Discrete Structures

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Q1// $x \leq 4$ $P(x)$

a) $P(0)$
$0 \leq 4$
True

b) $P(4)$
$4 \leq 4$
True

c) $P(6)$
$6 \leq 4$
False

Q2// $P(x)$ = The word x contains the letter a.

P(orange) = Word orange contains letter a.
= True

P(Lemon) = False

P(true) = False

P(False) = True

Q3// $Q(x,y)$ = x is the capital of y?

a) Q(Denver, colorado)

Denver is the capital of colorado = True

b) Q(Detroit, Michigan)

Detroit is the capital of michigan. = True False

c) Q(Massachusetts, Boston)

Massachusetts is the capital of Boston = False

d) Q(New York, New York)

Newyork is the capital of Newyork = False

Q4// $x := 1 \quad ;) \quad x > 1$

i) $x = 0$

$x \not> 1$

$= 0$

ii) $x = 1$

$1 \not> 1$

$= 0$

iii) $x = 2$

$2 > 1$

1 True

Q5// P(x) = x spends more than five hours every weekend in class.

where x consist of all students.

a) $\exists x\, P(x)$

There exist a student who spends more than five hours every weekend in class.

b) $\forall x\, P(x)$

All the students spends more than five hours every weekend in class.

c) $\exists x \neg P(x)$

There exist a student who does not spend more than five hours every weekenday in class.

d) $\forall x \neg P(x)$

All the students does not spend more them five hours every week day in class.

Q6// $N(x)$ = x has visited North Dakota
where x are students in your school

a) $\exists x\, N(x)$

There exist a student in your school who has visited North Dakota.

b) $\forall x\, N(x)$

All the students in your school has visited North Dakota.

c) $\neg \exists x\, N(x)$

All the students in your school has visited North Dakota

d) $\exists x \neg N(x)$

Their exist a student in your school who has not visited north Dakota.

e) $\neg \forall x\, N(x)$

Their exist a student in your school who has visited North Dakota.

f) $\forall x \neg N(x)$

All the students in your school has not visited North Dakota.

Q7// $C(x)$ = x is a comedian

$F(x)$ = x is funny

where x is all People

a) $\forall x (C(x) \rightarrow F(x))$

$\forall x C(x) \rightarrow \forall x F(x)$

If everyone is a comedian then everyone is funny.

b) $\forall x (C(x) \wedge F(x))$

$\forall x (I(x) \wedge \forall F(x)$

All People are comedian and all are funny.

c) $\exists x (C(x) \rightarrow F(x))$

$\exists x C(x) \rightarrow \exists x F(x)$

If there exist a comedian then there is a Person who is funny.

d) $\exists x (C(x) \wedge F(x))$

$\exists x C(x) \wedge \exists x F(x)$

There exist a comedian and their exists a person who is funny.

② 8// $R(x)$ = x is a Rabbit

$H(x)$ = x hops

Where x all animals.

a) $\forall x (R(x) \rightarrow H(x))$

$\forall x\, R(x) \rightarrow \forall x\, H(x)$

If all rabbits then all rabbits hop.

b) $\forall x (R(x) \wedge H(x))$

$\forall x\, R(x) \wedge \forall x (H(x))$

All animals are rabbits and all animals hops.

c) $\exists x (R(x) \rightarrow H(x))$

$\exists x\, R(x) \rightarrow \exists x H(x)$

If there exist a Rabbit then their exists an animal which hops.

d) $\exists x (R(x) \wedge H(x))$

$\exists x\, R(x) \wedge \exists x H(x)$

There exist a Rabbit and their exists a Rabbit who hops.

Q9// $P(x)$ = x can speak Russian

$J(x)$ = x knows the computer language C++.

where x is all students all your school.

a) $\exists x (P(x) \wedge Q(x))$

b) $\exists x (P(x) \wedge \neg Q(x))$

c) $\forall x (P(x) \vee Q(x))$

d) $\neg \exists x (P(x) \vee Q(x))$

Q10// $C(x)$ = x has a cat

$D(x)$ = x has a dog

$F(x)$ = x has a Ferret

where x are all students in your class.

a) $\exists x (C(x) \wedge D(x) \wedge F(x))$

b) $\forall x (C(x) \vee D(x) \vee F(x))$

c) $\exists x (C(x) \wedge \neg D(x) \wedge F(x))$

d) $\neg \exists x (C(x) \wedge D(x) \wedge F(x))$

e) $(\exists x C(x)) \wedge (\exists x (Dx)) \wedge (\exists x F(x))$

Q11

$P(x) = x = x^2$

x are all integers

a) $P(0)$

$0 = (0)^2$

$0 = 0$ True

b) $P(1)$

$1 = 1^2$

$1 = 1$ True

c) $P(2)$

$2 = 2^2$

$2 \neq 4$ False

d) $P(-1)$ $-1 = (-1)^x$

$-1 \neq 1$ False

e) $\exists x P(x)$

$P(1) = 1 = 1^2$

$1 = 1$ True (one True)

