Bangabandhu Sheikh Mujibur Rahman Science and Technology University,Gopalganj

Department of Computer Science and Engineering

3^{rd} Year 1^{st} Semester B.Sc. (Engg.) Examination 2014

Course No.: MAT304 Title:(Linear Algebra)

Full Marks: 70 Time: 04 Hours

N.B.

1) Answer SIX questions taking any THREE from each sections.
2) All questions are of equal values.
3) Use separate answer script for each section.

Section A

1.	(a)	Define consistent and inconsistent. Solve the following system of linear equations using Gaussian elimination: $x+2y-5z=-1, -3x+y-2z=-7, 5x+3y-4z=2$	$5^{2/3}$
	(b)	Define matrix. Find the inverse of the matrix $A=\begin{bmatrix}2 & -1 & 3\\ 4 & 5 & -1\\ 3 & 3 & 2\end{bmatrix}$	6
2.	(a)	Let V be a vector space over an arbitrary field F. Then prove that $\alpha \in F \text{ and } v \in V, (-\alpha)v=\alpha(-v)=-\alpha v$ For any scalar	3
	(b)	Define linear combination. The non-zero vectors $v_1, v_2, \ldots\ldots v_n$ in a vector space V are linearly dependent if and only if the vectors v_k is a linear combination of the preceding vectors $v_1, v_2 \ldots\ldots v_{k-1}$.	5
	(c)	Find a basis and the dimension of the subspace W generated by (1,4,-1,3), (2,1,-3,-1) and (0,2,1,-5).	$3^{2/3}$
3.	(a)	Show that the functions $f(t)=\sin t, g(t)=\cos t, h(t)=t$, from R into R are linearly independent.	4
	(b)	Let u and w be subspaces of a vector space V. Show that u+w is a vector space of V.	$2^{2/3}$
	(c)	Define Basis and Dimension. Prove that the vectors (1,2,0),(0,5,7),(-1,1,3) form a basis for R^3	5
4.	(a)	Define linear transformation. If T:U $\rightarrow$ V linear transformation then (i)The kernel of T is a subspace of U (ii) The image of T is a subspace of V.	6
	(b)	Define Rank and Nullity. Show that the following transformation defined a linear operator on R^3 such that $T(x,y,z)=(x+y,-x-y,z)$	$5^{2/3}$

Bangabandhu Sheikh Mujibur Rahman Science and Technology University,Gopalganj

Department of Computer Science and Engineering

3^{rd} Year 1^{st} Semester B.Sc. (Engg.) Examination 2014

Course No.: MAT304 Title: Linear Algebra

Full Marks: 70 Time: 04 Hours

		Section ' B	
5.	(a)	Let $R^2 \to R^2$ be the linear operator defined by $T(x,y) = (5x + 7y, -3x - y)$ and $S(x,y) = (5x + 7y, -3x - y)$. Find the formula defining properties, S+T, ST, TS, S^2, T^2	$6^{2/3}$
	(b)	Let $R^2 \to R^2$ be the linear operator defined by $T(x,y) = (5x + 7y, -3x + y)$. Find the matrix of T with respect to the basis $F = (e_1, e_2)$ ={(1,0),(0,1)} and $S = (u_1, u_2)$ ={(2,1),(3,5}	5
6.	(a)	Define linear functional and the dual space of a vector space.	3
	(b)	Define an annihilator of a subset of a vector space . Prove that the set of all annihilators of is a subspace of W is a subspace of V' .	4
	(c)	Let W be the subspace of $\mathbb{R}^4$ spanned by (1,2, −3,4) , (1,3, −2,6) and (1,4, −1,8) . Find a basis of all annihilators of W.	$4^{2/3}$
7.	(a)	Define a norm and an inner product in a vector space. Prove that $\|\langle u,v\rangle\| \leq \|u\| \, \|v\|$ for any vectors $u,v \in V$	5
	(b)	Let R^2 have the Eucliden inner product use the Gram-Schmidt Orthogonalization process to transform the basis $S = \{u_1, u_2\}$ in to a orthonormal basis $u_1 = (1,-2)$, $u_2 = (2,3)$	$6^{2/3}$
8.	(a)	Define adjoint operator. Let f and g be polynomials. For any square matrix A and scalar k prove that $(fg)(A) = f(A)g(A)$	5
	(b)	Define minimal Polynomial. Find the minimal polynomial $m(\lambda)$ of the matrix $A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$	$6^{2/3}$