(1)

Assignment # 4

a) $(z, =)$

Solution:-

i) let a is an integer from z $(a, =)$ then $(a, a) \Rightarrow (a = a)$

let $a = 2$

$(2 = 2)$

The following is reflexive.

ii) let $(a, =)$

$\forall a \ (a,a) \in Z \wedge (a,a) \in Z \rightarrow a = a \in Z$

let $a = 2$

$(2, 2,) \wedge (2, 2) \rightarrow 2 = 2$

The following is Anti-symetric.

iii) $\forall a \ (a,a) \in z \wedge (a,a) \in_z \rightarrow (a,a) \in z$

let $a = 3$

$(3,3) \wedge (3,3) \rightarrow (3,3)$

This is Transitive.

Hence, its POSET.

②

b) $(Z, \neq)$

Solution:-

i) $\forall a$ let a is integer from Z

$(a, \neq)$

So,

it mean there is no tupple of (a,a) in set

Hence not Reflexive.

This is not a POSET.

c) $(Z, \geq)$

Solution:-

let a is integer from Z

i) $(a, \geq) \Rightarrow$ Condition

$(a \geq a)$ so it Reflexive

$(2 \geq 2)$ True.

ii) For Anti-Symmetric

$\{(a,b), \geq\}$

let a, b are integer from Z

$\forall a,b \ (a,b) \in Z \land (b,a) \in Z \rightarrow a=b$

So, $(a \geq b) \land (b \geq a) \rightarrow a = b$

$T \land T \rightarrow T$

Hence Anti-Symmetric

③

iii) For Transitive:-

let a, b, c integer from Z
$\forall a, b, c \quad (a, b) \in Z \land (b, c) \in Z \rightarrow (a, c)$
$(a \geq b) \land (b \geq c) \rightarrow (a \geq c)$
$T \land T \rightarrow T$

Hence POSET.

d) $(Z, \leq)$

Solution:-

i) For Reflexive

let a is integer from Z.
$(a, \leq)$
So, $(a \leq a)$
Hence Reflexive

ii) For Anti-Symmetric

$\forall a, b \quad (a, b) \in Z \land (b, a) \in Z \rightarrow a = b$

Condition :- $(a \leq b) \land (b \leq a) \rightarrow a \leq b \,/\, a = b$
By-Default for anti-symmetric

iii) For Transitive

$\forall a, b, c \quad (a, b) \in Z \land (b, a) \in Z \rightarrow (a, c) \in Z$
$((a, b, c), \leq)$
$(a \leq b) \land (b \leq c) \rightarrow (a \leq c)$ | $T \land T \rightarrow T$
Hence Transitive

④

e) (z, /)

Solution:-

i) For Reflexive:-

let a is integer from z.

(a, /)

$\forall a$ (a,a) is (a /a)

let a = 4, (4/4) is Reflexive

ii) For Anti-symmetric:-

$\forall a,b$ (a / b) $\in$ z $\wedge$ (b/a) $\in$ z $\rightarrow$ a / b $\in$ z

let a = 2 (2,/4) $\wedge$ (4/2) $\rightarrow$ a / b
b = 4

T $\wedge$ F $\rightarrow$

By Default Anti-symmetric.

iii) For Transitive:

$\forall a,b,c$ (a / b) $\in$ z $\wedge$ (b / c) $\in$ z a/c $\in$ z

let a = 2
b = 4 (2 / 4) $\wedge$ (4 / 6) $\rightarrow$ (2 / 6)
c = 6
Hence Transitive

This is POSET.

Q2:-) a) a is not shorter then b.

Solution:

i) a is not shorter then b

it can be a = b,

So, (a = b) R is Reflexive

5

ii) For Anti-Symmetric:-

$\forall a,b \ (a,b) \in (S) \wedge (b,a) \in S \rightarrow a=b \in S$

So, Example a is equal to b in height

then $T \wedge T \rightarrow T$ Anti-Symmetric.

if Example a is greater then b

$T \wedge F \rightarrow F$

still Anti-Symmetric.

iii) For Transitive:-

$\forall a,b,c \ (a,b) \in S \wedge (b,c) \in S \rightarrow (a,c) \in S$

if a is greater then b and b is greater then c, Hence defenetelly a is greater then c.

