Topic to discuss

Operators in Interpolation

Forward difference operators (Δ)

Backward difference operators (∇)

Central difference operators (δ)

Forward shift operators (E)

Backward shift operators (E^{-1})

Averaging Operator (μ)

Operators in Interpolation

x	y
2	4
4	10
6	12
8	18

Definition : Let $y = f(x)$ be a given function and 'h' is the length of interval and has fixed value.

All x_i are equidistance.

such that,
$$x_1 = x_0 + h$$
$$x_2 = x_1 + h = x_0 + 2h$$
$$x_3 = x_2 + h = x_0 + 3h$$
$$\vdots$$
$$x_n = x_0 + nh$$

And, $y_0 = f(x_0)$, $y_1 = f(x_1)$ .... $y_n = f(x_n)$

1) Forward difference Operators (Δ) :

It is denoted by Δ.

and defined as,

$$\boxed{\Delta f(x) = f(x+h) - f(x)}$$

We have,

$$y_0 = f(x_0)$$

$$\Delta y_0 = \Delta f(x_0)$$

$$= f(x_0+h) - f(x_0)$$

$$= f(x_1) - f(x_0)$$

$$\boxed{\Delta y_0 = y_1 - y_0}$$

2) Backward difference Operators

It is denoted by ∇.

and defined as,

$$\boxed{\nabla f(x) = f(x) - f(x-h)}$$

we have,

$$y_1 = f(x_1)$$

$$\nabla y_1 = \nabla f(x_1)$$

$$= f(x_1) - f(x_1 - h)$$

$$= f(x_1) - f(x_0 + h - h)$$

$$= f(x_1) - f(x_0)$$

$$\boxed{\nabla y_1 = y_1 - y_0}$$

3) Central difference Operator

It is denoted by δ.

and defined as,

$$\boxed{\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)}$$

4) Forward Shift Operator :

It is denoted by E.

and defined as,

$$\boxed{E f(x) = f(x+h)}$$

Also,

$$\begin{aligned} E^2 f(x) &= E\left(E f(x)\right) \\ &= E\left(f(x+h)\right) \\ &= f(x+h+h) \\ &= f(x+2h) \end{aligned}$$

5) Backward Shift Operator :

It is denoted by E^{-1}.

and defined as,

$$\boxed{E^{-1} f(x) = f(x-h)}$$

Also,

$$E^{-2} f(x) = f(x-2h)$$

$$E^{\frac{1}{2}} f(x) = f\left(x+\frac{h}{2}\right)$$

$$y_0 = f(x_0)$$

$$E y_0 = E f(x_0)$$

$$= f(x_0+h)$$

$$= f(x_1)$$

$$\boxed{E y_0 = y_1}$$

6) Averaging Operator

It is denoted as μ.

and defined as,

$$\boxed{\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]}$$

Summary

1) Forward difference, $\Delta f(x) = f(x+h) - f(x)$

2) Backward difference, $\nabla f(x) = f(x) - f(x-h)$

3) Central difference, $\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$

4) Forward shift, $E f(x) = f(x+h)$

5) Backward shift, $E^{-1} f(x) = f(x-h)$

6) Averaging operator, $\mu f(x) = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$

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