Bangabandhu Sheikh Mujibur Rahman Science and Technology University
Department of Computer Science and Engineering
2nd Year 2nd Semester B.Sc. Engineering Examination-2014

Course No: CSE 260 — Course Title: Design and Analysis of Algorithm
Full Marks: 70 — Time: 4 hours

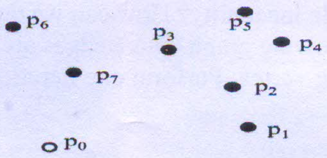
b[c[A[j]]]

N.B.
i) Answer **SIX** questions, taking any **THREE** from each section.
ii) All questions are of equal values.
iii) Use separate answer script for each section.

SECTION-A

Q.1 (a) What do you mean by Complexity of an algorithm? Describe O(n), Ω(n), θ(n) and ω(n) with proper example? — 4

(b) What is a convex hull? Given a set of points in the following figure, solve convex hull problem based on graham's scan algorithm. — $3\frac{2}{3}$



(c) Prove that complexity of quick sort algorithm is nlgn — 4

Q.2 (a) What are the minimum and maximum numbers of elements in a heap of height H? — 2

(b) Depict the operation of BUILD-MIN-HEAP on the array A = < 7, 17, 12, 14, 6, 4, 2, 1, 3, 8, 9, 10, 4, 25>. If we delete the root node what would be the array A? — 4

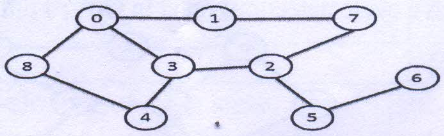
(c) What is counting sort? Illustrate the operation of counting sort on the array B = <5, 0, 2, 1, 1, 2, 3, 2, 4>. — 4

(d) What are meant by monotonically increasing and decreasing? — $1\frac{2}{3}$

Q.3 (a) For each of the following statements, say whether it is True or False. If it is True, give a brief explanation; if it is False, give a simple counter example. — 3

i. Bellman-Ford shortest path algorithm is an example of greedy algorithm.
ii. There are n(n-1)/2 edges in an *n*-vertex complete graph.
iii. Suppose all edge weights are same. Then the shortest path from A to B is unique.

(b) — 5



Describe DFS and BFS traversal of the graph step by starting form 4.

(c) Why topological sort needed? Give practical example of topological sort? — $3\frac{2}{3}$

Q.4 (a) How can a cross edge be detected in a directed graph when DFS is used? — 2

(b) What is Strongly Connected Components (SCC)? How you find SCC from DFS. — 3

(c) What is the property of bipartite graph? How can we detect it using BFS? — 4

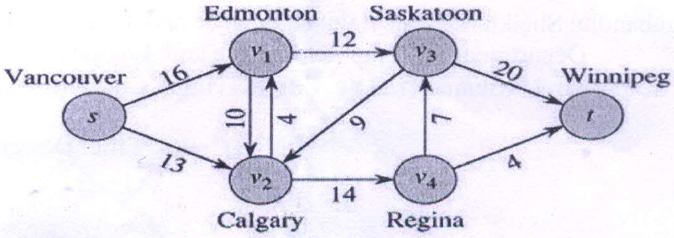
(d) Consider a directed graph in which the nodes are weighted (with positive weights), while the edges aren't. The weight of a path is now defined as the sum of the weights of the nodes that it uses. Given source and destination nodes, design algorithm that will determine the shortest path between them. — $2\frac{2}{3}$

SECTION-B

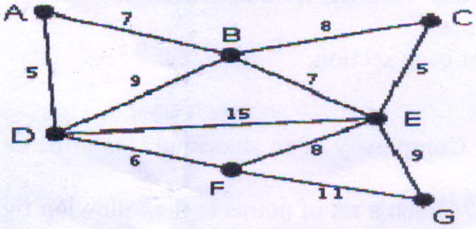
Q.5 (a) Compute the LCS table for the sequence X = <B,C,B,D,A,B> and Y = <B,D,C,A,B> — 4