Transitive ∴ its POSET

b) a weights more than b

Solution:-

i) If a is a person who is heavier then b, if b is a and ~~no~~ no one is heavier then its self.

so not reflexive

Hence, not POSET.

c) a is a brother of b.

Solution:-

i) For reflexive, if a is brother of b, if b is 'a', so, No one is a brother of itself.

not reflexive

Hence, not POSET.

d) Solution:

i) For Reflexive:

a is person, b is also a person so it has a common friend.

not Reflexive

e) a and b are enemies

Solution:-

Not Reflexive because no one is enemy of it self.

not POSET.

Q3:) Solution:

a) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

i) all diagonal are 1.
so, reflexive.

ii) As $M_{21} = 1$ & $M_{12} = 1$ But $2 \neq 1$

$T \wedge T \nrightarrow F$.

So it is not Anti-symetric

b) $\begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

Solution:

i) Diagonal is 1 so Reflexive.

ii) e.g $M_{14} = 0$ & $M_{41} = 1$

$F \wedge T \rightarrow F$

So, Anti-symmetric.

iii) For transitive:

As $M_{21} = 0$ and $M_{13} = 1$ then $M_{23} = 1$

$F \wedge T \nrightarrow T$

not transitive.

Q4) let $S = \{1,2,3,4\}$
to lexicographic order on the usual less then relation.
a) Find all pairs in $S*S$ less then $(2,3)$
b) Draw the Hasse diagram of the Poset $(S*S, \leq)$

<u>Solution:</u>

a) $S*S = \{(1,1),(1,2),(1,3)(1,4)$
$(2,1),(2,2)(2,3)(2,4)$
$(3,1),(3,2)(3,3)(3,4)$
$(4,1)(4,2)(4,3)(4,4)\}$

i) $S*S$ less then $(a<b) \Rightarrow \{(1,2)(1,3)$
$(1,4)(2,3)$
$(2,4)(3,4)\}$

ii) $S*S$ less then $(2,3) \Rightarrow \{(1,2)(1,3)$
$(1,1)(1,4)$
$(2,2)\}$

b) Draw Hass diagram of Poset $(S*S, \leq)$ less then.

<u>Solution:</u>

9

Solution

(4,4)
(4,3)
(4,2)
(4,1)
(3,4)
(3,3)
(3,2)
(3,1)
(2,4)
(2,3)
(2,2)
(2,1)
(1,4)
(1,3)
(1,2)
(1,1)

Q5) Solution

Powerset => $\{ \{\}, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}$
$\{a,c\}, \{a,d\}, \{b,c\}, \{b,c,d\}$
$\{c,d\}$ $\{b,d\}$ $\{a,b,d\}$ $\{a,c,d\}$
$\{a,b,c,d\}$ $\{a,b,c\}$ $\}$

(10)

Relation of subset of Pow(s) x pow(s) =>

=> {({ }, { }), ({a}, {a}), ({a}, {a,b}) ({a}, {a,c})
({a}, {a,d}), ({a}, {a,b,c}), ({a}, {a,b,d})
({a} {a,c,d}), ({a} {a,b,c,d}), ({b} {b})
({b}, {a,b}) ({b} {a,b,c}) ({b} {a,b,d}) ({b}, {a,b,c})
({b}, {a,b,d}), ({b}, {b,c,d}), ({b}, {a,b,c,d})
, ({c}, {c}) ({c} {a,c}) ({c}, {a,c}) ({c} {b,d})
({c} {c,d}) ({c} {a,b,c}) ({c} {a,c,d})
({c} {b,c,d}) ({c} {a,b,c,d}) ({d}, {d})
({d} {a,d}) ({d} {b,d}) ({d} {c,d}) ({d} {a,b,d})
({d}, {a,c,d}), ({d} {b,c,d}) ({d}, {a,b,c,d})

P.T.O

Hasse Diagram: