K.E are conserved.**



From the diagram $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

And according to **conservation of momentum**

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

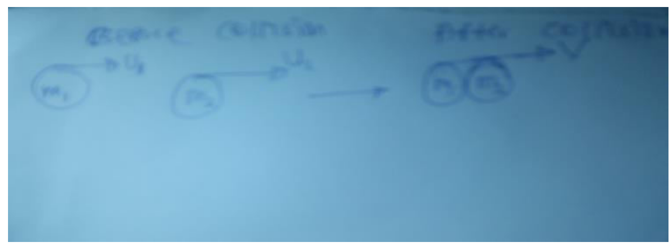
2. Inelastic Collision : here, the total K.E after the collision is less than the total K.E after collision. **i.e the K.E is not conserved**

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$\frac{1}{2}m_1u_1 + \frac{1}{2}m_2u_2 = \frac{1}{2}m_1v_1 + \frac{1}{2}m_2v_2 + Q$$

Q is loss of K.E

iii. Totally Inelastic Collision : it occurs where two colliding masses stick together after collision.



Let V be the common velocity, K_i and K_f are the K.E before(initial K.E) and after(final K.E) collision respectively.

$$m_1u_1 + m_2u_2 = (m_1 + m_2)V$$

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}(m_1 + m_2)V^2$$

If either m_2 is at rest, $u_2 = 0$m/s …. thus,

$$m_1u_1 + = (m_1 + m_2)V$$

$$K_i = \frac{1}{2}m_1u_1^2 \text{ and } K_f = \frac{1}{2}(m_1 + m_2)V^2$$

Taking the ratio K_f/K_i and simplifying

$$K_f/K_i = K = \frac{m_1}{m_1 + m_2}$$

K ratio gives the percentage of the original K.E that is lost

EXAMPLE 8 : a 4kg ball moving at 8m/s collides with a stationary ball of mass 12kg and they stick together. Calculate the final velocity and the kinetic energy lost in the impact.

SOLUTION : $m_1 = 4kg$, $m_2 = 12kg$, $u_1 = 8m/s$

$U_2 = 0$(stationary) $\mathbf{m_1u_1 + m_2u_2 = (m_1 + m_2)V}$

$4 \times 8 + 12\times 0 = (4 + 12)V$, $32 = 16V$, $V=2m/s$

K.E loss = K.E before collision – K.E after collision

K.E before = $\frac{1}{2} m_1u_1^2 = \frac{1}{2} \times 12\times 8^2 = 128j$

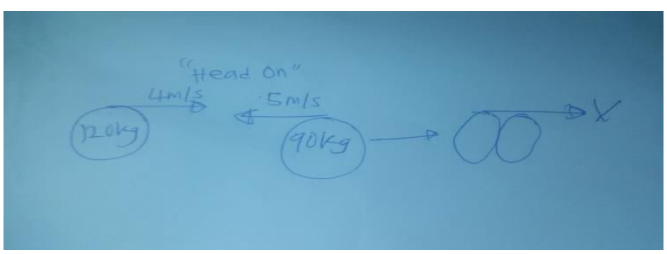
K.E after == $\frac{1}{2}(m_1 + m_2)V^2 = \frac{1}{2} (4 + 12)2^2 = 32j$

K.E loss = 128 – 32 = 96j

EXAMPLE 9 : a football of mass 90kg running back and moving with a speed of 5m/s, is tackled head on by a line backer of mass 120kg running 4m/s. they stick together. How

Fast are they moving after the collision.

SOLUTION :



$\mathbf{m_1u_1 + m_2u_2 = (m_1 + m_2)V}$

$120 \times 4 - 90\times5 = (120 + 90)V$, $30 =210V$, $V=0.14m/s$

EXAMPLE 10 : a 18k fish moving horizontally to the right at 3.2m/s swallows a 2kg fish that is swimming to the left at 7.4m/s. what is the speed of the large fish immediately after it lunch if the forces exerted on the fishes by the water can be neglected. **V = 2.14m/s**

EXAMPLE 11 : a 1kg ball with a speed 4.5m/s strikes a 2kg stationary ball. (a) what are the speeds of the ball after the collision? (b) what percentage of the initial kinetic energy do they have after the collision? (c) what is the total momentum after collision.

SOLUTION : $m_1 = 1kg$, $m_2 = 2kg$, $u_1=4.5$, $u_2=0$

a. $\mathbf{m_1u_1 + m_2u_2 = (m_1 + m_2)V}$

$1 \times 4.5 + 2\times0 = (1 + 2)V$, $4 = 3V$, $V = 1.5m/s$

b. $\frac{K_f}{K_i} = \frac{m_1}{m_1 + m_2} \times 100\%$

Where m_1 is the moving mass

$= \frac{1}{1 + 2} \times 100\% = 33\%$

c. p after : $\mathbf{(m_1 + m_2)V = 1.5(1+2) = 4.5kg,/s}$

RECOIL OF A GUN

Here, the velocity of the gun must negative

Momenta of the gun(g) and bullet(b) before collision = 0 respectively.

$\mathbf{m_bv_b = -m_gv_g}$

EXAMPLE 12 : a police man holds a riffle of mass 6kg loosely in his hands, so as to let it recoil freely when fired. He fires a bullet of mass 2.5g horizontally with a velocity of 250m/s relative to the ground (a) what is the recoil velocity of the riffle (b) what are the final momenta of the bullet and riffle.

SOLUTION : $m_g = 6kg$, $m_b = 2.5g = 0.0025kg$

$V_b = 250m/s$, $v_g = ?$

a. $\mathbf{m_b v_b = -m_g v_g}$, $0.0025 \times 250 = 6 \times v_g$,

$v_g = -0.1042$m/s

b. $p_g = m_g v_g = 6 \times -0.1042 = -0.625$kgm/s

$p_b = m_b v_b = 0.0025 \times 250 = 0.625$kgm/s

EXAMPLE 13 : a bullet of mass 10g when fired from a gun of mass 10kg, has a muzzle velocity of 720m/s. (i) what is the velocity of recoil of the gun and (ii) what is the total energy of the gun and bullet.

SOLUTION : (i) **answer : -0.72m/s**

(ii) $\frac{1}{2}m_b v_b^2 + \frac{1}{2}m_g v_g^2 = \frac{1}{2}\times0.01\times720^2 + \frac{1}{2}\times10\times0.72^2 = 2594.592$j

COEFFICIENT OF RESTITUTION(e)

A measure of the elasticity of collision is called the coefficient of restitution(e)

$$\mathbf{e} = \frac{\textit{relative velocity after collision}}{\text{relative velocity before collision}}$$

$$\mathbf{e} = \frac{\mathbf{v_1 - v_2}}{\mathbf{u_1 - u_2}}$$

for perfectly elastic collision, e < 1

for perfectly inelastic collision, e = 0

for most collision, 0 < e < 1

EXAMPLE 14 : a rock which is initially moving to the right at 5m/s strikes a soft ball which is also moving to the right but at 3m/s. after collision, the rock and the ball move to the right with a velocity of 3.8m/s and 2.2m/s respectively. What is the coefficient of restitution?

SOLUTION :

$$\mathbf{e} = \frac{\mathbf{v_1 - v_2}}{\mathbf{u_1 - u_2}} = \frac{\mathbf{3.8 - 2.2}}{\mathbf{5 - 3}} \mathbf{= 0.8}$$

e has no unit, it is dimensionless

ROCKET PROPULSION

$$\mathbf{V = V_e \ln\{m_T/(m_T - m_f)\}}$$

m_T = total mass, V_e = exhaust velocity

m_f = mass of fuel, V = burnout velocity

$$\mathbf{a} = \left(\frac{\mathbf{m_i}}{\mathbf{t}}\right)\mathbf{V \div m_T}$$

a = initial acceleration,

m_i/t = mass per second

EXAMPLE 15 : A single stage stationary in free space has a total mass of 4×10^5kg of which 2.4×10^5kg is fuel. If the velocity of the exhaust gases relative to the rocket is 2km/hr when the rocket engine is fired, what is the final velocity of the rocket at burnout.

SOLUTION : V_e =2km/hr = 0.5556m/s

$M_f = 2.4 \times 10^5$kg

$V = V_e \ln\{m_T/(m_T - m_f)\}$

$V = 0.5556\ln\{4 \times 10^5/(4 \times 10^5 - 2.4 \times 10^5)\}$

$V = 0.5556\ln\{2.5\} = 0.51$m/s

EXAMPLE 16 : a rocket is fired in deep space, where gravity is negligible. If the rocket has an initial mass of 7000kg and ejects a gas at a relative velocity of magnitude 2000m/s, how much gas must it eject in the first second to have an initial acceleration of 25m/s^2.

SOLUTION : m_T = 7000kg, V = 2000m/s

M_i = ? , t = 1s(first second)

$$a = \left(\frac{m_i}{t}\right)V \div m_T$$

$$25 = \left(\frac{m_i}{1}\right) 2000 \div 7000$$

$25 = 2000m_i \div 7000$, $m_i = 175/2 = 87.5kg$

EXAMPLE 17 : A rocket in deep outer space turns on its engine and ejects 1 percent(1%) of its mass per second with an ejection velocity of 2200m/s. what is the initial acceleration of the rocket.

SOLUTION : 1% is $m_i = m_T/100 = 0.01m_T$ kg

t = 1s(per second)

$$a = \left(\frac{m_i}{t}\right) V \div m_T$$

$$a = \left(\frac{0.01\, m_T}{1}\right) 2200 \div m_T$$

$$a = \frac{0.01\cancel{m}_T \times 2200}{\cancel{m}_T} = 22m/s^2$$

EXAMPLE 18 : a 3×10^3kg shuttle craft containing 50kg of fuel is located in deep space where the force of gravity can be

Neglected. If the fuel is consumed at a rate of 5kg/s with a constant exhaust velocity of 150m/s. what is : (a) the thrust exerted on the shuttle craft? (b) the initial acceleration of the shuttle craft?

SOLUTION : $m_{shuttle} = 3 \times 10^3$kg

$M_i/t = 5kg/s$, $V = 150m/s$, $m_f = 50kg$

a. thrust = force = $m_T\, a$

$m_T = 3 \times 10^3 + 50 = 3050kg$

$$a = \left(\frac{m_i}{t}\right) V \div m_T$$

$a = 5 \times 150 \div 3050 = 0.246m/s^2$

$F = m_T a = 3050 \times 0.25 = 750N$

EXAMPLE 19 : a rocket is in outer space far from any planet , when the rocket engine is turned on. In the first second of firing, the

Rocket ejects $^1/_{120}$ of its mass with a relative

speed of 2400m/s. what is the rocket's initial acceleration?

HINT : t = 1s(first second) , $m_i = m_T/120$,

V = 240m/s,

$$a = \left(\frac{m_i}{t}\right) V \div m_T$$

answer : $20m/s^2$

EXAMPLE 20 : a Saturn V rocket had a mass 2.45×10^6kg , 65% of which was fuel. In the absence of gravity and starting at rest, what would be the maximum velocity attained(burnout velocity) ? the fuel exhaust velocity was 3100m/s.

SOLUTION : $m = 2.45 \times 10^6$kg, $V_e = 3100m/s$

$M_f = \frac{65}{100} \times 2.45 \times 10^6 = 1592500kg$,

$V = V_e \ln\{m_T/(m_T - m_f)\}$

Answer : **3254.4m/s**

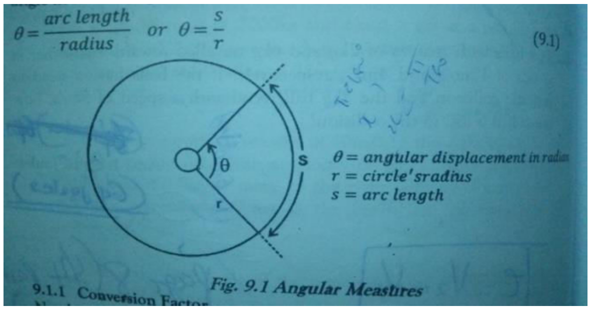
CHAPTER 9

ROTATIONAL MOTION

There are two types of motion : translational motion(or linear dynamics) and the other rotational motion (or rotational dynamics) which is the subject of this chapter.

Angular momentum : angles can be

measured either in degree or in radian.



Conversion Factor

π radians = 180^0

Angular Velocity(w)

$w = \frac{\theta}{t}$, $w = 2\pi f$, $f = \frac{n}{t}$, $T = \frac{1}{f}$

RELATIONSHIP BETWEEN ANGULAR AND LINEAR VELOCITIES

$V = \frac{s}{t}$, $s = \theta r$ (substitute), $v = \frac{\theta r}{t}$, $v = rw$

ANGULAR ACCELERATION (α)

$a_t = r\alpha$, a_t is tangential acceleration,

The centripetal acceleration is given by $a_c = \frac{v^2}{r}$

the total linear acceleration is given by

$a = \sqrt{(a_t^2 + a_c^2)}$, $\theta = \tan^{-1}\left(\frac{a_t}{a_c}\right)$

θ is the direction.

