Digital Image Processing

Image Compression

Recap

Some definitions

- Compression ratio
- Fidelity criteria

Data Redundancy

- Coding
- Interpixel
- Psychovisual

Compression techniques

- Loss-less and Lossy
- Symmetric and Asymmetric

Variable length coding

- Huffman Coding
- Information theoretic analysis

    Entropy

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Kraft's inequality

A uniquely decodable code with the codeword lengths $l_1, \ldots, l_N$ exists if and only if

$$\sum_{i=1}^{N} 2^{-l_i} \leq 1$$

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Lower Bound

Given that we have a memoryless source X_j and that we code one symbol at a time with a prefix code. Then the mean codeword length $\bar{l}$ (which is equal to the rate) is bounded by

$$\bar{l} \geq -\sum_{i=1}^{L} p_i \cdot \log_2 p_i = H(X_j)$$

Shannon's Coding Theorem

$H(X_j)$ is called the *entropy* of the source.

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$$
\begin{aligned}
H(X_j) - \bar{l} &= -\sum_{i=1}^{L} p_i \cdot \log p_i - \sum_{i=1}^{L} p_i \cdot l_i = \sum_{i=1}^{L} p_i \cdot (\log \frac{1}{p_i} - l_i) \\
&= \sum_{i=1}^{L} p_i \cdot (\log \frac{1}{p_i} - \log 2^{l_i}) = \sum_{i=1}^{L} p_i \cdot \log \frac{2^{-l_i}}{p_i} \\
&\le \frac{1}{\ln 2} \sum_{i=1}^{L} p_i \cdot (\frac{2^{-l_i}}{p_i} - 1) = \frac{1}{\ln 2} (\sum_{i=1}^{L} 2^{-l_i} - \sum_{i=1}^{L} p_i) \\
&\le \frac{1}{\ln 2} (1 - 1) = 0
\end{aligned}
$$

where we used the fact that $\ln x \le x - 1$ and Kraft's inequality.

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Efficiency of Huffman Coding

$$H(z)/L(z)$$

Variants of Huffman Coding

- Higher order estimate of entropy
- Truncated Huffman Coding
- Dynamic or Adaptive Huffman Coding

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Arithmetic Coding

Basic Idea:

a) Like Huffman coding requires prior knowledge of probabilities

b) Unlike Huffman coding, which assigns variable length codes to symbols arithmetic coding assigns codes to a variable group of symbols i.e. the message.

c) There is no one-to-one correspondence between the symbol and its corresponding code word.

d) The code word itself defines a real number within the half-open interval [0,1) and as more symbols are added, the interval is divided into smaller and smaller subintervals, based on the probabilities of the added symbols.

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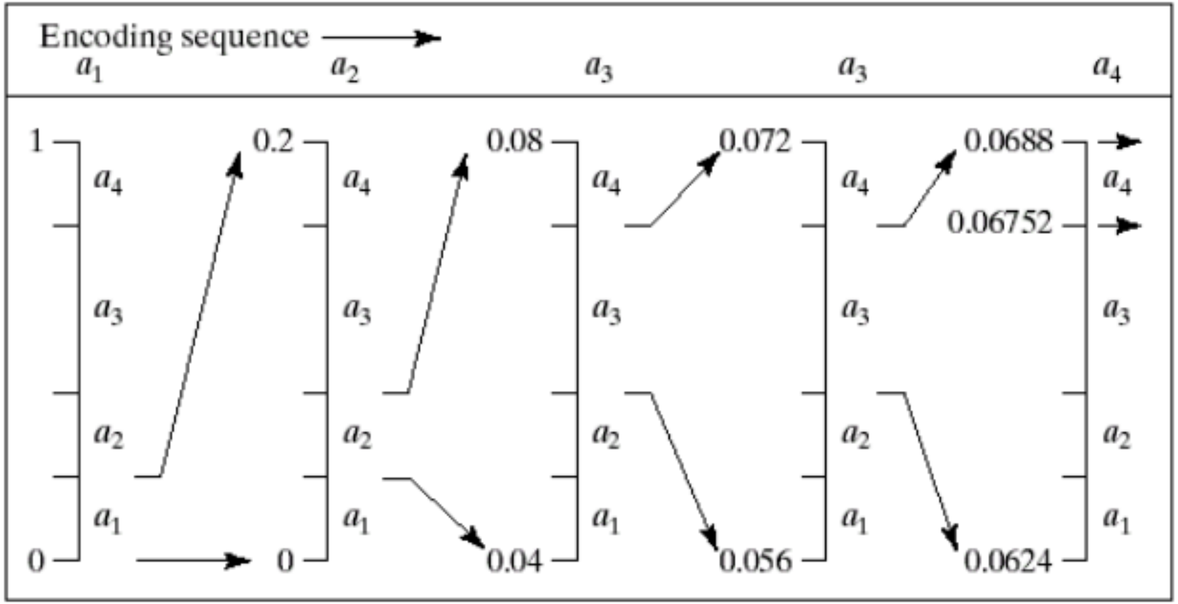
Arithmetic Coding

Source Symbol	Probability	Initial Subinterval
a_1	0.2	[0.0, 0.2)
a_2	0.2	[0.2, 0.4)
a_3	0.4	[0.4, 0.8)
a_4	0.2	[0.8, 1.0)

End of message or length of message is known.

