PROJECT 2

Explicit Compressible Euler Solver

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GROUP 10

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Van Leer Method

$$\bar{F}_{i+\frac{1}{2}} = A_{i+\frac{1}{2}} \left[\rho_i a_i M_i^{+} \begin{pmatrix} 1 \\ u \\ h_o \end{pmatrix}_i + \rho_{i+1} a_{i+1} M_{i+1}^{-} \begin{pmatrix} 1 \\ u \\ h_o \end{pmatrix}_{i+1} \right], \qquad |M| < 1$$

$$\bar{F}_{i+\frac{1}{2}} = \bar{F}_{i+1}, \qquad M_i < -1 \; M_{i+1} < -1$$

$$\bar{F}_{i+\frac{1}{2}} = \bar{F}_i, \qquad M_i > 1 \quad M_{i+1} > 1$$

$$C_{VL}^{+} = \alpha_i^{+} \, (1+\beta)\mathrm{M_i} - 0.25 \times \beta_i (1 + M_i^2)$$

$$C_{VL}^{-} = \alpha_{i+1}^{-} \, (1+\beta_{i+1})\mathrm{M_{i+1}} + 0.25 \times \, (\beta_{i+1} 1 + M_{i+1}^2)$$

$$C^{+} = C_{VL}^{+} \; ; C^{-} = C_{VL}^{-} \qquad M_i^{\pm} = \pm \frac{1}{4} (M_i \pm 1)^2$$

Feature

The Van Leer flux formulation does not preserve stationary contact discontinuties

AUSM Method

ADDITION TERM FOR FLUX

$$C^{+} = \max[0, C_{VL}^{+} + C_{VL}^{-}]$$

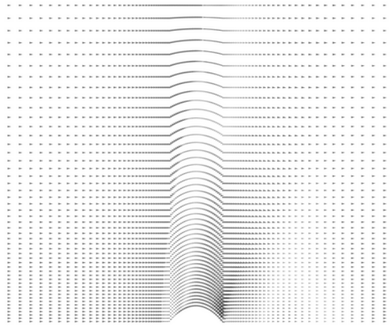
$$C^{-} = min[0, C_{VL}^{+} + C_{VL}^{-}]$$

Feature

AUSM preserves stationary contact discontinuous but gives rise to non monotonicity

Parameters Used:

CFL = 0.2 [CFL < 0.5]
convergence limit = $10^{(-6)}$
Temperature = 300 K
Mach=0.4, 0.8, 1.4
Pressure = 0.01 atmosphere
Ideal Gas Assumptions: P=ρRT



GRID PREVIEW

Time Stepping

$$\frac{\Delta V_i}{\Delta t_i} = \frac{1}{CFL} \times \left[\sum_K A_K \lambda_{max,K} \right]$$

$$\lambda_{max,K} = |u_k| + a$$

$$u_K = u_i n_x + v_i n_y$$

$$K = i + \frac{1}{2}, i - \frac{1}{2}, j + \frac{1}{2}, j - \frac{1}{2}$$

Boundary Condition:

Subsonic

Inflow

$$\begin{bmatrix} u_g \\ v_g \\ T_g \\ P_g \end{bmatrix} = \begin{bmatrix} u_\infty \\ v_{\infty=0} \\ T_\infty \\ P_I \end{bmatrix}$$

Outflow

$$\begin{bmatrix} u_g \\ v_g \\ T_g \\ P_g \end{bmatrix} = \begin{bmatrix} u_I \\ v_I \\ T_I \\ P_\infty \end{bmatrix}$$

Supersonic

Inflow

$$\begin{bmatrix} u_g \\ v_g \\ T_g \\ P_g \end{bmatrix} = \begin{bmatrix} u_\infty \\ v_\infty = 0 \\ T_\infty \\ P_\infty \end{bmatrix}$$

Outflow

$$\begin{bmatrix} u_g \\ v_g \\ T_g \\ P_g \end{bmatrix} = \begin{bmatrix} u_I \\ v_I \\ T_I \\ P_I \end{bmatrix}$$

