Topic to discuss

Gauss-Jordan Method

Numerical Problem

Homework Problem.

Gauss - Jordan Method

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Gauss elimination Method

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

or

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Gauss - Jordan Method

Q: Solve the system of Equation using Gauss-Jordan method.

$$2x_1 + 4x_2 - 6x_3 = -8$$
$$x_1 + 3x_2 + x_3 = 10$$
$$2x_1 - 4x_2 - 2x_3 = -12$$

Using Gauss-Jordan method.

Solution :

Normalise the first and third equation by diving it by 2

The result is :

$$x_1 + 2x_2 - 3x_3 = -4$$
$$x_1 + 3x_2 + x_3 = 10$$
$$x_1 - 2x_2 - x_2 = -6$$

Now, it can be written in the following matrix form, $AX = B$.

where, $A = \begin{bmatrix} 1 & 2 & -3 \\ 1 & 3 & 1 \\ 1 & -2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x_1 \\ \alpha_2 \\ x_3 \end{bmatrix}$, $B = \begin{bmatrix} -4 \\ 10 \\ -6 \end{bmatrix}$

So, augumented matrix from this can be,

$$C = [A \mid B] = \left[\begin{array}{ccc|c} 1 & 2 & -3 & -4 \\ 1 & 3 & 1 & 10 \\ 1 & -2 & -1 & -6 \end{array}\right]$$

1) $\begin{bmatrix} \checkmark & & \\ 0 & & \\ 0 & & \end{bmatrix}$

2) $\begin{bmatrix} & 0 & \\ & \checkmark & \\ & 0 & \end{bmatrix}$

3) $\begin{bmatrix} & & 0 \\ & & 0 \\ & & \checkmark \end{bmatrix}$

Now, lets do row operation to eliminate x_1 from second and third row,

$$R_2 \to R_2 - R_1 \quad \& \quad R_3 \to R_3 - R_1$$

$$C = \left[\begin{array}{ccc|c} 1 & 2 & -3 & -4 \\ 0 & 1 & 4 & 14 \\ 0 & -4 & 2 & -2 \end{array}\right]$$

Again apply row operation,

$$R_1 \to R_1 - 2R_2 \; ; \quad R_3 \to R_3 + 4R_2$$

$$C = \left[\begin{array}{ccc|c} 1 & 0 & -11 & -32 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 18 & 54 \end{array}\right]$$

$$R_3 \rightarrow \frac{R_3}{18} \qquad \therefore C = \left[\begin{array}{ccc|c} 1 & 0 & -11 & -32 \\ 0 & 1 & 4 & 14 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

Apply row operation,

$$R_1 \rightarrow R_1 + 11 R_3$$

$$R_2 \rightarrow R_2 - 4 R_3$$

So,

$$C = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$$C = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

So,

$$x_1 = 1$$

$$x_2 = 2$$

$$x_3 = 3$$

Homework Problem

Q: Solve the system of Equation using Gauss-Jordan Method

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7$$

Solution : $x = 1$

$y = 1$

$z = 1$

Solution: lets rearrange the system of equation as –

$$x + y + 5z = 7$$

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

Now, it can be written in the following matrix form,

$$AX = B$$

where,

$$A = \begin{bmatrix} 1 & 1 & 5 \\ 10 & 1 & 1 \\ 2 & 10 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 12 \\ 13 \end{bmatrix}$$

So, augumented matrix from this can be written as,

$$C = [A|B] = \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \end{array}\right]$$

Apply row operation on R_2 and R_3

$$R_2 \rightarrow R_2 - 10R_1 \quad \& \quad R_3 \rightarrow R_3 - 2R_1$$

$$C = \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & -9 & -49 & -58 \\ 0 & 8 & -9 & -1 \end{array}\right]$$

Again apply row Operations on R_1 and R_3

$$R_1 \longrightarrow 9R_1 + R_2 \quad \& \quad R_3 \longrightarrow 9R_3 + 8R_2$$

We get,

$$\left[\begin{array}{ccc|c} 9 & 0 & -4 & 5 \\ 0 & -9 & -49 & -58 \\ 0 & 0 & -473 & 473 \end{array}\right]$$

- $9R_3 \rightarrow 0 \quad 72 \quad -81 \quad -9$
- $8R_2 \rightarrow 0 \quad -72 \quad -392 \quad -464$

Add : $0 \quad 0 \quad -473 \quad -473$

Apply, $R_3 \longrightarrow \dfrac{R_3}{-473}$, we get

$$\left[\begin{array}{ccc|c} 9 & 0 & -4 & 5 \\ 0 & -9 & -49 & -58 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Again, apply row operation on R_1 & R_2.

$$R_1 \longrightarrow R_1 + 4R_3 \quad \& \quad R_2 \longrightarrow R_2 + 49R_3$$

We get,

$$\left[\begin{array}{ccc|c} 9 & 0 & 0 & 9 \\ 0 & -9 & 0 & -9 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

Now, we can write,

$9x = 9$	$-9y = -9$	$z = 1$
$\therefore x = 1$	$\therefore y = 1$	

So, $x = 1$, $y = 1$ & $z = 1$ is the solution.