Image restoration and reconstruction

1. Basic concepts about image degradation/restoration
2. Noise models
3. Spatial filter techniques for restoration

Image Restoration

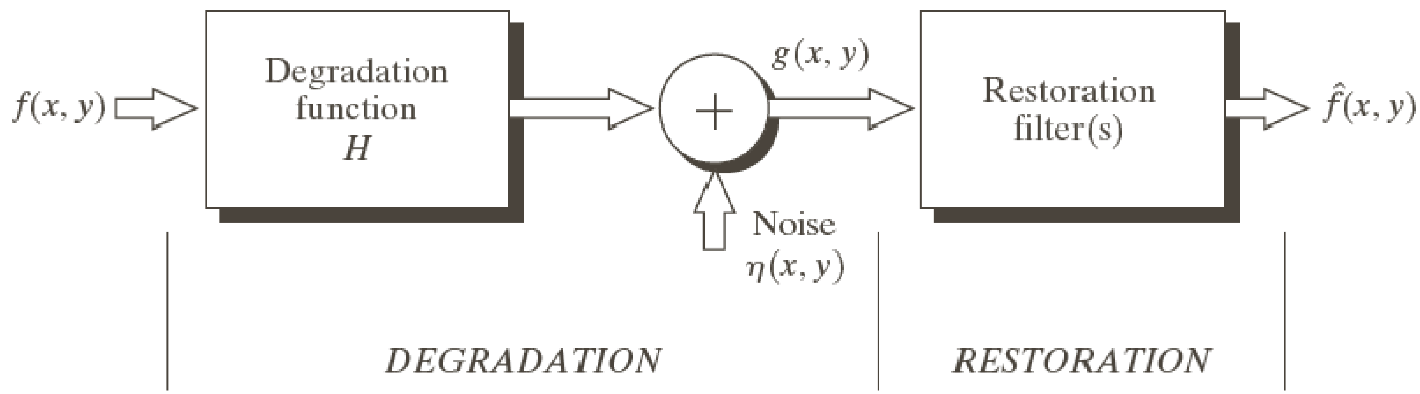
- Image restoration is to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon

- Image enhancement vs. image restoration
  - Enhancement is for vision
  - Restoration is to recover the original image
  - There is overlap of the techniques used

- Image restored is an approximation of the original image
  - Criteria for the goodness

The model of Image Degradation

FIGURE 5.1 A model of the image degradation/restoration process.



$$g(x, y) = h(x, y) * f(x, y) + \eta(x, y)$$

$$G(u, v) = H(u, v) F(u, v) + N(u, v)$$

Noise models

- Noise often arise during image acquisition/transformation
  - Caused by many factors
  - Spatial noise
  - Frequency noise
- Some important noise probability density functions
  - Gaussian noise
  - Rayleigh noise
  - Erlang (gamma) noise
  - Exponential noise
  - Uniform
  - Impulse
- Periodic noise

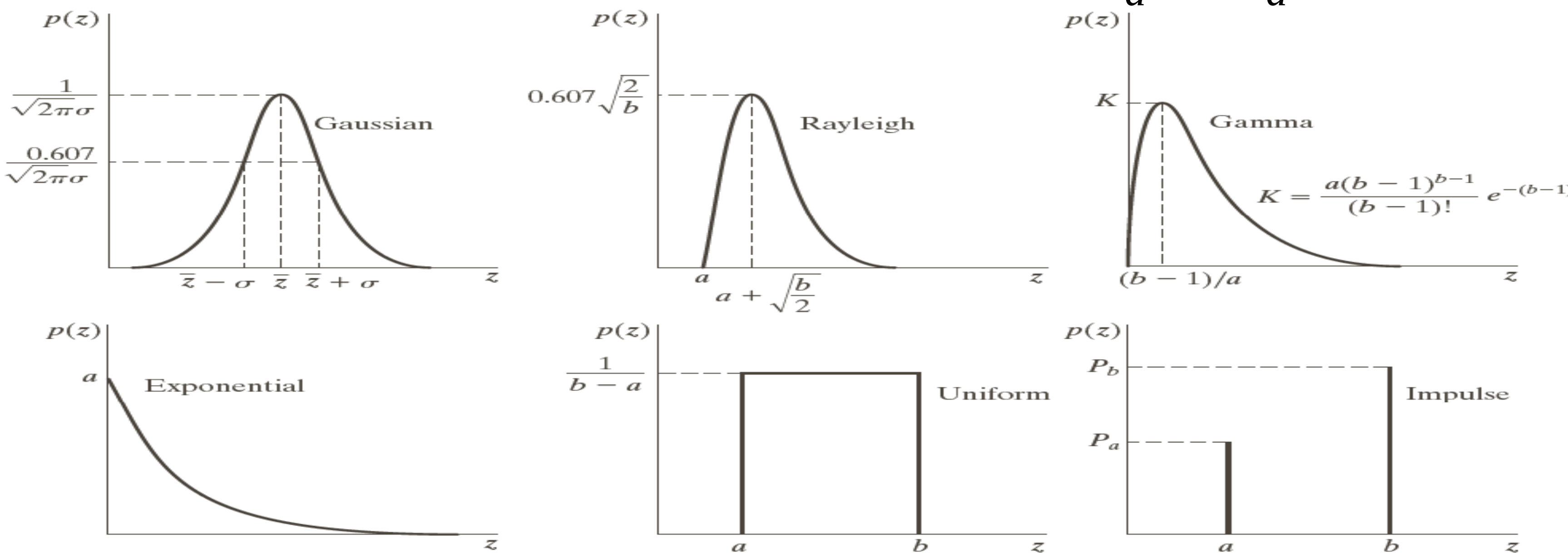
$$p(z)=\frac{1}{\sqrt{2\pi\sigma}}e^{-\frac{(z-\overline{z})^2}{2\sigma^2}}$$

$$p(z)=\begin{cases}\frac{2}{b}(z-a)e^{-(z-a)^2/b}, & z\geq a\\ 0, & z<a\end{cases}$$

$$\overline{z}=a+\sqrt{\pi b/4},\sigma^2=\frac{b(4-\pi)}{4}$$

$$p(z)=\begin{cases}\frac{a^b z^{b-1}}{(b-1)!}e^{-az}, & z\geq a\\ 0, & z<a\end{cases},$$

$$\overline{z}=\frac{b}{a},\sigma^2=\frac{b}{a^2}$$



a b c
d e f

FIGURE 5.2 Some important probability density functions.

