Elements of Probability

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Introduction

- The concept of the probability of a particular event of an experiment is subject to various meanings or interpretations

- There could be one of the two meanings of a geologist's quote "*there is a 60 percent chance of oil in a certain region*"

- The geologist feels that, over a long run, in 60 percent of the regions whose environmental conditions are very similar to the region under consideration, there will be oil (frequency interpretation of probability)

- The geologist believes that it is more likely that region will contain oil than it will not (subjective interpretation of probability)

Introduction

- In frequency interpretation, the probability of a given outcome of an experiment is considered as being a "property" of that outcome

- In the subjective interpretation, the probability of an outcome is not thought of as being a property of the outcome but rather is considered a statement about the beliefs of the person who is quoting the probability, concerning the chance that the outcome will occur.

- Mathematics of probability are same for both the interpretations

Sample Space and Events

- Experiment is a procedure whose outcome is not predictable with certainty in advance
- The set of all possible outcomes of an experiment is known as the **sample space** of the experiment and is denoted by $\mathcal{S}$

Sample Space and Events

- If the outcome of an experiment consists in the determination of the sex of a newborn child, then

$$\mathcal{S} = \{\mathrm{g}, \mathrm{b}\}$$

  - The outcome g means that the child is a girl and b that it is a boy.

Sample Space and Events

- If the experiment consists of the running of a race among the seven horses having post positions 1, 2, 3, 4, 5, 6, 7

$$\mathcal{S} = \{\text{all orderings of } (1, 2, 3, 4, 5, 6, 7)\}$$

  - The outcome $(2, 3, 1, 6, 5, 4, 7) \to$ the number 2 horse is first, then the number 3 horse, then the number 1 horse, and so on.

Sample Space and Events

- Suppose we are interested in determining the amount of dosage that must be given to a patient until that patient reacts positively
    - One possible sample space for this experiment is

$$\mathcal{S} = \{x : 0 < x < \infty\}$$

    - The outcome (dosage) x can take any value between 0 to ∞ (theoretically!)

Sample Space and Events

- Any subset E of the sample space is known as **an event**, i.e, an event is a set consisting of possible outcomes of the experiment

- If the outcome of the experiment is contained in E, then we say that E has occurred

  - For $\mathcal{S} = \{\mathrm{g}, \mathrm{b}\}$, the event that the child is a girl is $\mathrm{E} = \{\mathrm{g}\}$

  - For $\mathcal{S} = \{\text{all orderings of } (1, 2, 3, 4, 5, 6, 7)\}$, the event E could be for the number 3 horse will win the race, i.e.

$$\mathrm{E} = \{\text{all outcomes in } \mathcal{S} \text{ starting with } 3\}$$

Sample Space and Events

- For any two events E and F of a sample space $\mathcal{S}$
  - The new event $\mathrm{E} \cup \mathrm{F}$ is called *the union of the events* E *and* F, which consists of all outcomes that are either in E or in F or in both E and F
- Let us define with horse racing example

$$
\begin{aligned}
\mathrm{E} &= \{\text{all outcomes starting with 6}\} \\
\mathrm{F} &= \{\text{all outcomes having 6 in the second position}\}
\end{aligned}
$$

  - $\mathrm{E} \cup \mathrm{F}$ is the event that the number 6 horse comes in either first or second

Sample Space and Events

- For any two events E and F
    - The new event EF or $\mathrm{E} \cap \mathrm{F}$ is called *the intersection of* E *and* F, which consists of all outcomes that are in both E and F
    - EF will occur only if both E and F occur
- For the example with required dosage, define

$$\mathrm{E} = \{\mathrm{x} : 0 < \mathrm{x} < 5\} \text{ and } \mathrm{F} = \{\mathrm{x} : 2 < \mathrm{x} < 5\}$$

- $\mathrm{EF} = \{\mathrm{x} : 2 < \mathrm{x} < 5\}$

Sample Space and Events

- Consider two events from horse racing example

$$E = \{\text{all outcomes ending with 5}\}$$
$$F = \{\text{all outcomes starting with 5}\}$$

- The event EF does not have any outcomes and hence cannot occur, i.e. $EF = \varphi$
    - $\varphi \rightarrow$ null set, which does not contain any outcome
- Two events E and F are said to be **mutually exclusive** if $EF = \varphi$

