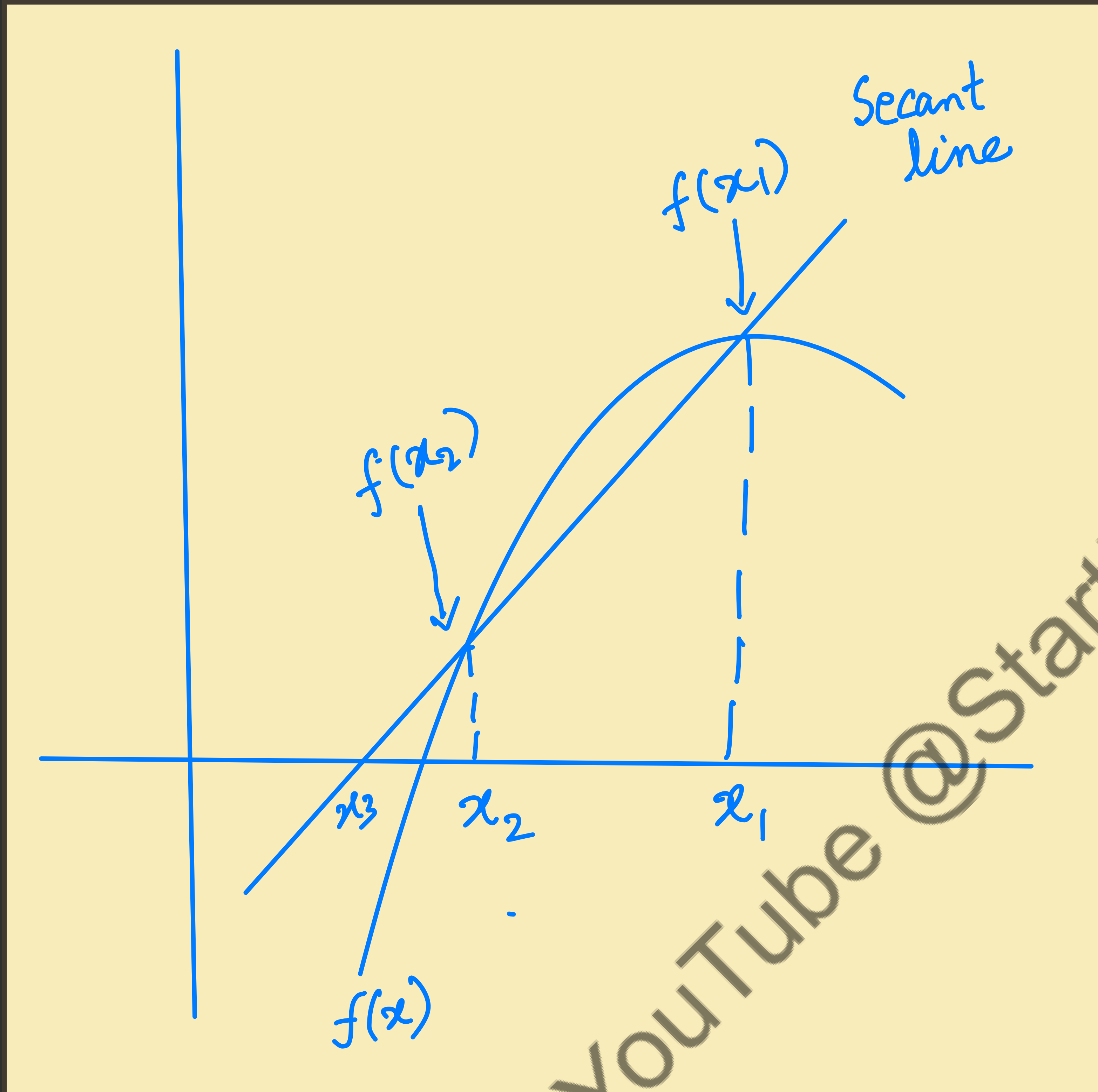
Topic to Discuss

- Secant Method Introduction.
- Numerical Problem
- Homework Problem.

Secant Method

Secant Method, like the false position and bisection methods, uses two initial estimates but does not require that they must bracket the root. For example, the secant method can use the points x_1 and x_2 as starting values, although they do not bracket the root.



$1^{st}: x_1, x_2 \Rightarrow x_3$

$2^{nd}: x_2, x_3 \Rightarrow x_4$

$3^{rd}: x_3, x_4 \Rightarrow x_5$

Slope of the secant line passing through x_1 & x_2 is given by,

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$ (General form)

We have, $(x_1, f(x_1))$ & $(x_2, f(x_2))$

So, $$\frac{y-f(x_1)}{f(x_2)-f(x_1)} = \frac{x-x_1}{x_2-x_1}$$

Putting, $y=0$ & $x = x_3$

So, $$\frac{-f(x_1)}{f(x_2)-f(x_1)} = \frac{x_3-x_1}{x_2-x_1}$$

$$\Rightarrow x_3 - x_1 = \frac{-f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$\Rightarrow x_3 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{x_1 f(x_2) - \cancel{x_1 f(x_1)} - x_2 f(x_1) + \cancel{x_1 f(x_1)}}{f(x_2) - f(x_1)}$$

$$\boxed{x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}}$$

The approximate value of root can be refined by repeating this procedure by replacing x_1 & x_2 by x_2 and x_3 respectively.

