

Q1 :-

Part 1

①

$$(P \vee Q) \vee (\neg P \wedge S) \rightarrow C_1$$

$$P \rightarrow C_2$$

$$Q \rightarrow C_3$$

$$\neg P \rightarrow C_4$$

$$S \rightarrow C_5$$

$$P \vee Q \rightarrow C_6$$

$$Q \rightarrow C_7$$

$$\neg P \rightarrow C_8$$

From $C_2 \wedge C_5 \wedge C_8 \rightarrow C_1$

From $C_1 \wedge C_7 \rightarrow \boxed{X}$

Argument is valid.

Part 2

$$(P \vee Q) \vee R \rightarrow C_1$$

$$Q \vee R \rightarrow C_2$$

$$R \rightarrow C_3$$

$$P \rightarrow C_4$$

$$Q \rightarrow C_5$$

$$R \rightarrow C_6$$

$$Q \wedge R \rightarrow C_7$$

$$P \vee Q \rightarrow C_8$$

$$P \vee R \rightarrow C_9$$

$$\boxed{X} \rightarrow C_{10}$$

valid argument

Part 3:

$\neg P \vee S \vee Q - C_1$

$\neg Q - C_2$

$\neg U - C_3$

$\neg T - C_4$

$\neg U \vee P - C_5$

$\neg S - C_6$

$\neg Q \vee S - C_7$

$\neg Q \vee P - C_8$

$S \vee Q - C_9$

$S - C_{10}$

using $C_10 \wedge C_5 \rightarrow X$

Argument is valid.

Part 4:

$\neg t - C_1$

$\neg I - C_2$

$\neg R - C_3$

$\neg U - C_4$

$\neg T - C_5$

$\neg U \vee P - C_6$

$\neg S - C_7$

$\neg Q - C_8$

$\neg S - C_9$

\boxed{X} using $C_1 \wedge C_9$
Argument is valid.

Part 5:

(3)

$$\begin{aligned}
 & \neg P \vee \neg t \vee \lambda v_s - (1) \\
 & \neg v \vee t - (2) \\
 & \neg u \vee p - (3) \\
 & \neg s - (4) \\
 & s - (5) \\
 & \neg r - (6)
 \end{aligned}$$

using (4) { (5)

argument is valid.

Q2

Part 1:

$$\exists x \exists y \forall z \neg P(x, y, z)$$

$$\begin{aligned}
 & = \left[(\neg P(1,1,1) \wedge \neg P(1,1,2) \wedge \neg P(1,1,3)) \vee (\neg P(1,2,1) \wedge \neg P(1,2,2) \wedge \neg P(1,2,3)) \right. \\
 & \quad \left. \vee (\neg P(1,3,1) \wedge \neg P(1,3,2) \wedge \neg P(1,3,3)) \right] \vee \left[(\neg P(2,1,1) \wedge \neg P(2,1,2) \wedge \neg P(2,1,3)) \right. \\
 & \quad \left. \vee (\neg P(2,2,1) \wedge \neg P(2,2,2) \wedge \neg P(2,2,3)) \right] \vee \left[(\neg P(2,3,1) \wedge \neg P(2,3,2) \wedge \neg P(2,3,3)) \right] \\
 & \quad \left. \vee [(\neg P(3,1,1) \wedge \neg P(3,1,2) \wedge \neg P(3,1,3)) \vee (\neg P(3,2,1) \wedge \neg P(3,2,2) \wedge \neg P(3,2,3))] \right] \\
 & \quad \left. \vee [(\neg P(3,3,1) \wedge \neg P(3,3,2) \wedge \neg P(3,3,3))] \right]
 \end{aligned}$$

