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2014

1.(a) Define with example (i) Singular Matrix (ii) Symmetric Matrix (iii) Hermitian Matrix (iv) Trace of a Matrix (v) Unitary Matrix.

Solⁿ:

(i) Singular Matrix: Let D be the Determinant of the square matrix A. Then if D=0, the matrix A is called Singular. example:

$$A = \begin{bmatrix} 3 & 6 \\ 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

(ii) Symmetric Matrix: If a square Matrix A is equal to its transpose matrix such that $a_{ij} = a_{ji}$ is said to be symmetric matrix $[A^T = A]$

example:

$$A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 6 \end{bmatrix}$$

(iii) Hermitian Matrix: If A is a square matrix over the complex field and $A = A^* = (\bar{A})^T$ $[a_{ij} = \bar{a}_{ji}]$; $i, j = 1, 2 \dots n$ then A is called Hermitian matrix.

example:

$$A = \begin{bmatrix} 2 & 2-3i & 3 \\ 2+3i & 5 & 1+i \\ 3 & 1-i & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & i & -1-2i \\ -i & 3 & 5+3i \\ -1+2i & 5-3i & -1 \end{bmatrix}$$

(iv) Trace of a Matrix : Let A be a square of order n, then the sum of the diagonal element of A is called the trace of A.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix}$$

$$\therefore tr(A) = a_{11} + a_{22} + \cdots + a_{nn}$$

example: $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ $\therefore tr(B) = 1+5+9 = 15$

(v) Unitary Matrix : If A be a complex square matrix, then A is called unitary matrix if $AA^* = A^*A = I$

example: $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$

(b) Solve the following equation with the help of inverse matrix.

$$3x+5y-7z=13, \quad 4x+y-12z=6, \quad 2x+9y-3z=20$$

Solⁿ: Matrix form of given eqn:

$$\begin{bmatrix} 3 & 5 & -7 \\ 4 & 1 & -12 \\ 2 & 9 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 6 \\ 20 \end{bmatrix}$$

Let, $A = \begin{bmatrix} 3 & 5 & -7 \\ 4 & 1 & -12 \\ 2 & 9 & -3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $L = \begin{bmatrix} 13 \\ 6 \\ 20 \end{bmatrix}$

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$\therefore AX = L$

$\Rightarrow A^{-1}AX = A^{-1}L$

$\Rightarrow IX = A^{-1}L \qquad \because A^{-1}A = I$

$\Rightarrow X = A^{-1}L \qquad \because IX = X$

Now finding A^{-1}. let the determinant of A is D

$\therefore D = 17 \neq 0$ so, A^{-1} exist.

Now, the co-fector of D are

$A_{11} = 105 \qquad A_{21} = -48 \qquad A_{31} = -53$

$A_{12} = -12 \qquad A_{22} = 5 \qquad A_{32} = 8$

$A_{13} = 34. \qquad A_{23} = -17 \qquad A_{33} = -17$

$$\therefore \text{Adj } A = \begin{bmatrix} 105 & -12 & 34 \\ -48 & 5 & -17 \\ -53 & 8 & -17 \end{bmatrix}^T = \begin{bmatrix} 105 & -48 & -53 \\ -12 & 5 & 8 \\ 34 & -17 & -17 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{\text{Adj } A}{D} = \frac{1}{17}\begin{bmatrix} 105 & -48 & -53 \\ -12 & 5 & 8 \\ 34 & -17 & -17 \end{bmatrix}$$

$$A^{-1}L = \frac{1}{17}\begin{bmatrix} 105 & -48 & -53 \\ -12 & 5 & 8 \\ 34 & -17 & -17 \end{bmatrix}\begin{bmatrix} 13 \\ 6 \\ 20 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} 1365-288-1060 \\ -156+30+160 \\ 442-102-340 \end{bmatrix}$$

$$= \frac{1}{17}\begin{bmatrix} 17 \\ 34 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \qquad \therefore X = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \text{ where } x=1, y=2, z=0$$

2. (a) Find the inverse matrix of the following matrix by row canonical form $A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & -2 & -2 \\ 3 & 1 & 2 \end{bmatrix}$

Soln:

$$[AI_3] = \left[\begin{array}{ccc:ccc} 2 & 1 & 5 & 1 & 0 & 0 \\ -1 & -2 & -2 & 0 & 1 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:ccc} -1 & -2 & -2 & 0 & 1 & 0 \\ 2 & 1 & 5 & 1 & 0 & 0 \\ 3 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$ Interchange r_1 and r_2

$$\sim \left[\begin{array}{ccc:ccc} 3 & 1 & 2 & 0 & 0 & 1 \\ 2 & 1 & 5 & 1 & 0 & 0 \\ -1 & -2 & -2 & 0 & 1 & 0 \end{array}\right]$$ Interchange r_1 and r_3

$r_1' \to r_1 - r_2$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & -3 & -1 & 0 & 1 \\ 2 & 1 & 5 & 1 & 0 & 0 \\ -1 & -2 & -2 & 0 & 1 & 0 \end{array}\right]$$

$r_2' \to r_2 - 2r_1$
$r_3' \to r_1 + r_3$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & -3 & -1 & 0 & 1 \\ 0 & 1 & 11 & 3 & 0 & -2 \\ 0 & -2 & -5 & -1 & 1 & 1 \end{array}\right]$$

$r_3' \to \frac{1}{17} \times (r_3 + 2r_2)$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & -3 & -1 & 0 & 1 \\ 0 & 1 & 11 & 3 & 0 & -2 \\ 0 & 0 & 1 & \frac{5}{17} & \frac{1}{17} & -\frac{3}{17} \end{array}\right]$$

$r_1' \to r_1 + 3r_1$
$r_2' \to r_2 - 11r_3$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & -2/17 & 3/17 & 8/17 \\ 0 & 1 & 0 & -4/17 & -1/17 & -1/17 \\ 0 & 0 & 1 & 5/17 & 1/17 & -3/17 \end{array}\right]$$

$$= \left[I_3 \; A^{-1} \right]$$

$$\therefore A^{-1} = \frac{1}{17}\begin{bmatrix} -2 & 3 & 8 \\ -4 & -11 & -1 \\ 5 & 11 & -3 \end{bmatrix} \quad \text{(Ans)}$$

(b) Define rank of a matrix. Reduce the following matrix to the normal (or canonical) form and hence obtain its rank.

Sol^n:

Rank of a Matrix: Let A be an $m \times n$ matrix and let A_r be the row echelon form of A. Then the rank of matrix A is the number of non zero rows of A_r.