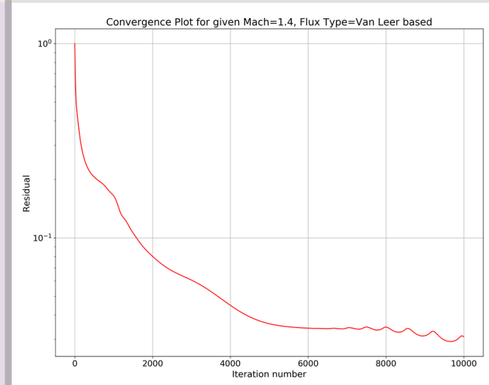
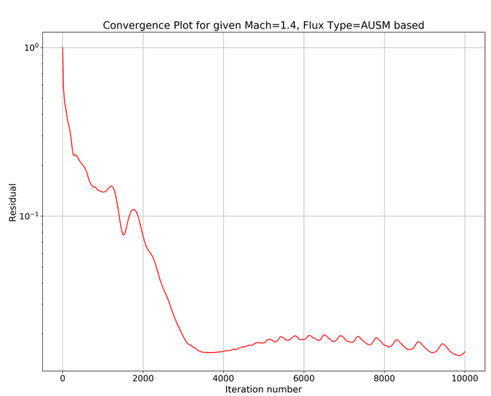
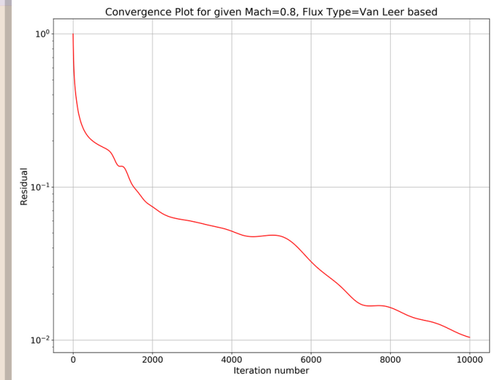
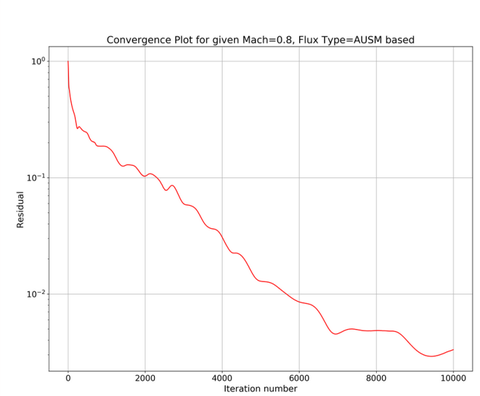
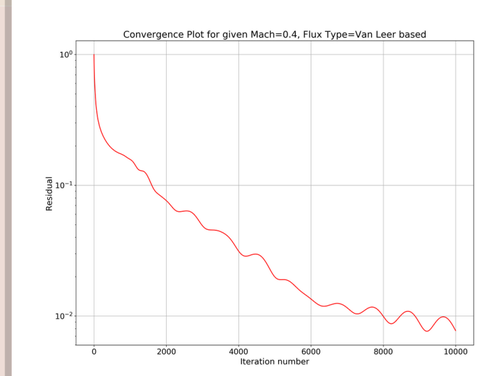
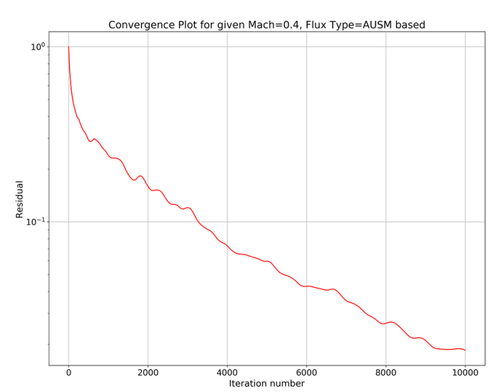
Initialization Condition:

Free-stream conditions are given for initialization

$$\begin{bmatrix} P \\ u \\ v \\ T \end{bmatrix} = \begin{bmatrix} \frac{P_\infty}{R \times T_\infty} \\ M_\infty \times a_\infty \\ 0 \\ T_\infty \end{bmatrix}$$

