**NOTE:** Read this please

1 - Take a print of this assignment (If there is no nearby printer then write it again in the same format along with the spaces provided for answer.
2 - some questions may need you to do some rough work. Do it on a blank sheet and write on the top of the sheet 'Rough Work'
3 - Solve it using pen
4 - Take snaps
5 - Make a pdf
6 - Submit

**Q1:** Use the principle of resolution to show that the hypothesis "Chohan works hard", "If Chohan works hard then he is a dull boy", "if Chohan is a dull boy, he will not get a job" imply the conclusion "Chohan will not get the job". **(Marks 3)**

**Solution:**

P1: $P$  
P2: $P \to q$  
P3: $q \to \neg r$  
C: $\neg r$

C1: $P$  
C2: $\neg P \lor q$  
C3: $\neg q \lor \neg r$  
C4: $r$  
C5: $q$  
C6: $\neg r$  
C7: $\square$

From $C_1$ and $C_2$  
From $C_3$ and $C_5$  
From $C_4$ and $C_6$

**Q2:** Write the negation of the following statements in English using the logical equivalence of $\neg \forall x\, P(x) = \exists x \neg P(x)$ and $\neg \exists x\, P(x) = \forall x \neg P(x)$. No credit will be given if you didn't use these logical equivalences. **(6 marks)**

a. $\forall x \forall y\, (P(x,y) \to \neg Q(x,y))$

Solution:

$$\exists x \exists y\, (P(x,y) \land Q(x,y))$$

b. $\exists x \forall y\, (P(x,y) \lor \neg Q(x,y))$

Solution:

$$\forall x \exists y\, (\neg P(x,y) \land Q(x,y))$$

Note: When you are done with simplification of the quantifiers then also use the equivalences of $P \to Q = \neg P \lor Q$ and Demorgan law to simplify further your answer. I will deduct marks if you ignore this.

Q3: Write a logical expression corresponding to the following using only the predicates, logical conjunctions, disjunction and nothing else. Assume the domain of x,y = {1,2,3}  **(Marks 9)**

$\sim \forall x \, \forall y \, P(x,y) = \exists x \, \neg \forall y \, P(x,y) = \exists x \, \exists y \, \neg P(x,y)$

$\Rightarrow [\neg P(1,1) \lor \neg P(1,2) \lor \neg P(1,3) \lor [\neg P(2,1) \lor \neg P(2,2) \lor \neg P(2,3)] \lor [\neg P(3,1) \lor \neg P(3,3)]]$

$\exists x \, \neg \forall y \, P(x,y) = \exists x \, \exists y \, \neg P(x,y)$

$\Rightarrow [\neg P(1,1) \, \neg P(1,2) \lor \neg P(1,3) \, \exists \lor [\neg P(2,1) \lor \neg P(2,2) \lor \neg P(2,3)] \lor [\neg P(3,1) \lor \neg P(3,2) \lor \neg P(3,3)]$

$\forall x \, \exists y \, \neg P(x,y) =$

$\Rightarrow [\neg P(1,1) \lor \neg P(1,2) \lor \neg P(1,3)] \cap [\neg P(2,1) \lor \neg P(2,2) \lor \neg P(2,3)] \cap [\neg P(3,1) \lor \neg P(3,2) \lor \neg P(3,3)]$

Q4: The following question relates to the inhabitants of the island of knights and knaves created by Smullyan. **The knights only speak the truth when they are happy. The knaves always speak lie regardless they are happy or sad.** You encounter two people A and B. Determine if possible what A and B are if they address you in the way. If you cannot determine what these two people are, can you draw any conclusions?  **(Marks 20)**

A says "The two of us are both knights" and
B says "A is a knave"

Solution:

P = A is a knight  i)  $P \land Q$          $\sim P$ = A is a knave          $P = T \quad \neg P = F$
Q = B is a knight  ii)  $\neg P$         $\sim Q$ = B is a knave          $Q = T \quad \neg Q = F$

Scenario 1: Assume all knights to be happy

CASE 1: _Knight_, _Knight_
1) $P \land Q = T$             $T \cap T = T$
2) $\neg P = T$
                               $\neg P \neq T$  Hence, Case does not hold.

