PHY 113

Week	Topic
1	Vibrations
2	Damped oscillation
3	Forced Damped Harmonic oscillation
4	Waves
5	Interference of Waves
6	Normal mode
7	Optics
8	Reflection at plane and curved surfaces
9	Refraction through plane surfaces
10	Refraction through curved surfaces
11	Optical Instruments
12	Dispersion and Aberration

VIBRATIONS

Periodic motion is a type of motion seperated in equal time intervals or regular cycle

Excamples

- A mass attached to a vibrating spring
- Swinging pendulum
- A mass attached to a string moving in circular motion
- A bouncing ball

At equilibrium position, net force is zero.

- Elasticity stores potential energy.
- Mass enables the system to have kinetic energy.

PROPERTIES OF OSCILLATORY MOTION

1) Amplitude (A) :- The maximum displacement of a body from the equilibrium position (metres).

2) Period (T) :- The time it takes a body to make one complete oscillation or cycle (seconds)

$$T = \frac{t}{n}, \quad T = \frac{2\pi}{\omega}, \quad T = 2\pi\sqrt{\frac{L}{g}}, \quad T = 2\pi\sqrt{\frac{m}{K}}$$

3) Frequency (f) :- The number of complete oscillations or cycles a body make in one second (Hertz).

It is the reciprocal of period

$$f = \frac{1}{T}, \; f = \frac{\omega}{2\pi}, \; f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

SIMPLE HARMONIC MOTION

Simple harmonic motion is the periodic motion of a point along a straight line, such that its acceleration is always towards a fixed point in that line and its proportional to its distance from that point.

EQUATION OF A SIMPLE HARMONIC MOTION

$$A = \sqrt{B_1^2 + B_2^2} \qquad v = \sqrt{v_1^2 + v_2^2}$$

where A is Amplitude and v is velocity.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \qquad [x(t) = B_1 \cos \omega t + B_2 \sin \omega t] \text{ or}$$

$x(t) = A\cos(\omega t + \phi)$ (General equation of a simple harmonic motion).

$$\phi \text{ (phase difference)} = \tan^{-1}\left(\frac{B_2}{B_1}\right)$$

VELOCITY AND ACCELERATION OF A

SINUSOIDAL OSCILLATION

$V = -\omega A$

$a = -\omega^2 x(t)$

$x = A$ (at maximum)

$v = \omega A$ (velocity) - maximum

$a = \omega^2 A$ (acceleration) - maximum

N.T - Velocity at any point is $v = \omega\sqrt{A^2 - x^2}$

$F = Ke$ or Kx (Hooke's law)

Energy in simple harmonic motion

$$E = \frac{1}{2} KA^2 \text{ or } \frac{1}{2} m\omega^2 A$$

Application of simple harmonic motion

1)- The simple pendulum
2)- The torsional oscillator
3)- Vibrating molecules.

Examples

1) A particle oscillates with simple harmonic motion, so that its displacement varies

according to the expression, $x = 5\cos\left(2t + \frac{\pi}{6}\right)$

where x is in centimetres and t in seconds.

At $t = 0$, find:

- The displacement of the particle.
- Its velocity
- Its acceleration
- Its period and amplitude.

Solution

(a) Displacement $= 5\cos\left(2t + \frac{\pi}{6}\right)$ at $t = 0$

$$x = 5\cos\frac{\pi}{6} = 5 \times 0.8660 = 4.33\text{cm}$$

(b) Velocity, $v = \frac{dx}{dt} = -10\sin\left(2t + \frac{\pi}{6}\right)$ at $t = 0$

$$= -10\sin\frac{\pi}{6} = -10 \times 0.5 = -5\text{cm/s}$$

(c) Acceleration, $a = \frac{d^2x}{dt^2} = -20\cos\left(2t + \frac{\pi}{6}\right)$ at $t = 0$

$$a = -20\cos\left(\frac{\pi}{6}\right) = -20 \times 0.8660$$
$$= -17.32\text{cm/s}^2.$$

2) A block attached to a spring executes simple harmonic motion in a horizontal plane with an amplitude of 0.25m. At a point 0.15m away from the equilibrium position the velocity of the block is 0.75m/s. What is the period of oscillation of the block.

Solution.

$v = \omega\sqrt{A^2 - x^2}$ (velocity at any point)

$v = 0.75m/s$, $A = 0.25m$, $x = 0.15m$

$$\omega = \frac{v}{\sqrt{A^2 - x^2}} = \frac{0.75}{\sqrt{0.25^2 - 0.15^2}} = 3.75 rad/s$$

Recall that $T = \frac{2\pi}{\omega}$

$$T = \frac{2\pi}{3.75} = 1.676s$$

3) Given that a particle moving with S.H.M has displacement given as $x = (6\cos 3t + 8\sin 3t)$ m. Find

i) Amplitude of this motion

ii) Natural angular frequency

iii) Period of oscillation
iv) Phase constant.

Solution

Recall that general equation of a motion is $x = B_1 \cos \omega t + B_2 \sin \omega t$

Comparing with $x = 6\cos 3t + 8\sin 3t$

$B_1 = 6$, $B_2 = 8$, $\omega = 3$

i) Amplitude $= \sqrt{B_1^2 + B_2^2}$

$= \sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100}$

$= 10$ metres.

ii) Natural angular frequency $(\omega) = 3$ rad/s

Angular velocity is the same as Natural angular frequency.

iii) Period of oscillation $(T) = \frac{2\pi}{\omega} = \frac{2 \times 3.142}{3}$

$= 2.1$ s

iv) Phase constant or phase difference

$\phi = \tan^{-1}\left(\frac{B_2}{B_1}\right) = \tan^{-1}\left(\frac{8}{6}\right) = 53°$.

4) A simple pendulum has a period of 6 seconds.

If the period is 7seconds when the length was increased by 1m, find the original length of the pendulum.

Solution

Recall $T = 2\pi\sqrt{\frac{L}{g}}$

$$6 = 2\pi\sqrt{\frac{L}{g}} \quad — \text{①}$$

$$7 = 2\pi\sqrt{\frac{L+1}{g}} \quad — \text{⑪}$$

Dividing ① and ⑪

$$\frac{6}{7} = \frac{\cancel{2\pi}\sqrt{\frac{L}{g}}}{\cancel{2\pi}\sqrt{\frac{L+1}{g}}}$$

$$\frac{6}{7} = \frac{\sqrt{\frac{L}{g}}}{\sqrt{\frac{L+1}{g}}}$$

$$\frac{6}{7} = \sqrt{\frac{L}{\cancel{g}}} \times \sqrt{\frac{\cancel{g}}{L+1}}$$

$$\frac{6}{7} = \sqrt{\frac{L}{L+1}}$$

Square both sides

$$\frac{L}{L+1} = \frac{36}{49}$$

$$36(L+1) = 49L$$

$$36L + 36 = 49L$$

$$13L = 36$$

$$L = \frac{36}{13} = 2.77$$

Original length = 2.77m.