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$

$c_2' \to 2c_1 + c_2$

$c_4' \to c_1 + c_4$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 8 & 1 & 5 \\ -2 & 13 & 2 & 3 \end{bmatrix}$$

$r_2' \to r_2 - 3r_1$

$r_3' \to r_3 + 2r_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 8 & 1 & 5 \\ 0 & 13 & 2 & 3 \end{bmatrix}$$

$c_2' \to c_2 - 7c_3$

$c_4' \to c_4 - 5c_3$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 2 & -7 \end{bmatrix}$$

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$$r_3' \to r_3 + r_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 3 & -7 \end{bmatrix}$$

$$c_3' \to c_3 + c_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 6 & 5 \\ 0 & 0 & -4 & -7 \end{bmatrix}$$

$$c_3' \to c_3/2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & -2 & -7 \end{bmatrix}$$

$$r_3' \to r_2 + r_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$c_3' \to c_3 - 3c_2$$
$$c_4' \to c_4 - 5c_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$c_4' \to c_4 + 2c_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sim [I_3 \; 0]$$

$\therefore$ rank$(A) = 3$ where $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

2015

1. (a) Define Symmetric matrix, Hermitian and Involutory matrix with example. Show that the matrix $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ is an idempotent matrix.

Sol^n:

Symmetric matrix: If a square matrix is equal to it's transpose matrix such that $a_{ij} = a_{ji}$ is said to be symmetric matrix.

example: $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 6 \\ 4 & 6 & 7 \end{bmatrix}$

Hermitian Matrix: If A is a complex square matrix and $A = (\bar{A})^T$ then A is called Hermitian matrix.

example: $A = \begin{bmatrix} 2 & 2-3i & 3 \\ 2+3i & 5 & 1+i \\ 3 & 1-i & 0 \end{bmatrix}$

Involutory Matrix:

A square matrix is called involutory if $A^2 = I$

example:

$$A = \begin{bmatrix} 4 & 3 \\ -5 & -4 \end{bmatrix}.$$

2nd part: Given, $A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$

$$\therefore A \cdot A = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1+3-5 & -3-9+15 & -5-15+25 \\ -1-3+5 & 3+9-15 & 5+15-25 \\ 1+3-5 & -3-9+15 & -5-15+25 \end{bmatrix}$$

$$\therefore A^2 = \begin{bmatrix} -1 & 3 & 5 \\ 1 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix} = A$$

$\because A^2 = A$, so, A is an idempotent matrix.

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(b) Solve the following system of linear equation by using matrix :

$$x-y+z=1$$
$$x+y-2z=0$$
$$2x-y-z=0$$

Solⁿ :

from the given eqⁿ :

$$\begin{bmatrix}1 & -1 & 1\\ 1 & 1 & -2\\ 2 & -1 & -1\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$$

let, $A = \begin{bmatrix}1 & -1 & 1\\ 1 & 1 & -2\\ 2 & -1 & -1\end{bmatrix}$, $X = \begin{bmatrix}x\\ y\\ z\end{bmatrix}$, $L = \begin{bmatrix}1\\ 0\\ 0\end{bmatrix}$

$\therefore AX = L$

$\Rightarrow A^{-1}AX = A^{-1}L$

$\Rightarrow IX = A^{-1}L$ $\quad [\because A^{-1}A = I]$

$\Rightarrow X = A^{-1}L$ $\quad [\because IX = X]$

Now finding A^{-1}

Let, the determinant of A is D

$\therefore D = -3 \neq 0$ so A^{-1} exist

$A_{11} = -3$ $\quad A_{21} = -2$ $\quad A_{31} = 1$

$A_{12} = -3$ $\quad A_{22} = -3$ $\quad A_{32} = 3$

$A_{13} = -3$ $\quad A_{23} = 1$ $\quad A_{33} = 2$

$$\therefore \text{Adj } A = \begin{bmatrix} -3 & -3 & -3 \\ -2 & -3 & 1 \\ 1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & 2 \\ -3 & -3 & 3 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{D} \text{Adj } A = \frac{1}{-3} \begin{bmatrix} -3 & -2 & 2 \\ 3 & -3 & 3 \\ -3 & 1 & 2 \end{bmatrix}$$

$$\therefore A^{-1}L = \frac{1}{-3} \begin{bmatrix} -3 & -2 & 2 \\ -3 & -3 & 3 \\ -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \frac{1}{-3} \begin{bmatrix} -3+0+0 \\ -3+0+0 \\ -3+0+0 \end{bmatrix}$$

$$= \frac{1}{-3} \begin{bmatrix} -3 \\ -3 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore x = 1, \ y = 1, \ z = 1$$

2) (b) Define rank of a matrix. Find the rank of the following matrix $A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$

Sol^n :

Rank of a matrix : Let A be a $m \times n$ matrix and let A_R be an row echelon form of A. Then the rank of A is number of non zero rows of A_R.

Finding rank :

$$A = \begin{bmatrix} -1 & 2 & 3 & -2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 2 & -5 & 1 & 2 \\ 3 & -8 & 5 & 2 \\ 5 & -12 & -1 & 6 \end{bmatrix} \quad r_1' \to (-1) r_1$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & -2 \\ 0 & -2 & 14 & -4 \\ 0 & -2 & 14 & -4 \end{bmatrix} \quad \begin{matrix} r_2' \to r_2 - 2r_1 \\ r_3' \to r_3 - 3r_1 \\ r_4' \to r_4 - 5r_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & -1 & 7 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} r_3' \to r_3 - 2r_2 \\ r_4' \to r_4 - 2r_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & -3 & 2 \\ 0 & 1 & -7 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_2' \to (-1) R_2$$

since the echelon form of matrix A have two non zero rows. so the rank of matrix A is 2

2016

1. (a) Define with example (i) Symmetric matrix, (ii) orthogonal matrix (iii) Unitary matrix (iv) Diagonal matrix (v) Singular matrix

Solⁿ: (i) — 2014(1, a, ii)
(iii) — 2014 (1, a, v)
(v) — 2014 (1, a, i)

(ii) Orthogonal matrix: A real matrix A is said to be orthogonal if $AA^T = A^TA = I$ $[A^T = A^{-1}]$

example: $\begin{bmatrix} \frac{1}{9} & \frac{8}{9} & -\frac{4}{9} \\ \frac{4}{9} & -\frac{4}{9} & -\frac{7}{9} \\ \frac{8}{9} & \frac{1}{9} & \frac{4}{9} \end{bmatrix}$

(iv) Diagonal Matrix : a square matrix whose a_{ij} element are non zero when $i=j$ and others element $(i \neq j)$ are zero is called diagonal matrix.

example : $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

(b) Solve the following equations by matrix method:

$x+2y+3z = 4$, $2x+3y+8z = 7$,

$x-y+9z = 1$

Sol^n :

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 1 & -1 & 9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$$

Let, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 8 \\ 1 & -1 & 9 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $L = \begin{bmatrix} 4 \\ 7 \\ 1 \end{bmatrix}$

$\therefore AX = L$

$\Rightarrow A^{-1}AX = L$

$\Rightarrow IX = A^{-1}L \quad [\because A^{-1}A = I]$

$\Rightarrow X = A^{-1}L$

Now find A^{-1}.

