Topic to discuss

Matrix Inversion Method

Numerical Problem

Homework Problem

Matrix Inversion Method

Another way to obtain the solution of an equation of type $Ax = B$ is by using matrix Inversion Method.

We have, $Ax = B$

so, $A^{-1} A X = A^{-1} B$

or, $X = A^{-1} B$

as, $A^{-1} A = I$, Identity matrix.

Steps to find Inverse of a matrix

1. Find the Determinant.
2. Find the cofactor Matrix
3. Find the adjoint (Transpose of cofactor matrix)
4. Compute the inverse

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

Q: Solve the system of Equation using matrix Inversion Method.

$$2x + y + z = 4$$
$$3x + 2y + 3z = 8$$
$$x + 4y + 9z = 14$$

Solution: Given system of equations can be written in the following matrix form,

$$AX = B$$

where, $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 4 \\ 8 \\ 14 \end{bmatrix}$

We have, $AX = B$

$$A^{-1}AX = A^{-1}B$$

$$\Rightarrow \quad X = A^{-1}B \qquad [A^{-1}A = I, \text{ Identity matrix}]$$

To find A^{-1}, we have to solve,

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A.$$

Since, $|A| = 2(18-12) - (27-3) + (12-2)$

$$= 2\times6 - 24 + 10$$

$$= -2$$

Co-factor matrix of $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$

$$= \begin{bmatrix} +\begin{vmatrix} 2 & 3 \\ 4 & 9 \end{vmatrix} & -\begin{vmatrix} 3 & 3 \\ 1 & 9 \end{vmatrix} & +\begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 4 & 9 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 1 & 9 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 4 \end{vmatrix} \\ +\begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} & +\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 6 & -24 & 10 \\ -5 & 17 & -7 \\ 1 & -3 & 1 \end{bmatrix}$$

$\operatorname{adj}(A)$ = co-factor matrix Transpose.

$$= \begin{bmatrix} 6 & -24 & 10 \\ -5 & 17 & -7 \\ 1 & -3 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 6 & -5 & 1 \\ -24 & 17 & -3 \\ 10 & -7 & 1 \end{bmatrix}$$

So, Now,

$$A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$$

$$= -\frac{1}{2}\begin{bmatrix} 6 & -5 & 1 \\ -24 & 17 & -3 \\ 10 & -7 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & \frac{5}{2} & -\frac{1}{2} \\ 12 & -\frac{17}{2} & \frac{3}{2} \\ -5 & \frac{7}{2} & -\frac{1}{2} \end{bmatrix}$$

We can write,

$$X = A^{-1}B$$

$$X = \begin{bmatrix} -3 & \frac{5}{2} & -\frac{1}{2} \\ 12 & -\frac{17}{2} & \frac{3}{2} \\ -5 & \frac{7}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 14 \end{bmatrix}$$

$$X = \begin{bmatrix} -3\times 4 + \frac{5}{2}\times 8 - \frac{1}{2}\times 14 \\ 12\times 4 - \frac{17}{2}\times 8 + \frac{3}{2}\times 14 \\ -5\times 4 + \frac{7}{2}\times 8 - \frac{1}{2}\times 14 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

So, $x = 1$, $y = 1$ & $z = 1$

Ans

Homework Problem

Solve the system of equations using matrix inversion method.

$$2x - 3y + 10z = 3$$
$$-x + 4y + 2z = 20$$
$$5x + 2y + z = -12$$

Solution: Given system of equations can be written in the following matrix form, $AX = B$

where,

$$A = \begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } Z = \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

we have,

$$AX = B$$

so, $$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B \qquad (\text{Since } AA^{-1} = I \text{, identity matrix})$$

we Know,

$$A^{-1} = \frac{1}{|A|}\operatorname{adj}(A)$$

So,

$$|A| = 2(4-4) + 3(-1-10) + 10(-2-20)$$

$$= -33 - 220$$

$$= -253$$

Since the co-factor of the matrix $\begin{bmatrix} 2 & -3 & 10 \\ -1 & 4 & 2 \\ 5 & 2 & 1 \end{bmatrix}$

is
$$\begin{bmatrix} +\begin{vmatrix} 4 & 2 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 5 & 1 \end{vmatrix} & +\begin{vmatrix} -1 & 4 \\ 5 & 2 \end{vmatrix} \\ -\begin{vmatrix} -3 & 10 \\ 2 & 1 \end{vmatrix} & +\begin{vmatrix} 2 & 10 \\ 5 & 1 \end{vmatrix} & -\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix} \\ +\begin{vmatrix} -3 & 10 \\ 4 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 10 \\ -1 & 2 \end{vmatrix} & +\begin{vmatrix} 2 & -3 \\ -1 & 4 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 0 & 11 & -22 \\ 23 & -48 & -19 \\ -46 & -14 & 5 \end{bmatrix}$$

Now,

adj (A) = Transpose of cofactor matrix

$$= \begin{bmatrix} 0 & 11 & -22 \\ 23 & -48 & -19 \\ -46 & -14 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 23 & -46 \\ 11 & -48 & -14 \\ -22 & -19 & 5 \end{bmatrix}$$

So, $A^{-1} = \frac{1}{|A|} \operatorname{adj}(A)$

$$= \frac{1}{-253}\begin{bmatrix} 0 & 23 & -46 \\ 11 & -48 & -14 \\ -22 & -19 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\frac{1}{11} & \frac{2}{11} \\ -\frac{1}{23} & \frac{48}{253} & \frac{14}{253} \\ \frac{22}{253} & \frac{19}{253} & -\frac{5}{253} \end{bmatrix}$$

Now, $X = A^{-1}B$

$$X = \begin{bmatrix} 0 & -\frac{1}{11} & \frac{2}{11} \\ -\frac{1}{23} & \frac{48}{253} & \frac{14}{253} \\ \frac{22}{253} & \frac{19}{253} & -\frac{5}{253} \end{bmatrix} \begin{bmatrix} 3 \\ 20 \\ -12 \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{20}{11} - \frac{24}{11} \\ -\frac{3}{23} + \frac{48}{253} \times 20 - \frac{14}{253} \times 12 \\ 3 \times \frac{22}{253} + 20 \times \frac{19}{253} + \frac{5}{253} \times 12 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 2 \end{bmatrix}$$

So, $x = -4$

$y = 3$

and $z = 2$ Ans.

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