# CSE 3318

Week of 07/23/2024

Instructor: Donna French

#### Prim vs Kruskal

You can't really ask which algorithm is better in a given case without considering specific details of problem you are trying to solve.

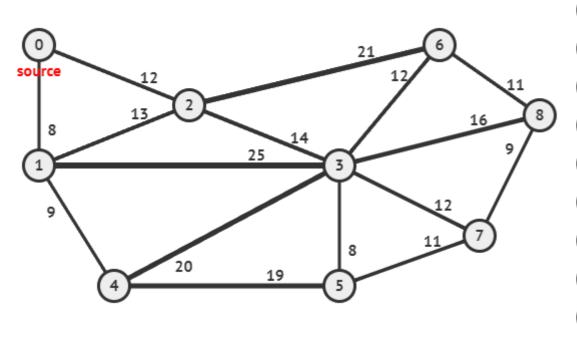
Use Prim's Algorithm when you have a graph with lots of edges.

Kruskal's Algorithm is general faster when you have a graph with fewer edges.

#### How many edges will our MST have?

How many vertices?

How many edges in the MST? 8



#### All edges

(0,1,8)

(3,5,8)

(4,1,9)

(7,8,9)

(5,7,11)

(6,8,11)

(0,2,12)

(3,6,12)

(3,7,12)

(1,2,13)

(2,3,14)

(3,8,16)

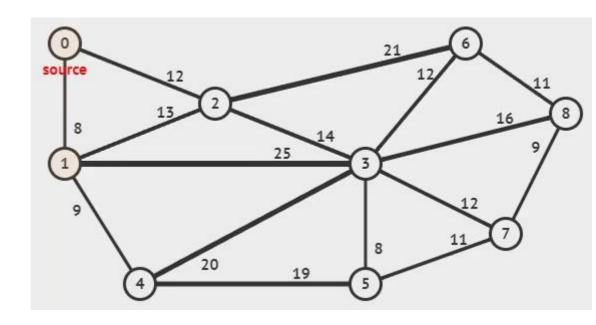
(4,5,19)

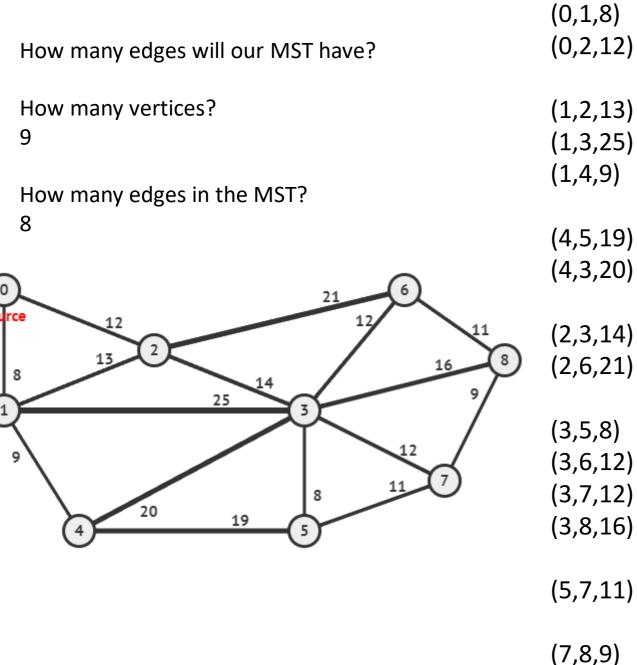
(3,4,20)

(2,6,21)

(1,3,25)

Kruskal will find all edges and sort them and then test each edge.





(0,2,12)

(1,3,25)

(1,4,9)

(4,5,19)

(4,3,20)

(2,3,14)

(2,6,21)

(3,5,8)

(3,6,12)

(3,7,12)

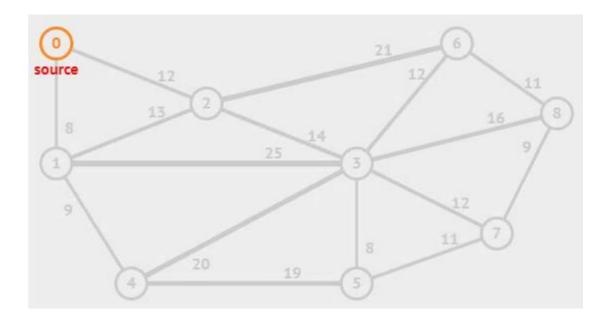
(3,8,16)

(5,7,11)

(7,8,9)

(8,6,11)

Prim's will start at a given vertex and connect the vertices until the MST is found



#### Prim vs Kruskal

#### Kruskal

builds a minimum spanning tree by adding one edge at a time. The next line is always the shortest (minimum weight) ONLY if it does NOT create a cycle.

#### **Prims**

builds a minimum spanning tree by adding one vertex at a time. The next vertex to be added is always the one nearest to a vertex already on the graph.

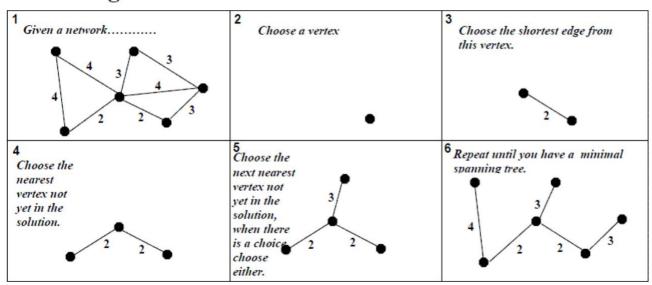
#### Prim vs Kruskal

Prim always joins a "new" vertex to an "old" vertex, so that every stage is a tree.

Kruskal allows both "new" to "new" and "old" to "old" to get connected, so it risks creating a circuit and must check for them every time.

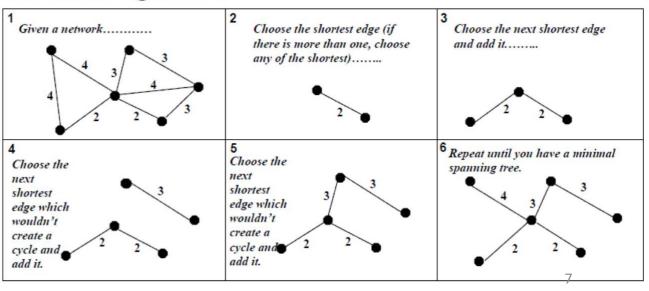
Kruskal has a larger complexity than Prim and could result in an incomplete/unconnected result if interrupted.

#### Prim's Algorithm

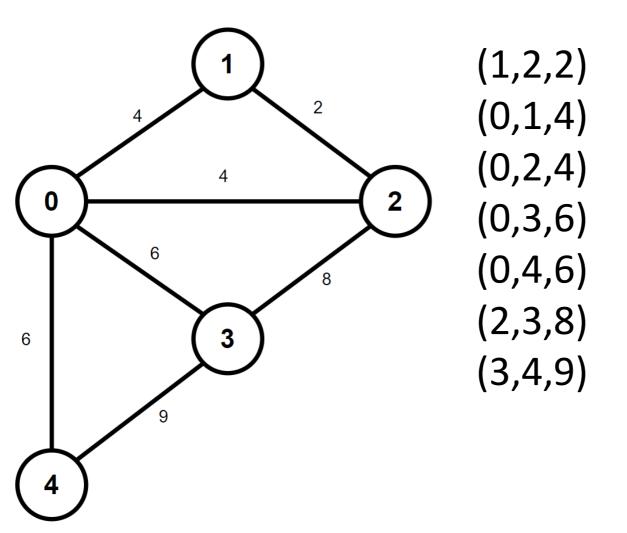


#### Prim vs Kruskal

#### Kruskal's Algorithm



### MST – Prim's Algorithm



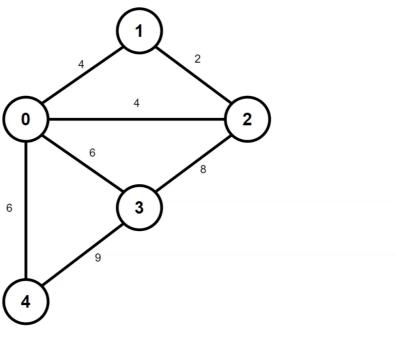
Add (0,1,4) to MST Add (1,2,2) to MST Remove (0,2,4) – causes cycle Add (0,3,6) to MST Add (0,4,6) to MST

Having 5 vertices means we need 4 edges so when we reach 4 edges in our MST, we are done.

The MST has a distance of 18

#### student@Maverick:/media/sf\_VM/CSE3318\$ ./a.out PrimGraphA.txt Enter starting vertex 0 (0,1,4)(0,2,4)(0,3,6)(0,4,6)Adding (0,1,4) to MST (1,2,2)(0,2,4)(0,3,6)(0,4,6)Adding (1,2,2) to MST (0,2,4)(0,3,6)(0,4,6)(2,3,8)Removing (0,2,4) - causes cycle (0,3,6)(0,4,6)(2,3,8)Adding (0,3,6) to MST (0,4,6)(2,3,8)(3,4,9)Adding (0,4,6) to MST MST distance = 18