Let the determinant of A is D

$\therefore D = 0$ so A^{-1} does not exist.

It can be said that can not find out the value of x, y, z without the existence of A^{-1}.

(2) a) Explain the following term the adjoint of a square matrix. Find the inverse of the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

Sol^n:

(1st part): Adjoint of A square matrix:

If A is a square matrix, then

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \quad ; \; i, j = 0, 1, 2 \ldots n$$

Let, D be the Determinant of the matrix A,

then, $$D = |A| = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$ ~~where~~

Let A_{ij} be the cofectors of determinat D.

Form the matrix $[A_{ij}]$ then the transpose of the matrix $[A_{ij}]$ is called the adjoint matrix of A.

$$\therefore \text{Adj } A = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & A_{22} & \cdots & A_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ A_{n1} & A_{n2} & \cdots & A_{nn} \end{bmatrix}^T = \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$$

(2nd part :)

Given $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$$[AI_3] = \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 2 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 0 & -3 & -3 & -2 & 0 & 1 \end{array}\right] \quad r_3' \to r_3 - 2r_1$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & \frac{2}{3} & 0 & \frac{1}{-3} \end{array}\right] \quad r_3' \to \left(-\frac{1}{3}\right) r_3$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & -1 & -\frac{4}{3} & 1 & \frac{2}{3} \\ 0 & 1 & 1 & \frac{2}{3} & 0 & \frac{1}{-3} \end{array}\right] \quad \begin{array}{l} r_1' \to r_1 - 2r_3 \\ r_2' \to r_2 - 2r_3 \end{array}$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 1 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & -1 & -\frac{4}{3} & 1 & \frac{2}{3} \\ 0 & 0 & 2 & 2 & -1 & -1 \end{array}\right] \quad r_3' \to r_3 - r_2$$

$$\sim \begin{bmatrix} 1 & 0 & 1 & : & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 1 & -1 & : & -\frac{4}{3} & 1 & \frac{2}{3} \\ 0 & 0 & 1 & : & 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad r_3' \rightarrow \left(\frac{1}{2}\right) \times r_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & : & -\frac{4}{3} & \frac{1}{2} & \frac{7}{6} \\ 1 & 0 & 0 & : & -\frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ 0 & 0 & 1 & : & 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \quad \begin{matrix} r_1' \rightarrow r_1 - r_3 \\ r_2' \rightarrow r_2 + r_3 \end{matrix}$$

$$= [I_3 A^{-1}]$$

$$\therefore A^{-1} = \begin{bmatrix} -\frac{4}{3} & \frac{1}{2} & \frac{7}{6} \\ -\frac{1}{3} & \frac{1}{2} & \frac{1}{6} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

(b) Define the rank of a matrix. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 4 & 10 & 18 \end{bmatrix}$

Sol^n: Rank of a matrix: Let A be an $m \times n$ matrix and let A_R be the row Echelon form of A. Then the rank r of the matrix A is the number of non zero rows of A_R.

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$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 4 & 10 & 18 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 8 \\ 0 & 0 & 2 \end{bmatrix} \quad r_3^1 \to r_3 - 2r_2$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \quad r_2^1 \to r_2 - 2r_1$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \quad r_3^1 \to \frac{1}{2} \times r_3$$

now this matrix in row echelon form and here this matrix has 3 non zero rows.

So, the rank of this matrix is 3

2017

1. (a) Define with example : (i) Matrix (ii) Rectangular Matrix (iii) Unitary Matrix (iv) Diagonal matrix (v) Singular matrix.

Solⁿ: (iii) — 2014 (1,a, v)
(iv) — 2016 (1, a, iv)
(v) — 2014 (1, a, i)

(i) Matrix : A Matrix is a rectangular array of numbers (real or complex) enclosed by a pair of brackets (or double vertical ~~rool~~ rolls) and the numbers in the array are called the entries or the elements of the matrix.

example:
$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

where the matrix has m rows and n column

(ii) Rectangular Matrix : When the number of rows and column of the array are not equal, then the matrix is called rectangular matrix.

example : $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

(b) Define symmetric and skew Symmetric matrix. Prove that every square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew Symmetric Matrix.

Solⁿ :

Symmetric Matrix : If a square matrix is equal to it's transpose matrix such that $a_{ij} = a_{ji}$ is said to be a Symmetric Matrix. In short way, we can be say that a square matrix A will be symmetrix if $A = A^T$

Skew Symmetric Matrix: If a square matrix is equal to the negative of it's transpose matrix such that $a_{ij} = -a_{ji}$ $[i \neq j]$ and $a_{ij} = 0\ [i = j]$ is called skew symmetric matrix. in short way, $A^T = -A$

<u>proof :</u> Let A be a square matrix and A^T be it's transpose matrix.

Then we have,

$$A = \frac{1}{2}(A + A^T) + \frac{1}{2}(A - A^T) = B + C \quad \cdots \quad (i)$$

where, $B = \frac{1}{2}(A + A^T)$, $C = \frac{1}{2}(A - A^T)$

Now, $B^T = \frac{1}{2}(A + A^T)^T$
$= \frac{1}{2}(A^T + (A^T)^T)$
$= \frac{1}{2}(A^T + A)$
$= \frac{1}{2}(A + A^T)$
$= B$

and $C^T = \frac{1}{2}(A - A^T)^T$
$= \frac{1}{2}(A^T - (A^T)^T)$
$= \frac{1}{2}(A^T - A)$
$= -\frac{1}{2}(A - A^T)$
$= -C$

Thus B is a symmetric and C is a skew symmetric matrix Thes it's clearly seen that a square matrix can be expressed as the sum of symmetric ~~ma~~ and skew-symmetric matrix.