Now, lets compare the equations of linear and rotational motion.

LINEAR	ROTATIONAL
$V = u + at$	$w = w_o + \alpha t$
$S = ut + 1/2at^2$	$\theta = w_0 t + 1/2\ \alpha t^2$
$V^2 = u^2 + 2as$	$W^2 = w_0^2 + 2\alpha\theta$

EXAMPLE 1 : a particle turns round a circle with acceleration of $5m/s^2$ towards the centre. Given the radius of the circle is 5/9m, find the angular velocity in revolution per minute.

$a_c = 5m/s^2$, $v^2 = ar = 5 \times 5/9$,

$v = \sqrt{5 \times 5/9}$ =1.67m/s , $v = wr$, $1.69 = w \times 5/9$

cross multiply, w = 3rad/s

note : it is also written as $\frac{3rad}{1sec}$

1 revolution means moving round a circle once (i.e 360^0) . thus :

1 revolution = 2π (i.e 360^0)

Hence, 1rev = 2π *(radians)*

Xrev = 3rad

By Cross multiplying we have,

$X = w = \frac{3}{2\pi}$ rev/sec

Now, lets convert 1seconds (in denominator) to minutes .

1min = 60 sec

X = 1sec cross multiply

X = 1/60 min. (substitute)

$W = \frac{3/2\pi\ rev}{\frac{1}{60}min} = \frac{3\ rev}{2\pi} \times \frac{60}{1\ min} = \frac{90}{\pi}$rev/min

EXAMPLE 2 : a particle is moving in a circle of

Radius 0.5m with an angular velocity and tangential velocity of 1.5rad/s and 0.75m/s respectively. If the angular acceleration is

$4ras/s^2$, determine the total linear acceleration of the particle.

SOLUTION : $a_c = \frac{v^2}{r} = \frac{0.75^2}{0.5} = 1.125m/s^2$

$a_t = r\alpha = 4 \times 0.5 = 2m/s^2$
$a = \sqrt{(a_t^2 + a_c^2)} = \sqrt{(2^2 + 1.125^2)} = 2.27m/s^2$

EXAMPLE 3 : a particle moving in a circle of radius 0.5m with an angular velocity of 1.5rad/s. determine the tangential velocity of the particle.

SOLUTION : $v = wr = 0.5 \times 1.5 = 0.75m/s$

EXAMPLE 4 : you measure the length of a distant car at an angular distance of 2^0 . if the car is actually 5m long , approximately how far away is the car?

SOLUTION: $s = 5m$, $\theta = 2^0$ lets convert to rad

$\pi = 180^0$

$X = 2^0$ $x = \pi/90$ rad

$S = r\theta$, $5 = r \times \pi/90$, cross multiply,

$r = 1.4 \times 10^2$ m

EXAMPLE 5 : during an acceleration, the angular speed of an engine increases from 700rpm to 3000rpm in 3sec. what is the average angular acceleration of the engine?

SOLUTION : $w_o = 700rpm$, $w = 3000rpm$, $t = 3s$

$\alpha = ?$, note : rpm means revolution per minute. Convert to rad/s. you can convert it the way we converted in example 1 of this

Chapter or use the shortcut (this shortcut can also be used in example 1 above to save time) .

$1\ rad/s = \frac{60}{2\pi}$ rev/min

$X = 3000$ $x = w = 314.4rad/s$

Do 700rpm the same way, you have

$X = w_o = 73.29rad/s$

$w = w_o + \alpha t$, $314.4 = 73.29 + \alpha \times 3$

$314.4 - 73.29 = 3\alpha$, $\alpha = 241.11/3 = 8.37rad/s^2$

EXAMPLE 6: the tangential speed of a particle on a rotating wheel is 3m/s. if the particle is 0.2m from the axis of rotation, how long will it take for the particle to go through one revolution.

SOLUTION: $v = 3m/s$, $r = 0.2$, $t = ?$

$n = 1$(one revolution) $v = wr$, $3 = w \times 0.2$

$w = 3/0.2 = 15rad/s$. but $w = 2\pi f$

$15 = 2 \times \pi \times f$, $f = 15/2\pi$ Hz, but $f = \frac{n}{t}$

$\frac{15}{2\pi} = \frac{1}{t}$, $t = \frac{2\pi}{15} = 0.42s$

TORQUE(T)

Torque on a particle is given by $T = I\alpha$

Where I is the moment of inertia. $I = mr^2$

MOMENT OF INERTIA OR ROTATIONAL INERTIA (I)

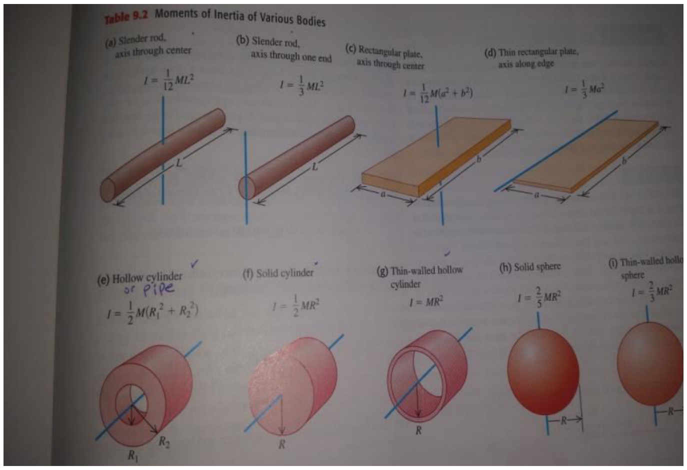
It is the tendency of a body to resist rotational motion. It is given as $I = mr^2$. its S.I unit is kgm^2. It is sometimes called the *second moment of inertia* . it depends on :

1. the perpendicular axis of rotation

2. the shape of the body

3. the distribution of mass within a body

MOMENT OF INERTIA OF VARIOUS BODIES



Considering the diagram by rows(i.e taking row 1 before row 2) :

a. slender rod, axis through centre, $I = \frac{1}{12}ML^2$

b. slender rod, axis through the end, $I = \frac{1}{3}ML^2$

c. rectangular plate, axis through centre,

$$I = \frac{1}{12}M(a^2 + b^2)$$

d. thin rectangular plate, axis along edge,

$$I = \frac{1}{3}Ma^2$$

e. hollow cylinder or pipe, $I = \frac{1}{2}M(R_1^2 + R_2^2)$

f. solid cylinder or disc, $I = \frac{1}{2}MR^2$

g. thin-walled hollow cylinder or hoop or ring

$$I = \frac{1}{2}MR^2$$

h. solid sphere about any diameter, $I = \frac{2}{5}MR^2$

i. thin-walled hollow sphere, $I = \frac{2}{3}MR^2$

Note : *if you do not know the shape of the body whose moment of inertia you are looking for use* **$I = mr^2$**

RADIUS OF GYRATION

Suppose it is possible to arrange the total mass m of a body in such a way that every particle is at a constant distance k from the axis of rotation, then the moment of inertia of the body about the body about the axis of rotation is given as $I = mk^2$ where k is the radius of gyration.

ROTATIONAL WORK, ENERGY AND POWER

Work, $w = T\boldsymbol{\theta}$, t is torque and $\boldsymbol{\theta}$ is angular displacement

ENERGY

Rotational kinetic energy, $K.E_r = \frac{1}{2}Iw^2$

Translational kinetic energy, $K.E_t = \frac{1}{2}mv^2$

Total kinetic energy = $\frac{1}{2}mv^2 + \frac{1}{2}Iw^2$

Work-energy theorem is given by :

$$(\frac{1}{2}Iw^2)_f - (\frac{1}{2}mv^2)_i$$

Where the subscripts f and i represent final and initial respectively.

NOTE : an object rolling down a plain has both rotational and translational kinetic energy.

Rotational power : $p = \frac{w}{t} = \frac{T\theta}{t} = Tw$

ANGULAR MOMENTUM(L)

$L = Iw = 2\pi fI$

CONSERVATION OF MOMENTUM

This states that the sum of the external torque acting on an object is zero. That is :

$\Delta L = L_f - L_i = 0$, $\Delta L = Iw_f - Iw_i = 0$, But $L_f = L_i$, thus $Iw_f = Iw_i$

MECHANICAL EQUILIBRIUM

A rigid body is said to be in mechanical equilibrium when both the rotational and translational equilibrium conditions are satisfied.

EXAMPLE 7 : a 2000kg ferris wheel accelerates from rest to an angular speed of 2rad/s in 12s. assume the ferris wheel as circular disc with a radius of 30m/s, what is the net torque on the wheel?

SOLUTION : w = 2rad/s , w_o = 0(rest) , t = 12s

$w = w_o + \alpha t$, $2 = 0 + \alpha\ 12$, $\alpha = 2/12 = 0.167 rad/s^2$

for a disc, $I = \frac{1}{2}mr^2 = \frac{1}{2} \times 2000 \times 30^2 = 900000 kg/m^2$

$T = I\alpha = 900000 \times 0.167 = 1.5 \times 10^5$ mN

EXAMPLE 8 : a cylindrical hoop of mass 10kg and radius 0.2m is accelerated by a motor from rest to an angular speed of 20rad/s during a 0.4s interval. (a) how much work is required ? (b) what is the power output of the motor?

SOLUTION: r = 0.2m, w_o = 0, w = 20rad/s, t=0.4s

For a cylindrical hoop, $I = mr^2 = 10 \times 0.2^2 = 0.4$

Work = $\frac{1}{2}Iw^2 = \frac{1}{2} \times 0.4 \times 20^2 = 80j$

b. $p = \frac{w}{t} = \frac{80}{0.4} = 200w$

EXAMPLE 9 : calculate the moment of inertia and radius of gyration of a uniform rod of length 2a about an axis through the centre perpendicular to the rod if the rod is of mass 20kg and length 6m.

SOLUTION : the moment of inertia of a uniform rod passing through the centre is given by:

$I = \frac{1}{12}mr^2 = \frac{1}{12} \times 20 \times 6^2 = 60kgm^2$

$I = mk^2$, $60 = 20 \times k^2$, $k = \sqrt{\frac{60}{20}} = 1.73m$

EXAMPLE 10 : calculate the radius of gyration of a uniform rod of mass 35kg and length 5m about an axis through one end and perpendicular to the rod.

SOLUTION : $I = \frac{1}{3}mr^2 = \frac{1}{3} \times 35 \times 5^2 = 291.67kgm^2$

$I = mk^2$, $291.67 = 35 \times k^2$, $k = \sqrt{\frac{291.67}{35}} = 2.89m$

EXAMPLE 11 : a flywheel of mass 1 tonne and diameter of gyration 2m is rotating once every second. Find its kinetic energy.(1 tonne = 1000kg)

SOLUTION : m = 1 tonne = 1000kg , d = 2m

$K = \frac{d}{2} = \frac{2}{2} = 1m$, f = n/t = 1/1 = 1Hz

$I = mk^2 = 1000 \times 1^2 = 1000kg/m^2$

$W = 2\pi f = 2 \times \pi \times 1 = 2\pi rad/s$

$K.E_r = \frac{1}{2}Iw^2 = \frac{1}{2} \times 1000 \times (2\pi)^2 = 2000\pi^2$ j

EXAMPLE 11 : a skater has a moment of inertia of $100kgm^2$ when his arms were outstretched And a moment or inertia of $75kgm^2$ when his arms are tucked in close to his chest. If he starts to spin at an angular speed of 2rev/sec, with his arms outstretched what will his angular speed be when they are tucked in ?

SOLUTION : law of conservation of momentum, $I_f w_f = I_o w_o$, $75 \times w_f = 100 \times 2$, w_f =2.7rev/s

EXAMPLE 12 : the moment of inertia of a thin cylindrical hoop is given as $I = mR^2$, calculate its radius of gyration . answer : K = R

EXAMPLE 13: A fixed 0.15kg solid disc pulley with a radius of 0.075m is acted on by a net torque of 6.4mN. what is the angular acceleration of the pulley ? **Use: T = Iα then $\frac{1}{2}$ mr^2 . ANS: 1.52 $\times 10^4$rad/m^2**

CHAPTER 10

TEMPERATURE AND THERMOMETRY

Temperature : temperature which is the degree of hotness or coldness of a body depends on the sense of touch. In this way, it is only assessed in an approximate way, relative hotness or coldness of bodies. Judgments made by use of sense of touch are very unreliable .

On a thermodynamic scale, the thermometric property x_{tr} is measured at the triple point of water, 273.16K, and measured again at an unknown temperature T as x_t , then by definition:

$$T = \frac{x_t}{x_{tr}} \times 273.16K$$

where x can be volume, V or pressure, P or temperature , T , or resistance, R

EXAMPLE 1 : I the resistance of a certain thermometer is 80.35ohms at the triple point of water. If the resistance is 86.26ohms at a certain temperature, find the temperature.

