Discrete Structures

Assignment

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Knights always lie
Knaves always speaks the truth
Spies can either lie or tell the truth.
Now, we encounter three people A, B and C.
We know two of them are knights and one is a knave.

Propositions:-

P_1 = A is a Knight , $\neg P_1$ = A is not a knight
P_2 = A is a knave , $\neg P_2$ = A is not a knave
P_3 = A is a spy , $\neg P_3$ = A is not a spy

q_1 = B is a knight , $\neg q_1$ = B is not a knight
q_2 = B is a knave , $\neg q_2$ = B is not a knave
q_3 = B is a spy. , $\neg q_3$ = B is not a spy.

R_1 = C is a knight , $\neg R_1$ = C is not a knight
R_2 = C is a knave , $\neg R_2$ = C is not a knave
R_3 = C is a spy , $\neg R_3$ = C is not a spy.

a) A: C is the knave ∴ R_2
B: A is the knight ∴ P_1
C: I am the spy. ∴ R_3

Case 1: Knight, Knight, Knave

i) $R_2 = F$
ii) $P_1 = F$
iii) $R_3 = T$

Now, put values
i) $T \neq F$

Hence, does not hold

Case 2: Knight, Knave, Knight

i) $R_2 = F$
ii) $P_1 = T$
iii) $R_3 = F$

Now, put values
i) $F = F$
ii) $T = T$
iii) $F = F$

Hence, it holds

Case 3: Knave, Knight, Knight

i) $R_2 = T$
ii) $P_1 = F$
iii) $R_3 = F$

Now, put values
i) $F \neq T$

Hence, does not hold

Hence, we have a result
A = Knight
B = Knave
C = Knight

(b) A: I am the knight
B: I am the knave
C: B is the knight

i) P_1
ii) q_2
iii) q_1

Case 1: Knight, Knight, Knave

i) $P_1 = F$
ii) $q_2 = F$
iii) $q_1 = T$

Now, put values

i) $F \neq T$

Hence, does not hold

Case 2: Knight, Knave, Knight

i) $P_1 = F$
$q_2 = T$
$q_1 = F$

Now, put values

i) $T \neq F$

Hence, does not hold

Case 3: Knave, Knight, Knight

i) $P_1 = F$
ii) $q_2 = T$
iii) $q_1 = T$

Now, put values

i) $F = F$
ii) $F \neq T$

Hence, does not hold

Here, we cannot determine which is a knight or a knave because there are only three cases. More cases are required to get the accurate result.

Case 4: Knight, Knave, Knave

i) $P_1 = F$
ii) $Q_2 = T$
iii) $Q_1 = T$

Now, put values

i) $T \neq F$

Hence, does not hold.

Case 5: Knave, Knight, Knave

i) $P_1 = T$
ii) $Q_2 = F$
iii) $Q_1 = T$

Now, Put values

i) $F \neq T$

Hence, does not hdd

Case 6: Knave, Knave, Knight

i) $P_1 = T$
ii) $Q_2 = T$
iii) $Q_1 = F$

Now, put values

i) $F \neq T$

Hence, does not hold

Case 7: Knave, Knave, Knave

i) $P_1 = T$
ii) $Q_2 = T$
iii) $Q_1 = T$

Now, put values

i) $F \neq T$

Hence, does not hold

Case 8: Knight, Knight, Knight

i) $P_1 = F$
ii) $q_2 = F$
iii) $q_1 = F$

Now, put values

i) $T \neq F$

Hence, does not hold

Result:-

We cannat identify which are the knights and knaves.

c) A: I am the knave
B: I am the knave
C: I am the knave

i) P_2
ii) q_2
iii) R_2

Case 1: Knight, Knight, Knave

i) $P_2 = F$
ii) $q_2 = F$
iii) $R_2 = T$

i) $F = F$
ii) $F = F$
iii) $T = T$

Hence, it holds.

Case 2: Knight, Knave, Knight

i) $P_2 = F$
ii) $q_2 = T$
iii) $R_2 = F$

Now, Put the values

i) $F = F$
ii) $T = T$
iii) $F = F$

Hence, it holds.

Case 3: Knave, Knight, Knight

$P_2 = T$

$q_2 = F$

$R_2 = F$

Now, Put values

i) $T = T$

ii) $F = F$

iii) $F = F$

Hence, it holds

Result: Evidence suggests that all the cases hold so there is no unique solution, so we can't determine who is who.

d) A: I am the knight

B: A is testing the Truth

C: I am the spy.

i) $P_1 = F$

ii) $q_1 = F$

iii) $R_3 = T$

Case 1: Knight, Knight, Knave

~~i) $P_2 = F$~~ Now Put values

i) $T \neq F$

Hence, does not hold.

Case 2: Knight, Knave, Knight

i) $P_1 = F$

ii) ~~$q_2 = T$~~

iii) $R_3 = F$

Now, Put values

i) $T \neq F$

Hence, does not hold.

Case 3: Knave, Knight, Knight

i) $P_1 = T$
ii) $q_3 = F$
iii) $R_3 = F$

Now, Put values

i) $T \neq F$

Hence, does not hold

Case 4: Knight, Knave, Knave

i) $P_1 = F$
ii) $q_3 = T$
iii) $R_3 = F$

Now, Put values

i) $T \neq F$

Hence, does not hold.

Case 5: Knave, Knight, Knave

i) $P_1 = T$
ii) $q_3 = F$
iii) $R_3 = T$

Now, Put values

i) $F \neq T$

Hence, does not hold.

Case 6: Knave, Knave, Knight

i) $P_1 = T$
ii) $q_3 = T$
iii) $R_3 = F$

Now, Put values

i) $F \neq T$

Hence, does not hold.

Case 7: Knight, Knight, Knight

i) $P_1 = F$

ii) $q_3 = F$

iii) $R_3 = F$

Now, Put values

i) $T \neq F$

Hence, does not hold.

Case 8: Knave, Knave, Knave

i) $P_1 = T$

ii) $q_3 = T$

iii) $R_3 = T$

Now, Put values

i) $F \neq T$

Hence, does not hold

Result: None of the cases holds, so we cannot identify that which one's are knaves and which one's are knights.