$$p(z)=\begin{cases}ae^{-az}, & z\geq a\\ 0, & z<a\end{cases}$$

$$\overline{z}=\frac{1}{a},\sigma^2=\frac{1}{a^2}$$

$$p(z)=\begin{cases}\frac{1}{b-1}, & a\leq z\leq b\\ 0, & otherwise\end{cases}$$

$$\overline{z}=\frac{a+b}{2},\sigma^2=\frac{(b-a)^2}{12}$$

$$p(z)=\begin{cases}P_a & a=z\\ P_b & z=b\\ 0 & otherwise\end{cases}$$

$$\overline{z}=\frac{a+b}{2},\sigma^2=\frac{(b-a)^2}{12}$$

Generate spatial noise of a given distribution

Theorem

Given CDF *F(z)*. Let w be the uniform random number generator on (0,1). Then the random number $z = F_z^{-1}(w)$ has the CDF *F(z)*

Example: Reyleigh's CDF is

$$F_z(w) = \begin{cases} 1 - e^{-(z-a)^2/b} & z \geq a \\ 0 & z < a \end{cases}$$

$$z = a + \sqrt{-b\ln(1-w)}$$

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Matlab example:
a = 50, b =10, M = 100, N = 100;
R = a + sqrt(-b*log(1-rand(M,N)));

MatLab example 2: Gaussian distribution mean a and std b
a = 10, b =10, M = 100, N = 100;
R = a + b*randn(M,N);
```

Add spatial noise to an image of

- Let $f(x, y)$ be an $M\ N$ image, and $N(x, y)$ be the random MN noise of the given distribution. Then the image with the spatial noise is $g(x, y) = f(x,y) + N(x,y)$

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MatLab example:
f = imread('moon.tif');
[M N] = size(f);
s = uint8(a + sqrt(-b*log(1-rand(M,N))));
fs =  y + s;
imshow(fs)
```

FIGURE 5.3 Test pattern used to illustrate the characteristics of the noise PDFs shown in Fig. 5.2.

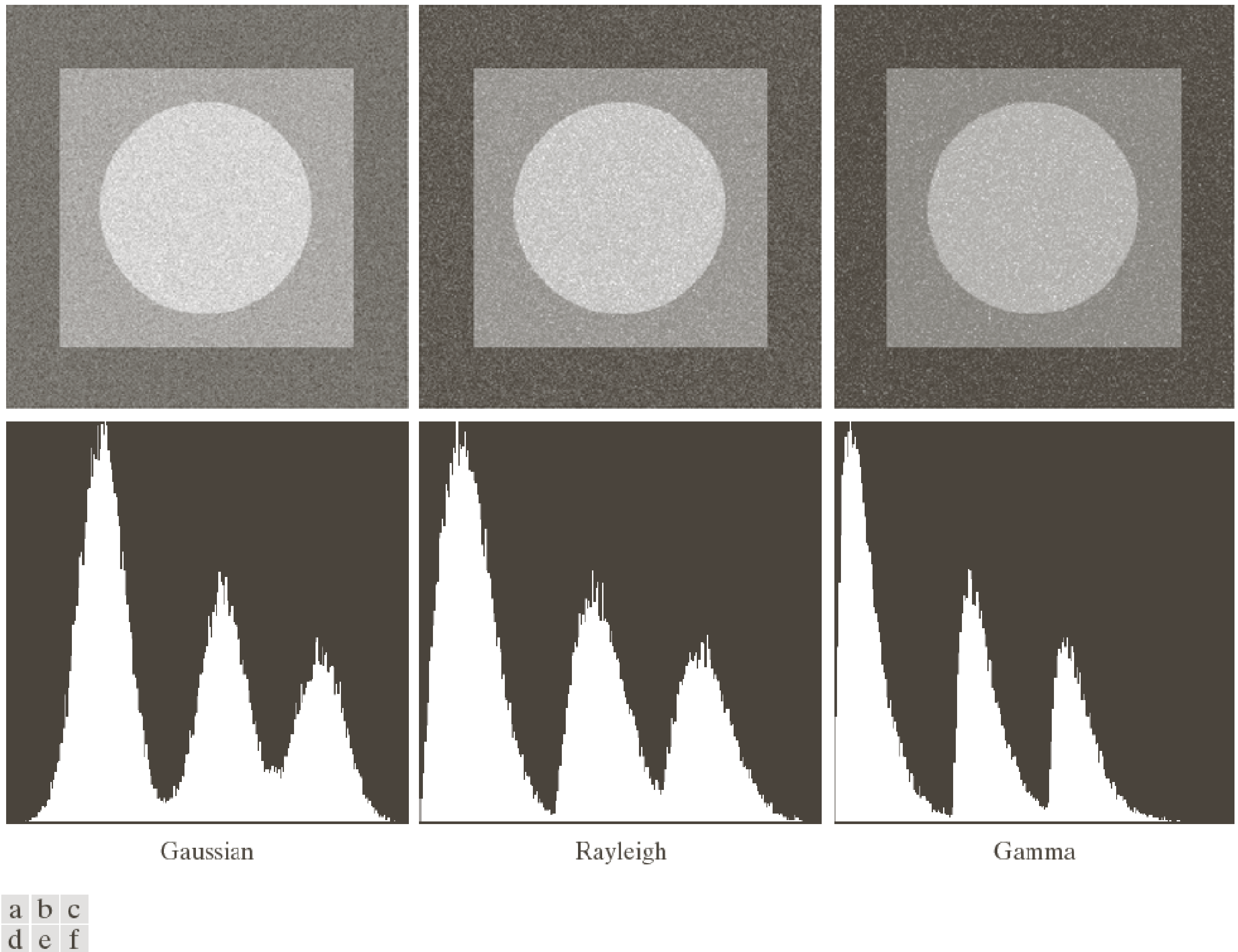
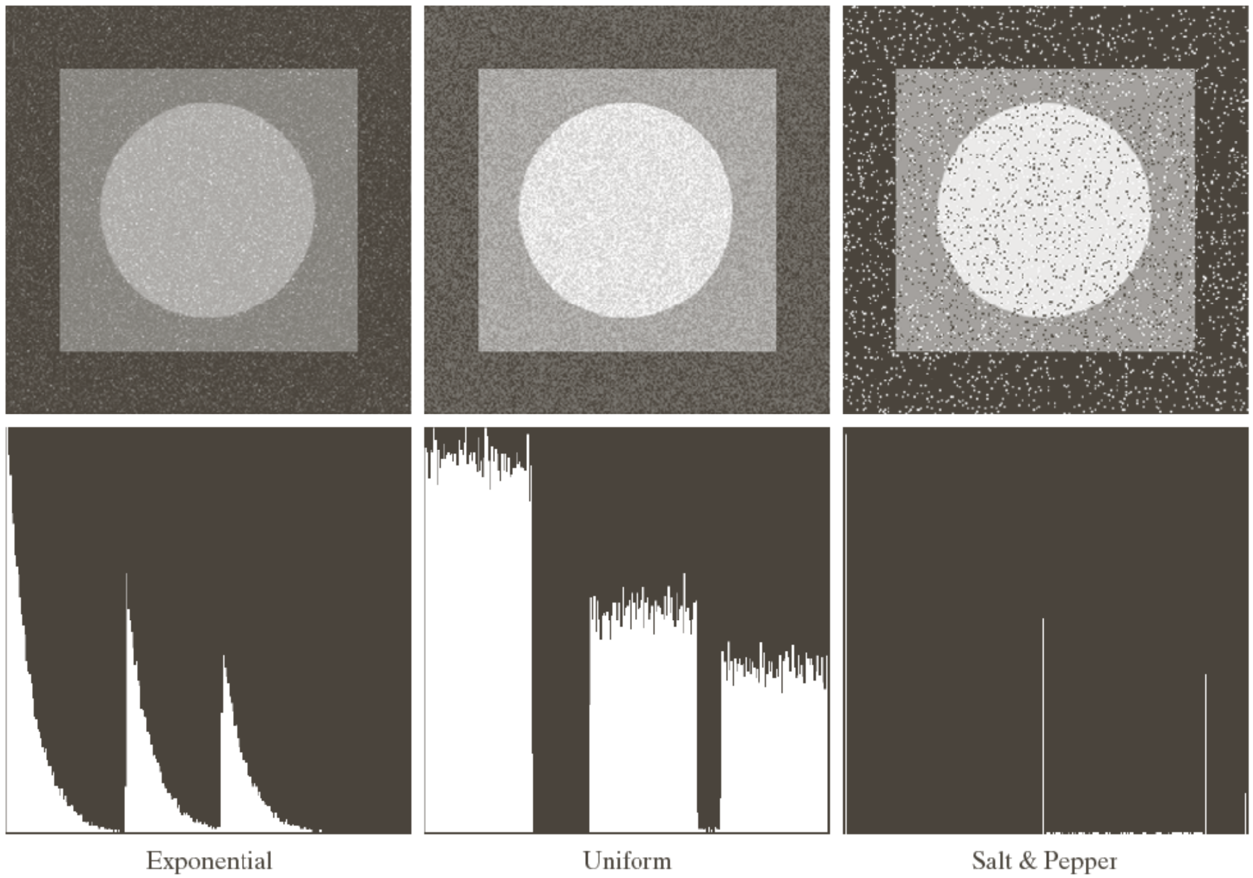


