Bangabandhu Sheikh Mujibur Rahman Science & Technology University
Department of Computer Science and Engineering
2^{nd} Year 1^{st} Semester B.Sc. (Eng.) Examination 2014
Course No.: MAT 204 Course Title: Matrices and Differential Equations
Full Marks: 70 Time: 04 Hours

N. B.
i) Answer **SIX** questions, taking any **THREE** from each section.
ii) All questions are of equal values.
iii) Use separate answer script for each section

Section-A

1. a. Define with example: (i) Singular matrix, (ii) Symmetric matrix , (iii) Hermitian matrix , (iv) Trace of a matrix, (v) Unitary matrix. **5**

   b. Solve the following equations with the help of inverse matrix **$6^{2/3}$**

$$\begin{aligned} 3x + 5y - 7z &= 13 \\ 4x + y - 12z &= 6 \\ 2x + 9y - 3z &= 20 \end{aligned}$$

2. a. Find the inverse matrix of the following matrix by row canonical method: **$5^{2/3}$**

$$A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & -2 & -2 \\ 3 & 1 & 2 \end{bmatrix}$$

   b. Define rank of a matrix. Reduce the following matrix to the normal (or canonical) form and hence obtain its rank. **6**

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

3. a. Define eigen values and eigen vectors. **4**

   b. Find the eigen values and eigen vectors of the following matrix: **$7^{2/3}$**

$$\begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix}$$

and also find the matrix P that diagonalize it.

4. a. State Cayley-Hamilton theorem. Find the characteristic equation of the matrix **6**

$$A \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

and verify Cayley-Hamilton theorem for it.

   b. Define similar matrices. Prove that similar matrices have the same characteristic polynomial and the same eigenvalue. **$5^{2/3}$**

Section-B:

5. a. Define linear differential equation. Find the DE of the family of curves $y=c(x-c)^2$ when c is arbitrary constants. $3^{2/3}$

b. Solve the initial value problem: $x\sin y\,dx+(x^2+1)\cos y\,dy=0,\ y(1)=\frac{\pi}{2}$. 4

c. Prove that if $M(x,y)dx + N(x,y)dy = 0$ is a homogeneous differential equation, then the change of variable $y = vx$ transform the homogeneous equation into a separable equation in the variable v *and* x. 4

6. a. Define integrating factor with examples. Solve the differential equation 4

$$(x^3 + xy^4)dx + 2y^3dy = 0$$

b. Solve the differential equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 4\sin\ln x$ 4

c. Define Bernoulli's differential equation. Solve the DE: $(1-x^2)\frac{dy}{dx}+xy=xy^2$. $3^{2/3}$

7. a. Find the solution of the following differential equation: $3+2^{2/3}$

i. $(D^3 - 7D - 6)y = (x+1)e^x$

ii. $(D^2 - DD' - 2D'^2)z = (y-1)e^x$.

b. Find the solution of the following partial differential equation: 3+3

i. $(D^3 - 4D^2D' + 4DD'^2)z = 4\sin(2x + y)$

ii. $(D+D' - 1)(D + 2D' - 3)z = 4 + 3x + 6y$

8. a. Find the complete integral of $q = -xp + p^2$ 4

b. Find the complete integral of $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$. 4

c. Solve the partial differential equation $(y - zx)p + (x + yz)q = x^2 + y^2$ $3^{2/3}$

Bangabandhu Sheikh Mujibur Rahman Science & Technology University
Department of Computer Science and Engineering
2nd Year 1st Semester B.Sc. (Eng.) Examination 2015

Course No.: MAT 204 Course Title: Matrices and Differential Equations

Full Marks: 70 Time: 04 Hours

N.B. 1) The Figures in the right margin indicate full marks.

2) Answer any **Three** from the each of the section of the following questions.

Section-A

1. a. Define Symmetric matrix, Hermitian and Involutory matrix with examples. Show that the matrix $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ is an idempotent matrix. **6.67**

   b. Solve the following system of linear equtions by using matrices: **5**

$$x - y + z = 1$$
$$x + y - 2z = 0$$
$$2x - y - z = 0$$

2. a. Define consistent and inconsistent system of linear equation. Show that the following system of linear equation is inconsistent: **5.67**

$$2x + 3y + 5z + t = 3$$
$$3x + 4y + 2z + 3t = -2$$
$$x + 2y + 8z - t = 8$$
$$7x + 9y + z + 8t = 0$$

   b. Define rank of a matrix. Find the rank of the following matrix: $A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$ **6**

3. a. Define eigenvalues and eigenvectors. **3**

   b. Find the eigenvalues and eigenvectors of the following matrix: **8.67**

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$$

   and also find the matrix P that diagonalizes the matrix A and find $P^{-1}AP$.