PAST QUESTION AND EXERCISES

1) A particle moving with simple harmonic m
has velocities 4cm/s and 3cm/s at distances
3cm and 4cm respectively from the equilib
position. What is the amplitude of the oscilla
- What is the velocity of the particle as it pass
the equilibrium position?

Solution.

Recall, $A = \sqrt{B_1^2 + B_2^2}$ and $v = \sqrt{v_1^2 + v_2^2}$

$$A = \sqrt{3^2 + 4^2} = \sqrt{9+16} = \sqrt{25}$$

$= 5cm$.

$$v = \sqrt{4^2 + 3^2} = \sqrt{9+16} = \sqrt{25}$$

$= 5 cm/s.$

2) The force acting on a mass of 4kg is given by $F = 36x$, where x is the displacement. Find the period.

Solution

From Hooke's law: $F = kx$

$k = 36$, $m = 4$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4}{36}}$$

$$T = 2\pi\sqrt{\frac{1}{9}} = 2\pi \times \frac{1}{3} = \frac{2\pi}{3}$$

3) A clown is rocking on a rocking chair in the dark. His glowing red nose moves back and forth a distance of 0.42m exactly 30 times a minute in a simple harmonic motion.

- What is the Amplitude of this motion.
- What is the period of this motion
- What is the frequency of this motion

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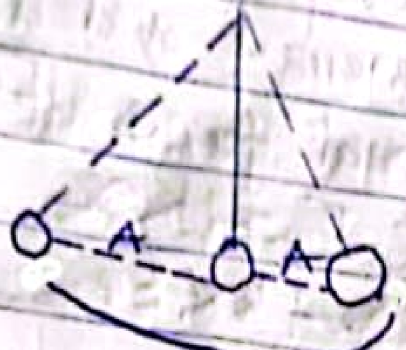
Solution.

- $x = 0.42m$, $n = 30$, $t = 1$ minute $= 60s$.

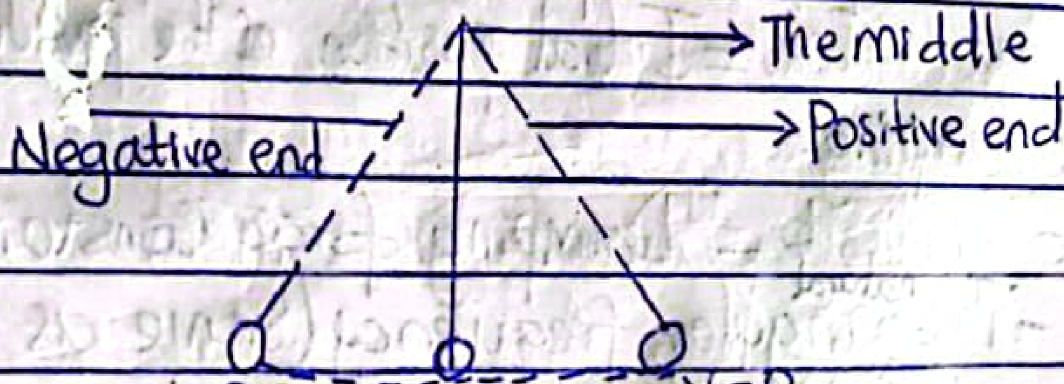


0.42m

$$\text{Amplitude} = \frac{0.42m}{2} = 0.21m.$$

- $$\text{Period} = \frac{t}{n} = \frac{60}{30} = 2 \text{ seconds.}$$

- $$\text{Frequency} = \frac{n}{t} = \frac{30}{60} = 0.5Hz$$



$V=0$	$V=max$	$V=0$
$a=max$	$a=0$	$a=max$
$K.E=0$	$K.E=max$	$K.E=0$
$P.E=Max$	$P.E=0$	$P.E=max$

DAMPED OSCILLATION

Damped oscillation is an oscillation whereby the amplitude of the vibrating body becomes progressively smaller due to loss of energy and dissipative forces.

The additional force $F = -bv$

Recall that $F = kx$

$$F = -kx - bv$$

$$-kx - bv = ma$$

$$-kx - b\left(\frac{dx}{dt}\right) = m\left(\frac{d^2x}{dt^2}\right)$$

$$-kx - b\dot{x} = m\ddot{x}$$

$$m\ddot{x} + b\dot{x} + kx = 0$$

- The amplitude of a damped oscillation is given by $A_d = C_o \exp\left(\frac{-b}{2m}\right) t \sin(\omega' t)$

where A_d = Amplitude of damped motion

C_o = Initial position of the particle before damping.

b = Damping force constant.

- The angular frequency (same as angular velocity) is $\omega' = \sqrt{W_o^2 - \frac{1}{4}\gamma^2}$

where W_0^2 = Natural angular frequency
W' = Damped angular frequency
γ = Damping force angular frequency
And $\gamma = \frac{b}{m}$ and $W_0 = \sqrt{\frac{k}{m}}$

$\gamma = \frac{b}{m}$ and $W_0^2 = \frac{k}{m}$

Therefore we have,

$$\omega' = \sqrt{\frac{k}{m} - \frac{1}{4}\left(\frac{b^2}{m^2}\right)}$$

$$\boxed{\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}}$$

formula for Damped angular freque

Cases of Damping

1) Critically damped system
$b = 2\sqrt{km}$ or $\left(\frac{k}{m} - \frac{b^2}{4m^2}\right) = 0$

2) Heavily damped system / Forcedly damp
$b > 2\sqrt{km}$ or $\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)$ is negativ

3) Under damped system/Lightly damped
$b < 2\sqrt{km}$ or $\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)$ is positive.

Energy of a damped Harmonic oscillator
- Know that Energy loss = Workdone against frictional force.

- $E(t) = \frac{1}{2} m\omega_0^2 C_0^2$

METHODS OF DESCRIBING A DAMPED OSCILLATOR
1) Logarithmic Decrement
2) Relaxation time or modulus of decay
3) Quality factor or Q - value of a Damped oscillator

- For Logarithmic Decrement
$$\delta = \log_e \frac{A_0}{A_1}$$

- For relaxation time or Modulus of decay

$t_r = \frac{2}{\gamma}$ $A_t = A_0 e^{-\frac{bt}{2m}}$

(Damping time) $t_d = \frac{1}{\gamma}$

So therefore Damping time = Relaxation time x 2

Note $e^{-1} = 0.363$ of its initial value A_0

- For Quality factor

$Q = \frac{W'}{\gamma}$ [Note that $2r = \gamma$]

Examples.

