L8: Environment Model

SWS3012: Structure and Interpretation of Computer Programs

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July 12, 2023

Outline

- State (3.1)
- Mutable Data (3.3)
- Environment Model (3.2)

Announcements

- New **game chapters** are out in Source Academy
  - Back to a weekly schedule

Outline

- State (3.1)
- Mutable Data (3.3)
- Environment Model (3.2)

Recap: Variable Declaration Statement

```
let name = expression;
```

- Declares a ***variable*** *`name`* in the current scope and initializes its value to the value of *`expression`*
- From now on, *`name`* will evaluate to the value of *`expression`*
- Note that from Source §3 onwards, **function parameters** are **variables**

Recap: Assignment Statement

```
name = expression;
```

- *name* is a **variable**; not evaluated
- *expression* is evaluated, then its value is ***assigned*** to the variable *name*
- From now on, *name* will evaluate to the value of *expression*

Assignment: Pros

- Assignment allows us to create objects with ***state***
- State allows objects to behave differently **over time**

Assignment: Cons

- Harder to reason about programs
  - Harder to debug
  - Harder to verify correctness

- **Substitution model of evaluation breaks down!**
  - Not powerful enough to explain state
  - Need a more sophisticated model — ***Environment Model***

Substitution Model Breaks Down

- Consider

```
function make_simplified_withdraw(balance) {
    return amount => {
        balance = balance - amount;
        return balance;
    }
}
```

- Use **substitution model** to evaluate

```
(make_simplified_withdraw(25))(20);
```

Substitution Model Breaks Down

- Use substitution model to evaluate

```
(make_simplified_withdraw(25))(20);
       ⇩
(amount => { balance = 25 - amount; return 25; })(20);
       ⇩
balance = 25 - 20; return 25; // WRONG!
```

- It returns 25, which is wrong!

Why Substitution Model Breaks Down?

- Substitution model considers a constant/variable as **just a name for a value**
  - Its value will not change
  - Therefore, one can be substituted for the other

https://sourceacademy.nus.edu.sg/sicpjs/3.
1

- But **assignment** considers a variable as a **"container" holding a value**
  - The contents of the container may be **changed over time**
  - The container is maintained in a structure called an ***environment***

Outline

- State (3.1)
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Mutable Data

- **Assignment** gives us the ability to create ***mutable*** data, i.e. data that can be modified
  - E.g. bank account

- In Source §1 and §2, all our data were ***immutable***. We had
  - Constructors, selectors, predicates, printers
  - But no ***mutators***

Mutable Pairs

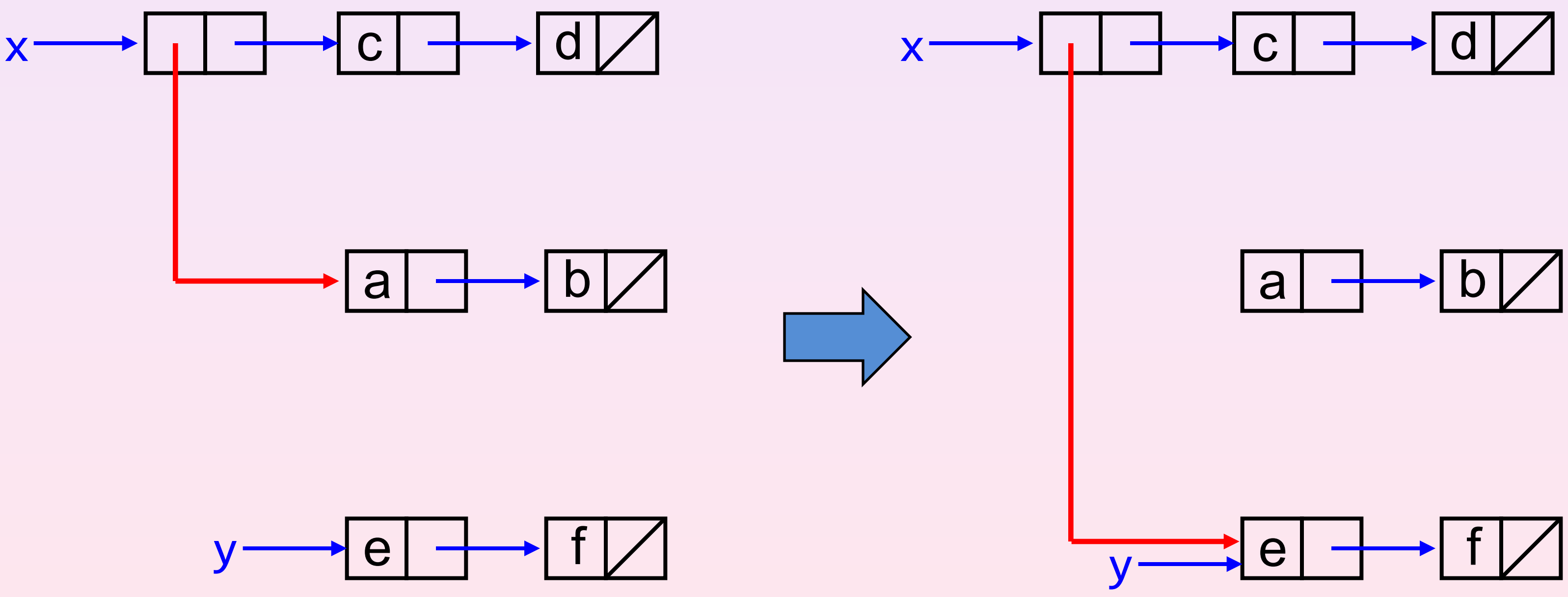
- Now we will allow ***mutators*** in order to create ***mutable data structures***

- After creating a pair with `pair`
  - The **head** can be changed using **`set_head`**
  - The **tail** can be changed using **`set_tail`**

- Mutating mutable pairs
  - `set_head(p, x)` changes **head** of pair `p` to `x`
  - `set_tail(p, x)` changes **tail** of pair `p` to `x`

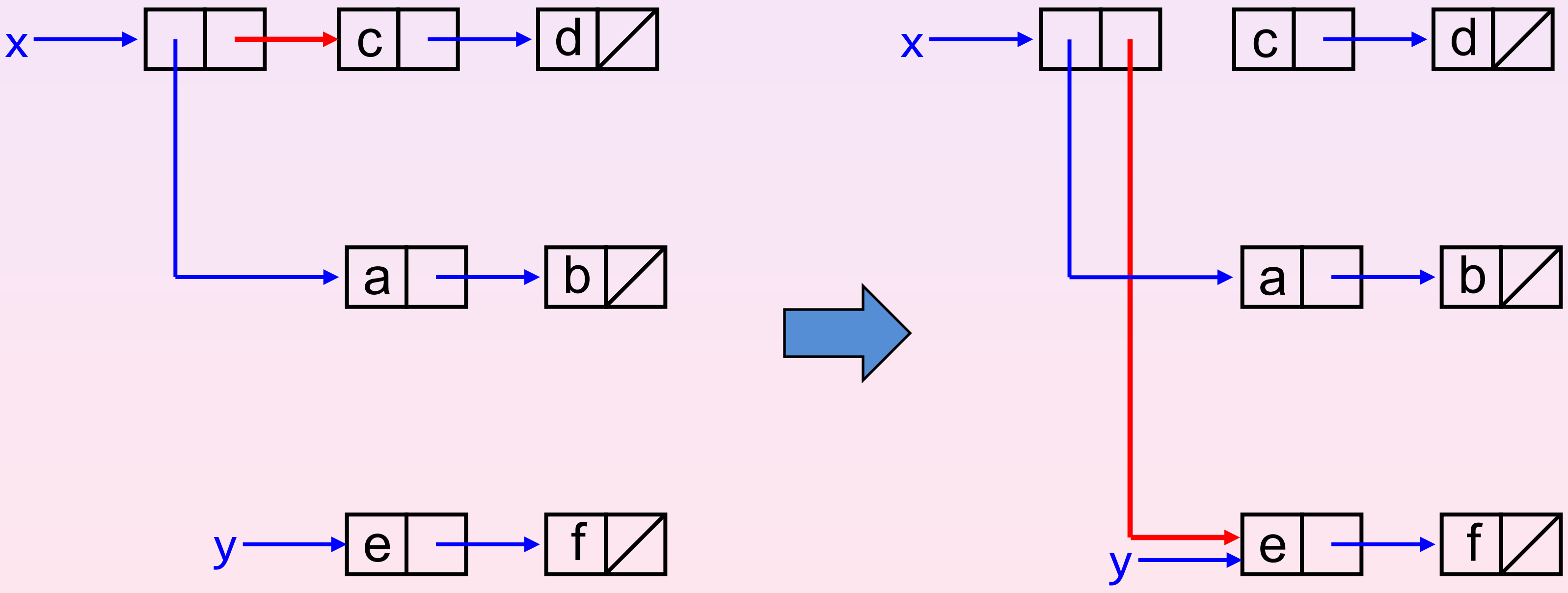
set_head Example

- Effect of `set_head(x, y)`



`set_tail` Example

- Effect of `set_tail(x, y)`



Be Careful with Mutators!

- **Example:**

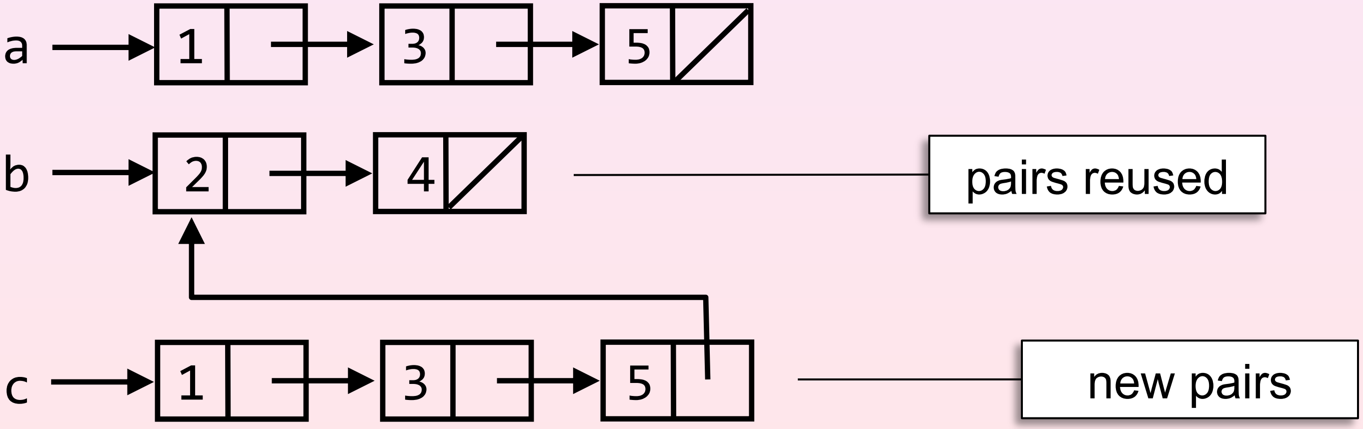
```
const a = list(1, 3, 5);
const b = list(2, 4);
const c = append(a, b);
c; ➔ [1, [3, [5, [2, [4, null]]]]]

set_head(b, 9);
b; ➔ [9, [4, null]
c; ➔ [1, [3, [5, [9, [4, null]]]]]
```

- Mutating b changes c as well !!!
- What is happening?

Recall the **append** function

```
function append(xs, ys) {
    return is_null(xs)
           ? ys
           : pair(head(xs), append(tail(xs), ys));
}
const a = list(1, 3, 5);
const b = list(2, 4);
const c = append(a, b);
```

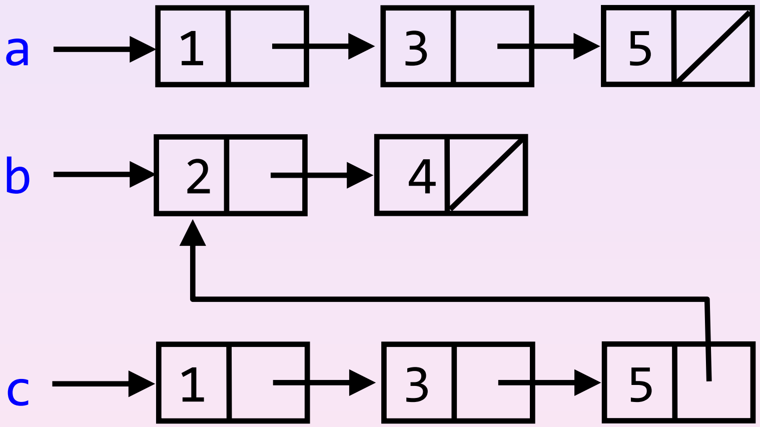


pairs reused

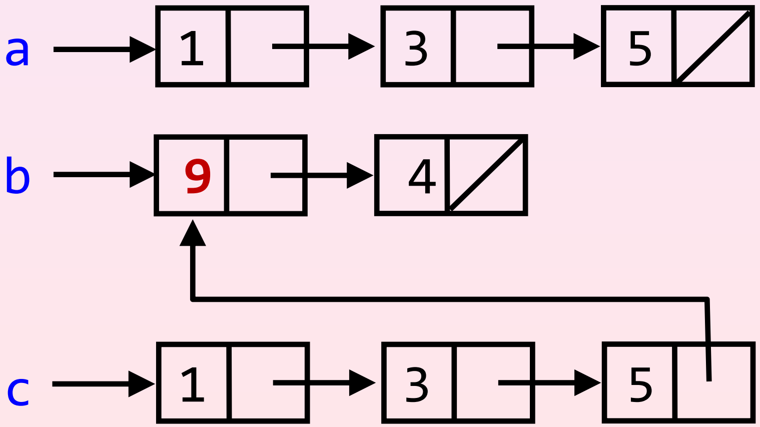
new pairs

Mutation and Sharing

- **Before** `set_head(b, 9)`



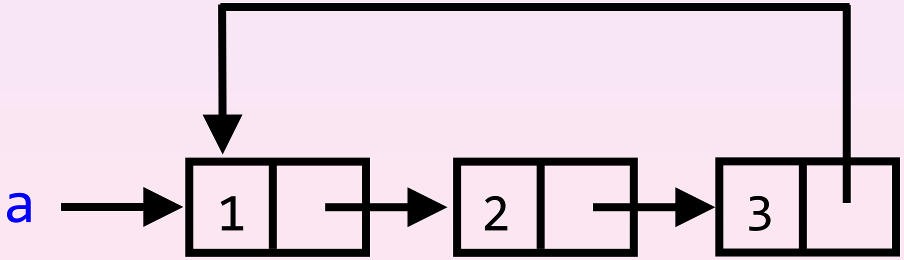
- **After** `set_head(b, 9)`



Be Careful with Mutators!

- Another example:

```
const a = list(1, 2, 3);
set_tail(tail(tail(a)), a);
```



Cyclical structure!

- What is `length(a)`?!

Mutable (“Destructive”) List Processing — Append

- **Wanted:**

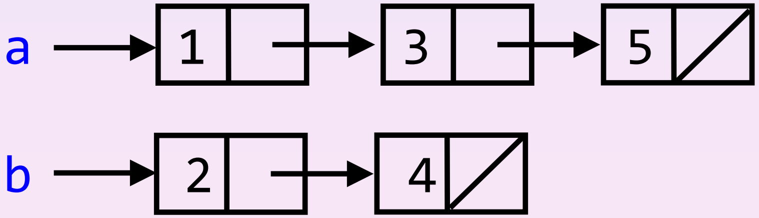
  A function to **append** two lists and return the result list

  - **No new pair must be created**
  - Result list is constructed from existing pairs of input lists

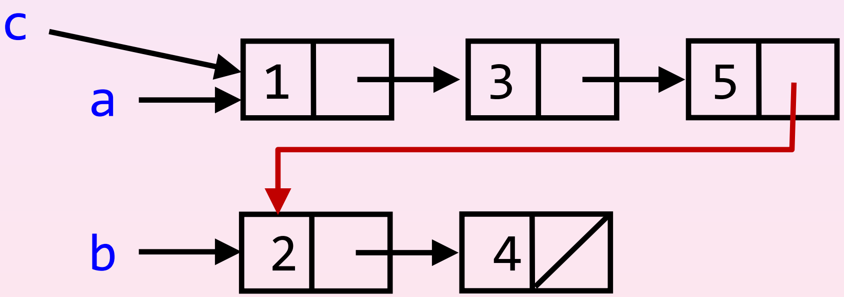
"Destructive" Append

- **Example:**

```
const a = list(1, 3, 5);
const b = list(2, 4);
```



```
const c = d_append(a, b);
```



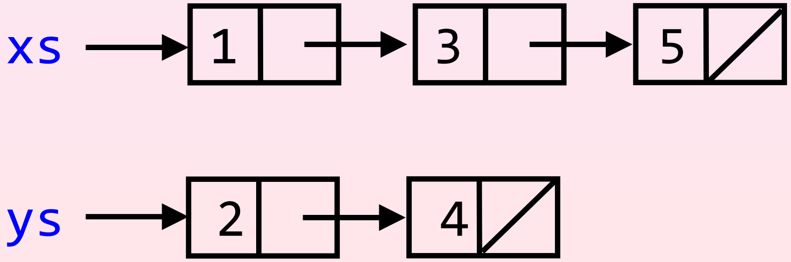
```
c; → [1, [3, [5, [2, [4, null]]]]]
a; → [1, [3, [5, [2, [4, null]]]]]
b; → [2, [4, null]]
```

“Destructive” Append

- **Implementation:**

[Show in Playground]

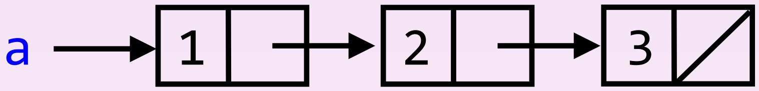
```
function d_append(xs, ys) {
    if (is_null(xs)) {
        return ys;
    } else {
        set_tail(xs, d_append(tail(xs), ys));
        return xs;
    }
}
```



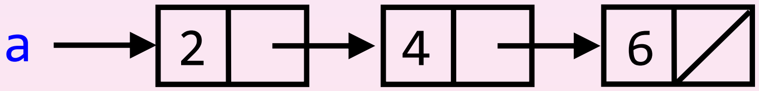
“Destructive” Map

- **Example:**

```
const a = list(1, 2, 3);
```



```
d_map(x => x * 2, a);
```



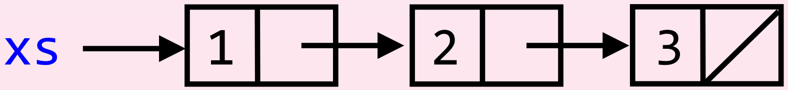
```
a; ➔ [2, [4, [6, []]]]
```

“Destructive” Map

- **Implementation:**

Show in Playground

```
function d_map(fun, xs) {
    if (!is_null(xs)) {
        set_head(xs, fun(head(xs)));
        d_map(fun, tail(xs));
    } else { }
}
```



Outline

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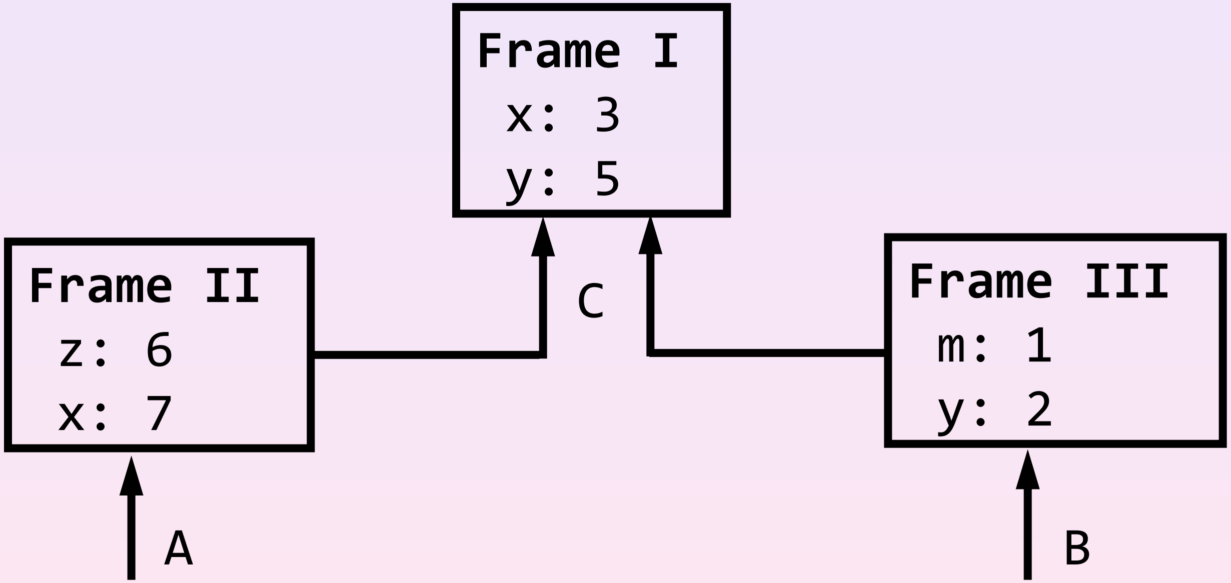
Environment

- An ***environment*** determines the context in which an expression should be evaluated
- Every **expression** is evaluated with respect to the **current** environment

Environments

- An ***environment*** is a ***sequence of frames***
- Each frame contains ***bindings*** of values to names
- A frame points to its ***enclosing environment***, the next one in the sequence
    - Unless the frame is the ***global frame***
- ***Extending*** an environment means ***adding*** a new frame "inside" the old one