#### MST – Prim's Algorithm

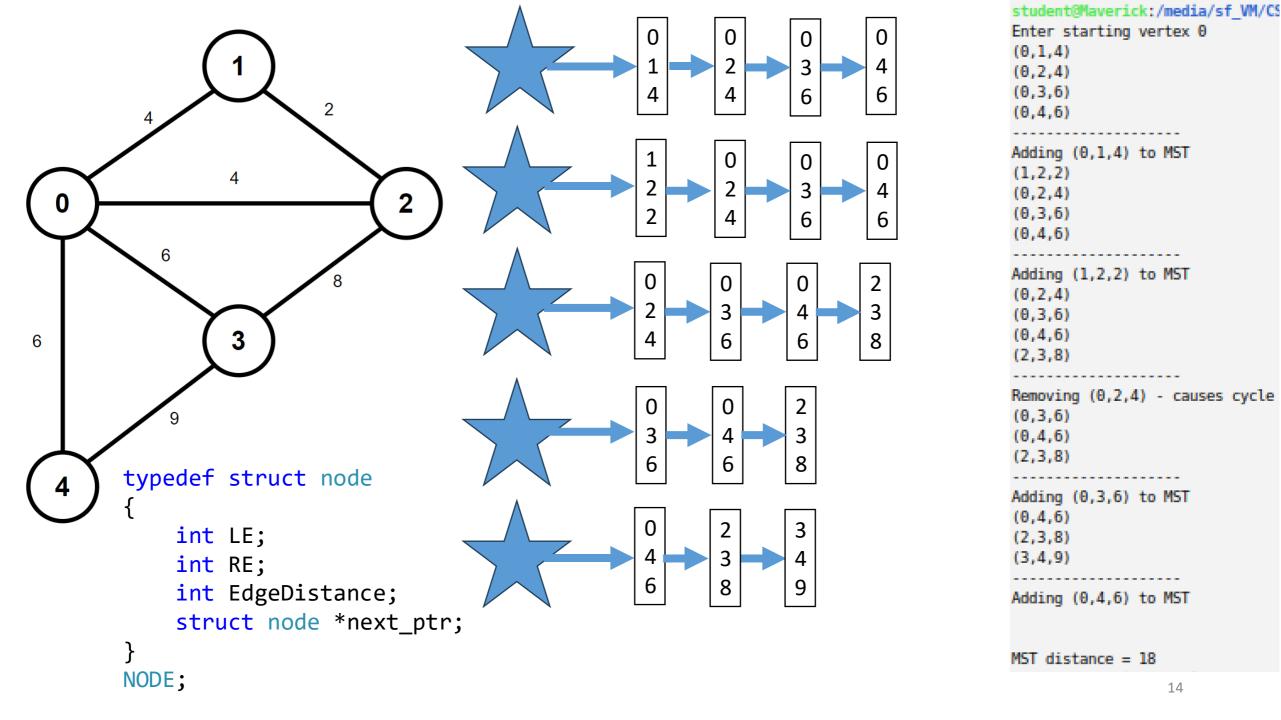


```
-1, 4, 4, 6, 6
                                                         4,-1,2,-1,-1
                                                         4,2,-1,8,-1
int main(int argc, char *argv[])
                                                         6, -1, 8, -1, 9
                                                         6, -1, -1, 9, -1
  int AdjacencyMatrix[MAX][MAX] = { };
  int StartingVertex = 0;
  int VertexCount = 0;
 VertexCount = ReadFile(argc, argv, AdjacencyMatrix);
 printf("Enter starting vertex ");
  scanf("%d", &StartingVertex);
 printf("\n\nMST distance = %d\n",
          Prims (AdjacencyMatrix, VertexCount, StartingVertex));
  return 0;
                                                                10
```

```
int row = 0, col = 0;
while (fgets(FileLine, sizeof(FileLine)-1, FH))
  token = strtok(FileLine, ",");
  while (token != NULL)
    AdjacencyMatrix[row][col] = atoi(token);
    token = strtok(NULL, ",");
    col++;
                                                     -1, 4, 4, 6, 6
  row++;
                                                     4,-1,2,-1,-1
  col = 0;
                                                     4,2,-1,8,-1
                                                     6, -1, 8, -1, 9
                                                    6, -1, -1, 9, -1
```

```
int Prims(int AdjacencyMatrix[][MAX], int VertexCount, int StartingVertex)
  NODE *LinkedListHead = NULL, *TempPtr = NULL, *NextVertex = NULL;
  int Visited[MAX] = {};
  int MSTLength = 0;
                                                   typedef struct node
  int EdgeCount = VertexCount - 1;
                                                      int LE;
  int CurrentVertex = StartingVertex;
                                                      int RE;
                                                      int EdgeDistance;
  Visited[StartingVertex] = 1;
                                                      struct node *next ptr;
  while(EdgeCount > 0)
                                                   NODE;
     // Create unvisited neighbor list
     // Find edge with the smallest distance that does not create a cycle
     // Add up distance, decrement edges, move current vertex
  return (MSTLength);
```

```
student@Maverick:/media/sf VM/CS
                                                                             Enter starting vertex 0
                                                                             (0,1,4)
                                                                             (0,2,4)
// Create unvisited neighbor list
                                                                             (0,3,6)
                                                                              (0,4,6)
for (int i = 0; i < VertexCount; i++)
                                                                             Adding (0,1,4) to MST
                                                                             (1,2,2)
                                                                             (0,2,4)
   if (Visited[i] == 0 &&
                                                                             (0,3,6)
                                                                              (0,4,6)
         AdjacencyMatrix[CurrentVertex][i] != -1)
                                                                             Adding (1,2,2) to MST
                                                                             (0,2,4)
         InsertNode(CurrentVertex, i,
                                                                             (0.3.6)
                                                                             (0,4,6)
                AdjacencyMatrix[CurrentVertex][i], &Lin[(2,3,8)
                                                                             Removing (0,2,4) - causes cycle
                                                                             (0,3,6)
                                                                             (0,4,6)
                                                                              (2.3.8)
                                                                             Adding (0,3,6) to MST
DisplayLinkedList(LinkedListHead);
                                                                             (0,4,6)
                                                                             (2,3,8)
                                                                             (3,4,9)
                                                                             Adding (0,4,6) to MST
                                                                             MST distance = 18
```



```
student@Maverick:/media/sf VM/CS
                                                                               Enter starting vertex 0
                                                                               (0.1.4)
// Find edge with the smallest distance that does not create (0,2,4)
                                                                               (0,3,6)
TempPtr = LinkedListHead;
                                                                               (0,4,6)
while (TempPtr != NULL && NextVertex == NULL)
                                                                               Adding (0,1,4) to MST
                                                                               (1,2,2)
                                                                               (0,2,4)
   if(Visited[TempPtr->LE] == 1 &&
                                                                               (0,3,6)
      Visited[TempPtr->RE] == 0)
                                                                               (0,4,6)
                                                                               Adding (1,2,2) to MST
                                                                               (0,2,4)
      NextVertex = TempPtr;
                                                                               (0.3.6)
                                                                               (0,4,6)
                                                                               (2,3,8)
   else
                                                                               Removing (0,2,4) - causes cycle
                                                                               (0,3,6)
      printf("Removing (%d,%d,%d) - causes cycle\n",
                                                                               (0,4,6)
                LinkedListHead->LE, LinkedListHead->RE,
                                                                               (2.3.8)
                LinkedListHead->EdgeDistance);
                                                                               Adding (0,3,6) to MST
                                                                               (0,4,6)
      DeleteNode (&LinkedListHead);
                                                                               (2.3.8)
      DisplayLinkedList(LinkedListHead);
                                                                               (3,4,9)
                                                                               Adding (0,4,6) to MST
   TempPtr = TempPtr->next ptr;
                                                                               MST distance = 18
```

```
MSTLength += NextVertex->EdgeDistance;
CurrentVertex = NextVertex->RE;
Visited[CurrentVertex] = 1;
EdgeCount--;
printf("Adding (%d,%d,%d) to MST\n",
       NextVertex->LE, NextVertex->RE,
       NextVertex->EdgeDistance);
NextVertex = NULL;
DeleteNode (&LinkedListHead);
```

## Prim's Algorithm

Implementing Prim using an array as the priority queue  $O(V^2)$ 

Implementing Prim using a binary heap as the priority queue  $O(E\log_2 V)$ 

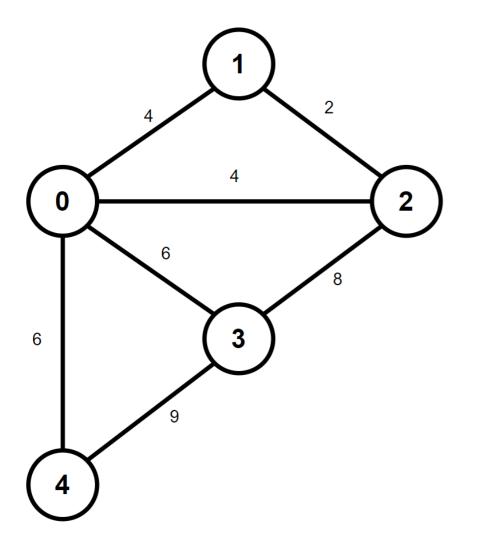
Implementing Prim using a Fibonacci heap as the priority queue  $O(V \log_2 V + E)$ 

So what is the runtime of using a linked list?

```
(V - 1)(V)
while (EdgeCount > 0) \bigvee - 1
     // Create unvisited neighbor list
     for (int i = 0; i < VertexCount; i++)
                                                     V^2 - V
     DisplayLinkedList(LinkedListHead);
     // Find the edge with the smallest distance
     TempPtr = LinkedListHead;
     while (TempPtr != NULL && NextVertex == NULL)
     //Housekeeping code and deleting the used node
```

18

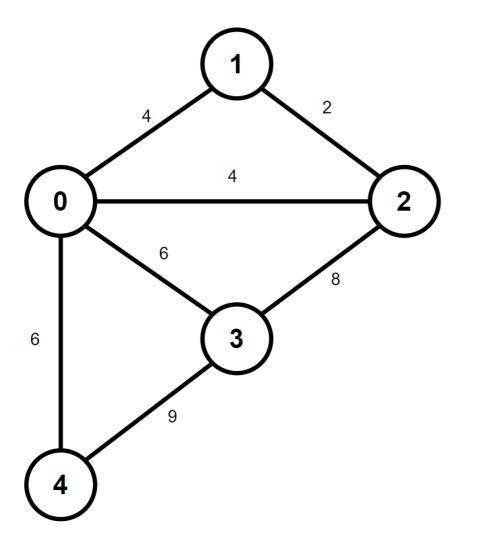
### MST – Kruskal's Algorithm



List all edges in order by weight and choose the minimum...

- (1,2,2)
- (0,1,4)
- (0,2,4)
- (0,3,6)
- (0,4,6)
- (2,3,8)
- (3,4,9)

### MST – Kruskal's Algorithm



List all edges in order by weight and choose the minimum...

(1,2,2) (0,1,4)

(0,1,4)

(0,2,4)

(0,3,6)

(0,4,6)

(2,3,8)

(3,4,9)

```
(1,2,2)
(0,1,4)
(0,2,4)
(0,3,6)
(0.4.6)
(2,3,8)
(3,4,9)
Adding (1,2,2) to MST
Adding (0,1,4) to MST
Removing (0,2,4) - causes cycle
(0,3,6)
(0,4,6)
(2,3,8)
(3.4.9)
Adding (0,3,6) to MST
Adding (0,4,6) to MST
MST distance = 18
                          20
```

```
for (int i = 0; i < VertexCount; i++)
  // Create neighbor list
  for(int j = 0; j < VertexCount; j++)</pre>
     if (Visited[j] == 0 && AdjacencyMatrix[i][j] != -1)
       InsertNode(i,j, AdjacencyMatrix[i][j], &LinkedListHead);
  Visited[i] = 1;
memset (Visited, 0, sizeof (Visited));
DisplayLinkedList(LinkedListHead);
```

```
while (EdgeCount > 0)
 // Find the edge with the smallest distance
 MSTLength += NextVertex->EdgeDistance;
 EdgeCount--;
 printf("Adding (%d,%d,%d) to MST\n",
         NextVertex->LE, NextVertex->RE, NextVertex->EdgeDistance);
 NextVertex = NULL;
 DeleteNode (&LinkedListHead);
```

```
TempPtr = LinkedListHead;
while (TempPtr != NULL && NextVertex == NULL)
  if(Visited[TempPtr->LE] == 1 &&
     Visited[TempPtr->RE] == 1)
    printf("Removing (%d,%d,%d) - causes cycle\n",
            LinkedListHead->LE,
            LinkedListHead->RE, LinkedListHead->EdgeDistance);
     DeleteNode (&LinkedListHead);
     DisplayLinkedList(LinkedListHead);
  else
    NextVertex = TempPtr;
    Visited[TempPtr->RE] = 1;
    Visited[TempPtr->LE] = 1;
  TempPtr = TempPtr->next ptr;
```

## Prim's Algorithm

What is the run time of Prim's Algorithm?

The big O of Prim's Algorithm will depend on how the algorithm is implemented

Which data structures are used and how those data structures are used.



## Prim's Algorithm

Implementing Prim using an array as the priority queue  $O(V^2)$ 

Implementing Prim using a binary heap as the priority queue  $O(E\log_2 V)$ 

Implementing Prim using a Fibonacci heap as the priority queue  $O(V \log_2 V + E)$ 

## Kruskal's Algorithm

What is the run time of Kruskal's Algorithm?

 $O(E \log_2 E)$ 

The most time consuming part of the algorithm is the sorting of the edges.

Kruskal's performance is mainly determined by the sorting step which dominates the overall complexity.

Space complexity is O(V + E)

Let's look at a familiar recursive problem

Calculate the *n*th Fibonacci value.

The recursive algorithm is

```
if n <= 1, then set f = n
else f = fib(n-1) + fib(n-2)
return f</pre>
```

n	Fibonacci
0	0
1	1
2	1
3	2
4	3
5	5
6	8
7	13
8	21
9	34
10	55
11	89

```
if n \le 1,
then set f = n
else f = fib(n-1) + fib(n-2)
```

return f

```
fib(5)
                           fib(3)
              fib(4)
        fib(3)
                 fib(2)
                       fib(2)
                                fib(1)
           fib(1)
   fib(2)
                if n <= 1
                                   fib(4)
       fib(0)
fib(1)
                    return n
                else
                    return fib(n-1) +
                             fib(n-2)
```

```
if n <= 1          fib(5)
    return n
else
    return fib(n-1) +
          fib(n-2)</pre>
```

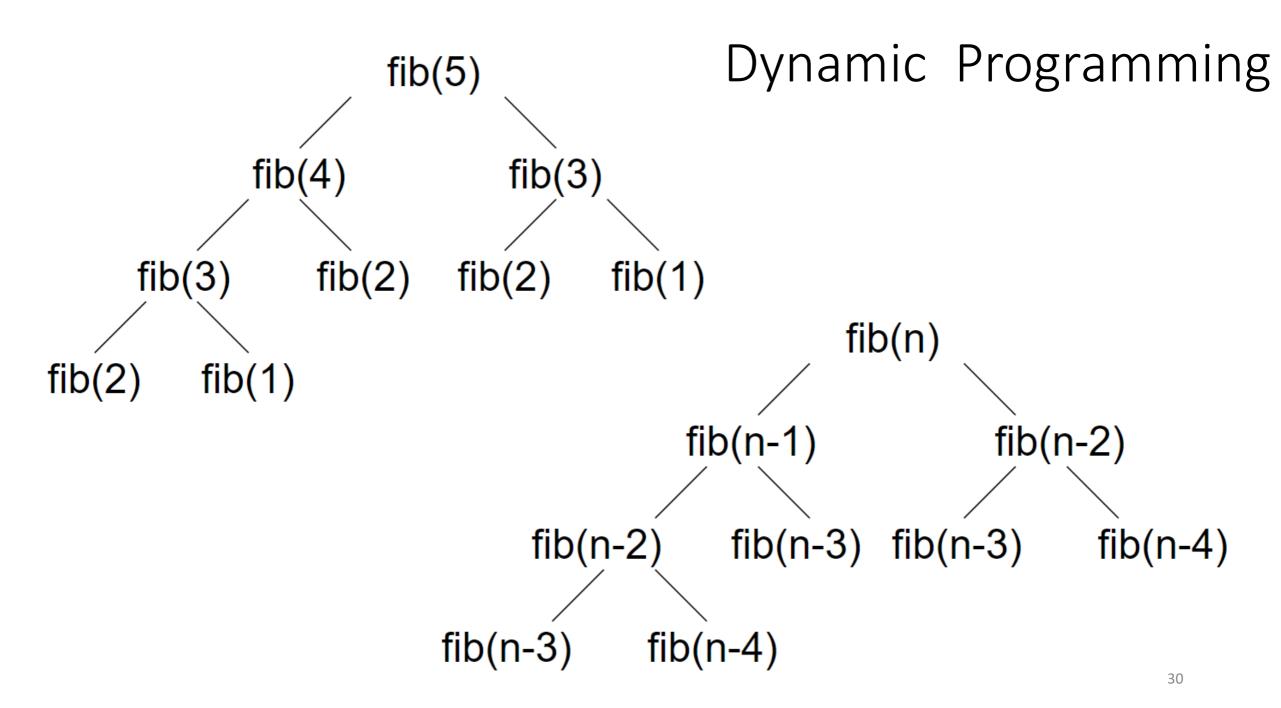
```
if n <= 1    fib(3)
    return n
else
    return fib(n-1) +
        fib(n-2)</pre>
```

