DIGITAL SIGNAL PROCESSING

FIR FILTER DESIGN

(Kaiser Windowing method)

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CONTENTS

1. ABSTRACT

Digital filters play an important role in the age of digital communication systems. Digital bandpass filter provides high attenuation to the unwanted frequency components in the stopband and offers ideally zero or significantly low attenuation to desired signal components in passband. The main objective of this project is to implement the Kaiser windowing method related to the given specifications. The rectangular method is also considered to compare the performances of the Kaiser method. Ideal bandpass filter responses are also taken into account to compare the quality of the filter output.

2. INTRODUCTION

A digital filter is a system that mathematically processes a sampled signal to reduce or enhance some aspects of that signal. The main goal of the filter is to remove the unwanted frequency components of the signal. The objective of this project is to design a digital band pass filter under the given specifications. The implementation of the filer is achieved using Matlab R2017b software.

Basically, there are two classical methods for designing non-recursive (FIR) filters. That is, by using the Fourier series in conjunction with a class of window functions and multivariable optimization method (weighted-Chebyshev method). In this implementation, the closed-form direct approach is used by following the Fourier series method with a Kaiser Windowing technique developed by James Kaiser. The characteristics of the Kaiser window can be adjusted by changing the parameters associated with α.

3. METHOD

3.1 Filter Implementation

3.1.1 Filter Specifications

The implementation of the digital band pass filter is done by using the Fourier series method with the Kaiser Windowing technique. The required parameters of the filter are determined using the specifications given in Table 1.

Table 1: Required Filter specifications.

Parameter	Value
Maximum pass band ripple - $\tilde{A}_p$	0.07dB
Minimum stop band attenuation - $\tilde{A}_a$	45 dB
Lower pass band edge - Ω_{p1}	1200 rad/s
Upper pass band edge - Ω_{p2}	1600 rad/s
Lower stop band edge - Ω_{a1}	1050 rad/s
Upper stop band edge - Ω_{a2}	1700 rad/s
Sampling frequency - Ω_s	4200 rad/s

3.1.2 Derivation of Filter Parameter

As shown in Table 2, the next parameters are determined using the above filter specifications.

Table 2: Derived Filter specifications.

Parameters	Derivation	Value
Sampling period - T	$= 2\pi/\Omega_s$	1.496 ms
Lower transition width - B_{t1}	$= \Omega_{p1} - \Omega_{a1}$	150 rad/s
Upper transition width - B_{t2}	$= \Omega_{a2} - \Omega_{p2}$	100 rad/s
Critical transition width - B_t	$= \min(B_{t1}, B_{t2})$	100 rad/s
Lower cutoff frequency - Ω_{c1}	$= \Omega_{p1} - B_t/2$	1150 rad/s
Upper cutoff frequency - Ω_{c2}	$= \Omega_{p2} + B_t/2$	1650 rad/s

3.1.3 Derivation of the Kaiser Window

There are several steps to implement the Kaiser window.

Step 1:

A parameter δ is defined such that the actual passband ripple (A_p) is less than or equal to the specified passband ripple ($\tilde{A}_p$) and the actual minimum stopband attenuation (A_a) is greater than or equal to the specified minimum stopband attenuation ($\tilde{A}_a$).

Define,

$$\delta = \min(\delta_p, \delta_a)$$

where, $\delta_p = \frac{10^{0.05A_p} - 1}{10^{0.05A_p} + 1}$ and $\delta_a = 10^{-0.05A_p}$

Step 2:

The actual stopband attenuation A_a and the actual passband ripple (A_p) can be calculated as,

$$A_a = -20\log(\delta) \quad \text{and} \quad A_p = 20\log\left(\frac{|1+\delta|}{|1-\delta|}\right)$$

Step 3:

Choose parameter α as,

$$\alpha = \begin{cases} 0 & ; \ for\ A_a \le 21 \\ 0.5842(A_a - 21)^{0.4} + 0.07886(A_a - 21) & ; \ for\ A_a < 21 \le 50 \\ 0.1102(A_a - 8.7) & ; \ for\ A_a > 50 \end{cases}$$

Step 4:

Choose parameter D as,

$$D = \begin{cases} 0.9222 & , \quad for\ A_a \le 21 \\ \dfrac{A_a - 7.95}{14.36} & , \quad for\ A_a > 21 \end{cases}$$

Then select the lowest odd value (N) that satisfies the following inequality,

$$N \ge \frac{\omega_s D}{B_t} + 1$$

Step 5:

Finally the Kaiser function $w_K(nT)$ is obtained by,

$$w_K(nT) = \begin{cases} \dfrac{I_0(\beta)}{I_0(\alpha)} & ; \ for\ |n| \leq (N-1)/2 \\ 0 & ; \quad otherwise \end{cases}$$

where

$$\beta = \alpha\sqrt{1-\left(\frac{2n}{N-1}\right)^2} \qquad and \qquad I_0(x) = 1 + \sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2$$

The above parameters calculated relative to our bandpass filter are shown in Table 3.

