PHY 113 MADE EASY

(VIBRATION, WAVES AND OPTICS)

(EXAMPLES $ EXERCISES SOLVED)

WITH PAST QUESTIONS

FOR FULL TIME, PART TIME, JUPEB AND DIPLOMA

IN GOD WE TRUST

"If you cannot learn the way we teach, then we will have to teach the way you learn"

CHAPTER 1

VIBRATIONS

PERIODIC OR OSCILLATORY MOTION

A vibration or oscillatory motions in general any motion which in general repeats in itself at regular intervals. Periodic or oscillatory motion is a type of motion separated in equal time intervals or regular circles. Examples:

1. The beating of the heart

2. The motion of the strings in a musical instrument e.g guitar

3. The motion of the balanced wheel of a watch

4. The motion of the pendulum of a clock

5. The motion of the prongs of a sounding turning fork

6. A mass attached to a string moving in circular motion

7. A bouncing ball

A mass spring system In the examples mentioned above, if there is the absence of frictional forces to remove mechanical energy from the system, the body will continue to oscillate while the restoring force at every time, draws the body towards the equilibrium position. At equilibrium position, the net force is zero. As the body moves towards equilibrium position, the speed increases and as it passes the equilibrium position, the speed decreases in such a way that there is a continual interchange of potential energy and kinetic energy due to the system having:

i. **Elasticity (springiness)** : This enables the system to store potential energy

Ii mass (inertia) : This enables the system to have kinetic energy.

PROPERTIES OF OSCILLATORY MOTION

i. Amplitude (A) : This is the maximum magnitude of displacement from the equilibrium position. For a pendulum, the amplitude is from the Centre of the swing to the swing, but for a circular motion, It is usually equal to the radius of the circle it is measured it meters(m)

ii. Period (T) : It is the time it takes a body to make one complete oscillation or circle. It is measured in seconds

$T = \frac{2\pi}{w}$, $T = \frac{t}{n}$

n= number of oscillations

t= time **$w = \frac{angular\ frequency}{velocity}$** **$w = 2\pi f$**

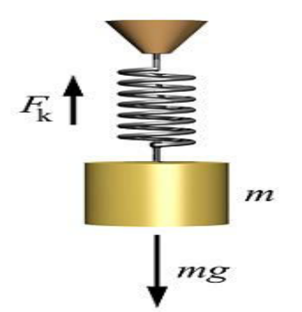
iii. Frequency(f) : it is the number of complete oscillation or cycle a body makes in one second. It is measured in hertz (Hz)

$F = \frac{1}{T}$, $F = \frac{n}{t}$, $F = \frac{w}{2\pi}$

SIMPLE HARMONIC MOTION (SHM)

It is the periodic motion of a body or particle along a straight line such that the acceleration of the body is directed towards a fixed point (or Centre of motion) and is also proportional to its displacement from that point .

CASE 1 : A MASS-SPRING SYSTEM.



Note: F_k in the diagram equals F_s

The restoring force acts along the axis of displacement and it is directly proportional to the displacement from the equilibrium position

i.eF_s is proportional to e (Hooks law). Therefore F= -Ke, where k= spring force constant. Note that the minus sign indicates that the force F_s always points in the direction to the displacement. **Here F=Ke**

$T=2\pi\sqrt{\frac{m}{k}}$ or $T=2\pi\sqrt{\frac{e}{g}}$

The second formula is used when the first formula fails

Energy = $\frac{1}{2}Ke^2$, w = $\sqrt{\frac{k}{m}}$

Example 1: A light is loaded with a mass of 50g and extends by 10cm. what is the period of small vertical oscillation if the acceleration due to gravity is 10m\s^2.

Note that to convert from gram to kg divide by 1000

m = 50kg = $\frac{50}{1000}$= 0.05kg

Note that to convert from cm to m divide by 100

e = 10cm = $\frac{10}{100}$ = 0.1m T=? g=10m\s^2

$T=2\pi\sqrt{\frac{e}{g}} = 2 \times \frac{22}{7} \times \sqrt{\frac{0.1}{10}} = \frac{44}{7} \times \sqrt{0.01}$

$= \frac{44}{7} \times 0.1 = 0.63$sec

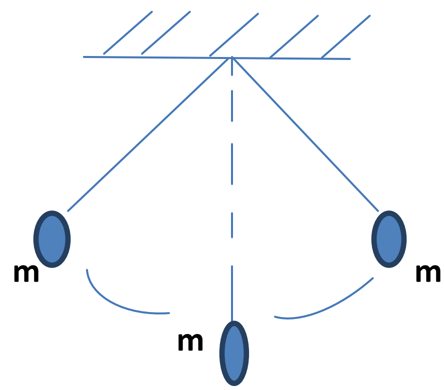
Example 2: A force of 5N compresses a spring by 4cm (a) find the force constant of the spring (b) find the elastic potential energy of the compressed spring

a. F = ke $k = \frac{f}{e} = \frac{5}{0.04}$ = 125N\m

b. P.E = $\frac{1}{2}ke^2 = \frac{1}{2} \times 125 \times (0.04)^2$

$= \frac{125 \times 0.0016}{2} = \frac{0.2}{2} = 0.1$j

CASE 2: THE SIMPLE PENDULUM. It consists of a body suspended from a light cord. When the body is pulled to oneside and released to swing past the equilibrium position and oscillate between the two maximum angular displacement.



Two forces acting on the pendulum at any given time are tension from the rope and gravity. Here $T = 2\pi\sqrt{\frac{l}{g}}$. Note that if only the length (l) and the period (t) are changing then use

$$\frac{T_1^2}{L_1} = \frac{T_2}{L_2}$$

Example 3: A simple pendulum has a period of 6 seconds. If the period is 7 seconds when the length was increased by 1m, find the original length of the pendulum

T_1 = 6sec let L_1 = L T_2 =7sec

If L_1 is increased by 1m then L_2 = L+ 1

Using $\frac{T_1^2}{L_1} = \frac{T_2^2}{L_2}$

$\frac{6^2}{L} = \frac{7^2}{L+1}$, $\frac{36}{L} = \frac{49}{L+1}$

Cross mulyiply

$36(L+1) = 49L$, $36L + 36 = 49L$

$36 = 49L - 36L$,

$36 = 13L$ (divide both sides by 13)

$\frac{36}{13} = \frac{13L}{13}$, $L = 2.77m$

If the question had said what is the new length then $L_2 = L+1 = 2.77+1 = 3.77m$. nowletsdo this the same way.

Example 4:A simple pendulum has a period of 7 seconds. When the length was shortened by 1m, the period is 6seconds. Find the original length of the pendulum

T_1 =7secs let $L_1 = L$ $T_2 = 6$ if L_1 is shortened i.e reduced by 1m then :

$L_2 = L-1$ using $\frac{T_1^2}{L_1} = \frac{T_2^2}{L_2}$

$\frac{7^2}{L} = \frac{6^2}{L-1}$, $\frac{49}{L} = \frac{36}{L-1}$ (cross multiply)

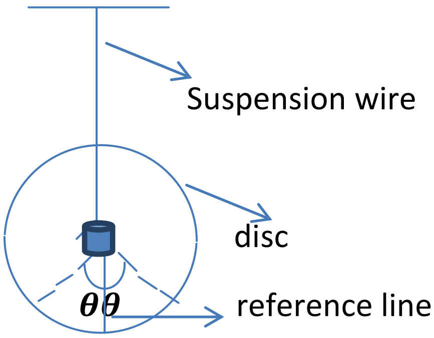
$49(L-1) = 36L$, $49L - 49 = 36L$

$49L - 36L = 49$, $13L = 49$

$\frac{13L}{13} = \frac{49}{13}$ $L = \frac{49}{13}$ $= 3.77m$

CASE 3: THE TORSIONAL OSCILLATOR

It consist of a disk-like mass suspended from a thin rod of wire. When the mass is twisted about the axis of the wire, the wire exert a torque on the mass, tending to rotate it back to its original position. If twisted and released the mass will rotate back and forth. This is the angular version of the bouncing mass hanging from a spring



$$\mathbf{\frac{Id^2\theta}{dt^2} + \frac{k\theta}{I} = 0 , w = \sqrt{\frac{k}{I}} \text{ and } T = 2\pi\sqrt{\frac{T}{K}}}$$

Where I is moment of inertia and θ is angular displacement.

CASE 4: VIBRATING MOLECULES

In molecular vibration we assume that each molecule can be treated as though it correspond to a spring. In the harmonic approximation, the spring obeys Hooke's law , where F=-KQ , Q is the position of atoms away from their equilibrium position with respect to a normal mode of vibration, and K is the force constant.

$\mathbf{\frac{d^2Q}{dt^2} + \frac{KQ}{\mu} = 0}$ where μ is the reduced mass

$\mathbf{w = \sqrt{\frac{K}{\mu}} , T = 2\pi\sqrt{\frac{\mu}{K}}}$

EQUATION OF A SIMPLE HARMONIC MOTION

Here we compare

1. **Equation of the form $\frac{d^2x}{dt^2} + w^2x = 0$**

Example 5: the equation of motion of a simple harmonic oscillator is $\frac{d^2x}{dt^2} = -9x$, where x is displacement and t is time. The period of oscillation is?

Re-write the equation $\frac{d^2x}{dt^2} + 9x = 0$

Compare with the equation $\frac{d^2x}{dt^2} + w^2x = 0$

By comparison, w^2 is the coefficient of x i.e what is attached to in both equations

:. $W^2 = 9$

Take square root of both sides

$\sqrt{w^2} = \sqrt{9}$

W = 3rads/sec

Recall $T = \frac{2\pi}{w} = \frac{2 \times 3.142}{3} = 2.09$sec

Where $\pi = \frac{22}{7}$ or 3.142

2. Equation of the form :

$X(t) = B_1 coswt + B_2 sinwt$

$W = \sqrt{\frac{K}{m}}$ $\quad$ $A = \sqrt{(B^2_1 + B^2_2)}$

Where A = amplitude

$\emptyset$ = **$\tan^{-1}(B_2/B_1)$** $\quad$ $\emptyset$ is the phase difference/constant

B_1 and B_2 are constants

Example : a SHM equation is given as

X = 6cos3t + 8sin3t, find the : (i) amplitude (ii) angular frequency (iii) period (iv) phase difference

SOLUTION :

Compare with the equation:

$X = B_1 coswt + B_2 sinwt$

X= 6cos3t + 8sin3t

:. $B_1 = 6$, $B_2 = 8$, w = 3rad/s

i. $A = \sqrt{(B^2_1 + B^2_2)}$

$A = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$m

ii. W = 3rad/sec

iii. $T = \frac{2\pi}{w} = \frac{2 \times 3.142}{3} = 2.1$sec

(iv) $\emptyset = \tan^{-1}(B_1/B_2)$

$\emptyset = \tan^{-1}(8/6) = \tan^{-1}(1.333)$

$= 53^0$

Equations in the form $X(t) = A\cos(wt + \emptyset)$

Where A is the amplitude w, t and $\emptyset$ have their usual meaning. X is the displacement .

Generally, when you are given a displacement equation, find velocity means you differentiate once while find acceleration means you differentiate twice

Example6 :A particle accelerates with simple harmonic motion, so that its displacement varies according to the equation, $x = 5\cos(2t+ \frac{\pi}{6})$ where x is in centimetres and t is in seconds. At t = 0, find (a) the displacement of the particles (b) its velocity (c) its acceleration (d) its period and amplitude.

SOLUTION:

(a) $X = 5\cos(2t+ \frac{\pi}{6})$, at t =0.

Note: $\pi = 180^0$

$X = 5\cos(2(0)+\frac{180}{6}) = 5\cos(0 + 30)$

X = 5cos30 = 4.33cm

(b) Velocity means differentiate once (using calculus)

Please try to learn **differentiation**

$X = 5\cos(2t+ \frac{\pi}{6})$

$\frac{dx}{dt} = -2 \times 5\sin(2t+ \frac{\pi}{6})$, at t = 0,

$V = \frac{dx}{dt} = -2 \times 5\sin(2(0)+ \frac{180}{6})$

V= -10sin30 = -5cm/s

(c) Acceleration means differentiate twice

(d) or differentiate the velocity you just got.

$V= -2 \times 5\sin(2t+ \frac{\pi}{6})$ therefore,

$a = -2 \times 2 \times 5\cos(2t+\frac{\pi}{6})$

$= -20\cos(2(0)+\frac{180}{6})$

$a = -20\cos30 = -17.3cm/s^2$

(e) Comparing the two equations, you wll observe that **w** is the coefficient of **t** i.e what is attached to **t.**

$X(t) = A\cos(wt + \emptyset)$

$X = 5\cos(2t+\frac{\pi}{6})$, thus $w = 2rad/s$

$T = \frac{2\pi}{w} = \frac{2\pi}{2} = \frac{2\times 3.142}{2} = 3.142s$

And Amplitude = 5m

SOME FORMULAE IN SIMPLE HARMONIC MOTION

$T = \frac{2\pi}{w}$, $w = 2\pi f$

$V = w\sqrt{(A^2 - x^2)}$, $a = -w^2x$

$K.E = \frac{1}{2}mv^2$, $P.E = \frac{1}{2}ke^2$

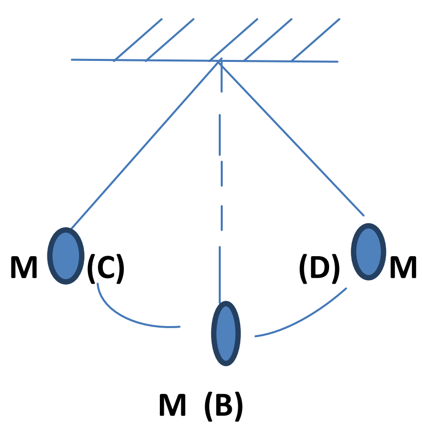
Where K.E and P.E are kinetic and potential energy respectively.

A is the amplitude.

V and a are velocity and acceleration respectively.

The TOTAL ENERGY OF THE SYSTEM OR THE MECHANICAL ENERGY OF THE SYSTEM , **E =** $\mathbf{\frac{1}{2}kA^2}$

$\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}ke^2$ if you are told to find E, use the left hand side formula ($\frac{1}{2}kA^2$), but if it fails , use the right hand side formula.



Consider the diagram above (case 2). The mass **M** is made to move to and fro, at the points **D** and **C** its velocity is minimum (v = 0m/s) and its acceleration is maximum (x = A). But at the point **B ,** (x = 0) , its velocity is maximum ($v = v_{max}$) and its acceleration is minimum. The distance **BD or BC** is called the amplitude . The point **B**is called the **EQUILIBRIUM POSITION.**

$V = w\sqrt{(A^2 - x^2)}$, at $x = 0$, $v = v_{max}$

Thus , $V = w\sqrt{(A^2 - 0^2)} = w\sqrt{A^2}$

Therefore, $\mathbf{v_{max} = -wA}$, v_{max} = maximum velocity

Also , $a = -w^2x$, at $x = A$, $a = a_{max}$

$\mathbf{a_{max} = -w^2A}$, a_{max} is maximum acceleration

The maximum velocity give the maximum K.E

$\mathbf{K.E_{max} = \frac{1}{2}mv^2_{max}}$

EXAMPLE 7: A block attached to a spring executes simple harmonic motion in a horizontal plane with an amplitude of 0.25m. At a point 0.15m away from the equilibrium position, the velocity of the block is 0.75m/s. what is the period of the block?

SOLUTION: A = 0.25m, x = 0.15m,

v =0.75m/s , T = ?

Recall : $V = w\sqrt{(A^2 - x^2)}$

$0.75 = w\sqrt{(0.25^2 - 0.15^2)}$

$0.75 = w\sqrt{(0.0625^2 - 0.0225^2)}$

$0.75 = w\sqrt{0.04}$

$0.75 = w \times 0.2$ (divide both sides by 0.2)

$w = \frac{0.75}{0.2} = 3.75rad/s$

Recall : $T = \frac{2\pi}{w} = \frac{2\times\pi}{3.75} = \frac{2\times 3.142}{3.75}$

= 1.676s

EXAMPLE 8 : A mass-spring system oscillates with an amplitude of 3.5cm. If the force constant of the spring is 250N/m and the mass is 0.5kg, determine (a)the mechanical energy of the system (b) the maximum speed of the mass (c) the maximum acceleration.

SOLUTION : note: the question says **MASS-SPRING SYSTEM i.e case 1**.

(a) A =3.5cm $=\frac{3.5}{100}$ = 0.035m

K = 250N/m m = 0.5kg

E = $\frac{1}{2}$ kA^2 = $\frac{1}{2} \times 250 \times 0.035^2$

$= \frac{0.30625}{2} =$ 0.153j

(b) **v_{max} =wA ,** recall : w = $\sqrt{\frac{k}{m}}$

(for mass-spring)

w = $\sqrt{\frac{250}{0.5}} =$ 22.36rad/s

v_{max} =wA = 22.36 × 0.035

v_{max} = 0.78m/s

(c) **a_{max} = w^2A**, **= 22.36^2**× 0.035

= 17.5m/s^2

EXAMPLE 9 : A system carrying out SHM vibrates at a frequency of 120Hz. If it has a velocity of 3.2m/s when its displacement from its point of equilibrium was 3.8mm. Find the maximum acceleration.

f = 120Hz, v =3.2m/s . NOTE : to convert from millimeter(mm) to meter(m), divide by 1000.

x = 3.8mm = 3.8/1000 =0.0038m

w = 2πf = 2× 3.142 × 120 = 754.03 rad/s

now, complete it yourself. HINT : find **A** using

V = w√ (A^2 – x^2) and substitute **A** into :

a_{max}= w^2A ……………**ANSWER : 3241.2m/s^2**

EXAMPLE10 : A 5kg block hung on a spring , causes a 10cm elongation of the spring . (a) What is the restoring force exerted on the block? (b) what is the spring constant ? (c) what force is required to stretch the spring 8.5cm horizontally ? (d) what will the spring elongation be when pulled by a force of 77.7N? (e) what is the period of this oscillation ?

SOLUTION : the question says SPRING i.e mass-spring system.

m = 5kg , e = 10cm = 10/100 = 0.1m ,

g = 9.8m/s^2

(a) F = mg 5 × 9.8 = 49N

(b) F = ke

49 = k × 0.1

K = 49/0.1 = 490N/m

(c) e =8.5cm = 8.5/100 = 0.085m, f = ?

F = ke = 490 × 0.085 =41.65N

(d) e = ? , F = 77.7N

F = ke

77.7 = 490 × e

e = 77.7/490 =0.1586m

(e) T = $2\pi\sqrt{\frac{m}{k}}$ = 2 × 3.142 × $\sqrt{\frac{5}{490}}$

T = 6.284 × $\sqrt{0.010204}$= 0.6347s

EXAMPLE 10: A punch bag of mass 0.60kg is struck so that it oscillates with SHM. The oscillation has a frequency of 2.6Hz and an amplitude of 0.45m. what is :

(a) The maximum velocity of the bag ?

(b) The maximum kinetic energy of the bag?

(c) What happens to the energy as the oscillations die away?

SOLUTION: m = 0.6kg , f = 2.6Hz , A = 0.4m

(a) w = 2πf = 2 × 3.142 × 2.6

= 16.34rad/s

$\mathbf{v_{max} = wA = 16.34} \times 0.45 = 7.35m/s$

(b)$\mathbf{K.E_{max} = \frac{1}{2} m v^2{}_{max} = \frac{1}{2} \times 0.6 \times 7.35^2}$

$\mathbf{= 16.2j}$

(c) it is transferred to the surroundings as heat.

EXAMPLE 11: A clown is rocking on a rocking chair in the dark . His glowing red nose moves back and front a distance of 0.42m exactly 30 times a minutes in a simple harmonic motion. (a) what is the amplitude of this motion ? (b) what is the period of this motion? (c) what is the frequency of this motion ?

SOLUTION : total distance = 0.43m

$f = \frac{30}{1min} = \frac{30}{60s} = 0.5Hz$

(a) $A = \frac{total\ distance}{2} = \frac{0.43}{2} = 0.21m$

(b) Recall: $T = \frac{1}{f} = \frac{1}{0.5} = 2s$

(c) Frequency has been found to be f =0.5Hz.

EXAMPLE12 : A particle moving with simple harmonic motion has velocities 4cm/s and 3cm/s at distances 3cm and 4cm respectively from the equilibrium position. What is the amplitude of the oscillation? what is the velocity of the particle as it passes the equilibrium position?

SOLUTION : $v_1 = 4cm/s$, $v_2 = 3cm/s$, $x_1 = 3cm$, $x_2 = 4cm$

$v_1 = w\sqrt{(A^2 - x_1^2)}$

$4 = w\sqrt{(A^2 - 3^2)}$

$4 = w\sqrt{(A^2 - 9)}$ ……………………………. (1)

Also $v_2 = w\sqrt{(A^2 - x_2^2)}$

$3 = w\sqrt{(A^2 - 4^2)}$

$3 = w\sqrt{(A^2 - 16)}$ ………………………….(2)

Equation (1) divided by equation (2) will give :

$\frac{4}{3} = \frac{\cancel{w}\sqrt{(A^2 - 9)}}{\cancel{w}\sqrt{(A^2 - 16)}}$

Square both sides

$\frac{4^2}{3^2} = \frac{\sqrt{(A^2 - 9)}^2}{\sqrt{(A^2 - 16)}^2}$

the square will cancel the square root.

$\frac{16}{9} = \frac{A^2 - 9}{A^2 - 16}$

Cross multiply

$16(A^2 - 16) = 9(A^2 - 9)^2$

$16A^2 - 256 = 9A^2 - 81$

$16A^2 - 9A^2 = 256 - 81$

$7A^2 = 175$ divide both sides by 7

$A^2 = 175/7 = 25$

$A^2 = 25$ take square root of both sides

$A = \sqrt{25} = 5cm$

the velocity of the particle as it passes the equilibrium position is v_{max}. butlets find w first.

put A = 5 into equation (1) to find w.

$4 = w\sqrt{(A^2 - 9)}$

$4 = w\sqrt{(5^2 - 9)}$

$4 = w\sqrt{(25 - 9)}$

$4 = w\sqrt{(16)}$

$4 = w \times 4$ divide both sides by 4

$w = 4/4 = 1rad/s$

$v_{max} = wA = 1 \times 5 = 5cm/s$

EXAMPLE13: How much would the time keeping of a pendulum clock be affected by taking it to the moon? Gravity on the moon is $1.6m/s^2$, compared with $10m/s^2$ on the earth.

SOLUTION :g_m = 1.6m/s^2 , g_e = 10m/s^2

$T = 2\pi\sqrt{\frac{l}{g}}$ remember that we said : if the period and the length are changing, use

Using $\frac{T_1^2}{L_1} = \frac{T_2^2}{L_2}$

Now , since it's the period and **g** that are changing use

$T_1\sqrt{g_1} = T_2\sqrt{g_2}$ in a similar way ,

$T_m\sqrt{g_m} = T_e\sqrt{g_e}$

$T_m\sqrt{1.6} = T_e\sqrt{10}$

$1.26T_m = 3.16T_e$

Divide both sides by 1.26

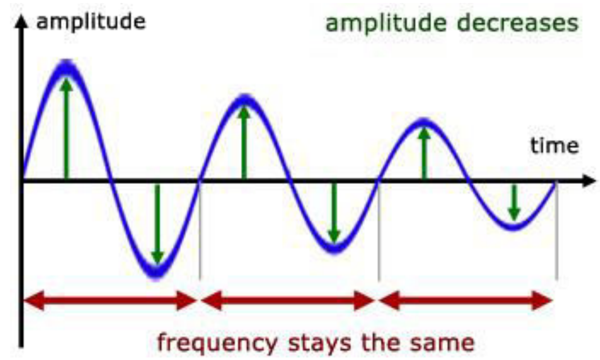
$\frac{1.26T_m}{1.26} = \frac{3.16T_e}{1.26}$

$T_m = 2.5T_e$ since it takes 2.5s of the period on earth, it is 2.5 times slower in the moon.

CHAPTER 2

DAMPED OSCILLATION

The amplitude of a vibrating body does not remain **constant** in practice, but becomes progressively smaller . Such a vibration is said to be damped. The **DECREASE** in amplitude is due to LOSS OF ENERGY, for example the mass **M** IN **CHAPTER 1 (CASE 2)** diminishes slowly owing to the viscosity of the air. See the diagram below.



The decrease in the amplitude caused by dissipative force is called **DAMPING** and the corresponding motion is called **DAMPED OSCILLATION.** In damped oscillation an additional force on the body due to friction, **F=-bV,** is introduced where **V=dx/dt** is the velocity and b is a constant that signifies the strength of the magnetic force(i.e damping constant) **. NOTE:** the minus (-) sign signifies that the force is always apposite in the direction to the velocity. Here, we use the general equation: $m\ddot{x}+b\dot{x}+kx=0$. Where x, $\dot{x}$ and $\ddot{x}$ are displacement, velocity and acceleration respectively.

$w^2=\frac{b}{m}$ **and** $w_o^2=\frac{k}{m}$ .where w is the damped angular frequency, w_o is the natural/initial angular frequency, m is the mass and k is the force/spring constant.