To prove the uniqueness of the representation of (i)
Let if possible $A = P + Q \quad \cdots \quad \cdots (ii)$
where P is symmetric and Q is skew symmetric and $P^T = P$, $Q^T = -Q$
Then, $A^T = (P+Q)^T = P^T + Q^T = P - Q \quad \cdots \quad (iii)$
Now, (ii) + (iii) $\Rightarrow A + A^T = 2P \quad \therefore P = \frac{1}{2}(A + A^T)$
(ii) − (iii) $\Rightarrow A - A^T = 2Q \quad \therefore Q = \frac{1}{2}(A - A^T)$

$\therefore$ This establish the uniqueness of (i) (proved)

(2) (a) Define Inverse of a matrix. Show that $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$ is the inverse of $\begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$.

Sol^n:

Inverse of a Matrix: A square matrix 'A' is said to be invertable if there exist a unique matrix B such that AB = BA = I where I is a unit matrix. We call such a matrix B the inverse of A & it's generally denoted by A^{-1}. If B is the inverse of A then A is also inverse of B.

Let, $A = \begin{bmatrix} 3 & -2 & -1 \\ -4 & 1 & -1 \\ 2 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ -2 & -4 & -5 \end{bmatrix}$

$$[AI_3] = \left[\begin{array}{ccc:ccc} 3 & -2 & -1 & 1 & 0 & 0 \\ -4 & 1 & -1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & -2 & -2 & 1 & 0 & -1 \\ 0 & 1 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} r_1' \to r_1 - r_3 \\ r_2' \to r_2 + 2r_3 \end{array}$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 & 1 & 2 \\ 2 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad r_1' \to r_1 + 2r_2$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & -4 & -5 \end{array}\right] \quad r_3' \to r_3 - 2r_1$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 2 & 5 & 7 \\ 0 & 0 & 1 & -2 & -4 & -5 \end{array}\right]$$

$$\simeq [I_3 \; A^{-1}]$$

$\therefore A^{-1} = B$ (showed)

(b) Define Idempotent and nilpoint matrix with an example. If A and B are nonsingular matrix, then show that $(AB)^{-1} = B^{-1}A^{-1}$

Solⁿ:

Idempotent matrix: If A is a square matrix, A is called idempotent if $A \cdot A = A^2 = A$

example: $\begin{bmatrix} -1 & 3 & 5 \\ 1 & 3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$, $\begin{bmatrix} 2 & -2 & 4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

Nilpoint Matrix: A matrix 'A' is called nilpoint matrix if $A^n = 0$ and $A^{-1} \neq 0$ $(n > 0)$

example: $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ for $n = 2$

<u>proof :</u>

since, $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1} = AIA^{-1} = AA^{-1} = I$

and $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}IB = B^{-1}B = I$

Thus $B^{-1}A^{-1}$ is the inverse of AB i.e $(AB)^{-1} = B^{-1}A^{-1}$

If A and B are two matrices such that $AB = I$ then $A = B^{-1}$ and $B = A^{-1}$

Therefore $A = B^{-1} = (A^{-1})^{-1}$ $\therefore A = (A^{-1})^{-1}$

$A^T (A^{-1})^T = (A^{-1}A)^T = (I)^T = I$ shows that $(A^{-1})^T = (A^T)^{-1}$

2018

1. (a) Define with example : (i) Matrix (ii) Rectangular Matrix (iii) Transposed matrix (iv) Diagonal Matrix (v) Singular matrix.

<u>Soln :</u> (i) — 2017 (1, a, i)
(ii) — 2017 (1, a, ii)
(iii) —
(iv) — 2016 (1, a, iv)
(v) — 2014 (1, a i)

(iii) <u>Transpose matrix :</u> If A is a matrix of $m \times n$ then by writing its rows as column and column as rows and obtained $n \times m$ matrix and it's called traspose of A (A^T)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad \therefore A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

b) Define order of a minor. Find the matrix X from the eqns $AX = B$ where $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$

Soln:

Here, $AX = B$

$\Rightarrow A^{-1}AX = A^{-1}B$

$\Rightarrow IX = A^{-1}B$

$\Rightarrow X = A^{-1}B$

Now finding A^{-1}. Let the determinat of A is D

$D = 3 \neq 0$ so, A^{-1} exist.

Now finding the co-fectors of D

$A_{11} = 2$ $\quad A_{21} = 1$ $\quad A_{31} = 1$

$A_{12} = -1$ $\quad A_{22} = 1$ $\quad A_{32} = 1$

$A_{13} = -1$ $\quad A_{23} = -2$ $\quad A_{33} = 1$

$$\therefore \text{Adj } A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 1 & -2 \\ 1 & 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{D} \text{Adj} A = \frac{1}{3} \begin{bmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{3} \begin{bmatrix} 2 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 4+1+7 \\ -2+1+7 \\ -2-2+7 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 12 \\ 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

(Ans)

<u>Order of a minor</u> : Let A be an $m\times n$ matrix and k be an integer with $0<k\leq m$, and $k\leq n$, $A(k\times k)$ minor of A, also called minor determinant of order k of A.

2. (a) What is rank of a matrix?

<u>Sol^n</u>: Let A be an $m\times n$ matrix and let A_r be the echelon form of A. Then the rank r of the matrix A is the number of non zero rows of A_r.

let take an example of 2×3 matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

here the number of non zero rows $= 2$

So, rank $= 2$

(b) Reduce the following matrix to its echelon form and find its rank.

$$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ -1 & 1 & 3 & 2 \\ 2 & 4 & -4 & 6 \\ 1 & 0 & -1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 3 & 1 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 1 & -1 \end{bmatrix} \qquad \begin{array}{l} r_2' \to r_2 + r_1 \\ r_3' \to r_3 - 2r_1 \\ r_4' \to r_4 - r_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 3 & 1 & 5 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{interchange } r_3 \text{ and } r_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 5 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} c_2 \to c_2 - 2c_1 \\ c_3 \to c_3 + 2c_1 \\ c_4 \to c_4 - 2c_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{array}{l} c_2' \to c_2 - 2c_3 \\ c_4' \to c_4 - 5c_3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -4 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad c_4' \to \left(\frac{1}{-6}\right) \times c_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad c_2' \to c_2 + 4c_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad c_3' \to c_3 - c_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad c_4' \to c_4 - c_3$$

$$\sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ the rank of A is 3

(c) Find the inverse of the following matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ -2 & 1 & 3 \end{bmatrix}$$