Example Environments



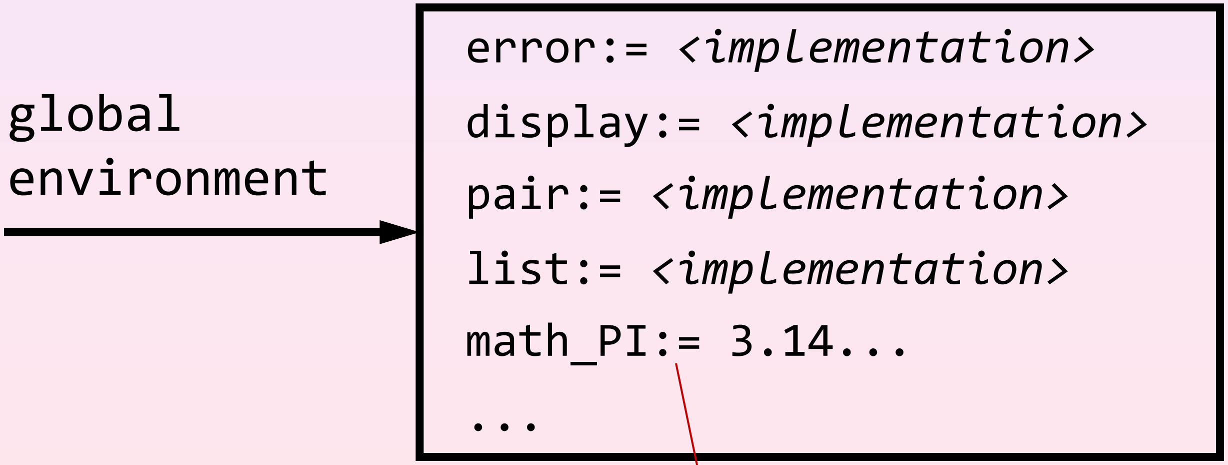
- **A**, **B**, **C** are environments
- Value of x in environment **A** is **7**
- Value of x in environment **B** is **3**
- x in Frame II ***shadows*** x in Frame I
- Value of m in environment **A**? ***unbound***

Accessing an Environment

- To evaluate a **name**, look up its value in the **current frame**
- If **not found in current frame**, then look in **enclosing environment**, and so on
- If **still not found**, then the name is said to be ***unbound*** in the environment

Global Environment

- The ***global environment*** consists of a single **frame** with bindings of **primitive** and **pre-declared** functions and constants
  - Functions’ names are bound to their implementations



global environment

```
error:= <implementation>
display:= <implementation>
pair:= <implementation>
list:= <implementation>
math_PI:= 3.14...
...
```

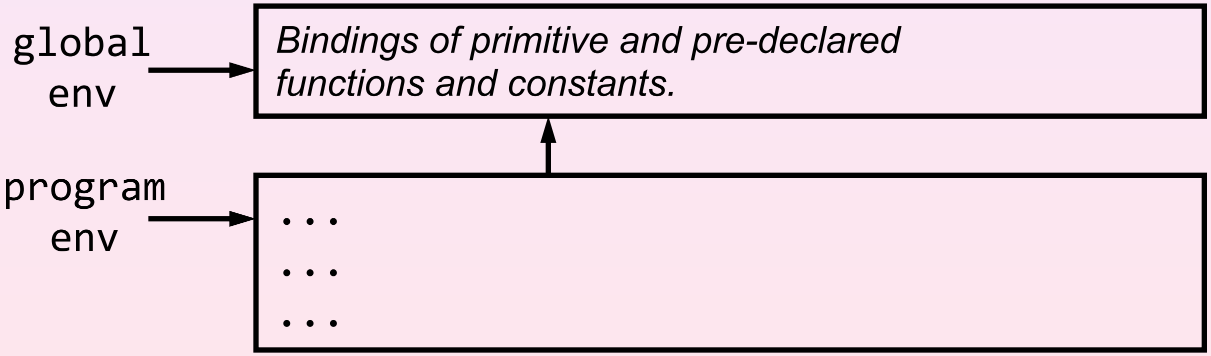
use := for constants, and : for variables

Evaluation of **Blocks**

- Evaluating a **block** `{ statements }` extends the environment by adding a **new frame**
  - New frame contains bindings of the **constants** and **variables** declared in the block
  - Initially, these **constants** and **variables** don't have associated values (**unassigned**)
  - When the **declarations** are evaluated, the **names** are **assigned** their values
- New **frame is not created** if the block has **no constant & variable declaration**
  - **There are no empty frames**

Evaluation of **Blocks**

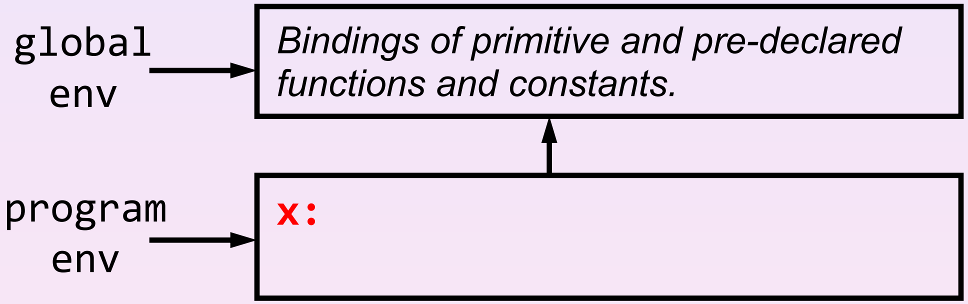
- **User program** is considered in an implicit **program block**
  - It directly extends the **global environment**
  - It is called the **program environment**


global
env
Bindings of primitive and pre-declared functions and constants.
program
env
...
...
...

Example for Evaluating Blocks

```
let x = 3;

{
    const x = 5;
    let y = 8;
}
```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program
env
x:

Example for Evaluating Blocks

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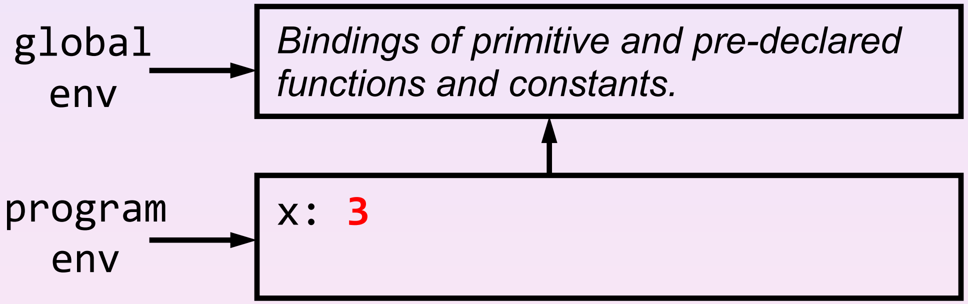

global env

Bindings of primitive and pre-declared functions and constants.

program env

x: 3

Example for Evaluating Blocks

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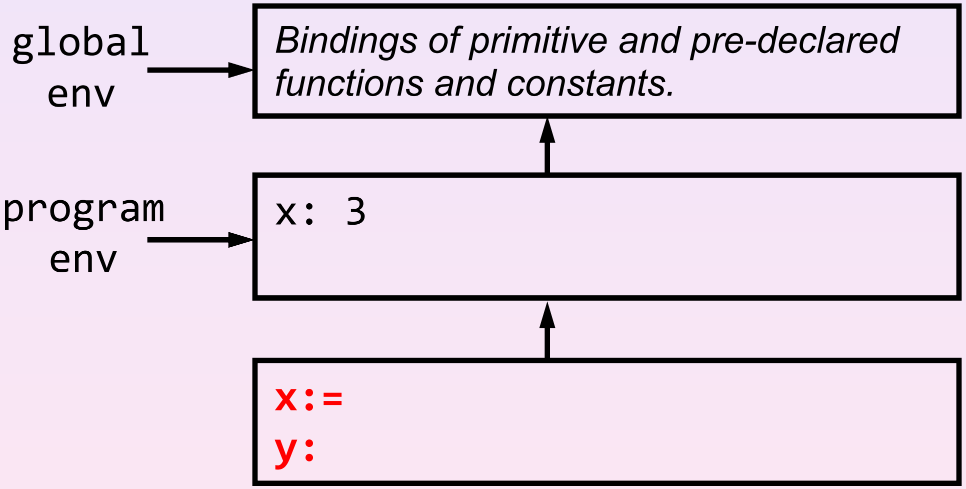
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Bindings of primitive and pre-declared functions and constants.
program
env
x: 3
x:=
y:

Example for Evaluating Blocks

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let x = 3;

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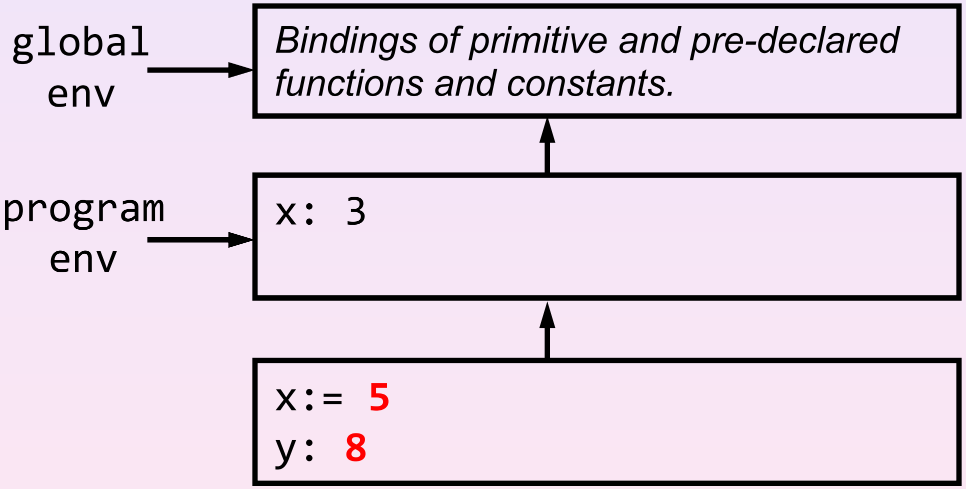
```


global env
Bindings of primitive and pre-declared functions and constants.
program env
x: 3
x:= 5
y: 8

Evaluation of **Declarations** and **Assignments**

- **Constant** / **variable** declaration statements:

```
const name = value;
let name = value;
```

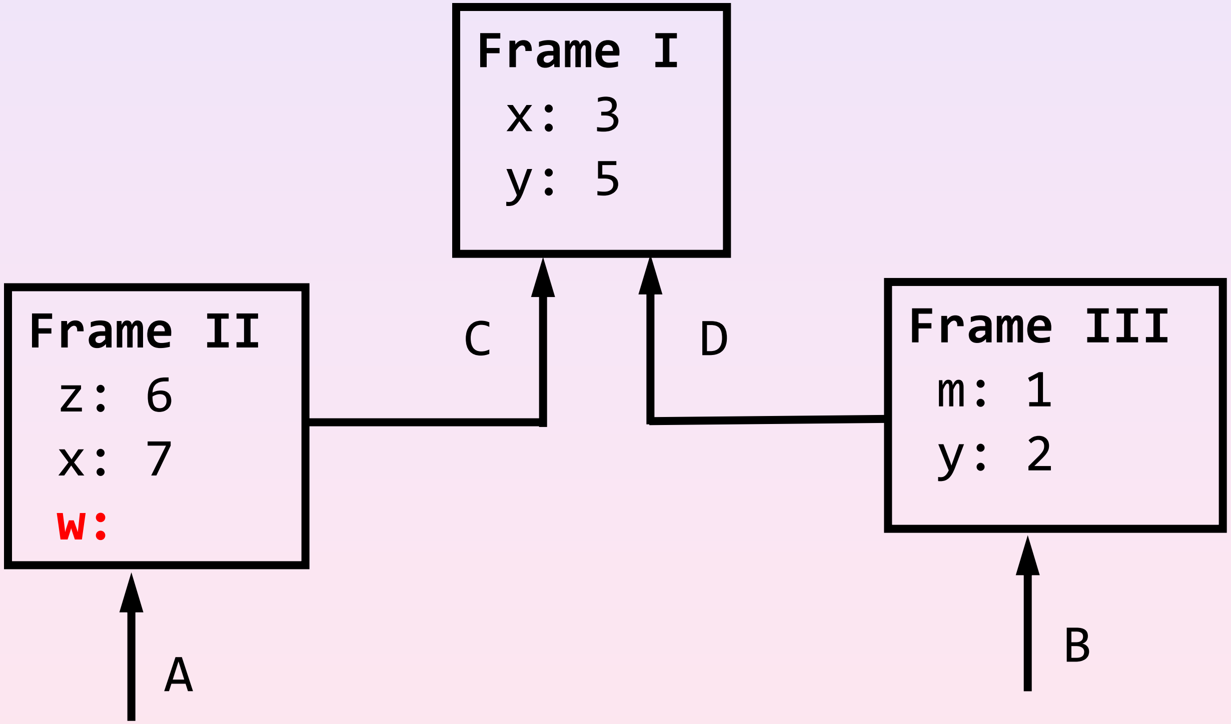
  - Changes binding of *name* to *value* in the **current frame**

- **Assignment** statement:

```
name = new_value;
```

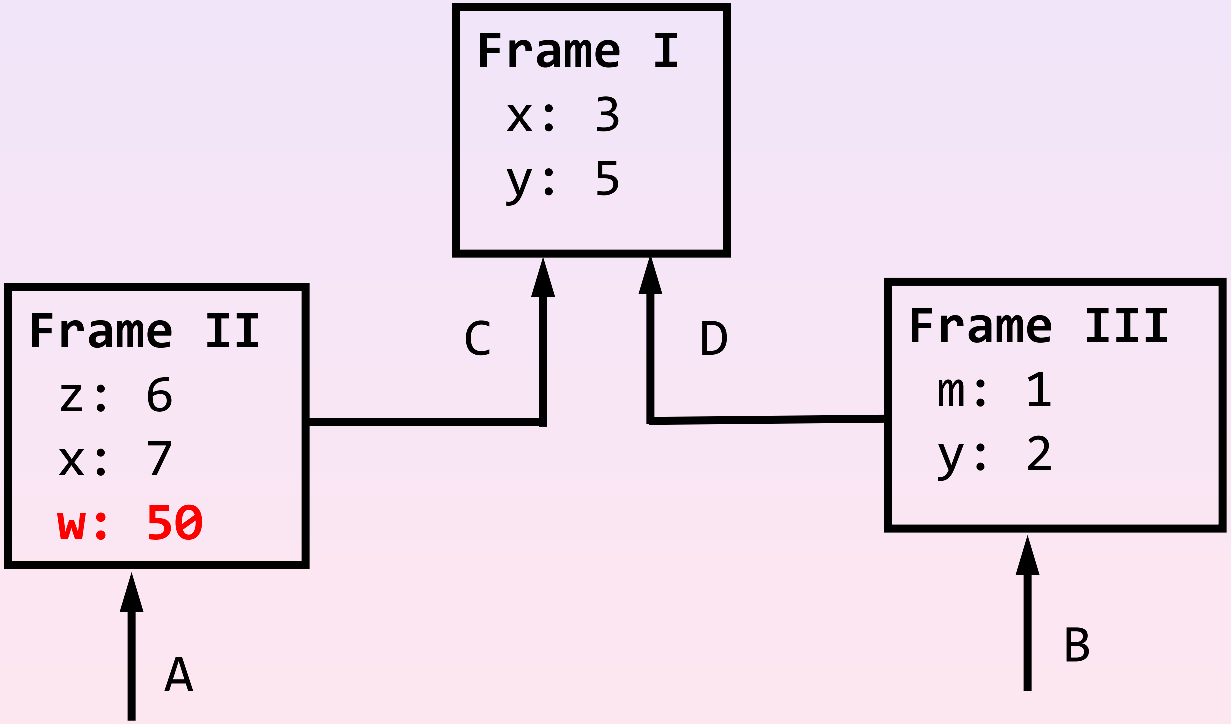
  - Searches for *name* beginning from **current frame**, then changes its binding to *new_value*
    - If *name* is a **constant**, signal an error
    - If *name* is **unbound** or **unassigned**, signal an error

Example Environments



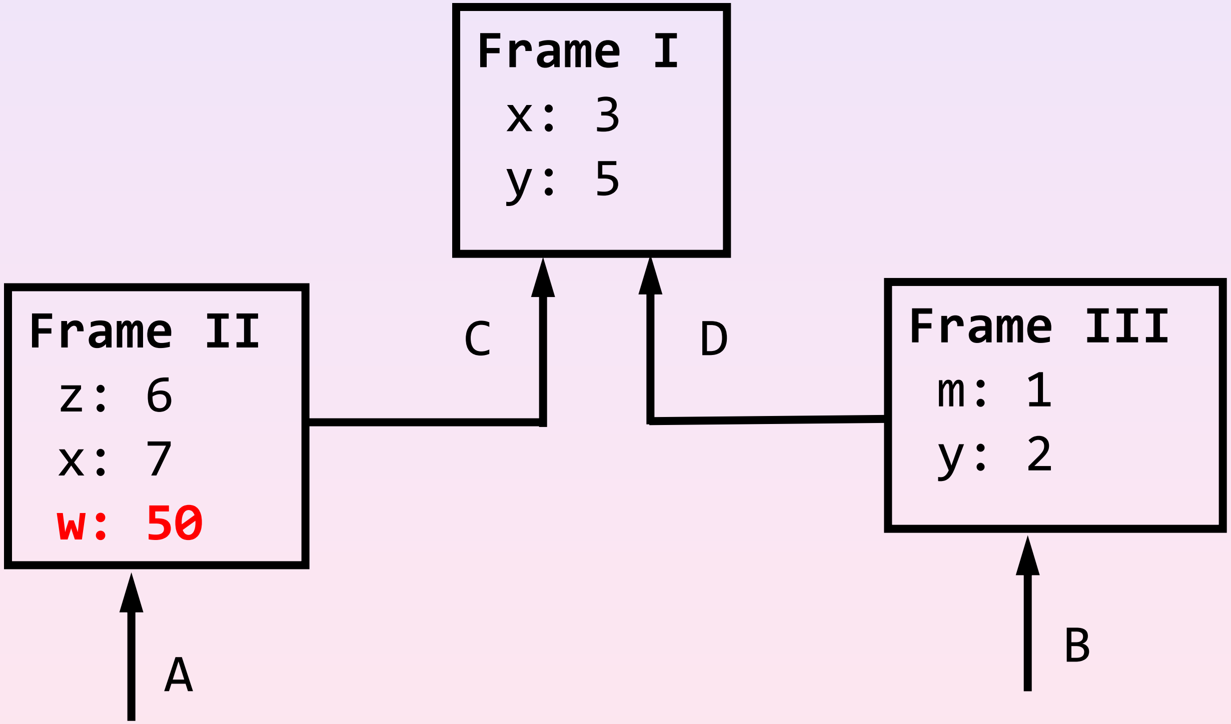
- Evaluate in **A**: `let w = 50;`

Example Environments



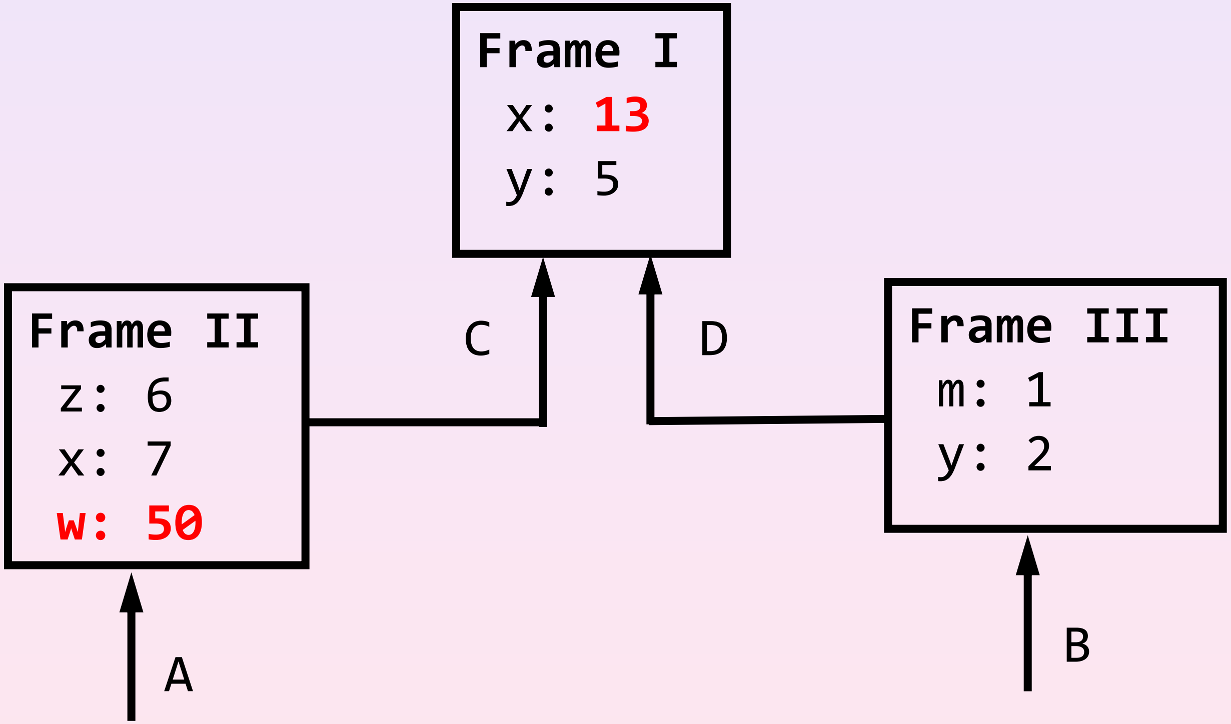
- Evaluate in **A**: `let w = 50;`
  - `w` in `Frame II` is assigned with the value **50**

Example Environments



- Evaluate in **B**: `x = x + 10;`

Example Environments



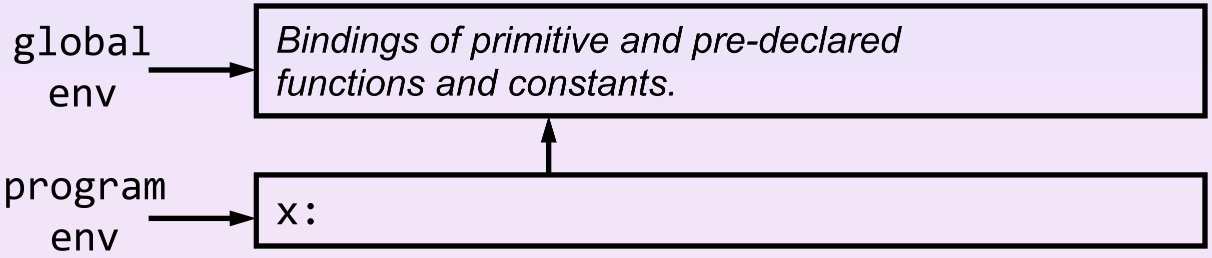
- Evaluate in **B**: `x = x + 10;`
  - The value of `x` in `Frame I` is updated from **3** to **13**

Block Example

