L3: Refresher on Elements and Functions

Structure and Interpretation of Computer Programs

Martin Henz

July 6, 2023

1. What is on offer?
   - Some goals
   - Today
   - Team today
   - Book
2. Module 1: Elements of programming
3. Module 2: Introduction to functions

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Some goals

- A taste of
  the *structure and interpretation of computer programs*

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- A taste of
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- Learn to write simple programs

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- Discover a mental model for computational processes

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- A taste of
  the *structure and interpretation of computer programs*
- Learn to write simple programs
- Discover a mental model for computational processes
- Fun with graphics and sound

What we are **not** doing

- Teach the language JavaScript
- Teach how to program websites
- Introduce object-oriented programming

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Components

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- **L**ecture: 10–12: slides available in Canvas

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- **P**ath: will be discussed during lecture

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- **P**ath: will be discussed during lecture
- **Q**uests: will be discussed in afternoon session: 2-6

This morning

- Modules 1 and 2 refresher
    - 10:05–10:45: Elements of programming
    - 10:45–11:00: break; getting to know each other
    - 11:00–11:45: Functional abstraction
    - 11:45–12:00: more getting to know each other

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Team

Instructor

Martin Henz, Associate Professor at the National University of Singapore

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Martin Henz, Associate Professor at the National University of Singapore

Teaching Assistant Brian will join at 2pm

- answering your questions
- clarifying concepts

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SICP JS

Online version

https://sourceacademy.nus.edu.sg/sicpjs

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PDF version

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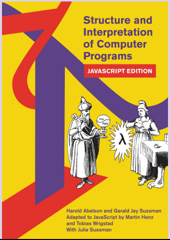
paper version



Processes

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Definition (from Wikipedia)

A process is a set of activities that interact to produce a result. The activities unfold according to patterns that *de*scribe or *pre*scribe the process.

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Examples

Processes are everywhere. They permeate our nature and culture:

- Galaxies and solar systems
- Metabolic pathways in our bodies
- Political parties, legislature, courts
- Industrial production

Computational processes

Definition

A *computational process* is a set of activities in a computer, designed to achieve a desired result.

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Our task

Here we are concerned about how this *design* happens.

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Here we are concerned about how this *design* happens.

Our design method

We use *programs* to prescribe how computational processes unfold.

What is programming?

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People in focus

As the complexity of computer systems increases, *communication* between affected people becomes more and more important.

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Programs as communication devices

Since programs prescribe the computational processes in these systems, they allow us to communicate their construction and operation.

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People in focus

As the complexity of computer systems increases, *communication* between affected people becomes more and more important.

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A central theme of SICP

Programming is *communicating computational processes*.

Our programming environment: Source Academy

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Website designed for SICP JS

Has just what you need for understanding the *structure and interpretation of computer programs*.

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Research

Source Academy is also a *educational research tool*: We want to study what happens when people like you learn how to program.

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Source Academy and Source Academy @ NUS

Source Academy (`https://sourceacademy.org`) is public.
We use mostly Source Academy @ NUS, which supports courses and requires login (`https://sourceacademy.nus.edu.sg`).

Elements of programming

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Every powerful language provides...

Elements of programming

Every powerful language provides...

- Primitive values

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Every powerful language provides...

- Primitive values
- Combination

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- Means of abstraction

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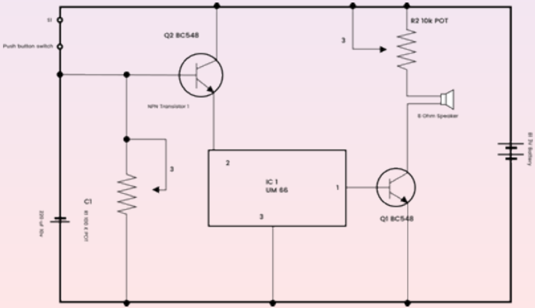

Primitive expressions (1.1.1)

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- Numerals: 0, -42, 486

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- Numerals use decimal notation

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- Numerals: 0, -42, 486
- Numerals use decimal notation
- Our interpreter can evaluate numerals, resulting in the numbers they represent

A detail

Expressions are not programs in Source

Instead they can be turned into programs using a semicolon.

