Topic to discuss

Jacobi iteration Method
Numerical Problem
Home work Problem

Jacobi iteration Method

We have system of linear equation,

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

Here the equations must satisfy the conditions

$$|a_1| > |b_1| + |c_1|$$
$$|b_1| > |a_1| + |c_1|$$
$$|c_1| > |a_1| + |b_1|$$

and then solving the equation,

$$x = \frac{1}{a_1}(d_1 - b_1 y - c_1 z)$$

$$y = \frac{1}{b_2}(d_2 - a_2 x - c_2 z)$$

$$z = \frac{1}{c_3}(d_3 - a_3 x - b_3 y)$$

Start the initial value of $x_0 = y_0 = z_0 = 0$

then iterate the same step.

Q: Using Jacobi Iteration Method, find the equation of the following system of linear equations correct upto two decimal places.

$$3x + y + 5z = 13$$
$$5x - 2y + z = 4$$
$$x + 6y - 2z = -1$$

Solution: first we make the system of equations diagonally dominant and rewrite them as,

$$5x - 2y + z = 4$$
$$x + 6y - 2z = -1$$
$$3x + y + 5z = 13$$

Now, the Jacobi iteration formula is given by,

$$x = \frac{4+2y-z}{5}, \quad y = \frac{-1+2z-x}{6}, \quad z = \frac{13-3x-y}{5}$$

Let, the initial approximation to the root be,

$$x_0 = y_0 = z_0 = 0$$

1st iteration,

$$x_1 = \frac{4+2\times 0-0}{5} = 0.8$$

$$y_1 = \frac{-1}{6} = -0.166$$

$$z_1 = \frac{13}{5} = 2.6$$

2nd iteration

$$x_2 = \frac{4+2\times(-0.166)-2.6}{5} = 0.213$$

$$y_2 = \frac{-1+2\times 2.6-0.8}{6} = 0.5667$$

$$z_2 = \frac{13-3\times 0.8-(-0.166)}{5} = 2.15334$$

3rd iteration

$$x_3 = \frac{4 + 2 \times 0.5667 - 2.15334}{5} = 0.596$$

$$y_3 = \frac{-1 + 2 \times 2.15334 - 0.213}{6} = 0.5156$$

$$z_3 = \frac{13 - 3 \times 0.213 - 0.5667}{5} = 2.359$$

4th iteration

$x_4 = 0.5346$

$y_4 = 0.5203$

$z_4 = 2.1392$

5th iteration

$x_5 = 0.5802$

$y_5 = 0.4573$

$z_5 = 2.1753$

6th iteration

$x_6 = 0.5479$

$y_6 = 0.4617$

$z_6 = 2.1604$

7th iteration

$x_7 = 0.5526$

$y_7 = 0.4621$

$z_7 = 2.1789$

8th iteration

$x_8 = 0.5491$

$y_8 = 0.4675$

$z_8 = 2.17602$

9th iteration

$x_9 = 0.5518$

$y_9 = 0.4672$

$z_9 = 2.1770$

So, the required solution of the equation is,

$x = 0.55$

$y = 0.46$

$z = 2.17$ Ans.

Homework Problem

Solve the equation using Jacobi Iteration Method.

$$3y - 2z = 3$$
$$2x - y + 4z = 27$$
$$4x - y - 3z = 3$$

Solution: Here, we can clearly see,

$$|3| > |2| + |0|$$
$$|4| > |2| + |1|$$
$$|4| > |3| + |1|$$

So, first we can make the system of equations diagonally dominant and rewrite them as,

$$4x - y - 3z = 3$$
$$3y - 2z = 3$$
$$2x - y + 4z = 27$$

Now, the Jacobi formula is given by,

$$x = \frac{3 + y + 3z}{4}$$

$$y = \frac{3 + 2z}{3}$$

$$z = \frac{27 - 2x + y}{4}$$

Lets take the initial approximation to the root be,

$$x_0 = y_0 = z_0 = 0$$

1st iteration :

$$x_1 = \frac{3+0+0}{4} = 0.75$$

$$y_1 = \frac{3+0}{3} = 1$$

$$z_1 = \frac{27+0+0}{4} = 6.75$$

2nd iteration

$$x_2 = \frac{3+1+3\times 6.75}{4} = 6.0625$$

$$y_2 = \frac{3+2\times 6.75}{3} = 5.5$$

$$z_2 = \frac{27-2\times 0.75+1}{4} = 6.625$$

3rd iteration

$$x_3 = \frac{3 + 5.5 + 3 \times 6.625}{4} = 7.09375$$

$$y_3 = \frac{3 + 2 \times 6.625}{3} = 5.416667$$

$$z_3 = \frac{27 - 2 \times 6.0625 + 5.5}{4} = 5.09375$$

4th iteration

$$x_4 = \frac{3 + 5.41667 + 3 \times 5.09375}{4} = 5.92448$$

$$y_4 = \frac{3 + 2 \times 5.09375}{3} = 4.3958$$

$$z_4 = \frac{27 - 2 \times 7.09375 + 5.41667}{4} = 4.5572$$

5th iteration,

$$x_5 = \frac{3 + 4.3958 + 3 \times 4.5572}{4} = 5.2669$$

$$y_5 = \frac{3 + 2 \times 4.5572}{3} = 4.03813$$

$$z_5 = \frac{27 - 2 \times 5.92448 + 4.3958}{4} = 4.88671$$

6th iteration

$$x_6 = 5.42459$$

$$y_6 = 4.25781$$

$$z_6 = 5.12609$$

7th iteration

$x_7 = 5.65902$

$y_7 = 4.41739$

$z_7 = 5.10216$

8th iteration

$x_8 = 5.68097$

$y_8 = 4.40144$

$z_8 = 5.02484$

9th iteration

$x_9 = 5.61899$

$y_9 = 4.34989$

$z_9 = 5.00988$

10th iteration

$$x_{10} = 5.59488$$

$$y_{10} = 4.33992$$

$$z_{10} = 5.02798$$

So, the solution of the equation can be

$$x = 5.59$$
$$y = 4.34$$
$$z = 5.03$$