Database Management Systems

Course Code: CSE309

2022

Chapter 14: Indexing and Hashing

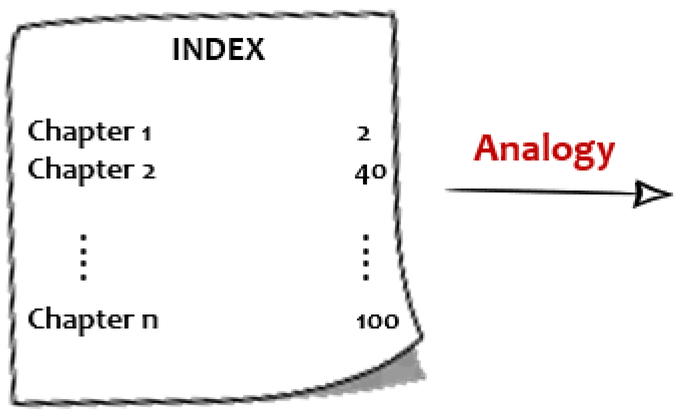
Database System Concepts, 7th Ed.

Outline

- Basic Concepts
- Ordered Indices
- B^+-Tree Index Files
- B-Tree Index Files
- Hashing
- Write-optimized indices
- Spatio-Temporal Indexing

Basic Concepts

- Indexing mechanisms used to speed up access to desired data.
  - E.g., author catalog in library
- An **index file** consists of records (called **index entries**) of the form:



Index page of a book

Search Key	Block pointer
1	Address of block 1
10	Address of block 2
21	Address of block 3

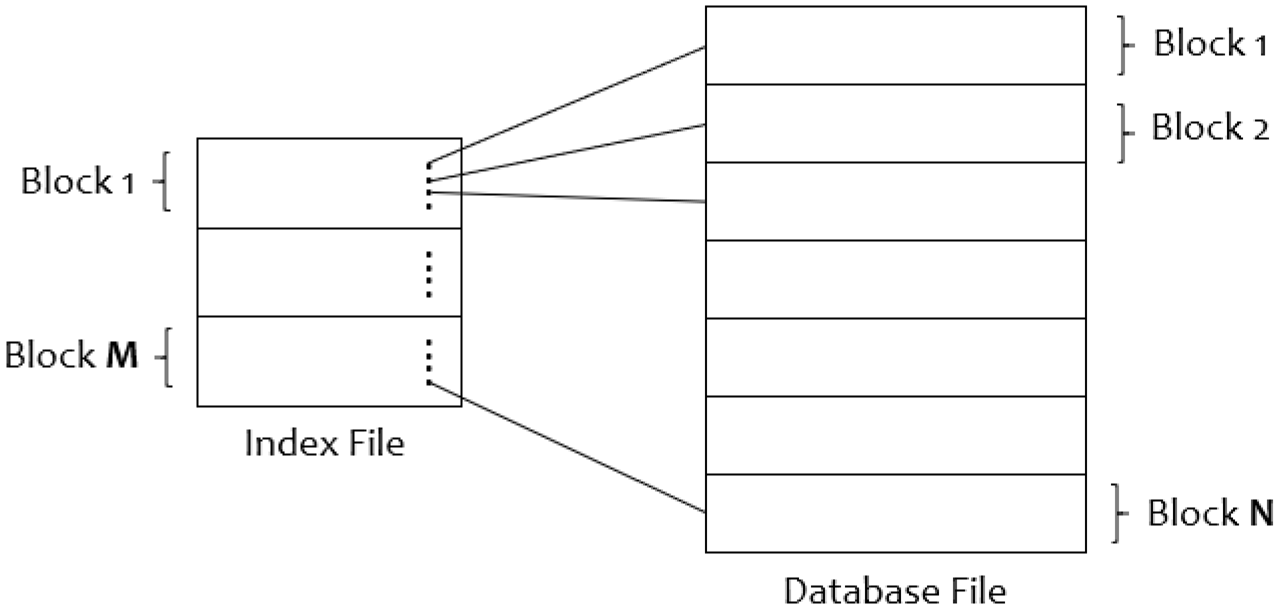
Index file of a database file

- **Search Key** - an attribute or a set of attributes used to look up records in a file.
- **Data Reference or Pointer -** contains a set of pointers holding the address of the disk block where that particular search key value can be found.

Basic Concepts (Cont.)

- Index files are typically much smaller than the original file

  No. of Index Blocks (M) << No. of database file blocks (N)



- Two basic kinds of indices:
  - **Ordered indices:** search keys are stored in sorted order
  - **Hash indices:** search keys are distributed uniformly across "buckets" using a "hash function".

Index Evaluation Metrics

- Access types: The types of access are supported efficiently. E.g.,
  - Records with a specified value in the attribute
  - Records with an attribute value falling in a specified range of values.
- Access time
- Insertion time
- Deletion time
- Space overhead

Types of Indices

- Two basic kinds of indices:
  - **Ordered indices:** search keys are stored in sorted order
  - **Hash indices:** search keys are distributed uniformly across "buckets" using a "hash function".

Ordered Indices

- In an **ordered index,** index entries are stored sorted on the search key value. Two main types of ordered indexing methods are:
- **Primary index:** in a sequentially ordered file, the index whose search key specifies the sequential order of the file.
  - Also called **clustering index**
  - The search key of a primary index is usually but not necessarily the primary key.
- **Secondary index**: an index whose search key specifies an order different from the sequential order of the file.
  - Also called **nonclustering index.**
- **Index-sequential file:** sequential file ordered on a search key, with a clustering index on the search key.
- The **indexing** is also further divided into two types.
  - Dense Index
  - Sparse Index

Dense Index Files

- **Dense index** — Index entry appears for every search-key value in the file.
- E.g. index on *ID* attribute of *instructor* relation

10101	
12121	
15151	
22222	
32343	
33456	
45565	
58583	
76543	
76766	
83821	
98345	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

Dense Index Files (Cont.)

- The index record contains the search-key value and a pointer to the first data record with that search-key value.
- The other records with the same search-key value would be sorted sequentially after the first record.
- Dense index on *dept_name*, with *instructor* file sorted on *dept_name*

Biology	
Comp. Sci.	
Elec. Eng.	
Finance	
History	
Music	
Physics	

76766	Crick	Biology	72000	
10101	Srinivasan	Comp. Sci.	65000	
45565	Katz	Comp. Sci.	75000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	
12121	Wu	Finance	90000	
76543	Singh	Finance	80000	
32343	El Said	History	60000	
58583	Califieri	History	62000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
33465	Gold	Physics	87000	

Sparse Index Files

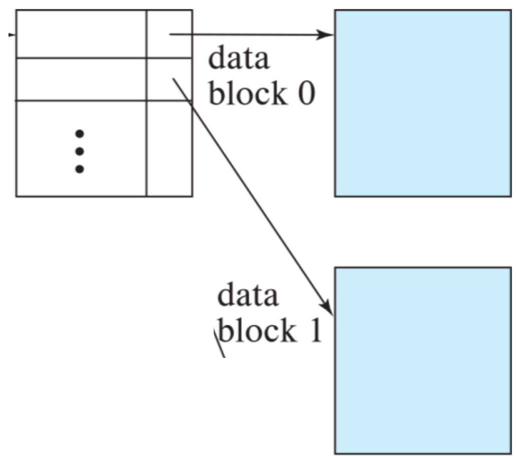
- **Sparse Index**: Index entry appears for only some of the search-key values in the file.
  - Applicable when records are sequentially ordered on search-key
- To locate a record with search-key value *K* we:
  - Find index record with largest search-key value < *K*
  - Search file sequentially starting at the record to which the index record points

10101	
32343	
76766	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
32343	El Said	History	60000	
33456	Gold	Physics	87000	
45565	Katz	Comp. Sci.	75000	
58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

Sparse Index Files (Cont.)

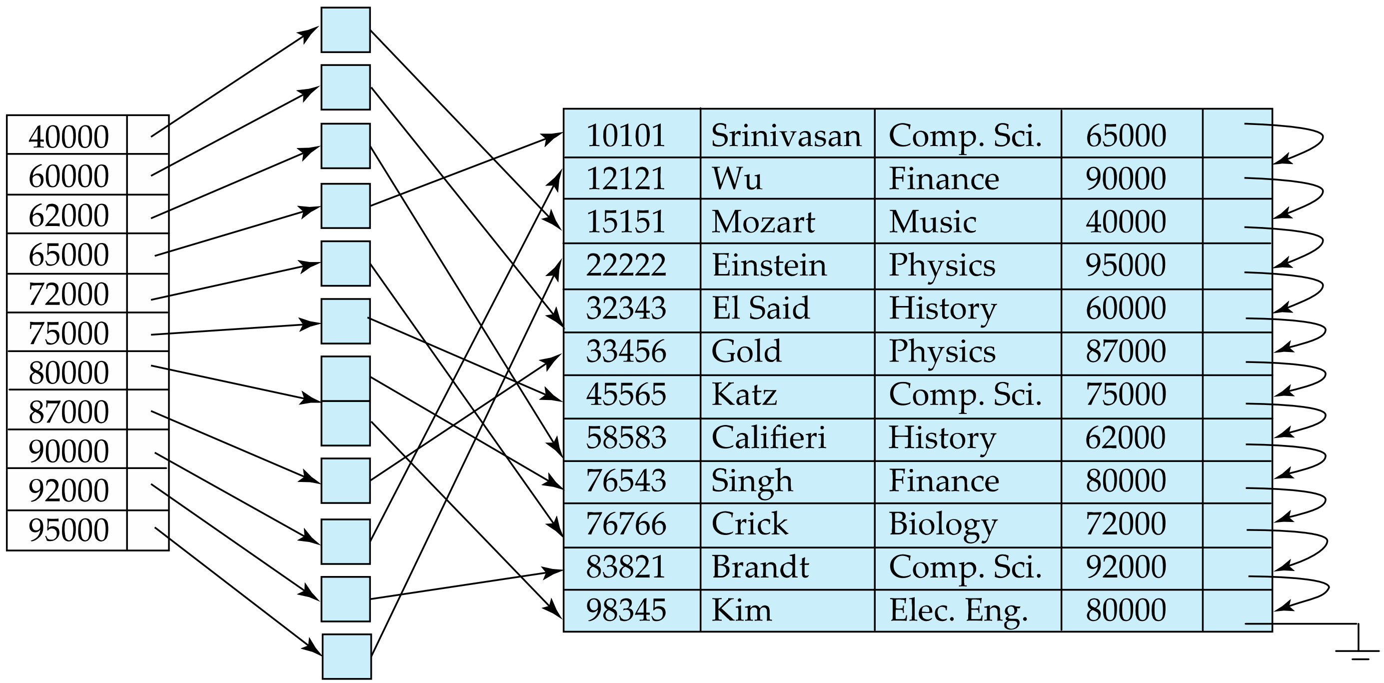
- Compared to dense indices:
  - Less space and less maintenance overhead for insertions and deletions.
  - Generally slower than dense index for locating records.
- **Good tradeoff**:
  - for clustered index: sparse index with an index entry for every block in file, corresponding to least search-key value in the block.