SOLUTION : R_t = 86.28ohms, R_{tr} = 80.35ohms

$$T = \frac{R_t}{R_{tr}} \times 273.16K = \frac{86.28}{80.35} \times 273.16K = 293.3K$$

DIFFERENT TEMPERATURE SCALE

1. Celsius scale : on this scale the fundamental interval is divided into 100 equal parts. Its ice point is 0^{o}c and its steam point is 100^{o}c.

2. Fahrenheit scale : the fundamental interval is made up of 180 divisions, with ice point being 32^{o}F and steam point being 212^{o}F.

3. Kelvin scale : its fundamental interval is divided into 100 parts, with ice point (triple point of water) being 273.16K and steam point

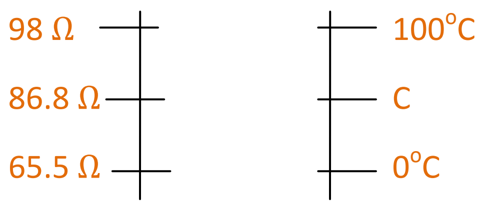
Being 373.16K

4. Rankine scale : it has a fundamental interval of 180 divisions with ice point being 492^{o}R and steam point being 672^{o}R. *it is used in engineering applications.*

In the following examples the formula we will be using is $\frac{middle\ value - down\ value}{upermost\ value - \ down\ value}$

EXAMPLE 2 : the resistance of a certain platinium thermometer is 65.5Ω at 0^{0}C and 98Ω at 100^{o}C. if the resistance is 86.8Ω when placed in hot water, find the temperature of hot water.

SOLUTION:



Using the formula : $\frac{middle\ value - down\ value}{upermost\ value - \ down\ value}$

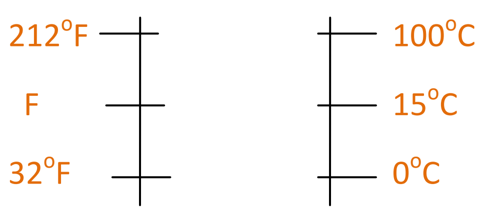
$\frac{86.6 - 65.5}{98 - 65.5} = \frac{C - 0}{100 - 0}$

$\frac{21.3}{32.5} = \frac{C}{100}$ always cross multiply

C = 2130/32.5 = 65.5^{o}C

EXAMPLE 3 : which of the following is the closest to 15^{o}C ? (a) 8.3^{o}F (b) 27^{o}F (c) 40^{o}F (d)50^{o}F

SOLUTION : it means convert 15^{o}C to **^{o}F**



$$\frac{F-32}{212-32} = \frac{12-0}{100-0}$$

$$\frac{F-32}{180} = \frac{15}{100}$$ always cross multiply

$100(F - 32) = 180 \times 15$ divide through by 100

$F - 32 = 27$, $F = 59^{o}F$

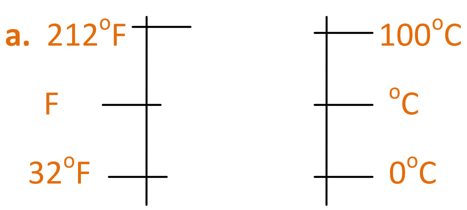
EXAMPLE 4 : convert (a)$50^{o}F$ and (b) $36^{o}R$ to degree Celsius. **Answers : (a) $10^{o}C$ (b) $-253.3^{o}C$**

EXAMPLE 5 : a person running a fever has a body temperature of $39.4^{o}C$. what is this temperature on the Fahrenheit scale ?

HINT : it means convert to ^{o}F . ANSWER:103^{O}

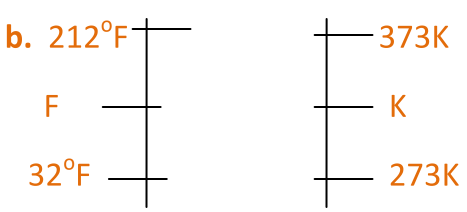
EXAMPLE 6 : derive the formula for interconversion of temperatures in the following cases. (a) from Fahrenheit to Celsius (b) from Fahrenheit to Kelvin (c) from Celsius to Kelvin (d) from Fahrenheit to rankine

SOLUTION :



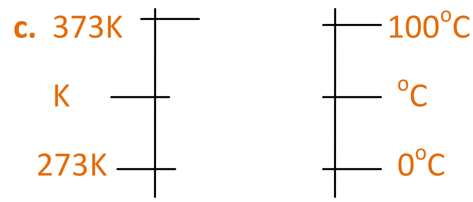
Using the same procedure in the previous examples, we get **$F = \frac{9}{5}(^{o}C + 32)$, and,**

$C = \frac{5}{9}(F - 32)$

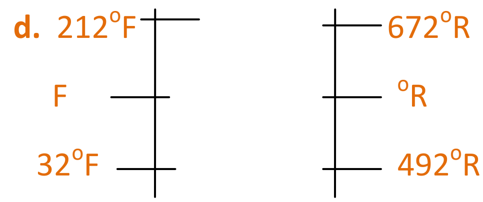


We get, **$K = \frac{5}{9}(F - 32) + 273$ and**

$F = \frac{9}{5}(K - 273) + 32$



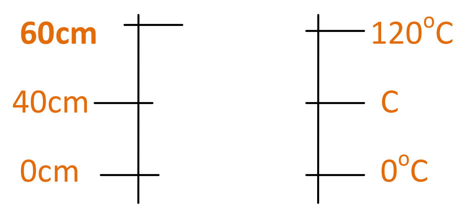
$^{o}C = K - 273$ and $K = {^{o}C} + 273$



$F = R - 460$ and $R = F + 460$

EXAMPLE 7 : a lagged copper rod of uniform cross-sectional area has a length of 60cm. the free ends of the rod are maintained at $120^{o}C$ and $0^{0}C$ respectively at a steady state. Calculate the temperature at a point 20cm from the high temperature end.

SOLUTION:



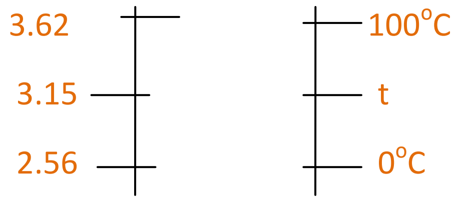
20cm from the high end means (in the diagram) (60 – 20) = 40cm

$\frac{40-0}{60-0} = \frac{C-0}{120-0}$, $\frac{2}{3} = \frac{C}{120}$, $C = 80^{o}C$

EXAMPLE 8 : a constant volume thermometer registers 180mmHg at $0^{o}C$ and 490mmHg at $100^{o}C$. Find the temperature when the pressure is 315mmHg. ***Answer : $43.54^{o}C$***

EXAMPLE 9 : A platinum wire has resistance of 2.56, 3.62 and 3.15ohms respectively at $0^{o}C$, 100 ^{o}C and $55^{o}C$ respectively. Calculate the difference between $55^{o}C$ and the corresponding platinum temperature.

SOLUTION :

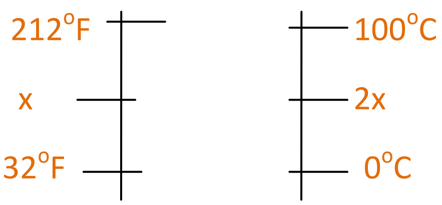


$\frac{3.15-2.56}{3.62-2.56} = \frac{t-0}{100-0}$, $\frac{0.59}{1.06} = \frac{t}{100}$, $t = 55.66^oC$

Difference = $55.66 - 55 = 0.66^oC$

EXAMPLE 10 : at what temperature will the Celsius scale read twice the Fahrenheit scale?

SOLUTION :



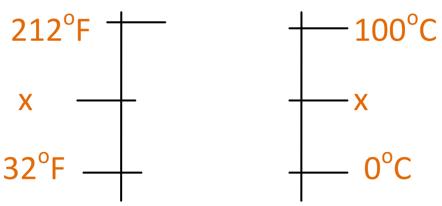
$\frac{x-32}{212-32} = \frac{2x-0}{100-0}$, $\frac{x-32}{180} = \frac{2x}{100}$,

$100(x - 32) = 360$, $100x - 3200 = 360x$, $x = -12.3^oC$

Thus, $2x = 2(-12.3) = -24.6^oC$

EXAMPLE 11 : at what temperature will the Celsius scale and Fahrenheit scale give the same reading.

SOLUTION:

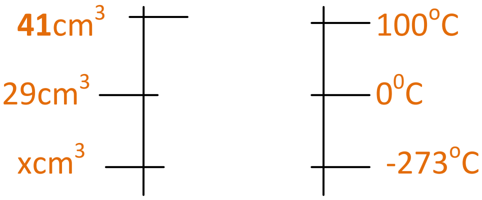


$\frac{x-32}{212-32} = \frac{x}{100-0}$, $\frac{x-32}{180} = \frac{x}{100}$

$100(x-32) = 180x$, $x = -40^oC$

EXAMPLE 12 : the volume of air at constant pressure in a gas is $29cm^3$ at 0^oC and $41cm^3$ at 100^oC. find the volume at absolute zero on this scale.

SOLUTION : ***absolute zero*** *means -273°C*

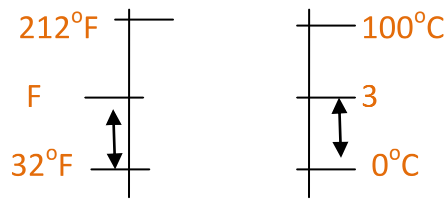


do it the same way……………………

answer = -3.76cm^3

EXAMPLE 13 : convert 3^o rise in Celsius temperature scale to Fahrenheit scale .

SOLUTION:



$\frac{F-32}{212-32} = \frac{3-0}{100-0}$, $\frac{F-32}{180} = \frac{3}{100}$,

$100(F - 32) = 540$, $F = 37.4^oF$

Thus for 3^o rise in Celsius temperature,

$F = (37.4 - 32) = 5.4^oF$

EXAMPLE 14 : the resistance R of a copper wire depends on temperature T via the equation:

$R = R_o[\ 1 + \alpha(T - T_o)\ ]$ where R_o is the resistance at temperature T_o . if the resistance increases by 10%, find the corresponding change in temperature ($\alpha = 3.8 \times 10^{-3}K^{-1}$).

SOLUTION : let $R_o = x$, $R = \frac{110x}{100}$ (10% increase)

Substitute into $R = R_o[\ 1 + \alpha(T - T_o)\ ]$

$\frac{110\cancel{x}}{100} = \cancel{x}[\ 1 + 3.8 \times 10^{-3}(T - T_o)]$

$1.1 = 1 + 3.8 \times 10^{-3}(T - T_o)$,

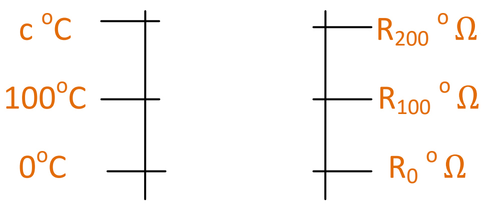
$1.1 - 1 = 3.8 \times 10^{-3}(T - T_o)$

$$\frac{0.1}{3.8 \times 10^{-3}} = (T - T_o)\ ,\ T - T_o = 26.3K$$

EXAMPLE 15 : the resistance R_t of a platinum wire at t^oC, measured on the gas scale is given by $R_t = R_o(1 + aT + bT^2)$ where $a = 3.8 \times 10^{-3} C^{-1}$ $b = -5.6 \times 10^{-7} C^{-1}$. What temperature will the platinum thermometer indicate when the

Temperature on the Celsius scale is 200^oC ?

SOLUTION :



$$\frac{c - 0}{100 - 0} = \frac{R_{200} - R_0}{R_{100} - R_0}$$

Since $R_t = R_o(1 + aT + bT^2)$, then

$R_{200} = R_o(1 + 200a + 200^2 b)$ and

$R_{100} = R_o(1 + 100a + 100^2 b)$

$$c = \frac{\{R_o(1 + 100a + 100^2 b) - R_o\} \times 100}{R_o(1 + 100a + 100^2 b) - R_o}$$

$$= \frac{\{R_o + 200R_o a + 200^2 R_o b - R_o\} \times 100}{R_o + 100R_o a + 100^2 R_o b - R_o}$$

$$= \frac{\{200R_o a + 200^2 R_o b\} \times 100}{100R_o a + 100^2 R_o b}$$

$$= \frac{200R_o\{a + 200b\} \times 100}{100R_o(a + 100b)}$$

$$= \frac{200(3.8 \times 10^{-3} - 200 \times -5.6 \times 10^{-7}}{3.8 \times 10^{-3} - 100 \times -5.6 \times 10^{-7}}$$

$c = 197^oC$

EXAMPLE 16 : the resistance of a platinum wire at 0^oC, $100\,^oC$ and $444.6\,^oC$ is found to be 5.5, 7.5 and 14.5ohms respectively. The resistance of a wire at a temperature t^oC is given by the equation $R_t = R_o(1 + \alpha t + \beta t^2)$. Find the values of α and .

