Pg# 22:- Enercise Q1 - Q10.

(i) $\boxed{e \to a}$ or $\boxed{\neg a \to \neg e}$

(ii) ~~If~~ m, then (e or p) $\to$ $\boxed{m \to (e \vee p)}$

(iii) g only if (r and not m and not b)

$\Rightarrow \boxed{g \to (r \wedge \neg m \wedge \neg b)}$ //

(iv) If w, then d or s $\to \boxed{w \to (d \vee s)}$ //

(v) $\boxed{c \to (a \wedge (b \vee p) \wedge r)}$ //

(vi) $\boxed{u \to (b32 \wedge g1 \wedge r1 \wedge h16) \vee (b64 \wedge g2 \wedge r2 \wedge h32)}$

(vii) a) $q \to p$

b) $q \wedge \neg p$

c) $q \to p$

d) $\neg q \to \neg p$

(viii) a) $r \wedge \neg p$

b) $(p \wedge r) \to q$

c) $\neg r \to \neg q$

d) $(\neg p \wedge r) \to q$

(ix) $p \leftrightarrow q, q \to r, \neg r \vee s, \neg p \to s, \neg s$

$\neg s = T \to s = F$

$p = T$

$q = T$

which implies $r = T \to \neg r = F$

Hence (iii) eq is false.

Result:-

Inconsistent. //

(x) $p \to \neg q \to$ (i)

$q \to r \to$ (ii)

$\neg r \to \neg p \to$ (iii)

first case:- Let $q = F, \neg q = T \xrightarrow{(i)} p = T$

$\neg p = F \xrightarrow{(iii)} \neg r = F \to r = T$

(ii) $q = F, r = T$

True.

second Case:-

Let $q = T, \neg q = F$

(ii) $r = T, \neg r = F$

(i) $p = F, \neg p = T$

(iii) $\neg r = F, \neg p = T$

Result:-

Consistent. //

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Sec:- BS(CS) 3-A

Pg # 34:- Q1 - Q10

1. Use truth tables to verify equivalences:-

a) $p \wedge T = p$

p	T	$p \wedge T$
T	T	T
F	T	F

b) $p \vee F = p$

p	F	$p \vee F$
T	F	T
F	F	F

c) $p \wedge F = F$

p	F	$p \wedge F$
T	F	F
F	F	F

d) $p \vee T = T$

p	T	$p \vee T$
T	T	T
F	T	T

e) $p \vee p = p$

p	$p \vee p$
T	T
F	F

f)

p	$p \wedge p$	$p \vee T$
T	T	T
F	F	T

Result:

a) $p \vee T = p$

b) $p \vee F = p$

c) $p \wedge F = F$

d) $p \vee T = T$

e) $p \vee p = p$

f) $p \wedge p = p$

(ii)

p	$\neg p$	$\neg(\neg p)$
T	F	T
F	T	F

Result:- The two propositions are logically equivalents only if their truth values agree for all possible combination as shown in the highlighted parts.

(iii) a)

p	q	$p \vee q$		$q \vee p$
T	T	T		T
T	F	T	$\Leftrightarrow$	T
F	T	T		T
F	F	F		F

logically equivalent

b)

p	q	$p \wedge q$		$q \wedge p$
T	T	T		T
T	F	F	$\Leftrightarrow$	F
F	T	F		F
F	F	F		F

logically equivalent

Result:-
The two propositions are logically equivalents only if their truth table values agree for all possible combination

---

Q#4 a) $(p \vee q) \vee r$ & $p \vee (q \vee r)$

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1 a)

p	q	r	$p \vee q$	$q \vee r$	$(p \vee q) \vee r$	$p \vee$
T	T	T	T	T	T	
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	T	T
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

b) $(p \wedge q) \wedge r$ & $p \wedge (q \wedge r)$

p	q	r	$p \wedge q$	$q \wedge r$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Result:

Verified.

Q#5:-

p	q	r	$q\vee r$	$p\wedge q$	$p\wedge r$	$p\wedge(q\vee r)$	$(p\wedge q)\vee$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Result:- $p\wedge(q\vee r)\equiv(p\wedge q)\vee(p\wedge r)$

Q#6:-

p	q	$\neg p$	$\neg q$	$(p\wedge q)$	$\neg(p\wedge q)$	$\neg p\vee\neg q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	F	T	T	F	T	T
F	T	T	F	F	T	T

Result:- Morgan law is verified.

is not rich or not happy

(b) Carlos will not bicycle & not run tomorrow.

