5/09/20

Week 8

Israel Nolazco

1) arriving late $p=0.6$, random sample of 10 orders show that 3 of fever arrive late. Should we reject in favor of $p<0.6$ Use binomial distribution

a) find prob of commiting type I error if the true proportion is $p=0.6$

$n=10$ $\quad X\leq 3$ $\quad H_0 \Rightarrow p=0.6$

$\alpha = P(\text{Type I error}) = P(X\leq 3 \text{ when } p=0.6)$

$= \sum_{X=0}^{3} b(X;3,0.6) \Rightarrow \boxed{0.0548 \text{ from table A.1}}$

b) Find prob of commiting typ II error if the true alternatives $p=0.3$, $p=0.4$ & $p=0.5$

$\beta = P(\text{Type II error}) = P(X\leq 3 \text{ when } p=0.3)$

$\beta = \sum_{X=4}^{10} b(X;10,0.3) = \sum_{X=0}^{10} b(x;10,0.3) - \sum_{X=0}^{3} b(X;10,0.3)$

*Note: We are looking for the prob of non rejection the null hypothesis ($H_0=0.6$) when alternative $H_1=0.3$ is true

Additionally we are looking of materrials arriving late in sample of 4 to 10 when $p=0.3$

)) Continue thus we subtracting

$$\sum_{x=0}^{10} b(x;10,0.3) - \sum_{x=0}^{3} b(x;10,0.3)$$

* Using table A!

$\beta = 1 - 0.6496 = 0.3504$

$\boxed{\beta = 0.3504 \text{ when } n=10 \quad p=0.3}$

$$\beta = \sum_{x=0}^{10} b(x;10,0.4) - \sum_{x=0}^{3} b(x;10,0.4)$$

$\boxed{\beta = 1 - 0.3823 \Rightarrow 0.6177}$ when $n=10$ $p=0.4$

$$\beta = \sum_{x=10}^{10} b(x;10,0.5) - \sum_{x=0}^{3} b(x;10,0.5)$$

$\boxed{\beta = 1 - 0.1719 = 0.8281 \text{ when } n=10 \quad p=0.5}$

2) college graduate $p=0.6$

Test hypothesis $n=15$, if # of college graduates anywhere from 6 to 12 we shall not reject the null hypothesis that $p=0.6$; otherwise shall conclude that $p \neq 0.6$

a) evaluate α assuming $p=0.6, n=15$

$\alpha = P(\text{Type I error})\ P(0 \leq X \geq 5 \text{ or } 13 \leq X \geq 15)$

$$\alpha = \sum_{x=0}^{5} b(x;15,0.6) + \sum_{x=13}^{15} b(x;15,0.6)$$

$$\sum_{x=0}^{5} b(x;15,0.6) \Rightarrow 0.0338$$

*from Table A.1

$$\sum_{x=13}^{15} b(x;15,0.6) = \sum_{x=0}^{15} b(x;15,0.6) - \sum_{x=0}^{12} b(x;15,0.6)$$

$$\sum_{x=13}^{15} b(x;15,0.6) = 1 - 0.9729 = 0.0271$$

$$\alpha = \sum_{x=0}^{5} b(x;15,0.6) + 0.0271 \Rightarrow 0.0338 + 0.0271 \Rightarrow$$

$$\boxed{\alpha = 0.0609}$$

b) evaluate β for alternative $p = 0.5$ & $p = 0.7$

*Note: We are looking at the null hypothesis $H_0 = 0.6$ when the alternative hypothesis $(H_1 = 0.5)$ is true

*we are also looking of a sample between 6 and 12

$\beta = P(\text{type II error}) = P(6 \leq X \geq 12 \text{ when } p = 0.5)$

$\beta = \sum_{X=6}^{12} b(X; 15, 0.5) = \sum_{X=0}^{12} b(X; 15, 0.5) - \sum_{X=0}^{5} b(X; 15, 0.5)$