Sol^n:

$$[A I_3] = \left[\begin{array}{ccc:ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ -2 & 1 & 3 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 1 & 9 & 2 & 0 & 1 \end{array}\right] \qquad r_3' \to r_3 + 2r_1$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 10 & 2 & -1 & 1 \end{array}\right] \quad R_3' \to R_3 - R_2$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 3 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & \frac{1}{5} & -\frac{1}{10} & \frac{1}{10} \end{array}\right] \quad R_3' \to R_3/10$$

$$\sim \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & \frac{2}{5} & \frac{3}{10} & -\frac{3}{10} \\ 0 & 1 & 0 & \frac{1}{5} & \frac{11}{10} & \frac{1}{10} \\ 0 & 0 & 1 & \frac{1}{5} & -\frac{1}{10} & \frac{1}{10} \end{array}\right] \quad \begin{array}{l} R_1' \to R_1 - 3R_3 \\ R_2' \to R_2 + R_3 \end{array}$$

$$\sim \left[I \; A^{-1} \right]$$

$$\therefore A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{3}{10} & \frac{-3}{10} \\ \frac{1}{5} & \frac{11}{10} & \frac{1}{10} \\ \frac{1}{5} & -\frac{1}{10} & \frac{1}{10} \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 3 & -3 \\ 2 & 11 & 1 \\ 2 & -1 & 1 \end{bmatrix}$$

3. (a) Define a Fourier Series and derive the Euler's Formula

Sol^n :

The trigonometric series :

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + \cdots\cdots + a_n \cos nx + \cdots\cdots$$

$$+ b_1 \sin x + b_2 \sin 2x + \cdots\cdots + b_n \sin nx + \cdots\cdots$$

$$= a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \quad \cdots \quad \cdots (i)$$

is a Fourier series if it's co-efficients a_0, a_n, b_n are given by the formulas.

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(v)\,dv$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(v)\cos nv\,dv \quad ; n = 1, 2, 3 \ldots$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(v)\sin nv\,dv \quad ; n = 1, 2, 3 \ldots$$

where, $f(x)$ is any single valued function defined on the interval $(-\pi, \pi)$.

The fourier series can also written:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where, $a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \quad ; n = 0, 1, 2 \ldots$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \quad ; n = 1, 2, 3 \ldots$

(b) State Dirichlets condition. Find the Fourier series representing $f(x) = x \quad ; 0 < x < 2\pi$

Sol^n : Suppose that

(i) $f(x)$ is defined and single valued except possibly at a finit number of points in $(-c, c)$

(ii) $f(x)$ is periodic with period $2c$

(iii) $f(x)$ and $f'(x)$ are piecewise continuous in $(-c, c)$

The fouries series :

$$a_0 + \sum_{n=1}^{\infty} \left(a_n \cos \frac{n\pi x}{c} + b_n \sin \frac{n\pi x}{c}\right)$$

with co-efficients

(a) $f(x)$ if x is a point of continuity

(b) $\frac{f(x+0) + f(x-0)}{2}$ if x is a point of discontinuity.

The condition i, ii, iii, impose on $f(x)$ are sufficient but not necessary. i.e. if the condition are satisfied ; The coverange is ~~gra~~ gurcanted, Howover if they are not satisfied the series may or not be converge.

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} x \, dx$$

$$= \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{2\pi} \cdot \frac{4\pi^2}{2}$$

$$= \pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \cos nx \, dx$$

$$= \frac{1}{\pi}\left[x \cdot \frac{\sin nx}{n} - 1 \cdot \left(-\frac{\cos nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{x \sin nx}{n} + \frac{\cos nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[0 + \frac{1}{n^2} - 0 - \frac{1}{n^2}\right]$$

$$= \frac{1}{\pi} \times 0 = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi}\int_0^{2\pi} x \sin nx \, dx$$

$$= \frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - 1 \cdot \left(-\frac{\sin nx}{n^2}\right)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{x \cos nx}{n} + \frac{\sin nx}{n^2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[-\frac{2\pi \cdot 1}{n} + 0 - 0 - 0\right]$$

$$= \frac{1}{\pi}\left(\frac{-2\pi}{n}\right)$$

$$= -\frac{2}{n}$$

$\therefore$ The required fourier series is ;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$= \pi + \sum_{n=1}^{\infty}\left(0 \times \cos nx + \left(-\frac{2}{n}\right)\sin nx\right)$$

$$= \pi - 2\sum_{n=1}^{\infty}\frac{1}{n}\sin nx$$

$$= \pi - 2\left(\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right) \text{ (Ans)}$$

4. (a) Define Fourier transformation, Fourier sine transformation and Fourier cosine transformation.

Sol^n :

<u>Fourier transformation</u> : The fourier transformation is a generalization of the complex fourier series in the limit as $L \to \infty$. Replace the discrete A_n with the continious $F(k)dk$ while letting $n/L \to k$ Then change. the sum to an integral and the eqn become

$$f(x) = \int_{-\infty}^{\infty} F(k)\, e^{2\pi ikx}\, dk$$

$$F(k) = \int_{-\infty}^{\infty} f(x)\, e^{-2\pi ikx}\, dx$$

Fourier Sine transformation and cosine transformation

the finite fourier sine transformation of $F(x)$; $0<x<l$

$$f_s(n) = \int_0^l F(x) \sin \frac{n\pi x}{l}\, dx$$

the finite fourier cosine transformation of $F(x)$; $0<x<l$

$$f_{(s)}(n) = \int_0^l F(x) \cos \frac{n\pi x}{l}\, dx$$

Infinite fourier sine transformation of $F(x)$; $0<x<\infty$

$$f_s(n) = \int_0^\infty F(x) \sin nx\, dx$$

Infinite Fourier cosine transformation of $f(x)$; $0<x<\infty$

$$f_s(n) = \int_0^\infty F(x) \cos nx\, dx$$

(b) State any two properties of Fourier transformation.

Soln :

i. can define fourier transformation and fourier cosine transformation in the following ways respectively.

1. $F_S\{f(x)\} = f_s(n) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin nx\, dx$

2. $F_c\{f(x)\} = f_c(n) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos nx\, dx$

which are called the linearity.

ii. Even and odd signals and spectra. If the signal $x(t)$ is an even/odd function of time, its spectrum $F\{f(x)\}$ is an even/odd function of frequency if $f(x) = f(-x)$ then $F\{f(x)\} = F\{-f(x)\}$ and,
if $f(x) = -f(-x)$ then $F\{f(x)\} = -F\{-f(x)\}$

c) What do you mean by convolution? state and prove the convolution theorem of Fourier transformation.