Part 2:

$$\begin{aligned}
 & \forall_n \exists_y \exists_z \top P(n, y, z) \\
 = & \left[(\top P(1, 1, 1) \vee \top P(1, 1, 2) \vee \top P(1, 1, 3)) \right] \vee \left[\top P(1, 2, 1) \vee \top P(1, 2, 2) \vee \top P(1, 2, 3) \right] \vee \\
 & \left[\top P(1, 3, 1) \vee \top P(1, 3, 2) \vee \top P(1, 3, 3) \right] \wedge \left[(\top P(2, 1, 1) \vee \top P(2, 1, 2) \vee \top P(2, 1, 3)) \vee \right. \\
 & \left. (\top P(2, 2, 1) \vee \top P(2, 2, 2) \vee \top P(2, 2, 3)) \vee (\top P(2, 3, 1) \vee \top P(2, 3, 2) \vee \top P(2, 3, 3)) \right] \wedge \\
 & \left[\top P(3, 1, 1) \vee \top P(3, 1, 2) \vee \top P(3, 1, 3) \right] \vee \left[(\top P(3, 2, 1) \vee \top P(3, 2, 2) \vee \top P(3, 2, 3)) \right. \\
 & \left. \vee (\top P(3, 3, 1) \vee \top P(3, 3, 2) \vee \top P(3, 3, 3)) \right]
 \end{aligned}$$

Part 3:

$$\begin{aligned}
 & \forall_n \forall_y \exists_z \top P(n, y, z) \\
 = & \left[(\top P(1, 1, 1) \wedge \top P(1, 1, 2) \wedge \top P(1, 1, 3)) \wedge (\top P(1, 2, 1) \wedge \top P(1, 2, 2) \wedge \top P(1, 2, 3)) \right. \\
 & \wedge \left. (\top P(1, 3, 1) \wedge \top P(1, 3, 2) \wedge \top P(1, 3, 3)) \right] \wedge \left[(\top P(2, 1, 1) \wedge \top P(2, 1, 2) \wedge \top P(2, 1, 3)) \wedge \right. \\
 & \left. (\top P(2, 2, 1) \wedge \top P(2, 2, 2) \wedge \top P(2, 2, 3)) \wedge (\top P(2, 3, 1) \wedge \top P(2, 3, 2) \wedge \top P(2, 3, 3)) \right] \wedge \\
 & \left[(\top P(3, 1, 1) \wedge \top P(3, 1, 2) \wedge \top P(3, 1, 3)) \wedge (\top P(3, 2, 1) \wedge \top P(3, 2, 2) \wedge \top P(3, 2, 3)) \right. \\
 & \left. \wedge (\top P(3, 3, 1) \wedge \top P(3, 3, 2) \wedge \top P(3, 3, 3)) \right]
 \end{aligned}$$

Part 4:

$$\begin{aligned}
 & \cancel{\forall_n \exists_y \exists_z \top P(n, y, z)} \vee \forall_n \forall_y \exists_z \top P(n, y, z) \\
 = & \left[(\top P(1, 1, 1) \wedge \top P(1, 1, 2) \wedge \top P(1, 1, 3)) \vee (\top P(1, 2, 1) \wedge \top P(1, 2, 2) \wedge \top P(1, 2, 3)) \right] \vee \\
 & \left[(\top P(1, 3, 1) \wedge \top P(1, 3, 2) \wedge \top P(1, 3, 3)) \right] \wedge \left[(\top P(2, 1, 1) \wedge \top P(2, 1, 2) \wedge \top P(2, 1, 3)) \vee \right. \\
 & \left. (\top P(2, 2, 1) \wedge \top P(2, 2, 2) \wedge \top P(2, 2, 3)) \vee (\top P(2, 3, 1) \wedge \top P(2, 3, 2) \wedge \top P(2, 3, 3)) \right] \vee \\
 & \left[(\top P(3, 1, 1) \wedge \top P(3, 1, 2) \wedge \top P(3, 1, 3)) \vee (\top P(3, 2, 1) \wedge \top P(3, 2, 2) \wedge \top P(3, 2, 3)) \right. \\
 & \left. \vee (\top P(3, 3, 1) \wedge \top P(3, 3, 2) \wedge \top P(3, 3, 3)) \right]
 \end{aligned}$$

Part 5:-

(5)

$$7 \exists_x \exists_y \forall_z P(x, y, z)$$

$$\begin{aligned} &= \{ (P(1,1,1) \wedge P(1,1,2) \wedge P(1,1,3)) \vee (P(1,2,1) \wedge P(1,2,2) \wedge P(1,2,3)) \vee \\ &\quad P(1,3,1) \wedge P(1,3,2) \wedge P(1,3,3)) \} \wedge \{ (P(1,1,1) \wedge P(1,1,2) \wedge P(1,1,3)) \vee \\ &\quad P(1,2,1) \wedge P(1,2,2) \wedge P(1,2,3)) \vee P(1,3,1) \wedge P(1,3,2) \wedge P(1,3,3)) \vee \\ &\quad \{ (P(1,1,1) \wedge P(1,1,2) \wedge P(1,1,3)) \vee \\ &\quad \vee (P(1,3,1) \wedge P(1,3,2) \wedge P(1,3,3)) \} \} \end{aligned}$$