Table 3: Kaiser Window parameters

Parameter	Value
δ_p	0.00403
δ_a	0.00562
δ	0.00403
Aa	47.895 dB
Ap	0.07 dB
α	4.3008
D	2.27817
N	119

3.1.4 Derivation of the Ideal impulse response

The ideal frequency response of the bandpass filter with cut-off frequencies Ω_{c1} and Ω_{c2} is deduced as,

$$H(e^{j\Omega t}) = \begin{cases} 1 & ; \ for -\Omega_{c2} \leq \Omega \leq -\Omega_{c1} \\ 1 & ; \ for\ \Omega_{c1} \leq \Omega \leq \Omega_{c2} \\ 0 & ; \ otherwise \end{cases}$$

After applying the inverses Fourier transform,

$$h(nT) = \frac{1}{\Omega}\int_{\frac{-\Omega_s}{2}}^{\frac{\Omega_s}{2}} H(e^{j\Omega t})\, e^{j\Omega nt} d\Omega$$

$$h(nT) = \begin{cases} \dfrac{2}{\Omega_s}(\Omega_{c2} - \Omega_{c1}) & ; \ for\ n = 0 \\ \dfrac{1}{n\pi}(\sin(\Omega_{c2} nT) - \sin(\Omega_{c1} nT)) & ; \ otherwise \end{cases}$$

3.1.5 Obtaining the Impulse response of windowed filter

The impulse response of the windowed filter can be obtained by multiplying the ideal impulse response $h(nT)$ with the Kaiser window function $w_K(nT)$.

$$h_W(nT) = w_K(nT)h(nT)$$

The z-transform of $h_W(nT)$ is evaluated as,

$$H_w(z) = Z[h_W(nT)] = Z[w_K(nT)h(nT)]$$

The shifted $H_w(z)$ for causality is performed as,

$$H'_w(z) = z^{-(N-1)/2}H_w(z)$$

3.2 Input signal Generation and Filter Operation

The input signal is constructed using 3 sinusoidal waves with different angular frequencies to evaluate the filter performances. The angular frequencies are generated as follows in Table 4.

$$x(nT) = \sum_{i=1}^{3} \sin(\Omega_i nT)$$

Frequency component	Derivation	Value
Ω_1	$\frac{\Omega_{c1}}{2}$	575 rad/s
Ω_2	$\frac{\Omega_{c1}+\Omega_{c1}}{2}$	1400 rad/s
Ω_3	$\frac{\Omega_{c2} + {\Omega_s}/{2}}{2}$	1875 rad/s

Table 4

The output signal can be obtained by taking the convolution of the filter impulse response with the input signal impulse. But considering the inconvenience of the convolution method, we convert both signals into Discrete Fourier Transform (using FFT in Matlab) and multiply them in the frequency domain. Thereafter the output signal can be obtained by getting the Inverse DFT (using IFFT in Matlab).

4. RESULTS

The characteristics and input/output results of the filter are demonstrated in both frequency and time domains using Matlab plots. It is better to study the filter performances and minimize the errors in the design process by displaying intermediate results in the code.

4.1 Frequency and Time domain Representation plots of the Filter

The impulse response of the Kaiser window is shown in figure I. The causal impulse response of the filter is obtained in figure II and the magnitude response of the digital filter obtained for the frequency range 0 to $\frac{\Omega s}{2}$ rads^{-1} is presented in figure III. The magnitude response of the digital filter for the frequencies in the passband and the magnitude response of the passband ripples are shown in figure IV and figure V respectively. The impulse response and the frequency response of the filter with the rectangular windowing technique are shown in figure VI and figure VII. Figure VIII displays the frequency domain representation of the filter using Kaiser and Rectangular windowing techniques in the same plot.

