Topic to discuss

Fixed point Method

Numerical Problem

Fixed point Method

Any function in the form of $f(x) = 0$ can be manipulated such that x is on the left hand side of the equation as shown below

$$x = \phi(x)$$

eg:- $x^2 + x - 2 = 0$ can be written as,

$$x = 2 - x^2 \quad \text{or}$$

$$x = x^2 + 2x - 2$$

$$x = \phi(x)$$

If $|\phi'(x)| < 1$ then

equation can be used as next iteration formula.

$$\boxed{x_{i+1} = \phi(x_i)}$$

This method of solution is also known as the method of successive approximations or method of direct substitution.

The algorithm is simple. The iteration process would be terminated when two successive approximations agree within some specified error.

Q:- Locate root of the equation $x^3 - 9x + 1 = 0$ using fixed point method.

Solution: Let $f(x) = x^3 - 9x + 1 = 0$

So,

$$f(0) = 0 - 9\times 0 + 1 = 1$$

$$f(1) = 1 - 9\times 1 + 1 = -7$$

$$f(2) = 8 - 18 + 1 = -9$$

$$f(3) = 3^3 - 9\times 3 + 1 = 1$$

So, root of equation lies between, 0 and 1

Lets assume, $x_0 = 0.5$

Lets reorganise the function as follows:

$$x^3 - 9x + 1 = 0$$

$$x^3 = 9x - 1$$

$$\therefore x = (9x-1)^{1/3}$$

So, $\phi(x) = (9x-1)^{1/3}$

hence, $\phi'(x) = \frac{1}{\cancel{3}} \times \frac{\cancel{9}\ 3}{(9x-1)^{2/3}}$

$$\phi'(x) = \frac{3}{(9x-1)^{2/3}}$$

$$\left|\phi'(x) \text{ at } x_0 = 0.5\right| = \frac{3}{(9x-1)^{2/3}} = \frac{3}{(9\times 0.5-1)^{2/3}} = 1.30$$

$$\phi'(x) \text{ at } x_0 = 0.5 > 1$$

Lets try another form,

$$x^3 - 9x + 1 = 0$$

$$x(x^2-9) + 1 = 0$$

$$x = \frac{-1}{x^2-9} = \frac{1}{9-x^2}$$

So, $$\phi(x) = \frac{1}{9-x^2}$$

So, $$\phi'(x) = \frac{-1 \times -2x}{(9-x^2)^2} = \frac{2x}{(9-x^2)^2}$$

$$\left| \phi'(x) \text{ at } x_0 = 0.5 \right| = \frac{2x}{(9-x^2)^2} = \frac{2 \times 0.5}{(9 - 0.5^2)^2}$$

$$= 0.01306 < 1$$

So,

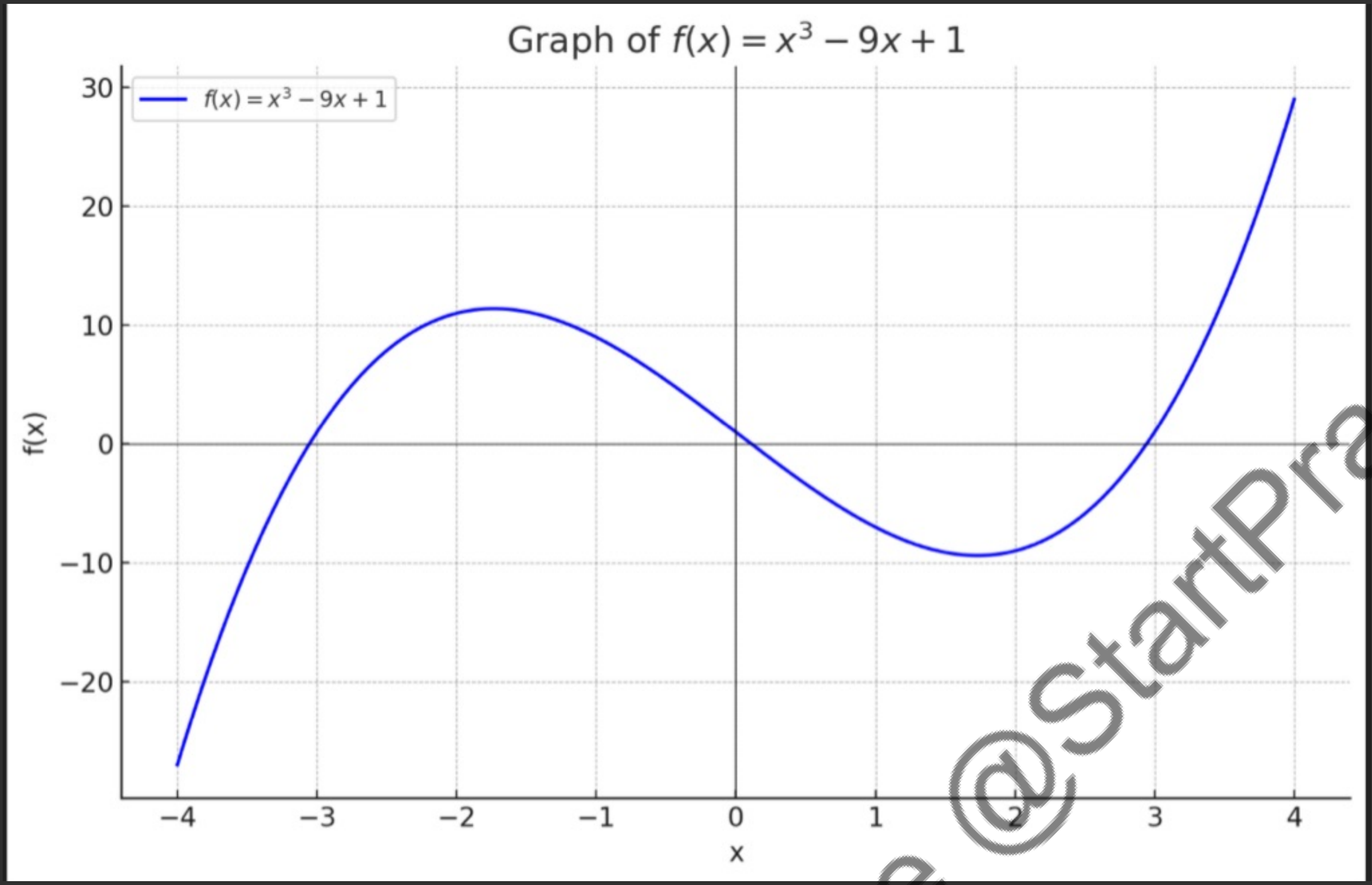
$$x = \frac{1}{9-x^2}$$

$$x_1 = \frac{1}{9-0.5^2} = 0.1142$$

$$x_2 = \frac{1}{9-0.1142^2} = 0.1112$$

$$x_3 = \frac{1}{9-0.1112^2} = 0.11126$$

$$x_4 = \frac{1}{9-0.11126^2} = 0.111264$$



Graph of $f(x) = x^3 - 9x + 1$
$f(x) = x^3 - 9x + 1$
30
20
10
0
−10
−20
f(x)
−4
−3
−2
−1
0
1
2
3
4
x