(b) Given p0 = 10, p1 = 15, p2 = 5, p3 = 15, p4= 10, p5 = 20, p6= 5, m[2,2] = 0, m[2,3] = 262, m[2,4] = 375, m[3,5] = 250, m[4,5] = 100 and m[5,5] = 0 calculate the value of m[2,5] using matrix chain multiplication. — 3

(c) Find the maximum flow using Ford-Fulkerson method for the following graph. — $4\frac{2}{3}$



Q.6 (a) Find the minimum spanning tree of the following graph using prim's algorithm. 6



(b) What do you meant by triangle inequality? How can we detect negative cycle by triangle inequality? 3

(c) Suppose you are give the following graph where edges are stored in ascending order (according to their cost). Suppose A is the starting vertex. Perform one iteration of Bellman-Ford algorithm and show d and Π values. $2\frac{2}{3}$

Q.7 (a) Consider the following matrix which corresponds to the initialized distance matrix of the all-pairs-shortest path algorithm: 1+4 +1

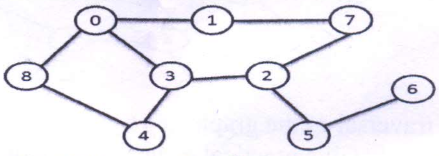
$$\begin{pmatrix} 0 & 2 & 4 & 3 \\ 3 & 0 & \infty & 3 \\ 5 & \infty & 0 & 3 \\ \infty & 1 & 4 & 0 \end{pmatrix}$$

(i) Draw the corresponding graph.
(ii) Execute two iteration of Floyd-Warshall algorithm.
(iii) What is the running time of the Floyd- Warshall algorithm?

(b) Answer true or false with proper justification. 2

(i) Bellman-Ford shortest path algorithm is an example of greedy algorithm.
(ii) Consider a graph G = (V, E), with negative weight edges. We can all pairs shortest paths (with negative cycle detection) on G in $O(|V|^3)$ time.

.(c) $3\frac{2}{3}$



Describe Dijkstra's algorithm from start node 2.

Q.8 (a) Generate the Huffman cod e for the following string. 5

AACABAABAAAABAADDDEEFFDEFFA

(b) What the recurrence relation for the following code. 4

```
function f(n) {

    if(n>1){
        for(i=1; i<=n;i=i*2)
            printf("running");
        f(n/2);
        f(n/3);
    }
    else  printf("stop");
}
```

(c) How can we detect a vertex as an articulation point when we run DFS algorithm? $2\frac{2}{3}$

Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Department of Computer Science and Engineering

2^{nd} Year 2^{nd} Semester B.Sc. Engineering Examination-2014

Course No. : CSE262, Title: Automata Theory

Full Marks: 70 Times: 3 Hours

N.B.:

i. Answer **SIX** questions, taking any **THREE** from each section.

ii. All questions are of equal values

iii. Use **separate answer script** for each section.

SECTION-A

Q.1 (a) Give the formal definition of DFA with example. — 4

(b) Design a DFA to accept the set of strings that either begin or end (or both) with 00. — 2

(c) Find the DFA that accepts all strings over {a,b} that has baba as a substring. — 2

(d) Converting the following NFA into DFA. — $3\frac{2}{3}$

	0	1
➔ P	{P}	{Q}
Q	{S}	{R}
S	{Q,R}	{S}
*R	{R}	{Q}

Q.2 (a) Formally describe the definition of nondeterministic finite automata (NFA). — 4

(b) What is ε- closure? Consider the following ε-NFA — 6

	E	a	b	c
➔ p	{r}	{p}	{q}	{r}
q	{p}	{q}	{r}	{}
*r	{q}	{r}	{}	{p}

(i) Compute the ε-closure of each state.

(ii) Give all the strings of length three or less accepted by the automation.

(iii) Convert the automation to a DFA

(c) Design the regular expression for all strings containing at least two 0's. — $1\frac{2}{3}$

Q.3 (a) Use the pumping lemma to show that the language B={ $a^n b^n c^n$ | n≥ 0 } is not context free. — 4

(b) Write Design the regular express for the following language. — $7\frac{2}{3}$

(i) The language of all strings in which the number of 0's is even.

