Topic to disscuss

- $\rightarrow \Delta \cdot \nabla = \Delta - \nabla$
- $\rightarrow (1+\Delta)(1-\nabla) = 1$
- $\rightarrow \mu^2 = 1 + \frac{1}{4}\delta^2$
- $\rightarrow \delta = \nabla \cdot E^{\frac{1}{2}}$
- $\rightarrow E = e^{hD}$, $D = \frac{d}{dx}$

Q:1 Prove that $\Delta \cdot \nabla = \Delta - \nabla$

Solution:- $\Delta f(x) = f(x+h) - f(x)$

$\nabla f(x) = f(x) - f(x-h)$

We can write,

$$\Delta f(x) - \nabla f(x) = f(x+h) - f(x) - f(x) + f(x-h)$$

$$(\Delta - \nabla) f(x) = f(x+h) - 2f(x) + f(x-h) \quad \text{--- } (1)$$

Also, $\Delta \cdot \nabla f(x) = \Delta\left(f(x) - f(x-h)\right)$

$$= \Delta f(x) - \Delta f(x-h)$$

$$= f(x+h) - f(x) - f(x-h+h) + f(x-h)$$

$$= f(x+h) - 2f(x) + f(x-h) \quad \text{--- } (2)$$

So, from eqn. — (1) & (11), we get.

$$(\Delta - \nabla) f(x) = \Delta \cdot \nabla f(x)$$

$$\therefore \Delta - \nabla = \Delta \cdot \nabla$$

(Proved)

Q:2 : Show that, $(1+\Delta)(1-\nabla) = 1$

Solution: $\Delta f(x) = f(x+h) - f(x)$

$$\Delta f(x) = Ef(x) - f(x)$$

$$\Delta f(x) = (E-1) f(x)$$

$$\Delta = E - 1$$

$$\boxed{E = \Delta + 1}$$

Now, $(1+\Delta)(1-\nabla) f(x) = (1+\Delta)[f(x) - \nabla f(x)]$

$$= E(f(x) - f(x) + f(x-h))$$

$$= E(f(x-h))$$

$$= f(x-h+h)$$

$$(1+\Delta)(1-\nabla) \cdot f(x) = f(x)$$

$$(1+\Delta)(1-\nabla) = 1$$

Proved

Q:3 : Prove that, $\mu^2 = 1 + \frac{1}{4}\delta^2$

Solution: We know,

$$\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)$$

$$= E^{\frac{1}{2}} f(x) - E^{-\frac{1}{2}} f(x)$$

$$\delta f(x) = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right) f(x)$$

$$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$$

squaring both side,

$$\delta^2 = \left(E^{\frac{1}{2}} - E^{-\frac{1}{2}}\right)^2$$

$$\delta^2 = E + E^{-1} - 2$$

$$E + E^{-1} = \delta^2 + 2 \quad \text{—— } 1$$

Again, we know,

$$\mu = \frac{1}{2}\left[f\left(x+\frac{h}{2}\right) + f\left(x-\frac{h}{2}\right)\right]$$

$$\mu = \frac{1}{2}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right]$$

Squaring both side,

$$\mu^2 = \frac{1}{4}\left[E^{\frac{1}{2}} + E^{-\frac{1}{2}}\right]^2$$

$$\mu^2 = \frac{1}{4}\left[E + E^{-1} + 2\right]$$

$$= \frac{1}{4}\left[\delta^2 + 2 + 2\right]$$

$$= \frac{1}{4}\left[\delta^2 + 4\right]$$

$$\boxed{\mu^2 = 1 + \frac{\delta^2}{4}}$$

Proved)

Q:4 Prove the relation, $\delta = \nabla . E^{\frac{1}{2}}$

Solution: $\delta f(x) = f\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$

$= E^{\frac{1}{2}} f(x) - E^{-\frac{1}{2}} f(x)$

$\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

$= E^{\frac{1}{2}}\left(1 - E^{-1}\right)$ $\quad [\because E^{-1} = 1 - \nabla]$

$= E^{\frac{1}{2}} \times \nabla$

$$\boxed{\delta = \nabla . E^{\frac{1}{2}}}$$

Q:5 Prove that, $E = e^{hD}$, $D = \frac{d}{dx}$

Solution: We know,

$$Ef(x) = f(x+h)$$

$$= f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots$$

(By tailors theorem)

$$= f(x) + hDf(x) + \frac{h^2}{2!} \times D^2 f(x) + \frac{h^3}{3!} D^3 f(x) + \cdots$$

$$Ef(x) = \left(1 + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \cdots\right) f(x)$$

$$E = 1 + hD + \frac{h^2D^2}{2!} + \frac{h^3D^3}{3!} + \cdots$$

$$E = e^{hD}$$

(Proved)

Connect with me

Start Practicing

i._am._arfin

Arfin Parween

Arfin Parween (Subscribe here too)

Arfin Parween

Arfin Parween