Part 3: 2D Transformation

1. **What do you understand by geometric transformation? Also define the following operation performed by it-**
   a. **Translation.**
   b. **Rotation.**
   c. **Scaling.**
   d. **Reflection.**
2. **Explain two dimensional Translation and Scaling with an example.**
3. **Obtain a transformation matrix for rotating an object about a specified local point.**
4. **Explain DDA line drawing algorithm.**
5. **Explain the steps in midpoint ellipse drawing algorithm.**
6. **What is polygon clipping? Explain Sutherland-Hodgeman algorithm for polygon.**
7. **Consider a triangle ABC whose coordinates are A[4,1], B[5,2], C[4,3]**
   a. **Reflect the given triangle about X axis.**
   b. **Reflect the given triangle about Y-axis.**
   c. **Reflect the given triangle about Y=X axis.**
   d. **Reflect the given triangle about X axis.**

8. **Why are homogeneous coordinate system used for transformation computation in computer graphics?**
9. **Magnify the triangle with vertices A(0,0),B(1,1) and C(5,2) to twice its size while keeping C(5,2) fixed?**
10. **Why are homogeneous coordinates needed in transformation matrices?**
11. **Derive the transformation matrix for rotation about origin by angle in an anticlockwise direction.**
12. **Derive the homogenous matrices to represent the following transformation**

Translation - Rotation about the origin – Scaling

Homogenous rotation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogenous scaling

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} xsf & 0 & 0 \\ 0 & ysf & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogenous translation

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

13. Discuss the difference between the rotation about the origin and the rotation about a local point.

Time for some trigonometry:

$$x_2 = r\cos(\theta + \varphi)$$

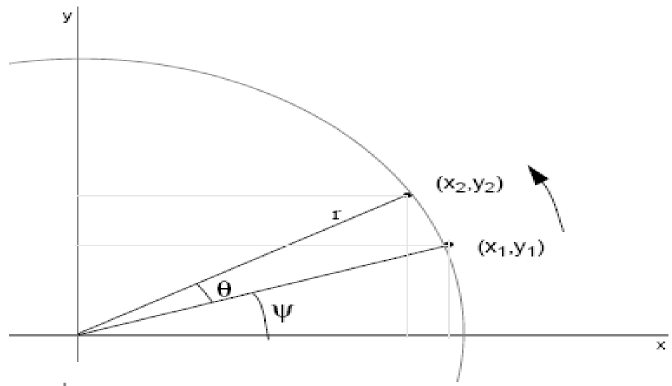
$$y_2 = r\sin(\theta + \varphi)$$

$$x_2 = r(\cos\theta\cos\varphi - \sin\theta\sin\varphi)$$

since $x_1 = r\cos\varphi$ and $y_1 = r\sin\theta$

$$x_2 = x_1\cos\theta - y_1\sin\theta$$

similarly: $y_2 = x_1\sin\theta + y_1\cos\theta$



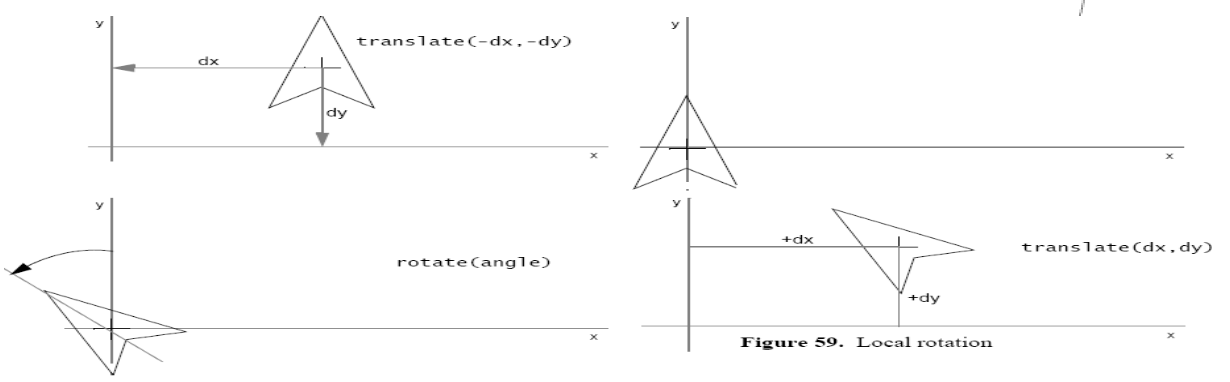


Figure 59. Local rotation

14. Rotate a triangle A(0,0), B(2,2) , C(4,2) about the origin and about P(-2,-2) by an angle of 45 degree.

Sol-

The given triangle ABC can be represented by a matrix, formed from the homogeneous coordinates of the vertices.