  - For unclustered index: sparse index on top of dense index (multilevel index)

Secondary Indices Example

- Secondary index on salary field of instructor



- Index record points to a bucket that contains pointers to all the actual records with that particular search-key value.
- Secondary indices have to be dense

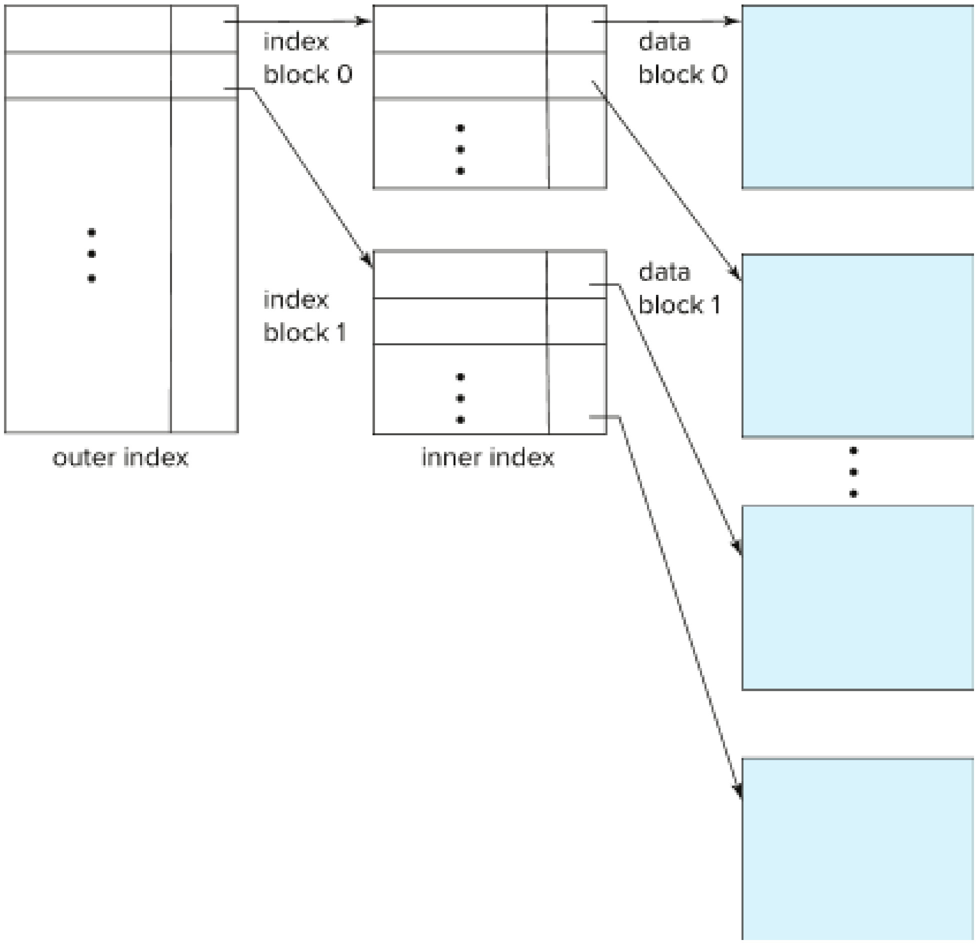
Secondary Indices (Cont.)

- Secondary index has a few drawbacks:
  - Index access takes longer, due to an extra level of indirection
  - If a search-key has very few or no duplicates, the whole block is allocated to its associated bucket, a lot of space would be wasted.

Multilevel Index

- If index does not fit in memory, access becomes expensive.
- Solution: treat index kept on disk as a sequential file and construct a sparse index on it.
  - outer index – a sparse index of the basic index
  - inner index – the basic index file
- If even outer index is too large to fit in main memory, yet another level of index can be created, and so on.
- Indices at all levels must be updated on insertion or deletion from the file.

Multilevel Index (Cont.)



Index Update: Deletion

Biology	
Comp. Sci.	
Elec. Eng.	
Finance	
History	
Music	
Physics	

76766	Crick	Biology	72000	
10101	Srinivasan	Comp. Sci.	65000	
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58583	Califieri	History	62000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
33465	Gold	Physics	87000	

- **Dense indices**
  - If deleted record was the only record in the file with its particular search-key value, the search-key is deleted from the index.
  - If the deleted record was the first record with the search-key value, the system updates the index entry to point to the next record.
  - Otherwise, no update is required in the index

Index Update: Deletion

10101	
32343	
76766	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
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33456	Gold	Physics	87000	
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58583	Califieri	History	62000	
76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

- **Sparse indices**
  - if an entry for the search key exists in the index, it is deleted by replacing the entry in the index with the next search-key value in the file (in search-key order).
  - If the next search-key value already has an index entry, the entry is deleted instead of being replaced.

Index Update: Insertion

Biology
Comp. Sci.
Elec. Eng.
Finance
History
Music
Physics

76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
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76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

- Perform a lookup using the search-key value of the record to be inserted
- **Dense indices**– if the search-key value does not appear in the index, insert it
  - Indices are maintained as sequential files
  - Need to create space for new entry, overflow blocks may be required

Index Update: Insertion

10101	
32343	
76766	

10101	Srinivasan	Comp. Sci.	65000	
12121	Wu	Finance	90000	
15151	Mozart	Music	40000	
22222	Einstein	Physics	95000	
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76543	Singh	Finance	80000	
76766	Crick	Biology	72000	
83821	Brandt	Comp. Sci.	92000	
98345	Kim	Elec. Eng.	80000	

- **Sparse indices -** We assume that the index stores an entry for each block. No change needs to be made to the index unless a new block is created.
  - If a new block is created, the first search-key value appearing in the new block is inserted into the index.

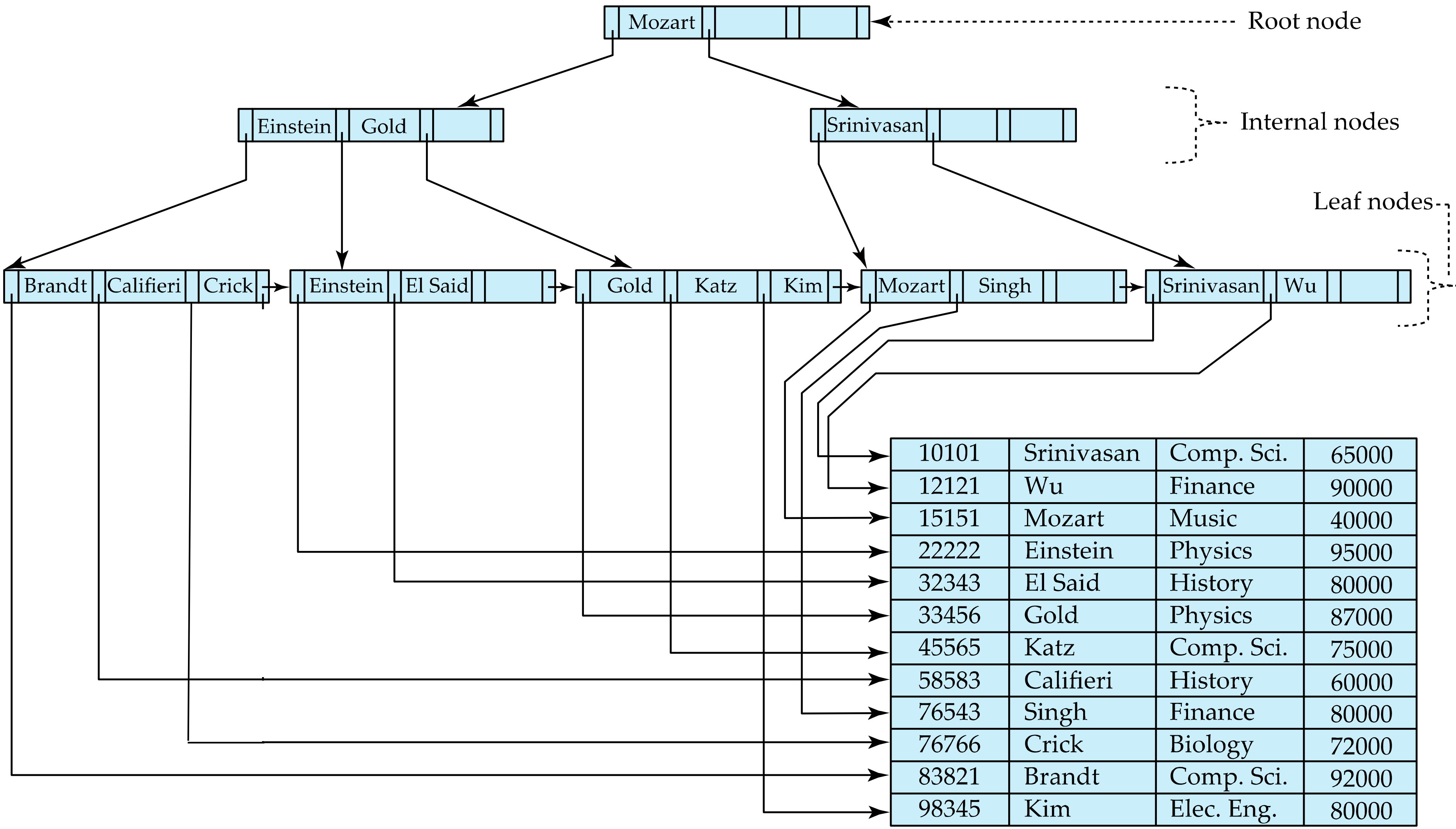
Indices on Multiple Keys

- **Composite search key**
  - E.g., index on *instructor* relation on attributes (*name, ID*)
  - Values are sorted lexicographically
    - E.g. (John, 12121) < (John, 13514) and
      (John, 13514) < (Peter, 11223)
  - Can query on just *name*, or on (*name, ID*)

B^+-Tree Index Files

- Disadvantage of indexed-sequential files
  - Performance degrades as file grows, since many overflow blocks get created.
  - Periodic reorganization of entire file is required.
- Advantage of B^+-tree index files:
  - Automatically reorganizes itself with small, local, changes, in the face of insertions and deletions.
  - Reorganization of entire file is not required to maintain performance.
- (Minor) disadvantage of B^+-trees:
  - Extra insertion and deletion overhead, space overhead.
- Advantages of B^+-trees outweigh disadvantages
  - B^+-trees are used extensively

Example of B+-Tree



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15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	80000
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76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

B⁺-Tree Index Files (Cont.)