Sample Space and Events

- For any event E, the event E^c is called the complement of E if E^c consists of all outcomes in the sample space $\mathcal{S}$ that are not in E
  - E^c will occur if and only if E does not occur, e.g. $\mathcal{S}^c = \varphi$
- For the example of determination of sex of a child
  - If $\mathrm{E} = \{\mathrm{g}\}$ then $\mathrm{E}^c = \{\mathrm{b}\}$

Sample Space and Events

- For any two events E and F, if all of the outcomes in E are also in F, then we say that E is contained in F and write $\mathrm{E} \subset \mathrm{F}$
- If $\mathrm{E} \subset \mathrm{F}$ and $\mathrm{F} \subset \mathrm{E}$ then we can write $\mathrm{E} = \mathrm{F}$
- For the required dosage example

$$\mathrm{F} = \{x : 0 < x < 10\}$$
$$\mathrm{E} = \{x : 2 < x < 8\}$$

  - $\mathrm{E} \subset \mathrm{F}$

Sample Space and Events

- Union of more than two events $\mathrm{E}_1, \ldots, \mathrm{E}_\mathrm{n}$

$$\mathrm{E}_1 \cup \mathrm{E}_2 \cup \cdots \cup \mathrm{E}_\mathrm{n} = \cup_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{E}_\mathrm{i}$$

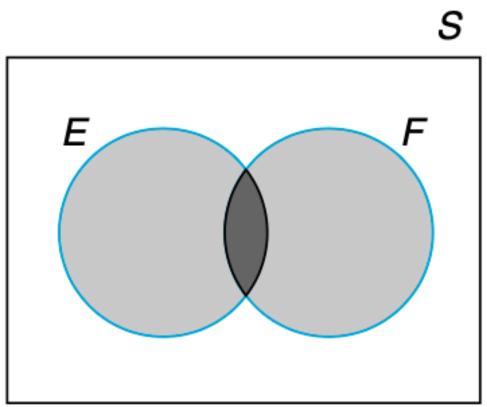
  - Union indicates at least one of the events E_i occurs

- Intersection of more than two events $\mathrm{E}_1, \ldots, \mathrm{E}_\mathrm{n}$

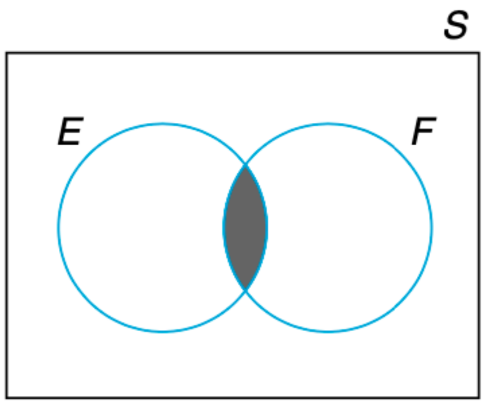
$$\mathrm{E}_1 \cap \mathrm{E}_2 \cap \cdots \cap \mathrm{E}_\mathrm{n} = \cap_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{E}_\mathrm{i}$$