FIGURE 5.4 Images and histograms resulting from adding Gaussian, Rayleigh, and gamma noise to the image in Fig. 5.3.



g	h	i
j	k	l

FIGURE 5.4 *(Continued)* Images and histograms resulting from adding exponential, uniform, and salt and pepper noise to the image in Fig. 5.3.

Estimation of Noise Parameters

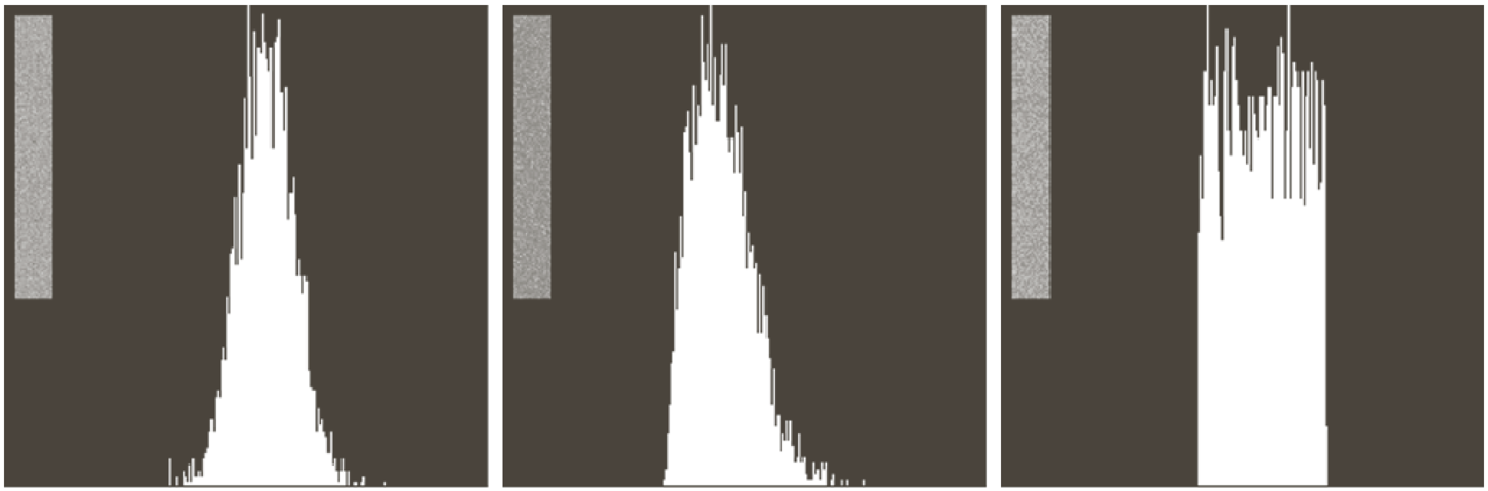
- Parameters of a PDF: mean, standard deviation, variance, moments about the mean
- The method of estimation
  - If possible, take a flat image the system and compute its parameter
  - If only images are available.

