Topic to discuss

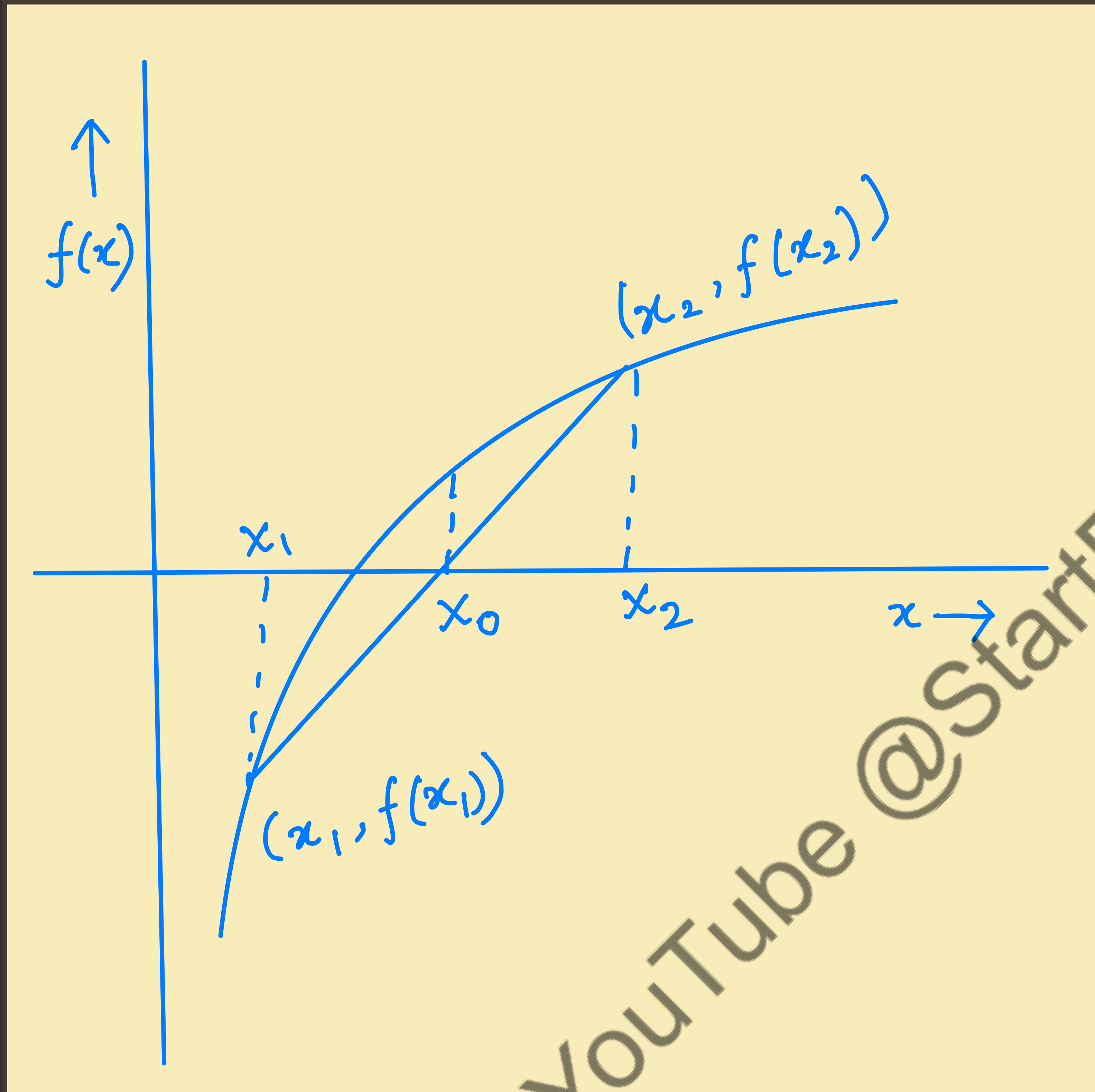
- False position Method or Regula Falsi (in Latin)
- Numerical Problem
- Homework Problems

False Position Method (Regula Falsi in Method)

Let us join the points x_1 and x_2 by a straight line. The point of intersection of this line with the x-axis (x_0) gives an improved estimate of the root and is called false position of the root.

This point then replaces one of the initial guesses that has a function value of the same sign as $f(x_0)$.

The process is repeated with the new values of x_1 and x_2.


f(x)
x
x1
x0
x2
(x1, f(x1))
(x2, f(x2))

We know that,

the equation of the line joining the points $(x_1, f(x_1))$ and $(x_2, f(x_2))$ is given by

$$\frac{y - f(x_1)}{f(x_2) - f(x_1)} = \frac{x - x_1}{x_2 - x_1}$$

[$\because$ Equation of line joining two points (x_1, y_1) & (x_2, y_2) is $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$]

Since the line intersects the x-axis at x_0,

when $x = x_0$, $y = 0$, we have,

$$\frac{-f(x_1)}{f(x_2) - f(x_1)} = \frac{x_0 - x_1}{x_2 - x_1}$$

or, $$x_0 - x_1 = \frac{-f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

or, $$x_0 = x_1 - \frac{f(x_1)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

or, $$x_0 = \frac{x_1 f(x_2) - \cancel{x_1 f(x_1)} - x_2 f(x_1) + \cancel{x_1 f(x_1)}}{f(x_2) - f(x_1)}$$

or, $$\boxed{x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}}$$

Q:- Find the smallest positive root of the given equation $3x - \cos x - 1 = 0$, correct up to three decimal places by Regula Falsi Method.

