MTH 101 QUICK REVIEW MATERIAL

Now I want you to read this carefully not intensely, but peacefully, because I want you to fully grasp everything I'm about to share with you through this material. So enjoy. MTH101 will be fun – Trust me :)

A Student gave me a question bank that comprises of 234 questions in total, there is no time to teach that in a short time, so I did a pie chart of it: Permutations & combinations, sequence & series and set theory dominate the content.

Other important areas include Binomial Expansion, Trigonometry and Partial fractions.

So with this been stated, I will clearly settle on 5 topics, the first dominate topics of course.

Don't worry I will hit the nail on the head :).

1.) PERMUTATION & COMBINATION

Permutation and Combination are quite similar, but different mathematically :–

$$^{n}P_{r} = \frac{n!}{(n-r)!} \Rightarrow \text{PERMUTATION}$$

$$^{n}C_{r} = \frac{n!}{r!(n-r)!} \Rightarrow \text{COMBINATION}$$

Now the major question is how to know which formula to use in a Permutation and Combination question.

know this: Permutation ⟶ order

Combination ⟶ No order

Once a question is given check if the question has order or no order if it's has order use Permutation formula, No order use Combination.

Examples

1) In how many ways can 3 people be selected as president, secretary, and Treasurer from a group of 10?

Solution →

Note: order matters in choosing roles, hence we use ${}^nP_r$

$${}^nP_r = \frac{n!}{(n-r)!}$$

n = total number of items

r = number of items selected/arranged

$n! = n(\text{factorial}) = n \times (n-1) \ldots$

2) In how many ways can you choose 3 students to form a Committee from 10?

Solution →

No order in a Committee, hence ${}^nC_r = \frac{n!}{r!(n-r)!}$

Now, from the question Bank

Q4: Evaluate ${}^{12}P_7$

this is clearly a permutation question, P stands for Permutation.

$${}^{12}P_7 = \frac{12!}{(12-7)!} = \frac{12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5!}{5!}$$

$$= 3,991,680$$

Q19: A board has 5 directors A, B, C, D, E. If Committees of 3 members each are to be constituted, how many Committees are possible?

Committees = No order

ANSWER is (D)

$${}^nC_r = \frac{n!}{r!(n-r)!} = \frac{5!}{3!(5-3)!} = \frac{5!}{3!2!} = 10$$

ANSWER is (B)

I Believe we are done with topic 1, use that information and apply it in your ^nP_r and ^nC_r question and see.

2.) Sequences & Series

1, 2, 3, 4 ... → this is a sequence
1 + 2 + 3 + 4 ... → = = = series

} Summing a sequence gives you a series.

1, 2, 3, 4, ⇒ Let a = first term = 1
d = difference = 1

second term = first term + difference
= 1 + 1 = 2 or $a + d = 2$

third term = second term + difference or $(a+d) + d = a + 2d$

⋮ ⋮

nth term = ... let me explain before dropping the famous formula.

NOTE:

n = position of a number

so 2, 6, 10, 14 ... } → sequence

we can clearly see number 10, but what position is it held in the sequence, 3, therefore $n = 3$.

Let say I'm looking for the 4th term = n_4, knowing fully well my difference is 4.

mathematically,

$d = 4$ ⇒ difference
$a = 2$ ⇒ first term

n_4 (4th term) = third term + difference
$= a + 2d + d$
$= a + 3d$
$= 2 + 3(4)$

$\therefore n_4 = 14$ ⇒ this read, the number is the 4th position is 14.

hence n means position of a number

so observe carefully. If we are looking for a term, let' say 4th term the difference is always 1 short of the term we are looking for

$n_4 = a + 3d$
↳ 3 difference — 3 (1 short of the term)
↳ 4th term — 4

$n_8 = a + 7d$
↳ 7 difference — 7 (1 short of the term)
↳ 8th term — 8

hence, n^{th} term means the last number in the sequence so we follow the steps 1 short of difference.