1) - A 0.5kg mass oscillates at the end of a spring with force constant 400N/m. Its initial amplitude of motion is 0.30m. When a damping force acts on the mass, its amplitude of motion decreases to 0.20m in 10s. Calculate the magnitude of the damping force constant

Solution

A_0 or $C_0 = 0.30m$, $A_d = 0.20m$, $t = 10$

$m = 0.5\,kg,\ k = 400\,N/m$

$$A_d = A_0 \exp\left(\frac{-b}{2m}\right)t$$

$$0.20 = 0.30 \exp\left(\frac{-10b}{2(0.5)}\right)$$

$$0.20 = 0.30 \exp\left(\frac{-10b}{1}\right)$$

$$\frac{0.20}{0.30} = \exp(-10b)$$

$$0.6667 = \exp(-10b)$$

$$\ln(0.6667) = -10b$$

$$b = \frac{\ln(0.6667)}{-10}$$

$$= 0.04054$$

$$b = 0.041\,kg/s$$

2) The equation of damped oscillation is given as $x = \exp(0.25t)\sin\left(\frac{4}{2}\right)t$ find the natural angular frequency

solution.

Recall that

$$x = C_0 \exp\left(\frac{b}{2m}\right)t \sin(\omega')t$$

Comparing equation.

$$A_0 = 1, \frac{\gamma}{2} = 0.25, \omega' = \frac{\pi}{2}$$

$$\gamma = 0.25 \times 2 = 0.5$$

Recall that $W' = \sqrt{W_0^2 - \frac{1}{4}\gamma^2}$

$$(W')^2 = W_0^2 - \frac{1}{4}\gamma^2$$

$$W_0^2 = (W')^2 + \frac{1}{4}\gamma^2$$

$$W_0^2 = \left(\frac{\pi}{2}\right)^2 + \frac{1}{4}(0.5^2)$$

$$W_0^2 = 2.4674 + 0.0625$$

$$W_0^2 = 2.5299$$

$$W_0 = 1.59 \text{ rad/s}.$$

3) A mass of 0.40kg moving at the end of a spring with force constant 350N/m be acted upon by a damping constant 9.0kg/s. Fi[nd] the angular frequency of the oscillating mass.

Solution

$m = 0.4kg$, $K = 350N/m$, $b = 9.0kg/s$

Recall

$$w' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$w' = \sqrt{\frac{350}{0.4} - \left(\frac{9}{2\times0.4}\right)^2}$$

$$w' = \sqrt{875 - 126.5625}$$

$$w' = 27.4 \text{rad/s}$$

4) The equation of damped oscillation [illegible] given in the form

$$x = 5\exp(-0.25t)\cos\left(\frac{\Delta}{2}\right)t$$

Find the particle velocity at time t=

solution

To find velocity, we differentiate t[illegible] displacement once. (using product rul[illegible]

$$\frac{dx}{dt} = \Big[5\exp(-0.25t) - \frac{\Delta}{2}\sin\frac{\Delta t}{2}$$

$$+ -(0.25)5\exp(-0.25t)\cos\left(\frac{4}{2}\right)t\Big]$$

Note $\sin 0 = 0$

So we have

$(-0.25)\ 5\exp(-0.25t) \cos(\frac{1}{2})t$

$= -0.25 \times 5 \exp(-0.25(0) \cos(\frac{1}{2})0)$

$= -1.25 \exp(0) \cos 0$

$= -1.25 m/s.$

Shortcut

Note any time the equation is Cos
The formula is $R[C_0]$
Remember $r = 1/2$, so $r = -0.25$

$R \times C_0 = -0.25 \times 5 = -1.25 m/s$

Particle velocity $= 1.25 m/s$

5) The equation for a damped harmonic oscillator is given as

$x = \left[10 \exp(-0.125t) \cos \frac{1}{2}t\right] m$

(a) - Th angular frequency w of the oscillator
(b) - The natural angular frequency
(c) - The initial energy per unit mass of the dampe

harmonic oscillator

d) The damped time

e) The nature of the oscillation

f) The quality factor

g) The particle velocity of the oscillation at time $t=0$?

Solution

Recall $x = [C_0 \exp(-\frac{1}{2}\gamma t)\cos(\omega' t + \phi)]$

$x = [10 \exp(-0.125t)\cos\frac{\pi}{2}t]\,m$

Comparing equation:

$C_0 = 10,\ \frac{1}{2}\gamma = 0.125,\ \gamma = 0.250$

$\omega' = \pi/2$

ⓐ $\omega' = \frac{\pi}{2} = 1.571\,rad/s$.

ⓑ $\omega' = \sqrt{\omega_0^2 - \frac{1}{4}\gamma^2}$

$(\omega')^2 = \omega_0^2 - \frac{1}{4}\gamma^2$

$\omega_0^2 = (\omega_1)^2 + \frac{1}{4}\gamma^2$

$\omega_0^2 = (1.571)^2 + \frac{1}{4}(0.250)^2$

$\omega_0^2 = 2.480525$

$W_0 = 1.5749 \, rad/s$.

(c) $E(t) = \frac{1}{2} m w_0^2 C_0^2$

$\frac{E(t)}{m} = \frac{1}{2} w_0^2 C_0^2$ (per unit mass)

$= \frac{1}{2} \times (1.5749)^2 \times 10^2$

$= 124.2 \, J/kg$

d) $T_d = \frac{1}{\gamma} = \frac{1}{0.25} = 4 \, sec$

e) $\left(W^2 - \frac{1}{4}\gamma^2\right)$ is positive

The motion is underdamped.

f) $Q = \frac{W'}{\gamma} = \frac{1.571}{0.250} = 6.284$

Q is dimensionless

g) Particle velocity $= R(C_0) = 0.125 \times 10 = 1.25 \, m/s$

PAST QUESTIONS AND EXERCISES

1) A simple pendulum of length 22m is set into oscillation with amplitude 0.05m after 5 min it has fallen to 0.025m calculate the relaxation time.