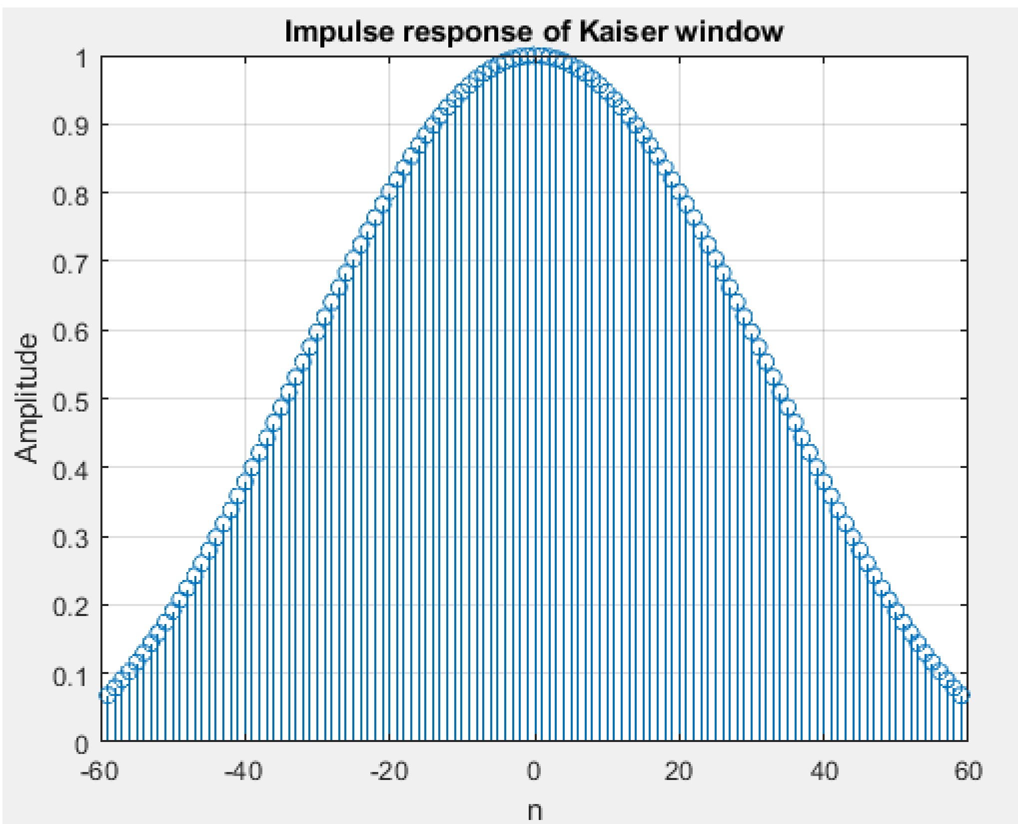


Figure I - Impulse response of the Kaiser window

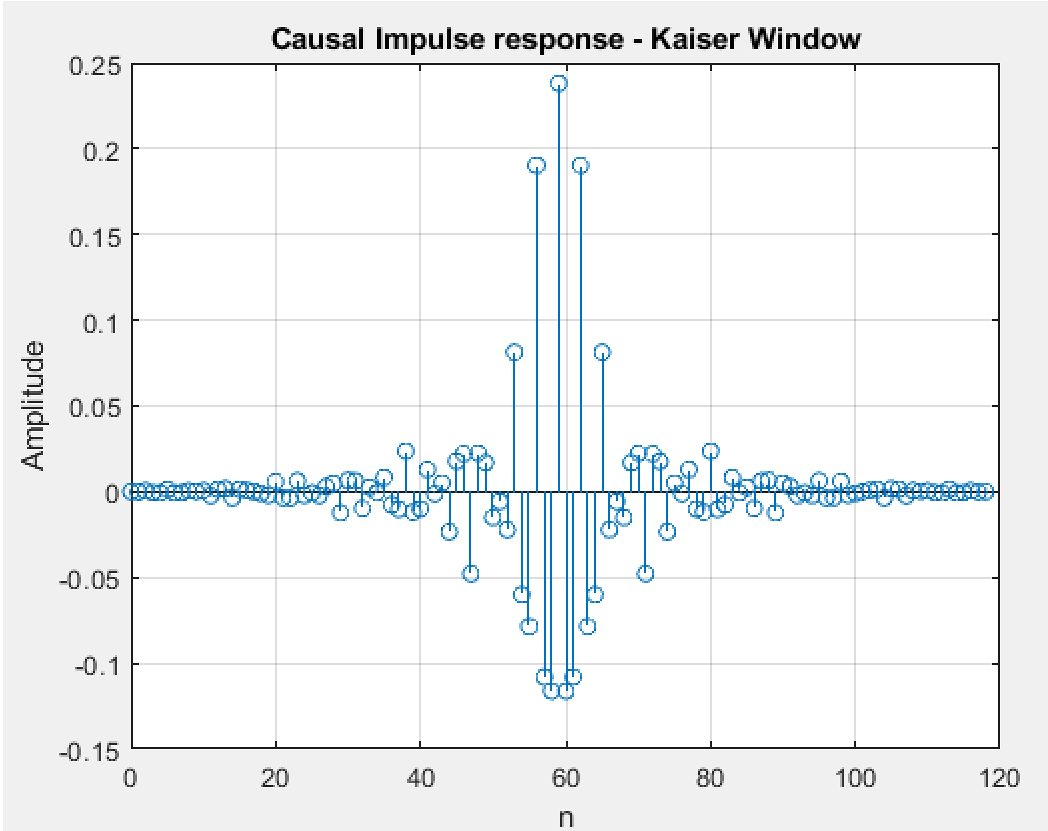


Figure II - Causal Impulse response of the filter

