Md. Ferdous · Follow
Feb 28, 2023

Maths behind Linear Prediction

I will start with basic loss calculation for linear prediction in this post. We know the loss equation is:

$$J(\boldsymbol{\theta}) = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} (y_i - \theta_1 - x_i\theta_2)^2$$

If we convert input/features and output/labels as a matrix, then we can write:

$$J(\boldsymbol{\theta}) = (\mathbf{y} - \mathbf{X}\boldsymbol{\theta})^T(\mathbf{y} - \mathbf{X}\boldsymbol{\theta}) = \sum_{i=1}^{n} (y_i - \mathbf{x}_i^T\boldsymbol{\theta})^2$$

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$$
\begin{aligned}
\mathrm{J}(\theta) &= (y - x\theta)^{\top}(y - x\theta) \\
&= y^{\top}y - y^{\top}x\theta - x^{\top}\theta^{\top}y + x^{\top}\theta^{\top}x\theta \\
&= y^{\top}y - y^{\top}x\theta - y^{\top}x\theta + x^{\top}\theta^{\top}x\theta \\
&= y^{\top}y - 2y^{\top}x\theta + x^{\top}\theta^{\top}x\theta
\end{aligned}
$$

Now, we will calculate partial derivatives with respect to theta.

$$
\begin{aligned}
\frac{\partial \mathrm{J}(\theta)}{\partial\theta} &= \frac{\partial}{\partial\theta}(y^{\top}y - 2y^{\top}x\theta + x^{\top}\theta^{\top}x\theta) \\
&= 0 - 2x^{\top}y + 2x^{\top}x\theta \\
&= -2x^{\top}y + 2x^{\top}x\theta
\end{aligned}
$$

Hence, $\frac{\partial A\theta}{\partial\theta} = A^{\top}$ and $\frac{\partial \theta^{\top}A\theta}{\partial\theta} = 2A^{\top}\theta$

As we want to calculate θ accurately, hence

$$
\frac{\partial \nabla(\theta)}{\partial\theta} = 0
$$

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$$\frac{\partial \mathrm{J}(\theta)}{\partial \theta} = 0$$

$$-2x^{\top}y + 2x^{\top}x\theta = 0$$

$$\Rightarrow \; 2x^{\top}x\theta = 2x^{\top}y$$

$$\Rightarrow \; \theta = \frac{x^{\top}y}{x^{\top}x}$$

$$\Rightarrow \; \theta = (x^{\top}x)^{-1}x^{\top}y$$

Now, we can calculate the correct theta using input/feature and output/label matrix. However, this simple linear prediction can lead to problems in poor data conditions. We need to add a regularizer to solve this issue; I will cover it in my next post.

Written by Md. Ferdous

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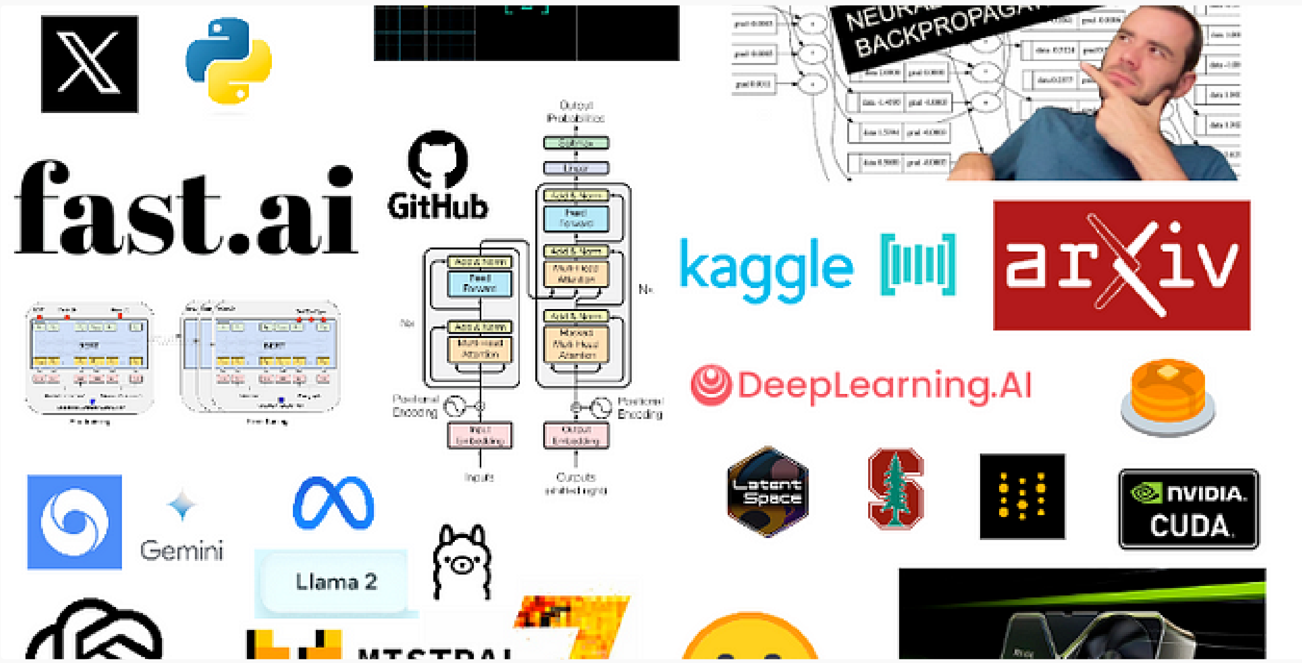