Solution

$A = 0.05m$, $A_d = 0.025m$, $t = 5min$
$= 5 \times 60 = 300$

$$0.025 = 0.05e^{-\frac{bt}{2m}}$$

$$0.025 = 0.05e^{-\frac{1}{2}\gamma t}$$

$$\frac{0.025}{0.05} = e^{-\frac{1}{2}\gamma t}$$

$$0.5 = e^{-\frac{1}{2}\gamma t}$$

$$\ln 0.5 = -\tfrac{1}{2}\gamma t$$

$$+0.69315 = +\tfrac{1}{2}\gamma t$$

$$\gamma(5 \times 60) = (0.69315 \times 2)$$

$$300\gamma = (0.69315 \times 2) = 1.3863$$

$$\gamma = 4.621 \times 10^{-3}$$

$$\text{Relaxation time} = \frac{2}{4.621 \times 10^{-3}} \quad \left(\frac{2}{\gamma}\right)$$

$$= 432.81 \text{ seconds.}$$

2) The equation given as $3\ddot{x}+12\dot{x}+39x = 0$ represents (a) critically damped (b) Lightly damped (c) Simple harmonic (d) forcedly damped (e) None of the above

solution

$m\ddot{x} + b\dot{x} + kx$ (General equation)

Comparing with $3\ddot{x}+12\dot{x}+39x$

$m = 3, b = 12, k = 39.$

$b = 12$, $2\sqrt{km} = 2\sqrt{39 \times 3} = 2\sqrt{117} = 21.63$

$12 < 21.63$

$\therefore b < 2\sqrt{km}$

So it is under damped or lightly dampe

FORCED DAMPED HARMONIC OSCILLATION

When the frequency of the driving force coincides with the natural frequency of the oscillator, the result in the phenomenon of RESONANCE.

MECHANICAL IMPEDANCE

$$Z^2m = R^2m + X^2m$$

$$X_m = \left(m\omega - \frac{k}{\omega}\right)$$

R_m (mechanical resistance) as;

$$R_m = m\gamma = b$$

Because recall that $\gamma = \frac{b}{m}$

So we can say

$$Z_m = \left(R^2m + \left(m\omega - \frac{k}{\omega}\right)^2\right)^{1/2}$$

RESONANCE FREQUENCY

Resonance frequency is denoted as ω_m

$$\omega_m = \omega_0\sqrt{1 - \frac{1}{2Q^2}}$$

ENERGY OF A FORCED HARMONIC OSCILLATOR

$$(\omega_0^2 - \omega^2)^2 = (\omega_0 + \omega)^2 (\omega_0 - \omega)^2$$

$$E = \frac{mF_0^2 \omega^2}{8m\omega_0^2} \cdot \frac{1}{(\omega_0 - \omega)^2 \cdot \frac{\gamma^2}{4}}$$

General equation of forced damped harmonic oscill

$$m\ddot{x} + b\dot{x} + kx = F\cos\omega t \quad , Q = \frac{\omega_0}{\gamma}$$

Examples

1) The equation of motion of a point mass i given as:

$$3\ddot{x} + 7\dot{x} + 10x = 10\sin(20t - \phi)$$

Find the mechanical impedance per unit ma

Solution

Comparing with $m\ddot{x} + b\dot{x} + kx = F\cos\omega t$

$$m = 3, \ b = 7, \ k = 10, \ \omega = 20$$

Mechanical impedance (Z_m)

$$= \left(R_m^2 + \left(m\omega - \frac{k}{\omega}\right)^2\right)^{1/2} \qquad R_m = b$$

$$= \left(7^2 + \left(3 \times 20 - \frac{10}{20}\right)^2\right)^{1/2}$$

$$Z_m = 59.55\ \Omega$$

But we are looking for impedance per unit mass

$$i.e = \frac{Z_m}{m} = \frac{59.55}{3} = 19.85\,\Omega/kg$$

2) The Equation of motion of a point mass is given as $3\ddot{x} + 7\dot{x} + 10x = 10\sin(20t$ find the resonance frequency

Solution

$$\omega_m = \omega_0 \sqrt{1 - \frac{1}{2Q^2}}$$

Comparing equation.

$m = 3, b = 7, k = 10, \omega = 20$

$$Q = \frac{\omega_0}{\gamma}, \text{ But } \gamma = \frac{b}{m} = \frac{7}{3} = 2.33333$$

$$\omega_0^2 = \frac{k}{m} = \frac{10}{3} = 3.33333$$

$$\omega_0 = \sqrt{3.33333} = 1.8257$$

$$Q = \frac{\omega_0}{\gamma} = \frac{1.8257}{2.3333} = 0.7825$$

But $\omega_m = \omega_0\sqrt{1-\frac{1}{2Q^2}}$

$\omega_m = 1.8257\sqrt{1-\frac{1}{2(0.7825)^2}}$

$\omega_m = 1.8257 \times 0.4282$

$\omega_m = 0.78\,rad/s$

3) The equation of motion of a point mass is given as $3\ddot{x} + 7\dot{x} + 10x = 10\sin(20t - \phi)$ Find the steady state period of oscillation

Solution.

Steady state of oscillation $= \frac{2\pi}{\omega}$

from the equation, $\omega = 20$

$= \frac{2\pi}{20} = 0.314\,sec.$

PAST QUESTIONS AND EXERCISE

- The equation of motion of a point mass $3\ddot{x} + 5\dot{x} + 12x = 5\cos(\pi r t + \phi)$ Find the value of r at maximum displacemen of the particle.

Solution.

Comparing equation.

$$W = \pi r, m = 3, b = 5, k = 12$$

But recall that

$$W_m = W_0\sqrt{1 - \frac{1}{2Q^2}}$$

$$W_0^2 = \frac{k}{m} = \frac{12}{3} = 4$$

$$w_0 = \sqrt{4} = 2.$$

$$Q = \frac{W_0}{Y} = \frac{2}{(b/m)} = \frac{2}{\left(\frac{5}{3}\right)} = \frac{6}{5} = 1.2$$

$$W_m = 2\sqrt{1 - \frac{1}{2(1.2)^2}}$$

$$W_m = 1.6159$$

But recall that

$w = \pi r$ (from the equation)

$$1.6159 = \pi r$$

$$r = \frac{1.6159}{\pi}$$ (Take it as $\frac{22}{7}$)

$$r = \frac{1.6159}{\left(\frac{22}{7}\right)} = 0.514\,Hz$$

- The equation of motion of a particle is given by $\ddot{x} + 6\dot{x} + 27x = 5\sin(\omega t + \phi)$. Determine the type of motion and determin the frequency at a steady state.

Solution.

Given $\omega = 2\pi f$

$$f = \frac{\omega}{2\pi}$$

- Simple harmonic motion.

WAVES

A wave is a disturbance from equilibrium that travels or propagates from one region of space to another.

Waves are classified as
- Mechanical waves
- Electromagnetic waves

- Mechanical wave propagates through a material medium.
  There are of two types:-
- Transverse waves:- The particle displacement is perpendicular to the direction of the wave propagation.
- Longitudinal waves:- In this case, The particle displacement is parallel to the direction of the wave propagation.