Take a strip image S. Determine the histogram of S. Let $p(z_i)$ denote the frequency of value z_i

$$\overline{z} = \sum_{i=0}^{L-1} z_i p(z_i)$$

$$\sigma = \sqrt{\sum_{i=0}^{L-1} (z_i - \overline{z})^2 p(z_i)}, \sigma^2 = \sum_{i=0}^{L-1} (z_i - \overline{z})^2 p(z_i)$$

$$\mu_n = \sum_{i=0}^{L-1} (z_i - \overline{z})^n p(z_i), \upsilon_n = \sum_{i=0}^{L-1} z_i^{\,n} p(z_i),$$



a b c

FIGURE 5.6 Histograms computed using small strips (shown as inserts) from (a) the Gaussian, (b) the Rayleigh, and (c) the uniform noisy images in Fig. 5.4.

Spatial filters based restoration technique

- When only additive random noise is present, spatial filter can be applied

$$g(x, y) = f(x, y) + \eta(x, y)$$
$$G(u, v) = F(u, v) + N(u, v)$$

- Mean filters
  - Arithmetic mean filter

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t)\in} g(s, t)$$

S_{xy} is the set of coordinates in a rectangle subimage window (neighborhood) of size $m \times n$ centered at (x, y)

Spatial filters based restoration technique

– Geometric mean filter

$$\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t) \right]^{\frac{1}{mn}}$$