$$\begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$R_{45^0} = \begin{bmatrix} cos45^0 & -sin45^0 & 0 \\ sin45^0 & cos45^0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate about P(-2,-2)

The rotation matrix is given by $R_{45^0}.P = T_V.R_{45^0}.T_{-V}$

$$R_{45^0}.P = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\sqrt{2}-2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$(A'B'C') = R_{45^0}.P.(ABC) = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -2 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 2\sqrt{2}-2 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 0 & 2 & 4 \\ 0 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -2 & \sqrt{2}-2 \\ 2\sqrt{2}-2 & 4\sqrt{2}-2 & 5\sqrt{2}-2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$A' = \left(-2, 2\sqrt{2}-2\right), B'\left(-2, 4\sqrt{2}-2\right), C'\left(\sqrt{2}-2, 5\sqrt{2}-2\right)$$

Draw the triangle ABC and the triangle A\, B\, C\

15. A unit square is transformed by 2 x 2 transformation matrix. The resulting position vector are:-

$$\begin{pmatrix} 0 & 2 & 8 & 6 \\ 0 & 3 & 4 & 1 \end{pmatrix}$$

, what is the transformation matrix?

Ans: Suppose the unit square have coordinates

(x , y) (x+1, y) (x+1, y+1) (x , y+1)

and let the transformation matrix be $\begin{pmatrix} a & c \\ b & d \end{pmatrix}$

So,

$$\begin{pmatrix} 0 & 2 & 8 & 6 \\ 0 & 3 & 4 & 1 \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \end{pmatrix} \begin{pmatrix} x & x+1 & x+1 & x \\ y & y & y+1 & y+1 \end{pmatrix}$$

$$= \begin{pmatrix} ax+cy & a(x+1)+cy & a(x+1)+c(y+1) & ax+c(y+1) \\ bx+by & b(x+1)+dy & b(x+1)+d(y+1) & bx+d(y+1) \end{pmatrix}$$

Now, $ax+cy=0$ and $bx+cy=0$

$a(x+1)+cy=2$ and $b(x+1)+dy=3$

$a(x+1)+c(y+1)= 8$ and $b(x+1)+d(y+1) = 4$

$ax+c(y+1) = 6$ and $bx+d(y+1)=1$

from this we get, $a=2$, $b=3$, $c=6$, $d=1$

Thus, the transformation matrix is $\begin{pmatrix} 2 & 6 \\ 3 & 1 \end{pmatrix}$

16. **Find the matrix that represents rotation of an object by 45o about the origin. a. What are the new coordinates of the point P (2, -4) after the rotation?**
17. **A triangle is defined by**

$$\begin{pmatrix} 2 & 4 & 4 \\ 2 & 2 & 4 \end{pmatrix}$$

Find the transformed coordinates after the following transformation

a. **90o rotation about origin.**
b. **Reflection about line y = -x.**

18. **Translate the square ABCD whose co-ordinate are A(0,0), b(3,0), C(3,3), D(0,3) by 2 units in both direction and then scale it by 1.5 units in x direction and 0.5 units in y direction.**

19. **Perform a 45o rotation of a triangle A(0,0, B(1,1), C(5,2)**
    a. **About the origin.**
    b. **About the point p(-1,-1)**

20. **Find the transformation matrix that transforms the square ABCD whose center is at (2, 2) is reduced to half of its size, with center still remaining at (2, 2). The coordinate of square ABCD are A (0, 0), B (0, 4), C (4, 4) and D (4, 0). Find the co-ordinate of new square.**

Ans. (HINT: - After scaling the square to half of its size, the new translated square will have center at (1, 1) so, translate again the new square by (1, 1), so that center again reach to (2, 2).)

21. **Consider the square A (1, 0), B (0, 0), C (0, 1), D (1, 1). Rotate the square ABCD by 45 degree clockwise about A (1, 0).**

HINT:- - First, translate the square by Tx= -1 and Ty=0. - Then rotate the square by 45 degree. - Again translate the square by TX=1 and Ty=0.

