Ex-1

(i) Find a differential equation from a straight line $y=mx$

$y=mx \cdots\cdots (i)$

$\Rightarrow \frac{dy}{dx}=m$

$\Rightarrow \frac{dy}{dx}=\frac{y}{x}$

$\Rightarrow y=x\frac{dy}{dx}$ (Ans)

(ii) From the differential equation of all circles passing through origin and having their centres on the x-axis.

Let, center $(a,0)$ and radious $=a$

$\therefore (x-a)^2+(y-0)^2=a^2$

$\Rightarrow x^2-2ax+a^2+y^2=a^2$

$\Rightarrow x^2+y^2=2ax$

$\Rightarrow 2x+2y\frac{dy}{dx}=2a$

$\Rightarrow 2x^2+2xy\frac{dy}{dx}=2ax$

$\Rightarrow 2x^2+2xy\frac{dy}{dx}=x^2+y^2$

$\Rightarrow x^2-y^2+2xy\frac{dy}{dx}=0$

(iii) Prove that, the differential equation of the circle $x^2+y^2+2fy=0$ is $(x^2-y^2)dy-2xydx=0$

given, $x^2+y^2+2fy=0$

$\Rightarrow 2x+2y\frac{dy}{dx}+2f\frac{dy}{dx}=0$

$\Rightarrow 2xy+2y^2\frac{dy}{dx}+2fy\frac{dy}{dx}=0$

$\Rightarrow 2xy+2y^2\frac{dy}{dx}-(x^2+y^2)\frac{dy}{dx}=0$

$\Rightarrow 2xy+2y^2\frac{dy}{dx}-x^2\frac{dy}{dx}-y^2\frac{dy}{dx}=0$

$\Rightarrow 2xy+y^2\frac{dy}{dx}-x^2\frac{dy}{dx}$

$\Rightarrow (x^2-y^2)\frac{dy}{dx}\cancel{+2x}=2xy$

$\Rightarrow (x^2-y^2)\frac{dy}{dx}=2xy$

$\Rightarrow (x^2-y^2)dy-2xy\,dx=0$ (Proved)

(iv) Show that $y=(x^2+c)e^{-x}$ is the general solution of the differential equation $\frac{dy}{dx}+y=2xe^{-x}$ where c is any arbitrary constant. Find particulate solution of this equation which satisfies $y(-1)=e+3$

First Part:

$y=(x^2+c)e^{-x}$

$\Rightarrow ye^{x}=x^2+c$ ......(i)

$\Rightarrow e^{x}\frac{dy}{dx}+ye^{x}=2x$

$\Rightarrow e^{x}\left[\frac{dy}{dx}+y\right]=2x$

$\Rightarrow \frac{dy}{dx}+y=2xe^{-x}$ (proved)

2nd Part: apply $y(-1) = e+3$ in eqn (i)

$\therefore x = -1, \; y = e+3$

$\therefore (e+3)e^{-1} = (-1)^2 + c$

$\Rightarrow 1 + \frac{3}{e} = 1 + c$

$\Rightarrow c = \frac{3}{e}$

$\therefore y = \left(x^2 + \frac{3}{e}\right)e^{-x}$; this is required equation

Ex-2:

(i) Find a differential equation from the relation $y = A\cos x + B\sin$

$$y = A\cos x + B\sin x$$

$$\Rightarrow \frac{dy}{dx} = -A\sin x + B\cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -A\cos x - B\sin x$$

$$\Rightarrow \frac{d^2y}{dx^2} = -(A\cos x + B\sin x)$$

$$\Rightarrow \frac{d^2y}{dx^2} = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

(Ans)

(ii) Show that, the differential equation of $Ax^2+By^2=1$ is
$$x\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=y\frac{dy}{dx}$$

$$Ax^2+By^2=1$$
$$\Rightarrow 2Ax+2By\frac{dy}{dx}=0$$
$$\Rightarrow By\frac{dy}{dx}=Ax$$
$$\Rightarrow \frac{y}{x}\frac{dy}{dx}=\frac{A}{B}$$
$$\Rightarrow \frac{y}{x}\frac{d^2y}{dx^2}+\frac{dy}{dx}\frac{d}{dx}\left(\frac{y}{x}\right)=0$$
$$\Rightarrow \frac{y}{x}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left[\frac{x\,dy/dx-y\cdot 1}{x^2}\right]=0$$
$$\Rightarrow \frac{y}{x}\frac{d^2y}{dx^2}+\frac{dy}{dx}\left[\frac{1}{x}\frac{dy}{dx}-\frac{y}{x^2}\right]=0$$
$$\Rightarrow \frac{y}{x}\frac{d^2y}{dx^2}+\frac{1}{x}\left(\frac{dy}{dx}\right)^2-\frac{y}{x^2}\frac{dy}{dx}=0$$
$$\Rightarrow \frac{y}{x}\frac{d^2y}{dx^2}+\frac{1}{x}\left(\frac{dy}{dx}\right)^2=\frac{y}{x^2}\frac{dy}{dx}$$
$$\Rightarrow x\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=y\frac{dy}{dx}$$ (showed)

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Ex-1: Solve: $2x\,dy - 2y\,dx - \sqrt{x^2+4y^2}\,dx = 0$

$$\Rightarrow 2x\,dy - \left(2y - \sqrt{x^2+4y^2}\right)dx = 0$$

$$\Rightarrow 2x\,dy = \left(2y + \sqrt{x^2+4y^2}\right)dx$$

$$\Rightarrow \frac{dy}{dx} = \frac{2y+\sqrt{x^2+4y^2}}{2x} \quad \cdots \cdots (i)$$

Now, put, $y = vx$, $\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(i) \Rightarrow v + x\frac{dv}{dx} = \frac{2vx + \sqrt{x^2+4v^2x^2}}{2x}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{2v+\sqrt{1+4v^2}}{2}$$

$$\Rightarrow v + x\frac{dv}{dx} = v + \frac{\sqrt{1+4v^2}}{2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\sqrt{1+4v^2}}{2}$$

$$\Rightarrow \frac{1}{\sqrt{1+4v^2}}\,dv = \frac{1}{2}\cancel{x}\,dx$$

$$\Rightarrow \int \frac{2}{\sqrt{1+4v^2}}\,dv = \int \cancel{x}\,dx$$

$$\Rightarrow \int \frac{\cancel{2}\,dv}{\cancel{2}\sqrt{(1/2)^2+v^2}} = \int x\,dx$$

$$\Rightarrow \ln\left(v+\sqrt{1/4+v^2}\right) = \ln x + \ln c$$

$$\Rightarrow \ln\left(v+\sqrt{1/4+v^2}\right) = \ln xc$$

$$\Rightarrow v+\sqrt{1/4+v^2} = xc$$

$$\Rightarrow \frac{y}{x} + \sqrt{1/4+\frac{y^2}{x^2}} = xc$$

সূত্র: $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \ln\left(x+\sqrt{a^2+x^2}\right)$

$$\Rightarrow \frac{y}{x} + \frac{\sqrt{x^2+4y^2}}{2x} = xc$$

$$\Rightarrow 2y + \sqrt{x^2+4y^2} = 2x^2c$$

Ex-2 (i) Solve: $(x^2+y^2)dx - 2xy\,dy = 0$

$$\Rightarrow (x^2+y^2)dx = 2xy\,dy$$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{2xy} \quad \cdots \quad \cdots (i)$$

Let, $y = vx$ $\quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(i) \Rightarrow v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{2x \cdot vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2-2v^2}{2v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1-v^2}{2v}$$

$$\Rightarrow \frac{2v}{1-v^2}dv = \frac{1}{x}dx$$

$$\Rightarrow -\frac{2v}{v^2-1}dv = \frac{1}{x}dx$$

$$\Rightarrow \int \frac{2v}{v^2-1}dv + \int \frac{1}{x}dx = 0$$

$$\Rightarrow \ln(v^2-1) + \ln x = \ln c$$

$$\Rightarrow (v^2-1)x = c$$

$$\Rightarrow \left(\frac{y^2}{x^2}-1\right)x = c$$

$$\Rightarrow \frac{y^2-x^2}{x^2}\cdot x = c$$

$$\Rightarrow y^2-x^2 = cx$$

(ii) Solve: $\frac{dy}{dx} = \frac{y(x-2y)}{x(x-3y)}$ ... ...(i)

let, $y=vx$ than, $\frac{dy}{dx} = v+x\frac{dv}{dx}$

$$(i) \Rightarrow v+x\frac{dv}{dx} = \frac{vx(x-2vx)}{x(x-3vx)}$$

$$\Rightarrow v+x\frac{dv}{dx} = \frac{vx^2-2v^2x^2}{x^2-3vx^2}$$

$$\Rightarrow v+x\frac{dv}{dx} = \frac{v-2v^2}{1-3v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-2v^2}{1-3v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v-2v^2-v+3v^2}{1-3v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{v^2}{1-3v}$$

$$\Rightarrow \frac{1-3v}{v^2}dv = \frac{1}{x}dx$$

$$\Rightarrow \int\left(\frac{1}{v^2}-\frac{3}{v}\right)dv = \int\frac{1}{x}dx$$

$$\Rightarrow -\frac{1}{v} - 3\ln v = \ln x + a$$

$$\Rightarrow \frac{1}{v} + 3\ln v + \ln x = a$$

$$\Rightarrow \cancel{3}\ln v^3 + \ln x = a - \frac{1}{v}$$

$$\Rightarrow \ln v^3 x = a - \frac{1}{v}$$

$$\Rightarrow \ln\left(\frac{y^3}{x^3} \cdot x\right) = a - \frac{1}{v}$$

$$\Rightarrow \frac{y^3}{x^2} = e^{a - x/y}$$

$$\Rightarrow \frac{y^3}{x^2} = e^{a} \cdot e^{-x/y}$$

$$\Rightarrow \frac{y^3}{x^2} = ce^{-x/y}$$

$$\Rightarrow y^3 = x^2 c e^{-x/y}$$

(iii) Solve: $(x^2 - xy + y^2)\,dx = xy\,dy$

$$\Rightarrow \frac{dy}{dx} = \frac{x^2 - xy + y^2}{xy} \quad \cdots \cdots (i)$$

Let, $y = vx$ $\quad \therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(i) \Rightarrow v + x\frac{dv}{dx} = \frac{x^2 - x \cdot vx + v^2x^2}{x \cdot vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1 - v + v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - v + v^2}{v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - v + v^2 - v^2}{v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1 - v}{v}$$

$$\Rightarrow \int \frac{v}{1-v}\,dv = \int \frac{1}{x}\,dx$$

$$\cancel{\Rightarrow \int \frac{1-v(1-v)}{}}$$

$$\Rightarrow \int \frac{v}{v-1}\,dv + \int \frac{1}{x}\,dx = 0$$

$$\Rightarrow \int \frac{v-1+1}{v-1}\,dv + \int \frac{1}{x}\,dx = 0$$

$$\Rightarrow \int \left(1+\frac{1}{v-1}\right)dv + \int \frac{1}{x}\,dx = 0$$

$$\Rightarrow v + \ln(v-1) + \ln x = c$$

$$\Rightarrow \ln(v-1) + \ln x = c - v$$

$$\Rightarrow \ln(v-1)x = c - \frac{y}{x}$$

$$\Rightarrow \ln\left(\frac{y}{x}-1\right)x = \frac{cx-y}{x}$$

$$\Rightarrow x\ln(y-x) = cx - y$$

$$\Rightarrow y + x\ln(y-x) = cx$$

Ex-3 (i) Solve $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$ ...... (i)

let, $y = vx$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

(i) $\Rightarrow v + x\frac{dv}{dx} = \frac{vx}{x} + \tan\frac{vx}{x}$

$$\Rightarrow v + x\frac{dv}{dx} = v + \tan v$$

$$\Rightarrow x\frac{dv}{dx} = \tan v$$

$$\Rightarrow \frac{1}{\tan v} dv = \frac{1}{x} dx$$

$$\Rightarrow \int \cot v \, dv = \int \frac{1}{x} dx$$

$$\Rightarrow \ln(\sin v) = \ln x + \ln c$$

$$\Rightarrow \ln\left(\sin \frac{y}{x}\right) = \ln(xc)$$

$$\Rightarrow \sin\left(\frac{y}{x}\right) = xc$$

(ii) Solve: $\left(x \sin \frac{y}{x} - y \cos \frac{y}{x}\right) dx + x \cos \frac{y}{x} dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{x \sin \frac{y}{x} - y \cos \frac{y}{x}}{-x \cos \frac{y}{x}} \quad \cdots \cdots \text{(i)}$$

let, $y = vx$ than $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\text{(i)} \Rightarrow v + x\frac{dv}{dx} = \frac{x \sin \frac{vx}{x} - \cancel{y} \cos \frac{vx}{x}}{-x \cos \frac{vx}{x}}$$

$$\Rightarrow v + x\frac{dv}{dx} = \cancel{\frac{x\sin v - y\cos v}{-x}} \; \frac{\sin v - v\cos v}{-\cos v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\sin v - v\cos v}{-\cos v} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\sin v - v\cos v + v\cos v}{-\cos v}$$

$$\Rightarrow x\frac{dv}{dx} = -\frac{\sin v}{\cos v}$$

$$\Rightarrow x\frac{dv}{dx} = -\tan v$$

$$\Rightarrow -\frac{1}{\tan v} dv = \frac{1}{x} dx$$

$$\Rightarrow -\cot v\,dv = \frac{1}{x}dx$$

$$\Rightarrow \int \cot v\,dv + \int \frac{1}{x}dx = 0$$

$$\Rightarrow \ln(\sin v) + \ln x = \ln c$$

$$\Rightarrow x\sin v = c$$

$$\Rightarrow x\sin\frac{y}{x} = c$$

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Ex-1: Solve $\frac{dy}{dx} = \frac{2x+y-2}{3x+y-3}$ ... ... (i)

since, $\left[\frac{3}{2} \neq \frac{1}{1}\right]$ let, $x = \alpha + h$ and $y = \beta + k$; h, k are constant

$\frac{dx}{d\alpha} = 1$ $\quad\quad$ $\frac{dy}{d\beta} = 1$

$\therefore dx = d\alpha$ $\quad\quad$ $dy = d\beta$

$\therefore \frac{dy}{dx} = \frac{d\beta}{d\alpha}$

Now from (i) $\Rightarrow$ $\frac{d\beta}{d\alpha} = \frac{2(\alpha+h)+\beta+k-2}{3(\alpha+h)+\beta+k-3}$

$\Rightarrow \frac{d\beta}{d\alpha} = \frac{2\alpha+\beta+2h+k-2}{3\alpha+\beta+3h+k-3}$ ... ... (ii)

Now, $2h+k-2=0$

$3h+k-3=0$

where, $h=1$, $k=0$

(ii) $\Rightarrow \frac{d\beta}{d\alpha} = \frac{2\alpha+\beta}{3\alpha+\beta}$ ... ...(iii)

let, $\beta = v\alpha$

$\therefore \frac{d\beta}{d\alpha} = v + \alpha\frac{dv}{d\alpha}$

(iii) $\Rightarrow v + \alpha\frac{dv}{d\alpha} = \frac{2\alpha+v\alpha}{3\alpha+v\alpha}$

$\Rightarrow v + \alpha\frac{dv}{d\alpha} = \frac{2+v}{3+v}$

$\Rightarrow \alpha\frac{dv}{d\alpha} = \frac{2+v}{3+v} - v$

$\Rightarrow \alpha\frac{dv}{d\alpha} = \frac{2+v-3v-v^2}{3+v}$

$\Rightarrow \alpha\frac{dv}{d\alpha} = \frac{-v^2-2v+2}{3+v}$

$\Rightarrow \frac{3+v}{-v^2-2v+2}dv = \frac{1}{\alpha}d\alpha$

$\Rightarrow \frac{3+v}{v^2+2v-2}dv + \frac{1}{\alpha}d\alpha = 0$

$\Rightarrow \int\frac{\frac{1}{2}(2v+2)+2}{v^2+2v-2}dv + \int\frac{1}{\alpha}d\alpha = 0$