```
let x = 3;

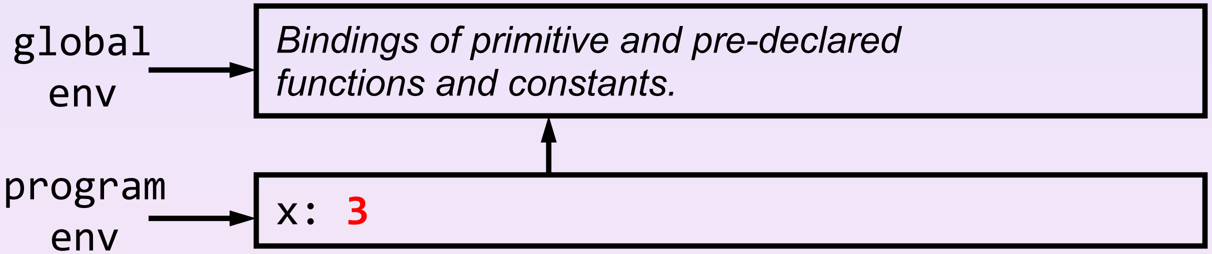
if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
        let x = 20 + y;
    }
    x = x + 1;
}
x;
```

Show in Playground


global
env
Bindings of primitive and pre-declared functions and constants.
program
env
x:

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```



global env

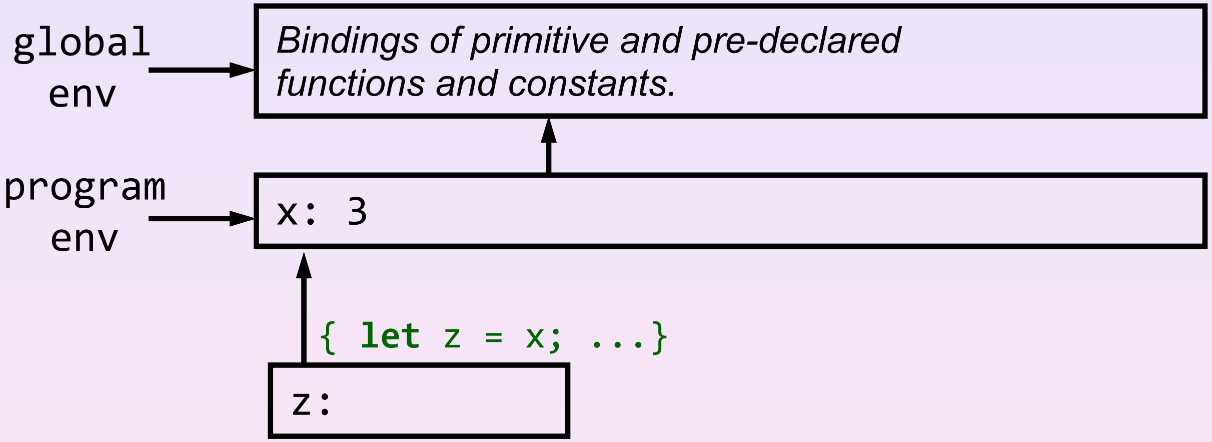
Bindings of primitive and pre-declared functions and constants.

program env

x: 3

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```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Bindings of primitive and pre-declared functions and constants.
program env
x: 3
{ let z = x; ...}
z:

```
let x = 3;

if (x <= 1) {
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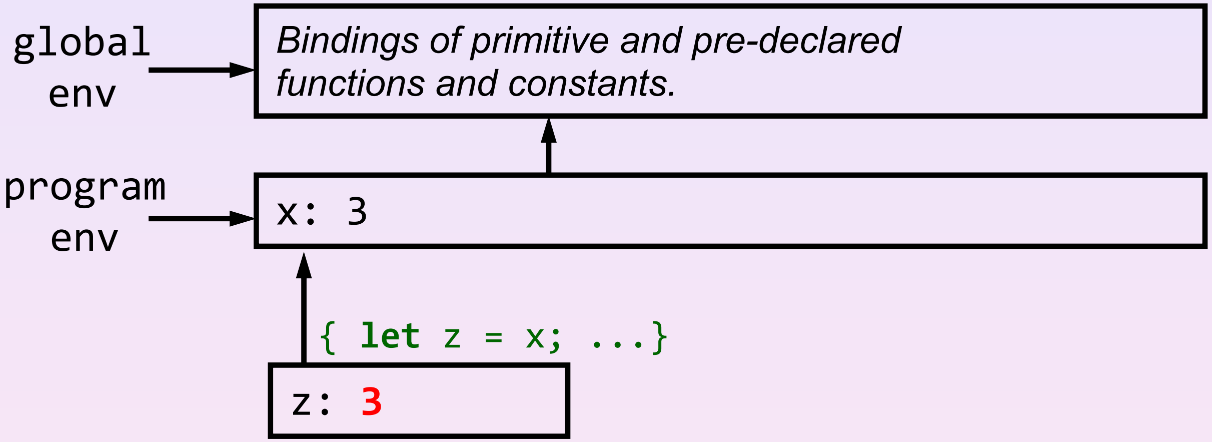
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```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Bindings of primitive and pre-declared functions and constants.
program env
x: 3
{ let z = x; ...}
z: 3

```
let x = 3;

if (x <= 1) {
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    x = y + 1;
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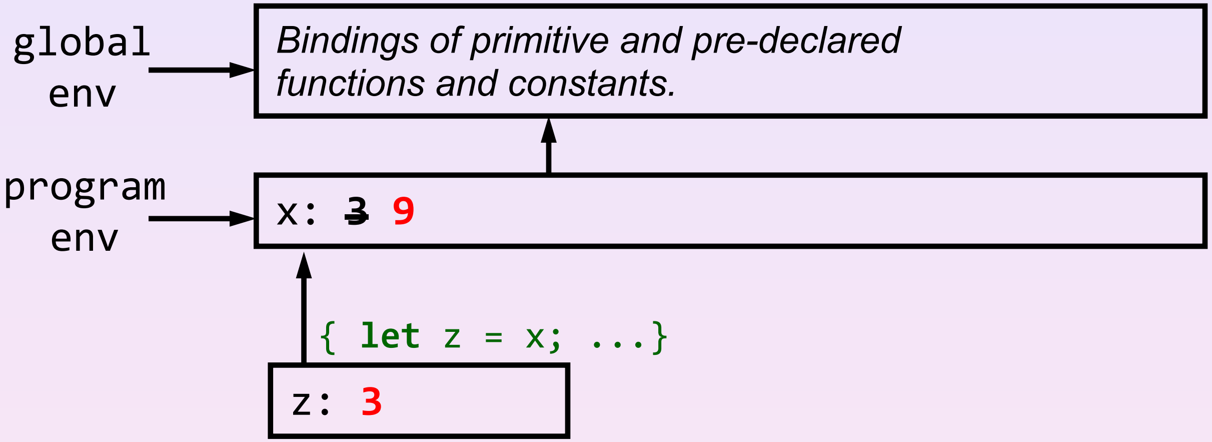
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```



global env

Bindings of primitive and pre-declared functions and constants.

program env

x: ~~3~~ 9

{ let z = x; ...}

z: 3

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
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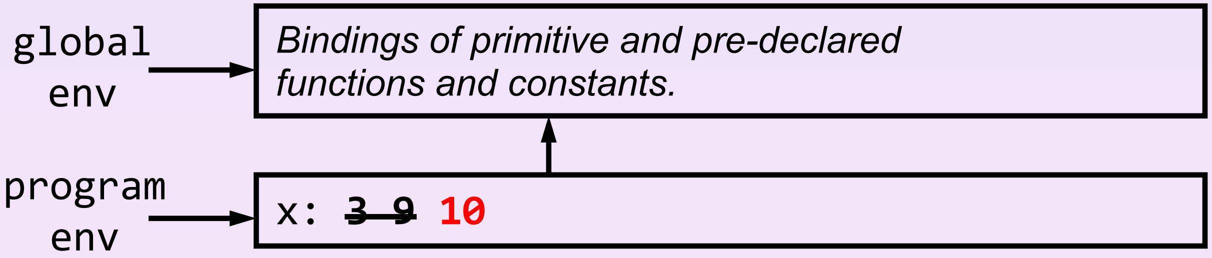
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```



global env

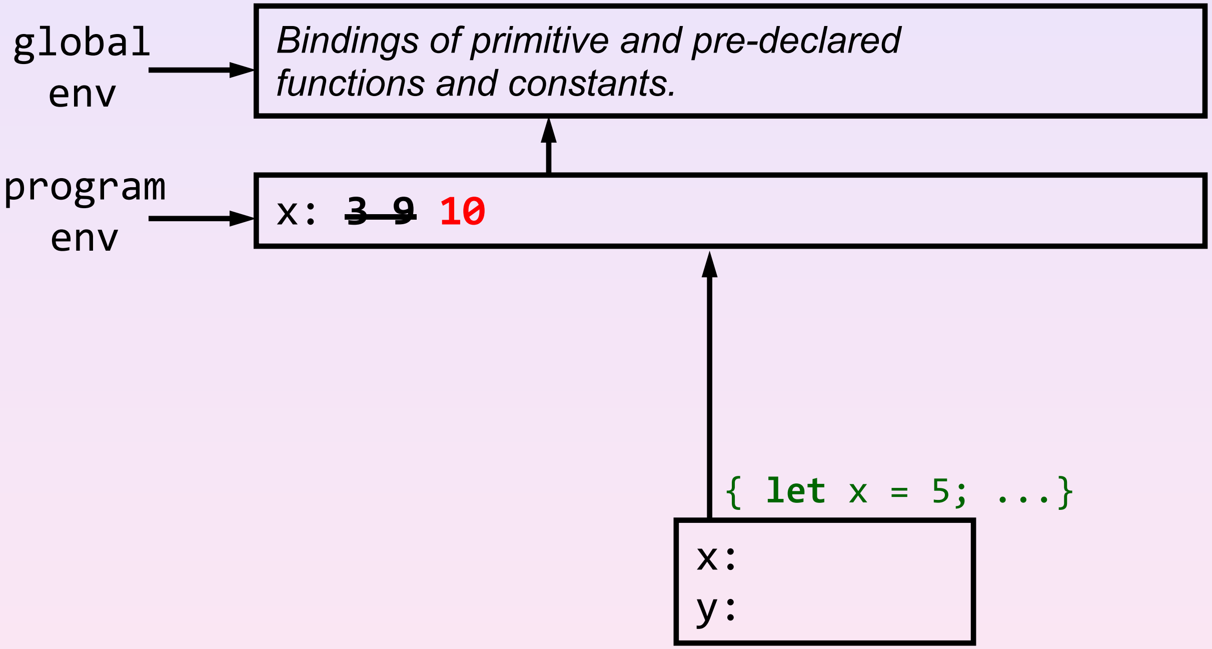
Bindings of primitive and pre-declared functions and constants.

program env

x: ~~3~~ ~~9~~ 10

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```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program env
x: ~~3 9~~ 10
{ let x = 5; ...}
x:
y:

```
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if (x <= 1) {
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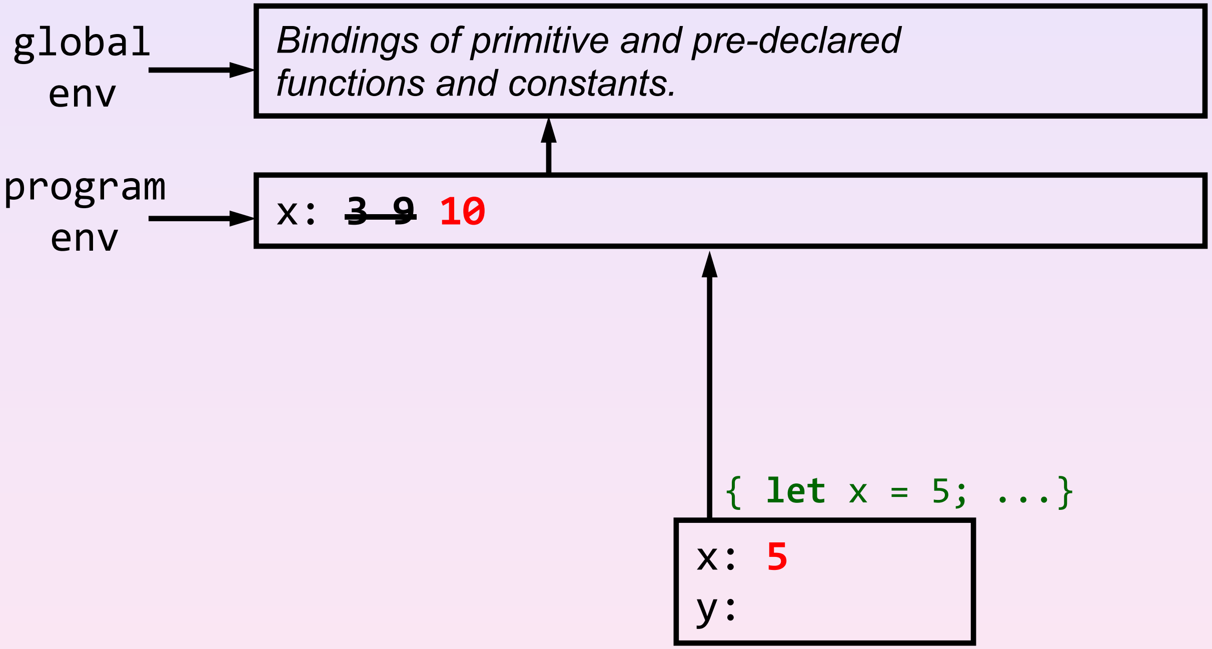
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```



global env

Bindings of primitive and pre-declared functions and constants.

program env

x: ~~3~~ ~~9~~ 10

{ let x = 5; ...}

x: 5
y:

```
let x = 3;

if (x <= 1) {
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    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
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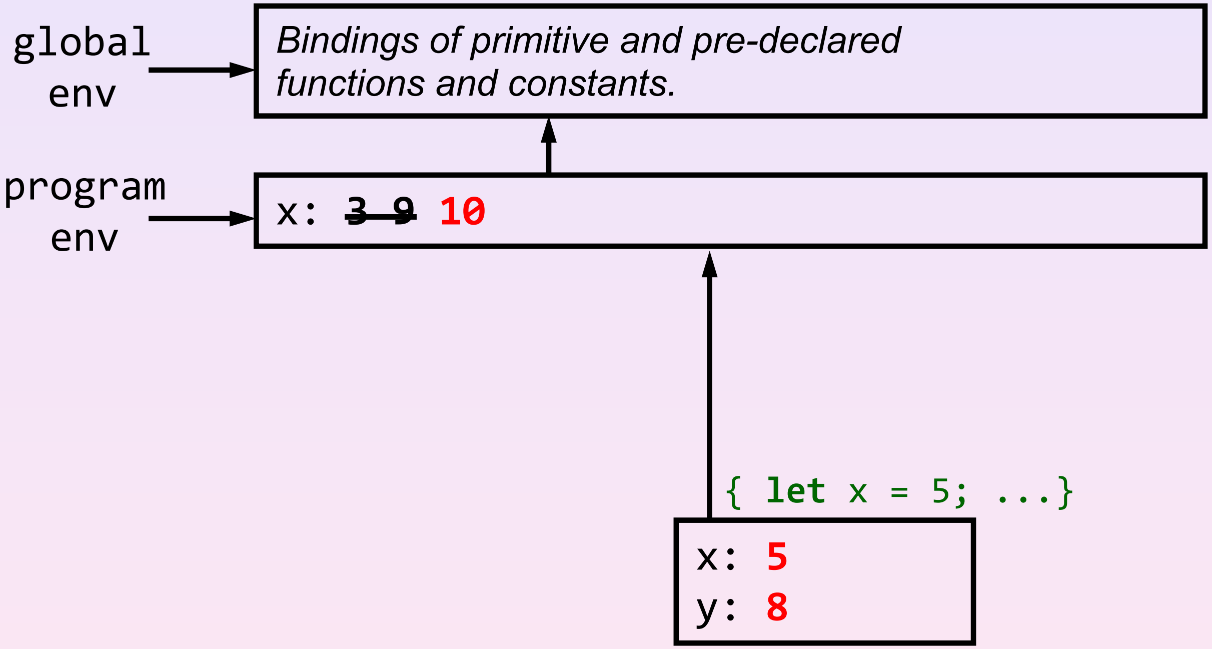
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```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x: ~~3~~ ~~9~~ 10
{ let x = 5; ...}
x: 5
y: 8

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let x = 3;

if (x <= 1) {
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    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```
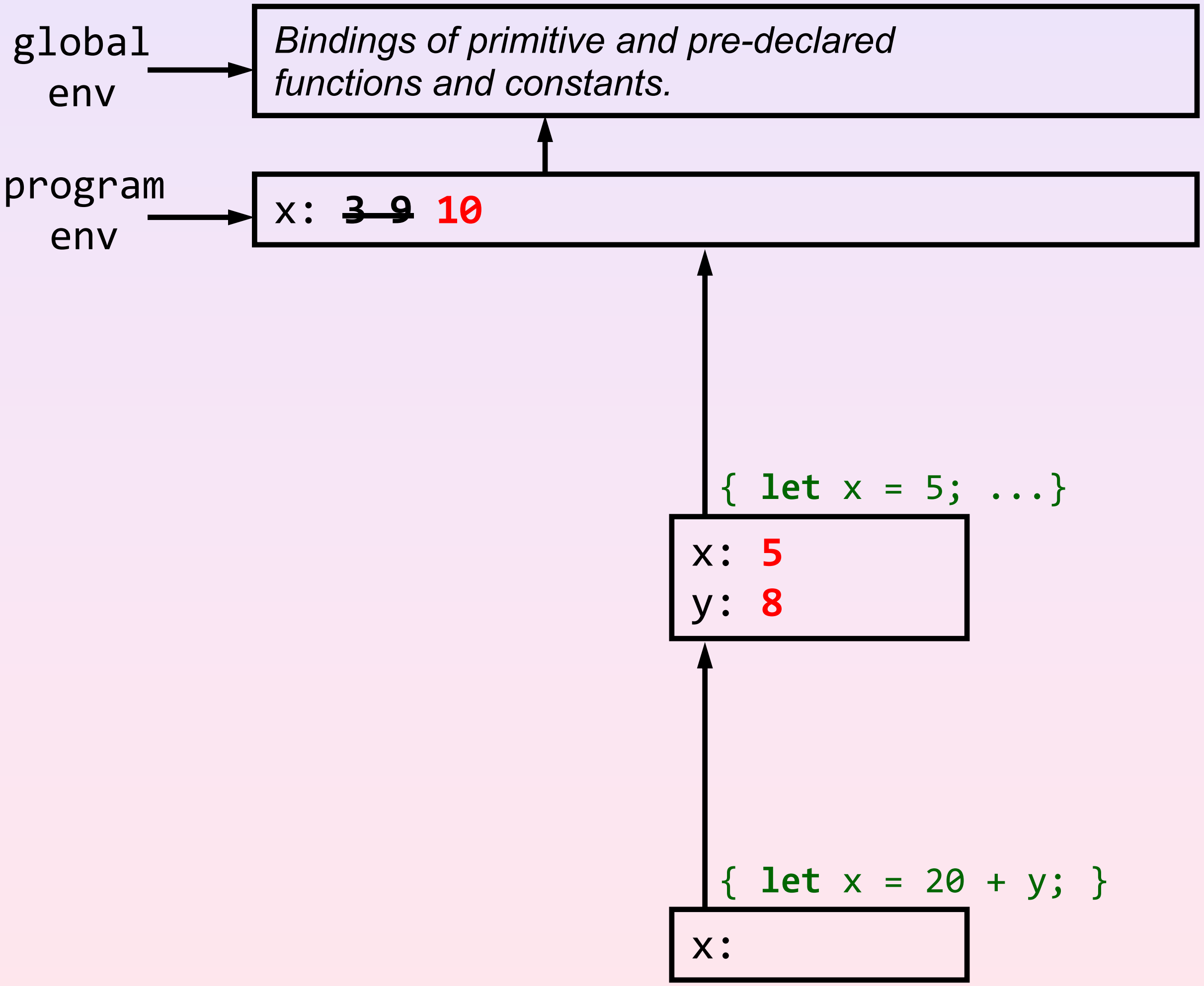

Environment Model



global env

Bindings of primitive and pre-declared functions and constants.

program env

x: ~~3 9~~ 10

{ let x = 5; ...}

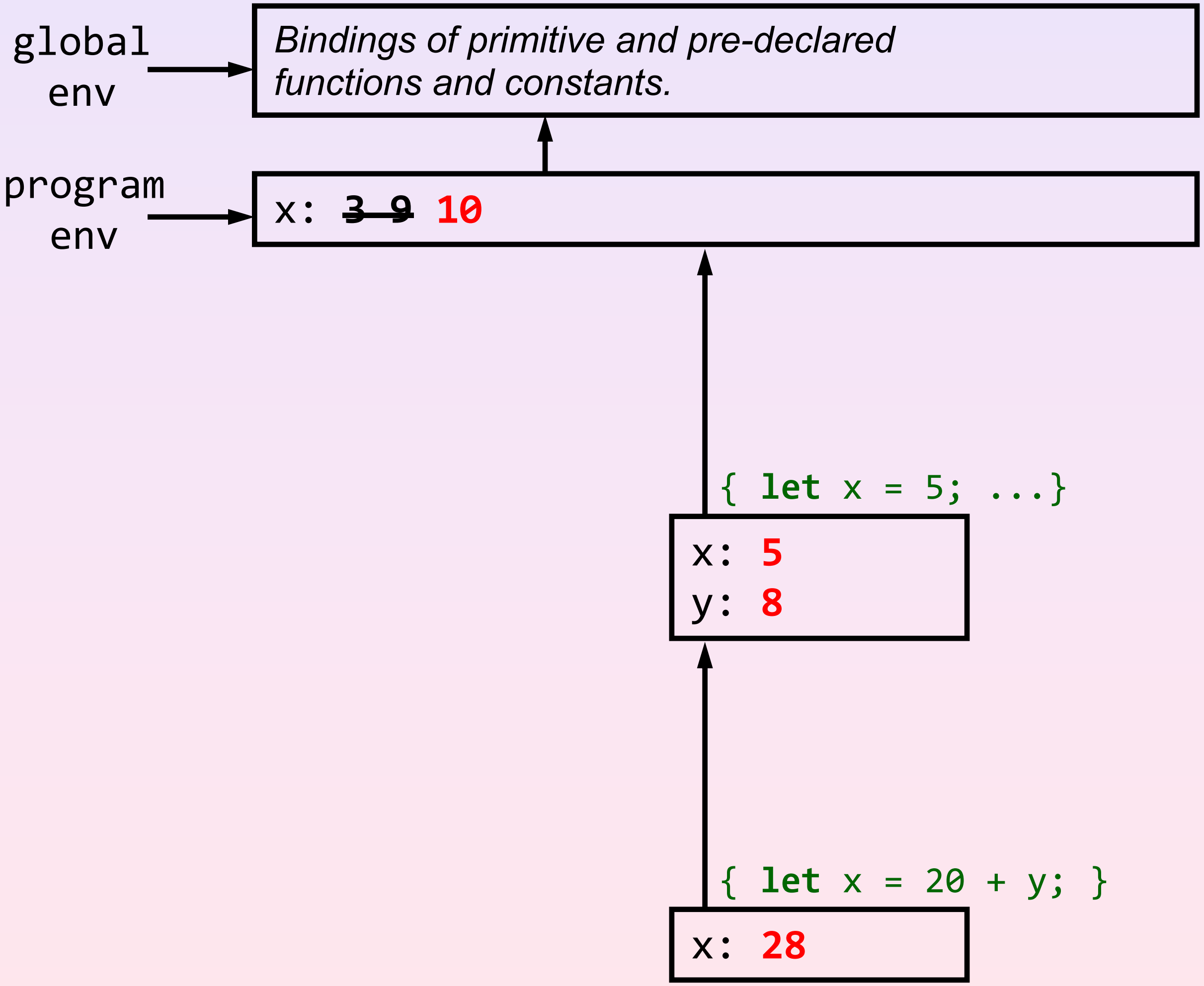
x: 5

y: 8

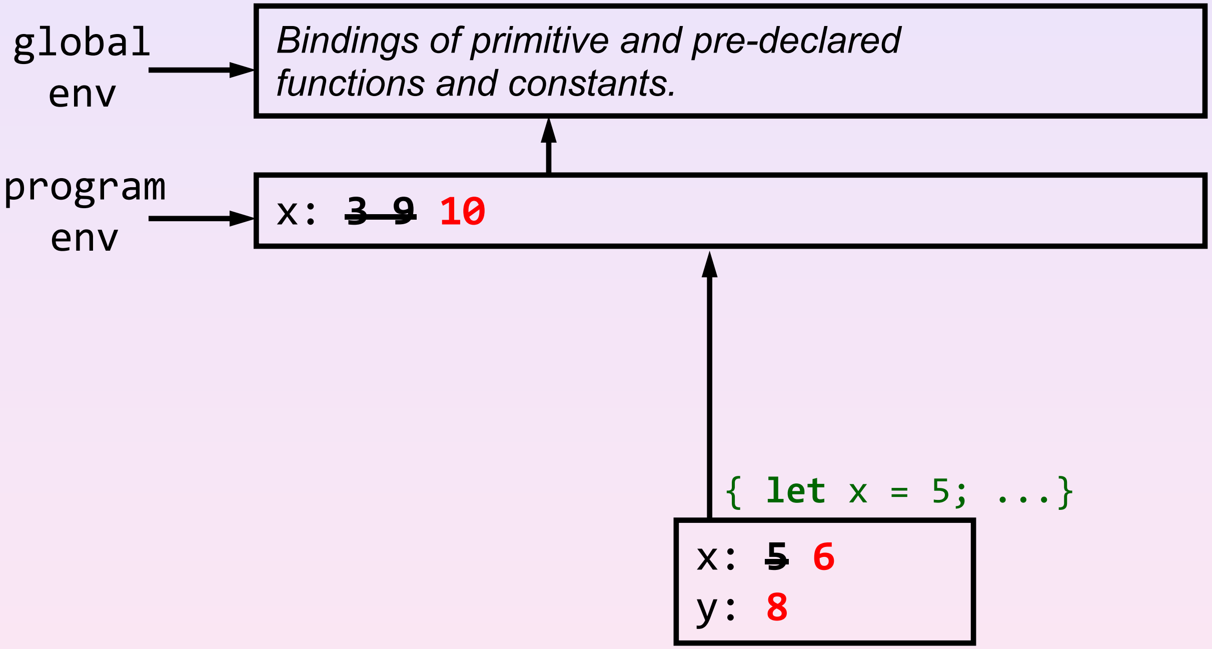
{ let x = 20 + y; }

x:

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```



