Trading in a Volatile Time Period

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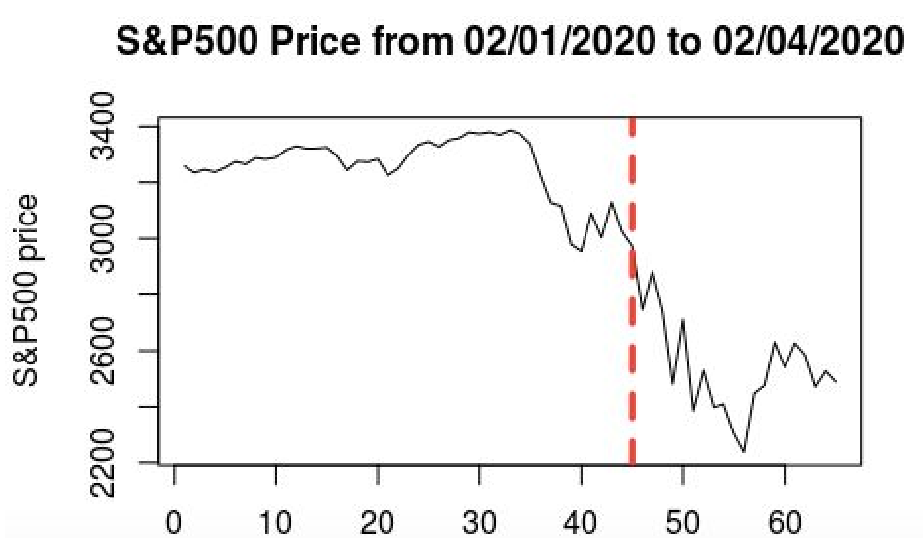
Due to the uncertainty introduced by the coronavirus, the US equity market has entered a period of extreme volatility after nearly 10 years of bull market. Equity market indices dropped significantly. However, there were big gains in the market due to long term investors buying under-priced stocks, Fed interventions, and expectation of government rescue packages. This project aims to construct a trading strategy making use of the volatile market environment.

Executive Summary

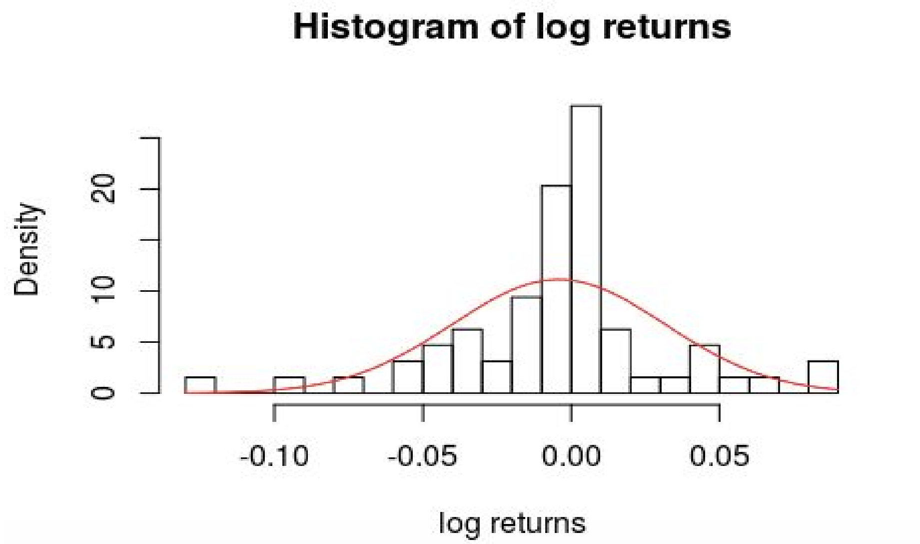
Overall, from looking at the distribution of the data and from the stark difference in the models when changing the training data period by even one day, we saw that this was a very volatile period for the S&P500. We found that the optimal band strategy was largely passive, and due to the differences in the price range of the training and testing data there was only one execution in the optimal band strategy. Though this did lead to a positive P&L, if this was delayed by even one day the P&L dropped significantly lower and to a somewhat insignificant amount. We produced a much more involved strategy using ad-hoc bands, and found that the end-of-period P&Ls for our ad-hoc strategies using $\sigma = 0.5$ and $\sigma = 1$ were significantly higher than our optimal band P&L. However, if we delayed execution by one day, the ad-hoc band P&Ls dropped to negative values in both cases. Therefore, if it were possible to execute trades exactly when instructed, it seems that an ad-hoc band strategy is more appropriate in volatile times. However if there is any delay in execution, an optimal band strategy appears to be the safer option.

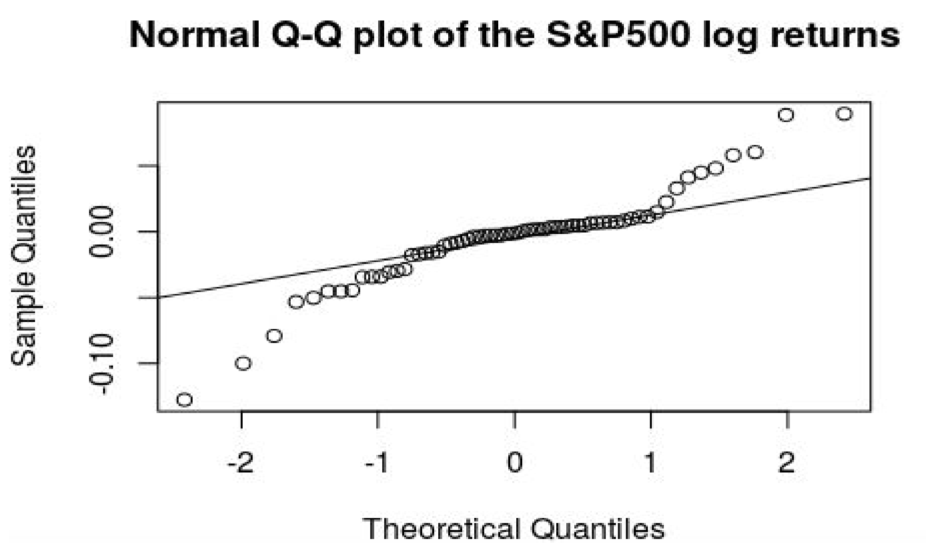
Data Preparation and Illustration



For this study, we used S&P500 data from January 2nd to March 5th to train our models, as although the markets only started to be significantly affected by coronavirus in late February, we required a longer time period in order to have enough data to train our models. However, this entire period represents an extremely volatile time, as the S&P500 reached its all-time high on February 19th, closing at 3386.15, yet by March 23rd it had fallen 34% to a low of 2237.40. The graph above demonstrates this extreme price movement. The red line indicates where our training period ends and testing period begins.

We began by looking at the log returns of the adjusted close prices, and found that the data was not normally distributed. If we look at the histogram we can see abnormally high frequencies around the centre of the distribution and also higher frequencies at the tails too. The normal quantile plot further supports the claim that the data is highly irregular and deviates significantly from a normal distribution. We can therefore see that this is a highly volatile period.





Fitting an AR(1) Model

For our portfolio formation period we used the data described above, and for our testing period we used March 6th to April 2nd, which followed the decreasing pattern of our training data until the low on March 23rd which was followed by slight gains.

During the portfolio training period we attempted to fit the S&P price as a linear function of time. This was what ultimately decided our training period, as we tried to include as much of the affected dates as possible whilst still using a period that showed some sort of linearity. Due to this we opted not to include the March 23rd low in the training period.