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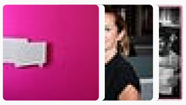

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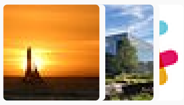

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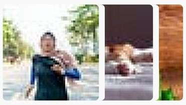

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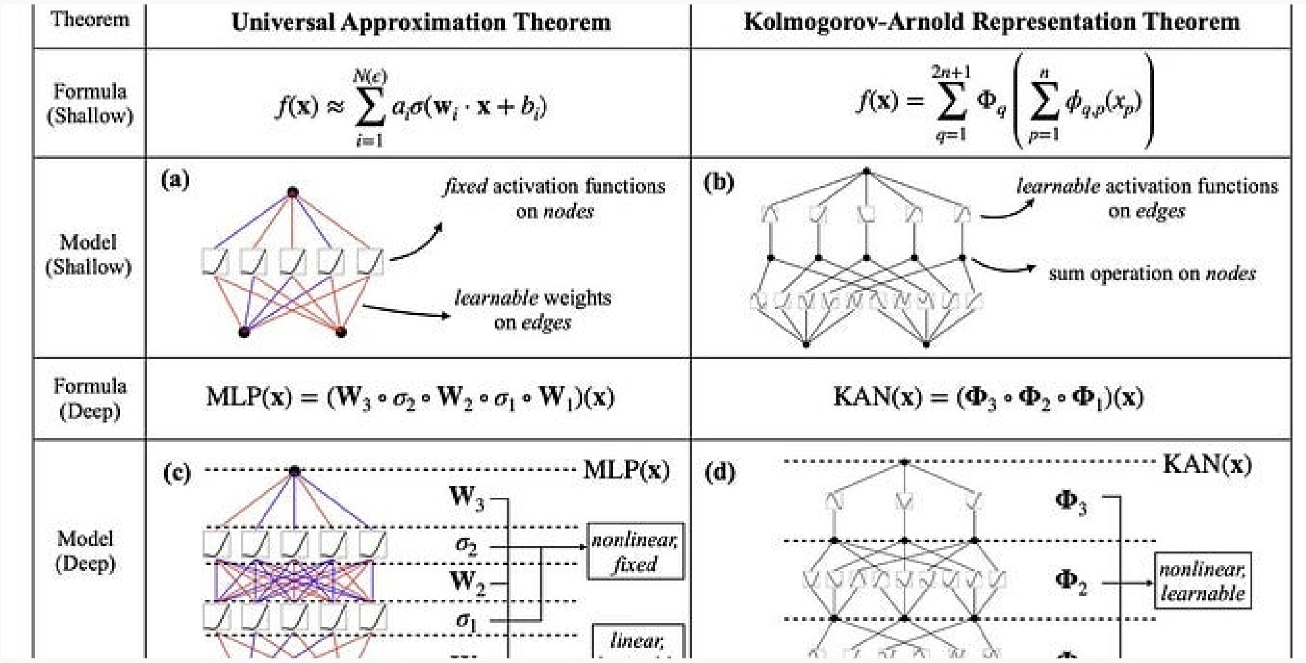