And $w^{/} = \sqrt{(w_o^2 - \frac{w4}{4})}$ or $w^{/} = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$w^{/}$ is the angular frequency of the system.

Example 1: A 0.4kg mass is moving on the end of a spring with force constant k=300N/m and acted upon by a damping force of F=-bV. If b=9.0kg/s ,what is the angular frequency of oscillation of the mass.

Solution: m=0.4kg, k=300N/m, b=9kg/s

Using :$\mathbf{w^{/}}=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}} = \sqrt{(\frac{300}{0.4}-\frac{9^2}{4(0.4)^2})}$

$w^{/}=\sqrt{(750-126.56)}$ =24.97rad/s

Example 2:A mass of 0.4kg moving at the end of a spring with force constant 350N/ is acted upon by a damping constant 9kg/s. Find the angular frequency of the oscillating mass.

m=0.4kg, k=350N/m,

Hint: $\mathbf{w^{/}}=\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$ **now proceed ANSWER=27.4rad/s**

NOW LET'S CONSIDER THESE EQUATIONS:

x=$C_O$exp($-\frac{bt}{2m}$)cos($w^{/}$t + Ø) OR

x=$C_O$exp($-\frac{bt}{2m}$)sin($w^{/}$t + Ø) where x and **C_O**are displacement and amplitude respectively.

Example 3: the equation of a damped oscillation is given as **x=exp($-0.25t$)sin($\frac{\pi}{2}$)t**. Find the natural angular frequency.

SOLUTION:

Compere **x=exp($-0.25t$)sin($\frac{\pi}{2}$)t**. with

x=$C_O$exp($-\frac{bt}{2m}$)sin($w^{/}$t + Ø)

$w^{/}=\frac{\pi}{2}$**rad/s ,** **$-\frac{b\cancel{t}}{2m}$= -0.25**$\cancel{t}$,

$\frac{b}{2m}$=0.25 (move 2 to the right hand side(RHS)

$\frac{b}{m}$=2×0.25 = 0.5

Recall: $w^2=\frac{b}{m}$=0.5 , $w=\sqrt{0.5}$rad/s

using :$\mathbf{w^{/}}=\sqrt{(\mathbf{w_o}^2-\frac{\mathbf{w4}}{\mathbf{4}})}$, $\frac{\pi}{2}=\sqrt{(w_o^2-\frac{(\sqrt{0.5})^4}{4})}$

square both sides to remove the square root

$\frac{\pi}{2}=\sqrt{(w_o^2-\ 0.0625\ )}$, $(\pi/2)^2=(w_o^2-\ 0.0625)$

$(\pi/2)^2+0.0625=w_o^2$

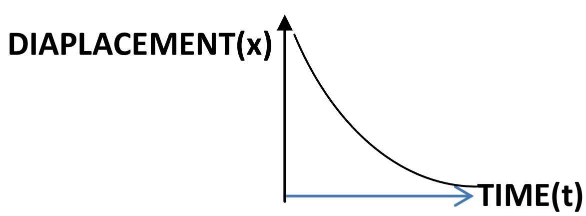
$2.4674+0.0625=w_o^2$

$2.5299=w_o^2$, w_o=1.59rad/s

CASES OF DAMPING

CASE 1: A CRITICALLY OR CENTRALLY DAMPED SYSTEM. This occurs when **b** is so large such that **$w^{/}$=0(i.e $w_o^2-\frac{w4}{4}$ =0)**. Using the equation

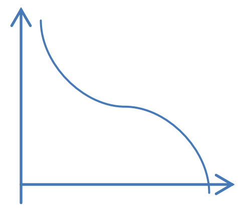
$w^{/}=\sqrt{(w_o^2-\frac{w4}{4})}$, put $w^{/}$=0 , we have **b=2$\sqrt{km}$**. Therefore, for a critically damped system, **b=2$\sqrt{km}$ and $w^{/}$=0.** In such condition, the system no longer oscillates but returns to its equilibrium position without oscillating when it is displaced. It can be illustrated by the graph below:



CASE 2: **HEAVILY** **OR OVER DAMPED SYSTEM.** It occurs when **b**is greater than **2$\sqrt{km}$**, thus **$\sqrt{(w_o^2-\frac{w4}{4})}$** is imaginary i.e **$w^{/}$**is negative. In this condition there is no oscillation**,** the system returns to its equilibrium position MORE

SLOWLY than in the case of critically damping. It results to a **DEAD SYSTEM**. See graph below.

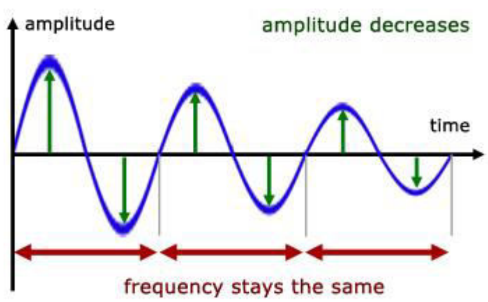
DISPLACEMENT(x)



TIME(t)

Here, **b**$>$**2**$\sqrt{km}$ **. and w**$^{/}< 0$

CASE 3: UNDER OR LIGHTLY DAMPED SYSTEM. This occurs when **b** is less than the critical value, **2**$\sqrt{km}$ **such that** $\sqrt{\frac{k}{m}-\frac{b^2}{4m^2}}$ is positive. In this case the system will oscillate with a steady decreasing amplitude shown below.



Here, **b**$< 2\sqrt{km}$ **and** **w**$^{/}> 0$.

EXAMPLE 4: the equation given as **3ẍ+12ẋ+39x =0.** is (a) simple harmonic (b)critically damped (c)lightly damped (d) none of the above.

SOLUTION: compare the equations

recall: **mẍ+bẋ+kx =0 and 3ẍ+12ẋ+39x=0**

m=3kg, b=12kg/s, k=39N/m

$2\sqrt{km}$ = $2\sqrt{39\times 3}$ = 21.63, but b=12kg

Since ,**b**$< 2\sqrt{km}$ then we say its lightly

damped.

EXAMPLE 5: A 0.4kg mass is moving on the end of a spring with force constant k=300N/m and its acted upon by a damping force of F=-bV. (a) if b= 9kg/s, what is the angular frequency of the mass? (b) For what value of b will the motion be critically damped ?

SOLUTION: =0.4kg, k=300N/m

(a) b=9kg/s,**w**$^{/}$=? We have done this before. Hint : **w**$^{/}=\sqrt{(w_o^2-\frac{w4}{4})}$

(b) For critically damped, **b=2**$\sqrt{km}$

b=2$\times\sqrt{300\times 0.4}$ **=21.9kg/s**

EXAMPLE 6: The equation of a damped harmonic oscillation is given by

x=3exp(-0.04t)cosπt, what is the nature of the oscillation?

SOLUTION: compere the equations

x=3exp(-0.04t)cosπ ,

x=C$_O$exp(-$\frac{bt}{2m}$**)cos(w$^{/}$t + Ø)**

w$^{/}$= π**= 3.142rad/s.** since **w**$^{/}>$ 0then we say the oscillation is lightly damped.

PARTICLE VELOCITY(V_P)

It is found by differentiating the given equation and substituting the given value of **t** (eg t=0 or t=1 e.t.c)the answer**.** but it will take too much time. Use the SHORTCUT: the equation can come in form of cos or sin. NOTE: this shortcut works only when t=0.

EXAMPLE7: The equation of a damped oscillation is given in the form **x=5exp(-**$0.25t$**)cos(**$\frac{\pi}{2}$**)**t. Find the particle velocity at t=0. SOLUTION :

just follow the arrows and multiply them.

This is how you do the ones in form of cos.

x=5exp($-0.25t$)cos($\frac{\pi}{2}$)t

V_p= 5×-0.25=-1.25m/s

EXAMPLE 8: The equation of a damped oscillation is given by **x=3exp($-0.5t$)sinπt**. Calculate the particle velocity.

SOLUTION: see how you do the ones in form of **sin.** Follow the arrows

x=3exp($-0.5t$)sinπt.

V_p=3× π= 9.42m/s

NOTE: if the question says find the MAGNITUDE OF THE PARTICLE VELOCITY, remove the negative sign in your answer.

EXAMPLE 9: The equation of a damped oscillation is given by **x=5exp($-0.25t$)cos$\frac{\pi}{2}$ t,** find the magnitude of the particle velocity at an oscillating point at time t=0.

SOLUTION :**x=5exp($-0.25t$)cos$\frac{\pi}{2}$ t,**

V_p=5× -0.25 = -1.25m/s. Therefore the magnitude or **V_p** is |**V_p**| =1.25m/s

METHODS OF DESCRIBING A DAMPED OSCILLATOR

1. LOGARITHMIC DECREMENT (σ) : It is used to measure the rate at which the amplitude dies away. **σ = $\log_e(A_o/A_1)$**

Where **A_o and A_1** are initial and final amplitude of the motion respectively.

2. RELAXATION TIME OR MODULUS OF DECAY (t_r): It is the time taken for the amplitude to decay to e^{-1} =0.363 of its initial value A_O .Recall : $w^2=\frac{b}{m}$,

$t_r=2/w^2=\frac{2m}{b}$. Half the relaxation time gives the damping time, **$t_d= 1/w^2=\frac{m}{b}$**

3. QUALITY FACTOR OR FIGURE OF MERIT (Q) : It is used to measure the rate at which the energy decays. It is defined as:

$$Q = \frac{\textit{enregy stored in the system}}{\textit{energy dissipated per radian}}$$

Q is dimensionless . $Q= w^{/}/w^2$

Where $w^{/}= \sqrt{(w_o^2-\frac{w4}{4})}$

NOTE: For a lightly damped oscillator, **Q**$\gg 1$. while for a heavily damped oscillator **Q**$\ll 1$.

ENERGY OF A DAMPED HARMONIC OSCILLATION

The damping force in a damped oscillator is **NON-CONSERVATIVE,** that is to say the mechanical energy of the system is not constant but decreases continuously, approaching zero after a long time. The mechanical energy decreases exponentially at a rate that is equal to that at which work is done against friction. i.e energy loss=work done against friction.

Energy at a given time, t is given as :

$E=\frac{1}{2}mw_o^2c_o^2e^{\frac{-bt}{2m}}$

Initial energy (i.e when t=0) is given as

$E_o=\frac{1}{2}mw_o^2c_o^2$

The initial energy per unit mass i.e (E_o/m) is given as (E_o/m) = $\frac{1}{2}w_o^2c_o^2$

DAMPING POWER (dE/dt) : It is the rate at which the damping force does work on the system. **(dE/dt) = $-bV^2$** . The negative sign indicates that energy continuously decreases though not at a uniform rate.

EXAMPLE 10: The equation of a damped harmonic oscillator is given as **x=10exp($-0.125t$)cos($\frac{\pi}{2}t$)** . calculate(a) The angular frequency $w^{/}$ of the oscillator (b) the natural angular frequency (c) the initial energy per unit mass of the damped oscillator (d) the damping time (e) the nature of the oscillator (f) the quality factor (f) the particle velocity of the oscillation at time t=0.

SOLUTION: compere the equations

x=10exp(-$0.125t$)cos($\frac{\pi}{2}t$)

x=$C_O$exp(-$\frac{bt}{2m}$)cos($w^/t + \emptyset$)

$w^/$ $= \frac{\pi}{2}$ =1.571 rad/s, c_o **=10**

$-\frac{bt}{2m} = -0.125t$, $\frac{b}{2m}$ =0.125

Move 2 to right hand side,

$\frac{b}{m} = 2 \times 0.125$ =0.25

Recall w^2= $\frac{b}{m}$ =0.25, thus

$w^/=\sqrt{0.25}$ =0.5rad/s

a. $w^/$=0.5rad/s

b. $w^/ = \sqrt{(w_o^2 - \frac{w4}{4})}$, $1.571 = \sqrt{(w_o^2 - \frac{0.5^4}{4})}$

$1.571 = \sqrt{(w_o^2 - 0.015625)}$

Take square of both sides,

$2.468041 = w_o^2 - 0.015625$

$w_o^2 = 2.468041 + 0.015625 = 2.484$

$w_o = \sqrt{2.484}$ =1.576rad/s

c. **$(E_o/m) = \frac{1}{2}w_o^2c_o^2$**

(E_o/m) $= \frac{1}{2} \times 1.576^2 \times 10^2$ =124.2j/kg

d. , $t_d = 1/w^2 = 1/0.5^2$ =0.4s

e. $w^/$=0.5rad/s, $w^/>0$, lightly damped

f. Q= **$w^/$**$/ w^2 = 1.571/0.5^2$ =6.284

g. **V_p=10$\times$ 0.125** =-1.25m/s

NOW LOOK AT THIS FORMULA :

A=$A_o$exp(-bt/2m), A and A_o are final and initial amplitudes respectively. **exp** means exponential, it is written as **e** in your calculator.

From the formula above, **$b = \frac{2m\ln(A_o/A)}{t}$,**

In is pronounced '**Lin**' and it is written as **In** in your calculator.

And **$t = \frac{2m\ln(A_o/A)}{b}$** **and**

$w^2 = \frac{2m\ln(A_o/A)}{t}$

EXAMPLE 11 : A 0.5kg mass oscillates at the end of a spring with force constant 400N/m. Its initial amplitude of motion is 0.30m. When a damping force acts on the mass , is amplitude of motion decreases to 0.2m in 10s. Calculate the magnitude of the damping force constant.

SOLUTION : b =? , k = 400N/m ,A_o=0.30m, A =0.2m , t =10s , m =0.5kg

$b = \frac{2m\ln(A_o/A)}{t}$, **=** $\frac{2\times 0.5\ln(0.3/0.2)}{10}$

$b = \frac{1 \times \ln(1.5)}{10}$ =0.041kg/s

EXAMPLE 12 : A simple pendulum of length 22m is set into oscillation with amplitude 0.05m, after 5min, it has fallen to 0.025m. Calculate the relaxation time.

SOLUTION :A_o = 0.05m,

t =5min =5 $\times$ 60 =300s

A =0.025m, t_r =? Recall **t_r=2/**w^2

$w^2 = \frac{2m\ln(A_o/A)}{t} = \frac{2\times \ln(0.05/0.025)}{300}$

$= \frac{2\ln 2}{300} = \frac{2\times 0.693}{300} = 0.00462$

$W = \sqrt{0.00462} = 0.0679$rad/s

$t_r = 2/\mathbf{w^2} = 2/0.0679^2 = 433.8$s

EXAMPLE 13 : Given that for a damped oscillation m = 250g, k = 85N/m, b =70g/s calculate (a) the period of the oscillation (b) the time taken for the amplitude of the damped to drop to half of its initial value (c) the time taken for the mechanical energy to drop to half of its initial value.

SOLUTION: m = 250g = 250 /1000 , k = 85N/m, b =70g/s =70 /1000 =0.07kg

(a) $T = 2\pi/w_o$, $w_o = \sqrt{\frac{k}{m}} = \sqrt{\frac{85}{0.25}} = 18.43$rad/s

Thus $T = \frac{2\times 3.142}{18.43} = 0.341$s

(b) T =? , let $\mathbf{A_o = A_1}$, thus $A = \mathbf{A_1}/2$(i.e half)

$\mathbf{t = \frac{2m\ln(A_o/A)}{b}}$

$= \frac{2\times 0.25\ln\{\cancel{A_1}/(\cancel{A_1}/2)\}}{0.07}$

$= \frac{\mathbf{0.5\ln 2}}{\mathbf{0.07}} = \frac{0.5\times 0.693}{\mathbf{0.07}} = 5$s

(c) $\frac{1}{2}\cancel{m w_o{}^2 c_o{}^2}\, e^{\frac{-bt}{2m}} = \frac{1}{2}(\frac{1}{2}\cancel{m w_o{}^2 c_o{}^2})$

$e^{\frac{-bt}{2m}} = \frac{1}{2}$, $1/e^{\frac{bt}{2m}} = \frac{1}{2}$ (cross multiply)

$2 = e^{\frac{bt}{2m}}$(take **ln** of both sides)

$\ln 2 = \ln e^{\frac{bt}{2m}}$

$\ln 2 = \frac{bt}{2m}\ln e$, (NOTE: $\ln e = 1$)

$\ln 2 = \frac{bt}{2m}$ (cross multiply)

$2m\ln 2 = bt$ (divide both sides by **b)**

$\frac{2m\ln 2}{b} = t$

$t = \frac{2m\ln 2}{b} = \frac{2\times 0.25\times 0.693}{0.07} = 5$s.

CHAPTER 3

FORCED DAMPED OSCILLATION

Forced Oscillation

Free Vibration**:** Damped or undamped are solely maintained by the energy stored up in the system with no subsequent supply of energy except at the start for setting up the vibrations. Moreover, in the case of damped vibrations, the energy of the system decays exponentially with time and after some time, the system comes to rest. However, the decay of vibration can be arrested and this can be maintained provided there is a driving force.

When the natural frequency of the driven oscillator is not the same as the frequency of the impressed force, the natural frequency of the impressed force dies out soon and it begins to oscillate with the frequency of the impressed periodic force. The vibrations, which are so maintained, are called **FORCED VIBRATIONS.** But when the frequency of the driving force coincides with the natural frequency of the oscillator (for the case of no damping), this results in the phenomenon of **RESONANCE.** Under this influence, either the displacement or velocity amplitude of the system goes on building or the system goes on receiving a certain amount of average power till any of those parameters assume its maximum value. The phenomenon of resonance has both positive and negative aspects. At the resonance condition, the system can build up a large displacement under a small driving force and this fact is utilized by pipes and resonant electric circuits which allow the turning frequency of our radios to the desired frequency. Moreover, amplitude motion is undesirable in the spring of automobile or in the crank shaft of its engine. A dissipative frictional force will reduce the response atresonance.

FPRCED OSCILLATIONS IN MASS-SPRING SYSTEM.

Here we use: $m\ddot{x}+b\dot{x}+kx = \frac{F_o\cos(pt + \emptyset)}{m}$

$w^2=\frac{b}{m}$ and $w_o^2 = \frac{k}{m}$

w = angular frequency of the driving force

$\frac{F_o\cos(pt + \emptyset)}{m}$ is the driving force

$F = \frac{F_o}{m}$ is the amplitude of the driving force

MECHANICAL IMPEDANCE(Z_m)

Its S.I unit is ohms(Ω)

$Z_m = \sqrt{\{b^2 + \left(mp - \frac{k}{p}\right)^2\}}$

EXAMPLE 1 : The equation of motion of a point mass is : **$3\ddot{x}+7\dot{x}+10x = 10\cos(20t - \emptyset)$.** Find the mechanical impedance.

SOLUTION: compare the equations :

$m\ddot{x}+b\dot{x}+kx = F\cos(pt + \emptyset)$ and

$3\ddot{x}+7\dot{x}+10x = 10\cos(20t - \emptyset)$

m = 3kg , b = 7kg/s, k = 10N/m, p = 20rad/s

$Z_m = \sqrt{\{7^2 + \left(3 \times 20 - \frac{10}{20}\right)^2\}}$

$Z_m = \sqrt{\{49 + (60 - 0.5)^2\}}$

$Z_m = \sqrt{\{49 + 3540.25\}} = 59.9\ \Omega$

But the question says " **find the mechanical impedance per unit mass"** it means find **Z_m** and divide it by **m.**

$\frac{Z_m}{m} = \frac{59.3}{3} = 19.97\ \Omega/kg$

NOTE : mechanical impedance is also called **MECHANICAL RESISTANCE**.

RESONANCE

Resonance frequency(w_m)

$w_m = \sqrt{(w_o^2 - \frac{w4}{2})}$ or $w_m = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}$

EXAMPLE 2: The equation of motion of a point mass is given as **$3\ddot{x}+7\dot{x}+10x = 10\sin(20t - \emptyset)$.** Find the resonance frequency.

SOLUTION: compare the equations :

$3\ddot{x}+7\dot{x}+10x = 10\sin(20t - \emptyset)$ with

$m\ddot{x}+b\dot{x}+kx = F\sin(pt + \emptyset)$

m = 3kg , b = 7kg/s, k = 10N/m, p = 20rad/s

$w_m = \sqrt{\frac{10}{3} - \frac{7^2}{2(3)^2}} = w_m = \sqrt{\frac{10}{3} - \frac{47}{18}}$

$w_m = \sqrt{3.333 - 2.72} = \sqrt{0.613} = 0.78$rad/s

Now do this. They are all done the same way.

EXAMPLE 3: The equation of a motion of a particle of mass 2kg is given by

$2\ddot{x}+4\dot{x}+7x = 6\sin(\pi t)$. Determine the resonance frequency.

HINT :$\mathbf{w_m} = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}}$ **ANSWER = 1.22rad/s**

EXAMPLE 4 : The equation of motion of a particle is given as : **$2\ddot{x}+4\dot{x}+7x = 6\sin(\pi t - \emptyset)$.** The maximum displacement of the periodic motion is obtained when the parameter **r**takes the value?

SOLUTION: at maximum displacement **$w_m = p$.**

Note that **$p = \pi r$**

m = 2kg , b = 4kg/s, k = 7N/m

$w_m = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}} = \sqrt{\frac{7}{2} - \frac{4^2}{2(2)^2}} = \sqrt{3.5 - 2}$

w_m = 1.224rad/s

since $w_m = p$ then p = 1.224rad/s

but $p = \pi r$ (π = 3.142)

$1.224 = 3.14 \times r$

$r = \frac{1.224}{3.142} =$ 0.3899Hz

STEADY STATE PERIOD(T_s) AND STEADY STATE FREQUENCY(F_s)

$T_s = \frac{2\pi}{p}$, $F_s = \frac{p}{2\pi}$, $F_s = 1/T_s$

EXAMPLE 5: The equation of motion of a point mass is given as:

$3\ddot{x}+7\dot{x}+10x = 10\sin(20t - \emptyset)$. Find the steady state period of oscillation.

SOLUTION : p = 20rad/s

$T_s = \frac{2\pi}{p} = \frac{2\times\pi}{20}$ = 0.314s

EXAMPLE 6 : The equation of motion of a particle is given by: **$\ddot{x}+6\dot{x}+27x = 5\sin(wt + \emptyset)$.** Determine the type of motion and determine the frequency at a steady state

SOLUTION :it is a STEADY STATE MOTION.

$F_s = \frac{p}{2\pi}$ **, but p = w,** therefore, $F_s = \frac{w}{2\pi}$,

EXAMPLE 7 : Referring to **example 6** above, find the angular frequency of the motion when the amplitude is maximum.

SOLUTION :**note:** the angular frequency of the motion when the amplitude is maximum is called the **resonance frequency.**

m =1kg , b = 6kg/s , k = 27N/m

$w_m = \sqrt{\frac{k}{m} - \frac{b^2}{2m^2}} = \sqrt{\frac{27}{1} - \frac{6^2}{2(1)^2}}$ = 3rad/s.

NO JESUS, NO SUCCESS,

Now , if you are not saved and you want to be forgiven of all your sins , say these words after me(say it from your heart): Lord Jesus, have mercy on me and forgive me of all my sins, even as I forgive everyone that has trespassed against me. I believe that you shed your blood and died for me and on the third day you resurrected that I may live. Lord Jesus , from this day, I accept you as my Lord and personal savior, in Jesus name, Amen. If you have just said those words, ***Congratulations****,* ***welcome to the body of Christ.*** *Look for a bible believing church and start attending, study your bible and pray daily*

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"If you cannot learn the way we teach then we will have to teach the way you learn"

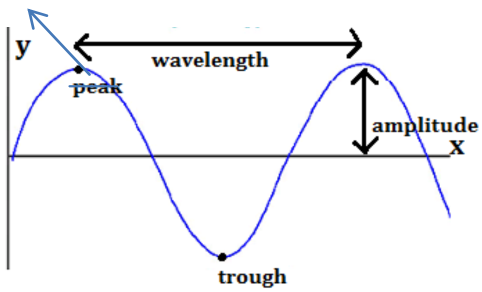
IN GOD WE TRUST

CHAPTER 4

WAVES

A wave is a disturbance which transfers energy from one point to another through a medium with the average position of the particles of the medium remaining constant.

crest



TYPES OF WAVES

(1) MECHANICAL WAVES :these are waves that requires a material medium for their propagation. E.g water waves, sound waves, waves in string or rope.

There are two types :

(i) TRANSVERSE WAVES :in the transverse waves, the particle displacement is perpendicular to the direction of the wave propagation. E.g waves produced in strings.

(ii) LONGITUDINAL WAVES : here, the particle displacement is parallel to the direction of wave propagation. E.g water waves

(2) ELECTROMAGNETIC WAVES :they do not require a physical medium for propagation and hence can travel through vacuum. E.g light waves, gamma-rays, x-rays

TERMS USED IN DISCRIBING A WAVE.

(1) PERIOD (T) :it is the time taken to complete one circle. It's unit is seconds(s)

(2) WAVELENGTH (λ) :it is the distance between two successive crests or trough. It's unit is meters(m)

(3) AMPLITUDE (A) :it is the maximum distance or displacement of the vibrating particles from their position of rest. It's unit is meters(m).