Soln: The function $H(x) = F * G = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(u)\, G(x-u)\, du$

is called the convolution or Faltung of two integrable functions F and G over the interval $(-\infty, \infty)$

The convolution Theorem for Fourier Transformation:

Statement: If $F\{f(x)\}$ and $F\{g(x)\}$ are the Fourier transforms of the functions $f(x)$ and $g(x)$ respectively then the Fourier transform of the convolution of $f(x)$ and $g(x)$ is the product of their Fourier transformations.

i.e. $F\{f(x) * g(x)\} = F\{f(x)\} \cdot F\{g(x)\}$

<u>Proof</u>:

By defination of convolution of two functions $f(x)$ and $g(x)$, we have

$$f(x) * g(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\, g(x-u)\, du$$

$$\therefore F\{f(x) * g(x)\} = F\left\{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\, g(x-u)\, du\right\}$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\; g(x-u)\, du\right] e^{ipx}\, dx$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(u)\left[\int_{-\infty}^{\infty} g(x-u)\, e^{ipx}\, dx\right] du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(u)\left[\int_{-\infty}^{\infty} g(x-u)\, e^{ip(x-u)} . e^{ipu}\, d(x-u)\right] du$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} f(u)\left[e^{ipu}\int_{-\infty}^{\infty} g(y)\, e^{ipy}\, dy\right] du$$

where $x-u=y$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\left[e^{ipu}\,\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} g(x)\, e^{ipx}\, dx\right] du$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\left[e^{ipu}\; F\{g(x)\}\right] du$$

$$= \left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(u)\, e^{ipu}\, du\right] F\{g(x)\}$$

$$= \left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\, e^{ipx}\, dx\right] F\{g(x)\}$$

$$= F\{f(x)\}\cdot F\{g(x)\}$$

Hence, $f(x) * g(x) = F\{f(x)\}\cdot F\{g(x)\}$ (proved)

5. (a) Define a unit vector. Find a unit vector parallel to the resultant of vectors $r_1 = 2\hat{i}+4\hat{j}-5\hat{k}$, $r_2 = \hat{i}+2\hat{j}+3\hat{k}$

Solⁿ:

Unit vector: A unit vector is a vector having unit magnitude then $\vec{A}$ is called unit vector. If $|\vec{A}| \neq 0$, $\frac{\vec{A}}{|\vec{A}|}$ is a unit vector having the same direction as $\vec{A}$. any vector $\vec{A}$ can be represented by a unit vector a in the direction of $\vec{A}$ multiplied by the magnitude of A.

$$\therefore \frac{\vec{A}}{|\vec{A}|} = \hat{a}$$

Here, $r_1 = 2\hat{i}+4\hat{j}-5\hat{k}$, $r_2 = \hat{i}+2\hat{j}+3\hat{k}$

Resultant, $\vec{R} = r_1 + r_2 = 2\hat{i}+4\hat{j}-5\hat{k}+\hat{i}+2\hat{j}+3\hat{k}$
$= 3\hat{i}+6\hat{j}-2K$

and $|\vec{R}| = \sqrt{3^2+6^2+(-2)^2} = 7$

A unit vector parallel to $\vec{R}$ is $\frac{\vec{R}}{|\vec{R}|} = \frac{3\hat{i}+6\hat{j}-2\hat{k}}{7} = \frac{3}{7}\hat{i}+\frac{6}{7}\hat{j}-\frac{2}{7}\hat{k}$

b) State and Prove Frenet-Serret Formula.

Sol^n: Frenet-Serret Formula's are:

(i) $\frac{d\vec{T}}{ds} = k\vec{N}$ (iii) $\frac{d\vec{N}}{ds} = \tau B - kT$

(ii) $\frac{d\vec{B}}{ds} = -\tau N$

where,

→ T is the unit vector tangent to the curve, pointing in the direction of motion.

→ N is the normal unit vector, the derivative of T with respect to the arclength parameter of the curve, devided by its length.

→ B is the Binomial unit vector, the cross product of T and N

Proof:

(i) Since $\vec{T}.\vec{T} = 1$ and $\vec{T}.\frac{d\vec{T}}{ds} = 0$ $\left[\frac{d\vec{T}}{ds} \text{ is perpendicular to } \vec{T}\right]$

If $\vec{N}$ is a unit vector in the direction $\frac{d\vec{T}}{ds}$, then $\frac{d\vec{T}}{ds} = k\vec{N}$

we call $\vec{N}$ the principal normal, k the curvature and $\rho = \frac{1}{k}$ the radious of curvature.

(ii) Let $\vec{B} = \vec{T}\times\vec{N}$ so that $\frac{d\vec{B}}{ds} = \vec{T}\times\frac{d\vec{N}}{ds} + \frac{d\vec{T}}{ds}\times N$

$$= k\vec{N}\times\vec{N}$$

$$= \vec{T}\times\frac{d\vec{N}}{ds}$$

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Then $\vec{T}\cdot\frac{d\vec{B}}{ds}=\vec{T}\cdot\vec{T}\times\frac{d\vec{N}}{ds}=0$

$\therefore \vec{T}$ is perpendicular to $\frac{d\vec{B}}{ds}$

But from $\vec{B}\cdot\vec{B}=1$ it follows that $\vec{B}\cdot\frac{d\vec{B}}{ds}=0$
so that $\frac{d\vec{B}}{ds}$ is perpendicular to $\vec{B}$ and is thus in the plane of $\vec{T}$ and $\vec{N}$

since $\frac{d\vec{B}}{ds}$ is in the plane of $\vec{T}$ and $\vec{N}$ and perpendicular to $\vec{T}$, it must be perellel to $\vec{N}$; then $\frac{d\vec{B}}{ds}=-\tau N$
we call $\vec{B}$ the binormal, τ the torsion, and $\sigma=\frac{1}{\tau}$ the radious of torsion.

(iii) Since, $\vec{T},\vec{N},\vec{B}$ form a reight handed system,
so do $\vec{N},\vec{B}$ and $\vec{T}$ $\quad[\vec{N}=\vec{B}\times\vec{T}]$

$$\therefore \frac{d\vec{N}}{ds}=\vec{B}\times\frac{dT}{ds}+\frac{d\vec{B}}{ds}\times\vec{T}$$

$$=\vec{B}\times k\vec{N}-\tau\vec{N}\times T$$

$$=-k\vec{T}+\tau B$$

$$=\tau B-kT$$

6. (a) Find the unit tangent vector to any point on the curve. $x=t^2+1$, $y=4t-3$, $z=2t^2-6t$

Solⁿ:

