50/no de elements of finite londer is a subgroup? As stated false (in) general (nonabelian group) Countenexample: infinite dihedral quoup reflections have order 2 but product of two reflections can be adnotation of infinite orden. Track if Gis abelian if x, y have finite orders m, n then (xy) lam (m,n) = e, so the tontion element torim a subgroup in abelian group. 1+0.13 2. Subgroup of index p where pris 0 smallest, prime divisor of 1191 is notimal of divisor 50h: Action on closets gives p: G->5p. The image has order dividing lail and dividing pl, so any won primetoddivision of lime 1/15 Epol By minimality of phothe image is a proup, so its londer 15 1 on p. 9t cannot be 1 (else H=G); solimpl= then [G: ken P.] = P and ken 9 CH fonces ken 9=H. Hence, His normal.

3. If gay be and ab = ba then (ab) 6 00 e. Koln . As wnitten, this is not valid. From 24=62 land commutation you cannot deduce (ab) = e (take a of infinite order pand bear as la countenexample). tonitail ent to 18 70009 Conrect simple case: It one b? = es andis ab = ba, Then lal 14/16/11/2/50 rabi 1 1em/(4,2)=14.0 This. (ab) 4 = e, thènce in partiquelan (ab) 6 need Historia hypothesis that = 4= 62=e,)

4. Thirteg: 2HA = norther x C H - Port X C B. S. The coset xH in al H has finite ronder dividing 1 G/H |= n. Thus (xH) = H, i.e. nxh & Hill 5. Unique subgroup of order pk for each kin implies normal sylow p-subgroup. solno The subgroup of order phis unique by hypothesis, so it is invariant under conjugation and hence normal.

6. Union of all conjugates of a priopert sub group H (finite G): sint nothing appropriate 5-Soling soubob transo up noitablements for e Claimin Uge A8 Higorio + Ginifrii to o solo! Proof à Let the distinct conjugates be min numberoneach has/HI elements and they all Tontain 14 the lidentisty 1501111 11. But m= [G: Na(H)] > 2 and HIX [GI), sometiments of the circulture 50 m(141-1) + 12/ G/ Minthus The runion is the union is H+ Grangway). is. Unique subgroup of orders pk for each kin quonpolue - quolpie l'omnor esilqui Ed supirons si d'à risbur to quampoine ent soisé mitagrifico respons tracinovas el 14 los ejentroque . Innor sonial bar