A B⁺-tree is a rooted tree satisfying the following properties:

- All paths from root to leaf are of the same length
- Non-leaf nodes other than root have between $\lceil n/2 \rceil$ and n children.
- A leaf node has between $\lceil (n-1)/2 \rceil$ and n–1 values
- Special cases for root:
  - If the root is not a leaf, it has at least 2 children.
  - If the root is a leaf (that is, there are no other nodes in the tree), it can have between 0 and (n–1) values.

B^+-Tree Node Structure

- Typical node

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

  - K_i are the search-key values
  - P_i are pointers to children (for non-leaf nodes) or pointers to records or buckets of records (for leaf nodes).
- The search-keys in a node are ordered

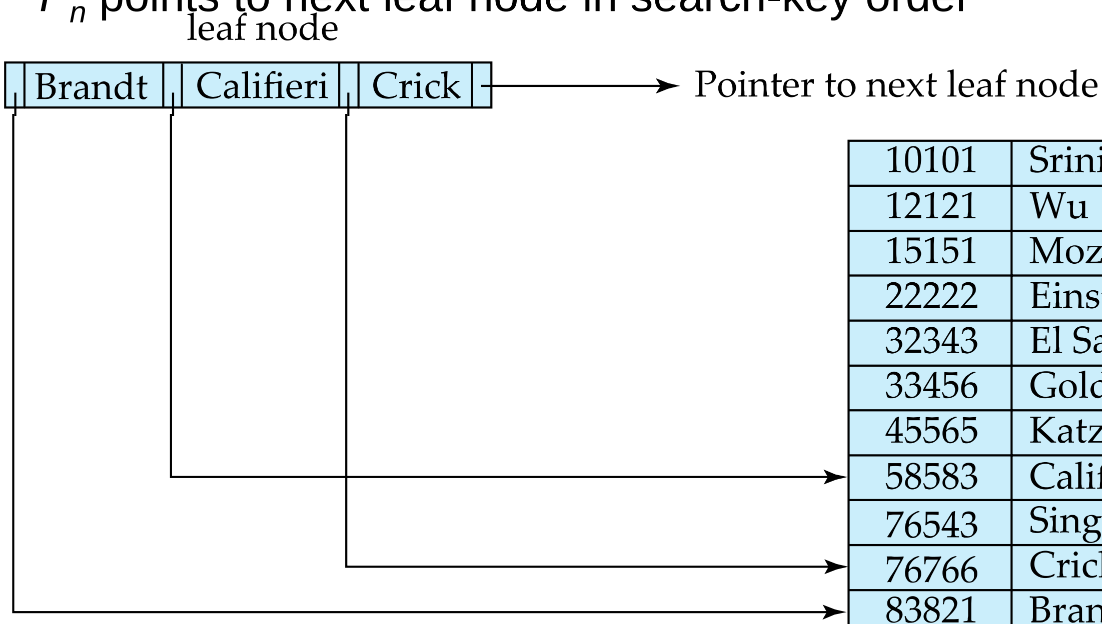
$$K_1 < K_2 < K_3 < \ldots < K_{n-1}$$

(Initially assume no duplicate keys, address duplicates later)

Leaf Nodes in B^+-Trees

Properties of a leaf node:

- For $i = 1, 2, \ldots, n-1$, pointer P_i points to a file record with search-key value K_i,
- If L_i, L_j are leaf nodes and $i < j$, L_i's search-key values are less than or equal to L_j's search-key values
- P_n points to next leaf node in search-key order



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76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
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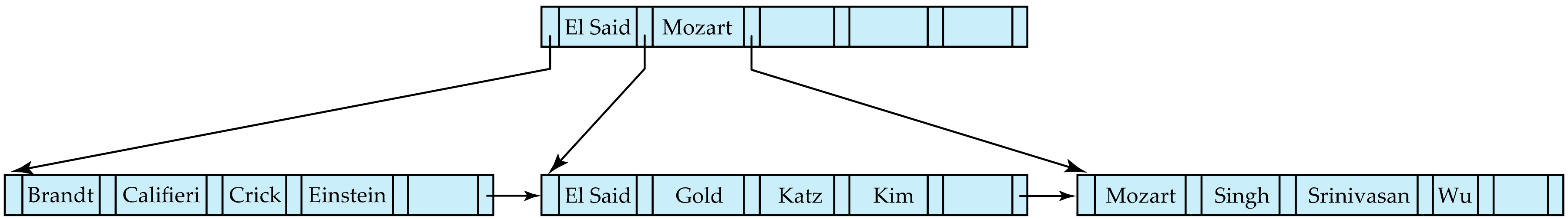
Non-Leaf Nodes in B$^+$-Trees

- Non leaf nodes form a multi-level sparse index on the leaf nodes. For a non-leaf node with *m* pointers:
  - All the search-keys in the subtree to which P_1 points are less than K_1
  - For $2 \le i \le n - 1$, all the search-keys in the subtree to which P_i points have values greater than or equal to K_{i-1} and less than K_i
  - All the search-keys in the subtree to which P_n points have values greater than or equal to K_{n-1}
  - General structure

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

Example of B$^+$-tree

- B$^+$-tree for *instructor* file ($n = 6$)



- Leaf nodes must have between 3 and 5 values ($\lceil (n-1)/2 \rceil$ and $n-1$, with $n = 6$).
- Non-leaf nodes other than root must have between 3 and 6 children ($\lceil (n/2) \rceil$ and n with $n = 6$).
- Root must have at least 2 children.

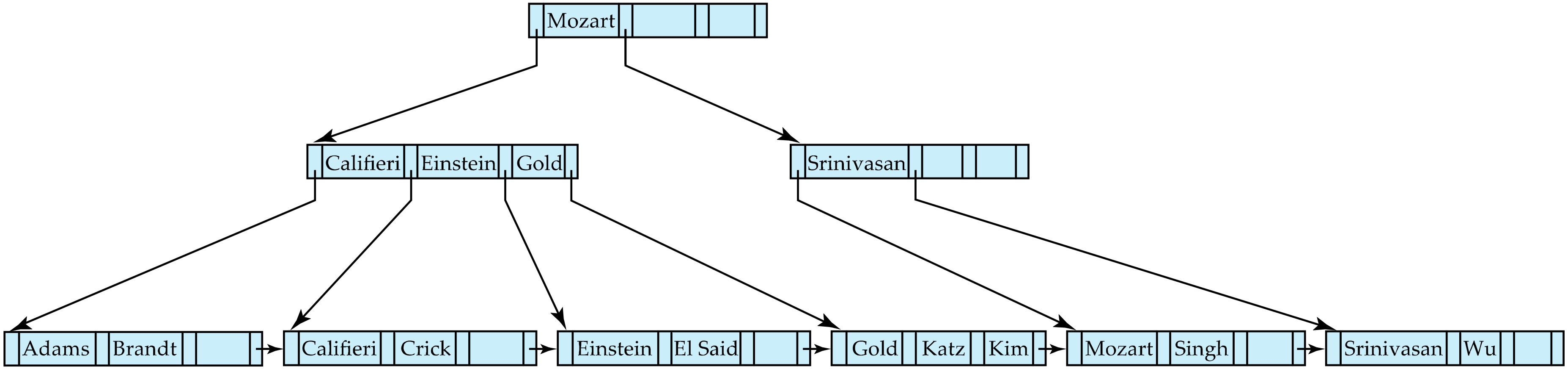
Observations about B+-trees

- Since the inter-node connections are done by pointers, "logically" close blocks need not be "physically" close.
- The non-leaf levels of the B+-tree form a hierarchy of sparse indices.
- The B+-tree contains a relatively small number of levels
    - Level below root has at least $2*\lceil n/2 \rceil$ values
    - Next level has at least $2*\lceil n/2 \rceil * \lceil n/2 \rceil$ values
    - .. etc.
  - If there are K search-key values in the file, the tree height is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$
  - thus searches can be conducted efficiently.
- Insertions and deletions to the main file can be handled efficiently, as the index can be restructured in logarithmic time (as we shall see).

Queries on B^+-Trees

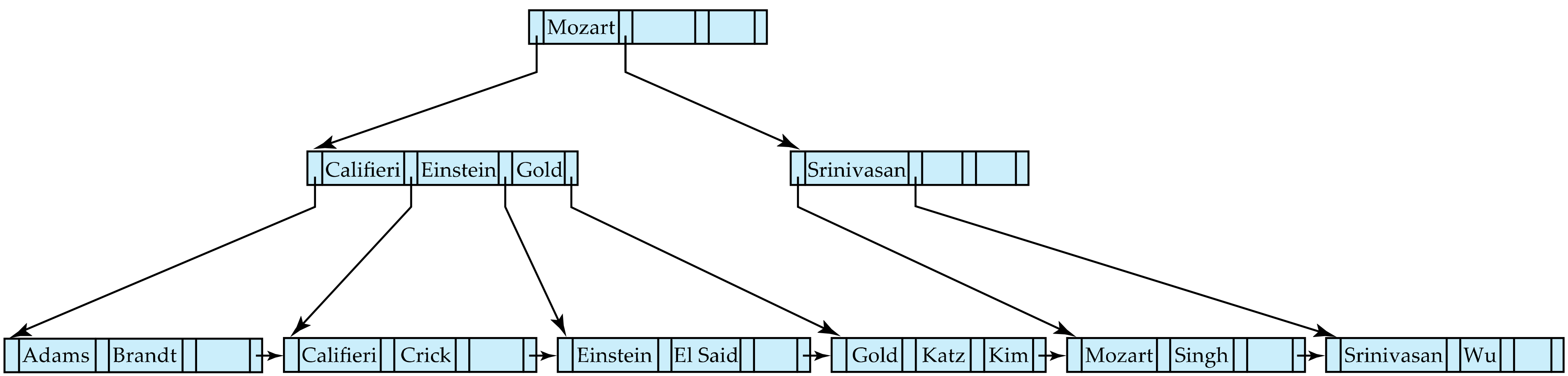
function *find*(*v*)

1. *C=root*
2. **while** (C is not a leaf node)
   1. Let *i* be least number s.t. $V \leq K_i$.
   2. **if** there is no such number *i then*
   3. Set *C* = *last non-null pointer in C*
   4. **else if** (v = C.K_i) Set C = P_{i+1}
   5. **else set** *C* = C.P_i
3. **if** for some *i*, $K_i = V$ **then** return C.P_i
4. **else** return null /* no record with search-key value *v* exists. */



Queries on B$^+$-Trees (Cont.)