A tangent vector to the curve at any point =

$$\frac{d\vec{r}}{dt} = \frac{d}{dt}\left[(t^2+1)\hat{i} + (4t-3)\hat{j} + (2t^2-6t)\hat{k}\right]$$

$$= 2t\,\hat{i} + 4\hat{j} + (4t-6)\hat{k}$$

the magnitude of the vector: $\left|\frac{d\vec{r}}{dt}\right| = \sqrt{(2t)^2+4^2+(4t-6)^2}$

the required unit tangent vector, $\vec{T} = \dfrac{2t\hat{i}+4\hat{j}+(4t-6)\hat{k}}{\sqrt{(2t)^2+4^2+(4t-6)^2}}$

$$= \frac{2t\hat{i}+4\hat{j}+(4t-6)\hat{k}}{\sqrt{20t^2-48t+52}}$$

since $\left|\frac{d\vec{r}}{dt}\right| = \frac{ds}{dt}$, $\vec{T} = \dfrac{d\vec{r}/dt}{ds/dt} = \dfrac{d\vec{r}}{ds}$

(b) Find the directional derivatives of $U=2xy-z^3$ at $(2,-1,1)$ in a direction towards $(3,1,-1)$

Solⁿ:

$$\vec{\nabla}U = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)(2xy-z^2)$$

$$= 2y\,\hat{i} + 2x\,\hat{j} - 2z\,\hat{k}$$

$$= -2\hat{i}+4\hat{j}-2\hat{k} \quad \text{at } (2,-1,1)$$

let, $P=(2,-1,1)$ and $Q=(3,1,-1)$

$\therefore$ the direction from $\vec{P}$ towards $\vec{Q}$: $\vec{PQ} = (3-2, 1+2, -1-1)$
$= (1,2,-2)$

$$\therefore \vec{PQ} = \hat{i}+2\hat{j}-2\hat{k}$$

$$\text{and } |\vec{PQ}| = \sqrt{1^2+2^2+(-2)^2} = 3$$

$$\therefore \hat{a} = \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{\hat{i}+2\hat{j}-2\hat{k}}{3} = \frac{1}{3}\hat{i}+\frac{2}{3}\hat{j}-\frac{2}{3}\hat{k}$$

$\therefore$ the required ~~differential der~~ directional derivative is

$$\vec{\nabla}\vec{U}.\vec{a} = (-2\hat{i}+4\hat{j}-2\hat{k})\left(\frac{1}{3}\hat{i}+\frac{2}{3}\hat{j}-\frac{2}{3}\hat{k}\right)$$
$$= -\frac{2}{3}+\frac{8}{3}+\frac{4}{3}$$
$$= \frac{-2+8+4}{3}$$
$$= \frac{10}{3}$$

since, this is positive, U is increasing in this direction.

7. (a) Define gradient, divergence and curl. prove that

$$\vec{\nabla}\cdot(\phi\vec{A}) = \vec{\nabla}\phi\cdot\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})$$

Sol^n: Gradient: Let $\phi(x,y,z)$ be defined and differentiable at each point (x,y,z) in a certain region of space (ϕ defined a differentiable scalar field).

$\therefore$ The gradient of φ : $\vec{\nabla}\varphi = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\varphi$

$$= \frac{\delta\varphi}{\delta x}\hat{i} + \frac{\delta\varphi}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}$$

<u>Divergence</u> : let $\vec{V}(x,y,z) = V_1\hat{i} + V_2\hat{j} + V_3\hat{k}$ be defined and differentiable at each point (x,y,z) in a certain region of space ($\vec{V}$ defines a differentiable vector field)

$\therefore$ the divergence of $\vec{V}$: $\vec{\nabla}\cdot\vec{V} = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$

$$= \frac{\delta V_1}{\delta x} + \frac{\delta V_2}{\delta y} + \frac{\delta V_3}{\delta z}$$

<u>Curl</u> : If $\vec{V}(x,y,z)$ is a differentiable vector field, then the curl or rotation of $\vec{V}$, written $\vec{\nabla}\times\vec{V}$

$\therefore$ $\vec{\nabla}\times\vec{V} = \left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\times(V_1\hat{i} + V_2\hat{j} + V_3\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ V_1 & V_2 & V_3 \end{vmatrix}$$

$$= \left(\frac{\delta V_3}{\delta y} - \frac{\delta V_2}{\delta z}\right)\hat{i} + \left(\frac{\delta V_1}{\delta z} - \frac{\delta V_3}{\delta x}\right)\hat{j} +$$

$$\left(\frac{\delta V_2}{\delta x} - \frac{\delta V_1}{\delta y}\right)\hat{k}$$

$$\boxed{\vec{\nabla}\cdot(\varphi\vec{A}) = (\vec{\nabla}\varphi)\cdot\vec{A} + \varphi(\vec{\nabla}\cdot\vec{A})}$$

proof : $\vec{\nabla}\cdot(\varphi\vec{A}) = \vec{\nabla}(\varphi A_1\hat{i} + \varphi A_2\hat{j} + \varphi A_3\hat{k})$

$$= \frac{\delta}{\delta x}(\varphi A_1) + \frac{\delta}{\delta y}(\varphi A_2) + \frac{\delta}{\delta z}(\varphi A_3)$$

$$= \frac{\delta\varphi}{\delta x}A_1 + \varphi\frac{dA_1}{\delta x} + \frac{\delta\varphi}{\delta y}A_2 + \varphi\frac{\delta A_2}{\delta y} + \frac{\delta\varphi}{\delta z}A_3 + \varphi\frac{\delta A_3}{\delta z}$$

$$= \frac{\delta\varphi}{\delta x}A_1 + \frac{\delta\varphi}{\delta y}A_2 + \frac{\delta\varphi}{\delta z}A_3 + \varphi\left(\frac{\delta A_1}{\delta x} + \frac{\delta A_2}{\delta y} + \frac{\delta A_3}{\delta z}\right)$$

$$= \left(\frac{\delta\varphi}{\delta x}\hat{i} + \frac{\delta\varphi}{\delta y}\hat{j} + \frac{\delta\varphi}{\delta z}\hat{k}\right)\cdot\left(A_1\hat{i} + A_2\hat{j} + A_3\hat{k}\right) +$$

$$\varphi\left(\frac{\delta}{\delta x}\hat{i} + \frac{\delta}{\delta y}\hat{j} + \frac{\delta}{\delta z}\hat{k}\right)\left(A_1\hat{i} + A_2\hat{j} + A_3\hat{k}\right)$$

$$= (\vec{\nabla}\varphi)\cdot\vec{A} + \varphi(\vec{\nabla}\cdot\vec{A}) \quad \text{(proved)}$$

(b) Define irrotational vector. Prove that, irrotational vector is conservative.