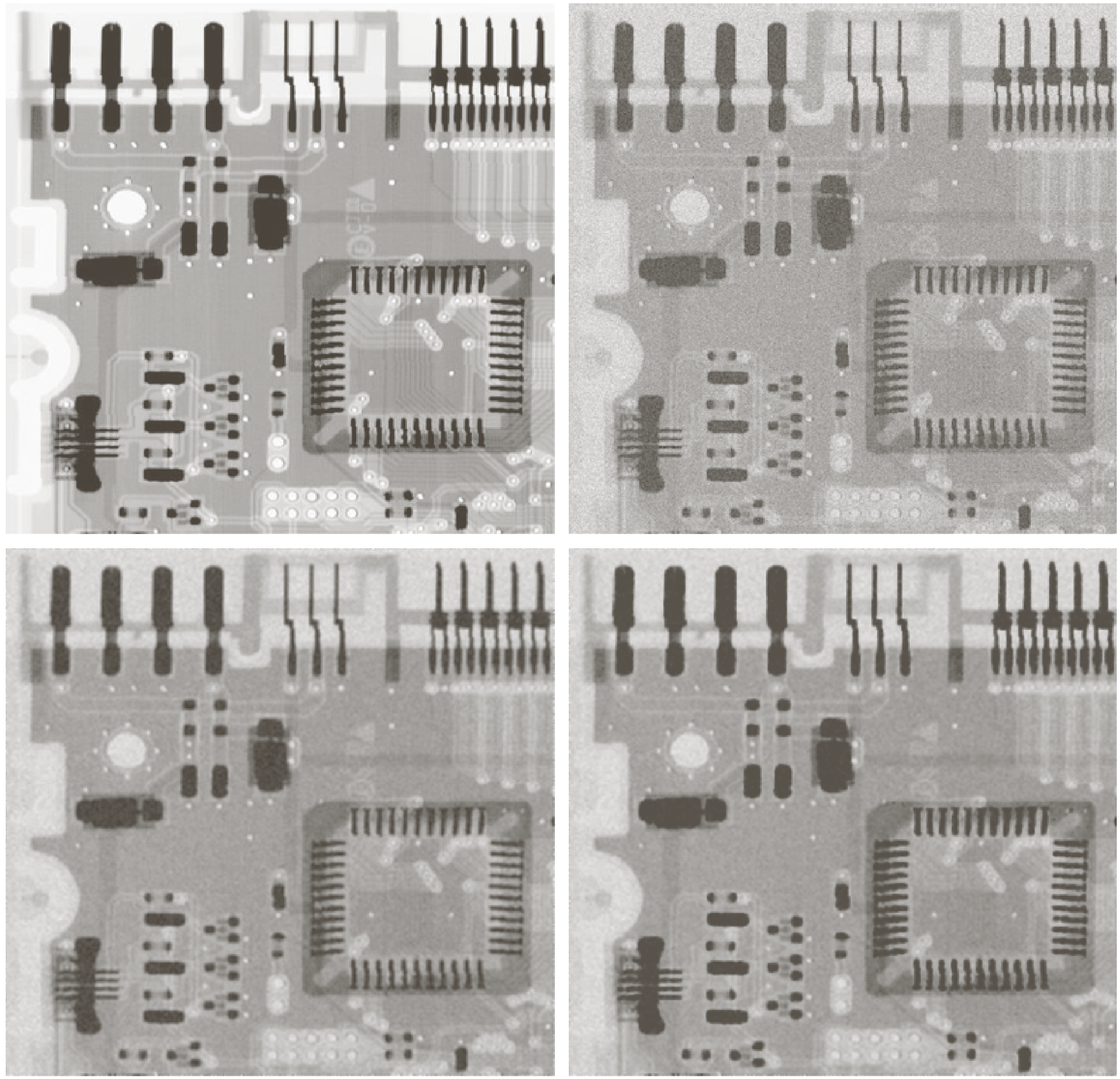
– Harmonic mean filter

$$\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}} \frac{1}{g(s,t)}}$$

– Contraharmonic mean filter

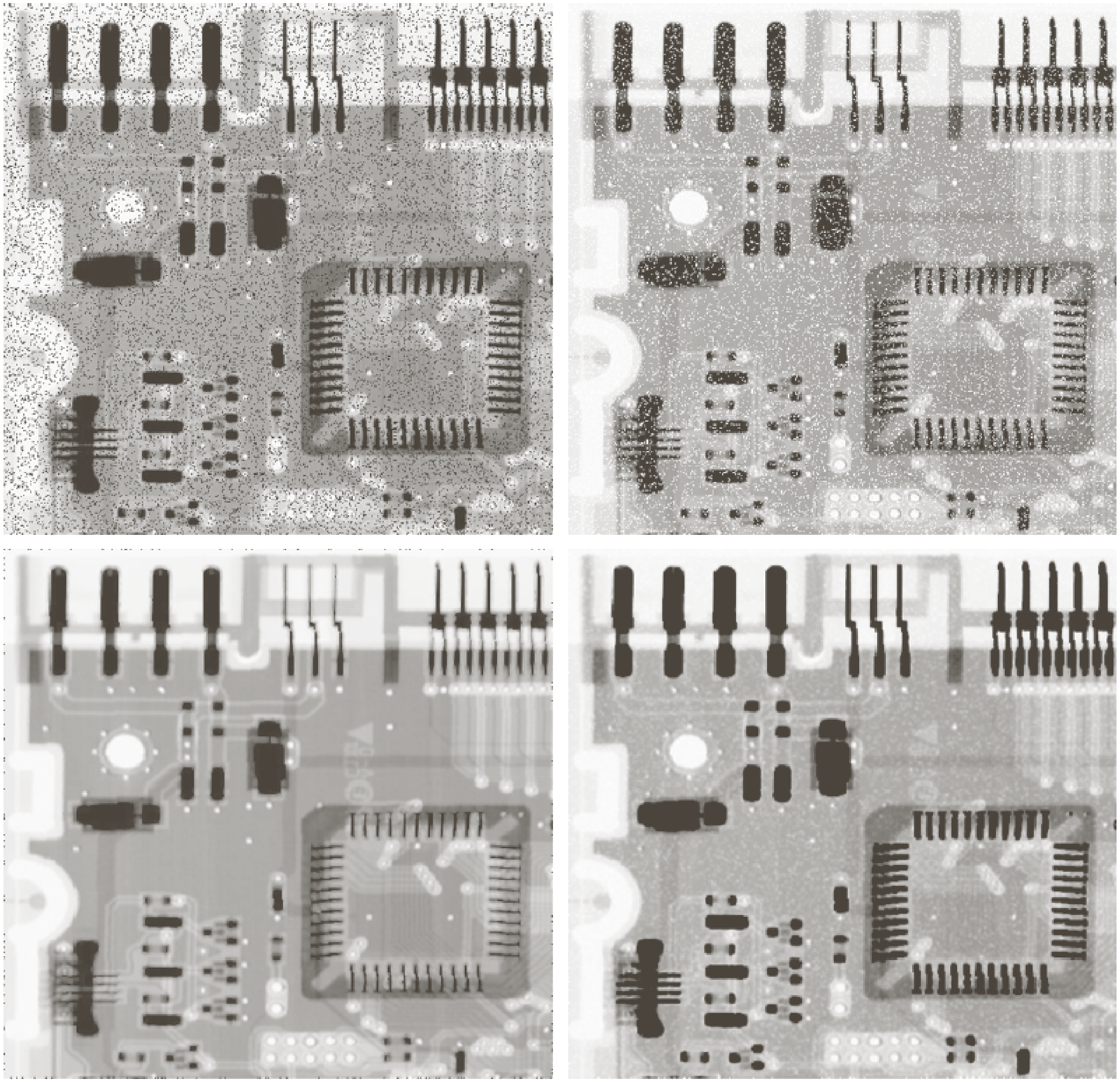
$$\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$$

where Q is called the order of the filter. This filter is good for reducing salt-and-pepper noise.



a b
c d

FIGURE 5.7
(a) X-ray image. (b) Image corrupted by additive Gaussian noise. (c) Result of filtering with an arithmetic mean filter of size 3×3. (d) Result of filtering with a geometric mean filter of the same size. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