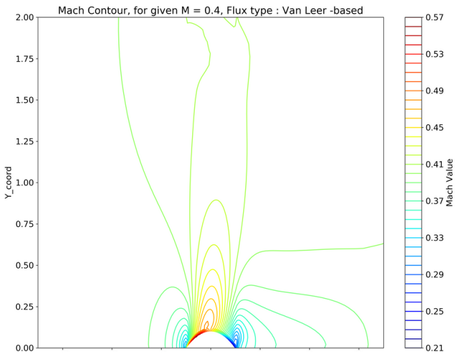
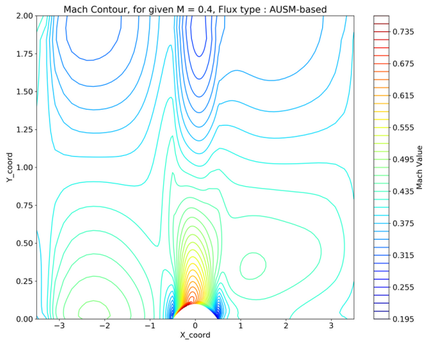
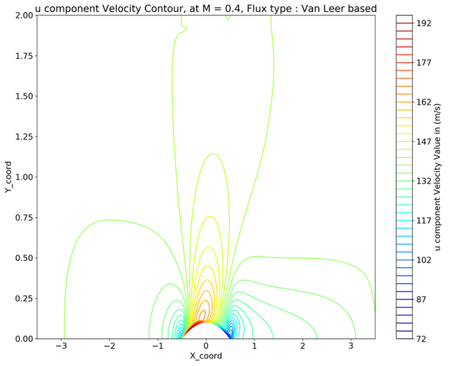
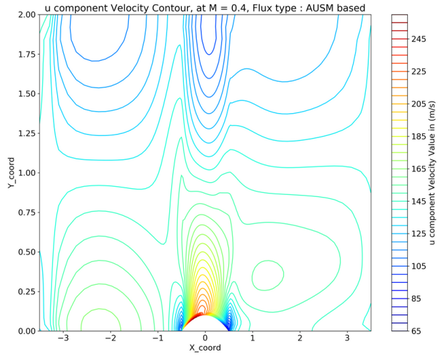
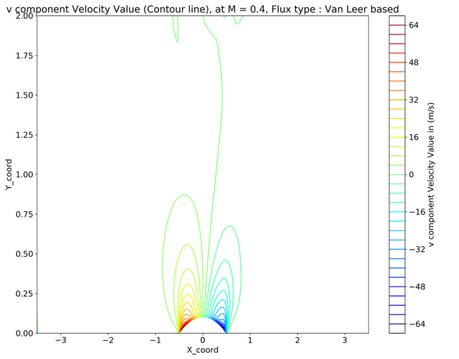
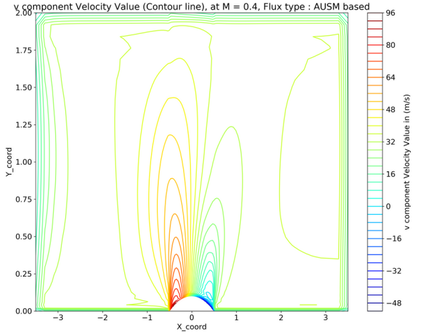
ANALYSIS

Convergence Histories

	VAN LEER	AUSM	Inference
M=1.4			• The Convergence for supersonic at lower iteration number is **faster** as compared to the subsonic case • The Convergence for Van Lear and AUSM is very similar **except having a larger difference in convergence at early number of iterations.**
M=0.8			• The Convergence for subsonic at lower iteration number is **slower** as compared to the supersonic case • The Convergence for Van Lear and AUSM is very similar except having a mild differences in convergence at an early number of iterations. **AUSM seems to reach convergence in more unsteady manner**.
M=0.4			• **Lower Mach** number leads to **less difference in AUSM and Van Leer convergence propagation**. • The Convergence for Van Lear and AUSM are extremely similar. It seems Van leer converge with more unsteady manner at lower Mach numbers

