Complex number

$\boxed{x+iy}$

$x\, y \in R$

$i = \sqrt{-1}$

$\{x+iy,\ x.y \in R\ i$

real and ing part of a complex num

$z = x+iy$

real part   img part

~~absolute~~

absolute value or modulus of a number:

$z = x+iy$

$|z| = \sqrt{x^2+y^2}$ → modulus term

if $z = 2+3i$; $|z| = \sqrt{2^2+3^2} \to \sqrt{13}$

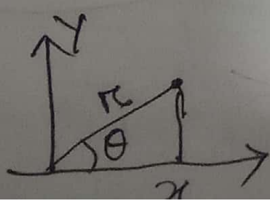
if $z = 2-3i$; $|z| = \sqrt{2^2+(-3)^2} = \sqrt{13}$

Arguments: of z

$z = x+iy$

$\theta = \tan^{-1}\frac{y}{x}$

polar form of complex number: (r, θ):



$x = r\cos\theta$

$y = r\sin\theta$

$$\text{mod } z = |z| = r = \sqrt{x^2+y^2}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{r\sin\theta}{r\cos\theta} = \frac{y}{x}$$

$$\therefore \theta = \tan^{-1}\frac{y}{x}$$

Addition:

$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

$$z_1 + z_2 = x_1 + iy_1 + x_2 + iy_2$$

$$= (x_1+x_2) + i(y_1+y_2)$$

Application

Show that $\boxed{\overline{z_1+z_2} = \overline{z_1} + \overline{z_2}}$ — এই type Qsn আসতে পারে।

Let, $z_1 = x_1 + iy_1$

$z_2 = x_2 + iy_2$

$$\overline{z_1+z_2} = \overline{x_1+iy_1+x_2+iy_2}$$

$$= \overline{(x_1+x_2)+i(y_1+y_2)}$$

$$= (x_1+x_2) - i(y_1+y_2)$$

$$= (x_1-iy_1) + (x_2-iy_2)$$

$$= \overline{z_1} + \overline{z_2}$$

Q. modulus & Arg নির্ণয় আসতে পারে।

$\frac{(1+2i)^2}{(2+i)^2}$ in the form $A+iB$. Also find the modulus & Argument.

$$\frac{(1+2i)^2}{(2+i)^2} = \frac{1+4i-4}{4+2i-1} = \frac{4i-3}{3+4i} = \frac{(4i-3)(3-4i)}{(3+4i)(3-4i)}$$

$$= \frac{-(4i-3)^2}{9-16i^2}$$

$$= \frac{-(4i-3)^2}{9+16} = \frac{-(16i^2-24i+9)}{25} = \frac{16+24i-9}{25} = \frac{7+24}{25}$$

$$= \frac{7}{25} + \frac{24}{25}i$$

$$\text{mod} = \sqrt{\left(\frac{7}{25}\right)^2 + \left(\frac{24}{25}\right)^2} = 1$$

$$\text{Arg} = \tan^{-1}\frac{24/25}{7/25} = \tan^{-1}\frac{24}{7}$$

<u>principal argu</u>

$\tan^{-1}\frac{y}{x}$

<u>general argument</u>:

$2n\pi + PA$

↳ principal argument

$= 2n\pi + \tan^{-1}\frac{y}{x}$

Find modulas and arguments

(i) $\frac{1-i}{1+i} \rightarrow$

(ii) $\left(\frac{1+i}{1-i}\right)^2$

Q. show that $|z-i| = |z+1|$ represent the straight line. which passing the origin with slope -1

let, $z = x+iy$

$|z-i| = |z+1|$

$\Rightarrow |x+iy-i| = |x+iy+1|$

$\Rightarrow |x+i(y-1)| = |x+1+iy|$

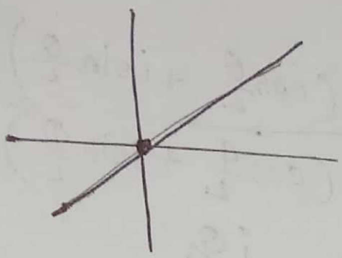
$\Rightarrow \sqrt{x^2+(y-1)^2} = \sqrt{(x+1)^2+y^2}$

