Topic to discuss

Gauss-seidel Method

Numerical Problem

Homework Problem.

Gauss - seidel Method

It is a improved version of Jacobi iteration method.

→ Jacobi method updates all the components of the solution vector simultaneously, using values from the previous iteration for all variables, which can lead to slower convergence.

→ Gauss - seidel method update each component sequentially, immediately using most recently updated values for calculation.

Q: Solve the system of equation using Gauss-seidel method.

$$-2x + 3y + 10z = 22$$
$$x + 10y - z = -22$$
$$10x + 2y + z = 9$$

Solution: We can rewrite each equation to isolate the variables x, y and z, and then iteratively update the values.

The system of equation is:

$$10x + 2y + z = 9$$
$$x + 10y - z = -22$$
$$-2x + 3y + 10z = 22$$

Rewriting each equation,

$$x = \frac{9 - 2y - z}{10} \quad , \quad y = \frac{-22 - x + z}{10} \quad , \quad z = \frac{22 + 2x - 3y}{10}$$

We start with an initial guess,

$$x_0 = y_0 = z_0 = 0$$

1st iteration,

$$x_1 = \frac{9}{10} = 0.9$$

$$y_1 = \frac{-22 - 0.9 + 0}{10} = -2.29$$

$$z_1 = \frac{22 + 2 \times 0.9 - 3 \times (-2.29)}{10} = 3.067$$

2nd iteration

$$x_2 = \frac{9 - 2y - 2}{10} = \frac{9 - 2\times(-2.29) - 3.067}{10} = 1.0513$$

$$y_2 = \frac{-22 - x + 2}{10} = \frac{-22 - 1.0513 + 3.067}{10} = -1.99843$$

$$z_2 = \frac{22 + 2x - 3y}{10} = \frac{22 + 2\times 1.0513 - 3\times(-1.99843)}{10}$$

$$= 3.0097789$$

3rd iteration,

$$x_3 = \frac{9-2y-z}{10} = \frac{9-(2x-1.998943)-3.0097789}{10}$$

$$= 0.998707$$

$$y_3 = \frac{-22-x+2z}{10} = \frac{-22-0.998707+3.009789}{10}$$

$$= -1.9988917$$

$$z_3 = \frac{22+2x-3y}{10} = \frac{22+2\times 0.998707-3\times(-1.9988917)}{10}$$

$$= 2.99940891$$

4th iteration

$$x_4 = \frac{9-2y-z}{10} = \frac{9-2(-1.99888917)-2.99940891}{10}$$

$$= 0.9998374$$

$$y_4 = \frac{-22-x+z}{10} = \frac{-22-0.999837+2.99940891}{10}$$

$$= -1.900042853$$

$$z_5 = \frac{22+2x-3y}{10} = \frac{22+2(0.9998374)-3(-1.90004285)}{10}$$

$$= 2.9699803$$

So, the required solution of system of linear equation is,

$x' = 0.999$, $y \simeq -1.9000$, $z \simeq 2.9699$

$x \simeq 1$, $y \simeq -2$, $z \simeq 3$

Homework Problem

Q: Solve the following system of equation using Gauss-seidel Method.

$$10x - y - z = 13$$

$$x + 10y + z = 36$$

$$x + y - 10z = -35$$

Solution :

The system of equation is

$$10x - y - z = 13$$

$$x + 10y + z = 36$$

$$x + y - 10z = -35$$

Rewriting each equation,

$$x = \frac{13 + y + z}{10}$$

$$y = \frac{36 - x - z}{10}$$

$$z = \frac{-35 - x - y}{-10}$$

Start from initial guess, $x_0 = y_0 = z_0 = 0$

1st iteration,

$$x_1 = \frac{13+0+0}{10} = 1.3$$

$$y_1 = \frac{36-1.3-0}{10} = 3.47$$

$$z_1 = \frac{-35-1.3-3.47}{-10} = 3.977$$

2nd iteration,

$$x_2 = \frac{13+3.47+3.977}{10} = 2.0447$$

$$y_2 = \frac{36-2.0447-3.977}{10} = 2.99783$$

$$z_2 = \frac{-35-2.0447-2.99783}{10}$$

$$= 4.004253$$

3rd iteration,

$$x_3 = \frac{13 + 2.99783 + 4.004253}{10}$$

$$= 2.0001083$$

$$y_3 = \frac{36 - 2.0001083 - 4.004253}{10}$$

$$= 2.99956$$

$$z_3 = \frac{-35 - 2.0001083 - 2.99956}{10}$$

$$= 4.000062817$$

4th iteration,

$$x_4 = \frac{13 + 2.99956 + 4.000062817}{10}$$

$$= 2.099962$$

$$y_4 = \frac{36 - 2.099962 - 4.0000628}{10}$$

$$= 2.989997$$

$$z_4 = \frac{-35 - 2.099962 - 2.989997}{10}$$

$$= 4.008996$$

So, the required solution of system of linear equation is,

$$x = 2 \quad , y = 3 \quad , z = 4$$

Ans