  - Intersection indicates all of E_i occur

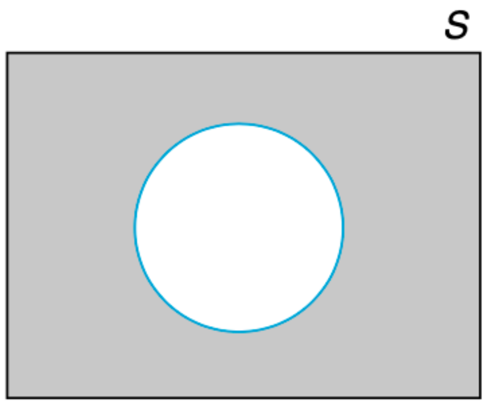
Venn Diagrams



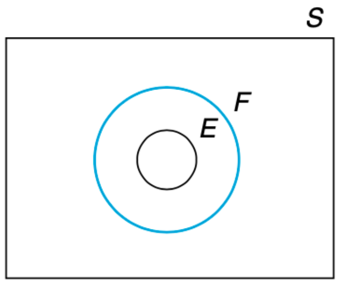
(a) Shaded region: $E \cup F$



(b) Shaded region: EF

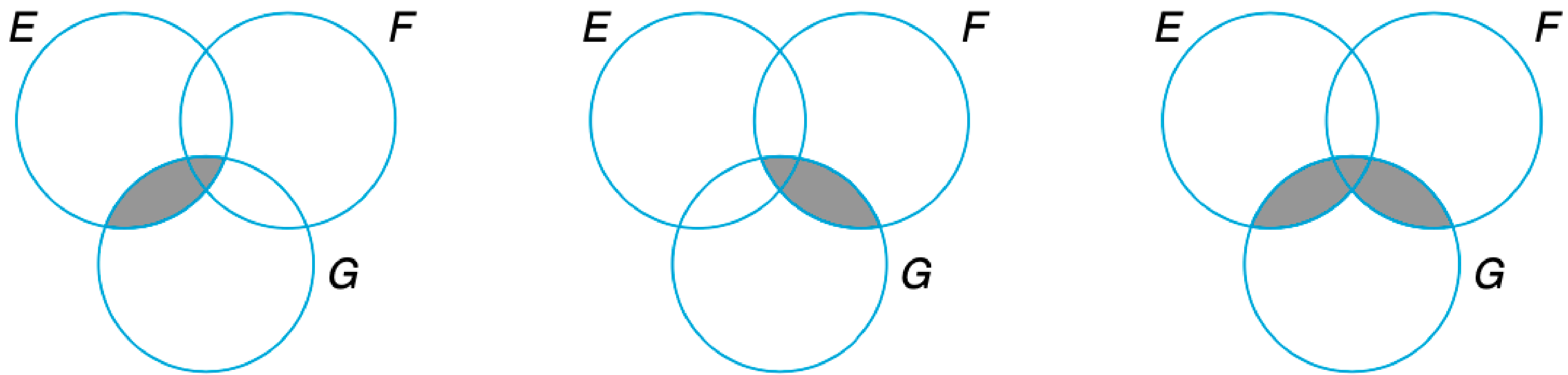


(c) Shaded region: E^c



$E \subset F$

Venn Diagrams



(a) Shaded region: EG (b) Shaded region: FG (c) Shaded region: $(E \cup F)G$

$(E \cup F)G = EG \cup FG$

The Algebra of Events

- *Commutative law*

$$E \cup F = F \cup E, \quad EF = FE$$

- *Associative law*

$$(E \cup F) \cup G = E \cup (F \cup G), \quad (EF)G = E(FG)$$

The Algebra of Events

- *Distributive law*

$$(E \cup F)G = EG \cup FG, \quad EF \cup G = (E \cup G)(F \cup G)$$

- *DeMorgan's law*

$$(E \cup F)^c = E^c F^c, \quad (EF)^c = E^c \cup F^c$$

Axioms of Probability

- If an experiment is continually repeated under the exact same conditions, then for any event E, the proportion of time that the outcome is contained in E approaches some constant value as the number of repetitions increases

- For instance, if a coin is continually flipped, then the proportion of flips resulting in heads will approach some value as the number of flips increases.

- This constant limiting frequency that we often have in mind when we speak of the probability of an event.

Axioms of Probability

- For each event E of an experiment having a sample space $\mathcal{S}$ there is a number, denoted by $\mathrm{P}(\mathrm{E})$, that is in accord with the following three axioms
  - *(Axiom I)* $0 \leq \mathrm{P}(\mathrm{E}) \leq 1$
  - *(Axiom II)* $\mathrm{P}(\mathcal{S}) = 1$
  - *(Axiom III)* For any sequence of mutually exclusive events $\mathrm{E}_1, \mathrm{E}_2, \ldots$

$$\mathrm{P}(\cup_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{E_i}) = \sum_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{P}(\mathrm{E_i}),\ \mathrm{i} = 1, 2, \ldots, \infty$$

Axioms of Probability

- From Axiom II

$$
\begin{aligned}
1 = P(\mathcal{S}) &= P(E \cup E^c) \\
&= P(E) + P(E^c)
\end{aligned}
$$

$$\Rightarrow \quad P(E) = 1 - P(E^c)$$

- It can be shown that for two events E and F

$$P(E \cup F) = P(E) + P(F) - P(EF)$$

Example 3.4a

- A total of 28 percent of American males smoke cigarettes, 7 percent smoke cigars, and 5 percent smoke both cigars and cigarettes.
- What percentage of males smoke neither cigars nor cigarettes?