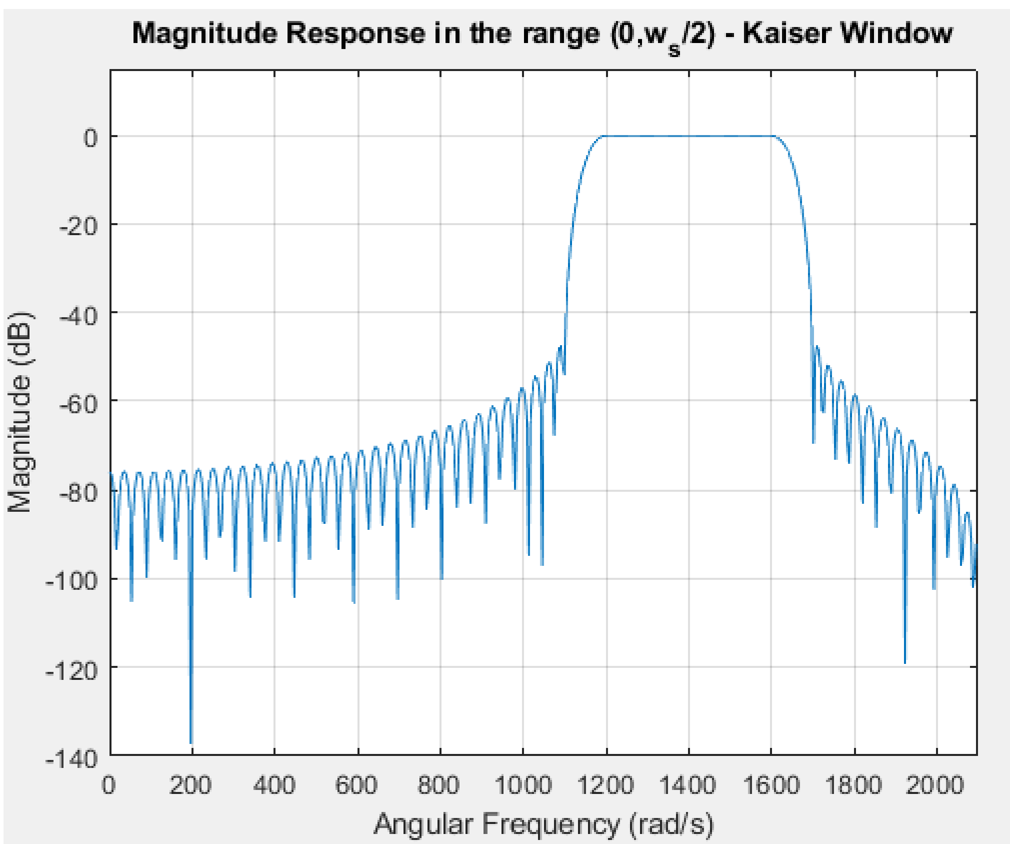


Figure III - Magnitude Response of the digital filter

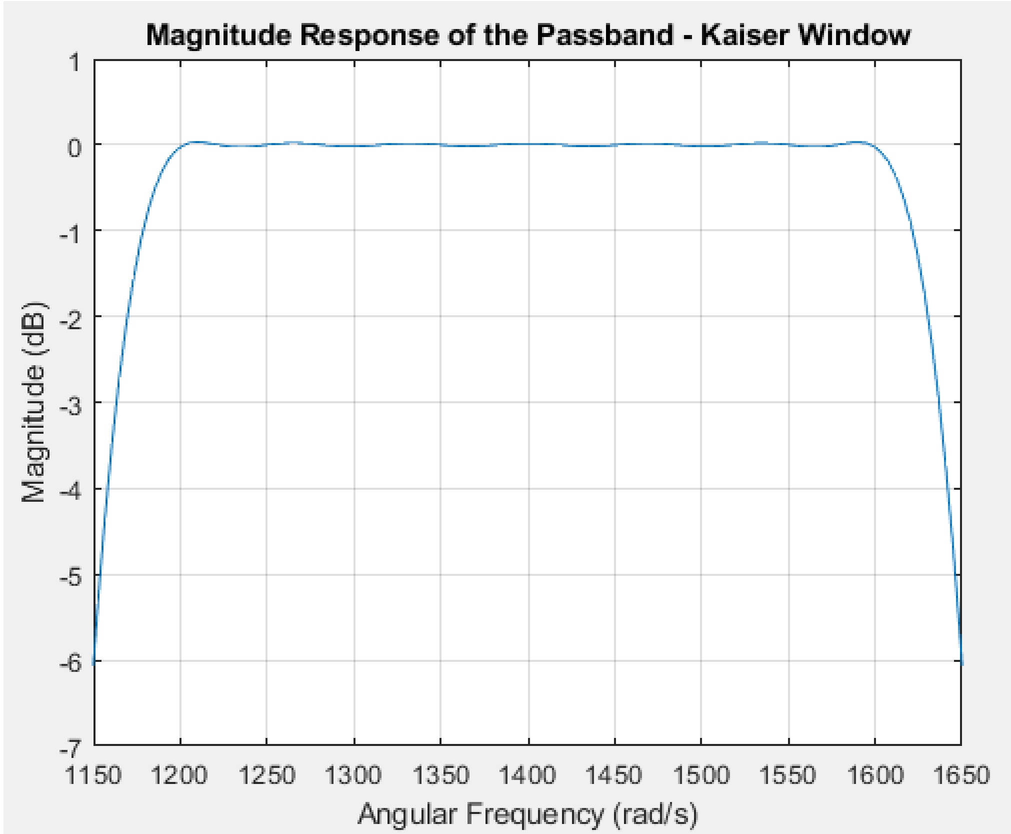


Figure IV - Magnitude Response of the Pasband