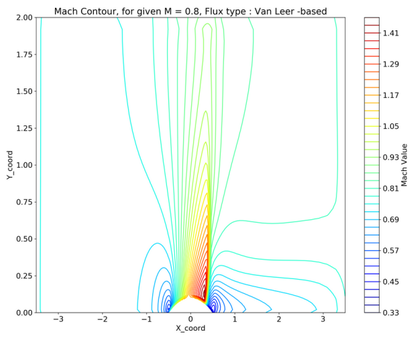
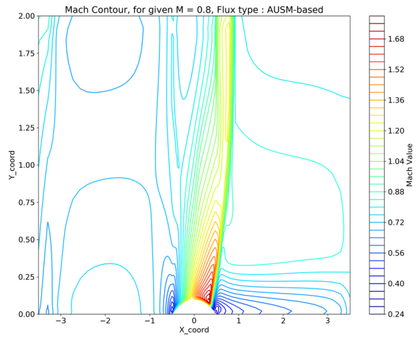
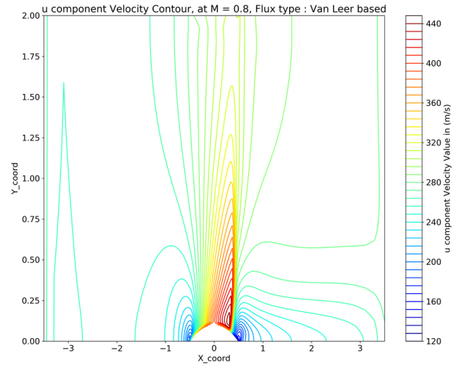
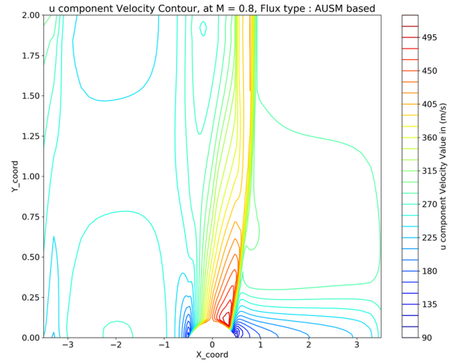
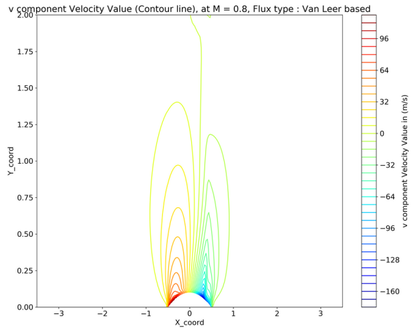
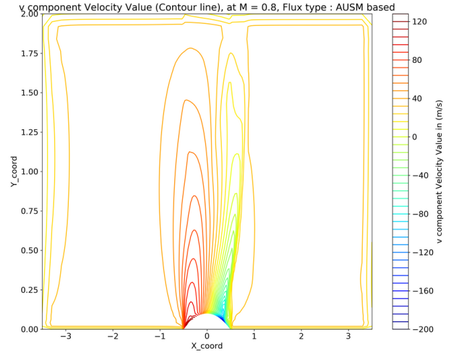
Analysis

Supersonic Case M=0.4 - Van Leer VS AUSM

	VAN LEER	AUSM	Inference
Mach			• AUSM and Van Leer have much different representations of contours for same configurations. • Mach number near the head of the bump is **higher and vanishes** as it moves further away. • Mach number near the corners of bump is diminishing and a gradient is observable.
U-Component			• The highest u-component value is present **near the head of the bump.** • The u-component near the head of the bump is higher and vanishes as it moves further away. • The u-component near the corners of the bump is **diminishing and a gradient** is observable.
V- Component			• The highest v-component value is present **only near the one corner side of the bump**. • The v-component near the head of the bump is higher and vanishes as it moves further away. • The v-component have **asymmetric gradient behavior** near the bump unlike U-component.