$\Rightarrow \frac{1}{2}\int\frac{(2v+2)dv}{v^2+2v-2} + 2\int\frac{dv}{(v+1)^2-(\sqrt{3})^2} + \int\frac{1}{\alpha}d\alpha = 0$

$\Rightarrow \frac{1}{2}\ln(v^2+2v-2) + \frac{2}{2\sqrt{3}}\ln\frac{v+1-\sqrt{3}}{v+1+\sqrt{3}} + \ln\alpha = \frac{1}{2}c$

$\Rightarrow \ln(v^2+2v-2) + \frac{2}{\sqrt{3}}\ln\frac{v+1-\sqrt{3}}{v+1+\sqrt{3}} + 2\ln\alpha = c$

$\Rightarrow \ln\left(\frac{\beta^2}{\alpha^2} + \frac{2\beta}{\alpha} - 2\right) + \frac{2}{\sqrt{3}}\ln\left(\frac{\beta/\alpha+1-\sqrt{3}}{\beta/\alpha+1+\sqrt{3}}\right) + \ln\alpha^2 = c$

$$\Rightarrow \ln\left(\frac{\beta^2+2\alpha\beta-2\alpha^2}{\alpha^2}\right)+\frac{2}{\sqrt{3}}\ln\left(\frac{\beta+\alpha-\sqrt{3}\alpha}{\beta+\alpha+\sqrt{3}\alpha}\right)+\cancel{\ln}\left(\frac{\beta^2}{\cancel{\alpha^2}}\right)\ln\alpha^2=c$$

$$\Rightarrow \ln\left(\frac{\beta^2+2\alpha\beta-2\alpha^2}{\alpha^2}\right)\cdot\left(\frac{\beta^2}{\cancel{\alpha^2}}\right)\alpha^2+\frac{2}{\sqrt{3}}\ln\left(\frac{\beta+\alpha-\sqrt{3}\alpha}{\beta+\alpha+\sqrt{3}\alpha}\right)=c$$

$$\Rightarrow \ln(\beta^2+2\alpha\beta-2\alpha^2)+\frac{2}{\sqrt{3}}\ln\left\{\frac{\beta+(1-\sqrt{3})\alpha}{\beta+(1+\sqrt{3})\alpha}\right\}=c$$

since, $x=\alpha+1$, $y=\beta+0$ $\therefore \alpha=x-1$, $\beta=y$

$$\therefore \ln\left\{y^2+2(x-1)y-2(x-1)^2\right\}+\frac{2}{\sqrt{3}}\ln\left\{\frac{y+(1-\sqrt{3})(x-1)}{y+(1+\sqrt{3})(x-1)}\right\}=c$$

Ex-2 (i) Solve: $(x+y+1)\,dx-(2x+2y+1)\,dy=0$

$\because \left[\frac{1}{2}=\frac{1}{2}\right]$

$$\Rightarrow (x+y+1)\,dx=(2x+2y+1)\,dy$$

$$\Rightarrow \frac{dy}{dx}=\frac{x+y+1}{2x+2y+1}$$

$$\Rightarrow \frac{dy}{dx}=\frac{x+y+1}{2(x+y)+1}\quad\cdots\cdots(i)$$

let, $x+y=v$

$$\Rightarrow 1+\frac{dy}{dx}=\frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx}=\frac{dv}{dx}-1\quad\cdots\cdots(ii)$$

Now from (i) and (ii)

$$\frac{dv}{dx}-1=\frac{v+1}{2v+1}$$

$$\Rightarrow \frac{dv}{dx}=\frac{v+1}{2v+1}+1$$

$$\Rightarrow \frac{dv}{dx}=\frac{v+1+2v+1}{2v+1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{3v+2}{2v+1}$$

$$\Rightarrow \frac{2v+1}{3v+2}\, dv = dx$$

$$\Rightarrow \int \left[\frac{\frac{2}{3}(3v+2) - \frac{1}{3}}{3v+2}\right] dv = \int dx$$

$$\Rightarrow \int \left[\frac{2}{3} - \frac{1}{3}\cdot\frac{1}{3v+2}\right] dv = \int dx$$

$$\Rightarrow \frac{2}{3}\int dv - \frac{1}{9}\int \frac{3}{3v+2}\, dv = \int dx$$

$$\Rightarrow \frac{2}{3}v - \frac{1}{9}\ln(3v+2) = x + \frac{1}{9}c$$

$$\Rightarrow 6v - \ln(3v+2) = 9x + c$$

$$\Rightarrow 6(x+y) - \ln(3v+2) = 9x + c$$

$$\Rightarrow 6y - 3x - \ln(3v+2) = c \Rightarrow 6y - 3x - \ln(3x+3y+2) = c$$

(ii) Solve : $\frac{dy}{dx} = \frac{6x-2y-7}{3x-y+4}$

$$\Rightarrow \frac{dy}{dx} = \frac{2(3x-y)-7}{3x-y+4} \quad \cdots\cdots (i)$$

let, $3x - y = v$

$$\Rightarrow 3 - \frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = 3 - \frac{dv}{dx} \quad \cdots (ii)$$

Now from (i) $\Rightarrow$

$$3 - \frac{dv}{dx} = \frac{2v-7}{v+4}$$

$$\Rightarrow \frac{dv}{dx} = 3 - \frac{2v-7}{v+4}$$

$$\Rightarrow \frac{dv}{dx} = \frac{3v+12-2v+7}{v+4}$$

$$\Rightarrow \frac{dv}{dx} = \frac{v+19}{v+4}$$

$$\Rightarrow \frac{v+4}{v+19}\, dv = dx$$

$$\Rightarrow \left[\frac{(v+19)-15}{v+19}\right] dv = dx$$

$$\Rightarrow \left[1-\frac{15}{v+19}\right] dv = dx$$

$$\Rightarrow \int dv - 15\int \frac{1}{v+19} = \int dx \quad \text{[integratting both side]}$$

$$\Rightarrow v - 15\ln(v+19) = x+c$$

$$\Rightarrow 3x-y-15\ln(v+19) = x+c$$

$$\Rightarrow 2x-y-15\ln(v+19) = c \quad \Rightarrow 2x-y-15\ln(3x-y+19) = c$$

(iii) Solve: $(2x+y+1)dx + (4x+2y-1)dy = 0$

$$\Rightarrow (2x+y+1)dx = -(4x+2y-1)dy$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2x+y+1}{4x+2y-1}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{2x+y+1}{2(2x+y)-1} \quad \cdots \quad \cdots (i)$$

let, $2x+y = v$

$$\Rightarrow 2+\frac{dy}{dx} = \frac{dv}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 2 \quad \cdots \quad \cdots (ii)$$

Now from (i) and (ii) $\Rightarrow$

$$\Rightarrow \frac{dv}{dx} - 2 = -\frac{v+1}{2v-1}$$

$$\Rightarrow \frac{dv}{dx} = 2 - \frac{v+1}{2v-1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{4v-2-v-1}{2v-1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{3v-3}{2v-1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{3(v-1)}{2v-1}$$

$$\Rightarrow \frac{2v-1}{v-1}\, dv = 3\, dx$$

$$\Rightarrow \left[\frac{2(v-1)+1}{v-1}\right] dv = 3dx$$

$$\Rightarrow \int \left[2 + \frac{1}{v-1}\right] dv = \int 3dx$$

$$\Rightarrow 2\int dv + \int \frac{dv}{v-1} = 3\int dx$$

$$\Rightarrow 2v + \ln(v-1) = 3x + c$$

$$\Rightarrow 2(3x+y) + \ln(3x-y-1) = 3x+c$$

$$\Rightarrow 3x + 2y + \ln(3x+y-1) = c$$

(Ans)

Exact Differential Equation

let, M. and N are function of x and y. If $Mdx+Ndy=0$ is obtained by differentiating $f(x,y)$ then $Mdx+Ndy=0$ is called exact differential equation.

$Mdx+Ndy=0$ will be exact :
if

$$\frac{\delta M}{\delta y}=\frac{\delta N}{\delta x}$$

Working role:

① যদি $\frac{\delta M}{\delta y}=\frac{\delta N}{dx}$ হয় ; then the equation is exact

$$\int_{y\ constant} Mdx+\int(N \text{ এর } x \text{ বর্জিত পদ})\,dy=c$$

② if $\frac{\delta M}{\delta y}\neq\frac{\delta N}{dx}$ (Not exact)

কিন্তু M ও N উভয়েই সমমাত্রিক এবং
$Mx+Ny\neq 0$ than I.F (Intregating factor) =
$\frac{1}{Mx+Ny}$. I.f দিয়ে গুন করলে exact হবে
than 1 নং নিয়মের মত ।

③ $f(xy)y\,dx+f(xy)x\,dy=0$ আকারের হলে, এবং Non-homogenous

$$I.F=\frac{1}{Mx-Ny}$$

④ ~~Md~~ $\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$ But $\frac{1}{N}\left(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}\right) = f(x)$ (কেবলমাত্র x এর ফাংশন)

তাহলে, $I.F. = e^{\int f(x)\,dx}$

④ $\frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$ But $\frac{1}{M}\left(\frac{\delta M}{\delta y} - \frac{\delta M}{\delta y}\right) = f(y)$ (কেবলমাত্র y এর ফাংশন)

then, $I.F. = e^{\int f(y)\,dy}$

Ex-1 : Solve:

$$(3x+2y-5)\,dx + (2x+3y-5)\,dy = 0$$

Here, $M = 3x+2y-5$ $\quad N = 2x+3y-5$

$\frac{\delta M}{\delta y} = 2$ $\quad \frac{\delta N}{\delta x} = 2$

since, $\frac{\delta M}{\partial y} = \frac{\delta N}{\delta x}$ so the equation is exact.

$$\therefore \int_{y\,constant} M\,dx + \int (N \text{ without } x)\,dy = c_1$$

$$\Rightarrow \int_{y\,constant} (3x+2y-5)\,dx + \int (3y-5)\,dy = c_1$$

$$\Rightarrow 3\int x\,dx + 2y\int dx - 5\int dx + 3\int y\,dy - 5\int dy = c_1$$

$$\Rightarrow 3\frac{x^2}{2} + 2yx - 5x + 3\frac{y^2}{2} - 5y = c_1$$

$$\Rightarrow 3x^2 + 4xy - 10x + 3y^2 - 10y = 2c_1$$

$$\Rightarrow 3x^2 + 4xy - 10x + 3y^2 - 10y = c$$

Ex-2 (i) $(x^3+3xy^2)\,dx + (y^3+3x^2y)\,dy = 0$

$M = x^3+3xy^2$ $\quad N = y^3+3x^2y$

$\frac{\delta M}{\delta y} = 6xy$ $\quad \frac{\delta N}{\delta x} = 6xy$

since, $\frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$ so, this equation is exact.

$$\int_{y\,constant} (x^3+3xy^2)\,dx + \int y^3\,dy = c_1$$

$$\Rightarrow \frac{x^4}{4} + \frac{3x^2y^2}{2} + \frac{y^4}{4} = c_1$$

$$\Rightarrow x^4 + 6x^2y^2 + y^4 = c$$

Ex – 2 (ii) Solve:

$$3x(xy-2)dx + (x^3+2y)dy = 0 \quad \ldots (i)$$

$$M = 3x(xy-2) = 3x^2y - 6x$$

$$\frac{\partial M}{\partial y} = 3x^2$$

$$N = x^3 + 2y$$

$$\frac{\delta N}{\delta x} = 3x^2$$

$$\therefore \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

$$\therefore \int_{y\text{ constant}} (3x^2y - 6x)\,dx + \int 2y\,dy = c_1$$

$$\Rightarrow \frac{3x^3y}{3} - 6\frac{x^2}{2} + 2\frac{y^2}{2} = c_1$$

$$\Rightarrow x^3y - 3x^2 + y^2 = c_1$$

$$\therefore x^3y - 3x^2 + y^2 = c$$

Ex-3

$$(1+e^{x/y})dx + e^{x/y}\left(1+\frac{x}{y}\right)dy = 0 \quad \cdots(i)$$

$$M = 1+e^{x/y}$$

$$\Rightarrow \frac{\delta M}{\delta y} = e^{x/y}\left(-\frac{x}{y^2}\right) = -\frac{xe^{x/y}}{y^2}$$

$$N = e^{x/y}\left(1-\frac{x}{y}\right)$$

$$\Rightarrow \frac{\delta N}{\delta x} = e^{x/y}\left(-\frac{1}{y}\right)\cancel{\left(\frac{1}{y}\right)} + e^{x/y}\cdot\left(\frac{1}{y}\right)\left(1-\frac{x}{y}\right)$$

$$= -\frac{e^{x/y}}{y} + \frac{e^{x/y}}{y} - \frac{x}{y}\cdot\frac{1}{y}e^{x/y}$$

$$= -\frac{xe^{x/y}}{y^2}$$

$$\because \frac{\delta M}{\delta y} = \frac{\delta N}{\delta x}$$

$$\therefore \int_{y\ \text{constant}} (1+e^{x/y})dx + \int 0\, dy = c_1$$

$$\cancel{\Rightarrow x + e^{x/y}\cdot\frac{1}{y}\cdot\frac{x^2}{2} = c_1}$$

$$\Rightarrow x + \frac{e^{x/y}}{1/y} = c_1$$

$$\Rightarrow x + ye^{x/y} = c$$

Ex-4: Solve: $(x^2y - 2xy^2)\,dx - (x^3 - 3x^2y)\,dy = 0 \quad \cdots(i)$

$$M = x^2y - 2xy^2 \qquad N = -(x^3 - 3x^2y)$$

$$\frac{\delta M}{dy} = x^2 - 4xy \qquad \frac{\delta N}{dx} = -3x^2 + 6xy$$

$$\because \frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$$

$$\therefore I.F = \frac{1}{Mx+Ny} = \frac{1}{(x^2y-2xy^2)x-(x^3-3x^2y)y}$$

$$= \frac{1}{x^3y-2x^2y^2-x^3y+3x^2y^2}$$

$$= \frac{1}{x^2y^2}$$

Multiplying (i) by $\frac{1}{x^2y^2}$

$$\frac{(x^2y-2xy^2)dx}{x^2y^2} - \frac{(x^3-3x^2y)dy}{x^2y^2} = 0$$

$$\Rightarrow \left(\frac{1}{y}-\frac{2}{x}\right)dx - \left(\frac{x}{y^2}-\frac{3}{y}\right)dy = 0 \quad \text{[It's exact]}$$

Now, $M = \frac{1}{y} - \frac{2}{x}$ and $N = -\left(\frac{x}{y^2} - \frac{3}{y}\right)$

$$\therefore \int_{y\text{ constant}} \left(\frac{1}{y}-\frac{2}{x}\right)dx + \int \frac{3}{y}\,dy = c_1$$

$$\Rightarrow \frac{x}{y} - 2\ln x + 3\ln y = c$$

~~Ex-4 $x^2y - 2xy^2$~~

V.V.I

Ex-5: $(1+xy)y\,dx + (1-xy)x\,dy = 0 \quad \cdots \text{(i)}$

$M = (1+xy)y \qquad N = (1-xy)x$

$\Rightarrow \dfrac{\delta M}{\delta y} = 1 + 2xy \qquad \dfrac{\delta N}{\delta x} = 1 - 2xy$

$\because \dfrac{\delta M}{\delta y} \neq \dfrac{\delta N}{\delta x}$; equation (i) is not exact

$$\therefore \text{I.F.} = \frac{1}{Mx - Ny}$$

$$= \frac{1}{(1+xy)yx - (1-xy)xy}$$

$$= \frac{1}{xy + x^2y^2 - xy + x^2y^2}$$

$$= \frac{1}{2x^2y^2}$$

Multiplying (i) by $\dfrac{1}{2x^2y^2}$

$$\frac{(1+xy)y\,dx}{2x^2y^2} + \frac{(1-xy)x\,dy}{2x^2y^2} = 0$$

$$\Rightarrow \frac{y+xy^2}{2x^2y^2}\,dx + \frac{x - x^2y}{2x^2y^2}\,dy = 0$$

$$\Rightarrow \left(\frac{1}{2x^2y} + \frac{1}{x}\right)dx + \left(\frac{1}{xy^2} - \frac{1}{y}\right)dy = 0$$

$$\therefore \int_{y\text{ constant}} \left(\frac{1}{x^2y} + \frac{1}{x}\right)dx + \int -\frac{1}{y}\,dy = C$$

$$\Rightarrow \left(-\frac{1}{xy}\right) + \ln x - \ln y = c$$

$$\therefore -\frac{1}{xy} + \ln x - \ln y = c$$

Ex-6 Solve

$$(12y+4y^3+6x^2)dx + 3(x+xy^2)dy = 0 \quad \cdots (i)$$

Here, $M = 12y+4y^3+6x^2$ $\quad N = 3x+3xy^2$

$$\frac{\delta M}{\delta y} = 12+12y^2 \qquad \frac{\delta N}{\delta x} = 3+3y^2$$

$\therefore \frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$; so the equation is not exact.