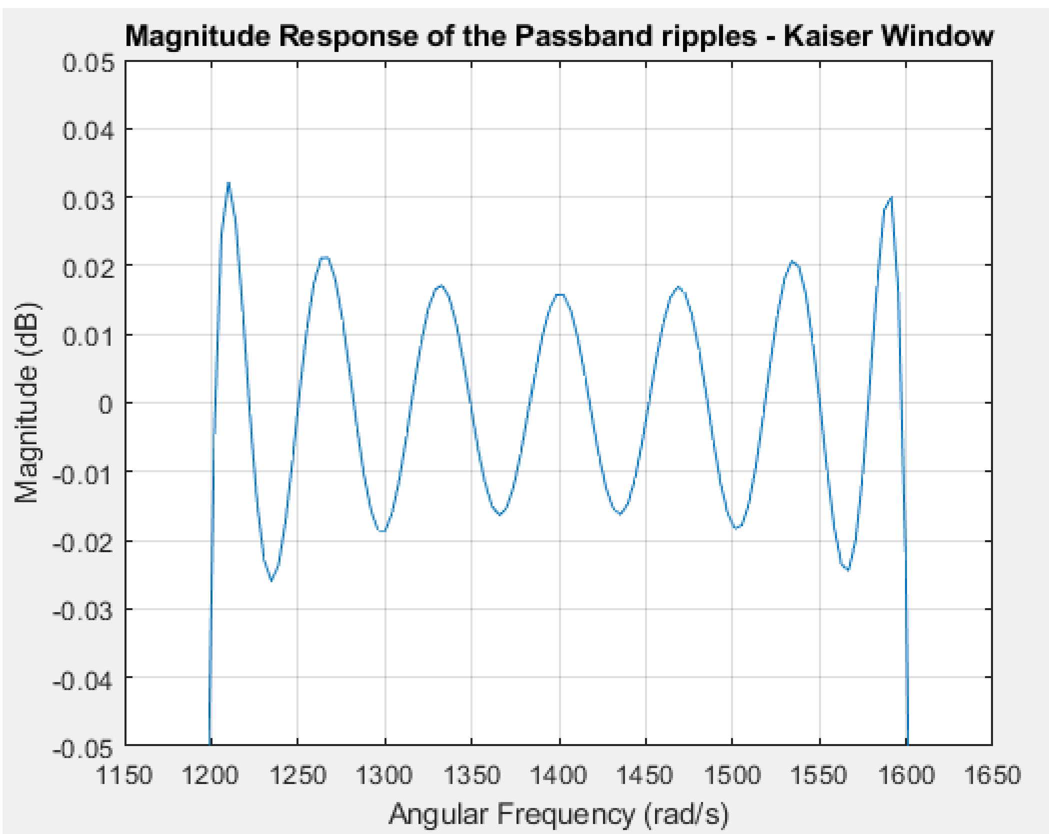


Figure V - Magnitude Response of the Pasband ripple

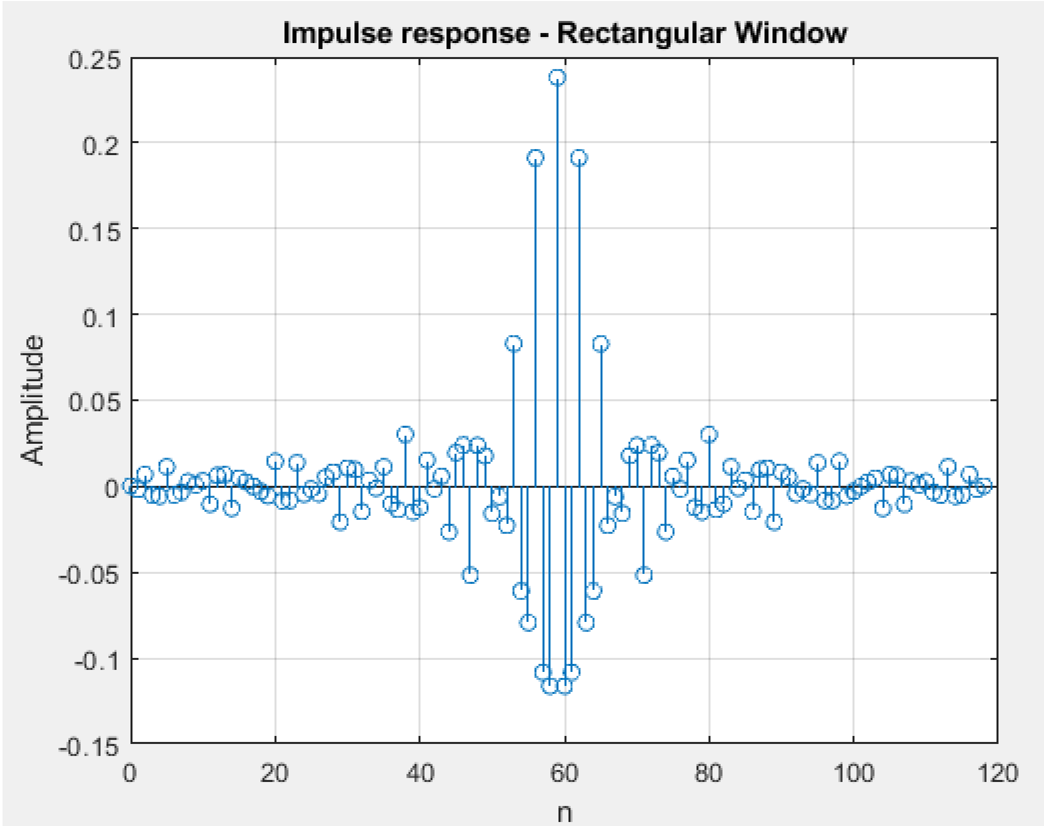


Figure VI – Impulse response of the filter using rectangular windowing technique