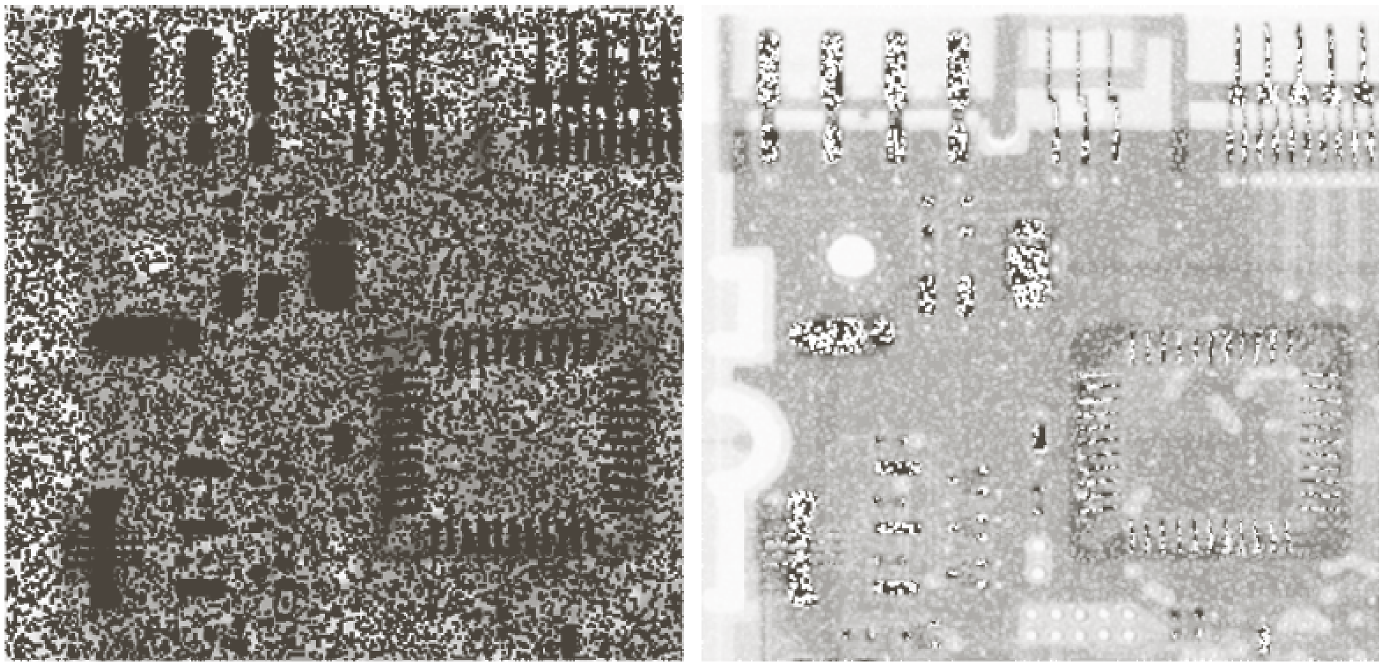


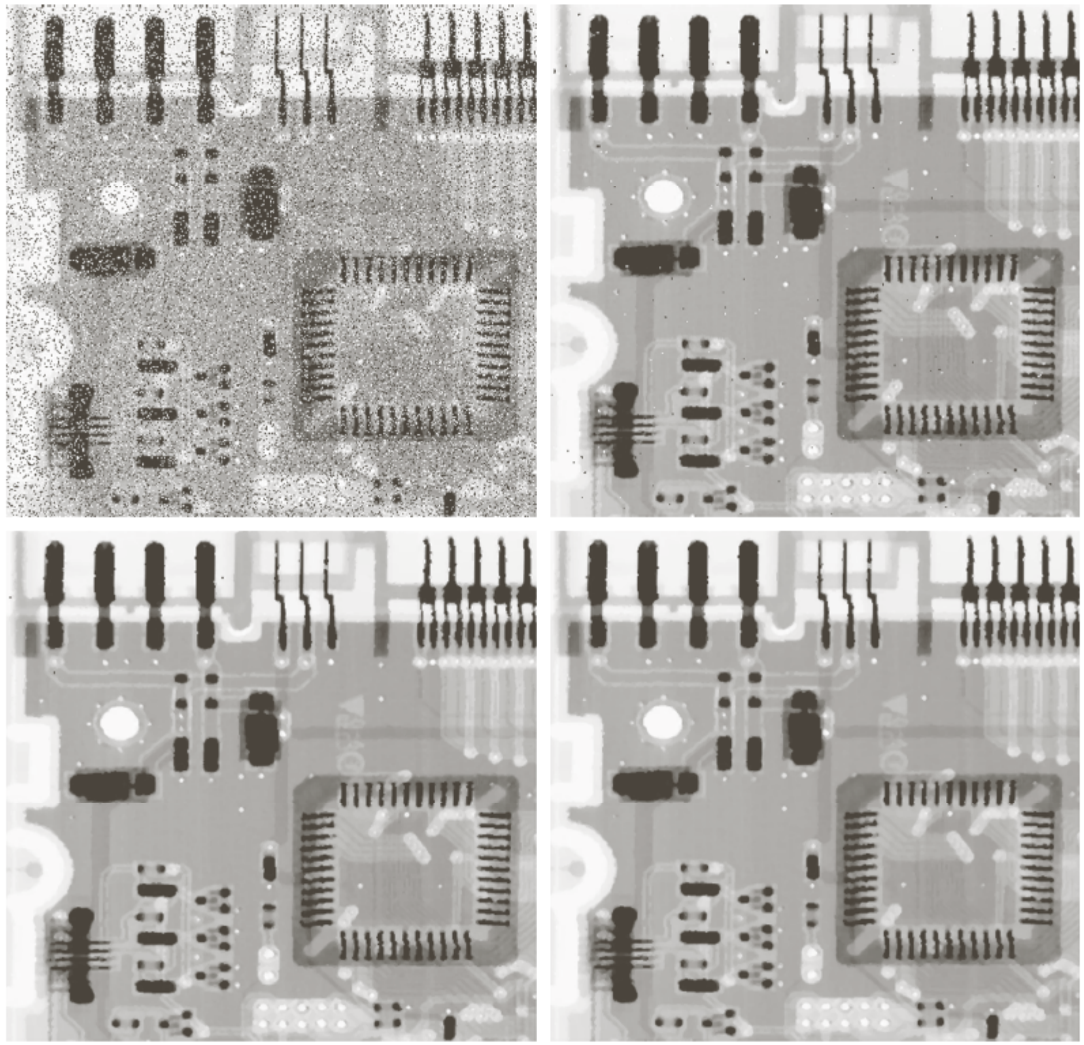
a	b
c	d

FIGURE 5.8
(a) Image corrupted by pepper noise with a probability of 0.1. (b) Image corrupted by salt noise with the same probability. (c) Result of filtering (a) with a 3×3 contraharmonic filter of order 1.5. (d) Result of filtering (b) with $Q = -1.5$.

a b

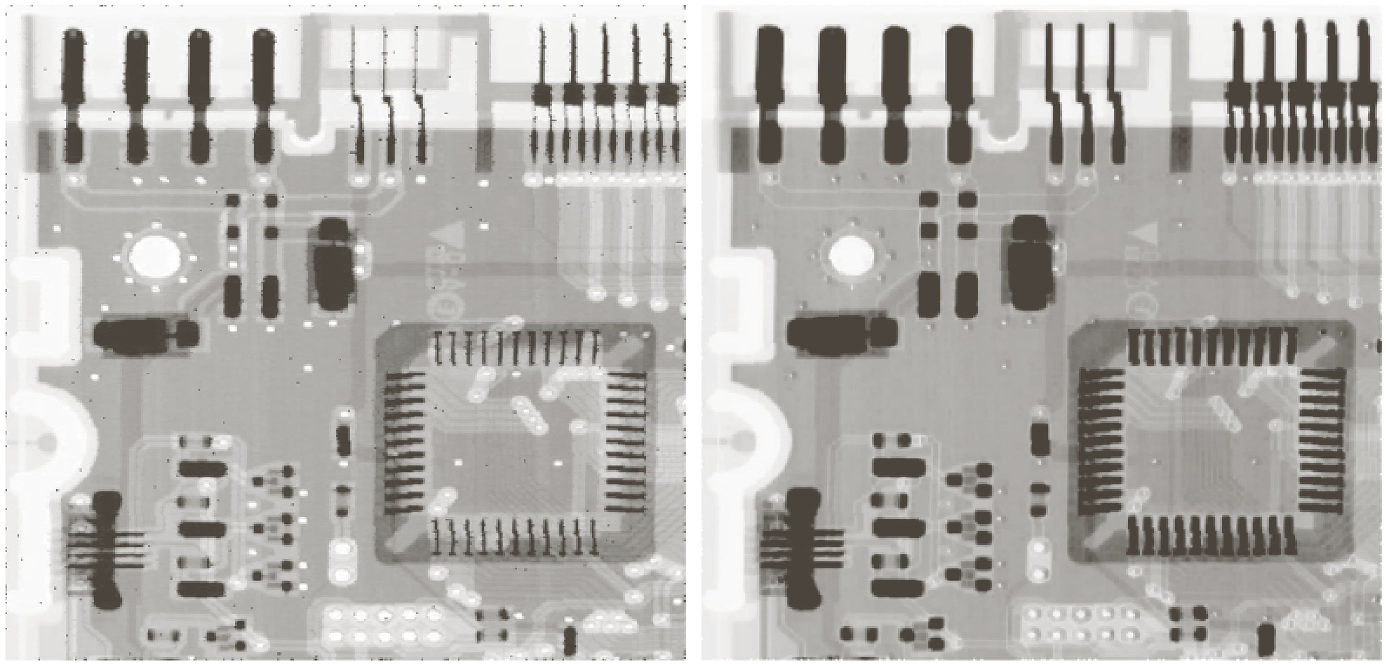
FIGURE 5.9 Results of selecting the wrong sign in contraharmonic filtering. (a) Result of filtering Fig. 5.8(a) with a contraharmonic filter of size 3×3 and $Q = -1.5$. (b) Result of filtering 5.8(b) with $Q = 1.5$.