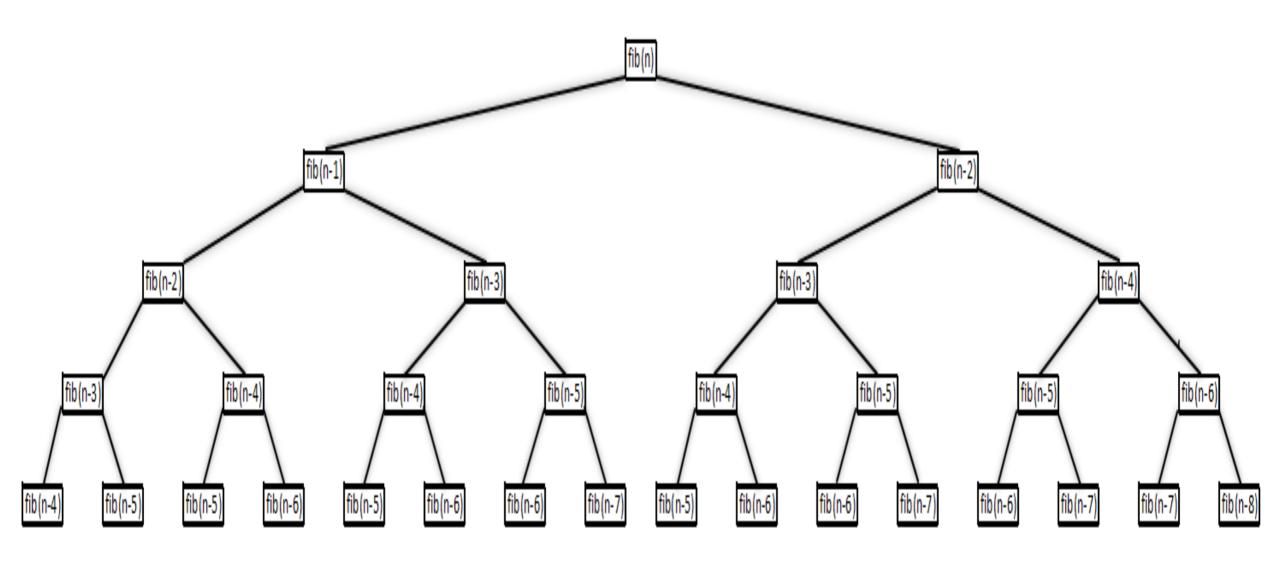
```
if n <= 1     fib(3)
    return n
else
    return fib(n-1) +
        fib(n-2)</pre>
```

```
if n <= 1    fib(2)
    return n
else
    return fib(n-1) +
        fib(n-2)</pre>
```

```
if n <= 1     fib(2)
    return n
else
    return fib(n-1) +
        fib(n-2)</pre>
```

```
if n <= 1          fib(1)
    return n
else
    return fib(n-1) +
          fib(n-2)</pre>
```





General, powerful algorithm design technique

Dynamic Programming is approximately "careful brute force"

Trying to get to polynomial time

Break a problem into subproblems, solve those subproblems and reuse those solutions.

Dynamic Programming (DP) is an algorithmic technique for solving an optimization problem by breaking it down into simpler subproblems and utilizing the fact that the optimal solution to the overall problem

depends upon the optimal solution to its subproblems.

There are two patterns used in solving DP problems

1. Tabulation – Bottom up

2. Memoization – Top Down

## MEMOIZATION









There are two patterns used in solving DP problems

Tabulation – Bottom up
 I will study the theory of Dynamic Programming in CSE3318, then I will practice some problems using DP and hence I will master Dynamic Programming.

#### 2. Memoization – Top Down

To master Dynamic Programming, I would have to practice DP problems and to practice problems, I would have to study some theory of Dynamic Programming from CSE3318.

#### **Tabulation Method – Bottom Up Dynamic Programming**

As the name itself suggests starting from the bottom and cumulating answers to the top.

Let's apply this method to the Fibonacci problem.

What is the bottom/base case of Fibonacci?

#### **Tabulation Method – Bottom Up Dynamic Programming**

What is the bottom/base case of Fibonacci?

Let's look at the classic definition of Fibonacci...

So for n = 0 or n = 1, n is always returned

```
if (n == 0 || n == 1)
{
    return n;
}
```

```
Fibonacci
n
0
6
        13
8
        21
9
        34
10
        55
11
        89
```

```
if (n == 0 | | n == 1)
   return n;
for (i = 2; i \le n; i++)
```

```
Fibonacci
n
6
        13
        21
9
        34
10
        55
11
        89
```

```
n Fib
if (n == 0 | | n == 1)
   return n;
                                   2 1 0 + 1
                                   3 \ 2 \ 1 + 1
                                   4 3 1+2
for (i = 2; i \le n; i++)
                                   5 \ 5 \ 2 + 3
                                   6 8 3 + 5
```

#### **Tabulation Method – Bottom Up Dynamic Programming**

```
Fib
if (n == 0 | | n == 1)
  return n;
int First = 0, Second = 1;
                                           2 1 0 + 1
for (i = 2; i \le n; i++)
                                           3 \ 2 \ 1 + 1
                                           4 3 1+2
  Fib = First + Second;
  First = Second;
                                           5 \ 5 \ 2 + 3
  Second = Fib;
                                           6 8 3 + 5
```

return Second;

```
n = 5
if (n == 0 | | n == 1)
                                                              Fib
                              1st 2nd
                                         Fib
  return n;
int First = 0, Second = 1;
                                                           2 1 0 + 1
for (i = 2; i \le n; i++)
                                                           3 2 1+1
                                                           4 \ 3 \ 1 + 2
  Fib = First + Second;
  First = Second;
                                                           5 \ 5 \ 2 + 3
  Second = Fib;
return Fib;
```

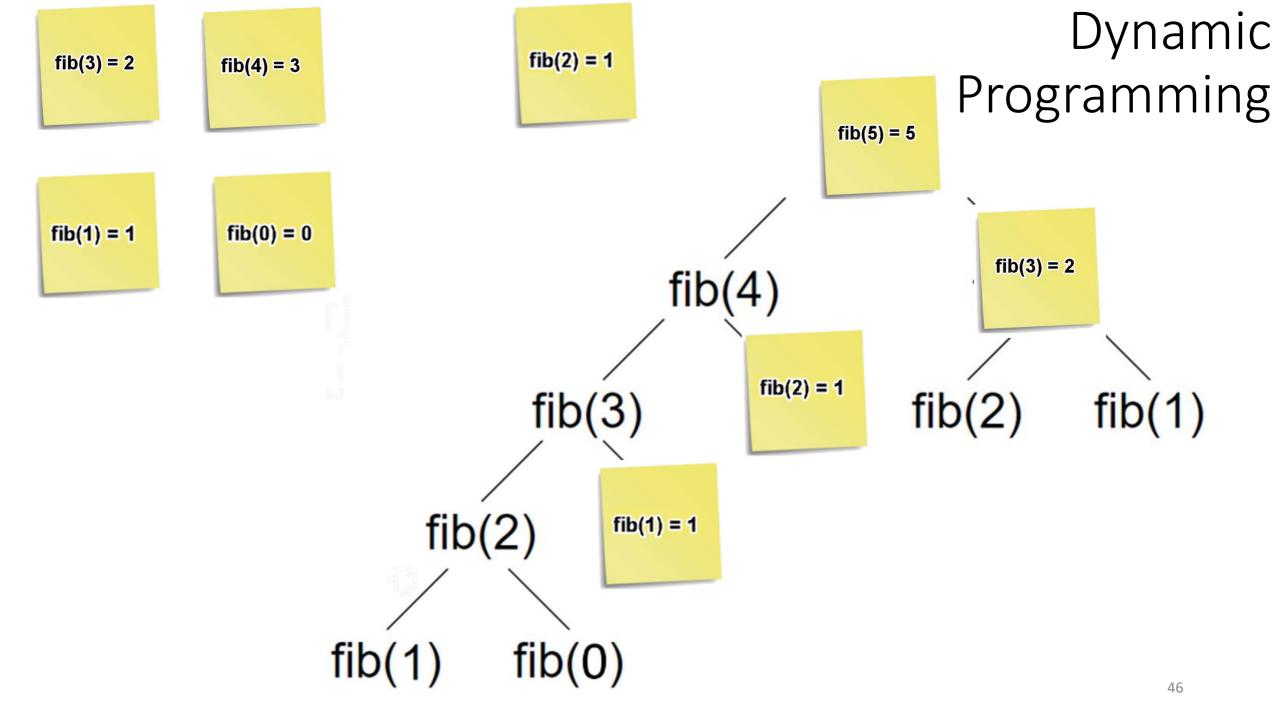
#### Memoization

In computing, memoization or memoisation is an optimization

technique used primarily to speed up computer programs by storing

the results of expensive function calls and returning the stored result

when the same inputs occur again.



The memoized program for a problem is like the recursive version with a small modification that it looks into a lookup table before computing solutions.

We initialize a lookup array with all initial values as NULL.

Whenever we need the solution to a subproblem, we first look into the lookup table.

If the precomputed value is there, then we return that value; otherwise, we calculate the value and put the result in the lookup table so that it can be reused later.

```
unsigned long long int fibonacci (unsigned int n)
                                            fib(5)
  if (n == 0 | | n == 1)
                                                 fib(3)
                                       fib(4)
    return n;
                                        fib(2) fib(2)
                                  fib(3)
                                                     fib(1)
  else
                              fib(2)
                                   fib(1)
    return fibonacci (n - 1) + fibonacci (n - 2);
```

```
unsigned long long Memo[100] = {};
unsigned long long fibonacci (unsigned long long n, unsigned long long Memo[])
  unsigned long long result = 0;
                                                                       fib(5)
  if (n == 0 | | n == 1)
                                                                              fib(3)
                                                                fib(4)
     result = n;
                                                          fib(3)
                                                                   fib(2)
                                                                          fib(2)
  else if (Memo[n] != ' \setminus 0')
                                                     fib(2) fib(1)
     result = Memo[n];
                                                       fib(0)
                                                fib(1)
  else
     result = fibonacci(n - 1, Memo) + fibonacci(n - 2, Memo);
     Memo[n] = result;
   return result;
                                                       fib(2) = 1
                                                                fib(3) = 2
                                                                        fib(4) = 3
                                                                                 fib(5) = 5
```

There are two main properties of a problem that suggests it can be solved using Dynamic Programming.

1. Overlapping Subproblems

2. Optimal Substructure

Overlapping Subproblems

Like Divide and Conquer, Dynamic Programming combines solutions to sub-problems.

Dynamic Programming is mainly used when solutions of same subproblems are needed again and again.

In dynamic programming, computed solutions to subproblems are stored/memoized so that these don't have to be recomputed.

Dynamic Programming is not useful when there are no common (overlapping) subproblems because there is no point storing the solutions if they are not needed again.

For example, Binary Search doesn't have common subproblems.

#### **Optimal Substructure**

A given problem has Optimal Substructure Property if the optimal solution of the given problem can be obtained by using optimal solutions of its subproblems.

Greedy Algorithms have the same property.

#### **Optimal Substructure**

A given problem has Optimal Substructure Property if the optimal solution of the given problem can be obtained by using optimal solutions of its subproblems.