ANSWERS : $\alpha = 3.62 \times 10^{-3} C^{-1}$, $\beta = 1.28 \times 10^{-7} C^{-1}$

TYPES OF THERMOMETER

1. Liquid-in-glass thermometer : it is constructed in such a way that the liquid contained in a small thin-walled glass bulb is capable of expanding along the tiny bore of a thick-walled hermetical sealed capillary tube. **This is based on the fact that matters expand when heated.** For a tube of uniform bore, the change in length of the liquid column when heated is proportional to its expansion.

for a liquid-in-glass thermometer, mercury is chosen because :

1. *it is easily seen, even in very low narrow bore*

2. it has a uniform coefficient of expansion over a wide range of temperature (between -39°C and 357°C , its freezing and boiling points respectively)

3. it does not wet glass

4. it is a good conductor of heat

5. it has a low specific heat

One of the weaknesses of mercury-in-glass thermometer is the low limit within which its readings are accurate (-39^oC and about 300^oC). The upper limit can however, be extended by 000^o by introduction of inert gas like **nitrogen** to the top of the mercury column. As the mercury expands, the pressure of the inert gas increases, thereby extending the boiling point of mercury. In this way, temperature up to 600^oC can be recorded. Reducing the lower limit by using mercury-in-glass, below -39^oC is difficult. Hence, for such lower temperature, another kind of liquid is used—**alcohol.** Alcohol has a freezing point of -114.9^oC. lower temperature up to -200^oC can be achieved by using a liquid called **pentane.**

2. Gas thermometer : mercury-in-glass thermometers are not very reliable when very accurate measurements are required. For purpose of standardization, gas thermometers are used. The thermometric substance is a gas and the thermometer is either of the constant

Pressure type, where the volume of a gas is measured at different temperatures at constant pressure, or with the volume kept constant as the pressure changes with temperature. This latter type – *constant volume gas thermometer* is more convenient and is widely used. **The advantages of this kind of thermometer include :**

(i) *gas thermometers are very sensitive because gas produce large proportionate increase in volume and pressure when heated .*

(ii) *since gas can be at high state of purity, the readings can be reproduced to a high degree of accuracy.*

(iii) a wide range of temperature can be covered. **Temperature up to 1500^oC can be measured with a constant volume gas thermometer.**

3. resistance thermometer : it is built on the principle of variation of the resistance of a metal conductor with changes in temperature. *One of the best is platinum resistance thermometer – it can cover a range from -200^oC to 1200^oC . its advantage is that it is convenient to use and even temperatures of very hot* objects (**e.g furnace)** can be taken at a distance using the leads. *Its main disadvantages lie in the fact that it does not respond rapidly to changing temperature.*

4. the thermoelectric thermometer : the function of this thermometer depends on the ***seebeck effect.*** *The seebeck effect shows that if two dissimilar metals are joined in series to make a complete circuit, then on heating one of the junctions, a current flows round the circuit.*

Current cannot flow unless there is an EMF.

5. pyrometers : it is a type of thermometer used in determining the temperatures in *furnace* by observing the radiation from it.

Unlike other types of thermometers, no part of the instrument is required to be in contact with hot body before the temperature could be recorded. **That is why they are known as RADIATION PYROMETERS.**

<u>**Types**</u>

1. total radiation thermometers : they are used where the radiation source is large such as that through open door of a furnace.

2. optical pyrometers : they are known as to record high temperatures by taking advantage of their response on the light of a small body such as a lamp filament.

<u>**Advantages**</u>

1. they can measure temperatures above the range of thermometers. Since they can measure temperatures quit above the melting point of metals.

2. they do not require being in contact with a hot body before the temperature can be known.

Disadvantages

Calibration of the pyrometer is usually required for the true temperature value of a body to be known.

CHAPTER 11

WORK DONE (W) BY SYSTEM OF EXPANDING GAS AND CALORIMENTRY

W = P dv (dv = change in volume(v))

$W = P(v_2 – v_1)$

EXAMPLE 1 : a gas is expanding against a constant pressure of 1atm from 10 to 16 litres, what is the work done by the gas

[take 1 lit. atm = 101.33j]

SOLUTION: **$W = P(v_2 – v_1)$**= 1(16 –10)= 6 lit. atm

But 1 lit. atm = 101.33j

6 lit. atm = x hence, x = 607.98j

HEAT

Heat is the energy transferred between a

system and its surroundings as a result of temperature difference only. The direction of this energy flow is always from the region of higher temperature to a region of lower temperature,. It is measured in **joules,** othere of its units are : calories (cal), and british thermal unit(BTU)**.**

The relationship between the various units of heat are :

1 Cal = 0.004BTU

1 BTU = 252 cal

1 Cal = 4.186j

1 Kcal = 4186j (or 4.2Kj)

Power, **p** = $\frac{energy\ or\ work}{time}$

EXAMPLE 2 : the rate at which energy is expended by a machine is 25.7W. if this energy is completely converted to heat energy, how many Kilocalories are expended in 5s.

SOLUTION : p = 25.7W, t = 5s

W = pt = 25.7(5) = 128.5j, convert to Kcal

1 Kcal = 4186j

X = 128.5j thus x = 3.07×10^{-2} Kcal

HEAT CAPACITY, SPECIFIC HEAT CAPACITY AND CALORIMETRY

The measurement of quantity of heat is called **calorimetry.**

The quantity of heat(Q) which a body contains is proportional to the mass of the body(m) and also the change in temperature

$(\theta_2 - \theta_1)$. Thus, **Q = mc($\theta_2 - \theta_1$).** If $(\theta_2 - \theta_1)$ is negative heat is lost but if positive heat is gained. Where **c** is the **specific heat capacity**.

Definition

The specific heat capacity, c , of a substance is the quantity of heat required to raise the temperature of a unit mass of that substance by 1°C or 1K. *Its S.I unit is* ***$jkg^{-1}\ {}^oC^{-1}$***

HEAT CAPACITY OF THERMAL CAPACITY (H)

It is the quantity of heat required to raise the entire mass of a body by 1°C of 1K

H = mc, thus Q = H($\theta_2 - \theta_1$).

EVAPORATION AND SUBLIMATION

Some factors hasten or retard this process. They are :

a. area of surface exposed (b) Humidity

c. air speed on the exposed surface

d. temperature of the atmosphere

the larger the surface area, the higher the surface temperature, the higher the air speed and the less the humility, the greater the evaporation.

*The saturated vapour pressure depends on **temperature only.*** Examples of substance that undergo sublimation are : camphor, iodine.

METHODS OF MEASURING THE QUANTITY OF HEAT

1. method of mixtures

2. method of cooling (3) electrical method

4. continuous flow method

5. method depending on latent heat.

*In all this methods, changes in temperatures are measured, except in **number 5** above.*

The fundamental principle underlying calorimetry is the law of conservation of energy. In any thermal process :

Heat gain = heat loss

LATENT HEAT

It is the heat supplied or removed which causes a change of state without a change of temperature. It is an invisible heat, hence the thermometer does not detect it. **It depends on the mass and nature of the substance.**

Latent heat of fusion : *it is also called heat of transformation or latent heat of transformation.* it is the quantity of heat required to convert a substance from its solid to its liquid state without a change in temperature. *When only unit mass of the substance is considered, the heat involved is known as **specific latent heat**.*

***Specific latent heat of fusion (L_f)** of a substance is the quantity of heat required to convert a unit mass of the solid at the melting point to its liquid form without a change in temperature.* **H = mL_f its S.I unit is j/kg**

The specific latent heat of vaporization(L_V) of a substance is the quantity of heat required to change unit mass of a substance from its liquid at the boiling point to vapour without a change in temperature. **H = mL_V . its S.I unit is j/kg**

NOTE : L_V is greater than L_f

EXAMPLE 3 : find the pressure increase of 1kg of water going over kainji falls, which is 51m high. Assume that all of the potential energy lost by the falling water is converted into heat energy, which is completely absorbed

by the falling water.

SOLUTION: m = 1kg, h = 51m, c_w = 4200j/kgK

The potential energy is completely converted into heat energy, thus: mgh = mc($\theta_2 - \theta_1$)

$1 \times 9.81 \times 51 = 1 \times 4200 \times (\theta_2 - \theta_1)$

$(\theta_2 - \theta_1) = \frac{1 \times 9.81 \times 51}{1 \times 4200} = 0.12^{\circ}C$

EXAMPLE 4 : a water fall is 500m high. If the water retains 65percent of the heat generated at the end of the fall, calculate the change in temperature due to the fall (specific heat capacity of water = 4200j/kgK)

SOLUTION : h = 500m, c = 4200j/kgK,

65% thus, $\frac{65}{100}\cancel{m}c(\theta_2 - \theta_1) = \cancel{m}gh$

$\frac{65}{100} \times 4200 \times (\theta_2 - \theta_1) = 10 \times 500$

Cross multiply, $(\theta_2 - \theta_1) = \frac{500000}{65 \times 4200} = 1.8^oC$

EXAMPLE 5 : a bathtub contains 70kg of water at 26°C. 10kg of water at 90°C is poured in. what is the final temperature of the mixture? Neglect heat losses to the air and to the bathtub.

SOLUTION: m_1=70kg, θ_1=26°C, m_2=10kg, T = ?

θ_2 = 90°C, note that $c_1 = c_2$ (i.e c of water)

Heat gained by 70kg of water = heat lost by 10kg of water, $m_1c_1(T - \theta_1) = m_2c_2(\theta_2 - T$

$\cancel{70} \times \cancel{c}(T - 26) = \cancel{10} \times \cancel{c}(90 - T)$

7T – 182 = 90 – T , 8T = 272 , T = 34°C

EXAMPLE 6 : a 60g of water at 90°C is poured into a container containing 20g of water at 30°C. the temperature of the mixture will be ? **answer = 75°C.** *it is done the same way.*

EXAMPLE 7 : an electric heater of 60w is used

to heat a metal block of mass 20kg for 5minute calculate the specific heat capacity of the metal block if the rise in temperature is 20°C.

SOLUTION : θ_1=20°C, m = 20kg, p = 60w,

t = 5min=300s

Energy = $mc(\theta_2 - \theta_1)$, pt = $mc(\theta_2 - \theta_1)$

$60 \times 300 = 20 \times c \times 20$, $c = \frac{6 \times 300}{20 \times 20}$ = 45j/kgK

EXAMPLE 8 : (a) calculate the total heat supplied when 10g ice at 0°C is heated to form water at 10°C. (b) calculate the total heat given out when 10g steam at 100°C condenses to form water at 45°C. (c_w = 4200j/kgK,

L_f = 336000j/kg, L_V = 2260000j/kg)

SOLUTION : m_g = 10g = 0.01kg,

a. the total heat supplied for the change from ice to water is : $Q = mL_f + mc(\theta_2 - \theta_1)$

$Q = 0.01\times 336000 + 0.01 \times 4200(10 - 0)$

Q = 3780j

b. the heat given out for the transformation from steam to water is : $Q = mL_V + mc(\theta_2 - \theta_1)$

$Q = 0.01\times 2260000 + 0.01 \times 4200(100 - 45)$

Q = 24910j

EXAMPLE 9 : 1kg of water in a beaker is heated from 25°C to 40°C in 20mins by an electric heater placed in hot water. Calculate the power of the heater. The water is replaced by an equal amount of glycerin and the temperature rises from 25°C to 40°C in 12mins. Calculate the specific heat capacity of glycerin. (specific heat capacity of water = 4.2kj/kgK)

SOLUTION : m =1kg, θ_1=25°C, θ_2= 40°C ,

t =20mins = 1200s, c = 4.2kj/K = 4.2×10^3j/kgK

$Q = mc(\theta_2 - \theta_1) = 1 \times 4.2\times10^3(40 - 25)$=63000j

$P = \frac{E}{t} = \frac{63000}{1200}$ = 52.5w

When t = 12min = 720s

E = pt = 52.5 × 720 = 37800j

$E = mc(\theta_2 - \theta_1)$, 37800 =1 × c × (40 – 25)

37800 = 15c, thus, c = 2520j/kgK

EXAMPLE 10 : a calorimeter has a specific heat capacity of 60j/K. 0.15kg of water at 20K is contained in the calorimeter. 1kg of the same liquid at 90K is added, and the final temperature of the mixture is 45K. calculate the specific heat capacity of the liquid.

SOLUTION : H_1 = 60j/K, θ_1 = 20K, m=1kg, θ_2=90K, T = 45K, c_L = ?