```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
     let x = 20 + y;
    }
    x = x + 1;
}
x;
```



global env

Bindings of primitive and pre-declared functions and constants.

program env

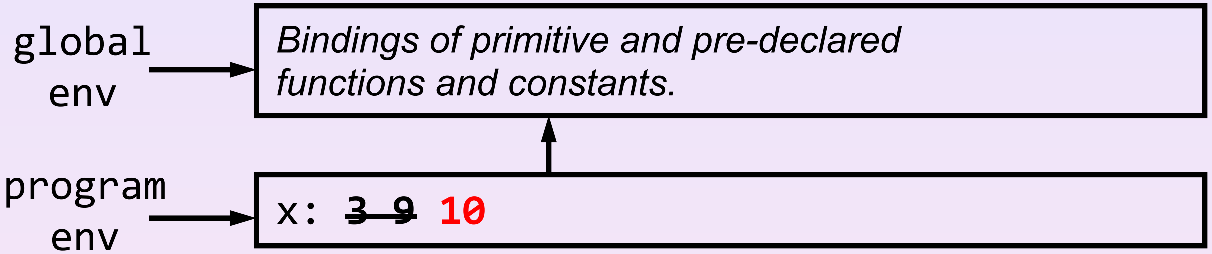
x: ~~3~~ ~~9~~ 10

{ let x = 5; ...}

x: ~~5~~ 6
y: 8

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```

Environment Model



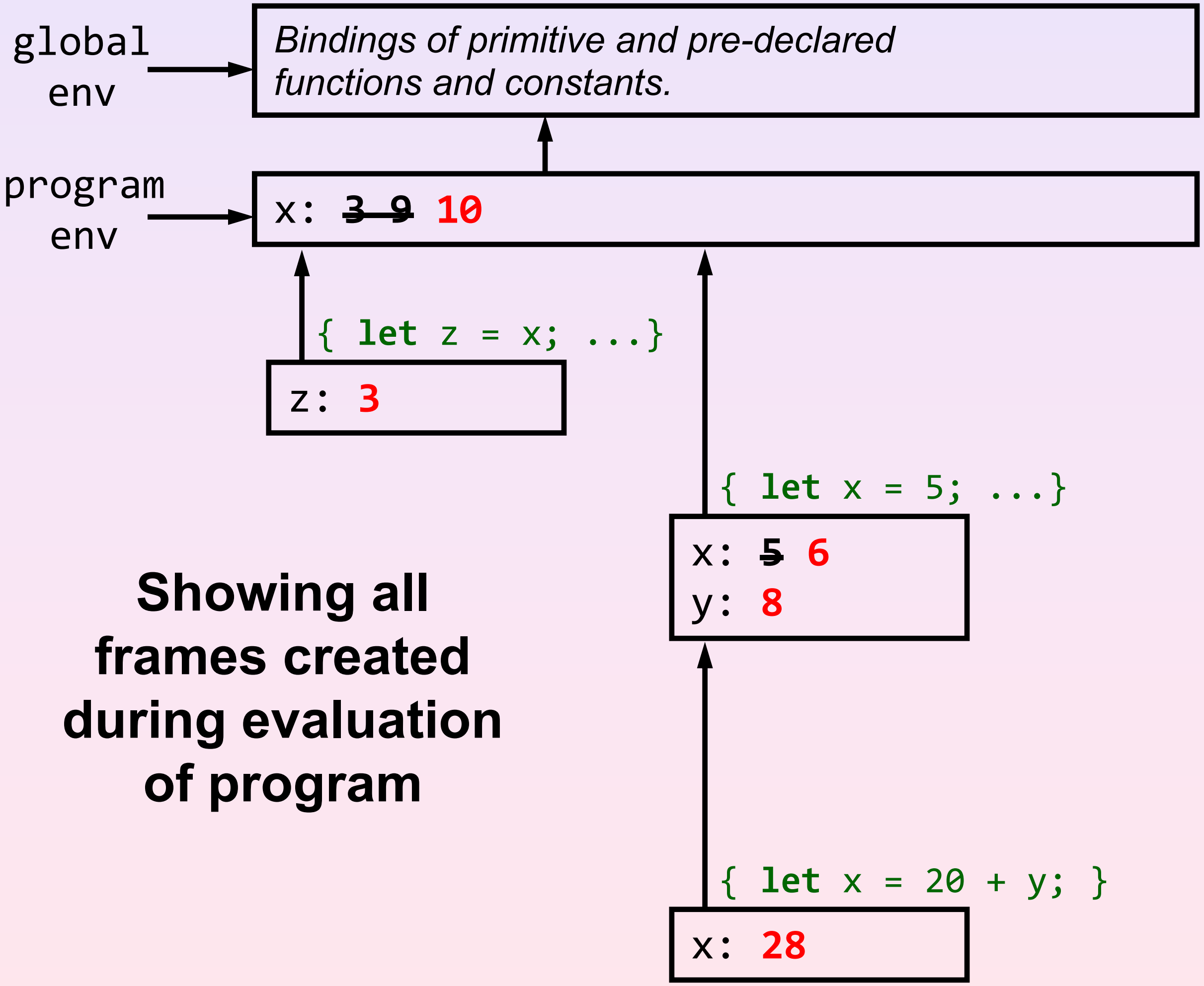
global env

Bindings of primitive and pre-declared functions and constants.

program env

x: ~~3~~ ~~9~~ 10

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```



Showing all frames created during evaluation of program

```
let x = 3;

if (x <= 1) {
    let y = x;
    x = y + 1;
} else {
    let z = x;
    x = z * z;
}
{
    x = x + 1;
}
{
    let x = 5;
    let y = 8;
    {
      let x = 20 + y;
    }
    x = x + 1;
}
x;
```

Evaluation of **Function Applications**

- Every **function application** extends the environment in which the ***function was created***
- The **new frame** contains bindings of **parameter variables** to actual arguments
  - New **frame is not created** if the function has **no parameter**
- The **function body block** is evaluated in this new environment

Evaluation of **Lambda Expressions**

- Lambda expression:

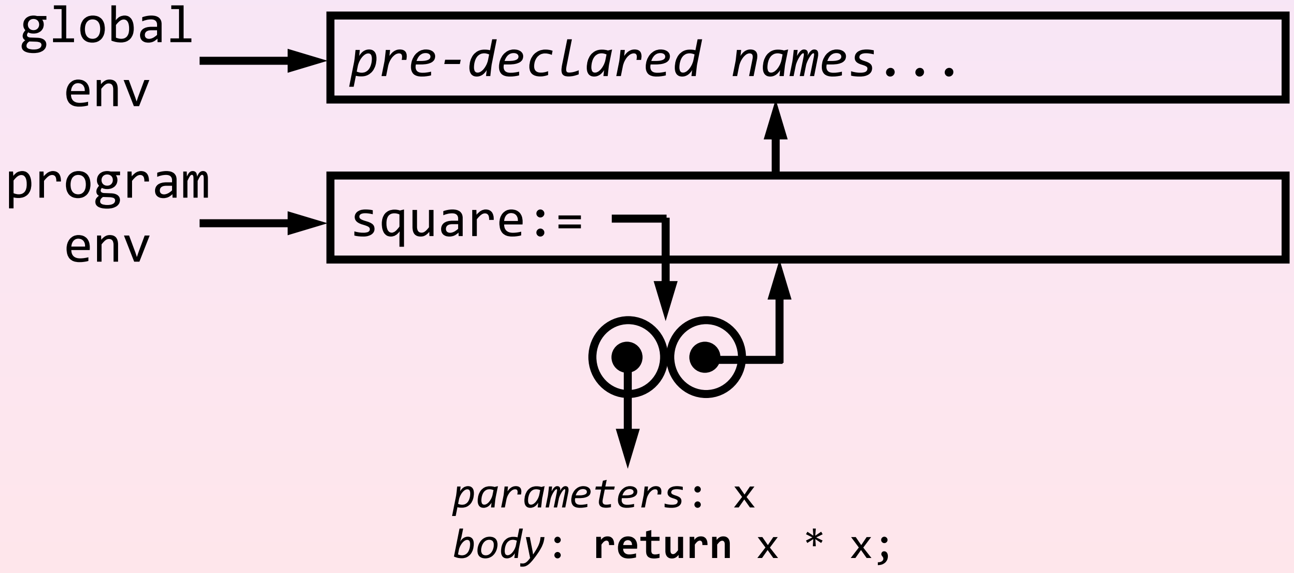
```
(var1, var2, ...) => { body }
```

  - Creates a ***function object***, represented as **two circles**
  - One points to *body* (text)
  - The other points to the environment **in which the function expression was evaluated**
  - A function therefore remembers the **environment in which it was created**

Example for Evaluating Lambda Expressions

```
const square = x => x * x;
square(5);
```

- After evaluating the constant declaration in **program env**:



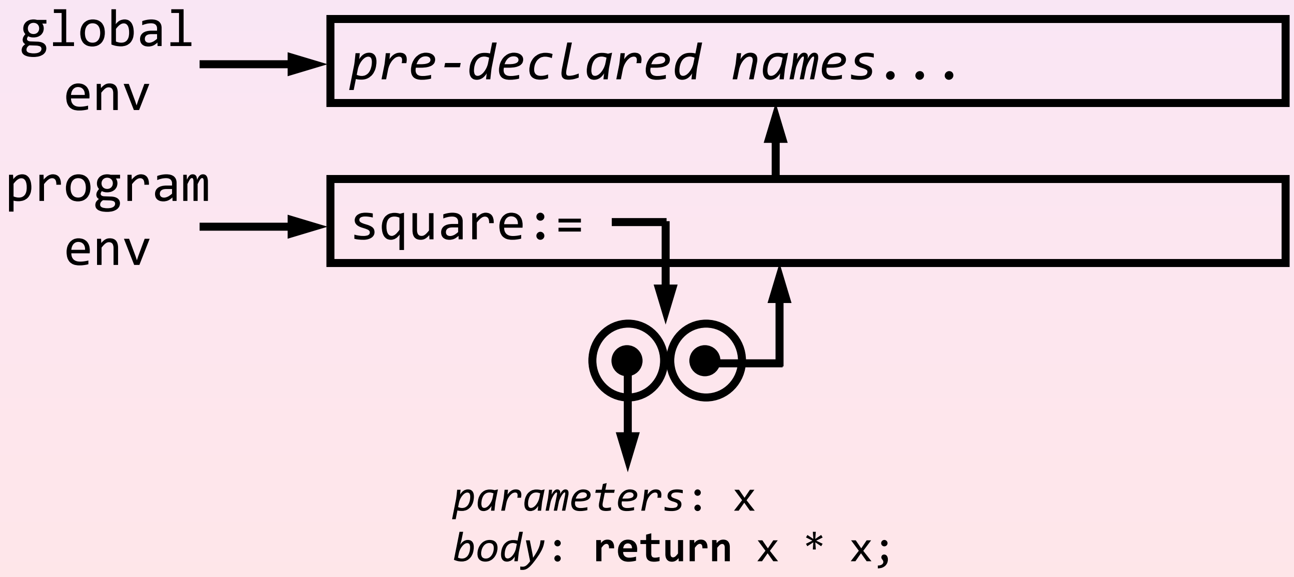
Example for Evaluating Function Declarations

```
function square(x) { return x * x; }
```

which is equivalent to*

```
const square = x => x * x;
```

- After evaluating the function declaration in **program env**:



Evaluation of **Function Applications**

- To evaluate a **function application**:

  ```
  fun(arg1, arg2, ..., argn);
  ```

- Evaluate the **subexpressions** in the **current environment**

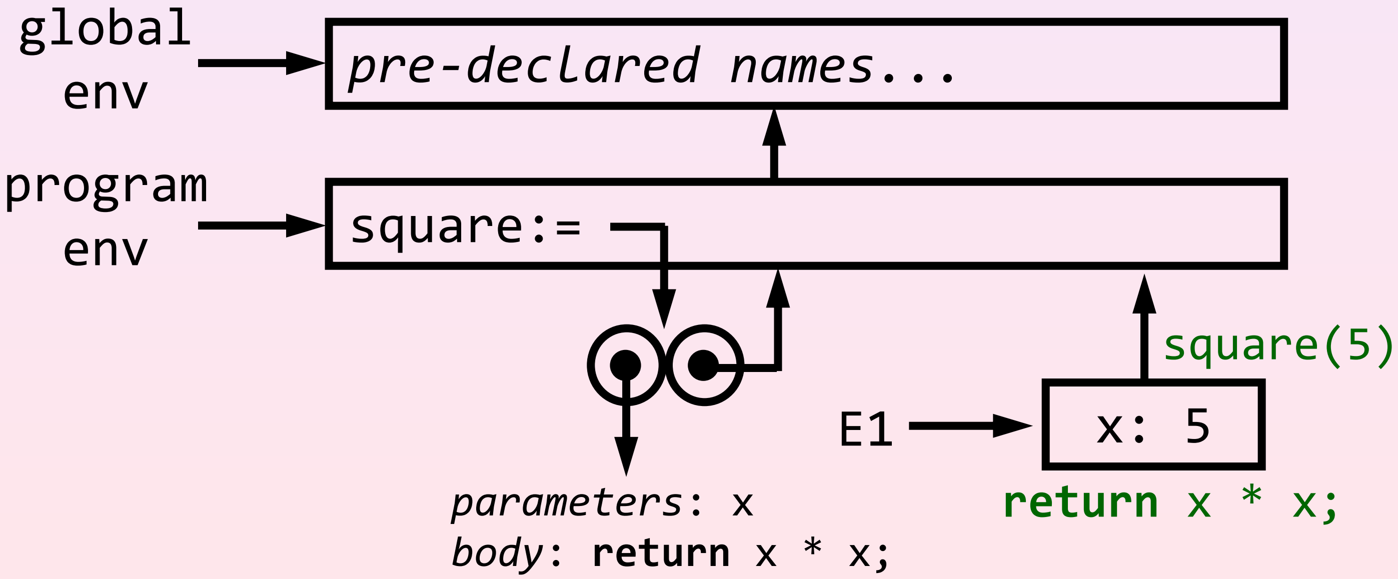
- Apply the value of the **function subexpression** (`fun`) to the values of the **argument subexpressions** (`arg1`, `arg2`, ..., `argn`)

Applying a Function Value to Argument Values

- Create a new frame that **points to the environment of the function**
  - **The environment of the function** is the environment in which the function was created
- In this new frame, **bind the formal parameters to the actual arguments**
- Evaluate the **body block** of the function in the new environment

Example: `square(5);`

- Evaluate subexpressions `square` and `5` in **program env**
- Create new frame that points to environment of `square`
- In this frame, bind value 5 to parameter `x`
- Evaluate body **`return`** `x * x;`



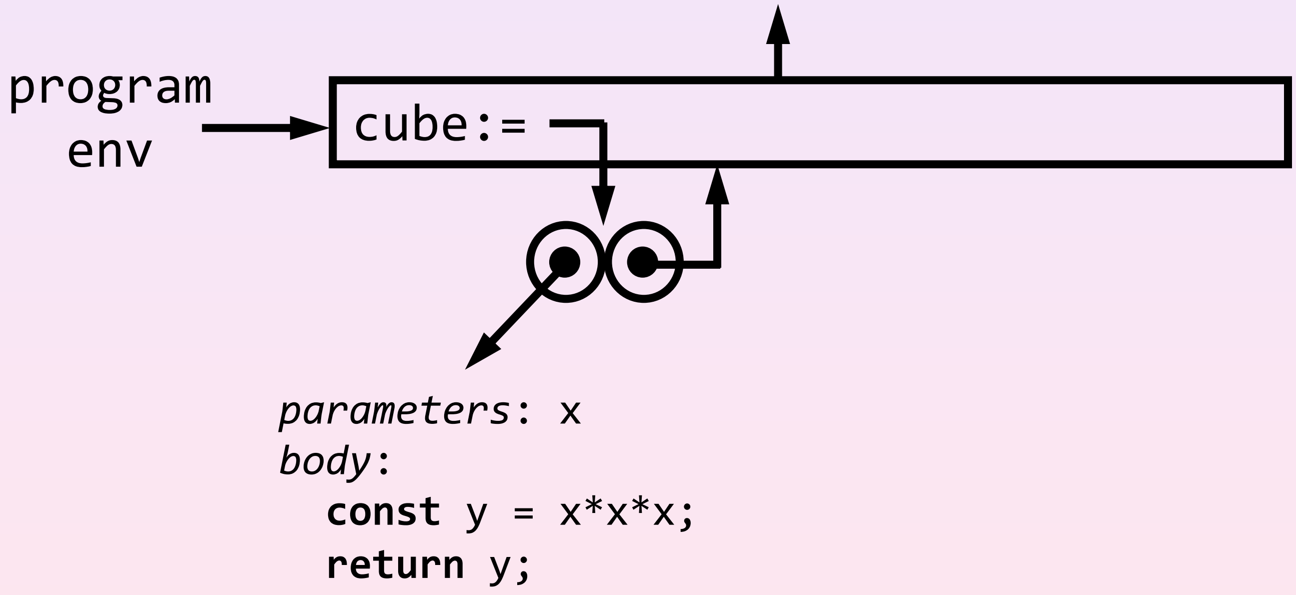
Example with Local Declaration

- The cube function has a **local constant** y