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Example

`486;` is a program.

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What process does the program prescribe?

A detail

Expressions are not programs in Source

Instead they can be turned into programs using a semicolon.

Example

`486;` is a program.

What process does the program prescribe?

The process is *trivial*:
There is nothing to do. The result is already explicit.

Means of Combination: Operators (1.1.1)

Examples

```
5 * 99;

25 - (4 + 2) * 3;
```

Means of Combination: Operators (1.1.1)

Examples

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5 * 99;

25 - (4 + 2) * 3;
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Notation as usual

operator between operands:
infix notation with *precedences*

Evaluating Operator Combinations

But exactly what happens...

...when we evaluate a program like this

```
(2 + 4 * 6) * (3 + 12);
```

???

Evaluation of expressions (1.1.3)

Demonstration: evaluate

```
(2 + 4 * 6) * (3 + 12);
```

Means of Abstraction: Naming (1.1.2)

Example

```
const size = 2;
5 * size;
```

Path 1: Expressions and names

See Source Academy https://sourceacademy.nus.edu.sg

Pre-declared names

Pre-declared constants

Source has a few names pre-declared. For example, the name `math_PI` refers to the constant π

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Predeclared functions

Source has *predeclared functions*.

Example: `math_sqrt` is the square root function.

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Example: `math_sqrt` is the square root function.

Function application expressions...

...use the usual mathematical notation:

```
math_sqrt(15);
```

applies `math_sqrt` to 15.

Import declarations

Import syntax

Source allows *import declarations*. Each of a list of names is imported from a given module. Example:

```
import { heart, show } from 'rune';
```

Now the name `heart` refers to a "rune" (shape) defined by the module `'rune'` and `show` refers to a rune function.

```
show(heart);  // shows heart rune in a tab
```

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```
show(heart);  // shows heart rune in a tab
```

Modules

The list of all modules is given in
https://source-academy.github.io/modules/documentation/

Quest 1A: Playing with Runes

See Source Academy https://sourceacademy.nus.edu.sg

1. What is on offer?
2. Module 1: Elements of programming
   - Expressions and names, 1.1.1 and 1.1.2
   - Predeclared names
   - Functional abstraction, 1.1.4
   - Predicates and conditional expressions, 1.1.6
3. Module 2: Introduction to functions

Means of Abstraction: Compound functions (1.1.4)

Example

```
function square(x) {
    return x * x;
}
```

Means of Abstraction: Compound functions (1.1.4)

Example

```
function square(x) {
    return x * x;
}
```

What does this statement mean?

This *function declaration* declares a name, here `square`. The value associated with `square` is a function that takes an argument `x` and produces (returns) the result of multiplying `x` by itself.

Review: Function application

Recall from the `math_sqrt` example

We apply a function by supplying its arguments in parentheses.

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Recall from the `math_sqrt` example

We apply a function by supplying its arguments in parentheses.

Example

```
function square(x) {
    return x * x;
}
square(7);
```

The core of the substitution model

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    return x * x;
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square(7);
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square(7);
```

Function application

To apply a compound function to arguments,
evaluate the return expression of the function
with each parameter *replaced* by the corresponding argument.

More examples

```
square(21);

square(2 + 5);

square(square(3));
```

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Boolean values (1.1.6)

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Predicates

A *predicate* is a function or an expression that returns or evaluates to a boolean value.

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Predicates

A *predicate* is a function or an expression that returns or evaluates to a boolean value.