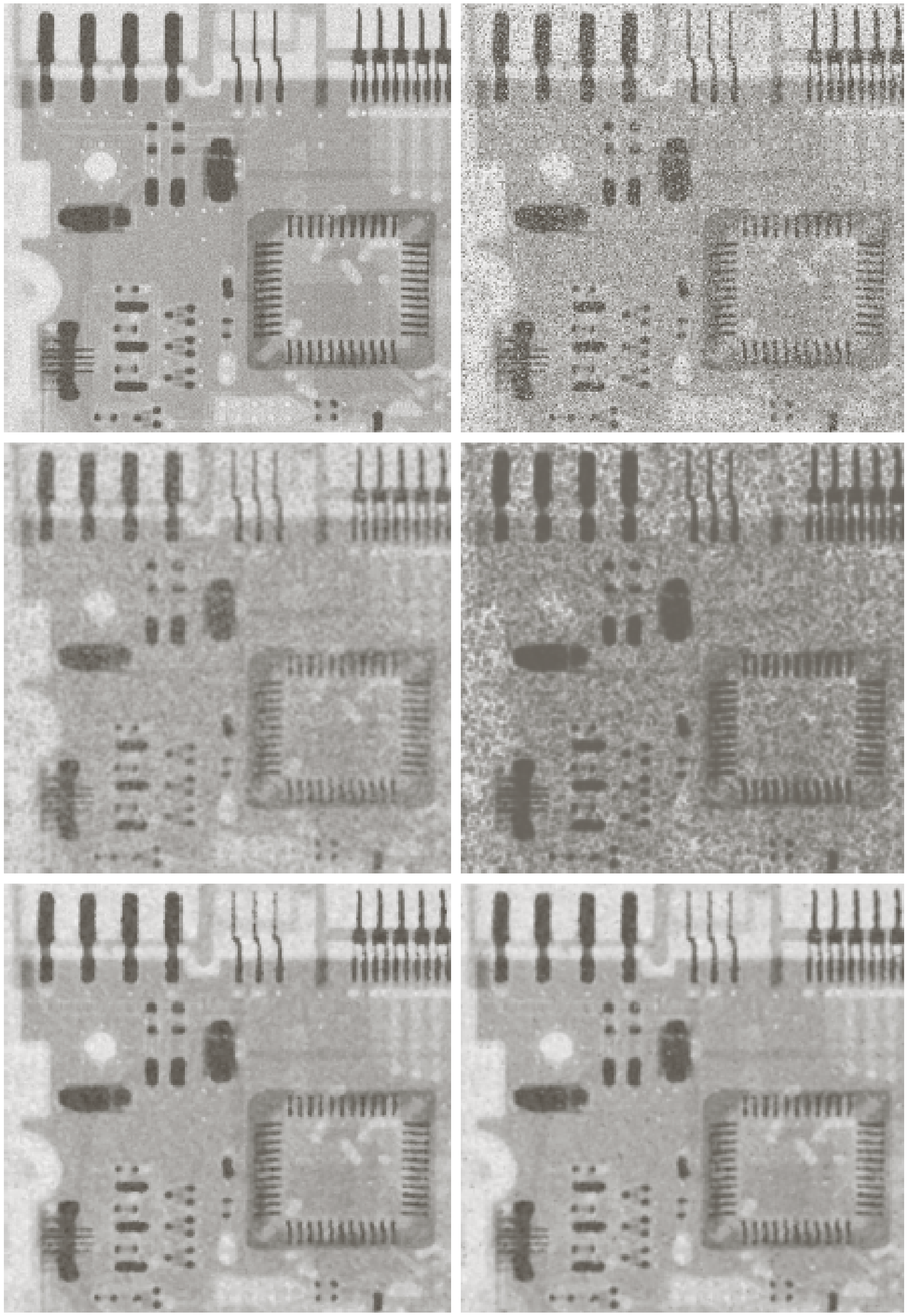
a	b
c	d

FIGURE 5.10
(a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.1$.
(b) Result of one pass with a median filter of size 3×3.
(c) Result of processing (b) with this filter.
(d) Result of processing (c) with the same filter.