Now, $\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x} = 12+12y^2-3-3y^2 = 9+9y^2$

$$\therefore \frac{1}{N}\left(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}\right) = \frac{9+9y^2}{3(x+xy^2)}$$

$$= \frac{9(1+y^2)}{3x(1+y^2)}$$

$$= \frac{3}{x}$$

$$\therefore I.F. = e^{\int \frac{3}{x} dx} = e^{\cancel{3x}} e^{3\ln x} = e^{\ln x^3} = x^3$$

Multiplying (i) by x^3

$$(12y+4y^3+6x^2)x^3 dx + (3x+3xy^2)x^3 dy = 0$$

$$\Rightarrow (12x^3y+4x^3y^3+6x^5)dx + (3x^4+3x^4y^2)dy = 0$$

$$\therefore \int_{y \text{ constant}} (12x^3y+4x^3y^3+6x^5)dx + \int 0 \, dy = c$$

$$\Rightarrow 12y\frac{x^4}{4} + 4y^3\frac{x^4}{4} + 6\frac{x^6}{6} = c$$

$$\Rightarrow 3x^4y + x^4y^4 + x^6 = c$$

Ex-7 : Solve :

$$y\ln y\,dx + (x-\ln y)\,dy = 0 \cdots\cdots (i)$$

Here, $M = y\ln y$ $\qquad N = x - \ln y$

$$\frac{\delta M}{\delta y} = y\cdot\frac{1}{y} + 1\cdot\ln y \qquad \frac{\delta N}{\delta x} = 1$$

$$\Rightarrow \frac{\delta M}{\delta y} = 1+\ln y$$

$\therefore \frac{\delta M}{\delta y} \neq \frac{\delta N}{\delta x}$, so equation (i) is not exact.

Now, $\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y} = 1-1-\ln y = -\ln y$

$$\therefore \frac{1}{M}\left(\frac{\delta N}{\delta x} - \frac{\delta M}{\delta y}\right) = \frac{-\ln y}{y\ln y} = \frac{-1}{y} = f(y)$$

$$\therefore \text{E.F.} = e^{\int f(y)\,dy} = e^{\int -\frac{1}{y}\,dy} = e^{-\ln y} = e^{\ln y^{-1}} = y^{-1} = \frac{1}{y}$$

Now Multiplying (i) by $\frac{1}{y}$

$$y\ln y\cdot\frac{1}{y}\,dx + (x-\ln y)\cdot\frac{1}{y}\,dy = 0$$

$$\Rightarrow \ln y\,dx + \left(\frac{x}{y} - \frac{\ln y}{y}\right)dy = 0$$

$$\therefore \int_{y\ \text{constant}} \ln y\,dx + \int -\frac{\ln y}{y}\,dy = 0$$

$$\Rightarrow x\ln y\,dx - \frac{1}{2}(\ln y)^2 = c$$

Let, $I = \int \frac{\ln y}{y}\,dy$

$= \ln y \int \frac{1}{y}\,dy - \int\left\{\frac{d}{dy}(\ln y)\int\frac{1}{y}\,dy\right\}dy$

$= \ln y\cdot\ln y - \int\frac{1}{y}\ln y\,dy$

$\Rightarrow I = (\ln y)^2 - I$

$\Rightarrow 2I = (\ln y)^2$

$\therefore I = \frac{(\ln y)^2}{2}$

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Ex-8: Find the Intregrating Factor to solve:

$$(4xy+3y^2-x)\,dx + x(x+2y)\,dy = 0 \quad \text{--- (1)}$$

Here, $M = 4xy+3y^2-x$ $\quad N = x^2+2xy$

$$\Rightarrow \frac{\delta M}{\delta y} = 4x+6y \qquad \frac{\delta N}{\delta x} = 2x+2y$$

Now, $\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x} = 4x+6y-2x-2y = 2x+4y$

$$\therefore \frac{1}{N}\left(\frac{\delta M}{\delta y} - \frac{\delta N}{\delta x}\right) = \frac{2x+4y}{x^2+2xy} = \frac{2(x+2y)}{x(x+2y)} = \frac{2}{x} = f(x)$$

$$\therefore I.F = e^{\int f(x)\,dx} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

(Ans)

Linear Differential Equation

If P and Q are only functions of x or constants then the differential equation of the form $\frac{dy}{dx}+Py=Q$ is called first order linear differential equation. To solve this equation multiply by $e^{\int Pdx}$

Working Role:

(i) প্রথমে সমীকরণকে $\frac{dy}{dx}+Py=Q$ আকারে সাজাতে হবে।

এখানে $\frac{dy}{dx}$ এর সহগ $=1$ এবং সর্বপদ Py আকারে থাকবে।

(ii) সর্বপদ y এর সহগ P দিয়ে I.F. তৈরি করতে হবে।

অর্থাৎ, $I.F = e^{\int Pdx}$

(iii) এখন সমীকরণটিকে I.F দ্বারা গুন করে এর সাপেক্ষে integrating করলে সমাধান পাওয়া যাবে।

অর্থাৎ, $e^{\int Pdx}\frac{dy}{dx} + e^{\int Pdx}Py = Qe^{\int Pdx}$

বা, $\boxed{\frac{d}{dx}\left[ye^{\int Pdx}\right] = Qe^{\int Pdx}}$ ...... একে x এর সাপেক্ষে integration করে পাই,

$$ye^{\int Pdx} = \int Qe^{\int Pdx}dx + c$$

Ex-1: Solved.

(i) $(1-x^2)\frac{dy}{dx} - xy = 1 \quad \ldots (i)$

$$\Rightarrow \frac{dy}{dx} - \frac{xy}{1-x^2} = \frac{1}{1-x^2} \quad \ldots (ii)$$

$$\therefore I.F. = e^{-\int \frac{x\,dx}{1-x^2}} = e^{\frac{1}{2}\int \frac{-2x\,dx}{1-x^2}} = e^{\frac{1}{2}\ln(1-x^2)} = e^{\ln(1-x^2)^{1/2}}$$

$$\therefore I.F = (1-x^2)^{1/2} = \sqrt{1-x^2}$$

Multiplying (ii) by $\sqrt{1-x^2}$

$$\cancel{\frac{\sqrt{1-x^2}}{1-x^2}\frac{dy}{dx} - (\sqrt{1-x^2})xy = \sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} - \frac{xy\sqrt{1-x^2}}{1-x^2} = \frac{\sqrt{1-x^2}}{1-x^2}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} - \frac{xy}{\sqrt{1-x^2}} = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{d}{dx}\left[y\sqrt{1-x^2}\right] = \frac{1}{\sqrt{1-x^2}}$$

integrating this w.r.t. to x:

$$y\sqrt{1-x^2} = \int \frac{1}{\sqrt{1-x^2}} + c$$

or, $y\sqrt{1-x^2} = \sin^{-1}x + c$

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(v.v.I)

(ii) $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$

$\Rightarrow \frac{dy}{dx}+\frac{y}{1+x^2}=\frac{\tan^{-1}x}{1+x^2}$ ... ...(i)

$\therefore$ I.F. $= e^{\int\frac{dx}{1+x^2}} = e^{\tan^{-1}x}$

multiplying (i) by $e^{\tan^{-1}x}$

$$\frac{d}{dx}[y\,e^{\tan^{-1}x}]=\frac{\tan^{-1}x\;e^{\tan^{-1}x}}{1+x^2}$$

integrating thire w.r.t x

$$y\,e^{\tan^{-1}x}=\int\frac{\tan^{-1}x\;e^{\tan^{-1}x}}{1+x^2}dx \quad \ldots \quad \ldots (ii)$$

in R.H.S, let $\tan^{-1}x=z$

$\Rightarrow \frac{dx}{1+x^2}=dz$

(ii) $\Rightarrow$ $y\,e^{\tan^{-1}x}=\int z\,e^z dz$

$= z\int e^z dz-\int\left\{\frac{d}{dz}(z)\int e^z dz\right\}dz$

$= ze^z-\int e^z dz$

$\Rightarrow y\,e^{\tan^{-1}x}=ze^z-e^z+c$

$\Rightarrow y\,e^{\tan^{-1}x}=\tan^{-1}x\,e^{\tan^{-1}x}-e^{\tan^{-1}x}+c$

$\Rightarrow y=\tan^{-1}x-1+c\,e^{-\tan^{-1}x}$

(iii) $(2+y^2)\,dx=(xy+2y+y^3)\,dy$

$\Rightarrow \frac{dx}{dy}=\frac{xy+2y+y^3}{2+y^2}$

$\Rightarrow \frac{dx}{dy}=\frac{xy}{2+y^2}+\frac{y(2+y^2)}{2+y^2}$

$$\Rightarrow \frac{dx}{dy} - \frac{xy}{2+y^2} = y \quad \cdots \quad \cdots (i)$$

$$\therefore I.F. = e^{\int -\frac{y\,dy}{2+y^2}} = e^{-\frac{1}{2}\int \frac{2y\,dy}{2+y^2}} = e^{-\frac{1}{2}\ln(2+y^2)}$$

$$\Rightarrow I.F = e^{\ln(2+y^2)^{-\frac{1}{2}}} = (2+y^2)^{-\frac{1}{2}} = \frac{1}{(2+y^2)^{1/2}} = \frac{1}{\sqrt{2+y^2}}$$

Multiplying (i) by $\frac{1}{\sqrt{2+y^2}}$

$$\frac{d}{dy}\left[x\frac{1}{\sqrt{2+y^2}}\right] = y\frac{1}{\sqrt{2+y^2}}$$

$$\Rightarrow x\frac{1}{\sqrt{2+y^2}} = \int \frac{y}{\sqrt{2+y^2}}$$; integrating both side w.r.t

$$\Rightarrow \frac{x}{\sqrt{2+y^2}} = \frac{1}{2}\int \frac{2y}{\sqrt{2+y^2}} +$$

$$\Rightarrow \frac{x}{\sqrt{2+y^2}} = \frac{1}{2}\cdot 2\sqrt{2+y^2} + c$$

$$\Rightarrow x = 2+y^2 + c\sqrt{2+y^2}$$

(iv) $y\,dx = (y - x - xy^2)\,dy$

$$\Rightarrow \frac{dx}{dy} = \frac{y - x - xy^2}{y}$$

$$\Rightarrow \frac{dx}{dy} = 1 - \frac{x(1+y^2)}{y}$$

$$\Rightarrow \frac{dx}{dy} + \left(y + \frac{1}{y}\right)x = 1 \quad \cdots \quad \cdots (i)$$

$$I.F. = e^{\int (y+\frac{1}{y})dy} = e^{y^2/2 + \ln y} = e^{y^2/2}\cdot e^{\ln y} = y e^{y^2/2}$$

multiplying (i) by $\boxed{y e^{y^2/2}}$

$$\frac{d}{dy}\left[x y e^{y^2/2}\right] = y e^{y^2/2}$$

$\Rightarrow \quad xye^{y^2/2} = \int ye^{y^2/2}\,dy$; integrating both side w.r.t. y

$\Rightarrow \quad xye^{y^2/2} = \cancel{y\int e^{y^2/2}dy - \int\left\{\frac{d}{dy}(y)\int e^{y^2/2}dy\right\}dy}\ \int e^{y^2/2}\,d\left(\frac{y^2}{2}\right)$

$\Rightarrow \quad xye^{y^2/2} = e^{y^2/2} + c$

$\Rightarrow xy = 1 + ce^{-y^2/2}$

Modeling With 1st Order D.E

Page-110 (2)

A population N grows according to the low $\frac{dN}{dt} = kN$, where K is a positive constant. Determine how long it takes the population to triple the size. Where the time t is measured in years. Find $\lim_{t\to\infty} N(t)$

1st Part: Given, $\frac{dN}{dt} = kN$

$$\Rightarrow \frac{dN}{N} = k\,dt \quad \dots \quad \dots (i)$$

Let, t=0, the population was $N=N_0$ and $t=T$, the populatio $N = 3N_0$.

Now intregreding (i) $\Rightarrow \int_{N_0}^{3N_0} \frac{dN}{N} = k\int_0^T dt$

$$\Rightarrow \ln[N]_{N_0}^{3N_0} = k[t]_0^T$$

$$\Rightarrow \ln 3N_0 - \ln N_0 = kT$$

$$\Rightarrow \ln \frac{3N_0}{N_0} = kT$$

$$\Rightarrow \ln 3 = kT$$

$$\Rightarrow T = \frac{\ln 3}{k}$$

$\therefore$ time, $T = \frac{\ln 3}{k}$ need to the popwation to triple the size

2nd Part: Again integreding (i)

$$\int \frac{dN}{N} = k\int dt$$

$$\Rightarrow \ln N = kt + \ln c$$

$$\Rightarrow \ln \frac{N}{c} = kt$$

$$\Rightarrow \frac{N}{c} = e^{kt}$$

$$\Rightarrow N = ce^{kt} \quad \dots \quad \dots (ii)$$

~~tet~~, when, t=0, $N=N_0$

Now from (ii) $\Rightarrow$ $N_0 = c e^0$ $\quad \therefore c = N_0$

Again from (ii) $\Rightarrow N = N_0 e^{kt}$

$\Rightarrow N(t) = N_0 e^{kt}$

$\therefore \lim_{t\to\infty} N(t) = \lim_{t\to\infty} N_0 e^{kt} = N_0 \lim_{t\to\infty} e^{kt} = N_0 e^{\infty} = \infty$

$\therefore$ After infinite time, the population will be infinite.

Page - 113 (4)

In a certain bacteria culture the rate of increase in the number of bacteria is proportional to the number present. If the number double in 4 hr, how many will be present in 12 hr? If the number 400 in 3 hr and 2000 in 10 hr. then find the number initially present.

$\rightarrow$ let, t hr after, the number of bacteria was x.

$\therefore \frac{dx}{dt} = kx$ [where k is constant] ... ...(i)

$\Rightarrow \int \frac{dx}{x} = \int k\,dt$

$\Rightarrow \ln x = kt + \ln c$

$\Rightarrow \ln \frac{x}{c} = kt$

$\Rightarrow \frac{x}{c} = e^{kt}$

$\therefore x = ce^{kt}$

when, $t = 0$, $x = n$ $\quad \therefore n = ce^0 \quad \therefore n = c$

or, $x = ne^{kt}$ ... ...(ii)

when, $t = 4$ $\quad x = 2n$

from (ii) $2n = ne^{4k}$ or, $e^{4k} = 2$

when, $t = 12$ hr, later ; from (ii)

$x = ne^{12k}$

$\Rightarrow x = n(e^{4k})^3$

$$\Rightarrow x = n(2)^3$$

$$\Rightarrow x = 8n$$

2nd Part:

if, $t=3$, $x=400$ and $t=10$, $x=2000$

$$\int_{400}^{2000} \frac{dx}{x} = k\int_{3}^{10} dt$$

$$\Rightarrow [\ln x]_{400}^{2000} = k[t]_{3}^{10}$$

$$\Rightarrow \ln \frac{2000}{400} = k[10-3]$$

$$\Rightarrow \ln 5 = 7k$$

$$\therefore k = \frac{\ln 5}{7} = \frac{1}{7}\ln 5 = \ln 5^{1/7}$$

Put, $k = \frac{\ln 5}{7}$ in eqn (ii) ; where, $t=3$, $n=400$

$$400 = n e^{\ln 5^{1/7} \cdot 3}$$

$$\cancel{\Rightarrow 400 = n 5^{1/7 \cdot 3}}$$

$$\cancel{\Rightarrow n =}$$

$$\Rightarrow 400 = n e^{3 \ln 5^{1/7}}$$

$$\Rightarrow 400 = n e^{\ln 5^{3/7}}$$

$$\Rightarrow 400 = n 5^{3/7}$$

$$\Rightarrow n = \frac{400}{5^{3/7}}$$

$$= 200.68 \approx 200$$

$\therefore$ Initially present 200

Page-116 (6)

The rate at which radioactive nuclei decay is proportional the number of such nuclei that are present in a given sample. Half of the original number of radioactive nuclei have undergone disintergration in a period of 1500 years. What percentage of the original radioactive nuclei will remain after 4500 years? and how many years will only one-tenth of the original number remain?