Examples

```
true;
false;
x >= 1;
```

Declare predicate with compound function

Example

```
function adult(x) {
    return x >= 18;
}
```

Conditional expressions

Purpose

We would like to check something, e.g. answer a yes/no question, and return a different value, depending on the answer

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Example:

```
adult(my_age)
? enter_club()
: go_to_movie()
```

Conditional expressions

Purpose

We would like to check something, e.g. answer a yes/no question, and return a different value, depending on the answer

Example:

```
adult(my_age)
? enter_club()
: go_to_movie()
```

Another example: abs function

Check if the number is greater or equal 0.
If yes, return the number unchanged,
and if no, return the number negated.

Quest 1B: Rune Trials

See Source Academy `https://sourceacademy.nus.edu.sg`

Racap: Elements of Programming (1.1)

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- Primitives: things like −42

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- Primitives: things like `-42`
- Combination: things like `-42 * 7`

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Racap: Elements of Programming (1.1)

- Primitives: things like `-42`
- Combination: things like `-42 * 7`
- Name abstraction: things like `const size = 2;`
- Functional abstraction: ...

A function can be a "black box"

Example

`math_sqrt`

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- Input: any number
- Output: a number whose square is the input number

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Function application

```
math_sqrt(15);
```

Means of Abstraction: Compound functions

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function square(x) {
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}
```

Means of Abstraction: Compound functions

Example

```
function square(x) {
    return x * x;
}
```

What does this statement mean?

Like constant declarations, this *function declaration* declares a name, here `square`. The value associated with the name is a function that takes an argument `x` and produces (returns) the result of multiplying `x` by itself.

Substitution model

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- Keep making small steps until the simplest possible form is reached

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- Reduce statement by statement, in the given order

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Substitution model

- Keep making small steps until the simplest possible form is reached
- Reduce statement by statement, in the given order
- Reduce operands of operator combinations and arguments of function applications, before applying the operator/function
- Reduce the predicate of conditional expressions until it is true or false
- If predicate is true/false, keep consequent/alternative
- When the arguments of a function application are *values*, replace the application by the return expression of the function, where parameters are replaced by the argument values.

A new predeclared combination: `stack_frac`

`stack_frac(r, heart, sail)`

splits available bounded heart occupies top fraction `r` of box and sail occupies remaining `1 - r` of box

Examples

`stack_frac(87 / 100, heart, sail)`

splits available bounded box such that heart occupies the top 87% of box and sail occupies remaining 13% of box

Examples

stack_frac(87 / 100, heart, sail)

splits available bounded box such that heart occupies the top 87% of box and sail occupies remaining 13% of box

Trisection of the heart

```
stack_frac(
    1 / 3,
    heart,
    stack_frac(
        1 / 2,
        heart,
        heart));
```

Can we define stackn in Source?

Trisection of the heart

```
stack_frac(1 / 3, heart,
  stack_frac(1 / 2, heart, heart));
```

Can we define stackn in Source?

Trisection of the heart

```
stack_frac(1 / 3, heart,
  stack_frac(1 / 2, heart, heart));
```

Quadrisection of the heart

```
stack_frac(1 / 4, heart,
  stack_frac(1 / 3, heart,
    stack_frac(1 / 2, heart,
                heart)));
```

Can we define stackn in Source?

Trisection of the heart

```
stack_frac(1 / 3, heart,
  stack_frac(1 / 2, heart, heart));
```

Quadrisection of the heart

```
stack_frac(1 / 4, heart,
  stack_frac(1 / 3, heart,
    stack_frac(1 / 2, heart,
                 heart)));
```

Can we generalise this idea?

A *Recursive* Function, first try

```
function stackn(n, rune) {
    return stack_frac(1 / n,
                      rune,
                      stackn(n - 1, rune));
}

stackn(3, heart);
```

A *Recursive* Function, first try

```
function stackn(n, rune) {
    return stack_frac(1 / n,
                      rune,
                      stackn(n - 1, rune));
}

stackn(3, heart);
```

Not working! Why?

A *Recursive* Function, first try

```
function stackn(n, rune) {
    return stack_frac(1 / n,
                      rune,
                      stackn(n - 1, rune));
}

stackn(3, heart);
```

Not working! Why? Computers will follow our orders

We need to *precisely* describe *how* a computational process should evolve.

The correct version