- Electromagnetic waves:- This type of wave does not require a physical medium for propagation and hence can travel through a vacuum. [Light waves is an example

PERIODIC WAVES

$$\omega = 2\pi f, \quad T = \frac{1}{F}, \quad T = \frac{2\pi}{\omega}, \quad v = f\lambda$$

WAVE FUNCTION FOR A SINUSOIDAL WAVE

$$\text{Phase velocity} = \frac{\omega}{K} \text{ or } \frac{dx}{dt}$$

The equation is $y(x,t) = A\sin(\omega t - Kx)$

VELOCITIES OF TRAVELING WAVES

- The phase velocity:- The rate at which phase of the wave propagates in space.

$$v = \frac{\omega}{K} \text{ or } \frac{dx}{dt}$$

- The particle velocity:- The simple harmonic velocity of the oscillation abo it's equilibrium position.

It is simply differentiating the equati

- Group Velocity (v_g) :- This is when waves of different frequencies, wavelengt[h] and velocities are super-imposed.

- $V = V_g + \lambda \frac{dv}{d\lambda} = C_1 + C_2 \lambda$

SPEED OF A TRANSVERSE WAV[E]

$$V = \sqrt{\frac{T}{\mu}}$$

where $\mu = \frac{\text{Mass}}{\text{Length}}$ (Linear mass density

SPEED OF LONGITUDINAL WAVE

- $V = \sqrt{\frac{B}{\rho}}$

where $B = \frac{1}{K}$ (where K is Compressibilit[y]

ρ - Density.

- $V = \sqrt{\frac{Y}{\rho}}$ where Y = Young's modulus

Examples.

1) - A fisherman notices that his boat is moving up and down periodically due to the waves on the surface of the water. It takes 2·0s for the boat to travel from its highest point to its lowest. The fisherman sees that the wave crests are spaced 7·0m apart. How fast are the waves travelling?

$T = 2s$, To and fro $- 2s \times 2 = 4s$

$F = \frac{1}{T} = \frac{1}{4} = 0{\cdot}25Hz$

But $v = f\lambda$

Note that λ (wave length is the distance two successive crest) $\lambda = 7m$

$v = f\lambda$

$= 0.25 \times 7 = 1{\cdot}75\ m/s$.

2) A wave source at the origin oscillate in SHM at a frequency of 5Hz and with amplitude A. The plane wave travels in the +x direction at a speed of 10m/s. Find an express for the displacement y at any point x at

a) $t=0$, b) $t=0.025$, c) $t=0.05$

Solution.

$$T=\frac{1}{f}=\frac{1}{5}=0.2sec$$

$$\lambda=\frac{v}{f}=\frac{10}{5}=2m$$

Recall that $y(x,t)=A\sin(\omega t-kx)$

$$=A\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$$

when $\omega=2\pi f$ and $f=\frac{1}{T}$

$\omega=2\pi\frac{1}{T}$ and $k=\frac{2\pi}{\lambda}$

The general equation.

$$A\sin 2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)$$

But $T=0.2s$, $\lambda=2m$

$$y=A\sin 2\pi\left(\frac{t}{0.2}-\frac{x}{2}\right)$$

$$y=A\sin\left[2\pi\left(\frac{t}{0.2}-\frac{x}{2}\right)\right]$$

- At $t = 0$

$$y = A\sin(w(0) - kx)$$

$$y = A\sin(0 - kx)$$

But $k = \frac{2\pi}{\lambda} = \frac{2\pi}{\cancel{2}}$ $k = \lambda$

$$y = A\sin(0 - \lambda x)$$

$$y = A\sin(-\lambda x)$$

B) At $t = 0.025sec$, $y = A\sin\left[\frac{\pi}{4} - \lambda x\right]$

c) At $t = 0.05sec$, $y = A\sin\left[\frac{\lambda}{2} - \lambda x\right]$

3) A wave is represented by the equation $y = 0.20\sin[0.40\pi(x - 60t)]$ in cm

Find

- The wavelength (λ)
- The frequency (f)
- The displacement at $x = 5.5cm$ and $t = 0.025$.

Solution:

i) $\frac{2\pi x}{\lambda} = \omega x = 0.4\pi x$

$\frac{2\pi x}{\lambda} = 0.4\pi x$

$\frac{2x}{\lambda} = 0.4x$

$2 = 0.4\lambda$

$\lambda = 5m.$

ii) $0.40\pi \times 60t = 2\pi ft$

$f = \frac{0.4 \times 60}{2} = 12Hz$

iii) At $x = 5.5cm$, $t = 0.02$

$y = A\sin[\omega t - kx]$

$= 0.20\sin[0.4\pi \times 0.02 \times 60 -$

$0.4\pi \times 5.5]$

$0.2\sin[2.2\pi - 0.48\pi]$

$= 0.2\sin[1.72\pi]$

4) A wave is described by $(x,t) = 0.1 \sin(3x - 10t)$, where x and y are in cm find the phase velocity

Solution

phase velocity $\cdot\ v = \frac{\omega}{K} = \frac{10}{3} = 3.3\text{cm/s}.$

Comparing equation $= A\sin(\omega t - kx)$

$\omega = 10,\ k = 3$

5) A wave is represented by the equation

- $y = 0.2 \sin[0.25\pi[x - 50t]]$ in cm.
- $x = 5 \exp(-0.75t) \sin \pi t$

solution.

- Differentiating

$V_p = 0.2 \times 0.25\pi \times 50 \cos[0.25\pi(x - 50t)]$

$V_p = 7.854 \cos[0.25\pi(x - 50t)]$

at $x = 5.5$cm and $t = 0.02$sec.

$V_p = 7.854 \cos[0.25\pi(5.5 - 50(0.02))$

$V_p = 7.8$ cm/s

- $V_p =$ $A[\pi] = 5\pi = 15.7$ m/s

6) The phase velocity of wave in certain medium is represented by $v = [C_1 + C_2\lambda]$ m/s where C_1 and C_2 are constants. What is the value of the group velocity if C_1 and C_2 are 10 and 15 respectively and what is the frequency if $n = 3k+2$.

Solution

Recall that $v_g = v - \lambda \frac{dv}{d\lambda}$

$$v = v_g + \lambda \frac{dv}{d\lambda} = C_1 + C_2\lambda$$

$\therefore v_g$ = group velocity $= C_1 = 10$ m/s

$\therefore C_2 = \frac{dv}{d\lambda} = 15$

$\left(\frac{v}{\lambda}\right) d = 15$

$v = 2\pi f \quad (w = 2\pi f)$

$$\lambda = \frac{v}{f}$$

$$= \left(\frac{2\pi f}{\lambda}\right) d = 15$$

$2\pi f \times \frac{f}{V} = \frac{2\pi f^2}{V}$

$V = c$ (speed of light)

$= \frac{2\pi f^2}{c}\left(d\right)$ formula for group velocity

$= \frac{2\pi f^2}{c}\left(d\right) = 15$

from the equation $n = 3k + 2$

$\frac{dn}{dk}$ same as $(d) = 3$.