$\Rightarrow x^2+y^2-2y+1 = x^2+2x+1+y^2$

$\Rightarrow 2y = -2x$

$\Rightarrow y = -x$

st. — $y = mx+c$
circle
elipse
parabola



here, slope $= -1$

$y = mx + c$

$y = -1(x) + 0$ so, its passing through the origin and its slope is -1 (showed)

Q. show that

(b) $|z+3i| = 3$; it represent a circle:

$\Rightarrow |x + i(y+3)| = 3$

$\Rightarrow \sqrt{x^2 + (y+3)^2} = 3$

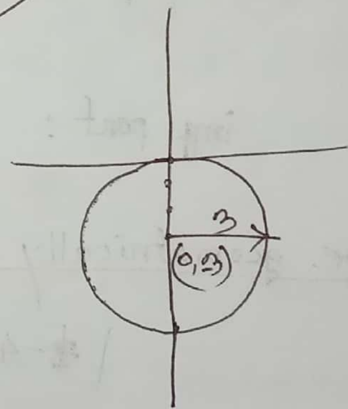
$\Rightarrow x^2 + (y+3)^2 = 3^2$

eqn of circle: $x^2 + y^2 = r^2$

center $(0, -3)$

radius $= 3$

(proved)



self practice: (i) $|z+i| = 2$, $r = 2$ $(0, -1)$

→ show represent circle.

(ii) $|z-3| - |z+3| = 4$ represent hyperbola.

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

(iii) $|z - 4i| + |z+4i| = 10$ represent elipse

(iv) Find the real and img part of $\frac{1+\cos\theta + i\sin\theta}{1+\cos\phi + i\sin\phi}$

$= \frac{2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\phi}{2} + 2i\sin\frac{\phi}{2}\cos\frac{\phi}{2}}$

by de moivres theorem

$$\boxed{\cos\theta + i\sin\theta = e^{i\theta}}$$

$$= \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\cos\frac{\phi}{2}\left(\cos\frac{\phi}{2} + i\sin\frac{\phi}{2}\right)}$$

$$= \frac{\cos\frac{\theta}{2}\, e^{i\theta/2}}{\cos\frac{\phi}{2}\, e^{i\frac{\phi}{2}}}$$

$$= \frac{\cos\theta/2}{\cos\frac{\phi}{2}}\, e^{i\left(\frac{\theta-\phi}{2}\right)}$$

$$= \frac{\cos\frac{\theta}{2}}{\cos\frac{\phi}{2}}\left(\cos\frac{\theta-\phi}{2} + i\sin\frac{\theta-\phi}{2}\right)$$

real part: $\frac{\cos\frac{\theta}{2}}{\cos\frac{\phi}{2}} \cdot \cos\frac{\theta-\phi}{2}$

img part: $\frac{\cos\frac{\theta}{2}}{\cos\frac{\phi}{2}} \cdot \sin\frac{\theta-\phi}{2}$

Q. describe geomatrically the region of the $|z-4| > |z|$

$$|z-4| > |z|$$

$$\Rightarrow |x+iy-4| > |x+iy|$$

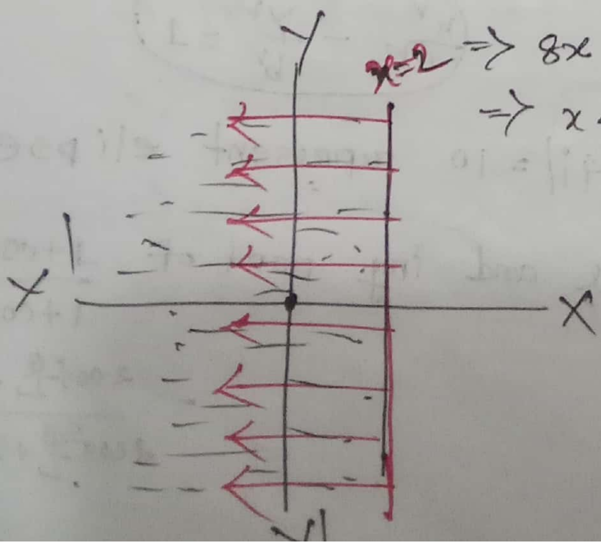
$$\Rightarrow \sqrt{(x-4)^2+y^2} > \sqrt{x^2+y^2}$$

$$\Rightarrow x^2-8x+16+y^2 > x^2+y^2$$

$$\Rightarrow -8x+16 > 0$$

$$\Rightarrow 8x < 16$$

$$\Rightarrow x < 2$$



Describe Geometrically :