Heat gained by liquid and calorimeter at 20K = heat lost by liquid at 90K

$H_1(T - \theta_1) + m_2c(T - \theta_2) = m_3c_L(\theta_3 - T)$

60(45 – 20) + 0.15c(45-20) = 1c(90-45)

1500+3.75c =45c, 1500=45c-3.75, c =36.4j/kgK

EXAMPLE 11 : how much heat must be added to 1kg of ice at 0^oC to convert it to steam at 100 ^oC than it is required to raise the temperature of 1kg of water from 0 ^oC to

100 ^oC ? (Sp latent heat of ice = 3.4×10^5j/kg)

SOLUTION : m = 1kg, $\theta_1 = 0^oC$, c = 4200j/kg^oC

The heat required to convert 1kg of ice at 0^oC to steam at 100^oC is: $Q = mL_f + mc(\theta_2 - \theta_1) + mL_V$

$Q = 1 \times 34000 + 1 \times 4200(100-0) + 1 \times 2200000$

$Q = 3.02 \times 10^6$j

Also, the heat required to convert 10kg of water at 0^oC to 100^oC is : $Q_1 = mc(\theta_2 - \theta_1)$

$Q_1 = 1 \times 4200(100 - 0) = 4.2 \times 10^5$j

Change in heat= $Q - Q_1 = (3.02 \times 10^6) - (4.2 \times 10^5)$

= **2.6** $\times 10^6$j

EXAMPLE 12 : how much heat is needed to change 10kg of ice at -20^oC to steam at 120^oC. take c_{ice} = 2100j/kg^oC, c_w = 4186j/kg^oC,

C_S =2010j/kg^oC, $L_f = 3.33 \times 10^5$j/kg

$L_V = 22.26 \times 10^5$j/kg.

Answer : $Q_{total} = 3.1 \times 10^7$j

EXAMPLE 13 : a 0.25kg cup at 20^oC is filled with 0.25kg of **boiling coffee** . the cup and the coffee came to thermal equilibrium at 80^oC. if no heat is lost, what is the specific heat capacity of the cup material? (Hint : consider the coffee to be essentially **boiling water**)

SOLUTION : m_1 = 0.25kg, $\theta_1 = 20^oC$, m_2 0.25kg $\theta_2 = 100^oC$(temperature of **boiling water**), T=80

$m_1c_1(T - \theta_1) = m_2c_2(\theta_2 - T)$

$0.25 \times c_1(80 - 20) = 0.25 \times 4200 (100 - 80)$

$15c_1 = 21000$, $c_1 = 21000/15 = 1.4 \times 10^3j/kg^oC$

EXAMPLE 14 : a heater supplies 240BTU of energy. What is this energy in joules?

SOLUTION : it means convert 240BTU to joules

1BTU = 252Cal

240 = x

X = 60480Cal, also

1Cal = 4.186j

60480 = x

X = 253169.28j, thus, 240BTU = 253169.28j

EXAMPLE 15 : a student eats a thanksgiving dinner that totaled 2800Kcal. He wants to use up that energy by lifting a 20Kg mass of 1m.

a. how many times must he lift the mass?

b. if he can lift the mass every 5sec, how long does this exercise take?(neglecting lowering)

SOLUTION : Q = 2800Kcal

1Kcal = 4186j

2800Cal = x

Q = x = 11720800j, m = 20kg, h = 1m

(a) energy = n(mgh)

11720800 = n (20 × 10 × 1)

n = $\frac{11720800}{200}$ =58604times, thus n ≅60,000times

(b) if $1_{(once)}$ = 5sec

Then, 60,000 = x

X = 300000sec, convert to hours by dividing by 3600, t = x = 83hr

CHAPTER 12

THERMAL PROPERTIES OF MATTER

EXPANSION : the addition of heat to any substance results to the expansion of solids, liquids and gases.

ADVANTAGES OF EXPANSION

Expansion effects is usefully applied in :

1. thermostat in order to maintain a steady temperature as in electric pressing iron, gas cooker and electric heating and cooling systems

2. riverting two metal plates

3. construction of automatic fire alarms.

4. the fitting of metal rim on metal wheels by first heating the rim and slipping it so that on cooling it contracts and fits firmly on the wheel

DISADVANTAGES OF EXPANSION

1. it deforms a bridge structure fixed at both ends when the weather is hot.

2. it can make thick glass tumblers break when hot liquids are poured into them.

3. it affects the oscillation of the pendulum clock and the balance wheel when there is a change in temperature.

TYPES OF EXPANSION

1. LINEAR EXPANSION : the **linear expansivity α,** of a substance is defined as the increase in length per unit length per degree rise in temperature.

$$\alpha = \frac{L_2 - L_1}{L_1(\theta_2 - \theta_1)}$$

L_2 and L_1 are final and initial lengths

Thus, $L_2 - L_1 = \alpha L_1(\theta_2 - \theta_1)$

WORK DONE BY LINEAR EXPANSION OF A SOLID

Strain = e/L_1, $L_2 - L_1 = \alpha L_1(\theta_2 - \theta_1)$

Substitute, thus: **strain = $\alpha L_1(\theta_2 - \theta_1)$**

Young's modulus, E = $\frac{stress}{strain} = \frac{F/A}{e/L}$

Thus, F = $AE\alpha(\theta_2 - \theta_1)$

Work, w = F × d

2. AREA OR SUPERFICIAL EXPANSION : the **area expansivity, β** of a solid is the increase in are per unit area per Kelvin increase in temperature.

$$\beta = \frac{A_2 - A_1}{A_2(\theta_2 - \theta_1)} \quad , \quad \beta = 2\alpha$$

Thus, $A_2 - A_1 = \beta A_1(\theta_2 - \theta_1)$

Note : $A = \pi r^2$ **and** $A = \pi d^2/4$

3. VOLUME OR CUBIC EXPANSION : the **volume expansivity,** γ is the increase in volume of a substance per unit volume per Kelvin rise in temperature.

$$\gamma = \frac{v_2 - v_1}{v_2(\theta_2 - \theta_1)} \quad , \quad \gamma = 2\alpha$$

$v_2 - v_1 = \gamma v_1(\theta_2 - \theta_1)$

NOTE : the relationship between γ *and* β *is* $\gamma = \frac{3\beta}{2}$

EXAMPLE 1 : (a) what is the increase in length of a steel girder that is 1500cm long at 5°C when its temperature rises to 25°C ? (b) how much force is associated with the expansion of the girder if its cross-sectional area is 3000cm^3? (young modulus for steel = 2×10^7N/cm^2, and the coefficient of linear expansion of steel is equal to 1.2×10^{-5}/°C)

SOLUTION : $L_2 - L_1$ = ? , L_1 = 1500cm, θ_1 = 5°C θ_2 = 25°C, from

$$\alpha = \frac{L_2 - L_1}{L_1(\theta_2 - \theta_1)} \qquad \text{thus, } L_2 - L_1 = \alpha L_1(\theta_2 - \theta_1)$$

$L_2 - L_1 = 1.2 \times 10^{-5} \times 1500(25 - 5) = 0.36\text{m}$

b. $F = AE\alpha(\theta_2 - \theta_1)$

$=3000\times2\times10^7\times1.2\times10^{-5}(25-5)=1.44\times10^7\text{N}$

EXAMPLE 2 : a steel rod of length 2000cm and uniform cross-section area 3×10^3cm^3 at 28°C is heated to 45°C. find the change in the length and the force due to expansion by the rod.

(young modulus for steel = 2×10^7N/cm^2, and the coefficient of linear expansion of steel is equal to 1.2×10^{-5}/°C)

ANSWERS : $L_2 - L_1$= 0.41cm, F = 1.22×10^7N

EXAMPLE 3 : what is the increase in length of a steel bar that is 1000cm long at 10°C when its temperature rises to 60°C. ($\alpha = 1.2 \times 10^{-5}$/°C)

ANSWER : 0.6cm

EXAMPLE 4 : referring to example 4 above, how much force is associated with the expansion of steel bar if its cross-sectional area is 100cm^2 and young modulus = 2×10^7N/cm^2

ANSWER : 1.2×10^6N

EXAMPLE 5 : a petrol station takes delivery of 10cm^3 of gasoline at a temperature of 0°C. if the delivery of the same mass of petroleum were made when the temperature was 30°C, determine the increase in volume of the liquid delivered.

(coefficient of volume expansion =9.6×10^{-4}/K)

SOLUTION : v_1 = 10cm^3 , $v_2 - v_1$ = ? , from

$$\gamma = \frac{v_2 - v_1}{v_1(\theta_2 - \theta_1)} \qquad \text{thus, } v_2 - v_1 = \gamma v_1(\theta_2 - \theta_1)$$

$v_2 - v_1 = 9.6 \times 10^{-4} \times 10 \times (30 - 0) = 0.29\text{m}^3$

EXAMPLE 6 : a square plate of side 15cm is made of a metal of linear expansivity 2×10^{-5}/K if the thickness of the plate is 5mm and the plate is heated from 25°C to 80°C, what is the cubical increase?

SOLUTION : α = 1.2 $\times$ $10^{-5}/^oC$, T = 5mm=0.005m

L =15cm=0.15m, for a square, area, A, = L^2

A = 0.15^2 = $0.0225m^2$, the volume V = A $\times$ T

V_1 = 0.0225 $\times$ 0.005 = 1.125 $\times$ $10^{-4}m^3$

$\gamma = 3\alpha = 3 \times 1.2 \times 10^{-5} = 6 \times 10^{-5}/K$

$v_2 - v_1 = 6 \times 10^{-5} \times 1.125 \times 10^{-5}(80 - 25)$

$= 3.7125 \times 10^{-7}\ m^3$

EXAMPLE 7 : if the length of a cylindrical solid metal is Lcm at 30°C and the linear expansivity is α, then the ratio of the new volume to the initial volume at 70°C is ?

SOLUTION :

$$\frac{V_2}{V_1} = \frac{\cancel{V_1}\{1 + \gamma(\theta_2 - \theta_1)\}}{\cancel{V_1}} = 1 + 3\alpha(70 - 30) = 1 + 120\alpha$$

EXAMPLE 8 : a wire of diameter 0.617mm and length 0.984m is suspended vertically from a rigid support and subjected to a tensile load of 8kg from its free end. If the wire is stretched by 1.3mm, find the young modulus of the wire.

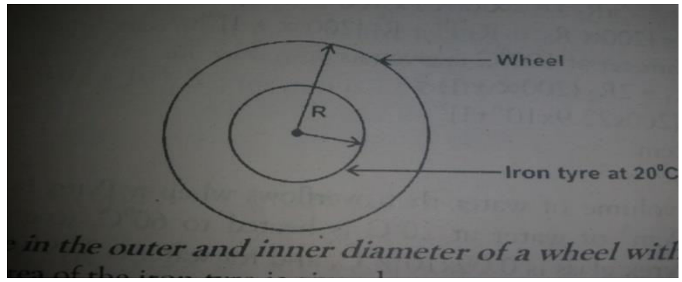
HINT: g = $10m/s^2$, f =mg, A= $\pi d^2/4$, E = $\frac{F/A}{e/L}$

Answer : 2 $\times$ $10^{11}N/m^2$

EXAMPLE 9 : the outside diameter of a wheel is 1m. an iron tyre for this wheel has an inside diameter of 0.992m at 20°C. to what

Temperature must the tyre **be heated** in order to fit over the wheel? (coeficient of linear expansion of iron = 1.2 $\times$ $10^{-5}/^oC$

SOLUTION : to fit over the wheel, the outside diameter becomes its $\mathbf{d_2}$. $\mathbf{d_2}$ **= 0.992m,** $\mathbf{d_2}$ **= 1m**



$A_1 = \frac{\pi d_1^2}{\mathbf{4}}$ and $A_1 = \frac{\pi d_1^2}{\mathbf{4}}$

Then use : $\beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)}$ **, ANS:** θ_2=694.75°C

EXAMPLE 10 : a lead rod is 100cm long. What must be the length of an iron rod if both must have equal expansion for all variations of temperature? (coefficient of linear expansion of lead is 27 $\times$ $10^{-6}/^oC$, for iron is 12 $\times$ $10^{-6}/^oC$)

SOLUTION : L_{1L} = 100cm, L_{1i} = ?