We then calculated

$$Y(t) \; = \; S(t) - S^{ave}(t)$$

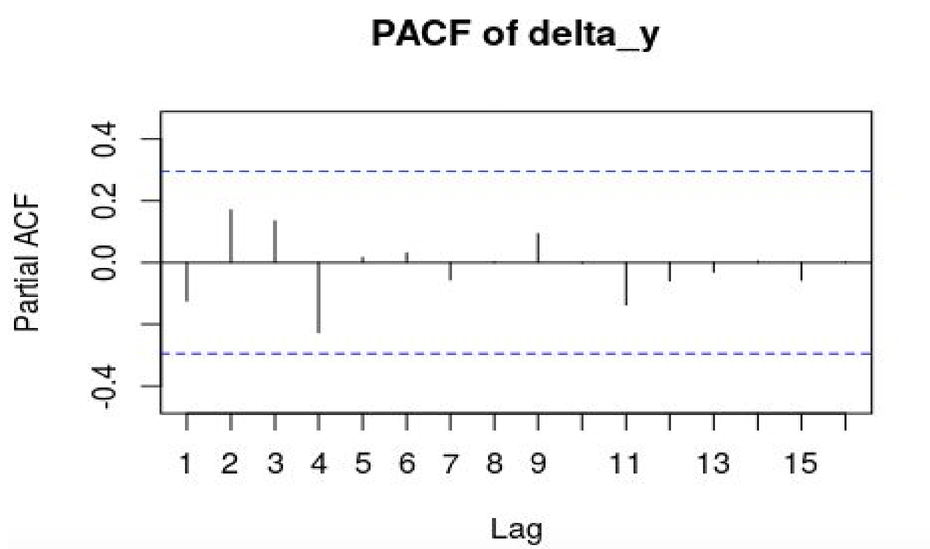
where S^{ave} (t) are the fitted values from our linear model, and Y(t) can be thought of as the deviation of the S&P500 price from the linear trend. We then fit an AR(1) model on

$$\Delta Y(t) = Y(t) - Y(t-1)\,,$$

Which gave us

$$\Delta Y_t = a + b\Delta Y_{t-1} + \varepsilon_t$$
$$a \; = \; -2.2214236, \quad b \; = \; -0.1241304$$

We do not have significant negative lag-1 autocorrelation in this case as this is given by the coefficient in the AR(1) model, and this is not close to -1. Furthermore, for an AR(1) model we should see the partial autocorrelations for lags 2, 3, etc. drop close to zero. From the graph below we can see that this is in fact the case and we can therefore conclude that an AR(1) model is a suitable fit.



From our AR(1) model we extracted the following continuous time mean-reverting model for Y

$$dY_t = \kappa(\theta - Y_t) + \sigma dW_t$$

Using

$$\kappa = 1 - b, \quad \theta = \frac{1}{\kappa} E[\Delta Y_t + \kappa Y_{t-1}]$$

Where b is the coefficient of the AR(1) model, and took σ to be the standard deviation of the Y(t) values.

Trading Strategy using Ad-Hoc Bands

We constructed an ad hoc bands trading strategy using the volatility of Y(t) as found in the previous question. Depending on a fixed value of sigma, we defined upper and lower bands around the 3-day rolling average of the S&P 500 index.

$$Y_u = S^{ave}(3) \ + \ \sigma * \sigma^{ave}(3)$$
$$Y_l = S^{ave}(3) \ - \ \sigma * \sigma^{ave}(3)$$