Ⓠ $Re\left(\frac{1}{z}\right) < 1$

| $Re\left(\frac{1}{z}\right) \rightarrow$ real number of $\frac{1}{z}$

$\Rightarrow Re\left(\frac{1}{x+iy}\right) < 1$

$\Rightarrow Re \frac{x-iy}{(x+iy)(x-iy)} < 1$

$\Rightarrow Re \frac{x-iy}{x^2+y^2} < 1$

$\Rightarrow \frac{x}{x^2+y^2} < 1$

$\Rightarrow x < x^2+y^2$

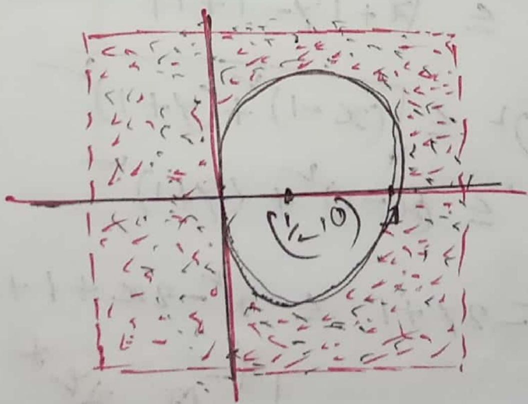
$\Rightarrow x^2+y^2-x > 0$

$\Rightarrow x^2 - 2x.\frac{1}{2} + \left(\frac{1}{2}\right)^2 + y^2 > \left(\frac{1}{2}\right)^2$

$\Rightarrow \left(x-\frac{1}{2}\right)^2 + y^2 > \left(\frac{1}{2}\right)^2$

circle : center : $\frac{1}{2}, 0$

radius : $\frac{1}{2}$



radius এর বাহিরে

11-04-2022

Q. Show that $|z-4i|+|z+4i|=10$ represent ellipse

$z=x+iy$

$|x+iy-4i|+|x+iy+4i|=10$

$\Rightarrow \sqrt{x^2+(y-4)^2}+\sqrt{x^2+(y+4)^2}=10$

$\Rightarrow \sqrt{x^2+(y-4)^2}=\left(10-\sqrt{x^2+(y-4)^2}\right)^2$

— — —

— —

$\Rightarrow 4y-25=-5\sqrt{x^2+y^2-8y+16}$

$\Rightarrow (4y-25)^2=25(x^2+y^2-8y+16)$

$\Rightarrow 25x^2+9y^2=225$

$\Rightarrow \frac{x^2}{9}+\frac{y^2}{25}=1$

Q. $|z+1-i|\le|z-1+i|$

$|x+iy+1-i|\le|x+iy-1+i|$

$\Rightarrow \sqrt{(x+1)+(y-1)^2}\le\sqrt{(x-1)^2+(y+1)^2}$

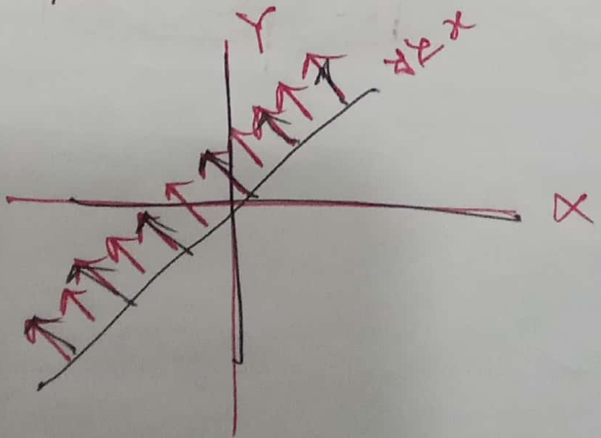
$\Rightarrow (x+1)+(y-1)^2\le(x-1)^2+(y+1)^2$

$\Rightarrow x^2+2x+1+y^2-2y+1\le x^2-2x+1+y^2+2y+1$

$\Rightarrow 4x\le 4y$

$\Rightarrow x\le y$

$\Rightarrow y\ge x$



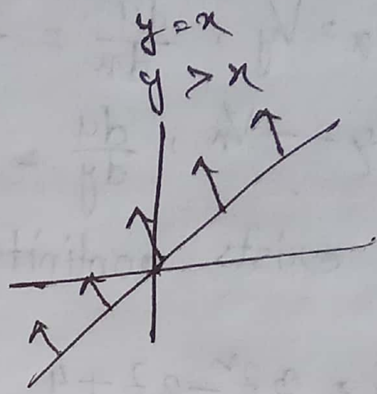
$|z+1-i| \le |z-1+i|$ find region.