CASE 2: _Knight_ , _Knave_
1) $P \land Q = T$
2) $\neg P = F$               $T \land F \neq T$
                               Hence, the case doesn't hold

CASE 3: _Knave_, _Knight_
1) $P \land Q = F$            $F \land T = F$
2) $\neg P = T$
                               $T = T$  Hence, it holds.

CASE 4: _Knave_, _Knave_
1) $P \land Q = F$           $F \land F = F$
2) $\neg P = F$
                               $T \neq F$  Hence, case does not hold

Conclusion: A is _Knave_, B is _Knight_

$$p = T$$
$$\neg p = F$$

$$v = T$$
$$\neg v = F$$

Scenario 2: Assume all knights to be sad (means the knights will speak lies now)

CASE 1: <u>Knight</u> , <u>Knight</u>
1) $p \wedge v = F$       $T \wedge \neg \top \neq F$   Hence, Case does not hold
2) $\neg p = F$

CASE 2: <u>Knight</u> , <u>Knave</u>
1) $p \wedge v = F$       $T \wedge F = F$
2) $\neg p = F$           $F = F$     Hence holds

CASE 3: <u>Knave</u> , <u>Knight</u>
1) $p \wedge v = F$       $F \wedge T = F$
2) $\neg p = F$           $T \neq F$     Hence, does not hold.

CASE 4: <u>Knave</u> , <u>knave</u>
1) $p \wedge v = F$       $F \wedge F = F$
2) $\neg p = F$           $T \neq F$     Hence, does not hold

Conclusion: A is <u>Knight</u> , B is <u>knave</u>

Q5: Assume that the statement "if it is sunny day then I will not go to beach" is in contrapositive form. Make the following forms of this statement using English sentences (**3 marks**)

Converse: If it is not a sunny day than I will go to the beach

Contrapositive: If I will go to the beach then it is not a sunny day.

Inverse: If i will not go to the beach then it is a sunny day.

# Rough Work

**1/** P = Chohan works hard

  ¬P = Chohan does not work hard

  Q = Chohan is a dull boy.

  ¬Q = Chohan is not a dull boy.

  r = Chohan will get a job

  ¬r = Chohan will not get a job.

**2/ a)** $\forall x \forall y (P(x,y) \rightarrow \neg Q(x,y))$

$\neg \forall x \forall y (P(x,y) \rightarrow \neg Q(x,y))$

$\exists x \exists y \neg (\neg P(x,y) \lor \neg Q(x,y))$

$\exists x \exists y (P(x,y) \land Q(x,y))$

**b)** $\exists x \forall y (P(x,y) \lor \neg Q(x,y))$

$\neg \exists x \forall y (P(x,y) \lor \neg Q(x,y))$

$\forall x \exists y \neg (P(x,y) \lor \neg Q(x,y))$

$\forall x \exists y (\neg P(x,y) \land Q(x,y))$

## 3//

$$= \neg \forall x \, \forall y \, P(x,y)$$

$$\exists x \, \neg \forall y \, P(x,y)$$

$$\exists x \, \exists y \, \neg P(x,y)$$

• $\exists x \left[ \neg P(x,1) \lor \neg P(x,2) \lor \neg P(x,3) \right]$

$\left[ \neg P(1,1) \lor \neg P(1,2) \lor \neg P(1,3) \right] \lor \left[ \neg P(2,1) \lor \neg P(2,2) \lor \neg P(2,3) \right] \lor$

$\left[ \neg P(3,1) \lor \neg P(3,2) \lor \neg P(3,3) \right]$

c) $\forall x \left[ \neg P(x,1) \lor \neg P(x,2) \lor \neg P(x,3) \right]$

$\left[ \neg P(1,1) \lor \neg P(1,2) \lor \neg P(1,3) \right] \land \left[ \neg P(2,1) \lor \neg P(2,2) \right.$

$\left. \lor \neg P(2,3) \right] \land \left[ \neg P(3,1) \lor \neg P(3,2) \lor \neg P(3,3) \right]$

## 4//

p = It is a sunny day

q = I will go to the beach

converse = $\neg p \to q$ ⇒

⇒ If it is not a sunny day then I will go to the beach.

contrapositive = $q \to \neg p$

⇒ If I will go to the beach than it is not a sunny day.

Inverse = $\neg q \to p$

⇒ I will not go to the beach than it is a sunny day.