```
function cube(x) {
    const y = x * x * x;
    return y;
}
cube(3);  ➔  27
```

cube Example

- After evaluating the declaration of function cube



program env

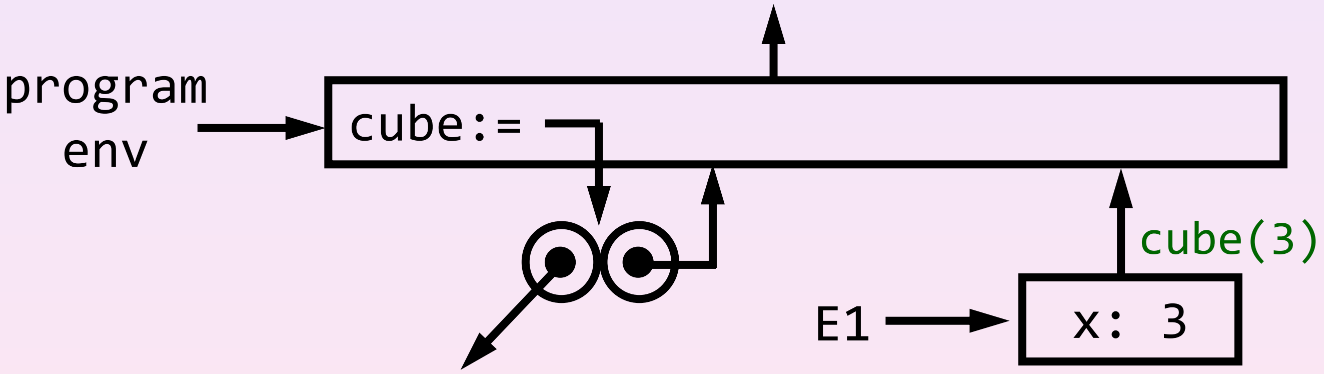
cube:=

parameters: x
body:
```
  const y = x*x*x;
  return y;
```

```
function cube(x) {
    const y = x * x * x;
    return y;
}
cube(3);
```

cube Example

- Evaluate cube(3) in **program env**

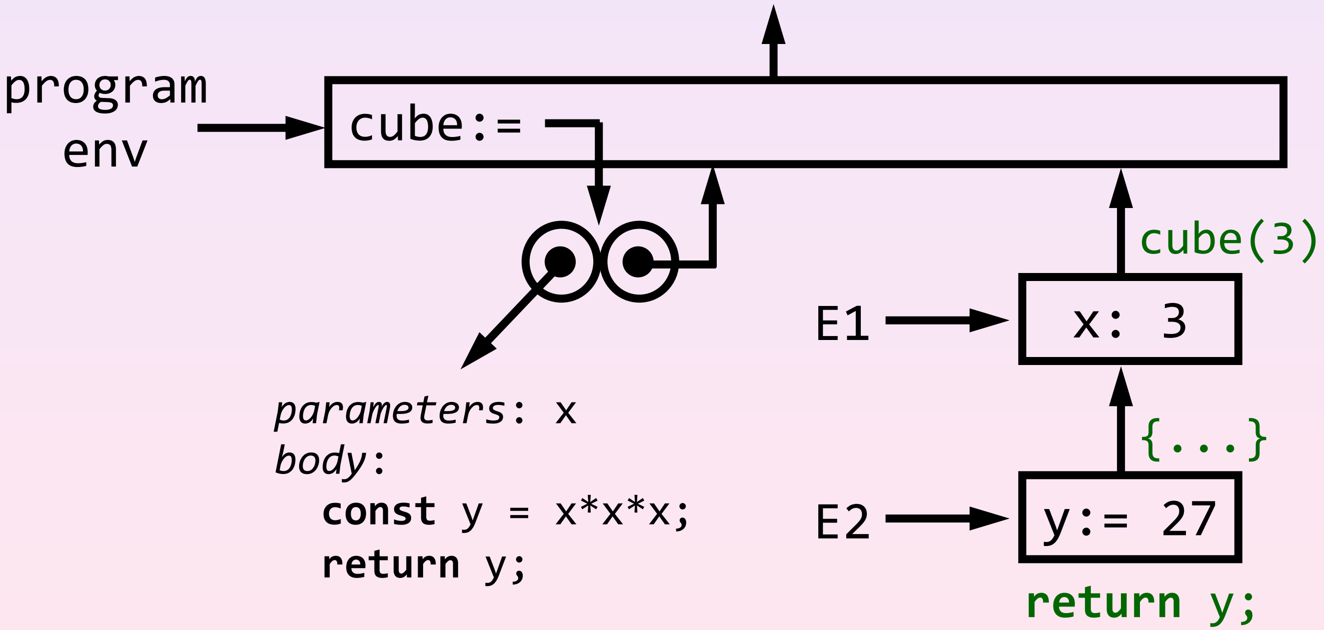


```
parameters: x
body:
  const y = x*x*x;
  return y;
```

```
function cube(x) {
    const y = x * x * x;
    return y;
}
cube(3);
```

cube Example

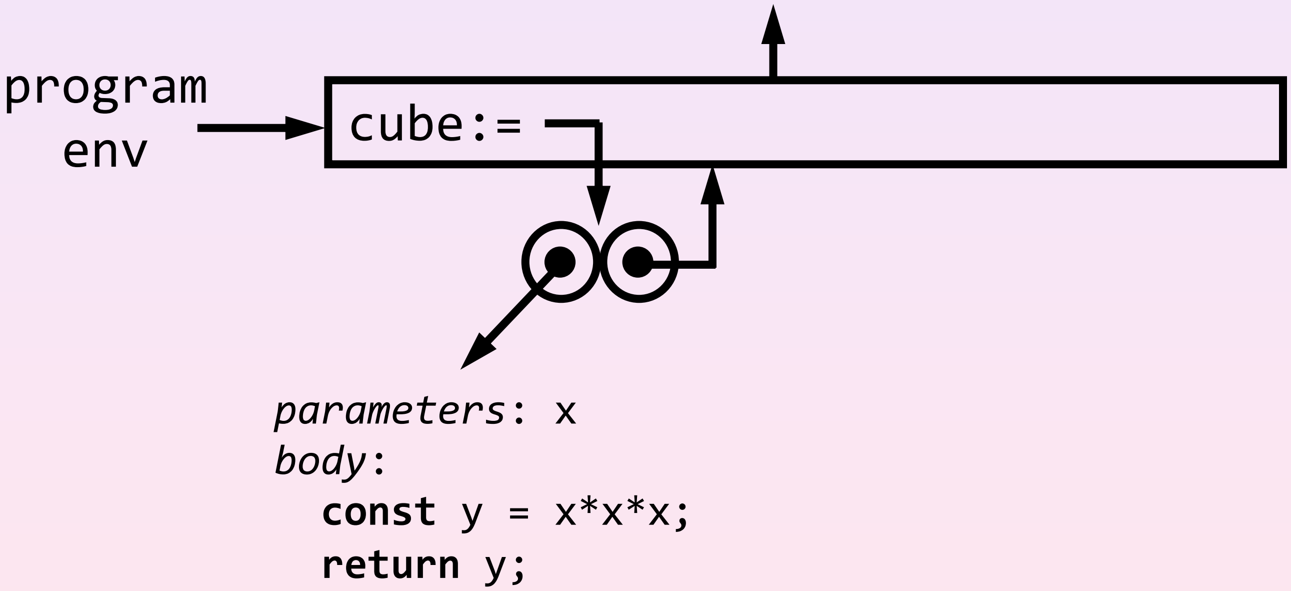
- Evaluate function **body block**



```
function cube(x) {
    const y = x * x * x;
    return y;
}
cube(3);
```

cube Example

- Environment after evaluation of `cube(3)`



program env

cube:=

parameters: x
body:

```
  const y = x*x*x;
  return y;
```

```
function cube(x) {
    const y = x * x * x;
    return y;
}
cube(3);
```

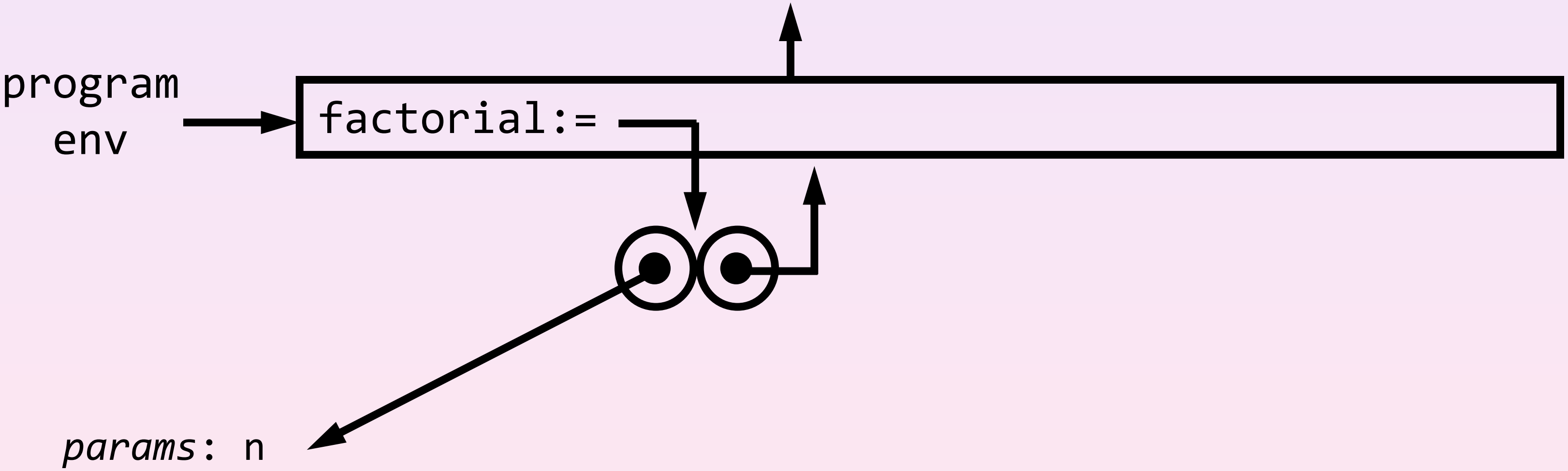
Example with Recursion

- Recursive `factorial` function

```
function factorial(n) {
    return n <= 1
           ? 1
           : n * factorial(n - 1);
}
factorial(2); → 2
```

factorial Example

- After evaluating the declaration of function factorial



program env

factorial:=

params: n

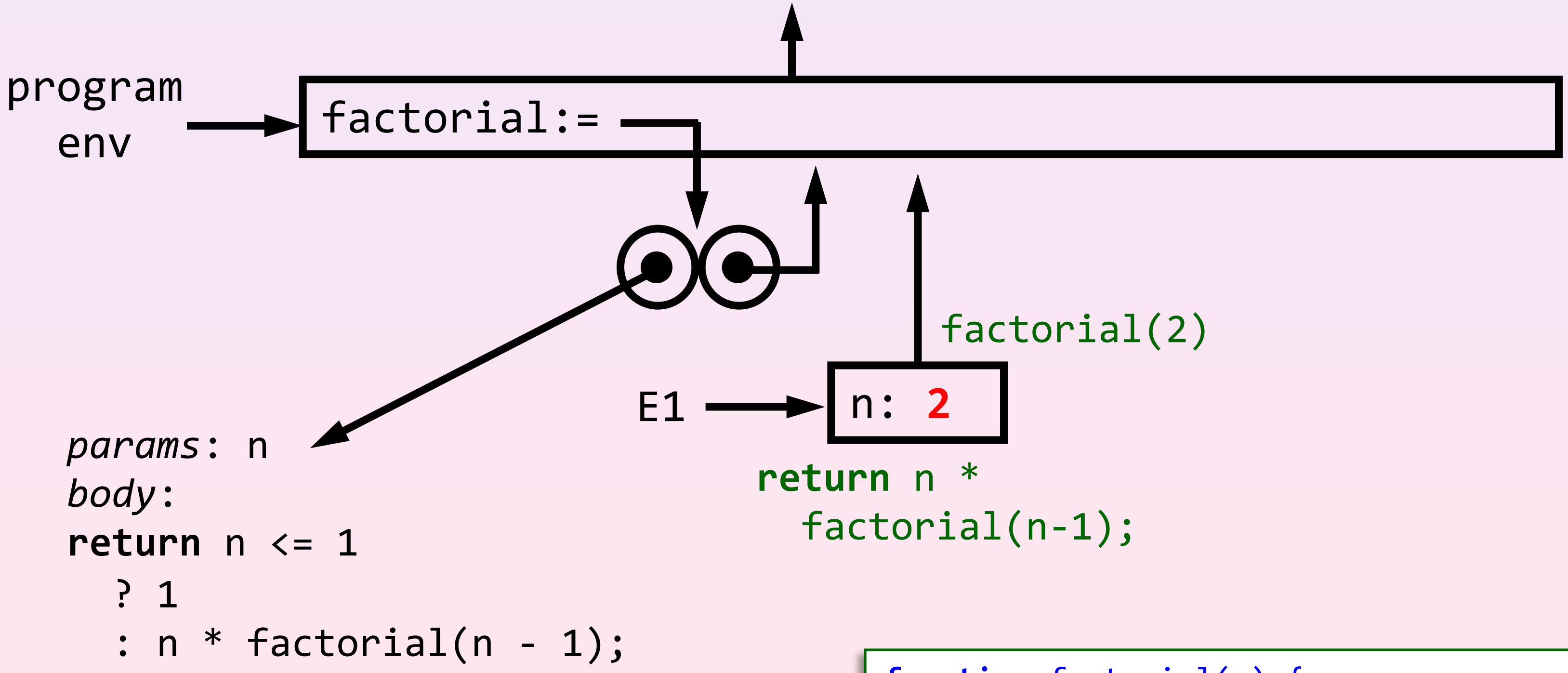
body:

```
return n <= 1
  ? 1
  : n * factorial(n - 1);
```

```
function factorial(n) {
    return n <= 1 ? 1 : n * factorial(n - 1);
}
factorial(2);
```

factorial Example

- Evaluate `factorial(2)` in **program env**



program env

factorial:=

E1

n: **2**

factorial(2)

```
return n *
  factorial(n-1);
```

params: n

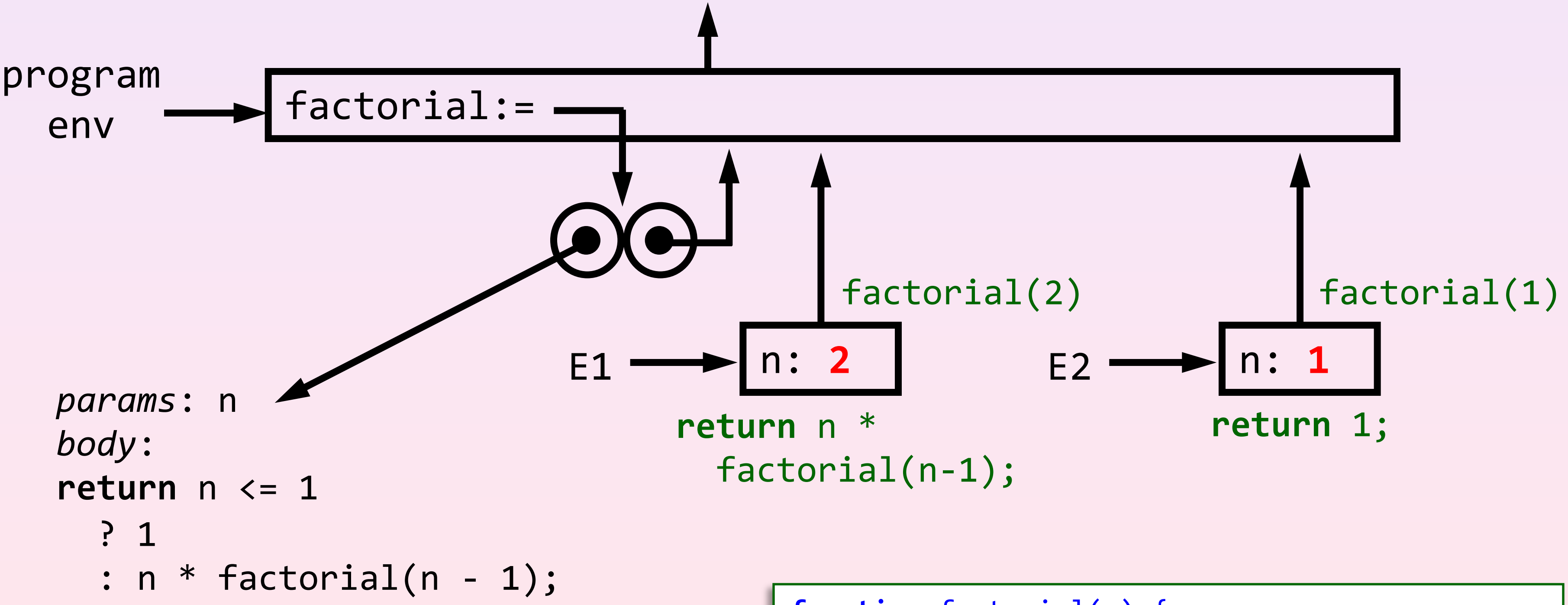
body:

```
return n <= 1
  ? 1
  : n * factorial(n - 1);
```

```
function factorial(n) {
    return n <= 1 ? 1 : n * factorial(n - 1);
}
factorial(2);
```

factorial Example

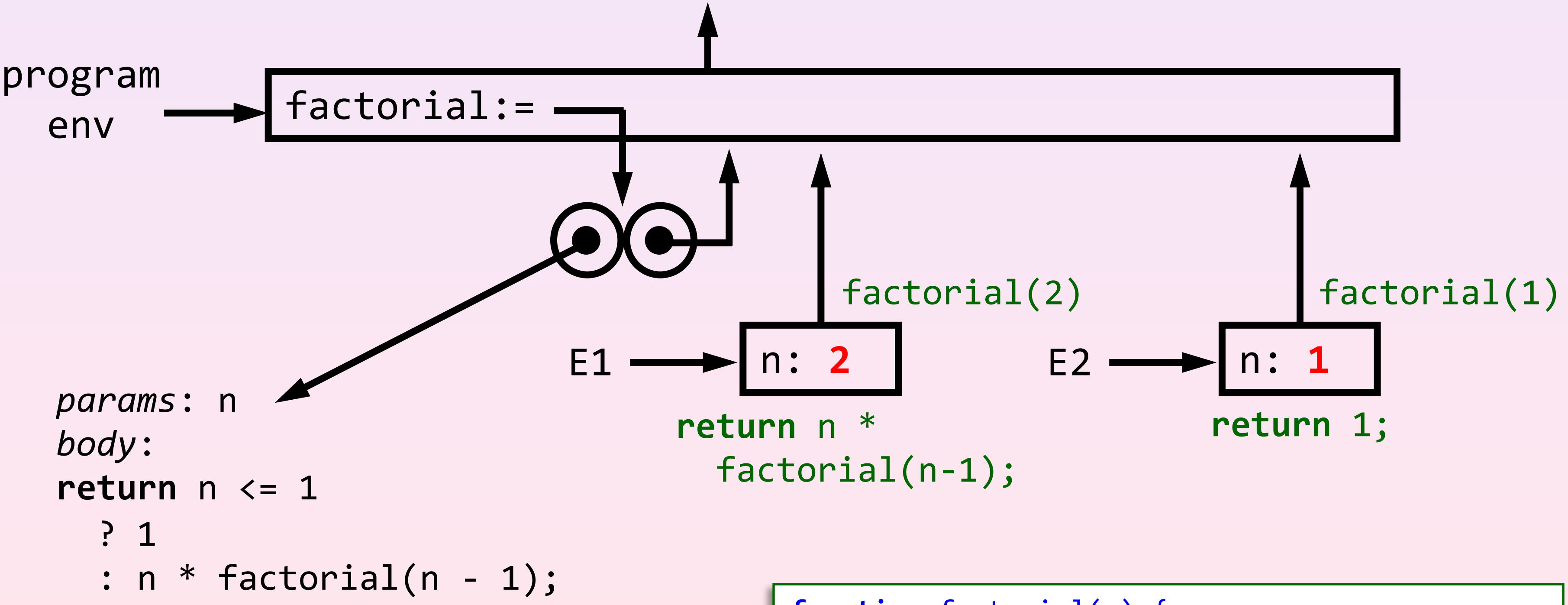
- Evaluate factorial(n-1) in **Environment E1**