(c) Mei does not walk & does not take the bus to class.

(d) Ibrahim is not smart, or not hard working.

QH8:- (a) Kwame will not take a job in industry & not go to graduate school.

(b) Yoshiko does not know Java or doesn't know Calculus.

(c) James is not young or not strong.

(d) Rita will not move to Oregon & will not move to Washington.

QH9:-

p	q	$p \wedge q$	$(p \wedge q) \to p$
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

p	q	$\neg p$	$p \to q$	$\neg p \to (p \to q)$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

p	q	$p \wedge q$	$p \to q$	$(p \wedge q) \to (p \to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

p	q	$p \to q$	$\neg(p \to q)$	$\neg(p \to q) \to p$
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

p	q	$p \to q$	$\neg(p \to q)$	$\neg q$	$\neg(p \to q) \to \neg q$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	F	T	T

Result: All conditional statements are tautologies.

Q# 10:-

p	q	$p \vee q$	$\neg p$	$\neg p \wedge (p \vee q)$	$[\neg p \wedge (p \vee q)] \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

p	q	r	$p \to q$	$q \to r$	$(p \to q) \wedge (q \to r)$	$p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

p	q	$p \to q$	$p \wedge (p \to q)$	$[p \wedge (p \to q)] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

p	q	r	$p \vee q$	$p \to r$	$q \to r$	$(p \vee q) \wedge (p \to r)$	$(p \vee q) \wedge (p \to r) \wedge (q \to r)$	$[(p \vee q) \wedge (p \to r) \wedge$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	F	T
F	F	T	F	T	T	F	F	T
F	F	F	F	T	T	F	F	T

Result: All conditional statements are tautologies.

Pg#53 Q1 – Q20:-

(I) a) true b) true c) false

(II) a) T b) F c) F d) T

(III) a) T b) F c) F d) F

(IV) a) $x=0$ b) $x=1$ c) $x=1$

(V) a) There exists a student that spends more than five hours every weekday in class.

(b) All students spend more than five hours every weekday in class.

(c) There exists a student that does not spend more than five hours every weekday in class.

(d) All students do not spend more than five hours every weekday in class.

(VI) a) There exists a student in your school who has visited North Dakota.

b) All students in your school have visited North Dakota.

c) There does not exist a student in your school who has visited North Dakota.

d) There exists a student in your school who has not visited North Dakota.

e) Not all students in your school have visited North Dakota.

f) All your students in your school have not visited North Dakota.

P
T
T
T
T
F
F
F
F

(VII) a) All comedians are funny.

b) Every person is a comedian and funny.

c) There exists a person such that, if the person is a comedian, then the person is funny.

d) There exists a person that is a comedian and funny.

(VIII) a) All rabbits hop b) All animals are rabbits and hop.

c) There exists an animal such that, if it is a rabbit, then it hops.

d) There exists an animal that is a rabbit and it hops.

e) Some rabbits hop.

(IX) a) $\exists x(P(x) \wedge Q(x))$ b) $\exists x(P(x) \wedge \neg Q(x))$ c) $\forall x(P(x) \vee Q(x))$

d) $\neg \exists x(P(x) \vee Q(x))$

(X) a) $\exists x(C(x) \wedge D(x) \wedge F(x))$ b) $\forall x(C(x) \vee D(x) \vee F(x))$

c) $\exists x(C(x) \wedge F(x) \wedge \neg(D(x)))$ d) $\neg \exists x(C(x) \wedge D(x) \wedge F(x))$

e) $(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$

(XI) a) T b) T c) F d) F e) T f) F

(XII) a) T b) T c) F d) T e) F f) T g) F

(XIII) a) T b) T c) T d) F

(XIV) a) T b) T c) T d) F

(XV) a) T b) F c) T d) F

(XVI) a) T b) F c) T d) F

(XVII) a) $P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4)$

b) $P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4)$

c) $\neg P(0) \vee \neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4)$

d) $\neg P(0) \wedge \neg P(1) \wedge \neg P(2) \wedge \neg P(3) \wedge \neg P(4)$

e) $\neg(P(0) \vee P(1) \vee P(2) \vee P(3) \vee P(4))$

f) $\neg(P(0) \wedge P(1) \wedge P(2) \wedge P(3) \wedge P(4))$

(XVIII) a) $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$

b) $P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$

c) $\neg P(-2) \vee \neg P(-1) \vee \neg P(0) \vee \neg P(1) \vee \neg P(2)$

d) $\neg P(-2) \wedge \neg P(-1) \wedge \neg P(0) \wedge \neg P(1) \wedge \neg P(2)$