```
function stackn(n, rune) {
    return n === 1
           ? rune
           : stack_frac(1 / n,
                        rune,
                        stackn(n - 1, rune));
}
```

The correct version

```
function stackn(n, rune) {
    return n === 1
           ? rune
           : stack_frac(1 / n,
                        rune,
                        stackn(n - 1, rune));
}
```

Observation

Solution for n computed using solution $n-1$,

solution for $n-1$ is computed using solution $n-2$, ...

until we reach trivial case.

"Wishful thinking"

```
function stackn(n, rune) {
    return n === 1
           ? rune
           : stack_frac(1 / n,
                        rune,
                        stackn(n - 1, rune));
}
```

"Wishful thinking"

```
function stackn(n, rune) {
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}
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Recipe for recursion

"Wishful thinking"

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function stackn(n, rune) {
    return n === 1
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Recipe for recursion

- Figure out trivial *base case*

"Wishful thinking"

```
function stackn(n, rune) {
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                        stackn(n - 1, rune));
}
```

Recipe for recursion

- Figure out trivial *base case*
- Assume you know how to solve problem for $n - 1$.

"Wishful thinking"

```
function stackn(n, rune) {
    return n === 1
           ? rune
           : stack_frac(1 / n,
                        rune,
                        stackn(n - 1, rune));
}
```

Recipe for recursion

- Figure out trivial *base case*
- Assume you know how to solve problem for $n - 1$. How can we solve problem for n?

Can we define repeat_pattern in Source?

Consider the function repeat_pattern in module rune

```
repeat_pattern(3, make_cross, sail)
// should lead to
make_cross(make_cross(make_cross(sail)))
```

Can we define repeat_pattern in Source?

Consider the function repeat_pattern in module rune

```
repeat_pattern(3, make_cross, sail)
// should lead to
make_cross(make_cross(make_cross(sail)))
```

Another example

```
function square(x) { return x * x; }
repeat_pattern(3, square, 2);
// should lead to
square(square(square(2)));
```

repeat_pattern, our first version

```
function repeat_pattern(n, pat, init) {
  return n === 0
         ? init
         : pat(repeat_pattern(n - 1, pat, init));
}
```

repeat_pattern, our first version

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function repeat_pattern(n, pat, init) {
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repeat_pattern, our first version

```
function repeat_pattern(n, pat, init) {
  return n === 0
         ? init
         : pat(repeat_pattern(n - 1, pat, init));
}
repeat_pattern(3, square, 2);
```

Recursive process

The applications of pat
accumulate as result of recursive calls.
They are *deferred* operations.

repeat_pattern, second version

```
function repeat_pattern(n, pat, rune) {
  return n === 0
         ? rune
         : repeat_pattern(n - 1, pat, pat(rune));
}
```

repeat_pattern, second version

```
function repeat_pattern(n, pat, rune) {
  return n === 0
         ? rune
         : repeat_pattern(n - 1, pat, pat(rune));
}

repeat_pattern(3, square, 2);
```

repeat_pattern, second version

```
function repeat_pattern(n, pat, rune) {
  return n === 0
         ? rune
         : repeat_pattern(n - 1, pat, pat(rune));
}

repeat_pattern(3, square, 2);
```

Difference

pat function is applied *before* the recursive call.

There is no deferred operation.

Summary

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  to generate complex runes.

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- **Abstractions** to conquer complexity:

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- **Abstractions** to conquer complexity:
  Naming and **functional abstraction**

Summary

- We started with primitive constants.
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- We introduced primitive combinations.
- No idea how the primitive combinations work!
  Example: `quarter_turn_right`
- Yet we can use primitives and combinations
  to generate complex runes.
- **Abstractions** to conquer complexity:
  Naming and **functional abstraction**
- **Recursion**: describe the solution to a problem
  by using a solution to a (slightly) smaller problem.
  ("wishful thinking")