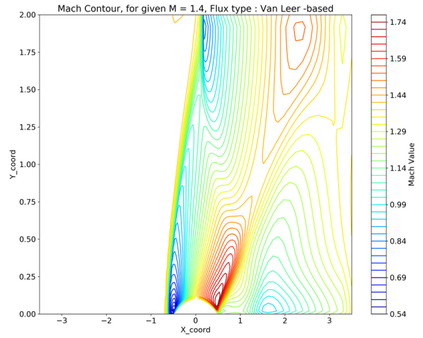
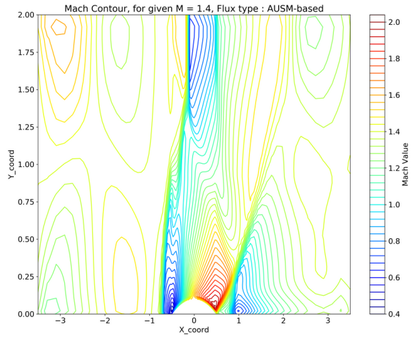
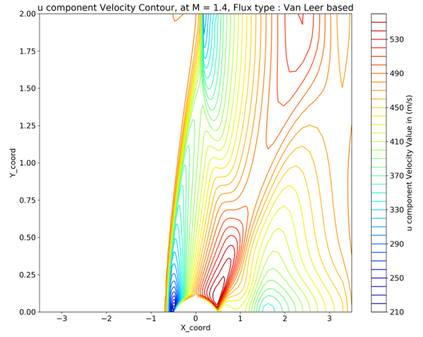
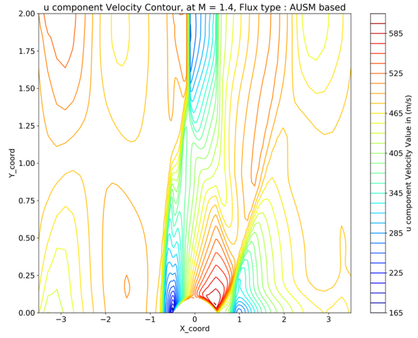
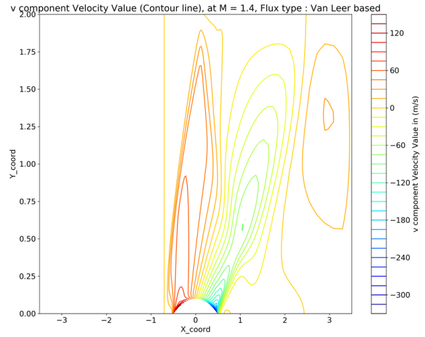
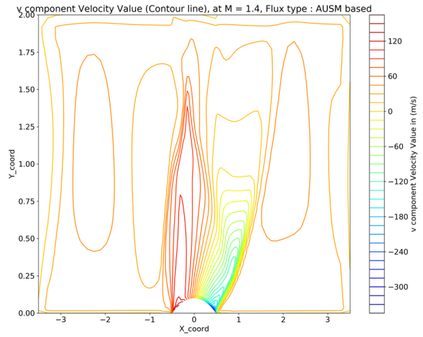
Analysis

Supersonic Case M=0.8 - Van Leer VS AUSM

	VAN LEER	AUSM	Inference
Mach			• As Mach increased, the **flow seems to be shifting right** side more (as compared to M=0.4) • Gradient in flow is observable as as Mach is doubled • Mach number near the corners still have same behavior. **AUSM is better in tracking contours**
U-Component			• As Mach increased, the U component seems to be shifting right side more (as compared to M=0.4) • Gradient in flow is observable as as Mach is doubled • U-component near the corners still have same behavior. **Contours are increased** both cases
V- Component			• As Mach increased, the V component value seems to be shifting left side more (as compared to M=0.4) • **Asymmetric Velocity Gradient** is observable as Mach is doubled near the bump • V-component have similar behavior. Contours are increased both cases

