

Histogram equalization

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Image processing

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Histogram

- ▶ The *histogram* of an L -valued image is a discrete function:

$$h(k) = n_k, \quad k \in [0, \dots, L - 1]$$

where n_k is the number of pixels with intensity k .

- ▶ Often it is preferable to consider the histogram normalized with respect to the number of pixels, $M \times N$:

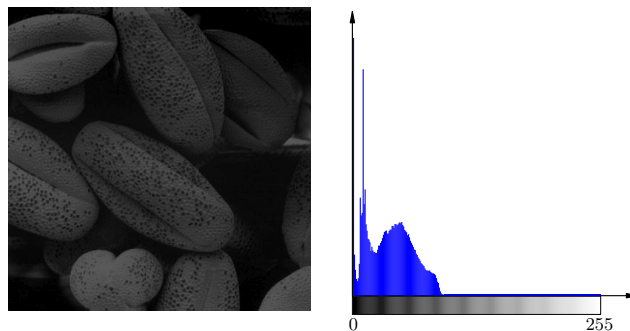
$$p(k) = \frac{n_k}{MN}$$

- ▶ M and N are the number of rows and columns of the image.
- ▶ The function $p(k)$ estimates the probability density of k ;
 - ▶ the sum $\sum_k p(k)$ is equal to 1.

Histogram based transformations

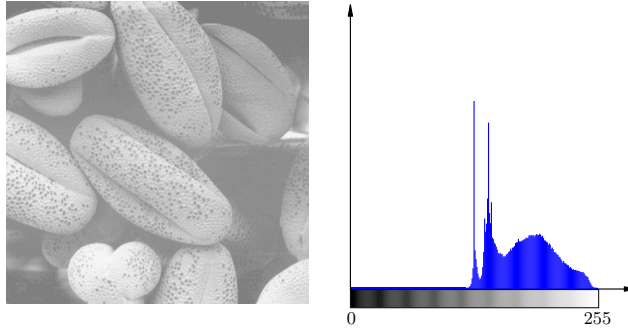
- ▶ The histogram provides an intuitive (visual) tool for evaluating some statistical properties of the image.
- ▶ Histogram based transformations are numerous:
 - ▶ enhancement,
 - ▶ compression,
 - ▶ segmentation;
- ▶ and can be easily implemented:
 - ▶ cheap;
 - ▶ dedicated hardware.

Dark image



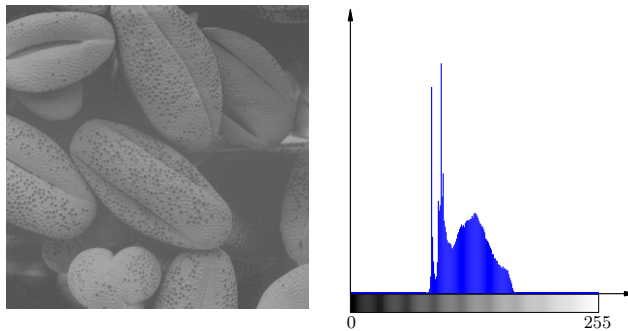
- ▶ The histogram components are localized to low intensity values.

Bright image



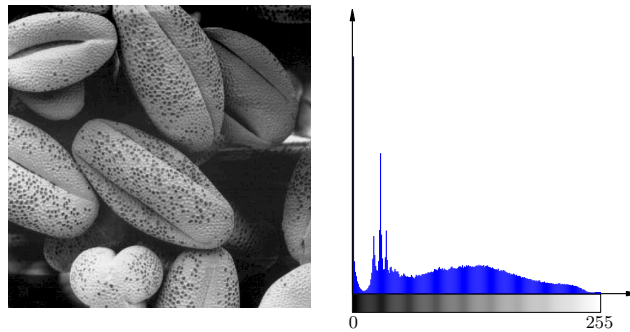
- The histogram components are localized to high intensity values.

Low contrast image



- The histogram components are localized in a narrow region of the intensity values.