Sol^n : Irrotational Vector : An irrotational vector field is a vector field where curl is equal to zero everywhere. If the domain is simply connected the vector field will be conservative or equal to the gradient of a function.

proof : Let $\vec{F}$ is a conservative field

Then we have to prove Curl $\vec{F} = \vec{\nabla}\times\vec{F} = 0$

$$\vec{F} = \vec{\nabla}\phi \quad ; \quad F \text{ is irrotational}$$

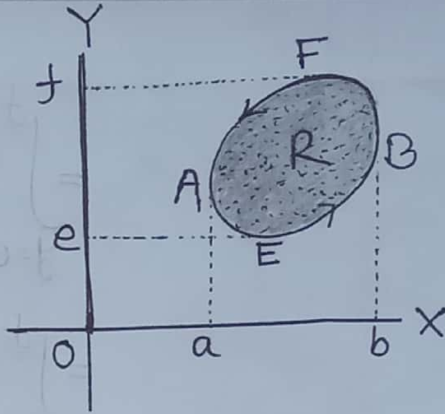
$$L.H.S = \vec{\nabla} \times \vec{F}$$
$$= \vec{\nabla} \times \vec{\nabla}\phi$$
$$= \vec{\nabla} \times \left(\frac{\delta\phi}{\delta x}\hat{i} + \frac{\delta\phi}{\delta y}\hat{j} + \frac{\delta\phi}{\delta z}\hat{k}\right)$$
$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ \frac{\delta\phi}{\delta x} & \frac{\delta\phi}{\delta y} & \frac{\delta\phi}{\delta z} \end{vmatrix}$$
$$= \left[\frac{\delta}{\delta y}\left(\frac{\delta\phi}{\delta z}\right) - \frac{\delta}{\delta z}\left(\frac{\delta\phi}{\delta y}\right)\right]\hat{i} + \left[\frac{\delta}{\delta z}\left(\frac{\delta\phi}{\delta x}\right) - \frac{\delta}{\delta x}\left(\frac{\delta\phi}{\delta z}\right)\right]\hat{j} +$$
$$\left[\frac{\delta}{\delta x}\left(\frac{\delta\phi}{\delta y}\right) - \frac{\delta}{\delta y}\left(\frac{\delta\phi}{\delta x}\right)\right]\hat{k}$$
$$= \left(\frac{\delta^2\phi}{\delta y\,\delta z} - \frac{\delta^2\phi}{\delta z\,\delta y}\right)\hat{i} + \left(\frac{\delta^2\phi}{\delta z\,\delta x} - \frac{\delta^2\phi}{\delta x\,\delta z}\right)\hat{j} + \left(\frac{\delta^2\phi}{\delta x\cdot\delta y} - \frac{\delta^2\phi}{\delta y\,\delta x}\right)\hat{k}.$$
$$= 0$$

since curl $\vec{F} = 0$, so the irrotational vector is conservative

8. (a) State and prove Green's Theorem in the plane.

Sol^n: Green's Theorem in a plane: If R is a closed region of xy plane bounded by a simple closed curve C and if M and N are continuous function of x and y having continuous derivative in R, then

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) dx\,dy$$



Proof :

Let the eqn of the curve AEB and AFB be $y = Y_1(x)$ and $y = Y_2(x)$ respectively.

$$\therefore \iint_R \frac{\delta M}{\delta y}\, dx\, dy = \int_{x=a}^{b} \int_{y_1 = Y_1(x)}^{Y_2(x)} \frac{\delta M}{\delta y}\, dy\, dx$$

$$= \int_{x=a}^{b} M(x, y) \Big|_{y=Y_1(x)}^{Y_2(x)} dx$$

$$= \int_{x=a}^{b} \left[M(x, Y_2) - M(x, Y_1) \right] dx$$

$$= -\int_{a}^{b} M(x, Y_1)\, dx - \int_{b}^{a} M(x, Y_2)\, dx$$

$$= -\oint_C M\, dx$$

$$\therefore \oint_C M\, dx = -\iint \frac{\delta M}{\delta y}\, dx\, dy \quad \cdots \quad \cdots (i)$$

Similarly, let the eqn of curve EAF and EBF be $x = X_1(y)$ and $x = X_2(y)$ respectively then

$$\iint_R \frac{\delta N}{\delta x}\, dx\, dy = \int_{y=e}^{f} \int_{x=X_1(y)}^{X_2(y)} \frac{\delta N}{\delta x}\, dx\, dy$$

$$= \int_{y=e}^{f} N(x, y) \Big|_{x=X_1(y)}^{X_2(y)} dy$$

$$= \int_{y=e}^{f} [N(x_2, y) - N(x_1, y)]\, dy$$

$$= \int_{f}^{e} N(x_1, y)\, dy + \int_{y=e}^{f} N(x_2, y)\, dy$$

$$= \oint_C N\, dy$$

$$\therefore \oint_C N\, dy = \iint_R \frac{\delta N}{\delta x}\, dx\, dy \quad \ldots \quad \ldots \quad \ldots (ii)$$

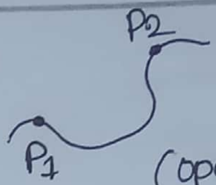
Adding (i) and (ii)

$$\oint_C M dx + N dy = \iint_R \left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) dx\, dy \quad \text{(proved)}$$

(b) Define line Integral. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\hat{i} - 5z\hat{j} + 10x\hat{k}$ along the curve $x = t^2 + 1$; $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$

Sol^n:

<u>Line Integral</u>: Let $\vec{r}$ be a position vector where $\vec{r} = x(u)\hat{i} + y(u)\hat{j} + z(u)\hat{k}$ and $\vec{r}(u)$ is the position vector of (x, y, z) defined a curve C joining points P_1 and P_2.



(open curve)

$$\therefore \int_{P_1}^{P_2} \vec{A}\cdot d\vec{r} = \int_C \vec{A}\cdot d\vec{r}$$

$$= \int_C A_1\,dx + A_2\,dy + A_3\,dz$$

(closed curve)

and if C is a closed curve,

$$\oint \vec{A}\cdot d\vec{r} = \oint A_1\,dx + A_2\,dy + A_3\,dz$$

Total work: $\int_C \vec{F}\cdot d\vec{r} = \int_C (3xy\,\hat{i} - 5z\,\hat{j} + 10x\,\hat{k})\cdot(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$$= \int_C 3xy\,dx - 5z\,dy + 10x\,dz$$

$$= \int_{t=1}^{2} 3(t^2+1)\,2t^2\cdot 2t\,dt - 5t^3\cdot 4t\,dt + 10(t^2+1)\,3t^2\,dt$$

$$= \int_{t=1}^{2} 12t^3(t^2+1)\,dt - 20t^4\,dt + 30t^2(t^2+1)\,dt$$

$$= \int_1^2 (12t^5 + 10t^4 + 12t^3 + 30t^2)\,dt$$

$$= 303$$

(Ans)

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