Let, time $= t$ and remain nuclei $= x$.

$\therefore \frac{dx}{dt} \propto x$

$\Rightarrow \frac{dx}{dt} = -kx$ $\quad \because$ nuclei was decreased

$\Rightarrow \frac{dx}{x} = -k\,dt$

$\Rightarrow \ln x = -kt + \ln c$

$\Rightarrow \ln \frac{x}{c} = -kt$

$\Rightarrow \frac{x}{c} = e^{-kt}$

$\Rightarrow x = ce^{-kt} \quad \ldots \quad \ldots (i)$

Let, when $t=0$, $x=m$ $\quad \therefore$ from (i) $\Rightarrow m=c$

Again from (i) $x = me^{-kt} \quad \ldots \quad \ldots (ii)$

when $t = 1500$, $x = \frac{m}{2}$

from (i) $\frac{m}{2} = me^{-kt}$

$\Rightarrow \frac{1}{2} = e^{-1500k}$

$\Rightarrow e^{-1500k} = \frac{1}{2}$

$\Rightarrow e^{-k} = \left(\frac{1}{2}\right)^{1/1500}$

from (ii) $x = m(e^{-k})^{t} = m\left[\left(\frac{1}{2}\right)^{1/1500}\right]^{t} \quad \ldots \quad \ldots (iii)$

when, $t = 4500$,

$$x = m\left[\left(\frac{1}{2}\right)^{1/1500}\right]^{4500}$$

$$= m\left[\left(\frac{1}{2}\right)^{3}\right]$$

$$= \frac{1}{8} m$$

$\therefore$ Remainig nuclei is $\frac{1}{8}$

2nd Part: let, $x = \frac{m}{10}$

Now, (iii) $\Rightarrow \frac{m}{10} = m\left[\left(\frac{1}{2}\right)^{1/1500}\right]^{t}$

$$\Rightarrow \frac{1}{10} = \left(\frac{1}{2}\right)^{\frac{t}{1500}}$$

$$\Rightarrow \ln\frac{1}{10} = \ln\left(\frac{1}{2}\right)^{\frac{t}{1500}}$$

$$\Rightarrow \ln\frac{1}{10} = \frac{t}{1500}\ln\left(\frac{1}{2}\right)$$

$$\Rightarrow t = \frac{\ln\frac{1}{10} \times 1500}{\ln\frac{1}{2}} = 4983$$

Page 119 - 8(i)

A body cools from 370°c to 330°c in 10 minute in air which maintained to 290°c what is the temperature after 40 minute

let, when time t, Temperature $T°C$ and the air temperature =

$\therefore$ Difference of Temperature $= T - 290$

Rate of change of temperature of the body: $\frac{dT}{dt} \propto T - 290$

$$\Rightarrow \frac{dT}{dt} = -k(T-290)$$ since, te is decrea

$$\Rightarrow \frac{dT}{T-290} = -k\,dt \quad \ldots \ldots (i$$

when, $t=0$, $T=370$ and when $t=10$, $T=330$

Now from (i) $\Rightarrow \int_{330}^{370} \frac{dT}{T-290} = \int_{10}^{0} -k\,dt$

$$\Rightarrow \ln[T-290]_{330}^{370} = K[t]_0^{10}$$

$$\Rightarrow \ln[370-290] - \ln[330-290] = 10K$$

$$\Rightarrow \ln 80 - \ln 40 = 10K$$

$$\Rightarrow \ln\frac{80}{40} = 10K$$

$$\Rightarrow \ln 2 = 10K$$

$$\therefore K = \frac{\ln 2}{10}$$

when, $t=0$; $T = 370°c$ let, the temperature of the body After 40 minutes will be x.

$$\therefore \int_x^{370} \frac{dT}{T-290} = -K\int_{40}^{0} dt$$

$$\Rightarrow \int_x^{370} \frac{dT}{T-290} = K\int_0^{40} dt$$

$$\Rightarrow \ln[T-290]_x^{370} = K[t]_0^{40}$$

$$\Rightarrow \ln[370-290] - \ln[x-290] = 40K$$

$$\Rightarrow \ln 80 - \ln[x-290] = \cancel{40}4 \cdot \frac{\ln 2}{\cancel{10}}$$

$$\Rightarrow \ln 80 - \ln[x-290] = 4\ln 2$$

$$\Rightarrow \ln\frac{80}{x-290} = \ln 2^4$$

$$\Rightarrow \frac{80}{x-290} = 2^4$$

$$\Rightarrow x-290 = \frac{80}{2^4}$$

$$\Rightarrow x = 290 + \frac{80}{2^4} = 290 + \frac{80}{16}$$

$$= 290 + 5 = 295°c$$

Page-121 8(ii) : If, when the temperature of the air is 20°c, a certain subtance cools from 100°c to 60°c in 10 minutes, find the temperature after 40 minutes.

→ let, when time $=t$ then, temperature $=T°c$

∴ The difference between the substance and air $= T-20$

∴ Rate of temperature: $\cancel{\frac{dT}{T-20}}$ $\frac{dT}{dt} = -k(T-20)$ | since Temp. is decreasing

$$\Rightarrow \frac{dT}{T-20} = -k\,dt \quad \text{--- (i)}$$

Here, when $t=0$, $T=100°c$

" $t=10$, $T=60°c$

from (i) $\Rightarrow \int_{100}^{60} \frac{dT}{T-20} = \int_0^{10} -k\,dt$

$$\Rightarrow \ln[T-20]_{100}^{60} = -k[t]_0^{10}$$

$$\Rightarrow \ln[60-20] - \ln[100-20] = -10K$$

$$\Rightarrow \ln 40 - \ln 80 = -10K$$

$$\Rightarrow \ln\frac{40}{80} = -10K$$

$$\Rightarrow \ln\frac{1}{2} = -10K$$

$$\Rightarrow K = \frac{\ln\frac{1}{2}}{-10}$$

Again, $t=0$, $T=100°c$

$t=40$, $T=T$

$$\therefore \text{(i)} \Rightarrow \int_{100}^{T} \frac{dT}{T-20} = \int_0^{40} -kt$$

$$\Rightarrow \ln[T-20]_{100}^{T} = -k[t]_0^{40}$$

$$\Rightarrow \ln[T-20]-\ln[100-20] = -\left(-\frac{\ln\frac{1}{2}}{10}\right)40$$

$$\Rightarrow \ln[T-20]-\ln 80 = 4\ln\frac{1}{2}$$

$$\Rightarrow \ln[T-20]-\ln 80 = \ln\left(\frac{1}{2}\right)^4$$

$$\Rightarrow \ln\frac{T-20}{80} = \ln\left(\frac{1}{2}\right)^4$$

$$\Rightarrow \frac{T-20}{80} = \left(\frac{1}{2}\right)^4$$

$$\Rightarrow T-20 = \frac{1}{16}\times 80$$

$$\Rightarrow T-20 = 5$$

$$\therefore T = 25°C$$

Page-125 (11)

A 30 volt electromotive force is applied to an LR series circuit in which the inductance in 0.2 henry and the registance is 50Ω find the current $i(t)$ if $I(0)=0$. determine the current after a long time.

Electric circuit differential eqn. is :

$$L\frac{dI}{dt}+RI = V(t) \quad \cdots\cdots (i)$$

Here, $L=0.2=\frac{1}{5}$, $R=50$ $V(t)=30$

$$(i) \Rightarrow \frac{1}{5}\frac{dI}{dt}+50I = 30$$

$$\Rightarrow \frac{dI}{dt}+250I = 150 \quad \cdots\cdots (ii)$$

Here, $I.F = e^{\int 250\,dt} = e^{250t}$

Now multiplying by e^{250t} $\Rightarrow$ (ii) $\Rightarrow$ ~~$Ie^{250t} = 150e^{250t}$~~ (ii

$$\frac{d}{dt}\left[Ie^{250t}\right] = 150e^{250t}$$

~~when, t=0, I=0~~

~~t=∞, I=I~~

integrating both side from $t=0$ to $t=t$

$$\left[i(t)e^{250t}\right]_0^t = 150\int_0^t e^{250t}\,dt$$

$$\Rightarrow i(t)e^{250t} = 150\left[e^{250t}\cdot\frac{1}{250}\right]_0^t$$

$$\Rightarrow i(t)e^{250t} = \frac{150}{250}\left(e^{250t}-e^{0}\right)$$

$$\Rightarrow i(t)e^{250t} = \frac{3}{5}\left(e^{250t}-1\right)$$

$$\Rightarrow i(t) = \frac{3}{5}\left(1-e^{-250t}\right)$$

when, $t\to\infty$ $\quad i(t) = \frac{3}{5}\left(1-e^{\infty}\right) = \frac{3}{5}(1-0) = \frac{3}{5}$

Page 126 (~~P~~. 12)

A 12 volt electromotive force is applicable to an LR series circuit in which the inductance is $\frac{1}{3}$ henry, the resistance is 10Ω. Find the current $f(t)$ if $f(0)=0$. determine the current after a long time.

Electric circuit differential eqn. is:

$$L\frac{dI}{dt} + RI = v(t) \quad \cdots\cdots (i)$$

here, $L=\frac{1}{3}$, $R=10\Omega$ $\quad v(t)=12$

$$(i)\Rightarrow \frac{1}{3}\frac{dI}{dt} + 10I = 12$$

$$\Rightarrow \frac{dI}{dt} + 30I = 36 \quad \cdots\cdots (ii)$$

$$\therefore I.F = e^{\int 30\,dt} = e^{30t}$$

Now multiplying by e^{30t}, eqn (ii)

$$\frac{d}{dt}\left(Ie^{30t}\right) = 36e^{30t} \quad \cdots\cdots (iii)$$

$\Rightarrow$

Integrating both side of (iii) from $t=0$ to $t=t$

$$\left[I(t)e^{30t}\right]_0^t = 36\int_0^t e^{30t}$$

$$\Rightarrow I(t)e^{30t} = \cancel{36\left[e^{30t}\right]_0^t}\ 36\left[e^{30t}\cdot\frac{1}{30}\right]_0^t$$

$$\Rightarrow I(t)\,e^{30t} = 36\left[\frac{e^{30t}}{30} - \frac{e^0}{30}\right]$$

$$\Rightarrow I(t)\,e^{30t} = \frac{36}{30}\left[e^{30t} - 1\right]$$

$$\Rightarrow I(t) = \frac{36}{30}\left[1 - \frac{1}{e^{30t}}\right] = \frac{6}{5}\left[1 - \frac{1}{e^{30t}}\right]$$

when, $t \to \infty$ $\quad I = \frac{6}{5}\left[1 - \frac{1}{e^{\infty}}\right] = \frac{6}{5}\left[1 - 0\right] = \frac{6}{5}$

Liniar Differential Eqn With constant Co-efficients

Let the Differential eqn be: $D^2y + P_1Dy + P_2y = 0$ ...... (i)

Let $y = e^{mx}$, $Dy = me^{mx}$, $D^2y = m^2e^{mx}$

Now from (i) $m^2e^{mx} + P_1me^{mx} + P_2e^{mx} = 0$

$$\Rightarrow (m^2 + P_1m + P_2)e^{mx} = 0$$

$$\Rightarrow m^2 + P_1m + P_2 = 0 \quad \text{(ii)} \quad [\because e^{mx} \neq 0]$$

Solve this eqn: we get the value of $\boxed{m_1, m_2}$

(i) $\boxed{\text{If } m_1 \neq m_2 \text{ and } m_1, m_2 \text{ are real: then: } y = c_1e^{m_1x} + c_2e^{m_2x}}$

(ii) $\boxed{\text{If } m_1 = m_2 \text{ and } m_1, m_2 \text{ are reall: then; } y = (c_1 + c_2x)e^{mx}}$

(iii) If m_1 and m_2 are complex: let, $m = a \pm ib$ then

$$y = e^{ax}\left[c_1\cos bx + c_2\sin bx\right]$$

Page - 165 Solve: $2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ ---- (i)

Let, $y = e^{mx}$ be the trial solution of eqn (i)

$\therefore \frac{dy}{dx} = me^{mx}$ $\quad \frac{d^2y}{dx^2} = m^2e^{mx}$

Now putting the values of y, $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ on eqn (i)

$2m^2e^{mx} - 3me^{mx} + e^{mx} = 0$

$\Rightarrow (2m^2 - 3m + 1)e^{mx} = 0$

$\Rightarrow 2m^2 - 3m + 1 = 0$

$\Rightarrow 2m^2 - 2m - m + 1 = 0$

$\Rightarrow 2m(m-1) - 1(m-1) = 0$

$\Rightarrow (m-1)(2m-1) = 0$ $\quad \therefore m = \frac{1}{2}, 1$

$\therefore$ ~~$y = e^{x/2}c_1 + c_2e^{x}$~~ $\therefore y = c_1e^{x/2} + c_2e^{x}$ [General Solu

Page - 165 - Solve: $(D^2 + 6D + 25)y = 0$ ---- (i)

Let, $y = e^{mx}$ be the trial Solution of (i)

$\therefore$ ~~$\frac{dy}{d}$~~ $Dy = me^{mx}$ $\quad D^2y = m^2e^{mx}$

Now putting the values of y, Dy, D^2y on (i)

$(i) \Rightarrow D^2y + 6Dy + 25y = 0$

$\Rightarrow m^2e^{mx} + 6me^{mx} + 25e^{mx} = 0$

$\Rightarrow (m^2 + 6m + 25)e^{mx} = 0$

$\Rightarrow m^2 + 6m + 25 = 0$

$\Rightarrow m = \frac{-6 \pm \sqrt{6^2 - 4.1.25}}{2}$

$= \frac{-6 \pm \sqrt{36 - 100}}{2}$

$= \frac{-6 \pm \sqrt{-64}}{2}$

$$= \frac{-6 \pm \sqrt{(8i)^2}}{2}$$

$$= \frac{-6 \pm 8i}{2}$$

$$= -3 \pm 4i$$

$\therefore$ General Solution is : $y = e^{-3x}[c_1 \cos 4x - c_2 \sin 4x]$

Page – 166 1(iii) Solve: $(D^3 - 5D^2 + 7D - 3)y = 0$ ... (i)

let, $y = e^{mx}$ be the trial solution of (i)

$\therefore Dy = me^{mx}$, $D^2y = m^2e^{mx}$, $D^3y = m^3e^{mx}$

Now from (i) $\Rightarrow m^3e^{mx} - 5m^2e^{mx} + 7me^{mx} - 3e^{mx} = 0$

$\Rightarrow (m^3 - 5m^2 + 7m - 3)e^{mx} = 0$

$\Rightarrow m^3 - 5m^2 + 7m - 3 = 0$

$\Rightarrow m^3 - m^2 - 4m^2 + 4m + 3m - 3 = 0$

$\Rightarrow m^2(m-1) - 4m(m-1) + 3(m-1) = 0$

$\Rightarrow (m-1)(m^2 - 4m + 3) = 0$

$\Rightarrow (m-1)(m^2 - m - 3m + 3) = 0$

$\Rightarrow (m-1)\{m(m-1) - 3(m-1)\} = 0$

$\Rightarrow (m-1)(m-1)(m-3)$

$\therefore m = 1, 1, 3$

$\therefore$ General Solution is: $(c_1 + c_2x)e^x + c_3e^{3x}$

Page 171 : 2(i) Solve: $\frac{d^3y}{dx^3}-\frac{d^2y}{dx^2}-8\frac{dy}{dx}+12y=0$

let, $y=e^{mx}$ be the trial solution of (i)

