

**Bangabandhu Sheikh Mujibur Rahman Science & Technology University**  
**Department of Computer Science and Engineering**  
2<sup>nd</sup> Year 1<sup>st</sup> Semester B.Sc. (Eng.) Examination 2014  
Course No.: MAT 204    Course Title: Matrices and Differential Equations  
Full Marks: 70    Time: 04 Hours

**N. B.**

- i) Answer **SIX** questions, taking any **THREE** from each section.
- ii) All questions are of equal values.
- iii) Use separate answer script for each section

**Section-A**

1.    a. Define with example: (i) Singular matrix, (ii) Symmetric matrix, (iii) Hermitian matrix, (iv) Trace of a matrix, (v) Unitary matrix. 5  
      b. Solve the following equations with the help of inverse matrix 6<sup>2/3</sup>  
$$\begin{aligned} 3x + 5y - 7z &= 13 \\ 4x + y - 12z &= 6 \\ 2x + 9y - 3z &= 20 \end{aligned}$$
  
2.    a. Find the inverse matrix of the following matrix by row canonical method: 5<sup>2/3</sup>  
$$A = \begin{bmatrix} 2 & 1 & 5 \\ -1 & -2 & -2 \\ 3 & 1 & 2 \end{bmatrix}$$
  
      b. Define rank of a matrix. Reduce the following matrix to the normal (or canonical) form and hence obtain its rank. 6  
$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$$
  
3.    a. Define eigen values and eigen vectors. 4  
      b. Find the eigen values and eigen vectors of the following matrix: 7<sup>2/3</sup>  
$$\begin{bmatrix} 2 & 4 \\ 3 & 13 \end{bmatrix}$$
  
          and also find the matrix P that diagonalize it.
  
4.    a. State Cayley-Hamilton theorem. Find the characteristic equation of the matrix 6  
$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$
  
          and verify Cayley-Hamilton theorem for it.  
      b. Define similar matrices. Prove that similar matrices have the same characteristic polynomial and the same eigenvalue. 5<sup>2/3</sup>

**Section-B:**

5. a. Define linear differential equation. Find the DE of the family of curves  $y=c(x-c)^2$  when  $c$  is arbitrary constants.  $3^{2/3}$
- b. Solve the initial value problem:  $x \sin y dx + (x^2 + 1) \cos y dy = 0$ ,  $y(1) = \frac{\pi}{2}$ . 4
- c. Prove that if  $M(x, y)dx + N(x, y)dy = 0$  is a homogeneous differential equation, then the change of variable  $y = vx$  transform the homogeneous equation into a separable equation in the variable  $v$  and  $x$ . 4
6. a. Define integrating factor with examples. Solve the differential equation 4  
$$(x^3 + xy^4)dx + 2y^3dy = 0$$
- b. Solve the differential equation  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 4 \sin \ln x$  4
- c. Define Bernoulli's differential equation. Solve the DE:  $(1 - x^2) \frac{dy}{dx} + xy = xy^2$ .  $3^{2/3}$
7. a. Find the solution of the following differential equation:  $3+2^{2/3}$   
i.  $(D^3 - 7D - 6)y = (x + 1)e^x$   
ii.  $(D^2 - DD' - 2D'^2)z = (y - 1)e^x$ .
- b. Find the solution of the following partial differential equation: 3+3  
i.  $(D^3 - 4D^2D' + 4DD'^2)z = 4 \sin(2x + y)$   
ii.  $(D + D' - 1)(D + 2D' - 3)z = 4 + 3x + 6y$
8. a. Find the complete integral of  $q = -xp + p^2$  4
- b. Find the complete integral of  $2p_1x_1x_3 + 3p_2x_3^2 + p_2^2p_3 = 0$ . 4
- c. Solve the partial differential equation  $(y - zx)p + (x + yz)q = x^2 + y^2$   $3^{2/3}$