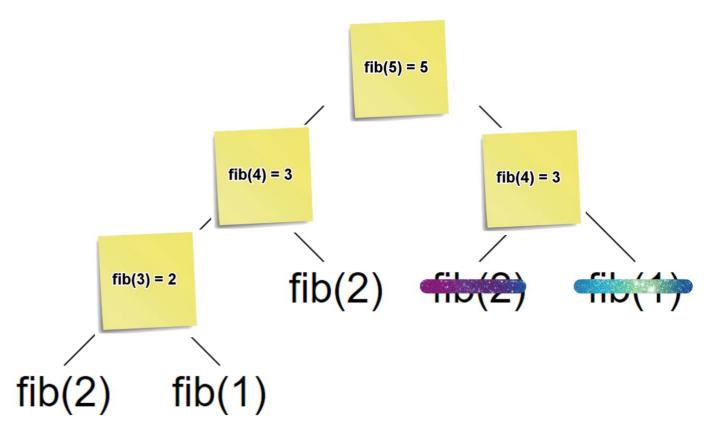
DP techniques exploit this property to split the problems into smaller subproblems and solve them instead.

After the subproblems become sufficiently small, solving them becomes trivial.

DP ≈ recursion + memoization

Memoize (take a memo) and re-use solutions to subproblems that help solve the problem.

For any given value k, fib(k) only recurses the first time it is called.



For any given value k, fib(k) only recurses the first time it is called.

Memoized calls cost  $\Theta(1)$ 

The number of non-memoized calls is n given that we are calculating fib(n).

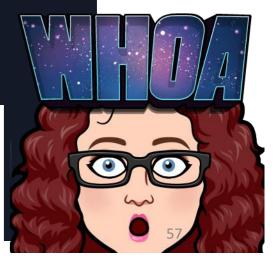
The amount of nonrecursive work per call is  $\Theta(1)$ .

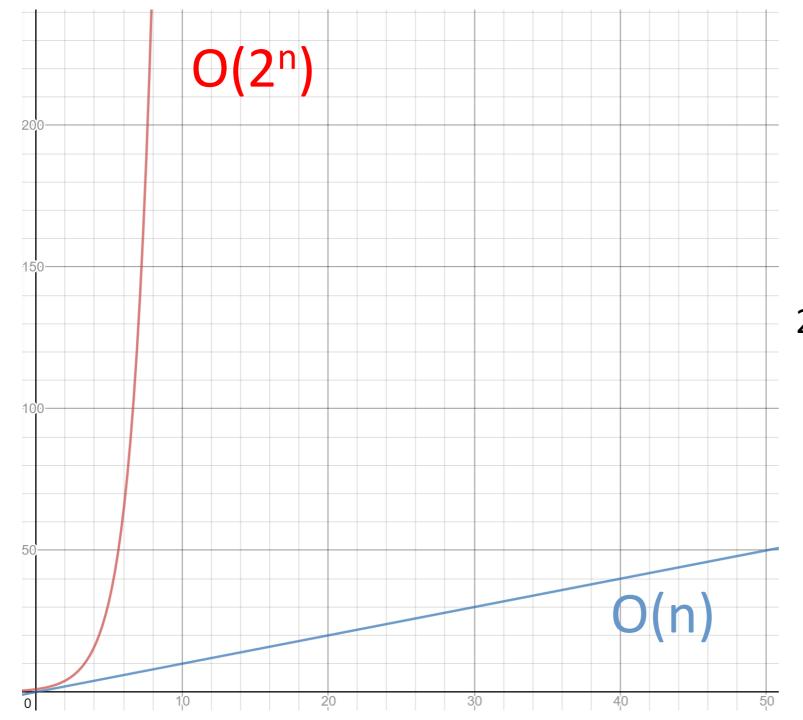
Total time of memoized version of Fibonacci is  $\Theta(n)$ 

student@maverick:/media/sf\_VM/CSE3318\$

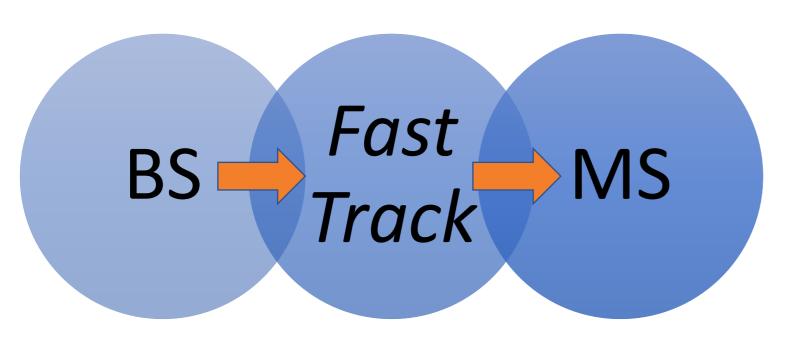
I







 $2^{50} = 1,125,899,906,842,624$ 





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# Fast Track

M.S. in Computer Science

Master of Software Engineering

M.S. in Computer Engineering



## Fast Track

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Undergraduate students admitted to the Fast Track program may complete up to three graduate level courses that will count towards both their bachelors and master's degrees.

Successful Fast Track students will be able to join the master's program without completing the traditional application process.

Students who successfully complete the undergraduate Fast Track requirements do not pay an application fee or take the GRE.

Meet with an undergraduate advisor to discuss the program or email donna.french@uta.edu to see if you qualify.

#### M.S. Fast Track - Computer Science

Students who meet the following conditions may apply for the Fast Track Program

- Must be within 30 hours of completing the bachelor's degree
- Must have completed at least 30 hours of study at UTA
- Must have an overall GPA of at least 3.25
- Must have completed the following prerequisite foundation coursework with a minimum GPA of 3.3
  - o CSE 2312 Computer Organization and Assembly Language Programming
  - o CSE 2315 Discrete Structures
  - o CSE 3315 Theoretical Concepts in Computer Science and Engineering
  - o CSE 3318 Algorithms & Data Structures
  - o CSE 3320 Operating Systems
  - o IE 3301 Engineering Probability or MATH 3313 Intro to Probability

## M.S. Fast Track - Computer Science

Undergraduate Fast Track students must take a minimum of 2 and a maximum of 3 of the following graduate courses

#### Required

o CSE 5311 – Design and Analysis of Algorithms

#### Choose 1 or 2 of the following

- o CSE 5301 Data Analysis & Modeling Techniques
- o CSE 5306 Distributed Systems
- o CSE 5317 Design and Construction of Compilers
- o CSE 5331 DBMS Models and Implementation Techniques
- o CSE 5360 Artificial Intelligence I
- o CSE 5365 Computer Graphics

## M.S. Fast Track - Computer Engineering

Students who meet the following conditions may apply for the Fast Track Program

- Must be within 30 hours of completing the bachelor's degree
- Must have completed at least 30 hours of study at UTA
- Must have an overall GPA of at least 3.25
- Must have completed the following prerequisite foundation coursework with a minimum GPA of 3.3
  - o CSE 2312 Computer Organization and Assembly Language Programming
  - o CSE 2315 Discrete Structures
  - o CSE 3318 Algorithms & Data Structures
  - o CSE 3320 Operating Systems
  - o CSE 3442 Embedded Systems I
  - o IE 3301 Engineering Probability or MATH 3313 Intro to Probability

## M.S. Fast Track - Computer Engineering

Undergraduate Fast Track students must take a minimum of 2 and a maximum of 3 of the following graduate courses

#### Choose 2 or 3 of the following

- o CSE 5342 Embedded Systems
- o CSE 5352 IoT and Networking
- o CSE 5354 Real-Time Operating Systems
- o CSE 5355 Electromechanical Systems and Sensors
- o CSE 5356 System on Chip (SoC) Design
- o CSE 5357 Advanced Digital Logic Design
- o CSE 5358 Microprocessor Systems

## M.S. Fast Track - Software Engineering

Students who meet the following conditions may apply for the Fast Track Program

- Must be within 30 hours of completing the bachelor's degree
- Must have completed at least 30 hours of study at UTA
- Must have an overall GPA of at least 3.25
- Must have completed the following prerequisite foundation coursework with a minimum GPA of 3.3
  - o CSE 2312 Computer Organization and Assembly Language Programming
  - o CSE 2315 Discrete Structures
  - o CSE 3310 Fundamental of Software Engineering
  - o CSE 3318 Algorithms & Data Structures
  - o CSE 3320 Operating Systems
  - o IE 3301 Engineering Probability or MATH 3313 Intro to Probability

## M.S. Fast Track - Software Engineering

Undergraduate Fast Track students must take a minimum of 2 and a maximum of 3 of the following graduate courses

#### Required

o CSE 5311 – Design and Analysis of Algorithms

#### Choose 1 or 2 of the following

- o CSE 5301 Data Analysis & Modeling Techniques
- o CSE 5321 Software Testing
- o CSE 5322 Software Design Patterns
- o CSE 5325 Software Engineering: Management, Maintenance, and Quality Assurance

If you need the definition of a word in the dictionary, how do you find it?

Do you start from page 1 and search until you find it? O(n)

Do you divide the dictionary in half and decide if you need to look in the right half or left half and keep repeating that process until you find the word?  $O(\log_2 n)$ 





#### **Dictionary**

Abstract Data Type - class of objects whose logical behavior is defined by a set of values and a set of operations

Maintains a set of items each with its own key

3 basic actions for a dictionary Insert item

Delete item

Search for item

any data structure that supports insert, search, and deletion is a "dictionary"

#### Dictionary

3 basic actions for a dictionary

Insert item

Overwrite any existing key

Delete item

Search for item

Return the item with the given key or report that it does not exist

For those that have studied C++, you might be wondering...

std::map

is that a dictionary?

Yes!

std::map is the C++ standard library implementation of a dictionary

A map is a set where each element is a pair, called a key/value pair.

The key is used for sorting and indexing the data and must be unique.

The value is the actual data.

Duplicate keys are *not* allowed—a single value can be associated with each key.

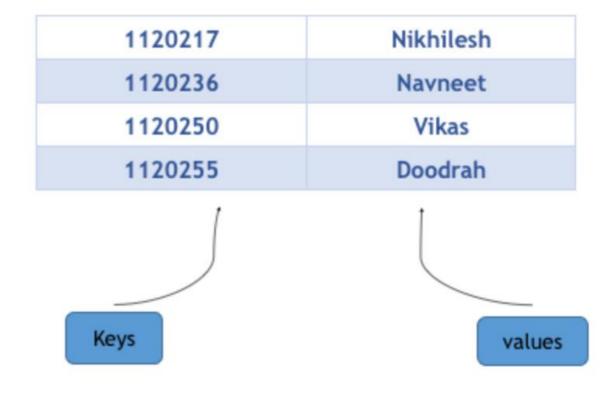
This is called a one-to-one mapping.

A map of students where **id number** is the key and **name** is the value can be represented graphically as

Notice that keys are arranged in ascending order.

Maps always arrange its keys in sorted order.

Here the keys are of string type; therefore, they are sorted lexicographically.



So what is a limitation of this implementation of a dictionary?



#### Dictionaries have a lot of uses

- Databases use them for lookups
- Spell check
- Username and password verification



- Network router
- Cryptography



A **hash table** is a data structure which implements an associative array abstract data type which is a structure that can map keys to values.

In hashing, large keys are converted into small keys by using a **hash function**. The hash function employs an algorithm to compute an index into the array. These small keys are then stored in this array which is called the **hash table**.

The idea of hashing is to distribute entries uniformly across an array.

Let's say we need to store student id's and their corresponding pin numbers.

1000012345	87345
1000023456	93727
1000046536	87323
1000085848	34522
1000100349	82234
1002002020	73721

We could store that information in an array of structures.

0	1	2	3	4	5	6	7	8	9
1000012345	1000023456	1000046536	1000085848	1000100349	1002002020				
87345	93727	87323	34522	82234	73721				

To find an item, we would perform a linear search – start at the beginning of the array and check each cell for a match.

Since this array has 10 cells, the performance would be based on the worst case which is the value we are looking for is in the last cell so O(10) or O(n) where n is the size of the array.

Instead of storing each student ID/pin in an array, let's use a **Hash Function** to determine which array element is used.

Given an input key, the hash function uses an algorithm to map the input key to an index in the array.

```
int GetHashCode(unsigned int student_id)
{
   return ((student_id) % MaxHashTableSize);
}
```

```
int GetHashCode(unsigned int student_id)
{
   return (student_id % MaxHashTableSize);
}
Input : 1000012345
Output : 1000012345 % 10 = 5
```

So we put 1000012345 and 87345 in array element 5.