For lead: $L_{2L} - L_{1L} = \alpha_L L_{1L}(\theta_2 - \theta_1)$

For iron: $L_{2i} - L_{1i} = \alpha_i L_{1i}(\theta_2 - \theta_1)$, equate them

$\alpha_L L_{1L}\cancel{(\theta_2 - \theta_1)} = \alpha_i L_{1i}\cancel{(\theta_2 - \theta_1)}$

$27 \times 10^{-6} \times 100 = 12 \times 10^{-6} \times L_i$

L_i = 225cm

EXAMPLE 11 : a circular hole in an aluminum plate is 2.54cm in diameter at 0°C. what is the diameter when the temperature of the plate is increased to 100°C? (linear expansivity of aluminum is 2.29 $\times$ $10^{-6}/^oC$)

Using : : $\beta = \frac{A_2 - A_1}{A_1(\theta_2 - \theta_1)}$ find A_2

Then using : $A_1 = \frac{\pi d_1^2}{4}$ find d_2, **ans : 2.552cm**

EXAMPLE 12 : calculate the volume of water that overflows when a pyrex beaker filled to the brim with $250cm^3$ of water at 20^oC is heated to 60^oC. (coefficient of volume expansion of pyrex glass is $0.09 \times 10^{-4}/^oC$, and for water is $2.1 \times 10^{-4}/^oC$)

SOLUTION : For the beaker :

$v_2 - v_1 = \gamma v_1(\theta_2 - \theta_1) = 0.09 \times 10^{-4} \times 250(60 - 20) = 0.09cm^3$. for water : $v_2 - v_1 = \gamma v_1(\theta_2 - \theta_1)$

$= 2.1 \times 10^{-4} \times 250(60 - 20) = 2.1cm^3$

The volume that overflows=2.1–0.09=$2.01cm^3$

EXAMPLE 13 : a wire of length 5m and uniform circular cross-sectional of radius 1.4mm was extended by 2mm by tension of 110N. calculate the average strain per unit volume.

average strain per unit volume = $\frac{1}{2}\left(\frac{F}{A} \times \frac{e}{L}\right)$

ANSWER : 3.57Kj

EXAMPLE 14 : an iron ball of diameter 15.23cm rest on a brass ring of internal diameter 15cm at a temperature of 20^oC. to what temperature must both be heated for the iron ball to pass through the ring ? ($\alpha_{iron} = 1.2 \times 10^{-5}/^oC$ and $\alpha_{brass} = 1.9 \times 10^{-5}/^oC$)

HINT : for the iron to pass through the ring, A_2 of brass must be equal to A_2 of iron. Just equate and solve. **Answer :** θ_2 = **2350.8°C**

EXAMPLE 15 : a brass rod is 1500cm long at 10^oC. to what temperature must the rod be heated to make it expand by 9mm?($\alpha_{brass} = 1.9 \times 10^{-5}/^oC$) **answer : 41.6°C.**

REAL AND APPARENT EXPANSION

Liquids have no definite shape of their own but simply conform to that of the containing vessel. On heating, the container and the liquid expand, thus the measured increase in

The volume of the liquid does not reflect the actual volume increase. In the expansion of a liquid, the containing vessel expands and makes the expansion of the liquid to appear less than it actually is. Thus we have two expansivities.

Apparent cubic expansivity (γ_a) of a liquid is the increase in volume per unit volume per degree rise in temperature when the liquid is heated in an expansible vessel.

Real or absolute cubic expansivity (γ_r) of a liquid is the increase in volume per unit volume per degree rise in temperature.

$$\gamma_a = \frac{\textbf{volume of liquid expelled}}{\textbf{Volume of liquid remaining} \times \textbf{temp. rise}}$$

$$\gamma_a = \frac{v_e}{v_r(\theta_2 - \theta_1)}$$ similarly

$$\gamma_a = \frac{m_e}{m_r(\theta_2 - \theta_1)}$$, **m = mass**

$$\gamma_r = \gamma_a + \gamma ,$$

γ = cubic expansivity of the container

EXAMPLE 16 : a density bottle weigh 16.5g when empty and 45.2g when filled with paraffin at 25^oC. when it had been heated to 80^oC and cooled again, it weighed 43.5g. calculate the coefficient of apparent and real expansion. (γ_{glass} = 0.000009/K)

SOLUTION : $m_b = 16.5$, $m_{b+p} = 45.2$,

M_{b+p} after heating = 43.5,

M_e = 45.2 – 43.5 = 1.7g

$M_r = 43.5 – 16.5 = 27g$

$$\gamma_a = \frac{m_e}{m_r(\theta_2 - \theta_1)} = \frac{1.7}{27\times(80-25)} = 0.0011/K$$

$\gamma = 3\alpha = 3 = 3 \times 0.000009 = 0.000027/K$

$\gamma_r = \gamma_a + \gamma = 0.0011 + 0.000027 = 0.001127/K$

EXAMPLE 17 : the mass of a gravity bottle and the liquid content inside is 65g at 30°C. when the bottle and its contents is heated to 100°C the mass of the bottle and its content become 58g. what is the real expansion of the liquid if the linear expansivity of the glass is 0.000008/K?

SOLUTION : $m_{b+L} = 65$, $m_b = 30$

M_{b+L} after heating = 58g

$M_e = 65 – 58 = 7g$, $m_r = 58 – 30 = 28g$

$$\gamma_a = \frac{m_e}{m_r(\theta_2 - \theta_1)} = \frac{7}{28\times(100-30)} = 3.57\times10^{-3}/K$$

$\gamma_r = \gamma_a + \gamma = 3.57\times10^{-3} + 0.000024 = 3.594\times10^{-3}/K$

VARIATION OF DENSITY (d) WITH TEMPERATURE

$$\gamma = \frac{d_1 - d_2}{d_2(\theta_2 - \theta_1)}$$

d_1 is density at lower temperature, and

d_2 is density at higher temperature

EXAMPLE 18 : what s the density of brass at 60°C if the density at 0 °C is 15.2g/cm³(linear expansivity of brass = 1.9 $\times10^{-5}$/K; cubical expansivity of brass = 5.7 $\times10^{-5}$/K)

SOLUTION : make d_2 subject of formula,

$$d_2 = \frac{d_1}{1 + \gamma(\theta_2 - \theta_1)} = \frac{15.2}{1 + 5.7\times10^{-5}(60-0)}$$

$= 15.1g/cm^3$

EXAMPLE 19 : the density of iron at 30°C is 4.8g/cm³. What is the density at 80°C if the linear expansivity of iron is 1.2 $\times10^{-5}$/K?

Answer : **4.79g/cm³**

CHAPTER 15

KINETIC THEORY AND THERMODYNAMICS

There are basically types of gas laws :

1. **Charles law:** it states that the volume of a given mass of gas at constant pressure increases by $\frac{1}{273}$ of its volume at 0°C for every degree centigrade rise in temperature.

$$V \propto T, \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

2. **boyle's law :** it states that the volume of a given mass of gas at constant temperature is inversely proportional to the pressure.

$$P \propto \frac{1}{v}, \quad p_1v_1 = p_2v_2$$

3. **pressure law :** the pressure of a given mass of gas at constant volume increases by $\frac{1}{273}$ of its pressure at 0°C for every degree rise in temperature.

$$P \propto T, \quad \frac{p_1}{T_1} = \frac{p_2}{T_2}$$

The ideal gas equation is given by : PV = nRT

$n = n_p/N_A$, n_p = number of particle and N_A is avogado's number = 6.02×10^{23} molecules.

EXAMPLE 1 : a fixed mass of gas occupying $6\times10^{-3}m^3$ and 27^oC is compressed at constant temperature until the pressure is doubled. What is the final volume?

SOLUTION : let $p_1 = p$, $p_2 = 2p$ (doubled)

$p_1v_1 = p_2v_2$, $\cancel{p} \times 6 \times 10^{-3} = 2\cancel{p} \times v_2$

$v_2 = 3 \times10^{-3}m^3$

EXAMPLE 2 : a tank of volume $0.5cm^3$ contains oxygen at an absolute pressure $1.5 \times 10^6 N/m^2$ and a temperature of 20^oC. assume that oxygen behaves like an ideal gas, how many moles of oxygen are in the tank? (molecular mass of oxygen = 32g, R = 8.31j/mole)

HINT : PV = nRT, **ANSWER : 308moles**

EXAMPLE 3 : an ideal gas in a container of volume $1000cm^3$ at 20^oC has a pressure of 1×10^4 N/m^2 . determine the number of gas molecules and the number of moles of gas in the container.

SOLUTION : $v = 1000cm^3 = 10^{-3}m^3$, n_p =?, n =?

$T = 20+273 = 293K$, $p = 1\times10^4 N/m^2$

$PV = nRT$, $1\times10^4 \times10^{-3} = n\times 8.31 \times 293$

$n = 4.1 \times 10^{-3}mol$,

$\mathbf{n = n_p/N_A}$, $4.1 \times 10^{-3} = n_p/6.02 \times10^{23}$

$\mathbf{n_p} = 2.4 \times10^{21}$molecules

the relationship between root mean square(r.m.s) speed and temperature is :

$$\frac{v_2}{v_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}}$$

EXAMPLE 4 : a molecule of gas has r.m.s speed of 500m/s at 20^oC. what is the rms speed at 80^oC?

SOLUTION : $v_1 = 500m/s$, $T_1 = 20 + 273 = 293K$

$T_2 = 80 + 273 = 353K$,

$$\frac{v_2}{v_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}}, \quad \frac{v_2}{500} = \frac{\sqrt{353}}{\sqrt{293}} \quad \text{cross multiply}$$

$v_2 = 548.8m/s \cong 550m/s$

EXAMPLE 5 : if the temperature of a gas increases from 20^oC to 40^oC, by what factor does the r.m.s speed increase ?

SOLUTION : convert temperature to kelvin

$$\frac{v_2}{v_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}}, \quad \frac{v_2}{v_1} = \frac{\sqrt{313}}{\sqrt{293}}, = 1.03 = \frac{103}{100}$$

thus, 3% increase

THERMODYNAMICS

It deals with the transfer or action (dynamics) of heat.

LAWS OF THERMODYNAMICS

1. Zeroth law : it states that if bodies A and B are in thermal equilibrium with a third body C (the thermometer) , then A and B are in thermal equilibrium with each other. **The essence of zeroth law is : *there exist a useful quantity called temperature.***

2. the first law : it states that the amount of heat supplied to a system is equal to the algebraic sum of the change in thermal energy of the system and the amount of external work done by the system. Thus **dQ = du + dw**

dQ = amount of heat supplied by the system

du = increase in internal energy of the system

dw = external work done by the system

this law establishes the relation between heat and work.

Also, **$R = c_p - c_v$ (mayer's formula)**

C_p = specific heat capacity at constant pressure

C_v = specific heat capacity at constant volume

R = *molar das constant*

EXAMPLE 6 : a system consists of 3kg of water. 25j of work is done on the system by stirring with a wheel, while 63j of heat is removed. What is the change in internal energy of the system.

SOLUTION : dw = -25j (done on the system)

dQ = -63j (heat removed) from dQ = du + dw

du = dQ – dw = -63 + 25 = -38j

since du < 0 then the internal energy of the system decreases.

EXAMPLE 7 : a refrigerator takes heat from its cold interior at a rate of 7.5KWnwhen the work required is done at a rate of 2.5KW. at what rate is heat exhausted to the kitchen.

SOLUTION : $dQ_p = du_p + dw_p = 7.5 + 2.5 = 10KW$

3. the second law : there are two conventional statements of this law : Clausius statements and Kelvin-plank statement. But at this level we will consider **Clausius statements** : *it says it is impossible for a self-acting machine, unaided by any external work agency, to transfer heat from a body at a lower temperature to a body at a higher temperature or heat cannot by itself pass from a cold to a hot body.*

Application of second law

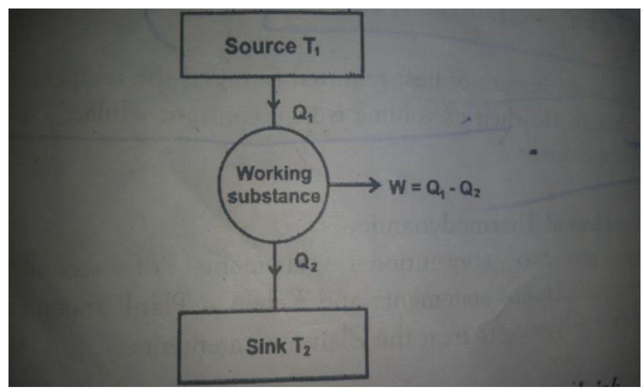
1. heat engines : it is a device which converts heat into work. It consists of three parts :

i. a source or high temperature reservoir at temperature T_1

ii. a sink or low temperature reservoir at temperature T_2

iii. a working substance

the working substance extracts heat Q_1 from source, does some work W and ejects remaining heat Q_2 to sink as shown below :



The efficiency (n) of heat engine is :

$$n = \frac{\text{work done}}{\text{heat taken from source}},$$

$$n = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

Note : in calculations, always multiply the formulae by 100.

2. carnot's circle as refrigerator : when the Carnot's engine works a s a refrigerator, it absorbs heat Q_2 from the sink. An amount of work W is done on it by some external means and rejects heat Q_1 to the source.