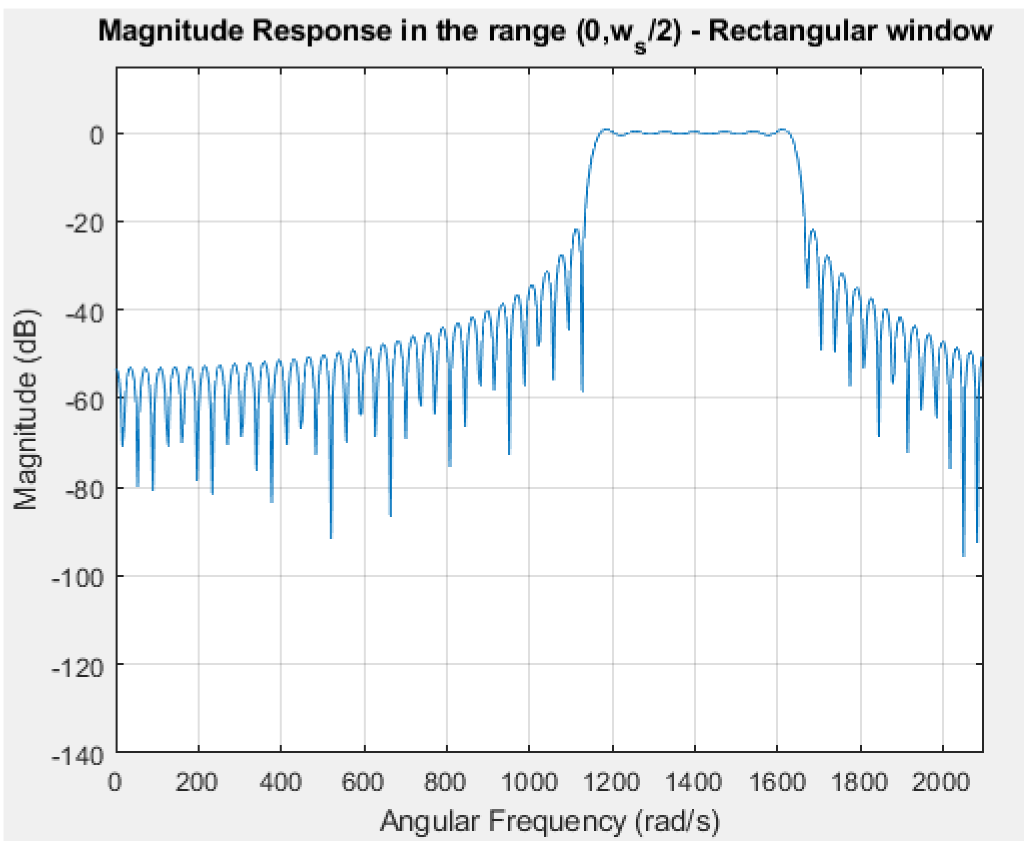
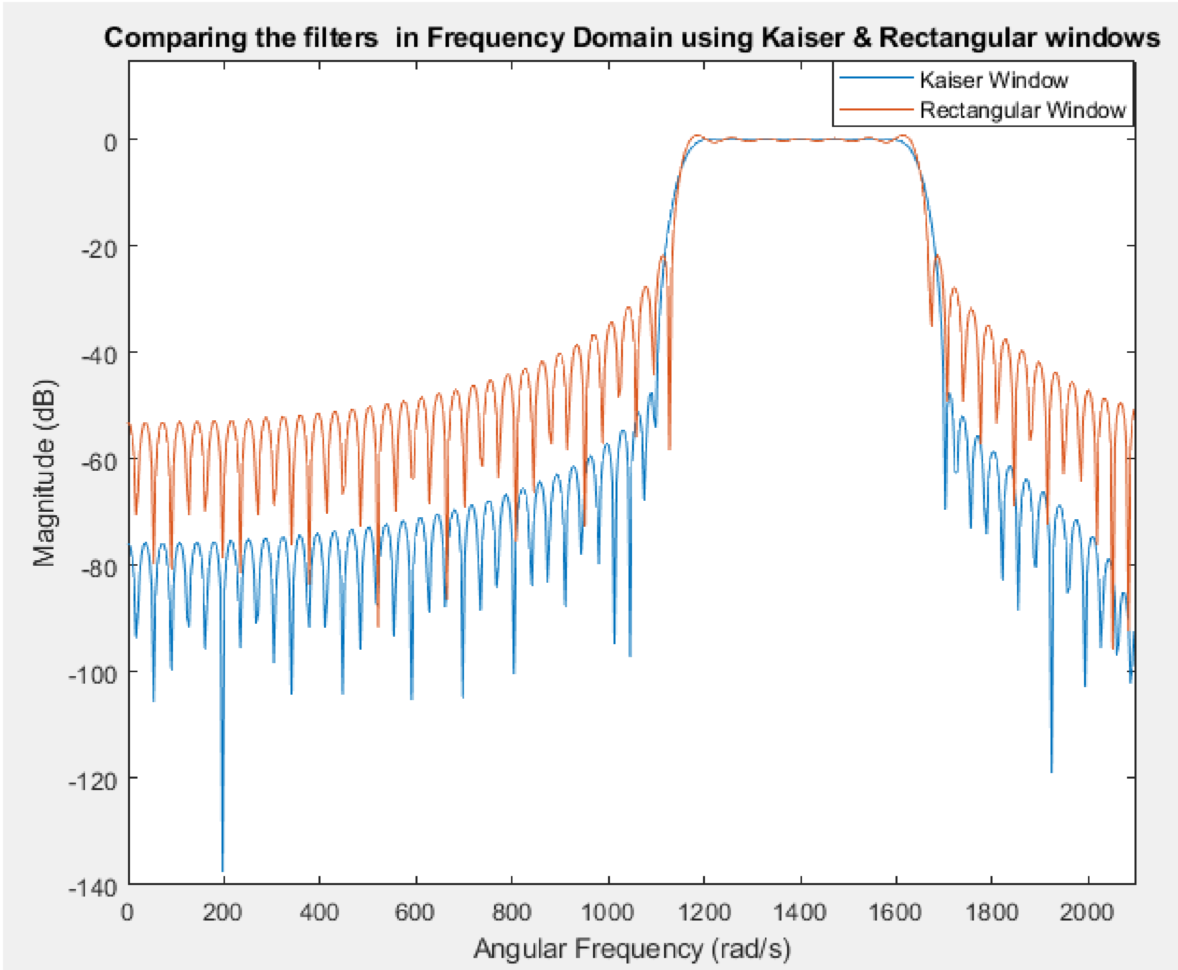


Figure VII - Frequency response of the filter using rectangular windowing technique