0	1	2	3	4	5	6	7	8	9
					1000012345				
					87345				

```
int GetHashCode(unsigned int student_id)
{
   return (student_id % MaxHashTableSize);
}
Input : 1000023456
Output : 1000023456 % 10 = 6
```

So we put 1000023456 and 93727 in array element 6.

0	1	2	3	4	5	6	7	8	9
					1000012345	1000023456			
					87345	93727			0.0

```
int GetHashCode(unsigned int student_id)
{
   return (student_id % MaxHashTableSize);
}
Input : 1000085848
Output : 1000085848 % 10 = 8
```

So we put 1000085848 and 34522 in array element 8.

0	1	2	3	4	5	6	7	8	9
					1000012345	1000023456		1000085848	
					87345	93727		34522	

```
int GetHashCode(unsigned int student_id)
{
   return (student_id % MaxHashTableSize);
}
Input : 1000100349
Output : 1000100349 % 10 = 9
```

So we put 1000100349 and 82234 in array element 9.

	0	1	2	3	4	5	6	7	8	9
						1000012345	1000023456		1000085848	1000100349
L						87345	93727		34522	82234

```
int GetHashCode(unsigned int student_id)
{
   return (student_id % MaxHashTableSize);
}
Input : 1002002020
Output : 1002002020 % 10 = 0
```

So we put 1002002020 and 73721 in array element 0.

0	1	2	3	4	5	6	7	8	9
1002002020					1000012345	1000023456		1000085848	1000100349
73721					87345	93727		34522	82234

85

Now, when we want to search for a student ID, we give the student id to the hash function and it returns which array element to look in.

```
int GetHashCode(unsigned int student_id)
{
    return (student_id % MaxHashTableSize);
}
Input : 1000012345
Output : 1000012345 % 10 = 5
```

Array Element 5 has student ID 1000012345 and pin 87345.

0	1	2	3	4	5	6	7	8	9
1002002020					1000012345	1000023456		1000085848	1000100349
73721					87345	93727		34522	<b>82234</b> 86

Array with linear search -O(n) where n is the size of the array.

0	1	2	3	4	5	6	7	8	9
1000012345	1000023456	1000046536	1000085848	1000100349	1002002020				
87345	93727	87323	34522	82234	73721				

#### Hash Table

Hash function returns the exact array element containing information – O(1)

0	1	2	3	4	5	6	7	8	9
1002002020					1000012345	1000023456		1000085848	1000100349
73721					87345	93727		34522	<b>82234</b> 87

### Hash Table — Hash Function

To achieve a good hashing mechanism, it is important to have a good hash function with the following basic requirements:

 Easy to compute - It should be easy to compute and must not become an algorithm in itself.

• Uniform distribution - It should provide a uniform distribution across the hash table and should not result in clustering.

 Minimize collisions: Collisions occur when pairs of elements are mapped to the same hash value. These should be avoided.

### Hash Table - Collisions

```
int GetHashCode(unsigned int student_id)
{
   return (student_id % MaxHashTableSize);
}
Input : 1000046536
Output : 1000046536 % 10 = 6
```

So we would put 1000046536 and 87323 in array element 6 but another record is already there.

0	1	2	3	4	5	6	7	8	9
1002002020					1000012345	1000023456		1000085848	1000100349
73721					87345	93727		34522	82234

89



### Hash Table — Hash Function

Irrespective of how good a hash function is, collisions are bound to occur.

Therefore, to maintain the performance of a hash table, it is important to manage collisions through various collision resolution techniques.

There are multiple techniques for dealing with collisions

Open Addressing

Chaining

#### **Open Addressing**

In open addressing, all item are stored in the hash table itself.

Each array cell contains either a record or NULL.

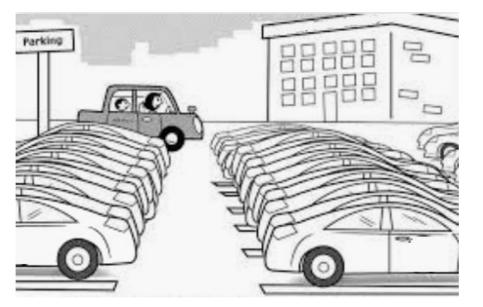
When inserting an item, if the array cell is already occupied, then open addressing takes the next cell.

#### **Open Addressing**

It keeps moving to the next cell until it finds an empty one.

If it reaches the end of the array, it starts over at the beginning and

keeps looking







Open Addressing



This method either requires enough array space for every item to be stored or it needs to be able to reallocate the array in order to expand.

When searching for an element, the hash function takes the search to the array cell indicated by the hash, but then must perform a linear search to ensure that the correct item was found.

The array cells of the hash table are examined one by one until the desired item is found or the end of the array is found (meaning the item is not present).

Open Addressing



#### **Linear Probing**

In linear probing, we linearly probe for the next open cell.

The main problem with linear probing is clustering, many consecutive elements form groups and it starts taking time to find a free slot or to search an element.

#### **Open Addressing**

#### **Quadratic Probing**

Takes the original hash index and adds successive values of an arbitrary quadratic polynomial until an open cell is found.

#### **Double Hashing**

Perform a second hash to find a new cell

#### **Open Addressing**

Linear probing is easy to compute and has the best cache performance but suffers from clustering.

Double hashing has poor cache performance but no clustering. Double hashing requires more computation time as two hash functions need to be computed.

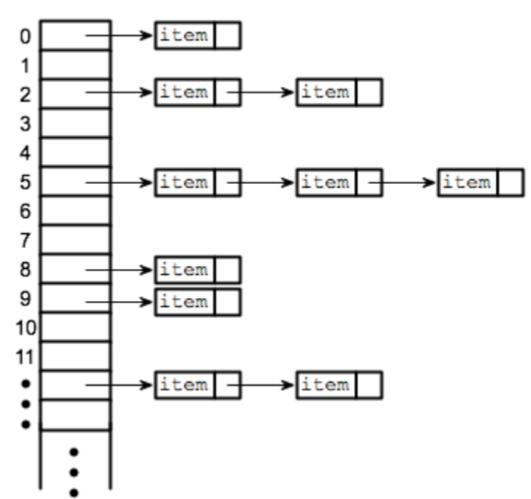
Quadratic probing lies between the two in terms of cache performance and clustering.

Separate chaining (open hashing)

Separate chaining is one of the most commonly used collision resolution techniques.

It is usually implemented using linked lists.

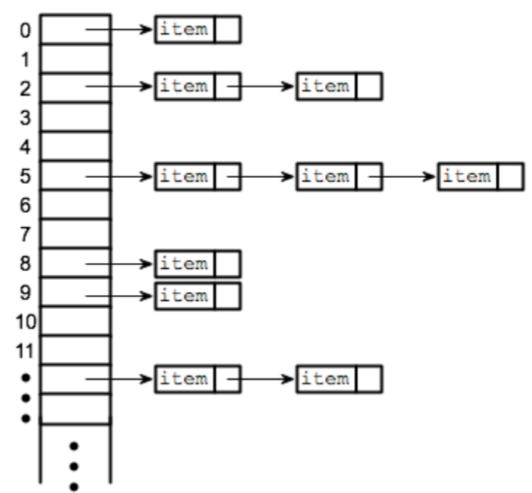
In separate chaining, each element of the hash table is a linked list.

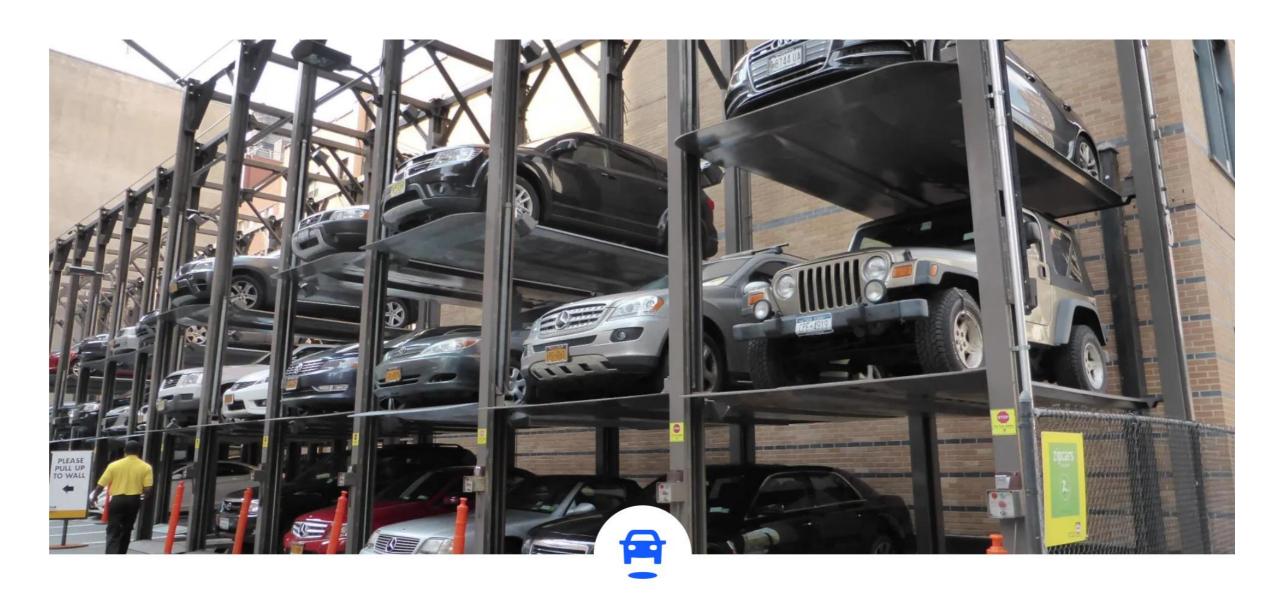


Separate chaining (open hashing)

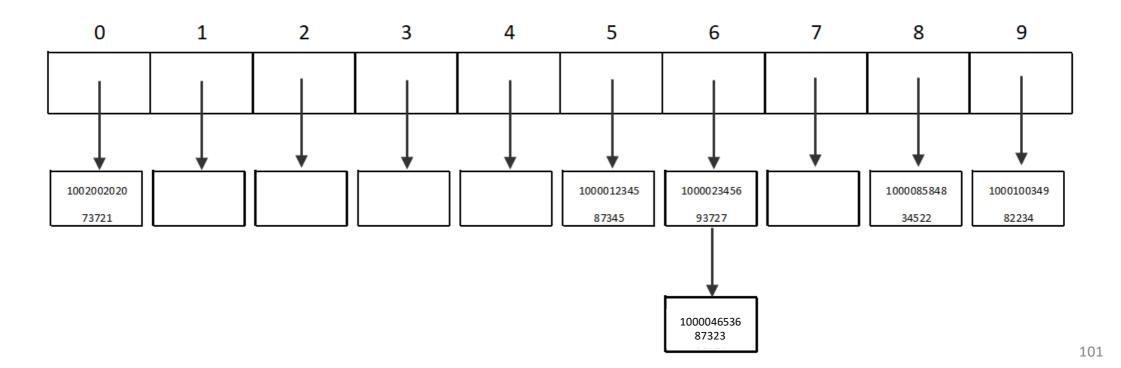
To store an element in the hash table you must insert it into a specific linked list.

If there is any collision (i.e. two different elements have same hash value) then store both elements in the same linked list.





Separate chaining turns each array element into its own linked list. Any collisions are handled by adding the new element on to the end of the linked list for that index.



For separate chaining, the worst-case scenario is when all the entries are inserted into the same linked list. The lookup procedure may have to scan all its entries, so the worst-case cost is proportional to the number (n) of entries in the table so O(n) which is the same as linear search.

A good hash function can prevent this worst-case scenario and make hash table look up always faster than linear search.

Hash tables, on average, are more efficient than search trees or any other table lookup structure. You can access an element in **O(1)** time. This is why hash tables are heavily used in many situations.

The capacity of the hash table is the size of the array.

The load factor is the number of keys stored in the hash table divided by the capacity.

The size should be chosen so that the load factor is less than 1.

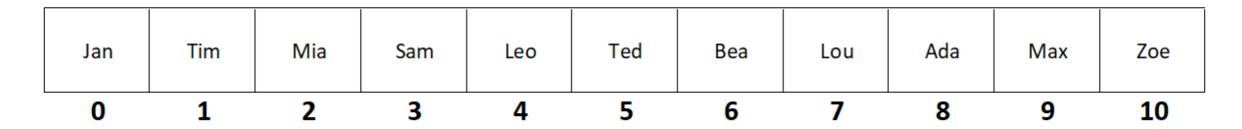
If we are NOT using separate chaining for conflict resolution, then what happens if our array size is 10 and we need to store 11 items?