- **Range queries** find all records with search key values in a given range
  - See book for details of **function** *findRange*(*lb, ub*) which returns set of all such records
  - Real implementations usually provide an iterator interface to fetch matching records one at a time, using a *next*() function



Queries on B^{+}-Trees (Cont.)

- If there are K search-key values in the file, the height of the tree is no more than $\lceil \log_{\lceil n/2 \rceil}(K) \rceil$.
- A node is generally the same size as a disk block, typically 4 kilobytes
  - and n is typically around 100 (40 bytes per index entry).
- With 1 million search key values and n = 100
  - at most $log_{50}(1,000,000) = 4$ nodes are accessed in a lookup traversal from root to leaf.
- Contrast this with a balanced binary tree with 1 million search key values — around 20 nodes are accessed in a lookup
  - above difference is significant since every node access may need a disk I/O, costing around 20 milliseconds

Non-Unique Keys

- If a search key a_i is not unique, create instead an index on a composite key (a_i, A_p), which is unique
  - A_p could be a primary key, record ID, or any other attribute that guarantees uniqueness
- Search for $a_i = v$ can be implemented by a range search on composite key, with range $(v, -\infty)$ to $(v, +\infty)$
- But more I/O operations are needed to fetch the actual records
  - If the index is clustering, all accesses are sequential
  - If the index is non-clustering, each record access may need an I/O operation

Updates on B$^+$-Trees: Insertion

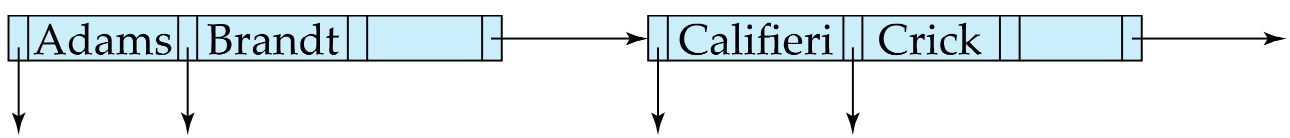
Assume record already added to the file. Let

- *pr* be pointer to the record, and let
- v be the search key value of the record

1. Find the leaf node in which the search-key value would appear
   1. If there is room in the leaf node, insert (v, *pr*) pair in the leaf node
   2. Otherwise, split the node (along with the new (*v, pr*) entry) as discussed in the next slide, and propagate updates to parent nodes.

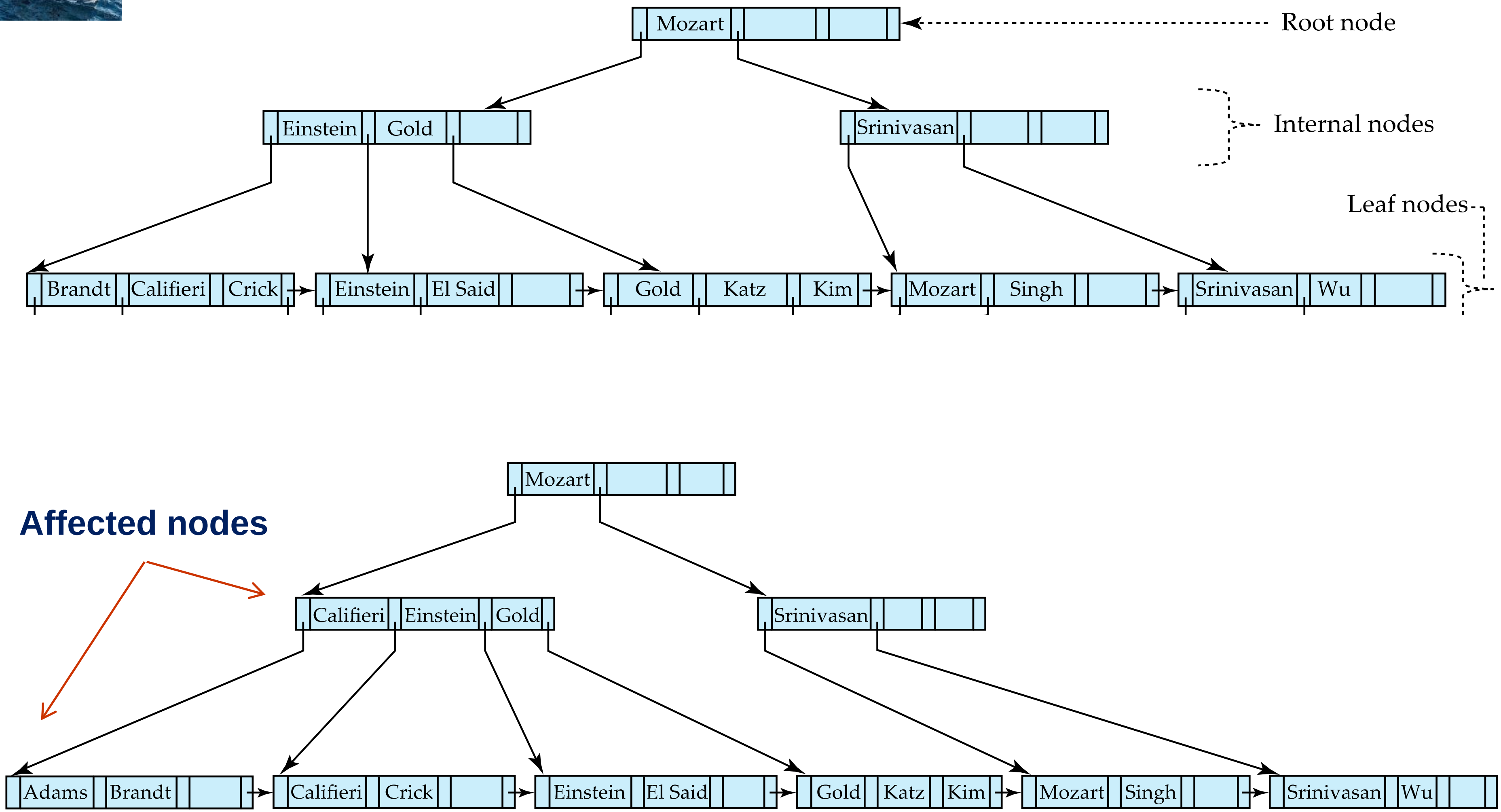
Updates on B+-Trees: Insertion (Cont.)

- Splitting a leaf node:
  - take the *n* (search-key value, pointer) pairs (including the one being inserted) in sorted order. Place the first $\lceil n/2 \rceil$ in the original node, and the rest in a new node.
  - let the new node be *p,* and let *k* be the least key value in *p.* Insert (*k,p*) in the parent of the node being split.
  - If the parent is full, split it and **propagate** the split further up.
- Splitting of nodes proceeds upwards till a node that is not full is found.
  - In the worst case the root node may be split increasing the height of the tree by 1.



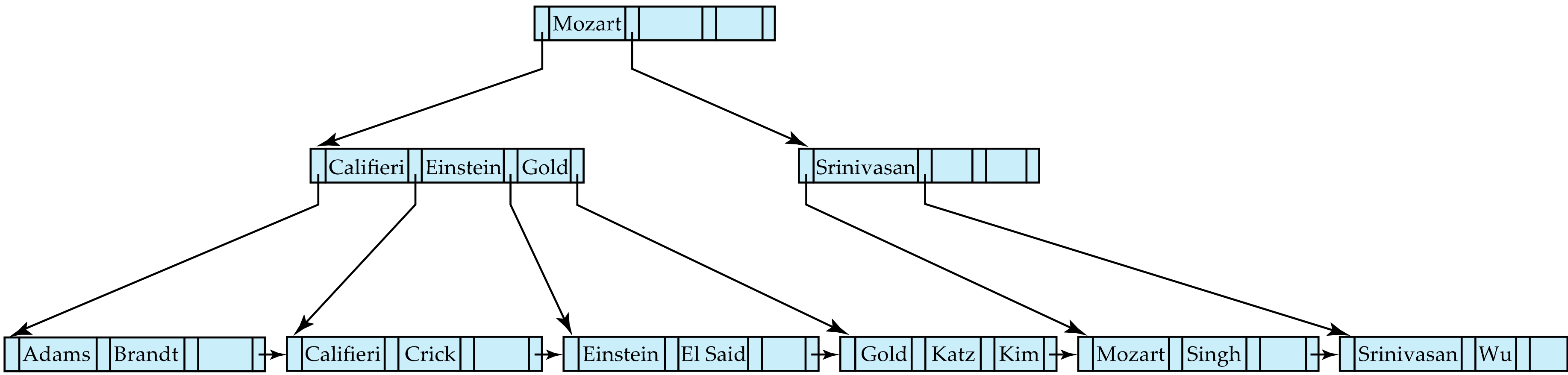
Result of splitting node containing Brandt, Califieri and Crick on inserting Adams
Next step: insert entry with (Califieri, pointer-to-new-node) into parent