Quest 2A: Rune Reading

See Source Academy `https://sourceacademy.nus.edu.sg`

Another example: Factorial 1.2.1

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Factorial

$n\,! = n \ \ (n-1)(n-2)\cdots 1$

Another example: Factorial 1.2.1

Factorial

$$n\,! = n \quad (n-1)(n-2)\cdots 1$$

Grouping

$$n\,! = n\ ((n-1)(n-2)\cdots 1)$$

Another example: Factorial 1.2.1

Factorial

$$n\,! = n \;\; (n-1)(n-2)\cdots 1$$

Grouping

$$n\,! = n\;((n-1)(n-2)\cdots 1)$$

Replacement

$$n\,! = n\;(n-1)\;!$$

Another example: Factorial 1.2.1

Factorial

$$n\,! = n\ \ (n-1)(n-2)\cdots 1$$

Grouping

$$n\,! = n\,((n-1)(n-2)\cdots 1)$$

Replacement

$$n\,! = n\,(n-1)\,!$$

Remember the base case

$$\begin{array}{ll} n\,! = 1 & \text{if } n = 1 \\ n\,! = n\,(n-1)\,! & \text{if } n > 1 \end{array}$$

Translation into Source

Remember the base case

$$
\begin{aligned}
n\,! &= 1 && \text{if } n = 1 \\
n\,! &= n\,(n-1)\,! && \text{if } n > 1
\end{aligned}
$$

Factorial in Source

```
function factorial(n) {
  return n === 1
         ? 1
         : n * factorial(n - 1);
}
```

Example execution using Substitution Model

```
function factorial(n) {
    return n === 1 ? 1 : n * factorial(n - 1);
}
```

```
factorial(4)
4 * factorial(3)
4 * (3 * factorial(2))
4 * (3 * (2 * factorial(1)))
4 * (3 * (2 * 1))
4 * (3 * 2)
4 * 6
24
```

Calculating 4!

```
factorial(4)
4 * factorial(3)
4 * (3 * factorial(2))
4 * (3 * (2 * factorial(1)))
4 * (3 * (2 * 1))
4 * (3 * 2)
4 * 6
24
```

Calculating 4!

```
factorial(4)
4 * factorial(3)
4 * (3 * factorial(2))
4 * (3 * (2 * factorial(1)))
4 * (3 * (2 * 1))
4 * (3 * 2)
4 * 6
24
```

Accumulating deferred operations: *recursive* process

Absence of deferred operations: *iterative* process

Just curious: Can we write an iterative factorial?

Just curious: Can we write an iterative factorial?

```
function factorial(n) {
  return iter(1, 1, n);
}
function iter(product, counter, n) {
  return counter > n
         ? product
         : iter(counter * product,
                counter + 1,
                n);
}
```

Rick the Rabbit

Rick the Rabbit

- Rick the rabbit needs to climb a flight of stairs.

Rick the Rabbit

- Rick the rabbit needs to climb a flight of stairs.
- Given: Rick can **hop** (1 step), **skip** (2 steps) or **jump** (3 steps).

Rick the Rabbit

- Rick the rabbit needs to climb a flight of stairs.
- Given: Rick can **hop** (1 step), **skip** (2 steps) or **jump** (3 steps).
- Let's consider the problem of how many different ways can Rick climb a flight of `n` stairs?

Rick the Rabbit

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip

- How about 3 stairs?

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip

- How about 3 stairs?
  hop hop hop

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip
- How about 3 stairs?
  hop hop hop
  skip hop

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip

- How about 3 stairs?
  hop hop hop
  skip hop
  hop skip

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip

- How about 3 stairs?
  hop hop hop
  skip hop
  hop skip
  jump

Rick the Rabbit

- Consider the case of `n = 2` (two stairs):
  hop hop
  skip
- How about 3 stairs?
  hop hop hop
  skip hop
  hop skip
  jump
- What about 0 stairs?
- What about -1 stairs?