$= \frac{2\pi f^2}{c}(3) = 15$

$= \frac{6\pi f^2}{c} = 15$

$c = (3.0 \times 10^8 m/s)$

$= \frac{6\pi f^2}{(3 \times 10^8)} = 15$

$= \frac{2\pi f^2}{10^8} = 15$

$f = \sqrt{\frac{15 \times 10^8}{2\pi}}$

$= 1.54 \times 10^4$ Hz

7) The linear mass density of a clothesline is 0.25kg/m. How much tension does Ko have to apply to produce the observed wave speed of 12m/s?

Solution:

$$V = \sqrt{\frac{T}{M}} = V^2 M = T$$

$$T = 12 \times 12 \times 0.25 = 36N$$

8) A ship uses sonar system to detect under water objects. The system emit under water sound waves and measure the time interval for the reflected wave return to the detector. Determine the speed of sound waves in water and find the wavelength of a wave having a frequency of 262HZ.

[Compressibility of water $= 45.8 \times 10^{-11}$ pa

[density of water $= 1 \times 10^{3}$ kg/m^3]

Solution

B(Bulk modulus) $= 1/K$

$$= \frac{1}{45.8\times10^{-11}} \text{ Pa}$$

$$v = \sqrt{\frac{1}{45.8\times10^{11}} \times \frac{1}{1\times10^{3}}}$$

$$= 1480 \text{ m/s}$$

$$\lambda = \frac{v}{f} = \frac{1480}{252} = 5.65.$$

INTERFERENCE OF WAVES.

Types of Interference

1) Constructive Interference
2) Destructive Interference

Mathematical representation

1) Phase constant $\frac{1}{2}\phi$
2) Amplitude of $y = 2A\cos\frac{1}{2}\phi$

$$y = 2A\cos 0 = 2A$$

The interference that produces the greatest possible amplitude is fully Construc Interference.

STANDING WAVES (STATIONARY WAVES).

Nodes (points of zero amplitude)

$$x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \frac{5\lambda}{2}.$$

Antinodes

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \frac{7\lambda}{4}, \frac{9\lambda}{4}$$

Example.

BEATS

The alteration of maximum and minimum sound intensities produced by the superposition of sound waves of slightly different frequencies.

DOPPLER EFFECT

The variation in perceived sound frequency due to motion of sound source is known as Doppler effect.

It is simply the observer behind source hears lower ~~than~~ and observer in front of source hears higher pitch.

- When the observer is moving towards a stationary source, the observed frequency

$$f_o = \frac{v'}{\lambda'} = \left(\frac{v+v_o}{v}\right) f_s$$

- When the observer is moving away from a stationary source, the observed frequency is,

$$f_o = \left(\frac{v-v_o}{v}\right) f_s.$$

Example:

1 A standing wave has nodes at $x = 0cm$, $x = 6.0cm$, $x = 12cm$ and $18cm$

a) What is the wavelength of the waves that are interfering to produce standing wave?

b) At what positions are ~~at~~ the antinode

Solution:

a) $x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, 2\lambda, \frac{5\lambda}{2}$

[for node]

$\frac{\lambda}{2} = 6cm$

$\lambda = 12cm$ ~~or~~ ~~[illegible]~~

$\lambda = 12cm$ or $\frac{3\lambda}{2} = 18cm$

$\lambda = 12cm$ (wavelength)

b) Antinodes $= \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$

$\frac{\lambda}{4} = \frac{12}{4} = 3cm$, $\frac{3\lambda}{4} = \frac{3 \times 12}{4} = 9cm$,

$\frac{5\lambda}{4} = \frac{5 \times 12}{4} = 15cm$, [So 3cm, 9cm, 15cm]

NORMAL MODES

The normal mode of an oscillating system is a pattern of motion restricted by boundary conditions in which all parts of the system move sinusiodally with the same frequency and in phase.

Adjacent nodes

$$f_1 = \frac{v}{2l}, f_2 = \frac{2v}{2l}, f_3 = \frac{3v}{2l}$$

But $f_1 = v/2l$

$$f_1 = v/2l, f_2 = 2f_1, f_3 = 3f_1$$

$$f_n = \frac{nv}{2l} = nf_1$$

Wavelength of various normal modes for a string of length l are

$$\lambda_n = \frac{2l}{n}, n = 1,2,3.$$

$$f_n = \frac{n}{2l}\sqrt{\frac{T}{M}}$$

$$v = f_1\lambda_1$$

$$v = f_1(2l)$$

for closed pipe

$$f_1 = \frac{v}{4l}$$

End Correction:

Closed pipe

$$\frac{\lambda}{4} = l + c$$

$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{v}{4(l+c)}$$

Open pipe

$$\frac{\lambda}{2} = l + c + c$$

$$\frac{\lambda}{2} = l + 2c$$

$$\lambda = 2(l + 2c$$

$$f = \frac{v}{\lambda} = \frac{v}{2(l+2c)}$$

Zero overtone – first harmonic

first overtone – Second harmonic

~~Fo~~ second overtone – Third harmonic,

Example-

A String on a violin has a fundamental frequency of 262 Hz. Calculate the frequency of the next two harmonics of the string-

$f_1 = 262\,Hz$, $f_n = nf_1$,

$f_2 = 2f_1 = 2 \times 262 = 524\,Hz$, $f_3 = 3 \times 262 = 786\,Hz$

OPTICS.

Optics is the study of light and vision.

Optics is divided into two

1) Physical optics
2) Geometric optics

RAY

Ray is a line drawn perpendicular to a series of wave fonts and pointing in the direction of propagation.

- Parallel ray and diverging ray

Laws of reflection.

1) The angle of incident is equal to the angle of reflection.
2) The normal line, the incident and the reflected ray lies in the same plane.

REFRACTION.

Refraction refers to a change in the direction of a wave experienced when it travels between media of different optical densities.

LAWS OF REFRACTION.

1) The incident ray, the refracted ray and

the normal to the surface at the point of incident all lies in the same plane.

2) Snell's law:- For a given pair of material, the ratio of the sine of angle of Incident is equal to the sine of angle of refraction.

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{n_b}{n_a}$$

$$\sin\theta_a n_a = n_b \sin\theta_b$$

REFRACTIVE INDEX

$n = \frac{c}{v}$ but $c = \lambda f$, $v = \lambda_m f$

Where λ = wavelength of vacuum

λ_m = wavelength of the medium

$$n = \frac{\lambda f}{\lambda_m f}$$

REFRACTIVE INDEX RELATIONSHIP

$${}_a n_b = \frac{1}{{}_b n_a}$$

But $_an_b = \frac{\sin\theta_a}{\sin\theta_b} = \frac{1}{_bn_a}$

$_an_b \times {_bn_c} = {_an_c}$

TOTAL INTERNAL REFLECTION

$\sin\theta_c = \frac{1}{n}$ (critical angle)

$\sin\theta = \frac{n_2}{n_1}$

Critical angle

EXAMPLES

1) Given the refractive index of air to glass $_an_g = 3/2$, of air to water $_an_w = 4/3$. What is the refractive index of water to glass?