B+-Tree Insertion



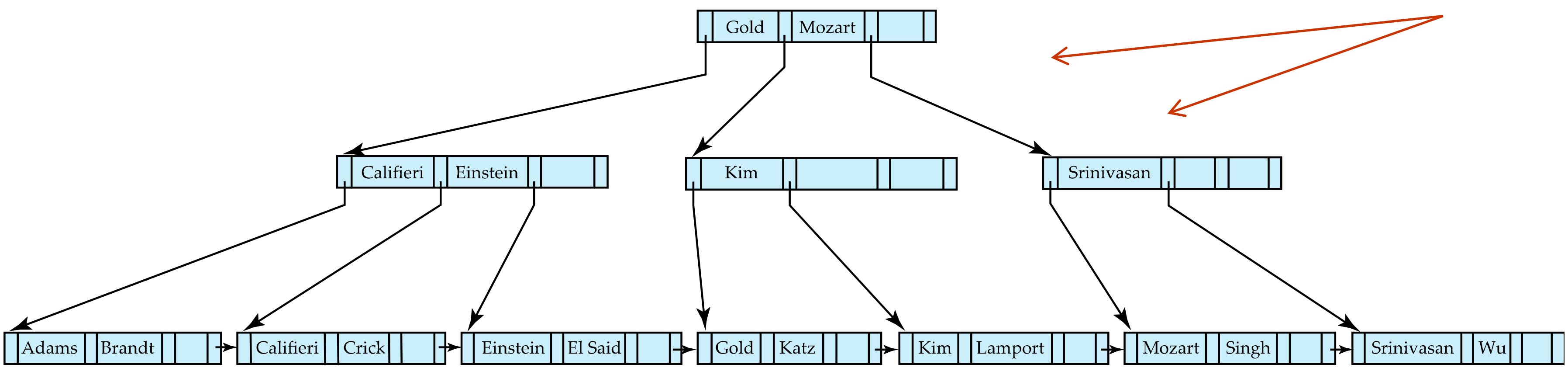
B+-Tree before and after insertion of “Adams”

B⁺-Tree Insertion



B⁺-Tree before and after insertion of "Lamport"

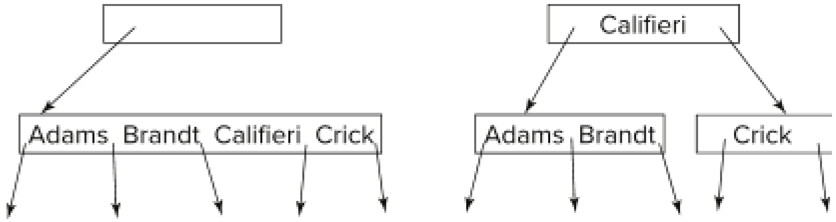
Affected nodes



Affected nodes

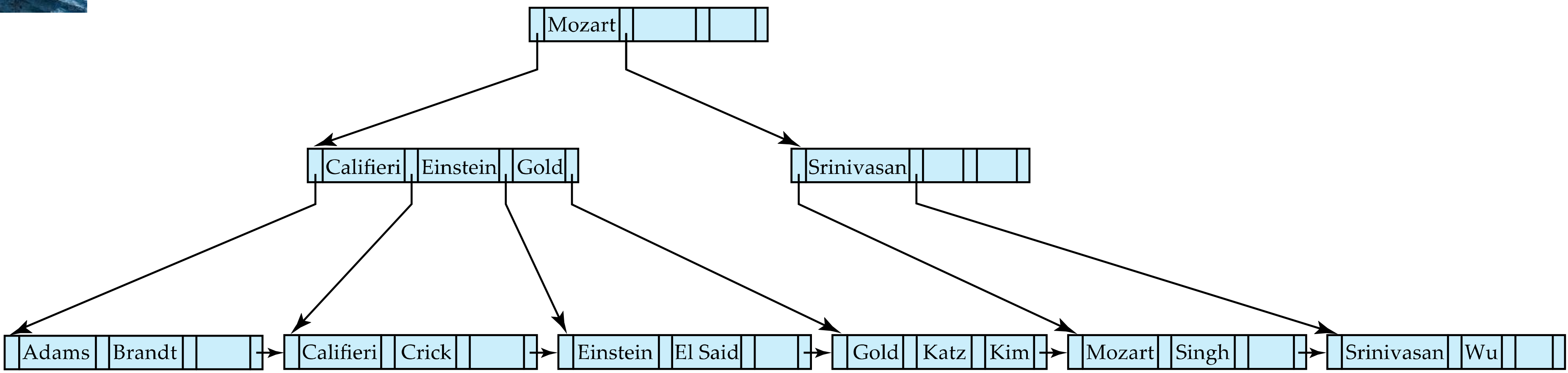
Insertion in B⁺-Trees (Cont.)

- Splitting a non-leaf node: when inserting (k,p) into an already full internal node N
  - Copy N to an in-memory area M with space for n+1 pointers and n keys
  - Insert (k,p) into M
  - Copy $P_1,K_1, \ldots, K_{\lceil n/2 \rceil -1},P_{\lceil n/2 \rceil}$ from M back into node N
  - Copy $P_{\lceil n/2 \rceil +1},K_{\lceil n/2 \rceil +1},\ldots,K_n,P_{n+1}$ from M into newly allocated node N'
  - Insert $(K_{\lceil n/2 \rceil},N')$ into parent N
- Example

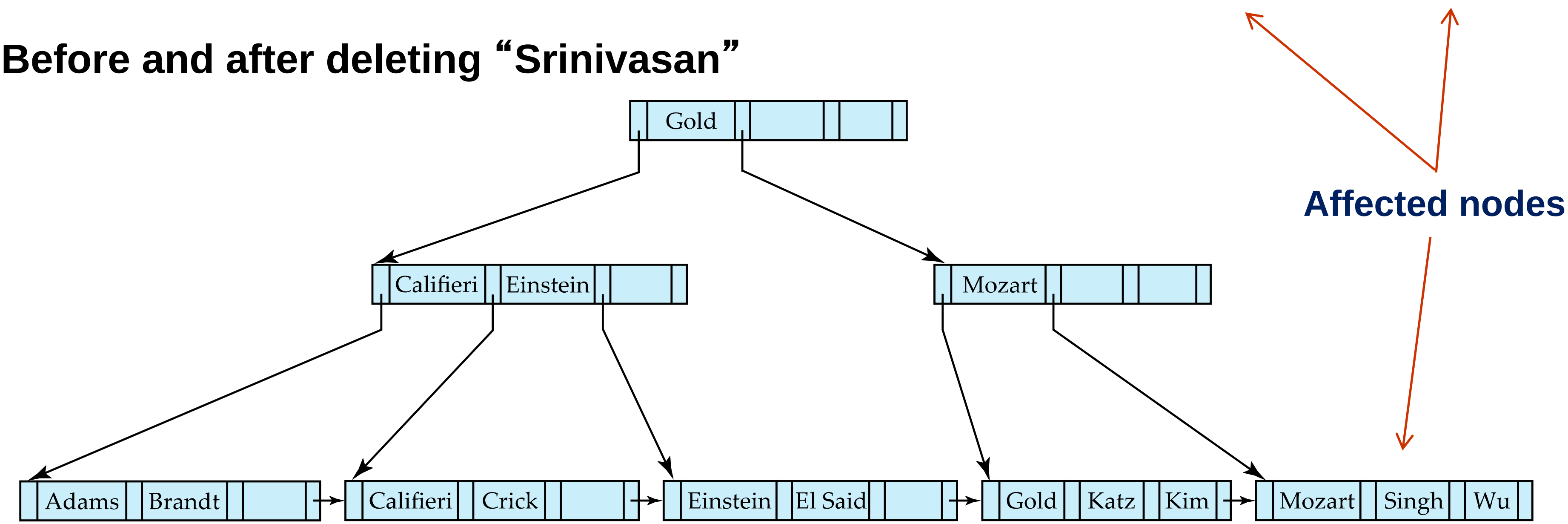


- **Read pseudocode in book!**

Examples of B⁺-Tree Deletion

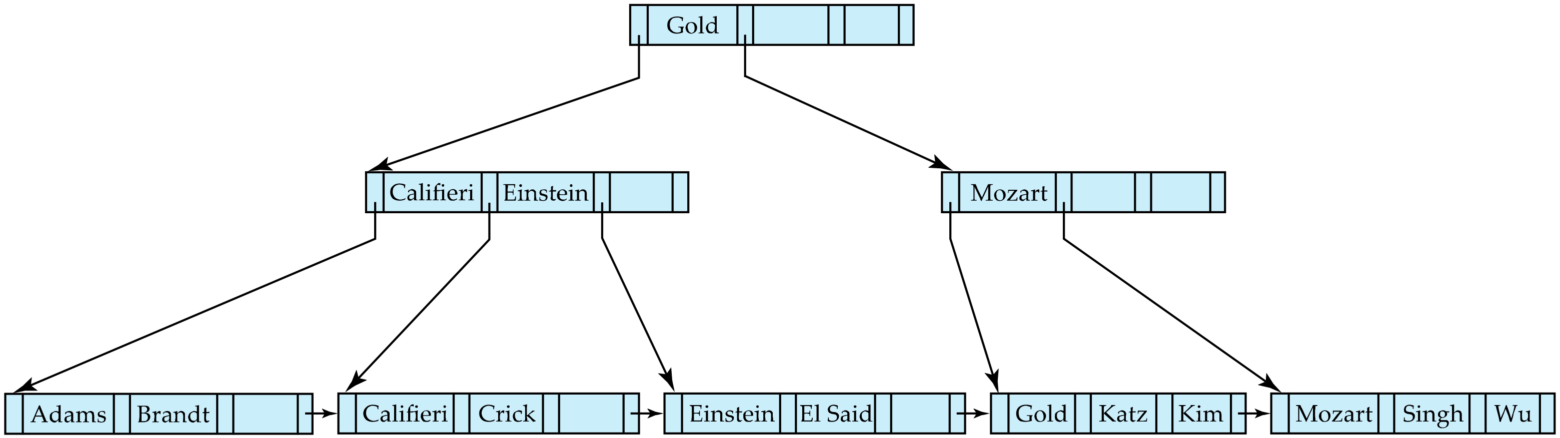


Before and after deleting "Srinivasan"