Source: Digital Image Processing, Gonzalez and Woods.

Arithmetic Coding



Final code 068

Source: Digital Image Processing, Gonzalez and Woods.

Arithmetic Decoding

Follows encoding procedure

- Code 068 may be converted to the real number 0.068, which falls in the first sub-interval [0,0.2) therefore first symbol is a1, and so on.

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Dictionary based methods

- Compressing multiple strings can be more efficient than compressing single symbols only (e.g. Huffman encoding).
- Strings of symbols are added to a dictionary. Later occurrences are referenced.
- Static dictionary: Entries are predefined and constant according to the application of the text
- Adaptive dictionary: Entries are taken from the text itself and created on-the-fly

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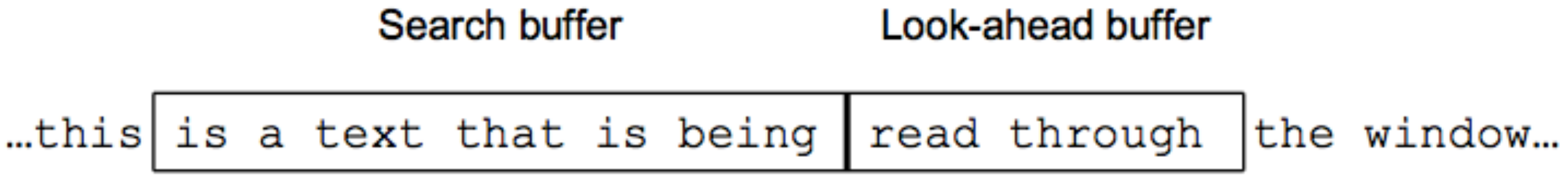
Dictionary based methods: LZ77

By Lempel and Ziv in 1977 about lossless compression with an adaptive dictionary.

- Runs through the text in a sliding window
- Two buffers are used - search (history) buffer and a look ahead buffer.
- The search buffer is used as dictionary
- Sizes of these buffers are parameters of the design



Source: http://jens.jm-s.de/comp/LZ77-JensMueller.pdf

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Dictionary based methods: LZ77

Encoding of the string:
abracadabrad

output tuple: (offset, length, symbol)

Search ⟷ | Look-ahead ⟷

	7	6	5	4	3	2	1							output
								a	b	r	a	c	ada...	(0,0,a)
							a	b	r	a	c	a	dab...	(0,0,b)
						a	b	r	a	c	a	d	abr...	(0,0,r)
					a	b	r	a	c	a	d	a	bra...	(3,1,c)
			a	b	r	a	c	a	d	a	b	r	ad	(2,1,d)
	a	b	r	a	c	a	d	a	b	r	a	d		(7,4,d)
...ac	a	d	a	b	r	a	d							

Source: http://jens.jm-s.de/comp/ LZ77-JensMueller.pdf

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Dictionary based methods: LZ77

Decoding

input		7	6	5	4	3	2	1
(0,0,a)								a
(0,0,b)							a	b
(0,0,r)						a	b	r
(3,1,c)				a	b	r	a	c
(2,1,d)		a	b	r	a	c	a	d
(7,4,d)	abrac	a	d	a	b	r	a	d

Source: http://jens.jm-s.de/comp/
LZ77-JensMueller.pdf

Dictionary based methods: LZW

Extended by Welch (Lempel, Ziv and Welch)

This coding scheme has been adopted in a variety of imaging file formats, such as the graphic interchange format (GIF), tagged image file format (TIFF) and the portable document format (PDF).

Dictionary based methods: LZW

Extended by Welch (Lempel, Ziv and Welch)

- Unlike Huffman coding and arithmetic coding, this coding scheme does not require a priori knowledge of the probabilities of the source symbols.
- The coding is based on a “dictionary” or “codebook” containing the source symbols to be encoded. The coding starts with an initial dictionary, which is enlarged with the arrival of new symbol sequences.
- There is no need to transmit the dictionary from the encoder to the decoder. The decoder builds an identical dictionary during the decoding process

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Dictionary based methods: LZW

Extended by Welch (Lempel, Ziv and Welch)

Example: 32 32 34 32 34 32 32 33 32 32 32 34

Consider a dictionary of size 256 locations (numbered 0 to 255) that contains entries corresponding to each pixel intensity value in the range 0-255.