$\Rightarrow |x+iy+1-i| \le |x+iy-1+i|$

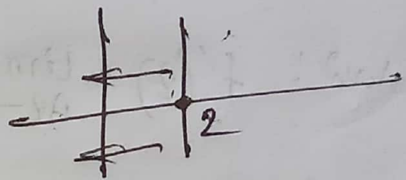
$\Rightarrow \sqrt{(x+1)^2+(y-1)^2} \le \sqrt{(x-1)^2+(y+1)^2}$

$\Rightarrow 4x \le 4y$

$\Rightarrow y \ge x$



$|x-4| > |z|$ it's region $x = 2$



Limit, continuity
Analytic function:

যেকোন point এর function যে analytic হবে তাকে তা analytic function.

continues curve
limiting "

— $f(z)$ defined

— $\lim_{z \to a} f(z) = f(z)$

Harmonic function: Laplace equation কে satisfy করলে তা harmonic function

laplace function/ equation: $\nabla + \frac{d}{dx^2} + \frac{d^2}{dy^2} = 0$

$u = x^2 + y$, $\frac{du}{dy}$

$\frac{du}{dx}$ $\qquad \frac{d^2u}{dx^2} + \frac{du}{dy^2} = 0$

~~cho~~ cauchy-Riemann equation: $f(z) = u(x,y) + iv(x,y)$

$z+1 \to$ analytic function কারণ সব point এই analytic তবে infinity নয়।

$f(z) = u(x,y) + iv(x,y)$ is analytic in a region R,

$u_x = V_y$, $\frac{du}{dx} = \frac{dv}{dy}$

$u_y = -V_x$, $\frac{du}{dy} = -\frac{dv}{dy}$

exists continity.

* $f(z) = 3z^2 - 2z + 4$ find the derivative of the function.

derivative law: $f'(z) = \lim_{\Delta z \to 0} \frac{f(z+\Delta z) - f(z)}{\Delta z}$

$f(z+\Delta z) - f(z)$

$= 6z\Delta z + 3(\Delta z)^2 - 2\Delta z$

$\therefore f'(z) = \lim_{\Delta z \to 0} \frac{6z\Delta z + 3(\Delta z)^2 - 2\Delta z}{\Delta z}$

$\Rightarrow \lim_{\Delta z \to 0} 6z + 3\Delta z - 2$

$\Rightarrow 6z - 2$

H.W. $f(z) = \frac{2z-1}{z+2i}$ where, $z = -1$ Ans: $\frac{5i}{(z+2i)(-z+2i)}$

$\therefore f'(z) = \lim_{\Delta z \to 0}$

17-05-22

Complex differentiation

Cauchy-Riemann equation

Analytic function → প্রত্যেক point এ derivative ~~exitts~~ exists করে সেটা যেকোন point এর z দিয়ে

C.R. equ^n → $f(z) = u + iv$

$$\frac{du}{dx} = \frac{dv}{dy}$$

$$\frac{du}{dy} = -\frac{dv}{dx}$$

Harmonic function :- $f(z) = u + iv$

$$\frac{d^2u}{dx^2} + \frac{d^2v}{dy^2} = 0$$

laplace equ^n → $\nabla u = 0$

$$\Rightarrow \frac{du}{dx} + \frac{du}{dy} + \frac{du}{dz} = 0$$

$$\Rightarrow \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} + \frac{d^2u}{dz^2} = 0$$

show that, $u = 2x(1-y)$ is harmonic.

$$\frac{du}{dx} = 2(1-y) \quad \Big| \quad \frac{du}{dy} = 2x$$

$$\Rightarrow \frac{d^2u}{dx^2} = 0 \quad \Big| \quad \Rightarrow \frac{d^2u}{dy^2} = 0$$

$$\therefore \frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$$ [showed]

agin find a function v such $f(z) = u + iv$ is analytic.