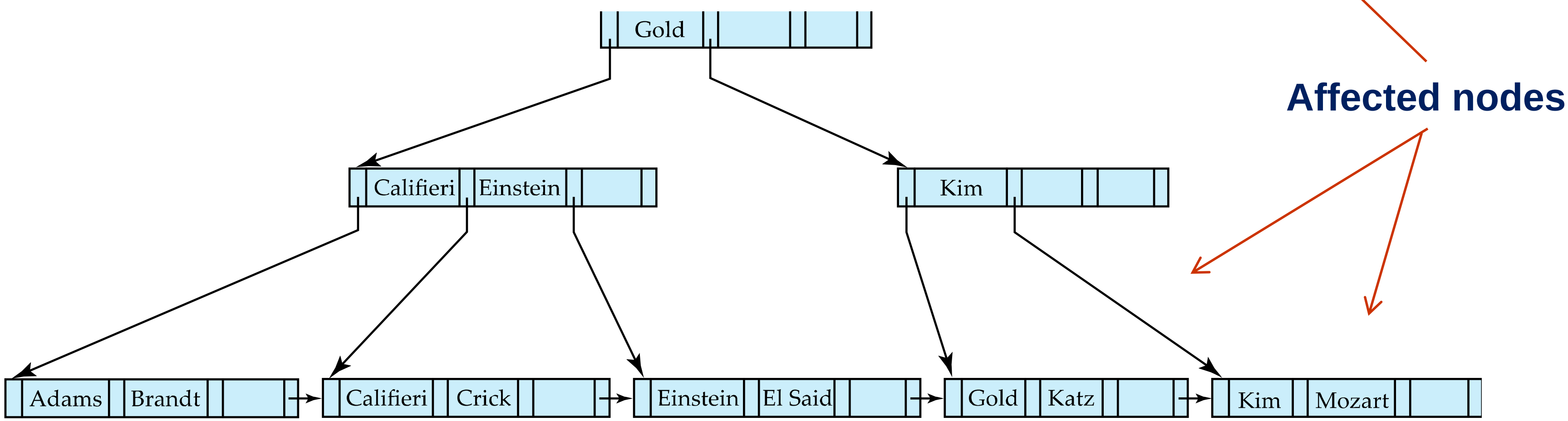


- Deleting "Srinivasan" causes **merging** of under-full leaves

Examples of B⁺-Tree Deletion (Cont.)

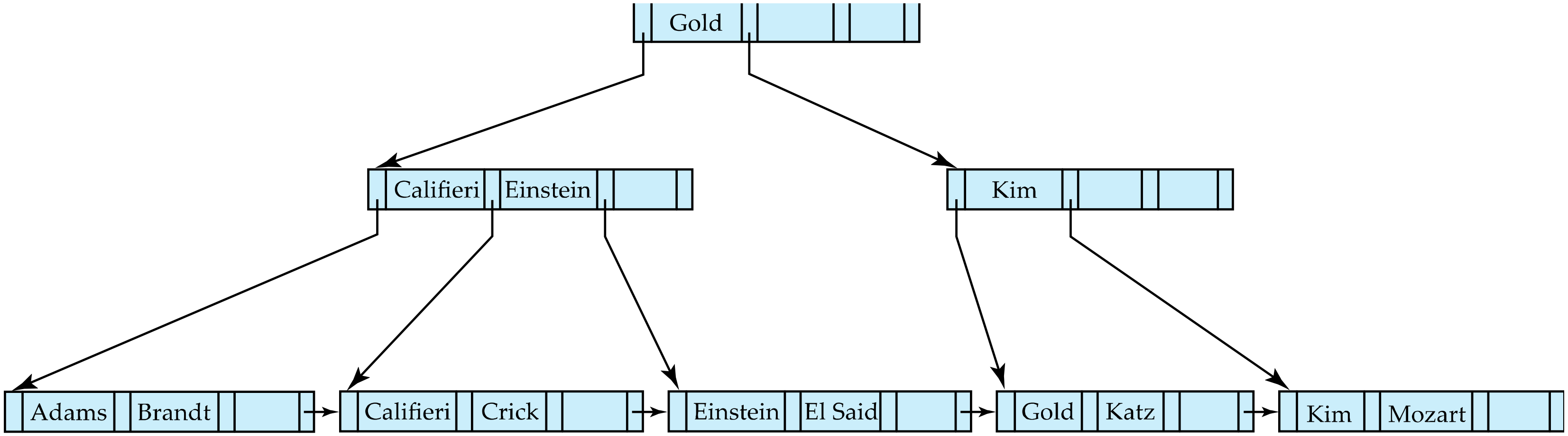


Before and after deleting "Singh" and "Wu"

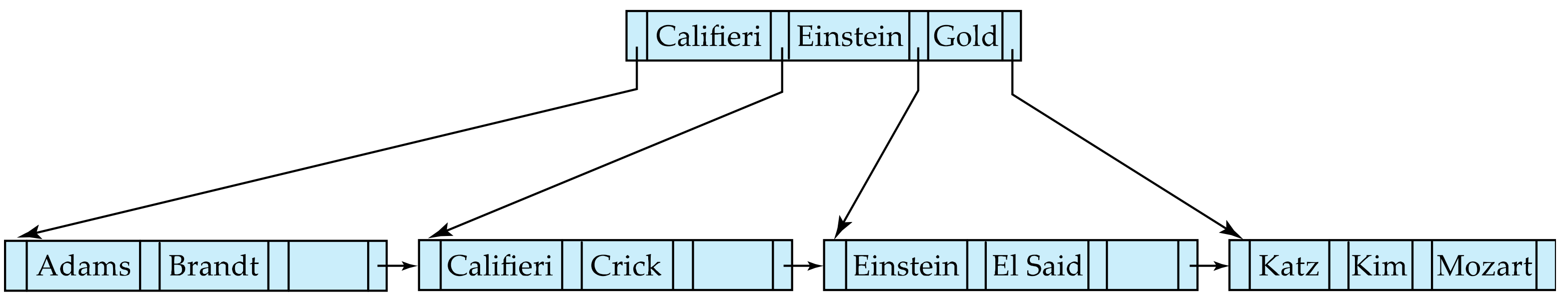


- Leaf containing Singh and Wu became underfull, and **borrowed a value** Kim from its left sibling
- Search-key value in the parent changes as a result

Example of B⁺-tree Deletion (Cont.)



Before and after deletion of "Gold"



- Node with Gold and Katz became underfull, and was merged with its sibling
- Parent node becomes underfull, and is merged with its sibling
  - Value separating two nodes (at the parent) is pulled down when merging
- Root node then has only one child, and is deleted

Updates on B^+-Trees: Deletion

Assume record already deleted from file. Let *V* be the search key value of the record, and *Pr* be the pointer to the record.

- Remove (*Pr, V*) from the leaf node
- If the node has too few entries due to the removal, and the entries in the node and a sibling fit into a single node, then ***merge siblings***:
  - Insert all the search-key values in the two nodes into a single node (the one on the left), and delete the other node.
  - Delete the pair (K_{i-1}, P_i), where P_i is the pointer to the deleted node, from its parent, recursively using the above procedure.

Updates on B$^+$-Trees: Deletion

- Otherwise, if the node has too few entries due to the removal, but the entries in the node and a sibling do not fit into a single node, then **redistribute pointers**:
  - Redistribute the pointers between the node and a sibling such that both have more than the minimum number of entries.
  - Update the corresponding search-key value in the parent of the node.
- The node deletions may cascade upwards till a node which has $\lceil n/2 \rceil$ or more pointers is found.
- If the root node has only one pointer after deletion, it is deleted and the sole child becomes the root.

Complexity of Updates

- Cost (in terms of number of I/O operations) of insertion and deletion of a single entry proportional to height of the tree
  - With K entries and maximum fanout of n, worst case complexity of insert/delete of an entry is $O(\log_{\lceil n/2 \rceil}(K))$
- In practice, number of I/O operations is less:
  - Internal nodes tend to be in buffer
  - Splits/merges are rare, most insert/delete operations only affect a leaf node
- Average node occupancy depends on insertion order
  - 2/3rds with random, ½ with insertion in sorted order

Non-Unique Search Keys

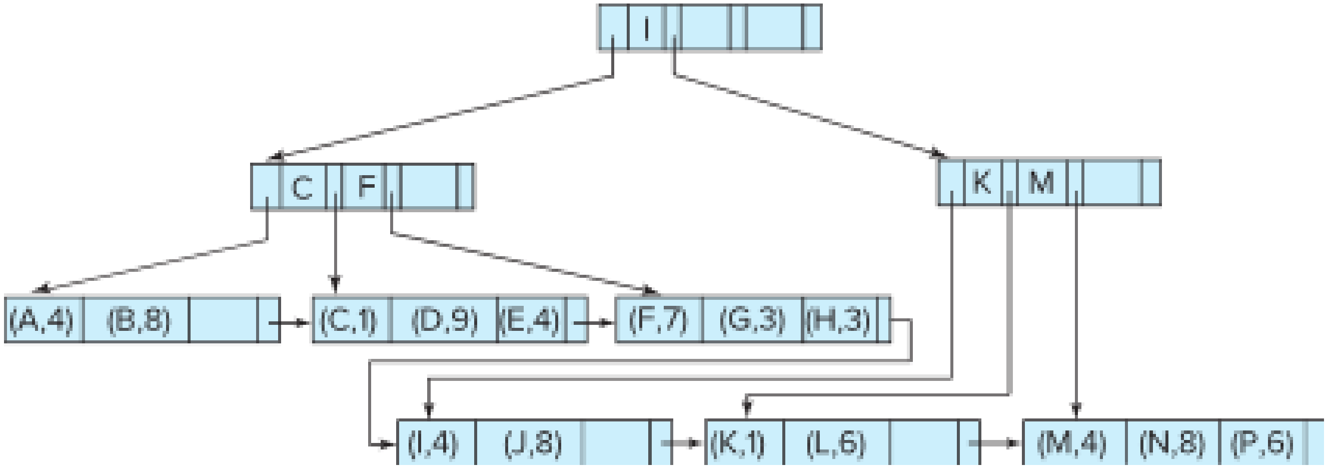
- Alternatives to scheme described earlier
  - Buckets on separate block (bad idea)
  - List of tuple pointers with each key
    - Extra code to handle long lists
    - Deletion of a tuple can be expensive if there are many duplicates on search key (why?)
      - Worst case complexity may be linear!
    - Low space overhead, no extra cost for queries
  - Make search key unique by adding a record-identifier
    - Extra storage overhead for keys
    - Simpler code for insertion/deletion
    - Widely used

B⁺-Tree File Organization

- B⁺-Tree File Organization:
  - Leaf nodes in a B⁺-tree file organization store records, instead of pointers
  - Helps keep data records clustered even when there are insertions/deletions/updates
- Leaf nodes are still required to be half full
  - Since records are larger than pointers, the maximum number of records that can be stored in a leaf node is less than the number of pointers in a nonleaf node.
- Insertion and deletion are handled in the same way as insertion and deletion of entries in a B⁺-tree index.