4. a. State Cayley-Hamilton. Find the characteristic equation of the matrix **7**

$$A = \begin{bmatrix} -4 & 5 & 5 \\ -5 & 6 & 5 \\ -5 & 5 & 6 \end{bmatrix}$$

   and verify Cayley-Hamilton theorem for it and also find the inverse of the matrix A by using Cayley-Hamilton

   b. Prove that any square matrix A and its transpose A^T have the same eigenvalues. **4.67**

Section-B

5. a. What do you mean by ordinary and partial differential equations. Find the differential equation of the family of curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where a and b are arbitrary constant and identify it. **4.67**

b. Solve the following initial value problem: $x\sin y\,dx + (x^2+1)\cos y\,dy = 0$; with initial condition $y(1)=\pi/2$. **4**

c. Define Bernoulli's equation. Solve the differential equation: $\frac{dy}{dx} + y = xy^3$. **3**

6. a. Define integrating factor with examples. Solve the differential equation: **3.67** 2

$$(x^2 + y^2 + 2x)dx + 2ydy = 0$$

b. Solve any two of the following differential equations: **4+4** 4

i) $(D^3 - D^2 - 6D)y = x^2 + 1$

ii) $(D^2 - 5D + 6)y = x^2e^{3x}$ $\quad e^{3x}\left(\frac{x^3}{3} - x^2 + 2x\right)$

iii) $(D^2 + 3D + 2)y = e^{2x}\sin x$

7. a. Solve the differential equation $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = 4\ln x$ $\quad \left[6\ln x - \frac{3}{?}\right]$ **3.67** 3.67

b. Find the complete integral of $2zx + pq = px^2 + 2qxy$ **4** 3

c. Find the complete integral of $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$ **4** 2

8. a. Solve the differential equation: $z(x + y)p + z(x - y)q = x^2 + y^2$ $\quad x^2 - y^2 - z^2$ **3.67** 3.67

b. Solve any two of the following differential equations: **4+4** 4+2

i. $\left(D^2 - 2DD' + D'^2\right)z = e^{x+2y} + x^3$ $\quad xy - \frac{z^2}{2}$

ii. $\left(D^2 + D'^2\right)z = \cos mx.\cos nx$

iii. $(D - 3D' - 2)^2z = 2e^{2x}\sin(3x + y)$

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Bangabandhu Sheikh Mujibur Rahman Science and Technology University
Department of Computer Science and Engineering
2nd Year 1st Semester B.Sc. Engineering Examination-2016

Course Title: Matrices and Differential Equations — **Course No: MAT 204**
Total Marks: 60 — **Time: 3 (three) Hours**

N.B.
i) Answer SIX questions taking any THREE from each Section
ii) All questions are of equal values.
iii) Use separate answer script for each section

Section-A

1. a) Define with example: (i) Symmetric matrix, (ii) Orthogonal matrix, (iii) Unitary matrix, (iv) Diagonal matrix and (v) Singular matrix. 5

   b) Solve the following equations by matrix method: 5
$$x + 2y + 3z = 4, \quad 2x + 3y + 8z = 7, \quad x - y + 9z = 1$$

2. a) Explain the following term the adjoin of a square matrix. Find the inverse of the following matrix: 5
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

   b) Define rank of a matrix. Find the rank of the matrix 5
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 4 & 10 & 18 \end{bmatrix}$$

3. a) Define eigen values and eigen vectors. 4

   b) Find the eigen values and eigen vectors of the following matrix: 6
$$A = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

4. a) Verify Cayley-Hamilton theorem for the matrix 5
$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

   b) Prove that If A is an $n \times n$ triangular matrix, then the elements of the principal diagonals are the characteristic roots of A. 5

Section-B

5. a) Write down the conditions for an equation to be non-linear differential equation with examples. 4

b) Solve the initial value problem $\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y3}$ with $y(1) = 2$. 6

6. a) What do you mean by Bernoulli's differential equation? State and prove the necessary condition for a differential equation of first degree being exact. 5

b) Solve the ordinary differential equation: 5

$$(2xy + 1)dx + (x^2 + 4y)dy = 0$$

7. a) Solve the equation: 5

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

b) The population of a city satisfies the differential equation: 5

$$\frac{dx}{dt} = \frac{1}{100}x - \frac{1}{10^8}x^2$$

Where t is measured in years. Find the population of the city at 2000. Where the population of the city on 1980 was 10^5.

8. a) Solve the equation: $xyp^2 + (x^2 + xy + y^2)p + x^2 + xy = 0$ 5

b) Solve the differential equation operationally: 5

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}$$

Bangabandhu Sheikh Mujibur Rahman Science & Technology University

Department of Computer Science & Engineering

2nd Year 1st Semester B.Sc. (Eng.) Examination- 2017

Course Title: Matrices and Differential Equations | **Course Code: MAT204**

Full Marks: 60 | **Time: 03 Hours**

N.B.

