Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Department of Computer Science and Engineering

1st Year 1st Semester B.Sc. Engineering Examination-2014

Course No: MAT 104
Full Marks: 70

Course Title: Algebra, Trigonometry and Vector Analysis
Time: 4 hours

N.B.
i) Answer **SIX** questions, taking any **THREE** from each section.
ii) All questions are of equal values.
iii) Use separate answer script for each section.

		SECTION-A	
Q.1	(a)	Define Power set, Complement set and Disjoint sets with an example.	3
	(b)	If $A = \{x \vert x\ is\ a\ multiple\ of\ 3\}$ and $B = \{x \vert x\ is\ even\}$ are subsets of the universal set $U = \{x \vert x\ are\ natural\ number \leq 15\}$ then verify $(A \cap B')' \cup B = A' \cup B$.	3
	(c)	Prove that if R is a relation on A, then $(R^{-1})^{-1} = R$.	$2\frac{2}{3}$
	(d)	If R be the set of real numbers, then in $R \times R$ draw the graph of the relation $G = \{(x, y) \vert x, y \in R\ and\ x^2 + y^2 \geq 4\}$ and $S = \{(x, y) \vert x, y \in R\ and\ y \geq \frac{4}{9}x^2\}$	3
Q.2	(a)	Define domain and range of a function. If $f(x) = \frac{x}{x-1}$ then find its domain and range. Also find its inverse function.	4
	(b)	Prove that the inverse of an invertible function is invertible.	3
	(c)	Prove that every polynomial of degree n has exactly n roots.	$4\frac{2}{3}$
Q.3	(a)	Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ given that one roots is $(-1 + i)$.	$3\frac{2}{3}$
	(b)	Using DeMoiver's theorem solve the equation $x^4 + x^2 + 1 = 0$.	4
	(c)	Prove that $\left[\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right]^n = (\sin\theta + i\cos\theta)^n$	4
Q.4	(a)	State and prove DeMoiver's theorem of trigonometry for positive integers.	4
	(b)	What do you mean by Gudermanian function? If $\cos^{-1}(u + iv) = \alpha + i\beta$, then prove that $(\cos\alpha)^2$ and $(\cosh\beta)^2$ are roots of the equation $x^2 - (1 + u^2 + v^2)x + u^2 = 0$.	4
	(c)	Define Hyperbolic function. Expand $\cos n\theta$ in term of $\cos\theta$.	$3\frac{2}{3}$

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SECTION-B

Q.5	(a)	State Gregory's series. Show that $\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \cdots$	4
	(b)	Sum to infinity the series: $1 + \cos x + \frac{\cos 2x}{2!} + \frac{\cos 3x}{3!} + \cdots to\ \infty$	4
	(c)	Find the sum of the following series: $\frac{3}{2.4} + \frac{3.4}{2.4.6} + \frac{3.4.5}{2.4.6.8} + \cdots\ \infty$	$3\frac{2}{3}$
Q.6	(a)	Define collinear and coplanar vectors. Examine whether the vectors 5a + 6b + 7c, 7a,-8b+9c and 3a +20 b + 5c (a, b, c being non-coplanar vectors) are linearly independent or dependent.	4
	(b)	A Force $\vec{F} = 3i + 2j - 4k$ is applied at the point $(1, -1, 2)$. Find the moment of the force about the point $(2, -1, 3)$.	3
	(c).	Define cross product with physical interpretation. Prove that $(\vec{A} \times \vec{B}).(\vec{B} \times \vec{C}) \times (\vec{C} \times \vec{A}) = (\vec{A}.\vec{B} \times \vec{C})^2$	$4\frac{2}{3}$
Q.7	(a)	If the vector $\vec{V} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational then find the values of constants a, b, c.	4
	(b)	The position vector of a particle at time t is $\vec{r} = \cos(t-1)\hat{\imath} + \sinh(t-1)\hat{\jmath} + \alpha t^3\hat{k}$. Find the condition imposed on α by requiring that at time t = 1, the acceleration normal to the position vector.	$3\frac{2}{3}$
	(c)	Define Directional derivative and Divergence. Write down the Geometrical interpretation of curl	4
Q.8	(a)	State and prove Green's theorem.	4
	(b)	Verify the divergence theorem $\vec{A} = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$.	4
	(c)	Define surface integrals. Evaluate $\int_s \int \vec{A}.\hat{n}ds$ where $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$ and s is the part of the plane $2x + 3y + 6z = 12$ which is located in the first octant.	$3\frac{2}{3}$