Topics to Discuss

- Bisection Method
- Numerical Problem
- Home work Problem with solution.

Bisection Method

(Also Known as binary chopping or half-interval method)

If $f(x)$ is real and continuous in the interval $a<x<b$, and $f(a)$ and $f(b)$ are of opposite signs, that is,

$$f(a)\, f(b) < 0$$

then there exists at least one real root in the interval between a and b.

(There may be more than one root in the interval).

Let $x_1 = a$ and $x_2 = b$. Let us also define another point x_0 to be the midpoint between a and b. That is,

$$x_0 = \frac{x_1 + x_2}{2}$$

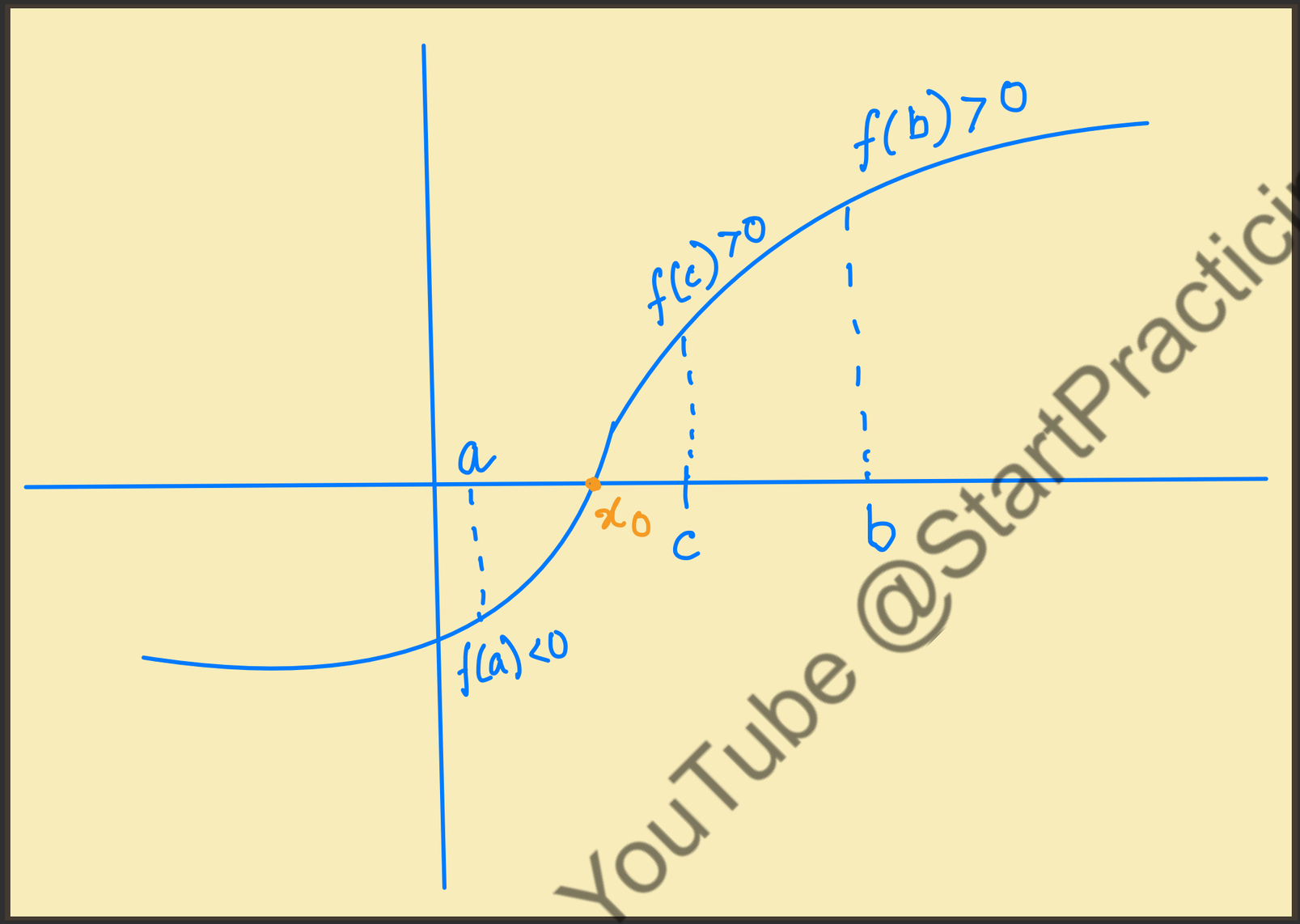
Now, there exists the following three conditions:

1) If $f(x_0) = 0$, we have a root at x_0.

2) If $f(x_0)\, f(x_1) < 0$, there is root between x_0 and x_1.

3) If $f(x_0) f(x_2) < 0$, there is a root between x_0 and x_2.

Illustration of bisection Method



* x_0 is the root of $f(x) = 0$
* a and b is the initial asumptions.
* c is mid point of a & b. and $f(c) > 0$

Q-1 : Find the root of the equation

$$x^3 - x^2 - 1 = 0$$

using bisection method.