(4) WAVE SPEED (V) :it is the distance covered by the wave divided by the time taken. It's unit is m/s

$V = \frac{d}{t}$

(5) FREQUENCY (f) :it is the number of circles or particle vibration per second. It's unit is hertz (Hz)

NOTE :a progressive wave is a wave which transfers energy travelling continuously from a source to another point.

SOME FORMULAE IN WAVES

$f = \frac{n}{t}$, $v = f\lambda$, $T = \frac{1}{f}$,

$T = \frac{t}{n}$, $\lambda = Tv$, $T = \frac{2\pi}{w}$

n = number of oscillation

EXAMPLE 1 : The speed of a wave is 344m/s. (a) What is the wavelength of that same wave with a frequency of 27.5Hz? (b) what is the frequency of the wave with a wavelength of 1.76m?

SOLUTION : v = 344m/s

(a) λ = ? , f = 27.5Hz

Using $v = f\lambda$

$344 = 27.5 \times \lambda$

$\lambda = \frac{344}{27.5} = 12.5$m

(b) f = ? , λ = 1.76m

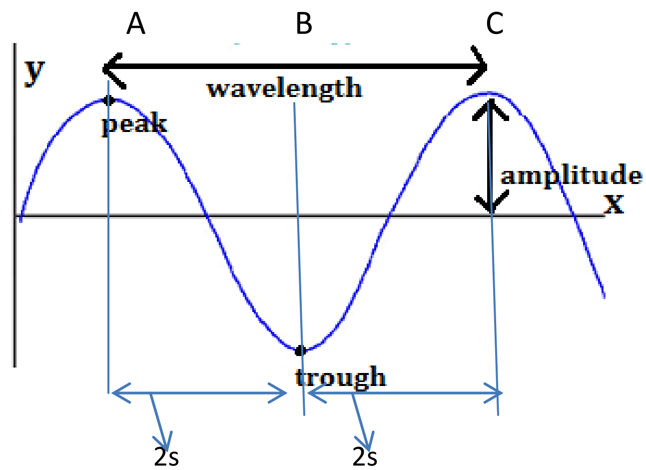
Using $v = f\lambda$

$344 = f \times 1.76$

$f = \frac{344}{1.76} = 195.5$Hz

EXAMPLE 2 :A fisherman notices that his boat is moving up and down periodically, owing to the waves on the surface of the water. It takes 2s for the boat to travel from its highest point to its lowest. The fisherman sees that the wave crests are spaced 7m apart. How fast are the waves traveling?

SOLUTION :



if it moves 2s from up (point A) to down (point B), the period , T = 2 + 2 = 4s

λ **= 7m, v = ?**

v = f , but T = $\frac{1}{f}$ **=** $\frac{1}{4}$ **= 0.25Hz**

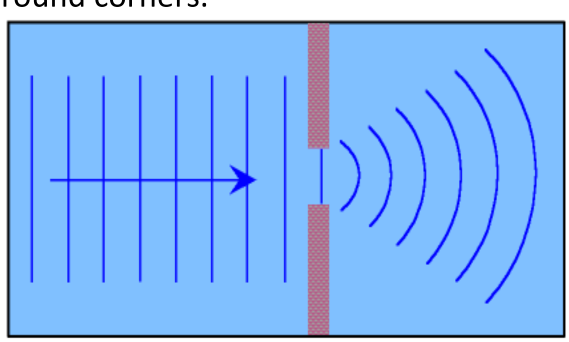
v = 0.25 $\times$ **7 = 1.75m/s**

PROPERTIES OF WAVES

(1) REFLECTION : it is the bouncing back of traveling waves (in the same medium) when they fall on obstacle.

(2) REFRACTION :it is the change of speed and direction of a waves when it enters another medium. (we will talk about reflection and refraction including their diagrams**in chapter 7**)

(3) DEFFRACTION :it is the spreading of wave when they pass through small openings or round corners.



(4) INTERFERENCE :it occurs when two waves of the same frequency, amplitude and wavelength traveling in the same direction meet or overlap or superpose. (we will talk about this in **chapter 5**).

POLARIZATION :it is the phenomenon whereby a wave whose vibration are only in one plane is produced. Such wave is said to be the **plane polarized** and the plane is called the **plane of polarization.**It is applicable to **ONLY transverse waves**.

MATHEMATICAL REPRESENTATION OF WAVE MOTION – PROGRESSIVE WAVE

Y = Asin($\theta \pm \phi$**)**

But θ **= wt,** and ϕ **= kx**

Where $\phi$is phase difference

Therefore, **Y = Asin(**$wt \pm kx$**) ………….(1)**

NOTE : The quantity **(**$wt \pm kx$**)** is called the **phase** measured in radians.

w = 2π**f ,** where w = angular velocity

K = $\frac{2\pi}{\lambda}$ **,** where k = wave number and its unit is **rad/m**

y = Asin2π**(**$\frac{t}{T} \pm \frac{x}{\lambda}$**) ………………………(2)**

where A = amplitude, t = time, T = period,

λ **= wavelength**

In the equation above use **only minus (-)** when the wave s traveling in the **positive x-direction.** Use **only plus (+)** when the wave is traveling in the **negative x-direction.**

EXAMPLE 3 :A wave is represented by the equation **y = 0.20sin{0.40**π**(x – 60t)}** in cm. Find : (a) the wavelength (b) the frequency (c) the displacement at x = 5.5cm and t = 0.02s.

SOLUTION :

(i) Compare the equations :
y = 0.20sin{0.40π(x – 60t)}
y = Asin2π($\frac{t}{T} - \frac{x}{\lambda}$)
therefore, $0.40\cancel{\pi x} = \frac{2\cancel{\pi x}}{\lambda}$
$\lambda = \frac{2}{0.4}$ = 5m
(ii) Again, by comparison,
$0.40\cancel{\pi} \times 60\cancel{t} = \frac{2\cancel{\pi t}}{\lambda}$
$0.4 \times 60 = \frac{2}{T}$
$T = \frac{2}{0.4 \times 60} = \frac{1}{12}$ s
Recall: $f = \frac{1}{T}$
$F = \frac{1}{1/12}$ = 12Hz
(iii) y = 0.20sin{0.40π(x – 60t)}
expand the inner bracket
y = 0.20sin{0.40πx – 24πt}
substitute x = 5.5cm and t = 0.02s

y = 0.20sin{0.40$\pi \times 5.5 - 24 \times \pi \times 0.02$}

y = 0.20sin{2.2– 0.48π}cm

EXAMPLE 4 :A wave is described by the equation **y = 2sin2π($\frac{t}{0.01} + \frac{x}{30}$)**where x is in cm and t is in sec. Find the : (i) wave's amplitude. (ii) Frequency (iii) wavelength (iv) speed and (v) direction.

SOLUTION : again, compare the equations

y = 2sin2π($\frac{t}{0.01} + \frac{x}{30}$)cm

y = Asin2π($\frac{t}{T} + \frac{x}{\lambda}$)

T = 0.01s, λ = 30cm

(i) A = 2cm
(ii) F = 1/T = 1/0.01 =100Hz
(iii) λ = 30cm
(iv) V = f λ = 100 $\times$ 30 = 3000cm/s
(v) Since the given equation is carrying plus (+) , its direction is the negative x-direction.

EXAMPLE 5 : The displacement y of air due to sound wave is given by **Y = 4sin($ax + bt$)m**

Where $a = \frac{4\pi rad/m}{3}$ and b = 440πrad/s. Find the : (a) wavelength (b) frequency (c) wave speed.

SOLUTION :

Y = 4sin($ax + bt$)m , substitute a and b into the equation.

Y = 4sin($\frac{4\pi x}{3} + 440\pi t$)m

Compare with y = Asin2π($\frac{t}{T} + \frac{x}{\lambda}$)

(i) $\frac{4\cancel{\pi x}}{3} = \frac{2\cancel{\pi x}}{\lambda}$
4λ = 3 $\times$ 2
λ = 6/2 = 1.5m
(ii) $440\cancel{\pi t} = \frac{2\cancel{\pi t}}{T}$

440T = 2

T = 2/440 =1/220

$f = \frac{1}{T} = \frac{1}{1/220}$ = 220Hz

(iii) V = f λ = 220 $\times$ 1.5 = 330m/s

EXAMPLE 6 :A wave moving along the x-axis is described by $y = 5e^{-i(x+5t)}$ were x is in meters and t is in seconds. Determine (a) the direction of the wave (b) the wave speed.

SOLUTION :

$y = 5e^{-i(x+5t)}$ (is in form of complex number) can also be written as

y = 5cos(x + 5t) – i5sin(x +5t)

using the real part, y = 5cos(x + 5t)

compare with y = Asin(wt + kx)

w = 5rad/s , k = 1rad/m

(a) Left (negative x-direction

(b) $W = 2\pi f$

$5 = 2\pi f$

Thus $f = \frac{5}{2\pi}$ Hz

$K = \frac{2\pi}{\lambda}$

$1 = \frac{2\pi}{\lambda}$

$\lambda = 2\pi$

$V = f\lambda = \frac{5}{\cancel{2\pi}} \times \cancel{2\pi} = 5m/s$

EXAMPLE 7 : Two waves sources separated by 2m apart vibrate in phase with frequency 200Hz and velocity800m/s. Calculate the phase difference at a point midway between them.

SOLUTION : x = 2m, f = 200Hz , v =800m/s

$\boldsymbol{\phi}$ **= kx =** $\frac{2\pi x}{\lambda}$($k = \frac{2\pi}{\lambda}$)

$V = f\lambda$

$800 = 200 \times \lambda$

$\lambda = 800/200 = 4m$

At midway between them, x = 2/2 = 1m

$\boldsymbol{\phi} = \frac{2\times\pi\times 1}{4} = \pi/2$

VELOCITIES OF TRAVELING WAVES

(1) PHASE VELOCITY (V) : It is the rate at which the phase of the wave propagates into space.It is the speed at which the waves travel.

phase velocity, v = $\frac{\mathbf{w}}{\mathbf{k}}$

NOTE : A medium in which the phase velocity depends on the frequency of the wave is known as a **dispersive medium (** $\frac{\mathbf{w}}{\mathbf{k}}$) is notconstant. A medium in which the velocity depends on the physical properties of the medium (i.e the elastic and inertia properties of a mechanical medium) then **(** $\frac{\mathbf{w}}{\mathbf{k}}$) is not constant, independent of frequency . Waves traveling through this medium will maintain a constant shape.

EXAMPLE 8 : Two points 60cm lie on axis propagated by wave of frequency 50Hz. If the phase difference between the points is 90^0 , find the phase velocity of the wave.

SOLUTION : x =60cm = 0.6m , f = 50Hz

$\boldsymbol{\phi} = 90^0$ convert to radians

$\boldsymbol{\pi}$**radian = 180^0**

X = 90^0

(just follow the arrow and cross multiply)

$X = \frac{90\pi}{180} = \frac{\pi}{2}$ rad

$\boldsymbol{\phi} = \frac{\pi}{2}$ rad

phase velocity, v = $\frac{\mathbf{w}}{\mathbf{k}}$

$\boldsymbol{\phi}$ **= kx**

$\frac{\pi}{2} = k \times 0.6$

Thus , $k = \frac{\pi}{0.6 \times 2} = \frac{\pi}{1.2}$ rad/m

$W = 2\pi f = 2\pi \times 50 = 100\pi$ rad/s

$v = \frac{w}{k} = \frac{100\pi}{\pi/1.2} = \frac{100\pi \times 1.2}{\pi} = 120m/s$

EXAMPLE 9 :A wave is described by

y = 0.1sin(3x – 10t), where x and y are in cm. Find the phase velocity.

SOLUTION : compare the equations

y = 0.1sin(3x – 10t)

y = Asin(wt – kx)

w = 10 rad/s, k = 3 rad/cm

$v = \frac{w}{k} = \frac{10}{3} = 3.3cm/s$

(2) PARTICLE VELOCITY (V_P) : It is the simple harmonic velocity (transverse velocity) of the oscillation about its equilibrium position.

$\frac{\partial y}{\partial t}$**(x,t) = V_P**

$\boldsymbol{maximum\ particle\ velocity}$**($V_{Pmax}$)**

V_{Pmax} = wA

When are told to find **V_P ,**simply differentiate the equation given with respect to **t** and substitute the values given.

EXAMPLE 10 : Find the particle velocity at t = 0 for the equation of motion given as x = 5exp(-0.75t)sinπt. Where x is in meters.

SOLUTION: use the shortcut we used in **chapter 2.**

V_P = 5 $\times$ π= 15.7m/s

EXAMPLE 11 :A wave is represented by the equation y = 0.2sin{0.25π(x – 50t)} in cm. Find the particle velocity.

SOLUTION :y = 0.2sin{0.25π(x – 50t)}

Differentiate with respect to **t**

$\frac{\partial y}{\partial t}$ = - 0.2 $\times$ 0.25π $\times$50cos{0.25π(x – 50t)}

$\frac{22}{7}$ 180^0

V_P = - 2.5πcos{0.25 $\times$ 180(5.5 – 50$\times$0.02)}

V_P = -2.5 $\times$ $\frac{22}{7}$ cos(162) = 7.47cm/s

EXAMPLE 12 : The equation of a transverse wave traveling along a stretched string is given as y = sin(10t – 4x), if the displacement at a point is zero, what is the ratio of the phase velocity of the wave to the particle velocity at the same point?

SOLUTION : compare

y = sin(10t – 4x)

y = Asin(wt –kx)

w = 10rad/s , k = 4rad/m

v = $\frac{w}{k}$ = $\frac{10}{4}$ = 2.5m/s

To find V_P we differentiate with respect to **t**

NOTE : the question says **if the displacement at a point is zero.** This means y = 0 .

y = sin(10t – 4x)

0 = sin(10t – 4x)

$\sin^{-1} 0$= (10t – 4x)

0 = (10t – 4x) ……….(1)

Next, we differentiate with respect to **t**

y = sin(10t – 4x)

$\frac{\partial y}{\partial t}$ = 10cos(10t – 4x) ……. (2)

Put (10t – 4x) = 0 in equation 1 into equation 2.

$\frac{\partial y}{\partial t}$ = 10cos(0) = 10m/s

V_P= 10m/s

Thus, the ratio V/V_P = 2.5/ 10 = 0.25

(3) GROUP VELOCITY (V_g) : This results when waves of different frequencies, wavelengths and velocities are super-imposed.
c = 3 $\times 10^8$m/s (speed of light in air)

$$V_g = V - \frac{2\pi f^2}{c}\lambda\frac{dn}{dk} \quad \text{……………………… (1)}$$

$$V = V_g + \frac{2\pi f^2}{c}\lambda\frac{dn}{dk} \quad \text{……………………………(2)}$$

Here, we compare equation two with the given equation

EXAMPLE 13 : The phase velocity of a wave in a certain medium is represented by v = (C_1+ $C_2\lambda$)

m/s where C_1 and C_2 are constants . What is the value if the group velocity if C_1 and C_2 are 10 and 15 respectively and what is the frequency if n = 3k+2

SOLUTION: C_1 = 10, C_2 =15

v = (C_1+ $C_2\lambda$) substitute

v = (10+ 15λ) compare with

$V = V_g + \frac{2\pi f^2}{c}\lambda \frac{dn}{dk}$

(i) V_g = 10m/s
(ii) n = 3k+2

thus, $\frac{dn}{dk}$ = 3(differentiation)

c = 3 $\times 10^8$m/s

π = 3.142

$\frac{2\pi f^2}{c}\lambda \frac{dn}{dk} = 15$

$$\frac{2\times 3.142 \times f^2 \times 3}{3 \times 10^8} = 15$$

$\frac{18.852 f^2}{3 \times 10^8} = 15$

cross multiply

$18.852 f^2 = 15 \times 3 \times 10^8$

$f^2 = \frac{15 \times 3 \times 10^8}{18.852}$

f^2 = 238701464

$f = \sqrt{238701464} = 1.54 \times 10^4$ Hz

SPEED OF A TRANSVERSE WAVE (V)

The physical quantities that determine the speed on transverse waves on a string are :

(i) The tension(T) on the string ; it is measured in Newtons
(ii) Its mass per unit length also called linear mass density (μ) . $\mu = \frac{m}{L}$

$V = \sqrt{\frac{T}{\mu}}$

EXAMPLE 14 : The linear mass density of clothesline is 0.25kg/m. How much tension does kate have to apply to produce the observed wave speed of 12m/s ?

SOLUTION : μ = 0.25kg/m T =? V = 12m/s

$V = \sqrt{\frac{T}{\mu}}$

$12 = \sqrt{\frac{T}{0.5}}$ square both sides

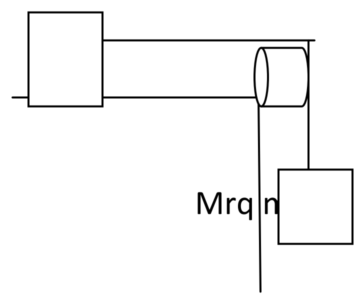
$12^2 = \frac{T}{0.5}$

Cross multiply

$144 \times 0.25 = T$

T = 36N

EXAMPLE 15: A string of length 10m and total mass 0.001kg is connected to a mass m as shown below



Suppose that the string has a very high elastic limit (meaning that it takes lots of pressure on the string before it will stretch) How much mass must you place on the string in other to produce a wave speed of 200m/s?

SOLUTION : L = 10m , m =0.001kg, m_2 = ?

$\mu = \frac{m}{L} = \frac{0.001}{10}$ = 0.0001kg/m

$V = \sqrt{\frac{T}{\mu}}$

$200 = \sqrt{\frac{T}{0.0001}}$ square both sides

$200^2 = \frac{T}{0.0001}$

$T = 200^2 \times 0.0001 = 4N$

But T=mg

$4 = m \times 9.8$

M =4/9.8 = 0.4082kg

EXAMPLE 16: A string has a total length of 5m and a total mass of 0.01kg. If the string has a tension of 4N applied to it. What is the speed of a transverse wave on the string.

SOLUTION : L = 5m, m = 0.01kg, T = 10N, V =?

$\mu = \frac{m}{L} = \frac{0.01}{5} = 0.002$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10}{0.002}} = \sqrt{5000} = 70.7m/s$

EXAMPLE 17: A wire 5000m long and has a mass of 1kg. The wire is under a tension of 30N. The wire is attached between two large trees. How long will it take for the pulse which is an upward pluck to travel there and back.

SOLUTION :

L = 5m, m = 1kg, T = 30N t = ?

$\mu = \frac{m}{L} = \frac{1}{5000} = 0.0002kg/m$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{30}{0.0002}} = 387.3m/s$

EXAMPLE 18 : A wire 5000m long and has a total mass of 1kg. the wire is under a tension of 30N. The wire is stretched between two large trees. How long will it take for the pulse which is an upward pluck to travel there and back.

SOLUTION:

L =5m, m = 1kg, T = 30N, t = ?

$\mu = \frac{m}{L} = \frac{1}{5000} = 0.0002$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{30}{0.0002}} = 387.3m/s$

$V = \frac{distance\ (to\ and\ fro)}{time}$

$387.3 = \frac{5000+5000}{t}$

$387.3 = \frac{10000}{t}$

$T = \frac{10000}{387.3} = 25.8s$

EXAMPLE 19: A uniform cord has a mass of 0.3kg. The cord passes over a pulley and supports 2kg object. Find the speed of a pulse travelling along this cord.(g = 9.8)

SOLUTION :

Diagram is same as the one in example 15.

M = 0.3kg, L = 6m, m =2kg, v =?

$\mu = \frac{m}{L} = \frac{0.3}{6} = 0.05kg/m$

$T = mg = 2 \times 9.8 = 19.6N$

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{19.6}{0.05}} = 19.8/s$

SPEED OF LONGITUDINAL WAVE(V)

Propagation speed of longitudinal as well as transverse depends on the mechanical properties of the medium.

$V = \sqrt{\frac{B}{D}}$, $B = \frac{1}{k}$

Where B, D and K are bulk modulus, density, and compressibility respectively. V is the speed of the longitudinal wave in the **fluid**. **B** is measured in Pascal (Pa) Thus, the speed of propagation in a fluid pulse in a fluid depends only on the **bulk modulus and density of the medium.** On the other hand, when a longitudinal wave propagates in a **SOLID ROD** bar whose sides are free to bulge and shrink, the speed is given by

$V = \sqrt{\frac{Y}{D}}$ where Y is young's modulus

EXAMPLE 20: A ship uses sonar system to detect under water objects. The system emits under water waves and measures the time interval for the reflected waves to return to the detector. Determine the speed of sound waves in water and find the wavelength of wave having a frequency of 262Hz. (compressibility of water = 45×10^{-11} pa^{-1})

(density of water = $1 \times 10^3 kg/m^3$)

SOLUTION :

$$B = \frac{1}{45 \times 10^{-11}} = 2183406114 \text{ Pa}$$

$$V = \sqrt{\frac{B}{D}} = \sqrt{\frac{2183406114}{1 \times 10\text{^}3}} = 1480 m/s$$

$V = f \lambda$

$1480 = 262 \lambda$

$\lambda = 1480/262 = 5.65m$

Now do this

EXAMPLE 21 : What is the speed of longitudinal waves in a lead rod. ($Y = 1.6 \times 10^{10}$Pa, $D = 11.3 \times 10^3 kg/m^3$)

HINT : $V = \sqrt{\frac{Y}{D}}$

Answer : $1.2 \times 10^3 m/s$

EXAMPLE 22 : In a liquid with density $9000kg/m^3$. Longitudinal waves with frequency 250Hz are found to have wavelength 8m. Calculate the bulk modulus of the liquid.

SOLUTION:

$V = f \lambda$

$V = 250 \times 8 = 2000 m/s$

$V = \sqrt{\frac{B}{D}}$

$2000 = \sqrt{\frac{B}{900}}$ square both sides

$2000^2 = \frac{B}{900}$

$B = 2000^2 \times 900 = 3.6 \times 10^9$Pa

EXAMLE 23 : The elastic limit of a piece of steel wire is 2.7×10^9 Pa. What is the maximum speed at which transverse wave pulses can propagate along the wire before this stress is exceeded?(density of steel is 7.86×10^3 kg/m^3)

HINT : $Y = 2.7 \times 10^9$ $V = \sqrt{\frac{Y}{D}}$

Answer = 586m/s

EXAMPLE 24 : what is the difference between the speed of longitudinal waves in air at 17^0C and their speed at 57^0C ?

SOLUTION:

V_{air} =340m/s(it is a constant),

$T_{air} = 17^0C = 17 + 273 = 290K$, V_2 = ?

$T_2 = 57^0C = 57 + 273 = 330K$

The relationship between speed and temperature is written as

$$\frac{V_{air}}{V_2} = \frac{\sqrt{T_{air}}}{\sqrt{T_2}}$$

$$\frac{340}{V_2} = \frac{\sqrt{290}}{\sqrt{330}}$$

$17.03V_2 = 18.17 \times 340$

$V_2 = 6177.8/17.03 = 362.7m/s$

Thus, difference = $V_2 - V_{air}$

$= 362.7 - 340 = 22.7m/s$

CHAPTER 5

SUPERPOSITIONOF WAVES

The principle of superposition of waves : When two or more waves overlap in space, the resultant displacement is the algebraic sum of the individual displacement of the overlapping waves, this is called the principle of superposition of waves. Waves that obey this principle are called **linear waves** and are generally characterized b having amplitude smaller than their wavelengths. Waves that violate the superposition principle are calle**d non linear waves** and are often characterized by large amplitudes. One consequence of the superposition principle is that two travelling waves can pass through each other without being destroyed or even altered.

INTERFERENCE OF WAVES

The combination of separate waves in the same region or space to produce a resultant wave is called interference. During this process, the principle of superposition holds. The resultant amplitude of the two waves at a point is the sum of the two amplitude at that point.

Constructive interference : when the vertical displacement of the two pulses are in the same direction, and the amplitude of the combined waveform is greater than that of either pulse, the situation is called constructive interference. Total constructive interference occurs when two waves of the same frequency and amplitude are exactly in phase(the crest of one wave is aligned with the crest of the other). The amplitude of the combined waveform is twice that of either individual wave.

Destructive interference : If one pulse has a negative displacement, the two pulses tend to cancel each other when they overlap, and the amplitude of the combined waveform is smaller than that of either pulse. This situation is called destructive interference. When these interfering pulses are completely out of phase (180^0 difference or crest coincide with trough) the waveforms momentarily disappear; that is the amplitude of the combined waveform is zero. This case is called **total destructive interference**.

MATHEMATHECAL REPRESENTATION

$Y(x,t) = (2A\cos\frac{\phi}{2})\sin(wt - kx + \frac{\phi}{2})$. This is the sinusoidal wave derived by summing the individual displacement of the two overlapping waves : $y_1 = A\sin(wt - kx)$ and

$y_2 = A\sin(wt - kx + \phi)$. The amplitude of the derived wave equation is $y = 2A\cos\frac{\phi}{2}$

ϕ is phase angle or phase constant

(a) when $\boldsymbol{\phi = 0}$ or 0rad, the combined waves are in phase , thus ,

$y = 2A\cos\frac{\phi}{2} = 2A\cos\frac{\mathbf{0}}{2} = 2A$.