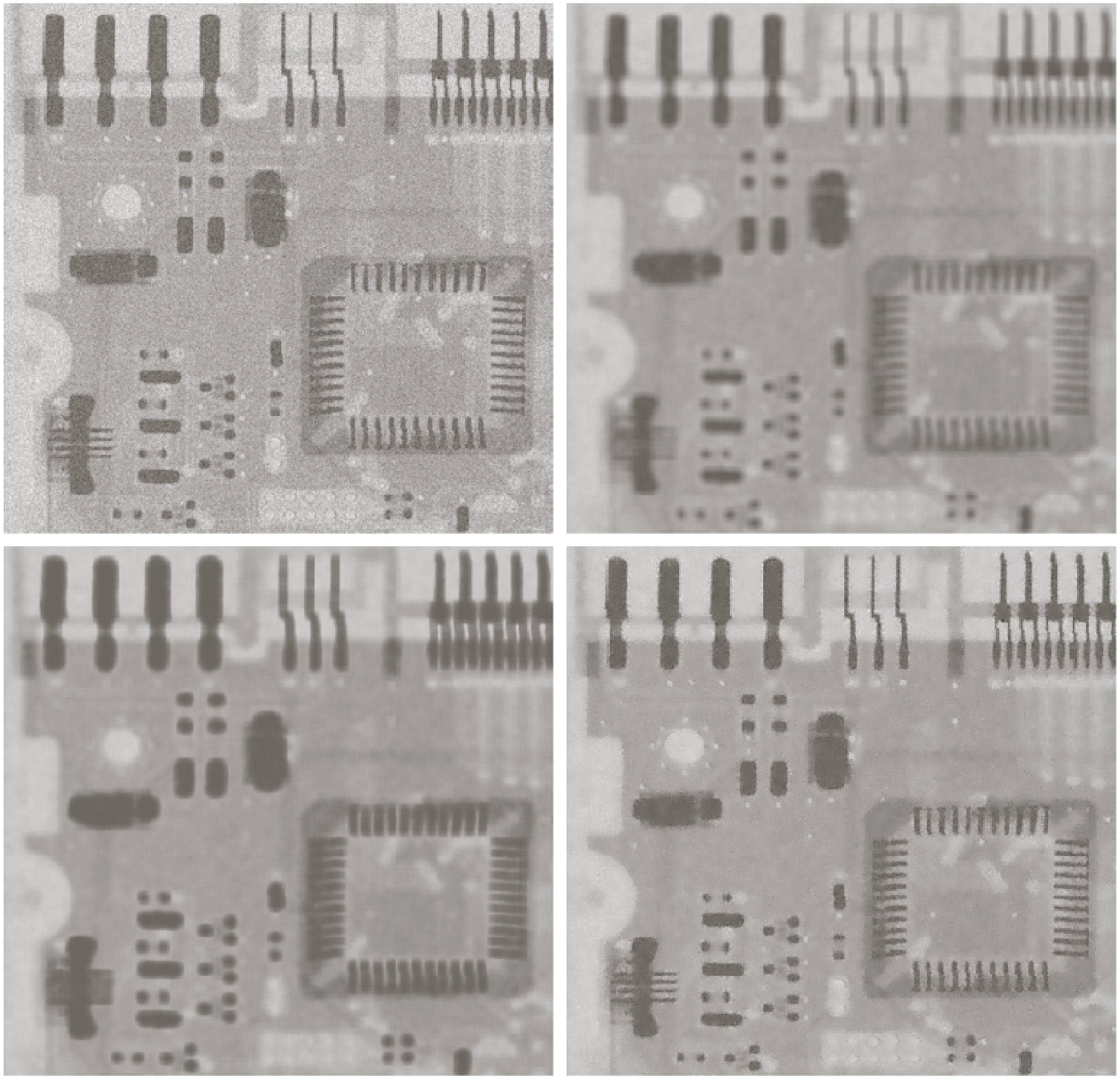
a b

FIGURE 5.11
(a) Result of filtering Fig. 5.8(a) with a max filter of size 3×3. (b) Result of filtering 5.8(b) with a min filter of the same size.



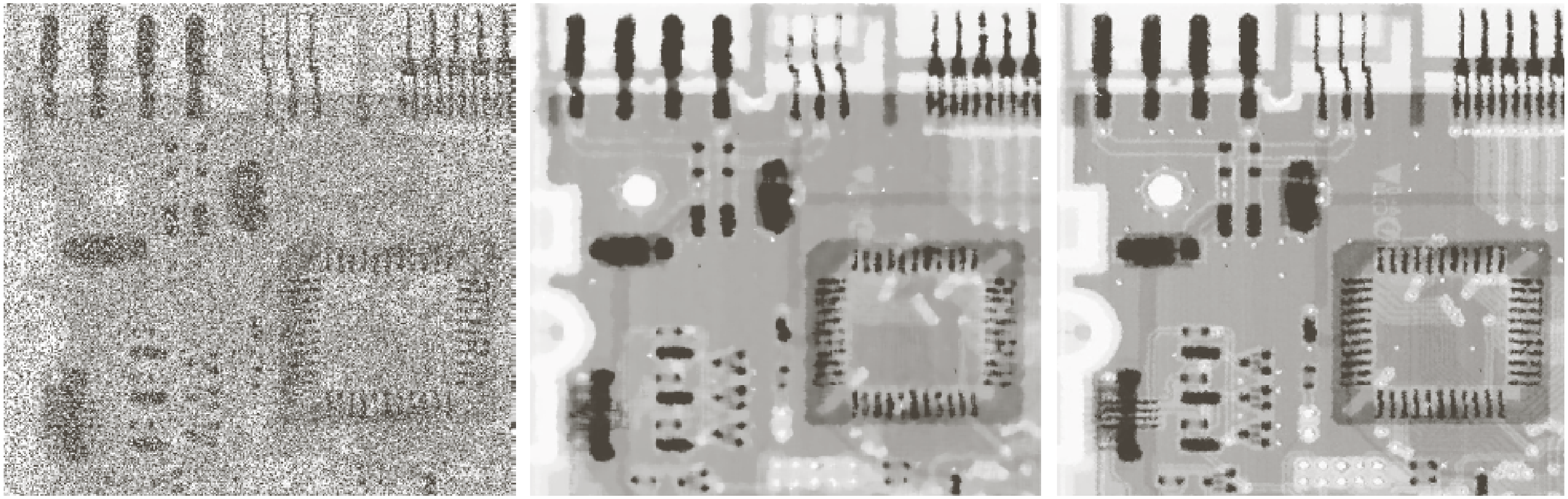
a b
c d
e f

FIGURE 5.12 (a) Image corrupted by additive uniform noise. (b) Image additionally corrupted by additive salt-and-pepper noise. Image (b) filtered with a 5×5; (c) arithmetic mean filter; (d) geometric mean filter; (e) median filter; and (f) alpha-trimmed mean filter with $d = 5$.



a	b
c	d

FIGURE 5.13 (a) Image corrupted by additive Gaussian noise of zero mean and variance 1000. (b) Result of arithmetic mean filtering. (c) Result of geometric mean filtering. (d) Result of adaptive noise reduction filtering. All filters were of size 7×7.



a b c

FIGURE 5.14 (a) Image corrupted by salt-and-pepper noise with probabilities $P_a = P_b = 0.25$. (b) Result of filtering with a 7×7 median filter. (c) Result of adaptive median filtering with $S_{max} = 7$.