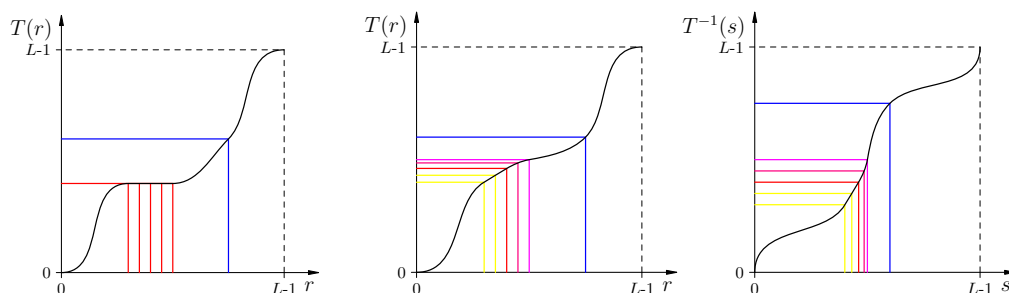
High contrast image



- ▶ The histogram components are distributed over all the intensity range.
- ▶ The distribution is almost uniform, with few peaks.
- ▶ If the distribution is uniform, the image tends to have a high dynamic range and the details are more easily perceived.
- ▶ This is the effect pursued by the histogram based transformations.

Monotonic transformations

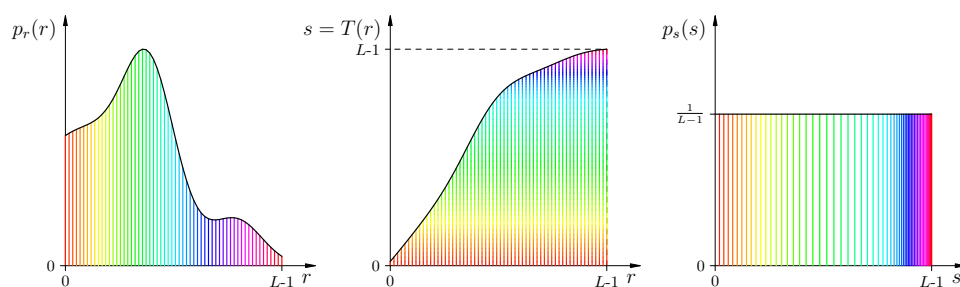
- ▶ In order to study the histogram transformations, it is useful to consider the (continuous) monotonic transforms on $[0, L-1]^2$:
 - ▶ $s = T(r)$, $0 \leq r \leq L-1$
 - ▶ $T(r_2) \geq T(r_1)$, $r_2 > r_1$
 - ▶ $0 \leq T(r) \leq L-1$, $0 \leq r \leq L-1$
- ▶ If T is strictly monotonically increasing, there is T^{-1} :
 - ▶ $r = T^{-1}(s)$, $0 \leq s \leq L-1$



Intensities as random variables

- ▶ The (continuous) intensities can be intended as random variables in $[0, L - 1]$.
- ▶ If $s = T(r)$ and $T(r)$ is continuous and differentiable:
 - ▶ $p_s(s) = p_r(r) \left| \frac{dr}{ds} \right|$
- ▶ In particular, the following transformation is interesting:
 - ▶ $s = T(r) = (L - 1) \int_0^r p_r(w) dw$
- ▶ Then:
 - ▶ $\frac{ds}{dr} = \frac{T(r)}{dr} = (L - 1) \frac{d}{dr} \left[\int_0^r p_r(w) dw \right] = (L - 1) p_r(r)$
- ▶ Hence:
 - ▶ $p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| = p_r(r) \left| \frac{1}{(L-1)p_r(r)} \right|$
 $= \frac{1}{L-1}, \quad 0 \leq s \leq L - 1$
- ▶ That is: s is uniform, independently of p_r .

Equalization



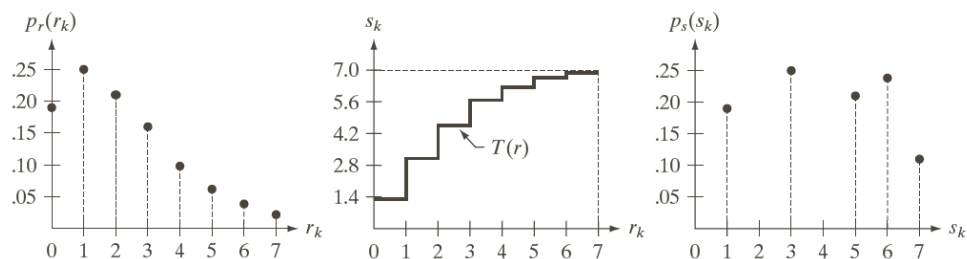
- ▶ The equalization transformation, $T(r)$, is steeper where r is more probable.
- ▶ It results in mapping intervals of r values with low probability into narrow intervals of $s = T(r)$.
- ▶ On the contrary, intervals of r values with high probability are mapped into large intervals of s .