Since the number of keys in the hash table is less than the capacity of the hash table, assuming that the keys are evenly distributed across indices, there will be few collisions, and most of the linked lists will be of length 1.

A few will be of length 2; a very few will be of length 3, and so on.

The probability that there is any linked list that is very much longer than the load factor is very small.

Separate Chaining	Open Addressing
Chaining is simpler to implement.	Open Addressing requires more computation.
Does not run out of room.	Array may fill up
Less sensitive to the hash function and load factor.	Requires extra care to avoid clustering and the load factor.
Mostly used when it is unknown how many and how frequently keys may be inserted or deleted.	Used when the frequency and number of keys is known.
Cache performance is not good as keys are stored using linked list.	Provides better cache performance as everything is stored in the same table.
Waste of space - some parts of hash table are never used	A cell can be used even if an input doesn't map to it.
Uses extra space for linked list pointers.	No links – everything in table
	105

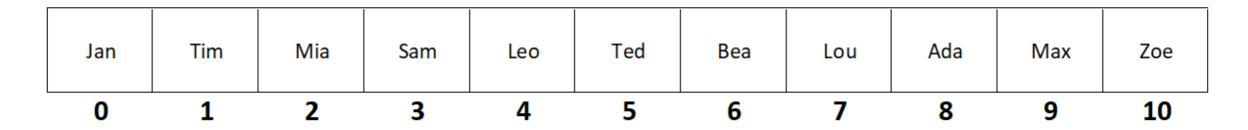


If we had this array of names and we wanted to look up name, how would we find it?

We would have to start at the first element and search.

What is the time complexity of a linear search? O(n)

What if we sorted and used a binary search?  $O(\log_2 n)$ 

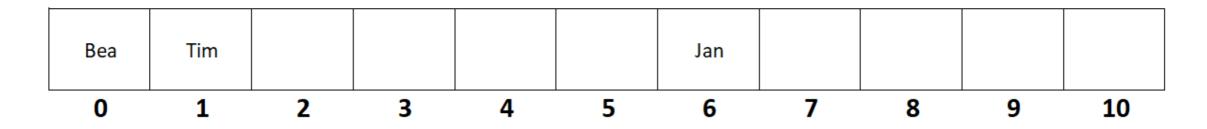


How about we use hashing?

$$Jan = J (74) + a (97) + n (110) = 281$$

What can we use for a hashing function?

How about ASCII values?



Tim = 
$$T(84) + i(105) + m(109) = 298$$
 Bea =  $B(66) + e(101) + a(97) = 264$ 

Now MOD by size of array

Now MOD by size of array

## Hashing

If we apply the hash function to the rest of the names, our hash table will look like...

Bea	Tim	Leo	Sam	Mia	Zoe	Jan	Lou	Max	Ada	Ted
0	1	2	3	4	5	6	7	8	9	10

Now, how do we look up a name?

# Hashing

		Bea	Tim	Leo	Sam	Mia	Zoe	Jan	Lou	Max	Ada	Ted
--	--	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----	-----

Let's say we want to find Ada.

We apply the hash function

$$Ada = A(65) + d(100) + a(97) = 262 \% 11 = 9$$

Time complexity?

Θ (1)

## Hash Functions/Algorithms

#### Numeric keys

Divide the key by the number of cells in the array and take the remainder (use mod)

#### Alphanumeric keys

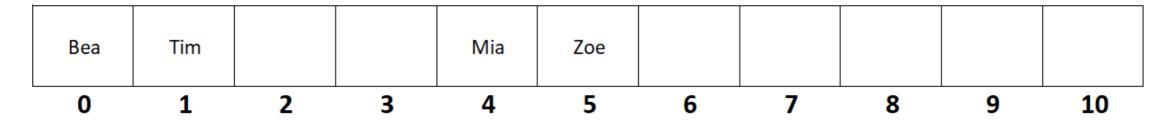
Divide the sum of the ASCII values by the number of cells in the array and take the remainder (use mod)

#### Folding Method

Rather than just adding the numbers of a phone number, for example, add the area code and then the prefix and then the exchange and use those 3 numbers as the array index.

215 673 2387 folds to become index 872.

Let's look at how Open Addressing handles collisions.

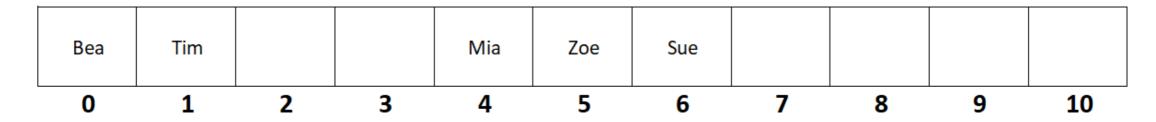


What happens when we try to add Sue?

$$S(83) + u(117) + e(101) = 301 \% 11 = 4$$

Mia is already in cell 4. So what does Open Addressing do with this collision?

Open Addressing with linear probing would add Sue at cell 6.

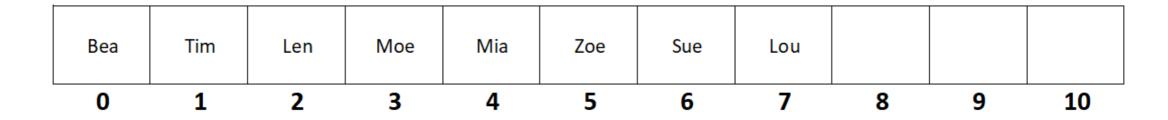


What happens when we try to add Len?

$$L(76) + e(101) + n(110) = 287 \% 11 = 1$$

Tim is already in cell 1. So what does Open Addressing do with this collision?

Open Addressing with linear probing would add Len at cell 2.



Moe would hash to cell 3 so no issue.

Lou would hash to cell 7 so no issue.

Rae would hash to cell 5 but Zoe is already there.

What happens with Open Addressing?

Open Addressing with linear probing would add Rae at cell 8.

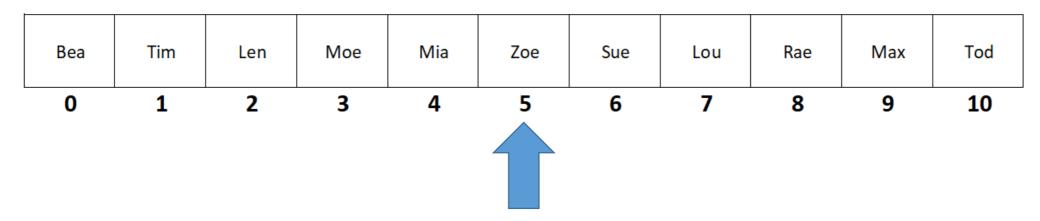
0	1	2	2	1	5	6	7	Q	<u> </u>	10
Bea	Tim	Len	Moe	Mia	Zoe	Sue	Lou	Rae	Max	Tod

Max would hash to cell 8 which is already occupied by Rae.

Open Addressing with linear probing would add Max at cell 9.

Tod would hash to cell 9 which is already occupied by Max.

Open Addressing with linear probing would add Tod at cell 10.



So how do we look up Rae?

We hash it

$$R(82) + a(97) + e(101) = 280 \% 11 = 5$$

So we would look for Rae at cell 5.

Let's do the same example but with Separate Chaining instead of Open Addressing.

Instead of storing the names in the actual array cells like with Open Addressing....

Bea	Tim		Mia	Zoe			10

When a key hashes to an array cell, a linked list is created.

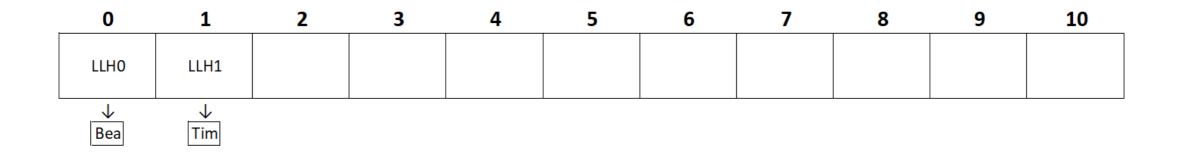
The linked list head is stored in the array – the array is now a char pointer array rather than just a char array.

Bea hashes to cell 0.

0	1	2	3	4	5	6	7	8	9	10
LLH0										



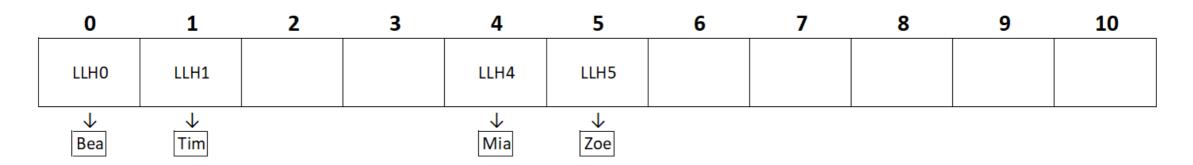
Tim hashes to cell 1.



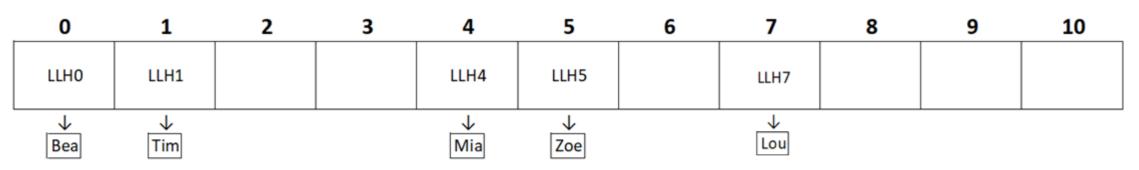
Mia hashes to cell 4.

0	1	2	3	4	5	6	7	8	9	10
LLH0	LLH1			LLH4						
↓ Bea	↓ Tim			↓ Mia						119

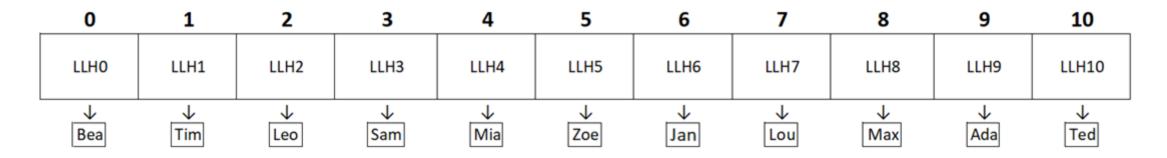
Zoe hashes to cell 5.



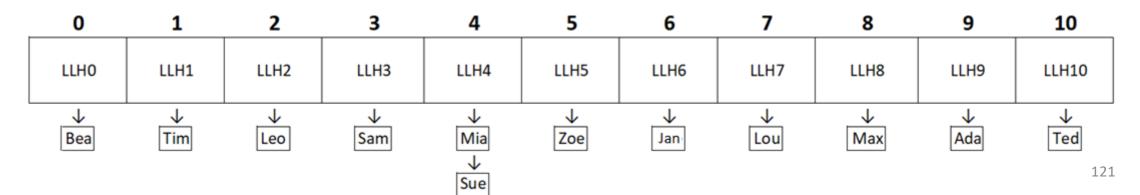
Lou hashes to cell 7.



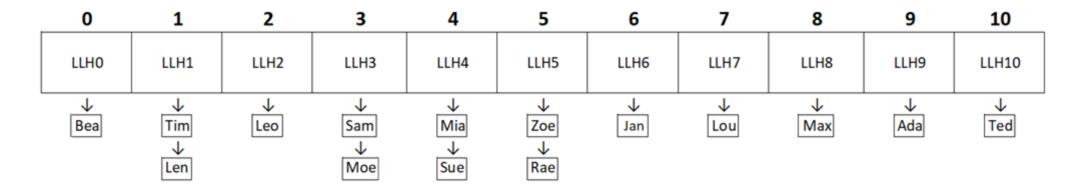
We can add the rest...



What happens when we try to add Sue which hashes to 4?

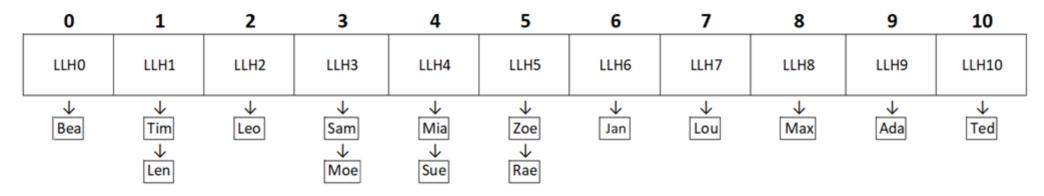


What happens when we add Len(1), Moe(3), Lou(7)



Any entries that hashes to match an existing entries gets added at the end of the linked list.