Odds of an event

- Odds of an event A is defined as

$$\mathrm{O(A)} = \frac{\mathrm{P(A)}}{\mathrm{P(A^c)}} = \frac{\mathrm{P(A)}}{1 - \mathrm{P(A)}} \;\Rightarrow\; \mathrm{P(A)} = \frac{\mathrm{O(A)}}{1 + \mathrm{O(A)}}$$

  - Thus the odds of an event A tells how much more likely it is that A occurs than that it does not occur
  - For $\mathrm{P(A)} = 2/3$, the odds of A is 2, i.e. $\mathrm{P(A)}$ is two times that of $\mathrm{P(A^c)}$

Equally likely outcomes

- Suppose all the outcomes of an experiment with sample space $\mathcal{S} = \{1, \ldots, N\}$ are equally likely, i.e.

$$P(1) = P(2) = \cdots = P(N) = p$$

$$\sum_{i=1}^{N} P(i) = 1 \;\Rightarrow\; p = \frac{1}{N} = P(i)\ \forall\, i$$

- For any event E,

$$P(E) = \frac{\text{number of outcomes in } E}{N}$$

Basic principle of counting

- Suppose that two experiments are to be performed
    - If "Experiment 1" can result in any one of m possible outcomes and
    - if, for each outcome of "Experiment 1", there are n possible outcomes of "Experiment 2",
    - then together there are $\mathrm{m} \times \mathrm{n}$ possible outcomes of the two experiments.
- For example, there will be 36 possible outcomes for an experiment with tossing 2 dice

Example 3.5a

- Two balls are "randomly drawn" from a bowl containing 6 white and 5 black balls.

- What is the probability that one of the drawn balls is white and the other black?

Generalized basic principle of counting

- If r experiments that are to be performed are such that
  - the first one may result in any of n_1 possible outcomes, and
  - if for each of these n_1 possible outcomes there are n_2 possible outcomes of the second experiment, and
  - if for each of the possible outcomes of the first two experiments there are n_3 possible outcomes of the third experiment, and so on
  - then there are a total of $n_1 \times n_2 \times \cdots \times n_r$ possible outcomes of the r experiments

Permutations

- The number of ways n distinct objects can be arranged in a linear order is

$$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$$

  - Each of these ordered arrangement is known as a *permutation*

- E.g. how many different ordered arrangements of the letters a, b, c are possible?

Permutations

- How many different ordered arrangements of the letters a, b, c are possible?
  - $\mathrm{abc, acb, bac, bca, cab, cba}$
  - There are $3! = 3 \times 2 \times 1 = 6$ possible permutations of three distinct objects

Example 3.5b

- Mr. Jones has 10 books that he is going to put on his bookshelf.
- Of these, 4 are mathematics books, 3 are chemistry books, 2 are history books, and 1 is a language book.
- Jones wants to arrange his books so that all the books dealing with the same subject are together on the shelf.
- How many different arrangements are possible?

Combinations

- Determine the number of different groups of r objects that could be formed from a total of n objects

- E.g. how many different group of three can be selected from five letters A, B, C, D, and E?

    - There are $5 \cdot 4 \cdot 3 = 60$ groups of three can be selected from five items when the order is relevant

    - Every group of three counted six times, e.g. ABC, ACB, BAC, BCA, CAB, and CBA

    - Total number of different groups that can be formed is $10 \ (= 60/6)$

Combinations

- In general, $n(n-1)(n-2)\cdots(n-r+1)$ represents the number of different ways that a group of r items could be selected from n items when the order of selection is relevant
  - Each group of r items will be counted $r!$ times in this count
- The number of different groups of r items that could be formed from a set of n items is

$$\frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} = \frac{n!}{r!(n-r)!} = \binom{n}{r}$$

  - $\binom{n}{r} \rightarrow$ the number of combinations of n objects taken r at a time

Example 3.5d

- A committee of size 5 is to be selected from a group of 6 men and 9 women.
- If the selection is made randomly, what is the probability that the committee consists of 3 men and 2 women?

