Johns Hopkins University
Department of Applied Mathematics & Statistics
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Fall 2015

EN 443
Financial Computing in C++
Assignment 3

due on Wed, Sep 30, 1:30pm

1. Write a function that reverse a string. The function should allocate and return a new sting (char * array), and may not manipulate the original string. The user would then be responsible for the deallocation. Please do not use any of the standard string functions.

   ```
   char * stringReverse(const char *st);     // allocates a new string, no manipulations on st
   ```

2. Implement a template function to perform a merge sort to sort an array. Merge sort consists of the following recursion steps.

   (a) If the array has zero, or one element, return the array. Otherwise, divide the array into two (almost) equal sub-arrays.

   (b) Sort each of the sub-arrays. This is done by applying recursively step 1, to each sub-array.

   (c) Merge the two sorted sub-arrays back into one array.

3. Cholesky decomposition of a symmetric positive definite matrix $\mathbb{A}$ is $\mathbb{A} = \mathbb{L}\mathbb{L}^T$, where $\mathbb{L}$ is a lower triangular matrix with strictly positive diagonal entries.

   For example of

$$\mathbb{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{12} & a_{22} & a_{23} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} = \mathbb{L}\mathbb{L}^T = \begin{pmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{pmatrix} \begin{pmatrix} l_{11} & l_{21} & l_{31} \\ 0 & l_{22} & l_{32} \\ 0 & 0 & l_{33} \end{pmatrix}.$$

   From here it's easy to see that $l_{11} = \sqrt{a_{11}}$, $l_{21} = \frac{a_{12}}{l_{11}}$, $l_{31} = \frac{a_{13}}{l_{11}}$ and so on.

   The general algorithm is:

$$\begin{aligned} l_{jj} &= \sqrt{a_{jj} - \sum_{k=1}^{j-1} l_{jk}^2}, \quad j = 1, .., n, \\ l_{ij} &= \frac{1}{l_{jj}} a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk}, \quad 1 \le j < i \le n. \end{aligned}$$

4. Calculate the implied volatility as a function of strike for MSFT stock priced \$28 from the following put options expiring in one year (i.e. T=1) assuming zero interest rate and no dividends

Strike	Price
17.5	\$0.28
20	\$0.48
22.5	\$0.81
25	\$1.41
27.5	\$2.42
30	\$3.8
32.5	\$5.56

For this problem you should write a function to find the implied volatility implementing the bisection method discussed in class. Recall that the Black-Scholes price for put option is given by

$$P = K\,\mathrm{e}^{-rT}\,\Phi(-d_-) - S_0\Phi(-d_+),$$

where Φ is the standard normal c.d.f., that was implemented in Assignment 3. Comment on the speed of convergence. How did you choose the initial bounds? Graph the implied volatility in Excel. For this question, you should submit both the Excel spreadsheet and your code.