f) $\forall x (Px)$

$-1 = (-1)^x$

$-1 \neq 1$

False, as one is false

Q12

$Q(x) = x+1 > 2x$

where x are integers

a) $Q(0)$

$0+1 > 2(0)$

$1 > 0$ True

b) $Q(-1)$

$-1+1 > 2(-1)$

$0 > -2$ True

c) $Q(1)$ $1+1 > 2(1)$

$2 > 2$ False

d) $\exists x \neg Q(x)$

$Q(x) = x+1 > 2x$

$\neg Q(x) = x+1 \leq 2x$

$Q(3) = 3+1 \leq 2(3)$

$= 4 \leq 6$ Hence, True

d) $\exists x\ Q(x)$

$Q(0) = 0+1 > 2(0)$

$1 > 0$

True

e) $\forall x\ Q(x)$

$Q(1)$ $1+1 > 2(1)$

$2 > 2$ Hence, False

f) $\exists x \neg Q(x)$

$Q(x) = x+1 > 2x$

$\neg Q(x) = x+1 \leq 2x$

$Q(3) = 3+1 \leq 2(3)$

$= 4 \leq 6$ True

g) $\forall x \neg Q(x)$

$Q(x) = x+1 > 2x$

$\neg Q(x) = x+1 \leq 2x$

$Q(0) = 0+1 \leq 2(0)$

$1 \leq 0$ Hence, False

Q13//

Where n are integers

a) $\forall_n (n+1 > n)$

$P(0) = 0+1 > 0 \Rightarrow 1 > 0$ True

$P(1) = 1+1 > 1 \Rightarrow 2 > 1$ True

$P(-1) = -1+1 > -1 \Rightarrow 0 > -1$ True

Hence, True

b) $\exists_n (2n = 3n)$

$P(0) = 2(0) = 3(0)$

$0 = 0$

Hence, True

c) $\exists_n (n = -n)$

$P(0) = 0 \Rightarrow -0$

$0 = 0$

Hence, True.

d) $\forall_n (n^2 \geq n)$

$P(0)$ $0 \geq 0$ True

$P(-1)$ $(-1)^2 \geq -1$ $1 \geq -1$ True

$P(1)$ $1 \geq 1$ True

Hence True.

Q14// where x all real numbers

a) $\exists x\,(x^3 = -1)$

$P(-1)\ (-1)^3 = -1$

$-1 = 1 \Rightarrow$ True

b) $\exists x\,(x^4 < x^2)$

$P(0.5)\quad 0.5^4 < 0.5^2$

$0.0625 < 0.25$ True

c) $\forall x\,((-x)^2 = x^2)$

$P(0) = \quad 0^2 = 0^2$

$0 = 0$ True

$P(1) = (-1)^2 = -1^2$

$1 = 1$ True

d) $\forall x\,(2x > x)$

$P(0) = 2(0) > 0$

$0 > 0$

False

Q15// where n is integer

a) $\forall n (n^2 \geq 0)$

$P(0)$ $0 \geq 0$ True

$P(1)$ $1 \geq 0$ True

$P(-1)$ $1 \geq 0$ True

So, True

b) $\exists n (n^2 = 2)$

$2^2 = 2$

$4 \neq 2$, False

c) $\forall n (n^2 \geq n)$

$P(2)$ $2^2 \geq 2$

$4 \geq 2$ True

$P(-3)$ $-3^2 \geq -3$

$9 \geq -3$ True

d) $\exists n (n^2 < 0)$

$P(0)$ $0 < 0$

False

$P(-1)$ $1 < 0$

False

So Hence, False

Q16// where x are real numbers

a) $\exists x (x^2 = 2)$

$P(\sqrt{2}) = (\sqrt{2})^2 = 2$

$2 = 2$ True.

b) $\exists x (x^2 = -1)$

$P(-1) = -1^2 = -1$

$1 = -1$

False

c) $P(\sqrt{-1}) = (\sqrt{-1})^2 = -1$

$1 \neq -1$ False

c) $\forall x (x^2 + 2 \geq 1)$

$P(0) = 0 + 2 \geq 1$

$2 \geq 1$

True

d) $\forall x (x^2 \neq x)$

$P(1) \quad 1^2 = 1$

$1 = 1$

False

Q17 //

where $x = \{0, 1, 2, 3, 4\}$

a) $\exists_x P(x)$

$P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$

b) $\forall x \, P(x)$

$P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

c) $\exists_x \neg P(x)$

$\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d) $\forall x \neg P(x)$

$\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e) $\neg \exists x \, P(x)$

$\neg (P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$

$\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

f) $\neg \forall x (P(x))$

$\neg (P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

$\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

Q18// where $x = \{-2, -1, 0, 1, 2\}$

a) $\exists_x P(x)$

$P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$

b) $\forall_x P(x)$

$P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$

c) $\exists_x \neg P(x)$

$\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$

d) $\forall_x \neg P(x)$

$\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$

e) $\neg \exists_x P(x)$

$\neg(P(2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$

$\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge P(2)$

f) $\neg \forall_x P(x)$

$\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

$\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$

Q19// $x = \{1, 2, 3, 4, 5\}$

a) $\exists_x P(x)$

$P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b) $\forall_x P(x)$

$P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c) $\neg \exists_x P(x)$

$\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

$\neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4) \wedge \neg P(5)$

d) $\neg \forall_x P(x)$

$\neg(P(1) \wedge P(2) \wedge P(3) P(4) \wedge P(5))$

$\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5)$

e) $\forall_x((x \neq 3) \rightarrow P(x)) \vee \exists x \neg P(x)$

$(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$

Q2.

$x = \{-5, -3, -1, 1, 3, 5\}$

a) $\exists_x P(x)$

$P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

b) $\forall_x P(x)$

$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

c) $\forall_x((x \neq 1) \rightarrow P(x))$

$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

d) $\exists_x((x \geq 0) \wedge P(x))$

$P(1) \vee P(3) \vee P(5)$

e) $\exists_x(\neg P(x)) \wedge \forall_x((x < 0) \rightarrow P(x))$

$(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-1))$