Solution

$_wn_g = {_wn_a} \times {_an_g}$

$_a n_g = 3/2$, $_a n_w = 4/3$, $_w n_a = 1/\frac{4}{3}$
$= 3/4$

$$_w n_g = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8}.$$

2) A light ray from a Helium-neon laser has a wavelength of 632.8nm and travels from air to Crown glass[n(crown glass) = 1.52]

a) What is the frequency of light in air
b) What is the frequency of light in glass
c) What is the wavelength of light in glass?

Solution.

$n_a = 1$, $n_g = 1.52$
$f = ?$ $f_m = ?$, $\lambda_m = ?$

ⓐ $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{632.8 \times 10^{-9}} = 4.74 \times 10^{14}$ Hz

ⓑ $F_m = F$ (frequency is same for all medium) $[= 4.74 \times 10^{14}$ Hz$]$

ⓒ $\lambda_m = \frac{\lambda}{n} = \frac{632.8\text{nm}}{1.52}$

$= \frac{632.8nm}{1.52} = 416nm.$

3) A swimmer is 1.5m underneath a pond of water. At what angle must the swimm[er] shine the beam of light towards the surface in order for a person on a distant bank to see it?

Solution.

θ_c is the critical angle

$$\theta_c = \sin^{-1}\left[\frac{n_2}{n_1}\right] = \sin^{-1}\left[\frac{1}{1.33}\right] = 48.8^{\circ}$$

n_2 = refractive index of air, it always 1

n_1 = refractive index of ~~glass~~ water

REFLECTION AT PLANE AND CURVED SURFACES

Mirrors are smooth reflecting surface usually made of polished metals or glass

TYPES OF MIRRORS

1) Plane mirrors:- Mirrors with a flat surface

Characteristics of images formed by plane Mirrors

- It is laterally inverted.
- It has infinite focus.
- It is upright and unmagnified.
- It is virtual
- Image distance(v) is equal to object distance (u)

Spherical mirrors

- Concave (Converging mirror)
- Convex (Diverging mirror)

2) Curved mirrors

- Vertex (V):- This is the center of the mirror surface.

- Principal or optical Axis:- The line that joints Center of Curvature to the vertex of the mirror.
- Centre of Curvature:- The point on the optical axis that correspond to the Center of the sphere of which the mirror form a section.
- Radius of Curvature:- This is the distance between the vertex and the center of Curvature (C).
- Principal focus (F):- The principal focus (F) is the point where parallel light rays close to the optical axis Converges to focus after reflection in the case of a Concave mirror
- focal length:- The distance from the prin focus to the vertex of the mirror.

TYPES OF RAY.

1) Parallel ray
2) Chief ray or radical ray
3) Focal ray

$$f = \frac{\text{radius of curvature}}{2}$$

f (focal length)

MIRROR EQUATION

$$\frac{1}{f} = \frac{u+v}{uv}$$

Magnification $(M) = \frac{-v}{u}$.

Image formed by Concave mirror and Convex mirror

$+v$ – ~~Ima~~ Real image

$-v$ – Virtual image

$+M$ – Upright or Erect

$-M$ – Inverted

$M > 1$ – enlarge or magnified.

$M < 1$ – reduced or diminished

Image formed by convex are VED [Virtual, Erect and Diminished]

Image formed by Concave are RIM [Real, Inverted and Magnified]

Radius for Convex is negative

Solution to 8.3

- $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, $f = r/2 = \frac{-45}{2} = -22.5$

$= \frac{-1}{22.5} - \frac{1}{15} = -1/9 = \frac{1}{v}$, $v = -9$.

$M = \frac{-v}{u}$, $M = \frac{-9}{15} = 0.6$ (Inverted)

REFRACTION THROUGH PLANE SURFACES

Refraction is the bending or changes in the direction of light as it moves from one medium to the other.

Laws of Refraction

1) The incident rays, the normal and the refracted rays all lies in the same point at the point of incident.

2). For two given media, $\frac{\sin i}{\sin r}$ is a constant where i is the angle of incidence and r is the angle of refraction.

Refractive index

$$n = \frac{\text{Velocity of light on medium 1}}{\text{Velocity of light on Medium 2}}$$

$$n = \frac{\text{Velocity of light in vacuum (c)}}{\text{velocity of light in medium (v)}}$$

$$\frac{\sin i}{\sin r} = {}_a n_g$$

$$n = \frac{\text{real dept (t)}}{\text{apparent dept (a)}}, \quad a = t - d$$

$${}_a n_w = \frac{t}{t-d}, \quad d = t\left(1 - \frac{1}{n_w}\right)$$

Apparent depth with more than two media

$$A = x - (d_1 + d_2 + d_3)$$

Real depth $x = x_1 + x_2 + x_3$

$$d_1 = x_1\left(1 - \frac{1}{n_1}\right), \quad d_2 = x_2\left(1 - \frac{1}{n_2}\right), \quad d_3 = x_3\left(1 - \frac{1}{n_3}\right)$$

NOTE!- $$\frac{n_a}{n_b} = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

ANGULAR DISPERSION.

$$\Delta d = (n_b - n_r)A$$

Examples

1) Consider a film of oil 8cm with refractive index $n = 1.48$ on water, $n = 1.33$, on top of a glass block 10cm with refractive index $n = 1.62$, contained in one medium. If an object is placed at the bottom of the glass block,

Calculate the apparent position of the object from above.

Solution.

$$D_{oil} = x_1\left(1-\frac{1}{n_1}\right) = 8\left(1-\frac{1}{1.48}\right) = 2.5946cm$$

$$D_{water} = x_2\left(1-\frac{1}{n_2}\right) = 6\left(1-\frac{1}{1.33}\right) = 1.4887cm$$

$$D_{glass} = x_3\left(1-\frac{1}{n_3}\right) = 10\left(1-\frac{1}{1.62}\right) = 3.827cm$$

Total displacement = $D_{oil} + D_{water} + D_{glass}$

$= (2.5946 + 1.4887 + 3.827)cm$

$= 7.9910cm.$

Apparent depth (A)

Recall that $a = t - d$

$a = (8+6+10) - 7.9910cm$

$= (24 - 7.9910)cm$

$A = 16.09cm.$

2) A certain prism of refracting angle 50° and refractive index 1.50 is immers in a certain liquid of refractive index

n_1:- If the angle of minimum deviation of parallel rays through the prism is 5.[illegible] Find the value of n_1

Solution.

$${}_L n_g = \frac{n_g}{n_L} = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$n_g = 1.5, A = 50^\circ, D = 5$

Substituting $\frac{1.5}{n_L} = \frac{\sin\left(\frac{50+5}{2}\right)}{\sin\left(\frac{50}{2}\right)}$

$$\frac{1.5}{n_L} = \frac{\sin 27.5}{\sin 25} = 1.0925$$

$$n_L = \frac{1.5}{1.0925} = 1.372.$$

REFRACTION THROUGH CURVED SURFACES

A lens is an optical system with two refracting surfaces.