Solution : We have, $y = f(x) = 3x - \cos x - 1 = 0$

To, find the initial root, we have to guess,

So, $f(0) = 3 \times 0 - \cos 0 - 1 = -2 < 0$

$f(1) = 3 \times 1 - \cos 1 - 1 = 1.459697 > 0$

x	0	1	0.578085	
$f(x)$	-2	1.459697	-0.103255	

1st iteration,

$x_1 = 0$ and $x_2 = 1$

$f(x_1) = -2$ $f(x_2) = 1.459697$

$$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0 \times 1.459697 - 1 \times (-2)}{1.459697 - (-2)}$$

$$= 0.578085$$

$f(x_0) = f(0.578085) = 3x - \cos x - 1$

$= -0.103255$

2nd iteration,

$x_1 = 0.578085$ and $x_2 = 1$

$f(x_1) = -0.103255$ $\quad f(x_2) = 1.459697$

$$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0.578085 \times 1.459697 - 1 \times (-0.103255)}{1.459697 - (-0.103255)}$$

$$= 0.605958$$

$$f(x_0) = f(0.605958) = 3x - \cos x - 1$$

$$= -0.0040813$$

x	0	1	0.578085	0.605958	0.60705
$f(x)$	-2	1.459697	-0.103255	-0.0040813	-0.0001606

3rd iteration,

$x_1 = 0.605958$ and $x_2 = 1$

$f(x_1) = -0.0040813$ $\quad f(x_2) = 1.459697$

$$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0.605958 \times 1.459697 - 1 \times (-0.0040813)}{1.459697 - (-0.0040813)}$$

$$= 0.60705$$

$f(x_0) = f(0.60705) = 3x - \cos x - 1$

$= -0.0001606$

x	0	1	0.578085	0.605958	0.60705	0.60709
$f(x)$	-2	1.459697	-0.103255	-0.0040813	-0.0001606	-0.0000300

4th iteration,

$x_1 = 0.60705$ and $x_2 = 1$

$f(x_1) = -0.0001606$ $\quad f(x_2) = 1.459697$

$$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0.60705 \times 1.459697 - 1 \times (-0.0001606)}{1.459697 - (-0.0001606)}$$

$$= 0.60709$$

$$f(x_0) = f(0.60709) = 3x - \cos x - 1$$

$$= -0.00003006$$

Hence the root of function. $3x - \cos x - 1 = 0$
is 0.607 (correct upto 3 decimal places).

Homework Problem

Q: Compute one root of $x + \ln x - 2 = 0$, Correct to two decimal places by using Regula Falsi method.

Solution: To find initial root, we have to guess,

$$f(1) = 1 + \ln 1 - 2 = -1$$
$$f(2) = 2 + \ln 2 - 2 = 0.6931$$

Now, 1st iteration,

$x_1 = 1$ and $x_2 = 2$

$f(x_1) = -1$ $\quad$ $f(x_2) = 0.6931$

$$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1 \times 0.6931 - 2 \times -1}{0.6931 - (-1)} = 1.59063$$

$$f(x_0) = f(1.59067) = x + \ln x - 2$$
$$= 1.59063 + \ln(1.59063) - 2$$
$$= 0.05476$$

<u>2nd iteration</u>,

$x_1 = 1.59063$ and $x_2 = 1$

$f(x_1) = 0.05476$ $f(x_2) = -1$

So, $$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1.59063 \times (-1) - 1 \times 0.05476}{-1 - 0.05476} = 1.559970$$

So, $$f(x_0) = f(1.55968) = x + \ln x - 2$$
$$= 0.0046371$$

3rd iteration,

$x_1 = 1.559970$ and $x_2 = 1$

$f(x_1) = 0.0046371$ $f(x_2) = -1$

So, $$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1.559970 \times (-1) - 1 \times 0.0046371}{-1 - 0.0046371}$$

$$= 1.557385$$

So, $f(x_0) = f(1.557385) = x + \ln x - 2$

$= 0.00039370$

4th iteration,

$x_1 = 1.557385$ and $x_2 = 1$

$f(x_1) = 0.00039370$ $f(x_2) = -1$

So, $$x_0 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1.557385 \times (-1) - 1 \times (0.00039370)}{-1 - 0.00039370}$$

$$= 1.55716$$

$f(x_0) = f(1.55716) = 0.000032917$

Hence the required root is 1.557 or we can say 1.56 (correct upto 2 significant figure)