Topic to discuss

Gauss Elimination Method

Numerical Problem

Homework Problem

Gauss Elimation Method

Gauss elimination method proposes a systematic strategy for reducing the system of equations to the upper triangular form using the forward elimination approach and then for obtaining values of unknown using the back substitution process.

The strategy, therefore comprises two phase

→ forward elimination phase.

→ Back substitution phase.

Q: Solve by Gauss Elimination method:

$$x + 2y + z = 0$$
$$2x + 2y + 3z = 3$$
$$-x - 3y = 2$$

Solution: Lets rearrange the given system of equation as –

$$x + 2y + z = 0$$
$$-x - 3y = 2$$
$$2x + 2y + 3z = 3$$

Now, it can be written in the following matrix form, $AX = B$

where,

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -3 & 0 \\ 2 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

Now, augemented matrix from this can be,

$$C = [A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ -1 & -3 & 0 & 2 \\ 2 & 2 & 3 & 3 \end{array}\right] \begin{array}{l} \longrightarrow R_1 \\ \longrightarrow R_2 \\ \longrightarrow R_3 \end{array}$$

Now, Lets perform row operation to convert it into upper triangular form or echelon form,

$$R_2 \rightarrow R_2 + R_1 \text{ and } R_3 \rightarrow R_3 - 2R_1$$

$$C = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & 1 & 2 \\ 0 & -2 & 1 & 3 \end{array}\right] \begin{array}{l} — R_1 \\ — R_2 \\ — R_3 \end{array}$$

Lets do another row Operation,

$$R_3 \rightarrow R_3 - 2R_2$$

$$C = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -1 & 1 & 2 \\ 0 & 0 & -1 & -1 \end{array}\right]$$

So, we can do back substitution here,

$-z = -1$	$-y + z = 2$	$x + 2y + z = 0$
$\therefore z = 1$	$y = -1$	$x = -2y - z$
		$= 1$

So, $x = 1$, $y = -1$ and $z = 1$

Homework Problem

Q: Solve the given system of equations by Gauss Elimination method

$$x + y + z = 9$$

$$2x - 3y + 4z = 13$$

$$3x + 4y + 5z = 40$$

Answer is, $x = 1$, $y = 3$ & $z = 5$

Solution: We have the given system of equation.

$$x + y + z = 9$$
$$2x - 3y + 4z = 13$$
$$3x + 4y + 5z = 40$$

Given system of equations can be expressed in the following matrix form.

$$AX = B$$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 9 \\ 13 \\ 40 \end{bmatrix}$

Now, augumented matrix from this matrix is represented as,

$$C = [A/B] = \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 2 & -3 & 4 & 13 \\ 3 & 4 & 5 & 40 \end{array}\right] \end{array} \begin{array}{l} — R_1 \\ — R_2 \\ — R_3 \end{array}$$

Now, lets perform row operation to convert it into upper triangular form or echelon form,

$$R_2' \longrightarrow R_2 - 2R_1 \quad \text{and} \quad R_3' \longrightarrow R_3 - 3R_1$$

so,

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 1 & 2 & 13 \end{array}\right] \begin{array}{l} — R_1' \\ — R_2' \\ — R_3' \end{array}$$

Lets do another row operation.

$$R_3'' \longrightarrow 5R_3' + R_2'$$

So,

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & 2 & -5 \\ 0 & 0 & 12 & 60 \end{array}\right]$$

from here we can write,

$12z = 60$

$\therefore z = \frac{60}{12} = 5$

$-5y + 2z = -5$

$y = \frac{5+2z}{5} = \frac{5+2\times 5}{5}$

$\therefore y = 3$

$x + y + z = 9$

$x = 9 - y - z$

$= 9 - 3 - 5$

$\therefore x = 1$

So, $x = 1$; $y = 3$ & $z = 5$