Equalization of a discrete random variable

- ▶ r_k is the intensity level in $0, \dots, L - 1$
 - ▶ $p_r(r_k) = \frac{n_k}{MN}$, $k = 0, 1, \dots, L - 1$
- ▶ p_r can be equalized by assigning the intensity s_k to those pixels having intensity r_k :
 - ▶ $s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j)$
 $= \frac{L-1}{MN} \sum_{j=0}^k n_j$, $k = 0, 1, \dots, L - 1$

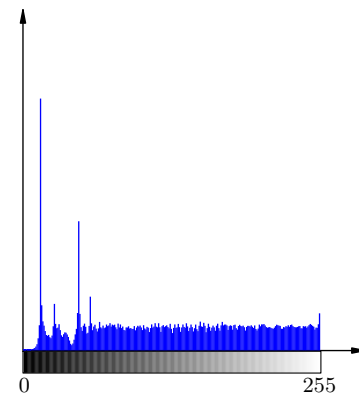
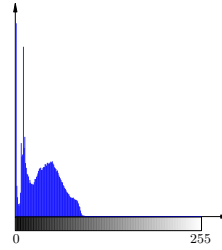
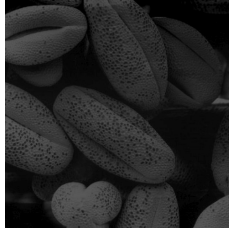
Equalization of a discrete random variable (2)

r_k	n_k	$p_r(r_k)$	$T(r_k)$	s_k	$p_s(s_k)$
$r_0 = 0$	790	0.19	1.33	1	0.19
$r_1 = 1$	1023	0.25	3.08	3	0.25
$r_2 = 2$	850	0.21	4.55	5	0.21
$r_3 = 3$	656	0.16	5.67	6	0.24
$r_4 = 4$	329	0.08	6.23	6	
$r_5 = 5$	245	0.06	6.65	7	0.11
$r_6 = 6$	122	0.03	6.86	7	
$r_7 = 7$	81	0.02	7.00	7	



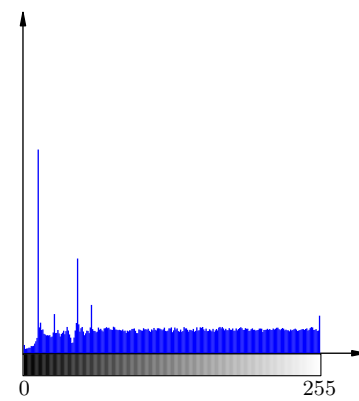
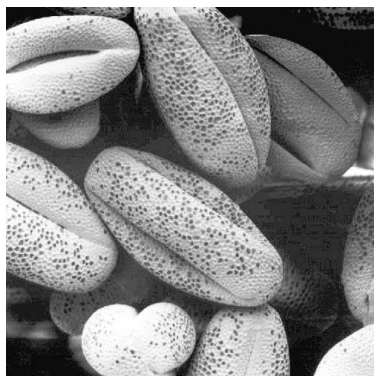
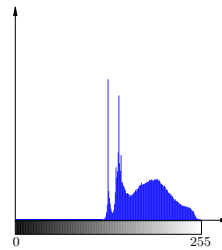
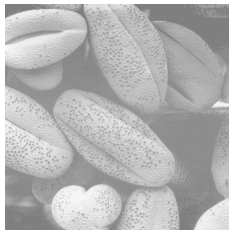
Examples

Dark image equalization



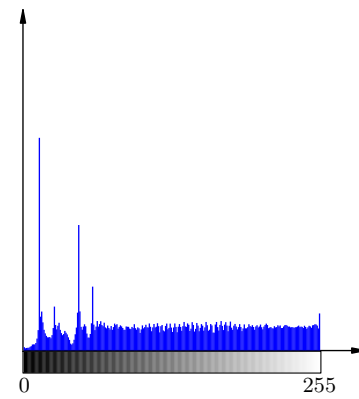
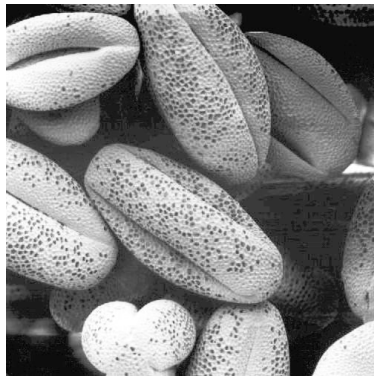
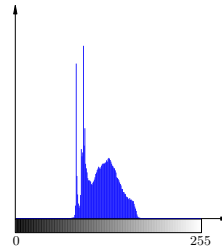
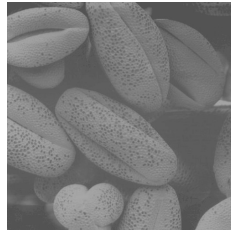
Examples (2)

Bright image equalization



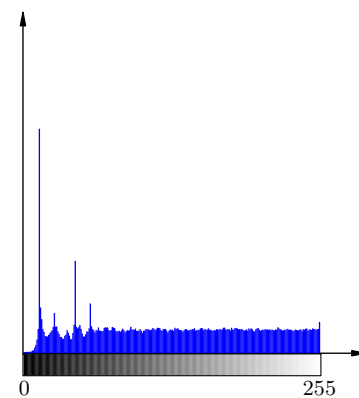
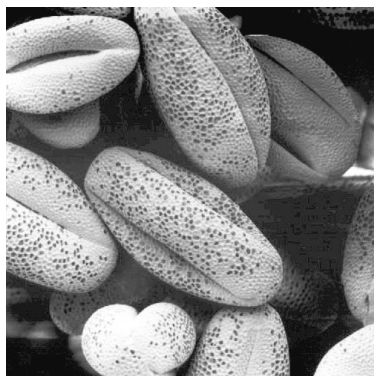
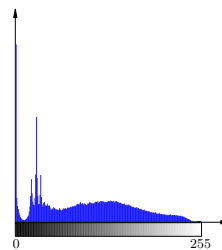
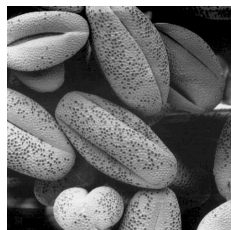
Examples (3)

Low contrast image equalization

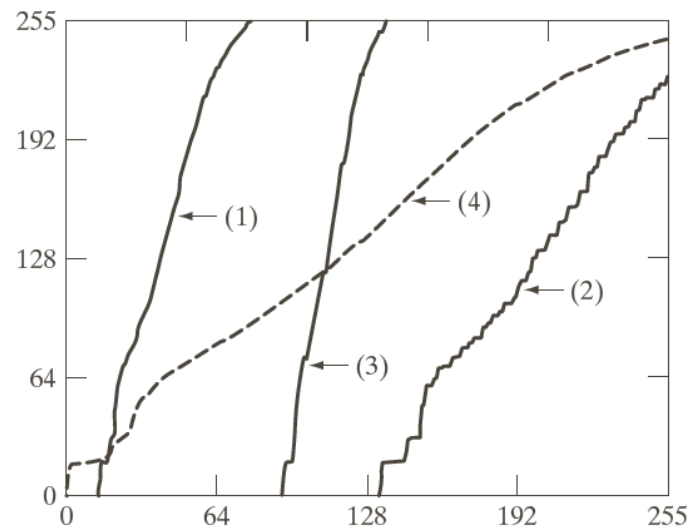


Examples (4)

High contrast image equalization



Examples (5)