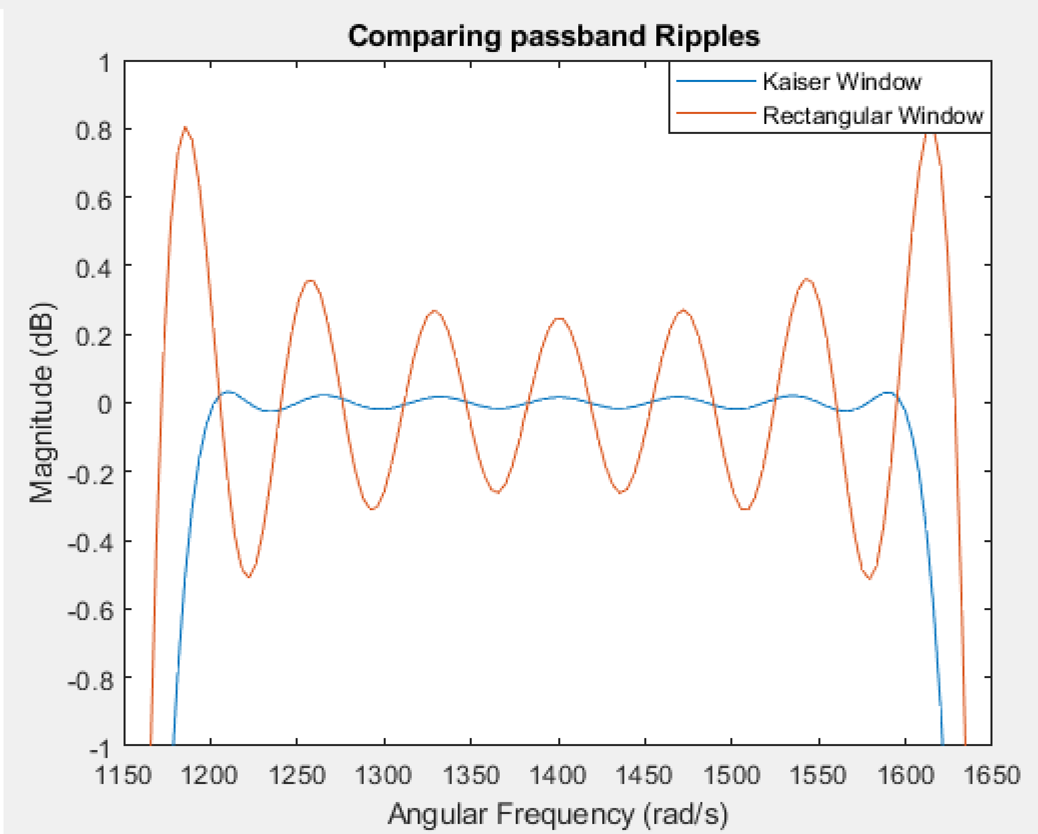


Figure VIII – Comparing the filters in Frequency Domain using Kaiser & Rectangular

4.2 Frequency and Time domain Representation plots of the Filter input and output signals.

The time and frequency domain representation of the input signal is given in figure IX. The dotted lines represent the continuous-time signal of the corresponding discrete-time signals plotted in same frequencies. Filter outputs and expected filter outputs (ideal filter responses) in the time domain and frequency domain are shown in figure X and figure XI respectively.

