

Topic to discuss

Part - 2

- Problems on Bisection Method.
- Homework Problem

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Q1: Using Bisection Method obtain root of the equation $e^x - 3x = 0$, correct upto 2 decimal places.

solution:- let $y = f(x) = e^x - 3x = 0$

To find initial root a & b , we have to guess the value of x .

$$\text{So, } f(1) = e^1 - 3 \times 1 = -0.281718$$

$$f(2) = e^2 - 3 \times 2 = 1.389056$$

$$f(1.5) = e^{1.5} - 3 \times 1.5 = -0.018310$$

x	1.5	2	1.75	1.625	
$f(x)$	-0.01831	1.38905	0.50460	0.203419	

1st iteration,

$$a = 1.5 \text{ and } b = 2$$

$$x_0 = \frac{a+b}{2} = \frac{1.5+2}{2} = 1.75$$

$$f(x_0) = f(1.75) = e^{1.75} - 3 \times 1.75 = 0.50460$$

2nd iteration,

$$a = 1.75 \text{ and } b = 1.5$$

$$x_0 = \frac{1.75+1.5}{2} = 1.625$$

$$f(x_0) = f(1.625) = e^{1.625} - 3 \times 1.625 = 0.203419$$

x	1.5	2	1.75	1.625	1.5625	1.53125
$f(x)$	-0.01831	1.38905	0.50460	0.203419	0.0832	0.03020

3rd iteration,

$$a = 1.625 \quad \text{and} \quad b = 1.5$$

$$x_0 = \frac{1.625 + 1.5}{2} = 1.5625$$

$$f(x_0) = f(1.5625) = e^x - 3x = 0.083233$$

4th iteration,

$$a = 1.5625 \quad \text{and} \quad b = 1.5$$

$$x_0 = \frac{1.5625 + 1.5}{2} = 1.53125$$

$$f(x_0) = f(1.53125) = 0.03020$$

x	1.5	1.5625	1.53125	1.515625	1.50781	
$f(x)$	-0.01831	0.0832	0.03020	0.005390	-0.006598	

5th iteration,

$$a = 1.53125 \text{ and } b = 1.5$$

$$x_0 = \frac{1.53125 + 1.5}{2} = 1.515625$$

$$f(x_0) = f(1.515625) = 0.005390$$

6th iteration,

$$a = 1.515625 \text{ and } b = 1.5$$

$$x_0 = \frac{1.515625 + 1.5}{2} = 1.50781$$

$$f(x_0) = e^x - 3x = -0.006598$$

x	1.5	1.5625	1.53125	1.515625	1.50781	1.5117
$f(x)$	-0.01831	0.0832	0.03020	0.005390	-0.066598	-0.0006403

7th iteration,

$$a = 1.50781 \text{ and } b = 1.515625$$

$$x_0 = \frac{1.50781 + 1.515625}{2} = 1.5117$$

$$f(x_0) = f(1.5117) = -0.0006403$$

The root of x lies between,
1.5117 and 1.515625

So, root of x is 1.51 (correct upto
2 decimal places)

Our initial assumption, $a = 1.5$, $b = 2$
 $f(a) = -0.0183$, $f(b) = 1.3890$

iteration	a	b	x_0	$f(x_0)$
1	1.5	2	1.75	0.50460
2	1.75	1.5	1.625	0.20341
3	1.625	1.5	1.5625	0.0832
4	1.5625	1.5	1.53125	0.03020
5	1.53125	1.5	1.515625	0.0053904
6	1.515625	1.5	1.5078125	-0.0065981
7	1.5078125	1.515625	1.511718	-0.0006384

Home work Problem

Q: Find the positive real root of $x \log_{10} x = 1.15$, using the bisection method. (perform 4 iteration)

Solution: let $y = f(x) = x \log_{10} x - 1.15$

$$\text{Now, } f(1) = -1.15$$

$$f(2) = -0.5479$$

$$f(3) = 0.28136$$

So $f(x) = 0$ has at least one root in $(2, 3)$

So, according to bisection Method,

we can take, $a = 2$ and $b = 3$

$$\text{So, } x_0 = \frac{2+3}{2} = 2.5$$

$$f(x_0) = f(2.5) = -0.15514$$

1st iteration,

$$a = 2.5 \quad \text{and} \quad b = 3$$

$$x_0 = \frac{2.5 + 3}{2} = 2.75$$

$$f(2.75) = 2.75 \times \log(2.75) - 1.15 = 0.05816$$

2nd iteration,

$$a = 2.5 \quad \text{and} \quad b = 2.75$$

$$x_0 = \frac{2.5 + 2.75}{2} = 2.625$$

$$f(2.625) = -0.0497$$



3rd iteration,

$$a = 2.625 \quad \text{and} \quad b = 2.75$$

$$x_0 = \frac{2.625 + 2.75}{2} = 2.6875$$

$$f(2.6875) = 2.6875 \times \log 2.6875 - 1.15 = 0.003874$$

4th iteration,

$$a = 2.6875 \quad \text{and} \quad b = 2.625$$

$$x_0 = \frac{2.6875 + 2.625}{2} = 2.65625$$

$$f(2.65625) = -0.02303$$

The positive root of function $x \log x = 1.15$
is lies between 2.625 and 2.65625
which approximates to 2.7
(correct upto 2 significant figures)

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