Formulation of Equations

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Let $u(x,t)$ be the population density of the prey. By conservation of mass,

$$u_t + \nabla \cdot J_u = R_u$$

where J_u is the flux and R_u is the local birth/death interactions. From Fick's Law, the diffusion term of J_u is given as $-D_u \nabla u$ (we will accept D_u as constant). If individuals drift with velocity field $V_u(x)$ then the advection term of J_u is given by $V_u u$. Thus, $J_u = V_u u - D_u u$.

We will assume logistic population growth that is $u_t = r(u)u$ where

$$r(u) = r_{max}\frac{K-u}{K}$$

where K is the carrying capacity. We may rescale K to 1 so that $u_t = u(1-u)$. The predation term is given by $-\alpha uv$ where v is the population density of the predator species. Then

$$R_u = u(1-u) - \alpha uv$$

Similarly,

$$R_v = \beta uv - \gamma v.$$

So our equations are

$$u_t - D_u \Delta u + V_u \cdot \nabla u = u(1-u) - \alpha uv \tag{1}$$
$$v_t - D_v \Delta v + V_v \cdot \nabla v = \beta uv - \gamma v \tag{2}$$