Example 3.5f

- A basketball team consists of 6 black and 6 white players.
- The players are to be paired in groups of two for the purpose of determining roommates.
- If the pairings are done at random, what is the probability that none of the black players will have a white roommate?

Example 3.5f

- A basketball team consists of 6 black and 6 white players. The players are to be paired in groups of two for the purpose of determining roommates. If the pairings are done at random, what is the probability that none of the black players will have a white roommate?

- The **first** pair can be selected from the 12 players is $\binom{12}{2}$ ways

- The **second** pair can be selected from the 12 players is $\binom{10}{2}$ ways

- The last pair (**sixth**) can be selected from the 12 players is $\binom{2}{2}$ ways

Example 3.5f

- A basketball team consists of 6 black and 6 white players. The players are to be paired in groups of two for the purpose of determining roommates. If the pairings are done at random, what is the probability that none of the black players will have a white roommate?

  - The number of ways 12 players can be divided into 6 pairs

$$\binom{12}{2}\binom{10}{2}\binom{8}{2}\binom{6}{2}\binom{4}{2}\binom{2}{2} = \frac{12!}{(2!)^6}$$

  - The number of ways 12 players can be divided into 6 unordered pairs

$$\frac{12!}{(2!)^6 6!} = \frac{12!}{2^6\, 6!}$$

Example 3.5f

- A basketball team consists of 6 black and 6 white players. The players are to be paired in groups of two for the purpose of determining roommates. If the pairings are done at random, what is the probability that none of the black players will have a white roommate?
- Number of pairs among only white players is $\frac{6!}{2^3\,3!}$, the same as for the black players
- The desired probability

$$\frac{[6!/(2^3\,3!)]^2}{[12!/(2^6\,6!)]} = \frac{5}{231} = 0.0216$$

Conditional probability

- Conditional probability is an important concept of probability theory
- It is useful to calculate probability when some partial information about the result of the experiment is available
- It is useful to recalculate the probability when some additional information is available
- Sometime it is easier to calculate conditional probability

Conditional probability

- Consider an experiment with rolling two fair dice, the sample space has 36 elements

$$\mathcal{S} = \{(\mathrm{i},\mathrm{j}),\ \mathrm{i} = 1,\ldots,6,\quad \mathrm{j} = 1,\ldots,6\}$$

- Since outcomes are equally likely, each outcome has the probability of (1/36) to occur

- The probability that sum of two dice equal to 8 is $(5/36)$

- What is the probability that sum of two dice equal to 8 provided the first die lands on 3?

Conditional probability

- Let E be the event that the sum of two dice is 8 and F denote the event that the first die lands on 3

- We want to calculate the probability of E given F, which is notationally denoted as $\mathrm{P}(\mathrm{E} \mid \mathrm{F})$, which is known as *conditional probability*

- The conditional probability of E given F is defined as

$$\mathrm{P}(\mathrm{E} \mid \mathrm{F}) = \frac{\mathrm{P}(\mathrm{EF})}{\mathrm{P}(\mathrm{F})}, \quad \text{provided } \mathrm{P}(\mathrm{F}) > 0$$

- The probabilities $\mathrm{P}(\mathrm{EF})$ and $\mathrm{P}(\mathrm{F})$ are unconditional probabilities and can be calculated using the sample space

Conditional probability

- What is the probability that sum of two dice equal to 8 provided the first die lands on 3?
- Let E be the event that the sum of two dice is 8 and F denote the event that the first die lands on 3

$$\mathrm{P}(\mathrm{E} \mid \mathrm{F}) = \frac{\mathrm{P}(\mathrm{EF})}{\mathrm{P}(\mathrm{F})} = \frac{1/36}{6/36} = \frac{1}{6}$$

- Conditional probability can also be calculated using "reduced sample space"

Example 3.6a

- A bin contains 5 defective (that immediately fail when put in use), 10 partially defective (that fail after a couple of hours of use), and 25 acceptable transistors.
- A transistor is chosen at random from the bin and put into use.
- If it does not immediately fail, what is the probability it is acceptable?

Example 3.6c

- Ms. Perez figures that there is a 30 percent chance that her company will set up a branch office in Phoenix.
- If it does, she is 60 percent certain that she will be made manager of this new operation.
- What is the probability that Perez will be a Phoenix branch office manager?