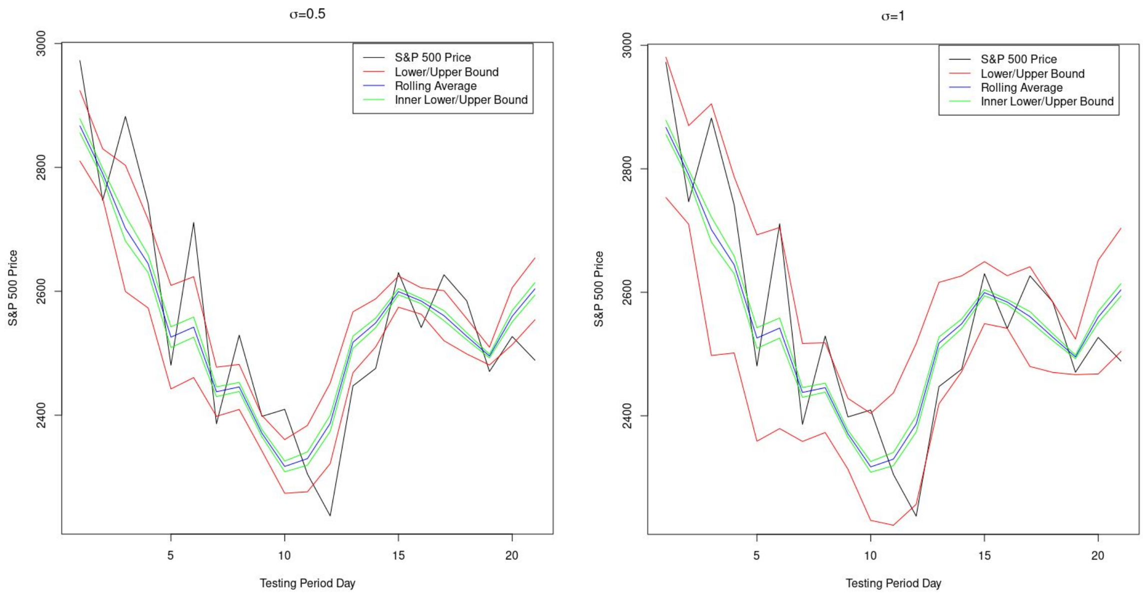
We also needed to define inner bands for closing our positions. As we did in class, we chose these to be within $\frac{1}{10}$ standard deviation of the rolling average.

$$Y_{iu} = S^{ave}(3) \ + \ \frac{1}{10} * \sigma^{ave}(3)$$
$$Y_{il} = S^{ave}(3) \ - \ \frac{1}{10} * \sigma^{ave}(3)$$

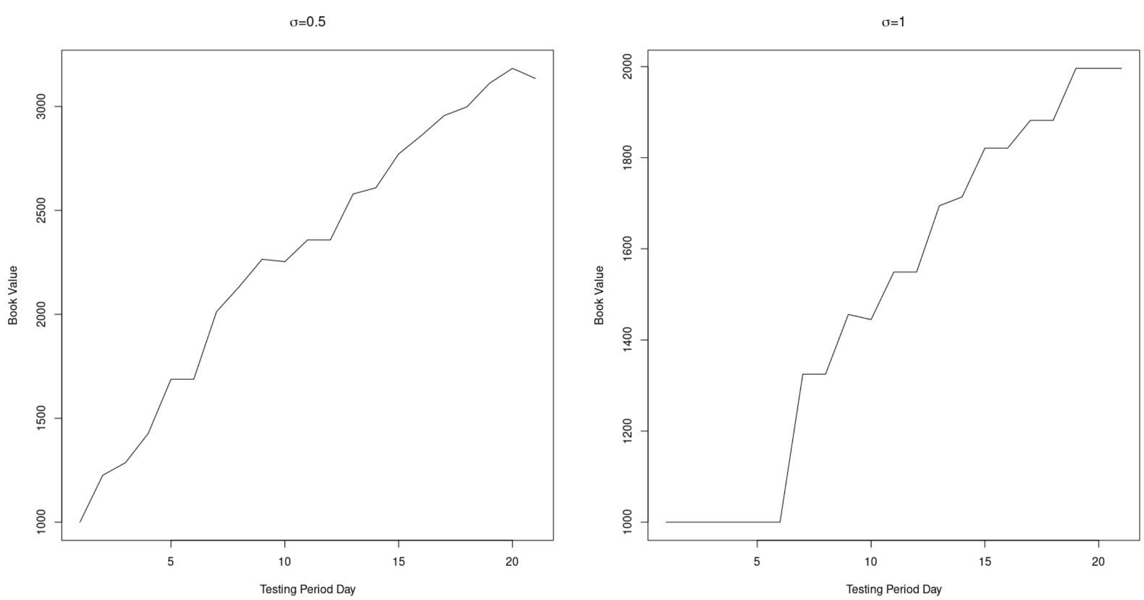
The trading strategy is defined as follows: we began with an initial wealth of $1000. For each day in the testing period, we opened a short position in the S&P 500 if the price was above Y_u, or we opened a long position in the S&P 500 if the price was below Y_l. We held one of these positions until the price crossed its respective inner band; if we held the short position, we closed it when the price fell below Y_{iu}, and if we held the long position, we closed it when the price rose above Y_{il}. If the price was in between the two outer bands and we didn't hold any positions, then we did nothing. If we held a position and the price had not yet reached its respective inner band, we did nothing by continuing to hold the position. Finally, if we had any open positions on the last day of the test period, we closed them regardless of the price.