e) $\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$

f) $\neg(P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

(XIX) a) $P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5)$

b) $P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5)$

c) $\neg(P(1) \vee P(2) \vee P(3) \vee P(4) \vee P(5))$

d) $\neg(P(1) \wedge P(2) \wedge P(3) \wedge P(4) \wedge P(5))$

e) $(P(1) \wedge P(2) \wedge P(4) \wedge P(5)) \vee (\neg P(1) \vee \neg P(2) \vee \neg P(3) \vee \neg P(4) \vee \neg P(5))$

(XX) a) $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

b) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

c) $P(-5) \wedge P(-3) \wedge P(-1) P(3) \wedge P(5)$

d) $P(1) \vee P(3) \vee P(5)$

e) $\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)$

$\rightarrow P(-5) \wedge P(-3) \wedge P(-1)$

e) $(\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)) \wedge (P(-5) \wedge P(-3) \wedge P(-1))$

64 Exercise Q1 - Q20 (Any 20 Questions)

Q1:- a) for every real number x there exists a real no. y such that x is less than y.

b) for every real number x and y, if x and y are positive, then their product is positive.

c) for every real number x and real number y, there exists a real number z such that $xy = z$.

Q2:- a) There exists a real number x such that for every real number y, the product of x and y is equal to y

b) For every real number x and every real number y, if x is nonnegative and y is negative, than the difference $x - y$ is positive.

c) For every real number x and every real number y, there exists a real number z such that the sum of y and z equals x.

Q3:- a) There is a student in your class who has sent a message to some student in your class.

b) There is a student in your class who has sent a message to every student in your class.

c) Every student in your class has sent a message to atleast one student in your class.

d) There is a student in your class who has been sent a message by every student in class.

e) Every student in your class has been sent a message ~~by~~ from atleast one student in your class.

f) Every student in the class has sent a message to every student in the class.

Q1(?):- a) $\forall x \exists y (A(x,y) \wedge \forall z ($ ...

b) $\exists$ ...

Q#4:- a) There is a student in your class that has taken a CS course.

b) There is a student in your class that has taken all CS courses.

c) All students in your class have taken a CS course.

d) There is a CS course that every student in your class has taken.

e) Every CS course has been taken by some student in class.

f) All students in your class have taken all CS courses.

Q5:- a) Student Sarah Smith has visited the website www.att.com.

b) There is a student in your school that has visited the website www.imbd.org

c) There is a website the José Orez has visited.

d) There is a website that Ashok Puri and Cidy Yoon have both visited.

e) There is a student in your school, beside David Belcher, that has visited all websites that David Belcher visited.

f) There are two different students in your class that have visited the same websites.

Q6:- a) Randy Goldberg is enrolled in class CS252.

b) There exists a student x that is enrolled in class Math695.

c) There exists a class y that Carol Sitea is enrolled in.

d) There exists a student x that is enrolled in both Math222 and CS252.

e) There exist two different students x and y such that if x is enrolled in class z then y is also enrolled in class z.

Q7:- a) Student Abdallah Hussain does not like Japanese.

b) There exists a student who likes Korean and all students like Mexican.

There exists a cuisine that Monique Arsenault or Jay Johnson likes

d) For every two students there exists a cuisine such that if the students are not the same students then they do not both like the same cuisine.

e) There exist two students such that for every cuisine the students both like the cuisine or the students both do not like the cuisine.

f) For every two students there exists a cuisine such that the students both like the cuisine or the students both do not like the cuisine.

Q8:- a) $\exists x \exists y Q(x,y)$, b) $\forall x \forall y \neg Q(x,y)$, c) $\exists x (Q(x, \text{Jeopardy}) \wedge Q(x, \text{Wheel of fortune}))$

d) $\forall y \exists x Q(x,y)$, e) $\exists x \exists z ((x \neq z) \wedge Q(x, \text{Jeopardy}) \wedge Q(z, \text{Jeopardy}))$

Q9:- (a) $\forall x L(x, \text{Jerry})$, b) $\forall x \exists y L(x,y)$, c) $\exists y \forall x L(x,y)$.