```
function factorial(n) {
    return n <= 1 ? 1 : n * factorial(n - 1);
}
factorial(2);
```

factorial Example

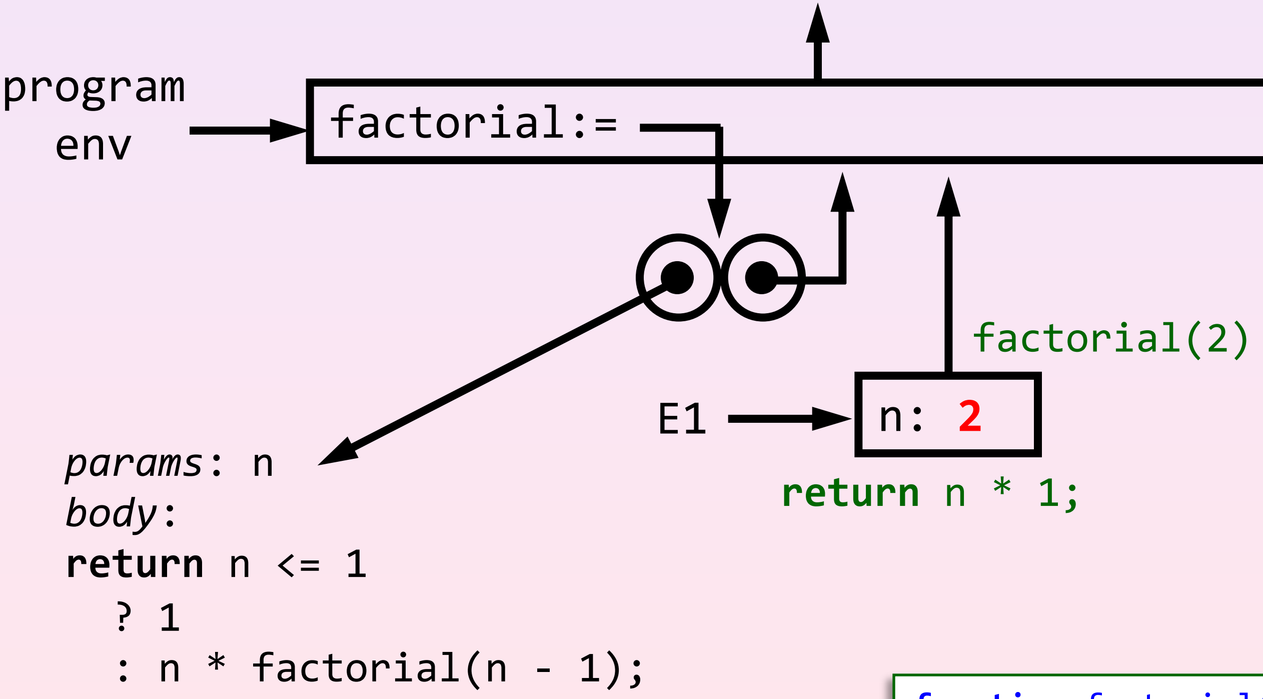
- Evaluate `return 1;` in **Environment E2**



```
function factorial(n) {
    return n <= 1 ? 1 : n * factorial(n - 1);
}
factorial(2);
```

factorial Example

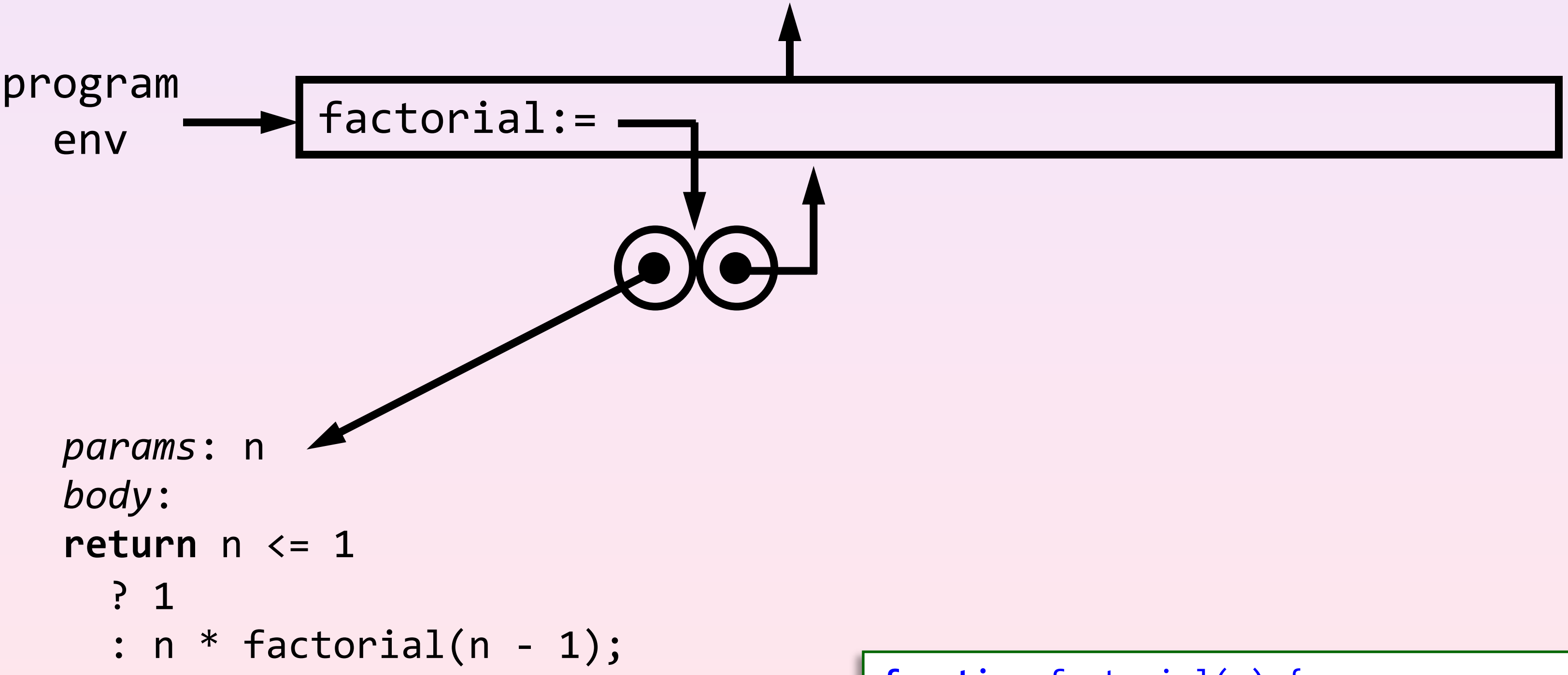
- Evaluate `return n * 1;` in **Environment E1**



```
function factorial(n) {
    return n <= 1 ? 1 : n * factorial(n - 1);
}
factorial(2);
```

factorial Example

- Environment after evaluation of factorial(2)



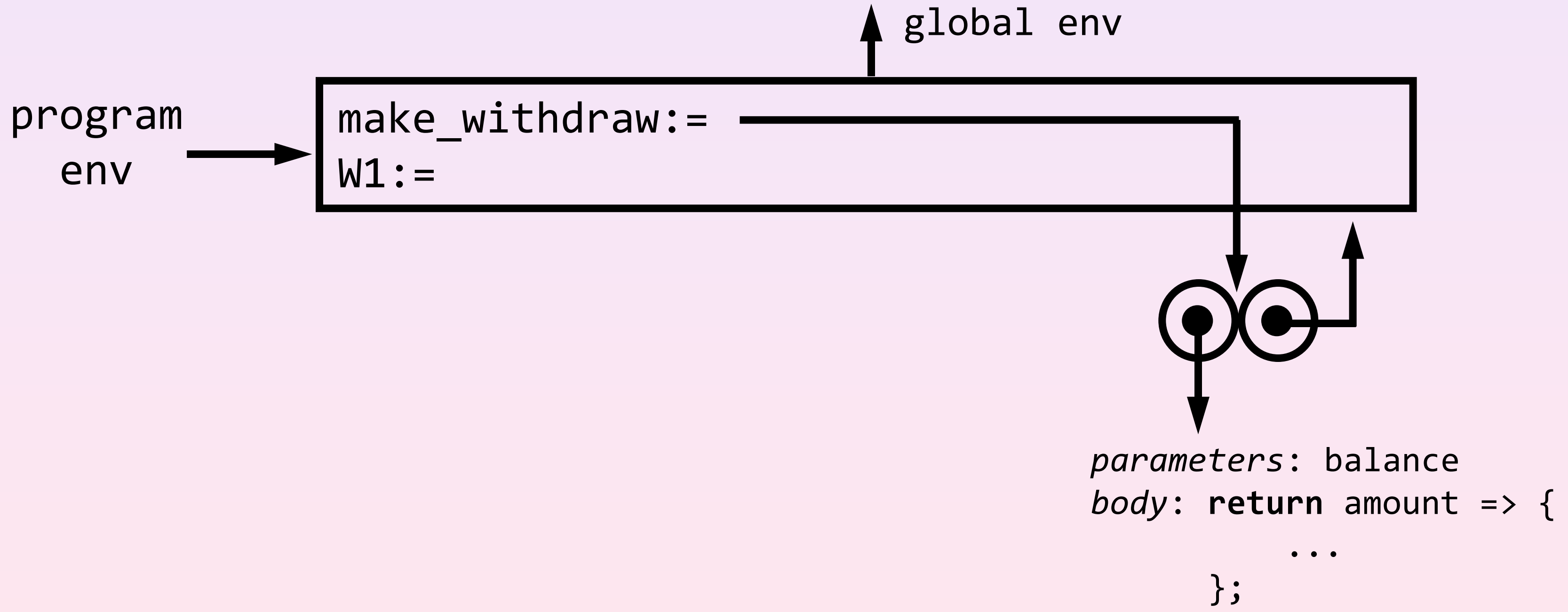
```
function factorial(n) {
    return n <= 1 ? 1 : n * factorial(n - 1);
}
factorial(2);
```

make_withdraw Example

```
function make_withdraw(balance) {
    return amount => {
        if (balance >= amount) {
            balance = balance - amount;
            return balance;
        } else {
            return "Insufficient funds";
        }
    };
}
const W1 = make_withdraw(100);
W1(40); → 60
```

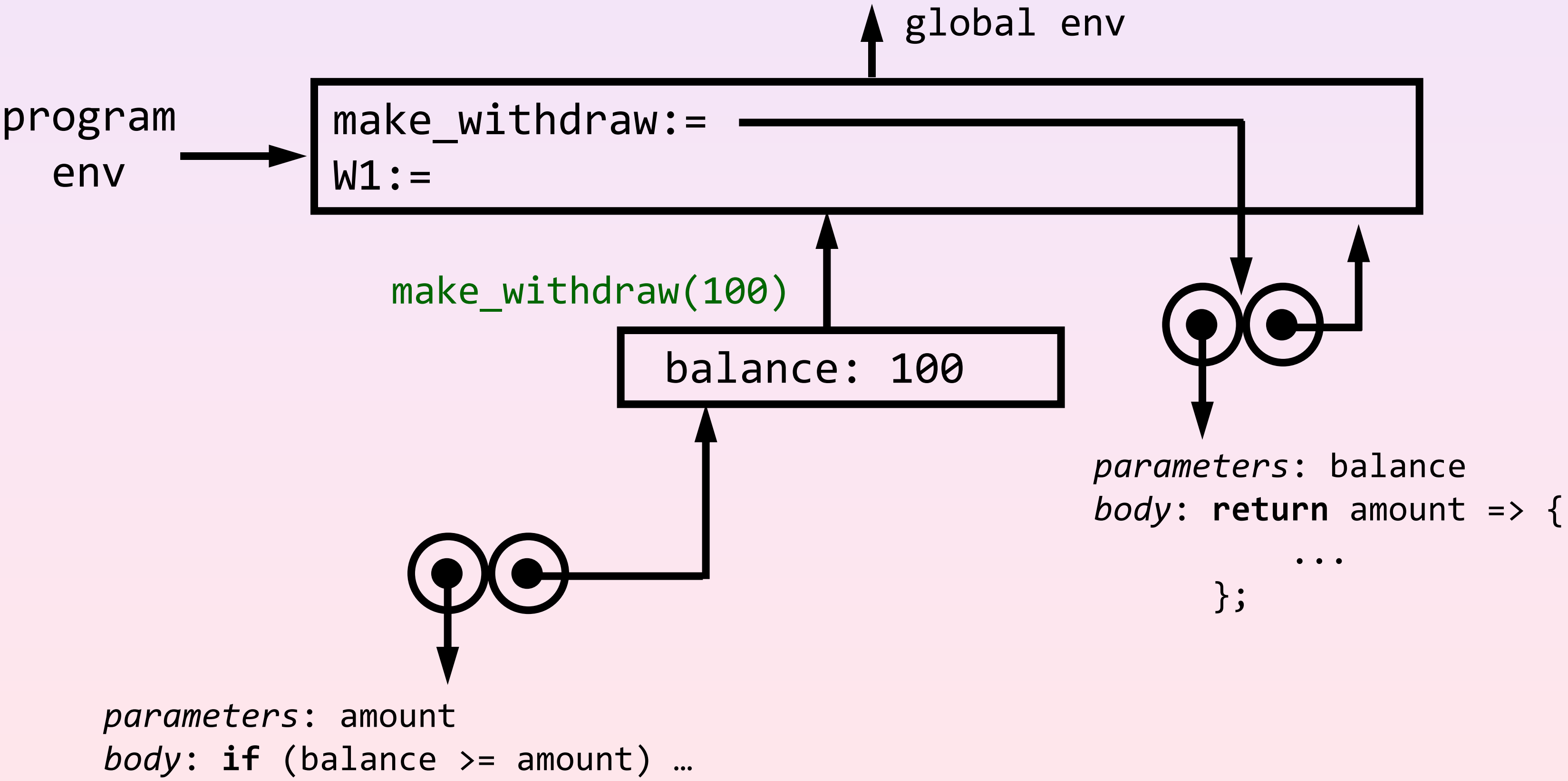
make_withdraw Example

- After declaring function make_withdraw



make_withdraw Example

- Evaluate make_withdraw(100)



global env

program env

make_withdraw:=
W1:=

make_withdraw(100)

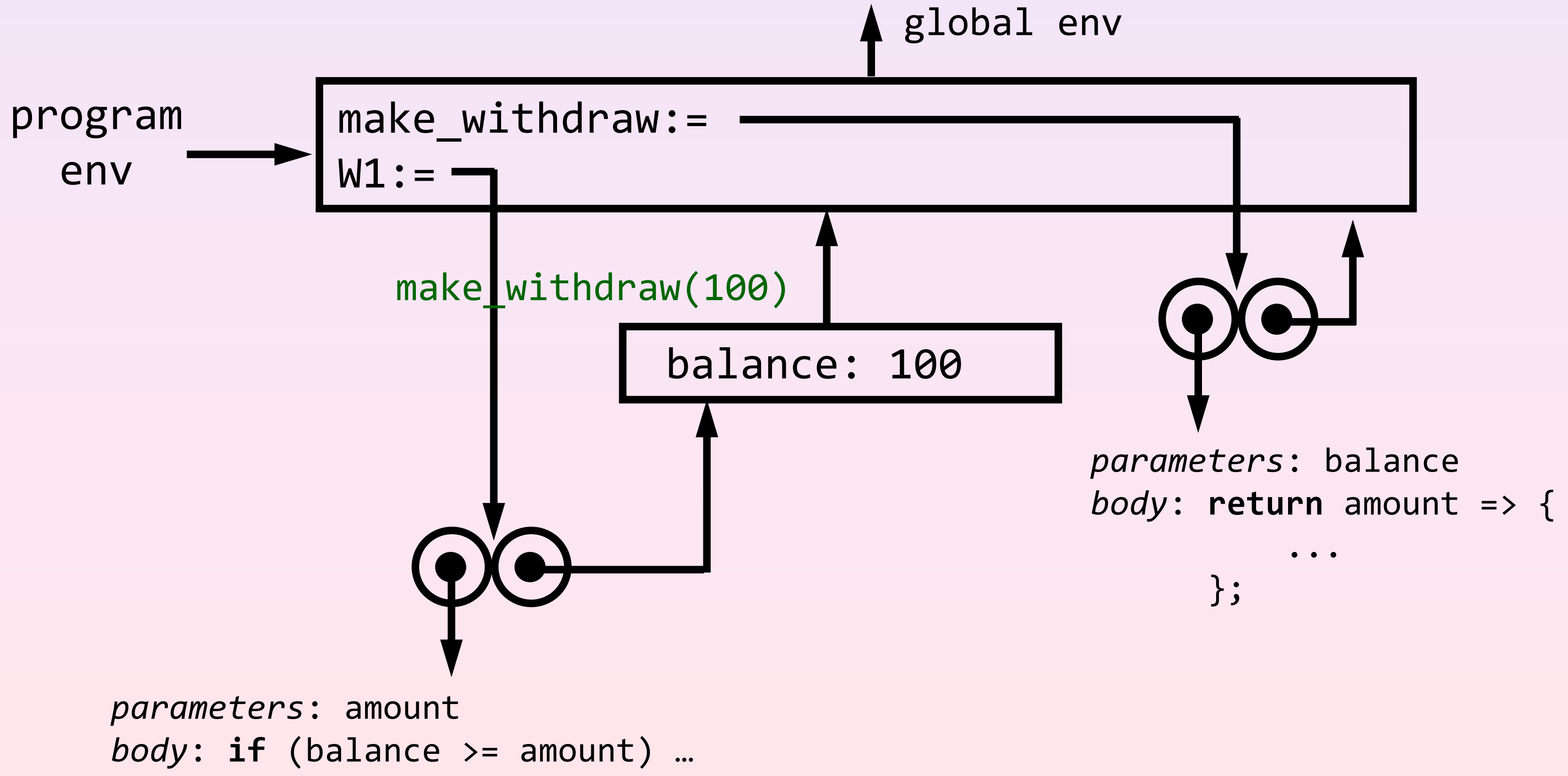
balance: 100

parameters: balance
body: **return** amount => {
...
};

parameters: amount
body: **if** (balance >= amount) …

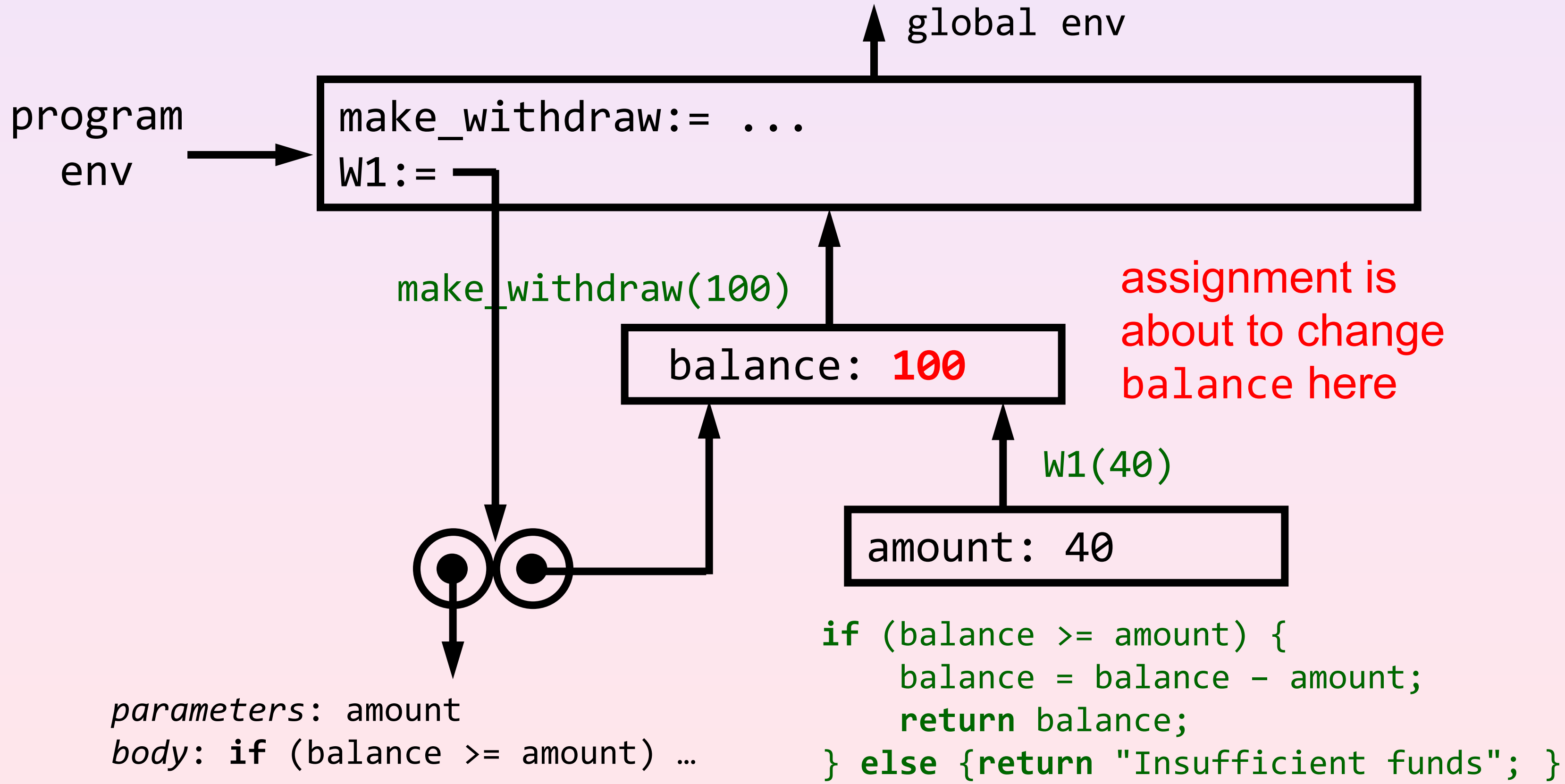
make_withdraw Example

- Evaluate `const W1 = make_withdraw(100);`



make_withdraw Example

- Evaluate W1(40)

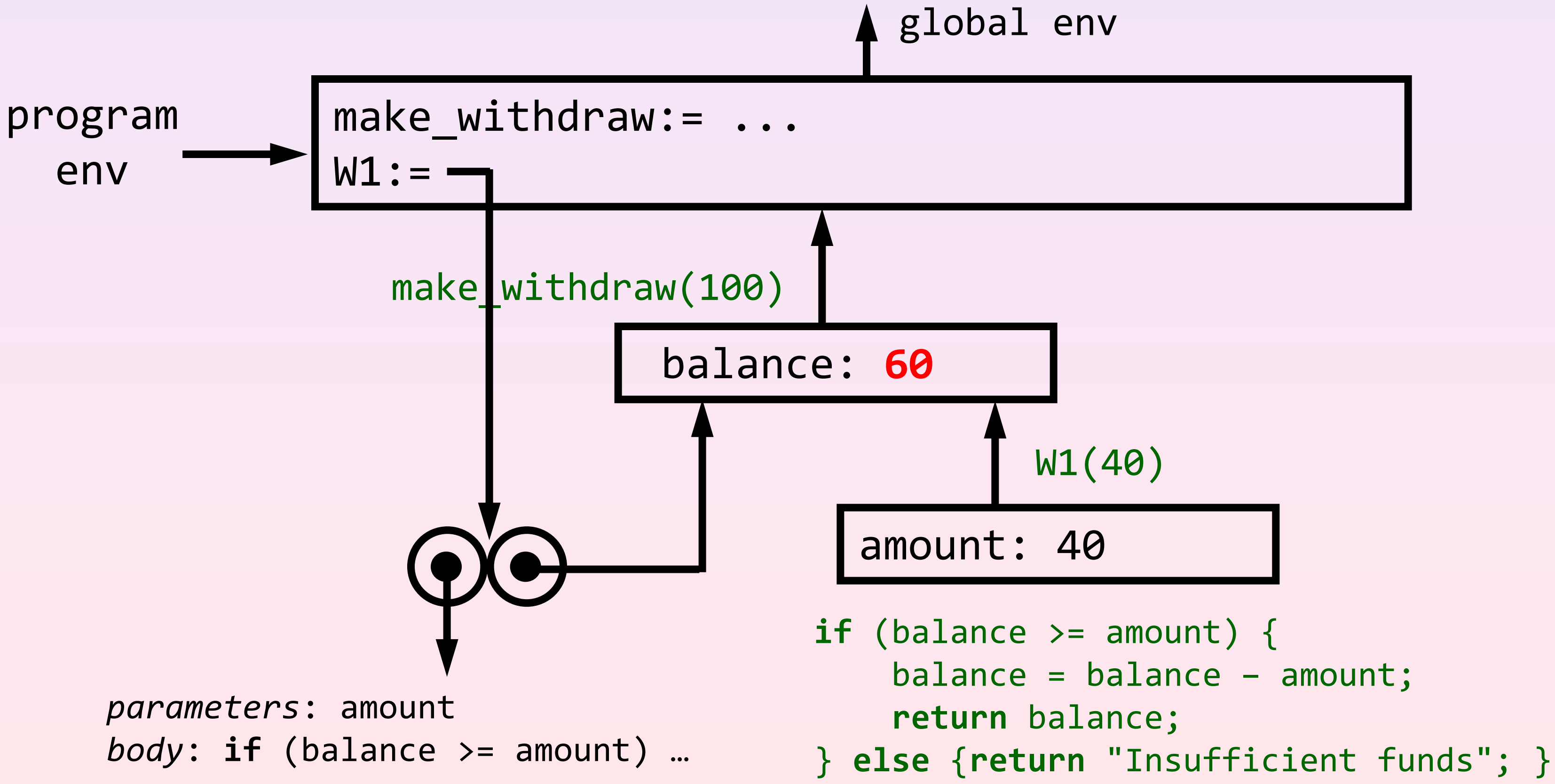


assignment is about to change balance here