This interference produce the greatest possible amplitude and is called **fully constructive interference**.

(b) if $\boldsymbol{\phi = \pi}$**rad** or 180^0, the combined waves are out of phase, thus,

$y = 2A\cos\frac{\phi}{2} = 2A\cos\frac{\mathbf{180}}{2} = 0$.

This is said to be fully destructive since it produces zero amplitude.

STANDING WAVES (STATIONARY WAVES)

It is a wave formed when two equal progressive waves having the same amplitude, frequency, wavelength and speed and also moving in opposite direction overlap or superpose. Most of them are formed as a result of **reflection** . Because the wave pattern does not appear to be moving in either direction, it is called a standing wave.

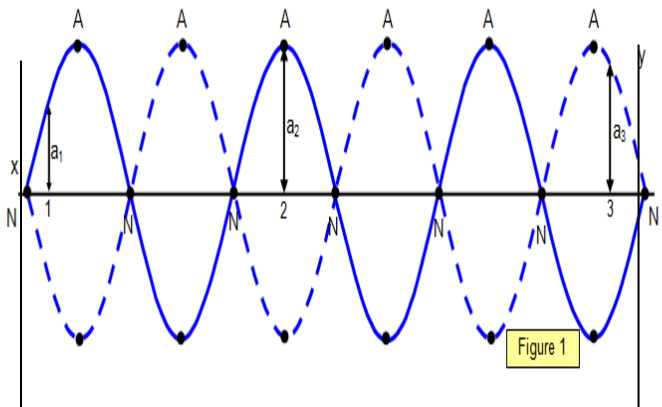


Figure 1

ANTINODE(A) : it is a point on a stationary wave where there is maximum displacement of the medium.

NODE(N): it is a point on a stationary wave where there is no movement of the medium.

NOTE : The distance between two adjacent node is $\frac{\lambda}{2}$, while the distance between two adjacent node is $\frac{\lambda}{2}$, but the distance between a node and an antinode is $\frac{\lambda}{4}$

From the diagram above,

y_1 = Asin(wt +kx) traveling to the left

and y_2 = -Asin(wt – kx) travelling to the right

If we add them using trigonometry we have:

$Y = y_1 + y_2$

Y = (2Asinkx)coswt

Where 2Asinkx is the amplitude

The positions of the nodes are

$\mathbf{X = 0, \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}}$ ………

The positions of the antinodes are

$\mathbf{X = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}}$ ……….

NOTE: a standing wave unlike a traveling wave does not transfer energy from one end to another because a standing wave is formed by two waves carrying equal amount of energy in opposite directions, resulting in a local flow of energy from each node to the adjacent antinodes and back, such that the average energy is zero at every point.

EXAMPLE 1: A standing wave has nodes at x = 0cm, x =6cm, x = 12cm, x = 18cm.

(a) what is the wavelength of the waves that are interfering to produce a standing wave?

(b) A what positions are the antinodes?

SOLITION:

Nodes 0, 6, 12, and 8.

(a) compare. $\mathbf{X = 0, \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}}$ ………

$6 = \frac{\lambda}{2}$

$\boldsymbol{\lambda} = \mathbf{6} \times 2 = 12$cm

(b) antinode, $\mathbf{X = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}}$ ……….

$\lambda = 12$cm

$X = \frac{12}{4}, 3\frac{12}{4}, 5\frac{12}{4}$ ……….

X = 3cm, 9cm, and 15cm

EXAMPLE 2 : Standing waves on a wire of length 4m described by y =(Asinkx)coswt with A = 3cm, w = 628rad/s, k = 1.25πrad/m and with the left end of the wire at x = 0. At what distances from the left end are ;

a. the nodes of the standing waves

b. the antinodes of the standing waves

SOLUTION:

K =1.25π, recall k = $\frac{2\pi}{\lambda}$

1.25$\cancel{\pi}$ = $\frac{2\cancel{\pi}}{\lambda}$

λ = $\frac{2}{1.25}$ = 1.6m

a. nodes X = 0, $\frac{\lambda}{2}$, λ , 3$\frac{\lambda}{2}$ ……… which is the same as X = 0 $\frac{\lambda}{2}$, 1 $\frac{\lambda}{2}$, 2$\frac{\lambda}{2}$, 3$\frac{\lambda}{2}$ ……… we wrote it in the form above because of the way they left their answer.

X = 0 $\frac{1.6}{2}$, 1 $\frac{1.6}{2}$, 2$\frac{1.6}{2}$, 3$\frac{1.6}{2}$ ………

X = 0(0.80), 1(0.8), 2(0.8), 3(0.8) …..

Following the pattern,

X = n(0.8),……(n =0, 1, 2, 3, 4, 5)

b. antinode : X = $\frac{\lambda}{4}$, 3$\frac{\lambda}{4}$, 5$\frac{\lambda}{4}$ ………. Which is same as X = 1 $\frac{\lambda}{4}$, 3$\frac{\lambda}{4}$, 5$\frac{\lambda}{4}$ ……….

X = 1 $\frac{1.6}{4}$, 3$\frac{1.6}{4}$, 5$\frac{1.6}{4}$ ……….

X = 1(0.4), 3(0.4), 5(0.4) …..

X = n(0.4), ………… (n = 1, 3, 5, 7, 9)

EXAMPLE 3: Adjacent nodes on a standing waves on a string are 12cm apart. A particle at an antinode oscillates in SHM with amplitude 2.5cm and period 0.5s. The string lies along the x-axis and is fixed at x = 0.

a. find the equation giving the displacement of a point on the string as a function of position and time.

b. find the speed of propagation of a transverse wave in the string.

c. find the amplitude at a point 3cm to the right of an antinode.

SOLUTION:
distance between adjacent antinodes, x

X = 12cm,

X = $\frac{\lambda}{2}$

12 = $\frac{\lambda}{2}$ cross multiply

λ = 24cm, A = 2.5cm, T = 0.5s

Thus f = 1/T = 1/0.5 =2Hz

K = $\frac{2\pi}{\lambda}$ = $\frac{2\times\pi}{24}$ = $\frac{\lambda}{12}$ rad/cm

W = 2πf = 2 × π × 2 = 4πrad/s

a. substitute into: y = (2Asinkx)coswt

y = (2 × 2.5 sin$\frac{\pi}{12}x$)cos4πt

y = 5 cos4πtSin$\frac{\pi}{12}x$

b. v = f λ , λ = 24cm = 0.24m

v = 2 × 0.24 = 0.48m/s

c. x = 3cm

amplitude = 2Asinkx = 2 × 2.5sin$\frac{\pi}{12}$ × 3

note: π = 180, amplitude = 5sin45 = 3.54cm

EXAMPLE 4 : If the superposition of sinusoidal waves (incident and reflected) travelling at a speed of 84m/s with amplitude of 1.5mm and frequency of 120Hzforms a standing wave on a string reflection at point x = 0

a. find the equation giving the displacement of a point as a function of position and time.

b. locate the points on the string that do not move.

c. find the amplitude, maximum transverse velocity, and maximum transverse acceleration at the point of maximum oscillation.

SOLUTION:

V =84m/s, f = 120Hz,

A = 1.5mm = 1.5/1000 =1.5×10^{-3}m

v = f λ

84 = 120 $\times$ λ

λ =84/120 = 0.7m

K = $\frac{2\pi}{\lambda}$ = K = $\frac{2 \times \pi}{0.7}$ = 8.97rad/m

W = 2πf = 2 × π × 120 = 754rad/s

a. substitute into y = (2Asinkx)coswt

y = $(2 \times 1.5 \times 10^{-3}$ sin8.97x)cos754t

y = $(3 \times 10^{-3}$ sin8.97x)cos754t

b. node : X = 0, $\frac{\lambda}{2}$, λ, $3\frac{\lambda}{2}$ ....

X = 0, $\frac{0.7}{2}$, 0.7 , $3\frac{0.7}{2}$ ....

X = 0, 0.35m, 0.7m, 1.05m

c. amplitude at the point of maximum oscillation = 2A = $2 \times 1.5 \times 10^{-3} = 3 \times 10^{3}$m

velocity, v or $\frac{\partial y}{\partial t}$ means differentiate(calculus) **the displacement equation once** . differentiating with respect to **t** i.e $\frac{\partial y}{\partial t}$ = v

v = $\frac{\partial y}{\partial t}$ = 3×10^{-3} sin(8.9x) × (-754) sin(754t)

v = (-2.26)sin(8.98x) × sin(754t)

acceleration, a, means differentiate velocity.

v = (-2.26)sin(8.98x) × sin(754t)

differentiating again we have

a = (-1.71×10^{3}) sin(8.98x) × cos(754t)

BEATS

It is the alteration of maximum and minimum sound intensities produced by the superposition of sound waves of slightly different frequencies.

The frequency at which loudness varies is called beat frequency(f_b)

$F_b = | f_1 - f_2 |$ and the corresponding period is called the beat period, T_b

T_b = 1/f_b note: T_b is also called *the inverse of the beat frequency.*

EXAMPLE 5: A violinist and a pianist simultaneously sound notes with frequencies 440Hz and 436Hz respectively.

a. what beat frequency will be heard?

b. what is the corresponding beat period?

SOLUTION: f_1 = 440Hz, f_2 = 436Hz

a. $F_b = | f_1 - f_2 | = | 440 - 436 | = 4$Hz

b. $T_b = 1/f_b = 1/4 = 0.25$s

EXAMPLE 6: A violinist tuning an instrument to a piano note of 264Hz detect three beats per seconds. What are the possible frequencies of the violin tone?

SOLUTION: f could be f_1 or f_2

F = 264Hz, f_b = 3Hz(three beats per seconds)

If f_1 = 264

$F_b = | f_1 - f_2 |$

0 = 264 – f_2

$F_2 = 264 - 3 = 261Hz$

Also, if $f_1 = 264Hz$

$3 = | f_1 - 264 |$; $f_1 = 264 + 3 = 267Hz$

Thus, the frequencies that will produce f_b of 3Hz when combined with f = 264 are **261Hz and 267Hz.**

THE DOPPLER EFFECT

It is the variation in perceived sound frequency due to the motion of sound source. The sound waves emitted by a moving source tend to bunch in front of the source and spread out behind the source.

Let v = speed of sound

CASE 1: <u>when the source is moving towards a stationary observer:</u>

V_s = velocity of the source

Here, velocity of the observer equals zero(stationary)

$$F_o = \frac{Vf_s}{V - V_s}$$

F_o is the observed frequency, f_s = is the frequency of the source.

NOTE : the source is the one producing the sound e.g a siren, horn e.t.c

CASE 2: <u>when the source is moving away from the observer:</u>

$$F_o = \frac{Vf_s}{V + V_s}$$

EXAMPLE 7: As a truck travelling at 96km/h approaches and passes a person standing along the highway, the driver sounds the horn. If the horn has a frequency of 400Hz, what are the frequencies of the sound waves heard by the person?

a. As the truck approaches and

b. After it has passed? (assumed that the speed of sound is 346m/s)

SOLUTION: $V_s = 96km/h = \frac{96 \times 1000}{3600} = 27m/s$

$F_s = 400Hz$, $V = 346m/s$,

a. this is case 1 (approaches or moving towards)

$$F_o = \frac{Vf_s}{V - V_s} = \frac{346 \times 400}{346 - 27} = 434Hz$$

b. this is case 2 (moving away or has passed)

$$F_o = \frac{Vf_s}{V + V_s} = \frac{346 \times 400}{346 + 27} = 371Hz$$

CASE 3 : <u>when the observer is moving towards a stationary source</u>

$$F_o = \frac{(V + V_o)f_s}{V}$$

V_o is the velocity of the observer

$V_s = 0m/s$ (stationary source)

CASE 4: <u>when the observer is moving away from a stationary source.</u>

$$F_o = \frac{(V + V_o)f_s}{V}$$

Here, $V_s = 0m/s$

EXAMPLE 8 : A sound source has a frequency of 500Hz if a listener moves at a speed of 30m/s towards the source. What is the frequency heard by the listener? (the speed of sound is 340m/s)

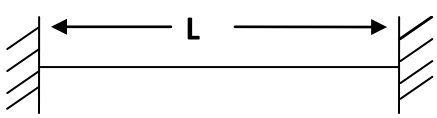
SOLUTION : f_s = 500Hz, V_o = 30m/s, V = 340m/s

$$F_o = \frac{(V + V_o) f_s}{V} = \frac{(340 + 30) \times 500}{340} = 544Hz$$

CHAPTER 6

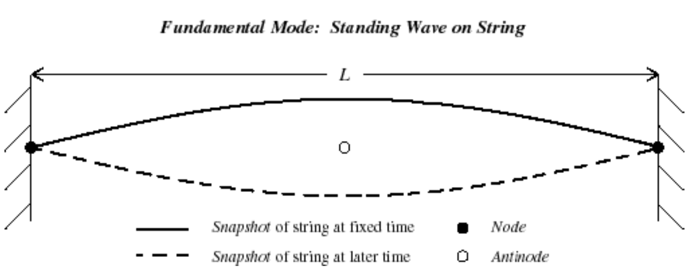
NORMAL MODES

Normal Modes For A String : A normal mode of an oscillating system is a pattern of motion restricted by boundary conditions in which all parts of the system move sinusoidally with the **same frequency and in phase**. A standing (stationary) wave is set up if the string of length L is fixed at both ends because of a continuous superposition of waves incident and reflected from both ends.



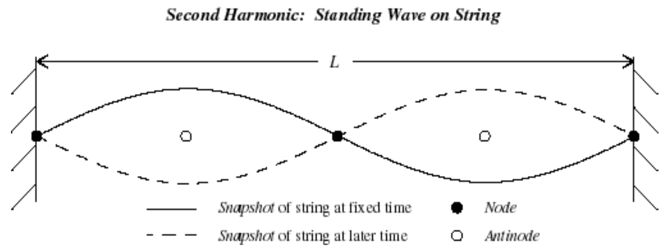
NOTE: waves formed in strings are **transverse and stationary.** The situation in which only certain frequencies oscillation are allowed is called **quantization.**

The diagram above shows the first normal mode if the string



$\mathbf{F_1 = \frac{V}{2L}}$, $\boldsymbol{\lambda_1 = 2L}$ f_1 is called the fundamental frequency or the first harmonics or zero overtone. And

λ = wavelength



$\mathbf{F_2 = \frac{V}{L}}$, $\boldsymbol{\lambda_2 = L}$

$\mathbf{F_2}$ is the frequency of the second harmonics or first overtone.

We also have the one for 3,4,5,…..e.t.c That is

F_3 = third harmonics or second overtone.

F_4 = forth harmonics or third overtone. E.t.c

Overtone is usually harmonics minus (-) one (1).

Generally, $\mathbf{F_n = nF_1}$ **where** $\mathbf{f_1 = \frac{V}{2L}}$

$\boldsymbol{and\ \ \lambda_n = \frac{2L}{n}}$ (n is the number of harmonics)

L is the length of the string.

$\mathbf{F_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}}$ **where** μ is the mass per unit length, $\frac{m}{L}$ of the string or **linear (mass) density** and T is tension in the string.

$\mathbf{V = \sqrt{\frac{T}{\mu}}}$ v is the velocity

NOTE: the two formula above is used when the question involves tension.

EXAMPLE 1: A string on a violin has a fundamental frequency of 262Hz. Calculate the frequency of the next two harmonics of the string.

SOLUTION: F_1 = 262Hz,

$F_n = nF_1$

$F_2 = 2 \times 262$ = 524Hz

Again, $F_n = nF_1$

$F_3 = 3 \times 262 = 786$Hz

EXAMPLE 2 : A high E string on a guitar measures 64cm in length. If its fundamental frequency is 330Hz, find the speed of the wave on the spring.

SOLUTION: L = 64cm = 0.64m, f_1 = 330Hz, V=?

$\lambda_n = \frac{2L}{n}$, n = 1

$\lambda_1 = \frac{2 \times 0.64}{1}$ = 1.28m

$v = f\lambda = 330 \times 1.28 = 422.4$m/s

EXAMPLE 3: A rope with a length of 2m is stretched between two supports with a tension that makes the speed of transverse waves 40m/s. Find the wavelength and frequency of : (a) the zero overtone (b) the first overtone (c) the third overtone.

SOLUTION: L = 2m, V = 40m/s

(a) λ_1 = ? , f_1 = ?

$\lambda_n = \frac{2L}{n}, \lambda_1 = \frac{2 \times 2}{1}$ = 4m

$F_1 = \frac{V}{2L} = \frac{40}{2 \times L}$ = 10Hz

(b) λ_2 = ? , f_2 = ?

$\lambda_n = \frac{2L}{n}, \lambda_2 = \frac{2 \times 2}{2}$ = 2m

$F_n = nF_1$, , $F_2 = 2 \times 10 = 20$Hz

(c) in a similar way.

$\lambda_n = \frac{2L}{n}, \lambda_3 = \frac{2 \times 2}{3}$ = 1.33m

$F_n = nF_1$, $F_3 = 3 \times 10 = 30$Hz

Now do this. Do not forget to convert.

EXAMPLE 4: calculate the frequency of the first overtone of a stretched string of length 60cm, if the velocity of sound produced is 330m/s. **answer : 550Hz**

EXAMPLE 5: will a standing wave be formed in a 4m length stretched string that transmits waves at a speed of 12m/s if it is driven at a frequency of (a) 15Hz or (b) 20Hz ? Give reasons for your answers

SOLUTION : L = 4m, V = 12m/s

(a) f = 15Hz, but $F_1 = \frac{V}{2L} = \frac{12}{2 \times 4}$ = 1.5Hz

$F_n = nF_1$, $15 = n \times 1.5$, n =15/1.5 = 10Hz

Yes, because 15Hz is the tenth harmonics

(b) f =20Hz, $F_n = nF_1$, $20 = n \times 1.5$, n =13.33

No, because 20Hz is not a harmonic(it gives a decimal number and not a whole number).

EXAMPLE 6: A string of mass 8g and length 1mis fixed at both ends. If the string is stretched by a load of 1.92kg and then released, find the fundamental frequency of the stationary wave produced.(g = 9.8m/s^2)

SOL : m = 8g = 0.008kg, L = 1m, m = 1.92kg

$\mu = \frac{m}{L} = \frac{0.008}{1}$ = 0.008kg/m

Tension, $T = mg = 1.92 \times 9.8 = 18.816$N

$F_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 1}\sqrt{\frac{18.816}{0.008}}$ = 24.2Hz

EXAMPLE 7 : A violin string with length 5m between fixed point has a linear mass density of 40g/m and a fundamental frequency of 20Hz. (a) calculate the tension in the string (b) calculate the frequency and wavelength of the second harmonic (c) calculate the frequency and wavelength of the second overtone.

SOLUTION : L = 5m, f_1 = 20Hz,

μ = 40g/m = 40/1000 = 0.04kg/m

(a) $F_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$, $20 = \frac{1}{2\times 5}\sqrt{\frac{T}{0.04}}$

$2 = \frac{1}{10} \times \frac{\sqrt{T}}{\sqrt{0.04}}$, $2 = \frac{1}{10} \times \frac{\sqrt{T}}{0.2}$, $20 = \frac{\sqrt{T}}{2}$

cross multiply, $40 = \sqrt{T}$,

take square of both sides : $T = 40^2 = 1600N$

(b) $f_2 = 2f_1 = 2\times 20 = 40Hz$, $\lambda_2 = \frac{2L}{n}$, $\lambda_1 = \frac{2\times 5}{2} = 5m$

(c) Hint : $f_3 = 3f_1$, $\lambda_n = \frac{2L}{n}$, **ANS: 60Hz, 3.3m**

EXAMPLE 8: Middle C string of a guitar has a fundamental frequency of 200Hz, and the first A string above the middle C string has a fundamental frequency of 350Hz. If the strings linear densities are equal, but the length of A string is only 64 percent of the length of C string, what is the ratio of their tensions? In other words, find $T_A:T_C$. where T_A and T_C are the tensions in string A and C.

SOLUTION: $f_{1C} = 200Hz$, $f_{1A} = 350Hz$, $\mu_{1C} = \mu$,

$\mu_{1A} = \mu$, $L_C = L$, $L_A = \frac{64L}{100} = 0.64L$

$F_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, make T subject of formula,

$T = 4\mu F_1^2L^2$, similarly, $T_C = 4\mu_C F_{1C}^2L_C^2$ then

$$\frac{T_A}{T_C} = \frac{4\mu_A F_{1A}^2L_A^2}{4\mu_C F_{1C}^2L_C^2}$$ substitute the values

$$\frac{T_A}{T_C} = \frac{\cancel{4}\times\cancel{\mu}\times 350^2\times(0.64\cancel{L})^2}{\cancel{4}\times\cancel{\mu}\times 200^2\times\cancel{L}^2}$$

$$\frac{T_A}{T_C} = \frac{350^2\times(0.64)^2}{200^2} = 1.254$$

LONGITUDINAL STATIONARY WAVES AND NORMAL MODES

When longitudinal waves propagates in a fluid in a pipe with infinite length, the waves are reflected from the ends in the same way transverse waves on a string are reflected at the ends. The superposition of waves traveling in opposite directions again forms a standing wave.

RESONANCE

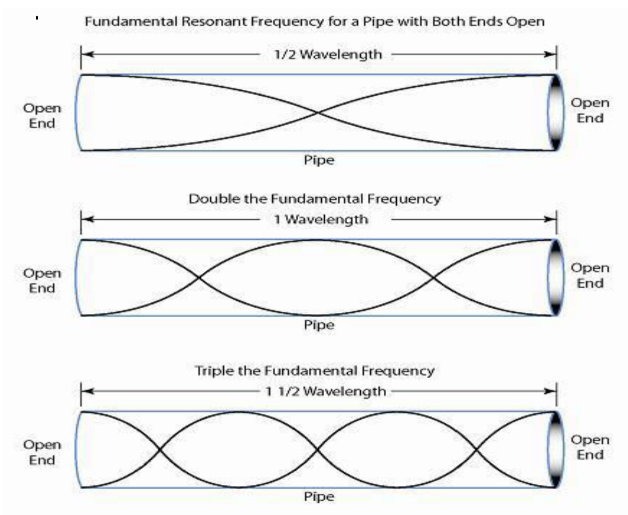
This is a condition in which a vibrating system vibrates with maximum amplitude to an alternating driving force. It exists when the frequency of the driving force coincides with the natural undamped oscillating frequency of the system. It also occurs when a periodic varying force is applied to a system with many normal modes.

RESONANCE IN PIPES

Any end of a pipe if open will contain column of air that when set into vibration will produce series of normal modes. The **displacement antinode(pressure node) and displacement node (pressure antinode)**, **refer to points with** maximum and zero displacement respectively.

Pipes 0pened at both ends

When air is blown into it from the button end, the column of air in the pipe is set into vibration and there is a series of possible normal modes. A stationary wave is set up in the air pipes, and as the two ends of the pipe are open, there must be displacement antinodes.



The first diagram shows the fundamental frequency, f_1 , or the zero overtone.

The second diagram shows the frequency of the second harmonics, f_1 , or first overtone.

The third diagram shows the frequency of the third harmonics, f_1 , or second overtone.

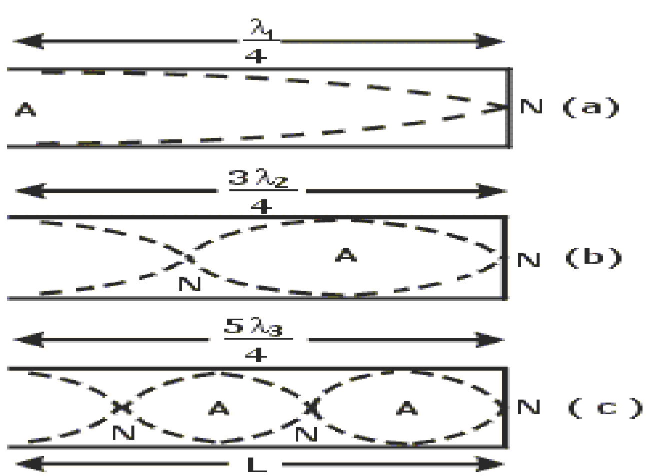
Generally, $F_n = nF_1$ where $f_1 = \frac{V}{2L}$

$and \quad \lambda_n = \frac{2L}{n}$ (n is the number of harmonics)

L is the length of the pipe.(It's similar to strings even the calculations are the same)

<u>Pipes Closed At One End</u>

In this case, the open end is a displacement node(pressure antinode).



The first diagram shows the first harmonics, f_1 , or zero overtone.

The third diagram shows the second harmonics, f_3 , or first overtone.

The first diagram shows the first harmonics, f_1 , or zero overtone.

The third diagram shows the fifth harmonics, f_5, or second overtone.

Generally, $F_n = nF_1$ where $f_1 = \frac{V}{4L}$

$and \quad \lambda_n = \frac{4L}{n}$ (n is the number of harmonics)

L is the length of the pipe.

Note : for pipes closed at one end, only odd harmonics are possible. i.e 1,3,5,............

