Topic to discuss

- Newton- Raphson method.
- Numerical Problem
- Homework Problem.

Newton-Raphson Method

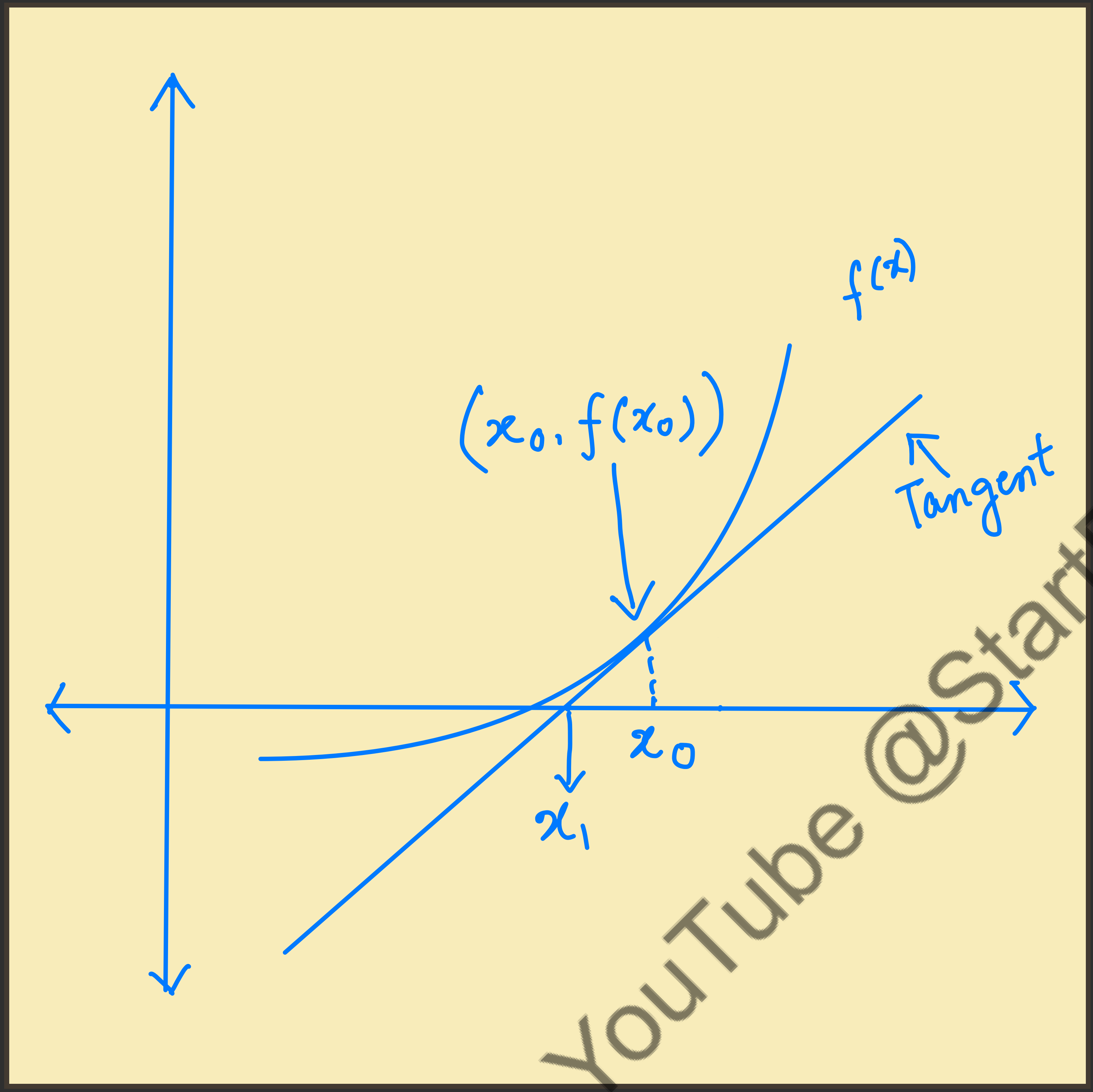
Consider a graph of $f(x)$ as shown.

Let us assume that x_0 is an approximate root of $f(x)=0$. Draw a tangent at the curve $f(x)$ at as shown.

The point of intersection of this tangent with the x-axis gives the second approximation to the root.

Let the point of intersection be be x_1

The point of the tangent is given by



f(x)
(x0, f(x0))
Tangent
x0
x1

We know the point-slope form of a straight-line is given by, $y - y_1 = m(x - x_1)$

So, here, the point we have is $(x_0, f(x_0))$

So the equation will be,

$$y - f(x_0) = f'(x_0)(x - x_0)$$

taking $y = 0$ and $x = x_1$

so,
$$-f(x_0) = f'(x_0)(x_1 - x_0)$$

$$x_1 - x_0 = -\frac{f(x_0)}{f'(x_0)}$$

$$\therefore \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Q: Find a real root of the equation $xe^x - 2 = 0$, correct to three decimal places using Newton-Raphson method.