So, the general formula would be,

$$\boxed{x_{i+1} = \frac{x_{i-1} f(x_i) - x_i f(x_{i-1})}{f(x_i) - f(x_{i-1})}}$$

$i = 1, 2, 3, 4 \ldots$

Q: Use the secant method to estimate the root of the equation, $x^2 - 4x - 10 = 0$
with the initial estimates of $x_1 = 4$ and $x_2 = 2$

solution : Given. $x_1 = 4$ & $x_2 = 2$

So, $f(x_1) = 4^2 - 4 \times 4 - 10 = -10$

$f(x_2) = 2^2 - 4 \times 2 - 10 = -14$

1st iteration,

$x_1 = 4$, $x_2 = 2$

$f(x_1) = -10$ & $f(x_2) = -14$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{4 \times (-14) - 2 \times -10}{-14 + 10} = 9$$

$f(x_3) = f(9) = 9^2 - 4 \times 9 - 10 = 35$

$\underline{2^{nd}\ \text{iteration}}$,

$x_2 = 2$, $x_3 = 9$

$f(x_2) = -14$ $f(x_3) = 35$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2 \times 35 - 9 \times (-14)}{35 - (-14)} = 4$$

$$f(x_4) = f(4) = x^2 - 4x - 10$$
$$= -10$$

3rd iteration,

$x_3 = 9$ $\quad$ $x_4 = 4$

$f(x_3) = 35$ $\quad$ $f(x_4) = -10$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{9 \times (-10) - 4 \times 35}{-10 - 35}$$

$$= 5.1111$$

$$f(x_5) = f(5.1111) = x^2 - 4x - 10$$
$$= -4.32098$$

4^{th} iteration

$x_4 = 4$ $\qquad$ $x_5 = 5.1111$

$f(x_4) = -10$ $\qquad$ $f(x_5) = -4.32098$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{4 \times (-4.32098) - 5.1111 \times (-10)}{-4.32098 - (-10)}$$

$$= 5.956494$$

$$f(x_6) = x^2 - 4x - 10 = 1.653845$$

5th iteration,

$x_5 = 5.1111$, $\quad x_6 = 5.956494$

$f(x_5) = -4.32098$, $\quad f(x_6) = 1.65384$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{5.111 \times 1.65384 - 5.956494 \times (-4.32098)}{1.65384 - (-4.32098)}$$

$$= 5.7224$$

$$f(x_7) = -0.1430$$

6th iteration,

$x_6 = 5.956494$, $x_7 = 5.7224$

$f(x_6) = 1.65384$, $f(x_7) = -0.1430$

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{5.956494 \times (-0.1430) - 5.7224 \times 1.65384}{-0.1430 - 1.65384}$$

$$= 5.741$$

$f(x_8) = x^2 - 4x - 10 = -0.003485$

Hence root of equation is 5.7

Homework Problem

Q: Find the real root of the equation, $x^3 - 2x - 5 = 0$ using secant method.

Solution: Let two initial approximation be,

$x_1 = 2$ and $x_2 = 3$

So,

$f(x_1) = 2^3 - 2\times 2 - 5 = -1$

$f(x_2) = 3^3 - 2\times 3 - 5 = 16$

1st iteration,

$x_1 = 2$ and $x_2 = 3$

$f(x_1) = -1$ $f(x_2) = 16$

So, $x_3 = \dfrac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$

$= \dfrac{2 \times 16 - 3 \times (-1)}{16 - (-1)}$

$= 2.05882$

$\therefore f(x_3) = f(2.05882) = x^3 - 2x - 5$

$= -0.39079$

2nd iteration,

$$x_2 = 3 \qquad \text{and} \qquad x_3 = 2.05885$$

$$f(x_2) = 16 \qquad\qquad f(x_3) = -0.39079$$

$$\therefore\ x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{3 \times (-0.39079) - 2.05885 \times 16}{-0.39079 - 16}$$

$$= 2.08128$$

$$f(x_4) = f(2.08128) = x^3 - 2x - 5$$

$$= -0.146926$$

3rd iteration,

$$x_3 = 2.05885 \qquad x_4 = 2.08128$$

$$f(x_3) = -0.39079 \qquad f(x_4) = -0.146926$$

So, $$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{2.05885 \times (-0.146926) - 2.08128 \times (-0.39079)}{-0.146926 - (-0.39079)}$$

$$= 2.09479$$

$$\therefore f(x_5) = f(2.09479) = x^3 - 2x - 5 = 0.002662$$

4th iteration,

$x_4 = 2.08128$, $\quad x_5 = 2.09479$

$f(x_4) = -0.146926$, $\quad f(x_5) = 0.002662$

So, $$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{2.08128 \times 0.002662 - 2.09479 \times (-0.146926)}{0.002662 - (-0.146926)}$$

$$= 2.0945$$

Hence the root of equation is 2.094
(correct upto 3 decimal places)