d) $\neg \exists x \forall y L(x,y)$, e) $\exists y \neg L(\text{Lydia}, y)$, f) $\exists y \forall x \neg L(x,y)$

g) $\exists x (\forall y L(y,x) \wedge \forall z ((\forall w L(w,z)) \to z = x))$,

h) $\exists y \exists z (L(\text{Lynn}, y) \wedge L(\text{Lynn}, z) \wedge y \neq z \wedge \forall w (L(\text{Lynn}, w) \to (w = y \vee w = z)))$

i) $\forall x L(x,x)$, j) $\exists x \forall y (L(x,y) \leftrightarrow x = y)$

Q10:- a) $\forall x F(x, \text{Fred})$, b) $\forall y F(\text{Evelyn}, y)$, c) $\forall x \exists y F(x,y)$

d) $\neg \exists x \forall y F(x,y)$, e) $\forall y \exists x F(x,y)$, f) $\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

g) $\exists y \exists z (F(\text{Nancy}, y) \wedge F(\text{Nancy}, z) \wedge y \neq z \wedge \forall w (F(\text{Nancy}, w) \to (w = y \vee w = z)))$

h) $\exists y (\forall x F(x,y) \wedge \forall z (\forall w F(w,z)) \to z = y)$, i) $\forall x \neg F(x,x)$,

j) $\exists x \exists y (L(x,y) \wedge \forall z (L(x,z) \to z = y \vee z = x$

Q14:- a) $\exists x A(x, \text{Hindi})$, b) $\forall x \exists y B(x,y)$

c) $\exists x [C(x, \text{Alaska}) \wedge \neg C(x, \text{Hawaii})]$, d) $\forall x \exists y D(x,y)$

e) $\exists x \exists y E(x,y)$, f) $\exists x \exists y (x \neq y \wedge F(x,y) \wedge \forall z (F(x,z) \to (x = z \vee y = z)))$

g) $\forall x \exists y \exists z G(x,y,z)$

Q16:- a) T b) F c) T d) F e) F

Q17:- a) $\forall x \exists y (A(x,y) \wedge \forall z (A(x,z) \to y = z))$

b) $\exists x (B(x) \to \forall y\, C(x,y))$, c) $\forall x \forall y (E(y, \text{edu}) \to D(x,y))$

d) $\exists x \exists z [x \neq z \wedge \forall y F(x,y) \wedge \forall y F(z,y) \wedge \forall w (\forall y F(w,y) \to (w = x \vee w = z))]$.

Q18:- a) $\forall y \exists x A(x,y)$, b) $\forall x \exists y [B(x,y) \to C(x)]$

c) $\forall y \exists x D(x,y) \leftrightarrow \exists z E(z)$, d) $\forall y \forall z \exists x \exists a [x \neq a \to (F(x,y,z) \wedge F(a,y,z))]$, e) $\forall x [H(x) \to \forall y\, G(x,y)] \wedge \neg \exists x [\neg H(x) \wedge \forall y\, G(x,y)]$.

Q19:- a) $\forall x \forall y ((x<0) \wedge (y<0) \to (x+y<0))$

b) $\exists x \exists y ((x>0) \wedge (y>0) \to (x-y \leq 0))$

c) $\forall x \forall y\ (x^2+y^2 \geq (x+y)^2)$

d) $\forall x \forall y\ (|x \cdot y| = |x| \cdot |y|)$

Q20:- a) $\forall x \forall y [(x<0 \wedge y<0) \to xy>0]$

b) $\forall x \forall y [(x>0\ y>0) \to \frac{x+y}{2} > 0]$

c) $\exists x \exists y [x<0 \wedge y<0 \wedge \neg(x-y<0)]$

d) $\forall x \forall y (|x+y| \leq |x|+|y|)$

Q21:- For all x, if x is positive integer, then there are four int- a, b, c, d such that the sum of their square is equal to x.

$\boxed{\forall x ((x>0) \to \exists a \exists b \exists c \exists d\, (a^2+b^2+c^2+d^2 = x))}$

Q22:- There exists an x, that is positive and for all three integers a, b and c, x is not the sum of the sq of 3 int.

$\boxed{\exists x [(x>0) \wedge \forall a \forall b \forall c (x \neq a^2+b^2+c^2)]}$

Q26:- a) F b) T c) F d) F e) T f) T g) T h) F i) F

Q27:- a) T b) T c) T d) T e) T f) F g) F h) T i) F

Q28:- a) T b) F c) T d) F e) T f) F g) T h) F.

i) F j) T