method-1 :

using chain Rule-

$$dv = \frac{dv}{dx} dx + \frac{dv}{dy} \cdot dy + \cancel{\frac{dv}{dz} \cdot dz} + c \quad \text{--- (1)}$$

From cauchy Riemann equation

we know, $\frac{du}{dx} = \frac{dv}{dy}$ and $\frac{du}{dy} = -\frac{dv}{dx}$

From (1):

$$dv = -2x\,dx + 2(1-y)\cdot dy + c$$

$$\Rightarrow \int dv = -\int 2x\cdot dx + 2\int 1-y\cdot dy + \int c$$

$$\Rightarrow V = -2\frac{x^2}{2} + 2\cdot\left(y - \frac{y^2}{2}\right) + c$$

$$\Rightarrow V = -x^2 + 2y - y^2 + c \quad \text{(Ans)}$$

$u = e^x(x\cos y - y\sin y)$, show that this is harmonic

$$\frac{du}{dx} = xe^x\cos y + e^x\cos y - ye^x\sin y$$

$$\frac{d^2u}{dx^2} = xe^x\cos y + e^x\cos y + e^x\cos y - ye^x\sin y$$

$$= xe^x\cos y + 2e^x\cos y - ye^x\sin y$$

$$\frac{du}{dy} = -xe^x\sin y \cancel{-e^x y\cos y} \cancel{-e^x\sin y}$$

$$\frac{d^2u}{dy^2} = -xe^x\cos y + e^x y\sin y - e^x\cos y - e^x\cos y$$

$$= -xe^x\cos y - 2e^x\cos y + e^x y\sin y$$

$$\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = \cancel{xe^x\cos y} + \cancel{2e^x\cos y} - \cancel{ye^x\sin y} - \cancel{xe^x\cos y} - \cancel{2e^x\cos y} + \cancel{e^x y\sin y}$$

$= 0$, so this equn is a harmonic function.

Find v.

$dv = \frac{dv}{dx} \cdot dx + \frac{dv}{dy} \cdot dy + c$

$\int dv = \int xe^x \cos y \cdot dx + \int (xe^x \sin y + e^x y \cos y + e^x \sin y) dy + c$

$\frac{du}{dx} = \frac{dv}{dy}$

$\frac{du}{dy} = -\frac{dv}{dx}$

$\int xe^x$
$= xe^x - \cancel{\int e^x} e^x$
$= e^x(x-1)$

$\int y \cos y$
$= y \sin y + \cancel{\sin y}$

$\Rightarrow V = \cos y \, e^x(x-1) + xe^x \cos y + e^x y \sin y + e^x \cos y - e^x \cos y + c$ (Ans)

and Find corresponding analytic function $f(z) = u + iv$ from it find, v.

so, solve, ~~milnes method~~ using by milnes method.

note: $u(x,y) = u(z,0)$

We get, $\frac{du}{dx} = xe^x \cos y - e^x y \sin y + e^x \cos y$

$u_1(z,0) = ze^z \cos 0 + e^z \cdot 0 + e^z \cos 0$

$= ze^z + e^z$

$\frac{du}{dy} = -xe^x \sin y - e^x y \cos y - e^x \sin y$

$u_2(z,0) = -ze^z \cdot 0 - e^z \cdot 0 - e^z \cdot 0$

$= 0$

$f(z) = \int \{u_1(z,0) - i u_2(z,0)\} \cdot dz$ [milnes formula]

$= \int (ze^z + e^z) dz$

$= ze^z - \cancel{e^z} + \cancel{e^z} + c$

$\boxed{f(z) = ze^z}$ this is analytic function.