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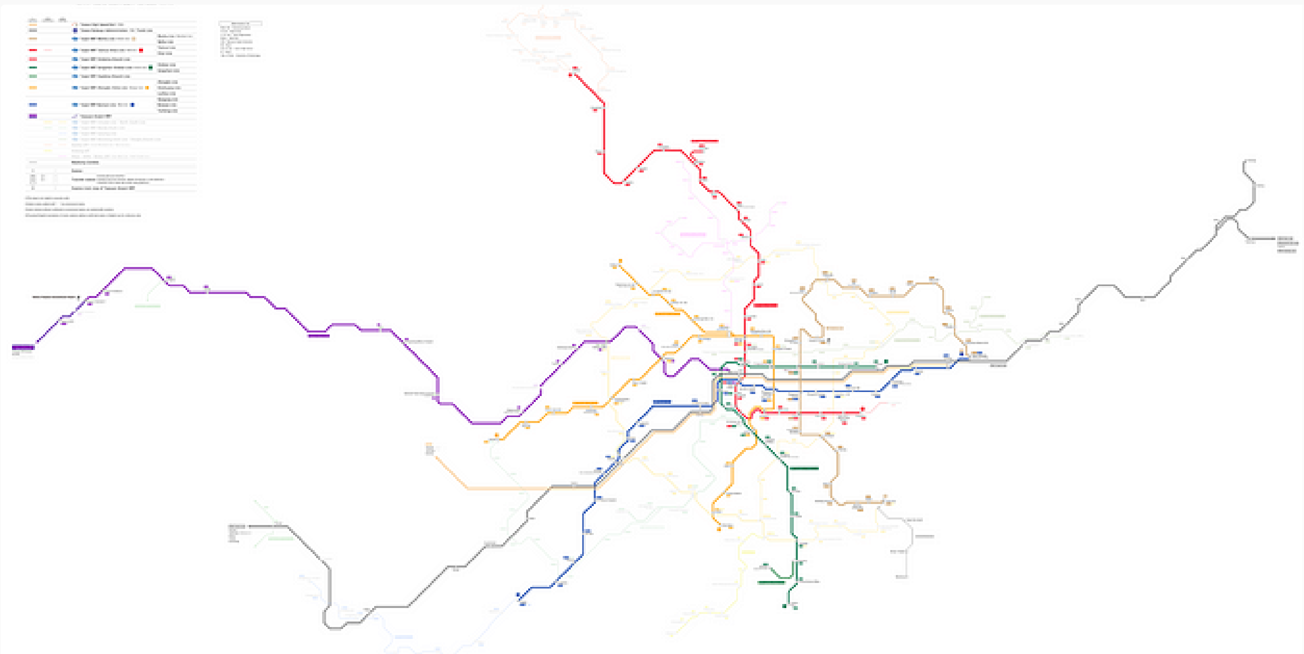

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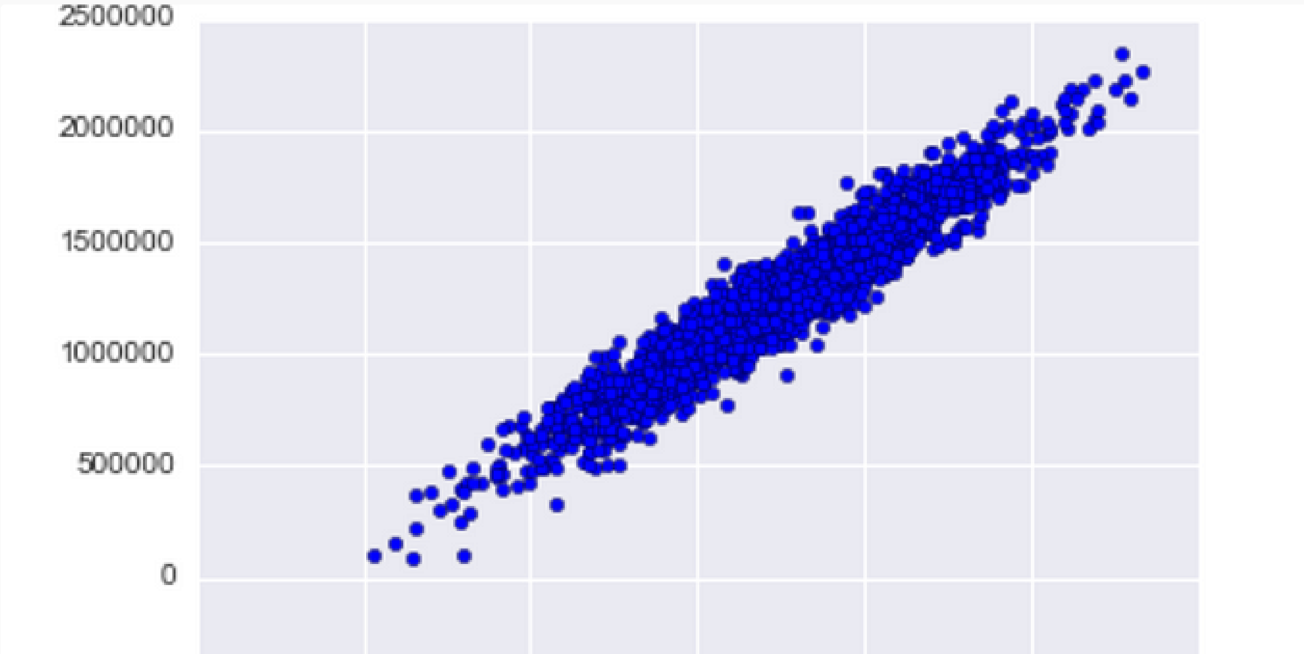

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