```
if (balance >= amount) {
    balance = balance - amount;
    return balance;
} else {return "Insufficient funds"; }
```

make_withdraw Example

- Evaluate W1(40)

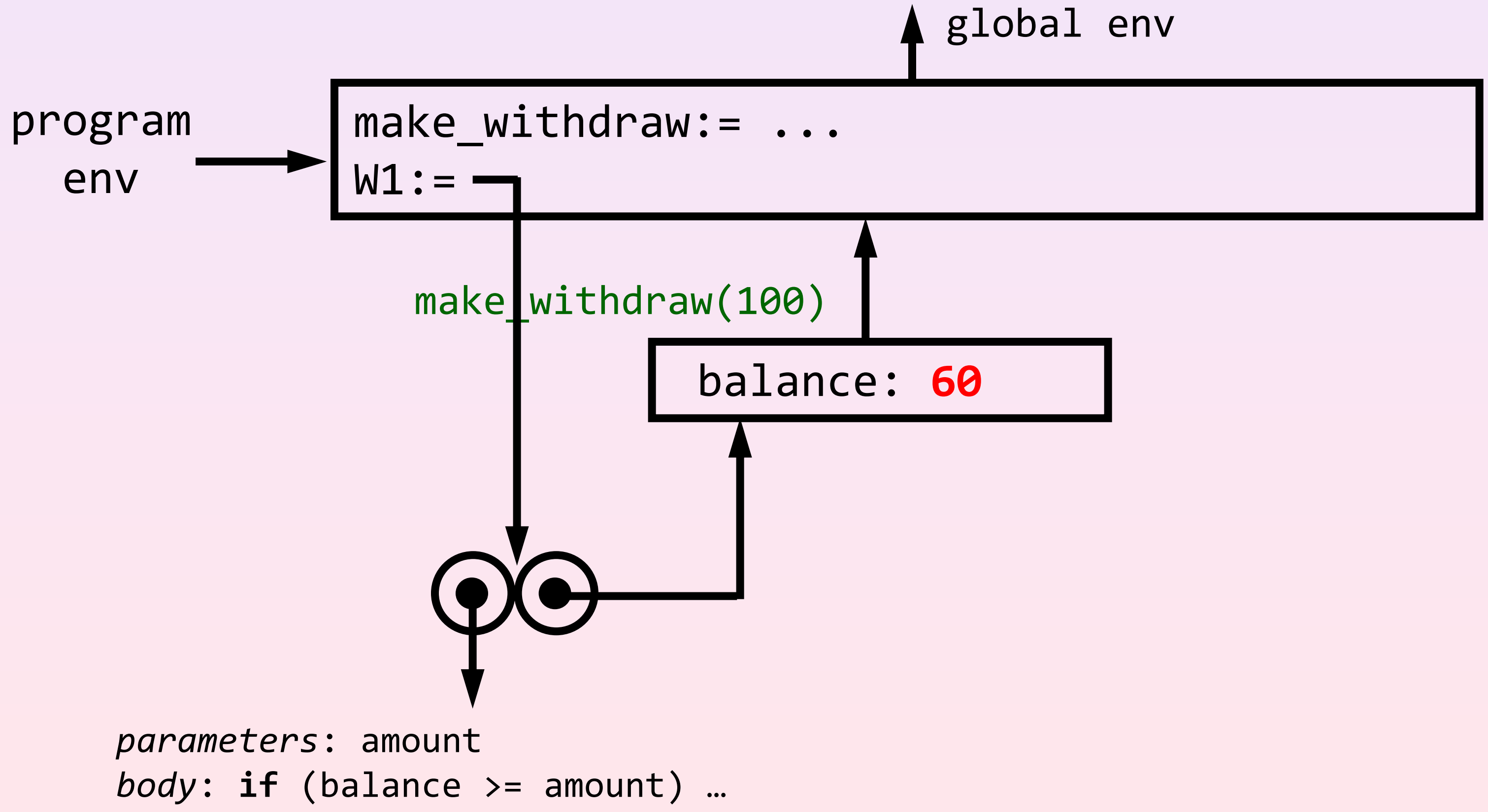


parameters: amount
body: **if** (balance >= amount) …

```
if (balance >= amount) {
    balance = balance - amount;
    return balance;
} else {return "Insufficient funds"; }
```

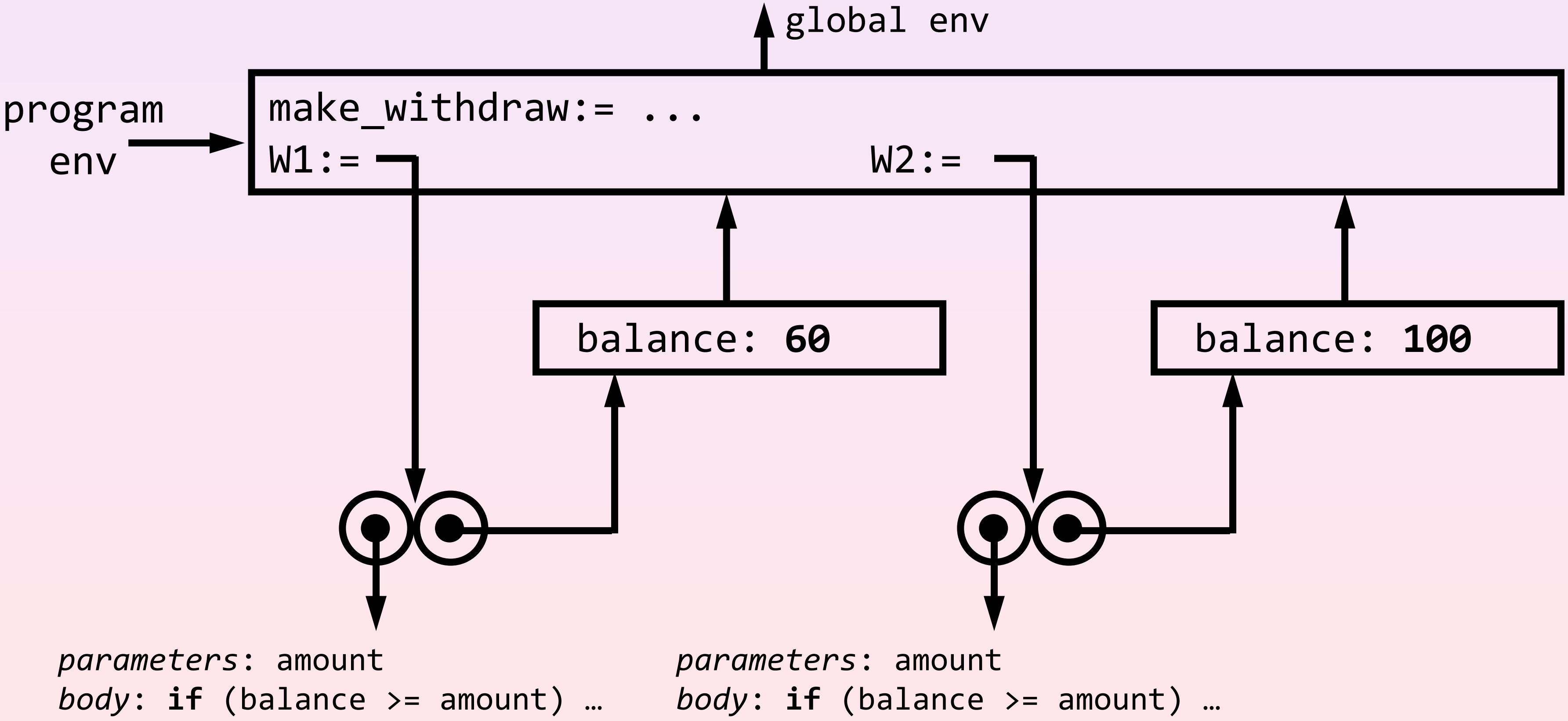
make_withdraw Example

- Environment after evaluating W1(40);



make_withdraw Example

- Environment after evaluating `const W2 = make_withdraw(100);`
- `W2` has its own state variable



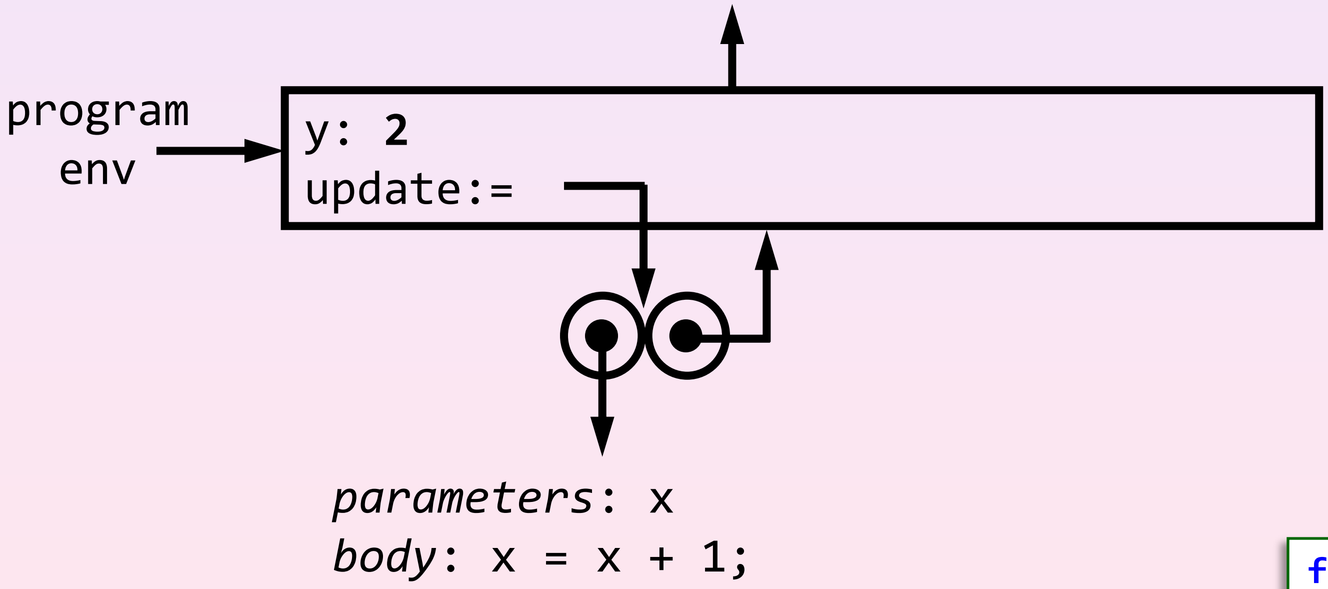
update Example

- What is the result of this program?

```
function update(x) {
    x = x + 1;
}
let y = 2;
update(y);
y; ➔ 2
```

update Example

- After evaluating declarations of function update and variable y



program env

y: **2**

update:=

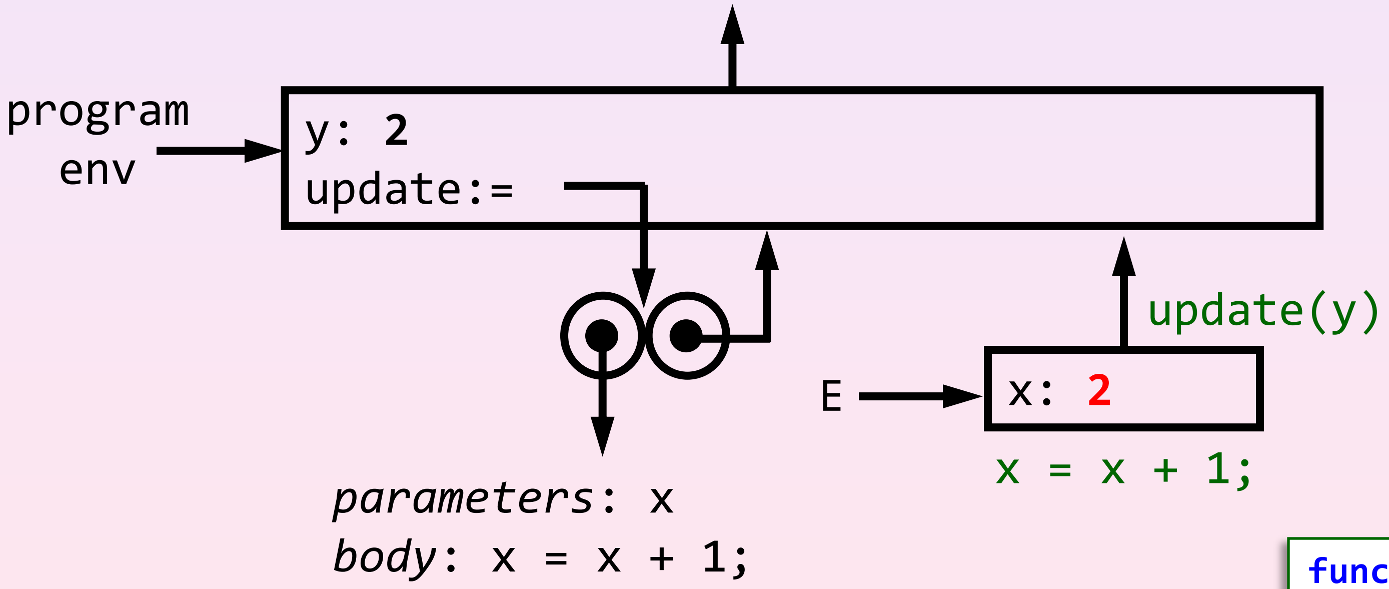
parameters: x

body: x = x + 1;

```
function update(x) {
    x = x + 1;
}
let y = 2;
update(y);
y;
```

update Example

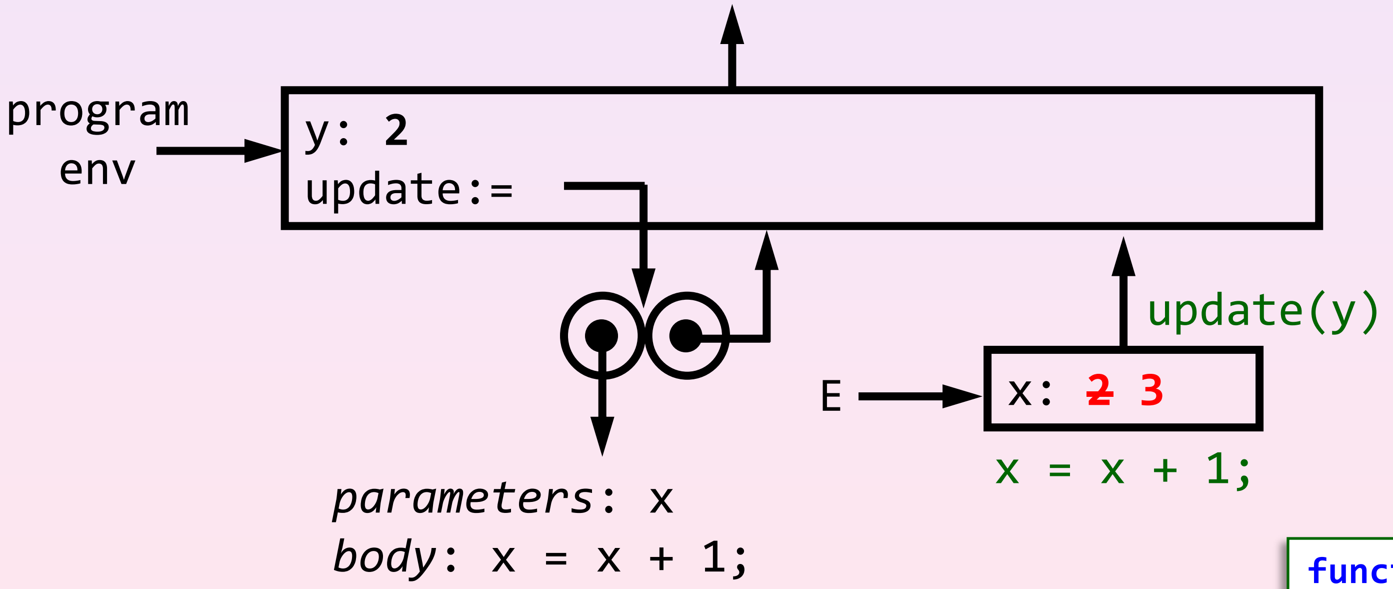
- Evaluate update(y) in **program env**



```
function update(x) {
    x = x + 1;
}
let y = 2;
update(y);
y;
```

update Example

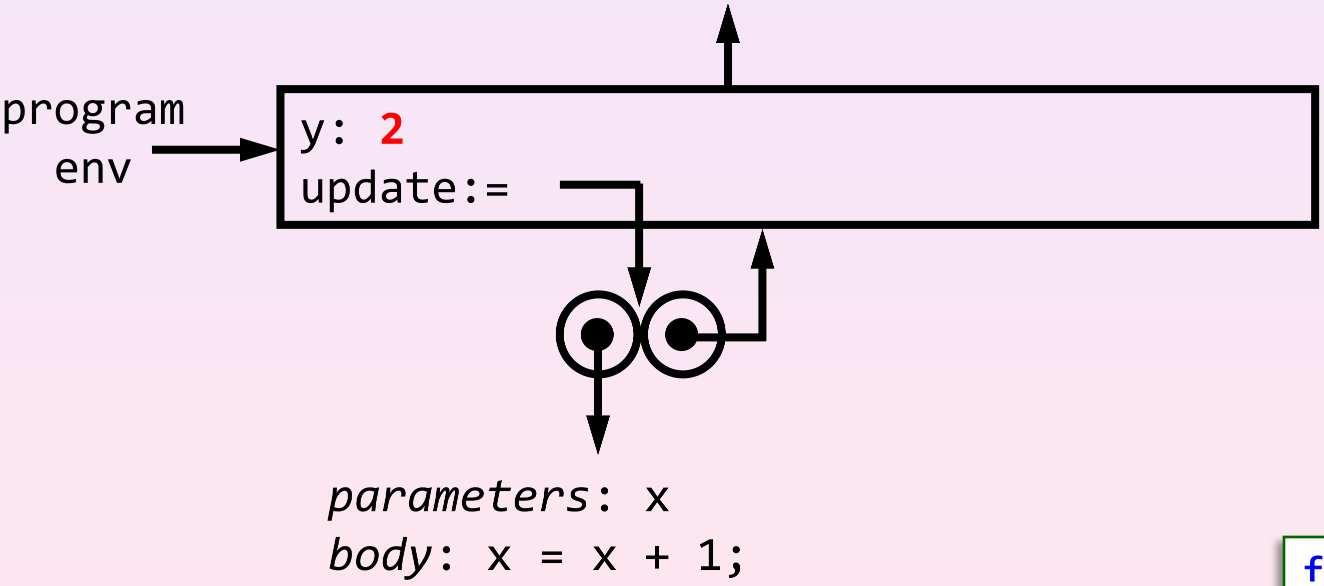
- Evaluate `x = x + 1;` in **Environment E**