(ii) The language of all strings whose no of 0's is divisible by five.

(iii) The language of all strings containing both 11 and 010 as substrings.

(iv) The language of all strings with at most one pair of consecutive 1's.

Q.4 (a) What is ambiguous grammar? Consider the grammar — 6

S = aS | aSbS | ε

This grammar is ambiguous. Show in particular that the string aab has two:

(i) Parse tree.

(ii) Leftmost derivations.

(iii) Rightmost derivations.

(b) Consider the grammar — $5\frac{2}{3}$

E = +EE | * EE | -EE | x | y

(i) Find leftmost and rightmost derivations, and a derivation tree for the string +*-xyxy.

(ii) Prove that this grammar is unambiguous.

Bangabandhu Sheikh Mujibur Rahman Science and Technology University
Department of Computer Science and Engineering
2^{nd} Year 2^{nd} Semester B.Sc. Engineering Examination-2014
Course No. : CSE270, Title: Concrete Mathematics and Numerical Analysis

Full Marks: 70 Times: 3 Hours

N.B.:
i. Answer **SIX** questions, taking any **THREE** from each section.
ii. All questions are of equal values
iii. Use **separate answer script** for each section.

Section- A

1.	a) Discuss the sources of errors in numerical analysis.	3
	b) What do you mean by significant digits? Write down the rules for significant digits.	3.67
	c) Round-off the following numbers correct to four significant digits: 3.26425, 4985561, 0.70035, 0.00032217, 30.0567, 3.14159.	3
	d) Which of the following numbers has the greatest precision? 4.3201, 4.32, 4.320106.	2
2.	a) Discuss the method of Bisection to find an approximate root of an equation f(x)=0.	5
	b) Use the Newton-Raphson method to estimate the root of $f(x) = e^{-x} - x$, employing an initial guess of $x_0 = 0$.	4
	c) Write down the advantages and disadvantages of Secant method.	2.67
3.	a) Discuss the method of False Position to find an approximate root of an equation f(x)=0.	5
	b) Drive the secant method to estimate the root of $f(x) = \ln x$. Start the computation with values of $x_l = x_{i-1} = 0.5$ *and* $x_u = x_i = 5.0$	4
	c) Write down some limitations of Newton-Raphson method.	2.67
4.	a) What is Regression? Discuss different types of Regression.	6
	b) Fit a second degree parabola to the following data taking y as dependent variable:	5.67

x	1	2	3	4	5	6	7	8	9
y	2	6	7	8	10	11	11	10	9

Section - B

5.	a) Establish Newton's divided difference formula for interpolation.	5
	b) Using Lagrange's interpolation formula find y(10) given that y(5) = 12 , y(6) = 13, y(9) = 14 and y(11) = 16.	4
	c) Write down the merits and demerits of lagranges interpolation formula.	2.67
6.	a) Derive a Newton-Cote's quadrature formula.	5.67
	b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using (i) Simpson's one-third rule (ii) Simpson's three-eighth rule (iii) Trapezoidal rule	6
7.	a) Evaluate $\int_3^7 \frac{dx}{1+x^2}$ using Gaussian quadrature with 3 points.	5
	b) Using Runge-Kutta method of 4[th] order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with y(0)=1 at x =0.2	5
	c) Define the local truncation error.	1.67
8.	a) Discuss Naive Gauss Elimination method for solving the system of n linear equations.	5.67
	b) Using Gauss-Seidel method solve the following system: $3x_1-0.1x_2-0.2x_3=7.85$ $0.1x_1+7x_2-0.3x_3=-19.3$ $0.3x_1-0.2x_2+10x_3=71.4$	6

	0	1	2	3
x =	5	6	9	11
y =	12	13	14	16.