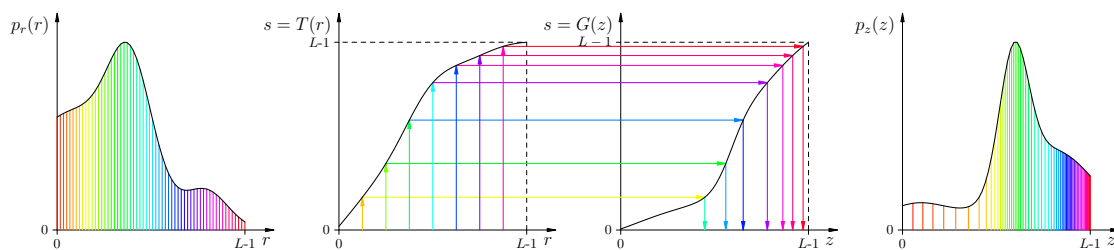
- ▶ The transformation of each image maps values from the range of the original images to the whole range of intensity levels.
- ▶ The transformation for (4) is close to the identity.

Histogram specification

- ▶ The histogram equalization is a basic procedure that allow to obtain a processed image with a specified intensity distribution.
- ▶ Sometimes, the distribution of the intensities of a scene is known to be not uniform.
- ▶ The possibility of obtaining a processed image with a given distribution is appreciable:
 - ▶ *Histogram matching*
- ▶ The problem can be formalized as follows:
 - ▶ given an input image, whose pixels are distributed with probability density p_r ,
 - ▶ given the desired intensity distribution, p_z ,
 - ▶ find the transformation F , such that $z = F(r)$.

Histogram specification (2)

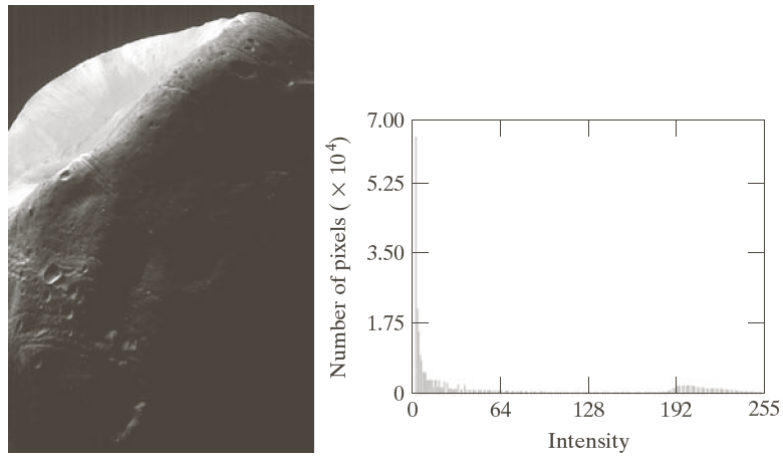
- ▶ Let s be a random variable such that:
 - ▶ $s = T(r) = (L-1) \int_0^r p_r(w) dw$
 - ▶ p_s is uniform
- ▶ Define a random variable z that satisfies:
 - ▶ $G(z) = (L-1) \int_0^z p_z(t) dt = s$
 - ▶ p_s is uniform
- ▶ Hence: $G(z) = s = T(r)$
- ▶ The desired mapping F , such that $z = F(r)$ can be obtained as:
 - ▶ $z = G^{-1}(T(r))$, i.e., $F = T \circ G^{-1}$



Histogram specification (3)