$\Rightarrow u + iv = (x+iy) e^{x+iy}$

$= (x+iy) \cdot e^x e^{iy}$

$= (x+iy) e^x (\cos y + i \sin y)$

$= (xe^x \cos y - e^x y \cos y) + i(xe^x \sin y + e^x y \cos y)$

$f(z) = u + iv$

$z = x + iy$

$e^{iy} = \cos y + i \sin y$

$\therefore V(x,y) = xe^x\sin y + e^x y\cos y$ (Ans)

Note: $u(x,y)$ function এর harmonic conjugate $v(x,y)$ function.

construct a analytic function $f(z)$ whose real part $e^x\cos y$. $f(z) = u+iv$.

real part, $u = e^x\cos y$, find v

$$\frac{du}{dx} = e^x\cos y, \quad \frac{du}{dx} = -e^x\sin y$$

From cauchy equ^n

$$\frac{du}{dx} = \frac{dv}{dy}$$

$$\frac{du}{dy} = -\frac{dv}{dx}$$

$$dv = \frac{dv}{dx}\cdot dx + \frac{dv}{dy}\cdot dy + c$$

$$\int dv = \int e^x\sin y \cdot dx + \int e^x\cos y \cdot dy$$

$$\Rightarrow v = e^x\sin y + \cancel{e^x\sin y}$$

$$= \cancel{2}e^x\sin y$$

$$\cancel{\therefore f(z) = e^x\cos y + i\{e^x(\sin y + \cos y)\}}$$ (Ans)

equation exact এর কারণ 2টা same function আসলো, 1টা function নিলেই হবে তাই $e^x\sin y$.

$\therefore u+iv = e^x\cos y + ie^x\sin y$ (Ans)

এর জন্য milnes method use করে অংক করা সহজ হয়।

or

Using milnes method:

$u_1(z,0) = e^z$

$u_2(z,0) = -e^z \sin 0 = 0$

$f(z) = \int \{u_1(z,0) - i(u_2(z,0))\} \cdot dz$

$\Rightarrow f(z) = \int e^z \cdot dz = e^z + c$ (Ans)

$\therefore u + iv = e^{x+iy}$

$= e^x \cdot e^{iy}$

$= e^x(\cos y + i\sin y) = e^x\cos y + ie^x \sin y$

$v(x,y) = e^x \sin y$

$f(z) = x^2 + axy + by^2 + i(cx^2 + dxy + y^2)$ find the values of the constants, a, b, c, d = ?

$u = x^2 + axy + by^2$

$v = cx^2 + dxy + y^2$

$\frac{du}{dx} = \frac{dv}{dy}$

$\Rightarrow 2x + ay = dx + 2y$

$\Rightarrow d = 2, a = 2$

$\frac{du}{dy} = -\frac{dv}{dx}$

$\Rightarrow ax + 2by = -2cx + dy$

$\Rightarrow a = -2c, 2b = -d$

$\Rightarrow c = -1, b = -1$

$\therefore a = 2, b = -1, c = -1, d = 2$ (Ans)

Singularities

যে point এ function analytic হবেনা

analytic হতে যে point এ fail করবে সেটি singular point.

$$f(z) = \frac{1}{(z-1)^2} \to \infty$$

$z-1$

classificatio of singularities

pole =

zeros =

finds pole: $\lim_{z \to z_0} (z-z_0)^m f(z) = A \neq 0$ (order)

$$f(z) = \frac{1}{(z-1)^2}$$

$z-1=0$

$z=1$

$\boxed{\therefore z_0 = 1}$

$\therefore$ pole $= \lim_{z \to 1} (z-1)^2 \frac{1}{(z-1)^2} = 1 \neq 0$

$\therefore z=1$ is a pole of order 2

Residues: → denoted by $\boxed{a_{-1}}$

Residues theorem: $f(z)$ analytic function of C (c is circle) radius and center

এই theorem use করে residues বের করব

$z=a$

$$f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n + \sum_{n=1}^{\infty} \frac{a_{-1}}{(z-a)^n}$$

$z=a$ is called residue of $f(z)$ at $z=a$

$\boxed{z = 2, 1, \pm i \text{ can be}}$

$a_{-1} = \lim_{z \to a} (z-a) f(z)$ → simple pole

$a_{-1} = \lim_{z \to a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} \{(z-a)^m f(z)\}$ double pole