Power of a lens $= \frac{1}{\text{focal length (m)}}$

Magnification (M) $= \frac{\text{image height or distance}}{\text{object height or distance}}$

$= \frac{h_i}{h_o} = \frac{(v)}{(u)}$

SIGN CONVENTION FOR THIN LENSES

focal length for

Converging lens (positive lens) +F

Diverging lens (negative lens) - F

OBJECT DISTANCE (u)

+u (Object in front of the lens)

- u (Object in behind the lens)

+v (Image is formed on the opposite (image) side of the Lens from the object)

-v (Image formed on the same (object) side of the Lens from the object (virtual image))

+m (when the image is upright with respect to the object).

$-M$ (when the image is inverted with respect to the object).

N.T - If the image falls behind the second lens, it is referred to as virtual object, and the object distance is taken to be negative (-).

LENS MAKERS EQUATION

$$\frac{1}{f}\left(\frac{n_1}{n_2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

for air to glass, $n_{air} = 1$.

$$\frac{1}{f}\left(n_g-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Example

1) A double convex lens made of glass ($n = 1.52$) has a radius of curvature of 50.0cm on the front side and 40cm on the backside. Find the power of the lens

Solution.

$n = 1.52$, $R_1 = 50.0cm$, $R_2 = -40.0cm$ ~~formed~~ (Behind), $P = ?$

$$P = \frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.52-1)\left[\frac{1}{0.50} - \frac{1}{-0.04}\right]$$

$$= 2.34m^{-1} \ (2.34D).$$

TWO LENSES IN CONTACT

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2}$$

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} - \frac{D}{F_1F_2}$$

where d = distance of seperation between the two lenses.

- Two lenses of focal length 9cm and −6cm are placed in contact. Calculate the focal length of the combination.

Solution.

Given that $f_1 = 9cm$, $F_2 = -6cm$

$$\frac{1}{F} = \frac{1}{F_1} + \frac{1}{F_2} = \frac{1}{9} + \frac{1}{-6} = \frac{1}{-18}$$

$$f = -18cm.$$

Optical Instruments

The Simple Microscope

A simple microscope or magnifying glass is an instrument used for looking at objects close to the eye and the image is usually formed at the least distance of vision (D) from the eye which is about 25cm.

$$M = \frac{25cm}{f} + 1$$

$$M = \frac{25cm}{f}$$

The Compound microscope

$$M = M_o \times M_e = -\left(\frac{251}{f_o f_e}\right)$$

The compound microscope consists of two converging lenses.

TELESCOPE

The purpose of a telescope is to make distant objects appear closer and therefore larger.

ASTRONOMICAL TELESCOPE

The astronomical telescope is another example of Combination of two Converging lenses, the objective lens and the eyepiece.

GALILEAN TELESCOPE

The Galilean telescope consists of a Converging lens of large focal length as the objective and a diverging lens of short focal length as the eyepiece.

REFLECTING TELESCOPES.

The largest telescopes are reflecting telescopes which use a Curved mirror as the objective.

TERRESTIAL TELESCOPE

A third Converging lens is a method of giving final erect images. It gives an image I which becomes a real object fo the lens L.

VISION

The human eye is a remarkable evolutionary achievement. Accomodation is the ability of the human eye to focus on

nearly and distant objects attributed
to the crystalline lens is called
accommodation.

DEFECTS OF VISION AND THEIR CORRECTIONS

1) Long Sightedness (hypermetropia) :- This
occurs when images of near objects are
formed behind the retina. A converging
lens is used to correct the defect.

2) Short sightedness (Myopia) :- This oc
when images of far objects are form
in front of the retina. A diverging le
is used to correct the defect.

3) Presbyopia (old sight)
4) Astigmatism.

DISPERSION AND ABERRATION

The seperation of white light into its component colours is called "dispersion".

Spectrum is the band of colours.

[ROYGBIV] – Red, orange, yellow, Green, Blue, indigo and violet.

$$\text{Dispersive power}(\omega) = \frac{\text{Angular dispersion}}{\text{Mean deviation}}$$

ABERRATIONS-

The departures of real images from the image predicted by the simple theory are called aberrations.

SPHERICAL ABERRATIONS

Spherical aberrations result from the fact that focal points of light rays far from the optical axis of a spherical lens (or mirro the focal points of rays are different from the focal points of rays of the same wavelength close to the princ axis.

CHROMATIC ABERRATION AN ACHROMATIC LENSES:

Because the refractive index of all transparent media varies with colour, a single lens forms not only one image of an object but a series of images for each colour of light present in the beam.

$$d = \frac{f_1 + f_2}{2}, \quad \frac{f_1}{f_2} = \frac{-w_1}{w_2}$$

Example:

The dispersive power of two glasses are in the ratio of 3:4 and they are used to make achromatic objective of focal length 40cm.

Compute:-

a) The focal length of the two lenses

b) The distance of seperation that will give an achromatic combination if the lenses are of the same material.

Solution.

Dispersive power is defined as

$$\frac{f_1}{f_2} = \frac{-w_1}{w_2}$$

$$\frac{f_1}{f_2} = -\frac{w_1}{w_2}$$

With the ratio of $w_1 : w_2 = 3:4$ and $F = 40cm$,

$$\frac{F_1}{F_2} = \frac{-3}{4}, \quad f_1 = \frac{-3}{4} \times F_2$$

But $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

By substituting eq (ii) into (iii) for f_1

$$\frac{1}{F} = \frac{-1}{\left(\frac{3}{4}f_2\right)} + \frac{1}{f_2} = \frac{-4}{3f_2} + \frac{1}{f_2}$$

$$\frac{1}{40} = \frac{-4+3}{3f_2} \qquad f_2 = -\frac{10}{3} cm$$

Substituting eq (iv) into (ii)

we have

$$f_1 = -3/4 \times f_2 = -3/4 \times \frac{-40}{30} = 10cm$$

$f_1 = 10cm$ and $f_2 = -13.33cm$

b) Since they are assumed to be made of the same material, the distance of the seperation that will permit achromatic combination is

$$d = \frac{f_1 + f_2}{2} \qquad d = \frac{10 + (-13.33)}{2} = \frac{-3.33}{2}$$