$\therefore \frac{dy}{dx}=me^{mx}$, $\frac{d^2y}{dx^2}=m^2e^{mx}$ $\frac{d^3y}{dx^3}=m^3e^{mx}$

from (i) $\Rightarrow m^3e^{mx}-m^2e^{mx}-8me^{mx}+12e^{mx}=0$

$\Rightarrow (m^3-m^2-8m+12)e^{mx}=0$

$\Rightarrow m^3-m^2-8m+12=0$

$\Rightarrow m^3-2m^2+m^2-2m-6m+12=0$

$\Rightarrow m^2(m-2)+m(m-2)-6(m-2)=0$

$\Rightarrow (m-2)(m^2+m-6)=0$

$\Rightarrow (m-2)(m^2+3m-2m-6)=0$

$\Rightarrow (m-2)\{m(m+3)-2(m+3)\}=0$

$\Rightarrow (m-2)(m+3)(m-2)=0$

$\therefore m=2,2,-3$

$\therefore$ General Solution: $(c_1+c_2x)e^{2x}+c_3e^{-3x}$

2(ii) Solve: $(D^4+2D^2-3)y=0$ ——— (i)

let $y=e^{mx}$ be the trial Solution of (i)

$\therefore Dy=me^{mx}$, $D^2y=m^2e^{mx}$, $D^4y=m^4e^{mx}$

(i) $\Rightarrow m^4e^{mx}+2m^2e^{mx}-3e^{mx}=0$

$\Rightarrow (m^4+2m^2-3)e^{mx}=0$

$\Rightarrow m^4+2m^2-3=0$

$\Rightarrow m^4+3m^2-m^2-3=0$

$\Rightarrow m^2(m^2+3)-1(m^2+3)=0$

$\Rightarrow (m^2+3)(m^2-1)=0$

$\Rightarrow m^2 = -3 \quad , \quad m^2 - 1 = 0$

$\Rightarrow m^2 = 3i^2 \qquad m^2 = 1$

$\therefore m = \pm\sqrt{3}i, \quad m = \pm 1$

$\therefore$ General Solution: $y = c_1 e^x + c_2 e^{-x} + [c_3 \cos\sqrt{3}x + c_4 \sin\sqrt{3}x] e^0$

$= c_1 e^x + c_2 e^{-x} + c_3 \cos\sqrt{3}x + c_4 \sin\sqrt{3}x$

~~Ex~~ - ③ Solve: $5\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 3y = 0$ ——— ①

Let, $y = e^{mx}$ be the trial solution of (i)

$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2 e^{mx}$

$(i) \Rightarrow 5m^2 e^{mx} - 2me^{mx} + 3e^{mx} = 0$

$\Rightarrow (5m^2 - 2m + 3) e^{mx} = 0$

$\Rightarrow 5m^2 - 2m + 3 = 0$

$\cancel{\Rightarrow 5m^2 - 5m + 3m + 3 = 0}$

$\cancel{\Rightarrow 5m(m-1) + 3}$

$m = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\cdot 5\cdot 3}}{2\cdot 5}$

$= \frac{2 \pm \sqrt{4 - 60}}{10}$

$= \frac{2 \pm \sqrt{-56}}{10}$

$= \frac{2 \pm \sqrt{56i^2}}{10}$

$= \frac{2 \pm 2i\sqrt{14}}{10} = \frac{1}{5} \pm \frac{i\sqrt{14}}{5}$

General Solution: $y = e^{x/5}\left[c_1 \cos\frac{\sqrt{14}}{5} + c_2 \sin\frac{\sqrt{14}}{5}\right]$

9(i) Find the Particular Solution of $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=0$ when $y(0)=0$ and $y'(0)=1$

Let, $y=e^{mx}$ be the trial solution of (i)

$\therefore \frac{dy}{dx}=me^{mx} \quad \frac{d^2y}{dx^2}=m^2e^{mx}$

(i) $m^2e^{mx}+3me^{mx}+2e^{mx}=0$

$\Rightarrow (m^2+3m+2)e^{mx}=0$

$\Rightarrow (m^2+3m+2)=0$

$\Rightarrow m^2+2m+m+2=0$

$\Rightarrow m(m+2)+(m+2)=0$

$\Rightarrow (m+2)(m+1)=0$

$\therefore m=-1,-2$

$\therefore y=c_1e^{-x}+c_2e^{-2x}$

$\therefore y(x)=c_1e^{-x}+c_2e^{-2x}$

$\therefore y(0)=c_1e^0+c_2e^0 \Rightarrow c_1+c_2=0 \quad \cdots\cdots (ii)$

and, $y'(x)=-c_1e^{-x}-2c_2e^{-2x}$

$\therefore y'(0)=-c_1-2c_2=1 \quad \cdots\cdots (iii)$

from (ii), $c_1=-c_2$

from (iii) $-(-c_2)-2c_2=1$

$\Rightarrow c_2-2c_2=1$

$\Rightarrow -c_2=1 \quad \therefore c_2=-1$ and $c_1=-(-1)=1$

$\therefore y=e^{-x}-e^{-2x}$

(Ans)

Page - 171   9 (ii) Find the Particular Solution of $\frac{d^2s}{dt^2} + 8\frac{ds}{dt} + 25s = 0$ —(i)

when $s(0) = 4$ and $s'(0) = -16$

Let, $s = e^{mt}$ be the trial solution of (i)

$\therefore \frac{ds}{dt} = me^{mt}$, $\frac{d^2s}{dt^2} = m^2e^{mt}$

$(i) \Rightarrow m^2e^{mt} + 8me^{mt} + 25e^{mt} = 0$

$\Rightarrow (m^2 + 8m + 25)e^{mt} = 0$

$\Rightarrow m^2 + 8m + 25 = 0$

$\Rightarrow m = \frac{-8 \pm \sqrt{8^2 - 4.1.25}}{2.1}$

$= \frac{-8 \pm \sqrt{64 - 100}}{2} = \frac{-8 \pm \sqrt{-36}}{2} = \frac{-8 \pm \sqrt{36i^2}}{2} = \frac{-8 \pm 6i}{2}$

$= -4 \pm 3i$

$\therefore$ General Solution: $s = e^{-4t}[c_1\cos 3t + c_2 \sin 3t]$ ---- -(ii)

$\therefore s(t) = e^{-4t}[c_1\cos 3t + c_2 \sin 3t]$

$\Rightarrow s(0) = c_1\cos 0 + c_2 \sin 0 = c_1 = 4$

Now, $s'(t) = -4e^{-4t}[c_1\cos 3t + c_2\sin 3t] +$
$e^{-4t}[-3c_1\sin 3t + 3c_2\cos 3t]$

$\therefore s'(0) = -4[c_1 + 0] + [0 + 3c_2] = -16$

$\Rightarrow -4c_1 + 3c_2 = -16$

$\Rightarrow -4 \times 4 + 3c_2 = -16$

$\Rightarrow -16 + 3c_2 = -16$

$\Rightarrow 3c_2 = 0 \quad \therefore c_2 = 0$

$(ii) \Rightarrow s = e^{-4t}[4\cos 3t + 0]$

$\therefore s(t) = 4e^{-4t}\cos 3t$

Liniar Differential Eqn. is not zero

$$(D^2+P_1D+P_2)y = R(x) \quad \cdots\cdots (i)$$

Let, $(D^2+P_1D+P_2)y = 0 \quad \cdots\cdots (ii)$

$\frac{1}{D} = \int dx$, $\frac{1}{D}x = \int x$

সূত্র:

$(1-D)^{-1} = 1+D+D^2+D^3+\cdots$

$(1+D)^{-1} = 1-D+D^2-D^3+\cdots$

$(1-D)^{-2} = 1+2D+3D^2+4D^3+$

$(1+D)^{-2} = 1-2D+3D^2-4D^3+$

Page-184

1(i) Solve: $\frac{d^2y}{dx^2}+9y=5x^2$ ..... (i) Now, $\frac{d^2y}{dx^2}+9y=0$ ..... (ii)

Let $y=e^{mx}$ be a trial solution of (ii)

$\therefore \frac{dy}{dx}=me^{mx}$ $\frac{d^2y}{dx^2}=m^2e^{mx}$

$(i) \Rightarrow m^2e^{mx}+9e^{mx}=0$

$\Rightarrow (m^2+9)e^{mx}=0$

$\Rightarrow m^2+9=0$

$\Rightarrow m^2=-9=9i^2$

$\therefore m=\pm 3i$

$\therefore y_c=c_1\cos 3x+c_2\sin 3x$

Now, (i) can be written as:

$$(D^2+9)y=5x^2$$

$$\therefore y_p=\frac{1}{D^2+9}5x^2$$

$$=\frac{1}{9(D^2/9+1)}5x^2$$

$$=\frac{1}{9}\left[1+\frac{D^2}{9}\right]^{-1}5x^2$$

$$=\frac{1}{9}\left[\cancel{5x^2}\ 1-\frac{D^2}{9}+\cdots\right]5x^2$$

$$=\frac{1}{9}\left[5x^2-\frac{1}{9}D^2(5x^2)+0\right]$$

$$=\frac{1}{9}\left[5x^2-\frac{5\cdot 2}{9}\right]$$

$$=\frac{1}{9}\left[5x^2-\frac{10}{9}\right]$$

$$\therefore y=y_c+y_p=c_1\cos 3x+c_2\sin cx+\frac{1}{9}\left[5x^2-\frac{10}{9}\right]$$

Solve

1(ii) $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 1+x+x^2 \quad \cdots \quad \cdots (i)$

Now, $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0 \quad \cdots \quad \cdots (ii)$

Let, $y = e^{mx}$ be a trial solution of (ii)

$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2e^{mx}$

$(ii) \Rightarrow m^2e^{mx} - 6me^{mx} + 9e^{mx} = 0$

$\Rightarrow (m^2 - 6m + 9)e^{mx} = 0$

$\Rightarrow m^2 - 6m + 9 = 0$

$\Rightarrow m^2 - 3m - 3m + 9 = 0$

$\Rightarrow m(m-3) - 3(m-3) = 0$

$\Rightarrow (m-3)(m-3) \quad \therefore m = 3, 3$

$\therefore y_c = (c_1 + c_2x)e^{3x}$

Now (i) can be written as:

$(D^2 - 6D + 9)y = 1 + x + x^2$

$\Rightarrow y_p = \frac{1}{D^2 - 6D + 9}(1+x+x^2)$

$= \frac{1}{(3-D)^2}(1+x+x^2)$

$= \frac{1}{9(1-\frac{D}{3})^2}(1+x+x^2)$

$= \frac{1}{9}\left(1 - \frac{D}{3}\right)^{-2}(1+x+x^2)$

$= \frac{1}{9}\left[1 + \frac{2D}{3} + 3\frac{D^2}{9} + \cdots\right)(1+x+x^2)$

$= \frac{1}{9}\left[(1+x+x^2) + \frac{2}{3}D(1+x+x^2) + \frac{1}{3}D^2(1+x+x^2) + 0\right]$

$= \frac{1}{9}\left[1+x+x^2 + \frac{2}{3}(1+2x) + \frac{1}{3}(2)\right]$

$= \frac{1}{9}\left[x^2 + x + \frac{4}{3}x + 1 + \frac{2}{3} + \frac{2}{3}\right]$

$= \frac{1}{9}\left[x^2 + \frac{7}{3}x + \frac{7}{3}\right] = \frac{1}{27}\left[3x^2 + 7x + 7\right]$

$\therefore y = y_c + y_p = (c_1 + c_2x)e^{3x} + \frac{1}{27}\left[3x^2 + 7x + 7\right]$

Page-190 solve: Find the particular solution of

3. $\frac{d^2y}{dx^2} - 9y = 3x$ where, $y(0)=0$ and $y'(0)=0$ ।

given, $\frac{d^2y}{dx^2} - 9y = 3x$ ... ...(i)

Now, $\frac{d^2y}{dx^2} - 9y = 0$ ... ...(ii), Let $y = e^{mx}$ be a solution of (ii)

$\therefore \frac{dy}{dx} = me^{mx}$, $\frac{d^2y}{dx^2} = m^2e^{mx}$

(ii) $\Rightarrow m^2e^{mx} - 9e^{mx} = 0$

$\Rightarrow (m^2-9)\, e^{mx} = 0$

$\Rightarrow m^2 - 9 = 0$ $\quad \therefore m = \pm 3$

$\therefore y_c = c_1e^{-3x} + c_2e^{3x}$

from, (i) $\Rightarrow (D^2-9)\,y = 3x$

$\Rightarrow y_P = \frac{1}{D^2-9}\, 3x$

$= \frac{1}{-9\left(1-\frac{D^2}{9}\right)}\, 3x$

$= -\frac{1}{9}\left[1-\frac{D^2}{9}\right]^{-1} 3x$

$= -\frac{1}{3}\left[1+\frac{D^2}{9}+\cdots\right] x$

$= -\frac{1}{3}[x+0] = -\frac{1}{3}x$

$\therefore y = y_c + y_P = c_1e^{-3x} + c_2e^{3x} - \frac{1}{3}x$ ... ...(iii)

$\Rightarrow y(x) = c_1e^{-3x} + c_2e^{3x} - \frac{1}{3}x$

$\therefore y(0) = c_1 + c_2 = 0$

$\therefore c_1 = -c_2$

and, $y'(x) = -3c_1e^{-3x} + 3c_2e^{3x} - \frac{1}{3}$

$\therefore y'(0) = -3c_1 + 3c_2 - \frac{1}{3} = 0$

$\Rightarrow -3(-c_2) + 3c_2 = \frac{1}{3}$

$\Rightarrow 3c_2 + 3c_2 = \frac{1}{3}$

$\Rightarrow 6c_2 = \frac{1}{3}$

$\therefore c_2 = \frac{1}{18}$ and $c_1 = -\frac{1}{18}$

(iii) $\Rightarrow y = -\frac{1}{18}e^{-3x} + \frac{1}{18}e^{3x} - \frac{1}{3}x$

(Ans)

1(i) Solve: $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$ ...... (i)

Let, $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$ ...... (ii)

Let, $y = e^{mx}$ be a trial solution of (ii)

$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2e^{mx}$

(ii) $\Rightarrow m^2e^{mx} - 3me^{mx} + 2e^{mx} = 0$

$\Rightarrow (m^2 - 3m + 2)e^{mx} = 0$

$\Rightarrow m^2 - 3m + 2 = 0$

$\Rightarrow m^2 - 2m - m + 2 = 0$

$\Rightarrow m(m-2) - (m-2) = 0$

$\Rightarrow (m-2)(m-1) = 0 \quad \therefore m = 1, 2$

$\therefore y_c = c_1e^{x} + c_2e^{2x}$

Now, (i) can be written as:

$(D^2 - 3D + 2)y = e^{5x}$

$\Rightarrow y_p = \frac{1}{(D^2 - 3D + 2)} e^{5x}$ $\boxed{D=5 \text{ বসিয়ে}}$

$= \frac{1}{5^2 - 3\cdot 5 + 2} e^{5x}$

$= \frac{1}{12} e^{5x}$

$\therefore y = y_c + y_p = c_1e^{x} + c_2e^{2x} + \frac{1}{12}e^{5x}$

1(ii) Solve: $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 2e^{x} + 10e^{5x}$ ...... (i)

Now, $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0$ ... (ii)

Let, $y = e^{mx}$ be a trial solution of (ii)

