

Chapter : Roots of Nonlinear Equations

- Bisection Method
- False position Method. (Regula Falsi in Latin)
- Newton Raphson Method
- Secant Method
- Muller's Method
- Fixed point Method
- Bairstow's Method



Topic to discuss

- Introduction
- Linear and Non-Linear Equations
- Algebraic, Polynomial and Transcendental Equation
- Methods to solve non-linear equation
(All methods discussion)



1. Introduction

Mathematical models can be formulated into equations of the form,

$$f(x) = 0$$

where x and $f(x)$ may be real, complex or vector quantities.

The values of x that satisfy function — ①

is called roots of equations

Since the $f(x)$ becomes zero at these values, they are also known as zeros of the function.

2. Linear and Non-Linear Equations

$$y = f(x)$$

→ $f(x)$ is a linear function, if the dependent variable y changes in direct proportion to the change in independent variable x .

eg: $y = 3x + 5$, is a linear function.

→ $f(x)$ is said to be non-linear, if the response of the dependent variable y is not in direct or exact proportion to change in independent variable x ,

eg: $x + 3xy + 6 = 0$ or
 $y = x^2 + 1$

Equation, $f(x)=0$ may belongs to one of the following types of equations

- ① Algebraic Equations
- ② Polynomial Equations
- ③ Transcendental Equations.

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Algebraic Equations

An equation of type $y=f(x)$ is said to be algebraic if it can be expressed in the form,

$$f_n y_n + f_{n-1} y_{n-1} + \dots + f_1 y_1 + f_0 = 0 \quad \text{--- ①}$$

where f_i is an i^{th} order polynomial in x .

Equation --- ① can be written as $f(x, y) = 0$

Here's some eg:

$$2x + 7y - 1 = 0 \quad (\text{Linear})$$

$$8x + 2xy - 81 = 0 \quad (\text{Non-linear})$$

$$x^3 + xy + y^2 = 0 \quad (\text{Non-linear})$$

Polynomial Equations

Polynomial Equations are a simple class of algebraic equations that are represented as follow:

$$a_n x^n + a_{n-1} x^{n-1} + \dots + \dots + a_1 x + a_0 = 0$$

This is called n^{th} degree polynomial and has n roots.

The roots may be

- Real and different
- Real and repeated
- Complex numbers.

Some specific examples of polynomial equations,

$$x^3 + 4x^2 - 3x + 8 = 0$$

$$x^2 + 2x + 3 = 0$$

$$(a_3x^3 + a_2x^2 + a_1x + a_0 = 0)$$

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Transcendental Equation

A non algebraic equation is called a transcendental equation. These including trigonometric, exponential and logarithmic functions

eg: $2 \cos x - x = 0$

$$e^x \tan x + 4x = 0$$

$$\log x^4 - 1 = 0$$

$$e^x + x = 0$$

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Methods to Solve Non-Linear Equation

1. Direct analytical methods
2. Trial and Error methods
3. Graphical Methods
4. Iterative Methods

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Direct Analytical Method :

for some equation, we can use direct analytical Methods to solve the equation.

eg:- $ax^2 + bx + c = 0$

We know that the solution of above equation is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trial and Error Method

This method involves a series of guesses of x , each time evaluating the function to see whether it is close to zero.



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Graphical Method

This method involves plotting the given function and determining the points where it crosses the x-axis. These points represent approximate values of the roots of the function.



Iterative Method

There are a number of iterative methods that have been tried and used successfully in various problem situations.

Iterative method can be grouped into two category.

- 1) Bracketing Method (Interpolation Method)
- 2) Open end Method (Extrapolation Method)

Bracketing Method (Interpolation Method)

Two popular methods under this category are :

- Bisection Method
- False position Method. (Regula Falsi in Latin)



Open End Method or Extrapolation Method

The following iterative methods fall under this category.

- Newton Raphson Method
- Secant Method
- Muller's Method
- Fixed point Method
- Bairstow's Method