EXAMPLE 9: On a day when the speed of sound was 345m/s, the fundamental frequency of a pipe closed at one end is 220Hz. (a) how long is the closed pipe ? (b) the second overtone of this pipe has the wavelength as the third harmonics of an open pipe, how long is the open pipe ? **SOLUTION :** V = 345m/s, f_1 = 220Hz

(a)$V = f_1 \lambda_1$, $345 = 220 \times \lambda_1$, λ=345/220=1.57

For pipes closed at one end, $\boldsymbol{\lambda_n} = \frac{4L}{n}$

$\lambda_1 = \frac{4L}{1}$, $\lambda_1 = 4L$,thus, $L = \lambda_1 /4$ = 1.57/4 = 0.39

(b) open pipe, $\lambda_3 = \lambda_3$ of closed pipe

$\frac{4L}{5} = \frac{2L_c}{3}$ put L=0.39cross multiply

4.68 = $10L_c$, L_c = 4.68/10 = 0.47m

EXAMPLE 10 : The first three natural frequencies of an organ pipe are 126Hz, 378Hz, and 630Hz. (a) is the pipe an open or closed pipe? (b) taking the speed of sound in air to be 340m/s, find the length of the pipe.

SOLUTION: $F_1 = 126$, $\mathbf{F_n = nF_1}$

(a) $F_2 = nf_1$, $378 = n \times 126$, $n = 378/126 = 3$

$F_3 = nf_1$, $630 = n \times 126$, $n = 630/126 = 5$

Since we have odd harmonics, it's a closed pipe.

(b) v = 340m/s; $v = f_1 \lambda_1$;

$340 = 126 \times \lambda_1$, $\lambda_1 = 340/126 = 2.698$m

$\lambda_1 = \frac{4L}{1}$; $\lambda_1 = 4L$; $2.698 = 4L$; L = 0.675m

EXAMPLE 11 : An open organ pipe has a length of 0.75m. What would be the length of a pipe closed at one end whose third harmonics is the same as the fundamental frequency of the open pipe?

SOLUTION: do it yourself. **Answer :1.1m**

EXAMPLE 12: Standing sound waves are produced in a pipe that is 0.8m long, open at one end, and closed at the other. For the fundamental, first overtone and fifth harmonics, where along the pipe(measured from the closed end) are: (a) the displacement antinodes (b) the pressure antinodes.

SOLUTION: L = 0.8m, (it's a pipe open at one end).

(a) using $\lambda_n = \frac{4L}{n}$; $\lambda_1 = \frac{4 \times 0.8}{1}$ = 3.2m

Displacement antinodes, x,

$X = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}$ ....(antinode formula)

$X = \frac{3.2}{4}, 3\frac{3.2}{4}, 5\frac{3.2}{4}$ ....

X = 0.8m, 2.4m, 4m…… the answer is only 0.8m since we are looking for only fundamental (first)

first overtone; $\lambda_n = \frac{4L}{n}$; $\lambda_3 = \frac{4 \times 0.8}{3}$ = 1.067m

$X = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}$ ....(antinode formula)

$X = \frac{1.067}{4}, 3\frac{1.067}{4}, 5\frac{1.067}{4}$ ....

X = 0.267m, 0.08m, 0.053m…. the answer is

0.267m, 0.08m, (Only the first two) since we are liking for first overtone.

fifth harmonics, $\lambda_n = \frac{4L}{n}$; $\lambda_5 = \frac{4 \times 0.8}{5}$ = 0.64m

$X = \frac{\lambda}{4}, 3\frac{\lambda}{4}, 5\frac{\lambda}{4}$ ....(antinode formula)

$X = \frac{0.64}{4}, 3\frac{0.64}{4}, 5\frac{0.64}{4}$ ....

X = 0.16m, 0.48m, 0.8m. that's the answer since we are looking for the first three.

(b) the pressure antinodes is same as the displacement node

For fundamental above, we found λ_1 = 3.2m

$X = 0, \frac{\lambda}{2}, \lambda, 3\frac{\lambda}{2}$ ....(node formula)

$X = 0, \frac{3.2}{2}, 3.2, 3\frac{3.2}{2}$ ....

X = 0m, 1.6m,3.2m……again we pick only 0m

from first overtone above, λ_3 = 1.063m

substituting into the node formula and picking the first two; **answer : 0m, 0.53m**

for the fifth harmonics above, λ_5 = 0.64m; substituting into the node formula picking the first three; **answer : 0m, 0.32m, 0.64m**

PIPES CLOSED AT BOTH ENDS

It uses the same **frequency and wavelength formulae as** strings. The only difference is just the diagram. The diagram in each case is like drawing a closed pipe and inserting the corresponding case for a string inside.

END CORRECTION

The air at the open end of a pipe is free to move and hence the vibrations at this end extends a little into the air outside the pipe. The displacement antinodes of the stationary wave due to any note is thus a distance C from the open end in practices.

CASE 1: <u>pipes closed at one end</u>

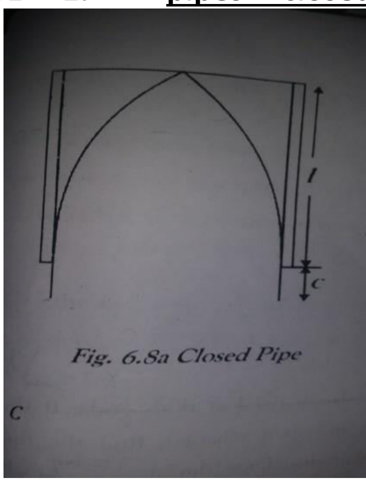


Fig. 6.8a Closed Pipe

Here, $f_n = \frac{nV}{4(L+C)}$ where V is velocity an C is end correction

EXAMPLE 13: A uniform pipe of length 16cm has a fundamental frequency of 500Hz when closed at one end. If the displacement antinodes occurs at a distance of 1cm from the open end, calculate the velocity of sound.

SOLUTION: L = 16cm = 0.16m,

C = 1cm = 0.001m , F_1 = 500Hz

$f_n = \frac{nV}{4(L+C)}$; $500 = \frac{1 \times V}{4(0.16+0.01)}$

V = 500 × 4(0.17); V = 340m/s

CASE 2 : <u>pipes opened at both ends</u>

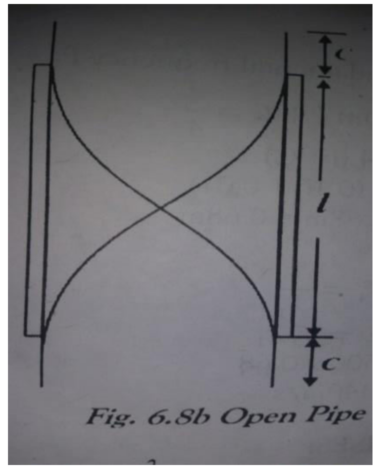


Fig. 6.8b Open Pipe

$f_n = \frac{nV}{2(L+2C)}$

EXAMPLE 14: A pipe of length 57cm has a fundamental frequency of 224Hz when open at both ends. If the displacement antinodes occur at a distance of 10cm from the open ends. Calculate the velocity.

Hints: C = 10cm,(convert) **answer : 345m/s**

EXAMPLE 15: An open pipe pipe 30cm long

and a closed pipe 23cmlong, both of the same diameters, have the same frequency when each of them is sounding its first overtone. What is the end correction of these pipes?

SOLUTION: L_o = 30cm

L_c = 23cm , $f_o = f_2$ and $f_c = f_3$

$f_3 = f_2$ (same) then,

$$\frac{2\cancel{V}}{2(L_o + 2C)} = \frac{3\cancel{V}}{4(L_c + C)}$$

$\frac{2}{2(30+2C)} = \frac{3}{4(23+C)}$ cross multiply

6(30 + 2C) = 8(23 + C)

180 + 12C = 184 + 8C

4 = 4C; C = 4/4 = 1cm.

CHAPTER 7

OPTICS

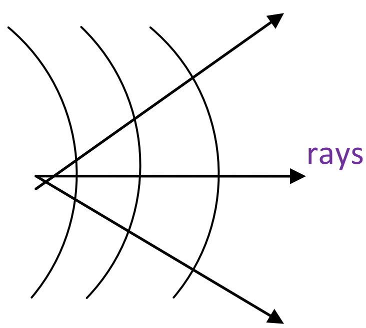
Optics is the study if light and vision. The vision of all living creature require light. Optics is divided into two broad areas:

(1) physical optics : it is the study of wave behavior of light, that is ; the study of those properties of light that rays cannot adequately explain.

(2) geometric optics : it involves the use of straight lines and angles in the investigation of optical properties such as reflection and refraction.

RAY

It is a line drawn perpendicular to a series of wave front and pointing in the direction of propagation.



NOTE: (1) a ray points in the direction of the energy flow of wave.

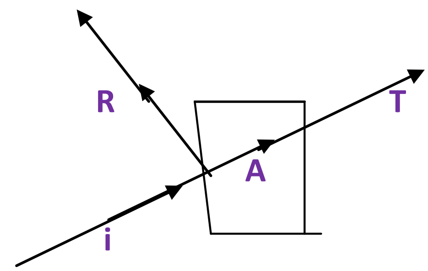
(2) rays are radii of spherical wave fronts

(3) they are straight lines perpendicular to the wave fronts.

(4) at a boundary surface between two or more material medium, the speed changes but the ray segment doesn't .

(5) when a ray is incident, i on a translucent material it is generally, partly reflected, R, partly transmitted, T and partly absorbed, A, such that

$$i = R + T + A$$



BEAM

A beam is a collection of rays.

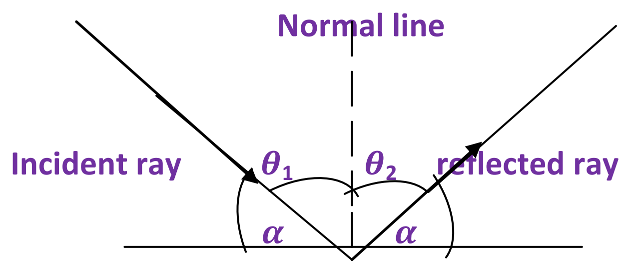
REFLECTION

See definition in chapter 4. It is divides into:

1. Regular Or Specular Reflection : it is formed when rays are reflected from smooth surface.

2. Irregular Or diffused reflection: it is formed when light rays is incidented on a non-parallel directions. Diffused reflection is the reason behind our ability to see illuminated objects like the moon.

LAWS OF REFLECTION



α is the glancing angle

1. the angle of incidence is equal to the

angle of reflection (i.e $\theta_1 = \theta_2$)

2. the normal line(perpendicular to the surface), the incident and the reflected ray at the point of incidence all lie on the same plane.

EXAMPLE 1 : the angle of incidence of light ray on a mirrored surface is 35° . what is the angle between the incident and reflected ray

SOLUTION: using the diagram above,

$\boldsymbol{\theta_1} = \boldsymbol{\theta_2} = 35^\circ$, the angle = 35 + 35 = 70°

EXAMPLE 2: a beam of light is incidented on a plane mirror at an angle 32° relative to the normal. What is the angle between the reflected ray and the surface?

SOLUTION: from the diagram above, $\boldsymbol{\theta_1} = \mathbf{32}$

$\alpha + \theta_1 = 90^0$, $\alpha = 90^0 - \theta_1$ 90 – 32 = 58°

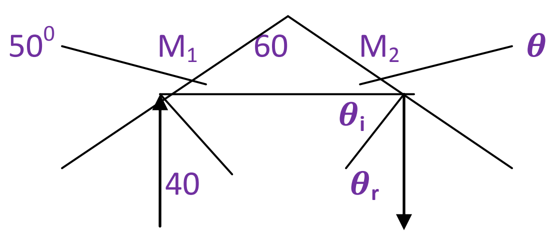
NOTE: when two mirrors are inclined at an angle $\boldsymbol{\theta}$ **to each** other , the number of images , **n** , formed is given as

$$\mathbf{n} = \frac{360}{\theta} - 1$$

lets consider another case:

EXAMPLE 3: Two plane upright mirrors touch along one edge where their planes make an angle of 60^0. If a beam of light is directed into one of the mirrors at an angle of incidence of 40° and is reflected into the other mirror, what will be the angle of reflection of the beam from the second mirror?

Solution:



From the diagram(triangle) $\theta + 50 + 60 = 180^0$

Thus $\theta = 180 - 50 - 60 = 70^0$

$\theta + \theta_i = 90^0$ (right angle)

Thus , $70 + \theta_i = 90^0$

$\theta_i = 20^0$ (incident angle)

But $\theta_I = \theta_r = 20^0$

EXAMPLE 4: two plane mirror m_1 and m_2 are placed together with edges touching each other at angle $\boldsymbol{\alpha}$**.** If a light ray is incidented on mirror m_1 at an angle 35^0 , what is the angle of reflection from the second mirror m_2.

Answer : 25⁰

EXAMPLE 5: the edges of two plane mirror m_1 and m_2 are inclined at an angle $\boldsymbol{\alpha}$ **to each other** . if a ray of light is incidented 35^0 on the first mirror, m_1 , for what value of $\boldsymbol{\alpha}$ will the angle of reflection from mirror m_2 equal to the angle of incidence in mirror m_1 ?

Answer : 70⁰

REFRACTION

See definition in chapter 4. Note: *when light is refracted, its velocity and wavelength are changed but its frequency remains constant.*

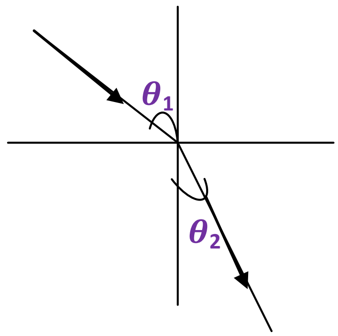
LAWS OF REFRACTION

1. the incident ray , the refracted ray and the normal to the surface, at the point of incidence all lie on the same plane.

2. For a monochromatic light and for a given pair of materials (1 and 2) on opposite sides of interface, the ratio of sine of angle of incidence to the sine of angle of refraction measured from the normal to the surface is equal the inverse of two indices of refraction (**snell's law).** Thus,

$$n = \frac{\sin \boldsymbol{\theta_1}}{\sin \boldsymbol{\theta_2}} \qquad \text{.............................. (1)}$$

refractive index ,n is the measure of the ability of a substance to bend light



Some formulae

$V = f\lambda$, since frequency is constant,

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

$${}_1n_2 = \frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

Equation 1 above is used when one refractive index(note plural is indices) is given.

EXAMPLE 6: a beam of light traveling in air is incident on a slab transparent material. The incident beam and the refracted beam make angle 40^0 and 30^0 to the normal respectively. Find the speed of light in the transparent medium.

SOLUTION: $n = \frac{sin40}{\sin 30} = 1.2855$. also recall

${}_1n_2 = \frac{V_1}{V_2}$, $V_1 = 3\times 10^8$ m/s (speed in air)

$1.2855 = \frac{3\times 10^8}{V_2}$, $V_2 = \frac{3\times 10^8}{1.2855} = 2.33\times 10^8$m/s

EXAMPLE 7: light passes from air to water.

If the angle of refraction is 20^0, what is the angle of incidence? $n_w = 1.33$

SOLUTION: hint : use the formula before the diagram above. **Answer = 27^0**

EXAMPLE 8: A light ray from a Helium-neon Lase has a wavelength of 632.8nm and travels from air to crown glass (n of crown glass, n_g = 1.52)

SOLUTION: $\lambda_1 = 632.8\text{nm} = 632.8 \times 10^{-9}$m

(a) $V_1 = 3\times 10^8$m/s ,

$$f = \frac{V_1}{\lambda_1} = \frac{3\times 10^8}{632.8 \times 10^{-9}} = 4.74 \times 10^{14}\text{Hz}$$

(b) $f = 4.74 \times 10^{14}$Hz because it doesn't change.

(c) $n = \frac{\lambda_1}{\lambda_2}$, $1.52 = \frac{632.8 \times 10^{-9}}{\lambda_2}$

$$\lambda_2 = \frac{632.8 \times 10^{-9}}{1.52} = 416 \times 10^{-9}\text{m}$$

If you were asked to leave your answer in nm,

You simply multiply the answer above by 10^9

Note : When two refractive indices are given

Use the formula: $n_1 \sin\theta_1 = n_2 \sin\theta_2$

EXAMPLE 9: a ray of light is incidented on a water-glass boundary at 30^0 i.e **$\theta_w = 30^0$** and if $n_w = 4/3$ while $n_g = 3/2$, find θ_g

SOLUTION: using **$n_1 \sin\theta_1 = n_2 \sin\theta_2$**

$\frac{4}{3}\sin 30 = \frac{3}{2}\sin\theta_2$ cross multiply

$8\sin 30 = 9\sin\theta_2$, $4 = 9\sin\theta_2$

Divide both sides by 9 , $\sin\theta_2 = \frac{4}{9}$

Take $\sin^{-1}$ of both sides, $\theta_2 = \sin^{-1}(\frac{4}{9}) = 26^0$

EXAMPLE 10: the speed of light in water is 75% of the speed of light in a vacuum. What is the value of its refractive index?

SOLUTION: let $V_{vacuum} = V$,

$V_{water} = \frac{75}{100}V = 0.75V$

But $n = \frac{V_1}{V_2} = \frac{\cancel{V}}{0.75\cancel{V}} = 1.33$

EXAMPLE 11: a glass of thickness 0.6cm has a refractive index of 1.55, calculate the time taken for the ray of light to pass through it.

SOLUTION: d = 0.6cm = 0.006m, n = 1.55, t =?

Note : the ray is coming from air. Thus, speed of the ray in air $V_1 = 3 \times 10^8$. velocity of the ray in glass V_2

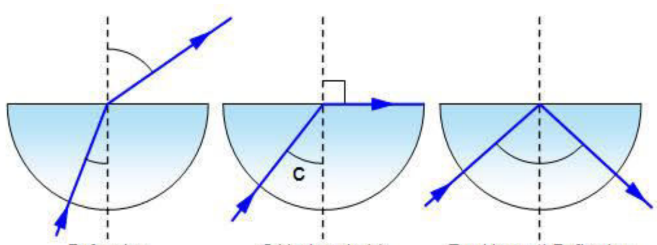
$n = \frac{V_1}{V_2}$, $1.55 = \frac{3 \times 10^8}{V_2}$,

Cross multiply, V_2 = 193548387.1m/s

$V_2 = \frac{d}{t}$, $193548387.1 = \frac{0.006}{t}$

$t = 3.1 \times 10^{-11}$ sec

TOTAL INTERNAL REFLECTION



Refraction Critical angle (c) Total internal Reflection

Critical angle(c) : it is the angle of incidence in the denser medium when the angle of refraction in the less dense medium is 90^0.

Total internal reflection: it is the reflection of the incident ray of light at the interface between the medium of incidence and another medium of lower refractive index when the angle of incidence in the denser medium exceeds the critical angle.

Conditions for total internal reflection to occur

1. light must be traveling from an optically more dense to an optically dense medium.

2. the angle of incidence in the denser medium must be greater than the critical angle.

3. Snell's law becomes invalid (does not hold)

From the middle diagram above, $\theta_1 = c$,

We know that the refractive index of air ,

$n_1 = 1$, $n_2 = n$, substituting into

$n_1 \sin \theta_1 = n_2 \sin \theta_2$, $\theta_2 = 90^0$, we have

$n = \frac{1}{\sin c}$ *we use this when one n is given*

*where **c** is the critical angle*

APPLICATIONS OF TOTAL INTERNAL REFLECTION

1. mirages 2. Fiber optics

3. field of view of a fish under water

EXAMPLE 12: what is the refracting index of a material if the critical angle of light passing from the material to air is 24.4^0 ?

SOLUTION: n = ? , c =24.4

$n = \frac{1}{\sin c} = \frac{1}{\sin 24.4} = 2.42$

EXAMPLE 13: an optical fiber is made of clear plastic with index of refraction n = 1.5. what is the minimum angle of incidence so that total internal reflection can occur?

SOLUTION: the minimum angle of incidence for total internal reflection can occur is the critical angle **c** . using $n = \frac{1}{\sin c}$

$1.5 = \frac{1}{\sin c}$, 1.5sin c = 1,

divide both sides by 1.5 , $\sin c = \frac{1}{1.5}$,

take $\sin^{-1}$ of both sides $c = 41.8^o$

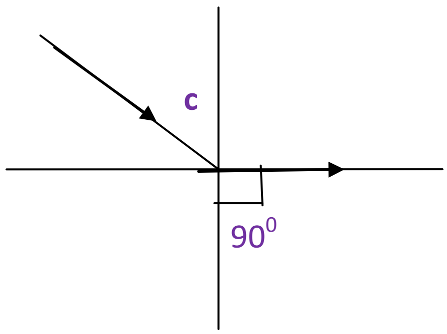
when two refractive indices are given, use

$n_1 \sin \theta_1 = n_2 \sin \theta_2$ where the angle in the second medium, $\theta_2 = 90^0$

EXAMPLE 14: what is the critical angle of light passing from a material of index of refraction 1.54 to a material of index of refraction

n = 1.33?

SOLUTION: $n_1 = 1.54$, $n_2 = 1.33$, c = ?

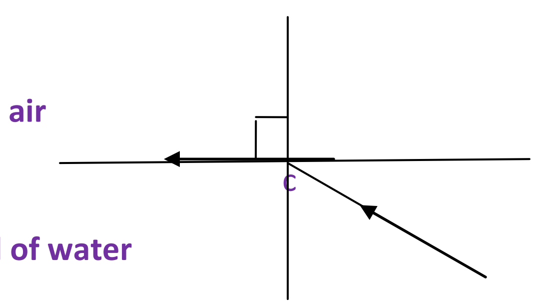


$n_1 \sin \theta_1 = n_2 \sin \theta_2$, 1.54sin c = 1.33sin 90

$\sin c = \frac{1.33 \sin 90}{1.54} = 0.864$, $c = \sin^{-1}(0.864) = 59.7^0$

EXAMPLE 15: a swimmer is 1.5m underneath a pond of water. At what angle must the swimmer shine the beam of light towards the surface in other for a person on a distant bank to see it, $n_w = 1.33$, $n_{air} = 1$

SOLUTION: notice that the swimmer is inside the pool of water . thus, the first medium is water and the second medium is air.



$n_1 \sin \theta_1 = n_2 \sin \theta_2$, 1.33sin c = 1sin 90

complete it .. **answer: c = 48.8^0**

<u>when three refractive indices are involved</u>

let **1 , 2** and **3** be three media(plural of medium)

${}_1n_2$ = refractive index from medium **1** to **2**

${}_1n_3$ = refractive index from medium **1** to **3**

${}_cn_b$ = refractive index from medium **c** to **b**

And so on……….

Let **a , g** and **w** represent air, glass and water respectively. Thus, ${}_an_e$ that is refractive index from **air** to medium **e.** note that :

${}_an_e = n_e$ and

${}_an_b = n_b$ and so on ………

generally , $\mathbf{{}_1n_2} = \frac{\mathbf{{}_an_2}}{\mathbf{{}_an_1}}$

EXAMPLE : 16 given the refractive index of air to glass ${}_an_g = 3/2$, of air to water

${}_an_w = 4/3$. what is the refractive index of water to glass?

SOLUTION: ${}_wn_g$ = ? ${}_wn_g = \frac{{}_an_g}{{}_an_w} = \frac{3/2}{4/3}$

using fraction we have $\frac{3 \times 3}{2 \times 4} = \frac{9}{8}$

CHAPTER 8

REFLECTION AT PLANE AND CURVE SURFACES

Mirrors are smooth reflecting surfaces usually made of polished metals or glass that have been coated with metallic substances e.g compound of tin, silver, mercury e.t.c . Even an uncoated piece of glass can act as a mirror

TYPES OF MIRRORS

1. <u>Plane mirrors :</u> these are mirrors with flat surface .

<u>Images Formed By A Plane Mirror</u>

The image formed by a plane mirror is:

i. *same size as the object*

ii. virtual (cannot be focused on a screen)

iii. literally inverted

iv. as far behind the mirror as the object as in the front of the mirror

v. is upright and unmagnified (M = 1)

vi. has an infinite focus.

NOTE: *a **virtual image** is one that cannot be caught on a screen. It is one through which the rays of do not actually pass through while a **real image** is one that can be caught on a screen. Light rays actually pass through a real image.*

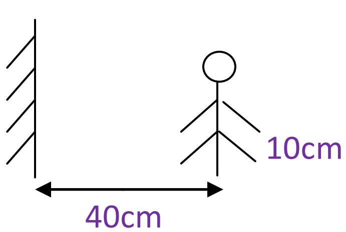
NOTE: *the magnification of an object in the front of a plane mirror is **ALWAYS EQUAL TO one(1).*** **U** and **V** are the object and image distance respectively, and **V = U .** $h_i = h_o$ i.e height of image is equal to height of object.

EXAMPLE 1: an object 10cm tall is placed 40cm from a plane mirror. Find :

(a) the distance from the object to the image

(b) the height of the image.