$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2e^{mx}$

(ii) $\Rightarrow m^2e^{mx} + 6me^{mx} + 5e^{mx} = 0$

$\Rightarrow (m^2 + 6m + 5)e^{mx} = 0$

$\Rightarrow m^2 + 6m + 5 = 0$

$\Rightarrow m^2 + 5m + m + 5 = 0$

$\Rightarrow m(m+5)+(m+5)=0$

$\Rightarrow (m+5)(m+1)=0 \qquad \therefore m=-1,-5$

$\therefore y_c = c_1e^{-x}+c_2e^{-5x}$

$(i) \Rightarrow (D^2+6D+5)y = 2e^{x}+10e^{5x}$

$\Rightarrow y_P = \frac{1}{D^2+6D+5}(2e^{x}+10e^{5x})$

$= \frac{1}{D^2+6D+5}2e^{x}+\frac{1}{D^2+6D+5}\cdot 10e^{5x}$

$= \frac{1}{1^2+6\cdot1+5}\cdot 2e^{x}+\frac{1}{5^2+6\cdot5+5}\cdot 10e^{5x}$

$= \frac{1}{6}e^{x}+\frac{1}{6}e^{5x}$

$\therefore y = y_c+y_P = c_1e^{-x}+c_2e^{-5x}+\frac{1}{6}e^{x}+\frac{1}{6}e^{5x}$

Right Rhnd side with x term of sin ax or cosax

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1(i) Solve : $(D^2+4)y = \sin 2x \sin x$ ......(i)

Let, $(D^2+4)y = 0$ ...... (ii)

Let, $y = e^{mx}$ be a trial solution of (ii)

$\therefore Dy = me^{mx} \quad D^2y = m^2e^{mx}$

$\therefore (ii) \Rightarrow m^2e^{mx} + 4e^{mx} = 0$

$\Rightarrow (m^2+4)e^{mx} = 0$

$\Rightarrow m^2+4 = 0$

$\Rightarrow m^2 = -4$

$\Rightarrow m^2 = 4i^2 \quad \therefore m = \pm 2i$

$\therefore y_c = c_1 \cos 2x + c_2 \sin 2x$

~~2 sinB sinD = cos(A−B)~~

$2\sin C \sin D = \cos(C-D) - \cos(C+D)$

$\Rightarrow \sin C \sin D = \frac{1}{2}[\cos(C-D) - \cos(C+D)]$

$(i) \Rightarrow y_p = \frac{1}{D^2+4} \sin 2x \sin x$

$= \frac{1}{D^2+4} \frac{1}{2}[\cos(2x-x) - \cos(2x+x)]$

$= \frac{1}{D^2+4} \frac{1}{2}[\cos x - \cos 3x]$

$= \frac{1}{2(D^2+4)} \cos x - \frac{1}{2(D^2+4)} \cos 3x$

$= \cancel{\frac{1}{2\{(-1)^2+4\}} \cos x - \frac{1}{2\{(-3)^2+4\}}}$

$= \frac{1}{2(-1^2+4)} \cos x - \frac{1}{2(-3^2+4)} \cos 3x$

$= \frac{1}{2(-1+4)} \cos x - \frac{1}{2(-9+4)} \cos 3x$

$= \frac{1}{2\times 3} \cos x - \frac{1}{2\times(-5)} \cos 3x$

$= \frac{1}{6} \cos x + \frac{1}{10} \cos 3x$

D= −(সহগের) power

$y = y_c + y_p = c_1 \cos 2x + c_2 \sin 2x + \frac{1}{6} \cos x + \frac{1}{10} \cos 3x$

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2 (ii) : Solve :- $\frac{d^2y}{dx^2}+y=\cos^2 x$ .... (i)

Let, $\frac{d^2y}{dx^2}+y=0$ .... (ii)

Now, $y=e^{mx}$ be a trial solution of (ii)

$\therefore \frac{dy}{dx}=me^{mx}$, $\frac{d^2y}{dx^2}=m^2e^{mx}$

(ii) $\Rightarrow m^2e^{mx}+e^{mx}=0$

$\Rightarrow (m^2+1)e^{mx}=0$

$\Rightarrow m^2+1=0$

$\Rightarrow m^2=-1$

$\Rightarrow m^2 = i^2$

$\therefore m=\pm i$

$\therefore y_c=c_1\cos x+c_2\sin x$

$2\cos^2 x=1+\cos 2x$

$\Rightarrow \cos^2 x=\frac{1}{2}(1+\cos 2x)$

(i) $\Rightarrow (D^2+1)y=\cos^2 x$

$\Rightarrow y_p=\frac{1}{D^2+1}\cos^2 x$

$=\frac{1}{D^2+1}\left\{\frac{1}{2}(1+\cos 2x)\right\}$

$=\frac{1}{2(D^2+1)}+\frac{1}{2(D^2+1)}\cos 2x$

$=\frac{1}{2(D^2+1)}\cdot e^{0x}+\frac{1}{2(D^2+1)}\cos 2x$

$=\frac{1}{2(0^2+1)}\cdot 1+\frac{1}{2(-2^2+1)}\cos 2x$

$=\frac{1}{2}+\frac{1}{2(-4+1)}\cos 2x$

$=\frac{1}{2}-\frac{1}{6}\cos 2x$

$\therefore y=y_c+y_p=c_1\cos x+c_2\sin x+\frac{1}{2}-\frac{1}{6}\cos 2x$

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3(i) Solve : $(D^3-3D^2+4D-2)y = e^x+\cos x$ ... ---(i)

$(D^3-3D^2+4D-2)y=0$ ... ---(ii)

Let, $y=e^{mx}$ be a trial solution of (ii)

$\therefore Dy = me^{mx}$, $D^2y = m^2e^{mx}$ $D^3y = m^3e^{mx}$

(ii) $\Rightarrow m^3e^{mx}-3m^2e^{mx}+4me^{mx}-2e^{mx}=0$

$\Rightarrow (m^3-3m^2+4m-2)e^{mx}=0$

$\Rightarrow m^3-3m^2+4m-2=0$

$\Rightarrow m^3-m^2-2m^2+2m+2m-2=0$

$\Rightarrow m^2(m-1)-2m(m-1)+2(m-1)=0$

$\Rightarrow (m-1)(m^2-2m+2)=0$

$m=1$ or, $m^2-2m+2=0$

$\therefore m = \frac{-(-2)\pm\sqrt{(-2)^2-4\cdot1\cdot2}}{2\cdot1}$

$= \frac{2\pm\sqrt{-4}}{2}$

$= \frac{2\pm\sqrt{4i^2}}{2}$

$= \frac{2\pm2i}{2}$

$= \cancel{1\pm2i}$ $1\pm i$

$\therefore y_c = e^x[c_1\cos x+c_2\sin x]+c_3e^x$

(i) $\Rightarrow y_p = \frac{1}{D^3-3D^2+4D-2}(e^x+\cos x)$

$= \frac{1}{D^3-3D^2+4D-2}e^x+\frac{1}{D^3-3D^2+4D-2}\cos x$

$= \frac{x}{3D^2-6D+4}e^x+\frac{1}{-D+3+4D-2}\cos x$

Here,
1st part: ~~Inde~~ Dif
2nd Part: $D^2=-$coff of x

$= \frac{x}{3-6+4}e^x+\frac{1}{1+3D}\cos x$

$= xe^x+\frac{1-3D}{1-9D^2}\cos x$

$= xe^x+\frac{1}{1+9}(1-3D)\cos x$

$= xe^x+\frac{1}{10}(\cos x+3\sin x)$

$\therefore y = y_c+y_p = e^x[c_1\cos x+c_2\sin x]+c_3e^x+xe^x+\frac{1}{10}(\cos x+3\sin x)$

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3 (ii) Solve: $(D^2-2D+1)y = x+\sin x \quad \cdots \quad (i)$

$$(D^2-2D+1)y=0 \quad \cdots \quad (ii)$$

Let, $y=e^{mx}$ be a trial solution of (ii)

$\therefore Dy = me^{mx}$, $D^2y = m^2e^{mx}$

(ii) $\Rightarrow m^2e^{mx} - 2me^{mx} + e^{mx} = 0$

$\Rightarrow (m^2-2m+1)e^{mx}=0$

$\Rightarrow m^2-2m+1=0$

$\Rightarrow (m-1)^2=0 \qquad \therefore m=1,1$

$\therefore y_c = \cancel{e^{mx}}\ (c_1+c_2x)e^x$

(i) $\Rightarrow \quad y_P = \dfrac{1}{D^2-2D+1}(x+\sin x)$

$$= \frac{1}{D^2-2D+1}x + \frac{1}{D^2-2D+1}\sin x$$

$$= \frac{1}{(D-1)^2}x + \frac{1}{-1-2D+1}\sin x$$

$$= (D-1)^{-2}x + \frac{1}{-2D}\sin x$$

$$= (1+2D+3D^2+\cdots\cdots]x - \frac{1}{2}\frac{1}{D}(\sin x)$$

$$= (x+2+0) - \frac{1}{2}\frac{D}{D^2}\sin x$$

$$= x+2-\frac{1}{2}\cdot\frac{1}{-1}D(\sin x)$$

$$= x+2+\frac{1}{2}\cos x$$

$$\therefore y = y_c + y_P = (c_1+c_2x)e^x + x + 2 + \frac{1}{2}\cos x$$

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1 (ii) Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^{2x}$ —— (i)

$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$ —— (ii)

Let, $y = e^{mx}$ be a trial solution of (ii)

$\therefore \frac{dy}{dx} = me^{mx}, \quad \frac{d^2y}{dx^2} = m^2 e^{mx}$

$(ii) \Rightarrow m^2 e^{mx} - 4me^{mx} + 4e^{mx} = 0$

$\Rightarrow (m^2 - 4m + 4)e^{mx} = 0$

$\Rightarrow m^2 - 4m + 4 = 0$

$\Rightarrow (m-2)^2 = 0 \qquad \therefore m = 2, 2$

$\therefore y_c = (c_1 + c_2 x)e^{2x}$

(i) can be written as:

$(D^2 - 4D + 4)y = x^2 e^{2x}$

$\Rightarrow y_p = \frac{1}{D^2 - 4D + 4} x^2 e^{2x}$

$= \frac{1}{(D-2)^2} x^2 e^{2x}$

$= e^{2x} \frac{1}{(D-2+2)^2} x^2$

e^{2x} -এর x এর স্থানে যোগ করে

$= e^{2x} \frac{1}{D^2} x^2$

$\frac{1}{D} = \int dx$

$= e^{2x} \frac{1}{D} \cdot \frac{1}{D} x^2$

$= e^{2x} \frac{1}{D} \int x^2 dx$

$= e^{2x} \frac{1}{D} \frac{x^3}{3}$

$= e^{2x} \int \frac{x^3}{3} dx$

$= e^{2x} \frac{x^4}{3 \times 4}$

$= \frac{1}{12} x^4 e^{2x}$

$\therefore y = (c_1 + c_2 x)e^{2x} + \frac{1}{12} x^4 e^{2x}$

Page-237 Solve: $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ . . . (i)

2(i) Let, $x=e^z$ $\therefore z=\ln x$ and $\frac{dz}{dx}=\frac{1}{x}$ . . . . . . (ii)

Now, $\frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}$

$\Rightarrow \frac{dy}{dx}=\frac{dy}{dx}\cdot\frac{1}{x}$

$\Rightarrow x\frac{dy}{dx}=\frac{dy}{dz}$

$\Rightarrow x\frac{dy}{dx}=Dy$ when $\frac{d}{dx}=D$

Again, $\frac{d^2y}{dx^2}=\frac{d}{dx}\left[\frac{dy}{dx}\right]$

$=\frac{d}{dx}\left[\frac{1}{x}\frac{dy}{dz}\right]$

$=\frac{1}{x}\frac{d}{dx}\left[\frac{dy}{dz}\right]+\frac{dy}{dx}\frac{d}{dx}\left[\frac{1}{x}\right]$

$=\frac{1}{x}\frac{d}{dx}\left[\frac{dy}{dz}\right]+\frac{dy}{dx}\left[-\frac{1}{x^2}\right]$

$=\frac{1}{x}\frac{d}{dx}\left[\frac{dy}{dz}\right]-\frac{1}{x^2}\frac{dy}{dx}$

$\cancel{=\frac{1}{x}\frac{d}{dz}\left[\frac{dy}{dx}\right]-\frac{1}{x^2}\frac{dy}{dx}}$

$=\frac{1}{x}\frac{d}{dz}\left[\frac{dy}{dz}\right]\frac{dz}{dx}-\frac{1}{x^2}\frac{dy}{dx}$

$=\frac{1}{x}\frac{d^2y}{dz^2}\cdot\frac{1}{x}-\left(\frac{1}{x^2}\frac{dy}{dx}\right)$

$=\frac{1}{x^2}\frac{d^2y}{dz^2}-\frac{1}{x^2}\frac{dy}{dx}$

$=\frac{1}{x^2}\left[\frac{d^2y}{dz^2}-\frac{dy}{dz}\right]$

$\Rightarrow x^2\frac{d^2y}{dx^2}=D^2y-Dy$

$=D(D-1)y$

Similarly, $x^3\frac{d^3y}{dx^3}=D(D-1)(D-2)y$

putting these values in (i)

$$D(D-1)(D-2)y + 3D(D-1)y - 2Dy + 2y = 0$$

$$\Rightarrow (D^3 - 2D^2 - D^2 + 2D) + (3D^2 - 3D)y - 2Dy + 2y = 0$$

$$\Rightarrow (D^3 - 2D^2 - D^2 + 2D + 3D^2 - 3D - 2D + 2)y = 0$$

$$\Rightarrow (D^3 - 3D + 2)y = 0 \quad \cdots \quad \cdots (iii)$$

Let, $y = e^{mz}$ be a trial solution of (iii)

$$\therefore Dy = me^{mz}, \; D^2y = m^2e^{mz}, \; D^3y = m^3e^{mz}$$

$$\therefore m^3e^{mz} - 3me^{mz} + 2e^{mz} = 0$$

$$\Rightarrow (m^3 - 3m + 2)e^{mz} = 0$$

$$\Rightarrow m^3 - 3m + 2 = 0$$

$$\Rightarrow m^3 - m^2 + m^2 - m - 2m + 2 = 0$$

$$\Rightarrow m^2(m-1) + m(m-1) - 2(m-1) = 0$$

$$\Rightarrow (m-1)(m^2 + m - 2) = 0$$

$$\therefore m = 1, 1, -2$$

$\therefore$ General Solution: $(c_1 + c_2 z)e^{z} + c_3 e^{-2z}$

$$y = (c_1 + c_2 \ln x)e^{\ln x} + c_3 e^{-2\ln x}$$

$$y = (c_1 + c_2 \ln x)x + c_3 x^{-2}$$

$$\therefore y = (c_1 + c_2 \ln x)x + \frac{c_3}{x^2}$$

Ex – 7(ii) Solve : $(1+2x)^2\frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$ —— (i)

Let, $1+2x = e^z$ $\therefore z = \ln(1+2x)$ $\therefore \frac{dz}{dx} = \frac{2}{1+2x}$ .... (ii)

Now, $\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$

$= \frac{dy}{dz}\left[\frac{2}{1+2x}\right]$ ... ...(iii)

$\Rightarrow (1+2x)\frac{dy}{dx} = 2Dy$ when, $D = \frac{d}{dz}$

Again, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$

$= \frac{d}{dx}\left[\frac{dy}{dz}\cdot\frac{2}{1+2x}\right]$

$= \frac{dy}{dz}\frac{d}{dx}\left[\frac{2}{1+2x}\right] + \frac{2}{1+2x}\frac{d}{dx}\left(\frac{dy}{dz}\right)$

$= \frac{dy}{dz}\left[-\frac{2\cdot 2}{(1+2x)^2}\right] + \frac{2}{1+2x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}$

$= -\frac{4}{(1+2x)^2}\frac{dy}{dz} + \frac{2}{1+2x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{2}{1+2x}$

