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Q1- Find the truncation error in the result of the following function for $x = 1/5$ when we use

(a) first three terms

(b) first four terms.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$$

Solution : (a) Truncation error when first three terms are used,

$$\text{Truncation Error} = e^x_{\text{approx}, 7\text{ terms}} - e^x_{\text{approx}, 3\text{ terms}}$$

$$= e^x_{\text{approx}, \text{ last } 4 \text{ terms}}$$

for $x = \frac{1}{5} = 0.2$

$$\text{Truncation error} = \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!}$$

$$= \frac{0.2^3}{3!} + \frac{0.2^4}{4!} + \frac{0.2^5}{5!} + \frac{0.2^6}{6!}$$

$$= 1.3334 \times 10^{-3} + 6.6667 \times 10^{-5} + 2.666 \times 10^{-6} + 8.8889 \times 10^{-8}$$

$$= 0.14027 \times 10^{-2}$$

Ans.

(b) Truncation error when first four terms are added, is 0.6942×10^{-4}.

Q2- Find the maximum absolute error in computing $u = \frac{x^3 y^3}{z}$, when $x = y = z = 0.1$ and $\Delta x = \Delta y = \Delta z = 0.002$.

Solution: for a function $u = f(x, y, z)$, the absolute error Δu can be approximated using the partial derivatives of u with respect to x, y and z as follow:

$$\Delta u = \left|\frac{\partial u}{\partial x}\right| \Delta x + \left|\frac{\partial u}{\partial y}\right| \Delta y + \left|\frac{\partial u}{\partial z}\right| \Delta z \quad \text{--- } (1)$$

Partial derivative w.r.t x,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{x^3 y^3}{z}\right) = \frac{3x^2 y^3}{z} \quad \text{--- } (2)$$

Partial derivative w.r.t y,

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{x^3y^3}{z}\right) = \frac{3x^3y^2}{z} \quad \text{—— (b)}$$

Partial derivative w.r.t z,

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(\frac{x^3y^3}{z}\right) = \frac{-x^3y^3}{z^2} \quad \text{—— (c)}$$

Now putting the values of ⓐ, ⓑ & ⓒ in eq — ①

$$\Delta u = \left|\frac{\partial u}{\partial x}\right| \Delta x + \left|\frac{\partial u}{\partial y}\right| \Delta y + \left|\frac{\partial u}{\partial z}\right| \Delta z$$

$$= \left|\frac{3x^2y^3}{z}\right| \times \Delta x + \left|\frac{3x^3y^2}{z}\right| \times \Delta y + \left|\frac{-x^3y^3}{z^2}\right| \times \Delta z$$

We have, $x = y = z = 0.1$ and $\Delta x = \Delta y = \Delta z = 0.002$

So, we get,

$$\Delta u = \frac{3 \times 0.1^2 \times 0.1^3}{0.1} \times 0.002 + \frac{3 \times 0.1^3 \times 0.1^2}{0.1} \times 0.002 + \frac{0.1^3 \times 0.1^3}{0.1^2} \times 0.002$$

$$= 6 \times 10^{-7} + 6 \times 10^{-7} + 2 \times 10^{-7}$$

$$= 14 \times 10^{-7} \text{ Ans.}$$

So the maximum absolute error in computing u is $\Delta u \approx 14 \times 10^{-7}$

and Maximum Relative Error, $= \frac{\Delta u}{u}$

We know, $u = \frac{x^3 y^3}{z} = \frac{0.1^3 \times 0.1^3}{0.1} = 1 \times 10^{-5}$

Thus, Maximum relative error $= \frac{\Delta u}{u}$

$$= \frac{14 \times 10^{-7}}{10^{-5}}$$

$$= 14 \times 10^{-2}$$

Ans.

Percentage error $= \frac{\Delta u}{u} \times 100 = (14 \times 10^{-2} \times 100)\%$

$= 14\%$

Q-3: Determine the absolute Error E_A of the following approximate number given their relative error $X_A = 67.84$, $E_R = 1\%$

Solution: We Know,

$$E_R = \frac{E_A}{X_a}$$

$$\Rightarrow \frac{1}{100} = \frac{E_A}{67.84}$$

$$\Rightarrow E_A = \frac{67.84}{100} = 0.6784$$

Ans

Homework Problem 1

Q-3: Find the absolute, relative and percentage error in computing $u = \frac{7xy}{z^3}$, when $x = y = z = 1$ and $\Delta x = \Delta y = \Delta z = 0.001$.

Ans: Absolute Error = 0.035

Relative Error = 0.005

Percentage Error = 0.5%

Solution of Homework Problem 1 :

For the function $u = f(x, y, z)$ the absolute error Δu can be approximated using the partial derivatives of u with respect to x, y and z.

$$\Delta u = \left|\frac{\partial u}{\partial x}\right| \Delta x + \left|\frac{\partial u}{\partial y}\right| \Delta y + \left|\frac{\partial u}{\partial z}\right| \Delta z \quad \text{—— (1)}$$

Partial derivatives w.r.t x, y & z are

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{7xy}{z^3}\right) = \frac{7y}{z^3} \quad \text{—— (a)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{7xy}{z^3}\right) = \frac{7x}{z^3} \quad \text{—— (b)}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(\frac{7xy}{z^3}\right) = -\frac{3 \times 7xy}{z^4} = -\frac{21xy}{z^4} \quad \text{—— (c)}$$