B+-Tree File Organization (Cont.)

- Example of B+-tree File Organization



- Good space utilization important since records use more space than pointers.
- To improve space utilization, involve more sibling nodes in redistribution during splits and merges
  - Involving 2 siblings in redistribution (to avoid split / merge where possible) results in each node having at least $\lfloor 2n/3 \rfloor$ entries

Other Issues in Indexing

- **Record relocation and secondary indices**
  - If a record moves, all secondary indices that store record pointers have to be updated
  - Node splits in B^+-tree file organizations become very expensive
  - *Solution*: use search key of B^+-tree file organization instead of record pointer in secondary index
    - Add record-id if B^+-tree file organization search key is non-unique
    - Extra traversal of file organization to locate record
      - Higher cost for queries, but node splits are cheap

Indexing Strings

- Variable length strings as keys
  - Variable fanout
  - Use space utilization as criterion for splitting, not number of pointers
- **Prefix compression**
  - Key values at internal nodes can be prefixes of full key
    - Keep enough characters to distinguish entries in the subtrees separated by the key value
      - E.g., “Silas” and “Silberschatz” can be separated by “Silb”
  - Keys in leaf node can be compressed by sharing common prefixes

Bulk Loading and Bottom-Up Build

- Inserting entries one-at-a-time into a B^+-tree requires $\geq$ 1 IO per entry
  - assuming leaf level does not fit in memory
  - can be very inefficient for loading a large number of entries at a time (**bulk loading**)
- Efficient alternative 1:
  - sort entries first (using efficient external-memory sort algorithms discussed later in Section 12.4)
  - insert in sorted order
    - insertion will go to existing page (or cause a split)
    - much improved IO performance, but most leaf nodes half full
- Efficient alternative 2: **Bottom-up B^+-tree construction**
  - As before sort entries
  - And then create tree layer-by-layer, starting with leaf level
    - details as an exercise
  - Implemented as part of bulk-load utility by most database systems

B-Tree Index Files

- Similar to B+-tree, but B-tree allows search-key values to appear only once; eliminates redundant storage of search keys.
- Search keys in nonleaf nodes appear nowhere else in the B-tree; an additional pointer field for each search key in a nonleaf node must be included.
- Generalized B-tree leaf node

P_1	K_1	P_2	…	P_{n-1}	K_{n-1}	P_n

(a)

P_1	B_1	K_1	P_2	B_2	K_2	…	P_{m-1}	B_{m-1}	K_{m-1}	P_m

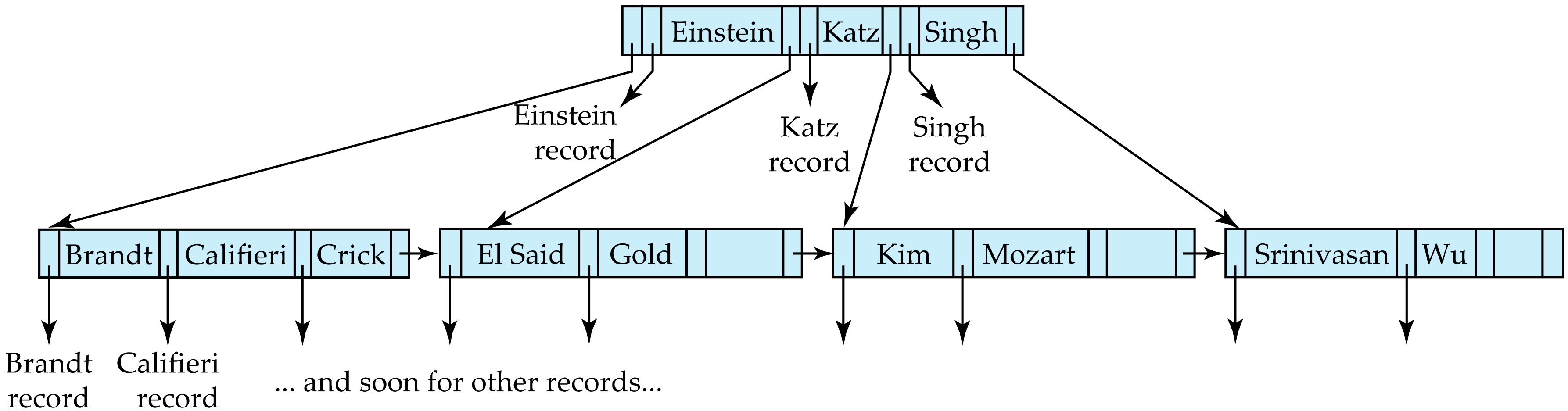
(b)

- Nonleaf node – pointers Bi are the bucket or file record pointers.

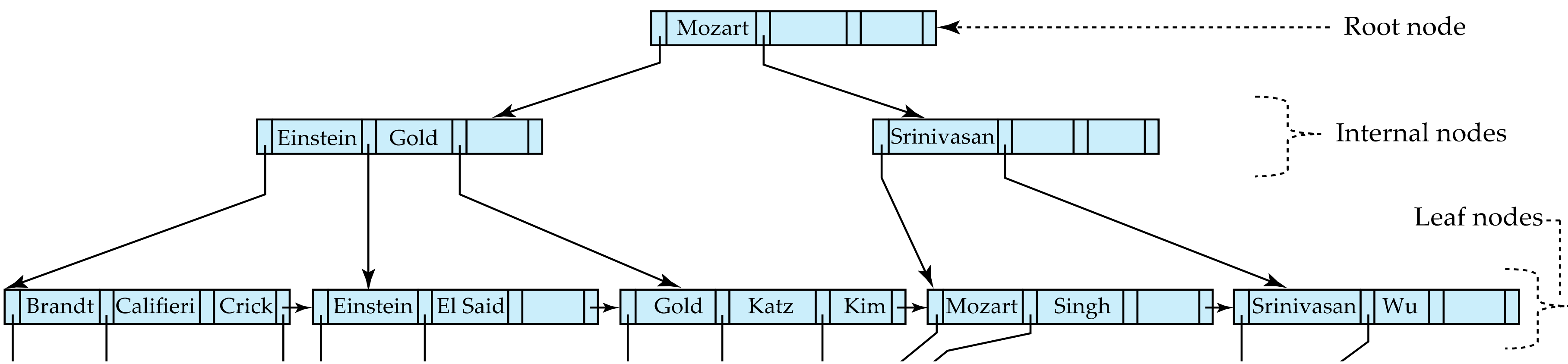
B-Tree Index Files (Cont.)

- Advantages of B-Tree indices:
  - May use less tree nodes than a corresponding B^+-Tree.
  - Sometimes possible to find search-key value before reaching leaf node.
- Disadvantages of B-Tree indices:
  - Only small fraction of all search-key values are found early
  - Non-leaf nodes are larger, so fan-out is reduced. Thus, B-Trees typically have greater depth than corresponding B^+-Tree
  - Insertion and deletion more complicated than in B^+-Trees
  - Implementation is harder than B^+-Trees.
- Typically, advantages of B-Trees do not out weigh disadvantages.

B-Tree Index File Example



B-tree (above) and B+-tree (below) on same data



Indexing on Flash

- Random I/O cost much lower on flash
  - 20 to 100 microseconds for read/write
- Writes are not in-place, and (eventually) require a more expensive erase
- Optimum page size therefore much smaller
- Bulk-loading still useful since it minimizes page erases
- Write-optimized tree structures (discussed later) have been adapted to minimize page writes for flash-optimized search trees

Indexing in Main Memory

- Random access in memory
  - Much cheaper than on disk/flash
  - But still expensive compared to cache read
  - Data structures that make best use of cache preferable
  - Binary search for a key value within a large B^+-tree node results in many cache misses
- B^+- trees with small nodes that fit in cache line are preferable to reduce cache misses
- Key idea: use large node size to optimize disk access, but structure data within a node using a tree with small node size, instead of using an array.

Hashing

Static Hashing

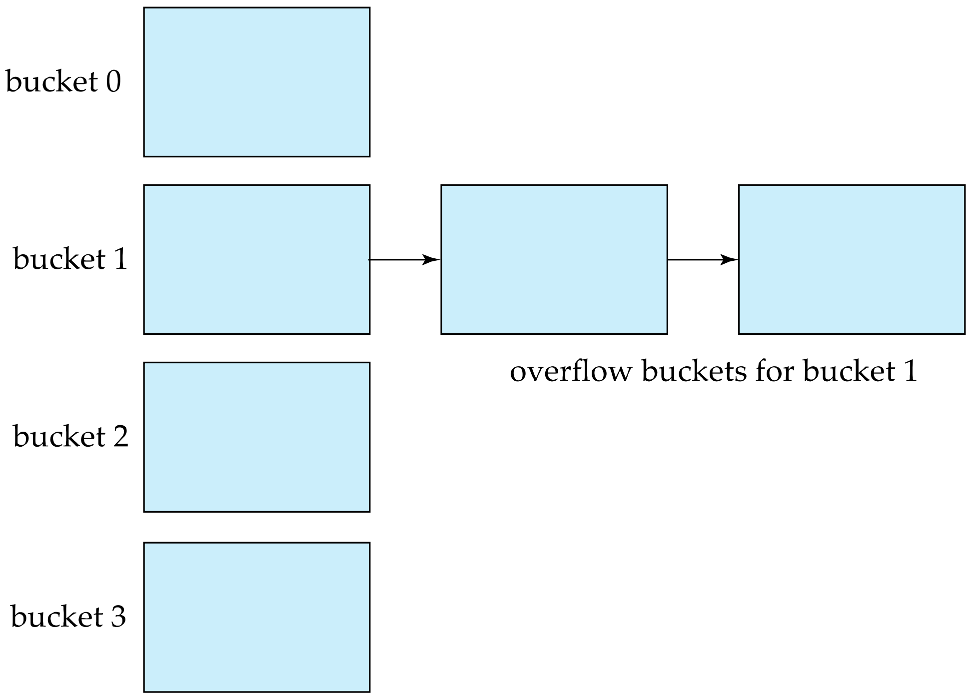
- A **bucket** is a unit of storage containing one or more entries (a bucket is typically a disk block).
  - we obtain the bucket of an entry from its search-key value using a **hash function**
- Hash function *h* is a function from the set of all search-key values *K* to the set of all bucket addresses *B.*
- Hash function is used to locate entries for access, insertion as well as deletion.
- Entries with different search-key values may be mapped to the same bucket; thus entire bucket has to be searched sequentially to locate an entry.
- In a **hash index**, buckets store entries with pointers to records
- In a **hash file-organization** buckets store records