The following results assumed no transaction costs.

The following figures illustrate the testing period data along with its 3 day rolling average, the upper and lower bands for two particular values of σ (0.5 and 1.0), and the inner bands, both set to $\frac{1}{10}$ standard deviation:



The following figures show the book values for each of those strategies:



For the $\sigma = 0.5$ strategy, the daily mean profit was \$106.75, the daily mean return was 5.71%, and the daily variance was 0.388% or \$9391.109. The Sharpe ratio was 34.28.

For the $\sigma = 1$ strategy, the daily mean profit was $49.81, the daily mean return was 3.46%, and the daily variance was 0.4499% or $7045.204. The Sharpe ratio was 14.85. Compare these to the buy and hold strategy, which would have resulted in a loss of -483.72 over the same time period.

When comparing these two strategies to each other, we observed that $\sigma = 0.5$ had a larger profit than $\sigma = 1$. To test this strategy further, we began by shorting 0.34 shares of the S&P 500 at the beginning of our test period. This gave us $1000 in cash to finance the strategy. At the end of the testing period, we made $2135.07 in profit, of which we used $837.26 to buy back the 0.34 shares of S&P 500, which had a price of $2488.65 on the last day of the testing period. Our final net profit was $1297.81, which is $1781.53 more than the buy and hold strategy.

Trading strategy using Optimal Bands

Our next step was to develop a trading strategy using optimal bands. Assuming a zero discounting factor and taking our investment horizon to be the end of the portfolio formation period, we were faced with the following optimal exiting and entering problems:

$$H(t, S(t)) = sup_{t<=T} E_{t,S(t)}[(S_{\tau\wedge T} - c)]$$
$$G(t, S(t)) = sup_{\eta<=T} E_{t,S(t)}[H(\eta \wedge T, S(\eta \wedge T)) - S(\eta \wedge T) - c)]$$

Where c is the transaction cost. From these we get the following HJB equations:

$$max\{(\partial_t + L)H(t, S(t)),\ S(t) - c - H(t, S(t))\} = 0, \quad H(T, S) = S - c$$
$$max\{(\partial_t + L)G(t, S(t)),\ (H(t, S(t)) - S(t) - c) - G(t, S(t))\} = 0, \quad G(T, S) = H(T, S) - S - C = -2C$$
$$where\ L = \kappa(\theta - S)\partial_S + \tfrac{1}{2}\sigma^2\partial^2_{SS}$$

And the associated free-boundary equations are:
Exiting:

$$(\partial_t + L)H(t, S) = 0,\ S < S^*$$
$$H(t, S) = S - c,\ S > S^*$$
$$\partial_s H(t, S) = 1,\ S = S^*$$

Entering:

$$(\partial_t + L)G(t, S) = 0,\ S > S^*$$
$$G(t, S) = H(t, S) - S - c,\ S < S^*$$
$$\partial_s G(t, S) = 1,\ S = S^*$$

We planned to solve these using the finite differences method, which first involved discretizing the equations as follows:

$$
\begin{aligned}
H(S_i, t_{j-1}) &= (1 - \tfrac{h_t\sigma^2}{h_s^2})H(S_i, t_j) + (\tfrac{h_t\sigma^2}{2h_s^2} - \tfrac{h_t}{2h_s}\kappa(\theta - s))H(S_{i-1}, t_j) + (\tfrac{h_t\sigma^2}{2h_s^2} + \tfrac{h_t}{2h_s}\kappa(\theta - s))H(S_{i+1}, t_j) \\
&= a_i H(S_i, t_j) + l_i\ H(S_{i-1}, t_j) + u_i H(S_{i+1}, t_j) \\
&\quad with\ H(T, S_i) = S_i - c, \quad H(t_j, 0) = 0, \ \ H(t_j, S_{max}) = S_{max} - c \\
h_t &= t_{j+1} - t_j = \tfrac{T}{N}(j+1) - \tfrac{T}{N}j = \tfrac{T}{N}, \quad h_s = S_{i+1} - S_i = \tfrac{S_{max}}{M}(i+1) - \tfrac{S_{max}}{M}i = \tfrac{S_{max}}{M}
\end{aligned}
$$