Rick the Rabbit

Rick the Rabbit

- If Rick hops, we have `n - 1` stairs remaining

Rick the Rabbit

- If Rick hops, we have `n - 1` stairs remaining
- If Rick skips, we have `n - 2` stairs remaining

Rick the Rabbit

- If Rick hops, we have `n - 1` stairs remaining
- If Rick skips, we have `n - 2` stairs remaining
- If Rick jumps, we have `n - 3` stairs remaining

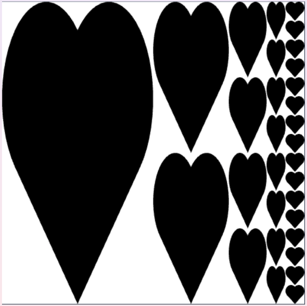
We now have a smaller problem (the number of stairs is decreasing)

Rick the Rabbit Source

```
function rabbit_ways(n) {
    return n < 0
           ? 0
           : n === 0
           ? 1
           : rabbit_ways(n - 1) // Rick hops
             +
             rabbit_ways(n - 2) // Rick skips
             +
             rabbit_ways(n - 3); // Rick jumps
}
```

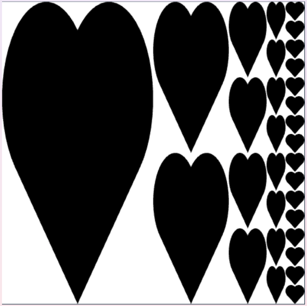
Example from Quest 2A “Rune Reading”: `fractal`

Define a function `fractal` that returns pictures like this:



Example from Quest 2A "Rune Reading": `fractal`

Define a function `fractal` that returns pictures like this:



when we call it like this: `fractal(heart, 5);`

Example from "Rune Reading": fractal, Solution 1

```
function fractal_1(rune, n) {
    return n === 1
        ? rune
        : beside(rune,
                 stack(fractal_1(rune, n - 1),
                       fractal_1(rune, n - 1)));
}
```

Tree recursion

```
function fractal_1(rune, n) {
    return n === 1
        ? rune
        : beside(rune,
                 stack(fractal_1(rune, n - 1),
                       fractal_1(rune, n - 1)));
}
```

Tree recursion

```
function fractal_1(rune, n) {
    return n === 1
        ? rune
        : beside(rune,
              stack(fractal_1(rune, n - 1),
                    fractal_1(rune, n - 1)));
}
```

Question

Can we implement this function with linear recursion?

Is this a good idea?

```
function fractal_2(rune, n) {
    const sub_frac = fractal_2(rune, n - 1);
    return n === 1
        ? rune
        : beside(rune, stack(sub_frac, sub_frac));
}
```

Is this a good idea?

```
function fractal_2(rune, n) {
    const sub_frac = fractal_2(rune, n - 1);
    return n === 1
        ? rune
        : beside(rune, stack(sub_frac, sub_frac));
}
```

Can we declare a const...

Is this a good idea?

```
function fractal_2(rune, n) {
    const sub_frac = fractal_2(rune, n - 1);
    return n === 1
        ? rune
        : beside(rune, stack(sub_frac, sub_frac));
}
```

Can we declare a const...

...**just for the alternative of the conditional**?

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

- Each branch of the conditional is a *block*.

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

- Each branch of the conditional is a *block*.
- A block can have local names, only visible inside the block.

Conditional *statements* (see SICP JS 1.3.2)

```
function fractal_3(rune, n) {
    if (n === 1) {
        return rune;
    } else {
        const f = fractal_3(rune, n - 1);
        return beside(rune, stack(f, f));
    }
}
```

- Each branch of the conditional is a *block*.
- A block can have local names, only visible inside the block.
- Remember to return a result *in each branch*.
  (Otherwise `undefined` is returned.)

A “divine” solution

```
function fractal_4(rune, n) {
    return n === 1
           ? rune
           : beside(rune,
                    fractal_4(stack(rune, rune),
                              n - 1));
}
```

Quest 2B: Beyond the Second Dimension

See Source Academy https://sourceacademy.nus.edu.sg