$n^{th} = a + (n-1)d$ Similar $n_8 = a + (8-1)d = a + 7d$
↓ the term ↓ the term

~~Hmm~~ All you could just cram it for all I care :)

$$\left.\begin{aligned} a_n &= a + (n-1)d \\ S_n &= \frac{n}{2}[2a + (n-1)d] \end{aligned}\right\} A.P$$

$$\left.\begin{aligned} a_n &= ar^{n-1} \\ S_n &= a \cdot \frac{1-r^n}{1-r} \text{ if } |r| < 1 \end{aligned}\right\} G.P$$

Q77: Find the 15th term and Sum of the first 26 terms of
21, 17, 13, 9, 5, ... → from the question Bank

Solution →

$a_{15} = a + 14d$
$= 21 + 14 \times (-4)$
$= -35$

$S_n = \frac{n}{2}(2a + (n-1)d)$
$= \frac{26}{2}(2 \times 21 + (25) \times -4)$
$= -754$ Option (C)

3.) SET THEORY & VENN DIAGRAM

Concepts to know:

* Set operations:

Union : $A \cup B$

Intersection : $A \cap B$

Complement : A'

Difference : $A - B$

Power set : $P(A) = 2^n$ e.g $A = \{a, b,\}$

$P(A) = 2^2 = 4$, hence the power set of A is 4. n = number of element.

Q45 : let $U = \{a, b, c, d, e, f, g, h\}$

$Q = \{c, f, g\}$ find Q'

Solution

$Q' = U - Q$ (meaning what is U that is not in Q)

$Q' = \{a, b, d, e, h\}$

4.) BINOMIAL EXPANSION

formula : $(x + a)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} a^r$

- the binomial formula is use to find the coefficient of a specific term.
- $\sum$ means summation of all term found in the binomial expansion
- r increases to n

from the question bank

Q31 : what is the coefficient of x^4 in the expansion of $(1+x)^5$?

Solution →

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1st step: $(x+a)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} a^r$

2nd step: Compare the expression $(1+x)^5$ with the binomial expression $(x+a)^n$ → 1st term → 2nd term

↓ → 2nd term

1st term

always take note that 1st term belong to 1st term position in the expression, like-wise 2nd term, I have seen many student missed that.

3rd step: $(1+x)^5 = \sum_{r=0}^{5} \binom{5}{0} (1)^{5-0} x^0$ } Proper positioning

Now we expand:

$$(1+x)^5 = \binom{5}{0}(1)^{5-0}x^0 + \binom{5}{1}(1)^{5-1}x^1 + \binom{5}{2}(1)^{5-2}x^2 + \binom{5}{3}(1)^{5-3}x^3 + \binom{5}{4}(1)^{5-4}x^4 + \binom{5}{5}(1)^{5-5}x^5$$

Note: the question says the coefficient of x^4, so we are only interested in when x is raised to the power of 4

$$\therefore \binom{5}{4}(1)^{5-4}x^4 = \binom{5}{4}1^1 x^4 = \binom{5}{4}x^4$$

$$\binom{5}{4} = \frac{5!}{4!(5-4)!} = \frac{5!}{4!} = \frac{5\times 4!}{4!} = 5.$$

Therefore, $5x^4$ the coefficient of x^4 is 5

ANSWER (C)

NOTE: No order, so we used Combination

Clax

5.) Trigonometry

Concepts to know:

$\sin^2 x + \cos^2 x = 1$

$\cos 2x = 1 - 2\sin^2 x$

$\sin 2x = 2\cos^2 x - 1$

Q117: Given that $\sin A = \frac{4}{5}$ and A is obtuse, Find $\cos 2A$

Solution

$\cos 2A = 1 - 2\sin^2 A$ → from the formula

Since $\sin A = \frac{4}{5}$, then $\sin^2 A = \left(\frac{4}{5}\right)^2$

$= \frac{16}{25}$

$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left[\frac{16}{25}\right]$

$= 1 - \frac{32}{25}$

$= \frac{-7}{25}$ OPTION (D)