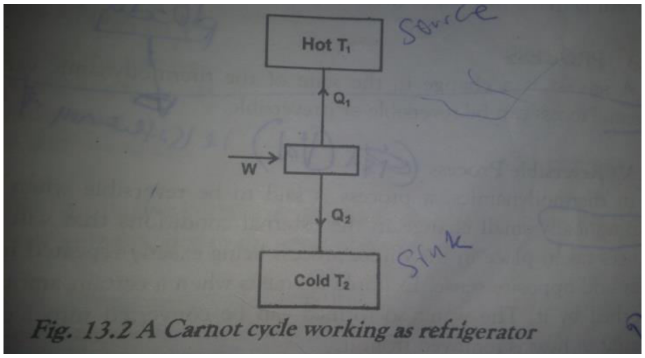


Fig. 13.2 A Carnot cycle working as refrigerator

The **coefficient of performance , k ,** is the ratio of the heat taken in from the cold body to the work needed to run the refrigerator.

$$K = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Also, in calculations multiply by 100

The relative efficiency is the ratio of the

Thermal efficiency to the carnot efficiency.

$n_{rel} = n_{th}/n_{carnot}$, also multiply by 100

3. thermal pump : it is a device that transfers energy from a low temperature reservoir to a high temperature one. To do this work must be done(according to second law) it will not happen on its own.

EXAMPLE 8 : a **carnot engine** is operated between two reservoir at temperature of 450K and 350K. if the engine receives 4200j of heat from the source in each cycle, calculate the amount of heat rejected to the sink in each cycle. Calculate the efficiency of the engine and the work done by the cycle in each cycle.

SOLUTUON : $T_2 = 350K$, $T_1 = 450K$, $Q_1 = 4200j$

$$\frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} \quad , \quad \frac{450 - 350}{450} = \frac{4200 - Q_2}{4200}$$

$$\frac{100}{450} = \frac{4200 - Q_2}{4200} \quad , \quad 420000 = 1890000 - 450Q_2$$

$Q_2 = 3267j$

$$n = \frac{T_1 - T_2}{T_1} \times 100 \, , \quad \frac{450 - 350}{450} \times 100 = 22.22\%$$

Work = $Q_1 - Q_2$ = 4200 – 3266 = 933j

EXAMPLE 9 : a carnot engine whose low temperature reservoir is 7^oC has an efficiency of 50%. It is designed to increase the efficiency to 70%. By how many degrees should the temperature of the high temperature reservoir be increased?

SOLUTION: convert temperatures to Kelvin

$$n = \frac{T_1 - T_2}{T_1} \times 100, \quad 50 = \frac{T_1 - 280}{T_1} \times 100$$

$50T_1 = 100T_1 - 28000$, $T_1 = 560K$

$$n = \frac{T_1 - T_2}{T_1} \times 100, \quad 70 = \frac{T_1 - 280}{T_1} \times 100$$

$70T_1 = 100T_1 - 28000$, $T_1 = 933.33K$

Thus, 933.33 - 560 = 373.3K = 100.3^oC

EXAMPLE 10 : find the efficiency of the carnot's engine working between the steam point and the ice point.

SOLUTION : $T_1 = 100^oC$ = 373K(i.e steam point)

$T_2 = 0^oC$ = 273K (i.e ice point) now proceed……

$$n = \frac{T_1 - T_2}{T_1} \times 100$$

answer : 26.81%

EXAMPLE 11 : a heat engine operates at a relative efficiency of 4%. If the temperature of the high-temperature and low-temperature reservoir are 400 and 100°C respectively, what are the carnot efficiency and the thermal efficiency.

SOLUTION : $n_{rel} = 45\%$, $T_1 = 400^oC = 673K$

$T_2 = 100^oC = 373K$

$$n_{carnot} = \frac{T_1 - T_2}{T_1} \times 100 = \frac{673-373}{673} \times 100 = 44.6\%$$

$$n_{rel} = \frac{n_{th}}{n_{carnot}} \times 100, \quad 45 = \frac{n_{th}}{44.6} \times 100$$

cross multiply , $n_{th} = 20.1\%$

PROCESSES

A process is a change in state of the thermodynamic variables of a system. Process can be reversible or irreversible.

1. REVERSIBLE PROCESS : in thermodynamics, a process is said to be reversible when there is an infinitesimally small change in the external

Conditions that will result in all changes taking place in the direct process being exactly repeated in the reversed order and opposite sense. E.G conversion of ice to water

2. IRREVERSIBLE PROCESS : it is a process which cannot be retracted in the opposite order by reversing the controlling factor. E.g all natural processes such as conduction, radiation, radioactive decay e.t.c

3. ISOTHERMAL PROCESS (du = 0) : *iso* processes are one in which one of the thermodynamic variables is held constant. Isothermal process involves the change in pressure and volume of a gas occurring at constant temperature or constant internal energy. **The work done in isothermal process is given as : w = RT $\log_e(v_2/v_1)$** Also, from first law, du = dQ – dw, when du=0, **dQ= dw**

Thus, the work done by an ideal gas is equal to the heat energy added to the gas.

EXAMPLE 12 : an ideal gas undergoes an isothermal process in doing 25j of work. What is the change in internal energy?

SOLUTION : no need to solve, for isothermal process du = 0

EXAMPLE 13 : A gas is compressed isothermally at 300K and 1 atm pressure from an initial volume of 0.5litres to final volume of 0.25litres in a tyre. Assuming the exit of the tyre is blocked, find the work done.(gas constant=8.31j/molK, 1atm =1.013× 10^5N/m).

SOLUTION : T = 300K, v_1 = 0.5L, v_2 = 0.25L

w = RT $\log_e(v_2/v_1)$ = 8.31× 300$\log_e(\frac{0.25}{0.5})$ =-1788j

divide by 1000, we have w = -1.728Kj

4. ADIABATIC PROCESS (dQ = 0) : it occurs when there is no heat change, i.e du = 0, between the gas and the surrounding. From

First law, dQ = du + dw, dQ = 0, **du = -dw**

Thermodynamic relations in adiabatic processes are given as :

$$\mathbf{P_1v_1^y = p_2v_2^y, \quad T_1v_1^{y-1} = T_2v_1^{y-1}}$$

$$\mathbf{\frac{P_1^{(1-\frac{1}{y})}}{T_1} = \frac{P_2^{(1-\frac{1}{y})}}{T_2}, \quad \text{where } y = \frac{c_p}{c_v}}$$

The work done is given by **dw = $nc_v(T_1 - T_2)$**

EXAMPLE 14 : Ten litres of nitrogen at 1 atm and 25°C is allowed to expand reversibly and adiabatically to 20 litres. What are the final pressure and temperature, given the ratio of the molar specific heat capacities is 1.4.

SOLUTION : v_1=10L, v_2=20L,

T_1 = 25+273=298K

P_1 = 1 atm, $y = 1.4$

$T_1 v_1^{y-1} = T_2 v_1^{y-1}$, $298 \times 10^{1.4-1} = T_2 \times 20^{1.4-1}$

$748.542 = 3.3142 T_2$, $T_2 = 225.8K$ also,

$P_1 v_1^y = p_2 v_2^y$, $1 \times 10^{1.4} = p_2 \times 20^{1.4}$, P_2=0.379atm

5. ISOBARIC PROCESS (dw = Pdv) : it is a process takes place at constant pressure. The work done here is **$w = P(v_2 - v_1)$**

6. ISOCHORIC PROCESS (dv = 0) : it is a process in which the volume remains constant. It is also called **isovolumetric or isometric process.** From first law, dQ = du + dw, dv = 0

dQ = du , *thus the increase in internal energy is equal to the heat supplied to the system*

EXAMPLE 15 : in an isometric process, the internal energy of the system decreases by 50j

a. what is the work done?

b. what is the heat exchange?

SOLUTION : dw = Pdv, dv = 0, thus w = 0

b. dQ = du + dw = -50 + 0 = -50j

thus, 50j of heat is removed from the system

7. ISOLATED PROCESS : a process is said to be isolated when there is no external work and into which there is no flow of heat. This implies that w = Q = 0. Thus, the first law reduces to **du = 0**. Therefore, the internal energy of an isolated system is **constant** .

THE KINETIC THEORY OF GASES

It attempts to explain the macroscopic properties **(p, v , T, n)** of a gas in terms of microscopic properties.

1. the temperature is a measure of the average kinetic energy of the molecules, i.e

$\frac{1}{2} Mv^2_{rms} = \frac{3}{2} K_B T$, m = mass of the molecule

K_B is Boltzmann constant = $R/N_A = 1.38 \times 10^{-23}$j

$V_{rms} = \sqrt{(3K_BT/M)} = \sqrt{(3RT/M)} = \sqrt{(3P/d)}$

d = density.

2. the total internal energy of an ideal monoatomic gas is given by :

$$K.E = U_{monoatomic} = \frac{3}{2} nRT = \frac{3}{2} N_A K_B T$$

the above is for translational motion.

$$K.E = U_{diatomic} = \frac{5}{2} nRT = \frac{5}{2} N_A K_B T$$

The above is for vibrational motion

$K.E = U_{diatomic} = NK_BT = nRT$ (rotational motion)

Where 3/2, 5/2……. Are degrees of freedom

EXAMPLE 16 : if one mole of a monoatomic gas has a total internal energy of 3.7×10^3j, what is the total internal energy of one mole of diatomic gas at the same temperature ?

SOLUTION : $U_{monoatomic} = \frac{3}{2} nRT = 3.7 \times 10^3$

$$nRT = \frac{3.7 \times 10^3 \times 2}{3} = 2466.67$$

$U_{diatomic} = \frac{5}{2} nRT = \frac{5}{2} \times 2466.67 = 6.2 \times 10^3$j

CHAPTER 14

HEAT TRANSFER

There are three processes by which hear is being transferred between two regions of different temperatures :

1.CONDUCTION : this process of heat transfer which needs a material medium, but the heat energy is transferred not by actual movement of the materials but simply by the interaction of materials with each other and exchanging energy in the process.

$$H = \frac{dQ}{dt} = \frac{KA(T_2 - T_1)}{L}, \quad H = \text{heat current}$$

$\frac{(T_2 - T_1)}{L}$ is temperature gradient

K is thermal conductivity, it is a constant – depending on the material. Its unit is

$Js^{-1}m^{-1o}C^{-1}$ or $Wm^{-1o}C^{-1}$

EXAMPLE 1 : one end of a 30cm long aluminium rod is exposed to a temperature of 500°C. while the other end is maintained at 20°C. the rod has a diameter 2.5cm. if heat is conducted through the rod at a rate 142Kcal/hr. calculate the thermal conductivity of aluminium.

SOLUTION :

$T_2 = 500^oC$, $T_2 = 20^oC$,

L =30cm=0.3m, H = 142Kcal/hr = 164.88j/s

d = 2.5cm = 0.025m.

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.025^2}{4} = 4.91 \times 10^{-4}m$$

$$= \frac{KA(T_2 - T_1)}{L}, \quad 164.9 = \frac{K\times 4.91 \times 10^{-4}(500-20)}{0.3}$$

$49.47 = 0.23568K$, $K = 209.9Wm^{-1o}C^{-1}$

EXAMPLE 2 : energy released by radioactivity within the earth is conducted outward as heat through oceans. Assume the average temperature gradient within the solid earth beneath the ocean to be 0.07°C/m and the average thermal conductivity to be 0.84j/ms°C and determine the rate of heat transfer per square meter.

SOLUTION :

$$\frac{(T_2 - T_1)}{L} = 0.07^oC/m, \quad \frac{dH}{dA} = ?, \quad K = 0.84j/ms^oC$$

$$\frac{dH}{dA} = \frac{K(T_2 - T_1)}{L} = 0.84 \times 0.07 = 5.88\times10^{-2} j/m^2s$$

EXAMPLE 3 : one end of a 50cm rod is exposed to a temperature of 600°C while the other end is maintained at 40°C. the rod has a diameter of 5cm. if heat is conducted through the rod at a rate of 150Kcal/hr. calculate the thermal conductivity of aluminium.

SOLUTION : $T_2 = 600$, $T_1 = 40$, L = 50cm = 0.5m

d = 5cm= 0.05m, H = 150Kcal/hr = 174.4j/s

$$A = \frac{\pi d^2}{4} = \frac{\pi \times 0.05^2}{4} = 1.96 \times 10^{-3}m^2$$

$$= \frac{KA(T_2 - T_1)}{L}, \quad 174.4 = \frac{K \times 1.96 \times 10^{-3}(600-40)}{0.3}$$

$87.5 = 1.0976K$, $K = 79.45Wm^{2o}C^{-1}$

2. CONDUCTION : it is a transfer of heat energy that involves the actual movement of the heated medium. It is applicable only to fluid.