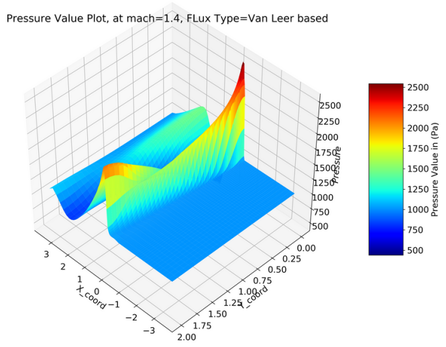
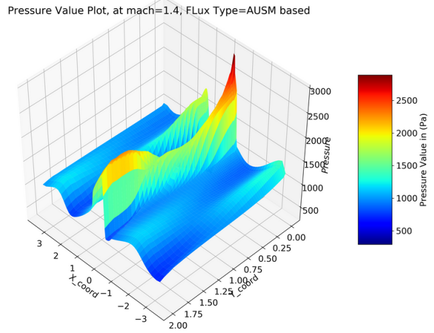
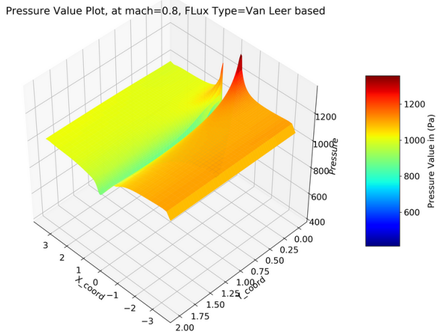
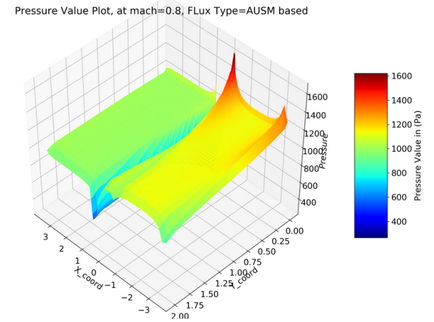
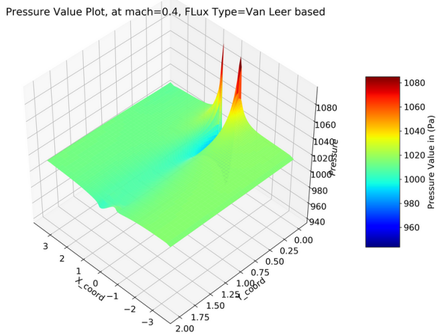
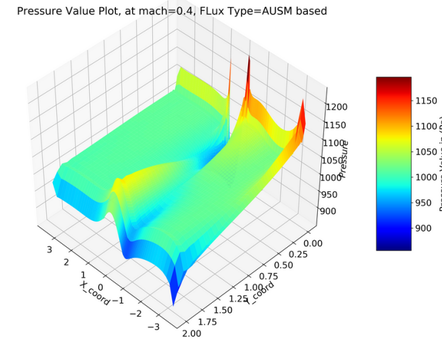
Analysis

Supersonic Case M=1.4 - Van Leer VS AUSM

	VAN LEER	AUSM	Inference
Mach			• As Mach reach supersonic, the **flow has shifted significantly to the right side** • The gradient inflow is remarkable as Mach is high • Mach numbers near the corners still have **asymmetric behavior. Shock can be with higher Mach**
U-Component			• As Mach reaches supersonic, the U component has shifted significantly to the right side. • **The gradient inflow is remarkable** as Mach is high • Mach number near the corners still have **significant asymmetric behavior unlike in subsonic case**
V- Component			• As Mach reaches supersonic, the V component has more intensity (more value near left corner) • The gradient inflow is remarkable as Mach is high • **V components have increased significantly** all over the domain. **AUSM is better in tracking contours**