$\beta = \sum_{X=0}^{12} b(X; 15, 0.5) - \sum_{X=0}^{5} b(X; 15, 0.5)$

$\beta = 0.9963 - 0.1509 = \boxed{0.8454 \text{ when } p = 0.5}$

$\beta = P(\text{Type II error}) = P(6 \leq X \geq 12 \text{ when } p = 0.7)$

$\beta = \sum_{X=6}^{12} b(X; 15, 0.7) = \sum_{X=0}^{12} b(X; 15, 0.7) - \sum_{X=0}^{5} b(X; 15, 0.7)$

$\beta = \sum_{X=0}^{12} b(X; 15, 0.7) - \sum_{X=0}^{5} b(X; 15, 0.7)$

$\beta = 0.8732 - 0.0037 \Rightarrow \boxed{0.8695 \text{ when } p = 0.7}$

c) is this a good procedure?

$\alpha = 0.0609$ $\beta_1 = 0.8454$ $\beta_2 = 0.8695$

*The probability of commiting Type Error I is very small which means the level of significant is too low. However Type Error II is high. Thus this is not a good test procedure.

3) new fish line $\mu = 15$ Kg $\sigma = 0.5$ Kg

alternative $\mu < 15$ Kg $n = 50$ lines critical region $\bar{x} < 14.9$

a) Find probability of Type I error when H_0 is true

$H_0 = \mu_0 = 15$ Kg, $\bar{x} < 14.9$ kg, $n = 50$ lines

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \Rightarrow \frac{14.9 - 15}{0.5 / \sqrt{50}} = \frac{-0.1}{0.0707} \Rightarrow$$

$z = -1.414$, $\alpha = P(\text{Type I error}) = P(\bar{x} < 14.9 \text{ when } \mu = 15)$

$\alpha = P(z < -1.414) \Rightarrow$ Per Table A.3 $z(1.41) = 0.9207$

$\alpha = z < -1.414 = 1 - z(1.41) = 1 - 0.927 = 0.0793$

$$\boxed{\alpha = 0.0793}$$

B) evaluate β for the alternatives $\mu = 14.8$ & $\mu = 14.9$

$\beta = P(\bar{x} \geq 14.9 \text{ when } \mu = 14.8)$

$$\beta = P\left(\frac{14.9 - 14.8}{0.5 / \sqrt{50}}\right) = \frac{0.1}{0.0707} = 1.41$$

$\beta = P(z \geq 1.41) = \beta = 1 - P(z < 1.41)$

$\beta = 1 - 0.9207 \Rightarrow \boxed{\beta = 0.0793}$

evaluate β for the alternative $\mu = 14.9$

$\beta = P(x \geq 14.9$ when $\mu = 14.9)$

$\beta = P\left(\frac{14.9 - 14.9}{0.0707}\right) = \beta = P\left(\frac{0}{0.0707}\right) = \beta = P(0)$

$\beta = P(Z \geq 0) \Rightarrow 1 - P(Z < 0) \Rightarrow 1 - 0.5 = 0.5$

$\boxed{\beta = 0.5}$

4) Female freshman height 162.5 cm

σ 6.9 $\quad n = 50$ females

is there a reason to believe that there has been a change in average height, if a random sample of 50 females has an average height of 165.2? Use P-value in your conclusion!

$$H_0 = \mu = 162.5$$
$$H_1 \neq \mu \neq 162.5$$

Note: No level of significance is given, so let us use a 97% confidence for this problem

$$\alpha = 100\% - 97\% = 3\% = 0.03$$

$\Rightarrow$ observed-value

$$-z_{\alpha/2} < z > z_{\alpha/2}$$

$$z_{\alpha/2} = z_{0.015} \Rightarrow z = 2.17$$

$$z(\text{observed value}) = \frac{165.2 - 162.5}{6.9/\sqrt{50}}$$

$$z(\text{obsrered value}) = 2.77$$

$$z(\text{observeb value}) > z_{\alpha/2} \quad \text{is true}$$
$$2.77 > 2.17$$

Then we reject H_0 ; conclude there is a reason to believe there has been a change in average height.

P-value?

$P(|z| > 2.77 = 2P(z < -2.77)$

$= 2(0.0028)$
$= 0.0056$

* Given that the P-value is less than the level of significance we reject the null hypothesis

∴ the average height is not 162.5