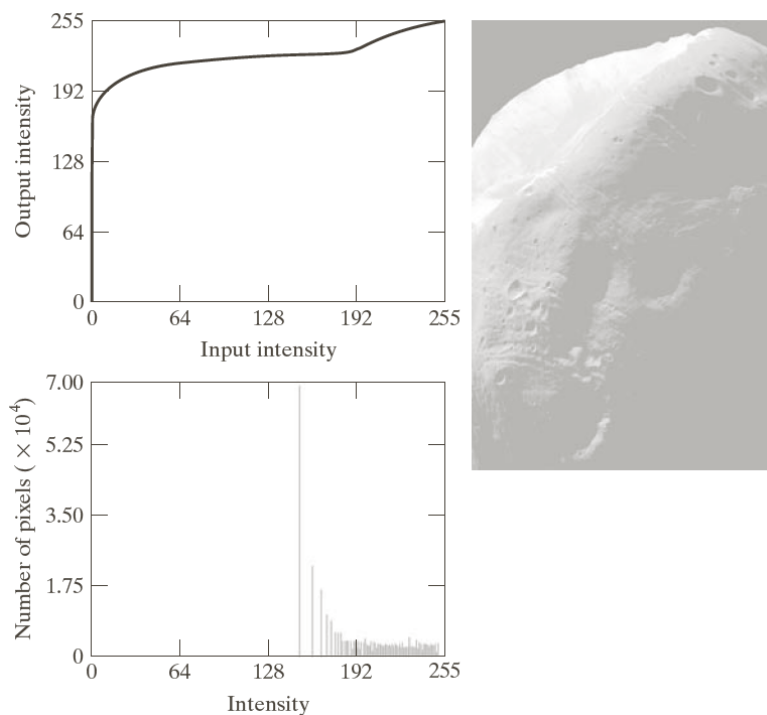
- ▶ When discrete random variables are considered, p_z can be specified by its histogram.
- ▶ The histogram matching procedure can be realized:
 1. obtain p_r from the input image;
 2. obtain the mapping T using the equalization relation;
 3. obtain the mapping G from the specified p_z ;
 4. build F by scanning T and finding the matching value in G ;
 5. apply the transformation F to the original image.
- ▶ In order to be invertible, G has to be strictly monotonic.
- ▶ In practical cases, this property is rarely satisfied.
- ▶ Some approximations should be allowed
 - ▶ e.g., the first matching value can be accepted.

Example

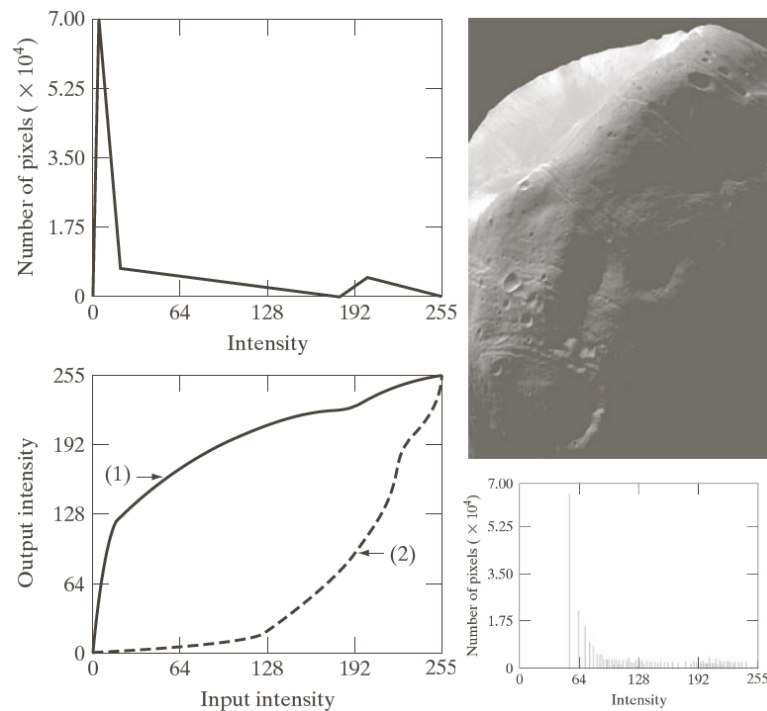


- Large concentration of pixels in the dark region of the histogram.

Example (2)



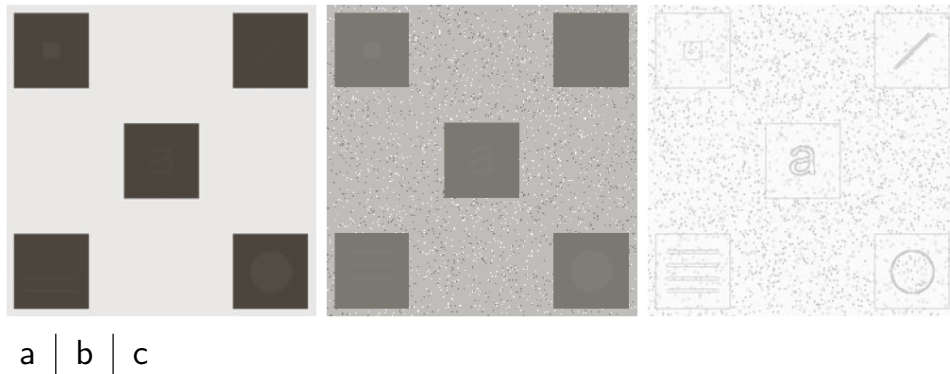
Example (3)



Local histogram processing

- ▶ Histogram equalization is a global approach.
- ▶ Local histogram equalization is realized selecting, for each pixel, a suitable neighborhood on which the histogram equalization (or matching) is computed.
 - ▶ More computational intensive, but neighboring pixels shares most of their neighborhoods.
- ▶ Non overlapping regions may produce “blocky” effect.

