

Issa
Odeh

1.
a) $p = 467, a = 2, A = 3, b = 5$

Alice sends key to Bob

$$\begin{aligned} A &= a^A \mod p \\ &= 2^3 \mod 467 \\ &= 8 \mod 467 \\ &= 8 \end{aligned}$$

Bob sends key to Alice

$$\begin{aligned} B &= a^b \mod p \\ &= 2^5 \mod 467 \\ &= 32 \mod 467 \\ &= 32 \end{aligned}$$

shared key for both

$$\begin{aligned} K &= a^{ab} \mod p \\ &= 2^{15} \mod 467 \\ &= 32768 \mod 467 \\ &= 78 \end{aligned}$$

Shared key received from Bob

$$\begin{aligned} K_a &= B^a \mod p \\ &= 32^3 \mod 467 \\ &= 32768 \mod 467 \\ &= 78 \end{aligned}$$

Shared key received from Alice

$$\begin{aligned} K_b &= 8^5 \mod p \\ &= 32768 \mod 467 \\ &= 78 \end{aligned}$$

$p = 467, a = 2, A = 400, b = 134$

b)

Alice sends key to Bob

$$\begin{aligned} A &= a^A \mod p \\ &= 2^{400} \mod 467 \\ &= 137 \end{aligned}$$

Bob sends key to Alice

$$\begin{aligned} B &= a^b \mod p \\ &= 2^{134} \mod 467 \\ &= 64 \end{aligned}$$

Shared key for both

$$\begin{aligned} K &= a^{ab} \mod p \\ &= 2^{400 \times 134} \mod 467 \\ &= 90 \end{aligned}$$

Shared key received by Bob

$$\begin{aligned} K_a &= B^a \mod p \\ &= 64^{400} \mod 467 \\ &= 90 \end{aligned}$$

Shared key received by Alice

$$\begin{aligned} K_b &= 137^{134} \mod 467 \\ &= 90 \end{aligned}$$

C) $p=467, a=2, a=2^{28}, b=57$

Alice sends key to Bob
 $= 2^{228} \mod 467$
 $= 394$

Bob's sends key to Alice
 $= 2^{57} \mod 467$
 $= 313$

Shared key For both

$$K = a^b \mod p$$

$$= 2^{228 \times 57} \mod 467$$

$$= 206$$

Shared key Received by Bob

$$= 313^{228} \mod 467$$

$$= 206$$

Shared key Received by Alice

$$= 394^{57} \mod 467$$

$$= 206$$

2. $p=467, a=2, x=105,$
 $K=2^{13}, m=33$

Bob's side:

$$B = 2^{105} \mod 467$$

$$= 444$$

Alice public key:

$$K_E = 2^{13} \mod 467$$

$$= 29$$

Alice creates a mask

$$K_m = 444^{29} \mod 467$$

$$= 292$$

Then Alice encrypts data

$$Y = x \cdot K_m \mod p$$

$$= 33 \times 292 \mod 467$$

$$= 9636$$

Public key now sent to bob

$$K_m = K_E^{-1} \mod p$$

$$= 29^{105} \mod 467$$

$$= 292$$

Bob's decrypts cipher text

$$x = y \cdot K_m^{-1} \mod p$$

$$= 9636 \cdot 292^{-1} \mod p$$

Decrypted = 33

b) $q = 467, a = 2, x = 105$
 $k, 123, m = 33$

Bob's side:
 $2^{105} \bmod 467$
 $B = 444$

Alice side:
 $k_E = 2^{123} \bmod 467$
 $= 125$

Alice creates mask for
 message:
 $k_m = 444^{123} \bmod 467$
 $= 278$

Alice encrypts data
 $y = 33 \cdot 278 \bmod 467$
 $= 9174$

public key sent to bob
 $k_m = 125^{105} \bmod 467$
 $= 278$

Then, bob decrypts ciphertext
 $x = 9636 \cdot 278^{-1} \bmod 467$
 $\boxed{x = 33}$

c) $q = 467, a = 2, x = 105$
 $k = 45, m = 248$

Bob's side:
 $2^{300} \bmod 467$
 $= 317$

Alice side:
 $k_E = 2^{45} \bmod 467$
 $= 80$

Alice creates mask for
 message:
 $k_m = 317^{45} \bmod 467$
 $= 12$

Alice encrypts data:
 $y = 248 \cdot 12 \bmod 467$
 $= 2976$

Then, bob decrypts ciphertext
 $x = 2976 \cdot 12^{-1} \bmod 467$
 $\boxed{x = 248}$

D). $q = 467, a = 2, X = 300, K = 47, M = 248$

Bob's side:

$$B = 2^{300} \bmod 467 \\ = 317$$

Alice's side:

$$= 2^{47} \bmod 467 \\ = 320$$

Alice creates mask for message:

$$K_m = 317^{47} \bmod 467 \\ = 74$$

Alice encrypts data

$$Y = 248 \cdot 74 \bmod 467 \\ = 18352$$

Then, Bob decrypts the ciphertext:

$$X = 18352 \cdot 74^{-1} \bmod 467 \\ \boxed{= 248}$$

3). one secure way against a MITM attack is to encrypt the Diffie-Hellman value with the other side's public key. All keys are stored on a server and are safe. So no, it is not vulnerable.

7.

22

y	y^2	$y^2 \bmod 11$
0	0	0
1	1	1
2	4	4
3	9	9
4	16	5
5	25	3
6	36	3
7	49	5
8	64	9
9	81	4
10	100	1

x	$x^3 + x + 6$	$(x^3 + x + 6) \bmod 11$	y_1	y_2
0	6	6	none	none
1	8	8	none	none
2	16	5	4	7
3	36	3	5	6
4	74	8	none	none
5	136	4	2	9
6	228	8	none	none
7	356	4	2	9
8	526	9	3	8
9	744	7	none	none
10	1016	4	2	-9

points are : $(2, 4), (2, 7), (3, 5), (3, 6), (5, 2), (5, 9),$
 $(7, 2), (7, 9), (8, 3), (8, 8), (10, 2), (10, 9)$

$$b) \quad y^2 = x^3 + x + 6 \pmod{11}$$

$$13 \quad p = (x_3, y_3)$$

$$\lambda = \frac{3x^2 + a}{2y_1} = \frac{3(2)^2 + 1}{2 \times 4} = \frac{13}{8} \pmod{11} = 5$$

$$\begin{aligned} x_3 &= \lambda^2 - x_1 - x_2 \\ &= 5^2 - 2 - 2 \\ &= 21 \end{aligned}$$

$$21 \pmod{11} = 10$$

$$\begin{aligned} y_3 &= \lambda(x_1 - x_3) - y_1 \\ &= 5(2 - 10) - 4 \\ &= -40 - 4 \\ &= -44 \\ &= -44 \pmod{11} = 0 \end{aligned}$$

$$\text{So, } P(2, 4) = (10, 0)$$

$$c) \quad \begin{matrix} P(2, 4) & \text{and} & Q(2, 7) \\ x_1 \ y_1 & & x_2 \ y_2 \end{matrix}$$

$$m = \frac{7 - 4}{2 - 2} = \frac{3}{0} = \infty$$