$y = y_0 + hf(x_0, y_0)$

$\frac{dy}{dx} = y.$

Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Department of Computer Science and Engineering

2^{nd} Year 2^{nd} Semester B.Sc. Engineering Examination-2014

Course No: MAT-256 Course Title: Linear Algebra

Total marks: 70 Time: 4 hours.

N.B.

i. Answer **SIX** questions, taking any **THREE** from each section.
ii. All questions are of equal values
iii. Use separate answer script for each section.

$$\begin{bmatrix} 3 & 11 & 1 \\ -11 & -5 & 19 \\ 12 & -9 & 9 \end{bmatrix}.$$

Section A

1. (a) Define linear equation. $5^{2/3}$

Solve the system of linear equations: $2x_1 + x_2 + 3x_3 = 0, x_1 + 2x_2 = 0, x_2 + x_3 = 0$ by Row- Echelon form.

(b) Define skew-symmetric Matrix and Hermition matrix. Find the inverse of the matrix 6

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{bmatrix}$$

53.

2. (a) Define vector space. Let $V = R^3$, show that W is not a subspace of V Where W= $\{(a,b,c) \mid a^2 + b^2 + c^2 \le 1\}$ i.e. W consists of those vectors of R^3 whose lengths do not exceed 1. 4

(b) Define Sub-space. Prove that the intersection of any family of subspaces of a vector space is subspace. 4

(c) Define linear independence. Determine whether the vectors: $3^{2/3}$

$v_1 = (1,-2,3), v_2 = (5,6,-1)$ and $v_3 = (3,2,1)$ form a linearly dependent or independent set.

3. (a) Define linear combination. Show that the matrix $\begin{bmatrix} 3 & -1 \\ 1 & -2 \end{bmatrix}$ is a linear combination of the matrices $A_1 = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$ and $A_3 = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$. $4^{2/3}$

(b) Define sum and direct sum. 2

(c) Define basis & dimension for vector space. Let, $v_1 = (1,2,1), v_2 = (2,9,0)$ and $v_3 = (3,2,1)$. Show that the set $S = \{v_1, v_2, v_3\}$ is a basis for R^3 5

4. (a) State and prove the first Fundamental theorem of sub space. 5

(b) Define linear transformation. Let U and V be vector space over the same field F. Let $\{u_1, u_2 \ldots\ldots\ldots\ldots u_n\}$ be a basis of U and $v_1, v_2 \ldots\ldots\ldots\ldots v_n$ be any vectors in V. Then prove that there exists a linear transformation $T : U \to V$ such that $T(u_1) = v_1, T(u_2) = v_2, \ldots\ldots\ldots, T(u_n) = v_n$. $6^{2/3}$

P.T.O

Section B

5. (a) Define Rank and Nullity. Show that the transformation defined by $T(x,y,z) = (x + y, -x - y, z)$ is a linear transformation. 4

(b) Let $R^2 \to R^2$ be the linear operator defined by $T(x,y) = (5x + 7y, -3x + y)$. Find the matrix of T with respect to the basis $F = (e_1, e_2) = \{(1,0),(0,1)\}$ and $S = (u_1, u_2) = \{(2,1),(3,5)\}$ $4^{2/3}$

(c) Define inner product on a real vector space. Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ be vectors in R^2 verify the weighted Euclidean inner product $\langle u, v \rangle = 3u_1v_1 + 2u_2v_2$ satisfies the four inner product space. 3

6. (a) What do you mean by matrix representation? 2

(b) Let $T : R^2 \to R^2$ be the linear operator defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ -2x_1 + 4x_2 \end{bmatrix}$ and $B = \{u_1, u_2\}$ be the basis, where $u_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Then find (i) $[T]_B$ and (ii) verify that $[T]_B [X]_B = [T(x)]_B$ for every vector x in R^2. 6

(c) Define linear functional. Let V be a vector space over the field F, then the mapping $f : V \to F$ define by $f(x) = 0$, for all $x \in V$ is a linear functional on V. $3^{2/3}$

7. (a) Define Orthogonality. Suppose S is the orthogonal set of non- zero vectors. Then prove that S is linearly independent. 5