Example



- (a) original image
- (b) equalized image
- (c) locally equalized image (3×3 neighborhood)

Histogram statistics

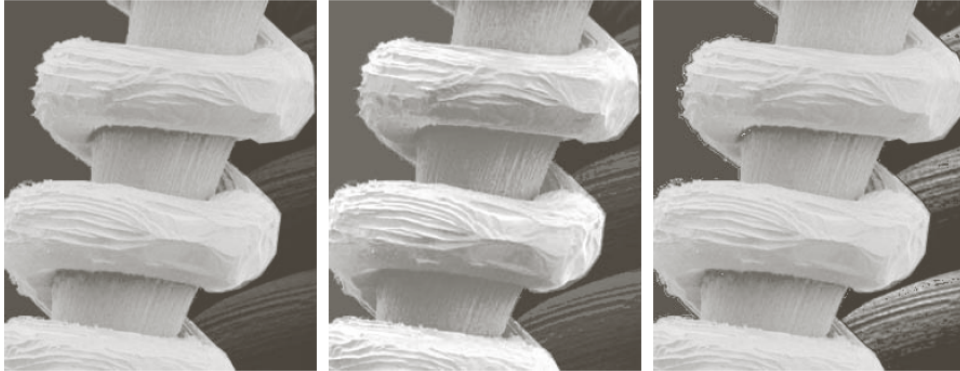
Some statistical indices can be easily computed from the histogram:

- ▶ Mean (average):
 - ▶ $m = \sum_{i=0}^{L-1} r_i p(r_i)$
- ▶ Variance:
 - ▶ $\sigma^2 = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$
 - ▶ Standard deviation: $\sigma = \sqrt{\sigma^2}$
- ▶ n -th moment:
 - ▶ $\mu_n = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$

Local statistical indices can be computed by bounding the histogram to a given neighborhood, S_{xy} :

- ▶ $m_{S_{xy}} = \sum_{i=0}^{L-1} r_i p_{S_{xy}}(r_i)$
- ▶ $\sigma_{S_{xy}}^2 = \sum_{i=0}^{L-1} (r_i - m_{S_{xy}})^2 p_{S_{xy}}(r_i)$

Example



a | b | c

- (a) original image
- (b) equalized image
- (c) local statistics enhanced image (3×3 neighborhood)

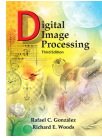
Example (2)

- ▶ Only dark regions need to be enhanced
 - ▶ $m_{S_{xy}} \leq k_0 m_G$
- ▶ Uniform regions have to be preserved
 - ▶ $\sigma_{S_{xy}} \geq k_1 \sigma_G$
- ▶ Low contrasted regions have to be enhanced
 - ▶ $\sigma_{S_{xy}} \leq k_2 \sigma_G$

$$g(x, y) = \begin{cases} E \cdot f(x, y) & \text{if } m_{S_{xy}} \leq k_0 m_G \\ & \text{AND } k_1 \sigma_G \leq \sigma_{S_{xy}} \leq k_2 \sigma_G \\ f(x, y) & \text{otherwise} \end{cases}$$

$$E = 4, k_0 = 0.4, k_1 = 0.02, k_2 = 0.4.$$

Homeworks and suggested readings



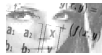
DIP, Sections 3.2, 3.3

- ▶ pp. 120–143



GIMP

- ▶ Colors
 - ▶ Info
 - ▶ Histogram
- ▶ Auto
 - ▶ Equalize



[http://www.imageprocessingbasics.com/
image-histogram-equalization/](http://www.imageprocessingbasics.com/image-histogram-equalization/)