Q3:- Part 1: Knights speak the truth ⑥

- Knaves speak the truth

- i) $P \wedge Q$
- ii) $\neg P$

Case 1:-

Knights, Knights

i) $T \wedge T = T$

$P = T \quad \neg P = F$

ii) $F \neq T$

$\neg V = T \quad \neg \neg V = F$

case does not hold.

Case 2:- Knight, Knave

i) $T \wedge F \neq T$

$P = T \quad \neg P = F$

case does not hold.

$\neg V = F \quad \neg \neg V = \cancel{T}$

Case 3:- Knave, Knight

$P = F \quad \neg P = T$

i) $F \wedge T \neq T$

$\neg V = T \quad \neg \neg V = F$

case does not hold.

Case 4:- Knave, Knave

$P = F \quad \neg P = T$

i) $F \wedge F \neq T$

$\neg V = F \quad \neg \neg V = T$

case does not hold.

Hence, we cannot tell that which one is a knight and which one is knave.

Part 2: Knights speak the ⁽⁷⁾ truth
Knave speaks the lie

Case 1:- Knight, Knight

- i) $P \wedge Q = T \rightarrow T \wedge T = T$ $P=T$ $\neg P=F$
ii) $\neg P=T \rightarrow F \neq T$ $Q=T$ $\neg Q=F$

case does not hold.

Case 2:- Knight, Knave

- i) $T \wedge F \neq T$ $P=T$ $\neg P=F$
ii) $\neg Q=F$ $Q=F$ $\neg Q=T$

case does not hold.

Case 3:- Knave, Knight

- i) $F \wedge T = F$ $P=F$ $\neg P=T$
ii) $\neg T=F$ $T=T$ $\neg T=F$

case holds.

Case 4:- Knave, Knave

- i) $F \wedge F = F$ $P=F$ $\neg P=T$
ii) $\neg T=F$ $T=F$ $\neg T=T$

case does not hold.

Hence,

A is a Knave

B is a ~~not~~ knight.

Part 3: Knights tells lies $\textcircled{8}$

Knaves tell lies

Case 1: Knight, Knight

$$P = T \quad \neg P = F$$

$$Q = T \quad \neg Q = F$$

i) $T \wedge T = F$

case does not hold

Case 2: Knight, Knave

i) $T \wedge F = F$

$$P = T \quad \neg P = F$$

ii) $F = F$

$$Q = F \quad \neg Q = T$$

case holds

Case 3:

Knave, Knight

i) $F \wedge T = F$

$$P = F \quad \neg P = T$$

ii) $T \neq F$

$$Q = T \quad \neg Q = F$$

case does not hold.

Case 4:

Knave, Knave

i) $F \wedge F = F$

$$P = F \quad \neg P = T$$

ii) $T \neq F$

$$Q = F \quad \neg Q = T$$

case does not hold

Hence, A is a knight

B is a knave.

Part 4:

Knights tell lies

(9)

Knave ~~tell~~
speaks truthCase 1:Knight, Knight

$$P = T \quad \neg P = F$$

$$\text{i) } T \wedge T = T$$

$$V = T \quad \neg V = F$$

case does not hold.

Case 2:Knight, Knave

$$P = T \quad \neg P = F$$

$$\text{i) } T \wedge F = F$$

$$V = F \quad \neg V = T$$

$$\text{ii) } F \neq T$$

case does not hold.

Case 3: Knave, Knight

$$P = F \quad \neg P = T$$

$$\text{i) } F \wedge F = F$$

$$V = T \quad \neg V = F$$

case does not hold

Case 4: Knave, Knave

$$P = F \quad \neg P = T$$

$$\text{i) } F \wedge F \neq T$$

$$V = F \quad \neg V = T$$

case does not hold

Hence,

we cannot determine who is who.