Solution :- Let , $y = f(x) = x^3 - x^2 - 1$

To find a and b, we have to first guess x.

$f(1) = 1^3 - 1^2 - 1 = -1 < 0$

$f(0) = 0 - 0 - 1 = -1 < 0$

$f(-1) = (-1)^3 - (-1)^2 - 1 = -3 < 0$

$f(2) = 2^3 - 2^2 - 1 = 3 > 0$

$f(1.5) = 1.5^3 - 1.5^2 - 1 = 0.125$

x	0	1	-1	2	1.5
$f(x)$	-1	-1	-3	3	0.125

x	1	1.5	1.25	1.375			
$f(x)$	−1	0.125	−0.6093	−0.2910			

So, $a = 1$ and $b = 1.5$

So $x_0 = \frac{1+1.5}{2}$

$= 1.25$

$f(x_0) = f(1.25) = 1.25^3 - 1.25^2 - 1 = -0.609375$

1st iteration

$a = 1.25$ and $b = 1.5$

$x_0 = \frac{1.25+1.5}{2} = 1.375$

$f(1.375) = -0.2910$

x	1	1.5	1.25	1.375	1.4375	1.46875	
$f(x)$	−1	0.125	−0.6093	−0.2910	−0.0959	0.0111	

2nd iteration,

$$a = 1.375 \quad \text{and} \quad b = 1.5$$

$$x_0 = \frac{1.375 + 1.5}{2} = 1.4375$$

$$f(1.4375) = -0.0959$$

3rd iteration,

$$a = 1.4375 \quad \text{and} \quad b = 1.5$$

$$x_0 = \frac{1.4375 + 1.5}{2} = 1.46875$$

$$f(1.46875) = 0.0111$$

x	1	1.5	1.25	1.375	1.4375	1.46875	1.453125
$f(x)$	−1	0.125	−0.6093	−0.29 10	−0.0959	0.0111	−0.04319

4th iteration,

$$a = 1.46875 \quad \text{and} \quad b = 1.4375$$

$$x_0 = \frac{1.46875 + 1.4375}{2} = 1.453125$$

$$f(1.453125) = -0.04319$$

5th iteration,

$$a = 1.453125 \quad \text{and} \quad b = 1.46875$$

$$x_0 = 1.4609$$

$$f(1.4609) = -0.01633$$

Homework Problem

Q-2: Find the root of the equation

$$x^2 - 4x - 10 = 0$$

using bisection Method.

Ans: The root lies between -1.735 and -1.7425

Solution: Let, $y = f(x) = x^2 - 4x - 10 = 0$

The first step is to guess two initial values of x.

Here is the chart of all the guess of x and their corresponding values of $f(x)$

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$	22	11	2	-5	-10	-13	-14	-13	-10

Let us take $a = -2$ and $b = -1$

Then, $x_0 = \frac{-2-1}{2} = -1.5$

$f(-1.5) = (-1.5)^2 - 4 \times (-1.5) - 10$

$= -1.75$

1st approximate iteration

$$a = -2 \quad \& \quad b = -1.5$$

$$x_0 = \frac{a+b}{2} = \frac{-2-1.5}{2} = -1.75$$

$$\therefore f(-1.75) = (-17.5)^2 - 4(1.75) - 10$$
$$= 0.0625$$

2nd iteration

$$a = -1.5 \quad \text{and} \quad b = -1.75$$

$$x_0 = \frac{-1.5-1.75}{2} = -1.625$$

$$f(-1.625) = (-1.625)^2 - 4(-1.625) - 10$$
$$= -0.8593$$

3rd iteration,

$$a = -1.625 \quad \text{and} \quad b = -1.75$$

$$x_0 = \frac{-1.625 - 1.75}{2} = -1.6875$$

$$f(-1.6875) = (-1.6875)^2 - 4(-1.6875) - 10$$

$$= -0.4023$$

4th iteration,

$$a = -1.6875 \quad \text{and} \quad b = -1.75$$

$$x_0 = \frac{-1.6875 - 1.75}{2} = -1.71875$$

$$f(-1.71827) = (-1.71875)^2 - 4(-1.71875) - 10$$

$$= -0.170898$$

5th iteration,

$$a = -1.71875 \quad \text{and} \quad b = -1.75$$

$$x_0 = \frac{-1.71875 \,.\, -1.75}{2} = -1.7343$$

$$f(-1.7343) = (-1.7343)^2 - (4 \times -1.7343) - 10$$

$$= -0.0544$$

6th iteration,

$$a = -1.7374 \quad \text{and} \quad b = -1.75$$

$$x_0 = \frac{-1.7374 - 1.75}{2} = -1.7437$$

$$f(-1.7437) = 0.01528$$

$f(-1.7343) = -0.0544$ and

$f(-1.7437) = 0.01528$

So the root of the equation lies between

-1.7343 to -1.7437 Ans.