Bayes' formula

- For two events E and F, we can write

$$\mathrm{P}(\mathrm{E} \mid \mathrm{F}) = \frac{\mathrm{P}(\mathrm{EF})}{\mathrm{P}(\mathrm{F})} \Rightarrow \mathrm{P}(\mathrm{EF}) = \mathrm{P}(\mathrm{F})\,\mathrm{P}(\mathrm{E} \mid \mathrm{F})$$

- We can also write

$$\mathrm{E} = \mathrm{EF} \cup \mathrm{EF}^{c}$$

- Since EF and EF^{c} are mutually exclusive

$$\mathrm{P}(\mathrm{E}) = \mathrm{P}(\mathrm{EF}) + \mathrm{P}(\mathrm{EF}^{c})$$

Bayes' formula

- $P(E)$ can be expressed in terms of weighted sum of conditional probabilities

$$
\begin{aligned}
P(E) &= P(EF) + P(EF^c) \\
&= P(E|F)P(F) + P(E|F^c)P(F^c) \\
&= P(E|F)P(F) + P(E|F^c)[1 - P(F)]
\end{aligned}
$$

Example 3.7a

- An insurance company believes that people can be divided into two classes - those that are accident prone and those that are not.
- Their statistics show that an accident-prone person will have an accident at some time within a fixed 1-year period with probability 0.4, whereas this probability decreases to 0.2 for a non-accident-prone person.
- If we assume that 30 percent of the population is accident prone, what is the probability that a new policy holder will have an accident within a year of purchasing a policy?

$\mathrm{A}_1 \rightarrow$ the policy holder will have an accident within a year of purchase

$\mathrm{A} \rightarrow$ the policy holder is an accident-prone

$$\mathrm{P}(\mathrm{A}_1) = \mathrm{P}(\mathrm{A}_1|\mathrm{A})\mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{A}_1|\mathrm{A}^c)\mathrm{P}(\mathrm{A}^c) = 0.26$$

Example 3.7a

- Suppose that a new policy holder has an accident within a year of purchasing his policy.

- What is the probability that he is accident prone?

$$P(A|A_1) = ?$$

Bayes' formula

- What is the probability that he is accident prone?

$$
\begin{aligned}
P(A|A_1) &= \frac{P(AA_1)}{P(A_1)} \\
&= \frac{P(A_1|A)\,P(A)}{P(A_1)} \\
&= \frac{(0.4)(0.3)}{(0.26)} = 0.46152
\end{aligned}
$$

Bayes' formula

- Let $F_1, F_2, \ldots, F_n$ are mutually exclusive events, such that $\cup_{i=1}^{n} F_i = \mathcal{S}$
- Define an event E in terms of F_i's as $E = \cup_{i=1}^{n} EF_i$
- We can write

$$
\begin{aligned}
P(E) &= \sum_{i=1}^{n} P(EF_i) \\
&= \sum_{i=1}^{n} P(E|F_i)P(F_i)
\end{aligned}
$$

Bayes' formula

- Given E, what is the probability that one of $\mathrm{F_i}$ will occur

$$
\begin{aligned}
\mathrm{P}(\mathrm{F_i}|\mathrm{E}) &= \frac{\mathrm{P}(\mathrm{F_i}\mathrm{E})}{\mathrm{P}(\mathrm{E})} \\
&= \frac{\mathrm{P}(\mathrm{E}|\mathrm{F_i})\mathrm{P}(\mathrm{F_i})}{\mathrm{P}(\mathrm{E})} \\
&= \frac{\mathrm{P}(\mathrm{E}|\mathrm{F_i})\mathrm{P}(\mathrm{F_i})}{\sum_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{P}(\mathrm{E}|\mathrm{F_j})\mathrm{P}(\mathrm{F_j})}
\end{aligned}
$$

    - This formula is known as Bayes' formula

Example 3.7f

- A plane is missing and it is presumed that it was equally likely to have gone down in any of three possible regions.
- Let $(1 - \alpha_i)$ be the probability the plane will be found upon a search of the $i\text{th}$ region when the plane is, in fact, in that region, $i = 1, 2, 3$. (*overlook probability*)
- What is the conditional probability that the plane is in the $i\text{th}$ region, given that a search of region 1 is unsuccessful, $i = 1, 2, 3$?