SOLITION:



a. u = 40cm, since V = U,

V = 40 cm (behind the mirror) ,

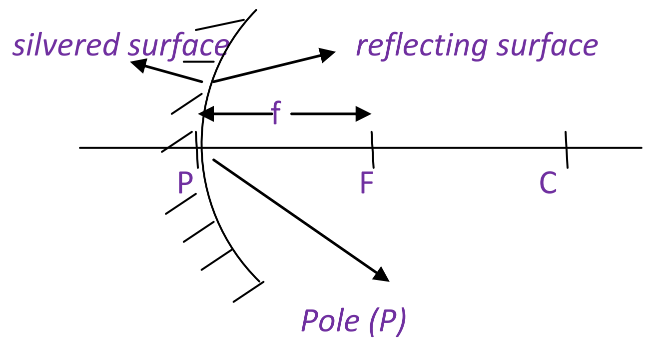
Thus, distance from U to V = 40 + 40 = 80cm

b. h_o = 10 cm, since $h_i = h_o$, then h_i = 10cm

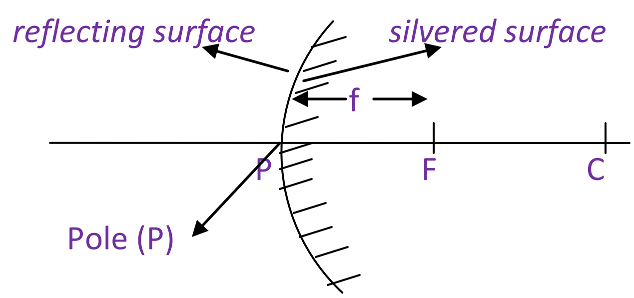
2. <u>spherical mirror :</u> it is a reflecting surface that has a spherical geometry.

<u>Types of spherical mirror</u>

i. ***concave mirrors or converging mirrors :*** it is a spherical mirror whose inside surface is the reflecting part and its outside surface is the silvered part.



ii. ***convex mirror or diverging mirror :*** it is a spherical mirror whose outside surface is the reflecting part and its inside surface is the silvered part.



<u>Parts of a spherical mirror</u>

i. aperture (A) :it is the width of the mirror

ii. ***pole (P) :*** it is the centre of the reflecting surface of the mirror.

iii. ***the centre of curvature (C) :*** it is the centre of the sphere where the mirror forms a part.

iv. ***the radius of curvature (r) :*** it is the distance **(pC).** It is the radius of the sphere of which the mirror forms a part.

v. the principal axis : it is the line (pC) from the pole to the centre of curvature.

vi. ***the principal focus (F) :*** it is a point on the principal axis where incident rays parallel and close to the principal axis converge (in a concave mirror) or from which they appear to diverge (in a convex mirror) after reflection.

NOTE: *the principal focus of a convex mirror is virtual because light ray do not actually pass through it.*

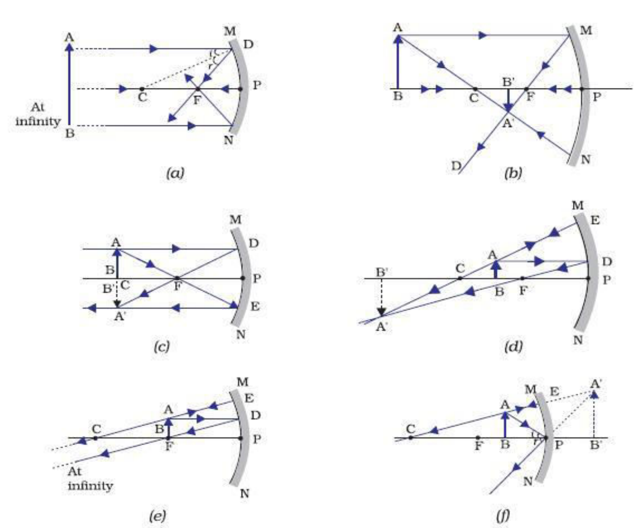
From the diagrams above :

f = focal length. r =radius of curvature;

$f = \frac{r}{2}$

<u>Images formed by spherical mirrors</u>

1. ***concave mirrors :*** the images formed by a concave mirrors depends on the position of the object. see ray diagram below



ii. convex mirrors : the images formed by a convex mirror is **always virtual, erect and diminished (** i.e **VED)** . *SPHERICAL MIRRORS IS ALSO CALLED CURVED MIRRORS.*

FORMULAE USED IN SPHERICAL MIRRORS

$f = \frac{r}{2}$, $\frac{1}{f} = \frac{1}{U} + \frac{1}{V}$ from the previous formula if we make f, u, v subject of the formula we will have: $f = \frac{vu}{u+v}$, $u = \frac{fv}{v-f}$, $v = \frac{fu}{u-f}$

where u and v are object and image distance respectively.

NOTE: (1) Generally, *v is taken to be negative if the image is erect/ virtual/upright.*

(2) *for convex mirror v and f are negative.*

(3) *all real images are inverted i.e v is positive*

(4) *for concave mirror* f *is positive but for a convex mirror* f *is negative.*

MAGNIFICATION (M) OF CURVED MIRRORS

When $|M| > 1$, *the image is magnified/enlarged.*

When $|M| < 1$, *the image is reduced/diminished*

when $|M| = 1$, the image Is true(real)

$$M = \frac{v}{u} = \frac{h_i}{h_o}$$

where h_i and h_o are the height of image and object respectively.

EXAMPLE 2: describe the image of a candle flame located 40cm from a concave spherical mirror of radius 64cm.

SOLUTION: u= 40cm, r = 64cm, $f = \frac{r}{2} = \frac{64}{2}$=32cm

$v = \frac{fu}{u-f} = \frac{32 \times 40}{40-32}$ = 162cm, $M = \frac{v}{u} = \frac{162}{40}$ = 4 since v is positive (162cm) , the image is **real** and **inverted.** Since M > **1,** then the image is **magnified/enlarged.** The answer is : *real, magnified and inverted.*

EXAMPLE 3 : an object 7cm high is placed 15cm from a convex spherical mirror of radius 45cm. describe its image, and give the value of the image distance V and magnification M.

SOLUTION: u = 15cm, r = 45cm

But $f = \frac{r}{2} = \frac{45}{2}$ = -22.5cm (convex mirror)

$v = \frac{fu}{u-f} = \frac{-22.5 \times 15}{15-(-22.5)}$ = -9cm, $M = \frac{v}{u} = \frac{9}{15} = 0.6$

thus, the image is virtual (v is negative) , upright and diminished(**M < 1**)

now, do the following questions :

EXAMPLE 4: what are the characteristics of an object placed 40cm from a convex mirror of radius 120cm?

Answers : v = -24cm , M = 0.6, image is :

Virtual, upright and diminished.

EXAMPLE 5: a convex mirror has a radius of curvature of 0.55m. calculate the position of the image of a man 10cm from the mirror.

Answer : v = -0.267m

EXAMPLE 6: a young lady look at her face in the front of a cosmetic concave mirror, the erect image formed is three times the face of the lady. What will the focal length of the mirror be , if she held the mirror 25cm from her face?

SOLUTION : f = ? , u = 25cm

M = 3 (i.e three times the face of the lady),

$M = \frac{v}{u}$, $3 = \frac{v}{25}$, v = -75cm (erect)

$f = \frac{vu}{u+v} = \frac{-75 \times 25}{-75+25}$ = 37.5cm

EXAMPLE 7: a mirror formed an erect image one-fifth the size of an object placed 15cm in front of it. If the mirror is spherical, what kind is it? And what is the radius of curvature?

SOLUTION: u = 15cm,

again, $M = \frac{1}{5}$ (one-fifth the size of an object…)

$M = \frac{v}{u}$, $\frac{1}{5} = \frac{v}{15}$, $v = \frac{15}{5}$ = -3cm (erect)

$f = \frac{vu}{u+v} = \frac{-3 \times 15}{-3+15} = \frac{-15}{4}$ cm

since f is negative, it is a convex mirror.

$f = \frac{r}{2}$, $r = 2f = 2 \times \frac{15}{4} = \frac{15}{2}$ cm

EXAMPLE 8: what is the focal length of a convex spherical mirror which produces an image one-sixth the size of an object located 12cm from the mirror.

SOLUTION: f = ? , $M = \frac{1}{6}$, u = 12cm

$M = \frac{v}{u}$, $\frac{1}{6} = \frac{v}{12}$, v = 12/6 = -2cm(convex)

EXAMPLE 9: if an object is 30cm in front of a convex mirror that has a focal length of 60cm, how far behind the mirror will the image appear to an observer ? how tall will the image be ?

SOLUTION: u = 30cm, f = -60cm

$v = \frac{fu}{u-f} = \frac{-60 \times 30}{30-(-60)}$ = -20cm

$M = \frac{v}{u} = \frac{h_i}{h_o}$

$\frac{20}{30} = \frac{h_i}{h_o}$ cross multiply

$20h_o = 30h_i$

$h_i = \frac{20h_o}{30} = \frac{2h_o}{3}$

Thus, the image will be $^2/_3$ (two-third) the height of the object.

EXAMPLE 10: a child looked at a reflecting Christmas tree that has a diameter of 9cmand sees an image of her face that is half the real size. How far is the child's face from the ball?

SOLUTION: d = 9cm, $M = \frac{1}{2}$

Note: the tree is **(like)** a convex mirror

$f = \frac{d}{4} = \frac{9}{4} = -2.25cm$

$M = \frac{v}{u}$

$\frac{1}{2} = \frac{v}{u}$, cross multiply , $v = \frac{-u}{2}$ cm (convex)

$\frac{1}{f} = \frac{1}{U} + \frac{1}{V}$, $\frac{1}{-2.25} = \frac{1}{u} + \frac{1}{-u/2}$

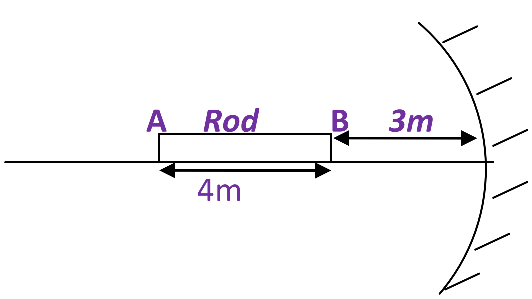
$\frac{1}{-2.25} = \frac{1}{u} - \frac{2}{u}$, $\frac{1}{-2.25} = \frac{1-2}{u}$

$\frac{1}{-2.25} = \frac{-1}{u}$, $u = 2.25cm$

Which is approximately equal to , u = 2.3cm

EXAMPLE 11 : A rod 4m long is placed along the principal axis of a concave mirror of focal length 2m. if the side nearer the mirror is 3m from it, find the length of the image.

SOLUTION: f = 2m .



From the diagram above, considering B, the end of the rod to be the object, then:

$V_B = \frac{u_B f}{u_B - f} = \frac{3 \times 2}{3 - 2} = 6m$

Considering A,

$U_A = 3 + 4 = 7m$

$V_A = \frac{u_A f}{u_A - f} = \frac{7 \times 2}{7 - 2} = 2.8m$

thus, the length of the image (is bigger value minus(-) smaller value)

length of image $= v_B - v_A = 6 - 2.8 = 2.3m$

EXAMPLE 12: A rod 10m long is placed along the principal axis of a convex mirror of focal

Length 4m, if the side nearer the mirror is 6m from it, find the length of the image.

Answer : **0.8m**

NO JESUS, NO SUCCESS,

Now , if you are not saved and you want to be forgiven of all your sins , say these words after me(say it from your heart): Lord Jesus, have mercy on me and forgive me of all my sins, even as I forgive everyone that has trespassed against me. I believe that you shed your blood and died for me and on the third day you resurrected that I may live. Lord Jesus , from this day, I accept you as my Lord and personal savior, in Jesus name, Amen. If you have just said those words, **Congratulations**, **welcome to the body of Christ.** Look for a bible believing church and start attending, study your bible and pray daily

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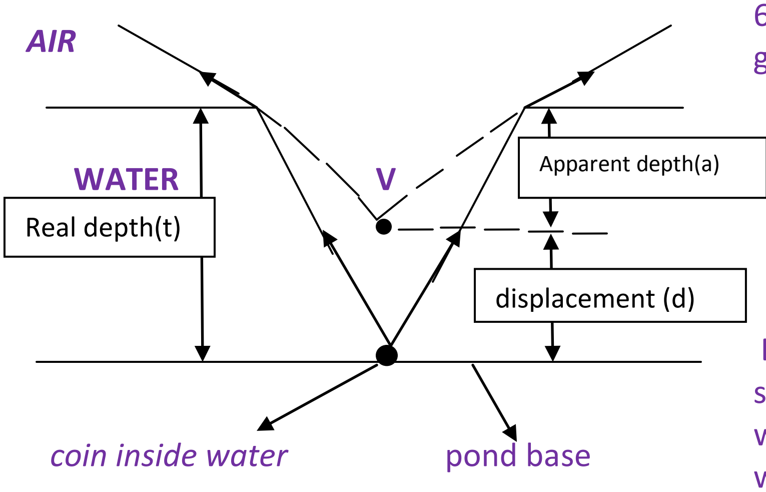
"If you cannot learn the way we teach then we will have to teach the way you learn"

IN GOD WE TRUST

CHAPTER 9

REFRACTION THROUGH PLANE SURFACES

Real And Apparent Depth: water in a pond appears to be only three-quarter of its real depth when viewed from above. A thick slab of glass appears to be only two-third of its real thickness when viewed vertically above.



it is obvious that rays from the coin at the bottom of water are refracted away when they leave the water and enter the eyes. They appear to be coming from a virtual, **V**, which is at an apparent depth, **a,** below the water, while the actual depth of the bottom, **t,** is referred to as the real depth. The refractive index will be given by :

$$\mathbf{n} = \frac{real\ depth\ (t)}{apparent\ depth\ (a)},\quad \mathbf{n} = \frac{t}{a},$$

from the diagram, a = t – d, thus

$$\mathbf{n} = \frac{t}{t-d}$$

also, making d the subject of the formula,

$$\mathbf{d = t}\left(1 - \frac{1}{n}\right),\ \textbf{d is displacement}$$

EXAMPLE 1 : A transparent rectangular block 5.0cm is placed on a black dot. The dot when viewed from above is seen 3cm from the top Of the block. Calculate the refractive index of the material of the block.

SOLUTION: t = 5cm (thickness or real dept)

a = 3cm, n = ?

$$n = \frac{t}{a} = \frac{5}{3}$$

EXAMPLE 2 : an object is placed at the bottom of a glass block 18cm thick, if it is displaced 6.1cm upwards, find the refractive index of the glass.

SOLUTION

t = 18cm, d = 6.1cm , n = ?

$$n = \frac{t}{t-d} = \frac{18}{18-6.1} = 1.5$$

EXAMPLE 3 : a microscope was focused on a scratch at the bottom of a jar. When a liquid was poured to a depth 8cm. The microscope was moved vertically upward though a distance 1.4cm to bring the scratch back into focus, find the refractive index of the liquid.

SOLUTION: t = 8cm , d = 1.4cm, n = ?

$$n = \frac{t}{t-d} = \frac{8}{8-1.4} = 1.2$$

EXAMPLE 4 : find the angle of incidence of an object placed at the bottom of a glass block 18cm thick displaced 4.4cm upward, if the angle of an observer vertically above is 45^o

SOLUTION: θ_1 = ?, t = 18cm, d = 4.4cm, θ_1 =45

$$_a n_g = \frac{t}{t-d} = \frac{18}{18-4.4} = 1.32$$

$$n = \frac{\sin\theta_1}{\sin\theta_2}$$

$$1.32 = \frac{\sin\theta_1}{\text{Sin } 45}$$ cross multiply

$\text{Sin }\theta_1 = 1.32\sin 45$

$\sin\theta_1 = 0.933$

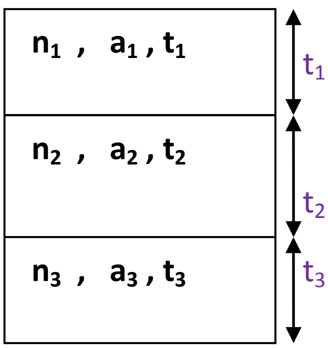
$\theta_1 = \sin^{-1}(0.933) = 68.97^0$

Now do this. It's the same way

EXAMPLE 5: an object at the bottom of a pool 20m deep was observed to be at 14cm position. Find the angles of incidence of the object, if the angle of an observer vertically above is 36^0

Answer = 57.1^0

APPARENT DEPTH WITH MORE THAN ONE MEDIA



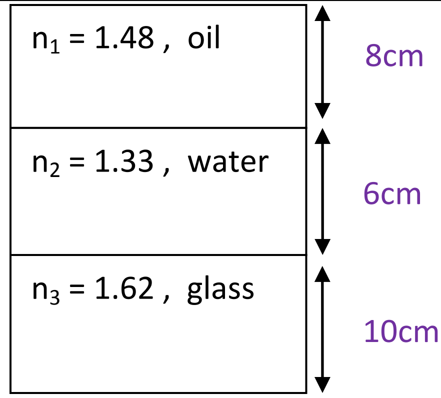
the total height, **$t_t = t_1 + t_2 + t_3$ ……..**

total displacement**, $d_t = d_1 + d_2 + d_3$ ……**

$$a_t = \frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}$$

EXAMPLE 6: consider a film of oil 8cm with refractive index n = 1.48 floating on top of water 6cm thick having refractive index n = 1.33 , on top of a glass block 10cm with refractive index n = 1.62, all contained in one medium. If an object is placed at the bottom of the glass block, calculate the apparent position of an object ***from above.***

SOLUTION:



$$a_t = \frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}$$

$$a_t = \frac{8}{1.48} + \frac{6}{1.33} + \frac{10}{1.62} = 16.09\text{cm}$$

*If you were told to look for the apparent position of the object **from below(i.e d_t)***

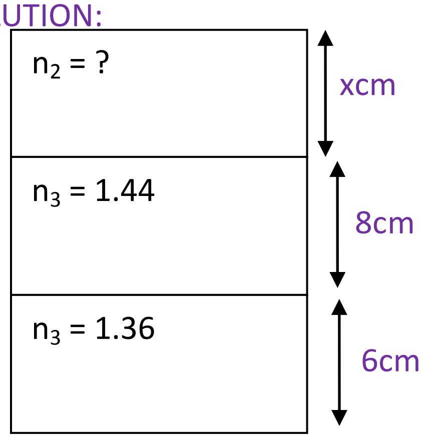
Then: $d_t = t_t - a_t$

But $t_t = t_1 + t_2 + t_3 = 8 + 6 + 10 = 24$cm

$d_t = t_t - a_t = 24 – 16.09 = 7.91$cm

EXAMPLE 7: consider water on top of flint glass of thickness 8cm, refractive index 1.44, resting on top of a crown glass 6cm thick and refractive index n = 1.36. if the displacement due to the combination is 6cm from below with total thickness being 18cm. compute the refractive index of water.

SOLUTION:



$d_t = 6$cm, $t_r = 18$cm,

But $t_t = t_1 + t_2 + t_3$

$18 = x + 8 + 6$, therefore, $x = t_1 = 4cm$

$a_t = t_t – d_t = 18 – 6 = 12cm$

$$a_t = \frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}$$

$$12 = \frac{4}{n_1} + \frac{8}{1.44} + \frac{6}{1.36}$$

$$\frac{4}{n_1} = 12 - \frac{8}{1.44} - \frac{6}{1.36}$$

$$\frac{4}{n_1} = 2.033$$

$2.033n_1 = 4$, $n_1 = 4/2.033 = 1.97$

EXAMPLE 8: If the thickness and refractive index of oil, water and glass together in a set up are respectively 4cm, 6cm, 5cm and 1.26, 1.33, n_g . Find the value of n_g if the apparent position of an object at the bottom is 12cm.

SOLUTION: $t_1 = 4cm$, $t_2 = 6cm$, $t_3 = 5cm$,

$n_1 = 1.26$, $n_2 = 1.33$, $n_3 = n_g$, $a_t = 12cm$

draw the diagram yourself

$t_t = t_1 + t_2 + t_3 = 4 + 6 + 5 = 15cm$

$$a_t = \frac{t_1}{n_1} + \frac{t_2}{n_2} + \frac{t_3}{n_3}$$

$$12 = \frac{4}{1.26} + \frac{6}{1.33} + \frac{5}{n_g}$$

$$\frac{5}{n_g} = 12 - \frac{4}{1.26} - \frac{6}{1.33}$$

$\frac{5}{n_g} = 4.314$, $n_g = 5/4.314 = 1.16$

EXAMPLE 9: Consider that block of flint and crown glasses are arranged on top of the other having thickness of 10cm and xcm with refractive index of 1.52 and 1.56 respectively. Calculate the thickness of the crown glass if the apparent position of an object from the bottom of the crown glass is 4cm. And also find the total thickness.

SOLUTION: $t_1 = 10cm$, $t_2 = xcm$, $n_1 = 1.52$

$n_2 = 1.56$, $d_t = 4cm$

$t_t = t_1 + t_2$, $t_t = x + 10$

$a_t = t_t - d_t = (x + 10) – 4 = x + 6$

$$a_t = \frac{t_1}{n_1} + \frac{t_2}{n_2}$$

$$x + 6 = \frac{10}{1.52} + \frac{x}{1.56}$$ collect like terms

$$\frac{x}{1} - \frac{x}{1.56} = \frac{10}{1.52} - \frac{6}{1}$$

$$\frac{1.56 - x}{1.56} = \frac{10 - 9.12}{1.52}$$

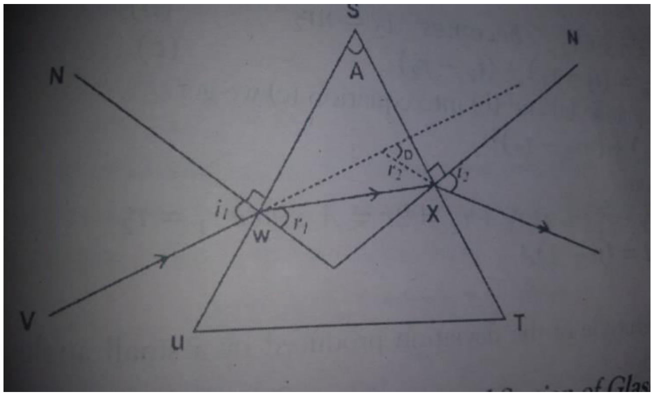
$$\frac{0.56x}{1.56} = \frac{0.88}{1.52}$$

$0.8512x = 1.3728$

$$x = t_2 = \frac{1.3728}{0.8512} = 1.6$$

REFRACTION THROUGH PRISM

Triangular Glass Prism



The prism deviates the incident ray through an angle known as angle of deviation(D). This is the angle between the incident ray and the emergent ray of the light passing through the prism.

$$\mathbf{A = r_1 + r_2}$$

$$\mathbf{d = i_1 + i_2 - (r_1 + r_2)}$$

as the angle of incidence is increased from zero, the deviation, d , begins to increase continuously to some minimum value D_{min} , of the light which occurs when the ray passes symmetrically through the prism and the angles made with the normal in the air and in the glass at W and X respectively are equal; i.e $i_1 = i_2 = I$ and $r_i = r_2 = r$

thus, $A = r + r = 2r$

$$\mathbf{A = 2r}$$

$$d = D_{min} = (i + i) – (r + r) = 2i – 2r$$

$$\mathbf{D_{min} = 2i – 2r}$$

$$i = \frac{D_{min} + A}{2}$$

$n = \frac{\sin i}{\sin r}$ substituting , we have :

$$\mathbf{n = \frac{\sin(A + D_{min})/2}{\sin (A/2)}}$$

EXAMPLE 10: Ray of light is refracted through a prism at minimum deviation. if the angle of refraction in a glass at the first face is 30^0 , what is the refracting angle of the prism?

SOLUTION: $r_1 = r_2 = 30^0$ (minimum deviation)

$A = 2r = 2 \times 30 = 60^0$

EXAMPLE 11 : A ray of light passes through a glass prism of refracting angle 50^0 . If the angle of incidence in the first face of the glass is 30^0. What is the angle of incidence in the second face if the prism has a refractive index of 1.63?

SOLUTION: $i_1 = 30^0$, r_2 = ? , n = 1.63

$A = 50^0$

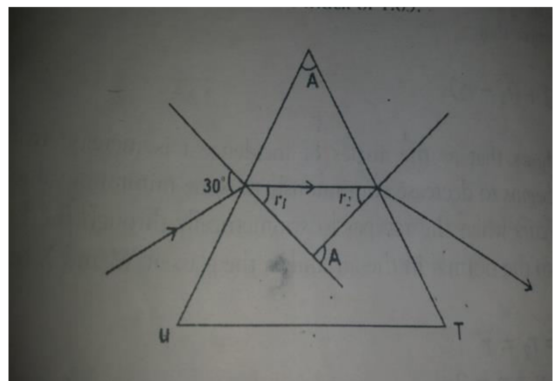


Fig. 9.9

$$n = \frac{\sin i_1}{\sin r_1}$$

$$1.63 = \frac{\sin 30}{\text{Sin } r_1}$$

$\text{Sin } r_1 = 0.30675$

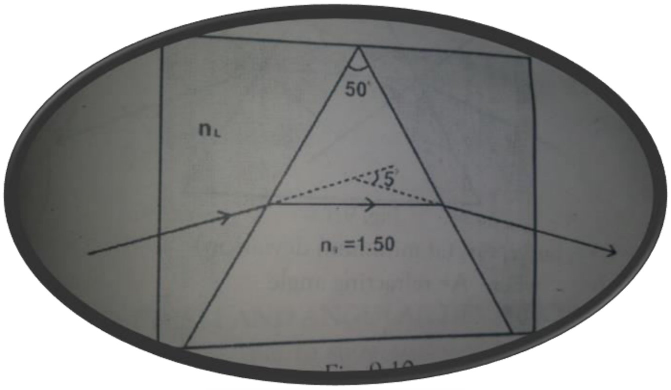
$r_1 = \sin^{-1}(0.30675) = 17.86^0$

$A = r_1 + r_2$

$50 = 17.86 + r_2$, $r_2 = 32.14^0$

EXAMPLE 12 : A certain prism of refracting angle 50^0 and refractive index 1.5 is immersed in a certain liquid of refractive index n_L . If the angle of minimum deviation of parallel rays through the prism is 5^0 , find the value of n_L .