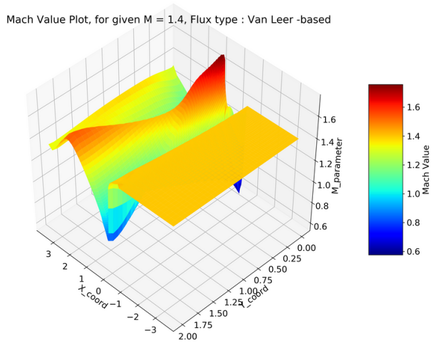
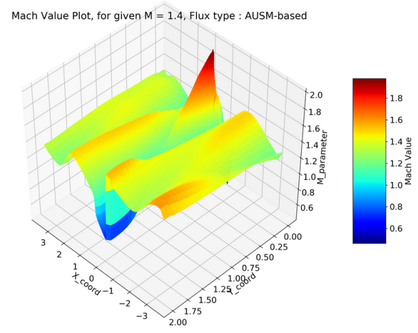
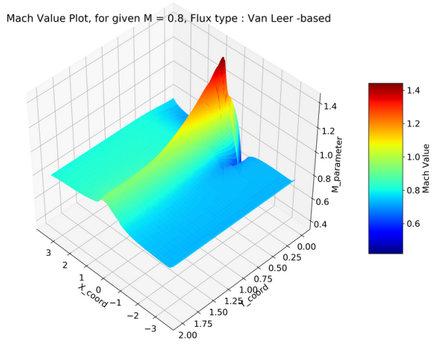
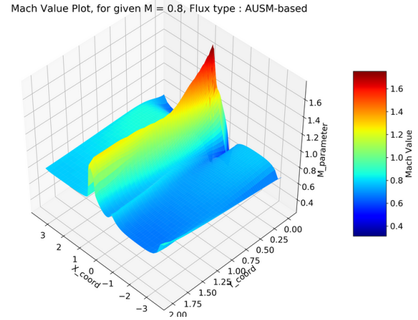
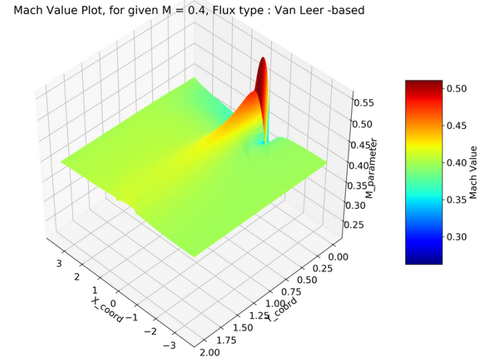
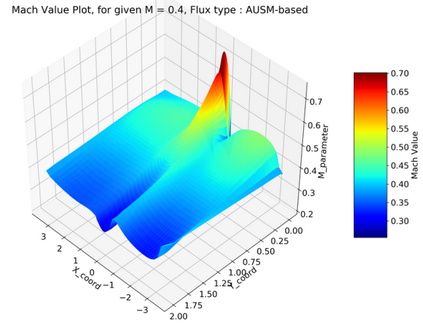
Analysis

Pressure Plot 3D - Van Leer VS AUSM

	VAN LEER	AUSM	Inference
M=1.4			• The 3D variation of Pressure is represented. This suggests **pressure be higher near the head of the bump for supersonic** configuration. • The pressure diminishes going away from the head of bump.
M=0.8			• Pressure has a **high range of variation over the domain** near sonic conditions (Mach=0.8) • Pressure has **slight asymmetric behavior** from the head of bump.
M=0.4			• Pressure to be higher at near the head of the bump for a smaller subsonic range of Mach number. • **Pressure again seems to be symmetric** (except near sonic Mach number where the feature is asymmetric)

Analysis

Mach Plot 3D - Van Leer VS AUSM

	VAN LEER	AUSM	Inference
M=1.4			• The 3D variation of Mach is represented. This suggests **Mach be higher near the head of the bump for supersonic** configuration along with some variation. • **The variations are more better traced by AUSM**
M=0.8			• Near the **sonic Mach range, the Mach near the bump can still become supersonic.** • AUSM is suggesting the Mach is higher as compared to Van Leer method.
M=0.4			• Near **subsonic Mach range, the Mach near the bump can reach close to sonic range.** • Mach is diminishing as we go further away from the head of bump.

Conclusion

- AUSM provides better identification of shock presence.
- Convergence can be validated by using higher iterations for higher Mach numbers (as seen from convergence histories)
- AUSM have oscillatory behavior in convergence (which reduces if Mach is kept to be smaller within the subsonic range)
- As the free stream Mach number increase, the head of the bump attains the sonic Mach number before subsonic freestream conditions.
- As Mach increases, the Contour plot suggests the flow is shifting more towards the right direction.
- Mach, u-Component, v-component velocity vectors are generated for a bump with given free stream conditions ranging from subsonic to supersonic Mach numbers.