Convention is said to be **natural or free** convention when it is due to **density differences** caused by thermal expansion and **forced convention** when artificial means is responsible for the circulation. If we consider a solid at temperature T_s with surface area A_s in contact with a fluid whose main body temperature is T_f, it was discovered that the rate of heat flow, H, by convention from the solid surface to the fluid flowing over it (or vice visa) is given as : $\mathbf{H = hA_s(T_s - T_f)}$. *where h is convention coefficient , its unit is j/sm² °C and depends on :*

1. nature of the surface (i.e flat, curved, horizontal of vertical)

2. nature of the fluid (liquid or gas)

3. fluid properties (density, viscosity, S.H.C and thermal conductivity) (4) fluid velocity

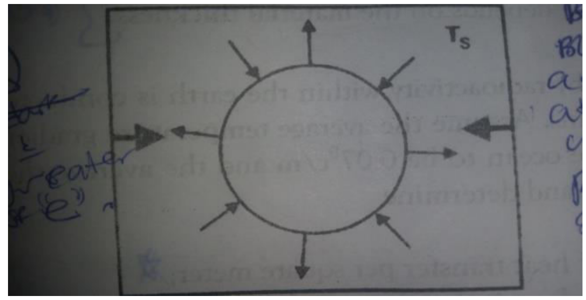
3. RADIATION : this mode of heat transfer requires no material medium, the energy simply moves through a vacuum in the form of electromagnetic waves. E.g heat flow from sun to the earth. *The intensity of radiant heat radiated by a surface depends on the* ***nature*** *as well as* ***the temperature of the surface.*** For a body of surface area A, and absolute temperature T, the rate at which radiant heat is emitted by the body is given as : $\mathbf{H = \sigma AeT^4}$

The formula above is **Stefan–Boltzmann's Law**

e is emissivity. Its value is between 0 and 1. **It depends on the nature of the surface.** The value of e is large for dark surfaces than for light ones. *For a dull black surface, e is close to 1.* For smooth copper surface e is about 0.3. the emissivity of human skin is about 0.70,

σ is Stefan-Boltzmann's constant = 5.67×10^{-8} w/m²K⁴.since all bodies absorb a different degree of thermal radiation incident on them, then the rate at which a body will absorb radiant energy when completely surrounded by an enclosure whose surface is maintained at absolute Kelvin temperature T_s is given as :

$$\mathbf{H = \sigma AeT_s^4}$$



the net rate of radiation from a body at temperature T with surroundings at temperature T_s is given as :

$\mathbf{H_{net} = Ae\sigma(T^4 - T_s^4)}$ **,** if T^4 is less than T_s^4, then H_{net} will be negative, indicating a net heat loss.

EXAMPLE 4 : a radiator has an emissivity of 0.70 and its exposed area is 1.5m². the temperature of the radiator is 100°C and the surrounding temperature is 20°C. what is the net heat flow rate from the body?

SOLUTION : $H_{net} = Ae\sigma(T^4 - T_s^4)$

$= 1.5\times0.7\times5.67\times10^{-8}(373^4 - 293^4) = 7.13\times10^2\text{W}$

EXAMPLE 5 : the operating temperature of a tungsten filament in an incandescent lamp is 2460K. and its emissivity is 0.35. find the surface area of the filament of a 100W lamp.

SOLUTION : $H = \sigma AeT^4$,

$100 = 5.67 \times 10^{-8} \times A \times 0.35 \times 2460^4$,

$100 = 726760.86A$, $A = 1.37 \times 10^{-4}\text{m}^2$

EXAMPLE 6 : (a) a sphere of radius 2cm with a black surface is cooled and then suspended in a large evacuated enclosure, the black walls of which are maintained at 27°C, if the rate of change of thermal energy of the sphere is 1.849W when its temperature is -83°C. calculate the value of Stefan's constant (b) if each square centimeter of the sun's surface radiates energy at the rate of $6.3 \times 10^3 W/cm^2$. Calculate the temperature of the sun's surface.

Assuming stefan's law applies to the radiation.

SOLUTION : e = 1(black surface), r=2cm=0.02m

$A = 4\pi r^2 = 4\pi(0.02)^2 = 16\pi \times 10^{-4} m^2$

$T = 27^oC = 300K$, $T_s = -83^oC = 190K$,

$H_{net} = Ae\sigma(T^4 - T_s^4)$

$1.848 = 16\pi \times 10^{-4} \times 1 \times \sigma (300^4 - 190^4)$,

$\sigma = 5.4 \times 10^{-8} W/m^2K^4$

(b) $\frac{H}{A} = 6.3 \times 10^3 W/cm^2 = 6.3 \times 10^7 W/m^2$

$\frac{H}{A} = \sigma e T_s^4$, $6.3 \times 10^7 = 1 \times 5.4 \times 10^{-8} \times T^4$

$T^4 = 1.166 \times 10^{15}$, $T = \sqrt[4]{(1.166 \times 10^{15})} = 5844.3K$

EXAMPLE 7 : suppose that your skin has an emissivity of 0.7 and that its exposed area is $0.27m^2$. how much net energy will be radiated per second from this area if the air temperature is 20^oC ? assume your skin temperature to be the same as normal body temperature, 37^oC.

Answer : H_{net} = -20W

CHAPTER 15

PHYSICAL STATE OF MATTER

Elasticity and Hook's law : if a body is deformed to its original size and shape when the external deforming force is removed, then the body is said to be elastic. The maximum deformation that an elastic body can undergo without a permanent change in its shape is called **the elastic limit.**

Hook's law states that the force applied to an elastic solid object is directly proportional to the extension, provided the elastic limit is not exceeded.

Thus F = ke

K depends on :

1. the size and shape of the body

2. the material of which the body is made

3. the temperature and pressure to which the body is subjected while the measurements are carried on.

4. in some cases, the direction in which the body is distended.

EXAMPLE 1 : a spring stretches by 7.5mm when a 50N weight is attached to it . (a) find the spring constant (b) if the 50N weight is removed and a 125N weight is attached in its place, find the stretched of the spring.

SOLUTION : e = 7.5mm = 0.0075m, F = 50N

a. F = ke, $50 = k \times 0.0075$, $k = 6.67 \times 10^3 N/m$

b. F = 125N, e = ? , F = ke , $125 = 6.67 \times 10^3 \times e$

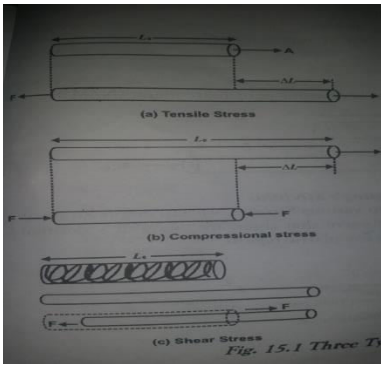
$e = 1.87 \times 10^{-2} m$

YOUNG'S MODULUS (E)

Hook's law can be restated as follows : **the stress is proportional to the strain as long as the elastic limit has not been exceeded.**

The proportionality constant is called the Young's elastic modulus of elasticity and depends on the material being deformed and the nature of the deformation. **The S.I unit if E is N/m^2**

$E = \frac{stress}{strain} = \frac{F/A}{e\backslash L}$, **note: e = ΔL**



NOTE : a tensile stress or tension tends to increase the length of an object; a compressional stress tends to shorten the length. A shear stress is produced when the force is applied tangentially to a surface area.

It an elastic object is subjected to various stresses, we can graph stress as a function of strain as show below

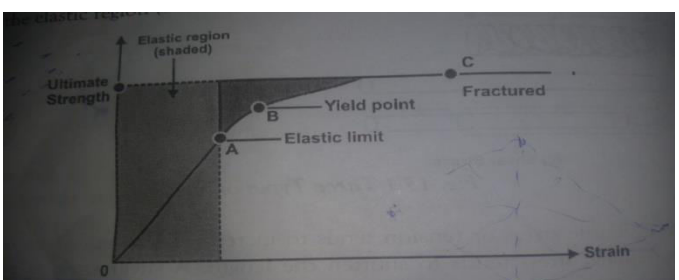


Fig. 15.3: Stress versus strain for an elastic solid

in the elastic region, the graph is a straight line and obeys Hook's law. The maximum stress that can be applied to the object without causing its permanent deformity is called **the elastic limit(it is labeled as A in the diagram).**

If the stress is increased beyond the elastic limit, the strain increases more rapidly than the stress(REGION **AB). **After the stress is removed the body does not regain its original shape and size, but shows a residual deformation. Thus, the region of elastic deformation is followed by the region of irreversible deformation, which is known as **plastic deformation.** In the portions of the plastic region where the curve slopes towards the horizontal axis, the object continues to enlongate, although the applied stress decreases. This behavior is known as **plastic flow.** At some point, the object breaks. Just before it breaks, the curve peaks, which gives the maximum stress that can be applied to the object without breaking it. This stress is called the **ultimate strength of the object.**

SHEAR MODULUS (G)

It is sometimes referred to as **modulus of rigidity.** A deformation in which the shape of an object but not its volume is changed.

G = $\frac{F/A}{\emptyset}$, **F is applied shear force,** A is surface area, Ø is shear angle. **The unit of G is N/m^2**

BULK MODULUS (B)

When force act uniformly over the entire surface of a body, the volume of the object change but its shape remains the same. We will only consider the forces which act perpendicular to the surface since they are the only forces which contribute to the volume changes. *Forces parallel to the surface only contribute to **shear stress.*** The **elastic modulus is called the bulk modulus.**

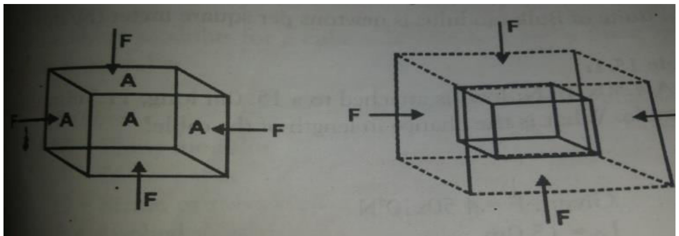


Fig. 15.4 Bulk Compression

B = 1/K, k is compressibility

EXAMPLE 2 : A 4.5×10^4N load is attached to a 15m long, 11.7mm diameter steel cable. What is the change in length of the cable.

HINT : $E = 20\times10^{10}N/m^2$(for steel)

$d=11.7mm=1.17\times10^{-2}m$, $A=\pi d^2/4=\pi\times(1.17\times10^{-2})^2/4$

$=1.07\times10^{-4}m^2$,**use this :** $E = \frac{F/A}{e\backslash L}$, **answer =** 3.14×10^{-2}m

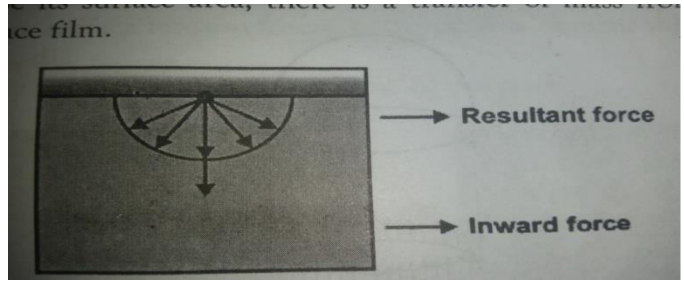
EXAMPLE 3 : a steel wire 2m in length and 2mm in diameter supports a 10kg mass. (a) what is the stress in the wire?

HINT : F = mg, stress = F/A, answer : $3.18\times10^7N/m^2$

SURFACE TENSION

It is the force acting along the surface of a liquid, causing the liquid surface to behave like a stretched elastic skin.

The particles in a thin layer near the surface of a liquid are subjected to the force acting from the other molecules of the liquid. **The resultant forces is directed inward normally to the surface.**



In the past questions below we will not write the options. We will only solve and write the answers

MOST DIFFICULT RECENT PAST QUESTIONS

NOTE : we have solved a lot of past questions as examples in this material.

1. what will the velocity of a stone 10m above the ground, if it is projected vertically upward from the top of a roof 35m with a velocity 10m/s.

SOLUTION : h_2= 10m, h_1 = 35m, $\theta = 90^o$

$v^2=(u\sin\theta)^2 - 2g(h_2 - h_1) = (10\sin90)^2 - 2\times10(10 - 35)$

$v = \sqrt{600} = 24.49m/s$

2. when a stone and a tennis ball of the same mass are thrown at a boy with the same velocity, he would prefer the tennis ball because the tennis ball will spend :

a. lesser time and lesser force when in contact with the boy (b) lesser time greater force when in contact with the boy (c) more time and greater force when in contact with the boy (d)more time and lesser force when in contact with the boy. **Answer is D**

3. find the resistance of a car of mass 521kg moving at 10m/s comes to rest in a distance of 350m.

SOLUTION : v = 0, u = 10m/s, s = 350m

$V^2 = u^2 - 2as$, $0^2 = 10^2 - 2\times350\times a$, a = 1/7m

F = ma = 521× 1/7 = 74.4N

4. what amount of work is done by a man who carries a load of 10kg on his head through a square distance of sides 1m, if he moves through ABCDA. **Answer is zero because work done in carrying a load around a closed loop or square is ZERO** regardless of the parameter given,

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