SOLUTION:



$A = 50^0$, $n = 1.5$, $D_{min} = 5$, $n_g = 1.5$

$${}_gn_L = \frac{n_g}{n_L} = \frac{\sin(A + D_{min})/2}{\sin(A/2)}$$

$$\frac{1.5}{n_L} = \frac{\sin(50 + 5)/2}{\text{Sin}(50/2)}$$

$$\frac{1.5}{n_L} = \frac{\sin 27.5}{\sin 25}$$ cross multiply

$n_L = 1.375$

EXAMPLE 13: a certain prism was found to produce a minimum deviation of 40^o and produce a deviation of 63^0 for two values of angle of incidence, namely 53^0 and 70^0 respectively. Calculate the refractive index of the material of the prism.

SOLUTION: $i_1 = 53$, $i_2 = 70$, $d = 63$, $D_{min} = 40$

$d = (i_1 + i_2) - (r_1 + r_2)$

$63 = (53 + 70) - A$

$A = 60^0$ but at minimum deviation:

$$n = \frac{\sin(A + D_{min})/2}{\sin(A/2)}$$

$$n = \frac{\sin(60 + 40)/2}{\sin(60/2)} = \frac{\sin 50}{\sin 30} = 1.582$$

SMALL ANGLED PRISM AND ANGULAR DEVIATION

For small angled prism, the deviation

d = (n -1). But for components of white lights, for example, red and blue lights, their individual deviation is expressed as :

$\mathbf{d_r = (n_r - 1)A}$ and $\mathbf{d_b = (n_b - 1)A}$

where : n_r and n_b are refractive indices of red and blue lights respectively. And d_r and d_b are the deviation of red and blue lights respectively.

The angular deviation, Δ**d**, is given as:

$\mathbf{\Delta d = (n_b - n_r)A}$

EXAMPLE 14 : the refractive of glass prism of refracting angle 6^0 is 1.55 and 1.52 for blue and red lights respectively, determine the angular dispersion produced by the prism.

SOLUTION: $n_b = 1.55$, $n_r = 1.52$, $A = 6^0$

$\mathbf{\Delta d = (n_b - n_r)A} = (1.55 - 1.52)6 = 0.18^0$

EXAMPLE 15 : the difference between the refractive indices of carbon bisulfide for blue and red light is 0.48. while the critical angle for red light at carbon bisulfide air interface is 49^0 . Calculate the critical range for carbon bisulfide for blue light.

SOLUTION:

Difference, $n_b - n_r = 0.48$

Note : anytime *critical angle is involved* the refractive index is : $n = \frac{1}{\sin C}$

$C_r = 49^0$, C_b = ?

$n_b - n_r = 0.48$

$$\frac{1}{\text{Sin } C_b} - \frac{1}{\sin C_r} = 0.48$$

$$\frac{1}{\text{Sin } C_b} = 0.48 + \frac{1}{\text{Sin } C_r}$$

$$\frac{1}{\text{Sin } C_b} = 0.48 + \frac{1}{\sin 49}$$

$$\frac{1}{\text{Sin } C_b} = 1.805$$

Cross multiply

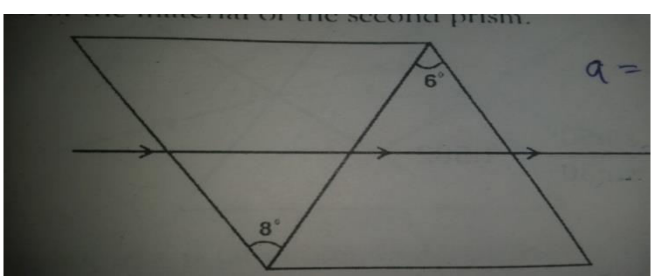
$1.805 \text{ Sin } C_b = 1$

$\text{Sin } C_b = 0.554$

$C_b = \sin^{-1}(0.554) = 33.64^0$

EXAMPLE 16: A glass prism of refracting angle 8^0 and a material of refractive index 1.64 held with its refracting angle downwards alongside another glass prism of angle 6^o which has its refracting angle pointing downwards. A narrow parallel beam passes through both prisms and is observed to emerge parallel to its original direction. Find the refractive index of the material of the second prism.

SOLUTION:



Since the emergent ray is parallel to the direction of the incident beam, it can be concluded that the deviation of the ray due to the prism A is equal to the deviation due to the prism B. Thus, the net deviation equals zero.

$d_{net} = d_A - d_B = 0$ therefore,

$d_A = d_B$. now lets substitute

$(n_A - 1)A_A = (n_B - 1)A_B$

$(1.64 - 1)8 = (n_B - 1)6$

$5.12 = (n_B - 1)6$

Divide both sides by 6

$n_B - 1 = \frac{5.12}{6}$

$n_B = 1.85$

CHAPTER 10

REFRACTION THROUGH CURVED SURFACES

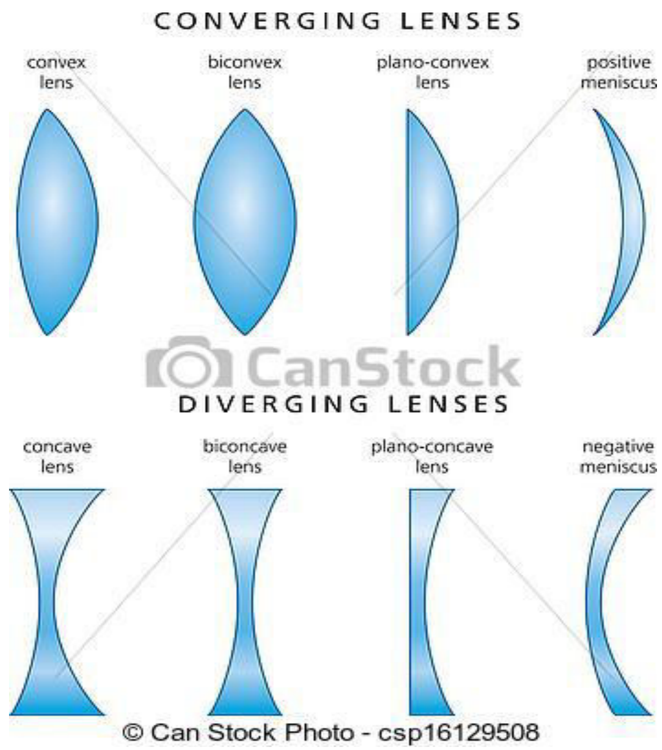
A lens is an optical system with two refracting surfaces (most commonly made of glass but sometimes plastic or crystals). **Or**

A lens is any transparent material with two faces, of which at least one is curved. A lens has two principal foci (plural of focus) since light may pass through a lens in either direction.

KINDS OF LENS

1. ***Convex Or Converging lens :*** it makes a parallel beam of light to converge to a point. Such lenses are thicker at the centre than at the edges.

2. ***Concave Or Diverging Lens :*** it makes a parallel beam of light to **appear** to diverge from a point. Such lenses are thinner at the centre than at the edges.



PARAMETERS OF A LENS

1. ***Optical centre (C)*** : it is the point through which rays of light pass without being deviated by the lens
2. ***Principal axis :*** it is the line passing through the optical centre of the lens and joining the centre of curvature of its surfaces.
3. ***Principal focus (F)*** : the principal focus of a converging lens is the point to which all rays parallel and close to the principal axis converge after refraction from the lens. But the principal focus of a diverging lens is the point from which all rays parallel and close to the principal axis **appear** to diverge after refraction through the lens
4. ***Focal length (f)*** : it is the distance between the optical centre and the principal focus of the lens.
5. ***Power (p)*** : the power of a lens is equal to the **reciprocal of the focal length.** It is measured in **dioptres (D)** *which is the same as* **m^{-1} (per metre)** when **f** is in ***meters.***

Convex lenses has positive power wile concave lenses has negative power.

For lenses **f** $\neq \frac{r}{2}$ as is it for mirrors.

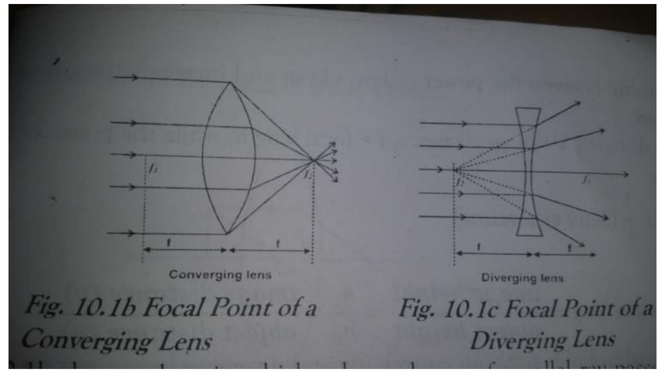


Fig. 10.1b Focal Point of a Converging Lens
Fig. 10.1c Focal Point of a Diverging Lens

P = $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ **(equation of a lens)**

f = $\frac{vu}{u+v}$**,** **u =** $\frac{fv}{v-f}$ **,** **v =** $\frac{fu}{u-f}$

Where u , v and f are object distance, image distance and focal length respectively.

A convex lens is also called a positive lens while a concave lens is also called a negative lens.

NOTE

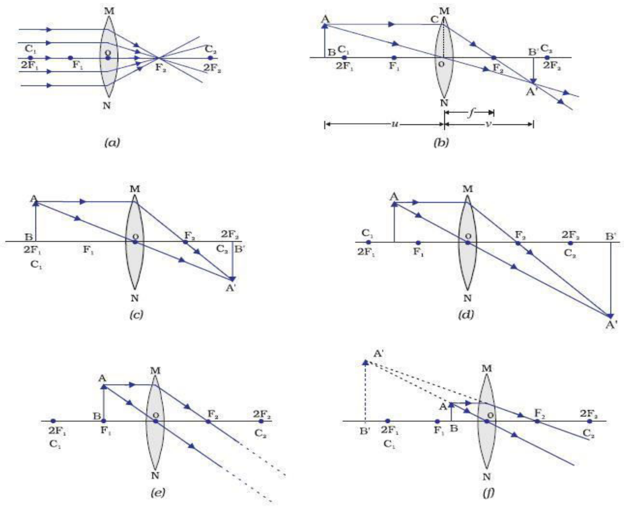
1. for a convex lens, f is positive while for a concave lens, f is negative.

2 an image is a real image when it is formed at the back (i.e behind) of the lens, but if it is formed at the front of the lens it is virtual(v is negative)

3. for a concave lens, v is negative

4. when an image is virtual, erect or upright, v is taken to be negative.

The image formed by a convex lens depend on the position of the object. see the ray diagram below.



The image formed by a concave lens is always VED i.e *virtual, erect and diminished.*

MAGNIFICATION

$$M = \frac{V}{u} = \frac{h_i}{h_i}$$

When :

$|M| > 1$, the image is enlarged/magnified

$|M| < 1$, the image is reduced/ diminished

$|M| = 1$, the image is true (real)

EXAMPLE 1 : a physics student used a diverging lens to examine an object. Determine the characteristics of the image of the object if it is placed 30cm in front of the lens of focal length 10cm.

SOLUTION: u = 30cm, f = -10cm

From : $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$v = \frac{fu}{u-f} = \frac{30 \times -10}{30 -(-10)} = -7.5\text{cm}$

since v is negative , it is virtual and erect

$M = \frac{v}{u} = \frac{7.5}{30} = 0.25$

$M < 1$, diminished

EXAMPLE 2 : A biconvex lens has a focal length 12cm. Where is the image formed ? and what are its characteristics for an object

(a) 18cm and (b) 5cm ?

SOLUTION: f = 12cm,

a. u =18cm

$$v = \frac{fu}{u-f} = \frac{18 \times 12}{18 - 12} = 36\text{cm}$$

$M = \frac{v}{u} = \frac{36}{18} = 2$

$M > 1$, thus , the image is magnified, real and inverted.

b. u = 5cm

$$v = \frac{fu}{u-f} = \frac{5 \times 12}{5 - 12} = -8.71\text{cm}$$

$M = \frac{v}{u} = \frac{8.71}{5} = 1.714$

$M > 1$, thus , the image is magnified, virtual and upright.

EXAMPLE 3 : what are the nature and focal length of the lens that will form a real image having one-third the dimensions of an object located 9cm from the lens?

SOLUTION: u = 9cm

$M = \frac{v}{u}$

$\frac{1}{3} = \frac{v}{9}$, v = 9/3 = 3cm

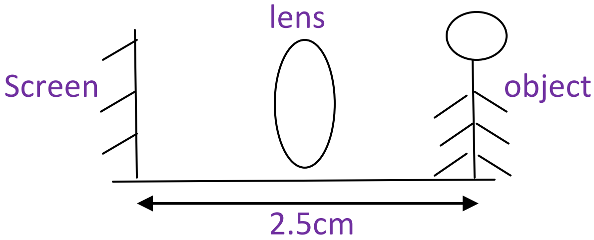
Since the question says the image is real, it must be a convex lens

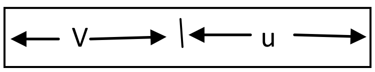
$f = \frac{vu}{u+v} = \frac{3 \times 9}{3+9} = 2.25\text{cm}$

Image is inverted real and diminished.

EXAMPLE 4 : a luminous object and a screen are 2.5cm apart. What are the position and focal length of the lens which will throw upon the screen an image of the object magnified 24times.

SOLUTION :



From the 

V + u = 2.5 ………………… (1)

Also, $M = \frac{v}{u}$

$24 = \frac{v}{u}$ cross multi~~ply~~

V = 24u ………………………… (2)

Put v = 24u into equation (1)

24u + u = 2.5

25u = 25 , u =2.5/25 = 0.1cm

Put u = 0.1cm into equation (2)

V = 24u = 24 × 0.1 = 2.4cm

$f = \frac{vu}{u+v} = \frac{2.4 \times 0.1}{2.4 + 0.1} = 0.096\text{cm}$

thus, the image is real, inverted and magnified.

EXAMPLE 5 : compute the focal length of the lens which will give an erect image 10cm from the lens when the object distance from the lens is (a) 200cm (b) very great?

SOLUTION: f = ? , v = -10cm

a. u = 200cm

$f = \frac{vu}{u+v} = \frac{-10 \times 200}{-10 + 200} = -10.53\text{cm}$

b. u = ∞ (infinity, i.e very large)

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

$\frac{1}{f} = \frac{1}{\infty} + \frac{1}{-10}$

$\frac{1}{f} = 0 - \frac{1}{10}$ (note $\frac{1}{\infty} = 0$)

$\frac{1}{f} = -\frac{1}{10}$ cross multiply

f = -10cm

LENS MAKERS EQUATION

When the lens is in air

$$P = \frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

when the lens is immersed in a medium of refractive index n_c then,

$$P = \frac{1}{f} = \left(\frac{n_g - 1}{n_c}\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where n_g is the refractive index of glass.

R_1 and R_2 are the two curved (refracting surfaces) .

NOTE

1. *for a convex lens, R_2 is negative*

2. *for a concave lens , R_1 is negative.*

3. *for a plane-convex lens, $R_2 = \infty$* (infinity)

EXAMPLE 6 : suppose a concave lens is considered to have absolute value of radii of curvature of 10cm and index of refraction of 1.52. what is the focal length of the lens.

SOLUTION: R_1 = -10cm and R_2 = 10cm

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{-10} - \frac{1}{10}\right)$$

$\frac{1}{f} = 0.52 \times \frac{-2}{10}$

$\frac{1}{f} = -0.104$ cross multiply

$f = 1/ -0.104 = -9.6cm$

EXAMPLE 7 : each face of a double convex lens has a radius of 20cm. The index of refraction of the glass is 1.5. Compute the focal length of this lens (a) in air (b) when it is immersed in carbon disulfide (n = 1.63)

SOLUTION: $R_1 = 20cm$, $R_2 = -20cm$

(a) $n_g = 1.5$

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{20} - \frac{1}{-20}\right)$$

$\frac{1}{f} = 0.5 \times \frac{2}{20}$

$\frac{1}{f} = 0.05$

$f = 1/0.05 = 20cm$

(b) $n_c = 1.65$

$$\frac{1}{f} = \left(\frac{n_g}{n_c} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = \left(\frac{1.5}{1.63} - 1\right)\left(\frac{1}{20} - \frac{1}{-20}\right)$$

$\frac{1}{f} = \frac{-13}{1630}$ cross multiply

$f = -125cm$

anytime you are required to look for ***power*** , before substituting R_1 and R_2 into the formula, convert them to meters(m) watch this !!!!!!!

EXAMPLE 8 : A double convex lens made of glass (n = 1.52) has a radius of curvature of 50cm on the front side and 40cm on the backside. Find the power of the lens.

SOLUTION: $R_1 = 50cm = 0.5m$,

$R_2 = -40cm = -0.4m$

$$p = \frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$p = \frac{1}{f} = (1.52 - 1)\left(\frac{1}{0.5} - \frac{1}{-0.4}\right)$$

$p = 0.52 (2 + 2.5)$

$p = 2.34m^{-1}$ or $p = 2.34D$

EXAMPLE 8 : A plane-convex lens of refractive index n = 1.56 is required to form a virtual image of an object with a magnification of 4. If the image distance is 15cm. Calculate the radius of the curved surface of the lens.

SOLUTION:

$M = \frac{v}{u}$

$4 = \frac{15}{u}$, $u = 15/4 = 3.75cm$

$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ **(equation of a lens)**

$f = \frac{vu}{u+v} = \frac{-15 \times 3.75}{-15 + 3.75} = 5cm$

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{5} = (1.56 - 1)\left(\frac{1}{R_1} - \frac{1}{\infty}\right)$$

$R_2 = \infty$ for plane-convex, note : $\frac{1}{\infty}$

$$\frac{1}{5} = 0.56\left(\frac{1}{R_1}\right)$$

$$\frac{1}{5} = \frac{0.56}{R_1}$$

$R_1 = 2.8cm$

EXAMPLE 9 : A convex-concave lens has faces radii 3cm and 4cm respectively, and is made of glass of refractive index f 1.6. Determine (a) the focal length and (b) the magnification of the image when the object is 28cm from the lens.

SOLUTION :

(a)Note: convex-concave

$R_1 = 3cm$ $R_2 = -4cm$

$$\frac{1}{f} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f} = (1.6 - 1)\left(\frac{1}{3} - \frac{1}{-4}\right)$$

$$\frac{1}{f} = 0.6 \times \frac{7}{12}$$

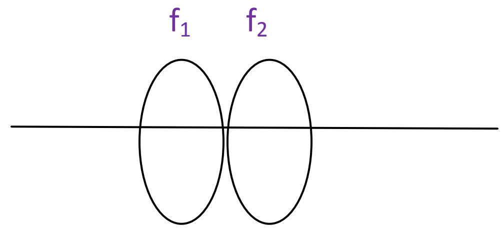
$f = 0.35cm$

(b) $v = \frac{fu}{u-f} = \frac{0.35 \times 28}{28 - 0.35} = 0.35cm$

$M = \frac{v}{u} = \frac{0.35}{28} = 0.0125$

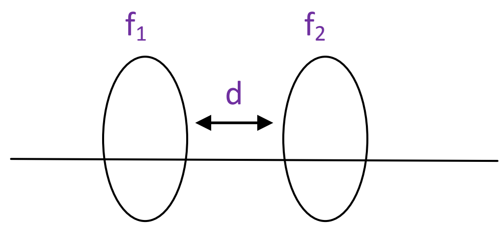
TWO LENSES IN CONTACT

In many optical instruments, there may be compound lenses, that is, two or more lenses in contact. These lenses may be separated by a distance, d , or may be so closed that the distance of separation may be neglected.



$$\mathbf{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

but if the lens are in separated by a distance, d,



$$\mathbf{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

where f is the focal length of the combination

EXAMPLE 10 : two lenses of focal length 9cm and -6cm are placed in contact. Calculate the focal length of the combination.

SOLUTION: $f_1 = 9cm$, $f_2 = -6cm$

$$\mathbf{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$$

$$\mathbf{\frac{1}{f} = \frac{1}{9} + \frac{1}{-6}}$$

$$\mathbf{\frac{1}{f} = \frac{1}{-18}}$$

f = -18cm

EXAMPLE 11 : two tin lenses of focal length +12cm and -30cm are in contact. Compute the focal length and power of the combination.

Hint : *first find focal length as above. Then*

$P = \frac{1}{f}$ … remember to convert to centimeter(cm) before finding power.

Answer : f = 20cm, p = 5m^{-1}or 5D

COMBINATION OF LENSES

Several optical systems such as compound microscope, telescope utilize more than one lens. Whenever more than one lens is used, the overall image produced may be determined by considering the lenses individually in sequence. That is to say, the image formed by the first lens becomes the object of the second lens and so on. If the first lens produces an image in the front of the other lens, the image is treated as a real object for the second lens. However, if the lenses are too close such that the image of the first is not formed in front of the second lens then a modification must be made in the sign convention. In this case, the image so formed from the first lens is treated as a virtual object for the second lens and the object distance for it is taken to be negative.

The total magnification for a compound lens system is the product of the individual magnification factors of the component lenses. i.e

$$\mathbf{M_{total} = M_1M_2}$$

EXAMPLE 12 : A laboratory scientist carried out an observation of an object by combining two similar lenses. Suppose an object is placed 20cm in front of lens L_1 which has focal length 15cm, lens L_2 with focal length 12cm is 20cm from L_1. What is the location of the final image and what are its characteristics.

SOLUTION :

$$v_1 = \frac{u_1f_1}{u_1 - f_1}$$

$$v_1 = \frac{20 \times 15}{20 - 15} = 60\text{cm (real image)}$$

$$M_1 = \frac{v_1}{u_1} = \frac{60}{20} = 3 \text{ (magnified)}$$

it is observed that the image of L_1 (which is now the object of L_2) is formed at the back of the lens L_2 . thus , u_2 is negative (negative object distance).

Lets now find the position of the final image

$$v_2 = \frac{u_2f_2}{u_2 - f_2}$$

$$v_2 = \frac{-40 \times 12}{-40 - 12} = 9.23\text{cm (real image)}$$

$$M_2 = \frac{v_2}{u_2} = \frac{9.23}{40} = 0.23 \text{ (diminished)}$$

$$\mathbf{M_{total} = M_1M_2} = 3 \times 0.23 = 0.69$$

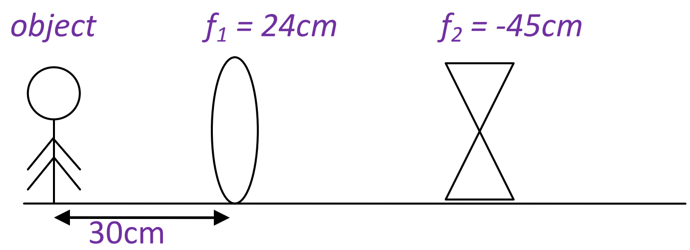
Hence, the final image is located on the left side of L_2 . The image is diminished, inverted and real.

EXAMPLE 13 : Light from an object passes through a tin converging lens with focal length24cm, if it is placed 30cm from the object and then through a thin diverging lens of focal length 45cm forming a real image 72cm from the diverging lens, calculate :

(a) the image distance due to the first lens (b) the distance between the lenses

(c) the magnification

SOLUTION : f_1 = 24cm, u_1 = 30cm



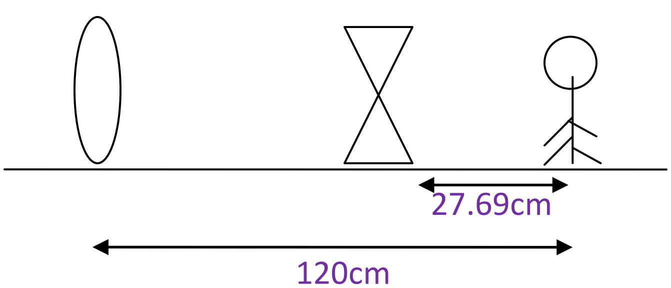
$V_2 = 72cm$

$$v_1 = \frac{u_1 f_1}{u_1 - f_1}$$

$$v_1 = \frac{30 \times 24}{30 - 24} = 120cm \text{ (real image)}$$

$$u_2 = \frac{v_2 f_2}{v_2 - f_2}$$

$$u_2 = \frac{72 \times -45}{-72 - (-45)} = -27.69cm \text{ (real image)}$$



(b) From the diagram , the distance between the lenses is (120 – 27.69) = 92.31cm

(c) $M_1 = \frac{v_1}{u_1} = \frac{120}{30} = 4cm$ (magnified)

$M_2 = \frac{v_2}{u_2} = \frac{72}{27.69} = 2.6cm$ (magnified)

$\mathbf{M_{total} = M_1 M_2} = 4 \times 2.6 = 10.4$

CHAPTER 11

OPICAL INSTRUMENT

Near point and far point : the nearest point at which an object can be clearly seen is called the *near point*. The farthest point at which an object can be clearly seen is called the *far point*.