1) Answer SIX questions, taking any THREE from each section.
2) All questions are of equal values.
3) Use separate answer script for each section.

Section-A

1. (a) Define with example: (i) Matrix (ii) Rectangular matrix (iii) Unitary matrix (iv) Diagonal matrix (v) Singular matrix. 5

   (b) Define symmetric and skew-symmetric matrix. Prove that Every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix. 5

2. (a) Define Inverse of a matrix. Show that $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$. 5

   A and B

   (b) Define idempotent and nilpotent matrix with an example. If, and are non-singular matrices, then show that $(AB)^{-1} = B^{-1}A^{-1}$. 5

3. (a) Define eigen values and eigen vectors. 4

   (b) Find the Characteristic root of the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$ 6

4. (a) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ 5

   (b) State and prove the Fundamental theorem of matrix. 5

Section-B

5. (a) Define order and degree of differential equations. Find the differential equation of all circles of radius c. 4

   (b) Solve the initial value problem $(y+2)dx + y(x+4)dy = 0$ with $y(-3) = -1$. 3

   (c) Solve the Ordinary Differential Equation, $\frac{dy}{dx} + \frac{y}{x} = x^3$. 3

6. (a) State and prove the necessary condition for an equation to be exact differential equation. 4

   (b) Solve the Ordinary Differential Equation 6

$$(2xy+1)dx + (x^2+4y)dy = 0$$

7. (a) Solve the equation 5

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0$$

   (b) Solve the initial value problem $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$ subject to $y(0) = 1$ and $y'(0) = 0$. 5

8. (a) Establish the general form of Charpit's method to solve the non-linear partial differential equations of order one. 5

   (b) Solve the differential equation $\frac{d^2y}{dx^2} - y = x^2\cos x$. 5

Bangabandhu Sheikh Mujibur Rahman Science & Technology University
Department of Computer Science & Engineering
2nd Year 1st Semester B.Sc. (Engg.) Examination- 2018

Course No.: MAT205 Course Title: Vector,Matrixs and Fourier Analysis
Full Marks: 60 Time: 03 Hours

N.B.
1) Answer SIX questions, taking any THREE from each section.
2) All questions are of equal values.
3) Use separate answer script for each section.

SECTION-A (30 Marks)

1. (a) Define with example: (i) Matrix (ii) Rectangular matrix (iii) Transposed matrix (iv) Diagonal matrix (v) Singular matrix. 5
   (b) Define order of a minor. Find the matrix X from the equations $AX = B$ where 5
   $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$.

2. (a) What is rank of a matrix? 2
   (b) Reduce the following matrix to its echelon form and find its rank. 4
   $$A = \begin{pmatrix} 1 & 2 & -2 & 3 \\ -1 & 1 & 3 & 2 \\ 2 & 4 & -4 & 6 \\ 1 & 0 & -1 & 2 \end{pmatrix}$$
   (c) Find the inverse of the matrix 4
   $$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ -2 & 1 & 3 \end{pmatrix}$$

3. (a) Define a Fourier series and derive the Euler's formula. 5
   (b) State dirichlets condition. Find the Fourier series representing $f(x) = x \quad 0 \langle x \langle 2\pi$. 5

4. (a) Define Fourier transformation, Fourier sine transformation and Fourier cosine transformation. 3
   (b) State any two properties of Fourier transformation. 2
   (c) What do you mean by convolution? State and prove the convolution theorem on Fourier transformation. 5

SECTION-B (30 Marks)

5. (a) Define a unit vector. Find a unit vector parallel to the resultant of vectors $r_1 = 2i + 4j - 5k, r_2 = i + 2j + 3k$. 4
   (b) State and prove Frenet-Serret formula. 6

6. (a) Find the unit tangent vector to any point on the curve 5
   $$x = t^2 + 1, \quad y = 4t - 3, \quad z = 2t^2 - 6t.$$
   (b) Find the directional derivatives of $U = 2xy - z^2$ at $(2,-1,-1)$ in a direction towards $(3,1,-1)$. 5

7. (a) Define gradient, divergence and curl. Prove that $\nabla.(\phi \mathrm{A}) = (\nabla\phi).\mathrm{A} + \phi(\nabla.\mathrm{A})$. 4
   (b) Define irrotational vector. Prove that, irrotatioal vector is conservative. 6

8. (a) State and prove Green's theorem in the plane. 5
   (b) Define line integral. Find the total work done in moving a particle in a force field given by $F = 3xyi - 5zj + 10xk$ along the curve$x = t^2 + 1; y = 2t^2; z = t^3$ from $t=1$ to $t=2$. 5