22. **Magnify the triangle with vertices A (0, 0), B (1, 1) and C (5, 2) to twice its size while keeping C (5, 2) fixed.**

Ans. HINT:-
1) First, translate the triangle by Tx= -5 and Ty=-2
2) Then magnify the triangle by twice its size
3) Again translate the triangle by Tx= 5 and Ty= 2.

23. **Consider a rectangle A (30, 10), B (60, 10), C (60, 30), D (30, 30). Work out a transformation to rotate the rectangle about point B by 60o anti clockwise. What will be the new coordinate of point D?**
24. **Show that two successive reflections about either of the coordinate's axes is equivalent to a single rotation about the coordinate origin.**

1) *Construct a matrix to rotate -22° about the* x-*axis.*

Using Equation 8.2 on page 108:

$$\mathbf{R}_x(-22^\circ) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos -22^\circ & \sin -22^\circ \\ 0 & -\sin -22^\circ & \cos -22^\circ \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & .927 & -.375 \\ 0 & .375 & .927 \end{bmatrix}$$

2) *Construct a matrix to rotate 30° about the* y-*axis.*

Using Equation 8.3 on page 108:

$$\mathbf{R}_y(30^\circ) = \begin{bmatrix} \cos 30^\circ & 0 & -\sin 30^\circ \\ 0 & 1 & 0 \\ \sin 30^\circ & 0 & \cos 30^\circ \end{bmatrix} \approx \begin{bmatrix} .866 & 0 & -.500 \\ 0 & 1 & 0 \\ .500 & 0 & .866 \end{bmatrix}$$

1. Find the matrix representation of a counter-clockwise rotation by θ degrees about the origin.

   The matrix for counter-clockwise rotation by θ *is given by*

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

   As a check, consider the point $\mathbf{p} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$.

   When rotated by $45°$ *counter-clockwise, we'd expect* $\mathbf{p}$ *to be translated to* $\begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}$.

   We have

$$\begin{aligned} \begin{bmatrix} \cos 45° & -\sin 45° & 0 \\ \sin 45° & \cos 45° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} &= \begin{bmatrix} \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ \sqrt{2} \\ 1 \end{bmatrix}, \end{aligned}$$

   as expected.

2. Find the translation matrix for given displacement vector $[\delta_x \ \delta_y \ 0]^T$

   The matrix for translation by displacement vector $\begin{bmatrix} \delta_x \\ \delta_y \\ 0 \end{bmatrix}$ *is given by*

$$\begin{bmatrix} 1 & 0 & \delta_x \\ 0 & 1 & \delta_y \\ 0 & 0 & 1 \end{bmatrix}.$$

<u>Multiple choice questions</u>

1. The basic transformations include

    a) Translation b) Rotation c) Scaling d) <u>**All of the above**</u>

2. The transformation in which an object is moved in a minimum distance path from one position to another is called

    a) Rotation b) Replacement <u>**c) Translation**</u> d) Scaling

3. The translation distances (dx, dy) is called as

    a) Translation vector b) Shift vector c<u>**) Both a and b**</u> d) Neither a nor b

4. The two-dimensional translation equation in the matrix form is

    <u>**a) P'=P+T**</u> b) P'=P-T c) P'=P*T d) P'=p

5. The transformation in which an object is moved from one position to another in circular path around a specified pivot point is called

    <u>**a) Rotation**</u> b) Shearing c) Translation d) Scaling

6. The transformation in which the dimension of an object are changed relative to a specified fixed point is called

    a) Rotation b) Reflection c) Translation <u>**d) Scaling**</u>

7. The transformation that produces a parallel mirror image of an object are called

    a) Rotation <u>**b) Reflection**</u> c) Translation d) Scaling

8. If a point (x, y) is reflected about an axis which is normal to the XY plane and passing through the origin, the reflected point (X,Y) is:-

    a) (x,-y) b) (-x, y) <u>**c) (-x,-y)**</u> d) (y, x)

9. Reflection of a point about x-axis, followed by a counter-clockwise rotation of 900, is equivalent to reflection about the line?

    a) x= -y b) x=0 <u>**c)x=y**</u> d) x + y=1

10. A circle, if scaled only in one direction becomes a:

    a) Hyperbola **b) Ellipse** c) Parabola d) remains a circle

11. (2, 4) is a point on a circle that has center at the origin. Which of the following points are also on circle?