With N being the size of our time grid, and M being the size of our price grid. We chose an S_{max} and S_{min} as appropriate maximums and minimums given our testing data. We used R to create a grid of values across our chosen time and price range for our value function H, and used this to find the optimal exiting price S^* for all times. We carried out a similar strategy for the entering problem:

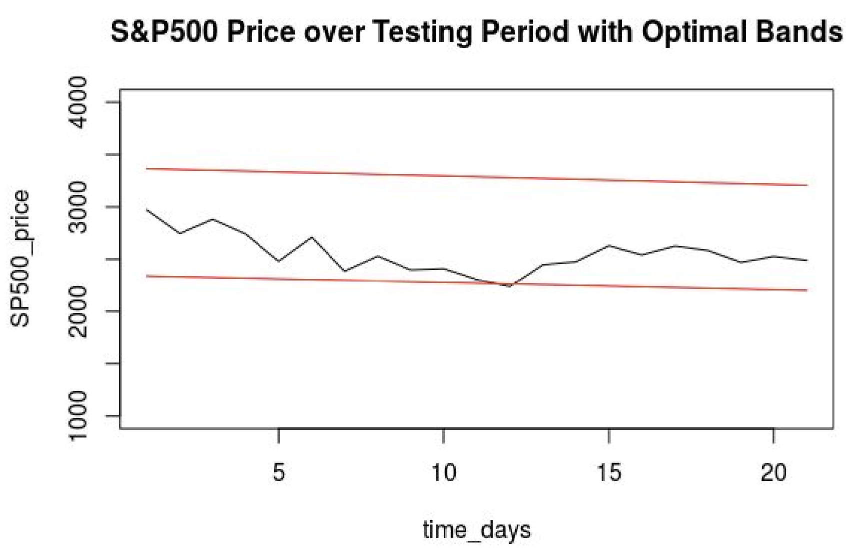
$$
\begin{aligned}
G(S_i, t_{j-1}) &= (1 - \tfrac{h_t\sigma^2}{h_s^2})G(S_i, t_j) + (\tfrac{h_t\sigma^2}{2h_s^2} - \tfrac{h_t}{2h_s}\kappa(\theta - s))G(S_{i-1}, t_j) + (\tfrac{h_t\sigma^2}{2h_s^2} + \tfrac{h_t}{2h_s}\kappa(\theta - s))G(S_{i+1}, t_j) \\
&= a_i G(S_i, t_j) + l_i G(S_{i-1}, t_j) + u_i G(S_{i+1}, t_j) \\
&\quad with\ G(T, S_i) = H(T, S_i) - S_i - c, \quad G(t_j, 0) = 0, \ \ H(t_j, S_{max}) = H(t_j, S_{max}) - S_{max} - c \\
h_t &= t_{j+1} - t_j = \tfrac{T}{N}(j+1) - \tfrac{T}{N}j = \tfrac{T}{N}, \quad h_s = S_{i+1} - S_i = \tfrac{S_{max}}{M}(i+1) - \tfrac{S_{max}}{M}i = \tfrac{S_{max}}{M}
\end{aligned}
$$

Since we had fit our continuous time mean-reverting model to Y and used the values of kappa and theta provided by this in our value functions above, we obtained optimal values for Y from the value function, and then used

$$S(t) = Y(t) + S^{ave}(t)$$

to find the optimal prices to use on our testing data.