$= -\frac{4}{(1+2x)^2}\frac{dy}{dz} + \frac{4}{(1+2x)^2}\frac{d}{dz}\left(\frac{dy}{dz}\right)$

$= \frac{4}{(1+2x)^2}\frac{d^2y}{dz^2} - \frac{4}{(1+2x)^2}\frac{dy}{dz}$

$= \frac{1}{(1+2x)^2}\left[4D^2y - 4Dy\right]$

$\Rightarrow (1+2x)^2\frac{d^2y}{dx^2} = 4D^2y - 4Dy$

$= 4(D-1)Dy$

Now from (i) $\Rightarrow$ $4(D-1)Dy - 6\cdot 2Dy + 16y = 8(e^z)^2$

$\Rightarrow (4D^2 - 4D)y - 12Dy + 16y = 8e^{2z}$

$\Rightarrow (D^2 - D)y - 3Dy + 4y = 2e^{2z}$

$\Rightarrow (D^2 - D - 3D + 4)y = 2e^{2z}$

$\Rightarrow (D^2 - 4D + 4)y = 2e^{2z}$ ... ...(iv)

Let, $(D^2-4D+4)y=0 \quad \cdots \quad \cdots (v)$

Let, $y=e^{mz}$ be a trial solution of (v)

$Dy = me^{mz}$, $D^2y = m^2e^{mz}$

(v) $\Rightarrow m^2e^{mz} - 4me^{mz} + 4e^{mz} = 0$

$\Rightarrow (m^2-4m+4)e^{mz} = 0$

$\Rightarrow m^2-4m+4 = 0$

$\therefore m = 2, 2$

$\therefore y_c = (c_1+c_2z)e^{2z} = [c_1+c_2\ln(1+2x)]e^{2\ln(1+2x)}$

$= [c_1+c_2\ln(1+2x)]\, e^{\ln(1+2x)^2}$

$= [c_1+c_2\ln(1+2x)](1+2x)^2$

$$y_p = \frac{1}{D^2-4D+4}\, 2e^{2z}$$

$$= \frac{z}{2D-4}\, 2e^{2z}$$

$$= \frac{z \cdot z}{2-0}\, 2e^{2z}$$

$$= \frac{z^2}{\cancel{2}}\, \cancel{2}e^{2z}$$

$= z^2e^{2z}$

$= \{\ln(1+2x)\}^2 e^{2\ln(1+2x)}$

$= 2(1+2x)\{\ln(1+2x)\}^2$

$\therefore y = y_c + y_p = [c_1+c_2\ln(1+2x)](1+2x)^2 + 2(1+2x)\{\ln(1+2x)\}^2$

Page-240 2(i) Solve: $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = x + x^2\ln x$ ... (i)

let, $x = e^z$ $\therefore z = \ln x$ $\frac{dz}{dx} = \frac{1}{x}$

Now, $\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$

$\Rightarrow \frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{1}{x}$

$\Rightarrow x\frac{dy}{dx} = \frac{dy}{dz}$

$\Rightarrow x\frac{dy}{dx} = Dy$ where, $D = \frac{d}{dz}$

Again, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left[\frac{1}{x}\frac{dy}{dz}\right]$

$= \frac{1}{x}\frac{d}{dx}\left(\frac{dy}{dz}\right) - \frac{1}{x^2}\frac{dy}{dz}$

$= \frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx} - \frac{1}{x^2}\frac{dy}{dz}$

$= \frac{1}{x}\frac{d^2y}{dz^2}\frac{1}{x} - \frac{1}{x^2}\frac{dy}{dz}$

$= \frac{1}{x^2}\frac{d^2y}{dz^2} - \frac{1}{x^2}\frac{dy}{dz}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{x^2}\left[D^2y - Dy\right]$

$\Rightarrow x^2\frac{d^2y}{dx^2} = D(D-1)y$

~~let, y = e^{mz} be a trial solution of D^2 4~~

(i) $\Rightarrow D(D-1)y - 3Dy + 4y = e^z + (e^z)^2 z$

$\Rightarrow D^2y - Dy - 3Dy + 4y = e^z + ze^{2z}$ ———— (ii)

~~let, y = e^{mz} be a trial solution of D^2~~

$\Rightarrow (D^2 - 4D + 4)y = e^z + ze^{2z}$ ... ... (ii)

let, $y = e^{mz}$ be a trial solution of $(D^2 - 4D + 4)y = 0$ ... (iii)

$\therefore Dy = me^{mz}$

$\Rightarrow (m^2-4m+4)e^{mz}=0$

$\Rightarrow m^2-4m+4=0$

$\Rightarrow (m-2)^2=0 \qquad \therefore m=2,2$

$\therefore y_c = [c_1+c_2 z]e^{2z}$

$= [c_1+c_2 \ln x]e^{2\ln x}$

$= [c_1+c_2 \ln x]x^2$

(ii) $\Rightarrow y_p = \frac{1}{D^2-4D+1}(e^z+ze^{2z})$

$= \frac{1}{(D-2)^2}(e^z+ze^{2z})$

$= \frac{1}{(D^2-2)^2}e^z + \frac{1}{(D-2)^2}ze^{2z}$

$= \frac{1}{(1+2)^2}e^z + e^{2z}\frac{1}{(D-2+2)^2}z$

$= \frac{1}{(-1)^2}e^z + e^{2z}\frac{1}{D^2}z$

$= e^z+e^{2z}\frac{1}{D}\frac{1}{D}z \qquad \frac{1}{D} \rightarrow$ integration

$= e^z+e^{2z}\frac{1}{D}\frac{z^2}{2}$

$= e^z+(e^z)^2\frac{z^3}{6}$

$= x+x^2\frac{(\ln x)^3}{6}$

$= x+\frac{1}{6}x^2(\ln x)^3$

$\therefore y = y_c+y_p = [c_1+c_2\ln x]x^2 + x+\frac{1}{6}x^2(\ln x)^3$

Page- 242 2(ii) Solve : $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\ln x$ ——(1)

let, $x=e^z \quad \therefore z=\ln x \quad \therefore \frac{dz}{dx}=\frac{1}{x}$

Now, $\frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}$

$=\frac{dy}{dz}\cdot\frac{1}{x}$

$\Rightarrow x\frac{dy}{dx}=Dy$ where, $D=\frac{d}{dz}$

Again, $\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$

$=\frac{d}{dx}\left[\frac{1}{x}\frac{dy}{dz}\right]$

$=\frac{1}{x}\frac{d}{dx}\left(\frac{dy}{dz}\right)-\frac{1}{x^2}\frac{dy}{dz}$

$=\frac{1}{x}\frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx}-\frac{1}{x^2}\frac{dy}{dz}$

$=\frac{1}{x}\frac{d^2y}{dz^2}\cdot\frac{1}{x}-\frac{1}{x^2}\frac{dy}{dz}$

$=\frac{1}{x^2}\frac{d^2y}{dz^2}-\frac{1}{x^2}\frac{dy}{dz}$

$\Rightarrow \frac{d^2y}{dx^2}=\frac{1}{x^2}\left[D^2y-Dy\right]$

$\Rightarrow x^2\frac{d^2y}{dx^2}=D(D-1)y$

Similarly, $x^3\frac{d^3y}{dx^3}=D(D-1)(D-2)y$

(i) $\Rightarrow D(D-1)(D-2)y+3D(D-1)y+Dy+y=e^z+z$

$\Rightarrow (D^3-D^2-2D^2+2D+3D^2-3D+D+1)\,y=e^z+z$

$\Rightarrow (D^3+1)\,y=e^z+z$ ... ... (ii)

Now, $(D^3+1)y=0$ ... ... (iii)

let $y=e^{mz}$ be a trial solution of (iii)

$\therefore Dy=me^{mz}$, $D^2y=m^2e^{mz}$, $D^3y=m^3e^{mz}$

(iii) $\Rightarrow (m^3+1)e^{mz}=0$

$\Rightarrow (m+1)(m^2-m+1)=0$

$\therefore m=-1,\ \frac{1}{2}(1\pm\sqrt{3}i)$

$\therefore y_c = c_1e^{-z} + e^{z/2}\left[c_2\cos\frac{\sqrt{3}}{2}z + c_2\sin\frac{\sqrt{3}}{2}z\right]$

$= c_1e^{-\ln x} + e^{\ln x/2}\left[c_2\cos\left(\frac{\sqrt{3}}{2}\right)\ln x + c_2\sin\left(\frac{\sqrt{3}}{2}\right)\ln x\right]$

$= c_1e^{\ln x^{-1}} + e^{\ln(x)^{\frac{1}{2}}}\left[c_2\cos\left(\frac{\sqrt{3}}{2}\right)\ln x + c_2\sin\left(\frac{\sqrt{3}}{2}\right)\ln x\right]$

$= c_1\cdot\frac{1}{x} + x^{\frac{1}{2}}\left[c_2\cos\left(\frac{\sqrt{3}}{2}\right)\ln x + c_2\sin\left(\frac{\sqrt{3}}{2}\right)\ln x\right]$

$y_p = \frac{1}{D^3+1}(e^z+z)$

$= \frac{1}{D^3+1}e^z + \frac{1}{D^3+1}z$

$= \frac{1}{1+1}e^z + (1+D^3)^{-1}z$

$= \frac{1}{2}e^z + (1-D^3+D^6-\cdots)z$

$= \frac{1}{2}e^z + (z-0)$

$= \frac{1}{2}e^z + z$

$= \frac{1}{2}e^{\ln x} + \ln x$

$= \frac{1}{2}x + \ln x$

$\therefore y = y_c + y_p = c_1\cdot\frac{1}{x} + x^{\frac{1}{2}}\left[c_2\cos\left(\frac{\sqrt{3}}{2}\right)\ln x + c_2\sin\left(\frac{\sqrt{3}}{2}\right)\ln x\right]$

$+\frac{1}{2}x + \ln x$

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Page-144 3(i) Solve: $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \cos(\ln x)$ ...(i)

let, $x = e^z$ $\therefore z = \ln x$ $\therefore \frac{dz}{dx} = \frac{1}{x}$

Now, $\frac{dy}{dx} = \frac{dy}{dz}\cdot\frac{dz}{dx}$

$= \frac{dy}{dz}\cdot\frac{1}{x}$

$\therefore x\frac{dy}{dx} = \frac{dy}{dz} = Dy$ where $D = \frac{d}{dz}$

Again, $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$

$= \frac{d}{dx}\left[\frac{1}{x}\frac{dy}{dz}\right]$

$= \cancel{\frac{dy}{dz}}\ \frac{1}{x}\frac{d}{dx}\left(\frac{dy}{dz}\right) - \frac{1}{x^2}\frac{dy}{dz}$

$= \frac{1}{x}\ \frac{d}{dz}\left(\frac{dy}{dz}\right)\frac{dz}{dx} - \frac{1}{x^2}\frac{dy}{dz}$

$= \frac{1}{x}\ \frac{d^2y}{dz^2}\cdot\frac{1}{x} - \frac{1}{x^2}\frac{dy}{dz}$

$= \frac{1}{x^2}\frac{d^2y}{dz^2} - \frac{1}{x^2}\frac{dy}{dz}$

$\Rightarrow x^2\frac{d^2y}{dx^2} = \frac{d^2y}{dz^2} - \frac{dy}{dz}$

$= D^2y - Dy$

$= D(D-1)y$

(i) $\Rightarrow D(D-1)y - 2Dy + 2y = \cos z$

$\Rightarrow (D^2 - D - 2D + 2)y = \cos z$

$\Rightarrow (D^2 - 3D + 2)y = \cos z$ ...(ii)

Now, $(D^2 - 3D + 2)y = 0$ ...(iii)

let, $y = e^{mz}$ be a trial Solution of (iii)

$\therefore Dy = me^{mz}$, $D^2y = m^2e^{mz}$

$(iii) \Rightarrow (m^2 - 3m + 2) e^{mz} = 0$

$\Rightarrow m^2 - 3m + 2 = 0$

$\therefore m = 1, 2$

$\therefore \cancel{y_c = c_1 x + c_2 x^2}$ $\quad y_c = c_1 e^{z} + c_2 e^{2z}$

$= c_1 e^{\ln x} + c_2 e^{\ln x^2}$

$= c_1 x + c_2 x^2$

Now from (ii) $\Rightarrow$

$$y_p = \frac{1}{D^2 - 3D + 2} \cos z \qquad D^2 = -1^2$$

$$= \frac{1}{-1^2 - 3D + 2} \cos z$$

$$= \frac{1}{1 - 3D} \cos z$$

$$= \frac{1 + 3D}{1 - 9D^2} \cos z$$

$$= \frac{1 + 3D}{1 - 9(-1^2)} \cos z$$

$$= \frac{1 + 3D}{10}$$

$$= \frac{1}{10}(1 + 3D) \cos z$$

$$= \frac{1}{10}[\cos z + 3D(\cos z)]$$

$$= \frac{1}{10}[\cos z + 3(-\sin z)]$$

$$= \frac{1}{10}(\cos z - 3 \sin z)$$

$$= \frac{1}{10}[\cos(\ln x) - 3 \sin(\ln x)]$$

$$\therefore y = y_c + y_p = c_1 x + c_2 x^2 + \frac{1}{10}[\cos(\ln x) - 3 \sin(\ln x)]$$

Page-247 $\quad$ 3(iii) $\quad$ 3(iii) $\quad$ solve:

$$x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x+\sin x \quad \text{---(i)}$$

let, $x=e^z \quad \therefore z=\ln x \quad \frac{dz}{dx}=\frac{1}{x}$

$$\therefore \frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}$$

$$=\frac{dy}{dz}\frac{1}{x}$$

$$\Rightarrow x\frac{dy}{dx}=\frac{dy}{dz}=Dy \quad \text{where } \frac{d}{dz}=D$$

$$\frac{d^2y}{dx^2}=\frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$=\frac{d}{dx}\left(\frac{1}{x}\frac{dy}{dz}\right)$$

$$=\frac{1}{x}\frac{d}{dx}\left(\frac{dy}{dz}\right)-\frac{1}{x^2}\frac{dy}{dz}$$

$$=\frac{1}{x}\frac{d}{dz}\frac{dy}{dz}\cdot\frac{dz}{dx}-\frac{1}{x^2}\frac{dy}{dz}$$

$$=\frac{1}{x}\frac{d^2y}{dz^2}\cdot\frac{1}{x}-\frac{1}{x^2}\frac{dy}{dz}$$

$$=\frac{1}{x^2}D^2y-\frac{1}{x^2}Dy$$

$$\Rightarrow x^2\frac{d^2y}{dx^2}=D^2y-Dy=D(D-1)y$$

$$(i)\Rightarrow D^2y-Dy+4Dy+2y=e^z+\sin e^z$$

$$\Rightarrow D^2y+3Dy+2y=e^z+\sin e^z$$

$$\Rightarrow (D^2+3D+2)y=e^z+\sin e^z \quad \ldots\ldots(ii)$$

Now, $(D^2+3D+2)y=0 \quad \ldots\ldots(iii)$

Let, $y=e^{mz}$ be a trial solution of (iii) $\therefore Dy=me^{mz}$

$$D^2y=m^2e^{mz}$$

$$(iii)\Rightarrow (m^2+3m+2)e^{mz}=0$$

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$\Rightarrow m^2+3m+2=0$

$\therefore m=-1,-2$

$\therefore y_c = c_1 e^{-z} + c_2 e^{-2z}$

$= c_1 e^{-\ln x} + c_2 e^{-2\ln x}$

$= c_1 e^{\ln x^{-1}} + c_2 e^{\ln x^{-2}}$

$= c_1 x^{-1} + c_2 \cancel{\ln} x^{-2}$

$= \frac{c_1}{x} + \frac{c_2}{x^2}$

(ii) $\Rightarrow y_p = \frac{1}{D^2+3D+2}(e^z + \sin e^z)$

$= \frac{1}{D^2+3D+2} e^z + \frac{1}{D^2+3D+2}\sin e^z$

$= \frac{1}{1^2+3\cdot 1+2} e^z + \frac{1}{(D+2)(D+1)}\sin e^z$

$= \frac{1}{6} e^z + \frac{1}{D+2}\cdot\frac{1}{D+1}(\sin e^z)$

$= \frac{1}{6} e^z + \frac{1}{D+2}\left[-e^{-z}\cos e^z\right]$

$= \frac{1}{6}(e^z) + (-e^{-2z}\sin e^z)$

$= \frac{x}{6} - {e^{z}}^{-2}\sin e^z$

$= \frac{x}{6} - x^{-2}\sin x$

$= \frac{x}{6} - \frac{\sin x}{x^2}$

$\therefore y = y_c + y_p = \frac{c_1}{x} + \frac{c_2}{x^2} + \frac{x}{6} - \frac{\sin x}{x^2}$