Example 3.7f

- $\mathrm{R_i} \rightarrow$ the event that plane in the region i
- $\mathrm{E} \rightarrow$ the event that the search in region 1 is unsuccessful

$$\mathrm{P}(\mathrm{R_1}|\mathrm{E}) = \frac{\mathrm{P}(\mathrm{R_1 E})}{\mathrm{P}(\mathrm{E})} = \frac{\mathrm{P}(\mathrm{E}|\mathrm{R_1})\mathrm{P}(\mathrm{R_1})}{\mathrm{P}(\mathrm{E})} = \frac{\alpha_1/3}{\mathrm{P}(\mathrm{E})}$$

- $\mathrm{P}(\mathrm{E}) = \sum_{\mathrm{i}=1}^{3} \mathrm{P}(\mathrm{E}|\mathrm{R_i})\mathrm{P}(\mathrm{R_i}) = (\alpha_1 + 2)/3$
- $\mathrm{P}(\mathrm{R_2}|\mathrm{E}) = ?$ and $\mathrm{P}(\mathrm{R_3}|\mathrm{E}) = ?$

Independent events

- Two events E and F are said to be independent if one of the following three conditions is true, otherwise the events are dependent

$$\mathrm{P}(\mathrm{EF}) = \mathrm{P}(\mathrm{E})\mathrm{P}(\mathrm{F})$$
$$\mathrm{P}(\mathrm{E}|\mathrm{F}) = \mathrm{P}(\mathrm{E})$$
$$\mathrm{P}(\mathrm{F}|\mathrm{E}) = \mathrm{P}(\mathrm{F})$$

- Three events E, F, and G are said to be independent if each pair of the events are independent and

$$\mathrm{P}(\mathrm{EFG}) = \mathrm{P}(\mathrm{E})\mathrm{P}(\mathrm{F})\mathrm{P}(\mathrm{G})$$

Independent events

- If E and F are independent then E and F^c are also be independent

$$\begin{aligned} \mathrm{P}(\mathrm{E}) &= \mathrm{P}(\mathrm{EF}) + \mathrm{P}(\mathrm{EF}^c) \\ &= \mathrm{P}(\mathrm{E})\mathrm{P}(\mathrm{F}) + \mathrm{P}(\mathrm{EF}^c) \\ \mathrm{P}(\mathrm{EF}^c) &= \mathrm{P}(\mathrm{E})[1 - \mathrm{P}(\mathrm{F})] = \mathrm{P}(\mathrm{E})\mathrm{P}(\mathrm{F}^c) \end{aligned}$$