Part 5:

Knight speaks the truth

(16)

Knave's tell lies

Case 1:-

Knight, Knight

$P = T$

$\neg P = F$

i) $T \wedge T = T$

$\neg V = T$

$\neg \neg V = F$

ii) $F \neq T$

case does not hold

Case 2:-

Knight, knave

$P = T$

$\neg P = F$

i) $T \wedge F \neq T$

$\neg V = F$

$\neg \neg V = T$

case does not hold

Case 3:-

Knave, knight

$P = F$

$\neg P = T$

i) $F \wedge T = F$

$\neg V = T$

$\neg \neg V = F$

case holds

Case 4:-

Knave, knave

$P = F$ $\neg P = T$

i) $F = F$

$\neg V = F$

ii) $T \neq F$

case does not hold

Hence,

A is a knave

B is a knight

Qn:

(11)

Part 1:

Implication: $\neg q \rightarrow \neg p \vee \neg v$

Inverse: $\neg v \rightarrow (\neg p \wedge \neg q)$

Converse: $\neg p \vee \neg v \rightarrow \neg q$

Contrapositive: $(\neg p \wedge \neg q) \rightarrow \neg v$

Part 2:

Implication: $\neg q \rightarrow (\neg p \wedge \neg q)$

Inverse: $\neg q \rightarrow (\neg p \vee \neg q)$

Converse: $(\neg p \wedge \neg q) \rightarrow \neg q$

Contrapositive: $\neg p \vee \neg q \rightarrow \neg q$

Part 3:

Implications: $\neg p \vee \neg q \rightarrow \neg q$

Inverse: $(\neg p \wedge \neg q) \rightarrow \neg q$

Converse:

Contrapositive: $\neg q \rightarrow (\neg p \vee \neg q)$

Part 4:

Implications: $\neg v \rightarrow (\neg p \vee \neg q)$

Inverse: $\neg v \rightarrow (\neg p \wedge \neg q)$

Converse: $(\neg p \vee \neg q) \rightarrow \neg v$

Contrapositive: $(\neg p \wedge \neg q) \rightarrow \neg v$

Part 5:-

Implications $(P \vee Q) \rightarrow Q$

(12)

Inverse $\sim (P \wedge Q) \rightarrow Q$

Converse $Q \rightarrow (P \vee Q)$

Contrapositive $\sim Q \rightarrow \sim P \wedge Q$

Q5:-

Part 1:-

$$\underline{(R_1 \cup R_2)^{-1}} = \{(1,1), (2,1), (1,2), (1,3), (2,3)\}$$

$$\underline{R_1 \cup R_2} \Rightarrow \{(2,2)\}$$

Part 2:-

$$\underline{(R_1 \cap R_2)^{-1}} = \{(1,2)\}$$

$$\underline{(R_1 \cap R_2)} = \{(1,1), (1,2), (2,2), (3,1), (3,2)\}$$

Part 3:-

$$\underline{(R_1 - R_2)^{-1}} = \{(2,1), (2,3)\}$$

$$\underline{(R_1 - R_2)} = \{(1,1), (2,1), (2,2), (3,1)\}$$

Part 4:-

$$\underline{(R_2 - R_1)^{-1}} = \{(1,1), (1,3)\}$$

$$\underline{(R_2 - R_1)} = \{(1,2), (2,1), (2,2), (3,2)\}$$

Part 5:-

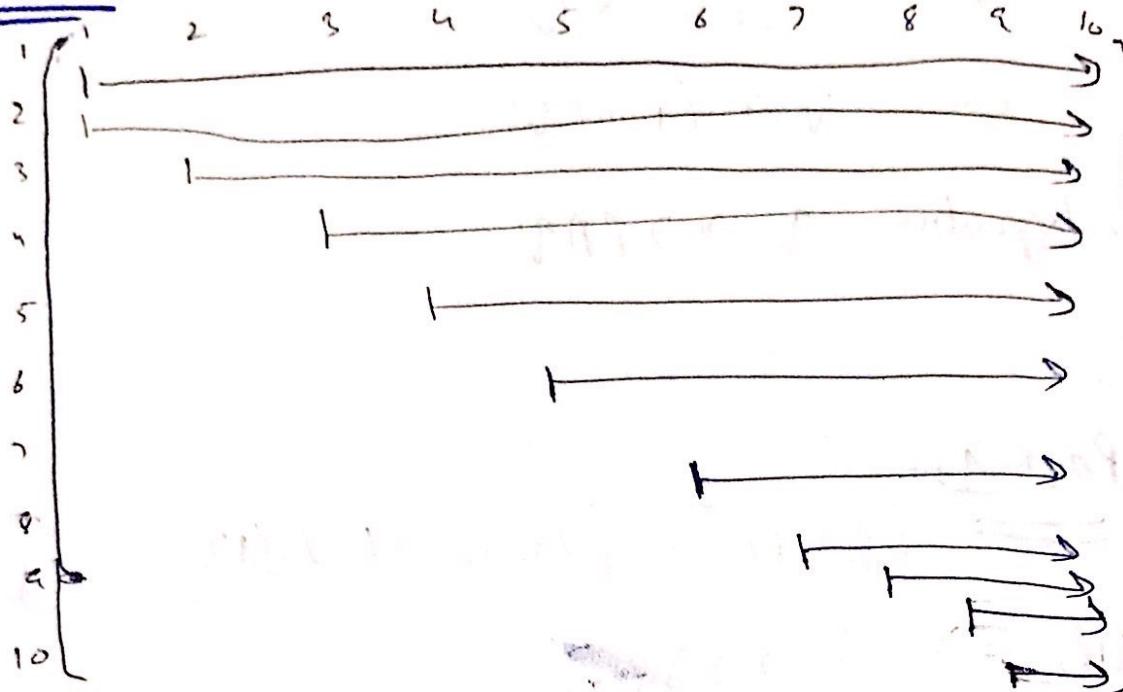
$$\underline{((R_1)^{-1})^{-1}} = \{(1,2), (2,1), (3,2)\}$$