When an item is hashed and needs to be added to the hash table,



we need to check if it is the first item to be added at that location.

If it is, then we create a linked list head, malloc a new node, put the data in the node and store the linked list head in the array.

If it is not the first, then we use the linked list head stored in the array cell and traverse to the end of the list and add a newly malloced node. 123

### Hash Table – Uses

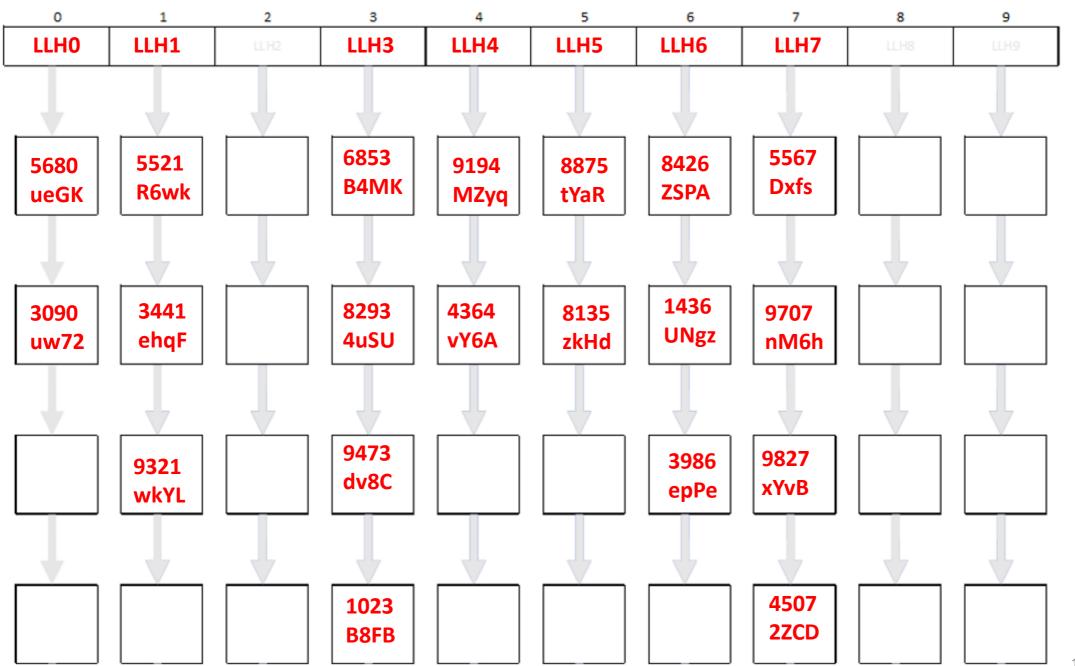
- Associative arrays: Hash tables are commonly used to implement many types of in-memory tables. They are used to implement associative arrays (arrays whose indices are arbitrary strings or other complicated objects).
- Database indexing: Hash tables may also be used as disk-based data structures and database indices (such as in dbm).
- Caches: Hash tables can be used to implement caches i.e. auxiliary data tables that are used to speed up the access to data, which is primarily stored in slower media.
- Object representation: Several dynamic languages, such as Perl, Python, JavaScript, and Ruby use hash tables to implement objects.
- Hash Functions are used in various algorithms to make their computing faster

Part A – Complete the hash version using the same values/Part A data

0	1	2	3	4	5	6	7	8	9
8000	7521	0762	5343	3624	9035	9466	8697	6708	2699
wyGK	wsaR	gL6h	FuMK	Gdgz	728C	ВТуq	uEPe	ycfs	xFuA

Part B - Create the hash table using the values from the Part B data. Use linear probing for resolving conflicts.

0	1	2	3	4	5	6	7	8	9
8000	8697	0762	5413	5343	8755	9466	3367	6708	2699
Uezx	epM4	nMsx	R63L	B4P3	4WF2	BtMZ	eh7d	Dx4e	ZSPX



```
Pichu|1.00|4.4|B|Tiny Mouse|Static|1|172
 2 Pikachu|1.04|13.2|B|Mouse|Static|2|025
                                                                               12 □{
  3 Raichu|2.07|66.1|B|Mouse|Static|3|026
                                                                               13
  4 Charmander | 2.00 | 18.7 | B | Lizard | Blaze | 1 | 004
                                                                               14
 5 Charmeleon | 3.7 | 41.9 | B | Flame | Blaze | 2 | | 005
  6 Charizard | 5.7 | 199.5 | B | Flame | Blaze | 3 | 006
                                                                               15
    Squirtle | 1.08 | 19.8 | B | Tiny Turtle | Torrent | 1 | 007
                                                                               16
 8 Wartortle|3.03|49.6|B|Turtle|Torrent|2|008
                                                                                17
 9 Blastoise | 5.03 | 188.5 | B | Shell fish | Torrent | 3 | 009
10 Igglybuff | 1.00 | 2.2 | B | Balloon | Cute Charm, Competitive | 1 | 174
                                                                               18
11 Jigglypuff | 1.08 | 12.1 | B | Balloon | Cute Charm, Competitive | 2 | 039
                                                                               19
12 Wigglytuff | 3.03 | 26.5 | B | Balloon | Cute Charm, Competitive | 3 | 040
                                                                                20
13 Ponyta|3.03|66.1|B|Fire Horse|Run Away, Flash Fire|1|077
                                                                                21
14 Rapidash | 5.07 | 209.4 | B | Fire Horse | Run Away, Flash Fire | 2 | 078
15 Psyduck | 2.07 | 43.2 | B | Duck | Damp, Cloud Nine | 1 | 054
                                                                                22
16 Golduck | 5.07 | 168.9 | B | Duck | Damp. Cloud Nine | 2 | 055
                                                                                23
17 Vaporeon | 3.03 | 63.9 | B | Bubble Jet | Water Absorb | 2 | 134
18 Eevee | 1.00 | 14.3 | B | Evolution | Run Away, Adaptability | 1 | 133
19 Jolteon 2.07 54.0 B Lightning Volt Absorb 3 135
20 Meowth | 1.04 | 9.2 | B | Scratch Cat | Pickup, Technician | 1 | 052
21 Persian 3.03 70.5 B Classy Cat Technician, Limber 2 053
22 Bulbasaur | 2.04 | 15.2 | B | Seed | Overgrow | 1 | 001
23 Ivysaur | 3.03 | 28.7 | B | Seed | Overgrow | 2 | 002
24 Venusaur | 6.07 | 220.5 | B | Seed | Overgrow | 3 | 003
25 Smoochum | 1.04 | 13.2 | F | Kiss | Oblivious, Forewarn | 1 | 238
26 Jynx | 4.07 | 89.5 | F | Human Shape | Oblivious, Forewarn | 2 | 124
27 Growlithe 2.04 | 41.9 | B | Puppy | Intimidate, Flash Fire | 1 | 058
28 Arcanine | 6.03 | 341.7 | B | Legendary | Intimidate, Flash Fire | 2 | 059
29 Finneon | 1.04 | 15.4 | B | Wing Fish | Swift Swim, Storm Drain | 1 | 456
30 Lumineon 3.11 52.9 B Neon Swift Swim, Storm Drain 2 457
31 Ditto|1.00|8.8|U|Transform|Limber|0|132
32 Nidoran | 1.04 | 15.4 | F | Poison Pin | Poison Point, Rivalry | 1 | 029
33 Midorina 12 071/1/ 11Fl Doigon, Den | Doigon, Doint Bixal ryl 21030
```

```
typedef struct Pokemon
    char *name;
    float height;
    float weight;
    char gender;
    char *category;
    char *abilities;
    int evolution;
    int national pokedex number;
    struct Pokemon *next ptr;
POKEMON;
```

### **Coding Assignment 6**

**CSE 3318** 

Coding Assignment 6 is over creating a hash table and using it to do the 3 basic dictionary actions - insert, delete and search.

Your hash table will be created using an integer array with separate chaining to resolve conflicts.

### Step 1

For this assignment, you need to choose a subject for your dictionary. For my version of the assignment, I chose Pokémon – you may not also use Pokémon – you need to pick your own subject. Pick a subject that allows you to gather enough information to fulfill the minimum requirements of the assignment.

#### **Structure**

You will need to create a struct to hold each entry of the dictionary. My Pokémon example is

```
typedef struct Pokemon
   char *name;
   float height;
   float weight;
   char gender;
   char *category;
   char *abilities;
   int evolution;
   int national pokedex number;
   struct Pokemon *next ptr;
POKEMON;
```

In this struct, the char pointers are used instead of char arrays since it is not known how much space those entries will need. Memory to hold those fields will be dynamically allocated using malloc() and the pointer returned by malloc() will be stored in the struct.

For your assignment, you will need a struct with a minimum of 5 fields – the parts shown in **bold** are required.

```
typedef struct YourStructName
{
    char *chararrayfield;
    float floatfield;
    char charfield;
    int intfield;
    struct YourStructName *next_ptr;
}
YOURSTRUCTNAME;
```

#### **Input File**

Your input file must contain a minimum of 30 pipe delimited records that represent your dictionary entries. Here's the first 10 lines of my Pokédex file.

```
Pichu|1.00|4.4|B|Tiny Mouse|Static|1|172
Pikachu|1.04|13.2|B|Mouse|Static|2|025
Raichu|2.07|66.1|B|Mouse|Static|3|026
Charmander|2.00|18.7|B|Lizard|Blaze|1|004
Charmeleon|3.7|41.9|B|Flame|Blaze|2||005
Charizard|5.7|199.5|B|Flame|Blaze|3|006
Squirtle|1.08|19.8|B|Tiny Turtle|Torrent|1|007
Wartortle|3.03|49.6|B|Turtle|Torrent|2|008
Blastoise|5.03|188.5|B|Shellfish|Torrent|3|009
Igglybuff|1.00|2.2|B|Balloon|Cute Charm, Competitive|1|174
```

The fields in each entry must be pipe delimited and must be listed in the same order as your struct. Save this as a text file named Code6\_xxxxxxxxxxxx\_InputFile.txt where xxxxxxxxxx is your student id.

### **Hashing Function**

Pick one of the methods discussed in class for creating your hash function (numeric keys, alphanumeric keys, folding). Pick which field(s) from your struct will be used for the hash function. Remember that whatever method you choose should result in a value that you can MOD with the hash table define to get an index that will always be within your array.

Write a small C program that prompts for input and calls your hash function and prints the result of the hash. You are proving that your hash function will work. Here's an example—your version may be slightly different depending on your input. THIS CODE IS ONLY FOR SHOWING THAT YOUR HASH FUNCTION WORKS. It is ONLY an example—modify it as needed to show that your hash function will work.

```
#include <stdio.h>
#define HASHTABLESIZE 10
int MyHashFunction(pass in value being used in hash)
   perform hash method
   return value % HASHTABLESIZE;
int main(void)
   char HashValue[20];
   printf("Enter value ");
   scanf("%s", HashValue);
   printf("The hash value for %s is %d\n'', HashValue, MyHashFunction(HashValue));
   return 0;
```

Compile your version and run and confirm that your function works. Save this code as Code6\_xxxxxxxxxx\_HashFunction.c where xxxxxxxxxx is your student id.

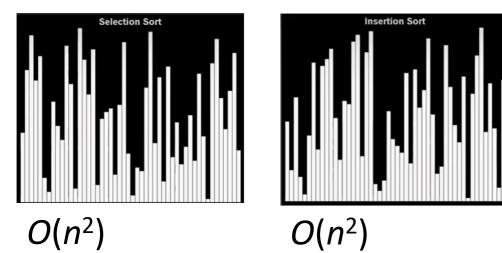
### Step 2

Submit your input file, struct and hashing function for approval. Submit these to me by email. If you want to zip the files, you can, but you can just send them as attachments. No pictures or screenshots will be accepted.

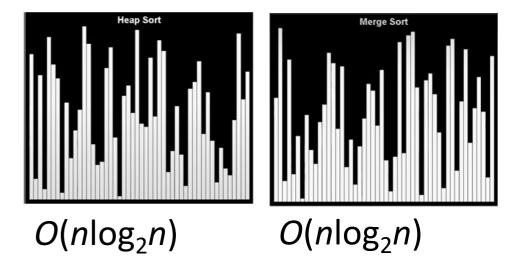
Three files must be submitted via email

```
Code6_xxxxxxxxxxxxx_struct.txt
Code6_xxxxxxxxxxx_InputFile.txt
Code6_xxxxxxxxxxx_HashFunction.c
```

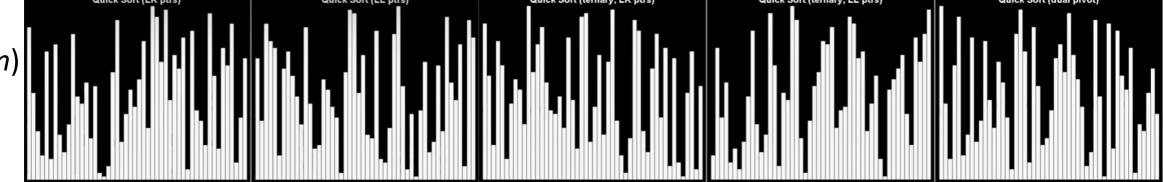
Once you get approval, you will receive the rest of the instructions for the assignment. If you do not get approval, any code you turn in for Coding Assignment 6 will be assigned a grade of 0 regardless.