Source: https://nptel.ac.in/courses/117/105/117105083/#

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Dictionary based methods: LZW

Extended by Welch (Lempel, Ziv and Welch)

Currently Recognized Sequence	Pixel being processed	Encoded Output	Dictionary Location (Code word)	Dictionary Entry
	32			
32	32	32	256	32-32
32	34	32	257	32-34
34	32	34	258	34-32
32	34			
32-34	32	257	259	32-34-32
32	32			
32-32	33	256	260	32-33
33	32	33	261	33-32
32	32			
32-32	32	256	262	32-32-32
32	34			
32-34		257		

Source: https://nptel.ac.in/courses/117/105/117105083/#

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Run Length Coding

Run: a string of the same symbol

Example

input: AAABBCCCCCCCCCAA
output: A3B2C9A2

compression ratio = 16/8 = 2

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Predictive Coding

Basic premise: Current pixel is similar to the previous pixel (coherence)

Differential Coding

$$d(x,y) = I(x,y) - I(x-1,y)$$

$d(x,y)$ prediction error which is to be encoded.

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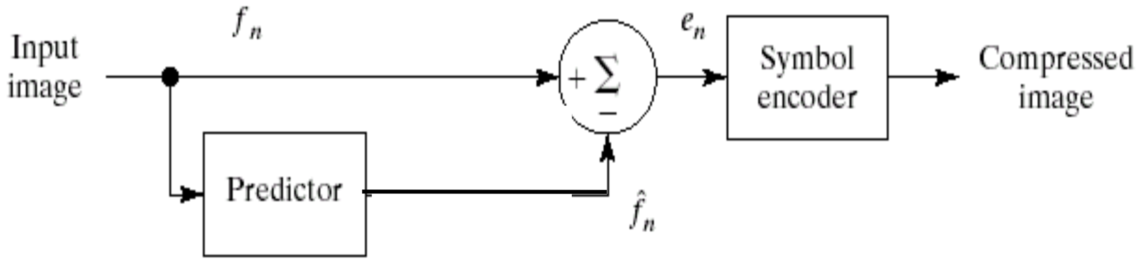
Predictive Coding

Compression

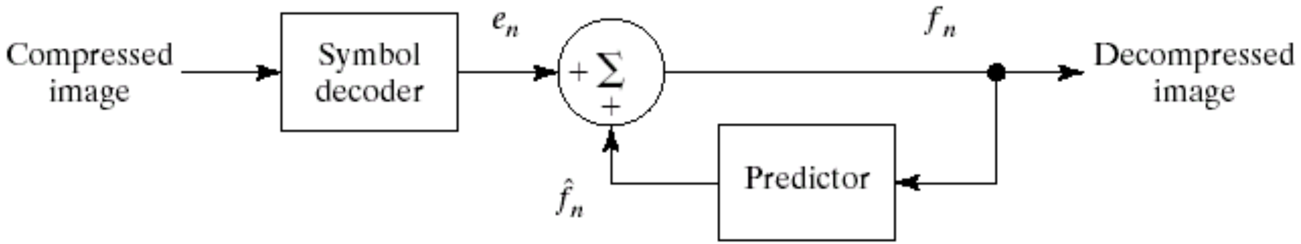


Source: Digital Image Processing, Gonzalez and Woods.

Image Compression

Predictive Coding

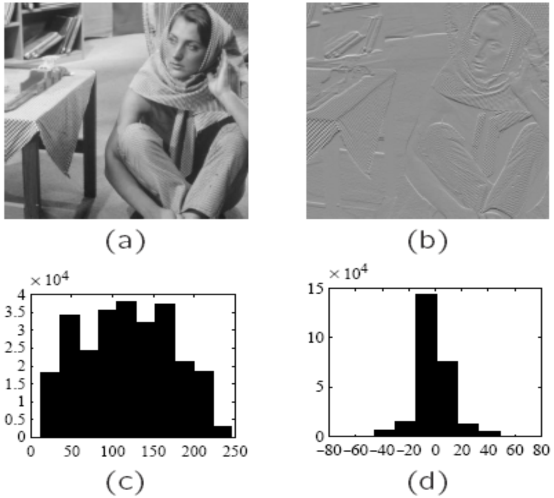
Decompression



Source: Digital Image Processing, Gonzalez and Woods.

Image Compression

Predictive Coding



Distributions for Original versus Derivative Images. (a,b): Original gray-level image and its partial derivative image; (c,d): Histograms for original and derivative images.