Handling of Bucket Overflows

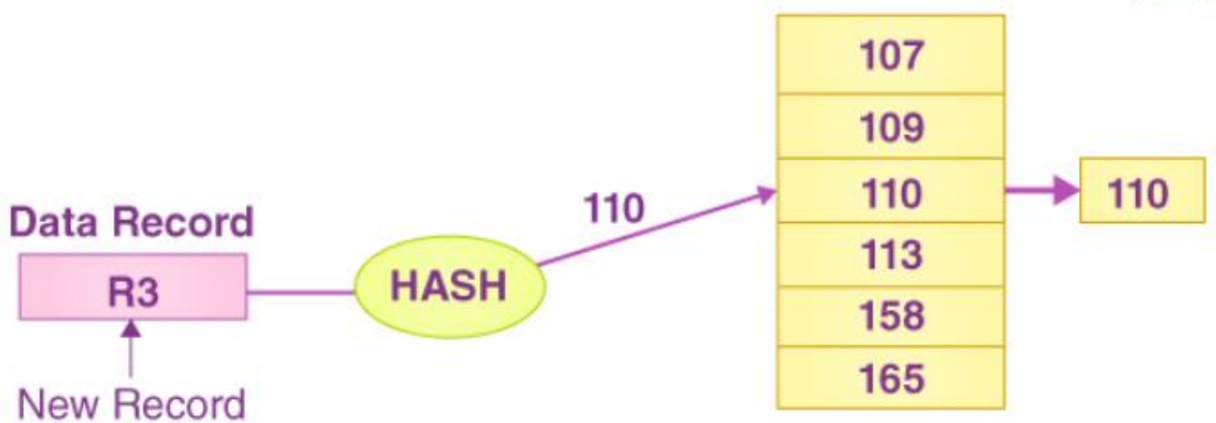
- Bucket overflow can occur because of
  - Insufficient buckets
  - Skew in distribution of records. This can occur due to two reasons:
    - multiple records have same search-key value
    - chosen hash function produces non-uniform distribution of key values
- Although the probability of bucket overflow can be reduced, it cannot be eliminated; it is handled by using ***overflow buckets***.

Handling of Bucket Overflows (Cont.)

- **Closed hashing -** When buckets are full, then a new data bucket is allocated for the same hash result and is linked after the previous one. This mechanism is known as **Overflow chaining**.

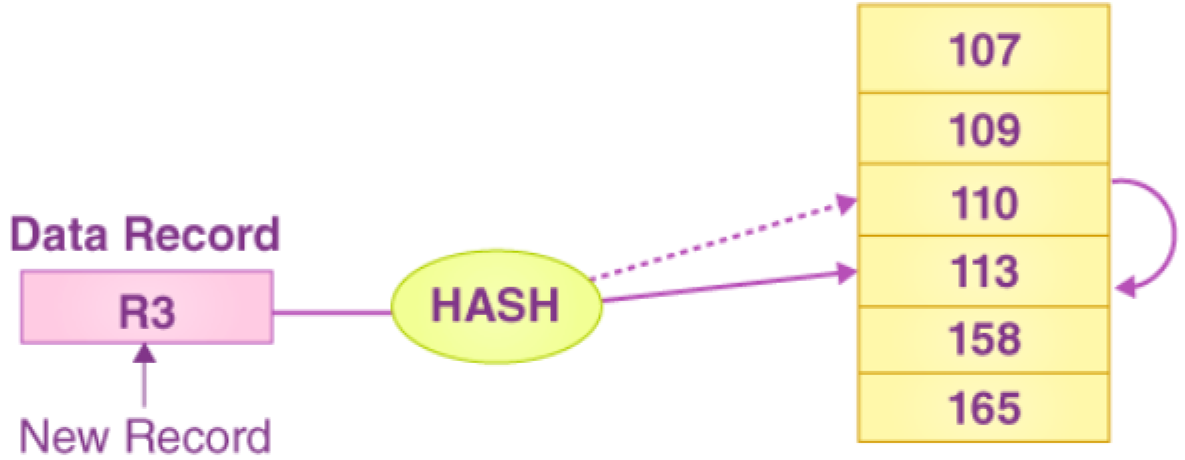


For example: Suppose R3 is a new address which needs to be inserted into the table, the hash function generates address as 110 for it. But this bucket is full to store the new data. In this case, a new bucket is inserted at the end of 110 buckets and is linked to it.

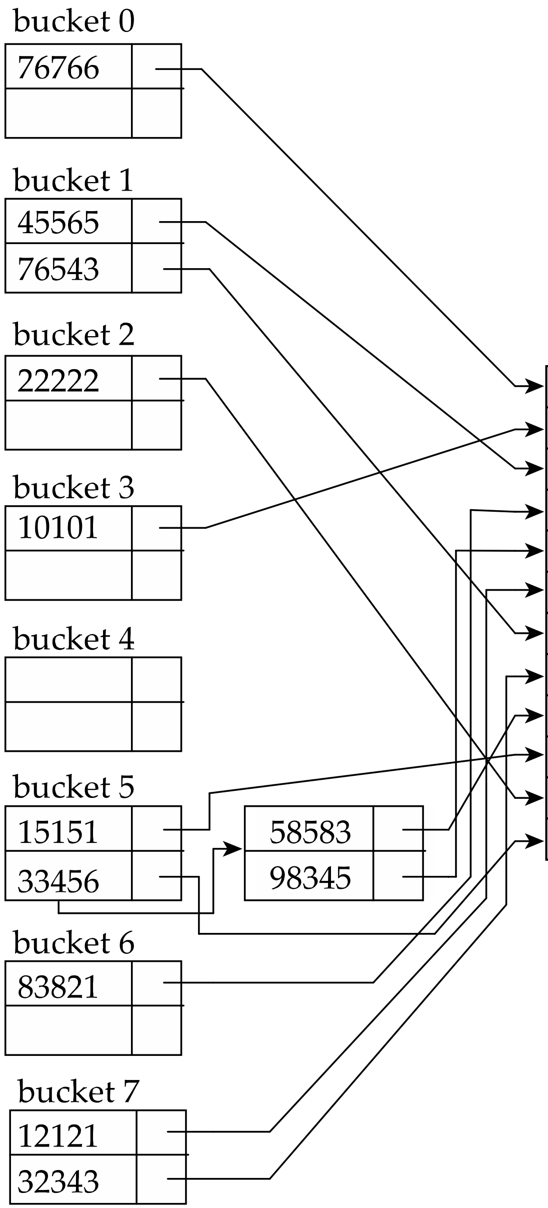


Handling of Bucket Overflows (Cont.)

- **Open Hashing** – When a hash function generates an address at which data is already stored, then the next bucket will be allocated to it. This mechanism is called as **Linear Probing**.
- For example: suppose R3 is a new address which needs to be inserted, the hash function generates address as 112 for R3. But the generated address is already full. So the system searches next available data bucket, 113 and assigns R3 to it.
- This method is not, is not suitable for database applications.



Example of Hash Index



76766	Crick	Biology	72000
10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000
12121	Wu	Finance	90000
76543	Singh	Finance	80000
32343	El Said	History	60000
58583	Califieri	History	62000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
33465	Gold	Physics	87000

hash index on *instructor*, on attribute *ID*

Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key
(See figure in next slide.)

- There are 10 buckets,
- The binary representation of the I^{th} character is assumed to be the integer *i.*
- The hash function returns the sum of the binary representations of the characters modulo 10
  - E.g. h(Music) = 1 h(History) = 2
    h(Physics) = 3 h(Elec. Eng.) = 3

Example of Hash File Organization

Hash file organization of *instructor* file, using *dept_name* as key.

bucket 0

bucket 1

15151	Mozart	Music	40000

bucket 2

32343	El Said	History	80000
58583	Califieri	History	60000

bucket 3

22222	Einstein	Physics	95000
33456	Gold	Physics	87000
98345	Kim	Elec. Eng.	80000

bucket 4

12121	Wu	Finance	90000
76543	Singh	Finance	80000

bucket 5

76766	Crick	Biology	72000

bucket 6

10101	Srinivasan	Comp. Sci.	65000
45565	Katz	Comp. Sci.	75000
83821	Brandt	Comp. Sci.	92000

bucket 7

Deficiencies of Static Hashing

- In static hashing, function *h* maps search-key values to a fixed set of *B* of bucket addresses. Databases grow or shrink with time.
  - If initial number of buckets is too small, and file grows, performance will degrade due to too much overflows.
  - If space is allocated for anticipated growth, a significant amount of space will be wasted initially (and buckets will be underfull).
  - If database shrinks, again space will be wasted.
- One solution: periodic re-organization of the file with a new hash function
  - Expensive, disrupts normal operations
- Better solution: allow the number of buckets to be modified dynamically.

Dynamic Hashing

- The dynamic hashing approach is used to solve problems like bucket overflow that can occur with static hashing. As the number of records increases or decreases, data buckets grow or shrink in this manner.
- Two such dynamic hashing techniques are: linear hashing and extendable hashing
- Linear Hashing
  - Do rehashing in an incremental manner
- Extendable Hashing
  - Tailored to disk based hashing, with buckets shared by multiple hash values
  - Doubling of # of entries in hash table, without doubling # of buckets

Comparison of Ordered Indexing and Hashing

- Cost of periodic re-organization
- Relative frequency of insertions and deletions
- Is it desirable to optimize average access time at the expense of worst-case access time?
- Expected type of queries:
  - Hashing is generally better at retrieving records having a specified value of the key.
  - If range queries are common, ordered indices are to be preferred
- In practice:
  - PostgreSQL supports hash indices, but discourages use due to poor performance
  - Oracle supports static hash organization, but not hash indices
  - SQLServer supports only B^+-trees

End of Chapter 14