For a normal eye, the near point is 25cm away from the eye. This is the limit to which the ciliary muscles can operate, for at this point they are strained to a maximum, and the accommodation is therefore the greatest. The distance from the near point to the eye is known as the *least distance of distinct vision.*

For a normal eye, the far point is at infinity. At this point the ciliary muscles are most relaxed and the lens is slim. The accommodation is also the least.

THE MICROSCOPE

A magnifying glass or microscope is an instrument used for looking at objects close to eye and the final image is usually formed at the least distance of distinct vision (D) from the eye.

1. The Simple Microscope : when a converging lens is used as a simple microscope, the object is placed between the principal focus and the optical centre of the lens. With this arrangement, a *magnified, virtual and erect* **(VEM)** *image is formed.* The lens is moved until the image is seen distinctly at 25cm from the normal eye.

$$\mathbf{M = \frac{D}{f} - 1}$$

Where D is the least distance of distinct vision

NOTE : if the question wants you to use

D = 25cm, use it but if it doesn't, it will give you what to use.

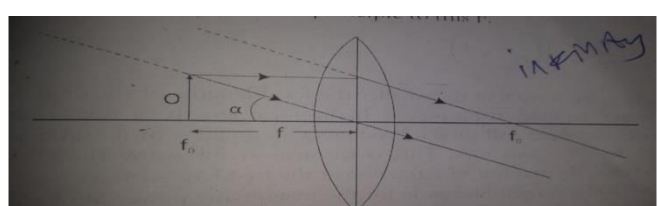


Fig. 11.2

EXAMPLE 1 : in a compound microscope, the image formed by the objective lens is at a distance 3cm from the eye lens. if the final image is at 25cm from the eye lens, calculate the focal length of the eye lens.

SOLUTION: u = 3cm,

v = -25cm(images formed by a microscope are virtual)

f = $\frac{vu}{u+v}$ = $\frac{3 \times -25}{3+(-25)}$ =3.41cm

2. <u>The Compound Microscope :</u> To produce a higher magnification than that obtained from a simple microscope, a combination of two convex lenses of short focal lengths is used in an arrangement called the compound microscope. The lens nearer to the object is known as the **objective lens** and the lens through which the final image is seen is known as the **eyepiece.** The eyepiece has a comparatively larger focal length than the objective lens. The objective lens produces a **real, inverted and magnified (RIM)** image while the eyepiece produces a **virtual, inverted and magnified (VIM)** image.

NOTE : *a concave lens cannot be used as a microscope because the image formed is always **diminished .***

The magnifying power of the instrument is given as : $\mathbf{M = \frac{f_e}{f_o}}$ or $\mathbf{M = \frac{h_e}{h_o}}$

where h_o is the height of object in objective lens

and h_e is the height of final image in the eyepiece.

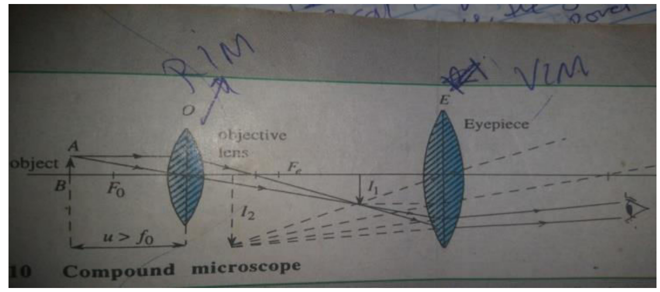
NOTE: the smaller the focal lengths of the

objective and eye lens, the greater the magnifying power.

Alternatively, the magnifying power, M of the instrument is :

$\mathbf{M = M_0 M_e}$, *where M_o and M_e are the magnifications of the objective lens and eye piece lens respectively. Also,*

$\mathbf{M_o = \frac{V}{f_o} - 1}$ **and** $\mathbf{M_e = \frac{D}{f_e} - 1}$



EXAMPLE 2 : a simple magnifying glass is used to view an object. At what distance from

From the lens must the object be placed so that an image 5 times the size of the object is produced 20cm from the lens?

SOLUTION : v = -20cm (same reason as above)

M = $\frac{v}{u}$, 5 = $\frac{20}{u}$, u = 4cm

EXAMPLE 3 : An object 5cm high is placed 15cm in front of a convex lens of 10cm focal length. A second convex lens also of 10cm focal length is placed 48cm behind the first lens. find:

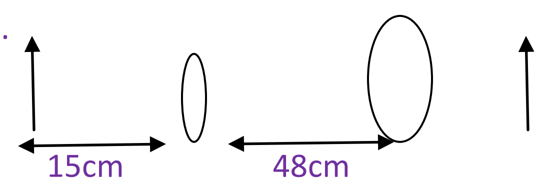
a. the image of the lens combination

b. the magnification of the system and

c. the height of the final image

SOLUTION: h_o = 5cm

a.



$v_1 = \frac{u_1 f_1}{u_1 - f_1}$ $v_1 = \frac{15 \times 10}{15 - 10} = 30cm$

the object distance from the second lens,

u_2 = 48 – 30 = 18cm

$v_2 = \frac{u_2 f_2}{u_2 - f_2}$ $v_2 = \frac{18 \times 10}{18 - 10} = 22.2cm$

thus, D = 22.2cm

b. $M_e = \frac{D}{f_e} - 1 = \frac{22.5}{10} - 1 = 1.25cm$

$M_o = \frac{V}{f_o} - 1 = \frac{30}{10} - 1 = 2$

$M = M_0 M_e = 2 \times 1.25 = 2.5$

c. $M = \frac{h_e}{h_o}$, $2.5 = \frac{h_e}{5}$, $h_e = 12.5cm$

EXAMPLE 4 : In a compound microscope, the objective and the eyepiece have focal lengths off 0.8cm and 2.5cm respectively. The real image formed by the objective is 16cmm from the objective. Determine the total magnification if the eye is held close to the eye piece and views the virtual image at a distance 25cm.

SOLUTION: $M = M_e M_o$

$$M = \left(\frac{D}{f_e} - 1\right)\left(\frac{V}{f_o} - 1\right) = \left(\frac{25}{2.5} - 1\right)\left(\frac{16}{0.8} - 1\right) = 1.71\times10^2$$

TELESCOPES

The purpose of a telescope is to make distant objects appear closer and therefore larger. The difference between a telescope and a microscope is that a telescope is used to view objects of large distances whereas a microscope is used to view very small objects at close distance. Another difference is that many telescopes use a curve mirror, not a lens, as an objective. A telescope is used to view distant objects such as the stars and the planets.

1. <u>The Astronomical Telescope:</u> it is the simplest telescope. The objective is a large converging lens of long focal length. Because the object is very far away, the image of the objective is for all intents and purposes located at the principal focus of the objective lens f_o . the eye piece is a converging lens of small focal length f_o and is placed such that the image of the first lens falls just within the principal focus of the second lens, thereby forming an **enlarged, virtual image.** That is, the eye piece acts as a simple magnifying glass. Since the first image falls approximately at the focal point f_o, the first lens and the second lens are placed such that the image of the first lens falls within the focal length of the second lens. The distance separating the lenses is approximately the sum of the two focal lengths. Hence, the overall length of the telescope is : $\mathbf{L = f_o + f_e}$

The astronomical telescope is not very good for viewing distant objects on the surface of the earth, because they are **all inverted.** The astronomical telescope can be converted to a terrestrial **telescope** by placing a third converging lens or erecting lens of focal length f, between the objective and the eye piece. The length or distance between objective lens and eye piece of a terrestrial telescope $\mathbf{L = f_o + 4f + f_e}$ **.** Telescope is usually adjusted so that the final image is formed at **infinity** so that the eye can be completely relaxed when viewing it. Such adjustment is called **normal adjustment.** The angular magnification (magnifying power)of a telescope is defined as :

$$\mathbf{M = \frac{f_o}{f_e} = \frac{d_o}{d_e}}$$

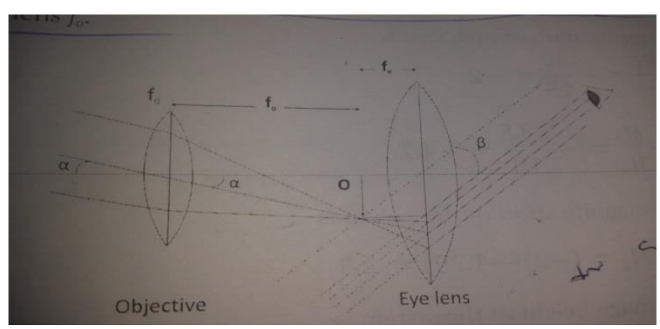
f_o is the objective focal length, f_e is the eyepiece focal length, d_o is the objective diameter, d_e is the eye piece diameter.

$\mathbf{L = f_e + f_o}$ and also $\mathbf{M = \frac{\beta}{\alpha}}$

α is the angle subtended by the objective lens

β is the angle subtended by the eye lens

NOTE: when the question involves two converging lens, you should know it is **an astronomical telescope**



EXAMPLE 5 : converging lenses of focal lengths 120cm and 10cm are used to construct an astronomical telescope. What is the distance between the lenses at its normal adjustment?

SOLUTION: $L = f_e + f_o = 120 + 10 = 130cm$

EXAMPLE 6 : A telescope consists of two thin converging lenses of focal lengths 0.5m and Xm separated by a distance of 0.54m. it is focused on the moon, which subtends an angle 0.8^0 at the objective. Find :

(a) the focal length of the second lens

(b) the angle subtended at the observer's eye by the image formed by the instrument

SOLUTION : $f_o = 0.5m$, $f_e = ?$, $L = 0.54m$,

$\alpha = 0.8^0$

a. $L = f_e + f_o$, $0.54 = f_e + 0.5$, $f_e = 0.04m$

b. $M = \frac{f_o}{f_e} = \frac{0.5}{0.04} = 12.5$

$M = \frac{\beta}{\alpha}$, $12.5 = \frac{\beta}{0.8}$, $\beta = 12.5 \times 10^0$

EXAMPLE 7 : A telescope objective focus has focal length 80cm an diameter 10cm.

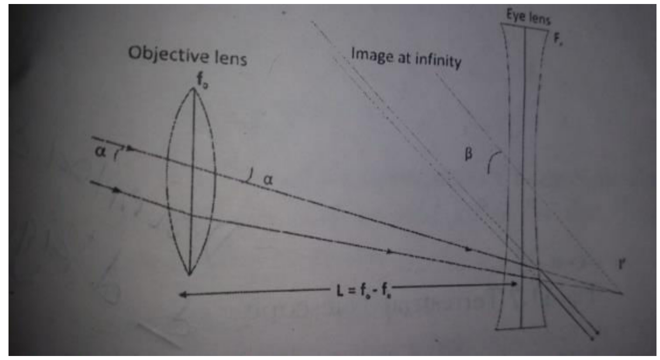
Calculate the focal length and minimum diameter of a simple eye lens for use with the telescope. If the magnifying power required is 20 and all the light transmitted by the objective from a distant point on the telescope axis is the fall on the eye piece.

SOLUTION : $f_o = 80cm$, $d_o = 10cm$, $f_e = ?$ $M=20$

$M = \frac{f_o}{f_e}$, $20 = \frac{80}{f_e}$, $f_e = 4cm$

$M = \frac{d_o}{d_e}$, $20 = \frac{10}{d_e}$, $d_e = 0.5cm$

2. <u>The Galilean Telescope :</u> The Galilean telescope consist of a converging lens of a large focal length for the objective and a diverging lens of short focal length for the eye piece. The advantage of a Galilean telescope is that it gives an **erect image** .



The objective lens forms a real image which acts as a virtual object for the lens. This then adjusted to give the final image at **infinity** . The length, L, of such telescope is $\mathbf{L = f_o - f_e}$ and it is therefore shorter than an astronomical telescope of similar magnification. One advantage of a Galilean telescope is that the **view is small.**

3. <u>Reflecting Telescopes :</u> the largest optical telescopes all use a concave objective mirror rather than a converging objective lens. The large apertures possible for reflecting telescopes both increase their ability to result and also enable fainter stars to be seen as they gather more light from a given source

4. <u>Terrestrial Telescope :</u> further method of giving final erect image is to use a third converging lens. The objective lens gives an image, I , which becomes a real object for the lens L. This lens is often called **the erecting lens** because it simply produces an inverted image of **I**. The eye lens is the used to magnify this image and so the final image at infinity is erect. Such a telescope is longer than the Galilean type because of the additional lens. The overall length of the telescope is,

$\mathbf{L = f_o + 4f + f_e}$

VISION

The ability for the human eye to focus on near and distant objects attributed to the crystalline lens is called **accommodation.** This is affected by an alteration focal length of the eye lens which is controlled by the ciliary muscles.

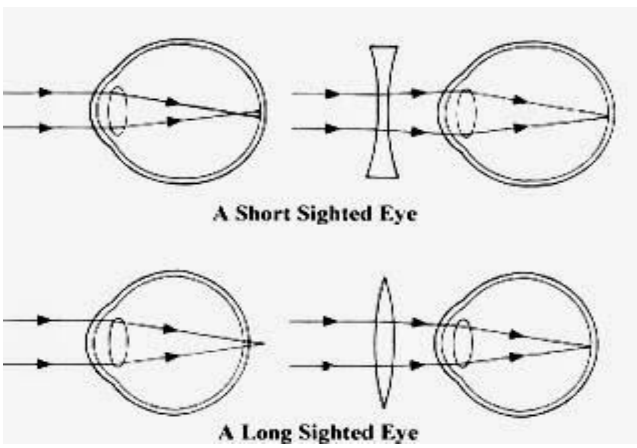
DEFECTS OF VISION AND THEIR CORRECTION

Normally, the eye can accommodate for clear vision of objects from infinity down to 25cm.

1. <u>Long Sightedness (hypermetropia) :</u> this occurs when images of near objects are formed behind the retina. It is caused by the eye ball becoming too short and the person with this defect can distinctly see far objects, but cannot see near objects clearly. The converging lens reduces divergence of ray so that they appear to and form the image on the retina. It is corrected with a convex lens.

2. Short Sightedness (myopia) : it occurs

When images of far objects are formed at the front of the retina. It is caused by the eye ball becoming too large and the person with the defect can see near objects very well but cannot see clearly distant objects. A diverging lens of sufficient power placed in the front of the eye lens will produce an image of the distant object at a sufficiently close distance so that the eye lens now form a clear image on the retina. It is corrected with a diverging lens.



3. Presbyopia (old sight) : it is a natural part of the aging process . it is due to hardening of the lens of the eye to focus light behind rather than on the retina when looking at **close objects** . it typically starts at around age 40. It is corrected with **a bifocal eyeglasses.**

4. Astigmatism : it is the result of inability of the **cornea** to properly focus an image on the retina. The result is a **blurred image.**

In addition to the **spherical lens power** used to correct short and long sightedness, astigmatism requires **an additional CYLINDRICAL LENS** power to correct the difference between the powers of the two principal meridians of the age.

CHAPTER 12

DISPERSION AND ABERRATION

The separation of white light into its component colours is called **dispersion.** The brand of colours is known as **spectrum**. The spectrum of visible light contains the colours: red, orange, yellow, green, blue, indigo, and violet (**ROYGBIV).** The white light is refracted such that each colour, or wavelength, is refracted by different angles as shown below.

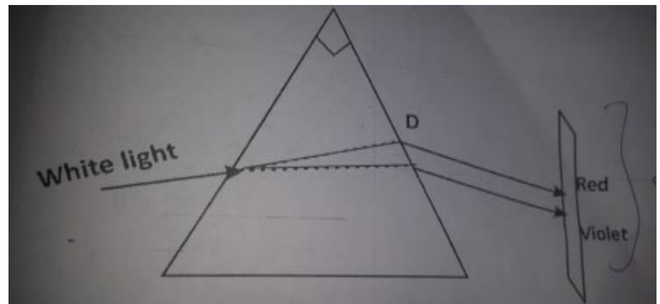


Fig. 12.1 Dispersion of White Light

The speed of light in vacuum is the same for all wavelengths, but the speed in the material substance is different for different wavelengths. Violet light which has the shortest wavelength is bent most, whereas red light with the longest wavelength is bent least. *The amount of dispersion depends on the difference between the refractive indices for violet and for red light.* **The wavelength decreases in the order ROYGBIV while the deviation increases in the order ROYGBIV.**

The dispersive power (of a material) depends only on the type of material of which a prism or lens is made and not on its shape. Dispersive power(**w**) is defined as :

$$w = \frac{angular\ dispersion}{mean\ deviation}$$

ABERRATIONS

The departures of real (imperfect) images from the image predicted by the simple theory are called **aberrations.** It can be classified as *chromatic aberrations* which involve wavelength – dependent imaging behavior, and *spherical aberrations.*

spherical aberrations : it results from the fact that the focal points of light rays far from the optical axis of a spherical lens (or mirror) are different from the focal points of light rays of the same wavelength close to the principal axis. In the case of mirrors used for very distant objects, spherical aberrations can be eliminated or at least minimized, by using parabolic surface rather than spherical surface.

Chromatic aberration and achromatic lenses:

The colouring effect of images as white source of light passes through a lens which is due to the different colours of light being brought to focus slightly at different points as a result of refraction is termed **chromatic aberration** of single lens. It can be corrected by several methods but the method of employing **two thin lenses in contact,** *one made of flint glass and the other made of crown glass is the commonest.* Such lenses are called ***achromatic lenses.*** For the two lenses, the ratio of their focal lengths is equal to the ratio of their dispersive power, since one lens is concave and the other convex a negative sign appears, thus :

$$\frac{f_1}{f_2} = -\frac{w_1}{w_2}$$

separated doublets : another method of obtaining an achromatic system is to employ two thin lenses made of the same glass and separated by a distance, d , equal to half of

the sum of their focal lengths, thus,

$$d = \frac{f_1 + f_2}{2}$$

EXAMPLE 1 : the dispersive power of two glasses are in the ratio 3:4 and they are used to make achromatic objective of focal length 40cm. calculate (a) the focal length of the two lenses (b) the distance of separation that will give an achromatic combination if the lenses are of the same material.

SOLUTION : $w_1 : w_2 = 3 : 4$, $f = 40$cm

$\frac{f_1}{f_2} = -\frac{w_1}{w_2}$, $\frac{f_1}{f_2} = \frac{-3}{4}$, $f_1 = \frac{-3f_2}{4}$ ....(1)

put equation (1) into : $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

$\frac{1}{40} = \frac{1}{(-3f_2/4)} + \frac{1}{f_2}$, thus, $f_2 = -40/3$cm

Put $f_2 = -40/3$ into (1) , $f_1 = -3f_2/4 = 10$cm

(b) $d = \frac{f_1 + f_2}{2} = \frac{10 + (-40/3)}{2} = -1.67$cm

distance cannot be negative, thus d = 1.67cm

EXAMPLE 2 : a lens of crown glass of dispersive power 0.064 has focal length of 45cm. calculate the focal length of a flint glass of dispersive power 0.044 which forms achromatic combination with the first. Also, find the resulting focal length from the combination.

SOLUTION : $w_1 = 0.064$, $w_2 = 0.004$, $f_1 = 45$cm

$\frac{f_1}{f_2} = -\frac{w_1}{w_2}$, $\frac{45}{f_2} = -\frac{0.064}{0.044}$, $f_2 = -31$cm

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ just substitute

ANS: f = -99.64cm (it is of a diverging lens).

MOST DIFFICULT RECENT PAST QUESTIONS

Please note that some of the questions we solved in the chapters of this material were past questions.

1. a spring has a spring constant of 52N/m. if the full length is 0.5m when a 1.5kg mass is hung from it. What is the equilibrium length when the 1.5kg is removed? (a) 1.12m (b) 0.21m (c) 0.31m (d) none

SOLUTION: f = ke, $1.5\times 10 = 52 \times e$, e = 15/52m

$L_o + e = 0.5$, $L_o = 0.5 - e$, $L_o = 0.5 - 15/52 = 0.21$m

2. two identical piano strings of length 0.75m are each turned exactly to 440Hz. The tension in one of the strings is increased by 1.0%. what is the beat frequency between their fundamentals?

(a) 5Hz (b) 4Hz (c) 3Hz (d) 1Hz (e) none of the above

SOLUTION : let $T_1 = T$, $T_2 = \frac{101T}{100}$ from $f_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

$\frac{f_1^{\,2}}{\sqrt{T_1}} = \frac{f_2^{\,2}}{\sqrt{T_2}}$, by substitution, $f_2 = 442.19$Hz

Beat frequency, $f_b = 442.19 - 440 = 2.2$Hz, **option E**

3. the first two successive positions of resonance occurred at lengths 15.3cm and 48.3cm respectively. Calculate the frequency of the source when the velocity of sound in air is 3.4×10^2m/s (a) 515.2Hz (b) 615.2Hz (c) 595.2Hz (d) 435.2Hz

SOLUTION : $f = \dfrac{v}{2(L_1 - L_2)} = \dfrac{3.4 \times 10^2}{2\,(0.483 - 0.153\,)} = 575.2$

4. a police siren emits a sinusoidal wave with frequency of 300Hz. The speed of the source is 340Hz. Find the wavelength of the wave if the siren ***is at rest in air.*** (a) 3.12m (b) 5.14m (c) 2.13m (d) 1.13m

SOLUTION :

$\lambda = v/f = 340/360 = 1.13$m , **D**

5. a body to two simple harmonic motion in the same direction with displacement **x** (in metres) given by : $x_1 = 9\cos 5t$ and $x_2 = 12\sin 5t$. calculate the amplitude of the resulting motion. (a)9m (b) 12m (c) 21m (d) 15m

SOLUTION: the two SHM are in the same direction, the resultant displacement of the body is their sum

$X = 9\cos 5t + 12\sin 5t$, $A = \sqrt{(B_1^2 + B_2^2)} = \sqrt{(9^2 + 12^2)} = 15$

6. the refractive index of a glass prism is 1.63. its refracting angle is equal to the angle of minimum deviation. Calculate its refracting angle.

SOLUTION: $n = \dfrac{\sin(A + D_{min})/2}{\text{Sin }(A/2)}$, $1.6 = \dfrac{\sin(A + A)/2}{\sin(A/2)}$

$1.6 = \dfrac{\sin A}{\text{Sin}(A/2)}$, note : $\sin A = 2\sin\frac{A}{2}\cos\frac{A}{2}$, substitute

$1.6 = \dfrac{2\cancel{\sin\frac{A}{2}}\cos\frac{A}{2}}{\cancel{\text{Sin}(A/2)}}$, $1.6 = 2\cos\frac{A}{2}$, A = `73.7^0

7. a particle exerts S.H.M with amplitude A. find the value of its displacement when its kinetic energy is $^3/_4$ of its maximum value.

SOLUTION: $\cancel{\frac{1}{2}m}v^2 = \frac{3}{4} \times \cancel{\frac{1}{2}m}v^2_{max}$, where $v_{max} = wA$

$V = \frac{3}{4}(wA)^2$, $\cancel{w^2}\sqrt{(A^2 - x^2)} = \frac{3}{4}\cancel{w^2}A^2$, $A^2 - x^2 = = \frac{3}{4}A^2$

$x^2 = A^2/4$,take square root, $x = A/2$

8. the force acting on a mass of 4kg is given y $F = 36x$,

Where x is the displacement, find the period.

SOLUTION: $F = 36x$, $F = Kx$, $k = 36$, $T = 2\pi\sqrt{\frac{m}{k}}$,

$T = 2\pi\sqrt{\frac{4}{36}} = 2\pi \times \frac{2}{6} = \frac{2\pi}{3}$.

9. a certain glass prism has a refractive index of 1.61 for red and violet light . If both colours pass through symmetrically and if the apex angle is 60^o , find the difference between the angles of minimum deviation of the two colours. (a) 5^0 (b) 8^o (c) 6^o (d)none

SOLUTION : using $n = \dfrac{\sin(A+D_{min})/2}{\text{Sin}(A/2)}$

red light, $1.6 = \dfrac{\sin(60 + D_{red})/2}{\text{Sin}(60/2)}$, $0.805 = \sin(60 + D_{red})/2$

(60 + D_{red})/2 = $\sin^{-1}(0.805)$, $D_{red} = 47.22^0$

For violet, $1.66 = \dfrac{\sin(60 + D_{vio})/2}{\text{Sin}(60/2)}$, $0.83 = \sin(60 + D_{vio})/2$

$(60 + D_{vio})/2 = \sin^{-1}(0.83)$, $D_{vio} = 52.198^0$

Difference = 52.198 – 47.22 = 5^o

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