Page 251 : Solve the cauchy Euler Equation: $x^3\frac{d^3y}{dx^3}-4x^2\frac{d^2y}{dx^2}+8x\frac{dy}{dx}-8y=4\ln x$ ——— (i)

3 (v)

Soln: Let, $x=e^z$

$\Rightarrow z=\ln x$ and $\frac{dz}{dx}=\frac{1}{x}$

$\therefore \frac{dy}{dx}=\frac{dy}{dz}\cdot\frac{dz}{dx}$

$=\frac{dy}{dz}\cdot\frac{1}{x}$

$\Rightarrow x\frac{dy}{dx}=\frac{dy}{dz}=Dy$ where, $\frac{d}{dz}=D$.

similarly, $x^2\frac{d^2y}{dy^2}=D(D-1)y$ and $x^3\frac{d^3y}{dx^3}=D(D-1)(D-2)y$

(i) $\Rightarrow$ $D(D-1)(D-2)y-4D(D-1)y+8Dy-8y=4z$

$\Rightarrow(D^3-D^2-2D^2+2D-4D^2+4D+8D-8)y=4z$

$\Rightarrow(D^3-7D^2+14D-8)y=4z$ ——— (ii)

Let, $(D^3-7D^2+14D-8)y=0$ . . . (iii)

Let $y=e^{mz}$ be the trial solution of (iii)

$\therefore Dy=me^{mz}$, $D^2y=m^2e^{mz}$, $D^3y=m^3e^{mz}$

Now, (iii) $\Rightarrow(m^3-7m^2+14m-8)e^{mz}=0$

$\Rightarrow m^3-7m^2+14m-8=0$ since, $e^{mz}\neq0$

$\therefore m=1,2,4$

$\therefore y_c=c_1e^z+c_2e^{2z}+c_3e^{4z}=c_1x+c_2x^2+c_3x^4$

from (ii) $\Rightarrow$ $y_p=\frac{1}{D^3-7D^2+14D-8}4z$

$=\frac{1}{-8\left[1-\frac{14D}{8}+\frac{7D^2}{8}-\frac{D^3}{8}\right]}4z$

$$= \frac{1}{-8\left[1-\left(\frac{7D}{4}-\frac{7D^2}{8}-\frac{D^3}{8}\right)\right]} 4z$$

$$= -\frac{1}{8}\left[1-\left(\frac{7D}{4}-\frac{7D^2}{8}+\frac{D^3}{8}\right)\right]^{-1} 4z$$

$$= -\frac{1}{8}\left[1+\left(\frac{7D}{4}-\frac{7D^2}{8}+\frac{D^3}{8}\right)+\cdots\cdots\right] 4z$$

$$= -\frac{1}{2}\left[1+\frac{7D}{4}-\frac{7D^2}{8}+\frac{D^3}{8}+\cdots\right] z$$

$$= -\frac{1}{2}\left[z+\frac{7}{4}(1)-0\right]$$

$$= -\frac{1}{2}\left[z+\frac{7}{4}\right]$$

$$= -\frac{1}{8}(4z+7)$$

$$\therefore y = y_c + y_p = c_1x + c_2x^2 + c_3x^4 - \frac{1}{8}(4z+7)$$

Page-523 Solve by the method of variation of Parameters:

$$\frac{d^2y}{dx^2} + 4y = 4\tan 2x \quad \text{——(i)}$$

Sol^n: Let, $y = e^{mx}$ be the trial solution of $\frac{d^2y}{dx^2} + 4y = 0$ ——(ii)

$\therefore \frac{dy}{dx} = me^{mx}$ and $\frac{d^2y}{dx^2} = m^2 e^{mx}$

$\therefore$ (ii) $\Rightarrow (m^2+4)e^{mx} = 0$

$\Rightarrow m^2+4 = 0$

$\Rightarrow m^2 = -4$

$\therefore m = \pm 2i$ $\qquad \therefore y_c = c_1\cos 2x + c_2 \sin 2x$

Let, the general solution of (i) be

$y = A\cos 2x + B\sin 2x$ where, A, B are function of x ---- (iii)

Differentiating (iii) w.r to x:

$y_1 = -2A\sin 2x + A_1\cos 2x + 2B\cos 2x + B_1 \sin 2x$ —(

here, A and B have chosen such that :--

$A_1 \cos 2x + B_1 \sin 2x = 0$

$\Rightarrow A_1\cos 2x = -B_1 \sin 2x$

$\therefore A_1 = -\frac{B_1 \sin 2x}{\cos 2x}$ ... ---(v)

(iv) $\Rightarrow y_1 = -2A\sin 2x + 2B\cos 2x$ ---- (vi)

Differentiating (vi) w.r. to x:

$y_2 = -4A\cos 2x - 2A_1\sin 2x - 4B\sin 2x + 2B_1 \cos 2x$ —(

Now putting the values of y, y_2 on (i)

$-4A\cos 2x - 2A_1 \sin 2x - 4B\sin 2x + 2B_1\cos 2x + 4(A\cos 2x + B\sin 2x) = 4t$

$\Rightarrow -2A_1 \sin 2x + 2B_1 \cos 2x = 4\tan 2x$

$$\Rightarrow -A_1 \sin 2x + B_1 \cos 2x = 2\tan 2x$$

$$\Rightarrow \frac{B_1 \sin 2x}{\cos 2x} \sin 2x + B_1 \cos 2x = 2\tan 2x \quad (\text{by } (V))$$

$$\Rightarrow \frac{B_1}{\cos 2x}\left[\sin^2 2x + \cos^2 2x\right] = 2\frac{\sin 2x}{\cos 2x}$$

$$\Rightarrow B_1 = 2\sin 2x$$

$$\Rightarrow \frac{dB}{dx} = 2\sin 2x$$

$$\Rightarrow B = \frac{-2\cos 2x}{2} + b \quad [\text{integrating w.r.t } x]$$

$$\Rightarrow B = -\cos 2x + b$$

$$(V) \Rightarrow A_1 = -\frac{B_1 \sin 2x}{\cos 2x}$$

$$= -\frac{2\sin 2x \sin 2x}{\cos 2x}$$

$$= \frac{-2(\sin^2 2x)}{\cos 2x}$$

$$= \frac{-2(1-\cos^2 2x)}{\cos 2x}$$

$$\frac{dA}{dx} = -2\sec 2x + 2\cos 2x$$

$$\Rightarrow A = -\ln\tan\left(\frac{\pi}{4}+x\right) + \sin 2x + a \quad [\text{integrating w.r.t } x]$$

put the value of A and B on (iii) $\Rightarrow$

$$y = \left[-\ln\tan\left(\frac{\pi}{4}+x\right) + \sin 2x + a\right]\cos 2x + \left[-\cos 2x + b\right]\sin 2x$$

$$= a\cos 2x + b\sin 2x - \cos 2x \ln\tan\left(\frac{\pi}{4}+x\right)$$

page-525: 1(ii) Solve by the method of variation of parameters.

$$\frac{d^2y}{dx^2} + y = \operatorname{cosec} x \quad \text{——— (i)}$$

Sol^n: Let $y = e^{mx}$ be a trial solution of $\frac{d^2y}{dx^2} + y = 0$ ——— (ii)

$\therefore \frac{dy}{dx} = me^{mx}$ and $\frac{d^2y}{dx^2} = m^2 e^{mx}$

$\therefore$ (ii) $\Rightarrow (m^2+1)e^{mx} = 0$

$\Rightarrow m^2+1 = 0 \qquad \because e^{mx} \neq 0$

$\Rightarrow m^2 = -1$

$\therefore m = \pm i$

$\therefore y_c = c_1 \cos x + c_2 \sin x$

Let the general solution of (i) by $y = A\cos x + B\sin x$ ——— (iii)

where A and B are function of x.

differentiate (iii) w.r.t x

(iii) $\Rightarrow y_1 = -A\sin x + A_1\cos x + B\cos x + B_1 \sin x$ ——— (iv)

here, A and B have chosen such that :

$A_1\cos x + B_1\sin x = 0$

$\Rightarrow A_1\cos x = -B_1\sin x$

$\Rightarrow A_1 = \frac{-B_1 \sin x}{\cos x}$ ——— (v)

(iv) $\Rightarrow y_1 = -A\sin x + B\cos x$

$\Rightarrow y_2 = -A\cos x - A_1\sin x - B\sin x + B_1\cos x$ [differentiate ~~integ~~ w.r.t x

Now, put the value of y and y_2 on (i)

(i) $\Rightarrow -A\cos x - A_1\sin x - B\sin x + B_1\cos x + A\cos x + B\sin x = \operatorname{cosec} x$

$\Rightarrow -A_1\sin x + B_1\cos x = \operatorname{cosec} x$

$$\Rightarrow \frac{B_1 \sin x}{\cos x} \cdot \sin x + B_1 \cos x = \operatorname{cosec} x$$

$$\Rightarrow \frac{B_1}{\cos x}\left[\sin^2 x + \cos^2 x\right] = \operatorname{cosec} x$$

$$\Rightarrow \frac{B_1}{\cos x} = \frac{1}{\sin x}$$

$$\Rightarrow B_1 = \frac{\cos x}{\sin x}$$

$$\Rightarrow B = \ln(\sin x) + b \quad [\text{integrating w.r. } x]$$

(V) $\Rightarrow A_1 = \frac{-B_1 \sin x}{\cos x} = \frac{-\frac{\cos x}{\sin x} \cdot \sin x}{\cos x} = -1$

$\therefore A = -x + a \qquad [\text{integrating w.r. } x]$

(III) $\Rightarrow y = (-x+a)\cos x + [\ln(\sin x) + b]\sin x$

$= a\cos x + b\sin x - x\cos x + \sin x \ln(\sin x)$ (Ans)

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1 (iii) By the method of variation of parameters to Solve : $(D^2+1)\,y = \sec x \tan x$ —— (1)

Let $y = e^{mx}$ be the trial solution of $(D^2+1)y = 0$ —— (ii)

$\therefore Dy = me^{mx}$ and $D^2y = m^2 e^{mx}$

$\therefore$ (ii) $\Rightarrow (m^2+1)e^{mx} = 0$

$\Rightarrow m^2 + 1 = 0$ since $e^{mx} \neq 0$

$\Rightarrow m = \pm i$

$\therefore y_c = c_1 \cos x + c_2 \sin x$ where c_1 and c_2 are arbitrary constant.

Let the general solution of (1) be $y = A\cos x + B\sin x$ —— (III)
where A and B are function of x

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$\therefore y_1 = -A\sin x + A_1\cos x + B\cos x + B_1\sin x$ —— (iv)

here A and B have chosen such that :

$$A_1\cos x + B_1\sin x = 0$$

$$\Rightarrow A_1 = \frac{-B_1\sin x}{\cos x} \quad \text{—— (v)}$$

(iv) $\Rightarrow y_1 = -A\sin x + B\cos x$

$D^2y = y_2 = -A\cos x - A_1\sin x - B\sin x + B_1\cos x$

Now put the values of y, y_2 on (i)

(i) $\Rightarrow -A\cos x - A_1\sin x - B\sin x + B_1\cos x + A\cos x + B\sin x = \sec x\tan x$

$\Rightarrow -A_1\sin x + B_1\cos x = \sec x\tan x$

$\Rightarrow \dfrac{B_1\sin x}{\cos x}\sin x + B_1\cos x = \sec x\tan x$

$\Rightarrow \dfrac{B_1}{\cos x}\left[\sin^2 x + \cos^2 x\right] = \sec x\,\dfrac{\tan x}{\cos x}$

$\Rightarrow B_1 = \tan x$

$\therefore B = -\ln(\cos x) + b$

(v) $\Rightarrow A_1 = \dfrac{-\tan x\sin x}{\cos x} = -\tan^2 x = -\sec^2 x + 1$

$\therefore A = -\tan x + x + a$

(iii) $\Rightarrow y = (-\tan x + x + a)\cos x + [-\ln(\cos x) + b]\sin x$

$\Rightarrow y = a\cos x + b\sin x - \sin x + x\cos x - \sin x\ln(\cos x)$

Page- 520 By the method of variation of parameter to solve: $(D^2-2D+1)y = e^x \ln x$ —— (i)

Let $y = e^{mx}$ be the trial solution of $(D^2-2D+1)y=0$ — (ii)

$\therefore Dy = me^{mx}$, $D^2y = m^2e^{mx}$

(ii) $\Rightarrow (m^2-2m+1)e^{mx}=0$

$\Rightarrow m^2-2m+1=0$ since $e^{mx} \neq 0$

$\Rightarrow (m-1)^2=0$

$\therefore m = 1, 1$

$\therefore y_c = (c_1+c_2x)e^x = c_1e^x + c_2xe^x$

Let the general solution of (i) be $y = Ae^x + Bxe^x$ —— (iii)

where A and B are function of x

$\therefore Dy = y_1 = Ae^x + A_1e^x + B(xe^x+e^x) + B_1xe^x$

$= Ae^x + A_1e^x + Bxe^x + Be^x + B_1xe^x$ —— (iv)

Here, A and B have chosen such that

$A_1e^x + B_1xe^x = 0$

$\Rightarrow A_1 = -B_1x$ —— (v)

(iv) $\Rightarrow y_1 = Ae^x + Be^x + Bxe^x$

$y_2 = Ae^x + A_1e^x + Be^x + B_1e^x + Bxe^x + Be^x + B_1xe^x$

Now put the values of y, y_1, y_2 on (i)

$Ae^x + A_1e^x + Be^x + B_1e^x + Bxe^x + Be^x + B_1xe^x - 2(Ae^x + \cancel{A_1e^x} + Bxe^x + Be^x + \cancel{B_1xe^x}) + Ae^x + Bxe^x = e^x \ln x$

$\Rightarrow A_1e^x + B_1e^x + B_1xe^x = e^x \ln x$

$\Rightarrow -B_1 x e^{x} + B_1 e^{x} + B_1 x e^{x} = e^{x} \ln x$

$\Rightarrow B_1 e^{x} = e^{x} \ln x$

$\Rightarrow B_1 = \ln x$ $\qquad \int \ln x = x\ln x -$

$\Rightarrow B = x \ln x - x$

(v) $\Rightarrow A_1 = -B_1 x$

$\Rightarrow A_1 = -\ln x \cdot x$

$\Rightarrow A_1 = -x \ln x$

$$\Rightarrow A = -\left[\ln x \int x\,dx - \int\left[\frac{d}{dx}(\ln x)\int x\,dx\right]dx\right]$$

$$= -\frac{x^2}{2}\ln x + \int \frac{1}{x}\cdot\frac{x^2}{2}\,dx$$

$$= -\frac{x^2}{2}\ln x + \frac{1}{2}\int x\,dx$$

$$= -\frac{x^2}{2}\ln x + \frac{x^2}{4} + a$$

Now put the values of A and B on (III)

$$y = \left[-\frac{x^2}{2}\ln x + \frac{x^2}{4} + a\right]e^{x} + \left[x\ln x - x + b\right]xe^{x}$$

$$= ae^{x} + bxe^{x} + \left(\frac{x^2}{4} - x^2\right)e^{x} - \left(\frac{x^2}{2} - x^2\right)e^{x}\ln x$$

$$= ae^{x} + bxe^{x} - \frac{3x^2}{4}e^{x} + \frac{x^2}{2}e^{x}\ln x$$

(Ans)