Chapter : Errors And Approximation

Topics to discuss

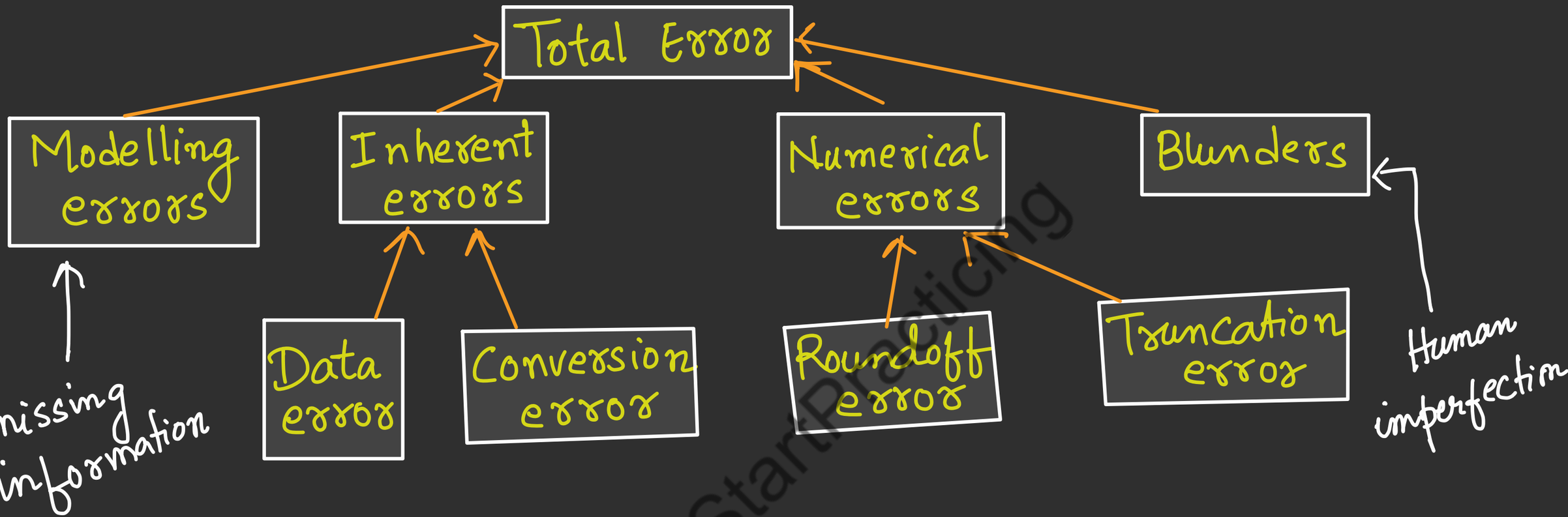
1) Errors

2) Types of Errors with definition.
   → Modelling, Inherent, Numerical, Truncation, Round off error etc.

3) Absolute, Relative & Percentage Error.

1) **Errors** : We are familiar with exact numbers and approximate numbers.

The difference between the exact solution and approximate solution is called error.



(1) Inherent Errors or Input Errors or intrinsic Errors

Inherent Error refers to the error that is naturally present in any numerical method or measurement due to the limitations of the method or the precision of the tools used.

It contains two components

a) Data Errors

b) Conversion Errors

a) **Data Errors** : Data Error refers to inaccuracies or inconsistencies present in the data used for analysis, computation or decision making. These errors can arise from various sources including :

→ Measurement Errors

→ Recording Errors

→ Input Errors

b) **Conversion Errors** : Conversion errors occurs when data is transformed from one format, type or system to another and inaccuracies or losses of information happen during this process.

Common scenarios where conversion errors might occur include:

→ Type conversion.

→ Unit conversion.

→ Format Conversion.

→ Encoding Conversion

(2) **Numerical Errors** : Numerical Errors are introduced during the process of implementation of a numerical method.

They come in two forms :

a) Round-off Errors

b) Truncation Errors

a) Round-off Errors : Roundoff errors occur when a fixed number of digits are used to represent exact numbers.

Since the number are stored at every stage of computation, roundoff error is introduced at the end of every arithemetic operation.

It can be done in 2 ways.

→ Chopping

→ Symmetric rounding.

a) Chopping : In chopping, Extra digits are dropped.

Chopping simply cuts off the extra digits without any adjustment.

b) Symmetric Round-off : In this method, the last retained significant digit is "round up" by 1 if the first discarded digit is larger or equal to 5. Otherwise the last retained digit is unchanged.

✯ Q:- Differences between Round-Off and Chopping

Ans: If a number is correct upto 'd' decimal places, we have to keep in mind that the digit in the d^{th} place of decimal is not exact but affected with roundoff error.

eg:- Lets say a number, 72.520457

eg: 72.520457	Round-off	Chopping
5 decimal places	72.52046	72.52045
4 decimal places	72 5205	72.5204
3 decimal places	72.520	72.520

b) Truncation Errors:

Truncation error is the error introduced when an infinite process is approximated by a finite one.

This type of error arise in numerical methods when an infinite series, iterative process or continuous function is truncated or cut off after a finite number of terms or steps.

Common source of truncation error:

→ Series approximation

→ Numerical Integration

→ Numerical Differentiation

③ **Modelling Errors** : This type of error arise because models are simplified representations of reality and cannot capture every detail of complexity.

Generally, it is arise due to incomplete knowledge or missing information.

④ **Blunders** : Blunders are errors that are caused due to human imperfection.

It can occur due to

→ lack of understanding

→ Wrong assumptions

→ Error in deriving mathematical equations.

(5) Absolute, Relative & Percentage Errors :

Absolute Error : The difference between true/exact value of a quantity and its approximate value is called Absolute Error.

Let x_t is true value and
x_a is approximate value.

Then, **True value (x_t) = Approximate value (x_a) + Error**

This error can be -ve or +ve depending on x_t and x_a.

So, Absolute Error (E_a) $= |x_t - x_a|$

Relative Errors :

Relative Errors (E_R) is defined as follows :

$$E_R = \frac{\text{Absolute Error } (E_a)}{\text{True Value } (x_t)}$$

$$E_R = \frac{|x_t - x_a|}{|x_t|}$$

Percentage Error :

$$\text{Percentage Error } (E_p) = 100 \times E_R$$

$$E_p = \frac{|x_t - x_a|}{|x_t|} \times 100.$$