Now putting value of equations ⓐ, ⓑ, ⓒ in equation ①

$$\Delta u = \left|\frac{\partial u}{\partial x}\right| \Delta x + \left|\frac{\partial u}{\partial y}\right| \Delta y + \left|\frac{\partial u}{\partial z}\right| \Delta z$$

$$= \frac{7y}{z^3} \Delta x + \frac{7x}{z^3} \Delta y + \frac{21xy}{z^4} \Delta z \qquad \left(\because \left|-\frac{21xy}{z^4}\right| = +\frac{21xy}{z^4}\right)$$

Now putting $x = y = z = 1$ and $\Delta x = \Delta y = \Delta z = 0.001$ in above equation we get,

$$\Delta u = \frac{7\times 1}{1} \times 0.001 + \frac{7\times 1}{1} \times 0.001 + \frac{21 \times 1 \times 1}{1} \times 0.001$$

$$= 7\times 10^{-3} + 7\times 10^{-3} + 21\times 10^{-3}$$

$$= 35\times 10^{-3} = 0.035$$

So, ① Absolute Error is 0.035 Ans

(2) Relative Error, $= \dfrac{\Delta u}{u}$

$$= \frac{0.035}{\frac{7xy}{z^3}} \quad \text{(substituting value of } x = y = z = 1\text{)}$$

$$= \frac{0.035}{7} = 0.005 = 5 \times 10^{-3} \text{ Ans}$$

(3) Percentage Error, $=$ Relative Error $\times$ 100%

$$= (0.005 \times 100)\%$$

$$= 0.5\% \text{ Ans}$$

Homework Problem - 2

Q-4 : Find the absolute, relative and percentage Error for the function, $u = \frac{5xy^2}{z^3}$ considered, $x = y = z = 1$ and $\Delta x = \Delta y = \Delta z = 0.001$

Answers :

Absolute Error $= 0.03$

Relative Error $= 0.006$

Percentage Error $= 0.6\%$

Solution of Homework Problem -2

We have, $u = \dfrac{5xy^2}{z^3}$

$$\Delta u = \left|\frac{\partial u}{\partial x}\right|\Delta x + \left|\frac{\partial u}{\partial y}\right|\Delta y + \left|\frac{\partial u}{\partial z}\right|\Delta z \quad \text{—— (1)}$$

Partial derivatives,

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left(\frac{5xy^2}{z^3}\right) = \frac{5y^2}{z^3} \quad \text{—— (a)}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\frac{5xy^2}{z^3}\right) = \frac{2 \times 5xy^3}{z^3} = \frac{10xy^3}{z^3} \quad \text{—— (b)}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(\frac{5xy^2}{z^3}\right) = -\frac{3 \times 5xy^2}{z^4} = \frac{-15xy^2}{z^4} \quad \text{—— (c)}$$

Putting values of ⓐ, ⓑ & ⓒ in equation ①,

we get,

$$\Delta u = \left|\frac{\partial u}{\partial x}\right|\Delta x + \left|\frac{\partial u}{\partial y}\right|\Delta y + \left|\frac{\partial u}{\partial z}\right|\Delta z \quad \text{——} ①$$

$$= \frac{5y^2}{z^3}\Delta x + \frac{10xy^3}{z^3}\Delta y + \frac{15xy^2}{z^4}\Delta z$$

We have, $x = y = z = 1$ & $\Delta x = \Delta y = \Delta z = 0.001$

$$\Delta u = \frac{5\times 1^2}{1^3}\times 0.001 + \frac{10\times 1\times 1^3}{1^3}\times 0.001 + \frac{15\times 1\times 1^2}{1^4}\times 0.001$$

$$= 5\times 10^{-3} + 10\times 10^{-3} + 15\times 10^{-3}$$

$$= 30\times 10^{-3}$$

$$= 0.03$$

Hence, Absolute Error, $\Delta u = 0.03$ Ans.

Now, Relative Error $= \dfrac{\Delta u}{u}$

$$= \frac{0.03}{\frac{5xy^2}{z^3}} \qquad \left[\because u = \frac{5xy^2}{z^3}\right]$$

$$= \frac{0.03}{5} \qquad [x = y = z = 1]$$

$$= 6 \times 10^{-3} \text{ or } 0.006 \text{ Ans.}$$

And Percentage Error $= (0.006 \times 100)\%$

$$= 0.6\% \text{ Ans.}$$

Homework Problem - 3

Q-3 : A civil engineer has measured the height of a 10 floor building as 2950 cm and the working height of each beam as 35 cm while the true values are 2945 cm and 30 cm, respectively. Compare their absolute and relative errors.

Solution : Measured height = 2950 cm

True height = 2945 cm

Absolute error in measuring the height of the buiding is,

$$e_a = |2950 - 2945| = 5 \text{ cm}$$

So, Relative Error is, $\dfrac{e_a}{\text{true value}}$

$$= \frac{5}{2945} = 0.0017$$

Percentage Error, $= (0.0017 \times 100)\%$

$$= 0.17\%$$

Now working with height of Each beam,

Measured height $= 35$ cm

True height $= 30$ cm

Absolute Error, $= |35 - 30| = 5$ cm

Relative Error $= \dfrac{5}{30} = 0.167$

Percentage Error $= (0.167 \times 100)\% = 16.7\%$

Although the absolute errors are the same, the relative errors differ by 100 times

It shows that there is something wrong in the measurement of height of the beam.

It should be done more accurately.