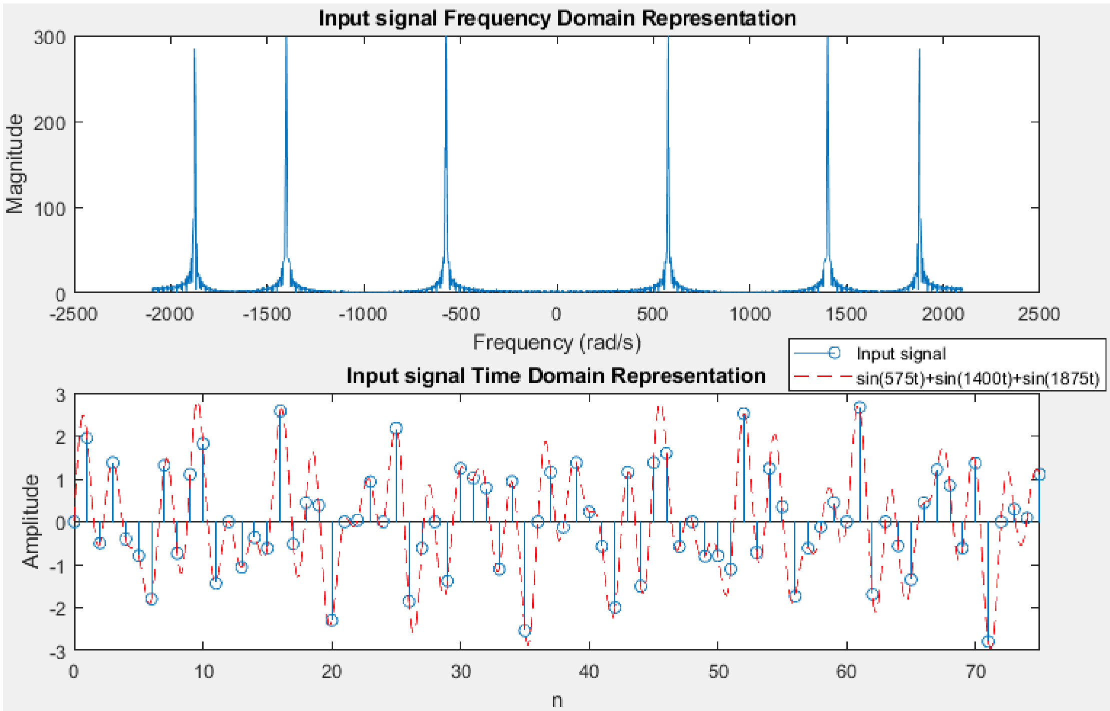


Figure IX - The Frequency and Time domain representation of the input signal

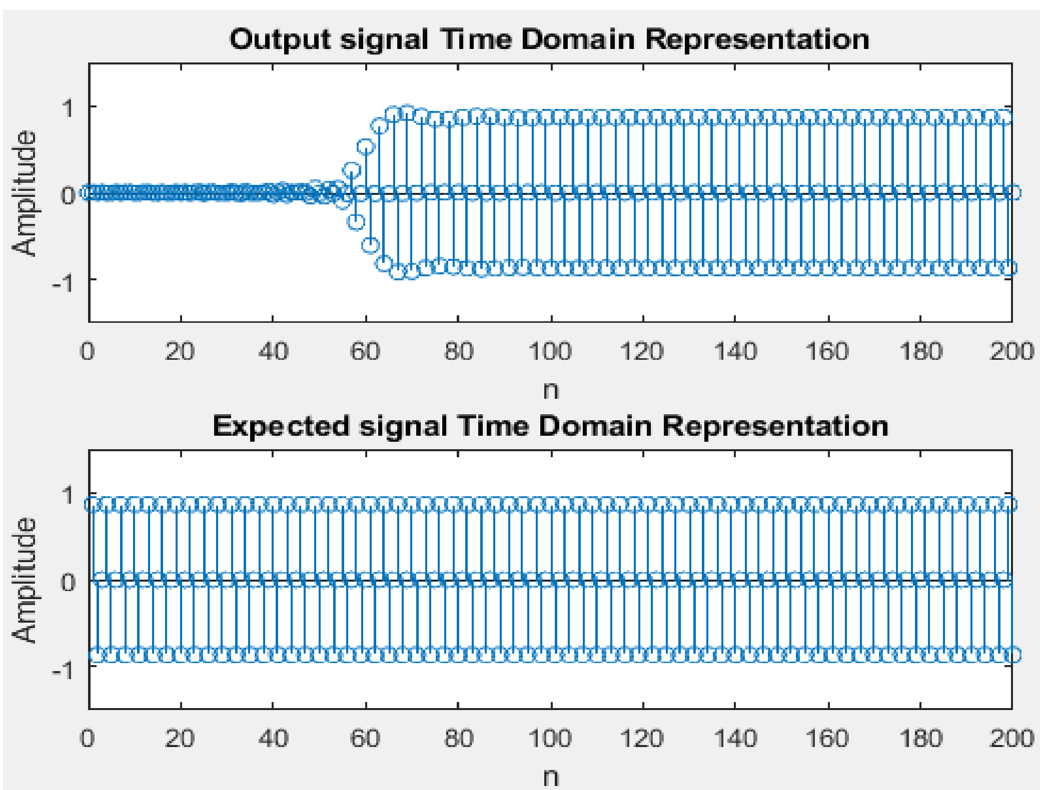


Figure X - Filter output and expected output signal in time domain

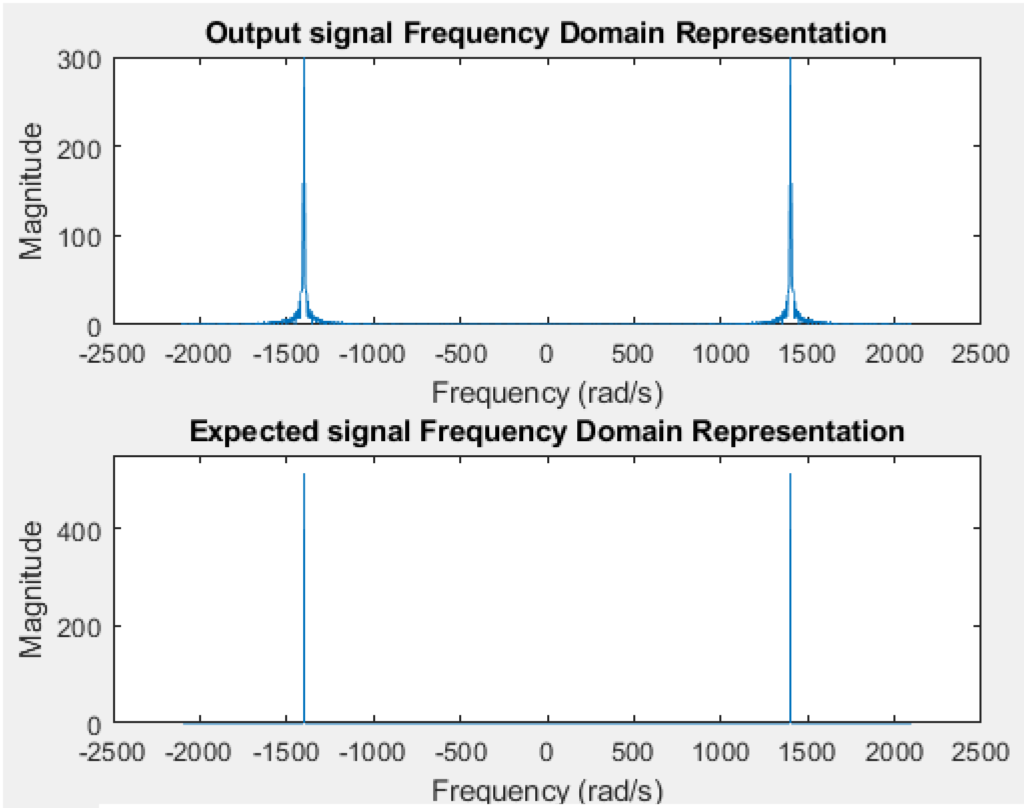


Figure XI – Filter output and expected output signal in frequency domain

5. DISCUSSION

Although our main purpose is to study the Kaiser Windowing method, here I have designed rectangular windowing method responses to analyze it more comparatively. The above Matlab plots show some significant differences between the filters obtained using the Kaiser window and the rectangular window, and it appears that a more optimal filtering effect can be obtained using the Kaiser window. As shown in Figures III and V, it is clear that the magnitude response of the filter has achieved a minimum stopband attenuation of approximately -45dB as well as a maximum passband ripple of 0.07dB. Although the two filters made using the two windowing techniques have very similar features in the impulse response (shown in Figures VI and II), there are some obvious differences in the frequency domain responses. Accordingly as shown in figure VIII, the magnitude response of the passband ripple in the Rectangular windowing method filter is significantly higher compared to the Kaiser windowing method filter, as well as the first sideband lobe is nearly -20dB. When the input signal in figure IX is inserted into the bandpass filter which has the cutoff frequencies in 1150 rad/s and 1650 rad/s, the 575 rad/s frequency component in the lower stopband region, and the 1875 rad/s frequency component in the upper stopband are almost attenuated while the 1400 rad/s component is preserved as in figure XI.

6. CONCLUSION

The above results show that the Kaiser Windowing technique can produce an output that is closer to the ideal filter output. It also has the ability to control the parameter values and limitations, so its flexibility is higher and the implementation complexity is less. The main drawback of this method is that the order of the filter is significantly higher. In a hardware implementation, high filter order means more unit delays, adders, and multipliers which implies more expensive in designing. In a software implementation, it means more computations per sample which implies a less efficient design.

7. REFERENCES

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