$\boxed{(z-1)^2 = 0, \; z = 1, 1}$

Q. Find Residues : $f(z) = \frac{z^2}{z^2+a^2}$

pole, $z^2+a^2=0$

$\Rightarrow z^2 = -a^2$

$\Rightarrow z^2 = a^2 i^2$

$\therefore z = \pm ai = ai, -ai$

at, $z=ai$, Residue $= \lim_{z\to ai} (z-ai)\frac{z^2}{z^2+a^2}$

$= \lim_{z\to ai} (z-ai)\frac{z^2}{(z+ai)(z-ai)}$

$= \lim_{z\to ai} \frac{z^2}{z+ai}$

$= \frac{a^2 i^2}{2ai}$

$= \frac{a i^3}{2 i^2}$

$= -\frac{ai}{2}$

Q. $f(z) \frac{z^2-2z}{(z+1)^2(z+4)}$

$(z+1)^2(z+4)=0 \quad \therefore z = -1, -1, -4$

for single pole:

$\lim_{z\to -4} (z+4)\frac{(z^2-2z)}{(z+1)^2(z+4)}$

$= \lim_{z\to -4} \frac{z^2-2z}{(z+1)^2}$

$= \frac{16+8}{9}$

$= \frac{24}{9}$

for double pole: $z=-1$

double order (2) $(m=2)$

Residue at $z=-1$, we get

$= \lim_{z\to -1} \frac{1}{(2-1)!} \frac{d^{2-1}}{dz^{2-1}}\left\{(z+1)^2 f(z)\right\}$

$= \lim_{z\to -1} \frac{d}{dz}(z+1)^2 \frac{z^2-2z}{(z+1)^2(z+4)}$

$= \lim_{z\to -1} \frac{d}{dz} \frac{z^2-2z}{z+4}$

$= \lim_{z\to -1} \frac{(z+4)(2z-2)-(z^2-2z)\cdot 1}{(z+4)^2}$

$= \frac{-12-3}{9} = -\frac{15}{9} = -\frac{5}{3}$

evaluate:

Q. $\oint \frac{e^{3z}}{z+\pi i}$; c is the circle $|z+1|=4$

cauchy integral are . Residue কিছুই বলা হয়নি।

Residue:

$$\frac{1}{2\pi i}\oint \frac{f(z)}{(z-a)} = \text{sum of the Residue}$$
$$= a_{-1} + b_{-1} + c_{-1}$$

$z+\pi i = 0$

$z = -\pi i$ pole:

$|z| = |-\pi i| = \pi = 3.1416 < 4$

yes its lie inside the circle . So its satisfy caucy

$\therefore$ at $z=-\pi i$ Residue $= \lim_{z\to -\pi i} (z+\pi i)\frac{e^{3z}}{z+\pi i}$

$= e^{-i3\pi}$

$= \cos 3\pi - i\sin 3\pi$

$= -1$

From cauchy residue: $\frac{1}{2\pi i}\oint \frac{f(z)}{z-a} = -1$

$= \oint \frac{f(z)}{z-a} = -2\pi i$

H.W: $\oint \frac{e^{3z}}{z-\pi i}$ (i) $|z-1| = 4$ (ii) $|z-2|+|z+2|=6$

(iv) Show that: $\frac{1}{2\pi i}\oint \frac{f'(z)}{f(z)} = -2$ where $f(z) = \frac{(z^2+1)^2}{(z^2+2z+2)^3}$

$|z| = 4$ $\boxed{\text{circle radius} = 4}$

$\boxed{\text{Argument theorem: (দেখতে হবে) proof}}$

$$\frac{1}{2\pi i}\oint_c \frac{f'(z)}{f(z)}\,dz = N - P \to \text{no. of poles}$$

N → no. of zeros

$(z^2+4)^2 = 0$

$z^2 = i, i, -i, -i$ zeros $= 2$ $(+i, -i)$

$z = i$ $|z| = 1 < 4$

$z = -i$ $|z| = 1 < 4$ (lies inside)

$(z^2+2z+2)^3=0$

$$z=\frac{-2\pm\sqrt{4-8}}{2}$$

$= -1\pm i$

$z=-1+i \quad |z|=\sqrt{2}<4$

$z=-i-i, \quad |z|=\sqrt{2}<4$

General Argument : $\frac{1}{2\pi i}\oint_C \frac{f'(z)}{f(z)}\,dz = N-P$

$= (2+2)-(3+3)$

$= 4-6$

$= -2$

$$\oint \frac{f'(z)}{f(z)}\,dz = -4\pi i$$

$\boxed{\text{H.W: problem 20, 21 (page 258)}}$