```
function update(x) {
    x = x + 1;
}
let y = 2;
update(y);
y;
```

update Example

- Evaluate y in **program env**



```
function update(x) {
    x = x + 1;
}
let y = 2;
update(y);
y;
```

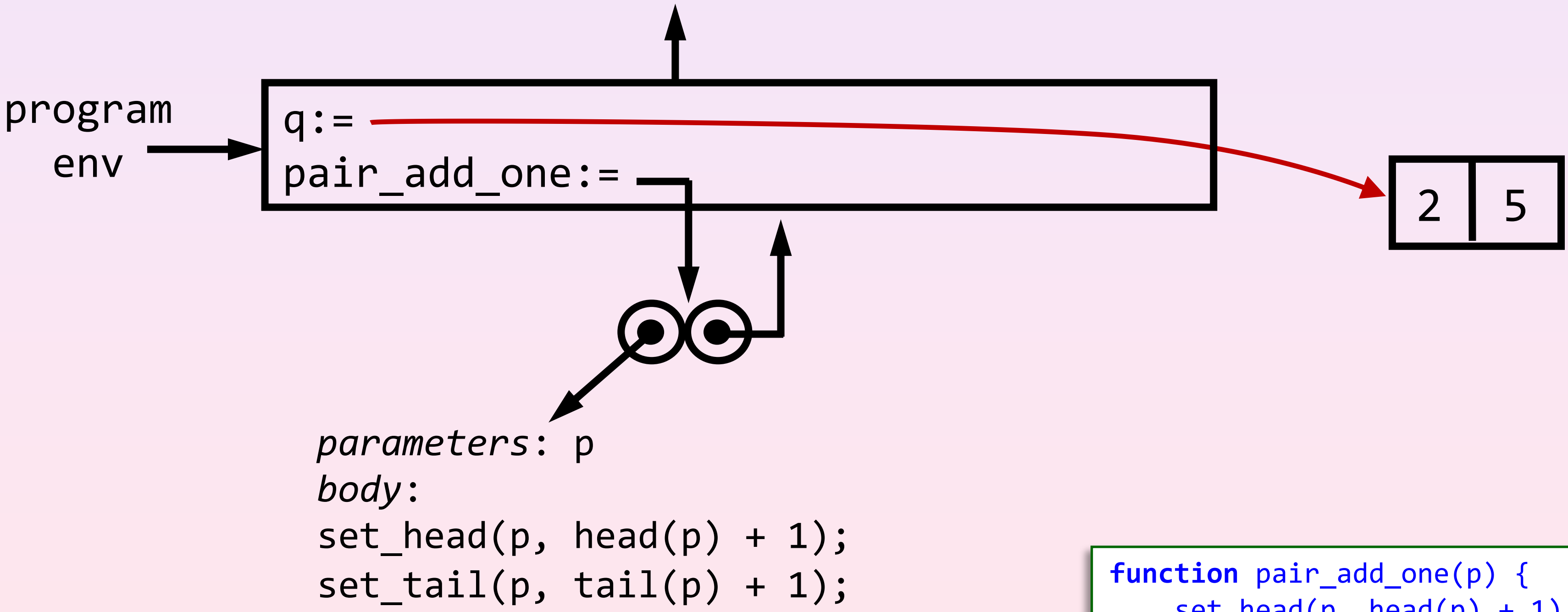
pair_add_one Example

- What is the result of this program?

```
function pair_add_one(p) {
    set_head(p, head(p) + 1);
    set_tail(p, tail(p) + 1);
}
const q = pair(2, 5);
pair_add_one(q);
q; →[3, 6]
```

pair_add_one Example

- After evaluating declarations of function pair_add_one and constant q

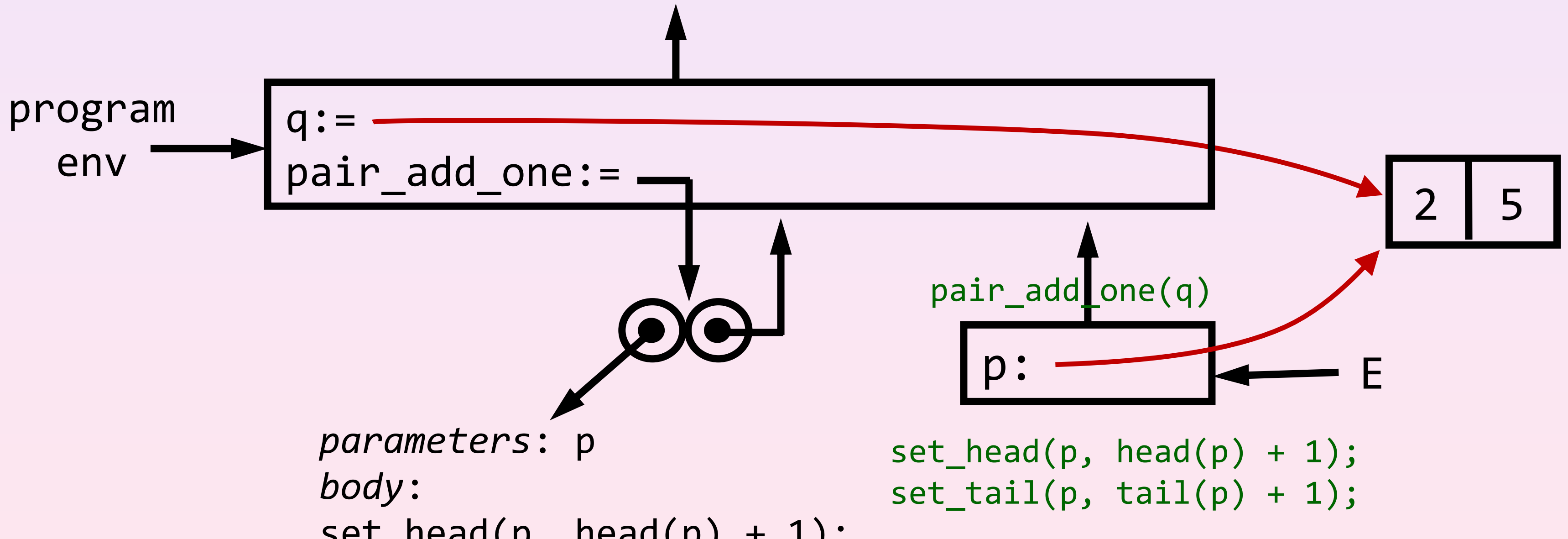


parameters: p
body:
```
set_head(p, head(p) + 1);
set_tail(p, tail(p) + 1);
```

```
function pair_add_one(p) {
    set_head(p, head(p) + 1);
    set_tail(p, tail(p) + 1);
}
const q = pair(2, 5);
pair_add_one(q);  q;
```

pair_add_one Example

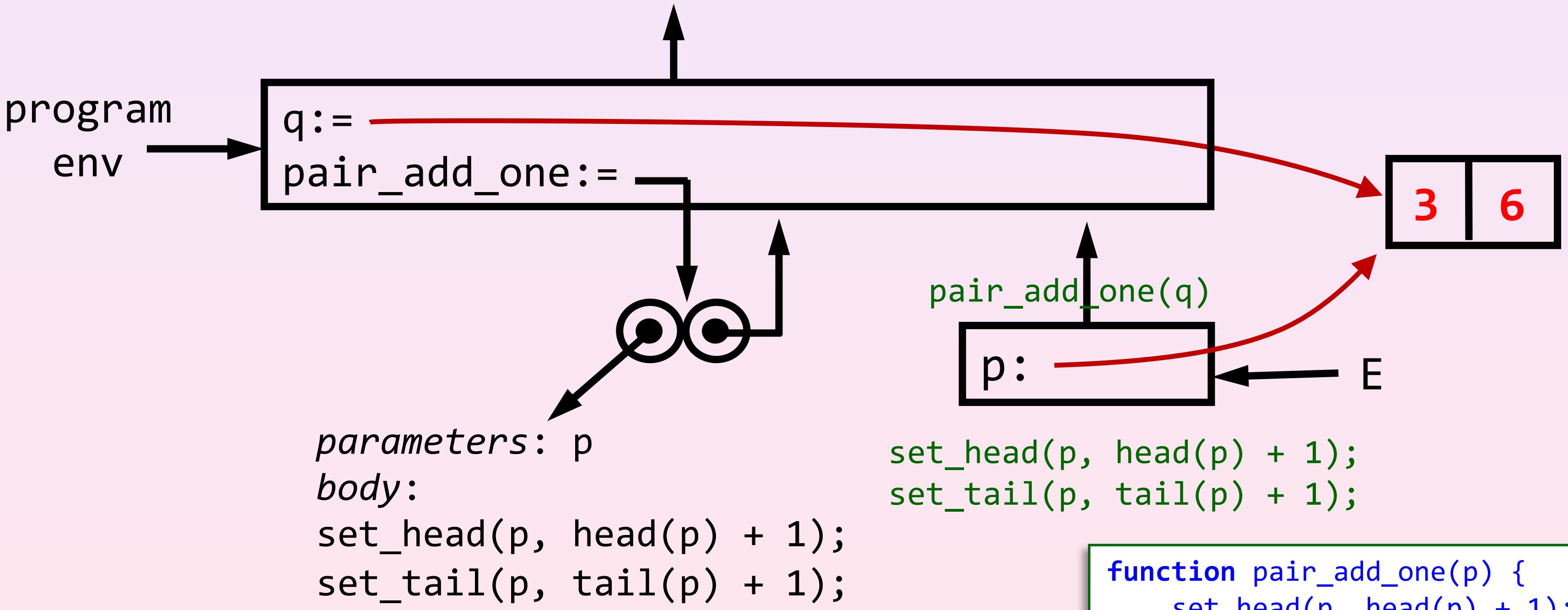
- Evaluate `pair_add_one(q)` in **program env**


program env
q:=
pair_add_one:=
2
5
pair_add_one(q)
p:
E
parameters: p
body:
set_head(p, head(p) + 1);
set_tail(p, tail(p) + 1);
set_head(p, head(p) + 1);
set_tail(p, tail(p) + 1);

```
function pair_add_one(p) {
    set_head(p, head(p) + 1);
    set_tail(p, tail(p) + 1);
}
const q = pair(2, 5);
pair_add_one(q);  q;
```

pair_add_one Example

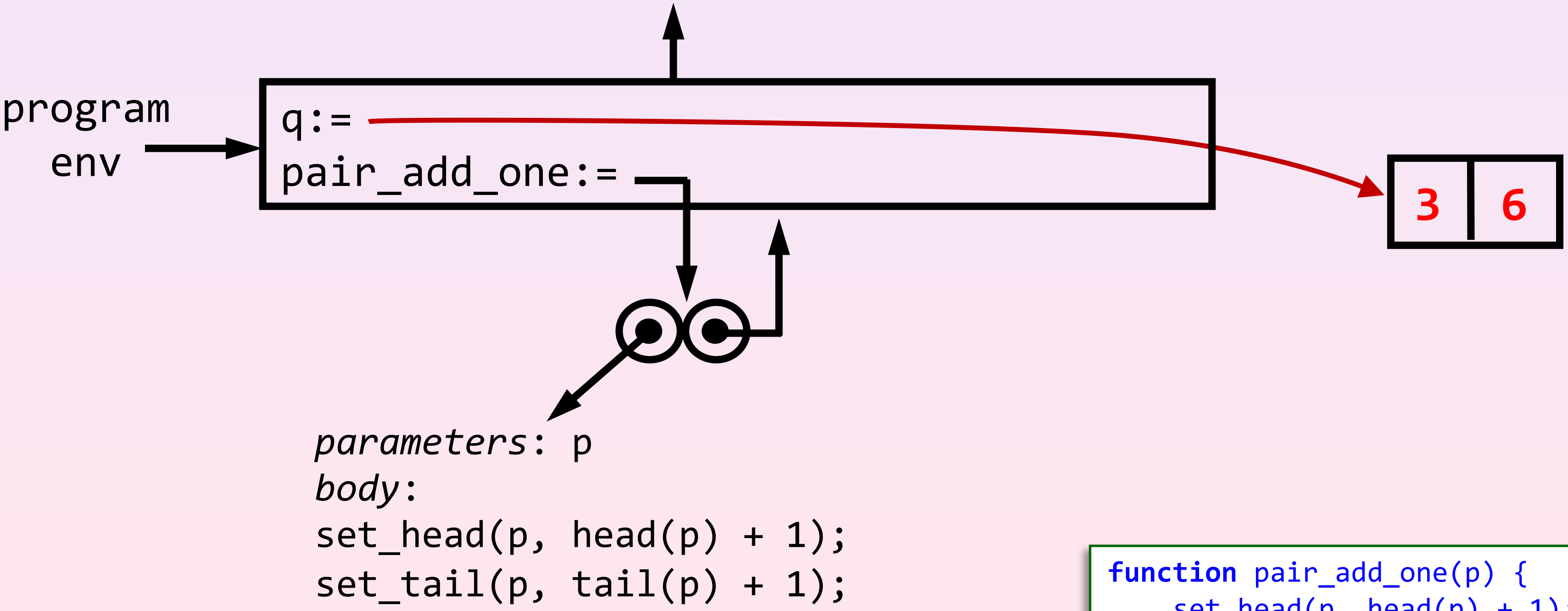
- Evaluate `set_head(p, head(p) + 1)` and `set_tail(p, tail(p) + 1)` in **Environment E**


program env
q:=
pair_add_one:=
3
6
pair_add_one(q)
p:
E
parameters: p
body:
set_head(p, head(p) + 1);
set_tail(p, tail(p) + 1);
set_head(p, head(p) + 1);
set_tail(p, tail(p) + 1);

```
function pair_add_one(p) {
    set_head(p, head(p) + 1);
    set_tail(p, tail(p) + 1);
}
const q = pair(2, 5);
pair_add_one(q);  q;
```

pair_add_one Example

- Evaluate q in **program env**

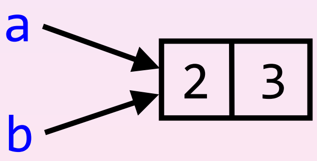


```
function pair_add_one(p) {
    set_head(p, head(p) + 1);
    set_tail(p, tail(p) + 1);
}
const q = pair(2, 5);
pair_add_one(q);  q;
```

Sharing and Identity

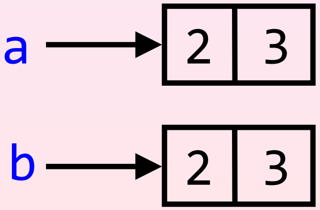
- In `pair_add_one` example, `q` and `p` **share** the same **pair**
- Checking whether **two names** are **sharing** the same **pair**
  - **Example:**

```
const a = pair(2, 3);
const b = a;
a === b; ➔ true
```



  - **Example:**

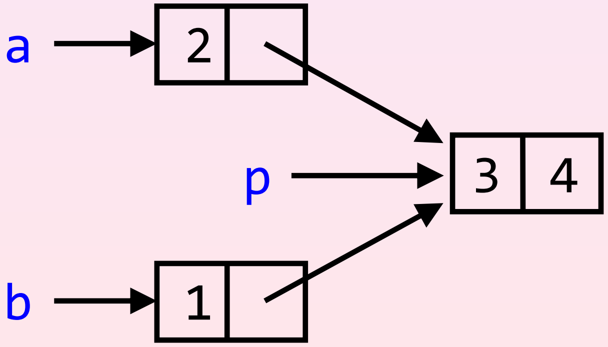
```
const a = pair(2, 3);
const b = pair(2, 3);
a === b; ➔ false
```



Sharing and Identity

- **Structures** sharing pairs
  - **Example:**

```
const p = pair(3, 4);
const a = pair(2, p);
const b = pair(1, p);
tail(a) === tail(b); ➔ true
```



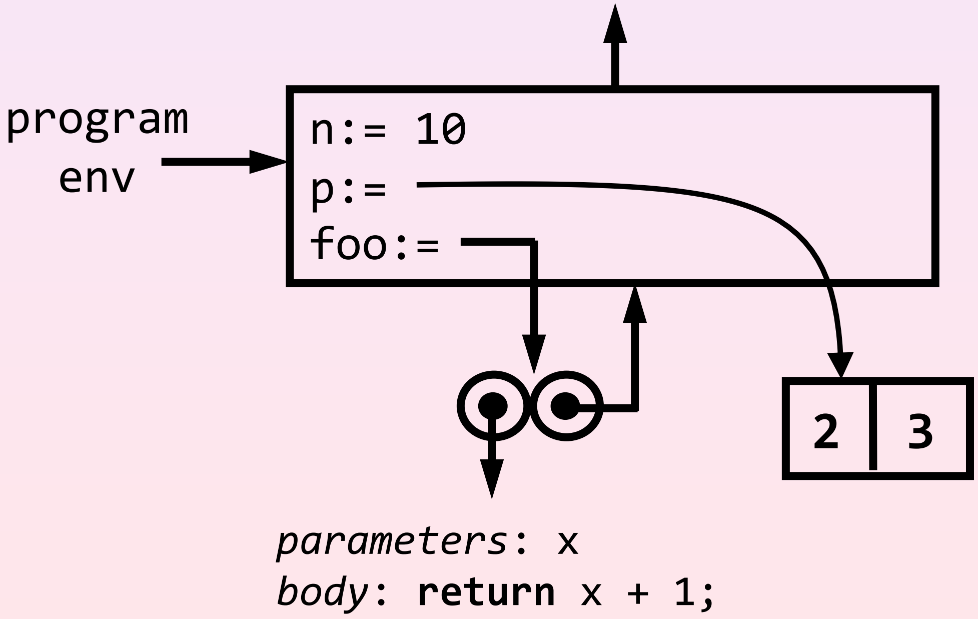
Identity in Source §3: ===

- For `a === b`, we say “is `a` ***identical*** to `b`”
  - `true`, `false`, `null`, `undefined` — each is identical to itself and nothing else
  - **Numbers** — two numbers are identical iff they have the same representation in the double-precision floating-point representation
  - **Strings** — two strings are identical if they have the same characters in the same order
  - **Functions** — functions are made by function expressions, and their creation bestows an identity upon them
  - **Pairs** — pairs are made by the `pair` function, and their creation bestows an identity upon them

Drawing Compound Structures

- **Primitive values** (e.g. numbers, strings, Boolean values, `null`) in bindings are drawn **inside** frames

- **Compound structures** (e.g. pairs, function objects) are drawn **outside** frames

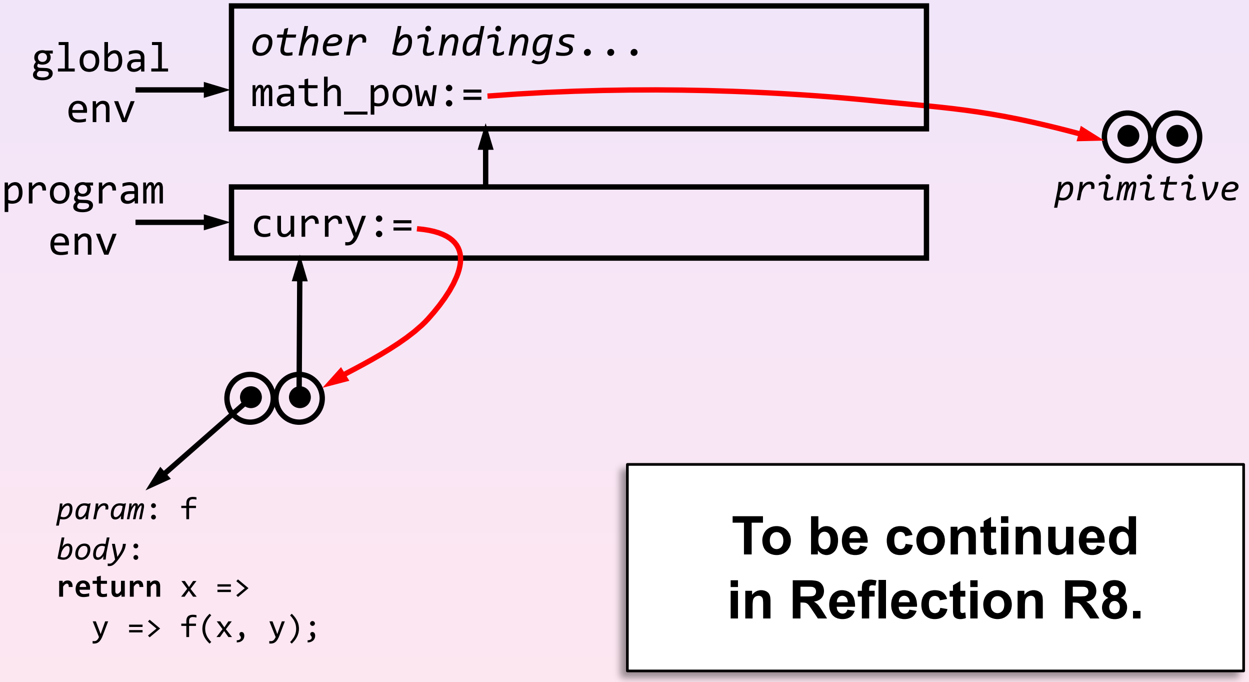


curry Example

- Consider this program

```
function curry(f) {
    return x => y => f(x, y);
}
curry(math_pow)(3)(4);
```

curry Example



```
function curry(f) {
    return x => y => f(x, y);
}
curry(math_pow)(3)(4);
```

To be continued in Reflection R8.

Summary

- **Assignment** allows us to create **state**
- **Substitution model** breaks down with **assignment**
- **Environment model** replaces **substitution model**
- When a **lambda expression** is evaluated, it creates a **function object** that remembers the **current environment**
- To evaluate a **function application**
  1) Create a **new frame** in the function's environment (right pointer of function object)
  2) Bind **parameters** in new frame
  3) Evaluate **body block** of function in the new environment