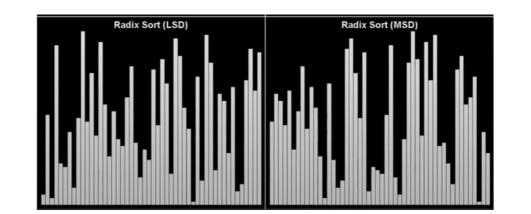


Sorts



 $O(n\log_2 n)$   $O(n^2)$ 







Let's start with a list of numbers to sort

645, 86, 182, 35, 601, 8

Step 1

All numbers to be sorted need to be the same length.

645, 86, 182, 35, 601, 8

How do we make 8 and 35 and 86 the same length as 648, 182 and 608?

645, 086, 182, 035, 601, 008

We add leading zeros/front pad our numbers with zeroes.

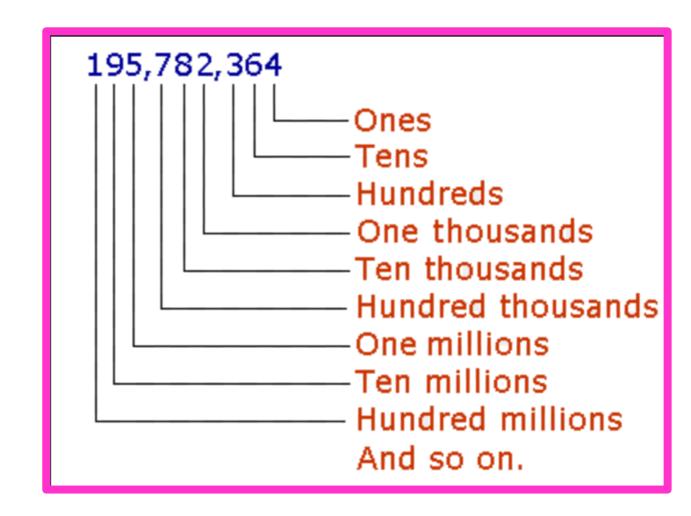
No difference in value between 001 and 1.

Big difference between 1 and 100 – add leading zeros to not change value.

64<mark>5</mark>, 08<mark>6</mark>, 18<mark>2</mark>, 03<mark>5</mark>, 60<mark>1</mark>, 00<mark>8</mark>

Now we arrange our list by place value starting with ones.

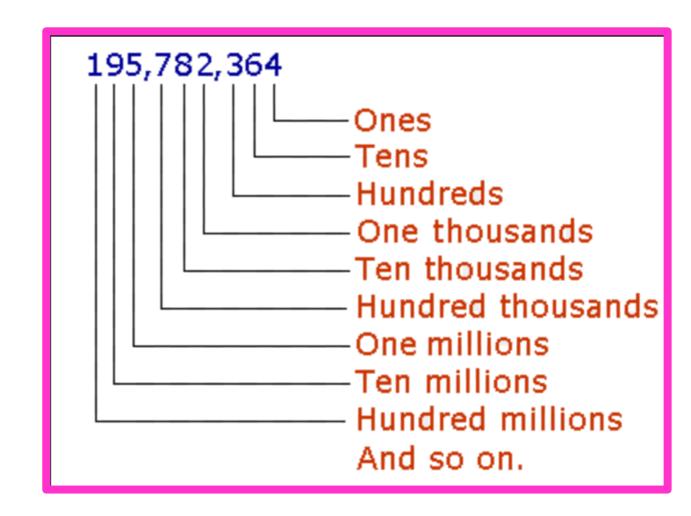
601, 182, 645, 035, 086, 008



6<mark>0</mark>1, 1<mark>8</mark>2, 6<mark>4</mark>5, 0<mark>3</mark>5, 0<mark>8</mark>6, 0<mark>0</mark>8

Now we arrange our list by place value tens.

601, 008, 035, 645, 182, 086

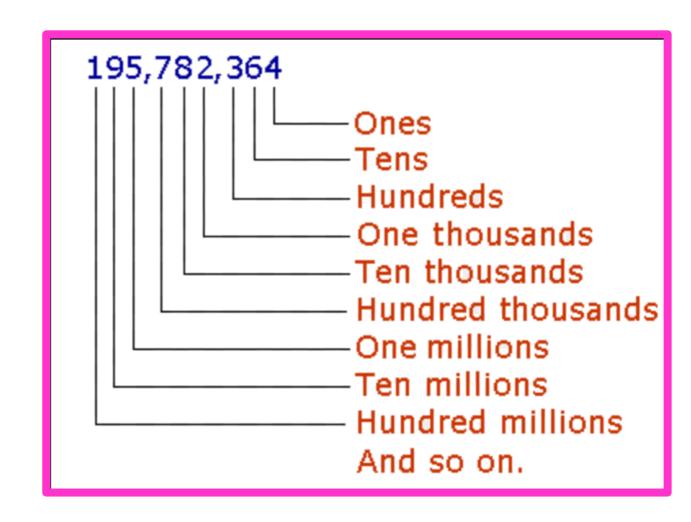


601, 008, 035, 645, 182, 086

Now we arrange our list by place value hundreds.

008, 035, 086, 182, 601, 645





645, 086, 182, 035, 601, 008

601, 182, 645, 035, 086, 008

601, 008, 035, 645, 182, 086

008, 035, 086, 182, 601, 645

### **Radix Definition**

The **base** of a system of numeration

So "Radix Sort" means a sort that uses the **base** of a system of numeration.

So can we use Radix Sort on numbers in bases other than 10?



Let's start with a list of binary numbers

110, 011, 101, 001, 100, 111, 010

110, 11, 101, 1, 100, 111, 10

Using ones, tens and hundreds place value is more applicable to the decimal system.

Add leading zeros

So let's talk about Most Significant Bit (MSB) and Least Significant Bit (LSB).

110, 011, 101, 001, 100, 111, 010

**MSB** 

the bit in a binary number which is of the greatest numerical value the leftmost/first bit of a number

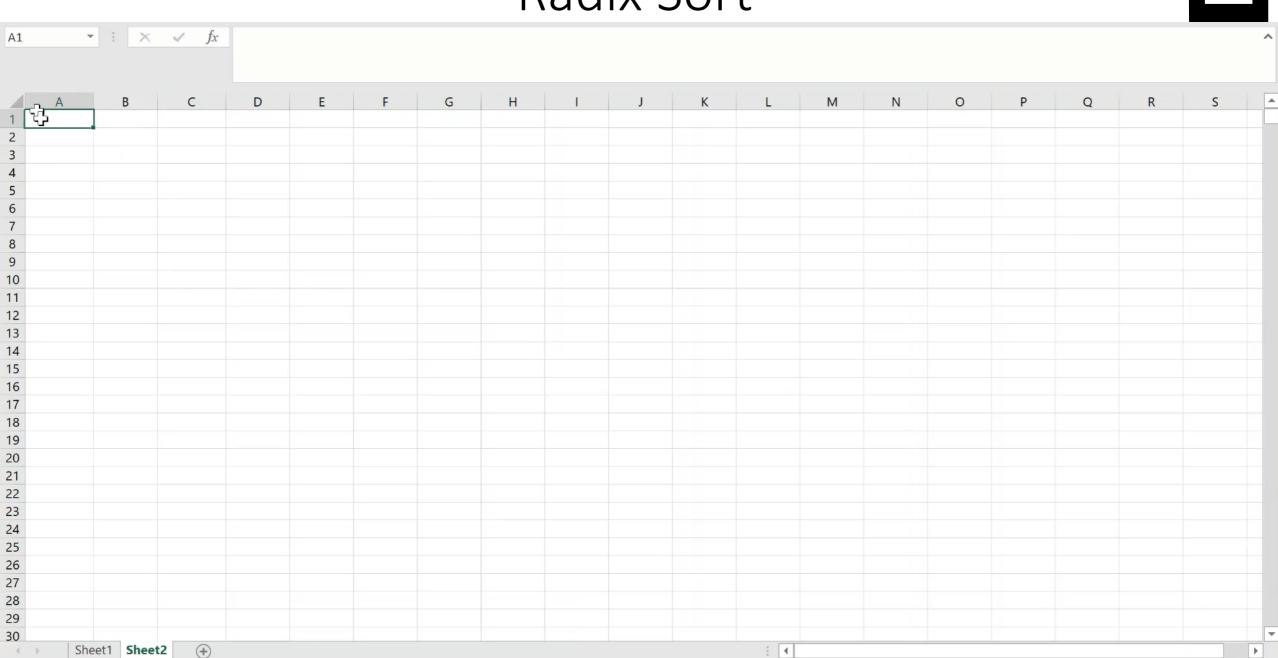
LSB

the bit in a binary number which is of the lowest numerical value. the rightmost/last bit of a number

Let's arrange them starting with LSB and work towards the MSB.

001, 010, 011, 100, 101, 110, 111





## Radix Sort Time Complexity

So what is the time complexity of Radix Sort?

Let's establish a few terms

d is the number of digits in the numbers in the list – use the number of digits in the biggest number in the list.

n is the number of numbers in the list

b is the base/radix we are using

## Radix Sort Time Complexity

For every number in our list, n, we checked each digit.

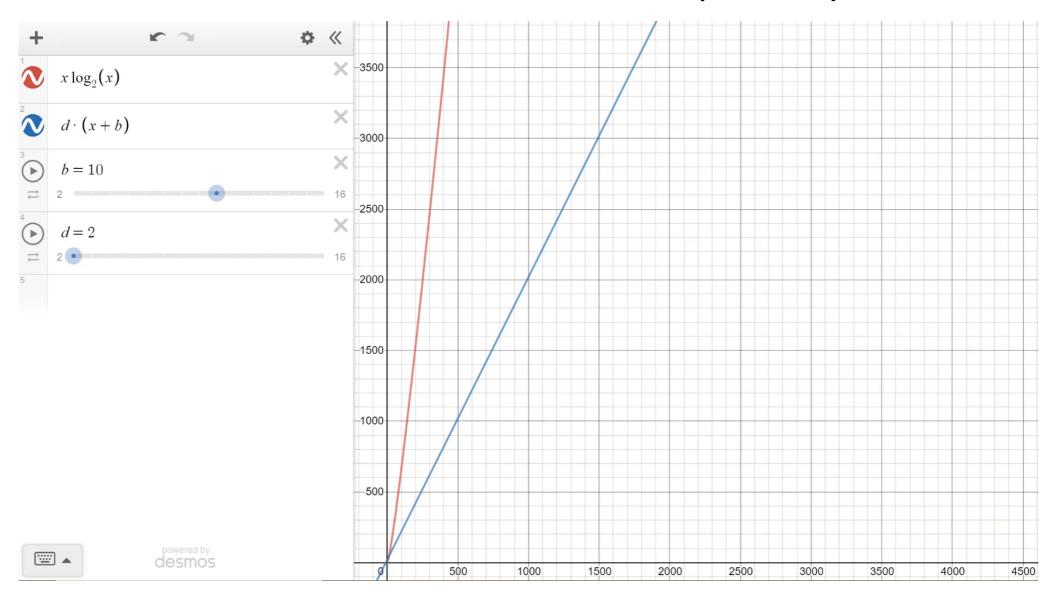
d\*n

When we had a base 10 number, we had to check 10 digits -> 0-9. When we had a base 2 number, we had to check 2 digits -> 0 and 1.

So the base influenced how much time we spent on each number.

$$O(d * (n+b))$$

# Radix Sort Time Complexity



Interesting Thoughts on Radix Sort

Radix Sort does not use any comparisons or swaps.

The time complexity of Radix Sort maintains its coefficient (number of digits) because of the significant impact of that value on the time complexity.

O(nk)

Even asymptotic notation does not wave away the impact of the number of digits.

So if Radix Sorts has a better run time than most other sorts, why isn't it used more?

The biggest limitation of Radix Sort is that it is limited to data that only consists of digits or letters.

Radix Sort also does not actually move any elements around in the storage container (like an array) – it makes a new version of the storage container; therefore, has a higher space complexity than some other "slower" sorts. O(n+k).