Given the extreme values of our data we found our optimal bands were not a very good fit for our testing data. In fact, we found that the data only hit the lower band; it did not hit the upper band across the entire testing period. Given the training data however this was



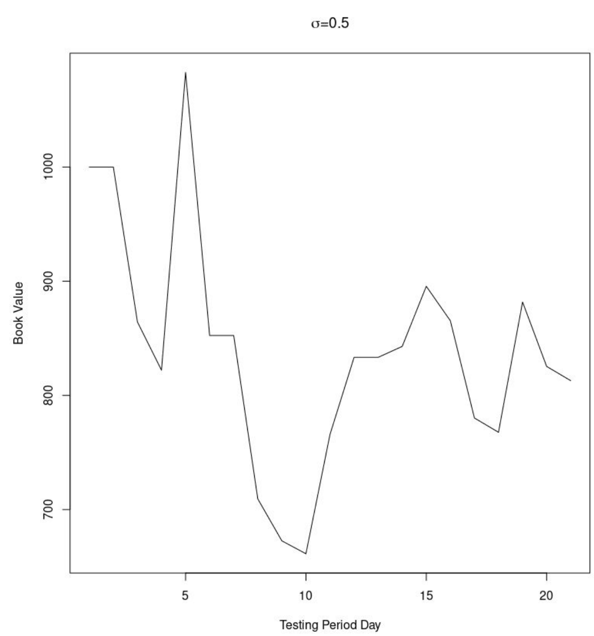
to be expected, as the training data generally covered a much higher range of values than the testing data. We also found that a strategy fell apart when attempting to include the extremely volatile data in our training period. If we included the extreme low of March 23rd, the bands went to straight lines. Therefore, in volatile times optimals bands clearly encourage very passive strategies.

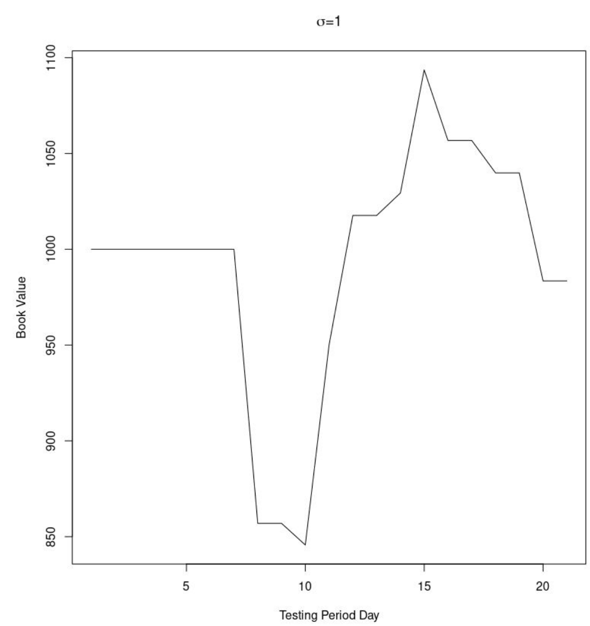
Using a zero transaction cost, the S&P500 price hit the lower band on day 12 of our training period. According to this we would buy S&P500 on day 12 at a price of $2237.40 per share. The price did not hit the upper band at any point so according to our strategy we would hold this to the end of our testing period, when we would close our position and sell our shares at $2488.65. This would give us a P&L of +$251.25 per share. With an initial capital of $1000 our end of period P&L would be $112.3

Delay in Execution

If we force trade execution to be delayed by a day, our strategy from problem 3 would suffer greatly. In both cases of $\sigma = 0.5$ and $\sigma = 1$, our final book value would finish lower than the initial $1000 wealth. In the first case, our final book value would be $812.90, which would be a loss of $187.10, and in the second case, our final book value would be $983.46, which would be a loss of $16.54.

The following figures show the book values over time in both cases:





Despite the poor performance, we can observe that both of these strategies would have still beaten the buy and hold strategy, which would have lost $483.72 over this same time period.

Regarding our strategy from problem 4, if we were to delay execution by one day, we would be buying S&P500 at $2447.33 a share, and would therefore be only making a P&L of +$41.32 per share when we close our position at day 21. This would give a P&L of +$16.88 using an initial capital of $1000, which is significantly lower than if we were executing as the bands instructed. However, this is still a positive result, therefore is a better performance in comparison to the ad-hoc band strategies.

We both contributed equally for this project, with Robbie focusing on the ad-hoc trading strategy and Issy focusing on the optimal trading strategy.