$$\underline{R_1^{-1}} = \{(1,1), (2,2), (3,1), (3,2)\}$$

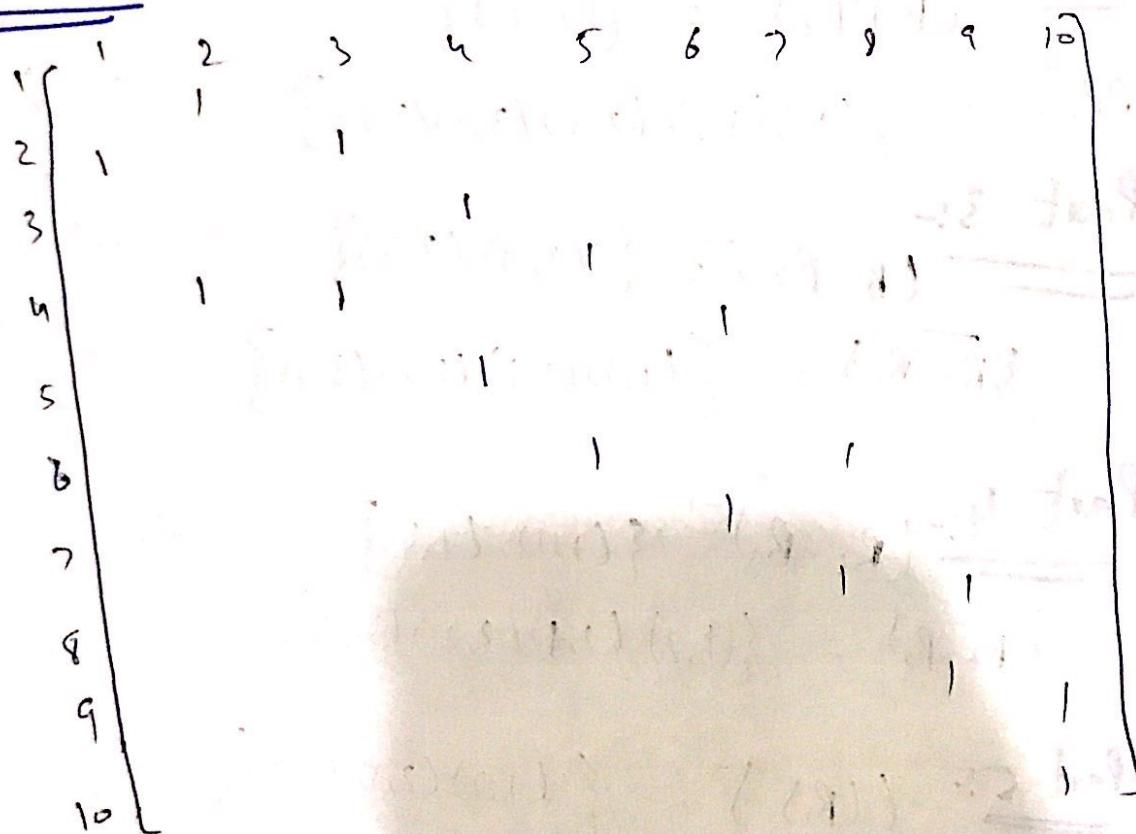
6261

(13)

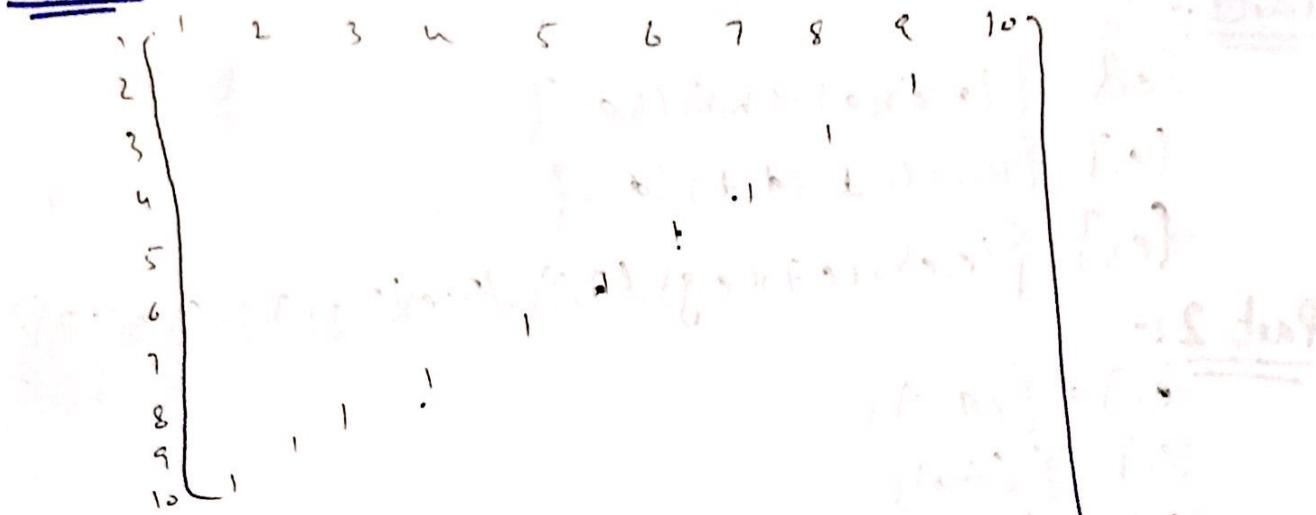
Part 1:



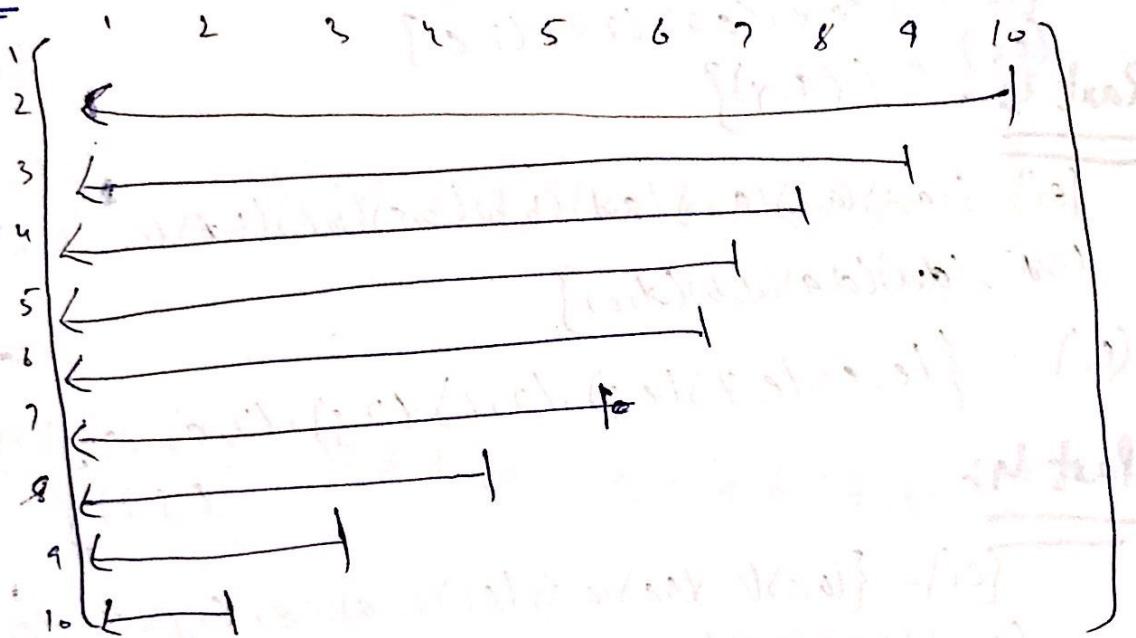
Part 2:



Part 3:



Part 4:



Part 5:



Q 7 :-

(15)

Part 1 :-

$$\{e_1\} = \{(a,a)(a,b)(b,b)(b,a)\}$$

$$\{e_2\} = \{(c,c)(c,d)(d,d)(d,c)\}$$

$$\{e_3\} = \{(e,e)(e,f)(e,g)(f,f)(f,e)(f,g)(g,g)(g,f)(g,e)(g,f)\}$$

Part 2 :-

$$\{e_1\} = \{(a,a)\}$$

$$\{e_2\} = \{(b,b)\}$$

$$\{e_3\} = \{(c,c)(c,d)(d,d)(d,c)\}$$

$$\{e_4\} = \{(e,e)(e,f)(f,f)(f,e)\}$$

$$\{e_5\} = \{(g,g)\}$$

$$\{e_1\} = \{(a,a)(a,b)(a,c)(a,d)(b,b)(b,c)(b,d)(c,c)(c,d)(c,a)(c,b)(c,f)(c,g)(d,d)(d,a)(d,b)(d,c)\}$$

$$\{e_2\} = \{(e,e)(e,f)(e,g)(f,f)(f,g)(g,g)(g,f)(g,e)\}$$

Part 3 :-

$$\{g,g\}$$

$$\{e_1\} = \{(a,a)(a,c)(a,e)(a,g)(c,c)(c,a)(c,e)(c,g)(e,e)(e,g)\}$$

$$(e,c)(e,g)(g,g)(g,c)(g,e)\}$$

$$\{e_2\} = \{(b,b)(b,d)(d,d)(d,b)\}$$

$$\{e_3\} = \{(f,f)\}$$

Part 4 :-

$$\{e_1\} = \{(a,a)\}$$

$$\{e_2\} = \{(b,b)\}$$

$$\{e_3\} = \{(c,c)(c,d)(c,e)(c,f)(c,g)(d,d)(d,f)(d,g)(e,e)(e,f)(e,g)(f,f)(f,g)(g,g)(g,f)(g,e)\}$$

$$(d,f)(d,g)(e,e)(e,f)(e,g)(f,f)(f,g)(g,g)(g,f)(g,e)\}$$

$$(f,f)(f,g)(g,g)(g,f)(g,d)(g,e)(g,f)$$

(Q8:-

(16)

Part 1:-

$$R = \{(a,a)(b,b)(c,c)(d,d)(e,e)(a,c)(a,b)(a,i)(a,d)(e,b)(e,c)(e,d)\}$$

Part 2:-

$$R = \{(a,a)(b,b)(c,c)(d,d)(e,e)(a,c)(b,c)(c,d)(e,e)(b,d)(b,e)\\(c,d)(e,e)\}$$

Part 3:-

$$R = \{(a,a)(b,b)(c,c)(d,d)(a,c)(b,c)(a,d)(b,d)(c,d)\}$$

Part 4:-

$$R = \{(a,a)(b,b)(c,c)(d,d)(a,b)(a,c)(a,d)(b,d)(c,d)\}$$

Part 5:-

$$R = \{(a,a),(b,b)(c,c)(d,d)(a,b)(a,d)(a,d)(b,c)(b,d)\}$$