Solution : Let $y = f(x) = xe^x - 2 = 0$

To find initial root, we have to guess,

$$f(0) = 0 \times e^0 - 2 = -2 < 0$$

$$f(1) = 1 \times e^1 - 2 = 0.718281 > 0$$

$\because$ $f(x) = xe^x - 2$

So, $f'(x) = xe^x + e^x$

$$= e^x(x+1)$$

1st iteration,

$$x_0 = 1$$

$$f(x_0) = 0.718281$$

$$f'(x_0) = e^x(x+1) = e^1(1+1) = 5.43656$$

$$\therefore \; x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 1 - \frac{0.718281}{5.43656}$$

$$= 0.867879$$

2nd iteration,

$$x_1 = 0.867879$$

$$f(x_1) = xe^x - 2 = 0.067163$$

$$f'(x_1) = e^x(x+1) = 4.449014$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.867879 - \frac{0.067163}{4.449014}$$

$$= 0.852782$$

3rd iteration,

$$x_2 = 0.852782$$

$$f(x_2) = xe^x - 2 = 0.00077080$$

$$f'(x_2) = e^x(x+1) = 4.346931$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.852782 - \frac{0.00077080}{4.346931}$$

$$= 0.85260$$

4th iteration,

$$x_3 = 0.85260$$

$$f(x_3) = xe^x - 2 = -0.000003572$$

$$f'(x_3) = e^x(x+1) = 4.345713$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.85260 - \frac{-0.000003572}{4.345713}$$

$$= 0.852600$$

Hence the root of the equation is,

0.85260

(Correct upto 4 deimal places)

Homework Problem

Q: Find the root of the equation $4e^{-x}\sin x = 1$ near 0.2 by Newton Raphson method correct to three decimal places.

Solution: We have $f(x) = 4e^{-x}\sin x - 1 = 0$

$\left(\frac{d}{dx}(1) = 0\right)$

So, $f'(x) = -4e^{-x}\sin x + 4e^{-x}\cos x$

$= 4e^{-x}(\cos x - \sin x)$

Now, $f(0) = 4e^{-0}\sin 0 - 1 = -1$

$f(1) = 4e^{-1}\sin 1 - 1 = 0.23823$

So, the value of x_0 must be between 0 and 1.

Taking $x_0 = 0.5$,

<u>First iteration</u>,

$x_0 = 0.5$

$f(x_0) = 4e^{-x}\sin x - 1 = 4e^{-0.5}\sin 0.5 - 1 = 0.16314$

$f'(x_0) = 4e^{-x}(\cos x - \sin x) = 4e^{-0.5}(\cos 0.5 - \sin 0.5) = 0.96597$

So,
$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 0.5 - \frac{0.16314}{0.96597}$$

$$= 0.3311$$

2nd iteration,

$$x_1 = 0.3311$$

$$f(x_1) = 4e^{-x}\sin x - 1 = -0.06618$$

$$f'(x_1) = 4e^{-x}(\cos x - \sin x) = 1.78269$$

so, $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.3311 + \frac{0.06618}{1.78269}$$

$$= 0.36822$$

3rd iteration,

$$x_2 = 0.36822$$

$$f(x_2) = 4e^{-x}\sin x - 1 \quad = -0.0036941$$

$$f'(x_2) = 4e^{-x}(\cos x - \sin x) \quad = 1.58602$$

So, $$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 0.37054$$

4th iteration,

$$x_3 = 0.37054$$

$$f(x_2) = 4e^{-x}\sin x - 1 \qquad = -0.0000049073$$

$$f'(x_2) = 4e^{-x}(\cos x - \sin x) \qquad = 1.574060$$

So, $$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.37054$$

Hence the root of x is 0.3705 Ans.

Some Useful derivatives

$\frac{d}{dx}(x^n)$	nx^{n-1}
$\frac{d}{dx}(e^x)$	e^x
$\frac{d}{dx}(\ln x)$	$\frac{1}{x}$
$\frac{d}{dx}(fg)$	$fg' + gf'$
$\frac{d}{dx}\left(\frac{f}{g}\right)$	$\frac{gf' - fg'}{g^2}$

$\frac{d}{dx}(\sin x)$	$\cos x$
$\frac{d}{dx}(\cos x)$	$-\sin x$
$\frac{d}{dx}(\tan x)$	$\sec^2 x$
$\frac{d}{dx}(a^x)$	$a^x \ln a$