(b) State and prove Gram-Schmidt orthonormalization process. $6^{2/3}$

8. (a) Define polynomials of matrices.. Let f and g be polynomials. For any square matrix A and scalar k prove that $(f + g)(A) = f(A) + g(A)$ 5

(b) Define minimal Polynomial. Find the minimal polynomial $m(\lambda)$ of the matrix $6^{2/3}$

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Bangabandhu Sheikh Mujibur Rahman Science and Technology University

Department of Computer Science and Engineering

2nd Year 2nd Semester B.Sc. Engineering Examination-2014

Course No: STA254 — Course Title: Theory of Statistics

Marks: 70 — Time: 3 hours

N.B.

i) Answer **SIX** questions, taking any **THREE** from each section.

ii) All questions are of equal values.

iii) Use separate answer script for each section.

Section-A

1. (a) What do you mean by linear orthogonal transformation? 2

   (b) If X_i $(i=1,2,3, \ldots, n)$ are independent $N(0,\sigma^2)$ and they are transformed to a new set of variables Y_i $(i=1,2,3, \ldots, n)$, by means of a linear orthogonal transformation, Then show that Y_i $(i=1,2,3, \ldots, n)$ are also independent $N(0,\sigma^2)$. 5

   (c) Write the form of χ^2 distribution. Is it a probability density function? Justify your answer. $4\frac{2}{3}$

2. (a) Show that the variance of χ^2 distribution is twice of its degrees of freedom. 4

   (b) What are the applications of χ^2 distribution? 3

   (c) Write down the applications and properties of 't' distribution. $4\frac{2}{3}$

3. (a) Show that $f(F)=\dfrac{\left(\gamma_1/\gamma_2\right)^{\gamma_1/2}}{\beta\left(\frac{\gamma_1}{2},\frac{\gamma_2}{2}\right)}\dfrac{F^{\frac{\gamma_1}{2}-1}}{\left(1+\frac{\gamma_1}{\gamma_2}F\right)^{\frac{\gamma_1+\gamma_2}{2}}}$; $0\le F<\infty$ is a probability density function. Also find its mean. 8

   (b) What are the applications of F distribution? $3\frac{2}{3}$

4. (a) Define with example parameter, statistic, estimator and estimate. $5\frac{2}{3}$

   (b) What do you mean by consistency and sufficiency? 3

   (c) Describe the method of maximum likelihood. 3

Section-B

5. (a) Suppose $x_1, x_2, \ldots x_n$ are *iid* $n(1,p)$. Find the maximum likelihood estimator (MLE) of p. 4

   (b) From the following density function, find out the maximum likelihood estimate of λ. 3

   $$f(x;\lambda)\frac{e^{-\lambda}\lambda^x}{x!};\lambda.>0$$

   (c) Show that maximum likelihood estimator is not always unbiased. $5\frac{2}{3}$

6. (a) What do you mean by statistical hypothesis? Explain various types of statistical hypothesis with example. 4

(b) Define (i) Type I and type II error (ii) Acceptance and Rejection region (iii) One sided and two sided test. $4\frac{2}{3}$

(c) Briefly describe the general procedure for performing a test of hypothesis. 3

7. (a) For 2×2 contingency table prove that chi-square test of independence gives 5

$$\chi^2 = \frac{N(ad-bc)^2}{(a+c)(b+d)(a+b)(c+d)}; \quad N = a+b+c+d .$$

(b) The following table shows the number of recruits taking a preliminary and a final test in car driving. 4

		Preliminary	
		Pass	Fail
Final	Pass	605	135
	Fail	195	65

Use chi-square test to discuss whether there is any association between the results of the preliminary and those of the final test. [$\chi^2_{0.05,1} = 3.841$]

(c) Describe how would you test the null hypothesis, $H_o:\ \sigma^2 = \sigma_o^2$. $2\frac{2}{3}$

8. (a) What do you mean by nonparametric test? What are the assumptions, advantages and disadvantages of nonparametric test? 7

(b) Describe the procedure of sign test. $4\frac{2}{3}$

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