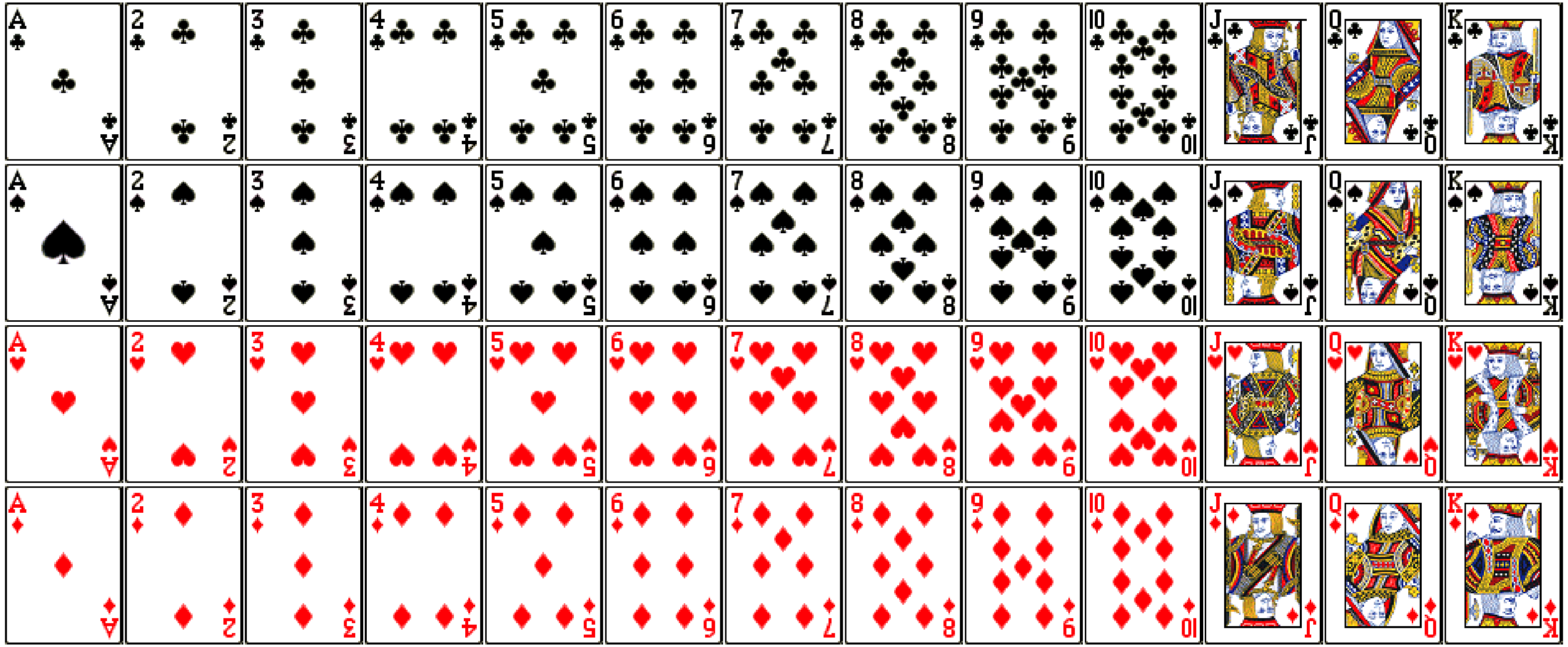
Independent events

- If E, F, and G are independent, then E is independent of any event formed from F and G

- E.g. E will be independent of $\mathrm{F} \cup \mathrm{G}$, i.e.,

$$\mathrm{P}(\mathrm{E}(\mathrm{F} \cup \mathrm{G})) = \mathrm{P}(\mathrm{E})\mathrm{P}(\mathrm{F} \cup \mathrm{G})$$

A pack of 52 cards



- Each card is a combination of color, type, and value
- Face cards (**J**, **K**, and **Q**) and Ace (**A**)

Example 3.8a

- A card is selected at random from an ordinary deck of 52 playing cards.
- If A is the event that the selected card is an ace and H is the event that it is a heart, then show A and H are independent.

Independence and mutually exclusive events

- Two events are said to be independent if the occurrence of one event does not affect the occurrence of the other
- Two events are said to be mutually exclusive if both events cannot happen simultaneously
- Two mutually exclusive events could be either dependent or independent

Independence and mutually exclusive events

- Let $\mathrm{A} = \{1, 2, 3\}$ and $\mathrm{B} = \{4, 5, 6\}$ be two events of an experiment rolling a fair die
  - A and B are mutually exclusive but not independent (Why?)
- Let $\mathrm{R} = \{\text{drawing a red card}\}$ and $\mathrm{A} = \{\text{drawing an Ace}\}$ are two events of an experiment of drawing a card from a pack of cards
  - R and A are independent, but they are not mutually exclusive

Problems

(Problem 1) A box contains three marbles — one red, one green, and one blue.

- Consider an experiment that consists of taking one marble from the box, then replacing it in the box and drawing a second marble from the box.
  - Describe the sample space.
  - Repeat for the case in which the second marble is drawn without first replacing the first marble.

Problems

(Problem 5) A system is composed of four components, each of which is either working or failed.

- Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector (x_1, x_2, x_3, x_4), where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.
    - How many outcomes are in the sample space of this experiment?
    - Let E be the event that components 1 and 3 are both failed. How many outcomes are contained in event E?

Problems

(Problem 29) You ask your neighbor to water a sickly plant while you were on a vacation. without water it will die with probability 0.8 and with water it will die with probability 0.15. You are 90 percent certain that your neighbor will remember to water the plant.

- What is the probability that plant will be alive when you return?
- If it is dead, what is the probability your neighbor forgot to water it?

Homeworks

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