a) (2,-4) b) (-2, 4) c) (-4,-2) **d) All of above**

12. Which technique of color CRT is used for production of realistic image

a) Beam penetration **b) Shadow mask** c) both a & b d) None of above

13. A composite transformation matrix can be made by determining the ________of matrix of the individual transformation

a) Sum **b) Product** c) Difference d) None of the above

14. Which of the following is not a rigid body transformation?

a) Translation b) Rotation **c) Shearing** d) Reflection

14. Forming products of transformation matrices is often referred as

a) Concatenation b) Composition **c) both a & b** d) None of above

15. Two consecutive translation transformation t1 and t2 are

a) Additive b) Multiplicative c) Subtractive d) none of above

16. Two consecutive rotation transformation r1 and r2 are

a) Additive **b) Multiplicative** c) Subtractive d) none of above

17. Two consecutive scaling transformation s1 and s2 are

a) Additive **b) Multiplicative** c) Subtractive d) none of above

Part 4: Viewing and Clipping

1. **Define window, viewport, user and screen coordinate. Derive window to viewport transformation matrix?**
2. **Explain Sutherland Hodgeman polygon clipping algorithm. Explain the disadvantage of it and how to rectify this disadvantage. Explain Two Dimensional Viewing.**
3. **At R be Rectangular window whose lower left head corner is at l(-3,1) and upper right head corner is at R(2,6).Find the region codes for the endpoints A(-4,2),B(-1,7), C(-1,5),D(3,8),G(1,-2),H(3,3),I(-4,7) and J(-2,10).**
4. **Using Cohen-Sutherland algorithm find the co-ordinates of the line joining (-1, 2) and (9, 7) which is visible in the rectangle (0, 0), (0, 5) and (10, 5), (10, 0).**

Multiple choice questions

1. The process of mapping a world window in world coordinate system to viewport are called

a) Transformation viewing b) Viewport c) Clipping window d) Screen coordinate system

2. The process of extracting a portion of a database or a picture inside or outside a specified region are called

a) Transformation b) Projection **c) Clipping** d) Mapping

3. The rectangle portion of the interface window that defines where the image will actually appear are called

a) Transformation viewing **b) View port** c) Clipping window d) Screen coordinate system

4. Coordinates of window are knows as .... a) Screen coordinates **b) World coordinates** c) Device coordinates d) Cartesian coordinates

5. Coordinates of viewport are known as ....

a) World coordinates b) Polar coordinates **c) Screen coordinates** d) Cartesian coordinates

6. The region against which an object is clipped is called a ....

a) Clip window b) Boundary c) Enclosing rectangle d) Clip square

7. The region code of a point within the window is ....

a) 1111 **b) 0000** c)1000 d)0001

8. According to Cohen-Sutherland algorithm, a line is completely outside the window if ....

a) **The region codes of line endpoints have a '1' in same bit position.**

b) The endpoints region code are nonzero values

c) If L bit and R bit are nonzero.

d) The region codes of line endpoints have a '0' in same bit position.

9. The region code of a point is 1001. The point is in the .... region of window.

a) Top right **b) Top left** c) Bottom left d) Bottom right

10. The result of logical AND operation with endpoint region codes is a nonzero value. Which of the following statement is true?

a) The line is completely inside the window

b) **The line is completely outside the window**

c) The line is partially inside the window

d) The line is already clipped

11. The left (L bit) bit of the region code of a point (X, Y) is '1' if ....

a) X > XWMIN **b) X< XWMIN** c) X< XWMAX d) X>XWMAX

12. The right bit (R bit) of the region code of a point (X, Y) is '1' if ....

a) X > XWMIN b) X< XWMIN c) X< XWMAX **d) X>XWMAX**

13. The Most Significant Bit of the region code of a point (X, Y) is '1' if ....

a) Y > YWMIN b) Y< YWMIN c) Y< YWMAX **d) Y>YWMAX**

14. The bottom bit of the region code of a point is '0' if ....

a) Y > YWMIN **b) Y< YWMIN** c) Y< YWMAX d) Y>YWMAX

Ans: Y< YWMIN

15. The .... algorithm divides a 2D space into 9 regions, of which only the middle part (viewport) is visible.

a) Cohen-Sutherland b) Liang Barsky c) Sutherland Hodegeman d) N-L-N

16. A method used to test lines for total clipping is equivalent to the ....

a) logical XOR b) logical OR **c) logical AND** d) both a & b